Relations

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

$n(A) = m, n(B) = n$

$\Rightarrow n(A \times B) = m \times n$

$\therefore$ Number of relations from A to B = $2^{n(A\times B)}$

= 2^{mn}

2. **Ans. (B)**

$R = \{1,2,3, \dots \dots, 30\} \to \{1,2,3, \dots, 30\}$

$R = \{(a, b): b = pq$, where $p, q \ \geq 3$ are numbers$\}$

No. of possible values of $a = 30$

If $p = 3, q = 3$			$\Rightarrow b = 9$
If $p = 3, q = 5$	or	$p = 5, q = 3$	$\Rightarrow b = 15$
If $p = 3, q = 7$	or	$p = 7, q = 3$	$\Rightarrow b = 21$
If $p = 5, q = 5$			$\Rightarrow b = 25$

So, number of possible values of $b = 4$

Hence, total number of relations = 30 × 4 = 120

3. **Ans. (B)**

Here, $p, p^n \in \{1,2,3,\dots\dots\dots,7\}$

Now p can take values 2, 3, 5 and 7.

$\Rightarrow R_1 = \{(p, p^n)\} : p$ is a prime and $n \geq 0$ is an integer$\}$

$\therefore$ Elements in R, can be,

$(2, 2^0), (2, 2^1), (2, 2^2)$

$(3, 3^0), (3, 3^1)$

$(5, 5^0), (5, 5^1)$

$(7, 7^0), (7, 7^1)$

$\Rightarrow R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or $1\}$

$\therefore$ Elements in R_2 can be,

$(2, 2^0), (2, 2^1)$

$(3, 3^0), (3, 3^1)$

$(5, 5^0), (5, 5^1)$

$(7, 7^0), (7, 7^1)$

$\Rightarrow R_1 - R_2 = \{(2, 2^2)\}$

So, number of elements in $R_1 - R_2 = 1$

4. **Ans. (A)**

$R : \{3,6,9,12\} \to \{3,6,9,12\}$

$A = \{3,6,9,12\}$

$R = \{(3,3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,12), (3,6)\}$

Hence, for all $a \in A; (a, a) \in R$

So, R is Reflexive.
Hence, $(6,12) \in R$ but $(12,6) \notin R$
So, R is not symmetric.
Also, for all $(a,b) \in R$ and $(b,c) \in R$; $(a,c) \in R$ where $a,b,c \in A$.
So, R is transitive.

5. Ans. (B)

R: $\{(\alpha,\beta); \alpha \perp \beta\}$ and $\alpha,\beta \in L$
$\Rightarrow$ For all lines $\in L$; a line is not perpendicular to it self
So, R is not reflexive.
$\Rightarrow$ Let $L_1, L_2 \in L$ and if $(L_1,L_2) \in L$ then $(L_2,L_1) \in L$.
So, R is symmetric.
$\Rightarrow$ Let $L_1, L_2, L_3 \in L$ and $(L_1,L_2) \in R$ and $(L_2,L_3) \in R$.
then, by definition of relation; $L_1 \perp L_2$ and $L_2 \perp L_3$
And hence $L_1 || L_3 \Rightarrow (L_1, L_3) \notin R$
So, R is not transitive.

6. Ans. (D)

Here $R = \{(x,y) : |x^2 - y^2| < 16\}$
and given $A = \{1,2,3,4,5\}$
$\therefore R = \{(1,2)(1,3)(1,4);(2,1)(2,2)(2,3)(2,4);(3,1)(3,2)$
$(3,3)(3,4);(4,1)(4,2)(4,3);(4,4)(4,5),(5,4)(5,5)\}$.

7. Ans. (A)

$A = \{1, 2, 3, 4, 5\}$; $R : A \to A$
$R = \{(x,y) : x, y \in A, x + y = 5\}$
$R = \{(1, 4), (4, 1), (2, 3), (3, 2)\}$
Here, $(1, 1) \notin R$. So R is not Reflexive
$\Rightarrow$ For all $(a,b) \in R$; $(b,a) \in R$ (where $(a, b \in A)$)
So, R is symmetric
$\Rightarrow$ Here, $(1, 4) \in R$ and $(4, 1) \in R$, but $(1, 1) \notin R$.
So, R is not transitive.

8. Ans. (C)

Since $x \nless x$, therefore R is not reflexive.
Also $x < y$ does not imply that $y < x$, so R is not symmetric.
Let xRy and yRz. Then $x < y$ and $y < z \Rightarrow x < z$ i.e., xRz. Hence R is transitive.

9. Ans. (A)

For any $a \in N$, we find that a is divisible by a, therefore R is reflexive but R is not symmetric, because aRb does not imply that bRa.

10. Ans. (A)

$xR_2y \Leftrightarrow x \geq y$	is not symmetric relation
$xR_3y \Leftrightarrow \frac{x}{y}$	is not symmetric relation
$xR_4y \Leftrightarrow x < y$	is not symmetric relation
$xR_1y \Leftrightarrow x^2 = y^2$	is reflexive, symmetric and transitive so equivalence relation

11. Ans. (A)

$1 + a.a = 1 + a^2 > 0, \forall\, a \in S, \ \therefore (a, a) \in R$

$\therefore$ R is reflexive

$(a, b) \in R \Rightarrow 1 + ab > 0 \ \Rightarrow \ 1 + ba > 0 \Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

$\therefore$ $(a, b) \in R$ and $(b, c) \in R$ need not imply $(a, c) \in R$

Example for transitive check:

Take $a = \frac{1}{2}, b = \frac{1}{4}, c = \frac{-5}{2}$

Hence, R is not transitive.

12. Ans. (D)

$R: N \times N \rightarrow N \times N$

$(a, b)R(c, d) \Rightarrow ad = bc$

$\Rightarrow$ Here, for all $(a, a) \in N \times N; (a, a)R(a, a) \Rightarrow a \times a = a \times a$

Which is true.

So, R is Reflexive.

$\Rightarrow$ If $(a, b)R(c, d) \Rightarrow ad = bc$

so, for $(c, d)R(a, b) \Rightarrow bc = ad$

Hence, R is symmetric.

$\Rightarrow$ If $(a, b)R(c, d)$ and $(c, d)R(e, f)$ then $ad = bc$ and $cf = de$

Taking product of both;

$ad \times cf = bc \times de$

$\therefore af = be$

$\therefore (a, b)R(e, f)$

So, R is transitive.

Hence, R is equivalence.

13. Ans. (B)

If R and S be two equivalence relations in a set A , then $R \cap S$ is an equivalence relation in A.

14. Ans. (D)

None of the given relations is symmetric relation.

Hence, none of these is equivalence.

15. Ans. (A)

w = set of words in English dictionary.

$R: W \times W \rightarrow W \times W$

$R = \{(x, y) \in W \times W : x$ and y have at least one letter in common

$\Rightarrow$ Here for $(x, y), x \in W$; if both words are same then, it will always have at least one letter in common

$\therefore (x, x) \in R$

So, R is reflexive.

$\Rightarrow$ If x and y are such that they have atleast one letter in common then $(x, y) \in R, x, y \in W$.

Then (y, x) will also have one letter in common. So, $(y, x) \in R$ also

Hence, R is symmetric.

$\Rightarrow$ For $x, y, z \in W$; if $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$. Is not always true. Because one letter can be common for x and y and a different letter can be common for y and z. But it is also possible that none of the letter of x and z is common.

So, R is not transitive.

For example:

Let x = man, y = ant, z = toy

$(x, y) \in R$ as 'a' is in common

$(y, z) \in R$ as 't' is in common

But $(x, z) \notin R$ as no letter is in common.

EXERCISE - S

1. **Ans. (6)**

$n(A) = 3, n(B) = 4.$

$\therefore n(A \times B) = 3 \times 4 = 12$

So, number of relations = $2^{n(A\times B)}$

$= 2^{12}$

$= 4^{6}$

Hence, n = 6

2. **Ans. (3)**

$A = \{1,2,3\}, R: A \to A$

$R = \{(a, b); |a^2 - b^2| \le 5\}$

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$

Range of $R = \{1,2,3\}$

$\therefore$ No. of elements in $R = 3$.

3. **Ans. (4)**

$A = \{2, 3, 4, 5, \ldots, 16\}$

$(a, b) \simeq (c, d) \Rightarrow ad = bc$

$(4, 3) \simeq (c, d) \Rightarrow 4d = 3c$

$\Rightarrow \dfrac{4}{3} = \dfrac{c}{d}$

$\dfrac{c}{d} = \dfrac{4}{3}$ & $c, d \in \{2, 3, \ldots\ldots, 16\}$

$(c, d) = \{(4, 3), (8, 6), (12, 9), (16, 12)\}$

No. of ordered pair = 4

4. **Ans. (7)**

$A = \{1,2,3\}, R: A \to A$

$R = \{(1,2), (2,3)\}$

To make R an equivalence relation,

$R = \{(1,2), (2,3), (2,1), (3,2), (1,1), (2,2), (3,3), (1,3), (3,1)\}$

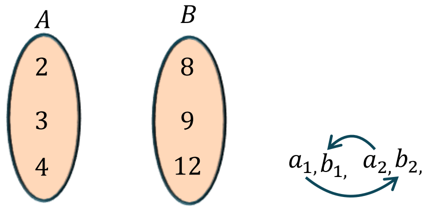
So, minimum number of ordered pairs to be added = 7

5. **Ans. (19)**

5 even numbers and 3 odd numbers

$\therefore \ {}^5C_1 \times {}^3C_1 + 4 = 19$

6. Ans. (36)



a_1 divides b_2

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

a_2 divides b_1

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

Total = $6 \times 6 = 36$

7. Ans. (7)

$R = [(-4, 4), (-3, 3), (3, -2), (0, 1), (0, 0), (1, 1), (4, 4), (3, 3)\}$

For reflexive, add $\Rightarrow (-2, -2), (-4, -4), (-3, -3)$

For symmetric, add $\Rightarrow (4, -4), (3, -3), (-2, 3), (1, 0)$

8. Ans. (13)

Given $R = \{(a, b), (b, c), (b, d)\}$

In order to make it equivalence relation as per given set, R must be

$\{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b),$

$(a, c), (a, d), (c, d), (d, c), (c, a), (d, a)\}$

There already given so 13 more to be added.

9. Ans. (1100)

$$A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i, j\}$$

$$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i, j\}$$

$$A=\sum_{j=1}^{10}\min(i,1)+\min(j,2)+\ldots\min(i,10)$$

$$=\underbrace{(1+1+1+\ldots+1)}_{19\text{ times}}+\underbrace{(2+2+2\ldots+2)}_{17\text{ times}}+\underbrace{(3+3+3\ldots+3)}_{15\text{ times}}+\quad \ldots(1)\ \ 1\text{ times}$$

$$B=\sum_{j=1}^{10}\max(i,1)+\max(j,2)+\ldots\max(i,10)$$

$$=\underbrace{(10+10+\ldots+10)}_{19\text{ times}}+\underbrace{(9+9+\ldots+9)}_{17\text{ times}}+\ldots+(1)\ 1\text{ times}$$

$A + B = 20(1 + 2 + 3 + \ldots +10)$

$= 20\times\dfrac{10\times11}{2} = 10 \times 110 = 1100$

10. Ans. (6)

$A = \{1, 2, 3, 4\}$

$R = \{(a, b), (c, d)\}$

$2a + 3b = 4c + 5d = \alpha$ let

$2a = \{2, 4, 6, 8\}$ $\quad\quad 4c = \{4, 8, 12, 16\}$

$3b = \{3, 6, 9, 12\}$ $\quad\quad 5d = \{5, 10, 15, 20\}$

$$2a + 3b = \begin{Bmatrix} 5,8,11,14 \\ 7,10,13,16 \\ 9,12,15,18 \\ 11,14,17,20 \end{Bmatrix} \qquad 4c + 5d \begin{Bmatrix} 9,14,19,24 \\ 13,18.... \\ 17,22.... \\ 21,26.... \end{Bmatrix}$$

Possible value of α = 9, 13, 14, 14, 17, 18

Pairs of $\{(a, b), (c, d)\} = 6$

EXERCISE - JEE (Main + Advanced) PYQ

1. Ans. (3)

$A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$

$\Rightarrow 2^{(x+2)(x^2-5x+6)} = 1 = 2^{\circ}$

$\Rightarrow (x + 2)(x^2 - 5x + 6) = 0$

$\Rightarrow (x + 2)(x - 2)(x - 3) = 0$

$\Rightarrow x = -2,2,3$

$A = \{-2,2,3\}$

Now $B = \{x \in z, -3 < 2x - 1 < 9\}$

$\Rightarrow -3 < 2x - 1 < 9$

$\Rightarrow -3 + 1 < 2x - 1 + 1 < 9 + 1$

$\Rightarrow -2 < 2x < 10$

$\Rightarrow \frac{-2}{2} < \frac{2x}{2} < \frac{10}{2}$

$\Rightarrow -1 < x < 5 \Rightarrow x = 0,1,2,3,4$ as $x \in z$

So, $B = \{0,1,2,3,4\}$

$n(A) = 3, n(B) = 5$

$n(A \times B) = 3 \times 5 = 15$

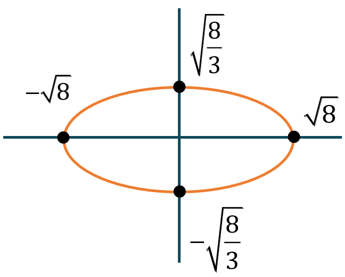
Total number of subsets of $A \times B = 2^{15}$

2. Ans. (2)

$R = \{(x, y) : x, y \in z, x^2 + 3y^2 \le 8\}$

For domain of R^{-1}

Collection of all integral of y's



For $x = 0$

$3y^2 \le 8$

$y^2 \le \frac{8}{3} \Rightarrow y = -1,0,1$

For $x = 1$

$1^2 + 3y^2 \le 8$

$3y^2 \le 7$

$y^2 \le \frac{7}{3} \Rightarrow y = -1, 0, 1$

For $x = 2$

$2^2 + 3y^2 \le 8$

$3y^2 \le 4$

$y^2 \le \frac{4}{3} \Rightarrow y = -1, 0, 1$

For $x = 3$

$3^2 + 3y^2 \le 8$

$3y^2 \le -1$ which is not possible for any value of y

So only possible values of y are $\{-1, 0, 1\}$ which will be the domain of R^{-1} (or range of R)

3. **Ans. (4)**

To check if R_1 is transitive

Take $a = 2+\sqrt{3}, b = 2-\sqrt{3}$ and $c = 1+2\sqrt{3}$

$a^2 + b^2 = 14 \in Q$ so $(a, b) \in R$

$b^2 + c^2 = 20 \in Q$ so $(b, c) \in R$

But $a^2 + c^2 = \left(2+\sqrt{3}\right)^2 + \left(1+2\sqrt{3}\right)^2$

$= 20 + 8\sqrt{3} \notin Q$

So $(a, c) \notin R$

R_1 is not transitive.

To check if R_2 is transitive

Take $a = 1, b = 3^{\frac{1}{4}}, c = \sqrt{2}$

$a^2 + b^2 = 1 + \sqrt{3} \notin Q$ so $(a, b) \in R$

$b^2 + c^2 = \sqrt{3} + 2 \notin Q$ so $(b, c) \in R$

But $a^2 + c^2 = 1 + 2 = 3 \in Q$ so $(a, c) \notin R$

R_2 is also not transitive.

4. **Ans. (4)**

$A = \{2, 3, 4, 5, ..., 30\}$

$(a, b) \simeq (c, d) \Rightarrow ad = bc$

$(4, 3) \simeq (c, d) \Rightarrow 4d = 3c$

$\Rightarrow \frac{4}{3} = \frac{c}{d}$

$\frac{c}{d} = \frac{4}{3}$ & $c, d \in \{2, 3, ..., 30\}$

$(c, d) = \{(4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)\}$

No. of ordered pair = 7

5. Ans. (4)

Check $R_1 = \{(x,y) : xy \geq 0,\ x, y \in R\}$

$\Rightarrow$ for reflexive $x.x \geq 0$ which is true

So, $(x,x) \in R_1 \forall x \in R$, it is Reflexive

$\Rightarrow$ for symmetric if

$(x,y) \in R_1 \Rightarrow x.y \geq 0$

So, $y.x \geq 0 \Rightarrow (y,x) \in R_1$ it is symmetric

$\Rightarrow$ for transitive let $x = 2,\ y = 0,\ z = -2$

$(x,y) \in R_1 \Rightarrow x.y = 0 \geq 0$

$(y,z) \in R_1 \Rightarrow y.x = 0 \geq 0$

But $(x,z) \notin R_1$ as $x.z = 2(-2) = -4 \leq 0$

R_1 is not equivalence relation

Check R_2

$(2,1) \in R_2$ as $2 \geq 1$

But $(1,2) \notin R_2$ as $1 \ngeq 2$

So, not symmetric

$\Rightarrow$ R_2 is not equivalence relation.

6. Ans. (4)

For R to be reflexive $\Rightarrow$ $x R x$

$\Rightarrow$ $3x + \alpha x = 7K \Rightarrow (3+\alpha)x = 7K$

$\Rightarrow$ $3 + \alpha = 7\lambda \Rightarrow \alpha = 7\lambda - 3 = 7N + 4, K, \lambda, N \in I$

$\therefore$ when α divided by 7, remainder is 4.

R to be symmetric $xRy \Rightarrow yRx$

$3x + \alpha y = 7N_1, 3y + \alpha x = 7N_2$

$\Rightarrow (3+\alpha)(x+y) = 7(N_1 + N_2) = 7N_3$

Which holds when $3 + \alpha$ is multiple of 7

$\therefore$ $\alpha = 7N + 4$ (as did earlier)

R to be transitive

$xRy \;\&\; yRz \Rightarrow xRz$.

$3x + \alpha y = 7N_1 \;\&\; 3y + \alpha z = 7N_2$ and $3x + \alpha z = 7N_3$

$\therefore$ $3x + 7N_2 - 3y = 7N_3$

$\therefore$ $7N_1 - \alpha y + 7N_2 - 3y = 7N_3$

$\therefore$ $7(N_1 + N_2) - (3+\alpha)y = 7N_3$

$\therefore$ $(3+\alpha)y = 7N$

Which is true again when $3+\alpha$ divisible by 7, i.e. when α divided by 7, remainder is 4.

7. **Ans. (2)**

Number of possible values of a = 60, for $b = pq$,

If $p = 3$, $q = 3, 5, 7, 11, 13, 17, 19$

If $p = 5$ $q = 5, 7, 11$

If $p = 7$ $q = 7$

Total cases = 60 × 11 = 660

8. **Ans. (2)**

$R_1 = \{(a,b) \in N \times N : |a-b| \le 13\}$

$R_2 = \{(a,b) \in N \times N : |a-b| \ne 13\}$.

For R_1 :

Reflexive relation

$(a,a) \in N \times N : |a-a| \le 13 \rightarrow$ it is reflexive

Symmetric relation

$(a,b) \in R_1, (b,a) \in R_1 : |b-a| \le 13 \rightarrow$ it is symmetric

Transitive relation

$(a,b) \in R_1, (b,c) \in R_1, (a,c) \in R_1$: $(1, 3) \in R_1$, $(3, 16) \in R_1$, but $(1, 16) \notin R_1$

It is not transitive.

For R_2 :

Reflexive relation

$(a,a) \in N \times N : |a-a| \ne 13$

Symmetric relation

If $(a,b) \in R$ is $|a-b| \ne 13 \ne |b-a|$

$(b,a) \in N \times N : |b-a| \ne 13$

Transitive relation

$(a,b) \in R_2, (b,c) \in R_2, (a,c) \in R_2$

$(1, 3) \in R_2$, $(3, 14) \in R_2$, but $(1, 14) \notin R_2$

It is not transitive.

9. **Ans. (8)**

Here, $p, p^n \in \{1, 2, \ldots 50\}$

Now p can take values

2,3,5,7,11,13,17,19,23,29,31,37,41,43 and 47.

∴ we can calculate no. of elements in R, as

$(2, 2^0)$, $(2, 2^1)$ .. $(2, 2^5)$

$(3, 3^0)$, ... $(3, 3^3)$

$(5, 5^0)$, ... $(5, 5^2)$

$(7, 7^0)$, ... $(7, 7^2)$

$(11, 11^0)$, ... $(11, 11^1)$

And rest for all other two elements each

$\therefore n(R_1) = 6 + 4 + 3 + 3 + (2 \times 11) = 38$

Similarly, for R_2

$(2, 2^0), (2, 2^1)$
$(47, 47^0), (47, 47^1)$
$\therefore n(R_2) = 2 \times 15 = 30$
$n(R_1 - R_2)$
$\therefore n(R_1) - n(R_2) = 38 - 30 = 8$ {as all the elements of R_2 are common to R_1}

10. Ans. (4)

$A = A_1 \cup A_2 \cup \ldots \ldots \cup A_k$
And $A_i \cap A_j = \phi$ i.e no two sets have any element in common.
Every pair of sets is disjoint sets.
Now $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i, 1 \le i \le k\}$.
So, for every A_i, R will have all the combination of (x, y) possible with elements of A_i
Let $A = \{1,2,3\}$
$A_1 = \{1\}, A_2 = \{2,3\}$

$$R = \{\underbrace{(1,1)}_{\substack{\text{Every element}\\ \text{of } A_1 \text{ is related}\\ \text{to } A_1}}, \underbrace{(2,2),(2,3),(3,2),(3,3)}_{\text{Every element } A_2 \text{ is related to } A_2}\}$$

$\rightarrow R$ will be an equivalence relation

11. Ans. (1)

Check for reflexivity:
As 3(*a* – *a*) + $\sqrt{7}$ = $\sqrt{7}$ which belongs to relation so relation is reflexive
Check for symmetric:
Take $a = \dfrac{\sqrt{7}}{3}, b = 0$
Now $(a, b) \in R$ but $(b, a) \notin R$
As $3(b - a) + \sqrt{7} = 0$ which is rational so relation is not symmetric.
<u>Check for Transitivity:</u>
Take (a, b) as $\left(\dfrac{\sqrt{7}}{3}, 1\right)$
& (b, c) as $\left(1, \dfrac{2\sqrt{7}}{3}\right)$
So now $(a, b) \in R$ & $(b, c) \in R$ but $(a, c) \notin$ R which means relation is not transitive

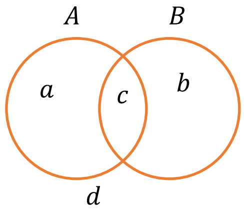
12. Ans. (1)

S = {1, 2, 3, ……10}
$P(S)$ = power set of S
$AR, B \Rightarrow (A \cap \vec{B}) \cup (\vec{A} \cap B) = \phi$
R_1 is reflexive, symmetric
For transitive

$(A \cap \vec{B}) \cup (\vec{A} \cap B) = \phi$; $\{a\} = \phi = \{b\}$ $A = B$

$(B \cap \vec{C}) \cup (\vec{B} \cap C) = \phi$ $\therefore B = C$

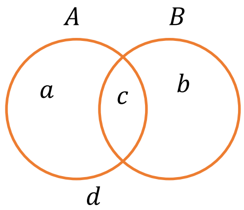
$\therefore A = C$ equivalence.



$R_2 \equiv A \cup \vec{B} = \vec{A} \cup B$

$R_2 \rightarrow$ Reflexive , symmetric

for transitive



$A \cup \vec{B} = \vec{A} \cup B \Rightarrow \{a, c, d\} = \{b, c, d\}$

$\{a\} = \{b\}$ $\therefore A = B$

$B \cup \vec{C} = \vec{B} \cup C \Rightarrow B = C$

$\therefore A = C$ $\therefore A \cup \vec{C} = \vec{A} \cup C$ $\therefore$ Equivalence

13. Ans. (2)

For relation $T = a^2 - b^2 = -I$

Then, (b, a) on relation R

$\Rightarrow b^2 - a^2 = -I$

$\therefore$ T is symmetric

$$S = \left\{(a,b) : a,b \in R - \{0\}, 2 + \frac{a}{b} > 0\right\}$$

$$2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2 \text{ , } \Rightarrow \frac{b}{a} < \frac{-1}{2}$$

If $(b, a) \in S$ then

$2 + \frac{b}{a}$ not necessarily positive

$\therefore$ S is not symmetric

14. Ans. (2)

Let $a_1 = 1 \Rightarrow 5$ choices of b_2

$a_1 = 3 \Rightarrow 4$ choices of b_2

$a_1 = 4 \Rightarrow 4$ choices of b_2

$a_1 = 6 \Rightarrow 2$ choices of b_2

$a_1 = 9 \Rightarrow 1$ choices of b_2

For (a_1, b_2) 16 ways .

Similarly, $b_1 = 2 \Rightarrow 4$ choices of a_2

$b_1 = 4 \Rightarrow 3$ choices of a_2

$b_1 = 5 \Rightarrow 2$ choices of a_2

$b_1 = 8 \Rightarrow 1$ choices of a_2

Required elements in $R = 160$

15. Ans. (4)

Reflexive : $(a, a) \Rightarrow$ gcd of $(a, a) = 1$

Which is not true for every $a \in Z$.

Symmetric:

Take $a = 2, b = 1 \Rightarrow$ gcd $(2, 1) = 1$

Also $2a = 4 \neq b$

Now when $a = 1, b = 2 \Rightarrow$ gcd $(1, 2) = 1$

Also now $2a = 2 = b$

Hence $a = 2b$

$\Rightarrow R$ is not Symmetric

Transitive:

Let $a = 14, b = 19, c = 21$

gcd $(a, b) = 1$

gcd $(b, c) = 1$

gcd $(a, c) = 7$

Hence not transitive

$\Rightarrow R$ is neither symmetric nor transitive.

16. Ans. (4)

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$R = \{(x, y) \in A \times A : x + y = 7\}$

$x + y = 7$

$y = 7 - x$

$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$(a, b) \in R \quad \Rightarrow (b, a) \in R$

$\Rightarrow$ Relation is symmetric