Fundamentals of Algebra

SOLUTIONS

Exercise-I (JEE Main Pattern)

SECTION–A

1. **Ans. (1)**

$y=\sqrt{6+y}$

$y^2 = 6 + y$

$y^2 - y - 6 = 0$

$y^2 - 3y + 2y - 6 = 0$

$y(y-3) + 2(y-3) = 0$

$(y-3)(y+2) = 0$

$y = 3\ , y = -2$ Rejected.

2. **Ans. (1)**

$x=8-\sqrt{60}$

$=\left(\sqrt{5}-\sqrt{3}\right)^2$

$\sqrt{x}=\sqrt{\left(\sqrt{5}-\sqrt{3}\right)^2}=\sqrt{5}-\sqrt{3}$

$=\frac{1}{2}\left[\sqrt{5}-\sqrt{3}+\frac{2}{\sqrt{5}-\sqrt{3}}\right]$

$=\frac{1}{2}\left[\sqrt{5}-\sqrt{3}+\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}\right]$

$=\frac{1}{2}\left[\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3}\right] \ =\sqrt{5}$

3. **Ans. (4)**

$\frac{4+3\sqrt{5}}{4-3\sqrt{5}}=a+b\sqrt{5}$

$=\frac{4+3\sqrt{5}}{4-3\sqrt{5}}\times\frac{\left(4+3\sqrt{5}\right)}{4+3\sqrt{5}}$

$=\frac{4^2+\left(3\sqrt{5}\right)^2+2\times4\times3\sqrt{5}}{16-45}$

$=\frac{16+45+24\sqrt{5}}{-29}$

$=\frac{61+24\sqrt{5}}{-29}$

$=-\frac{61}{29}-\frac{24\sqrt{5}}{29}$

So, $a=-\frac{61}{29}$, $b=-\frac{24}{29}$

4. **Ans. (4)**

$x = 5 + 2\sqrt{6}$

$\sqrt{x} = \sqrt{5 + 2\sqrt{6}}$

$= \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2 + 2\sqrt{2}\sqrt{3}}$

$= \sqrt{\left(\sqrt{3} + \sqrt{2}\right)^2}$

$= \sqrt{3} + \sqrt{2}$

5. **Ans. (1)**

$\frac{4}{2 + \sqrt{3} + \sqrt{7}} \times \frac{\left(\left(2 + \sqrt{3}\right) - \sqrt{7}\right)}{\left(2 + \sqrt{3}\right) - \sqrt{7}}$

$= \frac{4\left(2 + \sqrt{3} - \sqrt{7}\right)}{\left(2 + \sqrt{3}\right)^2 - \left(\sqrt{7}\right)^2}$

$= \frac{4\left(2 + \sqrt{3} - \sqrt{7}\right)}{4 + 3 + 4\sqrt{3} - 7}$

$= \frac{2}{\sqrt{3}} + 1 - \sqrt{\frac{7}{3}}$

$= 1 + \sqrt{\frac{4}{3}} - \sqrt{\frac{7}{3}}$

On comparing

$\sqrt{a} = 1, \ \sqrt{b} = \sqrt{\frac{4}{3}}, \ \sqrt{c} = \sqrt{\frac{7}{3}}$

$a = 1, \ b = \frac{4}{3}, \ c = \frac{7}{3}$

6. **Ans. (1)**

$x^{\frac{1}{(a-b)(a-c)}} \cdot x^{\frac{1}{(b-c)(b-a)}} \cdot x^{\frac{1}{(c-a)(c-b)}}$

$x^{\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}}$

$x^{\frac{-(b-c)-(c-a)-(a-b)}{(a-b)(b-c)(c-a)}}$

$x^{\frac{-b+c-c+a-a+b}{(a-b)(b-c)(c-a)}}$

$x^0 = 1$

7. **Ans. (3)**

$$\left(2d^2e^{-1}\right)^3\left(\frac{d^3}{e}\right)^{-2}$$

$= 8\,d^6e^{-3}d^{-6}e^2$

$= 8\,d^{6-6}e^{-3+2}$

$= 8\,d^0\,e^{-1}$

$8\,e^{-1}$

8. **Ans. (2)**

$$a = x + \frac{1}{x}$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x\cdot\frac{1}{x}\left(x + \frac{1}{x}\right)$$

$= a^3 - 3(a)$

$= a^3 - 3a$

9. **Ans. (4)**

$$\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$$

$$4^{\frac{2x+\frac{1}{2}}{3}} = \left(\frac{1}{2}\right)^5$$

$$2^{\frac{2}{3}\left(2x+\frac{1}{2}\right)} = \left(\frac{1}{2}\right)^5$$

$$\frac{2}{3}\left(2x + \frac{1}{2}\right) = -5$$

$4x + 1 = -15$

$4x = -16$

$x = -4$

10. **Ans. (4)**

$$\left(5+2\sqrt{6}\right)^{x^2-3} + \left(5-2\sqrt{6}\right)^{x^2-3} = 10$$

let $\left(5+2\sqrt{6}\right)^{x^2-3} = t$

then $\left(5-2\sqrt{6}\right)^{x^2-3} = \frac{1}{t}$

$$t + \frac{1}{t} = 10$$

$t^2 - 10t + 1 = 0$

$$t = \frac{10 \pm \sqrt{100-4}}{2}$$

$= 5 \pm 2\sqrt{6}$

$$\left(5+2\sqrt{6}\right)^{x^2-3}=5+2\sqrt{6} \text{ or } \left(5+2\sqrt{6}\right)^{x^2-3}=5-2\sqrt{6}$$

$x^2-3=1$	$x^2-3=-1$
$x^2=4$	$x^2=2$
$x=\pm 2$	$x=\pm\sqrt{2}$

11. **Ans. (3)**

$$3^{2x^2}-2\cdot 3^{x^2+x+6}+3^{2(x+6)}=0$$

$$\left(3^{x^2}\right)^2-23^{x^2}\cdot 3^{x+6}+\left(3^{x+6}\right)^2=0$$

$$\left(3^{x^2}-3^{x+6}\right)^2=0$$

$$3^{x^2}=3^{x+6}$$

$$x^2=x+6$$

$$x^2-x-6=0$$

$$x^2-3x+2x-6=0$$

$$x(x-3)+2(x-3)=0$$

$$(x+2)(x-3)=0$$

$$x=-2, x=3$$

12. **Ans. (3)**

Multiple of 5 = 300 – 19 = 281

Multiple of 11 = 136 – 9 = 127

Multiple of 55 = 27 – 1 = 26

Total no of multiples of 5 or 11

= 281 + 127 – 26

= 382

13. **Ans. (2)**

$A=\{(x,y)|xy=8 \text{ and } x,y\in z\}$

(1, 8)	(8, 1)	(2, 4)	(4, 2)
(–1, –8)	(–8, –1)	(–2, –4)	(–4, –2)

Total 8 pairs.

14. **Ans. (3)**

$$\frac{x-2}{x+2}-\frac{2x-3}{4x-1}>0$$

$$\frac{(x-2)(4x-1)-(x+2)(2x-3)}{(x+2)(4x-1)}>0$$

$$\frac{4x^2-9x+2-(2x^2+x-6)}{(x+2)(4x-1)}>0$$

$$\frac{2x^2-10x+8}{(x+2)(4x-1)}>0$$

$$\frac{x^2-4x-x+4}{(x+2)(4x-1)}>0$$

$$\frac{(x-4)(x-1)}{(x+2)(4x-1)} > 0$$

+ −2 − $\frac{1}{4}$ + 1 − 4 +

$$(-\infty,-2)\cup\left(\frac{1}{4},1\right)\cup(4,\infty)$$

15. Ans. (4)

$$\frac{x^2-4}{x^2-5x+6} \le 0$$

$$\frac{(x-2)(x+2)}{(x-2)(x-3)} \le 0$$

$$\frac{x+2}{x-3} \le 0 \qquad (x \neq 2)$$

+ −2 − 3 +

$x \in [-2,3)-\{2\}$

16. Ans. (4)

$$\frac{(x+3)^2(x-1)^9(x+1)^5}{(x-3)(x-5)^4(x-6)^5} \le 0$$

Let $x \neq 3, 5, 6$

$\Rightarrow (x+3)^2$ and $(x-5)^4$ are always positive

$\Rightarrow (x+3)^2, (x-5)^4$ can be cross multiplied

$$\Rightarrow \frac{(x-1)^9(x+1)^5}{(x-3)(x-6)^5} \le 0$$

+ −1 − 1 + 3 − 6 +

$\Rightarrow x \in [-1,1] \cup (3,6)$

We have to exclude $x = 3,5$

$\Rightarrow x \in [-1,1] \cup (3,5) \cup (5,6)$

Integers contained are $x = -1,0,1,4$

At $x = -3$, L.H.S = 0

Inequality becomes $0 \le 0$

We have to include $x = -3$ in the final answer.

So, possible integral values of $x = -1,0,1,4,-3$

17. Ans. (1)

$\left(\sqrt{2}+1\right)^{x}+\left(\sqrt{2}-1\right)^{x}-2\sqrt{2}=0$

$(\sqrt{2}+1).(\sqrt{2}-1)=1$

$\Rightarrow\left(\sqrt{2}-1\right)=\dfrac{1}{\left(\sqrt{2}+1\right)}$

Let $\left(\sqrt{2}+1\right)^{x}=t$

$\Rightarrow\left(\sqrt{2}-1\right)^{x}=\left(\dfrac{1}{\sqrt{2}+1}\right)^{x}=\dfrac{1}{\left(\sqrt{2}+1\right)^{x}}=\dfrac{1}{t}$

$\Rightarrow t+\dfrac{1}{t}=2\sqrt{2}$ $\quad\Rightarrow t^{2}-2\sqrt{2}t+1=0$ $\quad\Rightarrow t_1t_2=1$

$\Rightarrow(\sqrt{2}+1)^{x_1}.(\sqrt{2}+1)^{x_2}=1$ $\quad\Rightarrow(\sqrt{2}+1)^{x_1+x_2}=1$ $\quad\Rightarrow x_1+x_2=0$

18. Ans. (1)

$x^2+y^2+4z^2-6x-2y-4z+11=0$

$\Rightarrow(x-3)^2+(y-1)^2+(2z-1)^2=0$

$\Rightarrow x=3, y=1, z=\dfrac{1}{2}$ $\quad\Rightarrow xyz=\dfrac{3}{2}$

19. Ans. (2)

$\dfrac{\sqrt{3}+4\sqrt{2}}{4\sqrt{2}-\sqrt{3}}=\dfrac{a+b\sqrt{6}}{c}$

$\Rightarrow\dfrac{\left(4\sqrt{2}+\sqrt{3}\right)^{2}}{\left(4\sqrt{2}\right)^{2}-\left(\sqrt{3}\right)^{2}}$ $\quad\Rightarrow\dfrac{35+8\sqrt{6}}{29}$

$\Rightarrow a=35, b=8, c=29$ $\quad\Rightarrow a+b+c=72$

20. Ans. (2)

$x^2+4y^2+z^2-2xy-2yz-zx=0$

$\Rightarrow\dfrac{1}{2}\left((x-2y)^{2}+(2y-z)^{2}+(x-z)^{2}\right)=0$

$\Rightarrow x-2y=0, 2y-z=0, x-z=0$

$\Rightarrow x=2y=z$ $\quad\Rightarrow x:y:z=2:1:2$

SECTION-B

1. Ans. (9.00)

$a-\dfrac{1}{a}=\dfrac{1}{2}$

$\Rightarrow a^{2}+\dfrac{1}{a^{2}}-2=\dfrac{1}{4}\Rightarrow a^{2}+\dfrac{1}{a^{2}}=\dfrac{9}{4}\Rightarrow 4a^{2}+\dfrac{4}{a^{2}}=9$

2. Ans. (4)

Let $p(x)=Ax^3+4x^2+13x+5$

$\Rightarrow p(1)=p(-2)$ $\quad\Rightarrow(A+B+9)=-8A+16-2B+5$

$\Rightarrow 9A+3B=12\Rightarrow 3A+B=4$

3. **Ans. (12.80)**

$9^x + 6^x = 2.4^x$

$\Rightarrow (3x)^2 + 2^x.3^x = 2.(2^x)^2 \quad \Rightarrow (3^x)^2 + 2^x.3^x - 2.(2^x)^2 = 0$

$\Rightarrow (3^x)^2 + 2.(2^x.3^x) - (2^x.3^x) - 2.(2^x)^2 = 0$

$\Rightarrow 3^x(3^x + 2.2^x) - 2^x(3^x + 2.2^x) = 0 \quad \Rightarrow (3^x - 2^x)(3^x + 2.2^x) = 0$

$\Rightarrow 3^x - 2^x = 0$ or $3^x + 2.2^x = 0$ (not possible)

$\Rightarrow 3^x = 2^x \Rightarrow x = 0 \quad \Rightarrow \frac{x^3+64}{5} = \frac{64}{5} = 12.80$

4. **Ans. (5.5)**

$a + b + c = 6$

$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 36$

$\Rightarrow 2(ab + bc + ca) = (36 - 14) \quad \Rightarrow ab + bc + ca = 11$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow 36 - 3abc = 6(14 - 11) \quad \Rightarrow 3abc = 18 \quad \Rightarrow abc = 6$

5. **Ans. (216.50)**

$x + y + z = 12$

$\Rightarrow x^2 + y^2 + z^2 = 96 \quad \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 36 \quad \Rightarrow xy + yz + zx = 36xyz$

$\Rightarrow (x + y + z)^2 = 144$

$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 144$

$\Rightarrow 96 + 2(xy + yz + zx) = 144 \quad \Rightarrow xy + yz + zx = 24$

$\Rightarrow (x^3 + y^3 + z^3) - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow (x^3 + y^3 + z^3) - 3xyz = (12)(96 - 24)$

$\Rightarrow (x^3 + y^3 + z^3) = 12 \times 72 + 3xyz$

$= 12 \times 72 + 3 \times \left(\frac{24}{36}\right)$

$= 12 \times 72 + 2 = 866 \quad \Rightarrow \frac{866}{4} = \frac{433}{2} = 216.50$

6. **Ans. (36)**

$x = \sqrt{\left(\sqrt{3}\right)^2 + 1^2 - 2 \times \sqrt{3 \times 1}}$

$\Rightarrow x = \sqrt{3} - 1$

$y = \sqrt{(5)^2 + 2^2 - 2 \times \sqrt{5} \times 2} \quad \Rightarrow y = (\sqrt{5} - 2)$

$\Rightarrow \left(\sqrt{5}x - \sqrt{3}y\right)^2 = \left(\sqrt{15} - \sqrt{5} - \sqrt{15} + 2\sqrt{3}\right)^2$

$= \left(2\sqrt{3} - \sqrt{5}\right)^2$

$= 12 + 5 - 4\sqrt{15}$

$= 17 - 4\sqrt{15}$

$\Rightarrow a = 17, b = 4, c = 15$

$\Rightarrow a + b + c = 36$

7. **Ans. (16)**

$(11x)^2 + (2y)^2 + (3z)^2 - 22x + 4y + 6z + 3 = 0$

$\Rightarrow (11x-1)^2 + (2y+1)^2 + (3z+1)^2 = 0$

$\Rightarrow 11x - 1 = 2y + 1 = 3z + 1 = 0 \quad \Rightarrow x = \frac{1}{11}, y = \frac{-1}{2}, z = \frac{-1}{3}$

$\Rightarrow x^{-1} - y^{-1} - z^{-1} = 11 + 2 + 3 = 16.$

8. **Ans. (4)**

$$\frac{p^2}{q^2} = \frac{(x+2)^2}{(x-2)^2}$$

Apply componendo and dividendo,

$$\Rightarrow \frac{p^2-q^2}{p^2+q^2} = \frac{(x+2)^2-(x-2)^2}{(x+2)^2+(x-2)^2} \Rightarrow \frac{p^2-q^2}{p^2+q^2} = \frac{8x}{2(x^2+4)} \Rightarrow \left(\frac{p^2-q^2}{p^2+q^2}\right)\left(\frac{x^2+4}{x}\right) = 4$$

9. **Ans. (1)**

$$\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}} = \frac{5}{1}$$

Apply componendo and dividendo,

$$\Rightarrow \frac{\left(3x+\sqrt{9x^2-5}\right)+\left(3x-\sqrt{9x^2-5}\right)}{\left(3x+\sqrt{9x^2-5}\right)-\left(3x-\sqrt{9x^2-5}\right)} = \frac{5+1}{5-1}$$

$$\Rightarrow \frac{6x}{2\sqrt{(9x^2-5)}} = \frac{3}{2}$$

$\Rightarrow 12x = 6\sqrt{9x^2-5} \Rightarrow 2x = \sqrt{9x^2-5}$

$\Rightarrow 4x^2 = 9x^2 - 5 \Rightarrow 5x^2 = 5$

$\Rightarrow x = \pm 1$

Put $x = -1$ in initial equation,

L.H.S. = $\frac{-3+2}{-3-2} = \frac{-1}{5} = \frac{1}{5}$

So, $x = -1$ is rejected

Only possible value of x is 1.

10. **Ans. (2)**

$$x = \frac{\sqrt{6}}{\left(\sqrt{3}-\sqrt{2}\right)+x}$$

$\Rightarrow x^2 + (\sqrt{3}-\sqrt{2})x - \sqrt{6} = 0$

$\Rightarrow x(x+\sqrt{3}) - \sqrt{2}(x+\sqrt{3}) = 0$

$\Rightarrow (x-\sqrt{2})(x+\sqrt{3}) = 0$

$\Rightarrow x = \sqrt{2}, x = -\sqrt{3}$

But x cannot be negative.

$\Rightarrow x = \sqrt{2} \Rightarrow x^2 = 2$

Exercise - II (JEE-Main & JEE Advanced PYQs)

1. **Ans. (B)**

$$y=\frac{12}{\left(3+2\sqrt{2}\right)+\sqrt{5}}\times\frac{\left(3+2\sqrt{2}\right)-\sqrt{5}}{\left(3+2\sqrt{2}\right)-\sqrt{5}}$$

$$\Rightarrow \frac{12\left(3+2\sqrt{2}-\sqrt{5}\right)}{\left(3+2\sqrt{2}\right)^2-5} \Rightarrow \frac{12\left(3+2\sqrt{2}-\sqrt{5}\right)}{9+8+12\sqrt{2}-5} \Rightarrow \frac{12\left(3+2\sqrt{2}-\sqrt{5}\right)}{12\left(\sqrt{2}+1\right)}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$\Rightarrow \frac{3\sqrt{2}+4-\sqrt{10}-3-2\sqrt{2}+\sqrt{5}}{2-1} \Rightarrow 1+\sqrt{5}+\sqrt{2}-\sqrt{10}$$

2. **Ans.** $x=-\frac{1}{4}, y=-\frac{1}{4}$

$x+y+\frac{x}{y}=\frac{1}{2}$ ...(i) $x<0, y<0$

$\Rightarrow (x+y)\frac{x}{y}=-\frac{1}{2}$ ...(ii)

$$\Rightarrow \frac{x^2}{y}+x=-\frac{1}{2} \Rightarrow \frac{x^2}{y}=-\left(x+\frac{1}{2}\right) \Rightarrow \frac{x^2}{y}=-\left(\frac{2x+1}{2}\right)$$

$\Rightarrow y=-\left(\frac{2x^2}{2x+1}\right)$ ...(iii)

Put value of 'y' in equation (i)

$$\Rightarrow x-\left(\frac{2x^2}{2x+1}\right)-\frac{x.(2x+1)}{2x^2}=\frac{1}{2} \qquad \Rightarrow \left[\frac{2x^2+x-2x^2}{(2x+1)}\right]-\left(\frac{2x+1}{2x}\right)=\frac{1}{2}$$

$$\Rightarrow \frac{x}{2x+1}-\frac{2x+1}{2x}=\frac{1}{2} \qquad \Rightarrow \frac{2x^2-(2x+1)^2}{2x(2x+1)}=\frac{1}{2}$$

$$\Rightarrow 2x^2-\left(4x^2+4x+1\right)=2x^2+x \qquad \Rightarrow 4x^2+5x-1=0$$

$\Rightarrow (4x+1)(x-1)=0$

$\Rightarrow x-1=0 \qquad 4x+1=0$

$\Rightarrow x=1 \qquad x=-\frac{1}{4}\ [\because x<0]$

Form equation (iii)

$$\Rightarrow y=-\left(\frac{2\times\frac{1}{16}}{-\frac{1}{2}+1}\right)$$

$$\Rightarrow y=-\frac{1}{4} \Rightarrow x-\frac{2}{x-1}=1-\frac{2}{x-1}$$

$x=1,\ x\neq 1$

So, no solution

3. **Ans. (A)**

$$\frac{x^2-x-2}{x-1}=\frac{x-1-2}{x-1}$$

$$\Rightarrow \frac{x^2-x-2}{x-1}=\frac{x-3}{x-1} \Rightarrow \frac{x^2-x-2}{x-1}-\frac{x-3}{x-1}=0$$

$$\Rightarrow \frac{x^2-x-2-x+3}{x-1}=0 \Rightarrow \frac{x^2-2x+1}{x-1}=0$$

$$\Rightarrow \frac{(x-1)^2}{x-1}=0$$

No root exists.

4. **Ans.** $x\in\left[1-\sqrt{5},1\right)\cup\left(2,1+\sqrt{5}\right]$

$x^2-3x+2>0$

$(x-2)(x-1)>0$

$$\underset{-\infty}{}\ \ +ve\ \underset{1}{|}\ -ve\ \underset{2}{|}\ +ve\ \ \underset{\infty}{}$$

$x=1\pm\sqrt{5}$

$x\in(-\infty,1)\cup(2,\infty)$

$x^2-2x-4\le 0$

$$x=\frac{2\pm\sqrt{4+16}}{2}$$

$$x=\frac{2\pm2\sqrt{5}}{2}$$

$x\in\left[1-\sqrt{5},1+\sqrt{5}\right]$

$\therefore$ solution

$x\in\left[1-\sqrt{5},1\right)\cup\left(2,1+\sqrt{5}\right)$

5. **Ans.** $(-2,-1)\cup\left(-\frac{2}{3},-\frac{1}{2}\right)$

$$\frac{2x}{2x^2+5x+2}>\frac{1}{x+1}$$

$$\Rightarrow \frac{2x}{(2x+1)(x+2)}>\frac{1}{x+1} \Rightarrow \frac{2x}{(2x+1)(x+2)}>\frac{1}{x+1}$$

$$\Rightarrow \frac{2x}{(2x+1)(x+2)}-\frac{1}{(x+1)}>0$$

$$\Rightarrow \frac{2x^2+2x-2x^2-5x-2}{(2x+1)(x+2)(x+1)}>0$$

$$\Rightarrow \frac{-3x-2}{(2x+1)(x+2)(x+1)}>0$$

$$\Rightarrow \frac{3x+2}{(2x+1)(x+2)(x+1)}<0$$

$$\underset{-\infty}{}\ \ +ve\ \underset{-2}{|}\ -ve\ \underset{-1}{|}\ +ve\ \underset{-\frac{2}{3}}{|}\ -ve\ \underset{-\frac{1}{3}}{|}\ +ve\ \ \underset{\infty}{}$$

$$\Rightarrow x\in(-2,-1)\cup\left(-\frac{2}{3},-\frac{1}{2}\right)$$

6. **Ans. (4)**

$|x-2|^2 + |x-2| - 2 = 0$

Case – I

$\Rightarrow x > 2 \qquad \Rightarrow (x-2)^2 + (x-2) - 2 = 0$

$\Rightarrow x^2 - 4x + 4 + x - 2 - 2 = 0 \qquad \Rightarrow x^2 - 3x = 0$

$\Rightarrow x(x-3) = 0 \qquad \Rightarrow x = 0,3$

$x = 3 \quad \because x > 2$

Case – II

$\Rightarrow x < 2$

$\Rightarrow (x-2)^2 - (x-2) - 2 = 0$

$\Rightarrow x^4 + 4x + 4 - x + 2 - 2 = 0$

$\Rightarrow x^2 - 5x + 4 = 0 \qquad \Rightarrow (x-4)(x-1) = 0$

$\Rightarrow x = 1,4$

$\Rightarrow x = 1 \qquad \because x < 2$

Sum of all real roots = 4

7. **Ans. (A)**

$|x|^2 - 3|x| + 2 = 0$

Case – I

$x > 0$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 1,2$

Case – II

$x < 0$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -1,-2$

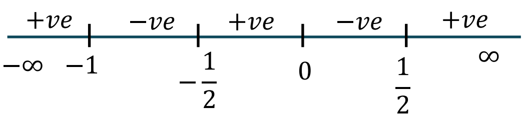
Number of real solution = 4

8. **Ans. (A,D)**

$$\frac{2x-1}{2x^3+3x^2+x} > 0$$

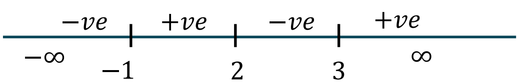
$$\Rightarrow \frac{2x-1}{x(2x^2+3x+1)} > 0 \Rightarrow \frac{2x-1}{x(2x+1)(x+1)} > 0$$

$$\Rightarrow x \in (-\infty,-1) \cup \left(-\frac{1}{2},0\right) \cup \left(\frac{1}{2},\infty\right)$$



9. **Ans. $x \in [-1, 2) \cup [3, \infty)$**

$$y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$$



$x \in [-1,2) \cup [3,\infty)$

10. Ans. (3)

$\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$

$\left|\sqrt{x}-2\right|+\left(\sqrt{x}\right)^2-4\sqrt{x}+2=0$

$\left|\sqrt{x}-2\right|^2+\left|\sqrt{x}-2\right|-2=0$

$\left|\sqrt{x}-2\right|=-2$ (not possible) or $\left|\sqrt{x}-2\right|=1$

$\sqrt{x}-2=1,-1$

$\sqrt{x}=3,1$

x = 9, 1

Sum = 10

11. Ans. (4)

Let $3^x = t$; t > 0

t(t – 1) + 2 = |t – 1| + |t – 2|

t^2 – t + 2 = |t – 1| + |t – 2|

Case-I : t < 1

t^2 – t + 2 = 1 – t + 2 – t

t^2 + 2 = 3 – t

t^2 + t – 1 = 0

$t=\frac{-1\pm\sqrt{5}}{2}$

$t=\frac{\sqrt{5}-1}{2}$ is only acceptable

Case-II : 1 ≤ t < 2

t^2 – t + 2 = t – 1 + 2 – t

t^2 – t + 1 = 0

D < 0 no real solution

Case-III : t ≥ 2

t^2 – t + 2 = t – 1 + t – 2

t^2 – 3t 5 = 0 ⇒ D < 0 no real solution

(4) Option

12. Ans. (2)

$9x^2$ – 18|x| + 5 = 0

$9|x|^2$ – 15|x| – 3|x| + 5 = 0 (∵ $x^2 = |x|^2$)

3|x| (3|x| – 5) – (3|x| – 5) = 0

$|x| = \frac{1}{3}, \frac{5}{3}$

$x = \pm\frac{1}{3}, \pm\frac{5}{3}$

Product of roots = $\frac{25}{81}$

13. **Ans. (4)**

$e^{4x} + e^{3x} - 4e^x + e^x + 1 = 0$

Divide by e^{2x}

$\Rightarrow e^{2x} + e^x - 4 + \frac{1}{e^x} + \frac{1}{e^{2x}} = 0 \qquad \Rightarrow \left(e^{2x} + \frac{1}{e^{2x}}\right) + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

$\Rightarrow \left(e^x + \frac{1}{e^x}\right)^2 - 2 + \left(e^x + \frac{1}{e^x}\right) - 4 = 0$

Let $e^x + \frac{1}{e^x} = t \Rightarrow (e^x - 1)^2 = 0 \Rightarrow x = 0.$

$\therefore$ Number of real roots = 1

14. **Ans. (2)**

$t^4 - t^3 - 4t^2 - t + 1 = 0,\ e^x = t > 0$

$\Rightarrow t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0 \qquad \Rightarrow \alpha^2 - \alpha - 6 = 0,\ \alpha = t + \frac{1}{t} \geq 2$

$\Rightarrow \alpha = 3, -2$ (reject)

$\Rightarrow t + \frac{1}{t} = 3$

$\Rightarrow$ The number of real roots = 2

15. **Ans. (2)**

Case-I

$x \leq 5$

$(x + 1)^2 - (x - 5) = \frac{27}{4}$

$(x + 1)^2 - (x + 1) - \frac{3}{4} = 0$

$x + 1 = \frac{3}{2}, -\frac{1}{2}$

$x = \frac{1}{2}, -\frac{3}{2}$

Case-II

$x > 5$

$(x + 1) + (x - 5) = \frac{27}{4}$

$(x + 1)^2 + (x + 1) - \frac{51}{4} = 0$

$x = \frac{-1 \pm \sqrt{52}}{2}$ (rejected as $x > 5$)

So, the equation have two real root.

16. Ans. (1)

$|x|^2 - |x| - 12 = 0$

$(|x| + 3)(|x| - 4) = 0$

$|x| = 4 \Rightarrow x = \pm 2$

17. Ans. (2)

$e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$

Let $f(x) = e^{2x}\left(e^{2x} + \frac{1}{e^{2x}} + 4\left(e^x + \frac{1}{e^x}\right) - 58\right)$

$e^x + \frac{1}{e^x}$

Let $h(t) = t^2 + 4t - 58 = 0$

$t = \frac{-4 \pm \sqrt{16 + 4.58}}{2}$

$\frac{-4 \pm 2\sqrt{62}}{2}$

$t_1 = -2 + 2\sqrt{62}$

$t_2 = -2 - 2\sqrt{62}$ (not possible)

$t \geq 2$

$e^x + \frac{1}{e^x} = -2 + 2\sqrt{62}$

$e^{2x} - (-2 + 2\sqrt{62})\, e^x + 1 = 0$

$(-2 + 2\sqrt{62}) - 4$

$4 + 4.62 - 8\sqrt{62} - 4$

$248 - 8\sqrt{62} > 0$

$\frac{-b}{2a} > 0$

both roots are positive

2 real roots

18. Ans. (1)

For $x \leq 3 \, or \, x \geq 5$

$x^2 - 8x + 15 - 2x + 7 = 0$

$x = 5 + \sqrt{3}$

For $3 < x < 5$, $x^2 - 8x + 15 + 2x - 7 = 0$

$x = 4$

Hence sum $= 9 + \sqrt{3}$

19. **Ans. (2)**

x |x| – 5 |x+2| + 6 = 0

C–1 :- x∈ [0,∞]

$x^2 - 5x - 4 = 0$

$$x=\frac{5\pm\sqrt{25+16}}{2}=\frac{5+\sqrt{41}}{2}$$

$$x=\frac{5\pm\sqrt{41}}{2}$$

C–2 :- :- x∈ [–2, 0)

$-x^2 - 5x - 4 = 0$

$x^2 + 5x + 4 = 0$

x = –1, –4

x = –1

C–3 : x∈ [–∞, –2)

$-x^2 + 5x + 16 = 0$

$x^2 - 5x - 16 = 0$

$$x=\frac{5\pm\sqrt{25+64}}{2}$$

$$\frac{5\pm\sqrt{89}}{2}$$

$$x=\frac{5-\sqrt{89}}{2}$$

20. **Ans. (2)**

$$3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0$$

$$3\left[\left(x+\frac{1}{x}\right)^2-2\right]-2\left(x+\frac{1}{x}\right)+5=0$$

Let $x+\frac{1}{x}=t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

3t(t – 1) + 1(t – 1) = 0

(t – 1) (3t + 1) = 0

$$t=1,-\frac{1}{3}$$

$x+\frac{1}{x}=1,-\frac{1}{3}\Rightarrow$ No solution.

21. **Ans. (2)**

Let $e^{2x}=t$

$\Rightarrow t^4-t^3-3t^2-t+1=0$ $\qquad$ $\Rightarrow t^2+\frac{1}{t^2}-\left(t+\frac{1}{t}\right)-3=0$

$\Rightarrow \left(t+\frac{1}{t}\right)^2-\left(t+\frac{1}{t}\right)-5=0$ $\qquad$ $\Rightarrow t+\frac{1}{t}=\frac{1+\sqrt{21}}{2}$

Two real values of t.