Limits

SOLUTIONS

EXERCISE - O

1. **Ans. (D)**

$$\lim_{x\to a}\frac{2x-\sqrt{x^2+3a^2}}{\sqrt{x+a}-\sqrt{2a}}=\sqrt{2}$$

$x = a + h$

$$\Rightarrow \lim_{h\to 0}\frac{2(a+h)-\sqrt{(a+h)^2+3a^2}}{\sqrt{a+h+a}-\sqrt{2a}}=\sqrt{2}\Rightarrow \lim_{h\to 0}\frac{2a+2h-\sqrt{4a^2+h^2+2ah}}{\sqrt{2a+h}-\sqrt{2a}}=\sqrt{2}$$

$$\Rightarrow \lim_{h\to 0}\frac{2a+2h-2a\sqrt{1+\frac{h^2}{4a^2}+\frac{h}{2a}}}{\sqrt{2a}\sqrt{1+\frac{h}{2a}}-\sqrt{2a}}=\sqrt{2}\Rightarrow \lim_{h\to 0}\frac{2a+2h-2a-2a\left(\frac{1}{2}\right)\left(\frac{h^2}{4a^2}+\frac{h}{2a}\right)}{\sqrt{2a}\left(1+\frac{1}{2}\cdot\frac{h}{2a}\right)-\sqrt{2a}}=\sqrt{2}$$

$$\Rightarrow \lim_{h\to 0}\frac{2h-\frac{h^2}{4a}-\frac{h}{2}}{\sqrt{2}.\frac{h}{4.\sqrt{a}}}=\sqrt{2}\Rightarrow \lim_{h\to 0}\frac{\left(\frac{3}{2}-\frac{h}{4a}\right)h}{\frac{h}{2\sqrt{2}\sqrt{a}}}=\sqrt{2}\Rightarrow\frac{\frac{3}{2}}{\frac{1}{2\sqrt{2}\sqrt{a}}}=\sqrt{2}$$

$$\Rightarrow\frac{3\left(2\sqrt{2}\right)\sqrt{a}}{2}=\sqrt{2}\Rightarrow\sqrt{a}=\frac{1}{3}$$

$$\Rightarrow a=\frac{1}{9}$$

2. **Ans. (A)**

$$\lim_{n\to\infty}\left(\sqrt{2n^2+n}-\lambda\sqrt{2n^2-n}\right)=\frac{1}{\sqrt{2}}\qquad\qquad(\infty-\infty)$$

Multiply in N^r and D^r by $\sqrt{2n^2+n}+\lambda\sqrt{2n^2-n}$

$$\Rightarrow \lim_{n\to\infty}\frac{\left(\sqrt{2n^2+n}-\lambda\sqrt{2n^2-n}\right)\left(\sqrt{2n^2+n}+\lambda\sqrt{2n^2-n}\right)}{\left(\sqrt{2n^2+n}+\lambda\sqrt{2n^2-n}\right)}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \lim_{n\to\infty}\frac{\left(2n^2+n\right)-\lambda^2\left(2n^2-n\right)}{\sqrt{2n^2+n}+\lambda\sqrt{2n^2-n}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \lim_{n\to\infty}\frac{\left(2-2\lambda^2\right)n^2+\left(1+\lambda^2\right)n}{|n|\left(\sqrt{2+\frac{1}{n}}+\lambda\sqrt{2-\frac{1}{n}}\right)}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \lim_{n\to\infty}\frac{\left(2-2\lambda^2\right)n^2+\left(1+\lambda^2\right)n}{n\left(\sqrt{2+\frac{1}{n}}+\lambda\sqrt{2-\frac{1}{n}}\right)}=\frac{1}{\sqrt{2}}$$

$2-2\lambda^2=0$	$\dfrac{1+\lambda^2}{\sqrt{2}+\lambda\sqrt{2}}=\dfrac{1}{\sqrt{2}}$
$\Rightarrow 2\lambda^2 = 2$	$1+\lambda^2=1+\lambda$
$\Rightarrow \lambda^2 = 1$	$\lambda^2 = \lambda$
$\Rightarrow \lambda = \pm 1$	$\lambda^2 - \lambda = 0$
	$\lambda(\lambda - 1) = 0$
	$\lambda = 0, \lambda = 1$

Common value of $\lambda = 1$

3. Ans. (B)

$a_n=\sum_{k=1}^{n}2k$	$b_n=\sum_{k=1}^{n}(2k-1)$
$a_n=2\sum_{k=1}^{n}k$	$b_n=2\sum_{k=1}^{n}k-\sum_{k=1}^{n}1$
$a_n=\dfrac{(2)(n)(n+1)}{2}$	$b_n=\dfrac{(2)(n)(n+1)}{2}-(n)$
$a_n=(n)(n+1)$	$b_n=n^2$

$$\Rightarrow \lim_{n\to\infty}\left(\sqrt{a_n}-\sqrt{b_n}\right)$$

$$\Rightarrow \lim_{n\to\infty}\left(\sqrt{(n)(n+1)}-\sqrt{n^2}\right)$$

$$=\lim_{n\to\infty}\frac{\left(\sqrt{(n)(n+1)}-\sqrt{n^2}\right)\times\left(\sqrt{(n)(n+1)}+\sqrt{n^2}\right)}{\left(\sqrt{(n)(n+1)}+\sqrt{n^2}\right)}$$

$$=\lim_{n\to\infty}\frac{n^2+n-n^2}{\sqrt{(n)(n+1)}+\sqrt{n^2}}=\lim_{n\to\infty}\frac{n}{|n|\left(\sqrt{1+\frac{1}{n}}+\sqrt{1}\right)}=\frac{1}{2}$$

4. Ans. (C)

$$\Rightarrow \lim_{x\to-1}\frac{\cos 2-\cos 2x}{x^2-|x|}$$

$$\Rightarrow \lim_{x\to-1}\frac{-2\sin\left(\frac{2+2x}{2}\right)\sin\left(\frac{2-2x}{2}\right)}{|x|^2-|x|}$$

$$\Rightarrow \lim_{x\to-1}\frac{-2\sin(x+1)\sin(1-x)}{|x|(|x|-1)}$$

$x=-1+h$

$$\Rightarrow \lim_{h\to0}\frac{-2\sin(-1+h+1)\sin(1+1-h)}{|-1+h|(|-1+h|-1)}$$

$$\Rightarrow \lim_{h\to0}\frac{-2\sin h\sin(2-h)}{|-1+h|(1-h-1)}$$

$\Rightarrow 2\sin 2$

5. Ans. (A)

$$\lim_{x\to 0}\frac{|\cos(\sin(3x))|-1}{x^2}$$

Modulus will open with positive sign because cos is positive at neighbourhood of $x = 0$

$$\lim_{x\to 0}\frac{\cos(\sin 3x)-1}{x^2} \qquad \left(\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right)$$

$$\lim_{x\to 0}\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{x^2\cdot\left(\frac{\sin 3x}{2}\right)^2}\cdot\left(\frac{\sin 3x}{2}\right)^2$$

$$-\lim_{x\to 0}\left(\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{\left(\frac{\sin 3x}{2}\right)^2}\right)\cdot\frac{\left(\frac{\sin 3x}{2}\right)^2}{x^2} = -\lim_{x\to 0}2\cdot\frac{\left(\frac{\sin 3x}{2}\right)^2}{\left(\frac{3x}{2}\right)^2}\cdot\left(\frac{3x}{2}\right)^2$$

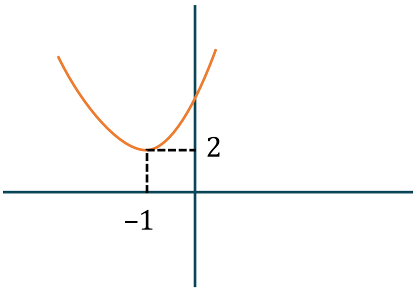
$$-\lim_{x\to 0}\frac{2\sin^2(3x)}{4\cdot(3x)^2}\cdot 9 \qquad \left(\because \lim_{x\to 0}\frac{\sin 3x}{3x}=1\right)$$

$$\frac{-2\times 9}{4} \Rightarrow -\frac{9}{2}$$

6. Ans. (C)

$$a=\min\{x^2+2x+3,\ x\in\mathbb{R}\}$$

$$a=2$$



$$b=\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$$

$$b=\frac{1}{2}$$

$$\Rightarrow \sum_{r=0}^{n}(2)^r\left(\frac{1}{2}\right)^{n-r}$$

$$=\sum_{r=0}^{n}(2)^r(2)^{r-n} = \frac{1}{2^n}\sum_{r=0}^{n}(2)^{2r}$$

$$=\frac{1}{2^n}\sum_{r=0}^{n}(4)^r = \frac{4^{n+1}-1}{2^n(4-1)} = \frac{4^{n+1}-1}{2^n(3)}$$

7. Ans. (B)

$$\lim_{x\to 0}(\cos x + a\sin bx)^{\frac{1}{x}} \qquad (1)^{\infty} \text{ form}$$

$$e^{\lim\limits_{x\to 0}\frac{1}{x}(\cos x + a\sin bx - 1)}$$

$$= e^{\lim\limits_{x\to 0}\frac{\left(1-\frac{x^2}{2!}\right)+a\left(bx-\frac{b^3x^3}{3!}\right)-1}{x}}$$

$$= e^{\lim\limits_{x\to 0}\frac{-\frac{x^2}{2}+abx-\frac{ab^3x^3}{6}}{x}}$$

$$= e^{\lim\limits_{x\to 0}-\frac{x}{2}+ab-\frac{ab^3x^2}{6}}$$

$$= e^{ab}$$

8. Ans. (A)

$$\lim_{x\to\infty}\left(\frac{1^{1/x}+2^{1/x}+3^{1/x}+.....+n^{1/x}}{n}\right)^{nx} \qquad (1)^{\infty} \text{ form}$$

$$= e^{\lim\limits_{x\to\infty} nx\left(\frac{1^{1/x}+2^{1/x}+...n^{1/x}}{n}-1\right)}$$

$$= e^{\lim\limits_{x\to\infty} nx\left(\frac{1^{1/x}+2^{1/x}+...n^{1/x}-n}{n}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{(1)^{1/x}-1+2^{1/x}-1+3^{1/x}-1+.......n^{1/x}-1}{1/x}\right)}$$

Replace x by $\frac{1}{t}$

$$\Rightarrow e^{\lim\limits_{t\to 0}\left(\frac{(1^t-1)+(2^t-1)+.........+(n^t-1)}{t}\right)}$$

$$= e^{\lim\limits_{t\to 0}\frac{1^t-1}{t}+\lim\limits_{t\to 0}\frac{2^t-1}{t}+..........+\lim\limits_{t\to 0}\frac{n^t-1}{t}}$$

$$= e^{\ell n1+\ell n2+.....+\ell nn}$$

$$= e^{\ell n(n!)}$$

$$= n!$$

9. Ans. (C)

$$\lim_{x\to\lambda}\left(2-\frac{\lambda}{x}\right)^{\lambda\tan\left(\frac{\pi x}{2\lambda}\right)} = \frac{1}{e}$$

$x = \lambda + h$

$$\Rightarrow \lim_{h\to 0}\left[2-\frac{\lambda}{\lambda+h}\right]^{\lambda\tan\left(\frac{\pi}{2}\left(\frac{\lambda+h}{\lambda}\right)\right)} = \frac{1}{e}$$

$$\Rightarrow \lim_{h\to 0}\left[\frac{2\lambda+2h-\lambda}{\lambda+h}\right]^{\lambda\tan\left(\frac{\pi}{2}+\frac{\pi h}{2\lambda}\right)} = \frac{1}{e}$$

$$\Rightarrow \lim_{h\to 0}\left(\frac{\lambda+2h}{\lambda+h}\right)^{-\lambda\cot\left(\frac{\pi h}{2\lambda}\right)} = \frac{1}{e}$$

$$\Rightarrow \lim_{h\to 0} e^{-\left(\frac{\lambda+2h}{\lambda+h}-1\right)\lambda\cot\frac{\pi h}{2\lambda}} = \frac{1}{e}$$

$$\Rightarrow \lim_{h\to 0} e^{\frac{-h}{(\lambda+h)}\times\frac{\lambda}{\tan\left(\frac{\pi h}{2\lambda}\right)}} = e^{-1}$$

$$\Rightarrow \lim_{h\to 0} e^{\frac{-\lambda}{(\lambda+h)}\times\frac{\frac{\pi h}{2\lambda}}{\tan\left(\frac{\pi h}{2\lambda}\right)}\times\frac{1}{\left(\frac{\pi}{2\lambda}\right)}} = e^{-1}$$

$$\Rightarrow e^{\frac{-\lambda}{\lambda}\times\frac{2\lambda}{\pi}} = e^{-1}$$

$$\Rightarrow \frac{-2\lambda}{\pi} = -1$$

$$\Rightarrow \lambda = \frac{\pi}{2}$$

10. Ans. (B)

$\frac{\tan x}{x} > 1$ for $x \to 0$

$\left\{\frac{\tan x}{x}\right\} = \frac{\tan x}{x} - 1$ for $x \to 0$

So, $\left[\frac{\tan x}{x}\right] = 1$

$$\lim_{x\to 0}\left(\left[f(x)\right]+x^2\right)^{\frac{1}{\{f(x)\}}}$$

$$\Rightarrow \lim_{x\to 0}\left(\left[\frac{\tan x}{x}\right]+x^2\right)^{\frac{1}{\{f(x)\}}}$$

$$= \lim_{x\to 0}\left(1+x^2\right)^{\frac{1}{\{f(x)\}}} \qquad = \lim_{x\to 0} e^{\left(1+x^2-1\right)\times\frac{1}{\{f(x)\}}}$$

$$= \lim_{x\to 0} e^{x^2\times\frac{1}{\left(\frac{\tan x}{x}-1\right)}} \qquad = \lim_{x\to 0} e^{x^2\left(\frac{x}{\tan x - x}\right)}$$

$$= \lim_{x\to 0}\frac{x^3}{e^{\tan x - x}}$$

$$= \lim_{x\to 0} e^{\frac{x^3}{\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+....\right)-x}}$$

$$= \lim_{x\to 0} e^{\frac{x^3}{\frac{x^3}{3}+\frac{2x^5}{15}+...}}$$

$$= \lim_{x\to 0} e^{\frac{1}{\frac{1}{3}+\frac{2x^2}{15}+...}}$$

$$= e^{\frac{1}{\frac{1}{3}+0}} = e^3$$

11. Ans. (D)

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$$

$$\Rightarrow \lim_{x\to 0}\frac{x+ax\left(1-\frac{x^2}{2!}\right)-b\left(x-\frac{x^3}{3!}\right)}{x^3}$$

$$\Rightarrow \lim_{x\to 0}\frac{x+ax-\frac{ax^3}{2}-bx+\frac{bx^3}{6}}{x^3}$$

$$\Rightarrow \lim_{x\to 0}\frac{x(1+a-b)+x^3\left(\frac{b}{6}-\frac{a}{2}\right)}{x^3}$$

Hence,

$1 + a - b = 0$ ...(1)

and $\frac{b}{6}-\frac{a}{2}=1$

$b - 3a = 6$ ...(2)

Add (1) & (2);

$\Rightarrow 1+a-3a=6$

$\Rightarrow a=-\frac{5}{2}$ and $1+a-b=0 \Rightarrow 1-\frac{5}{2}=b \Rightarrow b=-\frac{3}{2}$

12. Ans. (A,B,C)

$$f(x)=\begin{cases}\sin x, & x\in I\\ 0, & \text{other wise}\end{cases}$$

$$g(x)=\begin{cases}(x^2+1), & x\neq 0,2\\ 4 & x=0\\ 5 & x=2\end{cases}$$

(A) $\lim_{x\to 0} g(f(x))$

$\lim_{x\to 0^+} g(f(0^+))\Rightarrow \lim_{x\to 0^+} g(0)=4$

$\lim_{x\to 0^-} g(f(0^-))\Rightarrow \lim_{x\to 0^+} g(0)=4$

(B) $\lim_{x\to 0} f(g(x))$

$\lim_{x\to 0^+} f(g(0^+))\Rightarrow \lim_{x\to 0^+} f(1^+)\Rightarrow 0$

$\lim_{x\to 0^-} f(g(0^-))\Rightarrow \lim_{x\to 0^+} f(1^+)\Rightarrow 0$

(C) $\lim_{x\to 1^+} f(g(1^+))\Rightarrow \lim_{x\to 1^+} f(2^+)\Rightarrow 0$

$\lim_{x\to 1^-} f(g(1^-))\Rightarrow \lim_{x\to 1^+} f(1^+)\Rightarrow 0$

(D) $\lim_{x\to 1^+} g(f(1^+))=g(0)=4$

$\lim_{x\to 1^-} g(f(1^-))=g(0)=4$

13. **Ans. (B,C)**

LHL

$$\lim_{x\to 0^-}\frac{\sin^{-1}\left(\sin(-h)\right)}{\cos^{-1}\left(\cos(-h)\right)}$$

$$\lim_{h\to 0}\frac{-h}{h}=-1$$

RHL

$$\lim_{x\to 0^+}\frac{\sin^{-1}\left(\sin(h)\right)}{\cos^{-1}\left(\cos(h)\right)}$$

$$\lim_{x\to 0^+}\frac{h}{h}=1\ ,\ \text{LHL}\neq\text{RHL}$$

A does not exist

$$B=\lim_{x\to 0^+}\frac{[|x|]}{x}=\frac{0}{0^+}=0$$

$$\lim_{x\to 0^-}\frac{[|-x|]}{x}=\lim_{x\to 0^-}\frac{0}{0^-}=0$$

B will exist and equal to zero

14. **Ans. (A,B)**

$$\lim_{x\to 1}\frac{e^{x-1}-x}{(x-1)^n}$$

Put $x\to 1+h$

$$\lim_{h\to 0}\frac{e^{1+h-1}-(1+h)}{(1+h-1)^n}$$

$$\lim_{h\to 0}\frac{e^h-1-h}{(h)^n}$$

$$\lim_{h\to 0}\frac{\left(1+\frac{h}{\lfloor 1}+\frac{h^2}{\lfloor 2}......\right)-1-h}{h^n}$$

$$\lim_{h\to 0}\frac{\left(\frac{h^2}{\lfloor 2}+\frac{h^3}{\lfloor 3}....\right)}{h^n}$$

$$\lim_{h\to 0}\frac{h^2\left(\frac{1}{2}+\frac{h}{\lfloor 3}......\right)}{h^n}$$

$$\lim_{h\to 0}h^{2-n}\left(\frac{1}{2}+\frac{h}{\lfloor 3}.....\right)$$

$2-n\geq 0$

$n\leq 2$

$n=1,2$

15. Ans. (B,C)

(A) $\lim\limits_{x\to 3^+}[x]-[2x-1]$ $\qquad [[x]]=[x]$

$\lim\limits_{x\to 3^+}[x]-[2x]+1$ $\qquad [x+I]=[x]+I$

3 – 6 + 1 = –2

$\lim\limits_{x\to 3^-}[x]-[2x]+1$

2 – 5 + 1 = - 2

(B) $\lim\limits_{x\to 1}[x]-x$

$\lim\limits_{x\to 1^+}[x]-x \Rightarrow \lim\limits_{x\to 1^+}(1-x)=0$

$\lim\limits_{x\to 1^-}[x]-x \Rightarrow \lim\limits_{x\to 1^-}(0-x)=-1$

Limit does not exist

(C) $\lim\limits_{x\to 0^+}(x-[x])^2-(-x-[-x])^2$

$\lim\limits_{x\to 0^+}(x-0)^2-(-x+1)^2=-1$

$\lim\limits_{x\to 0^-}(x+1)^2-(-x-0)^2=1$

Limit does not exist

(D) RHL = $\lim\limits_{x\to 0^+}\dfrac{\tan(\text{sgn}(x))}{\text{sgn}(x)}$

$\dfrac{\tan(1)}{1}\Rightarrow \tan(1)$

LHL = $\lim\limits_{x\to 0^-}\dfrac{\tan(-1)}{(-1)}=\tan(1)$

LHL = RHL

16. Ans. (B)

$f(x)=\lim\limits_{n\to\infty}\left(\cos\dfrac{x}{\sqrt{n}}\right)^n$

$\Rightarrow f(x)=\lim\limits_{n\to\infty}\left(\cos\dfrac{x}{\sqrt{n}}\right)^n$ $\qquad (1^\infty)$

$\Rightarrow f(x)=e^{\left(\lim\limits_{n\to\infty} n\left(\cos\frac{x}{\sqrt{n}}-1\right)\right)}$

Replace $n\to\dfrac{1}{n}$

$f(x)=e^{\left(\lim\limits_{n\to 0}\frac{\cos\sqrt{n}x-1}{n}\right)}$

$\Rightarrow e^{\left(\lim\limits_{n\to 0}\frac{1}{n}\left(\frac{\cos\sqrt{n}x-1}{(\sqrt{n}x)^2}\right)(\sqrt{n}x)^2\right)}$

$\Rightarrow e^{-\frac{x^2}{2}}$ $\qquad \left(\because \lim\limits_{f(x)\to 0}\dfrac{\cos f(x)-1}{(f(x))^2}=-\dfrac{1}{2}\right)$

17. Ans. (A)

$$g(x) = -x^{4b}$$

$$b = \lim_{x\to\infty}\left(\frac{x^2+x+1-x^2-1}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}\right)$$

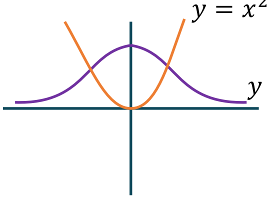
$$b = \lim_{x\to\infty}\left(\frac{x}{x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}\right)$$

$$b = \lim_{x\to\infty}\left(\frac{x}{x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)}\right) = \frac{1}{2}$$

$$g(x) = -x^{4\left(\frac{1}{2}\right)}$$

$$g(x) = -x^2$$

18. Ans. (A)



$$f(x)+g(x)=0$$

$$e^{\frac{-x^2}{2}} - x^2 = 0$$

$$e^{\frac{-x^2}{2}} = x^2$$

No. of solutions = 2

19. Ans. (–1.00)

$$\lim_{x\to 0^+}\frac{\sin x + ae^x + be^{-x} + c\ell n(1+x)}{x^3}$$

$$\lim_{x\to 0^+}\frac{\left(x-\frac{x^3}{\lfloor 3}...\right)+a\left(1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}...\right)+b\left(1-\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}-\frac{x^3}{\lfloor 3}...\right)+c\left(x-\frac{x^2}{2}+\frac{x^3}{3}...\right)}{x^3}$$

$$\lim_{x\to 0^+}\frac{(a+b)+x(1+a-b+c)+x^2\left(\frac{a}{2}+\frac{b}{2}-\frac{c}{2}\right)+x^3\left(-\frac{1}{6}+\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right)......}{x^3}$$

$a + b = 0,$ $\quad 1 + a - b + c = 0,$ $\frac{a}{2}+\frac{b}{2}-\frac{c}{2}=0$

$$a = \frac{-1}{2},\ b = \frac{1}{2},\ c = 0$$

$$a - b + c = -1$$

20. Ans. (–0.33)

$$\lim_{x\to 0^+}\frac{x^3\left(-\frac{1}{6}+\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right)}{x^3}$$

$$\Rightarrow -\frac{1}{6}+\frac{a}{6}-\frac{b}{6}+\frac{c}{3}$$

$$\Rightarrow -0.33$$

21. Ans. (0)

$$\lim_{x\to 0^+} x.f(x) = \frac{a+b-c}{2} = 0$$

22. Ans. (B)

(P) $$\lim_{x\to 0}\frac{\tan[e^2]x^2-\tan[-e^2]x^2}{\sin^2(x)}$$

$$\because\ \tan A+\tan B=\frac{\sin(A+B)}{\cos A\cos B}$$

$$\lim_{x\to 0}\frac{\tan(7x^2)+\tan(8x^2)}{\sin^2 x}$$

$$\lim_{x\to 0}\frac{\sin(7x^2+8x^2)}{\cos(7x^2)\cos(8x^2).\sin^2(x)}$$

$$\lim_{x\to 0}\frac{\sin(15x^2)}{\frac{\sin^2(x)}{x^2}.x^2}\Rightarrow\lim_{x\to 0}\frac{\sin(15x^2)}{15x^2}.15$$

$$\Rightarrow 15$$

(Q) $$\min(t^2+4t+6)$$

$$\min((t+2)^2+2)=2,\ \frac{\sin x}{x}<1$$

$$\lim_{x\to 0}\left[\frac{2\sin x}{x}\right]=1$$

(R) $$\lim_{x\to 0}\frac{(1+x^2)^{\frac{1}{3}}-(1-2x)^{\frac{1}{4}}}{x+x^2}$$

$$\lim_{x\to 0}\frac{\left(1+\frac{1}{3}(x^2)...\right)-\left(1+\frac{1}{4}(-2x)+\frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{\underline{|2}}(4x^2)...\right)}{x+x^2}$$

$$\lim_{x\to 0}\frac{\frac{x}{3}+\frac{1}{2}+\frac{12x}{32}}{1+x}=\frac{1}{2}$$

(S) $\lim_{x\to 0}\frac{2-(1+\cos x)}{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$

$\lim_{x\to 0}\frac{1-\cos x}{(\sin^2 x)(2\sqrt{2})}$

$\lim_{x\to 0}\frac{2\sin^2\left(\frac{x}{2}\right)}{4\sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)(2\sqrt{2})}=\frac{1}{4\sqrt{2}}$

$\Rightarrow \frac{\sqrt{2}}{8}$

23. Ans. (P $\rightarrow$ 1,3; Q $\rightarrow$ 1,3; R $\rightarrow$ 2; S $\rightarrow$ 2)

(P) $\lim_{t\to 0^+} g(t)+\lim_{t\to 2^-} g(t)$

$(-2)+(2)=0$

$\lim_{t\to 2^+} g(t)=0$

1,3 will match

(Q) $\lim_{t\to 0^-} g(t)+g(2)$

$(-1)+(1)=0$

1,3 will match

(R) $\lim_{t\to 0^+} g(t)=-2$

$\lim_{t\to 0^-} g(t)=-1$

2 will match

(S) $\lim_{t\to 2^+} g(t)=0$

$\lim_{t\to 2^-} g(t)=2$

2 will match

24. Ans. (P $\rightarrow$ 3; Q $\rightarrow$ 4; R $\rightarrow$ 1; S $\rightarrow$ 2)

(P) $\frac{1}{2}\lim_{n\to\infty} 2n.\sin\left(\frac{\pi}{4n}\right)\cos\left(\frac{\pi}{4n}\right)$

$\frac{1}{2}\lim_{n\to\infty} n.\frac{\sin\left(\frac{\pi}{2n}\right)}{\frac{\pi}{2n}}.\frac{\pi}{2n}$

$\frac{1}{2}\lim_{n\to\infty}(n)\left(\frac{\pi}{2n}\right)\Rightarrow\frac{\pi}{4}$

3 will match

(Q) $\lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$\lim_{x\to 0}\frac{\sin\left(\frac{\pi x}{180}\right)\left(\frac{\pi}{180}\right)}{\left(\frac{\pi x}{180}\right)} \Rightarrow \frac{\pi}{180}$

4 will match

(R) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)$

$\lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$

$\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$\lim_{x\to 0}\tan\frac{x}{2}=0$

(S) $x\to\frac{\pi}{2}-h$

$\lim_{h\to 0}\frac{1+\cos 2\left(\frac{\pi}{2}-h\right)}{\left(\pi-2\left(\frac{\pi}{2}-h\right)\right)^2}$

$\lim_{h\to 0}\frac{1-\cos 2h}{4h^2}\Rightarrow\lim_{h\to 0}\frac{2\sin^2 h}{4h^2}=\frac{1}{2}$

2 will match

25. Ans. (P→ 2; Q→1; R→2; S→3)

(P) $\lim_{x\to\infty}\frac{2^x}{2^x+1}$

$\lim_{x\to\infty}\frac{2^x}{2^x\left(1+2^{-x}\right)}=1$

2 will match

(Q) $\lim_{x\to 2}\frac{2\sin\left(e^{x-2}-1\right)}{\log(x-1)}$

$\Rightarrow \lim_{x\to 2}\frac{2\sin\left(e^{x-2}-1\right)}{\left(e^{x-2}-1\right)}\times\frac{\left(e^{x-2}-1\right)}{\log(x-1)}$

$\Rightarrow \lim_{x\to 2}\frac{2\left(e^{x-2}-1\right)}{\log(x-1)}$ $\left(\because \lim_{f(x)\to 0}\frac{\sin f(x)}{f(x)}=1\right)$

$$\Rightarrow \lim_{x\to 2}\frac{2\left(e^{x-2}-1\right)}{(x-2)}\frac{(x-2)}{\log(x-1)}$$

$$\Rightarrow \lim_{x\to 2}\frac{2(x-2)}{\log(x-1)} \qquad \left(\because \lim_{f(x)\to 0}\frac{\left(e^{f(x)}-1\right)}{f(x)}=1\right)$$

$$\Rightarrow \lim_{x\to 2}\frac{2(x-2)}{\log(1+x-2)} \qquad \left(\because \lim_{f(x)\to 0}\frac{\log(1+f(x))}{f(x)}=1\right)$$

$$\Rightarrow 2$$

(R) $\lim_{x\to e}\frac{(\ell nx-1)}{x-e}\times e$

$$\Rightarrow \lim_{x\to e}\frac{(\ell nx-\ell ne)}{x-e}\times e$$

$$\Rightarrow \lim_{x\to e}\frac{\ell n\left(\frac{x}{e}\right)}{(x-e)}\times e$$

$$\Rightarrow \lim_{x\to e}\frac{\ell n\left(1+\frac{x}{e}-1\right)}{e\left(\frac{x}{e}-1\right)}\times e=1$$

(S) $\lim_{x\to 0}\frac{x\left(5^x-1\right)}{(1-\cos x)4\ell n5}$

$$\Rightarrow \lim_{x\to 0}\frac{\left(5^x-1\right)}{x}\frac{x^2}{(1-\cos x)4\ell n5}$$

$$\Rightarrow \lim_{x\to 0}\frac{(\ell n5)(2)}{4\ell n5} \qquad \left(\because \lim_{f(x)\to 0}\frac{a^{f(x)}-1}{f(x)}=\ell na\right)$$

$$\Rightarrow \frac{1}{2} \qquad \left(\because \lim_{f(x)\to 0}\frac{1-\cos f(x)}{(f(x))^2}=\frac{1}{2}\right)$$

EXERCISE - S

1. Ans. (100)

$$\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+...+(x+100)^{10}}{x^{10}+10^{10}}$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^{10}\left(1+\frac{1}{x}\right)^{10}+x^{10}\left(1+\frac{2}{x}\right)^{10}+....+x^{10}\left(1+\frac{100}{x}\right)^{10}}{x^{10}\left[1+\frac{10^{10}}{x^{10}}\right]}$$

$$\lim_{x\to\infty}\frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+....+\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left[1+\frac{10^{10}}{x^{10}}\right]}$$

$$=\frac{1+1+...+1}{1+0} \qquad \left\{\because \frac{1}{x}\to 0,\frac{2}{x}\to 0,...,\frac{100}{x}\to 0\right\}$$

$$=100$$

2. Ans. (4)

$$1-\frac{1}{^{k+1}C_2}$$

$$=1-\frac{1}{\left(\frac{(k+1)k}{2!}\right)}$$

$$=1-\frac{2}{(k+1)k}$$

$$=\frac{k^2+k-2}{k(k+1)}$$

$$=\frac{(k+2)(k-1)}{k(k+1)}$$

$$\Rightarrow P_n=\prod_{k=2}^{n}\left(\frac{(k+2)(k-1)}{k(k+1)}\right)$$

$$=\frac{4.1}{2.3}\times\frac{5.2}{3.4}\times\frac{6.3}{4.5}\times.....\times\frac{(n+2)(n-1)}{n(n+1)}$$

$$\Rightarrow P_n=\frac{1}{3}\frac{(n+2)}{n}$$

$$\Rightarrow \lim_{n\to\infty}P_n=\lim_{n\to\infty}\frac{1}{3}\frac{\left(1+\frac{2}{n}\right)}{1}=\frac{1}{3}=\frac{a}{b}$$

$$\Rightarrow a+b=1+3=4$$

3. Ans. (0)

$$\lim_{x\to 0}\left(\left[\frac{-5\sin x}{x}\right]+\left[\frac{6\sin x}{x}\right]\right)$$

$\frac{\sin x}{x}=1^-$ as $x \to 0$

$\Rightarrow \frac{-5\sin x}{x}=-5^-$ as $x \to 0$

And $\frac{6\sin x}{x}=6^-$ as $x \to 0$

$\lim_{x\to 0}\left[-5^-\right]+\left[6^-\right]$

$= -5 + 5$

$=0$

4. Ans. (7)

$$\lim_{x\to 5^+} f(x)=\lim_{x\to 5^+}\frac{\sin\{x\}}{x^2+ax+b}$$

$$\Rightarrow \frac{\sin\{5^+\}}{25+5a+b}=\frac{0}{25+5a+b}$$

But given it is non-zero

$\Rightarrow 25 + 5a + b = 0$ ...(i)

So that we get $\frac{0}{0}$ form

$$\lim_{x\to 3^+} f(x)=\frac{\sin\{3^+\}}{9+3a+b}=\frac{0}{9+3a+b}$$

It is not zero

$\Rightarrow 9 + 3a + b = 0$ ...(ii)

Solving (i) & (ii) we get

$a = -8, b = 15$

$\Rightarrow a + b = 7$

5. Ans. (5)

$$\lim_{n\to\infty}\left(4^n+5^n\right)^{\frac{1}{n}}$$

$$\Rightarrow \lim_{n\to\infty}\left[5^n\left(\frac{4^n}{5^n}+1\right)\right]^{\frac{1}{n}}$$

$$= \lim_{n\to\infty}5\left[1+\left(\frac{4}{5}\right)^n\right]^{\frac{1}{n}}$$

$$= 5e^{\lim_{n\to\infty}\frac{1}{n}\times\left[\left(1+\left(\frac{4}{5}\right)^n\right)-1\right]}$$

$$= 5e^{\lim_{n\to\infty}\frac{1}{n}\times\frac{4^n}{5^n}}$$

$= 5e^{0\times 0}$

$= 5$

6. Ans. (2)

$$\lim_{x\to1}\frac{x^2-x\ell nx+\ell nx-1}{x-1}$$

$$=\lim_{x\to1}\frac{x^2-1-x\ell n+\ell nx}{x-1}$$

$$=\lim_{x\to1}\frac{(x-1)(x+1)-\ell nx(x-1)}{(x-1)}$$

$$=\lim_{x\to1}\frac{(x-1)[x+1-\ell n\,x]}{(x-1)}$$

$$=\lim_{x\to1}(x+1-\ell nx)$$

$=1+1-\ell n1$

$=2$

7. Ans. (1)

$\lim\limits_{x\to0}\ell n(1+\sin^2 x).\cot(\ell n^2(1+x))$

$$\Rightarrow \lim_{x\to0}\frac{\ell n(1+\sin^2 x)}{\sin^2 x}\times\sin^2 x\times\frac{\ell n^2(1+x)}{\tan(\ell n^2(1+x))\times\ell n^2(1+x)}$$

$\{\because \sin^2 x\to 0 \text{and } \ell n(1+x)\to 0 \text{ as } x\to 0\}$

$$=\lim_{x\to0}\frac{\sin^2 x}{\ell n^2(1+x)}$$

$$=\lim_{x\to0}\frac{\sin^2 x}{x^2}\times\left(\frac{x}{\ell n(1+x)}\right)^2$$

$=1\times1=1$ $\quad\left\{\because \lim\limits_{x\to0}\frac{\sin x}{x}=1 \text{ and } \lim\limits_{x\to0}\frac{\ell n(1+x)}{x}=1\right\}$

8. Ans. (167)

$$\lim_{x\to\infty}\frac{a(2x^3-x^2)+b(x^3+5x^2-1)-c(3x^3+x^2)}{a(5x^4-x)-bx^4+c(4x^4+1)+2x^2+5x}=1$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^3(2a+b-3c)+x^2(-a+5b-c)-b}{x^4(5a-b+4c)+2x^2+x(-a+5)+c}$$

Coefficient of x^4 should be 0

Coefficient of x^3 should be 0

$\Rightarrow$ coefficient of x^2 in numerator and denominator is equal.

$5a-b+4c=0$ ...(i)

$2a+b-3c=0$ ...(ii)

$-a+5b-c=2$

$\Rightarrow a-5b+c+2=0$ ...(iii)

Add (i) + (ii)

$7a+c=0\Rightarrow c=-(7a)$

5(ii) + (iii) $\Rightarrow 11a-14c+2=0$

$\Rightarrow\ 11a - 14(-7a) + 2 = 0$

$\Rightarrow 109a = -2$

$\Rightarrow\ a = \frac{-2}{109}$

$\Rightarrow\ c = \frac{14}{109}$

$\Rightarrow\ b = 5\left(\frac{-2}{109}\right) + 4 \times \frac{14}{109}$

$= \frac{46}{109}$

$\Rightarrow\ a + b + c = \frac{-2}{109} + \frac{46}{109} + \frac{14}{109}$

$\Rightarrow\ \frac{58}{109} = \frac{p}{q}$

$\Rightarrow p + q = 58 + 109 = 167$

9. Ans. (4)

$$\lim_{x\to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1} = \lim_{x\to 0} \frac{2^x - 1}{x} . \frac{x}{\sqrt{(1+x)} - 1}$$

$$= \lim_{x\to 0} \frac{2^x - 1}{x} . \frac{x}{(1+x) - 1} . \left(\sqrt{1+x} + 1\right)$$

$= \ 2ln2 = ln4$

10. Ans. (1)

$$L = \lim_{x\to\infty} x^2 \left(\frac{\sqrt{x+2}}{x} - \sqrt[3]{\frac{x+3}{x}} \right)$$

$$= \lim_{x\to\infty} x^2 \left(\left(1 + \frac{2}{x}\right)^{\frac{1}{2}} - \left(1 + \frac{3}{x}\right)^{\frac{1}{3}} \right)$$

$$= \lim_{x\to\infty} x^2 \left[\left(1 + \frac{1}{2}\left(\frac{2}{x}\right) + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{1 \times 2} . \left(\frac{2}{x}\right)^2 + ... \right) - \left(1 + \frac{1}{3} \times \frac{3}{x} + \frac{\frac{1}{3}\left(\frac{1}{3} - 1\right)}{1 \times 2} \left(\frac{3}{x}\right)^2 + ... \right) \right]$$

$$= \lim_{x\to\infty} x^2 \left[\left(1 + \frac{1}{x} - \frac{1}{2x^2} + ..\right) - \left(1 + \frac{1}{x} - \frac{1}{x^2} + ..\right) \right]$$

$$\Rightarrow \lim_{x\to\infty} x^2 \left[\frac{1}{2x^2} + ... \right] = \frac{1}{2}$$

$\Rightarrow 2L = 1$

EXERCISE - JEE (Main) PYQ

1. Ans. (2)

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)$$

$$\lim_{x\to 0^+} x\left(\frac{1+2+3+.....+15}{x}\right)-\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+.....+\left\{\frac{15}{x}\right\}\right) \qquad \because 0\le\left\{\frac{r}{x}\right\}<1$$

$$0\le x\left\{\frac{r}{x}\right\}< x$$

$$\lim_{x\to 0^+} x\left(\frac{1+2+3+......+15}{x}\right) = \frac{15.16}{2} = 120$$

2. Ans. (1)

$$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$$

$$= \lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$=\lim_{y\to 0}\frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}=\frac{1}{4\sqrt{2}}$$

3. Ans. (1)

$$\lim_{x\to 0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$$

$$x\to 0^-$$

$$[x]=-1 \Rightarrow \lim_{x\to 0^-}\frac{x(-x-1)\sin(-1)}{-x}=-\sin 1$$

$$|x|=-x$$

4. Ans. (4)

$$\lim_{x\to 1^+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$= \lim_{x\to 1^+}\frac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\frac{\pi}{2}(-1)\right)$$

$$= \lim_{x\to 1^+}\left(1-\frac{\sin(x-1)}{(x-1)}\right)(-1) = (1-1)(-1) = 0$$

5. Ans. (4)

R.H.L. = $\lim_{x\to 0^+}\dfrac{\tan(\pi\sin^2 x)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$

(as $x \to 0^+ \Rightarrow [x] = 0$)

$= \lim_{x\to 0^+}\dfrac{\tan(\pi\sin^2 x)+x^2}{x^2}$

$= \lim_{x\to 0^+}\dfrac{\tan(\pi\sin^2 x)}{(\pi\sin^2 x)}+1=\pi+1$

L.H.L. = $\lim_{x\to 0^-}\dfrac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

(as $x \to 0^- \Rightarrow [x] = -1$)

$\lim_{x\to 0+}\dfrac{\tan\left(\pi\sin^2 x\right)}{\pi\sin^2 x}\cdot\dfrac{\pi\sin^2 x}{x^2}+\left(-1+\dfrac{\sin x}{x}\right)^2 \Rightarrow \pi$

R.H.L. $\neq$ L.H.L.

6. Ans. (3)

$\lim_{x\to\pi/4}\dfrac{\cot^3 x-\tan x}{\cos\left(x+\dfrac{\pi}{4}\right)}$

$\lim_{x\to\pi/4}\dfrac{(1-\tan^4 x)}{\cos(x+\pi/4)}$

$2\lim_{x\to\pi/4}\dfrac{(1-\tan^2 x)}{\cos(x+\pi/4)}$

$2\lim_{x\to\pi/4}\dfrac{\cos^2 x-\sin^2 x}{\dfrac{\cos x-\sin x}{\sqrt{2}}}\dfrac{1}{\cos^2 x}$

$4\sqrt{2}\lim_{x\to\pi/4}(\cos x+\sin x)=8$

7. Ans. (3)

$\lim_{x\to 1^-}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}\times\dfrac{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}$

$\lim_{x\to 1^-}\dfrac{2\left(\dfrac{\pi}{2}-\sin^{-1}x\right)}{\sqrt{1-x}\cdot\left(\sqrt{\pi}+\sqrt{2\sin^{-1}x}\right)}$

$\lim_{x\to 1^-}\dfrac{2\cos^{-1}x}{\sqrt{1-x}}\cdot\dfrac{1}{2\sqrt{\pi}}$

Put $x = \cos\theta$

$\lim_{\theta\to 0^+}\dfrac{2\theta}{\sqrt{2}\sin\left(\dfrac{\theta}{2}\right)}\cdot\dfrac{1}{2\sqrt{\pi}}=\sqrt{\dfrac{2}{\pi}}$

8. Ans. (2)

$$\lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$$

$$=\frac{(1)^2.\left(2\sqrt{2}\right)}{\frac{1}{2}}=4\sqrt{2}$$

9. Ans. (4)

$$\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$$

$$\Rightarrow \lim_{x\to 1}(x+1)(x^2+1)=\frac{k^2+k^2+k^2}{2k}$$

$\Rightarrow k = 8/3$

10. Ans. (1)

$$\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$$

$1 - a + b = 0$ ...(i)

$2 - a = 5$ ...(ii)

$\Rightarrow a + b = -7.$

11. Ans. (36)

$$\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}\Rightarrow \lim_{x\to 2}\frac{3^{2x}-12.3^x+27}{3^{x/2}-3}$$

$$=\lim_{x\to 2}\frac{(3^x-9)(3^x-3)}{(3^{x/2}-3)}$$

$$=\lim_{x\to 2}\frac{(3^{x/2}+3)(3^{x/2}-3)(3^x-3)}{(3^{x/2}-3)}$$

= 36

12. Ans. (4)

$$e^{\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}-1\right)\frac{1}{x^2}}=e^{\lim\limits_{x\to 0}\left(\frac{-4x^2}{7x^2+2}\right)\frac{1}{x^2}}$$

$$=e^{\lim\limits_{x\to 0}\left(\frac{-4}{7x^2+2}\right)}$$

$$=e^{-2}$$

13. Ans. (40)

$$\lim_{x\to 1}\frac{x+x^2+.......+x^2-n}{x-1}=820$$

$$\Rightarrow \lim_{x\to 1}\left(\frac{x-1}{x-1}+\frac{x^2-1}{x-1}+......\frac{x^n-1}{x-1}\right)=820$$

$\Rightarrow 1 + 2 + \ldots\ldots + n = 820$

$\Rightarrow n(n + 1) = 2 \times 820$

$\Rightarrow n(n + 1) = 40 \times 41$

Since $n \in$ N, so $\boxed{n=40}$

14. Ans. (4)

$$\lim_{x\to 0}\left\{\tan\left(\frac{\pi}{4}+x\right)\right\}^{1/x}$$

$$= e^{\lim_{x\to 0}\frac{1}{x}\left\{\tan\left(\frac{\pi}{4}+x\right)-1\right\}}$$

$$= e^{\lim_{x\to 0}\left(\frac{1+\tan x-1+\tan x}{x(1-\tan x)}\right)}$$

$$= e^{\lim_{x\to 0}\frac{2\tan x}{x(1-\tan x)}}$$

$= e^2$

15. Ans. (1)

$$\lim_{n\to\alpha}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r(r+1)}\right)\right)$$

$$=\lim_{n\to\alpha}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)\right)$$

$$=\tan\left(\lim_{n\to\alpha}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$$

$$=\tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$$

$$=\tan\left(\frac{\pi}{4}\right)=1$$

16. Ans. (4)

$$\lim_{x\to 0^+}\frac{\cos^{-1}x}{(1-x^2)}\times\frac{\sin^{-1}x}{x}=\frac{\pi}{2}$$

17. Ans. (2)

$$\lim_{x\to\infty}\left(\sqrt{x^2-x+1}\right)-ax=b \qquad (\infty-\infty)$$

$\Rightarrow a > 0$

$$\text{Now, }\lim_{x\to\infty}\frac{(x^2-x+1-a^2x^2)}{\sqrt{x^2-x+1}+ax}=b$$

$$\Rightarrow\ \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}=b$$

$$\Rightarrow\ \lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$$

$\Rightarrow 1 - a^2 = 0 \Rightarrow\ a = 1$

$$\text{Now, }\lim_{x\to\infty}\frac{-x+1}{x\left(\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a\right)}=b$$

$$\Rightarrow\ \frac{-1}{1+a}=b\Rightarrow b=-\frac{1}{2}$$

$$(a,b)=\left(1,-\frac{1}{2}\right)$$

18. Ans. (3)

$$\lim_{x\to 0}\frac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$$

$$\lim_{x\to 0}\frac{1-\cos\left(2\pi\cos^4 x\right)}{2x^4}$$

$$\lim_{x\to 0}\frac{1-\cos\left(2\pi-2\pi\cos^4 x\right)}{\left[2\pi\left(1-\cos^4 x\right)\right]^2}4\pi^2.\frac{\sin^4 x}{2x^4}\left(1+\cos^2 x\right)^2$$

$$=\frac{1}{2}.4\pi^2.\frac{1}{2}(2)^2=4\pi^2$$

19. Ans. (1)

$$\lim_{x\to\frac{\pi}{2}}\tan^2 x\left[\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right]=\lim_{x\to\frac{\pi}{2}}\frac{\tan^2 x\,[\sin^2 x-3\sin x+2]}{\sqrt{9}+\sqrt{9}}$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\tan^2 x(\sin x-1)(\sin x-2)}{6}$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\tan^2 x(1-\sin x)$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\frac{\sin^2 x\,(1-\sin x)}{(1-\sin x)(1+\sin x)}=\frac{1}{12}$$

20. Ans. (3)

$$\lim_{x\to 0}\frac{\cos\left(\sin x\right)-\cos x}{x^4};\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\left(\frac{2.\sin\left(\frac{x+\sin x}{2}\right).\sin\left(\frac{x-\sin x}{2}\right)}{x^4}\right)$$

$$\lim_{x\to 0}2\left(\frac{\sin\left(\frac{x+\sin x}{2}\right)}{\left(\frac{x+\sin x}{2}\right)}\right)\left(\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right)\frac{\left(\frac{x+\sin x}{2}\right)}{x^4}\left(\frac{x-\sin x}{2}\right)$$

$$\lim_{x\to 0}\left(\frac{x^2-\sin^2 x}{2x^4}\right):\left(\frac{0}{0}\right)$$

Apply L-Hopital Rule :

$$\lim_{x\to 0}\frac{2x-2\sin x\cos x}{2.4x^3}$$

$$\lim_{x\to 0}\frac{2x-\sin 2x}{8x^3};\frac{0}{0}$$: Again apply L-Hopital rule

$$\lim_{x\to 0}\frac{2-2\cos\left(2x\right)}{8\left(3\right)x^2}$$

$$\lim_{x\to 0}\frac{2\left(1-\cos\left(2x\right)\right)}{24\left(4x^2\right)}\times 4\Rightarrow\frac{2}{24}\times\frac{1}{2}\times 4\Rightarrow\frac{1}{6}$$

21. Ans. (4)

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)-x}{1-\tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sqrt{1-x^2}-x}{1-\left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}(-x)=-\frac{1}{\sqrt{2}}$$

22. Ans. (2)

$$\lim_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$$

$$\lim_{x\to\infty}x^3\times\left\{\frac{x^3\left\{\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right\}}{x^6\left\{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right\}}\right\}$$

$$=\frac{\left(2\sqrt{3}\right)^6+0}{2^6+0}=3^3=(27)$$

23. Ans. (3)

$$\lim_{n\to\infty}\frac{0+3+6+9+.....n\text{ terms}}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{n\to\infty}\frac{3n(n-1)}{2\left(\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}\right)}$$

$$=\frac{3}{2\left(\sqrt{2}-1\right)}=\frac{3}{2}\left(\sqrt{2}+1\right)$$

24. Ans. (1)

$$\lim_{x\to a}\left([x-5]-[2x+2]\right)=0$$

$$\lim_{x\to a}\left([x]-5-[2x]-2\right)=0$$

$$\lim_{x\to a}\left([x]-[2x]\right)=7$$

$[a]-[2a]=7$

$a\in I,\ \ a=-7$

$a \notin I,\ \ a = I + f$

Now, $[a] - [2a] = 7$

$-I \ - [2f] = 7$

Case-I: $f \in \left(0, \frac{1}{2}\right)$

$2f \in (0, 1)$

$-I = 7$

$I = -7 \Rightarrow a \in (-7, -6.5)$

Case-II: $f \in \left(\frac{1}{2}, 1\right)$

$2f \in (1, 2)$

$-I - 1 = 7$

$I = -8 \Rightarrow a \in (-7.5, -7)$

Hence, $a \in (-7.5, -6.5)$

25. **Ans. (2)**

$$\lim_{t\to 0}\left(1^{\operatorname{cosec}^2 t} + 2^{\operatorname{cosec}^2 t} + \ldots\ldots\ldots + n^{\operatorname{cosec}^2 t}\right)^{\sin^2 t}$$

$$= \lim_{t\to 0} n\left(\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t} + \left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t} + \ldots\ldots\ldots + 1\right)^{\sin^2 t}$$

$= n$

26. **Ans. (3)**

$$\lim_{x\to 0}\frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{\frac{(1-\cos 2x)}{4x^2}\times 4x^2} = 17$$

On expansion,

$$\lim_{x\to 0}\frac{\left(1 + ax + \frac{a^2x^2}{2}\right) - \left(1 - \frac{b^2x^2}{2}\right) - \frac{cx}{2}(1 - cx)}{2x^2} = 17$$

$$\lim_{x\to 0}\frac{\left(a - \frac{c}{2}\right)x + x^2\left(\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2}\right)}{2x^2} = 17$$

For limit to be exist a – $\frac{c}{2} = 0$

$a = \frac{c}{2}$

and $\frac{a^2 + b^2 + c^2}{4} = 17$

$a^2 + b^2 + 4a^2 = 17 \times 4$

$5a^2 + b^2 = 68$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (A,C)**

$$\lim_{x\to 0}\frac{a-a\left(1-\frac{x^2}{a^2}\right)^{\frac{1}{2}}-\frac{x^2}{4}}{x^4}=\lim_{x\to 0}\frac{a-a\left(1-\frac{x^2}{2a^2}-\frac{1}{8}\frac{x^4}{a^4}\right)-\frac{x^2}{4}}{x^4}$$

$a = 2$, (coefficient of $x^2 = 0$)

$\therefore L = \frac{1}{64}$.

2. **Ans. (D)**

$$\lim_{x\to 0}\left[1+x\ell n(1+b^2)\right]^{\frac{1}{x}}=e^{\lim\limits_{x\to 0}\frac{x\ell n(1+b^2)}{x}}=1+b^2$$

Hence $1 + b^2 = 2b\sin^2\theta$

$\Rightarrow \sin^2\theta=\frac{1}{2}\left(b+\frac{1}{b}\right)\ge 1$

$\therefore \sin^2\theta = 1 \Rightarrow \sin\theta=\pm 1 \Rightarrow \theta = \pm\frac{\pi}{2}$

3. **Ans. (B)**

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{(1-a)x^2+x(1-a-b)+1-b}{x+1}\right)=4$$

$$\Rightarrow \lim_{x\to\infty}\frac{\left((1-a).x+1-a-b+\left(\frac{1-b}{x}\right)\right)}{1+\frac{1}{x}}=4$$

for limit to exist finitely

$1 - a = 0$ and $1 - a - b = 4$

$\Rightarrow a = 1$ and $b = -4$.

4. **Ans. (B)**

$$\left(\left(1+\frac{a}{3}\right)-1\right)x^2+\left(\left(1+\frac{a}{2}\right)-1\right)x+\left(1+\frac{a}{6}-1\right)=0$$

$a\left(\frac{x^2}{3}+\frac{x}{2}+\frac{1}{6}\right)=0 \Rightarrow 2x^2 + 3x + 1 = 0$

$\Rightarrow x=-\frac{1}{2},-1 \Rightarrow \lim\limits_{a\to 0^+}\alpha(a)$ and $\lim\limits_{a\to 0^+}\beta(a)$ are $-\frac{1}{2}$ and -1

5. **Ans. (0)**

$$\lim_{x\to 1}\left\{\frac{\sin(x-1)+a(1-x)}{(x-1)+\sin(x-1)}\right\}^{\frac{(1+\sqrt{x})(1-\sqrt{x})}{1-\sqrt{x}}}=\frac{1}{4}$$

$$\lim_{x\to 1}\left\{\frac{\frac{\sin(x-1)}{(x-1)}-a}{1+\frac{\sin(x-1)}{(x-1)}}\right\}^{1+\sqrt{x}}=\frac{1}{4}$$

$\Rightarrow \left(\frac{1-a}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow (a-1)^2 = 1 \Rightarrow a = 2$ or 0

but for $a = 2$ base of above limit approaches $-\frac{1}{2}$ and exponent approaches to 2 and since base cannot be negative hence limit does not exist.

6. Ans. (2)

$$\lim_{\alpha\to 0} \frac{e\left(e^{\cos\alpha^n - 1} - 1\right)}{\left(\cos\alpha^n - 1\right)} \frac{\left(\cos\alpha^n - 1\right)}{\left(\alpha^n\right)^2} \alpha^{2n-m}$$

$$= -\frac{e}{2} \therefore 2n = m \Rightarrow \frac{m}{n} = 2$$

7. Ans. (7)

If $\alpha \neq 1$, then $\lim\limits_{x\to 0} \frac{x\sin\beta x}{\frac{\alpha x - \sin x}{x}} = 0$

$$\therefore \alpha = 1 \Rightarrow \lim_{x\to 0} \frac{\beta x^3 \frac{\sin\beta x}{\beta x}}{x^3\left(\frac{x-\sin x}{x^3}\right)} = \frac{\beta}{1/6}$$

$\Rightarrow 6\beta = 1 \Rightarrow \beta = \frac{1}{6}$

$6(\alpha + \beta) = 7$

8. Ans. (A,C)

$$f(x) = \begin{cases} (1-x)\cos\frac{1}{1-x} & , \quad x < 1 \\ -(1+x)\cos\frac{1}{1-x} & , \quad x > 1 \end{cases}$$

$\lim\limits_{x\to 1^+} f(x) = d.n.e, \lim\limits_{x\to 1^-} f(x) = 0$

9. Ans. (D)

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{(x+j)-(x+j-1)}{1+(x+j)(x+j-1)}\right)$$

$$f_n(x) = \sum_{j=1}^{n} [\tan^{-1}(x+j) - \tan^{-1}(x+j-1)]$$

$f_n(x) = \tan^{-1}(x+n) - \tan^{-1}x$

$\therefore \tan(f_n(x)) = \tan[\tan^{-1}(x+n) - \tan^{-1}x]$

$\tan(f_n(x)) = \frac{(x+n)-x}{1+x(x+n)}$

$\tan(f_n(x)) = \dfrac{n}{1+x^2+nx}$

$\therefore\ \sec^2(f_n(x)) = 1 + \tan^2(f_n(x))$

$\sec^2(f_n(x)) = 1+\left(\dfrac{n}{1+x^2+nx}\right)^2$

$\lim\limits_{x\to\infty}\sec^2(f_n(x))=\lim\limits_{x\to\infty}1+\left(\dfrac{n}{1+x^2+nx}\right)^2 = 1$

10. Ans. (B,D)

P -1 :

$\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{\sqrt{|h|}}$ = exist and finite

(B) $f(x)=x^{2/3},\lim\limits_{h\to 0}\dfrac{h^{2/3}-0}{\sqrt{|h|}}=\lim\limits_{h\to 0}\dfrac{|h|^{2/3}}{\sqrt{|h|}}=0$

(D) $f(x)=|x|,\lim\limits_{h\to 0}\dfrac{|h|-0}{\sqrt{|h|}}\Rightarrow\lim\limits_{h\to 0}\sqrt{|h|}=0$

P-2 :

$\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h^2}$ = exist and finite

(A) $f(x) = x|x|$, $\lim\limits_{h\to 0}\dfrac{h|h|-0}{h^2}=\begin{cases} RHL=\lim\limits_{h\to 0}\dfrac{h^2}{h^2}=1\\ LHL=\lim\limits_{h\to 0}\dfrac{-h^2}{h^2}=-1\end{cases}$

(C) $f(x) = \sin x$ $\quad\lim\limits_{h\to 0}\dfrac{\sin h-0}{h^2}=DNE$

11. Ans. (1.00)

$\lim\limits_{x-0^+}\dfrac{e^{\left(\frac{\ell n(1-x)}{x}\right)}-\frac{1}{e}}{x^a}$

$= \lim\limits_{x-0^+}\dfrac{1}{e}\ \dfrac{e^{\left(1+\frac{\ell n(1-x)}{x}\right)}-1}{x^a}$

$=\dfrac{1}{e}\lim\limits_{x-0^+}\dfrac{1+\dfrac{\ell n(1-x)}{x}}{x^a}$

$=\dfrac{1}{e}\lim\limits_{x-0^+}\dfrac{\ell n(1-x)+x}{x^{(a+1)}}$

$=\dfrac{1}{e}\lim\limits_{x-0^+}\dfrac{\left(-x-\dfrac{x^2}{2}-\dfrac{x^3}{3}-.......\right)+x}{x^{a+1}}$

Thus, $a = 1$

12. Ans. (8)

$$\lim_{x\to\frac{\pi}{2}}\frac{4\sqrt{2}\cdot 2\sin 2x\cos x}{2\sin 2x\sin\frac{3x}{2}+\left(\cos\frac{5x}{2}-\cos\frac{3x}{2}\right)-\sqrt{2}(1+\cos 2x)}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{16\sqrt{2}\sin x\cos^2 x}{2\sin 2x\left(\sin\frac{3x}{2}-\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{16\sqrt{2}\sin x\cos^2 x}{4\sin x\cos x\left(2\cos x\cdot\sin\frac{x}{2}\right)-2\sqrt{2}\cos^2 x}$$

$$\lim_{x\to\frac{\pi}{2}}\frac{16\sqrt{2}\sin x}{8\sin x\cdot\sin\frac{x}{2}-2\sqrt{2}}=8$$

13. Ans. (0.50)

$$\lim_{x\to\alpha^+}\frac{2\ell n\left(\sqrt{x}-\sqrt{\alpha}\right)}{\ell n\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}\quad\left(\frac{0}{0}\text{form}\right)$$

$\therefore$ Using L'opital rule,

$$=2\lim_{x\to\alpha^+}\frac{\left(\frac{1}{\sqrt{x}-\sqrt{\alpha}}\right).\frac{1}{2\sqrt{x}}}{\left(\frac{1}{e^{\sqrt{x}}-e^{\sqrt{\alpha}}}\right).e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}}=\frac{2}{e^{\sqrt{\alpha}}}\lim_{x\to\alpha^+}\frac{\left(e^{\sqrt{x}}-e^{\sqrt{\alpha}}\right)}{\left(\sqrt{x}-\sqrt{\alpha}\right)}\quad\left(\frac{0}{0}\right)$$

$$=\frac{2}{e^{\sqrt{\alpha}}}\lim_{x\to\alpha^+}\frac{\left(e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}-0\right)}{\left(\frac{1}{2\sqrt{x}}-0\right)}=2$$

so, $\lim_{x\to\alpha^+} f\left(g\left(x\right)\right)=\lim_{x\to\alpha^+} f\left(2\right)$

$$=f\left(2\right)=\sin\frac{\pi}{6}=\frac{1}{2}$$

= 0.50

14. Ans. (5)

$$\beta=\lim_{x\to 0}\frac{e^{x^3}-\left(1-x^3\right)^{1/3}}{\frac{x\sin^2 x}{x^2}x^2}+\frac{\left(\left(1-x^2\right)^{1/2}-1\right)\sin x}{x\frac{\sin^2 x}{x^2}x^2}$$

use expansion

$$\beta=\lim_{x\to 0}\frac{\left(1+x^3\right)-\left(1-\frac{x^3}{3}\right)}{x^3}+\lim_{x\to 0}\frac{\left(\left(1-\frac{x^2}{2}\right)-1\right)}{x^2}\frac{\sin x}{x}$$

$$\beta=\lim_{x\to 0}\frac{4x^3}{3x^3}+\lim_{x\to 0}\frac{-x^2}{2x^2}$$

$$\beta=\frac{4}{3}-\frac{1}{2}=\frac{5}{6}$$

$6\beta = 5$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (2)

$$\lim_{x\to 0}\frac{\left(1+x^2\right)^{\frac{1}{3}}-\left(1-2x\right)^{\frac{1}{4}}}{x\left(1+x\right)}$$

$$\Rightarrow \lim_{x\to 0}\frac{1+\frac{x^2}{3}-\left[1+\frac{1}{4}(-2x)\right]}{x\left(1+x\right)}$$

$$\Rightarrow \lim_{x\to 0}\frac{1+\frac{x^2}{3}-1+\frac{x}{2}}{x\left(1+x\right)}$$

$$\Rightarrow \lim_{x\to 0}\frac{x\left(\frac{1}{2}+\frac{x}{3}\right)}{x\left(1+x\right)}\Rightarrow\frac{1}{2}$$

2. Ans. (1)

$$\lim_{x\to -\infty}\left(\sqrt{x^2-2x-1}-\sqrt{x^2-7x+3}\right) \qquad (\infty-\infty)$$

$$\lim_{x\to -\infty}\frac{\left(x^2-2x-1\right)-\left(x^2-7x+3\right)}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+3}}$$

$$\lim_{x\to -\infty}\frac{2x-4}{\sqrt{x^2-2x-1}+\sqrt{x^2-7x+3}}$$

$$\lim_{x\to -\infty}\frac{x\left(5-\frac{4}{x}\right)}{|x|\left(\sqrt{1-\frac{2}{x}-\frac{1}{x^2}}+\sqrt{1-\frac{7}{x}+\frac{3}{x^2}}\right)}$$

$$=-\frac{5}{2}$$

3. Ans. (3)

$$U_n=\frac{n!}{(n+2)!}$$

$$\Rightarrow U_n=\frac{n!}{(n+2)(n+1)n!}$$

$$\Rightarrow U_n=\frac{1}{(n+2)(n+1)}$$

$$\Rightarrow U_n=\frac{1}{n+1}-\frac{1}{n+2}$$

$$=S_n=\sum_{r=1}^{n}\left(\frac{1}{r+1}-\frac{1}{r+2}\right)$$

$T_1 = \frac{1}{2} - \frac{1}{3}$

$T_2 = \frac{1}{3} - \frac{1}{4}$

$\vdots$

$T_{n-1} = \frac{1}{n} - \frac{1}{n+1}$

$T_n = \frac{1}{n+1} - \frac{1}{n+2}$

$S_n = \frac{1}{2} - \frac{1}{n+2}$

$\Rightarrow \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{n+2}\right) = \frac{1}{2}$

4. Ans. (4)

$f(x) = \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + \ldots + \left[\frac{10\sin 10x}{x}\right]$

$\Rightarrow \lim_{x\to 0} f(x)$

$$\Rightarrow \lim_{x\to 0}\left[\left[\frac{x - \frac{x^3}{3!}}{x}\right] + \left[\frac{2\left(2x - \frac{8x^3}{3!}\right)}{x}\right] + \ldots\ldots + \left[\frac{10\left(10x - \frac{1000x^3}{3!}\right)}{x}\right]\right]$$

[0 + 3 + 8 + 15 +………………+ 99]

$s = \sum_{r=1}^{10}\left(r^2 - 1\right)$

$s = \frac{(10)(11)(21)}{6} - 10$

$s = 35 \times 11 - 10$

$s = 385 - 10$

$s = 375$

5. Ans. (4)

$$\lim_{x\to 0}\left(1 + \frac{f(x) + x^2}{x^2}\right)^{\frac{1}{x}}$$

Power of the expression is tending towards ∞ so base must tend toward 1 to get limit finite.

So, $\frac{f(x) + x^2}{x^2}$ must be tending towards zero.

For that $f(x) = ax^3 - x^2$

$$\Rightarrow \lim_{x\to 0}\left(1 + \frac{ax^3 - x^2 + x^2}{x^2}\right)^{\frac{1}{x}} = e^2$$

$\Rightarrow \lim_{x\to 0}(1+ax)^{\frac{1}{x}} = e^2$

$\Rightarrow \lim_{x\to 0} e^{(1+ax-1)\frac{1}{x}} = e^2$

$\Rightarrow \lim_{x\to 0} e^{ax\times\frac{1}{x}} = e^2$

$\Rightarrow \lim_{x\to 0} e^{a} = e^2 \Rightarrow a = 2$

$\Rightarrow f(x) = 2x^3 - x^2$

$\Rightarrow f(2) = 2(2)^3 - (2)^2$

$= 16 - 4$

$= 12$

6. **Ans. (4)**

$$\lim_{n\to\infty} \frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}}$$

$$\Rightarrow L = \lim_{n\to\infty} \frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}}$$

$$\Rightarrow \ell n L = \lim_{n\to\infty} \ell n\left[\frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}}\right]$$

$$= \lim_{n\to\infty}\left[n - n^2 \ell n\left(1+\frac{1}{n}\right)\right]$$

$$= \lim_{n\to\infty}\left[n - n^2\left(\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3}....\right)\right]$$

$$= \lim_{n\to\infty}\left[n - \left(n - \frac{1}{2} + \frac{1}{3n}...\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{2} + \frac{1}{3n}.....\right]$$

$$\Rightarrow \ell n L = \frac{1}{2}$$

$$\Rightarrow L = e^{\frac{1}{2}} = \sqrt{e}$$

7. **Ans. (2)**

$$\lim_{x\to a^-}\left(\frac{|x|^3}{a} - \left[\frac{x}{a}\right]^3\right)\ (a > 0)$$

$$x = a - h = \lim_{h\to 0}\left(\frac{|a-h|^3}{a} - \left[\frac{a-h}{a}\right]^3\right) = \lim_{h\to 0}\left(\frac{|a|^3}{a} - \left[1-\frac{h}{a}\right]^3\right) = a^2 - 0 = a^2$$

8. Ans. (3)

$$\lim_{n\to\infty} \cos\frac{x}{2}\cos\frac{x}{2^2}\cos\frac{x}{2^3}\cos\frac{x}{2^4}.......\cos\frac{x}{2^n}$$

$$= \lim_{n\to\infty} \frac{\sin x}{2^n \sin\frac{x}{2^n}} \qquad \left(\because \lim_{n\to\infty}\frac{x}{2^n} = 0\right)$$

$$= \frac{\sin x}{x}$$

9. Ans. (3)

$$\sin\theta < \theta < \tan\theta\,,\ \theta \in \left(0,\ \frac{\pi}{2}\right)$$

$$\frac{\sin\theta}{\theta} < 1 < \frac{\tan\theta}{\theta}$$

$$\frac{n\sin\theta}{\theta} < n < \frac{n\tan\theta}{\theta}$$

$$\lim_{\theta\to 0}\left(\left[\frac{n\sin\theta}{\theta}\right]+\left[\frac{n\tan\theta}{\theta}\right]\right)\ ;\ n\in N$$

$$\text{L.H.L.} = \lim_{\theta\to 0^-}\left(\left[\frac{n\sin\theta}{\theta}\right]+\left[\frac{n\tan\theta}{\theta}\right]\right) = n-1+n = 2n-1$$

$$\text{R.H.L.} = \lim_{\theta\to 0^+}\left(\left[\frac{n\sin\theta}{\theta}\right]+\left[\frac{n\tan\theta}{\theta}\right]\right) = n-1+n = 2n-1$$

$\therefore$ L.H.L. = R.H.L = $2n - 1$

10. Ans. (1)

$$\text{R.H.L.} = \lim_{x\to 0^+}\left[(1-e^x)\frac{\sin x}{x}\right]$$

when $x \in (0, h)$ and $h \to 0$ then $(1 - e^x) \in (-1, 0)$ and $\frac{\sin x}{x} < 1$

So $-1 < (1 - e^x)\ \frac{\sin x}{x} < 0$; $\lim_{x\to 0^+}\left[(1-e^x)\frac{\sin x}{x}\right] = -1$

$$\text{L.H.L.} = \lim_{x\to 0^-}\left[(1-e^x)\frac{\sin x}{-x}\right] = \lim_{x\to 0^-}\left[(e^x-1)\frac{\sin x}{x}\right]$$

when $x \in (-h, 0)$ and $h \to 0$, then $e^x - 1 \in (-1, 0)$ and $\frac{\sin x}{x} < 1$

So $-1 < (e^x - 1)\ \frac{\sin x}{x} < 0$

So $\lim_{x\to 0^-}\left[(e^x-1)\frac{\sin x}{x}\right] = -1$

L.H.L. = R.H.L. = -1

11. Ans. (2)

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}=\lim_{x\to 0}\frac{\cos\left(x-\frac{x^3}{3!}+\ldots\right)-\cos x}{x^4}$$

$$=\lim_{x\to 0}\frac{\left[1-\frac{\left(x-\frac{x^3}{6}+\ldots\right)^2}{2!}+\frac{\left(x-\frac{x^3}{6}+\ldots\right)^4}{4!}-\ldots\ldots\right]-\left[1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\right]}{x^4}$$

$$=\lim_{x\to 0}\frac{x^4\left(\frac{1}{2}.\frac{2}{6}+\frac{1}{24}\right)-\frac{x^4}{24}+\ldots\ldots}{x^4}=\frac{1}{6}+\frac{1}{24}-\frac{1}{24}=\frac{1}{6}$$

12. Ans. (2)

$$\lim_{t\to 1}\frac{t-t^t}{1-t+\ell nt}$$

$$=\lim_{t\to 1}\frac{1-t^t(1+\ell nt)}{-1+\frac{1}{t}}\ \text{(LH. rule)}$$

$$=\lim_{t\to 1}\frac{-t^{t-1}-t^t(1+\ell nt)^2}{-\frac{1}{t^2}}\ \text{(LH. rule)}$$

$$=\frac{-1-1}{-1}=2$$

13. Ans. (3)

$$\lim_{x\to\frac{\pi}{2}}\tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$$

$$=\lim_{x\to\frac{\pi}{2}}\tan^2 x\left(\frac{2\sin^2 x+3\sin x+4-(\sin^2 x+6\sin x+2)}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}\right)$$

$$=\lim_{x\to\frac{\pi}{2}}\tan^2 x\frac{(\sin^2 x-3\sin x+2)}{\sqrt{2+3+4}+\sqrt{1+6+2}}.$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{1}{6}\left[\frac{\sin^2 x-3\sin x+2}{\cos^2 x}\right]\quad\left(\frac{0}{0}\text{ form}\right)\left(\frac{0}{0}\ \text{i}\right)\text{ (use L'Hospital rule)}$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\frac{2\sin x\cos x-3\cos x}{2\cos x(-\sin x)}=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\frac{2\sin x-3}{-2\sin x}=\left(\frac{1}{6}\right)\left(\frac{1}{2}\right)=\frac{1}{12}$$

14. Ans. (3)

α, β are the roots of the equation $ax^2 + bx + c = 0$

$\because\ ax^2 + bx + c = a(x-\alpha)\ (x-\beta)$

$$\lim_{x\to\alpha}\left(1+ax^2+bx+c\right)^{\frac{1}{x-\alpha}}=e^{\lim_{x\to\alpha}\frac{ax^2+bx+c}{x-\alpha}}=e^{\lim_{x\to\alpha}\frac{a(x-\alpha)(x-\beta)}{(x-\alpha)}}=e^{a(\alpha-\beta)}$$

15. **Ans. (2)**

$$\lim_{x\to\infty} \frac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}} - \left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}, n \in N$$

$$\lim_{x\to\infty} \frac{(2)^{\frac{x^n}{e^x}} - (3)^{\frac{x^n}{e^x}}}{\frac{x^n}{e^x}}$$

when $x \to \infty$, $\frac{x^n}{e^x} \to 0$

Put $\frac{x^n}{e^x} = t$

$$\lim_{t\to 0}\left(\frac{2^t - 3^t}{t}\right) = \ell n2 - \ell n3 = \ell n\left(\frac{2}{3}\right)$$

16. **Ans. (2)**

$$\lim_{y\to 0}\left(\lim_{x\to\infty} \frac{\exp\left(x\ell n\left(1+\frac{ay}{x}\right)\right) - \exp\left(x\ell n\left(1+\frac{by}{x}\right)\right)}{y}\right)$$

$$= \lim_{y\to 0}\left(\lim_{x\to\infty} \frac{\left(1+\frac{ay}{x}\right)^x - \left(1+\frac{by}{x}\right)^x}{y}\right)$$

by expansion

$$= \lim_{y\to 0}\left(\lim_{x\to\infty} \frac{\left(1+ay+\frac{x(x-1)}{2!}.\frac{a^2y^2}{x^2}+.....\right) - \left(1+by+\frac{x(x-1)}{2!}.\frac{b^2y^2}{x^2}+....\right)}{y}\right)$$

$$= \lim_{y\to 0}\left[\frac{y(a-b)+\frac{y^2}{2}(a^2-b^2)+.....}{y}\right] = a - b$$

17. **Ans. (3)**

$$f(x) = \lim_{t\to 0}\left(\frac{2x}{\pi}\cot^{-1}\frac{x}{t^2}\right)$$

Case-I : when $x = 0$

$f(x) = 0$

Case-II : when $x > 0$

$$f(x) = \lim_{t\to 0}\left(\frac{2x}{\pi}\cot^{-1}\frac{x}{t^2}\right) = \frac{2x}{\pi}\times\cot^{-1}(\infty) = .\ \frac{2x}{\pi}\ 0 = 0$$

Case-III when $x < 0$

$$f(x) = \lim_{t\to 0}\left(\frac{2x}{\pi}\cot^{-1}\frac{x}{t^2}\right) = \frac{2x}{\pi} \times \cot^{-1}(-\infty) = \frac{2x}{\pi}.\pi = 2x \Rightarrow f(x) = 2x$$

18. Ans. (3)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sin^{-1}(1-\{x\}\,.\,\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,(1-\{x\})} = \lim_{h\to 0} \frac{\sin^{-1}(1-h)\;.\;\cos^{-1}(1-h)}{\sqrt{2h}\;\;(1-h)}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(1-h)\;\sin^{-1}\sqrt{h(2-h)}}{(1-h)\sqrt{2h}} = \lim_{h\to 0} \frac{\frac{\pi}{2}}{1} \frac{\sin^{-1}\sqrt{h(2-h)}}{\sqrt{2h-h^2}\sqrt{2h}} \sqrt{2h-h^2}$$

$$= \lim_{h\to 0} \frac{\pi}{2}\,.\,1\,.\,\sqrt{1-\frac{h}{2}} = \frac{\pi}{2}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin^{-1}(1-\{x\})\;\cos^{-1}(1-\{x\})}{\sqrt{2\;\{x\}}\;\;(1-\{x\})} = \lim_{h\to 0} \frac{\sin^{-1}h\;\;.\;\;\cos^{-1}h}{\sqrt{2(1-h)h}} = \frac{\frac{\pi}{2}}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

19. Ans. (4)

$$\lim_{x\to\infty}\left(x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right)\right)$$

$$\lim_{x\to\infty}\left(\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}+\sin\left(\frac{1}{x^2}\right)\right) = 1 + 0 = 1$$

20. Ans. (3)

$$\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2} = \lim_{x\to 0} \frac{1-\cos^2 x\cos 2x}{(1+\cos x\sqrt{\cos\;2x})x^2}$$

$$= \lim_{x\to 0} \frac{1}{(1+\cos x\sqrt{\cos 2x})}\;.\;\lim_{x\to 0} \frac{1-\cos^2 x\cos 2x}{x^2}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{1-\left(\frac{1+\cos 2x}{2}\right)\cos 2x}{x^2} = \frac{1}{2}\lim_{x\to 0} \frac{2-\cos 2x-(\cos 2x)^2}{2x^2}$$

$$= \frac{1}{4}\lim_{x\to 0} -\frac{1}{x^2}\;(\cos 2x + 2)\,(\cos 2x - 1) = \frac{1}{4}\lim_{x\to 0}(\cos 2x + 2).\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$$

$$= \frac{(1+2)}{4}\,.\,\lim_{x\to 0}\frac{2\sin^2 x}{x^2} = \frac{3}{4}.2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = \frac{3}{2} \Rightarrow \text{limit} = \frac{3}{2}+\frac{1}{2} = 2$$

SECTION–B

1. Ans. (2)

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots\ldots\ldots\ldots+\frac{1}{\sqrt{n^2+2n}}\right)$$

using sandwitch theorem

$$\frac{1}{\sqrt{n^2}} \le \frac{1}{n}$$

$$\frac{1}{\sqrt{n^2+1}} \le \frac{1}{n}$$

$\vdots \quad \vdots$

$$\frac{1}{\sqrt{n^2+2n}} \le \frac{1}{n}$$

adding all these inequilities

$$\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots\ldots\ldots\ldots+\frac{1}{\sqrt{n^2+2n}} \le \frac{2n}{n}$$

Taking both side $\lim\limits_{n\to\infty}$

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots\ldots\ldots\ldots+\frac{1}{\sqrt{n^2+2n}}\right) = 2$$

2. **Ans. (1)**

$$\lim_{x\to0^+} x^2\left[\frac{1}{x^2}\right] = \lim_{x\to0^+} x^2\left(\frac{1}{x^2}-\left\{\frac{1}{x^2}\right\}\right) \Rightarrow \lim_{x\to0^+}\left(1-x^2\left\{\frac{1}{x^2}\right\}\right) = 1$$

similarly $\lim\limits_{x\to0^-} f(x) = 1$

3. **Ans. (2)**

$$\lim_{x\to0}\left(\frac{\sin^{-1}x - \tan^{-1}x}{x^3}+\frac{84x\frac{\pi}{8}}{\sin\pi x}\right) = \lim_{x\to0}\frac{\left[x+\frac{1^2}{3!}x^3+\ldots\right]-\left[x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\right]}{x^3} + \frac{84}{8}$$

$$=\lim_{x\to0}\frac{x^3\left(\frac{1}{3!}+\frac{1}{3}\right)+x^5(\ \ )+\ldots}{x^3}+\frac{21}{2}=11$$

4. **Ans. (6)**

$$\lim_{x\to0}\frac{x^3}{\sqrt{a+x}\,(bx-\sin x)} = 1 \Rightarrow \lim_{x\to0}\frac{1}{\sqrt{a+x}}\ \lim_{x\to0}\frac{x^3}{bx-\sin x} = 1$$

$$\Rightarrow \frac{1}{\sqrt{a}}.\lim_{x\to0}\frac{x^3}{bx-\left[x-\frac{x^3}{3\,!}+\frac{x^5}{5\,!}-\ldots\ldots\right]} = 1 \Rightarrow \frac{1}{\sqrt{a}}.\lim_{x\to0}\frac{x^3}{(b-1)\ x\ +\ \frac{x^3}{3\,!}-\frac{x^5}{5\,!}+\ldots} = 1$$

If limit exists, then $b - 1 = 0 \Rightarrow b = 1$

$$\text{so } \frac{1}{\sqrt{a}}.\lim_{x\to0}\frac{x^3}{x^3\left[\frac{1}{6}-\frac{x^2}{120}+\ldots\right]} = 1 \Rightarrow \frac{1}{\sqrt{a}} \times \frac{1}{\frac{1}{6}} = 1 \Rightarrow a = 36$$

so, $a = 36$, $b = 1$

5. **Ans. (4)**

$$f(x) = \sum_{\lambda=1}^{n}\left(x-\frac{5}{\lambda}\right)\left(x-\frac{4}{\lambda+1}\right)$$

$$f(0) = \sum_{\lambda=1}^{n}\left(-\frac{5}{\lambda}\right)\left(-\frac{4}{\lambda+1}\right) \Rightarrow f(0) = \sum_{\lambda=1}^{n}\left(\frac{20}{(\lambda)(\lambda+1)}\right) \Rightarrow f(0) = 20\sum_{\lambda=1}^{n}\left(\frac{1}{\lambda}-\frac{1}{\lambda+1}\right)$$

$$\Rightarrow f(0) = 20\left(1-\frac{1}{n+1}\right) \Rightarrow f(0) = \frac{20n}{n+1}$$

Now $\lim\limits_{n\to\infty} f(0) = \lim\limits_{n\to\infty}\frac{20n}{n+1} = \lim\limits_{n\to\infty}\frac{20}{1+\frac{1}{n}} = \frac{20}{1+0} = 20$

6. Ans. (2)

LHL = $\lim\limits_{h\to 0^+} f(0-h)$ $\lim\limits_{h\to 0^+} = f(-h) = \lim\limits_{h\to 0^+}(-1)^{[h^2]} = 1$

RHL = $\lim\limits_{h\to 0^+} f(0+h) = \lim\limits_{h\to 0^+} f(h) = \lim\limits_{h\to 0^+}\left(\lim\limits_{n\to\infty}\frac{1}{1+h^n}\right) = 1$

7. Ans. (1)

$\because\ (1+x)^{\frac{1}{x}} = e\left[1-\frac{x}{2}+\frac{11}{24}x^2-.....\right]$

Now $\ell = \lim\limits_{x\to 0}\frac{e-(1+x)^{\frac{1}{x}}}{\tan x} = \lim\limits_{x\to 0}\frac{e-e\left[1-\frac{x}{2}+\frac{11}{24}x^2-.....\right]}{x+\frac{x^3}{3}+\frac{2x^5}{15}+.....} = \frac{e}{2}$

8. Ans. (6)

$$\lim_{x\to 0}\frac{e^{-nx}+e^{nx}-2\cos\frac{nx}{2}-kx^2}{(\sin x-\tan x)}$$

$$= \lim_{x\to 0}\frac{2\left[1+\frac{n^2x^2}{2!}+\frac{n^4x^4}{4!}+.....\right]-2\left[1-\frac{n^2x^2}{4.2!}+\frac{n^4x^4}{16.4!}-........\right]-kx^2}{\left(x-\frac{x^3}{3!}+........\right)-\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+.......\right)}$$

$$= \lim_{x\to 0}\frac{x^2\left(n^2+\frac{n^2}{4}-k\right)+x^4\left(\frac{2n^4}{4!}-\frac{2n^4}{16.4!}\right)}{x^3\left(-\frac{1}{3!}-\frac{1}{3}\right)+.........}$$

limit exists, if coff. of x^2 is zero.

$\Rightarrow n^2 + \frac{n^2}{4} - k = 0 \Rightarrow 4k = 5n^2$

so the possible value match that is $n = 2$, $k = 5$

9. Ans. (4)

$$\lim_{n\to\infty}\frac{\sum\limits_{r=1}^{n} r^2(n-r+1)}{\sum\limits_{r=1}^{n} r^3}$$

$$= \lim_{n\to\infty}\frac{\sum\limits_{r=1}^{n}(n+1)r^2-\sum\limits_{r=1}^{n} r^3}{\sum\limits_{r=1}^{n} r^3} = \lim_{n\to\infty}\left(\frac{(n+1)\frac{(n)(n+1)(2n+1)}{6}}{\frac{n^2(n+1)^2}{4}}-1\right) = \frac{1/3}{1/4} - 1 = \frac{4}{3}-1=\frac{1}{3}$$

10. Ans. (9)

$$\lim_{n\to\infty} \frac{n^{98}}{n^{x-1}\left({}^{x}C_1 - \frac{{}^{x}C_2}{2n} + \frac{{}^{x}C_3}{6n^2} - \ldots\right)} = \frac{1}{99}$$

the limit obviously exists if $x - 1 = 98$

JEE (Advanced) Practice Paper

1. Ans. (A)

$$f(x) = \frac{x^2 - 9x + 20}{x - [x]}$$

$$\lim_{x\to 5^-} \frac{x^2 - 9x + 20}{x - [x]} = \frac{25 - 45 + 20}{1} = 0$$

$$\lim_{x\to 5^+} \frac{x^2 - 9x + 20}{x - [x]} = \lim_{h\to 0} \frac{(5+h)^2 - 9(5+h) + 20}{5 + h - [5+h]} = \lim_{h\to 0} \frac{25 + 10h + h^2 - 45 - 9h + 20}{h}$$

$$= \lim_{h\to 0} \frac{h^2 + h}{h} = \lim_{h\to 0} \frac{h(h+1)}{h} = 1$$

$$\because \lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$$

So, $\lim_{x\to 5} f(x)$ does not exist

2. Ans. (A)

Let $f(x) = \dfrac{\cos 2 - \cos 2x}{x^2 - |x|}$

(A) $\lim_{x\to -1} f(x)$

for $x = -1$ $\quad |x| = -x$

$f(x) = \dfrac{\cos 2 - \cos 2x}{x^2 + x}$ Now $\lim_{x\to -1} \dfrac{\cos 2 - \cos 2x}{x^2 + x}$ (form)

$$= \lim_{x\to -1} \frac{2\sin 2x}{2x + 1} = 2\sin 2$$

(B) $\lim_{x\to 1} \dfrac{\cos 2 - \cos 2x}{x^2 - x}$ $\quad (\frac{0}{0}$ form)

$$= \lim_{x\to 1} \frac{2\sin 2x}{2x - 1} = 2\sin 2$$

3. Ans. (D)

For $\lim_{x\to 0} \dfrac{1 + a\cos x}{x^2}$

for $\left(\frac{0}{0}\right)$ form

$1 + a = 0 \Rightarrow a = -1$

for $\lim_{x\to 0} \dfrac{b\sin x}{x^3}$ $\dfrac{b}{x^2} = \lim_{x\to 0} b = 0$

Now, $\ell = \lim_{x\to 0} \dfrac{1 + a\cos x}{x^2} - \lim_{x\to 0} \dfrac{b\sin x}{x^3} = \lim_{x\to 0} \dfrac{1 - \cos x}{x^2} = \lim_{x\to 0} \dfrac{2\sin^2 \frac{x}{2}}{x^2}$

$$= \lim_{x \to 0} \frac{2}{4}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = \frac{1}{2}.(1)^2 = \frac{1}{2}$$

$\therefore (a, b) = (-1, 0)$ and $\ell = \frac{1}{2}$

4. Ans. (C)

$$f(x) = \frac{|x + \pi|}{\sin x}$$

(A) $f(-\pi^+) = \lim_{h\to 0} \frac{|-\pi+h+\pi|}{\sin(-\pi+h)} = \lim_{h\to 0} \frac{|h|}{-\sin h} = -1$

(B) $f(-\pi^-) = \lim_{h\to 0} \frac{|-\pi-h+\pi|}{\sin(-\pi-h)} = \lim_{h\to 0} \frac{|h|}{\sin h} = 1$

(C) $f(-\pi^+) \neq f(-\pi^-)$ so, $\lim_{x\to -\pi} f(x)$ does not exist

(D) for $\lim_{x \to \pi} f(x)$

$$\text{LHL} = \lim_{x\to\pi^-} \frac{|x + \pi|}{\sin x} = \lim_{h\to 0} \frac{2\pi - h}{\sin h} = \frac{2\pi}{0} = \infty$$

$$\text{RHL} = \lim_{x\to\pi^+} \frac{|x + \pi|}{\sin x} = \lim_{h\to 0} \frac{2\pi + h}{-\sin h} = -\frac{2\pi}{0} = -\infty$$

LHL $\neq$ RHL so, $\lim_{x\to\pi} f(x)$ does not exist.

5. Ans. (A,C)

(A) $$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}$$

$$\lim_{n\to\infty}\frac{1}{2}\sum_{r=1}^{n}\frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)}$$

$$\lim_{n\to\infty}\frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right) = \lim_{n\to\infty}\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}.......\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$$\lim_{n\to\infty}\frac{1}{2}\left(1-\frac{1}{2n+1}\right) \Rightarrow \frac{1}{2}(1-0) = \frac{1}{2}$$

(B) $$\lim_{x\to\infty}\frac{(3x^2)(4x^2)-(2x-1)(2x+1)(3x^2+x+2)}{4x^2(2x+1)}$$

$$\lim_{x\to\infty}\frac{12x^4-(12x^4+4x^3+8x^2-3x^2-x-2)}{8x^3+4x^2}$$

$$\lim_{x\to\infty}\frac{12x^4-(12x^4+4x^3+8x^2-3x^2-x-2)}{8x^3+4x^2}$$

$$\lim_{x\to\infty}\frac{-4x^3-5x^2+x+2}{8x^3+4x^2}$$

$$\lim_{x\to\infty}\frac{x^3\left(-4-\frac{5}{x}+\frac{1}{x^2}+\frac{2}{x^3}\right)}{x^3\left(8+\frac{4}{x}\right)} \Rightarrow \frac{-4}{8} \Rightarrow -\frac{1}{2}$$

(C) $\lim_{n\to\infty}\frac{1}{n^2}\frac{(n(n+1))}{2}$

$\lim_{n\to\infty}\frac{n^2\left(1\left(1+\frac{1}{n}\right)\right)}{2n^2}\Rightarrow\frac{1}{2}$

(D) $\lim_{n\to\infty}\frac{(n+2)(n+1)!+(n+1)!}{(n+2)(n+1)!-(n+1)!}$

$\lim_{n\to\infty}\frac{(n+1)!(n+2+1)}{(n+1)!(n+2-1)}$

$\lim_{n\to\infty}\frac{n+3}{n+1}\Rightarrow\lim_{n\to\infty}\frac{n\left(1+\frac{3}{n}\right)}{n\left(1+\frac{1}{n}\right)}=1$

6. **Ans. (A,C)**

(A) $\lim_{x\to\infty}\sqrt{x^2+2x}-x$

$\lim_{x\to\infty}\frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x}$

$\Rightarrow\lim_{x\to\infty}\frac{2x}{x\left(\sqrt{1+\frac{2}{x}}+1\right)}$

$\Rightarrow\lim_{x\to\infty}\frac{2}{\left(\sqrt{1+\frac{2}{x}}+1\right)}\Rightarrow\frac{2}{2}=1$

(B) $\lim_{x\to\infty}x+\sqrt{x^2+2x}$

limit does not exist

(C) $\lim_{x\to-\infty}x+\sqrt{x^2+2x}$

$\lim_{x\to-\infty}\frac{x^2-(x^2+2x)}{x-\sqrt{x^2+2x}}$

Replace $x\to -x$

$\lim_{x\to+\infty}\frac{2x}{-x-\sqrt{x^2-2x}}$

$\lim_{x\to\infty}\frac{2x}{-x\left(1+\sqrt{1-\frac{2}{x}}\right)}=-1$

(D) $\lim_{x\to-\infty}\sqrt{x^2+2x}-x$

Replace x by $-x$

$\lim_{x\to\infty}\sqrt{x^2-2x}+x$

Limit does not exist

7. Ans. (B,C,D)

(A) $\lim_{x\to 1^+}([x])^{\frac{1}{x-1}}$ (1^∞)

$e^{\left(\lim_{x\to1^+}\frac{1}{x-1}([x]-1)\right)}$

$e^{\left(\lim_{x\to1^+}\frac{0}{x-1}\right)}\Rightarrow e^0$

$\Rightarrow 1$

(B) $\lim_{x\to3}\frac{(x^2-9)-|x-3|}{(x-3)}$

RHL = $\lim_{x\to3^+}\frac{(x^2-9)-(x-3)}{(x-3)}\Rightarrow\lim_{x\to3^+}\frac{(x+3)-1}{(1)}=5$

LHL = $\lim_{x\to3^-}\frac{(x^2-9)+(x-3)}{(x-3)}\Rightarrow\lim_{x\to3^-}\frac{(x+3)+1}{(1)}=7$

LHL $\neq$ RHL limit does not exist

(C) $\lim_{x\to0^+}(x)^{\ell n(x)}$

$\lim_{x\to0^+}e^{\ell n(x)\ell n(x)}$

$\lim_{x\to0^+}e^{(\ell nx)^2}$

Limit does not exist

(D) $\lim_{x\to0^+}\left(\frac{1-\cos(\sin^2 x)}{x^2}\right)^{\frac{\ell n(1-2x^2)}{\sin^2 x}}=\lim_{x\to0^+}(f(x))^{g(x)}$ say,

$\lim_{x\to0^+}f(x)=\lim_{x\to0^+}\left(\frac{1-\cos(\sin^2 x)}{x^2}\right)$

$=\lim_{x\to0^+}\left(\frac{1-\cos(\sin^2 x)}{(\sin^2 x)^2}\right)\cdot\frac{(\sin^2 x)(\sin^2 x)}{x^2}$

$=\frac{1}{2}\times1\times(\sin 0)^2=0$

$\lim_{x\to0^+}g(x)=\lim_{x\to0^+}\frac{\ell n(1-2x^2)}{\sin^2 x}$

$=\lim_{x\to0^+}\frac{\ell n(1-2x^2)}{(-2x^2)}\frac{(-2x^2)}{\sin^2 x}$

$=-2$ $\left(\text{using }\lim_{f(x)\to0}\frac{\ell n(1+f(x))}{f(x)}=1,\ \lim_{f(x)\to0}\frac{\sin f(x)}{f(x)}=1\right)$

Now, $\lim_{x\to0^+}(f(x))^{g(x)}=(0)^{-2}$

Which is undefined.

So, limit doesn't exist.

8. Ans. (A,D)

For $\lim_{x\to 0}\frac{1+a\cos x}{x^2}$

for $\left(\frac{0}{0}\right)$ form

$1 + a = 0 \Rightarrow a = -1$

for $\lim_{x\to 0}\frac{b\sin x}{x^3}\ \frac{b}{x^2} = \lim_{x\to 0}\ b = 0$

Now $\ell = \lim_{x\to 0}\frac{1+a\cos x}{x^2} - \lim_{x\to 0}\frac{b\sin x}{x^3} = \lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}$

$= \lim_{x\to 0}\frac{2}{4}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = \frac{1}{2}.(1)^2 = \frac{1}{2}$

$\therefore (a, b) = (-1, 0)$ and $\ell = \frac{1}{2}$

9. Ans. (B)

10. Ans. (C)

Solution for Q.9 & Q.10

Let $a_n = x$ so question will be $\lim_{x\to 0}\frac{2^{2x}-2^{1+x}\cdot 3^x+3^{2x}}{\cos x+1-e^x-e^{-x}} = -\frac{\alpha}{3}\ell n^{\beta}\left(\frac{2}{3}\right)$

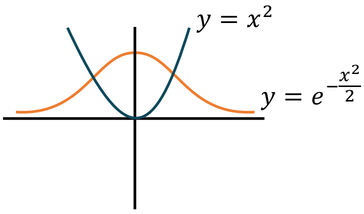
$\Rightarrow \lim_{x\to 0}\frac{\left(2^x-3^x\right)^2}{\cos x+1-e^x-e^{-x}} = -\frac{\alpha}{3}\ell n^{\beta}\left(\frac{2}{3}\right)$

$\Rightarrow \lim_{x\to 0}\frac{\frac{\left(2^x-3^x\right)^2}{x^2}}{\frac{\left(\cos x+1-e^x-e^{-x}\right)}{x^2}} = -\frac{\alpha}{3}\ell n^{\beta}\left(\frac{2}{3}\right)$

$\Rightarrow -\frac{2}{3}\left(\ln\frac{2}{3}\right)^2 = -\frac{\alpha}{3}\ell n^{\beta}\left(\frac{2}{3}\right)$

So, $\alpha = 2$ and $\beta = 2$

11. Ans (1)



$f(x) + g(x) = 0$

$e^{\frac{-x^2}{2}} - x^2 = 0$

$e^{\frac{-x^2}{2}} = x^2$

No. of solutions = 2

12. Ans. (B)

$$f(x)=\lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$$

$$\Rightarrow f(x)=\lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$$

$$\Rightarrow f(x)=e^{\left(\lim\limits_{n\to\infty} n\left(\cos\frac{x}{\sqrt{n}}1\right)\right)}$$

Replace $n\to\dfrac{1}{n}$

$$f(x)=e^{\left(\lim\limits_{n\to 0}\frac{\cos\sqrt{n}x-1}{n}\right)}$$

$$\Rightarrow e^{\left(\lim\limits_{n\to 0}\left(\frac{\cos\sqrt{n}x-1}{\left(\sqrt{n}x\right)^2}\right)\left(\sqrt{n}x\right)^2\right)}$$

$$\Rightarrow e^{-\frac{x^2}{2}} \qquad \left(\because \lim_{f(x)\to 0}\frac{\cos f(x)-1}{\left(f(x)\right)^2}=-\frac{1}{2}\right)$$

13. Ans. (2)

$$\ell im_{x\to 0^+} \quad f(x) = \ell im_{x\to 0^+} \quad \frac{\sin^{-1}(1-\{x\} \ . \ \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \quad (1-\{x\})} = \ell im_{h\to 0} \frac{\sin^{-1}(1-h) \ . \ \cos^{-1}(1-h)}{\sqrt{2h} \quad (1-h)}$$

$$= \ell im_{h\to 0} \frac{\sin^{-1}(1-h) \ \sin^{-1}\sqrt{h \ (2-h)}}{(1-h) \ \sqrt{2h}} = \ell im_{h\to 0} \frac{\frac{\pi}{2}}{1} \ \frac{\sin^{-1}\sqrt{h(2-h)}}{\sqrt{2h-h^2} \ \sqrt{2h}} \sqrt{2h-h^2}$$

$$= \ell im_{h\to 0} \frac{\pi}{2} \ . \ 1 \ . \ \sqrt{1-\frac{h}{2}}=\frac{\pi}{2}$$

$$\ell im_{x\to 0^-} \quad f(x) = \ell im_{x\to 0^-} \frac{\sin^{-1}(1-\{x\}) \ \cos^{-1}(1-\{x\})}{\sqrt{2 \ \{x\}} \quad (1-\{x\})} = \ell im_{h\to 0} \frac{\sin^{-1}h \ . \ \cos^{-1}h}{\sqrt{2(1-h)} \quad h} = \frac{\frac{\pi}{2}}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

14. Ans. (1)

$$\ell im_{x\to\infty}\left(x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right)\right)$$

$$\ell im_{x\to\infty}\left(\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}+\sin\left(\frac{1}{x^2}\right)\right) = 1 + 0 = 1$$

15. Ans. (2)

$$\ell im_{x\to 0}\frac{1-\cos x \ \sqrt{\cos \ 2x}}{x^2}$$

$$=\ell im_{x\to 0}\frac{1-\cos^2 x \ \ \cos 2x}{(1+\cos x \ \ \sqrt{\cos \ 2x} \ )x^2}$$

$$=\ell im_{x\to 0}\frac{1}{(1+\cos x \ \ \sqrt{\cos \ 2x} \ )} . \ell im_{x\to 0}\frac{1-\cos^2 x \ \ \cos 2x}{x^2}$$

$$= \frac{1}{2}\lim_{x\to 0}\frac{1-\left(\frac{1+\cos 2x}{2}\right)\cos 2x}{x^2} = \frac{1}{2}\lim_{x\to 0}\frac{2-\cos 2x-(\cos 2x)^2}{2x^2}$$

$$= \frac{1}{4}\lim_{x\to 0} - \frac{1}{x^2}(\cos 2x + 2)(\cos 2x - 1) = \frac{1}{4}\lim_{x\to 0}(\cos 2x + 2).\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$$

$$= \frac{(1+2)}{4}.\lim_{x\to 0}\frac{2\sin^2 x}{x^2} = \frac{3}{4}.2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = \frac{3}{2} \Rightarrow \text{limit} = \frac{3}{2}+\frac{1}{2} = 2$$

16. Ans. (1)

$\lim_{x\to\infty} f(x)$ exist and is finite & non zero

$$\lim_{x\to\infty}\left[f(x)+\frac{3f(x)-1}{f^2(x)}\right] = 3 \Rightarrow \lim_{x\to\infty} f(x) + \frac{3\lim_{x\to\infty} f(x)-1}{\left[\lim_{x\to\infty} f(x)\right]^2} = 3$$

Let $\lim_{x\to\infty} f(x) = A$

$$A + \frac{3A-1}{A^2} = 3 \Rightarrow A = 1$$

So, $\lim_{x\to\infty} f(x) = 1$

17. Ans. (0)

$\lim_{x\to 0} f(g(h(x)))$

L.H.L. $x \to 0^-$

$\lim_{x\to 0^-} h(x) = 0^+$

$\lim_{x\to 0^+} f(g(x))$

then $\lim_{x\to 0^+} g(x) = 1^+$

$\lim_{x\to 0^+} f(x) = 1 - 1 = 0$

R.H.L. $x \to 0^+$

$h(x) = 0^+$

so $\lim_{x\to 0^+} f(g(x)) = 0$

L.H.L. = R.H.L. = 0

18. Ans. (1)

$\lim_{x\to 0} g(f(x))$

L.H.L. = $\lim_{x\to 0^-} g[f(x)]$

$\lim_{x\to 0^-} g(sinx) = \lim_{h\to 0} g(\sin h) = \lim_{h\to 0}(\sin^2 h + 1) = 1$

R.H.L. = $\lim_{x\to 0^+} g[f(x)] = \lim_{x\to 0^+} g(\sin x) = \lim_{h\to 0} g(\sin h) = \lim_{h\to 0}(\sin^2 h + 1) = 1$

L.H.L. = R.H.L. = 1

So, $\lim_{x\to 0} g[f(x)] = 1$