Logarithm

SOLUTIONS

Exercise-I (JEE Main Pattern)

SECTION–A

1. **Ans. (1)**

$a^4b^5 = 1$

$\Rightarrow \log_a(a^4b^5) = \log_a(1) \qquad \Rightarrow \log_a a^4 + \log_a b^5 = 0$

$\Rightarrow 5\log_a b = -4 \Rightarrow \log_a b = \frac{-4}{5} \qquad ...(1)$

Now, $\log_a(a^5b^4)$

$= \log_a a^5 + \log_a b^4$

$= 5 + 4\log_a b$

$= 5 + 4\left(\frac{-4}{5}\right) \qquad$ [from (1)]

$= 5 - \frac{16}{5} = \frac{9}{5}$

2. **Ans. (1)**

$\log_p \log_p \sqrt[p]{\sqrt[p]{\sqrt[p]{......\sqrt[p]{p}}}} = \log_p \log_p (p)^{\frac{1}{p^n}} = \log_p p^{-n} = -n$

Hence, expression is independent of p.

3. **Ans. (3)**

$\log_2 7 = \frac{p}{q}$

Let $p, q \in I,\ q \neq 0$

$\Rightarrow 7 = 2^{\frac{p}{q}} \Rightarrow 7^q = 2^p$

LHS = 7, 49, 343....... $\frac{1}{7}, \frac{1}{49}, \frac{1}{343}$.......

RHS = 1, 2, 4, 8...... $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

LHS can not be equal to RHS in any case

4. **Ans. (2)**

$\left(a^{\log_3 7}\right)^{\log_3 7} + \left(b^{\log_7 11}\right)^{\log_7 11} + \left(c^{\log_{11} 25}\right)^{\log_{11} 25}$

$= (27)^{\log_3 7} + (49)^{\log_7 11} + \left(\sqrt{11}\right)^{\log_{11} 25}$

$= (7)^{\log_3 27} + (11)^{\log_7 49} + (25)^{\log_{11}\left(\sqrt{11}\right)}$

$= 7^3 + 11^2 + 25^{\frac{1}{2}}$

$= 343 + 121 + 5 = 469$

5. Ans. (1)

$2^a = 3 \Rightarrow a\log 2 = \log 3$ ...(1)

$9^b = 4 \Rightarrow b\log 9 = \log 4$ ...(2)

(1) × (2) $\Rightarrow$ $ab\log 2\cdot\log 9 = \log\cdot\log 4$

$\Rightarrow ab\cdot\log 2(2\log 3) = (\log 3)(2\log 2) \quad \Rightarrow ab = 1$

6. Ans. (4)

$$\log_{10}\left(\frac{\log 3}{\log 2}\right) + \log_{10}\left(\frac{\log 4}{\log 3}\right) \ldots\ldots\ldots\ldots \log_{10}\left(\frac{\log 1024}{\log 1023}\right)$$

$$= \log_{10}\left(\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} \times \ldots\ldots \frac{\log 1024}{\log 1023}\right)$$

$$= \log_{10}\left(\frac{\log 1024}{\log 2}\right) = \log_{10}\left(\frac{\log 2^{10}}{\log 2}\right) = \log_{10}(10) = 1$$

7. Ans. (2)

Let $\log_2(x^2+1) = a$ & $\log_{13}(x^2+1) = b$

$a + b = ab$

$$\frac{1}{a} + \frac{1}{b} = 1 \Rightarrow \frac{1}{\log_2(x^2+1)} + \frac{1}{\log_{13}(x^2+1)} = 1$$

$\log_{(x^2+1)} 2 + \log_{(x^2+1)} 13 = 1$

$\log_{x^2+1}(26) = 1 \Rightarrow 26 = x^2 + 1$

$\therefore\ x^2 = 25$

$\log_7(x^2+24) = \log_7 49 = \log_7 7^2 = 2$

8. Ans. (3)

$$\log_9\left(\frac{a^4b^3}{c}\right) = \frac{1}{2}\log_3\left(\frac{a^4b^3}{c}\right)$$

$$\log_9\left(\frac{a^4b^3}{c}\right) = \frac{1}{2}\left(\underbrace{4\log_3 a}_{\text{(i)}} + \underbrace{3\log_3 b}_{\text{(ii)}} - \underbrace{\log_3 c}_{\text{(iii)}}\right)$$

(i) $\log_3 a = p$

(ii) $\log_b 9 = \frac{2}{p^2} \Rightarrow \log_b 3 = \frac{1}{p^2} \Rightarrow \log_3 b = p^2$

(iii) $\log_b c = p \Rightarrow \frac{\log_3 c}{\log_3 b} = p \Rightarrow \log_3 c = p^3$ (using (ii))

$$\therefore\ \log_9\left(\frac{a^4b^3}{c}\right) = \frac{1}{2}(4p + 3p^2 - p^3)$$

$$= \frac{-1}{2}p^3 + \frac{3}{2}p^2 + 2p$$

$\therefore\ \alpha = -\frac{1}{2};\quad \beta = \frac{3}{2};\quad \gamma = 2;\quad \delta = 0$

$\alpha + \beta + \gamma + \delta = 3$

9. Ans. (1)

$60^a = 3 \Rightarrow a = \log_{60} 3$

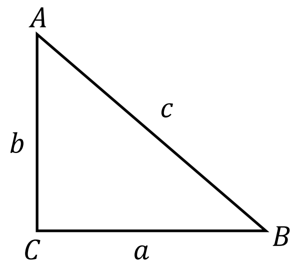
$60^b = 5 \Rightarrow b = \log_{60} 5$

$\therefore\ a + b = \log_{60} 3 + \log_{60} 5 = \log_{60} 15$

$\therefore\ 12^{\frac{1-\log_{60} 15}{2(1-\log_{60} 5)}} = 12^{\frac{\log_{60} 60 - \log_{60} 15}{2(\log_{60} 60 - \log_{60} 5)}}$

$12^{\frac{\log_{60} 4}{2\log_{60} 12}} = 12^{\log_{12} 2} = 2$

10. Ans. (2)



$c^2 = a^2 + b^2$

$\frac{\log_{b+c} a + \log_{c-b} a}{\log_{b+c} a \cdot \log_{c-b} a}$ $(b + c \neq 1, c - b \neq 1)$

$= \frac{\log_{b+c} a}{\log_{b+c} a \cdot \log_{c-b} a} + \frac{\log_{c-b} a}{\log_{b+c} a \cdot \log_{c-b} a}$

$= \frac{1}{\log_{c-b} a} + \frac{1}{\log_{b+c} a}$

$= \log_a (c-b) + \log_a (b+c)$

$= \log_a \left(c^2 - b^2\right) = \log_a a^2 = 2$

11. Ans. (4)

$(\log_2 x)^2 + 4(\log_2 x) - 1 = 0$

$\Rightarrow \log_2 x = \frac{-4 \pm \sqrt{16+4}}{2} = -2 \pm \sqrt{5}$ $\quad \Rightarrow \alpha = 2^{-2+\sqrt{5}} \Rightarrow \beta = 2^{-2-\sqrt{5}}$

$\Rightarrow \log_\beta \alpha = \log_{2^{-2-\sqrt{5}}} \left(2^{-2+\sqrt{5}}\right) = \frac{-2+\sqrt{5}}{-2-\sqrt{5}} = \frac{\left(-2+\sqrt{5}\right)^2}{-1} = 4\sqrt{5} - 9$

$\Rightarrow \log_\alpha \beta = \frac{1}{\log_\beta \alpha} = \frac{1}{4\sqrt{5}-9} = \frac{4\sqrt{5}+9}{-1} = -4\sqrt{5} - 9$

$\Rightarrow \log_\alpha \beta + \log_\beta \alpha = -4\sqrt{5} - 9 + 4\sqrt{5} - 9 = -18$

12. Ans. (3)

Let $4 + \log_3 (x) = N \Rightarrow \log_2 N = 3 \Rightarrow N = 2^3$

$N = 4 + \log_3 x = 8$

$\log_3 x = 4 \Rightarrow x = 81$

Sum of digits of $x = 9$

13. Ans. (3)

Using $\log_{10} p + \log_{10} r - \log_{10} s$

$= \log_{10} \left(\frac{pr}{s}\right)$

$\log \left(\frac{x(x+2)}{5x+4}\right) = 0$

$\Rightarrow x^2 + 2x = 5x + 4 \quad \Rightarrow x^2 - 3x - 4 = 0$

$\Rightarrow x = 4,\ \ x = -1$ (reject)

14. Ans. (4)

$x^{(1+\log_{10} x)} = 10^5 \cdot x$

Taking log on both sides to base 10

$(1+\log_{10} x)(\log_{10} x) = 5 + \log_{10} x$

$\log_{10} x = t \Rightarrow t(1+t) = 5+t$

$\Rightarrow t^2 + t = 5 + t \qquad \Rightarrow t = 5^{\frac{1}{2}} \text{ or } t = -5^{\frac{1}{2}}$

$\Rightarrow \log_{10} x = 5^{\frac{1}{2}}; \log_{10} x = -5^{\frac{1}{2}} \qquad \Rightarrow x = 10^{5^{\frac{1}{2}}}; x = 10^{-5^{\frac{1}{2}}}$

Product = 10^0 = 1

15. Ans. (1)

Take log of base e on both side

$\frac{3}{2} + 2(\log x)^2 = 4\log x$

$t = \log x$ ...(1)

$2t^2 - 4t + \frac{3}{2} = 0$ < $t_1 \rightarrow t_1 = \log x_1$; $t_2 \rightarrow t_2 = \log x_2$

$t_1 + t_2 = 2$

$\log x_1 + \log x_2 = 2$ (form equation (1))

$\log(x_1 x_2) = 2 \Rightarrow x_1 x_2 = e^2$

16. Ans. (1)

$\log_a\left(1-\sqrt{1+x}\right) = \frac{1}{2}\log_a\left(3-\sqrt{1+x}\right)$

$\Rightarrow 2\log_a\left(1-\sqrt{1+x}\right) = \log_a\left(3-\sqrt{1+x}\right) \Rightarrow \left(1-\sqrt{1+x}\right)^2 = 3-\sqrt{1+x}$

$\Rightarrow 2 + x - 2\sqrt{1+x} = 3 - \sqrt{1+x} \quad \Rightarrow (x-1)^2 = 1+x$

$\Rightarrow x^2 - 3x = 0 \Rightarrow x = 0$ (Rejected)

Or $x = 3$ (Rejected)

17. Ans. (2)

$\sqrt{(1+\log_3 x^2)(2+\log_9 x)} = \log_9 x^3 \qquad \Rightarrow \sqrt{(1+2\log_3 x)\left(2+\frac{1}{2}\log_3 x\right)} = \frac{3}{2}\log_3 x$

LHS > 0 $\Rightarrow$ RHS > 0 therefore $\log_3 x > 0$.

$\Rightarrow (1+2\log_3 x)\left(2+\frac{1}{2}\log_3 x\right) = \left(\frac{3}{2}\log_3 x\right)^2$

Let $\log_3 x = t(>0)$

$\Rightarrow (1+2t)\left(2+\frac{t}{2}\right) = \frac{9t^2}{4}$

$\Rightarrow 5t^2 - 18t - 8 = 0 \Rightarrow t = 4 \text{ or } t = \frac{-2}{5}$ (Rejected)

$\Rightarrow x = 81$.

18. **Ans. (4)**

$$\log_{10}\left(\frac{1}{2^x+x-1}\right)=x(\log_{10}5-1)$$

$$\Rightarrow -\log_{10}(2^x+x-1)=x(\log_{10}5-\log_{10}10)$$

$$\Rightarrow \log_{10}(2^x+x-1)=\log_{10}2^x \quad \Rightarrow 2^x+x-1=2^x \Rightarrow x=1$$

19. **Ans. (4)**

$$x^3-3x^2-4x+8=(x-3)^3$$

$$\Rightarrow 6x^2-31x+35=0 \quad \Rightarrow (3x-5)(2x-7)=0$$

$$\therefore\ x=\frac{5}{3}\ \text{(Rejected) or } x=\frac{7}{2}$$

20. **Ans. (1)**

$$3^{\log_3(\log_2 x)^2}=\log_2 x-(\log_2 x)^2+1$$

Let $\log_2 x=t$

$$\Rightarrow t^2=t-t^2+1 \qquad \Rightarrow 2t^2-t-1=0$$

$$t=1 \text{ or } t=\frac{-1}{2}\ \text{(Rejected)}$$

$$\therefore\ x=2$$

21. **Ans. (3)**

$$\sqrt{\log_{10}(-x)}=\log_{10}|x|$$

For log to be defined $x<0 \Rightarrow |x|=-x$

$$\Rightarrow \sqrt{\log_{10}(-x)}=\log_{10}(-x) \qquad \Rightarrow \log_{10}(-x)=0 \text{ or } \log_{10}(-x)=1$$

$$\Rightarrow x=-1 \text{ or } x=-10$$

22. **Ans. (2)**

$$\left(7^{x^2}\right)^2-2\cdot7^{x^2}\cdot7^{x+12}+\left(7^{x+12}\right)^2=0$$

$$\Rightarrow \left(7^{x^2}-7^{x+12}\right)^2=0 \qquad \Rightarrow x^2=x+12$$

$$\Rightarrow x^2-x-12=0 \Rightarrow x_1+x_2=1$$

23. **Ans. (1)**

$$\log_{0.3}(x-1)<\frac{1}{2}\log_{0.3}(x-1);\quad x>1$$

$$\Rightarrow \log_{0.3}(x-1)<0$$

$$\Rightarrow x-1>1 \Rightarrow x>2$$

$$\therefore\ x\in(2,\infty)$$

24. Ans. (1)

$(\log_5 x)^2 + (\log_5 x) - 2 < 0 \quad \Rightarrow (\log_5 x + 2)(\log_5 x - 1) < 0$

$\Rightarrow -2 < \log_5 x < 1 \quad \Rightarrow x \in \left(\frac{1}{25}, 5\right)$

25. Ans. (3)

Let $y = \sqrt{56 + \sqrt{56 + \sqrt{56 + \ldots\ldots\infty}}}$; $y > 0$

$\Rightarrow y^2 = 56 + y$

$\Rightarrow y^2 - y - 56 = 0 \Rightarrow y = 8$, $y = -7$ (Rejected)

$\Rightarrow \because x = \log_2 y \Rightarrow x = 3$

SECTION-B

1. Ans. (12)

$$A = \log_{10}\left(\frac{ab + \sqrt{(ab)^2 - 4(a+b)}}{2}\right) + \log_{10}\left(\frac{ab - \sqrt{(ab)^2 - 4(a+b)}}{2}\right)$$

$$A = \log_{10}\left(\frac{4(a+b)}{4}\right) = \log_{10}(a+b)$$

$A = \log_{10} 100 = 2$

$B = \left(2^{1+\log_6 3}\right)\left(3^{\log_6 3}\right)$

$B = 2^1 \cdot (2.3)^{\log_6 3} = 6$

$A.B = 2 \times 6 = 12.$

2. Ans. (6)

$\log_{200} 5^A + \log_{200} 2^B = C$

$\Rightarrow 5^A \cdot 2^B = (200)^C \quad \Rightarrow 5^A \cdot 2^B = 5^{2C} \cdot 2^{3C}$

$\Rightarrow A = 2C, B = 3C$

Since, HCF $(A, B, C) = 1$

So, $C = 1 \Rightarrow A = 2, B = 3$

Hence, $A + B + C = 6$.

3. Ans. (0)

$$\log_{(1+x)}(1-x) - 1 - \frac{2}{\log_{(1+x)}(1-x)} = 0$$

Let $\log_{(1+x)}(1-x) = t$

$\Rightarrow t^2 - t - 2 = 0 \Rightarrow t = 2$ or $t = -1$

$\therefore \frac{\log(1-x)}{\log(1+x)} = 2$ or $\frac{\log(1-x)}{\log(1+x)} = -1$

$\Rightarrow 1 - x = (1+x)^2$ or $(1-x)(1+x) = 1$

$\Rightarrow x^2 + 3x = 0$ or $x^2 = 0 \Rightarrow x = 0$ or $x = -3$ (both rejected)

4. **Ans. (8.00)**

$$2\log_2(\log_2 x)-\log_2\left(\frac{3}{2}+\log_2 x\right)=1$$

Let $\log_2 x=t(>0)$ $\Rightarrow \log_2\left(\dfrac{t^2}{\frac{3}{2}+t}\right)=1$

$\Rightarrow t^2-2t-3=0\Rightarrow t=3$ or $t=-1$ (Rejected)

$\therefore\ x=8$

5. **Ans. (10.00)**

$4^{\log_{16}4}+9^{\log_3 9}=10^{\log_x 83} \Rightarrow 4^{\log_4 2}+3^{\log_3 81}=10^{\log_x 83}$

$\Rightarrow 2+81=10^{\log_x 83} \Rightarrow \log_{10}83=\log_x 83 \Rightarrow x=10$

Exercise - II (JEE Main PYQs)

1. **Ans. (2)**

$x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$

$\log_2 2^{(x+1)}-\log_2(3+2^x)^2+\log_2(10-2^{-x})=0$

$$\log_2\left(\frac{2^{x+1}(10-2^{-x})}{(3+2^x)^2}\right)=0$$

$$\frac{2(10.2^x-1)}{(3+2^x)^2}=1$$

$\Rightarrow 20.2^x-2=9+2^{2x}+6.2^x$

$\therefore (2^x)^2-14(2^x)+11=0$

Roots are 2^{x_1} & 2^{x_2}

$\therefore 2^{x_1}.2^{x_2}=11$

$x_1+x_2=log_2(11)$

2. **Ans. (1)**

$\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$

$\log_{(x+1)}(2x+5)(x+1)+2\log_{(2x+5)}(x+1)=4$

$\log_{(x+1)}(2x+5)+1+2\log_{(2x+5)}(x+1)=4$

Put $\log_{(x+1)}(2x+5)=t$

$t+\dfrac{2}{t}=3 \Rightarrow t^2-3t+2=0$

$t=1,2$

$\log_{(x+1)}(2x+5)=1$ & $\log_{(x+1)}(2x+5)=2$

$x+1=2x+5$ & $2x+5=(x+1)^2$

$x=-4$ (rejected) $x^2=4\Rightarrow x=2,-2$ (rejected)

So, $x=2$

No. of solution = 1

Exercise - III (JEE Advanced Pattern)

SECTION-I

1. Ans. (A,B,C,D)

$loga^x = b$

$\Rightarrow x$ must be positive and $a > 0, a \neq 1$

$\Rightarrow x = a^b$

(A) let $a = \sqrt{2}, b = log_{\sqrt{2}}3$

$\Rightarrow a^b = \sqrt{2}^{log_{\sqrt{2}}3}$

$= 3$

(B) Let $a = 2, b = \log_2 3$

$\Rightarrow a^b = 2^{\log_2 3} = 3$

(C) Let $a = \sqrt{3}, b = \log_{\sqrt{3}} 3$

$\Rightarrow a^b = \sqrt{3}^{\log_{\sqrt{3}} 3} = 3$

(D) Let $a = 2, b = 3$

$\Rightarrow a^b = 8$

2. Ans. (A,B,D)

(A) $7^{\frac{1}{7}} = \left(7^2\right)^{\frac{1}{14}} = 49^{\frac{1}{14}} \qquad \Rightarrow 49^{\frac{1}{14}} > 42^{\frac{1}{14}} > 1$

(B) $\frac{\log_3 5}{\log_9 7}.\log_{11} 13$

$\Rightarrow \frac{\log_3 5}{\log_9 7} \times \log_3 9 \times \log_{11} 13 \qquad \Rightarrow 2\log_7 5.\log_{11} 13 > 0 > -2$

(C) $99 + 70\sqrt{2} = 99 + 2 \times 35\sqrt{2}$

$\Rightarrow 99 + 2 \times 7 \times 5\sqrt{2} \qquad \Rightarrow \left(5\sqrt{2}\right)^7 + 7^2 + 2 \times 7 \times 5\sqrt{2}$

$\Rightarrow \left(5\sqrt{2} + 7\right)^2 \qquad \Rightarrow \sqrt{99 + 70\sqrt{2}} + \sqrt{99 - 70\sqrt{2}}$

$\Rightarrow \left(5\sqrt{2} + 7\right) + \left(5\sqrt{2} - 7\right) \qquad \Rightarrow 10\sqrt{2}$

(D) $\frac{1}{\log_4 3} + \frac{1}{\log_7 3}$

$= \log_3 4 + \log_3 7$

$= \log_3 28 > \log_3 27 \qquad \Rightarrow \log_3 28 > 3$

3. Ans. (A,B,C,D)

$N = (\log_2 24)(\log_2 96) - (\log_2 192).(\log_2 12)$

$= (\log_2 8 \times 3)(\log_2 32 \times 3) - (\log_2 64 \times 3)(\log_2 4 \times 3)$

$= (\log_2 8 \times 3)(\log_2 32 \times 3) - (\log_2 64 \times 3)(\log_2 4 \times 3)$

$= (3 + \log_2 3)(5 + \log_2 3) - (6 + \log_2 3)(2 + \log_2 3)$

$= 15 + 8\log_2 3 + (\log_2 3)^2 - 12 - 8(\log_2 3) - (\log_2 3)^2$

$= 15 - 12 = 3$

4. **Ans. (B,C)**

If $\log_{\beta}\alpha = x$

Then $\log_{\alpha}\beta = \dfrac{1}{(\log_{\beta}\alpha)} = \dfrac{1}{x}$

$x^2 + \dfrac{1}{x^2} = 79$

$\Rightarrow x^2 + \dfrac{1}{x^2} + 2 = 81 \qquad \Rightarrow \left(x + \dfrac{1}{x}\right)^2 = 81$

$\Rightarrow x + \dfrac{1}{x} = \pm 9 \qquad \Rightarrow \left(\log_{\beta}\alpha + \log_{\alpha}\beta\right) = \pm 9$

5. **Ans. (B,C)**

$\dfrac{1}{\log_a(ax)} + \dfrac{2}{\log_a x} + \dfrac{3}{\log_a(a^2x)} = 0$

$\Rightarrow \dfrac{1}{1+\log_a x} + \dfrac{2}{\log_a x} + \dfrac{3}{2+\log_a x} = 0$

$\Rightarrow$ Let $\log_a x = t \qquad \Rightarrow \dfrac{1}{1+t} + \dfrac{2}{t} + \dfrac{3}{2+t} = 0$

$\Rightarrow t(2+t) + 2(1+t)(2+t) + 3t(1+t) = 0$

$\Rightarrow 6t^2 + 11t + 4 = 0 \quad \Rightarrow 6t^2 + 3t + 8t + 4 = 0$

$\Rightarrow (3t+4)(2t+1) = 0$

$\Rightarrow t = \dfrac{-4}{3}, \dfrac{-1}{2} \qquad \Rightarrow \log_a x = \dfrac{-4}{3}, \dfrac{-1}{2} \qquad \Rightarrow x = a^{\frac{-4}{3}}$ or $a^{\frac{-1}{2}}$

6. **Ans. (A,C)**

$5^{\log_a x} + 5x^{\log_a 5} = 3$

$\Rightarrow 6.\left(5^{\log_{ax}}\right) = 3 \quad \Rightarrow 5^{\log_a x} = \dfrac{1}{2}$

$\Rightarrow x^{\log_a 5} = \dfrac{1}{2} \quad \Rightarrow x = \left(\dfrac{1}{2}\right)^{\frac{1}{\log_a 5}}$

$\Rightarrow x = \left(2^{-1}\right)^{\log_5 a} \quad \Rightarrow 2^{-(\log_5 a)}$

7. **Ans. (A,B,C)**

Take $\log_2$ both sides

$\Rightarrow \left(\dfrac{3}{4}(\log_2 x)^2 + \log_2 x - \dfrac{5}{4}\right)\log_2 x = \dfrac{1}{2}$

Let $\log_2 x = t$

$\Rightarrow \left(3t^2 + 4t - 5\right)t = 2 \qquad \Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0$

$\Rightarrow (t-1)\left(3t^2 + 7t + 2\right) = 0 \qquad \Rightarrow (t-1)(t+2)(3t+1) = 0$

$\Rightarrow t = \dfrac{-1}{3}, -2, 1 \qquad \Rightarrow x = 2^{\frac{-1}{3}}, 2^{-2}, 2^{+1}$

8. Ans. (A,B,C,D)

$$\frac{\log_2 16}{\log_2 x^2}+\frac{\log_2 64}{\log_2 2x}=3$$

$$\Rightarrow \frac{4}{2\log_2 x}+\frac{6}{1+\log_2 x}=3$$

Put $\log_2 x = t$

$$\Rightarrow \frac{2}{t}+\frac{6}{1+t}=3 \qquad \Rightarrow 2t+2+6t=3t+3t^2$$

$$\Rightarrow 3t^2-5t-2=0 \qquad \Rightarrow 3t^2-6t+t-2=0$$

$$\Rightarrow (3t+1)(t-2)=0 \qquad \Rightarrow t=\frac{-1}{3},2$$

$$\Rightarrow \log_2 x=\frac{-1}{3},2 \qquad \Rightarrow x=4,2^{\frac{-1}{3}}$$

9. Ans. (A,B,C)

Take $\log_3$ both side

$$\left((\log_3 x)^2-\frac{9}{2}\log_3 x+5\right)(\log_3 x)=\log_3\left(3\sqrt{3}\right)$$

Put $\log_3 x = t$

$$\Rightarrow t\left(t^2-\frac{9t}{2}+5\right)=\frac{3}{2} \Rightarrow t\left(2t^2-9t+10\right)=3$$

$$\Rightarrow 2t^3-9t^2+10t-3=0 \Rightarrow (t-1)(t-3)(2t-1)=0$$

$$\Rightarrow \log_3 x=1,3,\frac{1}{2} \Rightarrow x=3,27,\sqrt{3}$$

10. Ans. (A,D)

$$2\left(\frac{1}{\log_c a}+\frac{1}{\log_c b}\right)=9\left(\frac{1}{\log_c ab}\right)$$

$$\Rightarrow \frac{2(\log_c b+\log_c a)}{(\log_c a.\log_c b)}=\frac{9}{(\log_c a+\log_c b)}$$

$$\Rightarrow 2(\log_c a+\log_c b)^2=9\log_c b.\log_c a$$

$$\Rightarrow 2(\log_c a)^2+4\log_c a.\log_c b-9\log_c a.\log_c b+2(\log_c b)^2=0$$

$$\Rightarrow 2(\log_c a)^2-5\log_c a\cdot\log_c b+2(\log_c b)^2=0$$

$$\Rightarrow (2\log_c a-\log_c b)(\log_c a-2\log_c b)=0$$

$$\Rightarrow 2\log_c a=\log_c b \text{ or } \log_c a=2\log_c b$$

$$\Rightarrow 2=\frac{\log_c b}{\log_c a},\ \frac{1}{2}=\frac{\log_c b}{\log_c a} \Rightarrow \log_a b=2,\frac{1}{2}$$

11. Ans. (A,B,C,D)

Let $\log_2(x-2)=p$, $\log_2\left(\frac{3}{x}\right)=q$

$\Rightarrow p^2+pq-2q^2=0 \Rightarrow p^2+2pq-pq-2q^2=0$

$\Rightarrow p(p-q)+2q(p-q)=0 \Rightarrow (p+2q)(p-q)=0$

$\Rightarrow p+2q=0$ or $p=q$ $\Rightarrow \log_2(x-2)+2\log_2\left(\frac{3}{x}\right)=0$ or $\log_2(x-2)=\log_2\left(\frac{3}{x}\right)$

$\Rightarrow \log_2\left(\frac{3^2(x-2)}{x^2}\right)=0$ or $(x-2)=\frac{3}{x}$ $\Rightarrow \frac{3^2(x-2)}{x^2}=1$ or $x^2-2x-3=0$

$\Rightarrow 9x-18=x^2$ or $(x-3)(x+1)=0$ $\Rightarrow x^2-9x+18=0$ or $x=3,-1 \Rightarrow x=3,6$

$\Rightarrow x=-1$ is rejected

12. Ans. (A,B)

$\log_3(xy)=2+\log_3 2$

$\Rightarrow \log_3 xy=\log_3 9+\log_3 2$

$\Rightarrow xy=18$ ...(i)

$\Rightarrow \log_{27}(x+y)=\frac{2}{3}$ $\Rightarrow x+y=(27)^{\frac{2}{3}}$

$\Rightarrow x+y=9$ ...(ii)

Form (i) and (ii)

$\Rightarrow x=6, y=3$ or $x=3, y=6$

13. Ans. (C,D)

$2^{x+y}=2^y.3^y$ $\Rightarrow 2^x.2^y=2^y.3^y$

$\Rightarrow 2^x=3^y$ ...(i)

$\Rightarrow 3^{x-1}=2^{y+1}$ $\Rightarrow \frac{3^x}{3}=(2^y).2$

$\Rightarrow 3^x=6.(2^y)$ ...(ii)

Multiplying (i) and (ii)

$6^x=6.6^y$ $\Rightarrow 6^{x-y}=6^1$ $\Rightarrow x-y=1$

14. Ans. (A,B)

$\left(\frac{1}{3}\right)^x$ is positive $\forall x \in R$

$\Rightarrow \left(\frac{1}{3}\right)^x \geq -1$ $\forall x \in R$

$\left(\frac{1}{3}\right)^x \leq 2$

$\Rightarrow 3^{-x}<2$

Taking log_3 both sides,

$\Rightarrow \log_3(3^{-x})<\log_3 2$ $\Rightarrow -x<\log_3 2$ $\Rightarrow x>-\log_3 2$

15. Ans. (B,C)

(A) $\log_2 3$ is greater than

$\log_{12} 10$ is less than

(B) $\log_6 5$ is less than

$\log_7 8$ is greater than

(C) $\log_3 26 < \log_3 27$

$\Rightarrow \log_3 26 < 3$

$\Rightarrow \log_2 9 > \log_2 8$

$\Rightarrow \log_2 9 > 3$

(D) $\log_{16} 15$ is less than

$\log_{10} 11$ is greater than

SECTION–II

16. Ans. (B)

$\log_{50} 30 = \frac{\log_{30}}{\log_{50}}$ $\qquad \frac{\log 5}{\log 2} = a$ ...(1)

$= \frac{\log 3 + \log 2 + \log 5}{\log 5 + \log 2 + \log 5}$ $\qquad \frac{\log 3}{\log 5} = b$ ...(2)

$= \frac{\log_2 3 + 1 + \log_2 5}{\log_2 5 + 1 + \log_2 5}$ $\qquad$ from (1) and (2)

$= \frac{ab + 1 + a}{2a + 1}$ $\qquad \log_2 3 = ab$

17. Ans. (B)

$$(\log_{10} 2 \log_{10} 5)x^2 - (\log_{10} 5 + \log_{10} 15 \log_{10} 2)x + \log_{10} 15 = 0$$

$$\Rightarrow \left(\frac{\log_2 2}{\log_2 10} \cdot \frac{\log_3 5}{\log_3 10}\right)x^2 - \left(\frac{\log_3 5}{\log_3 10} + \frac{\log_3 15}{\log_3 10}\frac{\log_2 2}{\log_2 10}\right)x + \frac{\log_3 15}{\log_3 10} = 0$$

$$\Rightarrow \left(\frac{\log_3 5}{\log_2 10 \log_3 10}\right)x^2 - \left(\frac{\log_3 5 \cdot \log_2 10 + \log_3 15}{\log_3 10 \log_2 10}\right)x + \frac{\log_3 15}{\log_3 10} = 0$$

$$\Rightarrow (\log_3 5)x^2 - (\log_3 5 \log_2 10 + \log_3 5 + 1)x + (\log_3 5 + 1)\log_2 10 = 0$$

$$\Rightarrow \frac{x^2}{\log_5 3} - \left(\frac{\log_2 2 + \log_2 5}{\log_5 3} + \frac{1}{\log_5 3} + 1\right)x + \left(\frac{1}{\log_5 3} + 1\right)(1 + \log_2 5) = 0$$

$$\Rightarrow \frac{x^2}{b} - \left(\frac{1+a}{b} + \frac{1}{b} + 1\right)x + \left(\frac{1}{b} + 1\right)(1+a) = 0$$

$$\Rightarrow x^2 - ((1+a) + (1+b))x + (1+b)(1+a) = 0$$

18. **Ans. (A,B)**

$\log_5 N = I_1 + f_1$

$I_1 = 2$

$\Rightarrow 0 \leq f_1 < 1$ $\quad \Rightarrow 2 \leq I_1 + f_1 < 3$ $\quad \Rightarrow 2 \leq \log_5 N < 3$

$\Rightarrow 25 \leq N < 125$ ...(1)

$\log_3 N = I_2 + f_2$, $I_2 = 3$

$\Rightarrow 0 \leq f_2 < 1$ $\quad \Rightarrow 3 \leq I_2 + f_2 < 4$ $\quad \Rightarrow 3 \leq \log_3 N < 4$

$\Rightarrow 3 \leq \log_3 N < 4$

$\Rightarrow 27 \leq N < 81$ ...(2)

Taking intersection of (1) & (2)

We get,

$27 \leq N < 81$

Maximum integral value of N = 80

So, required values of N is 79, 80.

19. **Ans. (A,D)**

$\log_5 N = I_1 + f_1$

$I_1 = 3$

$\Rightarrow 0 \leq f_1 < 1$ $\quad \Rightarrow 3 \leq I_1 + f_1 < 4$ $\quad \Rightarrow 3 \leq \log_5 N < 4$

$\Rightarrow 125 \leq N < 625$ ...(1)

$\log_3 N = I_2 + f_2, I_2 = 4$

$\Rightarrow 0 \leq f_2 < 1$ $\quad \Rightarrow 4 \leq I_2 + f_2 < 5$ $\quad \Rightarrow 4 \leq \log_3 N < 5$

$\Rightarrow 81 \leq N < 243$ ...(2)

Taking intersection of (1) & (2),

We get,

$125 \leq N < 243$

Number of possible integral value of $N = (243 - 125) = 118$

So, required value of $N = 118, 119$.

20. **Ans. (12.00)**

Let

$\log_{225} x = p \log_{64} y = q$

$p + q = 4$ and $\frac{1}{p} - \frac{1}{q} = 1$

$p + q = 4$ $\quad q - p = pq$

$q = 4 - p$ $\quad 4 - p - p = p(4 - p)$

$4 - 2p = 4p - p^2$

$p^2 - 6p + 4 = 0 \begin{cases} p_1 \\ p_2 \end{cases}$

$p_1 + p_2 = 6$

$\log_{225} x_1 + \log_{225} x_2 = 6$

$\log_{225} x_1 x_2 = 6 \Rightarrow x_1 x_2 = (225)^6$

$q - p = pq$

$q - (4 - q) = (4 - q)(q)$

$q - 4 + q = 4q - q^2$

$q^2 - 2q - 4 = 0 \begin{cases} q_1 \\ q_2 \end{cases}$ $\qquad q_1 + q_2 = 2$

$\log_{64} y_1 + \log_{64} y_2 = 2$

$y_1 y_2 = (64)^2$

$\log_{30}(x_1 x_2 y_1 y_2) \Rightarrow \log_{30}((15)^{12}(2^6)^2) \Rightarrow \log_{30}(30)^{12} \Rightarrow 12$

21. Ans. (18.00)

$x_1 y_1 x_2 y_2$

$\Rightarrow (225)^6(64)^2 \qquad \Rightarrow (30)^{12}$

$\log_{10}(30)^{12} \Rightarrow 12(\log_{10} 3 + \log_{10} 10)$

$\Rightarrow 12(0.477 + 1) \qquad \Rightarrow 12(1.4771) \qquad \Rightarrow 17.72$

Ans. 18

SECTION–III

22. Ans. (D)

(P) $2\ell n\left(2^x - 5\right) = \ell n 2 + \ell n\left(2^x - \frac{7}{2}\right)$

$\ell n\left(2^x - 5\right)^2 - \ell n\left(2^x - \frac{7}{2}\right) = \ell n 2 \quad \Rightarrow \quad \ell n\left(\frac{\left(2^x - 5\right)^2}{2^x - \frac{7}{2}}\right) = \ell n 2$

$\Rightarrow \left(2^x - 5\right)^2 = 2\left(2^x - \frac{7}{2}\right)$ $\qquad$ Let $2^x = t$

$\left(t - 5\right)^2 = 2\left(t - \frac{7}{2}\right)$

$t^2 + 25 - 10t = 2t - 7$

$t^2 - 12t + 32 = 0$

$t = 4, 8$

$2^x = 4, 8 \Rightarrow x = 2, 3$

$x = 2$ will be rejected because

$\ell n(2^x - 5) \Rightarrow \ell n\,(2^2 - 5)$

$\ell n(-1)$ not possible

(Q) Take log b are 10 both sides

$\log_{10}\left(\frac{5x}{2}\right) \cdot \log_{10} x = \log_{10} x \cdot \log_{10} 10$

$\left(\log_{10} 5x - \log_{10} 2\right)\log_{10} x = \log_{10} x$

$\log_{10} x\left(\log_{10} 5x - \log_{10} 2 - 1\right) = 0$

$\log_{10} x = 0 \qquad \log_{10} 5x - \log_{10} 2 - 1 = 0$

$x = 1 \qquad \log_{10} 5x = \log_{10} 2 + \log_{10} 10$

$\log_{10} 5x = \log_{10} 20$

$5x = 20$

$\boxed{x = 4}$

$\Rightarrow$ sum $\Rightarrow 1 + 4 \Rightarrow 5$

(R) $\alpha = 2^{\frac{\log_3(\log_3 5)}{\log_3 2}}$

$$\alpha = 2^{\frac{\frac{\log(\log_3 5)}{\log 3}}{\frac{\log 2}{\log 3}}}$$

$\alpha = 2^{\log_2(\log_3 5)}$

$\alpha = \log_3 5$

$\alpha = \log_3 5$

$\beta^{\alpha} = 9 \Rightarrow \beta^{\log_3 5} = 9$

$(\log_3 5)(\log_3 \beta) = \log_3 9$

$\log_3 \beta = \frac{2}{\log_3 5}$

$\log_3 \beta = 2\log_5 3$

$\beta = 3^{\log_5 9}$

$\beta^{\alpha^2} = \left(3^{\log_5 9}\right)^{(\log_3 5)^2} \Rightarrow 3^{(\log_5 9)\cdot\log_3 5\cdot\log_3 5}$

$\Rightarrow 3^{2(\log_5 3)(\log_3 5)(\log_3 5)}$

$\Rightarrow 3^{2\ \log_3 5} \quad \Rightarrow 3^{\log_3 (5)^2} \quad \Rightarrow 25$

$\frac{1}{5}\left(\beta^{\alpha^2}\right) \Rightarrow 5$

(S) $\log_{\sqrt{3}}\left(\sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}\right)$

Let $\alpha = \sqrt{6+\sqrt{6+\sqrt{6+...\infty}}}$

$\alpha = \sqrt{6+\alpha}$

$\alpha^2 = 6 + \alpha$

$\alpha^2 - \alpha - 6 = 0$ (α cannot be negative)

$\boxed{\alpha = 3}$

$\log_{\sqrt{3}} \alpha \Rightarrow \log_{\sqrt{3}} 3 = 2$

23. Ans. (A)

(P) $(25)^{\frac{2x^2-5x-9}{4}} = \left(\sqrt{11}\right)^{3\log_{11} 5}$

$(25)^{\frac{2x^2-5x-9}{4}} = (5)^{\frac{3}{2}}$

$(5)^{\frac{2x^2-5x-9}{2}} = (5)^{\frac{3}{2}}$

$2x^2 - 5x - 9 = 3$

$2x^2 - 5x - 12 = 0$

$2x^2 - 8x + 3x - 12 = 0$

$2 \times (x-4) + 3(x-4) = 0 \Rightarrow x = -\frac{3}{2} \quad x = 4$

(R) $\sqrt{\log^2_{\frac{1}{2}} 32} \Rightarrow \sqrt{(-\log_2 32)^2}$

$\Rightarrow \sqrt{(-\log_2 2^5)^2} \Rightarrow \sqrt{(-5)^2} \Rightarrow 5$

(S) $\log(\log 9) - \log(\log 3) + \log(\log 49) - \log(\log 7) = \log k$

$\Rightarrow \log\left(\frac{\log 9}{\log 3}\right) + \log\left(\frac{\log 49}{\log 7}\right) = \log k$

$\Rightarrow \log\left(\frac{2\log 3}{\log 3}\right) + \log\left(\frac{2\log 7}{\log 7}\right) = \log k$

$\log 2 + \log 2 = \log k \Rightarrow \log 4 = \log k$

$\boxed{k = 4}$ $3k \Rightarrow 12$ Ans.

(Q) $\sqrt{|x-4|^{x-2}} = \sqrt[4]{|x-4|^{x+2}}$

$\Rightarrow |x-4|^{\frac{x-2}{2}} = |x-4|^{\frac{x+2}{4}} \Rightarrow a^x = a^y$; $a > 0$ & $a \neq 1$

Case-I

$\Rightarrow x = y$ $\Rightarrow \frac{x-2}{2} = \frac{x+2}{4}$

$\Rightarrow 2x - 4 = x + 2$ $\Rightarrow x = 6$

Case-II

$a = 0$ & $x, y > 0$

$\Rightarrow |x-4| = 0$; at $x = 4$

$\Rightarrow x = 4$; $\frac{x-2}{2} = \frac{4-2}{2} = 1 > 0$

; $\frac{x+2}{4} = \frac{6}{4} = \frac{3}{2} > 0$

Case-III

$|x-4| = 1$

$\Rightarrow x - 4 = \pm 1$ $\Rightarrow x - 4 = 1, x = 5$

$\Rightarrow x - 4 = -1, x = 3$

Sum of values of x = 6 + 5 + 3 + 4 = 18.

SECTION–IV

24. Ans. (A → r; B → s; C → p; D → q)

(A) $x = 0.\bar{6}$

$\Rightarrow x = 0.6\bar{6}$ $\Rightarrow x = 0.6 + 0.0\bar{6}$ $\Rightarrow 10x = 6 + 0.\bar{6}$

$\Rightarrow 10x = 6 + x$ $\Rightarrow 9x = 6$ $\Rightarrow x = \frac{2}{3}$

Now, antilog of $\left(0.\bar{6} = \frac{2}{3}\right)$ to the base 27

$= (27)^{0.\bar{6}}$

$= (27)^{\frac{2}{3}} = 3^{3 \times \frac{2}{3}} = 9$

(B) $\log_2 2008 = \log_2(251 \times 8)$

$= \log_2 251 + 3$

We know that,

$128 < 251 < 256$

$\Rightarrow \log_2 128 < \log_2 251 < \log_2 256$

$\Rightarrow 7 < \log_2 251 < 8$

$\Rightarrow 10 < \log_2 251 + 3 < 11$

$\Rightarrow [\log_2 2008] = [3 + \log_2 251] = 10$

Characteristic should be 10.

(C) $\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$

$\Rightarrow 4\log_b 5 \times \log_e 2 = 4\log_{10} 2 \times \log_e 10 \quad \Rightarrow \log_e 2 \times \log_b 5 = \frac{\log 2}{\log 10} \times \frac{\log 10}{\log e}$

$\Rightarrow \log_e 2 \times \log_b 5 = \frac{\log 2}{\log e} = \log_e 2 \quad \Rightarrow \log_b 5 = 1 \Rightarrow b = 5$

(D) Let, $N = \left(\frac{5}{6}\right)^{100}$

$\Rightarrow \log_{10} N = 100\log_{10}\left(\frac{5}{6}\right)$

$= 100[\log_{10} 5 - \log_{10} 6]$

$= 100[0.6989 - 0.3010 - 0.4771]$

$= (-0.0792) \times 100$

$\Rightarrow \log_{10} N = -7.92$

$= -7.92 + 8 - 8$

$= -8 + 0.08$

$\text{Char}_{10} N = -8$

Number of noughts after decimal = $|\text{Char}_{10} N + 1| = |-8 + 1| = 7$.

Exercise - IV (JEE Advanced PYQs)

1. **Ans. (B)**

$\frac{1}{2} log_2(x - 1) = log_2(x - 3) \Rightarrow \sqrt{x - 1} = x - 3$

$(x - 1) = x^2 - 6x + 9 \quad \Rightarrow x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0 \quad$ but $x \neq 2$

$\therefore x = 5$

2. Ans. (C)

$(2x)^{\ell n 2} = (3y)^{\ell n 3}$

$\Rightarrow \ell n 2 \ \ \ell n(2x) = \ell n 3 \, \ell n(3y) \ \ = \ell n 3 \ (\ell n 3 + \ell ny)$ ...(1)

also $3^{\ell n x} = 2^{\ell n y}$

$\Rightarrow \ell n x \, \ell n 3 = \ell n y \, \ell n 2$ ...(2)

by (1) $\Rightarrow \ \ell n 2 \, \ell n(2x) \ = \ell n 3 \ (\ell n 3 + \ell n y) \Rightarrow \ell n 2 . \ell n(2x) = \ell n 3 \left\{\ell n3 + \frac{\ell nx \ell n3}{\ell n2}\right\}$

$\Rightarrow \ell n^2 2 \, \ell n2x = \ell n^2 3 \, (\ell n2 + \ell nx) \Rightarrow (\ell n^2 2 - \ell n^2 3)\, (\ell n2x) = 0 \Rightarrow \ell n2x = 0 \ \Rightarrow \ x = \frac{1}{2}$

3. Ans. (4.00)

Let $\sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \ldots\ldots}} = t \Rightarrow \sqrt{4 - \frac{1}{3\sqrt{2}} t} = t \Rightarrow 4 - \frac{1}{3\sqrt{2}} t = t^2$

$\Rightarrow t^2 + \frac{1}{3\sqrt{2}} t - 4 = 0 \ \Rightarrow \ 3\sqrt{2} t^2 + t - 12\sqrt{2} = 0 \ \Rightarrow t = \frac{-1 \pm \sqrt{1 + 4 \times 3\sqrt{2} \times 12\sqrt{2}}}{2 \times 3\sqrt{2}} = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$t = \frac{16}{6\sqrt{2}}, \frac{-18}{6\sqrt{2}} \Rightarrow t = \frac{8}{3\sqrt{2}}, \frac{-3}{\sqrt{2}}$ and $\frac{-3}{\sqrt{2}}$ is rejected

so $6 + log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}}\right) = 6 + log_{3/2}\left(\frac{4}{9}\right) = 6 + log_{3/2}\left(\left(\frac{2}{3}\right)^2\right) = 6 - 2 = 4$

4. Ans. (A,B,C)

$3^x = 4^{x-1} \Rightarrow \ x = (x-1)\, log_3 4 \ \Rightarrow \ x(1 - 2\, log_3 2) = -2\, log_3 2$

$x = \frac{2\, log_3 2}{2\, log_3 2 - 1}$

Again $x\, log_2 3 = (x-1) \cdot 2 \quad \Rightarrow x(log_2 3 - 2) \ = \ -2 \ \Rightarrow \ x = \frac{2}{2 - log_2 3}$

$x = \frac{1}{1 - \frac{1}{2} log_2 3} = \frac{1}{1 - log_4 3}$

5. Ans. (8.00)

$log_2 9^{\frac{2}{log_2 (log_2 9)}} \times 7^{\frac{1/2}{log_4 7}}$

$= (log_2 9)^{2 log^2_{log_2 9}} \times 7^{\frac{1}{2} log_7 4}$

$= 4 \times 2 = 8$

6. Ans. (1.00)

$x^{16(\log_5 x)^3 - 68 \log_5 x} = 5^{-16}$

Take log to the base 5 on both sides and put $\log_5 x = t$

$16t^4 - 68t^2 + 16 = 0 \quad \Rightarrow 4t^4 - 17t^2 + 4 = 0 \begin{cases} t_1 \\ t_2 \\ t_3 \\ t_4 \end{cases}$

$t_1 + t_2 + t_3 + t_4 = 0$

$\log_5 x_1 + \log_5 x_2 + \log_5 x_3 + \log_5 x_4 = 0$

$x_1 x_2 x_3 x_4 = 1$