EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. If $'a'$ and $'b'$ are the greatest and least values of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$, then

(A) $a - b = \frac{9}{4}$ (B) $a - b = \frac{9}{8}$ (C) $a - b = \frac{1}{12}$ (D) $a + b = \frac{9}{4}$

MMM001

2. The two parts of 100 for which the sum of double of first and square of second part is minimum, are

(A) 50, 50 (B) 99, 1 (C) 98, 2 (D) None of these

MMM002

3. $f(x)$ is cubic polynomial which has local maximum at $x = -1$, If $f(2) = 18$, $f(1) = -1$ and $f'(x)$ has local minima at $x = 0$, then

(A) the distance between $(-1, 2)$ and $(a, f(a))$, where $x = a$ is the point of local minima is $2\sqrt{5}$.

(B) $f(x)$ is increasing for $x \in [1, 2\sqrt{5})$

(C) $f(x)$ has local maxima at $x = 1$

(D) $f(0) = 5$

MMM003

4. Let $f(x) = \begin{cases} \sin\frac{\pi x}{2} & , 0 \le x < 1 \\ 3 - 2x & , x \ge 1 \end{cases}$ then :

(A) $f(x)$ has local maxima at $x = 1$

(B) $f(x)$ has local minima at $x = 1$

(C) $f(x)$ does not have any local extrema at $x = 1$

(D) $f(x)$ has a global minimum at $x = 1$

MMM004

5. Values of a and b such that $f(x) = \frac{a}{x} + bx$ has a minimum at point (1, 6), are

(A) $a = 3, b = -3$ (B) $a = b = 3$ (C) $a = 6, b = 0$ (D) $a = -3, b = -3$

MMM005

6. The point(s) of minimum of the function, $f(x) = 4x^3 - x|x - 2|, x \in [0, 3]$ is:

(A) $x = 0$ (B) $x = 1/3$ (C) $x = 1/2$ (D) $x = 2$

MMM006

7. The minimum value of the sum of sum and product of two numbers when their difference is '2' is

(A) 2 (B) – 2 (C) 0 (D) 1

MMM007

8. If $f(x) = x^3 + ax^2 + bx + c$ is minimum at $x = 3$ and maximum at $x = -1$, then-

(A) $a = -3, b = -9, c = 0$ (B) $a = 3, b = 9, c = 0$

(C) $a = -3, b = -9, c \in R$ (D) none of these

MMM008

9. Difference between the greatest and the least values of the function $f(x) = x(\ell n\, x - 2)$ on $[1, e^2]$ is

(A) 2 (B) e (C) e^2 (D) 1

MMM009

10. If $f(x) = |x| + |x-1| + |x-2|$, then-

(A) f(x) has minima at $x = 1$

(B) $f(x)$ has maxima at $x = 0$

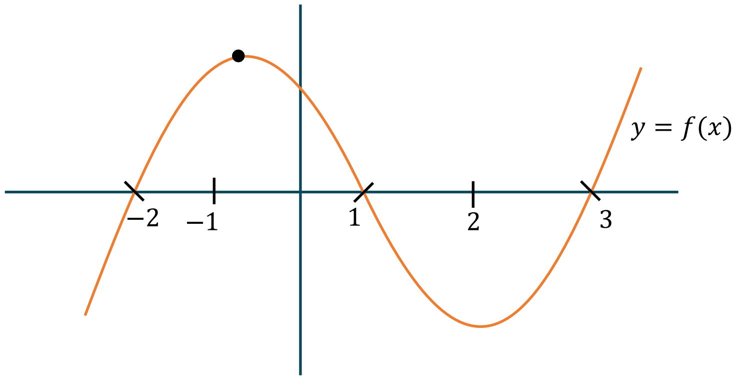
(C) Equation $f(x) = 1$ has 1 solution

(D) None of these

MMM010

MULTIPLE CORRECT TYPE QUESTIONS

11. Graph of $y = f(x)$ is given, then



(A) $y = H(x) = \max\{f(t) : t \le x\}\ \forall\ x \in R$ has a local maximum as well as a local minimum

(B) $y = |f(x)|$ has 5 point of extrema

(C) $y = f(|x|)$ has 5 points of extrema

(D) $y = f(|x|)$ has 3 points of extrema

MMM029

12. Let $f(x,y) = \sqrt{x^2+y^2} + \sqrt{x^2+y^2-2x+1} + \sqrt{x^2+y^2-2y+1} + \sqrt{x^2+y^2-6x-8y+25}\ \forall x, y \in R$, then-

(A) Minimum value of $f(x,y) = 5+\sqrt{2}$

(B) Minimum value of $f(x,y) = 5-\sqrt{2}$

(C) Minimum value occurs of $f(x,y)$ for $x = \frac{3}{7}$

(D) Minimum value occurs of $f(x,y)$ for $y = \frac{4}{7}$

MMM030

13. If $f(x) = \frac{x}{1 + x\tan x}$, $x \in \left(0, \frac{\pi}{2}\right)$, then

(A) $f(x)$ has exactly one point of minima

(B) $f(x)$ has exactly one point of maxima

(C) $f(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$

(D) maxima occurs at x_0 where $x_0 = \cos x_0$

MMM031

14. If the function $y = f(x)$ is represented as,

$x = \phi(t) = t^5 - 5t^3 - 20t + 7$

$y = \psi(t) = 4t^3 - 3t^2 - 18t + 3$ $(|t| < 2)$, then :

(A) $y_{max} = 12$ (B) $y_{max} = 14$ (C) $y_{min} = -67/4$ (D) $y_{min} = -69/4$

MMM032

15. Let $f(x)=\min\left\{|x|,\sqrt{1-x^2}\right\};x\in[-1,1]$ and $g(x)=\max\left\{|x|,\sqrt{1-x^2}\right\};x\in[-1,1]$.

Which of the following statement(s) is (are) correct ?

(A) number of local minima of $f(x)$ is 1
(B) number of local minima of $f(x)$ is 3
(C) $x=0$ is a point of local maxima for $g(x)$
(D) number of local maxima of $g(x)$ is 3

MMM033

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 16 to 18

Let $f(x)=1+a^2x-x^3$ where 'a' is real number, point of minima of $f(x)$ must lie between $[A,B]$ where A and B is the minimum and maximum value of $\frac{x^2+3x+1}{x^2+x+1}$ for all $x\in R$.

On the basis of above information, answer the following questions-

16. Find the value of A

(A) –1 (B) $-\frac{1}{2}$ (C) –2 (D) None of these

MMM034

17. Find the value of B

(A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{5}{4}$ (D) None of these

MMM035

18. If $a>0$, then find the point of local minima

(A) $\frac{-a}{\sqrt{3}}$ (B) $\frac{a}{\sqrt{3}}$ (C) $\frac{a}{\sqrt{2}}$ (D) None of these

MMM036

Paragraph for Question No. 19 to 20

Let $f:R\to R, g:R\to R, h:R\to R$

$f(x)=x^2e^x, g(x)=x^3-x^2+x$ and $h(x)=x+2$.

19. Choose the correct statement (s)

(A) The local minimum value of $f(g(x))$ is 0
(B) The local minimum value of $f(g(x))$ is an odd number
(C) The local minimum value of $f(g(x))$ does not exist
(D) The local minimum value of $f(g(x))$ is an even number

MMM037

20. Choose the correct statement (s)

(A) The local minimum value of $h(f(x))$ is 0
(B) The local minimum value of $h(f(x))$ is an odd number
(C) The local minimum value of $h(f(x))$ is 2
(D) The local minimum value of $h(f(x))$ is an even number

MMM038

Paragraph for Question No. 21 to 23

Consider $f(x) = x^3(x-2)^2(x-1)$.

21. If least value of $f(x)$ is at $x = a$, then the value of $(3a-2)$ is

MMM039

22. If least value of $f(x)$ is L, then the value of [L] is
(where [.] represents greatest integer function)

MMM040

23. If maxima and minima of $f(x)$ occurs at $x_i, i \in N$, then $\left[\sum_{i=1}^{n} x_i^2\right]$ is
(where [.] represents greatest integer function)

MMM041

MATRIX MATCH TYPE QUESTION

24.

	Column-I		Column-II
(A)	The number of point (s) of maxima of $f(x) = x^2 + \dfrac{1}{x^2}$ is	(P)	0
(B)	$(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ is maximum at $x =$	(Q)	2
(C)	If $[a, b], (b < 1)$ is largest interval in which $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 19$ is strictly increasing then $\dfrac{a}{b}$ is	(R)	$\dfrac{8}{3}$
(D)	If $a + b = 8, a, b > 0$ then minimum value of $\dfrac{a^3+b^3}{48}$ is	(S)	–1

MMM042

25.

	Column-I		Column-II
(A)	A rectangle is inscribed in an equilateral triangle of side $4cm$. Square of maximum area of such a rectangle is	(P)	65
(B)	The volume of a rectangular closed box is 72 and the base sides are in the ratio 1 : 2. The least total surface area is	(Q)	45
(C)	If x and y are two positive numbers such that $x + y = 60$ and x^3y is maximum then value of x is	(R)	12
(D)	The sides of a rectangle of greatest perimeter which is inscribed in a semicircle of radius $\sqrt{5}$ are a and b. Then $a^3 + b^3 =$	(S)	108

MMM043

EXERCISE - S

1. The maximum value of function $f(x) = x^3 - 12x^2 + 36x + 17$ in the interval [1, 10] is

MMM011

2. Maximum possible area that can be enclosed by a wire of length 20 cm by bending it in form of a circular sector is

MMM012

3. Two vertices of a rectangle are on the positive x-axis. The other two vertices lie on the lines $y = 4x$ and $y = -5x + 6$. Then the maximum area of the rectangle is A then find $100A$

MMM013

4. P is a point on positive x-axis, Q is a point on the positive y-axis and 'O' is the origin. If the line passing through P and Q is tangent to the curve $y = 3 - x^2$ then the minimum area of the triangle OPQ, is

MMM014

5. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, per tree the output drop by 10 times the number of additional trees. Number of trees that should be added to the existing orchard for maximising the output of the trees, is

MMM015

6. The graph of the derivative $f'(x)$ of a continuous function $f(x)$ in (0, 9). If

(i) f is strictly increasing in the interval $(a, b]$; $[c, d]$; $[e, f)$ and strictly decreasing in $[p,\ q]$; $[r,\ s]$.

(ii) f has a local minima at $x = x_1$ and $x = x_2$.

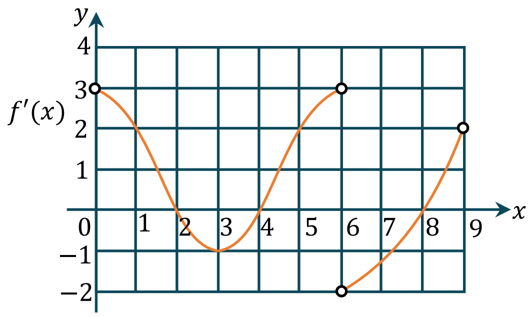
(iii) $f''(x) > 0$ in (l, m) ; (n, t)

(iv) f has inflection point at $x = k$

(v) number of critical points of $y = f(x)$ is 'w'.

Find the value of $(a + b + c + d + e) + (p + q + r + s) + (l + m + n) + (x_1 + x_2) + (k + w)$.

MMM044



7. Let $P(x)$ be a polynomial of degree 5 having extremum at $x = -1, 1$ and $\lim_{x \to 0}\left(\frac{P(x)}{x^3} - 2\right) = 4$. If M and m are the maximum and minimum value of the function $y = P'(x)$ on the set $A = \{x | x^2 + 6 \leq 5x\}$ then find $\frac{m}{M}$.

MMM045

8. Of all the lines tangent to the graph of the curve $y = \frac{6}{x^2 + 3}$, then the sum of the minimum and maximum slopes of these tangent lines.

MMM046

9. A running track of 440 ft. is to be laid out enclosing a football field, the shape of which is a rectangle with semi-circle at each end. If the area of the rectangular portion is to be maximum, then the perimeter of the rectangular portion is

Use : $\pi \approx 22/7$.

MMM047

10. The value of 'a' for which $f(x) = x^3 + 3(a - 7)x^2 + 3(a^2 - 9)x - 1$ have a positive point of maximum lies in the interval $(a_1, a_2) \cup (a_3, a_4)$. Find the value of $a_2 + 11a_3 + 70a_4$.

MMM048

EXERCISE - JEE (Main) PYQ

1. A helicopter is flying along the curve given by $y - x^{\frac{3}{2}} = 7, (x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

[JEE (Main) 2019]

(1) $\frac{1}{2}$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$ (3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (4) $\frac{\sqrt{5}}{6}$

MMM018

2. Let a_1, a_2, a_3,......be an A. P. with $a_6 = 2$. Then the common difference of this A. P., which maximises the produce $a_1 a_4 a_5$, is : **[JEE (Main) 2019]**

(1) $\frac{6}{5}$ (2) $\frac{8}{5}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

MMM019

3. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x \to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true ? **[JEE (Main) 2020]**

(1) f is an odd function

(2) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

(3) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f.

(4) $f(1) - 4f(-1) = 4$

MMM020

4. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10, f(1) = -6, f(x)$ has a critical point at $x = -1$ and $f'(x)$ has a critical point at $x = 1$. Then $f(x)$ has a local minima at x = ______.

[JEE (Main) 2020]

MMM021

5. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness the melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is : **[JEE (Main) 2020]**

(1) $\frac{1}{36\pi}$ (2) $\frac{5}{6\pi}$ (3) $\frac{1}{18\pi}$ (4) $\frac{1}{54\pi}$

MMM022

6. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :

(1) An isosceles triangle with base equal to $2r$. **[JEE (Main) 2021]**

(2) An equilateral triangle of height $\frac{2r}{3}$.

(3) An equilateral triangle having each of its side of length $\sqrt{3}r$.

(4) A right angle triangle having two of its sides of length $2r$ and r.

MMM023

7. The sum of all the local minimum values of the twice differentiable function $f : R \to R$ defined by $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is : **[JEE (Main) 2021]**

(1) –22 (2) 5 (3) –27 (4) 0

MMM024

8. The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1, 2]$ is : **[JEE (Main) 2021]**

(1) $\frac{\sqrt{17}+3}{2}$ (2) $\frac{\sqrt{17}+5}{2}$ (3) 5 (4) $\frac{9-\sqrt{17}}{2}$

MMM025

9. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point $P(-2, 0)$ and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of $y(x)$ is : **[JEE (Main) 2022]**

(1) $\frac{27}{4}$ (2) $\frac{29}{4}$ (3) $\frac{37}{4}$ (4) $\frac{9}{2}$

MMM026

10. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is _______. **[JEE (Main) 2022]**

MMM027

11. Let $x = 2$ be a local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12, x \in (-4, 4)$. If M is local maximum value of the function f in (–4, 4), then M = **[JEE (Main) 2023]**

(1) $12\sqrt{6} - \frac{33}{2}$ (2) $12\sqrt{6} - \frac{31}{2}$ (3) $18\sqrt{6} - \frac{33}{2}$ (4) $18\sqrt{6} - \frac{31}{2}$

MMM028

12. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to **[JEE (Main) 2023]**

(1) 675 (2) 1025 (3) 800 (4) 900

MMM067

13. $\max\limits_{0 \le x \le \pi}\left\{x - 2\sin x\cos x + \frac{1}{3}\sin 3x\right\} =$ **[JEE (Main) 2023]**

(1) $\frac{5\pi + 2 + 3\sqrt{3}}{6}$ (2) $\frac{\pi + 2 - 3\sqrt{3}}{6}$ (3) π (4) 0

MMM068

14. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2, a \neq 2b$ have a common extreme point, then $a + 2b + 7$ is equal to **[JEE (Main) 2023]**

(1) 4 (2) $\frac{3}{2}$ (3) 3 (4) 6

MMM069

EXERCISE - JEE (Advanced) PYQ

1. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote respectively, the absolute maximum of f, g and h on [0, 1], then

(A) $a = b$ and $c \neq b$ (B) $a = c$ and $a \neq b$ (C) $a \neq b$ and $c \neq b$ (D) $a = b = c$

[JEE (Advanced) 2010]

MMM049

2. Let f be a function defined on $\boldsymbol{R}$ (the set of all real numbers) such that

$f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x \in R$.

If g is a function defined on R with values in the interval $(0,\infty)$ such that

$f(x) = \ell n(g(x))$, for all $x \in R$,

then the number of points in R at which g has a local maximum is **[JEE (Advanced) 2010]**

MMM050

3. Let $f: IR \to IR$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is **[JEE (Advanced) 2012]**

MMM051

4. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is **[JEE (Advanced) 2012]**

MMM052

5. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio of 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are **[JEE (Advanced) 2013]**

(A) 24 (B) 32 (C) 45 (D) 60

MMM053

6. The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at $x =$ **[JEE (Advanced) 2013]**

(A) –2 (B) $\frac{-2}{3}$ (C) 2 (D) $\frac{2}{3}$

MMM054

7. A cylindrical container is to be made from certain solid material with the following constraints. It has a fixed inner volume of $V\ mm^3$, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10mm, then the value of $\frac{V}{250\pi}$ is **[JEE (Advanced) 2015]**

MMM055

8. The least value of $\alpha \in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is - **[JEE (Advanced) 2016]**

(A) $\frac{1}{64}$ (B) $\frac{1}{32}$ (C) $\frac{1}{27}$ (D) $\frac{1}{25}$

MMM056

9. Let $f : \mathbb{R} \to (0, \infty)$ and $g : \mathbb{R} \to \mathbb{R}$ be twice differentiable function such that f'' and g'' are continuous functions on $\mathbb{R}$. Suppose $f'(2) = g(2) = 0$, $f''(2) \neq 0$ and $g'(2) \neq 0$. If $\lim\limits_{x \to 2} \dfrac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

[JEE (Advanced) 2016]

(A) f has a local minimum at $x = 2$ (B) f has a local maximum at $x = 2$

(C) $f''(2) > f(2)$ (D) $f(x) - f''(x) = 0$ for at least one $x \in \mathbb{R}$

MMM057

10. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then **[JEE (Advanced) 2017]**

(A) $f'(x) = 0$ at exactly three points in $(-\pi, \pi)$

(B) $f(x)$ attains its maximum at $x = 0$

(C) $f(x)$ attains its minimum at $x = 0$

(D) $f'(x) = 0$ at more than three points in $(-\pi, \pi)$

MMM058

11. For every twice differentiable function $f : \mathbb{R} \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE ? **[JEE (Advanced) 2018]**

(A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s).

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$

(C) $\lim\limits_{x \to \infty} f(x) = 1$

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

MMM059

12. Let $f(x) = \dfrac{\sin \pi x}{x^2}$, $x > 0$

Let $x_1 < x_2 < x_3 < \ldots < x_n <\ldots$ be all the points of local maximum of f

and $y_1 < y_2 < y_3 < \ldots < y_n < \ldots$ be all the points of local minimum of f.

Then which of the following options is/are correct ? **[JEE (Advanced) 2019]**

(1) $|x_n - y_n| > 1$ for every n (2) $x_1 < y_1$

(3) $x_n \in \left(2n, 2n + \dfrac{1}{2}\right)$ for every n (4) $x_{n+1} - x_n > 2$ for every n

MMM060

13. Consider all rectangles lying in the region

$$\left\{(x, y) \in \mathbb{R} \times \mathbb{R} : 0 \leq x \leq \frac{\pi}{2} \text{ and } 0 \leq y \leq 2\sin(2x)\right\}$$

and having one side on the x-axis. The area of the rectangle which has the maximum perimeter among all such rectangles, is **[JEE (Advanced) 2020]**

(A) $\dfrac{3\pi}{2}$ (B) π (C) $\dfrac{\pi}{2\sqrt{3}}$ (D) $\dfrac{\pi\sqrt{3}}{2}$

MMM061

14. Let the function $f : (0, \pi) \to \mathbb{R}$ be defined by

$f(\theta) = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^4$

Suppose the function f has a local minimum at θ precisely when $\theta \in \{\lambda_1\pi, ..., \lambda_r\pi\}$,

where $0 < \lambda_1 < .... < \lambda_r < 1$. Then the value of $\lambda_1 + ... + \lambda_r$ is _____ **[JEE (Advanced) 2020]**

MMM062

Comprehension for Questions Nos. 15 and 16

Let $f_1 : (0, \infty) \to \mathbb{R}$ and $f_2 : (0, \infty) \to \mathbb{R}$ be defined by $f_1(x) = \int_0^x \prod_{j=1}^{21} (t-j)^j \, dt$, $x > 0$

and

$f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, x > 0,$

where, for any positive integer n and real numbers $a_1, a_2, ..., a_n$, $\prod_{i=1}^{n} a_i$ denotes the product of $a_1, a_2, ..., a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function $f_i, i = 1, 2$, in the interval $(0, \infty)$

15. The value of $2m_1 + 3n_1 + m_1n_1$ is _____. **[JEE (Advanced) 2021]**

MMM063

16. The value of $6m_2 + 4n_2 + 8m_2n_2$ is _____. **[JEE (Advanced) 2021]**

MMM064

17. Let $\alpha = \sum_{k=1}^{\infty} \sin^{2k}\left(\frac{\pi}{6}\right)$.

Let $g : [0, 1] \to \mathbb{R}$ be the function defined by

$g(x) = 2^{\alpha x} + 2^{\alpha(1-x)}$

Then, which of the following statements is/are TRUE ? **[JEE (Advanced) 2022]**

(A) The minimum value of $g(x)$ is $2^{\frac{7}{6}}$

(B) The maximum value of $g(x)$ is $1 + 2^{\frac{1}{3}}$

(C) The function $g(x)$ attains its maximum at more than one point

(D) The function $g(x)$ attains its minimum at more than one point

MMM065

18. Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1, x_2, x_3 \{0,1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0, 0, 0) and (1, 1, 1) is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1, \ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1, \ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

[JEE (Advanced) 2023]

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

MMM066

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0, if none of the bubbles is darkened.
  Negative Marks : –1 in all other cases.

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1. If $(x-a)^{2m}(x-b)^{2n+1}$, where m and n are positive integers and $a > b$, is the derivative of a function f, then-

(1) $x = a$ gives neither a maximum, nor a minimum
(2) $x = a$ gives a maximum
(3) $x = b$ gives neither a maximum nor a minimum
(4) None of these

MMM070

2. The cost of running a bus from A to B, is Rs. $\left(av + \frac{b}{v}\right)$, where $v\ km/h$ is the average speed of the bus. When the bus travels at 30 km/h, the cost comes out to be Rs. 75 while at 40 km/h, it is Rs. 65. Then the most economical speed (in km/h) of the bus is :

(1) 40 (2) 60 (3) 45 (4) 50

MMM071

3. For $x \in \left(0, \frac{\pi}{2}\right)$

(1) $(2sinx + tanx) > (3x)$
(2) $(2sinx + tanx) < (3x)$
(3) $\lim_{x \to 0^+} \left[\frac{3x}{2\sin x + \tan x}\right] = 1$, where [.] denote the GIF.
(4) Nothing can be say

MMM072

4. If $a = (100)^{\frac{1}{100}}$ and $b = (101)^{\frac{1}{101}}$ then

(1) $a = b$ (2) $a > b$ (3) $a < b$ (4) none of these

MMM073

5. The maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b is

(1) $2(ab)$ (2) $\frac{1}{2}(a+b)^2$ (3) $\frac{1}{2}(a^2+b^2)$ (4) none of these

MMM074

6. Let $f(x)=\dfrac{\tan^n x}{1+\sum_{r=1}^{2n}\tan^r x}$, $n\in N$, where $x\in\left[0,\dfrac{\pi}{2}\right)$

(1) $f(x)$ is bounded and it takes both of it's bounds and the range of $f(x)$ contains exactly one integral point.

(2) $f(x)$ is bounded and it takes both of it's bounds and the range of $f(x)$ contains more than one integral point.

(3) $f(x)$ is bounded but minimum and maximum does not exists.

(4) $f(x)$ is not bounded as the upper bound does not exist.

MMM075

7. If $f(x)=\begin{cases} e^{x+1}-e^x & x\le 0 \\ e^{1-x}-1 & 0<x<1 \\ x+\ell nx & x\ge 1\end{cases}$, then -

(1) x = 0 is point of local maxima, x = 1 is neither local maxima nor local minima.

(2) x = 1 is point of local minima, x = 0 is point of local maxima

(3) x = 0 and x = 1 both are points of local maxima

(4) x = 0 and x = 1 both are points of local minima

MMM076

8. A running track of 440 m is to be laid out enclosing a football field, the shape of which is a rectangle with semi circle at two opposite end. If the area of the rectangular portion is to be maximum, then the length of its sides is

(1) $120\,m$, $\dfrac{220}{\pi}\,m$ (2) $110\,m$, $\dfrac{\pi}{200}\,m$ (3) $110\,m$, $\dfrac{220}{\pi}\,m$ (4) $125\,m$, $\dfrac{220}{\pi}\,m$

MMM077

9. The combined resistance R of two resistors R_1 & R_2 ($R_1, R_2 > 0$) is given by, $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}$.

If $R_1 + R_2$ = constant. The combined resistance R will be maximum, when

(1) $R_1 = R_2$ (2) $R_1 = 2R_2$ (3) $3R_1 = 2R_2$ (4) $2R_1 = 3R_2$

MMM078

10. The greatest, the least values of the function, $f(x) = 2-\sqrt{1+2x+x^2}$, $x\in[-2, 1]$ are respectively

(1) 2, 1 (2) 2, – 1 (3) 2, 0 (4) –2, 3

MMM079

11. The dimensions of the rectangle of maximum area that can be inscribed in the ellipse $\left(\dfrac{x}{4}\right)^2+\left(\dfrac{y}{3}\right)^2=1$ are

(1) $\sqrt{8},\sqrt{2}$ (2) $4,3$ (3) $2\sqrt{8},3\sqrt{2}$ (4) $\sqrt{2},\sqrt{6}$

MMM080

12. If (a, b) be the point on the curve $y=\left|x^2-4x+3\right|$ which is nearest to the circle $x^2+y^2-4x-4y+7=0$, then $(a + b)$ is equal to -

(1) $\dfrac{7}{4}$ (2) 0 (3) 3 (4) 2

MMM081

13. By the post office regulations, the combined length & girth of a parcel must not exceed 3 metre. If the volume of the biggest cylindrical (right circular) packet that can be sent by the parcel post is $V\ cu.m$, then the value of $(4\pi V)$ is

(1) 2 (2) 3 (3) 4 (4) 5

MMM082

14. A rectangle is inscribed in an equilateral triangle of side 4 cm. Square of maximum area of such a rectangle is

(1) 10 (2) 15 (3) 20 (4) 12

MMM083

15. The complete set of values of the parameter 'a' for which the point of minimum of the function $f(x)=1+a^2x-x^3$ satisfies the inequality $\frac{x^2+x+2}{x^2+5x+6}<0$ is

(1) $(-3\sqrt{3},\ -2\sqrt{3})\cup(2\sqrt{3},\ 3\sqrt{3})$ (2) $(-3\sqrt{3},\ -2\sqrt{3})$

(3) $(-3\sqrt{3},\ -2\sqrt{3})$ (4) $(-3\sqrt{2},\ 2\sqrt{3})$

MMM084

16. If $f(x)=sin^3x+\lambda sin^2x\ ;\ -\frac{\pi}{2}<x<\frac{\pi}{2}$, then the interval in which λ should lie in order that $f(x)$ has exactly one minima and one maxima

(1) $\left(-\frac{3}{2},\frac{3}{2}\right)$ (2) $\left(-\frac{2}{3},\frac{2}{3}\right)-\{0\}$ (3) R (4) none of these

MMM085

17. In a regular triangular prism the distance from the centre of one base to one of the vertices of the other base is ℓ. The altitude of the prism for which the volume is greatest, is :

(1) $\frac{\ell}{2}$ (2) $\frac{\ell}{\sqrt{3}}$ (3) $\frac{\ell}{3}$ (4) $\frac{\ell}{4}$

MMM086

18. Let ABC is given triangle having respective sides a, b, c. D, E, F are points of the sides BC, CA, AB respectively so that $AFDE$ is a parallelogram. The maximum area of the parallelogram is

(1) $\frac{1}{4}bc\sin A$ (2) $\frac{1}{2}\ bc\sin A$ (3) $bc\sin A$ (4) $\frac{1}{8}bc\sin A$

MMM087

19. Least value of the function, $f(x) = 2^{x^2} - 1 + \frac{2}{2^{x^2}+1}$ is:

(1) 3 (2) 2 (3) 1 (4) 0

MMM088

20. The value of 'a' for which $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$ have a positive point of maximum lies in the interval $(a_1,\ a_2)\cup(a_3,\ a_4)$. Find the value of $a_2 +\ 11a_3 +\ 70a_4$.

(1) 320 (2) 340 (3) 350 (4) 380

MMM089

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4, if only correct answer is given.
  Zero Marks : 0, if no answer is given.
  Negative Marks : -1 for incorrect answer

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1. Maximum value of $\left(\sqrt{-3+4x-x^2}+4\right)^2+(x-5)^2$ (where $1 \le x \le 3$) is

MMM090

2. Find the total number of integral values of the parameter 'k' for which the equation $x^4+4x^3-8x^2+k=0$ has all roots real.

MMM091

3. If the function $f(x)=2x^3-9ax^2+12a^2x+1$, where $a>0$ attains its maximum and minimum at p and q respectively such that $p^2=q$, then a equals

MMM092

4. Find the absolute minimum value of $f\left(x\right)=8^x+8^{-x}-4\left(4^x+4^{-x}\right), \forall\ x \in R$

MMM093

5. A tangent to the curve y = 1 – x^2 is drawn so that the abscissa x_0 of the point of tangency belongs to the interval (0, 1]. The tangent at x_0 meets the x–axis and y–axis at A & B respectively. If the minimum area of the triangle OAB, where O is the origin is $\frac{\lambda\sqrt{\mu}}{9}$ then find the value of $\lambda - \mu$

MMM094

6. The three sides of a trapezium are equal each being 6 cms long. Let Δ cm^2 be the maximum area of the trapezium. The value of $\sqrt{3}\ \Delta$ is :

MMM095

7. If x and y are two positive numbers such that $x + y = 60$ and x^3y is maximum then value of x is

MMM096

8. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone which has 10 cm radius, is

MMM097

9. If the sum of the lengths of the hypotenuse and another side of a right angled triangle is given, if the area of the triangle is maximum when the angle between these sides is $\frac{\pi}{k}$, then value of k –

MMM098

10. If $a + b = 8$, $a, b > 0$ then minimum value of $\frac{a^3+b^3}{48}$ is $3k$, then the value of k is

MMM099

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-I

- This section contains **SIX** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 In all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1. If $f(x) = \begin{cases} -\sqrt{1-x^2} & , \quad 0 \le x \le 1 \\ -x & , \quad x > 1 \end{cases}$, then

(A) Maximum of $f(x)$ exist at $x = 1$
(B) Maximum of $f(x)$ doesn't exists
(C) Minimum of $f^{-1}(x)$ exist at $x = -1$
(D) Minimum of $f^{-1}(x)$ exist at $x = 1$

MMM100

2. Let $A(p, q)$ and $B(h, k)$ are points on the curve $4x^2 + 9y^2 = 1$, which are nearest and farthest from the line $72 + 8x = 9y$ respectively, then -

(A) $p+q=-\frac{1}{5}$
(B) $p+q=\frac{1}{5}$
(C) $h+k=-\frac{1}{5}$
(D) $h+k=\frac{1}{5}$

MMM101

3. Let $f(x)=ax^3+bx^2+cx+1$ have extrema at $x = \alpha, \beta$ such that $\alpha\beta < 0$ and $f(\alpha).f(\beta)<0$, then which of the following can be true for the equation $f(x) = 0$?

(A) three equal roots
(B) three distinct real roots
(C) one positive root if $f(\alpha) < 0$ and $f(\beta) > 0$
(D) one negative root if $f(\alpha) > 0$ and $f(\beta) < 0$.

MMM102

4. If $f(x) = tan^{-1}x - \left(\frac{1}{2}\right)\ell n\, x$. Then

(A) the greatest value of $f(x)$ on $\left[1/\sqrt{3}, \sqrt{3}\right]$ is $\pi/6 + (1/4)\ \ell n\ 3$

(B) the least value of $f(x)$ on $\left[1/\sqrt{3}, \sqrt{3}\right]$ is $\pi/3 - (1/4)\ \ell n\ 3$

(C) $f(x)$ decreases on $(0, \infty)$

(D) $f(x)$ increases on $(-\infty, 0)$

MMM103

5. If P is a point on the curve $5x^2 + 3xy + y^2 = 2$ and O is the origin, then OP has

(A) minimum value $\frac{1}{2}$
(B) minimum value $\frac{2}{\sqrt{11}}$
(C) maximum value $\sqrt{11}$
(D) maximum value 2

MMM104

6. Suppose velocity of waves of wave length λ in the Atlantic ocean is $k\sqrt{\left\{\left(\frac{\lambda}{a}\right)+\left(\frac{a}{\lambda}\right)\right\}}$, where k and a are constants. If the minimum velocity attained by the waves is in the form of $k_1\sqrt{k_2}$ where k_1 and k_2 are independent constants, then which of the following options are correct -

(A) minimum velocity is independent of a
(B) minimum velocity is dependent on k
(C) $k_2 = 2$
(D) $k_2 = 3$

MMM105

SECTION–II

- This section contains **TWO** paragraph.
- Based on each paragraph, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u> :

  Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 in all other cases.

Comprehension # 1 (Q. No. 7 - 9)

Let $P(x)$ be a polynomial of degree 4 and it vanishes at $x = 0$. Given $P(-1) = 55$ and $P(x)$ has relative maximum/relative minimum at $x = 1, 2, 3$.

7. Area of the triangle formed by extremum points of $P(x)$, is

(A) 1
(B) 2
(C) 3
(D) 4

MMM106

8. If the number of negative integers in the range of $P(x)$ is λ, then λ equals–

(A) 6
(B) 7
(C) 8
(D) 9

MMM107

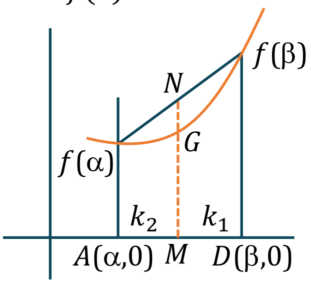
9. If $P(x) + \mu = 0$ has four distinct roots. then μ lies in interval

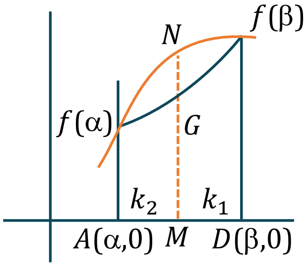
(A) (1, 2) (B) (8, 9) (C) (3, 4) (D) (–5, 7)

MMM108

Comprehension # 2 (Q. No. 10 - 12)

For a double differentiable function $f(x)$ if $f''(x) \geq 0$ then $f(x)$ is concave upward and if $f''(x) \leq 0$ then $f(x)$ is concave downward





Here $M\left(\frac{k_1\alpha+k_2\beta}{k_1+k_2}, 0\right)$

If $f(x)$ is a concave upward in $[a, b]$ and $\alpha, \beta \in [a, b]$ then $\frac{k_1 f(\alpha)+k_2 f(\beta)}{k_1+k_2} \geq f\left(\frac{k_1\alpha+k_2\beta}{k_1+k_2}\right)$, where $k_1, k_2 \in R^+$

If $f(x)$ is a concave downward in $[a, b]$ and $\alpha, \beta \in [a, b]$ then $\frac{k_1 f(\alpha)+k_2 f(\beta)}{k_1+k_2} \leq f\left(\frac{k_1\alpha+k_2\beta}{k_1+k_2}\right)$, where $k_1, k_2 \in R^+$

then answer the following

10. Which of the following is true

(A) $\frac{\sin\alpha+\sin\beta}{2} > \sin\left(\frac{\alpha+\beta}{2}\right)$; α, β $(0, \pi)$ (B) $\frac{\sin\alpha+\sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right)$; $\alpha, \beta \in (\pi, 2\pi)$

(C) $\frac{\sin\alpha+\sin\beta}{2} < \sin\left(\frac{\alpha+\beta}{2}\right)$; $\alpha, \beta \in (0, \pi)$ (D) None of these

MMM109

11. Which of the following is true

(A) $\frac{2^\alpha+2^{\beta+1}}{3} \leq 2^{\frac{\alpha+2\beta}{3}}$

(B) $\frac{2\ell n\ \alpha+\ell n\beta}{3} \geq \ell n\left(\frac{2\alpha+\beta}{3}\right)$

(C) $\frac{\tan^{-1}\alpha+\tan^{-1}\beta}{2} \leq \tan^{-1}\left(\frac{\alpha+\beta}{2}\right)$ $a, b \in R^-$

(D) $\frac{e^\alpha+2e^\beta}{3} \geq e^{\frac{\alpha+2\beta}{3}}$

MMM110

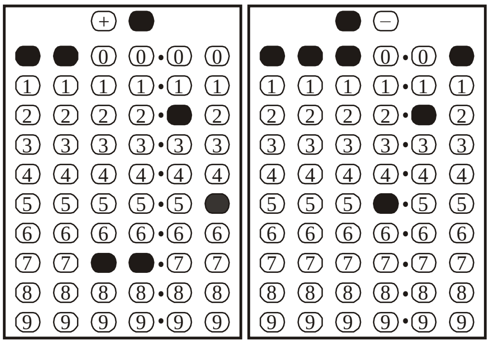
12. Let α, β and γ are three distinct real numbers and $f''(x) < 0$. Also $f(x)$ is increasing function and let $A = \frac{f^{-1}(\alpha)+f^{-1}(\beta)+f^{-1}(\gamma)}{3}$ and $B = f^{-1}\left(\frac{\alpha+\beta+\gamma}{3}\right)$, then order relation between A and B is ?

(A) $A > B$ (B) $A < B$ (C) $A = B$ (D) none of these

MMM111

SECTION–III

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/ rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.
  For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

13. John has 'x' children by his first wife and Anglina has 'x + 1' children by her first husband. They both marry and have their own children. The whole family has 24 children. It is given that the children of the same parents don't fight. Then find then maximum number of fights that can take place in the family.

MMM112

14. If the volume of the largest cylinder that can be inscribed in a sphere of radius 'r' cm is in the form of $\frac{4\pi r^3}{a\sqrt{b}}$, then the value of ab

MMM113

15. A given quantity of metal is to be casted into a half cylinder i.e. with a rectangular base and semi-circular ends. The total surface is to be minimum, if the ratio of the height of the cylinder to the diameter of the semi-circular ends is give by $\pi/(\pi + k)$, then the value of k is.

MMM114

16. The fuel charges for running a train are proportional to the square of the speed generated in m.p.h. and costs Rs. 48/- per hour at 16 mph. What is the most economical speed if the fixed charges i.e. salaries etc. amount to Rs. 300/- per hour.

MMM115

17. If $f(x)$ is a polynomial of degree 6, which satisfies $\lim_{x\to 0}\left(1 + \frac{f(x)}{x^3}\right)^{1/x} = e^2$ and has local maximum at x = 1 and local minimum at x = 0 and x = 2, then the value of $\left(\frac{5}{9}\right)^4 f\left(\frac{18}{5}\right)$ is :

MMM116

18. For any acute angled ΔABC, the maximum value of $\frac{\sin A}{A} + \frac{\sin B}{B} + \frac{\sin C}{C}$ is $\frac{a\sqrt{b}}{c\pi}$, then $\frac{a+2b}{c+1}$

MMM117

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	B	A	B	B	B	C	B	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,B,D	A,C,D	B,D	B,D	B,C,D	A	B	A	A,D	C,D
Que.	21	22	23	24			25			
Ans.	0	–1	6	$A\to P;B\to S;C\to Q;D\to R$			$A\to R;B\to S;C\to Q;D\to P$			

EXERCISE - S

1.	177	**2.**	25	**3.**	80	**4.**	4	**5.**	15
6.	74	**7.**	6	**8.**	0	**9.**	360	**10.**	320

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	2	3	3	3	3	1	1	5
Que.	11	12	13	14						
Ans.	1	3	1	4						

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	1	5	9	A,C	A,B	4	C	A,D	B,D
Que.	11	12	13	14	15	16	17	18		
Ans.	A,B,D	1,3,4	C	0.50	57	6	A,B,C	A		

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	2	1	2	2	1	1	3	1	3
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	3	3	3	4	1	4	2	1	3	1
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	36	4	2	10	1	81	45	5	3	3

JEE (Advanced) Practice Paper

Section-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**
	A.	A,C	A,D	B,C,D	A,B,C	B,D	A,B,C
Section-II	**Q.**	**7**	**8**	**9**	**10**	**11**	**12**
	A.	A	D	B	C	D	A
Section-III	**Q.**	**13**	**14**	**15**	**16**	**17**	**18**
	A.	191	9	2	40	32	5

Important Notes

Important Notes