Functions

SOLUTIONS

EXERCISE - O

1. Ans. (B)

$P(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

$\Rightarrow P(1) = 1, P(2) = 2, P(3) = 3, P(4) = 4, P(5) = 5, P(6) = 6$

Let $P(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6) + x$

according to given conditions.

$\Rightarrow P(7) = (7-1)(7-2)(7-3)(7-4)(7-5)(7-6) + 7$

$\Rightarrow P(7) = 727$

2. Ans. (D)

$\Rightarrow 2^x + 2^y = 2$

$\Rightarrow 2^y = 2 - 2^x$

$\Rightarrow 2^y > 0$ (exponential function)

$\Rightarrow 2 - 2^x > 0$

$\Rightarrow 2^x < 2 \Rightarrow x < 1$

$\Rightarrow x \in (-\infty, 1)$

3. Ans. (B)

Given : $f(x) = \log_2\left(\dfrac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$

Here, denominator is

$\Rightarrow \sqrt{x+2}+\sqrt{2-x} > 0$

Also,

$\Rightarrow x + 2 \geq 0$ and $2 - x \geq 0$

$\Rightarrow x \geq -2$ $\qquad x \leq 2$

$\therefore x \in [-2, 2]$

Let $g(x) = \sqrt{x+2} + \sqrt{2-x}$

$\Rightarrow g'(x) = \dfrac{1}{2\sqrt{x+2}} - \dfrac{1}{2\sqrt{2-x}} = 0$ [Diff. and equating to zero]

$\Rightarrow \dfrac{\sqrt{2-x}-\sqrt{2+x}}{2\sqrt{x+2}\sqrt{2-x}} = 0$

$\Rightarrow \sqrt{2-x} - \sqrt{2+x} = 0$

$\Rightarrow \sqrt{2-x} = \sqrt{2+x}$

$\Rightarrow 2 - x = 2 + x$ (squaring)

Now $f(0) = 2\sqrt{2}$

$\Rightarrow g(-2) = 2$

$\Rightarrow g(2) = 2$

$\therefore 2 \leq g(x) \leq 2\sqrt{2}$

$$\Rightarrow \frac{1}{2\sqrt{2}} \le \frac{1}{g(x)} \le \frac{1}{2}$$

$$\Rightarrow \frac{4}{2\sqrt{2}} \le \frac{4}{g(x)} \le \frac{4}{2}$$

$$\Rightarrow \frac{2}{\sqrt{2}} \le \frac{4}{g(x)} \le 2$$

$$\Rightarrow \log_2 \sqrt{2} \le \log_2 \frac{4}{g(x)} \le \log_2 2$$

$\therefore$ Range is $\left[\log_2 \sqrt{2}, \log_2 2\right] = \left[\frac{1}{2}, 1\right]$

4. **Ans. (B)**

$$f(x) = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1} \qquad x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow f(x) = \frac{1 + \sin x - \cos x}{\sin x - 1 + \cos x}$$

$$\Rightarrow f(x) = \frac{(1 - \cos x) + \sin x}{\sin x - (1 - \cos x)}$$

$$\Rightarrow f(x) = \frac{2\sin^2 \frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2} - 2\sin^2\left(\frac{x}{2}\right)}$$

$$\Rightarrow f(x) = \frac{\sin\frac{x}{2} + \cos\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}} \Rightarrow \frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}$$

$$\Rightarrow f(x) = \frac{\cos\frac{x}{2}\left(1 + \tan\frac{x}{2}\right)}{\cos\frac{x}{2}\left(1 - \tan\frac{x}{2}\right)} \Rightarrow \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$\Rightarrow f(x) = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2} \in \left(0, \frac{\pi}{4}\right)$$

$$\Rightarrow \frac{\pi}{4} + \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\Rightarrow \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \in (1, \infty)$$

Range $f(x)$ will be $(1, \infty)$

5. Ans. (D)

$f(x)=\frac{x}{1-x}$ also $x_0 = \alpha$

$\Rightarrow x_1 = f(x_0) = f(\alpha) = \frac{\alpha}{1-\alpha}$

$\Rightarrow x_2 = f(x_1) = f\left(\frac{\alpha}{1-\alpha}\right) = \frac{\frac{\alpha}{1-\alpha}}{1-\frac{\alpha}{1-\alpha}} = \frac{\alpha}{1-2\alpha}$

$\Rightarrow x_3 = f(x_2) = f\left(\frac{\alpha}{1-2\alpha}\right) = \frac{\frac{\alpha}{1-2\alpha}}{1-\frac{\alpha}{1-2\alpha}} = \frac{\alpha}{1-3\alpha}$

Similarly

$\Rightarrow x_{2011} = \frac{\alpha}{1-2011\alpha} = \frac{-1}{2012}$

$\Rightarrow 2012\alpha = 2011\alpha - 1$

$\boxed{\alpha = -1}$

6. Ans. (A)

$f_3(x) = [(2013)^{-1}]^{\log_{2013}\log_x 2012}$

$\Rightarrow f_3(x) = (\log_x 2012)\log_{2013}(2013)^{-1}$

$\Rightarrow f_3(x) = (\log_x 2012)^{-1} \Rightarrow \log_{2012} x$

$\Rightarrow f_2(x) = 2012^{\log_{2012} x} = x^{\log_{2012} 2012}$

$\Rightarrow f_2(x) = x$

$\Rightarrow f_1(x) = 2^x$

where x can be $(0,1) \cup (1,\infty)$

but $\log_x 2012 > 0$

Case – I

$\Rightarrow 0 < x < 1$

$\Rightarrow \log_x 2012 > 0$

$\Rightarrow 2012 < x^0$

$\Rightarrow 2012 < 1$

$\therefore$ no Solution

Case – II

$\Rightarrow x > 1$

$\Rightarrow \log_x 2012 > 0$

$\Rightarrow 2012 > x^0$

$\Rightarrow 2012 > 1$

$\therefore$ Complete set Solution

$\therefore$ for $x > 1$

2^x lies in $(2,\infty)$

7. Ans. (C)

Domain:- $2x + 15 \neq 0 \Rightarrow x \neq -\frac{15}{2}$

Let $f(x) = y = \frac{x+10}{2x+15}$

$\Rightarrow 2xy + 15y = x + 10$

$\Rightarrow 15y - 10 = x - 2xy$

$\Rightarrow x = \frac{15y-10}{1-2y}$

$\Rightarrow 1 - 2y \neq 0$

$\Rightarrow \boxed{y \neq \frac{1}{2}}$

Range = $R - \left\{\frac{1}{2}\right\}$ = Codomain

$\therefore f(x)$ is onto

$\Rightarrow f(x) = \frac{x+10}{2x+15}$

$\Rightarrow f'(x) = \frac{(2x+15)1-(x+10)(2)}{(2x+15)^2}$

$\Rightarrow = \frac{2x+15-2x-20}{(2x+15)^2}$

$= \frac{-5}{(2x+15)^2} < 0$

$\therefore f(x)$ is decreasing

$f(x)$ is one-one

$\therefore$ one-one and onto.

8. Ans. (D)

$f(x) = \frac{2x^2-5x+3}{8x^2+9x+11}$

$\Rightarrow f(x) = \frac{(x-1)(2x-3)}{8x^2+9x+11}$

$\Rightarrow f(1) = f\left(\frac{3}{2}\right) = 0$

Here, many one

$\Rightarrow y = \frac{2x^2-5x+3}{8x^2+9x+11}$

$\Rightarrow 8yx^2 + 9xy + 11y = 2x^2 - 5x + 3$

$\Rightarrow (8y-2)x^2 + (9y+5)x + 11y - 3 = 0$

$D \equiv (9y+5)^2 - 4(8y-2)(11y-3) \geq 0$

$81y^2 - 4(88y^2) + 90y + 46(4)y + 1 \geq 0$

$271y^2 - 274y - 1 \leq 0$

$\Rightarrow \alpha \leq y \leq \beta$

Where α, β are roots of $271y^2 - 274y - 1 = 0$

So, into

9. Ans. (B)

$$(f-g)(x)=f(x)-g(x)=\begin{cases} x+3+x & x\in Q \\ 4x-x-\sqrt{5} & x\notin Q \end{cases}$$

$$\Rightarrow (f-g)(x)=\begin{cases} 2x+3 & x\in Q \\ 3x-\sqrt{5} & x\notin Q \end{cases}$$

$$\Rightarrow (f-g)\left(\frac{-3}{2}\right)=0=(f-g)\left(\frac{\sqrt{5}}{3}\right)$$ many – one

Also, $(f-g)(x)=3-\sqrt{5}$ is not possible. As $3x-\sqrt{5}$ will not allow $x=1$

So, into

10. Ans. (B)

$f(x)=\sqrt{x-2}+\sqrt{4-x}$

$\Rightarrow x\geq 2$ & $4-x\leq 0$

Domain $x\in[2,4]$

For one – one $x\in[3,4]$

$\Rightarrow [f(x)]^2=x-2+4-x+2\sqrt{-x^2+6x-8}$

$\Rightarrow \{f(x)\}^2=2+2\sqrt{1-(x-3)^2}$

$\Rightarrow \{f(x)\}^2$ max at $x=3$

$\Rightarrow \{f(x)\}^2_{max}=2+2=4$

$\Rightarrow \{f(x)\}_{max}=2$

And $\{f(x)\}^2_{min}$ at $x=2,4$

$\Rightarrow \{f(x)\}^2_{min}=2$

$\{f(x)\}_{min}=\sqrt{2}$

Range $y=f(x)\in[\sqrt{2},2]$

11. Ans. (B)

$$f(x)=\frac{\sin([x]\pi)}{x^2+2x+3}+2x-1+\sqrt{x^2-x+\frac{1}{4}}$$

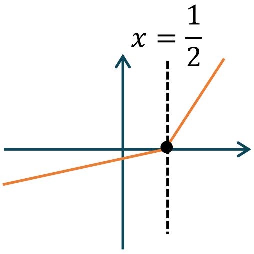
As $\sin(n\pi)=0$ $\quad$ $[x]=$ integer

$$\Rightarrow f(x)=2x-1+\left|x-\frac{1}{2}\right|$$

$$\Rightarrow f(x)=\begin{cases} 3x-\frac{3}{2} & x\geq\frac{1}{2} \\ x-\frac{1}{2} & x<\frac{1}{2} \end{cases}$$

Domain $\in R$

Range $\in R$



Clearly one – one point

12. Ans. (D)

$$y=f(x)=\begin{cases}0+(x-1)^1=x-1 & x\in(1,2)\\ 1+(x-2)^2=x^2-4x+5 & x\in(2,3)\end{cases}$$

Now, $y=x-1\Rightarrow x=y+1$

& $y=1+(x-2)^2\Rightarrow x=2+\sqrt{y-1}$

$f^{-1}(x)=2+\sqrt{x-1}, x\in[1,2]$

13. Ans. (C)

$f(x)=1+\frac{3}{2}\sqrt{x}$ $\qquad$ $g(x)=f^{-1}(x)$

$f(x)$ & f^{-1} intersect at $y=x$

$\Rightarrow f(x)=x\Rightarrow 1+\frac{3}{2}\sqrt{x}=x\Rightarrow 2x-3\sqrt{x}-2=0$

$\Rightarrow \sqrt{x}=2,\frac{-1}{2}$

$\therefore x=4$

$\Rightarrow f(4)=4$

14. Ans. (C)

(A) $f(x)=|x-2|+(x+2)sgn(x+2)$

$\Rightarrow f(-x)=|-x-2|+(-x+2)sgn(-x+2)$

$\Rightarrow f(x)\neq -f(-x)$

(B) $f(x)=\frac{1}{x(e^x-1)}+\frac{1}{2x}$

$\Rightarrow f(-x)=\frac{1}{-xe^{-x}-1}-\frac{1}{2x}$

$=-\frac{1}{x}\left(\frac{e^x}{1-e^x}+\frac{1}{2}\right)$

$\Rightarrow f(x)\neq -f(-x)$

(C) $f(x)=\log\left(\sin x+\sqrt{1+\sin^2 x}\right)$

$\Rightarrow f(-x)=\log\left(-\sin x+\sqrt{1+\sin^2 x}\right)$

$\Rightarrow f(x)+f(-x)=\log(1+\sin^2 x-\sin^2 x)$

$\Rightarrow \log 1=0$

$\Rightarrow f(x)=-f(x)$ $\qquad$ So Odd function.

(D) $f(x)=e^{-4x}(e^{2x}-1)^4$

$\Rightarrow f(-x)=e^{4x}(e^{-2x}-1)^4$

$=e^{4x}\frac{\left(1-e^{2x}\right)^4}{e^{8x}}$

$=e^{-4x}(e^{2x}-1)^4=f(x)$ even function

15. Ans. (A)

$f(10+x) = f(10-x) \quad \forall x \in R$ ...(1)

$\Rightarrow f(20+x) = -f(20-x)$ ...(2)

Put $x = x - 10$ in eq. (1)

$\Rightarrow f(x) = f(20-x)$ ...(3)

$\Rightarrow f(x) = -f(20+x)$ ...(4)

Put $x = x - 20$

$\Rightarrow f(x-20) = -f(x)$ ...(5)

$\Rightarrow f(20+x) = f(x-20)$

$\Rightarrow f(40+x) = f(x)$ Periodic

in eq. (4) Put $x = -x$

$\Rightarrow f(-x) = -f(20-x)$

Now from eq (3) $f(20-x) = f(x)$

$\Rightarrow f(-x) = -f(x)$

odd function.

16. Ans. (B)

$f(x) = 2x - \left\{\frac{x}{\pi}\right\}$

$\Rightarrow g(x) = \cos x$

$\Rightarrow gof(x) = \cos\left(2x - \left\{\frac{x}{\pi}\right\}\right)$

$\cos 2x.\cos\left\{\frac{x}{\pi}\right\} + \sin(2x).\sin\left\{\frac{x}{\pi}\right\}$

↓ ↓ ↓ ↓

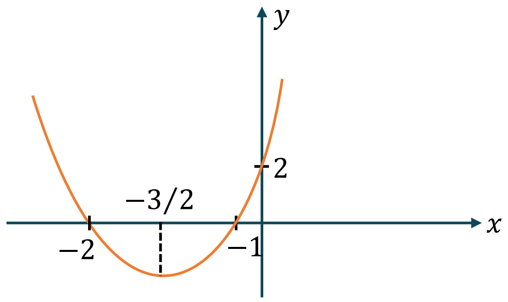
π π π π

Period of overall expression is π

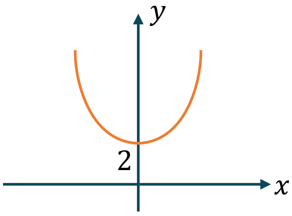
17. Ans. (A,C)

$f(x) = x^2 + 3x + 2$

graph of $f(x)$

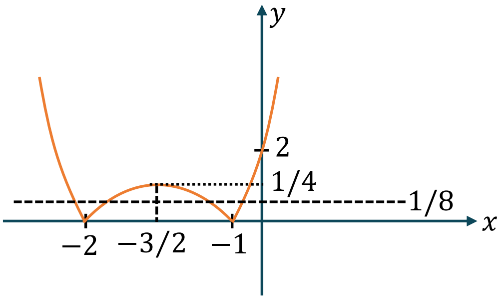


graph of $f(|x|)$



$f(|x|) = 2$

only one solution

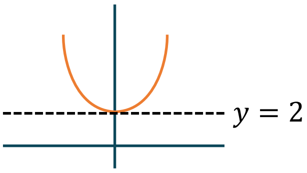


$|f(x)| = 0.125$

Number of solution $\Rightarrow 4$

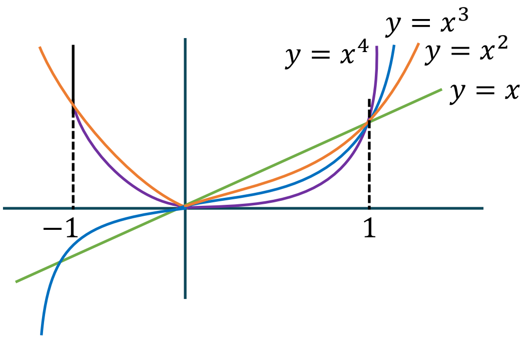
graph of $|f(|x|)|$

Combination of $f(|x|)$ of $|f(x)|$



18. Ans. (A,B,C)

$f(x) = \max\{x, x^2, x^3, x^4\}$



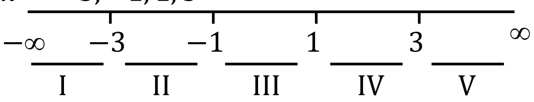
$$\Rightarrow f(x) = \begin{cases} x^4 & x \le -1 \\ x^2 & -1 < x \le 0 \\ x & 0 < x \le 1 \\ x^4 & x \ge 1 \end{cases}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{2}$$

19. Ans. (B,C,D)

$f(x) = |x+3| - |x+1| - |x-1| + |x-3|$

$x = -3, -1, 1, 3$



Case-I $x \le -3$

$\Rightarrow f(x) = -(x+3) + (x+1) + (x-1) - (x-3)$

$\Rightarrow f(x) = 0$

Case-II $-3 < x \le -1$

$\Rightarrow f(x) = x+3+x+1+x-1-x+3$

$\Rightarrow f(x) = 2x+6$

$\Rightarrow f(-1) = -2+6 = 4$ Maximum

Case-III $-1 < x \le 1$

$\Rightarrow f(x) = x+3-(x+1)+x-1-x+3$

$\Rightarrow f(x) = 4$

Case-IV $1 < x \le 3$

$\Rightarrow f(x) = x+3-x-1-x+1-x+3$

$\Rightarrow f(x) = -2x+6$

$\Rightarrow f(3) = -6+6 = 0$

$\Rightarrow f(1) = -2+6 = 4$

maximum of $f(x) = 4$

Case-V $x > 3$

$\Rightarrow f(x) = x+3-(x+1)-(x-1)+x-3$

$\Rightarrow f(x) = 0$

Answers

(B) Maximum value of $f(x)$ 1 & 4

(C) $f(x) = 4$ has infinite solution

(D) $f(x) = 0$ has infinite solution

Correct answers (B,C,D)

20. Ans. (C,D)

(A) Let $f: N \to N$

$f(x) = x+1$

then function is one-one but not onto

$y = x+1$

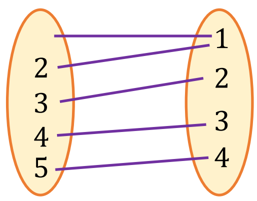
$x = y-1$

$f^{-1}(x) = x-1$

there is no element correspond to 1 in natural numbers

So (A) option is not correct

(B) Function is onto but not one-one

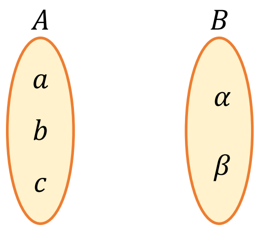


So option (B) is not correct

(C) Composition of two injective function is injective so if gof is injective then f must be injective

So option (C) is correct

(D) Numbers of function $= (2)^3 = 8$



Option (D) is correct

Answer (C,D)

21. **Ans. (A,B,C)**

$$f(x)=\begin{cases} x^2 & 0<x<2 \\ 2x-3 & 2\le x<3 \\ x+2 & x\ge 3 \end{cases}$$

(A) $\Rightarrow f\left(f\left(f(3/2)\right)\right)$

$\Rightarrow f\left(f(9/4)\right)$

$\Rightarrow f\left(2\cdot\left(\frac{9}{4}\right)-3\right)$

$\Rightarrow f\left(\frac{3}{2}\right)=\frac{9}{4}$ correct

(B) $1+f\left\{f\left\{f\left(\frac{5}{2}\right)\right\}\right\}$

$\Rightarrow 1+f\left\{f\left(2\times\frac{5}{2}-3\right)\right\}$

$\Rightarrow 1+f\{f(2)\}$

$\Rightarrow 1+f(2\times 2-3)$

$\Rightarrow 1+f(1)$

$=1+1^2=2=f\left(\frac{5}{2}\right)$ correct

(C) $f\{f(1)\}=f(1)=1$ correct

22. **Ans. (A,D)**

$\{x\}=\frac{2}{3}$

$\left[x+\left\{x+\left[x+\left\{x+\text{.......}100\text{ times}\right\}\right]\right\}\right]=5$

Let $\left[x+\left\{x+\text{.......}100\text{ times}\right\}\right]=I$ (where I is integer)

$\Rightarrow \left[x+\left\{x+I\right\}\right]=5$

$\Rightarrow \left[x+\frac{2}{3}\right]=5$

$\Rightarrow \left[[x]+\{x\}+\frac{2}{3}\right]=5$

$\Rightarrow \left[[x]+\frac{2}{3}+\frac{2}{3}\right]=5$

$\Rightarrow \left[[x]+\frac{4}{3}\right]=5$

$\Rightarrow \left[[x]+1+\frac{1}{3}\right]=5$

$\Rightarrow [x]+1+\left[\frac{1}{3}\right]=5$

$\Rightarrow [x]+1=5$

$\Rightarrow [x]+1=5$

$\Rightarrow [x]=4$

$4 \le x < 5$

$\boxed{x=4+\frac{2}{3}=\frac{14}{3}}$

23. **Ans. (B,C)**

Let $y = x + 1 = f(x)$ where x is even

$f(2) = 2 + 1 = 3$

$f(4) = 4 + 1 = 5$

$\because \quad y = x + 1$

$x = y - 1$

$f^{-1}(x) = x - 1$

$f^{-1}(3) = 2, \qquad f^{-1}(5) = 2$

$\Rightarrow \ f(1) \neq 2, f(3) = 2$ or $f(5) = 2$

$\Rightarrow \ f^{-1}(2) = 3$ or $f^{-1}(2) = 5$

Answer (B,C)

24. **Ans. (A,B,C)**

$f\left(x+\frac{1}{2}\right)+f\left(x-\frac{1}{2}\right)=f(x) \qquad \forall x \in R$ ...(1)

Put $x=x+\frac{1}{2}$

$\Rightarrow \ f(x+1)+f(x)=f\left(x+\frac{1}{2}\right)$

$\Rightarrow \ f(x+1)-f\left(x+\frac{1}{2}\right)=-f(x)$ ...(2)

add eq. (1) and (2)

$\Rightarrow \ f\left(x-\frac{1}{2}\right)=-f(x+1)$ ...(3)

Put $x = x - \frac{1}{2}$ in eq. (1)

$\Rightarrow f(x) + f(x-1) = f\left(x - \frac{1}{2}\right)$

$\Rightarrow f(x-1) - f\left(x - \frac{1}{2}\right) = -f(x)$ ...(4)

add equation (1) and (4)

$\Rightarrow f\left(x + \frac{1}{2}\right) + f(x-1) = 0$

Put $x \to x - 1$

$\Rightarrow f\left(x - \frac{1}{2}\right) = -f(x-2)$ ...(5)

Now From eq. (3) and eq. (5)

$f(x+1) = f(x-2)$

$x \to x - 1$

$f(x) = f(x-3)$

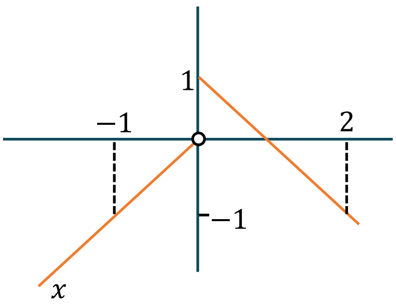
$x \to x + 3$

$\boxed{f(x+3) = f(x)}$

time period of $f(x)$=3

25. Ans. (A)

$$f(x) = \begin{cases} x & x < 0 \\ 1 - x & x \geq 0 \end{cases} \qquad g(x) = \begin{cases} x^2 & x < -1 \\ 2x+3 & -1 \leq x \leq 1 \\ x & x > 1 \end{cases}$$



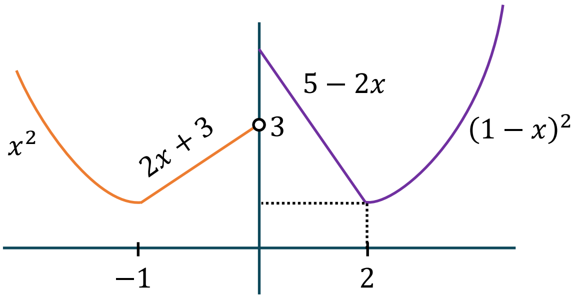
Range of $f(x) = (-\infty, 1]$

26. Ans. (C)

$$g(x) = \begin{cases} (f(x))^2 & f(x) < -1 \\ 2f(x)+3 & -1 \leq f(x) \leq 1 \\ f(x) & f(x) > 1 \end{cases}$$

$$g(x) = \begin{cases} x^2 & x < -1 \\ (1-x)^2 & x > 2 \\ 2x+3 & -1 < x < 0 \\ 2(1-x)+3 & 0 \leq x \leq 2 \end{cases}$$

graph of $g(f(x))$



Clearly Range of $g(f(x)) = [1,\infty)$

27. **Ans. (A)**

$f(2-x) = f(2+x)$ ...(i)

$f(x) = f(20-x)$ ...(ii)

Put $x \to 2-x$

$\Rightarrow f(x) = f(4-x)$ ...(iii)

from (ii) & (iii)

$f(4-x) = f(20-x)$

Put $4-x = t$

$f(t) = f(t-16)$

time period of $f(x) = 16$

28. **Ans. (D)**

$f(x) = f(4-x)$

Symmetric about $x = 2$

Time period of $f(x)$ is 16

So $f(x) = f(x+16)$

So $f(x)$ is symmetric about $x = 18, 34$ and so on

29. **Ans. (A)**

(I) $y = \dfrac{\cos^2 x + \cos x + 2}{\cos^2 x + \cos x + 1}$

$$\Rightarrow y = \frac{\cos^2 x + \cos x + 1 + 1}{\cos^2 x + \cos x + 1}$$

$$\Rightarrow y = 1 + \frac{1}{\cos^2 x + \cos x + 1}$$

$$\Rightarrow y = 1 + \frac{1}{\left(\cos x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$\Rightarrow \cos x + \frac{1}{2} \in \left[-\frac{1}{2}, \frac{3}{2}\right]$$

$$\Rightarrow \left(\cos x + \frac{1}{2}\right)^2 \in \left[0, \frac{9}{4}\right]$$

$$\Rightarrow 1+\frac{1}{\left(\cos x+\frac{1}{2}\right)^2+\frac{3}{4}} \in \left[\frac{4}{3},\frac{7}{3}\right]$$

Range $\in \left[\frac{4}{3},\frac{7}{3}\right]$

Correct answers – Q

(II) $$y=\left|\frac{\left(\sqrt{\cos x}-\sqrt{\sin x}\right)\left(\sqrt{\cos x}+\sqrt{\sin x}\right)}{3(\cos x+\sin x)}\right|$$

$$\Rightarrow y=\left|\frac{\cos x-\sin x}{3(\cos x+\sin x)}\right|$$

$$\Rightarrow y=\left|\frac{1-\tan x}{3(1+\tan x)}\right|$$

$$\Rightarrow y=\left|\frac{1}{3}\tan\left(\frac{\pi}{4}-x\right)\right|$$

Range $\in [0,\infty)$

correct answer – (s)

(III) $$y=\frac{7}{3\left(x^6+2x^4+3x^2+1\right)}$$

$\Rightarrow$ polynomial has only non negative terms

$\Rightarrow x^6+2x^4+3x^2+1 \geq 1$

$$\Rightarrow \frac{1}{x^6+2x^4+3x^2+1} \in (0,1]$$

$$\Rightarrow \frac{7}{3\left(x^6+2x^4+3x^2+1\right)} \in \left(0,\frac{7}{3}\right]$$

correct answer (p)

(IV) $y=\log_8(x^2+2x+2)$

$\Rightarrow y=\log_8((x+1)^2+1)$

$\Rightarrow (x+1)^2+1 \geq 1$

$\Rightarrow \log_8((x+1)^2+1) \geq \log_8 1$

$\Rightarrow \log_8((x+1)^2+1) \geq 0$

Range $\in [0,\infty)$

correct answer – (s)

30. Ans. (A → P,S ; B → R ; C → Q,S ; D → T)

(A) Domain of $f(x)=\ln\{x\}$

$f(x)$ to be defined $\{x\} \in (0,1)$

$\Rightarrow x \neq 0$ and $x \neq I$

$\therefore$ domain of $f(x)$ consists no integer point.

(B) Domain of $f(x)=\sqrt{\sec(\sin x)}+\sqrt{\left[x+\frac{1}{x}\right]}+\sqrt{10-[x]^2}$

I II III

(I) $\sqrt{\sec(\sin x)}$ to be defined

$\sec(\sin x) > 0$

$\because\ -1 < \sin x < 1, x \in R$

$\sec(-\theta) = \sec\theta$

$\therefore\ x \in R$

(II) $\sqrt{\left[x+\frac{1}{x}\right]}$ to be defined

$x+\frac{1}{x}\geq 0,\ x+\frac{1}{x}\geq 2, x \in R^+$

(III) $\sqrt{10-[x]^2}$ to be defined

$10 - [x]^2 \geq 0$

$[x]^2 - 10 \leq 0$

$[x]^2 \leq 10$

$[x] \in [-\sqrt{10}, \sqrt{10}]$

$[x] = \{-3, -2, -1, 0, 1, 2, 3\}$

$x \in [-3, 4)$

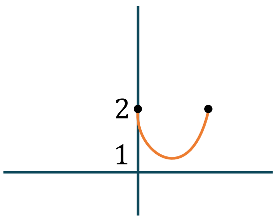
Domain of $f(x)$ = (domain of I) $\cap$ (domain of II) $\cap$ (domain of III)

$= x \in (0,4)$

$x \in \{1, 2, 3\}$

3 integers in domain.

(C) Range of $f(x) = x^2 - 2x + 2,\ x \in [0, 2]$



Range of $f(x) = [1, 2]$

2 integer points in range.

(D) Range of $f(x)=\sqrt{25-[x]^2}$

$\Rightarrow y=\sqrt{25-[x]^2}$

$\Rightarrow y^2 = 25 - [x]^2$

$\Rightarrow [x]^2 = 25 - y^2$

$\Rightarrow 25 - y^2 \geq 0$

$\Rightarrow y^2 - 25 \leq 0$

$\Rightarrow y \in [-5,\ 5]$

but it is square root function.

$y \in [0, 5]$

Range of $f(x)$ contains more than 3 integer point.

EXERCISE - S

1. **Ans. (6)**

$f(x)=\sqrt{\log_{1/2}|3-x|}$

$\log_{1/2}|3-x|\geq 0$ $\quad\quad$ $|3-x|>0$

$|3-x|\leq 1, x\neq 3$

$|x-3|\leq 1$

$-1\leq x-3\leq 1$

$2\leq x\leq 4, x=2,4$

Sum = 6

2. **Ans. (1002.50)**

$f(x)=\frac{9^x}{9^x+3}, f(1-x)=\frac{9^{1-x}}{9^{1-x}+3}$

$\Rightarrow f(x)+f(1-x)=\frac{9^x}{9^x+3}+\frac{9.9^{-x}}{\frac{9}{9^x}+3}$

$=\frac{9^x}{9^x+3}+\frac{9}{3\left(3+9^x\right)}=1$

$\left.\begin{array}{l} f\left(\frac{1}{2006}\right)+f\left(\frac{2005}{2006}\right)=1 \\ 2f\left(\frac{1003}{2006}\right)=1 \end{array}\right]$ All pairs

= 1002.5

3. **Ans. (2)**

$[x-5][x-3]+2<[x-5]+2[x-3]$

$\Rightarrow [x-5]=a$

$\Rightarrow [x-3]=b$

$\Rightarrow ab+2<a+2b$

$\Rightarrow ab+2-a-2b<0$

$\Rightarrow (a-2)(b-1)<0$

$\Rightarrow a-2<0$ or $a-2>0$

and $b-1<0$ or $b-1>0$

Case-1

$a<2$ and $b>1$

$[x-5]<2$ and $[x-3]>1$

$[x]<7$ and $[x]>4$

$[x]=5$ or $[x]=6$

Case-2

$\Rightarrow a>2$

$\Rightarrow b<1$

$\Rightarrow [x-5]>2$

$\Rightarrow [x-3]<1$

$\Rightarrow [x]>7$

$\Rightarrow [x]<4$

Not possible

possible integral values of x is $5, 6 \rightarrow 2$ values.

4. Ans. (11)

$$f(x,n)=\sum_{k=1}^{n}\log_x\left(\frac{k}{x}\right)$$

$$\Rightarrow f(x,10)=\sum_{k=1}^{10}\log_x\left(\frac{k}{x}\right)=\sum_{k=1}^{10}\log_x k-1$$

$= \log_x(1.2.3.4.......10) - 10$

$= \log_x(10!) - 10$

$$\Rightarrow f(x,11)=\sum_{k=1}^{11}\log_x\left(\frac{k}{x}\right)=\sum_{k=1}^{11}\log_x k-1$$

$= \log_x(1.2.3.......11) - 11$

$= \log_x(11!) - 11$

$\Rightarrow \log_x 10! - 10 = \log_x 11! - 11$

$$\Rightarrow x=\frac{11!}{10!}=11$$

5. Ans. (6)

$$f(x)=a\log\left(\frac{1+x}{1-x}\right)+bx^3+c\sin x+5$$

$$\Rightarrow f(-x)=-a\log\left(\frac{1+x}{1-x}\right)-bx^3-c\sin x+5$$

$\Rightarrow f(x) + f(-x) = 10$

$$\Rightarrow f(\log_3 2)+f\left(\log_3\left(\frac{1}{2}\right)\right)=10$$

$$\Rightarrow f\left(\log_3\left(\frac{1}{2}\right)\right)=10-4=6$$

6. Ans. (5)

$f(x) = px + q, f\left(f(f(x))\right) = 8x + 21$

$\Rightarrow f(f(x)) = p(px + q) + q$

$= p^2x + pq + q$

$\Rightarrow f\left(f(f(x))\right) = p^2(px + q) + pq + q$

$= p^3x + p^2q + pq + q = 8x + 21$

$\Rightarrow p^3 = 8, p^2q + pq + q = 21$

$\Rightarrow p = 2, 4q + 2q + q = 21$

$\Rightarrow q = 3$

$\Rightarrow p + q = 5$

7. Ans. (6)

$$f(x)=\frac{2x-1}{x+3},\ f^{-1}=\frac{ax+b}{c-x}$$

$$\Rightarrow y=\frac{2x-1}{x+3}$$

$\Rightarrow xy + 3y = 2x - 1$

$\Rightarrow x(y-2) = -1-3y$

$\Rightarrow x = \dfrac{-1-3y}{y-2}$

$\Rightarrow f^{-1}(x) = \dfrac{1+3x}{2-x}$ $\quad a = 3, b = 1$

$\Rightarrow c = 2$

$\Rightarrow a+b+c = 6$

8. Ans. (401)

$f(2+x) = f(2-x)$

Put $2-x = t$

$\Rightarrow f(4-t) = f(t)$

$\Rightarrow f(4-x) = f(x)$ ...(1)

$\Rightarrow f(7+x) = f(7-x)$

Put $7+x = t$

$\Rightarrow f(t) = f(7-t+7)$

$\Rightarrow f(x) = f(14-x)$ ...(2)

From eq. (1) & (2)

$\Rightarrow f(4-x) = f(14-x)$

Put $4-x = t$

$\Rightarrow f(t) = f(10+t)$

$\Rightarrow f(x) = f(10+x)$, period of function = 10

$\Rightarrow f(0) = f(10)$

$\Rightarrow f(2+x) = f(2-x)$

put $x = 2$ $\quad f(4) = f(0)$

$\Rightarrow f(4), f(10)$

$[-1000, 1000]$ total periodic intervals = 200

$\Rightarrow 400 + 1$

= 401

9. Ans. (11)

$f(x,y) = \max(x,y) + \min(x,y)$, $g(x,y) = \max(x,y) - \min(x,y)$

$\Rightarrow g\left(\dfrac{-2}{3}, \dfrac{-3}{2}\right) = \dfrac{-2}{3} - \dfrac{-3}{2} = \dfrac{3}{2} - \dfrac{2}{3} = \dfrac{9-4}{6} = \dfrac{5}{6}$

$\Rightarrow g(-3,-4) = -3-(-4) = 1$

$\Rightarrow f\left(\dfrac{5}{6}, 1\right) = 1 + \dfrac{5}{6}$

$= \dfrac{11}{6}$

10. Ans. (15)

$$f(x)=\begin{cases}2^{-x}-1, & -2\le x\le 0\\ 2^{x}-1, & 0\le x\le 2\\ 2^{4-x}-1, & 2\le x\le 4\end{cases}$$

$f(x)=1$	$f(x)=1$
$2^x-1=1$	$2^{4-x}-1=1$
$2^x=2$	$2^{4-x}=2$
$x=1$	$4-x=1$
	$x=3$

two solutions in one period

$[0,20]\quad\Rightarrow 5$ period

Total solutions in five periods = $5\times 2=10$

$[-8,0]=2$ period

Solution in two periods = $2\times 2=4$

In interval $[-10,-8]$ one solution

Total solutions = 10 + 4 + 1 = 15

EXERCISE - JEE (Main) PYQ

1. Ans. (1)

Given $f_1(x)=\frac{1}{x}$, $f_2(x)=1-x$ and $f_3(x)=\frac{1}{1-x}$

$\Rightarrow (f_2\circ J\circ f_1)(x)=f_3(x)$

$\Rightarrow f_2\circ(J(f_1(x)))=f_3(x)$

$\Rightarrow f_2\circ\left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}$

$\Rightarrow 1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$

$\Rightarrow J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{-x}{1-x}=\frac{x}{x-1}$

Now $x\to\frac{1}{x}$

$\Rightarrow J(x)=\frac{\frac{1}{x}}{\frac{1}{x}-1}=\frac{1}{1-x}=f_3(x)$

2. Ans. (2)

From the given functional equation :

$\Rightarrow f(x)=2^x \qquad \forall x\in N$

$\Rightarrow 2^{a+1}+2^{a+2}+....+2^{a+10}=16(2^{10}-1)$

$\Rightarrow 2^a(2+2^2+....+2^{10})=16(2^{10}-1)$

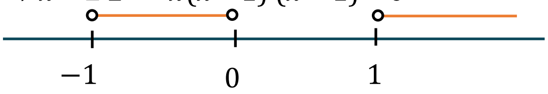
$\Rightarrow 2^a.\frac{2.(2^{10}-1)}{1}=16(2^{10}-1)$

$\Rightarrow 2^{a+1}=16=2^4\Rightarrow a=3$

3. Ans. (3)

$4 - x^2 \neq 0$; $x^3 - x > 0$

$\Rightarrow x = \pm 2 \quad x(x-1)(x+1) > 0$



$\Rightarrow D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

4. Ans. (2)

$g(x) = x^2 + x - 1$

$\Rightarrow g(f(x)) = 4x^2 - 10x + 5$

$\Rightarrow (2x-2)^2 + (2-2x) - 1$

$\Rightarrow (2-2x)^2 + (2-2x) - 1$

$\Rightarrow f(x) = 2 - 2x$

$\Rightarrow f\left(\frac{5}{4}\right) = \frac{-1}{2}$

5. Ans. (4)

$$f(x) = \begin{cases} \dfrac{x}{x^2+1} & ; \quad x \in (1,2) \\ \dfrac{2x}{x^2+1} & ; \quad x \in [2,3) \end{cases}$$

$f(x)$ is decreasing function

$\Rightarrow f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(4) Option

6. Ans. (2)

$$f(x) = \frac{2\left(2^x + 2^{-x}\right) + \left(3^x + 3^{-x}\right)}{2} \geq 3$$

(A.M $\geq$ G.M)

7. Ans. (3)

$$f(x) = y = \frac{8^{4x} - 1}{8^{4x} + 1} = 1 - \frac{2}{8^{4x} + 1}$$

so, $8^{4x} + 1 = \frac{2}{1-y} \Rightarrow 8^{4x} = \frac{1+y}{1-y}$

$$\Rightarrow x = \ell n\left(\frac{1+y}{1-y}\right) \times \frac{1}{4\ell n 8} = f^{-1}(y)$$

Hence, $f^{-1}(x) = \frac{1}{4}\log_8 e \ell n\left(\frac{1+x}{1-x}\right)$

8. Ans. (1)

$f(x+y) = f(x) + f(y)$

$\Rightarrow f(n) = nf(1)$

$\Rightarrow f(n) = 2n$

$\Rightarrow g(n) = \sum_{k=1}^{n-1} 2n = 2\left(\frac{(n-1)n}{2}\right) = n(n-1)$

$\Rightarrow g(n) = 20 \Rightarrow n(n-1) = 20$

$\Rightarrow n = 5$

9. Ans. (2)

$$f(x)=\frac{a-x}{a+x}\quad x\in R-\{-a\}\to R$$

$$\Rightarrow f(f(x))=\frac{a-f(x)}{a+f(x)}=\frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}$$

$$\Rightarrow f(f(x))=\frac{(a^2-a)+x(a+1)}{(a^2+a)+x(a-1)}=x$$

$\Rightarrow (a^2-a)+x(a+1)=(a^2+a)x+x^2(a-1)$

$\Rightarrow a(a-1)+x(1-a^2)-x^2(a-1)=0$

$\Rightarrow a=1$

$$\Rightarrow f(x)=\frac{1-x}{1+x},$$

$$\Rightarrow f\left(\frac{-1}{2}\right)=\frac{1+\frac{1}{2}}{1-\frac{1}{2}}=3$$

10. Ans. (5.00)

$f(x+y)=f(x)\,f(y)$

put $x=y=1\quad f(2)=\left(f(1)\right)^2=3^2$

put $x=2,y=1 f(3)=\left(f(1)\right)^3=3^3$

⋮

Similarly $f(x)=3^x$

$$\Rightarrow\sum_{i=1}^{n}f(i)=363\Rightarrow\sum_{i=1}^{n}3^i=363$$

$(3+3^2+...+3^n)=363$

$$\Rightarrow\frac{3(3^n-1)}{2}=363$$

$3^n-1=242\Rightarrow 3^n=243$

$\Rightarrow n=5$

11. Ans. (4)

$$f(x)=\log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$$

$$\Rightarrow f(x)=\log_{\sqrt{5}}\left[3+2\cos\left(\frac{\pi}{4}\right)\cos(x)-2\sin\left(\frac{3\pi}{4}\right)\sin(x)\right]$$

$$\Rightarrow f(x)=\log_{\sqrt{5}}\left[3+\sqrt{2}(\cos x-\sin x)\right]$$

Since $-\sqrt{2}\le\cos x-\sin x\le\sqrt{2}$

$$\Rightarrow\log_{\sqrt{5}}\left[3+\sqrt{2}(-\sqrt{2})\le f(x)\le\log_{\sqrt{5}}\left[3+\sqrt{2}(\sqrt{2})\right]\right]$$

$$\Rightarrow\log_{\sqrt{5}}(1)\le f(x)\le\log_{\sqrt{5}}(5)$$

So Range of $f(x)$ is $[0,2]$

Option (4)

12. **Ans. (26)**

$kf(k) + 2 = \lambda(x-2)(x-3)(x-4)(x-5)$ ...(1)

put $x = 0$

we get $\lambda = \dfrac{1}{60}$

Now put λ in equation (1)

$\Rightarrow kf(k) + 2 = \dfrac{1}{60}(x-2)(x-3)(x-4)(x-5)$

Put $x = 10$

$\Rightarrow 10f(10) + 2 = \dfrac{1}{60}(8)(7)(6)(5)$

$\Rightarrow 52 - 10f(10) = 52 - 26 = 26$

13. **Ans. (3)**

Given $y = 5^{(\log_a x)} = f(x)$

Interchanging x & y for inverse

$\Rightarrow x = 5^{(\log_a y)} = y^{(\log_a 5)}$

option (1) or option (2)

Further, from given relation

$\Rightarrow \log_5 y = \log_a x$

$\Rightarrow x = a^{(\log_5 y)} = y^{(\log_5 a)}$

$\Rightarrow x = y^{\left(\frac{1}{\log_a 5}\right)} = f^{-1}(y)$

option (3)

14. **Ans. (3)**

$f(x) = y = \dfrac{x-2}{x-3}$

$\therefore x = \dfrac{3y-2}{y-1}$

$\therefore f^{-1}(x) = \dfrac{3x-2}{x-1}$

& $g(x) = y = 2x - 3$

$\therefore x = \dfrac{y+3}{2}$

$\therefore g^{-1}(x) = \dfrac{x+3}{2}$

$\because f^{-1}(x) + g^{-1}(x) = \dfrac{13}{2}$

$\therefore x^2 - 5x + 6 = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$\therefore$ sum of roots

$x_1 + x_2 = 5$

15. Ans. (3)

For domain,

$$\frac{|[x]|-2}{|[x]|-3}\geq 0$$

Case I : When $|[x]|-2\geq 0$

and $|[x]|-3>0$

$\therefore\ x\in(-\infty,-3)\cup[4,\infty)$ ...(1)

Case II : When $|[x]|-2\leq 0$

and $|[x]|-3<0$

$\therefore\ x\in[-2,3)$ ...(2)

So, from (1) and (2)

we get

Domain of function

$=(-\infty,-3)\cup[-2,3)\cup[4,\infty)$

$\therefore\ (a+b+c)=-3+(-2)+3=-2\ (a<b<c)$

$\Rightarrow$ Option (3) is correct.

16. Ans. (2)

$f:\{1,3,5,7,\ldots..99\}\rightarrow\{2,4,6,8,\ldots.,100\}$

$f(3)\geq f(9)\geq f(15)\geq\ldots\ldots.f(99)$ ... (1)

3, 9, 15,.....99 $\Rightarrow$ 17 numbers

for condition one we have ${}^{50}C_{17}\times 1$ way rest 33 elements 33 !

$={}^{50}C_{17}\times 33!$

$={}^{50}C_{33}\times 33!$

$={}^{50}P_{33}$.

17. Ans. (25)

Let $f(x)=x^2+bx+p$

$\Rightarrow\ f(1)=\frac{1}{3}\Rightarrow 1+b+p=\frac{1}{3}$ ...(1)

Assume common root be α

$f(\alpha)=0\ \&\ f(f(f(f(\alpha))))=0$

$\Rightarrow f\Big(f\big(f(0)\big)\Big)=0$

$\Rightarrow f\big(f(p)\big)=0$

$\Rightarrow f(p^2+bp+p)=0$

$\Rightarrow f\big(p(p+b+1)\big)=0$

$\Rightarrow f\left(\frac{p}{3}\right)=0$

$\Rightarrow\frac{p^2}{9}+b\cdot\frac{p}{3}+p=0$

$\Rightarrow\frac{p}{9}+\frac{b}{3}+1=0$

$\Rightarrow p + 3b + 9 = 0$ ...(2)

From (1) & (2) $\Rightarrow p = \frac{7}{2}$

Now, $f(-3) = 9 - 3b + p$

$= 9 - (-p - 9) + p$

$= 18 + 2p = 18 + 2 \times \frac{7}{2} = 25$

18. Ans. (2)

$A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3, 4, 5, 6\}$

Here $f(3)$ can be 2, 3, 4, 5, 6

$\Rightarrow f(3) = 2, (f(1), f(2)) \rightarrow (1,1) \rightarrow$ 6 cases

$\Rightarrow f(3) = 3, (f(1), f(2)) \rightarrow (1,2), (2,1)$

$\rightarrow 2 \times 6 = 12$ cases

$\Rightarrow f(3) = 4, (f(1), f(2)) \rightarrow (1,3), (3,1), (2,2)$

$\rightarrow 3 \times 6 = 18$ cases

$\Rightarrow f(3) = 5, (f(1), f(2)) \rightarrow (1,4), (4,1), (2,3), (3,2)$

$\rightarrow 4 \times 6 = 24$ cases

$\Rightarrow f(3) = 6, (f(1), f(2)) \rightarrow (1,5), (5,1), (2,4), (4,2), (3,3)$

$\rightarrow 5 \times 6 = 30$ cases

Total number of cases $= 6 + 12 + 18 + 24 + 30 = 90$

19. Ans. (2)

$f(x) - f(x/3) = x/3$

$\Rightarrow f(x/3) - f(x/3^2) = x/3^2$ .... on adding

$\Rightarrow f(x) - \lim_{n\to\infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} .... \infty\right)$

$\Rightarrow f(x) - f(0) = \frac{x}{2}$

$\Rightarrow f(8) = 7 ; f(0) = 3$

$\Rightarrow f(x) = x/2 + 3$

$\Rightarrow f(14) = 10$

20. Ans. (4)

$f(x) = x - 1$; $g(x) = \frac{x^2}{x^2-1}$

$f(g(x)) = g(x) - 1$

$= \frac{x^2}{x^2-1} - 1 = \frac{x^2 - x^2 + 1}{x^2 - 1}$

$f(g(x)) = \frac{1}{x^2-1}$; $x \neq \pm 1$, even function

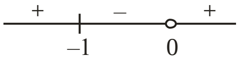
$\rightarrow$ Hence $f(g(x))$ is many one function

$\Rightarrow y = \frac{1}{x^2 - 1}$

$\Rightarrow y \cdot x^2 - y = 1$

$\Rightarrow x^2 = \left(\frac{1+y}{y}\right)$

$\Rightarrow \left(\frac{1+y}{y} \geq 0\right)$



Range:- $y \in (-\infty, -1] \cup (0, \infty)$

Hence, Range $\neq$ Co-domain $\Rightarrow f(g(x))$ is into function

21. Ans. (31)

$2f(a) + 3f(c) = f(d) - f(b)$

Using fundamental principle of counting

Number of one-one function is 31

22. Ans. (2)

$$f(x) = \left[2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$$

$$\Rightarrow f(x) = \left[\left(2-x^{25}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$$

$$= (4 - x^{50})^{\frac{1}{50}}$$

$$\Rightarrow f(f(x)) = \left(4-\left(\left(4-x^{50}\right)^{1/50}\right)^{50}\right)^{1/50} = x$$

$\Rightarrow g(x) = f\big(f(f(x))\big) + f(f(x))$

$= f(x) + x$

$\Rightarrow g(1) = f(1) + 1 = 3^{\frac{1}{50}} + 1$

$[g(1)] = [3^{\frac{1}{50}} + 1] = 2$

23. Ans. (20)

$$D_f : \frac{6x^2+5x+1}{2x-1} > 0, \frac{2x^2-3x+4}{3x-5} \geq -1, \frac{2x^2-3x+4}{3x-5} \leq 1$$

$$D_f : \left(\frac{-1}{2}, \frac{-1}{3}\right) \cup \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$$

24. Ans. (180)

Total no. of onto function provided $f(a) \neq 1$

= Total no. of onto function – No. of onto function when $f(a) = 1$

= $\frac{5!}{2!3!} \times 4! - \left(\frac{4!}{2!2!} \times 3! + 4!\right) = 180$

25. Ans. (4)

Given for $x \geq 2$

$f(1) + 2f(2) + ... + xf(x) = x(x+1)f(x)$

replace x by $x + 1$

$\Rightarrow x(x+1)f(x) + (x+1)f(x+1)$

$= (x+1)(x+2)f(x+1)$

$$\Rightarrow \frac{x}{f(x+1)}+\frac{1}{f(x)}=\frac{(x+2)}{f(x)}$$

$$\Rightarrow x\,f(x) = (x+1)\,f(x+1) = \frac{1}{2}, x \geq 2$$

$$f(2)=\frac{1}{4},\ f(3)=\frac{1}{6}$$

Now $f(2022)=\dfrac{1}{4044}$

$$f(2028)=\frac{1}{4056}$$

So, $\dfrac{1}{f(2022)}+\dfrac{1}{f(2028)}$ = 4044 + 4056 = 8100

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (A)**

Given $f(x) = x^2; g(x) = \sin x$

$fogogof(x) = \sin^2(\sin x^2)$

and $gogof(x) = \sin(\sin x^2)$

given $fogogof(x) = gogof(x)$

$\Rightarrow \sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow \sin(\sin x^2) = 0$ or 1 (rejected)

$\sin(\sin x^2) = 0 \;\; \Rightarrow \;\; x^2 = n\pi$

$\Rightarrow \; x = \pm\sqrt{n\pi};\ x \in \{0, 1, 2, 3, ...\}$

2. **Ans. (B)**

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow f'(x) = 6(x^2 - 5x + 6) = 6(x-2)(x-3)$

$\because\ f(x)$ is non monotonic in $x \in [0,3]$

$\Rightarrow\ f(x)$ is not one-one

$f(x)$ is increasing in $x \in [0,2)$ and decreasing in $x \in (2,3]$

$f(0) = 1, f(2) = 29\ \&\ f(3) = 28$

$\therefore$ Range of $f(x)$ is $[1,29]$

$\Rightarrow\ f(x)$ is onto.

3. **Ans. (A,B,C)**

$f(x) = \left(\log(\sec x + \tan x)\right)^3$

$f(-x) = \left(\log(\sec x - \tan x)\right)^3$

$= \log\left[\left(\dfrac{1}{\sec x + \tan x}\right)\right]^3$

$= -\log(\sec x + \tan x)^3$

$\therefore\ f$ is odd function

$$f'(x)=3(\log(\sec x+\tan x))^2\frac{(\sec x\tan x+\sec^2 x)}{(\sec x+\tan x)}$$

$$f'(x)=3\sec x(\log(\sec x+\tan x))^2$$

$f'(x) > 0$

$\therefore$ f is one-one & onto

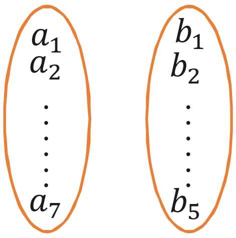
4. **Ans. (119)**

$n(X) = 5$

$n(Y) = 7$

$\alpha \rightarrow$ Number of one-one function = ${}^7C_5 \times 5!$

$\beta \rightarrow$ Number of onto function Y to X



1, 1, 1, 1, 3   1, 1, 1, 2, 2

$$\frac{7!}{3!4!}\times 5!+\frac{7!}{(2!)^3 3!}\times 5! = ({}^7C_3+3.{}^7C_3)5! = 4\times{}^7C_3\times 5!$$

$$\frac{\beta-\alpha}{5!}=4\times{}^7C_3-{}^7C_5=4\times 35-21=119$$

5. **Ans. (C)**

$f(x)$ is a non-periodic, continuous and odd function

$$\Rightarrow f(x)=\begin{cases}-x^2+x\sin x, & x<0\\ x^2-x\sin x, & x\geq 0\end{cases}$$

$$\Rightarrow f(-\infty) = \lim_{x\to-\infty}(-x^2)\left(1-\frac{\sin x}{x}\right)=-\infty$$

$$\Rightarrow f(\infty) = \lim_{x\to\infty}x^2\left(1-\frac{\sin x}{x}\right)=\infty$$

$\Rightarrow$ Range of $f(x) = R$

$\Rightarrow$ $f(x)$ is an onto function ...(1)

$$f'(x)=\begin{cases}-2x+\sin x+x\cos x, & x<0\\ 2x-\sin x-x\cos x, & x\geq 0\end{cases}$$

For $(0,\infty)$

$f'(x) = (x-\sin x) + x(1-\cos x)$

always $+ve$ always $+ve$

or 0 or 0

$\Rightarrow$ $f'(x) > 0$

$\Rightarrow$ $f'(x) \geq 0,\ \forall x\in(-\infty,\infty)$

equality at $x = 0$

$\Rightarrow$ $f(x)$ is one –one function ...(2)

From (1) & (2), $f(x)$ is both one-one & onto.

6. **Ans. (19.00)**

$$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$$

$$=\frac{4^x}{4^x+2}+\frac{4/4^x}{\frac{4}{4^x}+2}$$

$$=\frac{4^x}{4^x+2}+\frac{4}{4+2.4^x}$$

$$=\frac{4^x}{4^x+2}+\frac{2}{2+4^x}=1$$

so, $f\left(\frac{1}{40}\right)+f\left(\frac{2}{40}\right)+...+f\left(\frac{39}{40}\right)-f\left(\frac{1}{2}\right)$

$$=19+f\left(\frac{1}{2}\right)-f\left(\frac{1}{2}\right)=19$$

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (1)**

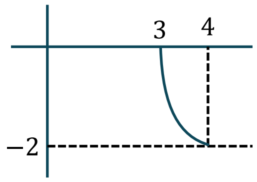
Put $x = 30,\ y=\frac{40}{30}$ then

$$f\left(30\times\frac{40}{30}\right)=\frac{f(30)}{40}\times 30$$

Hence,

$f(40) = 15$

2. **Ans. (1)**



$$f(x)=\sqrt{(\sqrt{x-3}-1)^2}-\sqrt{(\sqrt{x-3}+1)^2}=\left|\sqrt{x-3}-1\right|-\left|\sqrt{x-3}+1\right|$$

$$f(x)=\begin{cases}-2\sqrt{x-3} & 3\le x<4\\ -2 & x\ge 4\end{cases}$$

$R_f = [-2, 0]$

3. **Ans. (2)**

$f(x)$ must be a perfect cube

$f(x) = (x-2)^3 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow a = -6, b = 12$

4. Ans. (1)

$f(g(x))$ and $g(f(x))$ are identity functions $\Rightarrow f^{-1}(x) = g(x)$

$f(x)$ is linear function $\Rightarrow f(x) = ax + b$

$\Rightarrow g(x) = \frac{x-b}{a}$

$f(0) = 4$

$a(0) + b = 4$

$b = 4$

$g(5) = 17$

$\frac{5-4}{a} = 17$

$a = \frac{1}{17}$

$f(2006) = \frac{1}{17} 2006 + 4$

$= 118 + 4$

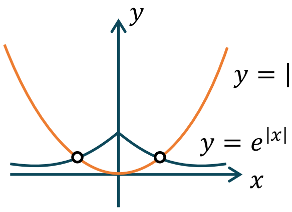
$= 122$

5. Ans. (2)

$f(x) = y = ln(x + \sqrt{x^2+1})$

$\Rightarrow e^y = \sqrt{x^2+1} + x;\ e^{-y} = \sqrt{x^2+1} - x$

$f^{-1}(x) = \frac{e^x - e^{-x}}{2}$



No. of solutions = 2

6. Ans. (4)

$f(x) = log_{1/2}\left(-\log_2\left(1 + \frac{1}{\sqrt[4]{x}}\right) - 1\right) \Rightarrow - log_2\left(1 + \frac{1}{x^{1/4}}\right) - 1 > 0$

$\Rightarrow -\infty < log_2\left(1 + \frac{1}{x^{1/4}}\right) < -1 \Rightarrow\ 0 < 1 + \frac{1}{x^{1/4}} < \frac{1}{2} \Rightarrow -1 < \frac{1}{x^{1/4}} < -\frac{1}{2}$

$\Rightarrow x \in \phi$ (null set)

7. Ans. (2)

$q^2 - 4pr = 0, p > 0 \Rightarrow f(x) = log(px^3 + (p+q)x^2 + (q+r)x + r)$

Let $g(x) = px^3 + (p+q)x^2 + (q+r)x + r \Rightarrow g(x) = (x+1)(px^2 + qx + r)$

Discriminant of $px^2 + qx + r$ is $q^2 - 4pr = 0$

Domain $(x+1)(px^2 + qx + r) > 0 \Rightarrow p(x+1)\left(x + \frac{q}{2p}\right)^2 > 0$

$\Rightarrow x \neq -\dfrac{q}{2p}$ and $x > -1$

$\therefore x \in R - [(-\infty, -1] \cup \left\{-\dfrac{q}{2p}\right\}$

8. **Ans. (3)**

$f(x) = \dfrac{x-[x]}{1+x-[x]} = \dfrac{\{x\}}{1+\{x\}} = 1 - \dfrac{1}{1+\{x\}}$

$\because \{x\} \in [0, 1) \Rightarrow f(x) \in \left[0, \dfrac{1}{2}\right)$

9. **Ans. (4)**

Here $(2 - \log_2 (16 \sin^2 x + 1) > 0 \Rightarrow 0 < 16 \sin^2 x + 1 < 4$

$\Rightarrow \dfrac{-1}{16} \leq \sin^2 x < \dfrac{3}{16} \Rightarrow 1 \leq 16 \sin^2 x + 1 \leq 4 \Rightarrow 0 \leq \log_2 (16 \sin^2 x + 1) < 2$

$\Rightarrow 2 \geq 2 - \log_2 (16 \sin^2 x + 1) > 0$

$\Rightarrow \log_{\sqrt{2}} 2 \geq \log_{\sqrt{2}} (2 - \log_2 (16 \sin^2 x + 1)) > -\infty \Rightarrow 2 \geq y > -\infty$

Hence range is $y \in (-\infty, 2]$

10. **Ans. (1)**

$f(6\{x\}^2 - 5\{x\} + 1) \Rightarrow f((3\{x\} - 1)(2\{x\} - 1)) \Rightarrow (3\{x\} - 1)(2\{x\} - 1) \leq 0$

or $\{x\} \in \left[\dfrac{1}{3}, \dfrac{1}{2}\right]$

$\therefore x \in \bigcup_{n \in I} \left[n + \dfrac{1}{3}, n + \dfrac{1}{2}\right]$

11. **Ans. (3)**

$f : (e, \infty) \to R \Rightarrow f(x) = \ell n(\ell n(\ell n\, x))$

$D : \ell n(\ell n\, x) > 0$ or $\ell n x > 1$ or $x > e$

$R : (-\infty, \infty) \Rightarrow$ one-one and onto function

12. **Ans. (3)**

$f(x) = 2[x] + \cos x$; $f(x) = \cos x$		$x \in [0, 1)$
$= 2 + \cos x$	$x \in [1, 2)$	
$= 4 + \cos x$	$x \in [2, 3)$	
$= 6 + \cos x$	$x \in [3, 4)$	
for	$x \in [0, 1)$	$f'(x) = -ve$
	$x \in [1, 2)$	$f'(x) = -ve$
	$x \in [2, 3)$	$f'(x) = -ve$
	$x \in [3, 4)$	$f'(x) = +ve$

$\Rightarrow$ function is not one-one

	if $x \in [0, 1)$	range : $[1, \cos 1)$
	$x \in [1, 2)$	range : $[2 + \cos 1, 2 + \cos 2)$

not onto function

13. **Ans. (3)**

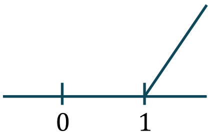
$x \in (2, 4) \Rightarrow \left[\frac{x}{2}\right] = 1$

so

$\Rightarrow f(x) = x - 1 \Rightarrow y = x - 1$

$\Rightarrow x = y + 1 \Rightarrow f^{-1}(x) = x + 1$

14. **Ans. (2)**



$f : [1, \infty) \to [1, \infty)$

$f(x) = 2^{x(x-10)}$

$x(x - 1)$ is strictly increasing in domain

$f(x) = 2^{x(x-1)}$ is one one & onto function so inverse is defined

$2^{x(x-1)} = y \Rightarrow x^2 - x = \log_2 y \Rightarrow x^2 - x - \log_2 y = 0$

$x = \left(\frac{1 \pm \sqrt{1 + 4\log_2 y}}{2}\right)$

–ve sign rejected as domain range of f^n is $[1, \infty)$

$f^{-1}(x) = \left(\frac{1 \pm \sqrt{1 + 4\log_2 x}}{2}\right)$

15. **Ans. (2)**

$f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

$1 - \sqrt{1 - x^2} \geq 0 \Rightarrow \sqrt{1 - x^2} \leq 1 \Rightarrow 0 \leq 1 - x^2 \leq 1 \Rightarrow x \in [-1, 1]$

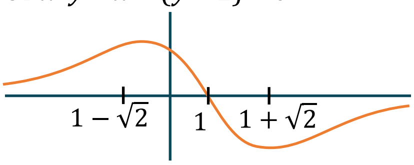
16. **Ans. (2)**

$f(x) = \frac{1 - x}{1 + x^2} \Rightarrow f'(x) = 0$ at $x = 1 \pm \sqrt{2}$

for $x \in \left[-\sqrt{2} + 1, 1 + \sqrt{2}\right]$ f is bijective function hence f is invertible.

$\frac{1 - x}{1 + x^2} = y$

Or $x^2y + x + (y - 1) = 0$



Or $x = \frac{-1 \pm \sqrt{1 - 4y(y - 1)}}{2y} = \frac{-1 \pm \sqrt{4y - 4y^2 + 1}}{2y}$

$$f^{-1}(x) = \begin{cases} \frac{-1 + \sqrt{4x - 4x^2 + 1}}{2x}, & x \neq 0 \\ 1 \quad , & x = 0 \end{cases} \qquad \text{as } f(1) = 0$$

17. Ans. (3)

$x = 1, \frac{-1\pm\sqrt{5}}{2}$

$\frac{x^3+1}{2} = 2\sqrt[3]{2x-1}$

Let $f(x) = \frac{x^3+1}{2} \Rightarrow f^{-1}(x) = \sqrt[3]{2x-1}$

Equation becomes $f(x) = f^{-1}(x)$

$\Rightarrow f(x) = x \Rightarrow \frac{x^3+1}{2} = x \Rightarrow x^3 - 2x + 1 = 0$

$\Rightarrow (x-1)(x^2+x-1) = 0 \Rightarrow x = 1, \frac{-1\pm\sqrt{5}}{2}$

Allter :

Let $y = \sqrt[3]{2x-1} \Rightarrow y^3 - 2x + 1 = 0$ and $x^3 - 2y + 1 = 0$

$\Rightarrow (y^3 - 2x + 1) - (x^3 - 2y + 1) = 0 \Rightarrow (y - x)(y^2 + xy + x^2 + 2) = 0$

$\Rightarrow y = x$ or $y^2 + xy + x^2 + 2 = 0 \Rightarrow y = x$ or $(x+y)^2 + x^2 + y^2 + 4 = 0$

Putting $y = x$ in $y = \sqrt[3]{2x-1}$, we get $y = x$

$x^3 - 2x + 1 = 0$

Which yields the values $x = 1, \frac{-1\pm\sqrt{5}}{2}$

18. Ans. (3)

(1) $f(x) = e^{1/2\,\ell n x} = \sqrt{x}, D: x > 0$

$g(x) = \sqrt{x}, D: x \geq 0$

(2) $\tan(\tan x)$ $\quad D: x \neq \pm(2n+1)\frac{\pi}{2}$ & $\tan x \neq \pm(2n+1)\frac{\pi}{2}$

$\cot(\cot x)$ $\quad D: x \neq \pm n\pi$ & $\cot x \neq \pm n\pi$

(3) $f(x) = cos^2x + sin^4x = cos^2x + (1 - cos^2x)^2 = 1 - cos^2x + cos^4x = sin^2x + cos^4x$

$g(x) = sin^2x + cos^4x$

(4) $f(x) = \frac{|x|}{x}$, $\quad D: x \neq 0 \Rightarrow g(x) = sgn(x), \quad D: x \in R$

19. Ans. (1)

$f(x) = \log_2[3x - 3[x]] = \log_2[3\{x\}]$

period 1.

so $0 \leq \{x\} < 1$

$0 \leq 3\{x\} < 3 \Rightarrow [3\{x\}] = 0, 1, 2$

so range $\{\log_2 1, \log_2 2\} = \{0, 1\}$.

20 Ans. (2)

$f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1 = \frac{x}{2}\left(\frac{e^x+1}{e^x-1}\right) + 1$

$f(-x) = -\frac{x}{2}\left(\frac{e^{-x}+1}{e^{-x}-1}\right) + 1 = \frac{x}{2}\left(\frac{e^x+1}{e^x-1}\right) + 1 = f(x)$, even function

SECTION–B

1. **Ans. (9)**

$$f(x)=\frac{\tan^2 x+8\tan x+15}{1+\tan^2 x}$$

$= \sin^2 x + 8\sin x\cos x + 15\cos^2 x$

$$\Rightarrow \frac{1-\cos 2x}{2}+4\sin 2x+15\left(\frac{1+\cos 2x}{2}\right)$$

$\Rightarrow 8 + 7\cos 2x + 4\sin 2x$

we know that

$7\cos 2x + 4\sin 2x \in \left[-\sqrt{7^2+4^2},\sqrt{7^2+4^2}\right]$

so $f(x) = 8 + 7\cos 2x + 4\sin 2x$

$\in \left[8-\sqrt{65},8+\sqrt{65}\right]$

Now calculate assuming $\sqrt{65}$ is slightly greater than 8.

2. **Ans. (0)**

$$f(x)=\frac{x^3+2x^2+3x+2}{x^3+2x^2+2x+1}=\frac{(x+1)(x^2+x+2)}{(x+1)(x^2+x+1)}$$

$$\Rightarrow\ y=\frac{x^2+x+2}{x^2+x+1}$$

$x \in \mathbb{R} - \{0\}$

$\Rightarrow n \neq 0 \ \Rightarrow y \neq 2$

Now $y=\dfrac{x^2+x+2}{x^2+x+1}$

$(y - 1)x^2 + (y - 1)x + y - 2 = 0$

$y - 1 \neq 0 \Rightarrow y \neq 1$

$x \in \mathbb{R} - \{0\}$

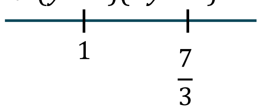
$\Rightarrow D \geq 0$

$\Rightarrow (y - 1)^2 - 4(y - 1)(y - 2) \geq 0$

$\Rightarrow (y - 1)(y - 1 - 4y + 8) \geq 0$

$\Rightarrow (y - 1)(7 - 3y) \geq 0$

$\Rightarrow (y - 1)(3y - 7) \leq 0$



$$\Rightarrow\ y\in\left(1,\frac{7}{3}\right]$$

$\Rightarrow y \neq 1 \ \&\ y \neq 2$

So No. of integers in Range = 0

3. **Ans. (15)**

Period of $e^{\tan\left\{\frac{x}{4}\right\}}$ is 4 $\Rightarrow$ $\cos\pi\left(\dfrac{(1-2[x])}{2}\right) = 0 \quad \forall\ x \in R$

Period of $\sin\left(\dfrac{\pi[x]}{2}\right)$ is 4

$\therefore$ Period of $f(x)$ is 4. For periodic function $f(x)$ range can be calculated for $x \in [0, 4]$

If $x \in [0, 1)$; $f(x) = \frac{x}{4}$, $f(x) \in \left[0, \frac{1}{4}\right)$; If $x \in [1, 2)$; $f(x) = \frac{x}{4} + 1$, $f(x) \in \left[\frac{5}{4}, \frac{3}{2}\right)$

If $x \in [2, 3)$; $f(x) = \frac{x}{4}$, $f(x) \in \left[\frac{2}{4}, \frac{3}{4}\right)$; If $x \in [3, 4)$; $f(x) = \frac{x}{4} - 1$, $f(x) \in \left[-\frac{1}{4}, 0\right)$

$\therefore$ Range $\in \left[-\frac{1}{4}, \frac{1}{4}\right) \cup \left[\frac{2}{4}, \frac{3}{4}\right) \cup \left[\frac{5}{4}, \frac{3}{2}\right)$

4. Ans. (2)

$$f(x) + f\left(\frac{1}{x}\right) = x \qquad ...(1)$$

put $x \to \frac{1}{x}$

$$f\left(\frac{1}{x}\right) + f(x) = \frac{1}{x} \qquad ...(2)$$

by (1) & (2)

$$x = \frac{1}{x}$$

$\Rightarrow x = \pm 1$

So domain of $f(x)$ is $\{-1, 1\}$

No. of elements in domain is $k = 2$

5. Ans. (2)

f & g are 2 distinct functions $[-1, 1] \to [0, 2]$ onto functions

So f & g are either $-x + 1$ or $x + 1$

Case-I:

$f(x) = -x + 1$; $g(x) = x + 1$

$$h(x) = \frac{f(x)}{g(x)} = \frac{1-x}{1+x} ; h(h(x)) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = x$$

$$h(1/x) = \frac{x-1}{x+1} ; h(h(1/x)] = \frac{1 - \frac{x-1}{x+1}}{1 + \frac{x-1}{x+1}} = \frac{1}{x}$$

$\left|h(h(x)) + h\left(h\left(\frac{1}{x}\right)\right)\right| = |x + 1/x| > 2$ as domain does not contain point $x = \pm 1$

Case-II:

$f(x) = 1 + x$; $g(x) = 1 - x$

$$h(x) = \frac{1+x}{1-x} ; h(h(x)) = \frac{1 + \frac{1+x}{1-x}}{1 - \frac{1+x}{1-x}} = -\frac{1}{x}$$

$h(h(1/x)) = -x$; $\left|h(h(x)) + h\left(h\left(\frac{1}{x}\right)\right)\right| = |-x - 1/x| = (x + 1/x) > 2$

6. Ans. (2)

$f^{-1}(x) = \dfrac{x-2}{2x-1} \Rightarrow f(x) = f^{-1}(x)$

$\Rightarrow f(f(x)) = x$

$\Rightarrow f(x) + 2f^{-1}(x) + 2 = f(f(x))$ reduces to

$\dfrac{3(x-2)}{2x-1} = x-2$

$\Rightarrow \dfrac{(x-2)}{(2x-1)}(3-2x+1)=0$

$\Rightarrow 2(x-2)(2-x) = 0 \Rightarrow x = 2$

7. Ans. (3)

$f(x)=\sec^2\left(\dfrac{\pi x}{6}\right)+\cot^2\left(\dfrac{\pi x}{6}\right)$

$=1+\tan^2\left(\dfrac{\pi x}{6}\right)+\cot^2\left(\dfrac{\pi x}{6}\right)$

$\downarrow \qquad \downarrow$

$T_1=\dfrac{\pi}{\pi/6}=6, \quad T_2=\dfrac{\pi}{\pi/6}=6$

L.C.M of T_1 and T_2 is 6

but $\tan^2\theta + \cot^2\theta$ repeats its values after interval of $\dfrac{\pi}{2}$

$\Rightarrow$ Period $T_1=\dfrac{\pi/2}{\pi/6}=3, T_2=3$

$\Rightarrow$ Period of $f(x)$ is 3.

8. Ans. (7)

$-x^2 + 4x + 11 \geq 0 \qquad \& -x^2 + 4x + 11 \neq 1$

$\Rightarrow x^2 - 4x - 11 \leq 0 \quad \& \quad x^2 - 4x - 10 \neq 0$

$\Rightarrow 2-\sqrt{15} \leq x \leq 2+\sqrt{15} \quad \& \quad x \neq -2, 6$

integral values of x are –1, 0, 1, 2, 3, 4, 5

9. Ans. (400)

$\sqrt[3]{1}, \sqrt[3]{2}, \ldots\ldots, \sqrt[3]{7}$ lies between 1 and 2

$\therefore \left[\sqrt[3]{1}\right], \left[\sqrt[3]{2}\right], \ldots, \left[\sqrt[3]{7}\right]$ are each equal to 1

$\sqrt[3]{8}, \sqrt[3]{9} \ldots\ldots \sqrt[3]{26}$ lies between 2 and 3

$\therefore \left[\sqrt[3]{8}\right], \left[\sqrt[3]{9}\right] \ldots\ldots \left[\sqrt[3]{26}\right]$ are each equal to 2

$\sqrt[3]{27}, \sqrt[3]{28}, \ldots\ldots, \sqrt[3]{63}$ lies between 3 & 4

$\therefore \left[\sqrt[3]{27}\right], \left[\sqrt[3]{28}\right], \ldots\ldots, \left[\sqrt[3]{63}\right]$ are each equal to 3

$\sqrt[3]{64}, \sqrt[3]{65}, \ldots\ldots, \sqrt[3]{124}$

lies between 4 & 5

$\therefore$ are each

equal to 4

$\Rightarrow$ Required number

= 1 × 7 + 2 × 19 + 3 × 37 + 4 × 61 = 400

10. Ans. (1)

$$\ell n\left(\sin\frac{\pi}{4}\sqrt{x}\right)\geq 0 \Rightarrow \sin\frac{\pi}{4}\sqrt{x}\geq 1$$

$$\Rightarrow \sin\left(\frac{\pi}{4}\sqrt{x}\right)=1$$

$$\Rightarrow \frac{\pi}{4}\sqrt{x}=2n\pi+\frac{\pi}{2}, n\in N$$

$$\Rightarrow \sqrt{x}=4\left(2n+\frac{1}{2}\right), n\in N$$

$\Rightarrow x = 4(4n + 1)^2, n \in N$

also $16 - x^2 \geq 0$

$\Rightarrow x \in [-4, 4]$

$x = 4(4n + 1)^2$

put $n = 0, x = 4,$

only $x = 4$ is possible.

JEE (Advanced) Practice Paper

1. Ans. (A,C)

$$f(x) = \sin\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)$$

$4 - x^2 > 0$ or $x \in (-2, 2)$ and $\dfrac{\sqrt{4-x^2}}{1-x} > 0$

$D : (-2, 1)$

$R : [-1, 1]$

2. Ans. (A,D)

(i) $f(x) = \sqrt{x-1} + 2\sqrt{3-x}$

$D : x - 1 \geq 0 \ \& \ 3 - x \geq 0 \Rightarrow x \in [1, 3]$

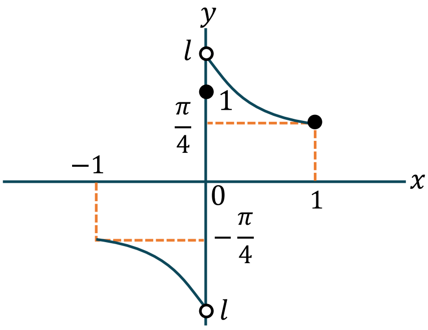
Range : $f'(x) = \dfrac{1}{2\sqrt{x-1}} - \dfrac{1}{\sqrt{3-x}} = 0$ or $f'(x) = 0$ at $x = \dfrac{7}{5}$

$f'\left(\dfrac{7^-}{5}\right) > 0$ & $f'\left(\dfrac{7^+}{5}\right) < 0 \Rightarrow$ maxima at $x = \dfrac{7}{5}$; Range : $\left[\sqrt{2}, \sqrt{10}\right]$

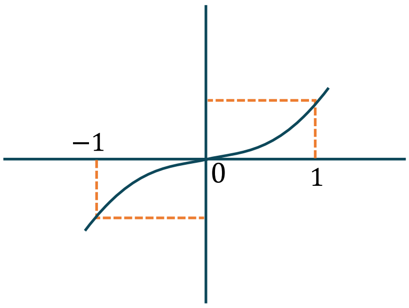
3. **Ans. (B,D)**

Domain $D \in [-1, 1]$

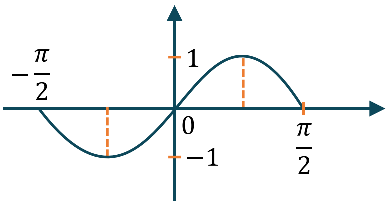
(A) $f(x) = \begin{cases} \tan^{-1}\dfrac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ many -one



(B) $g(x) = x^3$ one - one



(C) $h(x) = \sin 2x$ many - one



(D) $k(x) = \sin\left(\dfrac{\pi x}{2}\right)$ one-one function

4. **Ans. (A,D)**

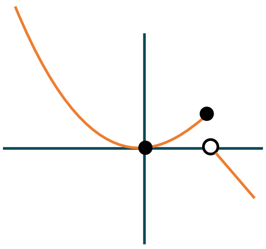
$f(x) = (x^3 - x)Q(x) + ax^2 + bx + c$

$f(0) = 1 = c$

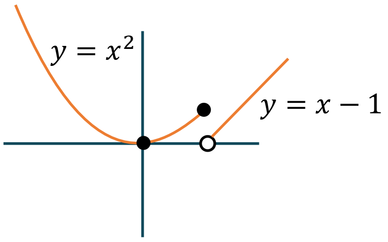
$f(1) = a + b + c = 3$

$f(-1) = a - b + c = -1 \Rightarrow a = 0, b = 2, c = 1 \Rightarrow g(x) = 2x + 1$

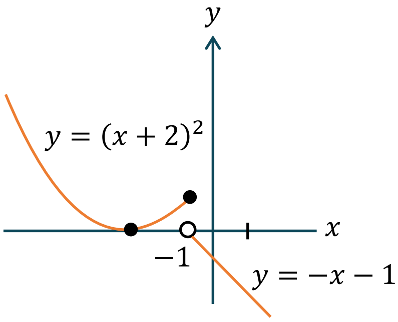
5. **Ans. (A,C,D)**



$y = f(x)$



$y = |f(x)|$



$$y = |f(x)| + f(x+2) = \begin{cases} x^2 + (x+2)^2 & x \le -1 \\ x^2 + (-x-1) & -1 < x \le 1 \\ x - 1 - x - 1 & x > 1 \end{cases};$$

$$y = |f(x)| + f(x+2) = \begin{cases} 2x^2 + 4x + 4 & x \le -1 \\ x^2 - x - 1 & -1 < x \le 1 \\ -2 & x > 1 \end{cases}$$

6. **Ans. (A,D)**

$$f(-x) = \begin{cases} 0 & x = 0 \\ -x^2 \sin\left(\frac{\pi}{x}\right) & x \in (-1,1) - \{0\} \\ -x\ |x| & |x| > 1 \end{cases} = -f(x) \quad \text{odd function}$$

7. **Ans. (A,C,D)**

$$g(x) = x^3 + tanx + \left[\frac{x^2+1}{P}\right] \Rightarrow g(-x) = (-x)^3 + tan(-x) + \left[\frac{(-x)^2+1}{P}\right]$$

$$\Rightarrow g(-x) = -x^3 - tanx + \left[\frac{x^2+1}{P}\right] \Rightarrow g(x) + g(-x) = 0$$

because $g(x)$ is a odd function

$$\therefore \left(-x^3 - \tan x + \left[\frac{x^2+1}{P}\right]\right) + \left(-x^3 - \tan x + \left[\frac{x^2+1}{P}\right]\right) = 0$$

$$\Rightarrow 2\left[\frac{(x^2+1)}{P}\right] = 0 \qquad \Rightarrow 0 \le \frac{x^2+1}{P} < 1$$

Now $x \in [-2, 2] \Rightarrow 0 < \dfrac{5}{P} < 1 \Rightarrow P > 5$

8. Ans. (A,C)

$f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x}{\pi}\right]+1}$

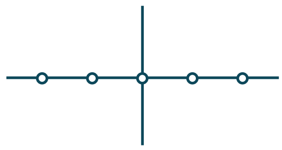
if $x = n\pi$, $f(n\pi) = 0$ if $x \neq n\pi$ $\left[\dfrac{-x}{\pi}\right] = -\left(1+\left[\dfrac{x}{\pi}\right]\right)$

$\therefore f(-x) = -f(x)$ odd function

$f(0) = f(\pi)$ hence many one

9. Ans. (A,B,C,D)

$f(x) = \dfrac{\sin(\pi[x])}{\{x\}} = 0, x \notin I$



(A) By graph fundamental period is one

(B) $f(-x) = 0 = f(x)$ $\therefore$ even function

(C) Range y $\in$ {0}

(D) y = sgn – 1, $x \notin I$

$y = sgn\left(\text{sgn}\dfrac{\{x\}}{\sqrt{\{x\}}}\right)(1) - 1 \quad \Rightarrow y = 1 - 1$

$y = 0, x \notin I$ Identical to $f(x)$

10. Ans. (B,D)

Clearly $h(x) = 2a$

so neither one-one nor onto

11. Ans. (A)

Number of one-one functions = 0

12. Ans. (C)

Total number of function = 3^5 = 243

Number of into function = 3 + 3(2^5 – 2) = 93

total number of onto function = 243 – 93 = 150

13. Ans. (D)

$\Rightarrow g(6) \leq g(7) \leq g(8)$

g(6)	g(7)	g(8)	No. of maps
1	1	1,2,3,4,5	5
	2	2,3,4,5	4
	3	3,4,5	3
	4	4,5	2
	5	5	1
2	2	2,3,4,5	4
	3	3,4,5	3
	4	4,5	2
	5	5	1
3	3	3,4,5	3
	4	4,5	2
	5	5	1
4	4	4,5	2
	5	5	1
5	5	5	1

14. **Ans. (17)**

$7x - x^2 - 6 \geq 0 \quad \Rightarrow \quad x^2 - 7(x) + 6 \leq 0$

$(x-1)(x-6) \leq 0 \quad \Rightarrow \quad x \in [1, 6]$ and $\sin x + \cos x \geq 0 \Rightarrow \left[1, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 6\right]$

15. **Ans. (17)**

$\left[x + \frac{1}{2}\right] = 2 \Rightarrow 2 \leq x + \frac{1}{2} < 3 \Rightarrow \frac{3}{2} \leq x < \frac{5}{2}$

16. **Ans. (1)**

$$f(x) = \begin{cases} |\sin 2x| & 0 \leq x < \frac{\pi}{2} \quad \Rightarrow \quad 0 \leq 2x < \pi \quad \Rightarrow \quad 0 \leq \sin 2x \leq 1 \\ 0 & \frac{\pi}{2} \leq x < \pi \\ |-\sin 2x| & \pi \leq x < \frac{3\pi}{2} \quad \Rightarrow \quad 2\pi \leq 2x < 3\pi \quad \Rightarrow \quad \theta \leq |-\sin 2x| \leq 1 \\ 0 & \frac{3\pi}{2} \leq x \leq 2\pi \end{cases}$$

so range [0, 1].

17. **Ans. (34)**

$f(-x) = -\left[ax^7 + bx^3 + cx\right] - 5 \quad ; \quad f(-x) = -[f(x) + 5] - 5$

$f(-x) = -f(x) = -10$ put $x = 7$

$f(7) = -17$ so $f(7) + 17\cos x = -17(\cos x - 1) \in [-34, 0]$

18 **Ans. (7)**

$f(x) = \frac{4a-7}{3}x^3 + (a-3)x^2 + x + 5 \quad \Rightarrow f'(x) = (4a-7)x^2 + 2(a-3)x + 1$

$D \leq 0$ for all $x \in R$

$4(a-3)^2 - 4(4a-7) \leq 0 \quad \Rightarrow a^2 + 9 - 6a - 4a + 7 \leq 0$

$a^2 - 10a + 16 \leq 0 \quad \Rightarrow (a-8)(a-2) \leq 0$

or $a \in [2, 8] \quad \Rightarrow f'(x)$ is always +ve for $a \in [2, 8]$

19. **Ans. (22)**

21 x

22 y

23 z

	case-I	**case-II**	**case-III**
$f(21) = x$	T	F	F
$f(22) \neq x$	F	T	F
$f(23) \neq y$	F	F	T

case-I $f(22) = x, f(23) = y$

then $f(21) = x$ is not true

case-II $f(23) = y, f(22) = z, f(21) = x$

not possible

case-III $f(22) = x, f(23) = z, f(21) = y$

$\therefore \quad f^{-1}(x) = 22$