Sequence and Series

SOLUTIONS

Exercise-I (JEE Main Pattern)

SECTION–A

1. **Ans. (4)**

$a_1, a_2, a_3, \ldots a_n$ AP

$a_n = a + (n-1)d$

$a_4 - a_7 + a_{10} = m$

$\Rightarrow a + 3d - a - 6d + a + 9d = m$

$\Rightarrow a + 6d = m$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$\Rightarrow S_{13} = \frac{13}{2}(2a + 12d) \Rightarrow S_{13} = \frac{13}{2}(2)(a+6d) \Rightarrow S_{13} = 13m$

2. **Ans. (1)**

a, b, c AP

$2b = a + c$

Square on both side

$\Rightarrow 4b^2 = (a+c)^2$

$\Rightarrow 4b^2 = a^2 + c^2 + 2ac$

$\Rightarrow 4b^2 = a^2 + c^2 - 2ac + 4ac$

$\Rightarrow 4b^2 - 4ac = (a-c)^2$

$\Rightarrow 4(b^2 - ac) = (a-c)^2$

3. **Ans. (2)**

$a_1, a_2, a_3 \ldots.$ AP

$$\frac{a_1 + a_2 + \ldots\ldots + a_p}{a_1 + a_2 + a_3 + \ldots. + a_q} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{\frac{p}{2}(2a_1 + (p-1)d)}{\frac{q}{2}(2a_1 + (q-1)d)} = \frac{p^2}{q^2} = \frac{p}{q}$$

$$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$$\Rightarrow \frac{a_1 + \left(\frac{P-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{2} \qquad \ldots(i)$$

$$\frac{a_6}{a_{21}} = \frac{a_1 + 5d}{a_1 + 20d} \qquad (a_n = a_1 + (n-1)d)$$

From equation (i)

$$\frac{p-1}{2}=5 \text{ and } \frac{q-1}{2}=20$$

p = 11   q = 41

put $p = 11$ and $q = 41$ in equation (i)

$$\frac{a_1+5d}{a_1+20d}=\frac{11}{41}$$

$$\frac{a_6}{a_{21}}=\frac{11}{41}$$

4. **Ans. (1)**

$S_{11} = S_{19}$ (given)

$$\frac{11}{2}(2a+10d)=\frac{19}{2}(2a+18d)$$

$11(a + 5d) = 19(a + 9d)$

$11a + 55d = 19a + 171d$

$-116d = 8a$

$-29d = 2a$

$$S_{30}=\frac{30}{2}(2a+29d)$$

From equation (i)

$$\Rightarrow S_{30}=\frac{30}{2}(-29d+29d)$$

$\Rightarrow S_{30} = 0$

5. **Ans. (3)**

Let there be $2n + 1$ stones. So, there will be n stones each side of middle stone

If the person started the job from middle,

Distance covered for first stone = 2×10 m

Similarly, distance covered for second stone = 2×20 m

So, the distance covered in bringing stones to centre form on side

$= 2[10 + 20 + 30 + \cdots + 10n]$

Distance covered in brining stones from both side $= 4[10 + 20 + 30 \ldots\ldots + 10n]$

Person started from middle the total distance covered

$4800 = 4(10 + 20 + 30 + \cdots . + 10n)$

$4800 = 40(1 + 2 + 3 + \cdots \ldots + n)$

$$4800=40\frac{(n)(n+1)}{2}$$

$240 = n(n + 1)$

$n^2 + n - 240 = 0$

$n^2 + 16n - 15n - 240 = 0$

$(n + 16)(n - 15) = 0$

$n = -16$ (Rejected)

$n = 15$

Total stone $= 2(15) + 1$

$= 31$

6. Ans. (1)

$a_1, a_2, a_3 \ldots\ldots a_n$ AP

$$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\frac{1}{a_3a_4}+\ldots..+\frac{1}{a_na_{n+1}}$$

Multiply and divided by d

$$\Rightarrow \frac{1}{d}\left(\frac{d}{a_1a_2}+\frac{d}{a_2a_3}+\frac{d}{a_3a_4}+\ldots..+\frac{d}{a_na_{n+1}}\right)$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+\frac{a_4-a_3}{a_3a_4}+\ldots.+\frac{a_{n+1}-a_n}{a_n.a_{n+1}}\right)$$

$$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\frac{1}{a_3}-\frac{1}{a_4}+\ldots.+\frac{1}{a_n}-\frac{1}{a_{n+1}}\right)$$

$$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{n+1}}\right)$$

If n approaches infinity then a_{n+1} also approaches infinity it means $\frac{1}{a_{n+1}}$ approaches – zero

$$\Rightarrow \frac{1}{a_1d}$$

7. Ans. (2)

Let common ratio of G.P is r

GP

$1, r, r^2, r^3, ^4 \ldots..$

$1 = r + r^2 + r^3 + r^4 + \ldots\ldots$

$$\Rightarrow 1=\frac{r}{1-r} \Rightarrow 1-r=r$$

$$\Rightarrow 2r = 1 \Rightarrow r=\frac{1}{2}$$

$T_4 = a.(r^3)$

$$\Rightarrow T_4 = (1)\left(\frac{1}{2}\right)^3 \Rightarrow T_4=\frac{1}{8}$$

8. Ans. (3)

$$a=\sum_{n=0}^{\infty} x^n$$

$a = 1 + x + x^2 + x^3 + \cdots$ upto ∞

$$a=\frac{1}{1-x}$$

$$\Rightarrow 1-x=\frac{1}{a} \Rightarrow x=1-\frac{1}{a} \qquad \ldots(i)$$

$$b=\sum_{n=0}^{\infty}(y)^n$$

$b = 1 + y + y^2 + y^3 \ldots..$ upto ∞

$b=\frac{1}{1-y}$

$\Rightarrow y=1-\frac{1}{b}$ ...(ii)

$C=\sum_{n=0}^{\infty}(xy)^n$

$c = 1 + (xy) + (xy)^2 + (xy)^3 + \cdots$ upto ∞

$C=\frac{1}{1-xy}$

$\Rightarrow xy=1-\frac{1}{C}$ ...(iii)

From equation (i), (ii) and (iii)

$\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)=1-\frac{1}{c}$

$\frac{(a-1)(b-1)}{ab}=\frac{c-1}{c}$

$\frac{ab-a-b+1}{ab}=\frac{c-1}{c}$

$\Rightarrow abc - ac - bc + c = abc - ab$

$\Rightarrow ab + c = ac + bc$

9. Ans. (2)

Let common ratio = r

First term = 1

$4T_2 + 5T_3$

$\Rightarrow (4)r + 5(r)^2$

$\Rightarrow 5r^2 + 4r$

$\Rightarrow 5\left(r^2+\frac{4}{5}r\right)$

$\Rightarrow 5\left(r^2+2\left(\frac{2}{5}\right)r+\left(\frac{2}{5}\right)^2\right)-\left(\frac{2}{5}\right)^2$

$\Rightarrow 5\left(r+\frac{2}{5}\right)^2-\left(\frac{2}{5}\right)^2$

For minimum value of $5\left(r+\frac{2}{5}\right)^2-\frac{4}{25}$ r should be equal to $-\frac{2}{5}$

10. Ans. (3)

$G=\sqrt{xy}$

$\Rightarrow \frac{1}{G^2-x^2}+\frac{1}{G^2+y^2}$

$\Rightarrow \frac{1}{xy-x^2}+\frac{1}{xy-y^2}$

$$\Rightarrow \frac{1}{x(y-x)}+\frac{1}{y(x-y)}$$

$$\Rightarrow \frac{1}{y-x}\left(\frac{1}{x}-\frac{1}{y}\right) \Rightarrow \frac{1}{y-x}\left(\frac{y-x}{xy}\right)$$

$$\Rightarrow \frac{1}{xy} \Rightarrow \frac{1}{G^2}$$

11. Ans. (2)

$$\frac{1+3+5+.....\text{upto } n \text{ terms}}{4+7+10+.......\text{upto } n \text{ terms}}=\frac{20}{7\log_{10}x}$$

$$\Rightarrow \frac{\left(\frac{n}{2}\right)(2+(n-1)(2))}{\left(\frac{n}{2}\right)(8+(n-1)(3))}=\frac{20}{7\log_{10}x}$$

$$\Rightarrow \frac{2n}{3n+5}=\frac{20}{7\log_{10}x} \qquad ...(i)$$

Now,

$$n=\log_{10}x+\log_{10}x^{\frac{1}{2}}+\log_{10}x^{\frac{1}{4}}+........+\infty$$

$$\Rightarrow n=\log^{n}{}_{10}x+\frac{1}{2}\log_{10}x+\frac{1}{4}\log_{10}x+........+\infty$$

$$\Rightarrow n=\log_{10}x\left(1+\frac{1}{2}+\frac{1}{4}+.....+\infty\right)$$

$$\Rightarrow n=\log_{10}x\left(\frac{1}{1-\frac{1}{2}}\right)$$

$$\Rightarrow n=2\log_{10}x \qquad ...(ii)$$

Put the value of n from equation (ii) in equation (i)

$$\frac{2(2\log_{10}x)}{3(2\log_{10}x)+5}=\frac{20}{7\log_{10}x}$$

Let $\log_{10}x=t$

$$\Rightarrow \frac{4t}{6t+5}=\frac{20}{7t}$$

$$\Rightarrow 28t^2=120t+100$$

$$\Rightarrow 7t^2=30t+25 \Rightarrow 7t^2-30t-25=0$$

$$\Rightarrow 7t^2-35t+5t-25=0 \Rightarrow 7t(t-5)+5(t-5)=0$$

$$\Rightarrow (7t+5)(t-5)=0 \Rightarrow t=-\frac{5}{7} \text{ or } t=5$$

$$\log_{10}x=5$$

$$\Rightarrow x=10^5$$

12. Ans. (4)

$a^2 + (\ln a^2)^2 + (\ln a^2)^3 + ... = 3\left(\ln a+(\ln a)^2+(\ln a)^3+(\ln a)^4+.......\right)$

$$\frac{\ell n\left(a^2\right)}{1-\ell na^2}=\frac{3\ell na}{1-\ell na}$$

$$\Rightarrow \frac{2\ell na}{1-2\ell na}=\frac{3\ell na}{1-\ell na}$$

$$\Rightarrow 2-2\ell na=3-6\ell na$$

$$\Rightarrow 4\ell na=1 \Rightarrow \ell na=\frac{1}{4}$$

$$\Rightarrow a=e^{\frac{1}{4}}$$

13. Ans. (2)

a, b, c HP

$$b=\frac{2ac}{a+c}$$

$$\frac{b+a}{b-a}+\frac{b+c}{b-c}$$

$$\Rightarrow \frac{\frac{2ac}{a+c}+a}{\frac{2ac}{a+c}-a}+\frac{\frac{2ac}{a+c}+c}{\frac{2ac}{a+c}-c}$$

$$\Rightarrow \frac{2ac+a^2+ac}{2ac-a^2+ac}+\frac{2ac+ac+c^2}{2ac-ac-c^2}$$

$$\Rightarrow \frac{3ac+a^2}{ac-a^2}+\frac{3ac+c^2}{ac-c^2} \Rightarrow \frac{a(3c+a)}{a(c-a)}+\frac{c(3a+c)}{c(a-c)}$$

$$\Rightarrow \frac{3c+a}{c-a}+\frac{3a+c}{a-c} \Rightarrow \frac{3c+a}{c-a}-\left(\frac{3a+c}{c-a}\right)$$

$$\Rightarrow \frac{3c+a-3a-c}{c-a} \Rightarrow \frac{2c-2a}{c-a}=2$$

14. Ans. (1)

$$x=\sum_{n=0}^{\infty}a^n$$

$= 1+a+a^2+.............\infty$

$$x=\frac{1}{1-a}\Rightarrow a=1-\frac{1}{x}$$

$$y=\sum_{n=0}^{\infty}b^n$$

$= 1+b+b^2+.........\infty$

$$y=\frac{1}{1-b}\Rightarrow b=1-\frac{1}{y}$$

$$z=\sum_{n=0}^{\infty}c^n$$

$= 1+c+c^2+..........\infty$

$z=\frac{1}{1-c}$

$c=1-\frac{1}{z}$

a,b,c in A.P.

$2b=a+c$

$2\left(1-\frac{1}{y}\right)=1-\frac{1}{x}+1-\frac{1}{z}$

$\frac{2}{y}=\frac{1}{x}+\frac{1}{z}$

So, x,y,z in H.p.

15. Ans. (1)

$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$ are positive real numbers

AM $\geq$ GM

$\frac{\frac{a}{b}+\frac{b}{c}+\frac{c}{a}}{3}\geq\left(\frac{a}{b}.\frac{b}{c}.\frac{c}{a}\right)^{\frac{1}{3}}$

$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 3$

16. Ans. (3)

$\frac{1}{a},\frac{1}{b},\frac{1}{b},\frac{1}{c},\frac{1}{c},\frac{1}{c}$

AM $\geq$ GM

$\frac{\frac{1}{a}+\frac{2}{b}+\frac{3}{c}}{6}\geq\left(\frac{1}{ab^2c^3}\right)^{\frac{1}{6}}$

$\frac{1}{a}+\frac{2}{b}+\frac{3}{c}\geq 6\times\frac{1}{2}$

$\frac{1}{a}+\frac{2}{b}+\frac{3}{c}\geq 3$

Minimum value = 3

17. Ans. (4)

$\left(5^{1+x}+5^{1-x}\right),\frac{a}{2},\left(25^x+25^{-x}\right)$

$\Rightarrow a=\left(5^{1+x}+5^{1-x}\right)+\left(25^x+25^{-x}\right)$

$5\left(5^x+5^{-x}\right)+5^{2x}+5^{-2x}$

$5^x+5^{-x}\geq 2$

Minimum value at $x=0$

$5^{2x}+5^{-2x}\geq 2$

Minimum value at $x=0$

Range of $[12,\infty)$

18. Ans. (2)

$S_n = 2+4+7+11+16+..............+n$ terms.

$S_n = 2+4+7+11+16+..............+t_n$

$S_n = \quad 2+4+7+11+................+t_{n-1}+t_n$

$0 = 2+\left(2+3+4+5+...................\left(t_n - t_{n-1}\right)\right)-t_n$

$t_n = 2+\left(2+3+4+.....................\left(t_n - t_{n-1}\right)\right)$

$= 2+\frac{n-1}{2}\left(4+\left(n-2\right)\right)$

$= 2+\frac{\left(n-1\right)\left(n+2\right)}{2}$

$t_n = 2+\frac{n^2+n-2}{2}$

$S_n = \sum t_n = \sum 1+\frac{\sum n^2}{2}+\frac{\sum n}{2}$

$= n+\frac{n\left(n+1\right)\left(2n+1\right)}{12}+\frac{n\left(n+1\right)}{4} = \frac{n\left(12+\left(n+1\right)\left(2n+1\right)+3\left(n+1\right)\right)}{12}$

$= \frac{n\left(12+2n^2+3n+1+3n+3\right)}{12} = \frac{n\left(2n^2+6n+16\right)}{12} = \frac{n\left(n^2+3n+8\right)}{6}$

19. Ans. (4)

$S_{11} = \frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+........11$ terms

$t_n = \frac{2n+1}{1^2+2^2+.......+n^2}$

$t_n = \frac{6\left(2n+1\right)}{n\left(n+1\right)\left(2n+1\right)}$

$S_{11} = \sum_{n=1}^{11} t_n$

$= 6\sum_{n=1}^{11} \frac{1}{n}-\frac{1}{n+1}$

$$= 6\begin{pmatrix} \frac{1}{1}-\frac{1}{2} \\ \frac{1}{2}-\frac{1}{3} \\ \vdots \\ \frac{1}{11}-\frac{1}{12} \end{pmatrix} = 6\left(1-\frac{1}{12}\right) = \frac{11}{2}$$

20. Ans. (3)

$$S_n = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + ...$$

$$t_n = \frac{1}{1+2+3+.....+n}$$

$$t_n = \frac{2}{n(n+1)}$$

$$S_n = \sum t_n$$

$$S_{10} = \sum_{n=1}^{10} \frac{2}{n(n+1)}$$

$$= 2\sum_{n=1}^{10} \frac{1}{n} - \frac{1}{n+1}$$

$$= 6\begin{pmatrix} \frac{1}{1} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{3} \\ \frac{1}{3} - \frac{1}{4} \\ \vdots \quad \vdots \\ \frac{1}{10} - \frac{1}{11} \end{pmatrix}$$

$$2\left(1 - \frac{1}{11}\right)$$

$$S_{10} = \frac{20}{11}$$

SECTION-B

1. Ans. (12)

Sum of all interior angles = $(n-2)\times 180°$

Now let smallest angle = θ

$\theta + (\theta + 4°) + ...n$ terms$=(n-2)\times 180°$ ...(1)

Now $\theta + (n-1)4° = 172°$

$\theta + 4n = 176$

$\theta = 176 - 4n$ ...(2)

From eq. (1) & (2)

$$\frac{n}{2}[\theta + 172°] = (n-2)\times 180$$

$\frac{n}{2}[176° - 4n + 172] = (n-2) \times 180$

$n(174 - 2n) = 180n - 360$

$2n^2 + 6n - 360 = 0$

$n^2 + 3n - 180 = 0$

$n^2 + 5n - 12n - 180 = 0$

$n(n+15) - 12(n+15) = 0$

$n = 12$

2. Ans. (27)

$1,\ 1+d,\ 1+2d$............$1+8d$

$\frac{9}{2}[2+5d] = 369$

$\Rightarrow\ 1+4d = 41$

$\Rightarrow\ d = 10$

First term of G.P. = $a = 1$

9th term = $ar^8 = 81$

$r^8 = 81$

$r = \sqrt{3}$

7th term = ar^6

= $1.(\sqrt{3})^6$

= 27

3. Ans. (22)

$a,\ a^2, a^2,\ a^3,\ a^{-1},\ a^{-3},\ a^{-4},$ (1, 1,..........15 times)

AM $\geq$ GM

$\frac{a + a^2 + a^2 + a^3 + 15 + a^{-3} + a^{-4}}{22} \geq (1)^{\frac{1}{22}}$

$a + 2a^2 + a^3 + 15 + a^{-1} + a^{-3} + a^{-4} \geq 22$

Minimum value = 22

4. Ans. (0.50)

$t_n = \frac{n}{1 + n^2 + n^4}$

$S_n = \sum \frac{n}{n^4 + n^2 + 1}$

$S_n = \frac{1}{2}\sum \frac{(n^2+n+1) - (n^2-n+1)}{(n^2+n+1)(n^2-n+1)}$

= $\frac{1}{2}\sum \frac{1}{n^2 - n + 1} - \frac{1}{n^2 + n + 1}$

$$= \frac{1}{2}\begin{pmatrix} \frac{1}{1}-\frac{1}{3} \\ \frac{1}{3}-\frac{1}{7} \\ \vdots \\ \frac{1}{n^2-n+1}-\frac{1}{n^2+n+1} \end{pmatrix}$$

$$S_n = \frac{1}{2}\left(1-\frac{1}{n^2+n+1}\right)$$

$n \to \infty$

$$S_\infty = \frac{1}{2}$$

5. **Ans.** $\left(\frac{1}{201}\right)$

AM $\geq$ GM

$$\frac{1+x+x^2+x^3+\ldots\ldots\ldots\ldots+x^{200}}{201} \geq \left(x^{1+2+3+\ldots\ldots200}\right)^{\frac{1}{201}}$$

$$\Rightarrow \frac{1+x+x^2+\ldots\ldots\ldots\ldots+x^{200}}{201} \geq x^{100}$$

$$\Rightarrow \frac{x^{100}}{1+x+x^2+\ldots\ldots+x^{200}} \leq \frac{1}{201}$$

So, greatest value = $\frac{1}{201}$

Exercise - II (JEE Main PYQs)

1. **Ans. (2)**

$\sum_{k=0}^{12} a_{4k+1} = 416$

$\Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$

$\Rightarrow a_1 + 24d = 32$ ...(i)

$a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ ...(ii)

from (i) & (ii)

$d = 1$ and $a_1 = 8$

$$\Rightarrow 140m = \sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17}\left[8+(r-1).1\right]^2$$

$$\Rightarrow 140m = \sum_{r=1}^{17}(r+7)^2 \Rightarrow 140m = \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2$$

$\Rightarrow$ $140m = \frac{24.25.49}{6} - \frac{7.8.15}{6}$

$\Rightarrow$ $140m = \frac{7.8.5}{6}[105 - 3]$

$\Rightarrow$ $140m = 280.17 \Rightarrow m = 34$

2. **Ans. (1)**

$B-2A=\sum_{r=1}^{40} T_r - 2\sum_{r=1}^{20} T_r$

$= \sum_{r=1}^{40} T_r - \sum_{r=1}^{20} T_r$

$B - 2A = (21^2 + 2.22^2 + 23^2 + 2.24^2 + \ldots\ldots + 40^2) - (1^2 + 2.2^2 + 3^2 + 2.4^2 \ldots\ldots + 20^2)$

$= 20[22 + 2.24 + 26 + 2.28 + \ldots. + 60]$

$$= 20\left[\underbrace{22+24+26.......60}_{20\text{ terms}} + \underbrace{24+28+.........+60}_{10\text{ terms}}\right]$$

$$= 20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]$$

$= 10[20.82 + 10.84] = 100[164 + 84] = 100.248$

So, $100\lambda = 100.248 \Rightarrow \lambda = 248$.

3. **Ans. (4)**

$S = a_1 + a_2 + \ldots\ldots + a_{30}$

$$S = \frac{30}{2}[a_1 + a_{30}]$$

$S = 15(a_1 + a_{30}) = 15\ (a_1 + a_1 + 29d)$

$T = a_1 + a_3 + \ldots\ldots + a_{29}$

$= (a_1) + (a_1 + 2d) \ldots\ldots + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + \ldots\ldots + 14)$

$T = 15a_1 + 210d$

Now use $S - 2T = 75$

$\Rightarrow 15\ (2a_1 + 29d) - 2\ (15a_1 + 210d) = 75$

$\Rightarrow d = 5$

Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$

Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

4. **Ans. (1)**

$$T_n = \frac{(3+(n-1)\times 3)(1^2+2^2+....+n^2)}{(2n+1)}$$

$$T_n = \frac{3n\left(\frac{n(n+1)(2n+1)}{6}\right)}{2n+1} = \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3+n^2) = \frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15\times 16\times 31}{6}\right]$$

$= 7820$

5. **Ans. (2)**

$$\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})} = \frac{1}{\left(x^m + \frac{1}{x^m}\right)\left(y^n + \frac{1}{y^n}\right)} \le \frac{1}{4}$$

using AM ≥ GM

6. **Ans. (1)**

$\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t, \gamma = \alpha t^2$

$\therefore \alpha x^2 + 2\alpha t x + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore t^2 - t - 1 = 0$ ...(1)

Now

$\alpha(\beta + \gamma) = \alpha^2(t + t^2) = (\alpha t)(\alpha(1 + t))$

$= (\alpha t)(\alpha t^2)$ $[1 + t = t^2]$

$= \beta\gamma$ $[\alpha t^2 = \gamma]$

7. **Ans. (3)**

a, b, c in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

$x = -r$ is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$\Rightarrow d.\frac{c}{a} - \frac{2ce}{b} + f = 0 \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

So, $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

8. **Ans. (3)**

$a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$

$\Rightarrow a_1 + a_{16} = 38$

So, $a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$

$= 2 \times 38 = 76$

9. **Ans. (2)**

$b = ar$

$c = ar^2$

$3a, 7b$ and $15\,c$ are in A.P.

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14(ar) = 3a + 15\,ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow (3r - 1)(5r - 3) = 0$

$r = \frac{1}{3}, \frac{3}{5}$.

Only acceptable value is $r = \frac{1}{3}$, because $r \in \left(0, \frac{1}{2}\right]$

$\therefore$ Common difference = $7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$

$\therefore$ 4th term = $15c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$.

10. Ans. (1)

Sum of the 40 terms of

3 + 4 + 8 + 9 + 13 + 14 + 18 + 19...

= (3 + 8 + 13 +...upto 20 term)

+ [4 + 9 + 15 +... upto 20 terms]

= 10 [{6 + 19 × 5} + {8 + 19 × 5}]

= 10 × 204 = 20 × 102

So, $20(102) = (102)m \Rightarrow m = 20$.

11. Ans. (1)

$a_1 + a_2 = 4$

$16 = a_3 + a_4 = r^2 a_1 + r^2 a_2$

$= r^2(a_1 + a_2)$

$= r^2(4)$

$\Rightarrow r^2 = 4 \Rightarrow r = \pm 2$

when $r = 2$	when $r = -2$
$a_1 + a_2 = 4$	$a_1 + a_2 = 4$
$a_1(1 + r) = 4$	$a_1(1 + r) = 4$
$a_1(1 + 2) = 4$	$a_1(1 - 2) = 4$
$a_1 = \frac{4}{3}$ (Rejected as $a_1 < 0$)	$a_1 = -4$

$\sum_{i=1}^{9} a_i = 4\lambda$

As $a_1 = -4$ and $r = -2$

$4\lambda = (-4)\left(\frac{(-2)^9 - 1}{-2-1}\right) = (-4) \times \frac{513}{3}$

$\Rightarrow \lambda = -171$

12. Ans. (3)

Let the A.P is

$a - 2d, a - d, a, a + d, a + 2d$

$\because$ sum = $25 \Rightarrow a = 5$

Product = 2520

$(25 - 4d^2)(25 - d^2) = 504$

$4d^4 - 125d^2 + 121 = 0$

$\Rightarrow d^2 = 1, \dfrac{121}{4}$

$\Rightarrow d = \pm 1, \pm\dfrac{11}{2}$

$d = \pm 1$ is rejected because none of the term can be $\dfrac{-1}{2}$.

$\Rightarrow d = \pm\dfrac{11}{2}$

$\Rightarrow$ AP will be $-6, -\dfrac{1}{2}, 5, \dfrac{21}{2}, 16$

Largest term is 16.

13. Ans. (504)

$$\frac{1}{4}\left(\sum_{n=1}^{7} 2n^3 + \sum_{n=1}^{7} 3n^2 + \sum_{n=1}^{7} n\right)$$

$$= \frac{1}{4}\left(2\left(\frac{7\times 8}{2}\right)^2 + 3\left(\frac{7\times 8\times 15}{6}\right) + \frac{7\times 8}{2}\right) = 504$$

Ans. 504.00

14. Ans. (14)

Here, $d_1 = 4, d_2 = 7$

So, $d = \text{LCM}(4, 7) = 28$

Common term are : 23, 51, 79, ... T_n

$T_n \le 407 \Rightarrow 23 + (n-1)28 \le 407$

$\Rightarrow n \le 14.71$

$n = 14$

15. Ans. (1)

$$2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot \ldots\ \infty$$

$$= 2^{\frac{1}{4}} \cdot 2^{\frac{2}{16}} \cdot 2^{\frac{3}{48}} \cdot 2^{\frac{4}{128}} \cdot \ldots\infty$$

$$= 2^{\frac{1}{4}} \cdot 2^{\frac{1}{8}} \cdot 2^{\frac{1}{16}} \cdot 2^{\frac{1}{32}} \cdot \ldots\infty$$

$$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\infty} = (2)^{\left(\frac{1/4}{1-1/2}\right)} = 2^{1/2}$$

16. Ans. (7)

$a_{n+2} = 2a_{n+1} + a_n$, let $\sum_{n=1}^{\infty} \dfrac{a_n}{8^n} = \text{P}$

Divide by 8^n we get

$$\Rightarrow \frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$$

$$\Rightarrow 64\frac{a_{n+2}}{8^{n+2}} = \frac{16a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$$

$$\Rightarrow 64\sum_{n=1}^{\infty}\frac{a_{n+2}}{8^{n+2}}=16\sum_{n=1}^{\infty}\frac{a_{n+1}}{8^{n+1}}+\sum_{n=1}^{\infty}\frac{a_n}{8^n}$$

$$\Rightarrow 64\left(P-\frac{a_1}{8}-\frac{a_2}{8^2}\right)=16\left(P-\frac{a_1}{8}\right)+P$$

$$\Rightarrow 64\left(P-\frac{1}{8}-\frac{1}{64}\right)=16\left(P-\frac{1}{8}\right)+P$$

$\Rightarrow$ 64P – 8 – 1 = 16 P – 2 + P

$\Rightarrow$ 47P = 7

17. **Ans. (5143)**

A = 4 – digit numbers divisible by 3

A = 1002, 1005, ..., 9999.

9999 = 1002 + $(n-1)3$

$\Rightarrow (n-1)3 = 8997 \Rightarrow n = 3000$

B = 4 – digit numbers divisible by 7

B = 1001, 1008, ..., 9996

$\Rightarrow$ 9996 = 1001 + $(n-1)7$

$\Rightarrow n = 1286$

$A \cap B$ = 1008, 1029, ..., 9996

9996 = 1008 + $(n-1)21$

$\Rightarrow n = 429$

So, no divisible by either 3 or 7

= 3000 + 1286 – 429 = 3857

total 4-digits numbers = 9000

required numbers = 9000 – 3857 = 5143

18. **Ans. (3)**

Let number are a, ar, ar^2, ar^3

$$a\frac{(r^4-1)}{r-1}=\frac{65}{12} \qquad ...(1)$$

$$\frac{1}{a}\frac{\left(\frac{1}{r^4}-1\right)}{\frac{1}{r}-1}=\frac{65}{18}$$

$$\frac{1}{ar^3}\left(\frac{1-r^3}{1-r}\right)=\frac{65}{18} \qquad ....(2)$$

$\frac{(1)}{(2)} \Rightarrow a^2r^3 = \frac{3}{2}$

and $a^3.r^3 = 1$

$ar = 1$

$(ar)^2.r = \frac{3}{2}$

$r = \frac{3}{2}, a = \frac{2}{3}$

α = third term = $ar^2 = \frac{2}{3}\times\frac{9}{4}=\frac{3}{2}$

$2\alpha = 3$

19. Ans. (4)

$\frac{2(1+2+3+...+y)}{3(1+2+3+...+y)}=\frac{4}{\log_{10}x} \quad \Rightarrow \log_{10}x = 6 \Rightarrow x = 10^6$

Now, $y=(\log_{10}x)+\left(\log_{10}x^{\frac{1}{3}}\right)+\left(\log_{10}x^{\frac{1}{9}}\right)+..\infty$

$=\left(1+\frac{1}{3}+\frac{1}{9}+...\infty\right)\log_{10}x \;=\left(\frac{1}{1-\frac{1}{3}}\right)\log_{10}x=9$

So, $(x,y) = (10^6,9)$

20. Ans. (27560)

$a_1 = b_1 = 1$

$a_2 = a_1 + 2$

$a_3 = a_2 + 2$

$a_4 = a_2 + 2$

$\vdots \qquad \vdots$

$a_{n-1}=a_{n-2}+2$

$a_n=a_{n-1}+2$

$+ \quad +$

$a_n=2(n-1)+a_1$

$=2n-2+1$

$=2n-1$

$\Rightarrow a_n = 2n - 1$

$b_2 = a_1 + b_1$

$b_3 = a_3 + b_2$

$b_4 = a_4 + b_3$

$\vdots \qquad \vdots$

$b_{n-1} = a_{n-1} + b_{n-2}$

$b_n = a_n + b_{n-1}$

$b_n=\sum_{r=1}^{n}a_r \;=\sum_{r=1}^{n}2r-1$

$b_n = n^2$

$\sum_{n=1}^{15}a_nb_n=\sum_{n=1}^{15}(2n-1)n^2$

$$= \sum_{n=1}^{15} (2n^3 - n^2)$$

$$= 2\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$$

Put $n = 15$

$$= \frac{2\times 225\times 16\times 16}{4} - \frac{15\times 16\times 31}{6} = 27560$$

21. Ans. (120)

$$T_n = \frac{2\sum_{r=1}^{n}(2r)^3 - (\sum_{r=1}^{2n} r^3)}{n(4n+3)}$$

$$\Rightarrow T_n = n$$

So, $\sum_{n=1}^{15} T_n = 120$

22. Ans. (4)

$$\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \frac{1}{2^3\cdot 3^8} + ... + \frac{1}{2^{10}\cdot 3} = \frac{K}{2^{10}\cdot 3^{10}}$$

$K = 2^9 + 2^8 \cdot 3 + 2^7 \cdot 3^2 + ... + 3^9$

$$= \frac{2^9\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\frac{3}{2} - 1} = 3^{10} - 2^{10}$$

Now, $3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5)$

$= (211)(275)$

$= (35 \times 6 + 1)(45 \times 6 + 5)$

$= 6\lambda + 5$

Remainder is 5.

23. Ans. (3)

$$A = \left(\frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + ...\infty\right)$$

$$A = \left(\frac{1}{2} + \frac{1}{2^3} + ...\infty\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + ...\infty\right)$$

$$A = \left(\frac{\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{1}{16}}{1 - \frac{1}{16}}\right)$$

$$\Rightarrow A = \frac{1}{2}\times\frac{4}{3} + \frac{1}{16}\times\frac{16}{15} \Rightarrow A = \frac{11}{15}$$

$$B = \frac{-1}{2} + \frac{1}{4^2} - \frac{-1}{2^3} + \frac{1}{4^4} + ...\infty$$

$$B=\left(\frac{-1}{2}+\frac{-1}{2^3}+\ldots\infty\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+\ldots\infty\right)$$

$$B=\frac{-\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}$$

$$\Rightarrow B=-\frac{1}{2}\times\frac{4}{3}+\frac{1}{16}\times\frac{16}{15}$$

$$B=-\frac{9}{15}$$

$$\frac{A}{B}=\frac{11}{15}\times\frac{15}{(-9)}$$

$$\frac{A}{B}=-\frac{11}{9}$$

24. Ans. (2)

$$T_r=\frac{(r^2+r+1)-(r^2-r+1)}{2(r^4+r^2+1)}$$

$$\Rightarrow T_r=\frac{1}{2}\left[\frac{1}{r^2-r+1}-\frac{1}{r^2+r+1}\right]$$

$$T_1=\frac{1}{2}\left[\frac{1}{1}-\frac{1}{3}\right]$$

$$T_2=\frac{1}{2}\left[\frac{1}{3}-\frac{1}{7}\right]$$

$$T_3=\frac{1}{2}\left[\frac{1}{7}-\frac{1}{13}\right]$$

$$\vdots$$

$$T_{10}=\frac{1}{2}\left[\frac{1}{91}-\frac{1}{111}\right]$$

$$\Rightarrow \sum_{r=1}^{10}T_r=\frac{1}{2}\left[1-\frac{1}{111}\right]=\frac{55}{111}$$

25. Ans. (5)

$$1^3+2^3+3^3\ldots..+n^3=\left(\frac{n(n+1)}{2}\right)^2$$

$$1\cdot3+2\cdot5+3\cdot7+\ldots\ldots+\text{n terms}=\sum_{r=1}^{n}r(2r+1)=\sum_{r=1}^{n}\left(2r^2+r\right)$$

$$=\frac{2\cdot n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)}{6}\left(2(2n+1)+3\right)$$

$= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3}$

$= \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2} \times \frac{(4n+5)}{3}} = \frac{9}{5}$

$\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3}$

$\Rightarrow$ 15n (n + 1) = 18 (4n + 5)

$\Rightarrow 15n^2 + 15n = 72n + 90$

$\Rightarrow 15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow (n-5)(5n+6) = 0$

$\Rightarrow n = \frac{-6}{5}$ or 5

$\Rightarrow$ n = 5.

26. **Ans. (12)**

$T_4 = 500$ where a = first term,

r = common ratio = $\frac{1}{m}, m \in N$

$ar^3 = 500$

$\frac{a}{m^3} = 500$

$S_n - S_{n-1} = ar^{n-1}$

$S_6 > S_5 + 1$ and $S_7 - S_6 < \frac{1}{2}$

$S_6 - S_5 > 1$ $\frac{a}{m^6} < \frac{1}{2}$

$ar^5 > 1$ $m^3 > 10^3$

$\frac{500}{m^2} > 1$ m > 10 ...(2)

$m^2 < 500$ ...(1)

From (1) and (2)

m = 11, 12, 13…………..,22

So number of possible values of m is 12

27. **Ans. (60)**

$a_4 . a_6 = 9 \Rightarrow (a_5)^2 = 9 \Rightarrow a_5 = 3$

& $a_5 + a_7 = 24 \Rightarrow a_5 + a_5r^2 = 24 \Rightarrow (1+r^2) = 8 \Rightarrow r = \sqrt{7}$

$\Rightarrow a = \frac{3}{49}$

$\Rightarrow a_1a_9 + a_2a_4a_9 + a_5 + a_7 = 9 + 27 + 3 + 21 = 60$

28. **Ans. (3)**

Option (3)

If $a_n = \frac{-2}{4n^2 - 16n + 15}$ then $a_1 + a_2 + \ldots\ldots a_{25}$

$$\Rightarrow \sum_{n=1}^{25} a_n = \sum \frac{-2}{4n^2 - 16n + 15}$$

$$= \sum \frac{-2}{4n^2 - 6n - 10n + 15}$$

$$= \sum \frac{-2}{2n(2n-3) - 5(2n-3)}$$

$$= \sum \frac{-2}{(2n-3)(2n-5)}$$

$$= \sum \frac{1}{2n-3} - \frac{1}{2n-5}$$

$$= \frac{1}{47} - \frac{1}{(-3)}$$

$$= \frac{50}{141}$$

Exercise - III (JEE Advanced Pattern)

SECTION-I

1. **Ans. (B,D)**

$a_1, a_2, \ldots a_n$ are in A.P.

$a_2 = a_1 + d$

$a_3 = a_1 + 2d$

$a_4 = a_1 + 3d$

$a_5 = a_1 + 4d$

(A) $a_1 + 2a_2 + a_3$

$\Rightarrow a_1 + 2(a_1 + d) + a_1 + 2d$

$\Rightarrow 4a_1 + 4d = 4a_2$

(B) $a_1 - 2a_2 + a_3$

$\Rightarrow a_1 - 2(a_1 + d) + a_1 + 2d$

$\Rightarrow 2a_1 + 2d - 2a_1 - 2d \Rightarrow 0$

(C) $a_1 + 3a_2 - 3a_3 - a_4$

$\Rightarrow a_1 + 3(a_1 + d) - 3(a_1 + 2d) - (a_1 + 3d)$

$\Rightarrow a_1 + 3a_1 + 3d - 3a_1 - 6d - a_1 - 3d$

$\Rightarrow -6d$

(D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5$

$\Rightarrow a_1 - 4(a_1 + d) + 6(a_1 + 2d) - 4(a_1 + 3d) + a_1 + 4d$

$\Rightarrow a_1 - 4a_1 - 4d + 6a_1 + 12d - 4a_1 - 12d + a_1 + 4d$

$\Rightarrow -6a_1 + 6a_1 \Rightarrow 0$

$\therefore$ option (B,D) are correct.

2. **Ans. (A,B,C,D)**

$a_1 = 25, b_1 = 75$

$a_{100} + b_{100} = 100$

$\Rightarrow a_1 + 99d_1 + b_1 + 99d_2 = 100$

$\Rightarrow a_1 + b_1 + 99(d_1 + d_2) = 100$

$\Rightarrow 100 + 99(d_1 + d_2) = 100$

$\Rightarrow d_1 + d_2 = 0$

$\Rightarrow d_1 = -d_2$

Now $\quad a_n + b_n = a_1 + (n-1)d_1 + b_1 + (n-1)d_2$

$a_n + b_n = a_1 + b_1 + (n-1)(d_1 + d_2)$

$a_n + b_n = a_1 + b_1 + (n-1) \times 0$

$a_n + b_n = 100$

$a_1 + b_1 = 100$

$a_2 + b_2 = 100$

$\vdots$

$a_n + b_n = 100$

$(a_1 + b_1), (a_2 + b_2), (a_3 + b_3)$...... are A.P.

$$\sum_{r=1}^{100}(a_r + b_r) = (a_1 + b_1) + (a_2 + b_2) + (a_3 + b_3)......(a_{100} + b_{100})$$

= 100 + 100 + 100 100

= 100 × 100 = 10000

$\therefore$ option (A,B,C,D) are correct.

3. **Ans. (A,B)**

x_1, x_2, x_3 are in A.P.

$\Rightarrow \quad x_2 = x_1 + d \Rightarrow x_1 = x_2 - d$

$\Rightarrow \quad x_3 \Rightarrow x_2 + d$

x_1, x_2, x_3 are roots of $x^3 - x^2 + \alpha x + \beta = 0$

$\Rightarrow \quad x_1 + x_2 + x_3 = 1$

$\Rightarrow \quad x_2 - d + x_2 + x_2 + d = 1$

$\Rightarrow \quad 3x_2 = 1$

$\Rightarrow \quad x_2 = \frac{1}{3}$

$\Rightarrow \quad x_1x_2 + x_2x_3 + x_3x_1 = \alpha$

$\Rightarrow \quad x_2(x_1 + x_3) + x_1x_3 = \alpha$

$\Rightarrow \quad x_2(1 - x_2) + (x_2 - d)(x_2 + d) = \alpha$

$\Rightarrow \quad \frac{1}{3}\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - d\right)\left(\frac{1}{3} + d\right) = \alpha$

$\Rightarrow \quad \frac{2}{9} + \frac{1}{9} - d^2 = \alpha$

$\Rightarrow \quad \frac{1}{3} - d^2 = \alpha$

$\Rightarrow \quad a^2 \geq 0 \Rightarrow \alpha \leq \frac{1}{3}$

$\Rightarrow \quad x_1x_2x_3 = -\beta$

$\Rightarrow \quad (x_2 - d)x_2(x_2 + d) = -\beta$

$\Rightarrow \quad \left(\frac{1}{3}-d\right)\frac{1}{3}\left(\frac{1}{3}+d\right)=-\beta$

$\Rightarrow \quad \left(\frac{1}{9}-d^2\right)\frac{1}{3}=-\beta$

$\Rightarrow \quad \frac{1}{27}-\frac{d^2}{3}=-\beta$

$\Rightarrow \quad \beta=\frac{d^2}{3}-\frac{1}{27}$

$\Rightarrow \quad \frac{d^2}{3}\geq 0 \quad \Rightarrow \quad \beta\geq-\frac{1}{27}$

$\therefore$ option (A,B) are correct.

4. **Ans. (A,B,C,D)**

$a_1 = a$

$\Rightarrow a_2 = ar$

$\Rightarrow a_3 = ar^2$

$\Rightarrow a_4 = ar^3$

(A) $a_1^3 = a^3$

$\Rightarrow a_2^3 = a^3r^3$

$\Rightarrow a_3^3 = a^3r^6$

$\Rightarrow a_4a^3 = a^3r^9$

Common ration = $\frac{a_2^3}{a_1^3}=\frac{a_3^3}{a_2^3}=\frac{a_4^3}{a_3^3}=r^3$

$\Rightarrow a_1^3, a_2^3, a_3^3$...... are in G.P.

(B) $\frac{a_2}{a_1}=\frac{ar}{a}=r$

$\Rightarrow \left(\frac{a_3}{a_2}\right)^2=\frac{ar^2}{ar}=r$

$\Rightarrow \left(\frac{a_4}{a_3}\right)^3=\frac{ar^3}{ar^2}=r$

$\Rightarrow \frac{a_2}{a_1},\left(\frac{a_3}{a_2}\right)^2,\left(\frac{a_4}{a_\rightarrow}\right)^3$....... are in G.P.

(C) $\ell na_1 = \ell na$

$\Rightarrow \ell na_2 = \ell n\, ar$

$\Rightarrow \ell na_3 = \ell n\, ar^2$

$\Rightarrow \ell na_4 = \ell n\, ar^3$

Common difference $= \ell n\, ar - \ell n\, a = \ell n\, r$

$= \ell n\, ar^2 - \ell n\, ar = \ell n\, r$

$= \ell n\, ar^3 - \ell n\, ar^2 = \ell n\, r$

$\Rightarrow \ell n\, a_1, \ell n\, a_2, \ell n\, a_3 \ldots$ A.P.

(D) $a_1 a_2 = a \times ar = a^2 r$

$\Rightarrow a_2 a_3 = (ar) \times (ar^2) = a^2 r^3$

$\Rightarrow a_3 a_4 = (ar^2) \times (ar^3) = a^2 a^5$

Common ration = $\frac{a^2 r^3}{a^2 r} = \frac{a^2 r^5}{a^2 r^3} = r^2$

$\Rightarrow a_1 a_2, a_2 a_3, a_3 a_4 \ldots.$ are in G.P.

option (A,B,C,D) are correct.

5. **Ans. (A,B,C)**

$16x^4 - mx^3 + (2m + 17)x^2 - mx + 16 = 0$

$x = \frac{1}{2}$ is the roots of this equation

$\Rightarrow 16 \times \left(\frac{1}{2}\right)^4 - m\left(\frac{1}{2}\right)^3 + (2m+17)\left(\frac{1}{2}\right)^2 - \frac{m}{2} + 16 = 0$

$\Rightarrow 1 - \frac{m}{8} + \frac{2m+17}{4} - \frac{m}{2} + 16 = 0$

$\Rightarrow \frac{136 - m + 4m + 34 - 4m}{8} = 0$

$\Rightarrow m = 170$

Now equation

$16x^4 - 170x^3 + 357x^2 - 170x + 16 = 0$ → α, β, γ, δ

Sum of roots = $\frac{170}{16} = \frac{85}{8}$

roots $\alpha = \frac{a}{r^3}$, $\beta = \frac{a}{r}$, $r = ar, \delta = ar^3$

$\alpha\beta\gamma\delta = 1$

$\Rightarrow \frac{a}{r^3} \times \frac{a}{r} \times ar \times ar^3 = 1$

$\Rightarrow a^4 = 1$

$\Rightarrow a = 1, -1$

When $a = 1$

Sum of roots = $\frac{85}{8}$

$\Rightarrow \frac{1}{r^3}+\frac{1}{r}+r+r^3=\frac{85}{8}$

$\Rightarrow \left(r+\frac{1}{r}\right)+\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=\frac{85}{8}$

$\Rightarrow r+\frac{1}{r}=t$, we get $8t^3-16-85=0$

$\Rightarrow t=\frac{5}{2}$ satisfies the above equation

$\Rightarrow r+\frac{1}{r}=\frac{5}{2}$, $2r^2-5r+2=0$

$\Rightarrow r=\frac{1}{2}, 2$

$\therefore$ Common ratio = $r^2=4$.

6. Ans. (A,B,C,D)

$2, a, b, c, 18$

$\Rightarrow 2a=b+2$ ...(1)

$\Rightarrow a+b+c=25$ ...(2)

$\Rightarrow c^2=18b$ ...(3)

from (1) & (2)

$\Rightarrow \frac{b+2}{2}+b+c=25 \quad \Rightarrow \quad 3b+2c=48$

from (3)

$\Rightarrow 3\left(\frac{c^2}{18}\right)+2c=48 \quad \Rightarrow \quad c^2+12c-288=0$

$\Rightarrow c=12, b=8, a=5$

$\therefore$ option (A,B,C,D) are correct.

7. Ans. (A,C,D)

$a=\lambda, b=\lambda r, c=\lambda r^2\ (a,b,c\in Z^+)$

$\log_6 a+\log_6 b+\log_6 c=6$

$\Rightarrow \log_6(abc)=6 \quad \Rightarrow \quad abc=6^6$

$\Rightarrow \lambda\times\lambda r\times\lambda r^2=6^6 \quad \Rightarrow \quad \lambda^3 r^3=6^6$

$\Rightarrow \lambda r=6^2=36 \quad \Rightarrow \quad b-a=$ perfect cube

$\Rightarrow \lambda r-\lambda=$ perfect cube

$\Rightarrow 36-\lambda=$ perfect cube

$\lambda=9, r=4$

$\Rightarrow a=9 \quad \Rightarrow \quad b=36 \Rightarrow \quad c=144$

$\Rightarrow a+b+c=9+36+144=189$

Now first term = 9

Common ratio = $\frac{5r}{2}=\frac{5\times 4}{2}=10$

$S=\frac{9(10^{10}-1)}{10-1}=10^{10}-1$

$S = 9999999999$

Sum of digit of $S = 9 + 9$ .... 10 times

$= 90$

Option (A,C,D) are correct.

8. **Ans. (B,C)**

HM = 2

$\Rightarrow \text{AM} = 2^x - \frac{7}{2}$

$\Rightarrow \text{GM} = 2^x - 5$

$(\text{GM})^2 = \text{AM} \times \text{HM}$ ...(1)

$\Rightarrow (2^x - 5)^2 = \left(2^x - \frac{7}{2}\right) \times 2$ ...(2)

$\Rightarrow 2^{2x} - 10 \cdot 2^x + 25 = 2^x \times 2 - 7$

$\Rightarrow 2^{2x} - 12 \cdot 2^x + 34 = 0$

$\Rightarrow (2^x - 8)(2^x - 4) = 0$

$x = 3\,, x = 2$

$\Rightarrow$ $x = 2$ doesn't satisfy equation (1)

$\Rightarrow x = 3$

$\text{GM} = (2)^3 - 5 = 3$

$\text{AM} = (2)^3 - \frac{7}{2} = \frac{9}{2}$

$\text{AM} = \frac{a+b}{2} = \frac{9}{2}$

$\Rightarrow a + b = 9$ ...(2)

$\text{GM} = \sqrt{ab} = 3$

$\Rightarrow ab = 9$ ...(3)

Now $(a+b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + b^2 = (a)^2 - 9 \times 2 = 63$

option (B,C) are correct.

9. **Ans. (B,C,D)**

$S = \sum_{n=1}^{\infty} (-1)^{n+1} \times \frac{n}{5^n}$

$S = \frac{1}{5} - \frac{2}{5^2} + \frac{3}{5^3} - \frac{4}{5^4} + \frac{5}{5^5}$ ....∞

$\frac{S}{5} = \frac{1}{5^2} - \frac{2}{5^3} + \frac{3}{5^4} - \frac{4}{5^5}$ ....∞

+

$S\left(1 + \frac{1}{5}\right) = \frac{1}{5} - \frac{1}{5^2} + \frac{1}{5^3} - \frac{1}{5^4}$ ....∞

$\Rightarrow S \times \frac{6}{5} = \frac{\frac{1}{5}}{1 + \frac{1}{5}} = \frac{1}{6}$ $\quad \Rightarrow S = \frac{5}{36}$

$a = 5\,, b = 36$

$\therefore$ option (B,C,D) are correct.

10. Ans. (A,B)

$$S=\sum_{r=2}^{100}\frac{2^r(2-r)}{r(r+1)(r+2)}$$

$$S=\sum_{r=2}^{100}\left[\frac{2^r}{r(r+1)}-\frac{2^{r+1}}{(r+1)(r+2)}\right]$$

$$S=\frac{2^2}{2\times3}-\frac{2^3}{3\times4}+\frac{2^3}{3\times4}-\frac{2^4}{4\times5}+\frac{2^4}{4\times5}-\frac{2^5}{5\times6}\ldots\ldots\ldots\frac{2^{100}}{100\times101}-\frac{2^{101}}{101\times102}$$

$$S=\frac{2}{3}-\frac{2^{101}}{101\times102}$$

$\therefore$ Option (A,B) are correct.

SECTION–II

11. Ans. (D)

$T_1=a$

$T_2=ar=4$ ...(1)

$$S_\infty=\frac{a}{1-r}=25$$

$\Rightarrow a=25(1-r)$ ...(2)

from (1) & (2)

$\Rightarrow 25(1-r)r=4$ $\Rightarrow 25r-25r^2=4$

$\Rightarrow 25r^2-25r+4=0$ $\Rightarrow 25r^2-20r-5r+4=0$

$\Rightarrow 5r(5r-4)-1(5r-4)=0$

$\Rightarrow (5r-1)(5r-4)=0$ $\Rightarrow r=\frac{1}{5},\frac{4}{5}$

$$\Rightarrow r=\frac{1}{5}\Rightarrow a=25\left(1-\frac{1}{5}\right)=20$$

$$\Rightarrow r=\frac{4}{5}\Rightarrow a=25\left(1-\frac{4}{5}\right)=5$$

Sum of all possible values of $a=20+5=25$.

12. Ans. (B)

$(r)_{\text{larger}}=\frac{4}{5}, a=5$

$S_n>15$

$$\Rightarrow \frac{a(1-r^n)}{1-r}>15\Rightarrow \frac{5\left[1-\left(\frac{4}{5}\right)^n\right]}{1-\frac{4}{5}}>15$$

$$\Rightarrow 1-\left(\frac{4}{5}\right)^n>\frac{3}{5}\Rightarrow \left(\frac{4}{5}\right)^n<\frac{2}{5}$$

$$\Rightarrow \frac{2^{2n}}{2}<\frac{5^n}{5}\Rightarrow 2^{2n-1}<5^{n-1}$$

smallest value of $n=5$.

13. Ans. (B,C,D)

Let two consecutive numbers are λ, $\lambda + 1$

A.M. of remaining numbers = $\frac{1+2+3...n-(\lambda+\lambda+1)}{n-2}$

$$\frac{105}{4}=\frac{\frac{n(n+1)}{2}-(2\lambda+1)}{n-2}$$

$\Rightarrow 2n^2 - 103n - 8\lambda + 206 = 0$

Since n and λ are integers so n must be even

Let $n = 2k$

$$\lambda=\frac{4k^2+103(1-k)}{4}$$

Since λ is an integer then $(1 - k)$ must be divisible by 4 so that $k = 1 + 4\mu$

We get

$\Rightarrow n = 2(1 + 4\mu) = 2 + 8\mu$

$\Rightarrow \lambda = 16\mu^2 - 95\mu + 1$

Now $1 \leq \lambda < n$

$\Rightarrow \mu = 6$

$n = 48 + 2 = 50, \lambda = 7$

$\because n$ (B,C,D) are correct.

14. Ans. (B,C)

$\lambda = 7$

$\lambda + 1 = 7 + 1 = 8$

removed numbers are 7, 8

$\because$ (B,C) are correct.

15. Ans. (C)

$T_1 + T_n = 66$

$a + ar^{n-1} = 66$ ...(1)

$T_2 \times T_{n-1} = 128$

$ar \times ar^{n-2} = 128$

$a^2r^{n-1} = 128$

$\Rightarrow r^{n-1}\frac{128}{a^2}$ ...(2)

from (1) & (2)

$a^2 - 66a + 128 = 0$

$\Rightarrow a = 2$ or $a - 64$

$\Rightarrow r^{n-1} = 32$ or $r^{n-1}=\frac{1}{32}$

When $r^{n-1} = 32$ (increasing G.P.)

$$S_n=\frac{a\left(r^n-1\right)}{r-1}=126$$

$\Rightarrow \dfrac{2(32r-1)}{r-1}=126$

$32\,r - 1 = 63\,r - 63$

$62 = 31\,r$

$r = 2, n = 6.$

16. Ans. (B)

T_6 (greatest term) = ar^5 = 2 × 2^5 = 64

T_1 (least term) = $a = 2$

Difference of greatest and least term = 64 – 2 = 62.

SECTION–III

17. Ans. (B)

(P) first A.P.

1, 3, 5, 7 …….. 157

Common diff. $(d_1) = 2$

second A.P.

2, 5, 8, 11 …… 161

Common diff. $(d_2) = 3$

Common A.P. have common diff = LCM (d_1, d_2)

$d = 6$

Common A.P. will be

5, 11, 17, 23 ……. 155

$\Rightarrow$ 155 = 5 + $(n - 1)$ 6

$\Rightarrow (n-1)6 = 150$

$\Rightarrow (n-1) = 25$

$\Rightarrow n = 26$

$\therefore$ (2) will be correct

(Q) $11x^2 - 4x - 2 = 0$ with roots a, b

$a+b=\dfrac{4}{11},\ ab=\dfrac{-2}{11}$

$\Rightarrow 10(1 + a + a^2 \dots \infty)(1 + b + b^2 \dots \infty)$

$\Rightarrow \dfrac{10\times1}{1-a}\times\dfrac{1}{1-b} \Rightarrow \dfrac{10}{1-b-a+ab}=\dfrac{10}{1-\dfrac{4}{11}-\dfrac{2}{11}}=\dfrac{10\times11}{5}=22$

$\therefore$ (3) will match

(R) Sum of first 20 terms = Sum of first 10 terms

$\dfrac{20}{2}[2a+19d]=\dfrac{10}{2}[2a+9d]$

$\Rightarrow 4a + 38d = 2a + 9d$

$\Rightarrow 2a + 29d = 0$

$\Rightarrow S_{30} = \frac{30}{2}[2a + (30-1)d]$

$\Rightarrow S_{30} = 15[2a + 29d] = 15 \times 0 = 0$

$\Rightarrow S_{30} = 0$

$\therefore$ (1), (2), (3), (4), (5) will match

(S) $T_n = \frac{2n+1}{n^2(n+1)^2} = \frac{(n+1)^2 - n^2}{n^2(n+1)^2}$

$T_n = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$S_n = \sum_{n=1}^{99} T_n = T_1 + T_2 + T_3 \ldots\ldots T_{99}$

$S_{99} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots \frac{1}{99^2} - \frac{1}{100^2}$

$S_{99} = 1 - \frac{1}{10000} = \frac{9999}{10000}$

$\therefore a = 9999, b = 10000$

$20|b - a| = 20|10000 - 9999| = 20 \times 1 = 20$

$\therefore$ (4) will match.

SECTION–IV

18. Ans. (A $\rightarrow$ p ; B $\rightarrow$ s ; C $\rightarrow$ r; D $\rightarrow$ q)

(A) Coefficient of x^{49} in

$(x-1)(x-3)(x-5)(x-7)\ldots\ldots(x-99)$

There are 50 brackets, for making x^{49}, we will multiply x 49 times and multiply the constant term 1 time, since the no. of possible constant terms which may be taken for 50

$S_n = (-1 - 3 - 5\ldots\ldots 99)x^{49}$

$= \frac{50 \times 100}{2(-1)} x^{49}$

$= -2500x^{49}$

$\therefore$ (p) will match

(B) $S_{2n} = 3\,S_n$

$\Rightarrow \frac{2n}{2}[2a + (2n-1)d] = 3 \times \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 4a + (4n-2)d = 6a + (3n-3)d$

$\Rightarrow 2a + d[3n - 3 - 4n + 2] = 0$

$\Rightarrow 2a - (n+1)d = 0$

$\Rightarrow 2a = (n+1)d$

$$\Rightarrow \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]}$$

$$\Rightarrow \frac{S_{3n}}{S_n} = \frac{3[(3n+1)d+(3n-1)d]}{(n+1)d+(n-1)d}$$

$$\Rightarrow \frac{S_{3n}}{S_n} = 3\left[\frac{4nd}{2nd}\right] = 6$$

$\therefore$ (s) will match

(C) $S_n = \sum_{r=2}^{n} \frac{1}{r^2-1}$

$$\Rightarrow S_n = \frac{1}{2}\sum_{r=2}^{n}\frac{2}{(r-1)(r+1)} = \frac{1}{2}\sum_{r=2}^{n}\left[\frac{(r+1)-(r-1)}{(r-1)(r+1)}\right]$$

$$\Rightarrow S_n = \frac{1}{2}\sum_{r-2}^{n}\left[\frac{1}{r-1} - \frac{1}{r+1}\right]$$

$$\Rightarrow S_n = \frac{1}{2}\left[1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6} \ldots\ldots \frac{1}{n-2} - \frac{1}{n} + \frac{1}{n-1} - \frac{1}{n+1}\right]$$

$$\Rightarrow S_n = \frac{1}{2}\left[\frac{3}{2} - \frac{1}{n} - \frac{1}{n+1}\right] \Rightarrow S_\infty = \frac{3}{4}$$

$\therefore$ (r) will match

(D) $\ell = \frac{a}{r}$

$\Rightarrow b = a$

$\Rightarrow h = ar$

volume = $\ell \times b \times h$

$$27 = \frac{a}{r} \times a \times ar = a^3$$

$a = 3$

total surface area = $2(\ell b + bh + h\ell)$

$$78 = 2\left(\frac{a}{r} \times a + a \times ar + \frac{a}{r} \times ar\right)$$

$$78 = 2 \times a^2\left[\frac{1}{r} + r + 1\right]$$

$$\frac{1}{r} + r + 1 = \frac{78}{18} = \frac{13}{3}$$

$$\Rightarrow \frac{r^2+r+1}{r} \times \frac{13}{3}$$

$\Rightarrow 3r^2 + 3r + 3 = 13r$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r(r-3) - 1(r-3) = 0$

$\Rightarrow (3r-1)(r-3) = 0$

$\Rightarrow \quad r = \frac{1}{3}, 3 \qquad \Rightarrow r = 3$

$\Rightarrow \quad \ell = \frac{3}{3} = 1 \qquad \Rightarrow b = 3$

$\Rightarrow \quad h = 9$

but $\ell > b > h$ given rejected

$\Rightarrow \quad r = \frac{1}{3} \qquad \Rightarrow \qquad r = \frac{1}{3}$

$\Rightarrow \quad \ell = 9$

$\Rightarrow \quad b = 3$

$\Rightarrow \quad h = 1$

$\therefore$ (q) will match.

Exercise - IV (JEE Advanced PYQs)

1. **Ans. (C)**

$S_n = cn^2$

$S_{n-1} = c(n-1)^2$

$T_n = S_n - S_{n-1} = c(2n-1)$

$T'_n = T_n^2$

$= c^2(4n^2 - 4n + 1)$

$S_n = \sum T_n'$

$= c^2\left[4\sum n^2 - 4\sum n + \sum 1\right]$

$= c^2\left[\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right]$

$\Rightarrow \sum T'_n = nc^2\left(\frac{4n^2-1}{3}\right)$

2. **Ans. (0)**

$a_1 = 15$

$27 - 2a_2 > 0$

$a_k = 2a_{k-1} - a_{k-2}\ \forall\ k = 3, 4, 5 \ldots 11$

$\frac{a_1^2 + a_2^2 + \ldots\ldots a_{11}^2}{11} = 90$

$a_{k-1} = \frac{a_k + a_{k-2}}{2}$

all $a_i (i = 1, 2, \ldots\ldots 11)$ are in A.P.

Let the numbers are

$(a_6 + 5d), (a_6 + 4d), \ldots\ldots, a_6, \ldots\ldots, (a_6 - 4d), (a_6 - 5d)$

$11a_6^2 + 110d^2 = 990$

$a_6 = 15 - 5d$

$a_6^2 + 10d^2 = 90$

$(15 - 5d)^2 + 10d^2 = 90 \Rightarrow 7d^2 - 30d + 27 = 0 \Rightarrow d = 3, \frac{9}{7}$

for $d = 3 \Rightarrow a_2 = 12$ (possible)

for $d = 9/7 \Rightarrow a_2 = 13.7$ (not possible since $a_2 < 13.5$)

$a_6 = 0 \Rightarrow \frac{a_1 + a_2 + ... + a_{11}}{11} = a_6 = 0$

3. Ans. (8)

As $a > 0$

and all the given terms are positive

hence considering A.M. ≥ G.M. for given numbers :

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^{8} + a^{10}}{7} \ge \left(a^{-5}.a^{-4}.a^{-3}.a^{-3}.a^{-3}.a^{8}.a^{10}\right)^{\frac{1}{7}}$$

$$\Rightarrow \frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^{8} + a^{10}}{7} \ge 1 \Rightarrow \left(a^{-5} + a^{-4} + 3a^{-3} + a^{8} + a^{10}\right)_{\min} = 7$$

where $a^{-5} = a^{-4} = a^{-3} = a^8 = a^{10}$ i.e. $a = 1$

$\Rightarrow (a^{-5} + a^{-4} + 3a^{-3} + a^8 + a^{10} + 1)_{\min} = 8$ when $a = 1$

4. Ans. (9 or 3)

(Comment : The information about the common difference i.e. <u>zero or non-zero</u> is not given in the question. Hence there are two possible answers)

Consider d ≠ 0 the solution is

$a_1, a_2, a_3, ..., a_{100} \rightarrow$ AP

$a_1 = 3$; $S_p = \sum_{i=1}^{p} a_i$ $1 \le n \le 20$

$m = 5n$

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a_1 + (m-1)d]}{\frac{n}{2}[2a_1 + (n-1)d]}$$

$$\frac{S_m}{S_n} = \frac{5[(2a_1 - d) + 5nd]}{[(2a_1 - d) + nd]}$$

for $\frac{S_m}{S_n}$ to be independent of n

$\therefore 2a_1 - d = 0 \Rightarrow d = 2a_1$

$\Rightarrow d = 6 \Rightarrow a_2 = 9$

If $d = 0 \Rightarrow a_2 = a_1 = 3$

5. **Ans. (D)**

a_1, a_2, a_3 .... be in H.P $\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$.... be in A.P.

in A.P. $T_1 = \frac{1}{a_1} = \frac{1}{5}$ and $T_{20} = \frac{1}{a_{20}} = \frac{1}{25}$

$\therefore T_{20} = T_1 + 19d$

$\frac{1}{25} = \frac{1}{5} + 19d \Rightarrow d = -\frac{4}{19\times 25}$

$T_n = T_1 + (n-1)d < 0$

$\Rightarrow \frac{1}{5} - \frac{(n-1).4}{19\times 25} < 0 \Rightarrow \frac{1}{5} < \frac{4(n-1)}{25\times 19}$

$\Rightarrow \frac{5\times 19}{4} + 1 < n \Rightarrow \frac{99}{4} < n$

$\Rightarrow$ least positive integer n is 25.

6. **Ans. (A,D)**

$S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2$ ....

$S_n = (3^2 - 1^2) + (4^2 - 2^2) +$ ....

$S_n = 2(1 + 2 + 3 + \ldots\ldots + 4n)$

$= \frac{2(4n)(4n+1)}{2}$

$S_n = 4n(4n+1)$

$S_n = 4n(4n + 1) = 1056$ is possible when $n = 8$

$4n(4n + 1) = 1088$ not possible

$4n(4n + 1) = 1120$ not possible

$4n(4n + 1) = 1332$ possible when $n = 9$.

7. **Ans. (5)**

When 1 and 2 are removed from numbers 1 to n then

we get maximum possible sum of remaining numbers and when $n-1, n$ are removed then we get minimum possible sum of remaining numbers.

$\Rightarrow \frac{n(n+1)}{2} - (2n-1) \le 1224 \le \frac{n(n+1)}{2} - 3$

$\Rightarrow \begin{cases} n^2 + n - 2454 \ge 0 \\ n^2 - 3n - 2446 \le 0 \end{cases} \Rightarrow \begin{cases} n \ge 50 \\ n \le 50 \end{cases} \Rightarrow n = 50$

Now let x and $x + 1$ be two consecutive numbers

$\Rightarrow \frac{50(50+1)}{2} - x - x - 1 = 1224$

$\Rightarrow x = 25$

$\Rightarrow 25^{th}$ and 26^{th} cards are removed from pack

$\Rightarrow k = 25 \Rightarrow k - 20 = 5$

8. **Ans. (4)**

Let a, b, c are a, ar, ar^2 where $r \in N$

also $\frac{a+b+c}{3} = b+2$

$\Rightarrow a + ar + ar^2 = 3(ar) + 6$

$\Rightarrow ar^2 - 2ar + a = 6$

$\Rightarrow (r-1)^2 = \frac{6}{a}$

$\because \frac{6}{a}$ must be perfect square & $a \in N$

$\therefore a$ can be 6 only.

$\Rightarrow r - 1 = \pm 1 \Rightarrow r = 2$

& $\frac{a^2+a-14}{a+1} = \frac{36+6-14}{7} = 4$ Ans.

9. **Ans. (9)**

$$\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11} \Rightarrow \frac{7(2a+6d)}{(2a+10d)} = 6$$

$\Rightarrow \quad 2a = 18d$

$\quad a = 9d$

also $130 < a + 6d < 140$

$\frac{26}{3} < d < \frac{28}{3} \Rightarrow d = 9$

10. **Ans. (B)**

If $\log_e b_1, log_e b_2, \ldots\ldots, \log_e b_{101} \rightarrow$ AP ; D $= \log_e 2$

$\Rightarrow b_1 b_2 b_3, \ldots\ldots, b_{101} \rightarrow$ GP ; $r = 2$

$\therefore b_1, 2b_1, 2^2 b_1, \ldots\ldots\ldots, 2^{100} b_1 \ldots\ldots GP$

$a_1 a_2 a_3 \ldots\ldots\ldots\ldots a_{101} \ldots\ldots\ldots$ AP

Given, $a_1 = b_1$ & $a_{51} = b_{51}$

$\Rightarrow \quad a_1 + 50\text{D} = 2^{50} b_1$

$\therefore \quad b_1 + 50\text{D} = 2^{50} b_1$ (As $b_1 = a_1$)

Now, $t = b_1(2^{51} - 1)$;

$\Rightarrow t < a_1 . 2^{51}$ ...(i)

$s = \frac{51}{2}(a_1 + a_1 + 50\text{D}) = \frac{51}{2}(a_1 + 2^{50} a_1)$

$s = \frac{51 a_1}{2} + \frac{51}{2} . 2^{50} a_1$

$\Rightarrow s > a_1 . 2^{51}$ ...(ii)

clearly $s > t$ (from equation (i) and (ii))

Also $a_{101} = a_1 + 100D$; $b_{101} = b_1 . 2^{100}$

$\therefore a_{101} = a_1 + 100\left(\frac{2^{50}a_1 - a_1}{50}\right)$;

$b_{101} = 2^{100}a_1$ ...(iii)

$a_{101} = a_1 + 2^{51}a_1 - 2\,a_1$

$\Rightarrow a_{101} = 2^{51}a_1 - a_1$

$\Rightarrow a_{101} < 2^{51}a_1$ ...(iv)

clearly $b_{101} > a_{101}$ (from equation (iii) and (iv))

11. Ans. (6)

where $d > 0, a > 0$

$\Rightarrow$ length of smallest

side = $a - d$

Now $(a+d)^2 = a^2 + (a-d)^2$

$\Rightarrow a(a - 4\,d) = 0$

$\therefore a = 4d$ ...(1)

(As $a = 0$ is rejected)

Also,

$\Rightarrow a(a-d) = 48$ ...(2)

$\therefore$ From (1) and (2), we get $a = 8, d = 2$

Hence, length of smallest side

$\Rightarrow (a-d) = (8-2) = 6$

12. Ans. (3748)

$X: 1, 6, 11, \ldots\ldots\ldots, 10086$

$Y: 9, 16, 23, \ldots\ldots\ldots, 14128$

$X \cap Y: 16, 51, 86, \ldots\ldots$

Let $m = n\,(X \cap Y)$

$\therefore 16 + (m-1) \times 35 \leq 10086$

$\Rightarrow m \leq 288.71$

$\Rightarrow m = 288$

$\therefore n\,(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 2018 + 2018 - 288 = 3748$

13. Ans. (8.00)

$$\frac{3^{y_1} + 3^{y_2} + 3^{y_3}}{3} \geq \left[3^{(y_1+y_2+y_3)}\right]^{\frac{1}{3}}$$

$\Rightarrow 3^{y_1} + 3^{y_2} + 3^{y_3} \geq 3^4$

$\Rightarrow \log_3(3^{y_1} + 3^{y_2} + 3^{y_3}) \geq 4 \;\Rightarrow m = 4$

Also, $\dfrac{x_1 + x_2 + x_3}{3} \geq \sqrt[3]{x_1x_2x_3}$

$\Rightarrow x_1x_2x_3 \leq 27$

$\Rightarrow \log_3 x_1 + \log_3 x_2 + \log_3 x_3 \leq 3$

$\Rightarrow M = 3$

Thus, $\log_2(m^3) + \log_3(M^2) = 6 + 2 = 8$

14. Ans. (1.00)

Given $2(a_1 + a_2 + \ldots\ldots + a_n) = b_1 + b_2 + \ldots\ldots + b_n$

$\Rightarrow 2 \times \frac{n}{2}(2c + (n-1)2) = c\left(\frac{2^n - 1}{2 - 1}\right)$

$\Rightarrow 2n^2 - 2n = c(2^n - 1 - 2n)$

$\Rightarrow c = \frac{2n^2 - 2n}{2^n - 1 - 2n} \in \mathbb{N}$

So, $2n^2 - 2n \geq 2^n - 1 - 2n$

$\Rightarrow 2n^2 + 1 \geq 2^n \Rightarrow n < 7$

$\Rightarrow n$ can be $1,2,3,\ldots.,$

Checking c against these values of n

we get $c = 12$ (when $n = 3$)

Hence number of such $c = 1$

15. Ans. (18900.00)

Given

$A_{51} - A_{50} = 1000 \Rightarrow \ell_{51}w_{51} - \ell_{50}w_{50} = 1000$

$\Rightarrow (\ell_1 + 50d_1)(w_1 + 50d_2) - (\ell_1 + 49d_1)(w_1 + 49d_2) = 1000$

$\Rightarrow (\ell_1 d_2 + w_1 d_1) = 10$ ....(1)

(As $d_1 d_2 = 10$)

$\therefore A_{100} - A_{90} = \ell_{100}w_{100} - \ell_{90}w_{90}$

$= (\ell_1 + 99d_1)(w_1 + 99d_2) - (\ell_1 + 89d_1)(w_1 + 89d_2)$

$= 10(\ell_1 d_2 + w_1 d_1) + (99^2 - 89^2)d_1 d_2$

$= 10(10) + \underbrace{(99-89)}_{=10}(99+89)(10)$

(As, $d_1 d_2 = 10$)

= 100(1 + 188) = 100 (189)

= 18900

16. Ans. (1219)

$S = 77 + 757 + 7557 + \ldots + 7\underbrace{5\ldots\ldots5}_{98}7$

$10S = \quad 770 + 7570 + \ldots + 75\ldots570 + 7\underbrace{55\ldots\ldots5}_{98}70$

$9S = -77 + \underbrace{13 + 13 + \ldots\ldots + 13}_{98\ times} + 7\underbrace{5\ldots\ldots5}_{98}70$

$= -77 + 13 \times 98 + 7\underbrace{5\ldots\ldots5}_{99}7 + 13$

$S = \frac{7\underbrace{5\ldots\ldots5}_{99}7 + 1210}{9}$

m = 1210

n = 9

m + n = 1219