EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. If the product of n matrices $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}......\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is equal to the matrix $\begin{bmatrix}1 & 378\\0 & 1\end{bmatrix}$ then the value of n is equal to -

(A) 26 (B) 27 (C) 377 (D) 378

MMT005

2. If $A = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$ and $(aI_2 + bA)^2 = A$, then -

(A) $a = b = \sqrt{2}$ (B) $a = b = 1/\sqrt{2}$ (C) $a = b = \sqrt{3}$ (D) $a = b = 1/\sqrt{3}$

MMT006

3. Which of the following is an orthogonal matrix -

(A) $\begin{bmatrix}6/7 & 2/7 & -3/7\\2/7 & 3/7 & 6/7\\3/7 & -6/7 & 2/7\end{bmatrix}$ (B) $\begin{bmatrix}6/7 & 2/7 & 3/7\\2/7 & -3/7 & 6/7\\3/7 & 6/7 & -2/7\end{bmatrix}$

(C) $\begin{bmatrix}-6/7 & -2/7 & -3/7\\2/7 & 3/7 & 6/7\\-3/7 & 6/7 & 2/7\end{bmatrix}$ (D) $\begin{bmatrix}6/7 & -2/7 & 3/7\\2/7 & 2/7 & -3/7\\-6/7 & 2/7 & 3/7\end{bmatrix}$

MMT009

4. Let the matrix A and B be defined as $A = \begin{bmatrix}3 & 2\\2 & 1\end{bmatrix}$ and $B = \begin{bmatrix}3 & 1\\7 & 3\end{bmatrix}$ then the value of Det$(2A^9B^{-1})$ is

(A) 2 (B) 1 (C) – 1 (D) – 2

MMT010

5. If $\begin{bmatrix}2 & 1\\7 & 4\end{bmatrix}A\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then matrix A equals -

(A) $\begin{bmatrix}7 & 5\\-11 & -8\end{bmatrix}$ (B) $\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}$ (C) $\begin{bmatrix}7 & 1\\34 & 5\end{bmatrix}$ (D) $\begin{bmatrix}5 & 3\\13 & 8\end{bmatrix}$

MMT011

6. If $A = \begin{bmatrix}0 & 5\\0 & 0\end{bmatrix}$ and $f(x) = 1 + x + x^2 + ... + x^{16}$, then $f(A) =$

(A) 0 (B) $\begin{bmatrix}1 & 5\\0 & 1\end{bmatrix}$ (C) $\begin{bmatrix}1 & 5\\0 & 0\end{bmatrix}$ (D) $\begin{bmatrix}0 & 5\\1 & 1\end{bmatrix}$

MMT012

7. If A and B are square matrices of same order and $AA^T = I$ then $(A^TBA)^{10}$ is equal to -

(A) $AB^{10}A^T$ (B) $A^TB^{10}A$ (C) $A^{10}B^{10}(A^T)^{10}$ (D) $10A^TBA$

MMT041

8. Let A, B, C, D be (not necessarily square) real matrices such that $A^T = BCD$; $B^T = CDA$; $C^T = DAB$ and $D^T = ABC$. For the matrix $S = ABCD$, which of the following is true

(A) $S^3 = S$ (B) $S^3 = S^4$ (C) $S = S^2$ (D) none of these

MMT042

9. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then -

(A) $a = 1, c = -1$ (B) $a = 2, c = -\frac{1}{2}$ (C) $a = -1, c = 1$ (D) $a = \frac{1}{2}, c = \frac{1}{2}$

MMT044

10. Let A be an invertible matrix then which of the following is incorrect:

(A) $|A^{-1}| = |A|^{-1}$ (B) $(A^2)^{-1} = (A^{-1})^2$

(C) $(A^T)^{-1} = (A^{-1})^T$ (D) none of these

MMT046

MULTIPLE CORRECT TYPE QUESTIONS

11. If A and B are different matrices satisfying $A^3 = B^3$ and $A^2B = B^2A$, then which of the following is/are incorrect-

(A) $\det(A^2 + B^2)$ must be zero

(B) $\det(A - B)$ must be zero

(C) $\det(A^2 + B^2)$ as well as $\det(A - B)$ must be zero

(D) At least one of $\det(A^2 + B^2)$ or $\det(A - B)$ must be zero

MMT047

12. If $A = \begin{bmatrix} 1 & 9 & -7 \\ i & \omega^n & 8 \\ 1 & 6 & \omega^{2n} \end{bmatrix}$, where $i = \sqrt{-1}$ and ω is complex cube root of unity, then $\operatorname{tr}(A)$ will be

(A) 1, if $n = 3k,\ k \in N$ (B) 3, if $n = 3k, k \in N$

(C) 0, if $n \neq 3k, k \in N$ (D) -1, if $n \neq 3k, k \in N$

MMT048

13. Let $A = \begin{bmatrix} 0 & \tan\theta & 2 \\ -\tan\theta & 0 & \sec\theta \\ -2 & -\sec\theta & 0 \end{bmatrix}$, where $\theta \neq (2n+1)\frac{\pi}{2}$ & $B = \sum_{r=1}^{5} A^{(2r-1)}$ then which of following statement is correct

(A) A is skew symmetric matrix

(B) B is skew symmetric matrix

(C) $(A + B)$ is skew symmetric matrix

(D) $(A + B)$ is symmetric matrix

MMT049

14. Let A be a square matrix with B as its adjoint & C be the adjoint of B. If $|A| = 1$ then $|C^{-1}B + BA|$ is equal to -

(A) $|A + B|$ (B) $|B + C|$ (C) $|A + C|$ (D) $|A + adjC|$

MMT051

15. Which of the following options is/are correct ?

(A) Let $A=\begin{bmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{bmatrix}$ are two matrices such that $AB = (AB)^{-1}$ and $AB \neq I$ (where I is an identity matrix of order 3 × 3). Then $tr\left(AB+(AB)^2+(AB)^3+.....+(AB)^{100}\right)$ equals 100.

(B) If A and B are square matrices of order 3, where $|A| = -2$ and $|B| = 1$, then $\left|\left(A^{-1}\right)adj\left(B^{-1}\right)adj\left(2A^{-1}\right)\right|$ equals 8.

(C) If $F(x) = \begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ then $F(x).F(y) = F(x + y)$ and $[F(x)]^{-1} = F(-x)$.

(D) Let $A = \begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$, then $A^3 = A$

MMT079

16. Consider the following statements

Statement-1 : Given $A = \begin{bmatrix}1 & 1 & 1\\ 2 & 4 & 1\\ 2 & 3 & 1\end{bmatrix}$, $B = \begin{bmatrix}2 & 3\\ 3 & 4\end{bmatrix}$. If $BPA = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$, then λ denotes sum of elements of P.

Statement-2 : Let μ denote the sum of elements of the matrix A satisfying the matrix equation, $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}.A.\begin{bmatrix}3 & 2\\ 5 & -3\end{bmatrix} = \begin{bmatrix}2 & 4\\ 3 & -1\end{bmatrix}$

Statement-3 : Given that $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 2 & 3\\ 1 & -1 & 3\end{bmatrix}$, $C=\begin{bmatrix}2 & 1 & 1\\ 2 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}$, $D=\begin{bmatrix}10\\ 13\\ 9\end{bmatrix}$ and that $Cb = D$. If $AX = b$, then ν denotes the sum of elements of X.

Then, which of the following options is/are correct ?

(A) $2\lambda - 19\mu = 3$ (B) $38\mu + 15\nu = 5$ (C) $10\nu + 8\mu = 12$ (D) $\lambda + 19\mu + 38\mu = 0$

MMT080

17. Which of the following options is/are correct ?

(A) Let $A=\begin{bmatrix}1 & \frac{3}{2}\\ 1 & 2\end{bmatrix}$, $B=\begin{bmatrix}4 & -3\\ -2 & 2\end{bmatrix}$ and $C_r=\begin{bmatrix}r.3^r & 2^r\\ 0 & (r-1)3^r\end{bmatrix}$ be 3 given matrices. Then $\sum_{r=1}^{50} tr.\left((AB)^r C_r\right) = 3(49.3^{50} + 1)$.

(B) Given $A = \begin{bmatrix}2 & 1\\ 2 & 1\end{bmatrix}$; $B = \begin{bmatrix}9 & 3\\ 3 & 1\end{bmatrix}$. I is a unit matrix of order 2. If $AX = A$, then $X = \begin{bmatrix}a & b\\ 2-2a & 1-2b\end{bmatrix}$ for $a, b \in R$

(C) Given $A = \begin{bmatrix}2 & 1\\ 2 & 1\end{bmatrix}$; $B = \begin{bmatrix}9 & 3\\ 3 & 1\end{bmatrix}$. I is a unit matrix of order 2. If $XA = I$, then X does not exist

(D) Given $A = \begin{bmatrix}2 & 1\\ 2 & 1\end{bmatrix}$; $B = \begin{bmatrix}9 & 3\\ 3 & 1\end{bmatrix}$. I is a unit matrix of order 2. If $XB = \boldsymbol{O}$ but $BX \neq \boldsymbol{O}$, then $X = \begin{bmatrix}a & -3a\\ c & -3c\end{bmatrix}$ $a, c \in R, 3a + c \neq 0$

MMT081

COMPREHENSION TYPE QUESTIONS

Paragraph for Question 18 to 20

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a,b,c \in \{0,1,2,\ldots\ldots, p-1\} \right\}$$

18. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and $det(A)$ divisible by p is -

(A) $(p-1)^2$ (B) $2(p-1)$ (C) $(p-1)^2+1$ (D) $2p-1$

MMT085

19. The number of A in T_p such that the trace of A is not divisible by p but $det(A)$ is divisible by p is -
[**Note :** The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$ (B) $p^3-(p-1)^2$
(C) $(p-1)^2$ (D) $(p-1)\,(p^2-2)$

MMT086

20. The number of A in T_p such that $det(A)$ is not divisible by p is -

(A) $2p^2$ (B) p^3-5p (C) p^3-3p (D) p^3-p^2

MMT087

MATCHING LIST TYPE QUESTION

21. Let $\boldsymbol{A}$ be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

List-I		**List-II**
(A) The number of matrices in A is	(P)	more than 2
(B) The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is	(Q)	12
(C) The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is	(R)	more than 4
(D) The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has infinitely many solutions, is	(S)	less than 7

(A) I-Q, II-P,R,S, III-P,S, IV-S (B) I-Q, II-Q,S, III-P, IV-S
(C) I-P,R, II-P,S, III-Q, IV-R,S (D) I-R,S, II-Q,S, III-P, IV-P,Q

MMT088

EXERCISE - S

1. A is a square matrix of order n.

ℓ= maximum number of distinct entries if A is a triangular matrix

m = maximum number of distinct entries if A is a diagonal matrix

p = minimum number of zeroes if A is a triangular matrix

If $\ell + 5 = p + 2m$, the value of n is

MMT017

2. If $\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$ then find $a + n$.

MMT018

3. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $P = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Such that $AP = P$ and $a + d = 5050$. If the value of $(ad - bc)$ is M, then value of $[M]$? (Where [ ] represents GIF)

MMT019

4. Given A and B are two non-singular matrices such that $B \neq I$, $A^5 = I$ and $AB^2 = BA$, then the least value of n for which $B^n = I$ is –

MMT020

5. Let X be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \in$ N then the minimum value of $\sum(cos^x\theta + sin^x\,\theta), \theta \in R$ is

MMT054

6. If a matrix $A = \begin{bmatrix} 1 & 3k+\frac{1}{3} \\ 0 & 1 \end{bmatrix}$, then the value of $\prod_{k=1}^{36}\begin{bmatrix} 1 & 3k+\frac{1}{3} \\ 0 & 1 \end{bmatrix}$ is equal to $\begin{bmatrix} 1 & p \\ q & 1 \end{bmatrix}$, then number of prime factors of $(p + q)$

MMT056

7. Let P be a 2 × 2 matrix such that $P\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $P^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. If p_1 and p_2 $(p_1 > p_2)$ are two values of p for which $\det(P - pI) = 0$, where I is an identity matrix of order 2, then $(5p_1 + 2p_2)$ is equal to

[**Note :** $\det(M)$ denotes determinant of square matrix M]

MMT058

8. Let A, B and $A + B$ are non-singular matrices of order 3 × 3 satisfying $A^{-1} + B^{-1} = (A+B)^{-1}$ and $|AB^{-1}| \in R$ then value of $\frac{|A|}{|B|}$ is

MMT089

9. If p, q, r are 3 real number satisfying the matrix equation, $\begin{bmatrix} p & q & r \end{bmatrix}\begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = [3\ 0\ 1]$, then $2p + q - r$ equals :-

MMT091

10. Let $P = \left[a_{ij}\right]$ be a 3 × 3 matrix and let $Q = \left[b_{ij}\right]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the number of positive divisors of determinant of the matrix Q is -

MMT152

EXERCISE - JEE (Main) PYQ

1. If $A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t-e^{-t}\sin t & -e^{-t}\sin t+e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$ Then A is- **[JEE (Main) 2019]**

(1) Invertible only if $t=\frac{\pi}{2}$ (2) not invertible for any $t \in R$

(3) invertible for all $t \in R$ (4) invertible only if $t = \pi$

MMT021

2. Let $A=\begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. It $AA^T = I_3$, then $|p|$ is : **[JEE (Main) 2019]**

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{1}{\sqrt{6}}$ (4) $\frac{1}{\sqrt{3}}$

MMT022

3. Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA^T) = 8$ and $det(AB^{-1}) = 8$, then det $(BA^{-1} B^T)$ is equal to :- **[JEE (Main) 2019]**

(1) 16 (2) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) 1

MMT023

4. If $A=\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$, $\det(A)$ lies in the interval:

[JEE (Main) 2019]

(1) $\left[\frac{5}{2},4\right)$ (2) $\left(\frac{3}{2},3\right]$ (3) $\left(0,\frac{3}{2}\right]$ (4) $\left(1,\frac{5}{2}\right]$

MMT024

5. The total number of matrices $A=\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}, (x, y \in R, x \neq y)$ for which $A^T A = 3I_3$ is :-

[JEE (Main) 2019]

(1) 6 (2) 2 (3) 3 (4) 4

MMT025

6. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to: **[JEE (Main) 2020]**

(1) A^3 (2) A (3) A^2 (4) I_3

MMT026

7. If $A=\begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to **[JEE (Main) 2020]**

(1) $4I - A$ (2) $A - 6I$ (3) $6I - A$ (4) $A - 4I$

MMT027

8. If the matrices $A=\begin{bmatrix}1 & 1 & 2\\ 1 & 3 & 4\\ 1 & -1 & 3\end{bmatrix}$, $B = adjA$ and $C = 3A$, then $\frac{|adjB|}{|C|}$ is equal to :

[JEE (Main) 2020]

(1) 72 (2) 2 (3) 8 (4) 16

MMT028

9. Let A be a 2×2 real matrix with entries from $\{0, 1\}$ and $|A| \neq 0$. Consider the following two statements:

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$,

where I_2 denotes 2×2 identity matrix and $tr(A)$ denotes the sum of the diagonal entries of A. Then: **[JEE (Main) 2020]**

(1) (P) is true and (Q) is false (2) Both (P) and (Q) are false

(3) Both (P) and (Q) are true (4) (P) is false and (Q) is true

MMT029

10. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively.

If $x_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, x_2=\begin{bmatrix}0\\2\\1\end{bmatrix}, x_3=\begin{bmatrix}0\\0\\1\end{bmatrix}, b_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $b_2=\begin{bmatrix}0\\2\\0\end{bmatrix}$ and $b_3=\begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to :

[JEE (Main) 2020]

(1) $\frac{1}{2}$ (2) 4 (3) $\frac{3}{2}$ (4) 2

MMT030

11. Let A = $\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}$, $a \in \mathbf{R}$ be written as $P + Q$ where P is a symmetric matrix and Q is skew symmetric matrix. If $det(Q) = 9$, then the modulus of the sum of all possible values of determinant of P is equal to: **[JEE (Main) 2021]**

(1) 36 (2) 24 (3) 45 (4) 18

MMT031

12. If $P=\begin{bmatrix}1 & 0\\ 1/2 & 1\end{bmatrix}$, then P^{50} is: **[JEE (Main) 2021]**

(1) $\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & 50\\ 0 & 1\end{bmatrix}$ (3) $\begin{bmatrix}1 & 25\\ 0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$

MMT032

13. If $A=\begin{bmatrix}0 & -\tan\left(\frac{\theta}{2}\right)\\ \tan\left(\frac{\theta}{2}\right) & 0\end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix}a & -b\\ b & a\end{bmatrix}$, then $13(a^2 + b^2)$ is equal to ______ .

[JEE (Main) 2021]

MMT033

14. Let A be a 3 × 3 matrix with $det(A) = 4$. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on $2A$, then $det(B)$ is equal to : **[JEE (Main) 2021]**

(1) 16 (2) 80 (3) 128 (4) 64

MMT034

15. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × l null matrix, has : **[JEE (Main) 2021]**

(1) no solution (2) exactly two solutions

(3) infinitely many solutions (4) a unique solution

MMT035

16. Let $A=\begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}, a,b \in \mathbb{R}$. If for some $n \in N$, $A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then $n + a + b$ is equal to.

[JEE (Main) 2022]

MMT036

17. Let $A=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $B=\begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$, then the value of $A'BA$ is: **[JEE (Main) 2022]**

(1) 1224 (2) 1042 (3) 540 (4) 539

MMT037

18. Let A be a 2 × 2 matrix with det (1) = –1 and det ((A + I) (Adj (1) + I)) = 4. Then the sum of the diagonal elements of A can be : **[JEE (Main) 2022]**

(1) –1 (2) 2 (3) 1 (4) $-\sqrt{2}$

MMT038

19. Let $S=\{\sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd}\}$

Let $a \in S$ and $A=\begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$

If $\sum_{a \in S} \det(adjA) = 100\lambda$, then λ is equal to **[JEE (Main) 2022]**

(1) 218 (2) 221 (3) 663 (4) 1717

MMT039

20. Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is

[JEE (Main) 2022]

(1) a non-identity symmetric matrix

(2) a skew-symmetric matrix

(3) neither symmetric nor skew-symmetric matrix

(4) an identify matrix

MMT040

21. If $A=\frac{1}{5!6!7!}\begin{bmatrix}5! & 6! & 7!\\ 6! & 7! & 8!\\ 7! & 8! & 9!\end{bmatrix}$, then $|adj(adj(2A))|$ is equal to : **[JEE (Main) 2023]**

(1) 2^8 (2) 2^{12} (3) 2^{20} (4) 2^{16}

MMT092

22. Let $A=\begin{pmatrix} m & n\\ p & q\end{pmatrix}, d=|A|\neq 0$ $|A-d(Adj\ A)|=0$. Then **[JEE (Main) 2023]**

(1) $(1+d)^2=(m+q)^2$ (2) $1+d^2=(m+q)^2$

(3) $(1+d)^2=m^2+q^2$ (4) $1+d^2=m^2+q^2$

MMT093

23. Let the determinant of a square matrix A of order m be $m - n$, where m and n satisfy $4m + n = 22$ and $17m + 4n = 93$. If det $(n\ adj(adj(mA)))$ = $3^a5^b6^c$. Then $a + b + c$ is equal to:

[JEE (Main) 2023]

(1) 96 (2) 101 (3) 109 (4) 84

MMT094

24. Let $A = [a_{ij}]_{2\times2}$ where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and $b = |A|$, then $3a^2+4b^2$ is equal to **[JEE (Main) 2023]**

(1) 7 (2) 14 (3) 3 (4) 4

MMT095

25. Let $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 4 & -1\\ 0 & 12 & -3\end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to:

[JEE (Main) 2023]

(1) 6144 (2) 4094 (3) 4097 (4) 2050

MMT096

26. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix}2 & 1\\ 3 & \frac{3}{2}\end{bmatrix}A=\begin{bmatrix}1 & 2\\ \alpha & \beta\end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to _____. **[JEE (Main) 2023]**

MMT097

27. Let $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$. If $B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50}B^n$ is equal to **[JEE (Main) 2023]**

(1) 100 (2) 50 (3) 75 (4) 125

MMT098

28. If $A=\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$, $A^{-1}=\alpha A+\beta I$ and $\alpha+\beta=-2$, then $4\alpha^2+\beta^2+\lambda^2$ is equal to: **[JEE (Main) 2023]**

(1) 12 (2) 10 (3) 19 (4) 14

MMT099

29. If P is a 3 × 3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then **[JEE (Main) 2023]**

(1) P is a singular matrix
(2) $|Adj\ P| > 1$
(3) $|Adj\ P| = \frac{1}{2}$
(4) $|Adj\ P| = 1$

MMT100

30. Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PQP^T$. If $P^T Q^{2007} P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to

[JEE (Main) 2023]

(1) 2007
(2) 2005
(3) 2006
(4) 2004

MMT101

EXERCISE - JEE (Advanced) PYQ

1. For 3×3 matrices M and N, which of the following statement(s) is (are) **NOT** correct?

[JEE (Advanced) 2013]

(A) $N^T MN$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(B) $MN - NM$ is skew symmetric for all symmetric matrices M and N

(C) MN is symmetric for all symmetric matrices M and N

(D) $(adjM)(adjN) = adj(MN)$ for all invertible matrices M and N

MMT059

2. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

[JEE (Advanced) 2014]

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

MMT060

3. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

[JEE (Advanced) 2014]

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

MMT061

4. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$, Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R, k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $det(Q) = \frac{k^2}{2}$, then-

[JEE (Advanced) 2016]

(A) $\alpha = 0, k = 8$

(B) $4\alpha - k + 8 = 0$

(C) $det(P\ adj(Q)) = 2^9$

(D) $det(Q\ adj(P)) = 2^{13}$

MMT062

5. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals

[JEE (Advanced) 2016]

(A) 52 (B) 103 (C) 201 (D) 205

MMT063

6. Which of the following is(are) NOT the square of a 3×3 matrix with real entries?

[JEE (Advanced) 2017]

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

MMT064

7. For a real number α, if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$ **[JEE (Advanced) 2017]**

MMT065

8. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution of each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$? **[JEE (Advanced) 2018]**

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

MMT066

9. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is ____ **[JEE (Advanced) 2018]**

MMT067

10. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$,

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real number, and I is the 2×2 identity matrix. If
α^* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and
β^* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$,
then the value of $\alpha^* + \beta^*$ is **[JEE (Advanced) 2019]**

(A) $-\frac{37}{16}$ (B) $-\frac{29}{16}$ (C) $-\frac{31}{16}$ (D) $-\frac{17}{16}$

MMT068

11. Let $M=\begin{bmatrix}0&1&a\\1&2&3\\3&b&1\end{bmatrix}$ and $adjM=\begin{bmatrix}-1&1&-1\\8&-6&2\\-5&3&-1\end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct? **[JEE (Advanced) 2019]**

(A) $a+b=3$

(B) $det(adjM^2)=81$

(C) $(adjM)^{-1}+adjM^{-1}=-M$

(D) If $M\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$, then $\alpha-\beta+\gamma=3$

MMT069

12. Let $P_1=I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, P_2=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}, P_3=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}, P_4=\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}, P_5=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix},$

$P_6=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$ and $X=\sum_{k=1}^{6}P_K\begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix}P_K^T$

where P_K^T denotes the transpose of the matrix P_K. Then which of the following options is/are correct? **[JEE (Advanced) 2019]**

(A) $X-30I$ is an invertible matrix

(B) The sum of diagonal entries of X is 18

(C) If $X\begin{bmatrix}1\\1\\1\end{bmatrix}=\alpha\begin{bmatrix}1\\1\\1\end{bmatrix}$, then $\alpha=30$

(D) X is a symmetric matrix

MMT070

13. Let $x\in\mathbb{R}$ and let $P=\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}$, $Q=\begin{bmatrix}2&x&x\\0&4&0\\x&x&6\end{bmatrix}$ and $R=PQP^{-1}$.

Then which of the following options is/are correct? **[JEE (Advanced) 2019]**

(A) For $x=1$, there exists a unit vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ for which $R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

(B) There exists a real number x such that $PQ=QP$

(C) $\det R=det\begin{bmatrix}2&x&x\\0&4&0\\x&x&5\end{bmatrix}+8$, for all $x\in\mathbb{R}$

(D) For $x=0$, if $R\begin{bmatrix}1\\a\\b\end{bmatrix}=6\begin{bmatrix}1\\a\\b\end{bmatrix}$, then $a+b=5$

MMT071

14. Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1}=adj(adj\ M)$, then which of the following statement is/are ALWAYS TRUE? **[JEE (Advanced) 2020]**

(A) $M=I$

(B) $det\ M=1$

(C) $M^2=I$

(D) $(adj\ M)^2=I$

MMT072

15. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2 ×2 matrix such that the trace of A is 3 and the trace of A^3 is −18, then the value of the determinant of A is ____

[JEE (Advanced) 2020]

MMT074

16. For any 3 × 3 matrix M, let $|M|$ denote the determinant of M. Let

$$E=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 8 & 13 & 18\end{bmatrix}, P=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix} \text{ and } F=\begin{bmatrix}1 & 3 & 2\\ 8 & 18 & 13\\ 2 & 4 & 3\end{bmatrix}$$

If Q is a nonsingular matrix of order 3 × 3, then which of the following statements is (are) **TRUE** ?

[JEE (Advanced) 2021]

(A) $F = PEP$ and $P^2=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

(B) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(C) $\left|(EF)^3\right| > |EF|^2$

(D) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

MMT075

17. For any 3 × 3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3 × 3 identity matrix. Let E and F be two 3 × 3 matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) **TRUE?** **[JEE (Advanced) 2021]**

(A) $|FE| = |I - FE||FGE|$

(B) $|I-FE|(I+FGE)=I$

(C) $EFG = GEF$

(D) $(I-FE)(I-FGE)=I$

MMT076

18. If $M=\begin{pmatrix}\frac{5}{2} & \frac{3}{2}\\ -\frac{3}{2} & -\frac{1}{2}\end{pmatrix}$, then which of the following matrices is equal to M^{2022} ?

[JEE (Advanced) 2022]

(A) $\begin{pmatrix}3034 & 3033\\ -3033 & -3032\end{pmatrix}$ (B) $\begin{pmatrix}3034 & -3033\\ 3033 & -3032\end{pmatrix}$ (C) $\begin{pmatrix}3033 & 3032\\ -3032 & -3031\end{pmatrix}$ (D) $\begin{pmatrix}3032 & 3031\\ -3031 & -3030\end{pmatrix}$

MMT077

19. Let β be a real number. Consider the matrix

$$A=\begin{pmatrix}\beta & 0 & 1\\ 2 & 1 & -2\\ 3 & 1 & -2\end{pmatrix}$$

If $A^7 - (\beta - 1)A^6 - \beta A^5$ is a singular matrix, then the value of 9β is ____. **[JEE (Advanced) 2022]**

MMT078

20. Let $M = (a_{ij})$, $i, j \in \{1, 2, 3\}$, be the 3 × 3 matrix such that $a_{ij} = 1$ if $j+1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is (are) true ? **[JEE (Advanced) 2023]**

(A) M is invertible

(B) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that $M\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$

(C) The set $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$, where $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(D) The matrix $(M - 2I)$ is invertible, where I is the 3 × 3 identity matrix

MMT102

21. Let $R = \left\{ \begin{pmatrix} a & 3 & b \\ c & 2 & d \\ 0 & 5 & 0 \end{pmatrix} : a, b, c, d \in \{0, 3, 5, 7, 11, 13, 17, 19\} \right\}$. Then the number of invertible matrices in R is **[JEE (Advanced) 2023]**

MMT103

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1 in all other cases.

1. Given $A=\begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$. If $xyz = 60$ and $8x + 4y + 3z = 20$ then $|A.\text{adj}A| =$

(1) $(64)^3$ (2) $(88)^3$ (3) $(68)^3$ (4) $(34)^3$

MMT104

2. Let A and B be two $n \times n$ matrices such that $A + B = AB$ then

(1) $AB = I_n$ (2) $A = I_n$ or $B = I_n$ (3) $AB = BA$ (4) $A = B$

MMT105

3. The number of diagonal matrix A of order n for which $A^3 = A$ is

(1) 1 (2) 0 (3) 2^n (4) 3^n

MMT106

4. If A and B are two square matrices such that $B=-A^{-1}BA$ then $(A + B)^2 =$

(1) 0 (2) $A^2 + 2AB + B^2$ (3) $A + B$ (4) $A^2 + B^2$

MMT107

5. Let $a_k = k.{}^{10}C_k$, $b_k = (10-k)\ {}^{10}C_k$ and $A_k = \begin{bmatrix} a_k & 0 \\ 0 & b_k \end{bmatrix}$. If $A=\sum_{k=1}^{9} A_k = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ than $a + b =$

(1) 10200 (2) 10220 (3) 10100 (4) 10230

MMT108

6. For all values of $\theta \in \left[0,\frac{\pi}{2}\right]$, the determinant of the matrix $\begin{bmatrix} -2 & \tan\theta+\sec^2\theta & 3 \\ -\sin\theta & \cos\theta & \sin\theta \\ -3 & -4 & 3 \end{bmatrix}$ always lies in the interval :

(1) (2) [3, 5] (3) (4, 6) (4) $\left(\frac{5}{2},\frac{19}{4}\right)$

MMT109

7. If A is a non–singular matrix and A^T denotes the transpose of A, then:

(1) $|A| \neq |A^T|$ (2) $|A.A^T| \neq |A|^2$

(3) $|A^T A| \neq |A^T|^2$ (4) $|A| + |A^T| \neq 0$

MMT110

8. For any 2 × 2 matrix A, if A (adj$A$) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ =

(1) 10 (2) 100 (3) 0 (4) 1000

MMT111

9. If $A=\left[a_{ij}\right]_{3\times3}$ is a scalar matrix with $a_{11}=a_{22}=a_{33}=2$ and $A(adjA) = kI_3$ then k is

(1) 1 (2) 8 (3) 0 (4) –1

MMT112

10. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$. If $10A^{10} + adj\left(A^{10}\right) = B$, then $b_1+b_2+b_3+b_4$ is equal to

(1) 91 (2) 92 (3) 111 (4) 112

MMT113

11. Matrix $\begin{bmatrix} a & b & (a\alpha-b) \\ b & c & (b\alpha-c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible if

(1) $\alpha = 1$ (2) a, b, c are in A.P.
(3) a, b, c are in G.P. (4) a, b, c are in H.P.

MMT114

12. If A and B are two invertible matrices such that $AB = C$ and $|A| = 2$, $|C| = -2$, then det(B) is

(1) 1 (2) 8 (3) 0 (4) –1

MMT115

13. If D is a determinant of order three and Δ is a determinant formed by the cofactors of determinant D ; then

(1) $\Delta = D^2$ (2) $\Delta = D^3$
(3) if $D = 9$, then Δ is perfect cube (4) $\Delta = D$

MMT116

14. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then value of A^{-1} is equal to :

(1) A (2) A^2 (3) A^3 (4) A^4

MMT117

15. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then $(5A - I)(A - I)$ =

(1) A^2 (2) A^3 (3) A^4 (4) A

MMT118

16. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2+k=0$, then

(1) $a - d = 0$ (2) $k = -|A|$ (3) $k = |A|$ (4) $k = a + d$

MMT119

17. Let $A = \begin{bmatrix} 3x^2 \\ 1 \\ 6x \end{bmatrix}$, $B = [a\ b\ c]$ and $C = \begin{bmatrix} (x+2)^2 & 5x^2 & 2x \\ 5x^2 & 2x & (x+1)^2 \\ 2x & (x+2)^2 & 5x^2 \end{bmatrix}$

Where a, b, c and $x \in R$, Given that $tr(AB) = tr(C)$, then the value of $(a + b + c)$.

(1) 7 (2) 2 (3) 1 (4) 4

MMT120

18. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ is a root of polynomial $x^3 - 6x^2 + 7x + k = 0$, then the value of k is

(1) 2 (2) 4 (3) –2 (4) 1

MMT121

19. Let $A = \begin{bmatrix} a & o & b \\ 1 & e & 1 \\ c & o & d \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ where $a, b, c, d, e \in \{0, 1\}$

then number of such matrix A for which system of equation $AX = O$ have unique solution.

(1) 16 (2) 6 (3) 5 (4) none

MMT122

20. If $AB = O$ for the matrices $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $\theta - \phi$ is

(1) an odd multiple of $\frac{\pi}{2}$ (2) an odd multiple of π

(3) an even multiple of $\frac{\pi}{2}$ (4) 0

MMT123

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4, if only correct answer is given.
Zero Marks : 0, if no answer is given.
Negative Marks : -1 for incorrect answer

1. Number of all possible skew symmetric matrices whose elements are taken from
0, 0, 0, 1, 1, 1, 1, –1, 1,–1,–1, are

MMT124

2. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $(a, b, c, d \neq 0)$ is a matrix such that $A^2 = A$ then $|A|$ must be equal to

MMT125

3. Let $A = \begin{bmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{bmatrix}$ be a matrix such that $|A| = -2$ then value of $\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ is

MMT126

4. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that $AB = B$ and $a + d = 3$ then $|A| =$

MMT127

5. If A is a non singular matrix satisfying $AB - BA = A$ and $|B| = 3$ then value of $|B - I| + |B + I|$ is

MMT128

6. Let A be a 3^{rd} order square matrix and B be its adjoint matrix such that $|A| = -2$ then $|AB + 3I_3| =$

MMT129

7. If A is a diagonal matrix of order 3×3 and commutative with every square matrix of order 3 under multiplication and $tr(A) = 12$, then value of $|A|^{\frac{1}{2}}$ is

MMT130

8. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ be a matrix and $A^8 = \lambda A + \mu I$, $\lambda, \mu \in I$ then $\lambda + \mu =$

MMT131

9. If A' denotes transpose of matrix A, $A'A = I$ and $\det A = 1$, then $\det(A - I)$ must be equal to

MMT132

10. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $X \neq O$ be a column matrix such that $AX = \lambda X$ then sum of square of all possible values of λ is

MMT133

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–I

- This section contains **SEVEN** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1, in all other cases

1. If $X=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, then value of X^n is, (where n is natural number)

(A) $\begin{bmatrix}3n & -4n\\ n & -n\end{bmatrix}$ (B) $\begin{bmatrix}2+n & 5-n\\ n & -n\end{bmatrix}$ (C) $\begin{bmatrix}3^n & (-4)^n\\ 1^n & (-1)^n\end{bmatrix}$ (D) $\begin{bmatrix}2n+1 & -4n\\ n & -(2n-1)\end{bmatrix}$

MMT134

2. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2+B^2=$
(A) $2AB$ (B) $2BA$ (C) $A + B$ (D) AB

MMT135

3. Find number of all possible ordered sets of two $(n \times n)$ matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ for which $\boldsymbol{AB} - \boldsymbol{BA} = \boldsymbol{I}$
(A) infinite (B) n^2 (C) $n!$ (D) zero

MMT136

4. If B, C are square matrices of order n and if $A = B + C$, $BC = CB$, $C^2=O$, then which of following is true for any positive integer N.
(A) $A^{N+1}=B^N\left(B+(N+1)C\right)$ (B) $A^N=B^N\left(B+(N+1)C\right)$
(C) $A^{N+1}=B\left(B+(N+1)C\right)$ (D) $A^{N+1}=B^N\left(B+(N+2)C\right)$

MMT137

5. Number of 3 × 3 non symmetric matrix A such that $A^T=A^2-I$ and $|A|\neq 0$, equals to
(A) 0 (B) 2 (C) 4 (D) Infinite

MMT138

6. Matrix A is such that $A^2=2A-I$, where I is the identity matrix. Then for $n \geq 2$, A^n =
(A) $nA-(n-1)I$ (B) $nA-I$ (C) $2^{n-1}A-(n-1)I$ (D) $2^{n-1}A-I$

MMT139

7. If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q=PAP^T$ and $x=P^TQ^{2005}P$, then x is equal to

(A) $\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$ (B) $\begin{bmatrix}4+2005\sqrt{3} & 6015\\ 2005 & 4-2005\sqrt{3}\end{bmatrix}$
(C) $\frac{1}{4}\begin{bmatrix}2+\sqrt{3} & 1\\ -1 & 2-\sqrt{3}\end{bmatrix}$ (D) $\frac{1}{4}\begin{bmatrix}2005 & 2-\sqrt{3}\\ 2+\sqrt{3} & 2005\end{bmatrix}$

MMT140

SECTION-II

- This section contains **SEVEN** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 In all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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8. Let $\theta = \frac{\pi}{5}$, $X = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, O is null maxtrix and I is an identity matrix of order 2 × 2, and if $I + X + X^2 + ... + X^n = O$, then n can be

(A) 9 (B) 19 (C) 4 (D) 29

MMT141

9. If A is a non-singular matrix and A^T denotes the transpose of A, then:

(A) $|A| \neq |A^T|$ (B) $|A.A^T| = |A|^2$ (C) $|A^T.A| = |A^T|^2$ (D) $|A| + |A^T| \neq 0$

MMT142

10. Let A, B, C, D be real matrices such that $A^T = BCD$; $B^T = CDA$; $C^T = DAB$ and $D^T = ABC$ for the matrix $M = ABCD$, then find M^{2016} ?

(A) M (B) M^2 (C) M^3 (D) M^4

MMT143

11. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two 2 × 2 matrix with real entries. If $\boldsymbol{AB} = \boldsymbol{O}$ and $tr(\boldsymbol{A}) = tr(\boldsymbol{B}) = 0$ then

(A) $\boldsymbol{A}$ and $\boldsymbol{B}$ are comutative w.r.t. operation of multiplication.
(B) $\boldsymbol{A}$ and $\boldsymbol{B}$ are not commutative w.r.t. operation of multiplication.
(C) $\boldsymbol{A}$ and $\boldsymbol{B}$ are both null matrices.
(D) $\boldsymbol{BA} = 0$

MMT144

12. If $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then

(A) $|A| = 2$ (B) A is non-singular

(C) Adj. $A = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$ (D) A is skew symmetric matrix

MMT145

13. If A and B are square matrices of order 3, then the true statement is/are (where I is unit matrix).
(A) det $(-A) = -\det A$
(B) If AB is singular then atleast one of A or B is singular
(C) det $(A + I) = 1 + \det A$
(D) det $(2A) = 2^3 \det A$

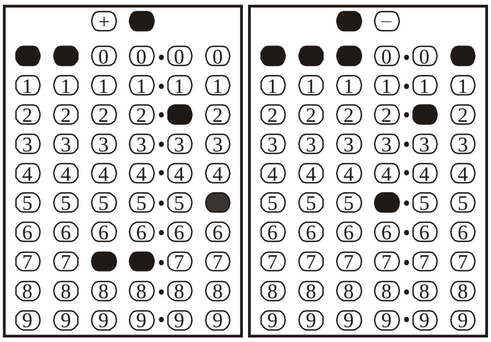
MMT146

14. Let $\boldsymbol{M}$ be a 3 × 3 non-singular matrix with $\det(\boldsymbol{M}) = 4$. If $\boldsymbol{M}^{-1}$ adj(adj $\boldsymbol{M}$) = $\boldsymbol{k}^2\boldsymbol{I}$, then the value of '$\boldsymbol{k}$' may be :
(A) +2 (B) 4 (C) –2 (D) –4

MMT147

SECTION–III

- This section contains **FOUR** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.
For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : + 4 If ONLY the correct numerical value is entered as answer.
Zero Marks : 0 In all other cases.

15. ***A***, is a (3 × 3) diagonal matrix having integral entries such that $\det(A) = 120$, number of such matrices is $10n$. Then n is :

MMT148

16. If A is a square matrix of order 3 and A' denotes transpose of matrix A, $A'A = I$ and $\det A = 1$, then det $(A - I)$ must be equal to

MMT149

17. Suppose A is a matrix such that $A^2 = A$ and $(I + A)^6 = I + kA$, then k is

MMT150

18. Given $A = \begin{bmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{bmatrix}$ For $\alpha \in R - \{a, b\}$, A^{-1} exists and $A^{-1} = A^2 - 5bA + cI$, when $\alpha = 1$. The value of $a + 5b + c$ is :

MMT151

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	A	D	A	B	B	A	A	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,B,C	B,C	A,B,C	A,B,D	A,C,D	A,B	A,B,C,D	D	C	D
Que.	21									
Ans.	A									

EXERCISE - S

1.	4	**2.**	200	**3.**	5049	**4.**	31	**5.**	2
6.	4	**7.**	8	**8.**	1	**9.**	–3	**10.**	14

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	1	2	2	4	1	2	3	4	4
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	1	1	13	4	3	24	4	2	2	1
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	4	1	1	4	3	5	1	4	4	2

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C,D	C,D	A,B	B,C	B	A,B	1	A,C,D	4	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,C,D	B,C,D	C,D	B,C,D	5	A,B,D	A,B,C	A	3	B,C
Que.	21									
Ans.	3780									

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3	3	4	4	2	2	4	1	2	4
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	3	4	1	3	2	3	1	1	2	1
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	11	0	4	2	6	1	8	1	0	6

JEE (Advanced) Practice Paper

Section-I	Q.	1	2	3	4	5	6	7
	A.	D	C	D	A	A	A	A
Section-II	Q.	8	9	10	11	12	13	14
	A.	A,B,D	B,C,D	B,D	A,D	B,C	A,B,D	A,C
Section-III	Q.	15	16	17	18			
	A.	36	0	63	17			