Matrices

SOLUTIONS

EXERCISE - O

1. Ans. (B)

$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2+3 \\ 0 & 1 \end{bmatrix}$

on multiplying the matrix we get

$= \begin{bmatrix} 1 & 1+2+.....+n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$

$n(n+1) = 378 \times 2 \Rightarrow n = 27$

2. Ans. (B)

$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$(aI_2 + bA)^2 = A$

$\Rightarrow a^2 + b^2A^2 + 2abI_2A = A$

$\Rightarrow a^2I_2 + b^2(-I_2) + 2ab\,A = A$

$\Rightarrow (a^2 - b^2)\,I_2 = A\,(1 - 2ab)$

$\Rightarrow \begin{bmatrix} a^2-b^2 & 0 \\ 0 & a^2-b^2 \end{bmatrix} = \begin{bmatrix} 0 & 1-2ab \\ 2ab-1 & 0 \end{bmatrix}$

$\therefore a^2 = b^2$ and $2ab = 1$

$\therefore a = b = \dfrac{1}{\sqrt{2}}$

3. Ans. (A)

for orthogonal matrix check $AA^T = I$

$\begin{bmatrix} 6/7 & 2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ 3/7 & -6/7 & 2/7 \end{bmatrix}\begin{bmatrix} 6/7 & 2/7 & 3/7 \\ 2/7 & 3/7 & -6/7 \\ -3/7 & 6/7 & 2/7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Ans. (D)

$|A| = -1, |B| = 2$

$|2A^9\,B^{-1}| = 2^2|A|^9\,\dfrac{1}{|B|} = -2$

5. Ans. (A)

Let $P = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$, $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$PAQ = R \Rightarrow A = P^{-1}RQ^{-1}$

$A = P^{-1}IQ^{-1} = P^{-1}Q^{-1} = \frac{1}{1}\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} \frac{1}{(-1)}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -11 & -8 \end{bmatrix}$

6. Ans. (B)

$F(x) = (1 + x + x^2 .... + x^{16})$

$F(A) = (I + A + A^2 + ... + A^{16})$

$A^2 = \begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow F(A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 5 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + ... \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow F(A) = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}$

7. Ans. (B)

$x = A^TBA$

$x^2 = A^TBA\,.A^TBA = A^TB^2A$

$x^{10} = A^TB^{10}A$

8. Ans. (A)

$A^T = BCD$

$AA^T = ABCD \Rightarrow AA^T = S \Rightarrow AA^T = S^T$

$\Rightarrow S = S^T$

$D^TC^TB^TA^T = ABC\,.DAB\,.CDA.BCD$

$(ABCD)^T = (ABCD)(ABCD)(ABCD)$

$S^T = S^3 \Rightarrow S = S^3 \Rightarrow S^2 = S^4$

9. Ans. (A)

$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$

$AA^{-1} = I$

$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & c \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & c+1 \\ 0 & 1 & 2c+2 \\ 4-4a & 3a-3 & ac+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

If $a = 1 \Rightarrow c = -1$

10. **Ans. (D)**

Option(A)

$AA^{-1} = I$

$|A||A^{-1}| = 1$

$|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

Option(B)

$(A^2)^{-1} = (AA)^{-1} = A^{-1}A^{-1} = (A^{-1})^2$

Option(C)

$AA^{-1} = I$

Transpose-

$(A^{-1})^T A^T = I$

$(A^{-1})^T A^T (A^T)^{-1} = I(A^T)^{-1}$

$(A^{-1})^T = (A^T)^{-1}$

11. **Ans. (A,B,C)**

$A^3 = B^3, A^2B = B^2A \Rightarrow \det A = \det B$

Now $(A^2 + B^2)(A - B) = A^3 - A^2B + B^2A - B^3$

$= A^3 - B^3 + B^2A - A^2B = 0$

$\therefore$ Det $(A^2 + B^2) = 0$ or det $(A - B) = 0$ as $A \neq B$

12. **Ans. (B, C)**

$tr(A) = 1 + \omega^n + \omega^{2n}$

Let $n = 3k$

$tr(A) = 1 + \omega^{3k} + \omega^{6k} = 1 + (\omega^3)^k + (\omega^3)^{2k}$

$= 1 + 1 + 1 = 3$

Let $n = 3k + 1$

$tr(A) = 1 + \omega^{3k+1}\omega^{6k+2}$

$= 1 + (\omega^3)^k\omega + (\omega^6)^k\omega^2$

$= 1 + \omega + \omega^2 = 0$

Let $n = 3k + 2$

$tr(A) = 1 + \omega^{3k+2} + \omega^{6k+4}$

$= 1 + (\omega^3)^k\omega^2 + (\omega^6)^k\omega^3\omega$

$= 1 + \omega^2 + \omega = 0$

13. **Ans. (A,B,C)**

(A, B) A is skew symmetric matrix $\Rightarrow A + A^T = 0$

$B = A + A^3 + A^5 + A^7 + A^7 + A^9$ ...(i)

$B^T = -A - (A^3) - (A^5) - A^7 - A^9$ ...(ii)

$B + B^T = 0 \Rightarrow B$ is skew symmetric matrix

(C) Let $C = A + B$ ...(i)

$C^T = A^T + B^T$

$C^T = -A - B$ ...(ii)

(i) + (ii) $\Rightarrow C + C^T = 0$

14. **Ans. (A,B,D)**

$B = adjA$ & $(adjA)A = |A|I = I$

$|C^{-1}B + BA| = |C^{-1}B + AB| = |C^{-1} + A||B| = |C^{-1} + A||A|^{n-1} = |C^{-1} + A|$

$C = adj(adjA) = |A|^{n-2} \times A = A$

$\Rightarrow C = A,\ \ C^{-1} = A^{-1} = adjA = B$

$\Rightarrow (C^{-1} + A) = |A + B| = |B + C|$

Now $adjC = adjA = |A + adjC|$

15. **Ans. (A,C,D)**

option A

$AB = (AB)^{-1} \Rightarrow (AB)^2 = I$

$$AB = \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix}$$

$$(AB)^2 = \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix}\begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 25x^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2+50x^2-10x & 25x^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad 25x^{25} = 1 \Rightarrow = \pm\frac{1}{5}$

$50x^2 - 2 = 0 \Rightarrow x = \pm\frac{1}{5}$

at $\quad x = \frac{1}{5}, AB = I\ but\ AB \neq I$

$\therefore \quad x = -\frac{1}{5}$

$$AB = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & -1 \end{bmatrix}$$

$\because \quad (AB)^2 = I \Rightarrow (AB)^3 = AB$

$(AB)^4 = (AB)^2 = I$

$\therefore \quad (AB)^{2n} = I$ and $(AB)^{2n-1} = AB, n \in N$

$\therefore \quad AB + (AB)^2 + (AB)^3 + \cdots + (AB)^{100}$

$= AB + I + AB + I + \cdots + I$

$= 50(AB) + 50I$

Tr. $(AB + (AB)^2 + \cdots + (AB)^{100})$ = Tr.$(50(AB) + 50I)$

$= Tr(50(AB)) + Tr.(50I)$

$= 50\,Tr.(AB) + 50\,Tr.(I)$

$= 50(-1 + 1 - 1) + 50(1 + 1 + 1)$

= – 50 + 150 = 100 Ans.

option B $\left|\left(A^{-1}\right)adj\left(B^{-1}\right)adj\left(2A^{-1}\right)\right|$

$= \left|\left(A^{-1}\right)adj\left(B^{-1}\right)adj\left(2A^{-1}\right)\right|$

$=\left|A^{-1}\right|\left|B^{-1}\right|^{2}\left|2A^{-1}\right|^{2}$

$=\frac{1}{|A|}\frac{1}{|B|^{2}}\frac{2^{6}}{|A|^{2}}$

$= \frac{1}{-2}\times\frac{1}{1}\times\frac{2^{6}}{4}=-8$

option C

$$F(x)\,F(y) = \begin{bmatrix} \cos x\cos y-\sin x\sin y & -\cos x\sin y-\sin x\cos y & 0 \\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} \cos x(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}=F(x+y)$$

Also : $F(0) = I$

$\because F(x)F(y) = F(x+y)$

put $y = -x$

$F(x)\,F(-x) = F(0) = I$

$\Rightarrow [F(x)]^{-1} = F(-x)$

option D

$\because\quad A^2 = I \qquad \therefore A^2 = I \Rightarrow x = 2, 4, 6,$ --------

$\therefore\quad \sum\left(\cos^{x}\theta+\sin^{x}\theta\right)$

$=\quad \frac{\cos^{2}\theta}{1-\cos^{2}\theta}+\frac{\sin^{2}\theta}{1-\sin^{2}\theta}$

$=\quad \cot^2\theta + \tan^2\theta \geq 2$

$\therefore$ Minimum value = 2

16. Ans. (A,B)

Statement 1. $BPA=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$P=B^{-1}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}A^{-1}$$

where $B^{-1}=\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}; A^{-1}=\begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$

$\Rightarrow P=\begin{bmatrix} -4 & 7 & 7 \\ 3 & -5 & 5 \end{bmatrix}$

Statement 2. $A=\left(\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\right)^{-1}\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}\left(\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}\right)^{-1}$

$=\frac{1}{19}\begin{bmatrix} 48 & -25 \\ -70 & 42 \end{bmatrix}$

Statement 3. $Ax = b$

$\Rightarrow x = A^{-1} b$

Also. $Cb = D$

$\Rightarrow b = C^{-1} D$

$$\therefore\ x = A^{-1}C^{-1}D = (CA)^{-1}D = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Where : $CA = \begin{bmatrix} 5 & 5 & 10 \\ 7 & 7 & 13 \\ 4 & 3 & 8 \end{bmatrix}; (CA)^{-1} = \begin{bmatrix} -17/5 & 2 & 1 \\ 4/5 & 0 & -1 \\ 7/5 & -1 & 0 \end{bmatrix}$

17. **Ans. (A,B,C,D)**

(A) $AB = \begin{bmatrix} 1 & 3/2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$$\sum_{r=1}^{50} tr\left((AB)^r C_r\right) = \sum_{r=1}^{50} tr(C_r)$$

$$= \sum_{r=1}^{50}\left(r.3^r + (r-1)3^r\right)$$

$$= \sum_{r=1}^{50}(2r-1)3^r = 3\left[(49)\left(3^{50}\right)+1\right]$$

For (B,C,D)

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(B) $AX = A \Rightarrow \begin{bmatrix} 2a+c & 2b+d \\ 2a+c & 2b+d \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow 2a + c = 2$ and $2b + d = 1$

$\Rightarrow c = 2 - 2a$ and $d = 1 - 2b$

$\therefore\ X = \begin{bmatrix} a & b \\ 2-2a & 1-2b \end{bmatrix}$

(C) $XA = I \Rightarrow \begin{bmatrix} 2a+2b & a+b \\ 2c+2d & c+d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow 2a + 2b = 1$ and $a + b = 0$

not possible, hence X does not exist.

(D) $XB = 0$ but $BX \neq 0$

$$XB = \begin{bmatrix} 9a+3b & 3a+b \\ 9c+3d & 3c+d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 3a + b = 0 \Rightarrow b = -3a$

and $3c + d = 0 \Rightarrow d = -3c$

$$BX = \begin{bmatrix} 9a+3c & 9b+3d \\ 3a+c & 3b+d \end{bmatrix} \neq 0$$

$\Rightarrow 3a + c \neq 0$ and $3b + d \neq 0$

$\therefore$ Required matrix is.

$x = \begin{bmatrix} a & -3a \\ c & -3c \end{bmatrix}$ where $3a + c \neq 0$

Solution Q.18, 19, 20

as A is symmetric $b = c$

$\det A = a^2 - b^2 = (a+b)(a-b)$

$a, b, c, \in \{0, 2, ...p-1\}$

no. of numbers of type

$np = 1$

$np + 1 = 1$

$np + 2 = 1 \qquad n \in I$

$\vdots$

$np + (p-1) = 1$

18. Ans. (D)

as $det(A)$ is divisible by $p \Rightarrow$ either $a + b$ divisible by p corresponding number of ways = $(p-1)$ [excluding zero] or $(a-b)$ is divisible by p corresponding number of ways = p

total number of ways = $2p - 1$

19. Ans. (C)

as $Tr(A)$ not divisible by $p \Rightarrow a \neq 0$

$det(A)$ is divisible by $p \Rightarrow a^2 - bc$ divisible by p

no. of ways of selection of a, b, c

$(p-1)\,[(p-1)\times 1] = (p-1)^2$

20. Ans. (D)

Total number of $A = p \times p \times p = p^3$

No. of A such that $det(A)$ divisible by p

= $(p-1)^2$ + no. of A in which $a = 0$

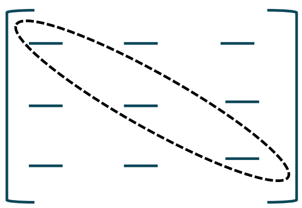
= $(p-1)^2 + p + p - 1$

= p^2

required no. = $p^3 - p^2$.

21. Ans. (A)

(A)



$0 \rightarrow 4$ times

$1 \rightarrow 5$ times

Matrix $\Rightarrow$ Symmetric

* only two zero in diagonal & one in upper triangle

$^3C_2\,^3C_1 = 9$

* All entry in diagonal is '1'

so two zero in upper triangle

$^3C_2 1 = 3$

(B) $\because \; A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

* $A = \begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 1 \end{bmatrix}$

either $b = 0$ or $c = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ 2 – matrix

* $A = \begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{bmatrix}$

either $a = 0$ or $c = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ 2 – matrix

* $\begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$

either $a = 0$ or $b = 0 \Rightarrow |A| \neq 0$

2 – matrix

* $\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$

if $a = b = 0 \Rightarrow |A| = 0$

$a = c = 0 \Rightarrow |A| = 0$

$b = c = 0 \Rightarrow |A| = 0$

so there is no matrix.

$\therefore$ only six matrix are possible

(C) & (D)

six matrix A for which $|A| = 0$

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow$ inconsistent

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow$ inconsistent

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow$ infinite sol^n

$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ inconsistent

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow \text{inconsistent}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \text{infinite sol}^n$$

EXERCISE - S

1. Ans. (4)

$\ell = 1 + 2 + 3 + ... + n \Rightarrow \ell = \frac{n(n+1)}{2} + 1$

$m = n + 1; p = \frac{n(n-1)}{2}$

$\ell + 5 = p + 2m$

$\frac{n(n+1)}{2} + 1 + 5 = \frac{n(n-1)}{2} + 2n + 2$

$n = 4$

2. Ans. (200)

$$\left[\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{I} + \underbrace{\begin{bmatrix} 0 & 2 & a \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}}_{A}\right]^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 2 & a \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & a \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & a \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow (I + A)^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow I + nA + {}^nC_2\, A^2 + {}^nC_3\, A^3 + ... + {}^nC_n\, A^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$$ (A^3, A^4....... is a null matrix)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + n\begin{bmatrix} 0 & 2 & a \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix} + \frac{n(n-1)}{2}\begin{bmatrix} 0 & 0 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2n & na + 4n(n-1) \\ 0 & 1 & 4n \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 2n = 18$

$\Rightarrow n = 9$

$9a + 36 \times 8 = 2007$

$9a = 1719$

$a = 191$

$a + n = 191 + 9 = 200$

3. Ans. (5049)

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\qquad P = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$AP = P$

$AP - P = 0$

$(A - I)P = 0$

If $\det(A - I) \neq 0$ then $P = 0$, which is not possible

So, $\det(A - I) = 0$

$\begin{vmatrix} a-1 & b \\ c & d-1 \end{vmatrix} = 0$

$ad + 1 - a - d - bc = 0$

$ad - bc = a + d - 1 = 5050 - 1 = 5049$

4. Ans. (31)

$\because BA = AB^2$

Pre-multiply by B, we get

$B^2A = (BA)B^2 = (AB^2)B^2 = AB^4$

Again pre-multiply by B

$B^3A = (BA)B^4 = (AB^2)B^4$

$\Rightarrow B^3A = AB^6$ In general $B^mA = AB^{2m}$ ...(1)

again $BA = AB^2$ post multiply by A^4

$\Rightarrow BA^5 = AB^2A^4 \Rightarrow B = AB^2A^4$

$\therefore B = A(B^2A)A^3 = A(AB^4)A^3$ (using (1))

$\therefore B = A^2(B^4A)A^2 = A^2\ (AB^8)A^2$

$\Rightarrow B = A^3(B^8A)A = A^3(AB^{16})A$

$\Rightarrow B = A^4(B^{16}A) = A^4.AB^{32} \Rightarrow B = B^{32}$

Pre-multiply by B^{-1}, we get $B^{31} = I$

5. Ans. (2)

$A^2 = I \Rightarrow x = 2, 4, 6, ...$

$x \geq 2$

$\sum(cos^x\theta + sin^x\theta) = \cos^2\theta + \sin^2\theta + \cos^4\theta + \sin^4\theta + \cos^6\theta + \sin^6\theta + ...$

$= \cos^2\theta + \cos^4\theta + \cos^6\theta + ...+ \sin^2\theta + \sin^4\theta + \sin^6\theta + ...$

$= \dfrac{\cos^2\theta}{1-\cos^2\theta} + \dfrac{\sin^2\theta}{1-\sin^2\theta}$

$= \cot^2\theta + \tan^2\theta = \tan^2\theta + \dfrac{1}{\tan^2\theta}$

Minimum value of $\left(\tan^2\theta + \dfrac{1}{\tan^2\theta}\right)$ is 2.

6. Ans. (4)

For $k = 1$, A = $\begin{bmatrix} 1 & \frac{10}{3} \\ 0 & 1 \end{bmatrix}$

For $k = 2$, A = $\begin{bmatrix} 1 & \frac{19}{3} \\ 0 & 1 \end{bmatrix}$

$$A_{k=1}A_{k=2} = \begin{bmatrix} 1 & \frac{10}{3} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{19}{3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{10}{3}+\frac{19}{3} \\ 0 & 1 \end{bmatrix}$$

$$A_{k=1}A_{k=2}A_{k=3} = \begin{bmatrix} 1 & \frac{10}{3}+\frac{19}{3} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{28}{3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{10}{3}+\frac{19}{3}+\frac{28}{3} \\ 0 & 1 \end{bmatrix} = \prod_{k=1}^{36} A = \begin{bmatrix} 1 & \sum_{k=1}^{36}\left(3k+\frac{1}{3}\right) \\ 0 & 1 \end{bmatrix}$$

$$\sum_{k=1}^{36}\left(3k+\frac{1}{3}\right) = \frac{3.36.37}{2} + \frac{36}{3} = 2010 \Rightarrow \prod_{k=1}^{36} A = \begin{bmatrix} 1 & 2010 \\ 0 & 1 \end{bmatrix}, p = 2010, q = 0$$

$p + q = 2010 = 2 \times 3 \times 5 \times 67$

7. Ans. (8)

Let $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\Rightarrow a - b = -1,\ c - d = 2$

Now, $P^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Rightarrow -a + 2b = 1,\ -c + 2d = 0$

On Solving

$a = -1, b = 0, c = 4, d = 2$

$|P - pI| = 0 \Rightarrow p^2 - p - 2 = 0$

$\Rightarrow p_1 = 2, p_2 = -1$

8. Ans. (1)

$A^{-1} + B^{-1} = (A+B)^{-1}$

$(A^{-1} + B^{-1}) = (A+B) = I$

$I + A^{-1}B + B^{-1}A + I = I$

$I + A^{-1}B + B^{-1}A = 0$

$I + A^{-1}B + (A^{-1}B)^{-1} = 0$

$(A^{-1}B) + (A^{-1}B)^{-1} = -I$

$(A^{-1}B) + I = -(A^{-1}B)$

$\left(A^{-1}B\right)^2+\left(A^{-1}B\right)+I=0$

$\left(A^{-1}B-I\right)\left(\left(A^{-1}B\right)^2 A^{-1}B+I\right)=0$

$\left(A^{-1}B\right)^3=I\rightarrow\left|A^{-1}B\right|=1$

9. **Ans. (–3)**

$$(p\ q\ r)\begin{pmatrix}3 & 4 & 1\\ 3 & 2 & 2\\ 2 & 0 & 3\end{pmatrix}=(3\ 0\ 1)$$

$\left[3p+3q+2r\ 4p+2q\ p+3q+2r\right]=\left[301\right]$

$3p + 3q + 2r = 3 \rightarrow$ (i)

$4p + 2q = 0 \rightarrow$ (ii)

$p + 3q + 2r = 1 \rightarrow$ (iii)

from eq. (iii)

$3q + 2r = 1 - p$

put the value of $3q + 2r$ in eq. (i)

we get

$3p + 1 - p = 3$

$2p = 2$

$p = 1$

Put the value of P in equation (ii)

$4(1) + 2q = 0$

$2q = -4$

$q = -2$

Put the value of P and Q in equation (i)

$3(1) + 3(-2) + 2r = 3$

$3 - 6 + 2r = 3$

$-6 = -2r$

$r = 3$

find $= 2p + q - r$

$= 2(1) - 2 - 3$

$= -3$

10. **Ans. (14)**

Given, $[P] = a_{ij}$ is a order 3 matrix.

$$\text{Let } Q=\begin{bmatrix}2^2a_{11} & 2^3a_{12} & 2^4a_{13}\\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23}\\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33}\end{bmatrix}$$

$$|Q|=\begin{vmatrix}2^2a_{11} & 2^3a_{12} & 2^4a_{13}\\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23}\\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33}\end{vmatrix}$$

$$\Rightarrow|Q|=2^2 2^3 2^4\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ 2a_{21} & 2a_{22} & 2a_{23}\\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33}\end{vmatrix}$$

$$\Rightarrow |Q| = 2^9 \times 2 \times 2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$\Rightarrow |Q| = 2^{12}|P|$

$\Rightarrow |Q| = 2^{13}$

So number of positive divisors = 13 + 1 = 14

EXERCISE - JEE (Main) PYQ

1. **Ans. (3)**

$$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\, t \in R = 5e^{-t} \neq 0\ \forall\, t \in R$

2. **Ans. (1)**

A is orthogonal matrix

$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \dfrac{1}{\sqrt{2}}$

3. **Ans. (2)**

$|A|^2.|B| = 8$ and $\dfrac{|A|}{|B|} = 8 \Rightarrow |A| = 4$ and $|B| = \dfrac{1}{2}$

$\therefore det(BA^{-1}.B^T) = \dfrac{1}{4} \times \dfrac{1}{4} = \dfrac{1}{16}$

4. **Ans. (2)**

$$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} = 2(1 + \sin^2\theta)$$

$\theta \in \left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right) \Rightarrow -\dfrac{1}{\sqrt{2}} < \sin\theta < \dfrac{1}{\sqrt{2}} \Rightarrow 0 \le \sin^2\theta < \dfrac{1}{2}$

$\therefore |A| \in [2, 3)$

5. **Ans. (4)**

$A^T A = 3I_3$

$$\begin{pmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$8x^2 = 3$

$6y^2 = 3$

$x^2 = 3/8$

$y^2 = 1/2$

$x=\pm\sqrt{\frac{3}{8}}; y=\pm\sqrt{\frac{1}{2}}$

6. Ans. (1)

$x^2 + x + 1 = 0$

$\alpha = \omega$

$\alpha^2 = \omega^2$

$$A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \Rightarrow A^4 = A^2 . A^2 = I_3$$

$A^{31} = A^{28} . A^3 = A^3.$

7. Ans. (2)

$A=\begin{pmatrix}2 & 2\\ 9 & 4\end{pmatrix}$; $|A| = 8 - 18 = -10$

$$A^{-1}=\frac{adjA}{|A|}=\frac{\begin{pmatrix}4 & -2\\ -9 & 2\end{pmatrix}}{-10}$$

$$10A^{-1}=\begin{pmatrix}-4 & 2\\ 9 & -2\end{pmatrix}=A-6I$$

8. Ans. (3)

$$A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$$

$\Rightarrow \quad |A| = 6$

$$\frac{|adjB|}{|c|}=\frac{|adj(adjA)|}{|3A|}=\frac{|A|^4}{3^3|A|}=\frac{|A|^3}{3^3}$$

$$=\frac{(6)^3}{(3)^3}=8$$

9. Ans. (4)

$|A| \neq 0$

For (P) : $A \neq I_2$

So, $A = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ or $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ or $\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$ or $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ or $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

$|A|$ can be – 1 or 1

So (P) is false.

For (Q); $|A| = 1$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow tr(A) = 2$

$\Rightarrow Q$ is true

10. Ans. (4)

$Ax_1 = b_1$

$Ax_2 = b_2$

$Ax_3 = b_3$

$$\Rightarrow |A| \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$\Rightarrow |A| = \frac{4}{2} = 2$

11. Ans. (1)

$A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, a \in R$

and $P = \frac{A + A^T}{2} = \begin{bmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix}$

and $Q = \frac{A - A^T}{2} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$

As, $det\ (Q) = 9$

$\Rightarrow (a-3)^2 = 36 \Rightarrow a = 3 \pm 6$

$\therefore \boxed{a = 9, -3}$

$\therefore \quad \det.(P) = \begin{vmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{vmatrix}$

$= 0 - \frac{(a-3)^2}{4} = 0$, for $a = -3$ $= 0 - \frac{(a-3)^2}{4} = -\frac{1}{4}(12)(12)$, for $a = 9$

$\therefore$ Modulus of the sum of all possible values of

$det.(P) = |-36| + |0| = 36$ Ans.

$\Rightarrow$ Option (1) is correct

12. Ans. (1)

$$P=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$$

$$P^2=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

$$P^3=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ \frac{3}{2} & 1\end{bmatrix}$$

$$P^4=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$$

$\vdots$

$$\therefore\ P^{50}=\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$$

13. Ans. (13)

$$a^2+b^2=|I_2+A||I_2-A|^{-1}$$

$$=\sec^2\frac{\theta}{2}\times\cos^2\frac{\theta}{2}=1$$

14. Ans. (4)

$|A| = 4$

$\Rightarrow |2A| = 2^3 \times 4 = 32$

$\because$ B is obtained by $R_2 \rightarrow 2R_2 + 5R_3$

$\Rightarrow |B| = 2 \times 32 = 64$

option (4)

15. Ans. (3)

Let $A^T = A$ and $B^T = -B$

$C = A^2B^2 - B^2A^2$

$C^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T = B^2A^2 - A^2B^2$

$C^T = -C$

C is skew symmetric.

So $det(C) = 0$

so system have infinite solutions.

16. Ans. (24)

$$A^2=\begin{bmatrix}1 & a & a\\ 0 & 1 & b\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & a & a\\ 0 & 1 & b\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2a & 2a+ab\\ 0 & 1 & 2b\\ 0 & 0 & 1\end{bmatrix}$$

$$A^2A=\begin{bmatrix}1 & 2a & 2a+ab\\ 0 & 1 & 2b\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & a & a\\ 0 & 1 & b\\ 0 & 0 & 1\end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3a & 3a+3ab \\ 0 & 1 & 3b \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 4a & 4a+6ab \\ 0 & 1 & 4b \\ 0 & 1 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & na & \frac{(n^2-n)}{2}ab+na \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix}$$

$na = 48, nb = 96$

$na + \frac{nab}{2}(n-1) = 2160$

$48 + 24b(n - 1) = 2160$

$48 + 24 \times 96 - 24b = 2160$

$b = 8$ and $a = 4, n = 12$

$n + a + b = 24$

17. Ans. (4)

$$A'BA = [1\ \ 1\ \ 1] \begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= [9^2 + 12^2 - 15^2 \quad -10^2 + 13^2 + 16^2 \quad 11^2 - 14^2 + 17^2] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$= [9^2 + 12^2 - 15^2 - 10^2 + 13^2 + 16^2 + 11^2 - 14^2 + 17^2] = [539]$

18. Ans. (2)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; ad – bc = –1

$|A+I||adj\ A+I| = 4$

$\Rightarrow ad - bc + a + d + 1 = 2 \text{ or } -2$

$a + d = 2 \text{ or } -2$

19. Ans. (2)

$S = \{\sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd}\}$

$= \left\{\sqrt{1}, \sqrt{3}, \sqrt{5} ....\sqrt{49}\right\}$, 25 terms

$|A| = 1 + a^2$

$$\sum_{a\in S} \det(adjA) = \sum_{a\in S} |A|^2 = \sum (1+a^2)^2$$

= 22100 = 100λ

$\lambda = 221$

20. Ans. (1)

$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$

$A^3 = -4A$

$A^4 = (-4I)(-4I) = (-4)^2 I$

$A^5 = (-4)^2 A, A^6 = (-4)^3 I$

$M = \sum_{k=1}^{10} A^{2k} = A^2 + A^4 + ... + A^{20}$

$= [-4 + (-4)^2 + (-4)^3 + ... + (-4)^{20}]I = -4\lambda I$

$\Rightarrow$ M is symmetric matrix

$N = \sum_{k=1}^{10} A^{2k-1} = A + A^3 + ... + A^{19}$

$= A[1 + (-4) + (-4)^2 + ... + (-4)^9] = \lambda A \Rightarrow$ skew symmetric

$\Rightarrow N^2$ is symmetric matrix

$\Rightarrow MN^2$ is non identity symmetric matrix

21. Ans. (4)

$|adj\, adj\,(2A)| = |2A|^{(n-1)^2}$

$= |2A|^4 = (2^3|A|)^4 = 2^{12}|A|^4 \Rightarrow 2^{16}$

$|A| = \frac{1}{5!6!7!} 5!6! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$

$R_3 \to R_3 \to R_2$

$R_2 \to R_2 \to R_1$

$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$

22. Ans. (1)

$A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \quad |A - d(adj\, A)| = 0$

$\Rightarrow |A - d(adj\, A)| = \left|\begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}\right| = \begin{vmatrix} m-qd & n(1+d) \\ p(1+d) & q-md \end{vmatrix} = 0$

$\Rightarrow (m-qd)(q-md) - np(1+d)^2 = 0 \Rightarrow mq - m^2d - q^2d + mqd^2 - np(1+d)^2 = 0$

$\Rightarrow (mq-np) + d^2(mq-np) - d(m^2+q^2+2np) = 0 \Rightarrow d + d^3 - d\left((m+q)^2 - 2d\right) = 0$

$\Rightarrow 1 + d^2 = (m+q)^2 - 2d \Rightarrow (1+d)^2 = (m+q)^2$

$\therefore$ Option (1) is correct.

23. **Ans. (1)**

$|A| = m - n$

$4m + n = 22$

$17m + 4n = 93$

$m = 5, n = 2$

$|A| = 3$

$\left|2\, adj\ (adj\ 5A)\right| = 2^5 |5A|^{16}$

$= 2^5 . 5^{80}\ |A|^{16} = 2^5 . 5^{80} . 3^{16} = 3^{11} . 5^{80} . 6^5$

$a + b + c = 96$

24. **Ans. (4)**

Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$A^2 = \begin{bmatrix} p^2 + qr & pq + qs \\ pr + rs & qs + s^2 \end{bmatrix}$

$\Rightarrow p^2 + qr = 1\ (1)\ \ pq + qs = 0 \Rightarrow q(p+s) = 0\ (3)$

$\Rightarrow s^2 + qr = 1\ (2)\ \ pr + rs = 0 \Rightarrow r(p+s) = 0\ (4)$

Equation (1) – equation (2)

$p^2 = s^2 \Rightarrow p + s = 0$

Now $3a^2 + 4b^2$

$= 3(p+s)^2 + 4(ps - qr)^2$

$= 3.0 + 4\ (-p^2 - qr)^2 = 4(p^2 + qr)^2 = 4$

25. **Ans. (3)**

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$

$\Rightarrow A^3 = A^4 = \ldots\ldots = A$

$(A+I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \ldots\ldots {}^{11}C_{10} A + {}^{11}C_{11} I$

$= \left({}^{11}C_0 + {}^{11}C_1 + \ldots\ldots {}^{11}C_{10}\right) A + I = \left(2^{11} - 1\right) A + I = 2047A + I$

$\therefore$ Sum of diagonal elements = 2047(1 + 4 – 3) + 3

= 4094 + 3 = 4097

26. **Ans. (5)**

$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

Now $ac - b^2 = 2$ and $2a + b = 1$

and $2b + c = 2$

solving all these above equations we get

$\frac{1-b}{2} \times \left(\frac{2-2b}{1}\right) - b^2 = 2$

$\Rightarrow (1-b)^2 - b^2 = 2 \Rightarrow 1 - 2b = 2$

$\Rightarrow b = -\frac{1}{2}$ and $a = \frac{3}{4}$ and $c = 3$

Hence $\alpha = 3a + \frac{3b}{2} = \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$

and $\beta = 3b + \frac{3c}{2} = -\frac{3}{2} + \frac{9}{2} = 3$

also $s = a + c = \frac{15}{4}$

$\therefore \frac{\beta s}{\alpha^2} = \frac{3\times 15}{4\times \frac{9}{4}} = 5$

27. Ans. (1)

Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$B = CAD$

$B^n = \underbrace{(CAD)(CAD)(CAD)....(CAD)}_{n-times}$

$\Rightarrow B^n = CA^nD$ ....(1)

$A^2 = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{2}{51} \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$

similarly $A^n = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$B^n = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & \frac{n}{51}+2 \\ -1 & -\frac{n}{51}-1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{n}{51}+1 & \frac{n}{51} \\ -\frac{n}{51} & 1-\frac{n}{51} \end{bmatrix}$

$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25+50 & 25 \\ -25 & -25+50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$

Sum of the elements = 100

28. Ans. (4)

$A=\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$

$A^{-1}=\alpha A+\beta I$

$\alpha + \beta = -2$

$A^{-1}=\dfrac{1}{10-5\lambda}\begin{bmatrix}10 & -5\\ -\lambda & 1\end{bmatrix}=\alpha\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}+\beta\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Comparing we get

$\lambda = 3$

$\alpha = \dfrac{1}{5}$

$\beta = \dfrac{-11}{5}$

$4\alpha^2+\beta^2+\lambda^2=14$

29. Ans. (4)

$P^T=aP+(a-1)I$

$\Rightarrow P=aP^T+(a-1)I \;\; \Rightarrow P^T-P=a(P-P^T)$

$\Rightarrow P=P^T$, as $a\neq -1$

Now, $P=aP+(a-1)I$

$\Rightarrow P=-I\Rightarrow |P|=1 \;\Rightarrow |Adj\ P|=1$

30. Ans. (2)

$PP^T=I$

$P^TQ^{2007}P \;=\; A^{2007}$

$=\begin{bmatrix}1 & 2007\\ 0 & 1\end{bmatrix}\Rightarrow 2a+b-3c-4d=2005$

EXERCISE - JEE (Advanced) PYQ

1. Ans. (C,D)

(A) Let $A = N^TMN$

$A^T = N^TM^TN$

$A = A^T$ when $M = M^T$

$A = -A^T$ when $M = -M^T$

(B) Let $B = MN - NM$ & $M^T = M$ & $N^T = N$

$B^T = N^TM^T - M^TN^T$

$B^T = NM - MN$

$B + B^T = 0$

(C) $M = M^T$ & $N = N^T$

$A = MN$

$A^T = N^T M^T \Rightarrow A^T = NM$

$A \neq A^T$

(D) $adj(MN) = (adj\ N)(adj\ M)$

Hence correct answer are C and D.

2. **Ans. (C,D)**

Let $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

(A) Given that $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix} \Rightarrow a = b = c = \alpha$(let)

$\Rightarrow M = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix} \Rightarrow |M| = 0 \Rightarrow$ Non-invertible

(B) Given that $[b\ c] = [a\ b] \Rightarrow a = b = c = \alpha$(let)

again $|M| = 0 \Rightarrow$ Non-invertible

(C) As given $M = \begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix} \Rightarrow |M| = ac \neq 0$

($\because$ a & c are non zero)

$\Rightarrow M$ is invertible

(D) $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow |M| = ac - b^2 \neq 0$

$\because$ ac is not equal to square of an integer

$\therefore$ M is invertible

3. **Ans. (A,B)**

$M^2 = N^4$

$M^2 - N^4 = 0$ $\qquad$ ($\because MN = NM$)

$(M + N^2)(M - N^2) = 0$

As $(M - N^2) \neq 0$, $|M + N^2| = 0$

$\Rightarrow |M \cdot (M + N^2)| = 0$

Option A is right

So we know $A \cdot B = 0$

when $|B| \neq 0 \Rightarrow (A) = 0$

$\Rightarrow (M^2 + MN^2)U = 0$ for some non-zero U.

So option B is also right.

4. **Ans. (B,C)**

$PQ = kI$

$|P| . |Q| = k^3$

$\Rightarrow |P| = 2k \neq 0 \Rightarrow P$ is an invertible matrix

$\because PQ = kI$ $\qquad$ $\therefore Q = kP^{-1}I$

$\therefore Q = \frac{adj.P}{2}$ $\qquad$ $\because q_{23} = -\frac{k}{8}$

$\therefore \frac{-(3\alpha + 4)}{2} = -\frac{k}{8} \Rightarrow k = 12\alpha + 16$

$\therefore |P| = 2k \Rightarrow k = 10 + 6\alpha$ $\qquad$ ...(i)

Put value of k in (i), we get $\alpha = -1, k = 4$

$\therefore 4\alpha - k + 8 = 0$

& $det\,(P(adj.Q)) = |P|\,|adj.Q|$

$$= 2k.\left(\frac{k^2}{2}\right)^2 = \frac{k^5}{2} = 2^9$$

5. Ans. (B)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \Rightarrow P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 16+32 & 8 & 1 \end{bmatrix}$$

So, $P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 16+32+48 & 12 & 1 \end{bmatrix}$

(from the symmetry)

$$P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ \frac{16.50.51}{2} & 200 & 1 \end{bmatrix}$$

As, $P^{50} - Q = I \Rightarrow q_{31} = \frac{16.50.51}{2}$

$q_{32} = 200$ and $q_{21} = 200$

$$\therefore \frac{q_{31}+q_{32}}{q_{21}} = \frac{16.50.51}{2.200} + 1 = 102 + 1 = 103$$

6. Ans. (A,B)

$A = B^2 \Rightarrow |A| = |B|^2 = +ve$

(A) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 1(-1) =$ negative

Matrix B can not be possible

(B) $\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1 =$ negative

Matrix B can not be possible

(C) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 =$ positive

Matrix B can be I

(D) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 1(1-0) =$ positive

Matrix B can be possible

Ex. $\begin{vmatrix}1&0&0\\0&1&-1\\0&2&-1\end{vmatrix}\begin{vmatrix}1&0&0\\0&1&-1\\0&2&-1\end{vmatrix}=\begin{vmatrix}1&0&0\\0&-1&0\\0&0&-1\end{vmatrix}$

7. **Ans. (1)**

$\Delta = 0$

$\Rightarrow 1(1-\alpha^2)-\alpha(\alpha-\alpha^3)+\alpha^2(\alpha^2-\alpha^2)=0$

$(1-\alpha^2)-\alpha^2+\alpha^4=0$

$(\alpha^2-1)^2=0 \Rightarrow \alpha=\pm 1$

but at $\alpha = 1$ No solution so rejected

at $\alpha = -1$ all three equations become

$x-y+z=1$ (coincident planes)

$\therefore 1+\alpha+\alpha^2=1$

8. **Ans. (A, C, D)**

We find $D=0$ & since no pair of planes are parallel, so there are infinite number of solutions.

Let $\alpha P_1+\lambda P_2=P_3$

$\Rightarrow P_1+7P_2=13P_3$

$\Rightarrow b_1+7b_2=13b_3$

(A) $D\neq 0 \Rightarrow$ unique solution for any b_1, b_2, b_3

(B) $D=0$ but $P_1+7P_2\neq 13P_3$

(C) As planes are parallel and there exist infinite ordered triplet for which they will be coincident although satisfying $b_1+7b_2=13b_3$, so system have infinite solutions.

(D) $D\neq 0$

9. **Ans. (4)**

$$\Delta=\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}$$

$$=\underbrace{(a_1b_2c_3+a_2b_3c_1+a_3b_1c_2)}_{x}-\underbrace{(a_3b_2c_1+a_2b_1c_3+a_1b_3c_2)}_{y}$$

Now if $x\leq 3$ and $y\geq -3$

the Δ can be maximum 6

But it is not possible

as $x=3 \Rightarrow$ each term of $x=1$

and $y=-3 \Rightarrow$ each term of $y=-1$

$\Rightarrow \prod_{i=1}^{3} a_ib_ic_i = 1$ and $\prod_{i=1}^{3} a_ib_ic_i=-1$

which is contradiction

so now next possibility is 4

which is obtained as

$$\begin{vmatrix}1&1&1\\-1&1&1\\1&-1&1\end{vmatrix}=1(1+1)-1(-1-1)+1(1-1)=4$$

10. Ans. (B)

Given $M = \alpha I + \beta M^{-1}$

$\Rightarrow M^2 - \alpha M - \beta I = O$

By putting values of M and M^2, we get

$\alpha(\theta) = 1 - 2\sin^2\theta\cos^2\theta = 1 - \frac{\sin^2 2\theta}{2} \geq \frac{1}{2}$

Also, $\beta(\theta) = -(\sin^4\theta\cos^4\theta + (1+\cos^2\theta)(1+\sin^2\theta))$

$= -(\sin^4\theta\cos^4\theta + 1 + \cos^2\theta + \sin^2\theta + \sin^2\theta\cos^2\theta)$

$= -(t^2 + t + 2), t = \frac{\sin^2 2\theta}{4} \in \left[0, \frac{1}{4}\right]$

$\Rightarrow \beta(\theta) \geq -\frac{37}{16}$

11. Ans. (A,C,D)

$(adjM)_{11} = 2 - 3b = -1 \Rightarrow b = 1$

Also, $(adjM)_{22} = -3a = -6 \Rightarrow a = 2$

Now, $det\ M = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -2$

$\Rightarrow det(adjM^2) = (detM^2)^2 = (detM)^4 = 16$

Also $M^{-1} = \frac{adjM}{detM}$

$\Rightarrow adjM = -2M^{-1} \Rightarrow (adjM)^{-1} = \frac{-1}{2}M$

And, $adj(M^{-1}) = (M^{-1})^{-1}\ det(M^{-1})$

$= \frac{1}{det\ M}M = \frac{-M}{2}$

Hence, $(adjM)^{-1} + adj(M^{-1}) = -M$

Further, $MX = b$

$\Rightarrow X = M^{-1}b = \frac{-adjM}{2}b$

$= \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\Rightarrow (\alpha, \beta, \gamma) = (1, -1, 1)$

12. Ans. (B,C,D)

Let $Q = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

$X = \sum_{k=1}^{6}(P_k Q P_K^T)$

$X^T = \sum_{k=1}^{6}(P_k Q P_K^T)^T = X$

X is symmetric

Let $R=\begin{bmatrix}1\\1\\1\end{bmatrix}$

$XR=\sum_{k=1}^{6}P_K Q P_K^T R.$ $\qquad [\because P_K^T R = R]$

$=\sum_{K=1}^{6}P_K QR.=\left(\sum_{K=1}^{6}P_K\right)QR$

$\sum_{K=1}^{6}P_K=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}$ $QR=\begin{bmatrix}6\\3\\6\end{bmatrix}\Rightarrow XR=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}\begin{bmatrix}6\\3\\6\end{bmatrix}=\begin{bmatrix}30\\30\\30\end{bmatrix}=30R \Rightarrow \alpha = 30.$

Trace X = Trace $\left(\sum_{K=1}^{6}P_K Q P_K^T\right)$

$=\sum_{K=1}^{6}$ Trace $(P_K Q P_K^T) = 6(\text{Trace } Q) = 18$

$X\begin{bmatrix}1\\1\\1\end{bmatrix}=30\begin{bmatrix}1\\1\\1\end{bmatrix}$

$\Rightarrow (X-30I)\begin{bmatrix}1\\1\\1\end{bmatrix}=O\Rightarrow |X-30I|=0$

$\Rightarrow X-30I$ is non-invertible

13. Ans. (C,D)

$det(R) = det(PQP^{-1}) = (det\ P)(detQ)\left(\frac{1}{\det P}\right)$

$= det\ Q = 48 - 4x^2$

Option-1 :

for $x = 1$ $det(R) = 44 \neq 0$

$\therefore$ for equation $R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

We will have trivial solution

$\alpha = \beta = \gamma = 0$

Option-2 :

$PQ = QP$

$PQP^{-1} = Q$

$R = Q$

No value of x.

Option-3 :

$\det\begin{bmatrix}2&x&x\\0&4&0\\x&x&5\end{bmatrix}+8$

$= (40-4x^2) + 8 = 48 - 4x^2 = det\ R\ \forall\ x \in R$

Option-4 :

$$R=\begin{bmatrix}2 & 1 & 2/3\\ 0 & 4 & 4/3\\ 0 & 0 & 6\end{bmatrix}$$

$$(R-6I)\begin{bmatrix}1\\ a\\ b\end{bmatrix}=O \ \Rightarrow -4+a+\frac{2b}{3}=0$$

$$-2a+\frac{4b}{3}=0$$

$\Rightarrow a = 2\ ,\ b = 3$

$a + b = 5$

14. **Ans. (B,C,D)**

$det\ (M) \neq 0$

$M^{-1} = adj(adj\ M)$

$M^{-1} = det(M).M$

$M^{-1}M = det(M).M^2$

$I = det(M).M^2$ ...(i)

$det(I) = \left(det(M)\right)^5$

$1 = det(M)$ ...(ii)

From (i) $I = M^2$

$(adj\ M)^2 = adj\ (M^2) = adj\ I = I$

15. **Ans. (5)**

M-I

Let $A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$$A^2=\begin{bmatrix}a^2+bc & ab+bd\\ ac+dc & bc+d^2\end{bmatrix}$$

$$A^3=\begin{bmatrix}a^3+2abc+bdc & a^2b+abd+b^2c+bd^2\\ a^2c+adc+bc^2+d^2c & abc+2bcd+d^3\end{bmatrix}$$

Given trace(A) = $a + d = 3$

and trace(A^3) = $a^3 + d^3 + 3abc + 3bcd = -18$

$\Rightarrow a^3 + d^3 + 3bc(a + d) = -18 \Rightarrow a^3 + d^3 + 9bc = -18$

$\Rightarrow (a + d)((a + d)^2 - 3ad) + 9bc = -18 \Rightarrow 3(9 - 3ad) + 9bc = -18$

$\Rightarrow ad - bc = 5$ = determinant of A

M-II

$A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$; $\Delta = ad - bc$

$|A - \lambda I| = (a - \lambda)(d - \lambda) - bc$

$= \lambda^2 - (a + d)\lambda + ad - bc$

$= \lambda^2 - 3\lambda + \Delta$

$\Rightarrow O = A^2 - 3A + \Delta I \Rightarrow A^2 = 3A - \Delta I \Rightarrow A^3 = 3A^2 - \Delta A$

$= 3(3A - \Delta I) - \Delta A = (9 - \Delta)A - 3\Delta I$

$= (9 - \Delta)\begin{bmatrix} a & b \\ c & d \end{bmatrix} - 3\Delta\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore$ trace $A^3 = (9 - \Delta)(a + d) - 6\Delta$

$\Rightarrow -18 = (9 - \Delta)(3) - 6\Delta = 27 - 9\Delta$

$\Rightarrow 9\Delta = 45 \Rightarrow \Delta = 5$

16. **Ans. (A,B,D)**

(A) $PEP = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{pmatrix}$

$P^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(B) $\left|EQ + PFQ^{-1}\right| = \left|EQ\right| + \left|PFQ^{-1}\right|$

$\left|E\right| = 0$ and $\left|F\right| = 0$ and $\left|Q\right| \neq 0$

$\left|EQ\right| = \left|E\right|\left|Q\right| = 0$, $\left|PFQ^{-1}\right| = \dfrac{|P||F|}{|Q|} = 0$

$T = EQ + PFQ^{-1}$

$TQ = EQ^2 + PF = EQ^2 + P^2\,EP = EQ^2 + EP = E(Q^2 + P)$

$\left|TQ\right| = \left|E\left(Q^2 + P\right)\right| \Rightarrow \left|T\right|\left|Q\right| = \left|E\right|\left|Q^2 + P\right| = 0 \Rightarrow \left|T\right| = 0$ (as $|Q| \neq 0$)

(C) $\left|(EF)^3\right| > \left|EF\right|^2$

Here 0 > 0 (false)

(D) as $P^2 = I \Rightarrow P^{-1} = P$ so $P^{-1}\,FP = PFP = PPEPP = E$

so $E + P^{-1}\,FP = E + E = 2E$

$P^{-1}\,EP + F \Rightarrow PEP + F = 2PEP$

$Tr(2PEP) = 2Tr(PEP) = 2Tr(EPP) = 2Tr(E)$

17. **Ans. (A,B,C)**

$|I - EF| \neq 0$; $G = (I - EF)^{-1} \Rightarrow G^{-1} = I - EF$

Now, $G.G^{-1} = I = G^{-1}\,G$

$\Rightarrow G\,(I - EF) = I = (I - EF)\,G$

$\Rightarrow G - GEF = I = G - EFG$

$\Rightarrow GEF = EFG$ [C is Correct]

$(I - FE)\,(I + FGE) = I + FGE - FE - FEFGE$

$= I + FGE - FE - F\,(G - I)\,E$

$= I + FGE - FE - FGE + FE$

$= I$ [(B) is Correct]

(So 'D' is Incorrect)

We have

$(I - FE)\,(I + FGE) = I$ .....(I)

Now

$FE(I + FGE)$

$= FE + FEFGE$

$= FE + F(G - I)E$

$= FE + FGE - FE$

$= FGE$

$\Rightarrow |FE|\,|I + FGE| = |FGE|$

$\Rightarrow |FE| \times \dfrac{1}{|I - FE|} = |FGE|$ (from (1))

$\Rightarrow |FE| = |I - FE|\,|FGE|$

(option (A) is correct)

18. **Ans. (A)**

$$M = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ \frac{-3}{2} & \frac{-1}{2} \end{bmatrix}$$

$$M = \begin{bmatrix} \frac{3}{2}+1 & \frac{3}{2} \\ \frac{-3}{2} & \frac{-3}{2}+1 \end{bmatrix}$$

$$M = I + \frac{3}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M^{2022} = \left(I + \frac{3}{2}A\right)^{2022} = I + 3033A$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 3033\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 3034 & 3033 \\ -3033 & -3032 \end{bmatrix}$$

19. **Ans. (3)**

$$A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix} \quad |A| = -1$$

$\Rightarrow |A^7 - (\beta - 1)A^6 - \beta A^5| = 0$

$\Rightarrow |A|^5\,|A^2 - (\beta - 1)A - \beta I| = 0$

$\Rightarrow |A|^5\,|(A^2 - \beta A) + A - \beta I| = 0$

$\Rightarrow |A|^5\,|A(A - \beta I) + I(A - \beta I)| = 0$

$|A|^5\,|(A + I)\,(A - \beta I)| = 0$

$$A+I=\begin{pmatrix}\beta+1 & 0 & 1\\ 2 & 2 & -2\\ 3 & 1 & -1\end{pmatrix}\Rightarrow|A+I|=-4\text{, Here }|A|\neq 0\ \&\ |A+I|\neq 0$$

$$A-\beta I=\begin{pmatrix}0 & 0 & 1\\ 2 & 1-\beta & -2\\ 3 & 1 & -2-\beta\end{pmatrix}$$

$$|A-\beta I|=2-3(1-\beta)=3\beta-1=0\Rightarrow\beta=\frac{1}{3}$$

$9\beta = 3$

20. **Ans. (B,C)**

$$M=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

$|M| = -1 + 1 = 0 \Rightarrow M$ is singular so non-invertible

(B) $M\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix}=\begin{bmatrix}-a_1\\ -a_2\\ -a_3\end{bmatrix}\Rightarrow\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix}=\begin{bmatrix}-a_1\\ -a_2\\ -a_3\end{bmatrix}$

$$\left.\begin{matrix}a_1+a_2+a_3=-a_1\\ a_1+a_3=-a_2\\ a_2=-a_3\end{matrix}\right\}\Rightarrow a_1=0 \text{ and } a_2+a_3=0 \text{ infinite solutions exists [B] is correct.}$$

Option (D)

$$M-2I=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}-2\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 1 & 1\\ 1 & -2 & 1\\ 0 & 1 & -2\end{bmatrix}$$

$|M-2I|=0 \Rightarrow$ [D] is wrong

Option (C) :

$$MX=0\Rightarrow\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$x+y+z=0$

$x+z=0$

$y=0$

$\therefore$ Infinite solution

[C] is correct

21. **Ans. (3780)**

Let us calculate when $|R| = 0$

Case-I $ad = bc = 0$

Now $ad = 0$

$\Rightarrow$ Total – (When none of a & d is 0)

$= 8^2 - 1 = 15$ ways

Similarly $bc = 0 \Rightarrow 15$ ways

$\therefore 15 \times 15 = 225$ ways of $ad = bc = 0$

Case-II $ad = bc \neq 0$

either $a = d = b = c$ OR $a \neq d, b \neq d$ but $ad = bc$

${}^7C_1 = 7$ ways ${}^7C_2 \times 2 \times 2 = 84$ ways

Total 91 ways

$\therefore |R| = 0$ in $225 + 91 = 316$ ways

$|R| \neq 0$ in $8^4 - 316 = 3780$

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (3)**

Here $|A| = xyz - (8x + 3z + 4y) + 28 = 60 - 20 + 28 = 68$

$\therefore |A.\text{adj}A| = |A|\,|\text{adj}A| = |A|\,.\,|A|^2 = 68^3$

2. **Ans. (3)**

$A + B = AB$

$\Rightarrow I_n - A - B + AB = I_n \Rightarrow (I_n - A)(I_n - B) = I_n$

$\Rightarrow (I_n - A)^{-1} = (I_n - B)$ $\therefore (I_n - B)(I_n - A) = I_n$

$\Rightarrow I_n - B - A + BA = I_n \Rightarrow A + B = BA$

$\therefore AB = BA$

3. **Ans. (4)**

Let $A = diag\left(a_1 a_2 a_3 .... a_n\right)$

$A^3 = diag\left(a_1^3, a_2^3, a_3^3, ... a_n^3\right)$

$\therefore A^3 = A$

$\therefore a_1^{\ 3} = a_1 \Rightarrow a_1 = 0, 1, -1$

$\therefore$ Total Number of diagonal matrix = 3^n

4. **Ans. (4)**

$\therefore B = -A^{-1}BA$ $\therefore AB = -BA$

$\Rightarrow AB + BA = 0$

$\therefore (A+B)^2 + A^2 + AB + BA + B^2 = A^2 + B^2$

5. **Ans. (2)**

$a + b = \sum_{k=1}^{9}(a_k + b_k) = \sum_{k=1}^{9}\left(k\,{}^{10}c_k + (10-k)\,{}^{10}c_k\right) = 10\sum_{k=1}^{9}{}^{10}C_k = 10(2^{10} - 2) = 10220$

6. **Ans. (2)**

Make $C_1 \to C_1 + C_3$ we get

$C_1 \to C_1 + C_3$

determinant = $f(\theta) = \begin{bmatrix} 1 & \tan\theta + \sec^2\theta & 3 \\ 0 & \cos\theta & \sin\theta \\ 0 & -4 & 3 \end{bmatrix}$

$= 3\cos\theta + 4\sin\theta \Rightarrow f'(\theta) = 0 \Rightarrow -3\sin\theta + 4\cos\theta = 0$

$\Rightarrow \tan\theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\frac{4}{3} \Rightarrow f(\theta)$ is $\uparrow$ for $\left[0, \tan^{-1}\frac{4}{3}\right]$

and $\downarrow$ for $\left[\tan^{-1}\frac{4}{3}, \frac{\pi}{2}\right]$

max $f(\theta)$ is at $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow$ max $f(\theta) = 3\left(\frac{3}{5}\right) + 4\left(\frac{4}{5}\right) = 5$

minimum $f(\theta)$ is at $\theta = 0$

$\Rightarrow$ min $f(\theta) = 3$

7. Ans. (4)

$|A| + |A^T| = 2|A| \neq 0$

8. Ans. (1)

$A\,(\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$A\,(\text{adj } A) = 10\begin{bmatrix} 1 & 0 \\ 1 & 10 \end{bmatrix}$

$A\,(\text{adj } A) = |A| I_n$

$\therefore |A| = 10$

9. Ans. (2)

$A(\text{Adj } A) = I_3 = kI_3 \Rightarrow k = |A| = 8$

10. Ans. (4)

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \Rightarrow A^{10} = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$

$adj\ A^{10} = \begin{bmatrix} 1 & 0 \\ -10 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 10 & 100 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 90 \\ 0 & 11 \end{bmatrix}$

$b_1 + b_2 + b_3 + b_4 = 22 + 90 = 112.$

11. Ans. (3)

Taking $C_3 \to C_3 - (C_1\alpha - C_2)$

we get

$$|A| = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha+1 \end{vmatrix} = (1-2\alpha)(ac-b^2)$$

$\therefore$ non-invertible if $\alpha = \frac{1}{2}$ and if a, b, c are in G.P.

12. Ans. (4)

$AB = C \Rightarrow \det(A)\det(B) = \det(C) \Rightarrow \det(B) = -1$

13. Ans. (1)

For nth order determinant $\Delta = |C_{ij}| = D^{n-1}$

(1) For 3^{rd} order determinant $\Delta = D^{3-1} = D^2$ ... (1)

(2) From (1) if $D = 0$ then $\Delta = 0$

(3) $D = 9 = 3^2$

$\Delta = (3^2)^2 = 3^4$ (Δ is not a perfect cube)

14. Ans. (3)

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adj\ (A)}{|A|}$$

$$A^3 = A^2.A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} = A^{-1}$$

15. Ans. (2)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix};\ A^3 = 5A^2 - 6AI + I^2$$

$$A^2 = \begin{bmatrix} 3 & 3 & 7 \\ 1 & 4 & 4 \\ 3 & 1 & 6 \end{bmatrix} \text{ and } A^3 = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix}$$

$$5A^2 - 6A + I = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix} = A^3$$

16. Ans. (3)

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, bc \neq 0$

Characteristic equation is $|A - xI| = 0$

$\begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = 0$

$(a - x)(d - x) - bc = 0$

$x^2 - x(a + d) + ad - bc = 0$

On comparing with the given equation $x^2 + k = 0$

$a + d = 0, k = ad - bc = |A|$

17. Ans. (1)

$AB = \begin{bmatrix} 3ax^2 & 3bx^2 & 3cx^2 \\ a & b & c \\ 6ax & 6bx & 6cx \end{bmatrix}$

Now, $tr(AB) = tr(C) \Rightarrow 3ax^2 + b + 6cx = (x+2)^2 + 2x + 5x^2$

$3ax^2 + 6c + b = 6x^2 + 6x + 4 \Rightarrow a = 2, b = 4, c = 1 \Rightarrow a + b + c = 7.$

18. Ans. (1)

$|A - \lambda I| = 0$

$\begin{bmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{bmatrix} = 0 \Rightarrow (1 - \lambda)[(2 - \lambda)(3 - \lambda)] + 2[-2(2 - \lambda)] = 0$

$(1 - \lambda)[\lambda^2 - 5\lambda + 6] + 4(\lambda - 2) = 0 \Rightarrow \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0$

$x^3 - 6x^2 + 7x + 2 = 0 \Rightarrow k = 2$

19. Ans. (2)

$|A| \neq 0$

$\Rightarrow a(ed - 0) + b(0 - ce)$

$|A| = aed - bce = e(ad - bc) \neq 0$

$e = 1$ and $ad - bc \neq 0$

$ad - bc = 1$ if $[ad = 1, bc = 0]$ Total = 3

$ad - bc = -1$ if $[ad = 0, bc = 1]$ Total = 3

Total = 3 + 3 = 6

20. Ans. (1)

$AB = 0$

$\Rightarrow \begin{bmatrix} \cos^2\theta\cos^2\phi + \sin\theta\sin\phi\cos\theta\cos\phi & \cos^2\theta\cos\phi\sin\phi + \sin^2\phi\cos\theta\sin\theta \\ \cos^2\phi\cos\theta\sin\theta + \sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin^2\phi \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi(\cos\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix} = 0 \Rightarrow \cos(\theta-\phi)\begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi \\ \cos\phi\sin\theta & \sin\theta\sin\phi \end{bmatrix} = 0$

$\Rightarrow \cos(\theta - \phi) = 0 \Rightarrow \theta - \phi = (2n+1)\frac{\pi}{2}$

SECTION–B

1. Ans. (11)

1×1 matrix [0] i.e.1

2×2 matrix $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$ i.e $2 \times 1 = 2$

3×3 matrix i.e $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ $2 \times 2 \times 2 = 8$

Total = 1 + 2 + 8 = 11

2. Ans. (0)

Here $A^2 = A \Rightarrow a^2 + bc = a, b(a + d) = b$

$c(a + d) = c, bc + d^2 = d$

$\therefore abcd \neq 0$

$\therefore a + d = 1 \Rightarrow bc = ad$

$\Rightarrow bc - ad = 0$

$\Rightarrow |A| = 0$ Ans.

3. Ans. (4)

Let $\Delta = \begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix} = (|A|)^2 = (-2)^2 = 4$ as it is a cofactor determinant of A

4. Ans. (2)

$AB = B \Rightarrow \begin{bmatrix} ap+bq \\ cp+dq \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$

$\Rightarrow ap + bq = p$

$cp + dq = q$

Eliminating p and q we get

$\Rightarrow ad - bc - (a + d) + 1 = 0 \Rightarrow ad - bc - 3 + 1 = 0$

$\Rightarrow ad - bc = 2 \Rightarrow |A| = 2$

5. Ans. (6)

$\therefore A$ is non singular $\therefore |A| \neq 0$

$\therefore AB - BA = A \Rightarrow AB = A + BA = A(I + B)$

$\Rightarrow |A|\,|B| = |A|\,|I + B| \Rightarrow |B| = |I + B|$

Similarly $|B| = |B - I|$

$\therefore |B - I| + |B - I| = 6$

6. Ans. (1)

$AB = A(adjA) = |A|I_3 = -2I_3$

$\therefore (AB+3I_3) = |-2I_3 + 3I_3| = |I_3| = 1$

7. Ans. (8)

Let A = diagonal (a, b, c), B is any other square matrix of order 3

$\therefore\ AB = BA \qquad \therefore\ a = b = c$

given $a + b + c = 12$

$\therefore\ a = b = c = 4$

$\therefore\ |A| = abc = 4.\,4.\,4 = 64$

$\therefore\ |A|^{\frac{1}{2}} = 8$

8. Ans. (1)

Here $A^2 = 3A - 2I \qquad \therefore A^8 = 255A - 254I$

$\therefore \lambda = 255, \mu = -254 \qquad \therefore \lambda + \mu = 1$

9. Ans. (0)

$A'A = I$

$|A - I| = |A - A'A|$

$\Rightarrow |A - I| = |A|\,|I - A'| \Rightarrow |A - I| = -1\,.\,|A' - I|$

$\Rightarrow |A - I| = -|A - I| \Rightarrow |A - I| = 0$

10. Ans. (6)

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0 \Rightarrow (-\lambda + 2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$(\lambda - 2)\ [(\lambda^2 - 1) - (-\lambda - 1)\ + 1(1 + \lambda\,] = 0$

$\Rightarrow (\lambda - 2)\,(\lambda^2 + 2\lambda + 1) = 0 \Rightarrow \lambda = 2, -1, -1$

JEE (Advanced) Practice Paper

1. Ans. (D)

$$X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

For $n = 2$ option A, B, C are not satisified. Hence option D is correct.

2. Ans. (C)

$AB = B$

Premultiply both sides by B

$BAB = B^2 \Rightarrow AB = B^2 \Rightarrow B = B^2$

Similarly

$BA = A \Rightarrow ABA = A^2 \Rightarrow BA = A^2$

$\Rightarrow A = A^2$

3. Ans. (D)

No such ordered pair is possible.

Assume $\boldsymbol{C = AB - BA}$

If $\boldsymbol{C = I}$

then trace $(\boldsymbol{C}) = 1 + 1 + \ldots\ldots + 1 = n$

But trace $(\mathbf{C}) = 0 \qquad$ (trace $(\boldsymbol{AB})$ = trace $(.\boldsymbol{BA})$)

Which is a contradiction

Hence no such ordered pair is possible.

4. Ans. (A)

$A^{N+1} = (B+C)^{N+1}$

We can expand $(B+C)^{N+1}$ like binomial expansion as $BC = CB$.

$(B+C)^{N+1}$ $\quad\quad BC = CB$.

$\therefore\ (B+C)^{N+1} = {}^{N+1}C_0 B^{N+1} + {}^{N+1}C_1 B^N C + {}^{N+1}C_2 B^{N-1} C^2 + \ldots\ldots\ldots + C^{N+1}$.

$= B^{N+1} + (N+1)B^N C + 0 + 0 \ldots\ldots + 0 = B^N (B + (N+1)C)$.

5. Ans. (A)

$A = (A^T)^2 - I = (A^2 - I)^2 - I$

$\Rightarrow A^4 - 2A^2 - A = O \Rightarrow A^3 - 2A - I = O \Rightarrow (A+I)(A^2 - A - I) = O \Rightarrow (A+I)(A^T - A) = O$

Because $|A+I| \neq 0 \Rightarrow A^T = A \Rightarrow A$ is symmetric which contradict given condition

$\Rightarrow$ no such matrix possible

6. Ans. (A)

$A^2 - 2A + I = 0 \Rightarrow (A-I)^2 = 0$

$A^n = (A - I + I)^n = {}^nC_0 (A-I)^n + \ldots\ldots + {}^nC_{n-2}(A-I)^2 \cdot I^{n-2} + {}^nC_{n-1}(A-I)\cdot I^{n-1} + {}^nC_n I^n$

$= 0 + 0 + \ldots\ldots\ldots + 0 + n(A-I) + I = nA - (n-1)I$

7. Ans. (A)

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $X = P^T Q^{2005} P$

We observe that $Q = PAP^T$

$Q^2 = (PAP^T)(PAP^T) = PA\,(P^T P)AP^T = PA(IA)P^T = PA^2P^T$

Proceeding in the same way, we get

$Q^{2005} = PA^{2005}P^T$

Also $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\quad A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

And proceeding in the same way

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$P^T P = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, $X = P^T Q^{2005} P = P^T\,(PA^{2005}P^T)P = (P^T P)A^{2005}(P^T P) = IA^{2005}I$

$= A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

8. **Ans. (A,B,D)**

$$X+X^2+\ldots\ldots X^n=\begin{bmatrix}\cos\frac{\pi}{5}+\cos\frac{2\pi}{5}+\ldots\ldots+\cos\frac{n\pi}{5} & -\left(\sin\frac{\pi}{5}+\ldots.+\sin\frac{n\pi}{5}\right)\\ \sin\frac{\pi}{5}+\sin\frac{2\pi}{5}+\ldots.+\cos\frac{n\pi}{5} & \cos\frac{\pi}{5}+\ldots.+\cos\frac{n\pi}{5}\end{bmatrix}$$

$$\sum_{r=1}^{n}\cos\frac{r\pi}{5}=-1 \text{ and } \sum_{r=1}^{n}\sin\frac{r\pi}{5}=0\Rightarrow n=9,19,29,\ldots..$$

9. **Ans. (B,C,D)**

$|A|+|A^T|=2|A|\neq 0$

10. **Ans. (B,D)**

$M = ABCD = A(BCD) = AA^T$

$M^3=(ABCD)(ABCD)(ABCD)=(ABC)(DAB)(CDA)(BCD)=D^TC^TB^TA^T=(BCD)^TA^T=AA^T=M.$

Similarly, $M^{9k}=M$.

11. **Ans. (A,D)**

Let $\boldsymbol{A}=\begin{bmatrix}a_1 & b_1\\ c_1 & -a_1\end{bmatrix}$ and $\boldsymbol{B}=\begin{bmatrix}a_2 & b_2\\ c_2 & -a_2\end{bmatrix}\Rightarrow \boldsymbol{AB}=\begin{bmatrix}a_1a_2+b_1c_2 & a_1b_2-b_1a_2\\ c_1a_2-a_1c_2 & c_1b_2+a_1a_2\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\Rightarrow a_1a_2+b_1c_2=a_1b_2-b_1a_2=c_1a_2-a_1c_2=c_1b_2+a_1a_2=0$

$\boldsymbol{BA}=\begin{bmatrix}a_2a_1+b_2c_1 & a_2b_1-b_2a_1\\ c_2a_1-a_2c_1 & c_2b_1+a_2a_1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

12. **Ans. (B,C)**

$$\text{adj } A = A^{-1}|A| = \frac{1}{2}\begin{vmatrix}1 & -1 & 0\\ 0 & -2 & 1\\ 0 & 0 & -1\end{vmatrix}=\begin{vmatrix}1/2 & -1/2 & 0\\ 0 & -1 & 1/2\\ 0 & 0 & -1/2\end{vmatrix}$$

Here $A^{-1}=\frac{adj\ A}{|A|}$, $|A^{-1}|=\frac{1}{|A|}$ and $|A|=\frac{1}{2}$

13. **Ans. (A,B,D)**

det $(-A)$ = $(-1)^n$ det (A) where n is order of square matrix.

(det $(-A)$ = $(-1)^n$ det (A)

14. **Ans. (A,C)**

$\boldsymbol{M}^{-1}$ adj(adj $\boldsymbol{M}$) = $\boldsymbol{k^2 I}$

Pre-multiplying by $\boldsymbol{M}$

adj(adj $\boldsymbol{M}$) = $\boldsymbol{k^2 M}$

det(adj(adj M)) = det($\boldsymbol{k^2 M}$)

= $\boldsymbol{k^6}$ det ($\boldsymbol{M}$)

(det $\boldsymbol{M}$)^4 = $\boldsymbol{k^6}$ (det $\boldsymbol{M}$)

(det $\boldsymbol{M}$)^3 = $\boldsymbol{k^6}$

det $\boldsymbol{M}$ = $\boldsymbol{k^2}\Rightarrow k^2=\alpha\Rightarrow k^2=4$

15. Ans. (36)

given $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \Rightarrow abc = 120 = 2^3 \times 3^1 \times 5^1$

Case – I All are Positive = ${}^5C_2 \times {}^3C_2 \times {}^3C_2 = 90$

Case - I one Positive and two negative

$= 3 \times ({}^5C_2 \times {}^3C_2 \times {}^3C_2) = 270$

So number of possible matrices = 90 + 270 = 360

= 90 + 270 = 360

16. Ans. (0)

$A'A = I$

$|A - I| = |A - A'A| \Rightarrow |A - I| = |A|\,|I - A'|$

$\Rightarrow |A - I| = -1 \,.\, |A' - I| \Rightarrow |A - I| = -|A - I| \Rightarrow |A - I| = 0$

17. Ans. (63)

$A^2 = A \Rightarrow A^{-1} A^2 = A^{-1}A \Rightarrow A = I$

$\therefore\ (I + A)^6 = (I + I)^6 = (2I)^6 = 64I = I + KI = (K+1)I$

$\therefore\ K + 1 = 64 \Rightarrow K = 63$

18. Ans. (17)

$A = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix}$; $|A| = \begin{vmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{vmatrix} \neq 0$

$6\alpha - 5\alpha^2 \neq = 0 \Rightarrow \alpha(6 - 5\alpha) \neq 0$

$\alpha = 0, 6/5$

$\therefore\ \alpha \in R - \{0, 6/5\}$

For $\alpha = 1$

$A = \begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix} \Rightarrow |A| = 6 - 5 = 1$; Adj $A = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix}$

$\therefore\ A^{-1} = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & 5 \\ 5 & -2 & 2 \end{vmatrix}$

By characteristic equation $|A - xI| = 0$

$$\begin{vmatrix} 2-x & 0 & -1 \\ 5 & 1-x & 0 \\ 0 & 1 & 3-x \end{vmatrix} = 0 \Rightarrow x^3 - 6x^2 + 11x - 1 = 0$$

By cayley hamilton theorem

$A^3 - 6A^2 + 11A = I \Rightarrow A^{-1} = A^2 - 6A + 11.I$