04 Set Theory and Number System

1. Set theory

1.1 Sets:

A set is a collection of well-defined objects which are distinct from each other. Sets are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by a, b, c,... etc.

If a is an element of a set A, then we write $a \in A$ (a belongs to A).

If a is **not** an element of a set A, then we write $a \notin A$ (a does not belong to A).

Ex. The collection of vowels in English alphabet is a set containing the elements a, e, i, o, u.

1.2 Methods to write a Set:

(i) Roster Method: In this method a set is described by listing elements, separated by commas and enclosing them by curly brackets.

Ex. The set of vowels of English Alphabet may be described as $\{a, e, i, o, u\}$.

(ii) Set Builder Form: In this case we write down a property or rule or proposition, which gives us all the element of the set.

$A = \{x : P(x)\}$

Ex. $A = \{x : x \in \mathbb{N}$ and $x = 2n$ for $n \in \mathbb{N}\}$

i.e. $A = \{2, 4, 6, ...\}$

Ex. $B = \{x^2 : x \in \mathbb{N}\}$

i.e. $B = \{1, 4, 9, ...\}$

Illustration 1:

The set $A = \{x : x \in \mathbb{R}, x^2 = 16$ and $2x = 6\}$ is equal to

(A) ϕ (B) $\{14, 3, 4\}$ (C) $\{3\}$ (D) $\{4\}$

Ans. (A)

Solution:

$x^2 = 16 \Rightarrow x = \pm 4$

$\Rightarrow 2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations.

Thus, $A = \phi$

1.3 Cardinality of a Finite Set:

The number of elements in a finite set is called the cardinality of the set A and is denoted $|A|$ or $n(A)$. It is also called cardinal number of the set.

Ex. $A = \{a, b, c, d\} \Rightarrow n(A) = 4$

1.4 Types of Sets

(i) **Null set or Empty set or Void set :** A set having no element in it is called an Empty set or a Null set or Void set. It is denoted by ϕ or { }

Ex. $A = \{x \in \mathbb{N} : 5 < x < 6\} = \phi$

A set consisting of atleast one element is called a non-empty set or a non-void set.

(ii) **Singleton set :** A set consisting of a single element is called a singleton set.

Ex. set {0}, is a singleton set

(iii) **Finite Set :** A set which has only finite number of elements is called a finite set.

Ex. $A = \{a, b, c\}$

(iv) **Infinite set :** A set which has an infinite number of elements is called an infinite set.

Ex. $A = \{1, 2, 3, 4, ...\}$ is an infinite set

(v) **Subset :** Let A and B be two sets, if every element of A is an element of B, then A is called a subset of B. If A is a subset of B, we write $A \subseteq B$

Ex : $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\} \Rightarrow A \subseteq B$

The symbol "$\Rightarrow$" stands for "implies"

Note : $(x \in A \Rightarrow x \in B) \Leftrightarrow A \subseteq B$

(vi) **Proper subset:** If A is a subset of B and $A \neq B$ then A is a proper subset of B and we write $A \subset B$

Note :

- Every set is a subset of itself i.e. $A \subseteq A$ for all A
- Empty set ϕ is a subset of every set
- The total number of subsets of a finite set containing n elements is 2^n

(vii) **Universal set:** A set consisting of all possible elements which occur in the discussion is called a Universal set and is denoted by U

Note : All sets are contained in the universal set

Ex. If $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 3, 5, 7\}$ then $U = \{1, 2, 3, 4, 5, 6, 7\}$ can be taken as the Universal set.

(viii) **Power set:** Let A be any set. The set of all subsets of A is called power set of A and is denoted by $P(A)$.

Note :

- If $A = \phi$ then $P(A)$ has one element.
- Power set of a given set is always non empty

Illustration 2:

If $A = \{1, 2\}$ then find its power set.

Solution:

$A = \{1, 2\}$ then $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Illustration 3:

If $A = \{x, y\}$, then the power set of A is-

(A) $\{x^y, y^x\}$ (B) $\{\phi, x, y\}$ (C) $\{\phi, \{x\} \{2y\}\}$ (D) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

Ans. (D)

Solution:

Clearly $P(A)$ = set of all subsets of A

$= \{\phi, \{x\}, \{y\}, \{x, y\}\}$

1.5 Some Operation on Sets

(i) Union of two sets : $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Ex. $A = \{1, 2, 3\}, B = \{2, 3, 4\}$ then $A \cup B = \{1, 2, 3, 4\}$

Note: $x \in (A \cup B) \Leftrightarrow x \in A$ or $x \in B$

(ii) Intersection of two sets : $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Ex. $A = \{1, 2, 3\}, B = \{2, 3, 4\}$ then $A \cap B = \{2, 3\}$

Note :

- $x \in (A \cap B) \Leftrightarrow x \in A$ and $x \in B$
- If $A \cap B = \phi$, then A, B are disjoint sets.

Ex. If $A = \{1, 2, 3\}, B = \{7, 8, 9\}$ then $A \cap B = \phi$

Illustration 4:

If $aN = \{ax : x \in N\}$, then the set $6N \cap 8N$ is equal to-

(A) $8N$ (B) $48N$ (C) $12N$ (D) $24N$

Ans. (D)

Solution :

$6N = \{6, 12, 18, 24, 30, ...\}$

$\Rightarrow 8N = \{8, 16, 24, 32, ...\}$

$\Rightarrow 6N \cap 8N = \{24, 48, ...\} = 24N$

Short cut Method

$6N \cap 8N = 24N$ [24 is the L.C.M. of 6 and 8]

(iii) Difference of two sets : $A - B$ or $A \backslash B = \{x : x \in A \text{ and } x \notin B\}$

Ex. $A = \{1, 2, 3\}, B = \{2, 3, 4\}$; $A - B = \{1\}$

Note : $x \in (A - B) \Leftrightarrow x \in A$ and $x \notin B$

(iv) Complement of a set : A' or A^c or $= \{x : x \notin A \text{ but } x \in U\} = U - A$

Ex. $U = \{1, 2, ..., 10\}, A = \{1, 2, 3, 4, 5\}$ then $A' = \{6, 7, 8, 9, 10\}$

Note :

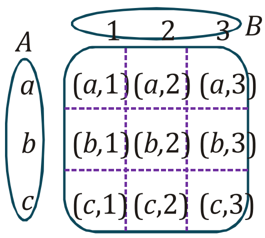
- $x \in A \Leftrightarrow x \notin A^c$
- $A \cap A' = \phi$ $\therefore A, A'$ are disjoint.
- $A \cup A' = U$
- $(A')' = A$.

(v) Cartesian Product of two sets

Cartesian product of two sets A and B, denoted as $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$

$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$

Ex: Cartesian product $A \times B$ when $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ is represented in the square grid



Note : $n(A \times B) = n(A) \times n(B)$

Illustration 5:

Find $A \times B$ when $A = \{x|x$ is prime number less than 5$\}$ and $B = \{1,2,3\}$

Solution:

$A = \{2,3\}$

$\Rightarrow B = \{1,2,3\}$

$\Rightarrow A \times B = \{(2,1), (2,2), (2,3), (3,1), (3,2),(3,3)\}$

1.6 Equality of two Sets :

Two sets A and B are said to be equal if every element of A is an element of B and every element of B is an element of A. If sets A and B are equal, we write $A = B$ and if A and B are not equal, then $A \neq B$

Ex. $A = \{1, 2, 6, 7\}$ and $B = \{6, 1, 2, 7\} \Rightarrow A = B$

Note : $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$

Illustration 6:

Find the pairs of equal sets (if any), give reasons:

$\Rightarrow A = \{0\}, B = \{x : x > 15$ and $x < 5\}$,

$\Rightarrow C = \{x : x - 5 = 0\}, D = \{x : x^2 = 25\}$,

$\Rightarrow E = \{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$.

Solution:

Since $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that, $A \neq B, A \neq C, A \neq D, A \neq E$.

Since $B = \phi$ but none of the other sets are empty. Therefore $B \neq C, B \neq D$ and $B \neq E$.

Also $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.

Since $E = \{5\}$, $C = E$. Further, $D = \{-5, 5\}$ and $E = \{5\}$, we find that, $D \neq E$.

Thus, the only pair of equal sets is C and E.

Illustration 7:

Show that $A \cup B = A \cap B$ implies $A = B$

Solution:

Let $a \in A$. Then $a \in A \cup B$. Since $A \cup B = A \cap B$, $a \in A \cap B$. So $a \in B$.

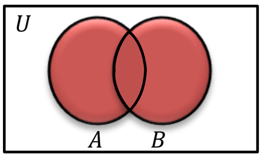
Therefore, $A \subseteq B$, Similarly, if $b \in B$, then $b \in A \cup B$. Since

$\Rightarrow A \cup B = A \cap B$, $b \in A \cap B$. So, $b \in A$. Therefore, $B \subseteq A$. Thus, $A = B$

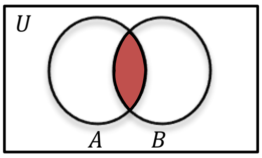
1.7 Venn Diagram

Venn diagram is a diagram representing sets pictorially as circles or closed curves within an enclosing rectangle/a closed curve (the universal set), common elements of the sets being represented by intersections of the circles/closed curves.

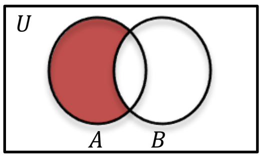
See the following Examples :



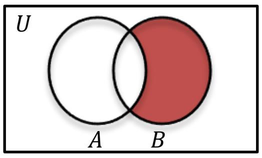
$A \cup B$



$A \cap B$

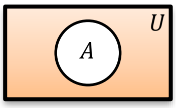


$A - B$

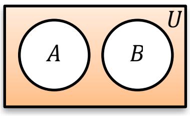


$B - A$

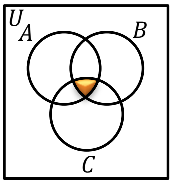
Clearly $(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$



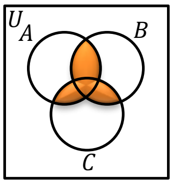
A'



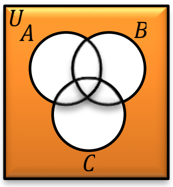
Disjoint Sets



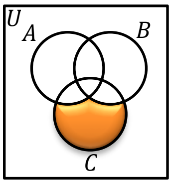
$(A \cap B \cap C)$



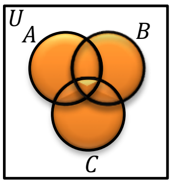
$(A \cap B) \cup (B \cap C) \cup (C \cap A)$



$(A \cup B \cup C)'$



$C - (A \cup B)$



$(A \cup B \cup C)$

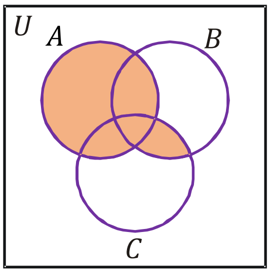
1.8 Some Important Laws

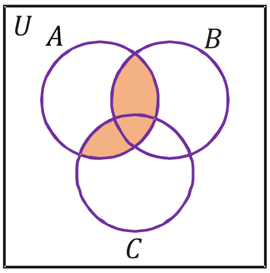
(i) **Commutative Law:** $A \cup B = B \cup A$; $A \cap B = B \cap A$

(ii) **Associative Law:** $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) **Distributive Law:**

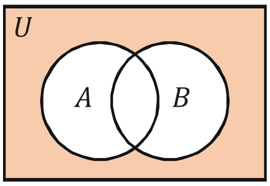
(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

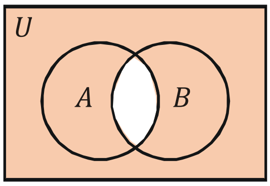




(iv) **De-Morgan's Law :**

(a) $(A \cup B)' = A' \cap B'$; (b) $(A \cap B)' = A' \cup B'$





1.9 Some Important Results on Cardinality of Sets

If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint sets.

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Illustration 8:

In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali ? How many can speak both Hindi and Bengali?

Ans. (250)

Solution:

Let A and B be the sets of persons who can speak Hindi and Bengali respectively.

then $n(A \cup B) = 1000, n(A) = 750, n(B) = 400$.

Number of persons whose can speak both Hindi and Bengali

$\Rightarrow n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$\Rightarrow 750 + 400 - 1000 = 150$

Number of persons who can speak Hindi only

$\Rightarrow n(A - B) = n(A) - n(A \cap B) = 750 - 150 = 600$

Number of persons who can speak Bengali only

$\Rightarrow n(B - A) = n(B) - n(A \cap B) = 400 - 150 = 250$

Illustration 9:

Each person in a group of 80 can speak either Hindi or English or both. If 55 persons can speak English and 40 can speak both, find the number of persons who can speak Hindi.

Ans. (65)

Solution:

Let E = set of persons who can speak English.

H = set of persons who can speak Hindi.

$\Rightarrow n(E) = 55, n(H) = x, n(E \cap H) = 40, n(H \cup E) = 80$

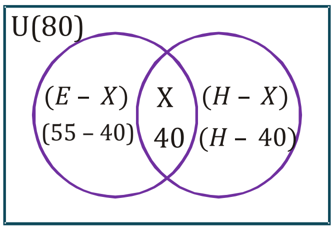
Using $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\Rightarrow 80 = x + 55 - 40$

$\Rightarrow x = 80 - 55 + 40 = 65$

Alternate:

Using the Venn diagram



Let $X = E \cap H$

$\Rightarrow n(U) = n(E - X) + n(X) + n(H - X)$

$\Rightarrow 80 = (55 - 40) + 40 + n(H) - 40$

$\Rightarrow n(H) = 80 - 15 = 65$

Illustration 10:

A group of members know at least one of the two languages, Hindi or Kannada. In the group, 150 members know Hindi and 80 members know Kannada and 70 of them know both. How many members are there in the group?

Ans. (160)

Solution:

Let H = set of persons who know Hindi.

K = set of persons who know Kannada.

$\Rightarrow n(H \cap K)$ = the number of persons who know both Hindi and Kannada is 70.

$\Rightarrow n(H \cup K) = n(H) + n(K) - n(H \cap K)$

$\Rightarrow 150 + 80 - 70 = 160$

Illustration 11:

In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics, and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry, and 3 had taken all the three subjects. Find the number of students who had taken.

(a) Only mathematics

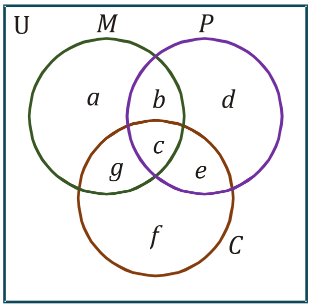
(b) Mathematics and physics, but not chemistry

(c) At least one of the three subjects

(d) Only one of the three subjects

Solution:

Let M denote the set of students who had taken mathematics. P the set of students who had taken physics and C the set of students who had taken chemistry. In the Venn diagram, let a, b, c, d, e, f, g denote the number of students in the respective regions.



Now, $n(M \cap P \cap C) = c = 3$

$\Rightarrow n(M \cap C) = g + c = 5 \Rightarrow g = 2$

$\Rightarrow n(M \cap P) = b + c = 9 \Rightarrow b = 6$

$\Rightarrow n(P \cap C) = e + c = 4 \Rightarrow e = 1$

$\Rightarrow n(M) = a + b + g + c = 15 \Rightarrow a = 4$

$\Rightarrow n(P) = b + c + d + e = 12 \Rightarrow d = 2$

$\Rightarrow n(C) = e + f + g + c = 11 \Rightarrow f = 5$

(a) The number of students who had taken only mathematics = $a = 4$.

(b) The number of students who had taken mathematics and physics, but not chemistry = $b = 6$

(c) The number of students who has taken at least one of the three subjects

$= a + b + c + d + e + f + g = 23$.

(d) The number of students who had taken only one of the three subjects = $a + d + f = 11$.

2. Theory Of Numbers

2.1 Types of Numbers:

(i) Natural numbers : The counting numbers 1, 2, 3, 4, ... are called natural numbers. The set of natural numbers is denoted by $\mathbb{N}$. Thus $\mathbb{N} = \{1, 2, 3, 4, ...\}$

(ii) Whole numbers : Natural numbers including zero are called whole numbers. The set of whole numbers is denoted by $\mathbb{W}$ or $\mathbb{N}_0$. Thus $\mathbb{W} = \{0, 1, 2, 3, 4, ...\}$

(iii) Integers : The numbers ... –3, –2, –1, 0, 1, 2, 3, ... are called integers and the set is denoted by $\mathbb{I}$ or $\mathbb{Z}$.

Thus ($\mathbb{I}$ or $\mathbb{Z}$) = {... –3, –2, –1, 0, 1, 2, 3, ...}

Note :

- Natural numbers are also called positive integers
  (some time denoted by $\mathbb{I}^+$ or $\mathbb{Z}^+$) = {1, 2, 3, ...}
- Whole numbers are also called non-negative integers
  (denoted by $\mathbb{W}$ or $\mathbb{I}_0^+$ or $\mathbb{Z}_0^+$) = {0, 1, 2, 3, ...}
- The set of negative integers, $\mathbb{I}$ or $\mathbb{Z}^-$ = {..., –3, –2, –1}
- The set of non-positive integers in $\mathbb{I}_0^-$ or $\mathbb{Z}_0^-$ = {..., –3, –2, –1, 0}
- Zero is neither positive nor negative but 0 is a member of the set of non-negative integers as well as of the set of non-positive integers.

(iv) Even integers : Integers which are divisible by 2 are called even integers.
e.g. 0, ±2, ±4, ...

(v) Odd integers : Integers which are not divisible by 2 are called as odd integers.
e.g. ±1, ±3, ±5, ±7, ...

(vi) Prime number : Natural number having exactly two positive divisors i.e. 1 and itself are called prime numbers.
e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

(vii) Composite number : Let 'a' be a natural number, 'a' is said to be composite if it has atleast three distinct positive divisors.

Note :

- 1 is the only natural number that is neither a prime number nor a composite number.
- 2 is the only prime number which is even.
- Numbers which are not prime are composite numbers (except 1).
- '4' is the smallest composite number.

(viii) Co-prime number: Two natural numbers (not necessarily prime) are coprime, if their H.C.F (Highest common factor) is 1.
e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) (15, 16) etc.
These numbers are also called relatively prime numbers.

Note :

- Two prime numbers are always co-prime but converse need not be true.
- Two consecutive natural numbers are always co-prime numbers.
- Two consecutive odd natural numbers are always co-prime numbers.

(ix) Rational numbers : The numbers which can be reduced in the form p/q where $p, q \in \mathbb{Z}$ and $q \neq 0$ and H.C.F $(p, q) = 1$, are called rational numbers. '$\mathbb{Q}$' represents their set.

Note :

- All integers are rational numbers with $q = 1$
- When numbers are expressed in reduced form of $\frac{p}{q}$, $q \neq 1$, the rational numbers are called fractions.
- Rational numbers when represented in decimal form are either 'terminating' or 'non-terminating but repeating'.
  e.g., 5/4 = 1.25 (terminating)
  5/3 = 1.6666 ... or $1.\overline{6}$ or $1.\dot{6}$ (non-terminating but repeating)
- $0.\overline{9} = 0.9999... = 1$

Illustration-12:

Express the following rational numbers in the form of p/q, (where $p, q \in \mathbb{Z}$)

(i) $0.1\overline{2}$ (ii) $1.5\overline{23}$

Solution:

(i) Let $x = 0.1222...$

$\Rightarrow 10x = 1.\overline{2}$ ... (i)

$\Rightarrow 100x = 12.\overline{2}$ ... (ii)

$\Rightarrow 90x = 11$

$\Rightarrow x = \frac{11}{90}$ (so x is a rational number)

(ii) Let $x = 1.5\overline{23}$

$\Rightarrow 10x = 15.\overline{23}$

$\Rightarrow 1000x = 1523.\overline{23}$

$\Rightarrow 990x = 1508$

$\Rightarrow x = \frac{1508}{990} = \frac{754}{495}$ (so x is a rational number)

(x) Irrational numbers : Numbers, which cannot be represented in p/q form as above. In decimal representation, they are neither terminating nor repeating.

e.g., $\sqrt{2}, (15)^{1/3}, \pi$ etc.

Note :

- $\pi \neq \frac{22}{7}$. Infact $\pi < \frac{22}{7}$

  $\left(\frac{22}{7} = 3.142857...\right)$ is only an approximate value of π in terms of rational numbers, taken for the sake of convenience. Actually $\pi = 3.14159265359...$

(xi) Real numbers: All rational and irrational numbers taken together form the set of real numbers, represented by $\mathbb{R}$. This is the largest set in the real world of numbers.

Note :

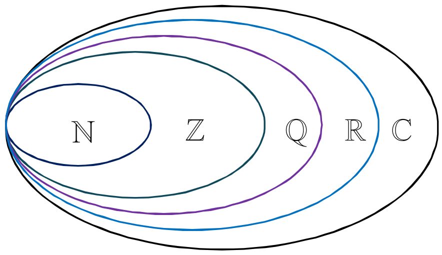
- Division by 0 is not defined.
- Integers are rational number, but converse need not be true.
- Sum of a rational number and an irrational number is always an irrational number.

  e.g. $5 + \sqrt{6}$
- The product of a non zero rational number and an irrational number will always be an irrational number.
- If $a \in \mathbb{Q}$ and $b \notin \mathbb{Q}$ then ab = rational number, only if $a = 0$.
- Sum, difference, product and quotient of two irrational numbers need not be an irrational number or we can say, result may be a rational number also.
- Sum, difference, product and quotient of two rational numbers is always a rational number.

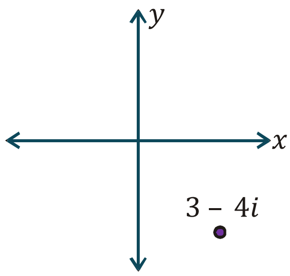
(xii) Complex number : A number z of the form $a + ib$ is called complex number, where $a, b \in \mathbb{R}$ and i stands for $\sqrt{-1}$. Here 'a' is called real part of z denoted by Re(z) and 'b' is called imaginary part of z denoted by Im(z).

The set of complex number is represented by $\mathbb{C}$.

It may be noted that $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$



Complex numbers can be represented on the **complex plane,** in the same way as a point (x, y) is plotted on the Cartesian plane. We can plot the number $x + iy$ by taking the real part 'x' as the horizontal coordinate and the imaginary part 'y' as the vertical coordinate. For example, in the adjacent diagram, the point $3 - 4i$ is shown on the complex plane.



When graphing complex numbers, the horizontal axis is often referred as the **real axis** and the vertical axis as the **imaginary axis**.

The complex plane also gives us a way to visualize the magnitude of a complex number.

The magnitude of a complex number is its distance from the origin when plotted on the complex plane. We use $|x + iy|$ to denote the distance between $x + iy$ and the origin when plotted on the complex plane. So, for example, we have $|3-4i| = \sqrt{3^2 + (-4)^2} = 5$. More generally, we have $|x + iy| = \sqrt{x^2 + y^2}$. '$i$' (iota) is called the imaginary unit. Also $i^0 = 1$; $i^2 = -1$; $i^3 = -i$; $i^4 = 1$ etc.

Conjugate of z :

If $z = a + ib$, $a, b \in \mathbb{R}$, then conjugate of z (denoted as $\bar{z}$), $\bar{z} = a - ib$.

Illustration 13:

Find the roots of following

(i) $x^2 + x + 1 = 0$ (ii) $x^4 = 1$

Solution:

(i) $x^2 + x + 1 = 0$

$\Rightarrow x = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\Rightarrow x = \dfrac{-1 \pm i\sqrt{3}}{2}$

$\Rightarrow x = -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2}$

(ii) $x^4 = 1$

$\Rightarrow x^2 = \pm 1$

$\Rightarrow x = 1, -1, i, -i$

Numbers to Remember :

Number	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Square	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400
Cube	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375	4096	4913	5832	6859	8000
Sq. Root	1.41	1.73	2	2.24	2.45	2.65	2.83	3	3.16	← rounded upto two places of decimal									

Note :

- Square of a real number is always non-negative. (i.e. $x^2 \geq 0$)
- Square root of a positive number is always positive. e.g. $\sqrt{4} = 2$

2.2 Divisibility Test

Divisibility of	Test
2	The digit at the unit's place of the number is divisible by 2.
3	The sum of digits of the number is divisible by 3.
4	The last two digits of the number together are divisible by 4.
5	The digit at the unit's place is either 0 or 5.
6	The digit at the unit's place of the number is divisible by 2 & the sum of all digits of the number is divisible by 3.
8	The last 3 digits of the number all together are divisible by 8.
9	The sum of all digits is divisible by 9.
11	The difference between the sum of the digits at even places and the sum of digits at odd places is 0 or multiple of 11. E.g. 1298, 1221, 123321, 12344321, 1234554321, 123456654321

Illustration-14:

Consider a number $N = 2\ 1\ P\ 5\ 3\ Q\ 4$

(i) Number of ordered pairs (P, Q) so that the number 'N' is divisible by 9, is

(A) 11 (B) 12 (C) 10 (D) 8

Ans. (A)

(ii) Number of values of Q so that the number 'N' is divisible by 8, is

(A) 4 (B) 3 (C) 2 (D) 6

Ans. (B)

Solution:

(i) Sum of digits = $P + Q + 15$

'N' is divisible by 9 if

$\Rightarrow P + Q + 15 = 18, 27$

$\Rightarrow P + Q = 3$ ...(i) or $P + Q = 12$ ...(ii)

From equation (i)

$$\Rightarrow \left.\begin{matrix} P=0, & Q=3 \\ P=1, & Q=2 \\ P=2, & Q=1 \\ P=3, & Q=0 \end{matrix}\right\} \text{Number of ordered pairs is 4}$$

From equation (ii)

$$\Rightarrow \left.\begin{matrix} P=3, & Q=9 \\ P=4, & Q=8 \\ \dots\dots\dots & \dots\dots\dots \\ P=8, & Q=4 \\ P=9, & Q=3 \end{matrix}\right\} \text{Number of ordered pairs is 7}$$

Total number of ordered pairs is 11

(ii) N is divisible by 8 if

$\Rightarrow Q = 0, 4, 8$

Number of values of Q is 3