EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. Let $P(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a polynomial such that $P(1) = 1; P(2) = 2; P(3) = 3; P(4) = 4; P(5) = 5$ and $P(6) = 6$, then find the value of $P(7)$.

(A) 700 (B) 727 (C) 650 (D) 725

MFN002

2. The domain of definition of the function, $y(x)$ is given by the equation, $2^x + 2^y = 2$ is -

(A) $0 < x \le 1$ (B) $0 \le x \le 1$ (C) $-\infty < x \le 0$ (D) $-\infty < x < 1$

MFN003

3. Range of function $f(x) = \log_2\left(\frac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$ is given by

(A) $(0, \infty)$ (B) $\left[\frac{1}{2}, 1\right]$ (C) $[1, 2]$ (D) $\left[\frac{1}{4}, 1\right]$

MFN004

4. Range of $f(x) = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$; $x \in \left(0, \frac{\pi}{2}\right)$ is-

(A) $(0,1)$ (B) $(1, \infty)$ (C) $(-1,0)$ (D) $(-\infty, -1)$

MFN005

5. Let $f(x) = \frac{x}{1-x}$ and let α be a real number. If $x_0 = \alpha, x_1 = f(x_0), x_2 = f(x_1)$,.......... & $x_{2011} = -\frac{1}{2012}$ then the value of α is -

(A) $\frac{2011}{2012}$ (B) 1 (C) 2011 (D) -1

MFN006

6. If $f_1(x) = 2^{f_2(x)}$, where $f_2(x) = 2012^{f_3(x)}$, where $f_3(x) = \left(\frac{1}{2013}\right)^{f_4(x)}$, where $f_4(x) = \log_{2013}\log_x 2012$, then the range of $f_1(x)$ is -

(A) $(2, \infty)$ (B) $(2012, \infty)$ (C) $(0, \infty)$ (D) $(-\infty, \infty)$

MFN007

7. Let $f: \mathbb{R} - \left\{\frac{-15}{2}\right\} \to \mathbb{R} - \left\{\frac{1}{2}\right\}$ be defined by $f(x) = \frac{x+10}{2x+15}$ then $f(x)$ is-

(A) one-one but not onto (B) many one but not-onto

(C) one-one and onto (D) many one and onto

MFN008

8. $f: \mathbb{R} \to \mathbb{R}$ $f(x) = \frac{2x^2 - 5x + 3}{8x^2 + 9x + 11}$, then f is -

(A) one-one onto (B) many-one onto (C) one-one into (D) many one into

MFN009

9. If functions $f(x)$ and $g(x)$ are defined on $\mathbb{R}\to\mathbb{R}$ such that $f(x)=\begin{cases} x+3 & , \quad x\in \text{rational} \\ 4x & , \quad x\in \text{irrational} \end{cases}$,

$g(x)=\begin{cases} x+\sqrt{5} & , \quad x\in \text{irrational} \\ -x & , \quad x\in \text{rational} \end{cases}$ then $(f-g)(x)$ is-

(A) one-one & onto
(B) neither one-one nor onto
(C) one-one but not onto
(D) onto but not one-one

MFN010

10. Let $f: X \to Y$ be a function such that $f(x)=\sqrt{x-2}+\sqrt{4-x}$, then the set of X and Y for which $f(x)$ is both injective as well as surjective, is -

(A) [2,4] and $\left[\sqrt{2},2\right]$
(B) [3,4] and $\left[\sqrt{2},2\right]$
(C) [2,4] and [1,2]
(D) [2,3] and [1,2]

MFN011

11. If $f: \mathbb{R} \to \mathbb{R}$ & $f(x)=\dfrac{\sin([x]\pi)}{x^2+2x+3}+2x-1+\sqrt{x(x-1)+\dfrac{1}{4}}$ (where $[x]$ denotes integral part of x), then $f(x)$ is -

(A) one-one but not onto
(B) one-one & onto
(C) onto but not one-one
(D) neither one-one nor onto

MFN012

12. $f(x) = [x-1] + \{x\}^{[x]}, x \in (1,3)$, then $f^{-1}(x)$ is –
(where [.] denotes greatest integer function and {.} denotes fractional part function)

(A) $\begin{bmatrix} x+1 & x\in(1,2) \\ 2+\sqrt{x-1} & x\in[2,3) \end{bmatrix}$
(B) $\begin{bmatrix} x-1 & x\in(1,2) \\ 2-\sqrt{x-1} & x\in[2,3) \end{bmatrix}$
(C) $\begin{bmatrix} x-1 & x\in(0,1) \\ 2-\sqrt{x-1} & x\in[1,2) \end{bmatrix}$
(D) $\begin{bmatrix} x+1 & x\in(0,1) \\ 2+\sqrt{x-1} & x\in[1,2) \end{bmatrix}$

MFN014

13. Let $f : (0,\infty) \to (1,\infty)$ be a function such that $f(x)=1+\dfrac{3}{2}\sqrt{x}$ and $g(x)$ is inverse of $f(x)$ then the point where $f(x)$ and $g(x)$ intersect is

(A) $\left(\dfrac{1}{4},\dfrac{1}{4}\right)$
(B) $(4, 2)$
(C) $(4, 4)$
(D) Does not exist

MFN015

14. Which of the following functions is an odd function ?

(A) $|x-2| + (x+2)\, sgn(x+2)$
(B) $\dfrac{1}{x(e^x-1)}+\dfrac{1}{2x}$
(C) $\log\left(\sin x+\sqrt{1+\sin^2 x}\right)$
(D) $e^{-4x}\left(e^{2x}-1\right)^4$

(where $sgn(x)$ denotes signum function of x)

MFN016

15. Let $f : \mathbb{R} \to \mathbb{R}$ be a real valued function such that $f(10+x) = f(10-x)\,\forall\, x \in \mathbb{R}$ and $f(20+x) = -f(20-x)\forall x \in \mathbb{R}$. Then which of the following statements is true -

(A) $f(x)$ is odd and periodic
(B) $f(x)$ is odd and aperiodic
(C) $f(x)$ is even and periodic
(D) $f(x)$ is even and aperiodic

MFN017

16. Let $f(x) = 2x - \left\{\frac{x}{\pi}\right\}$ and $g(x) = \cos x$, where {.} denotes fractional part function, then period of $gof(x)$ is -

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{3\pi}{2}$ (D) $\frac{\pi}{4}$

MFN020

MULTIPLE CORRECT TYPE QUESTIONS

17. Let $f(x) = x^2 + 3x + 2$, then number of solutions to -

(A) $f(|x|) = 2$ is 1 (B) $f(|x|) = 2$ is 3
(C) $|f(x)| = 0.125$ is 4 (D) $|f(|x|)| = 0.125$ is 8

MFN061

18. For each real x, let $f(x) = max\{x, x^2, x^3, x^4\}$, then $f(x)$ is -

(A) x^4 for $x \leq -1$ (B) x^2 for $-1 < x \leq 0$ (C) $f\left(\frac{1}{2}\right)=\frac{1}{2}$ (D) $f\left(\frac{1}{2}\right)=\frac{1}{4}$

MFN062

19. For the function $f(x) = |x + 3| - |x + 1| - |x - 1| + |x - 3|$, identify correct option(s)

(A) Range of $f(x)$ is $(-\infty, 4]$ (B) maximum value of $f(x)$ is 4
(C) $f(x) = 4$ has infinite solutions (D) $f(x) = 0$ has infinite solutions

MFN063

20. Which of the following statement(s) is(are) correct ?

(A) If f is a one-one mapping from set A to A, then f is onto.
(B) If f is an onto mapping from set A to A, then f is one-one
(C) Let f and g be two functions defined from $\mathbb{R} \rightarrow \mathbb{R}$ such that gof is injective, then f must be injective.
(D) If set A contains 3 elements while set B contains 2 elements, then total number of functions from A to B is 8.

MFN064

21. Let $f(x)=\begin{cases} x^2 & ; \quad 0<x<2 \\ 2x-3 & ; \quad 2\leq x<3 \\ x+2 & ; \quad x\geq 3 \end{cases}$ Then :-

(A) $f\left\{f\left(f\left(\frac{3}{2}\right)\right)\right\}=f\left(\frac{3}{2}\right)$ (B) $1+f\left\{f\left(f\left(\frac{5}{2}\right)\right)\right\}=f\left(\frac{5}{2}\right)$
(C) $f\{f(1)\} = f(1) = 1$ (D) $f\{f(1)\} = 2$

MFN066

22. If $\{x\}=\frac{2}{3}$ & $\left[x+\left\{x+\left[x+\left\{x+.......100 \text{ times}\right\}\right]\right\}\right]=5$, then -

(A) $x=\frac{14}{3}$ (B) $[x] = 5$ (C) $x=\frac{17}{3}$ (D) $[x] = 4$

(where [.] & {.} denotes greatest integer function & fractional part function respectively)

MFN067

23. Let $f:\{1,2,3,4,5\} \to \{1,2,3,4,5\}$ is such that $f(x)$ is a one-one function satisfying following condition $f(x) = x + 1$ if and only if x is even (i.e. $f(3) \neq 4, f(4) = 5$ etc). Then $f^{-1}(2)$ can be-

(A) 1 (B) 3

(C) 5 (D) 2

MFN068

24. If $f\left(x+\frac{1}{2}\right)+f\left(x-\frac{1}{2}\right)=f(x)\ \forall x\in\mathbb{R}$, then period of function $f(x)$ is greater than-

(A) 1 (B) $\frac{3}{2}$

(C) 2 (D) 3

MFN070

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 25 to 26

Let $f(x)=\begin{cases} x & ; \quad x<0 \\ 1-x & ; \quad x\geq 0\end{cases}$ & $g(x)=\begin{cases} x^2 & ; \quad x<-1 \\ 2x+3 & ; \quad -1\leq x\leq 1 \\ x & ; \quad x>1\end{cases}$

On the basis of above information, answer the following questions :

25. Range of $f(x)$ is -

(A) $(-\infty, 1]$ (B) $(-\infty, \infty)$

(C) $(-\infty, 0]$ (D) $(-\infty, 2]$

MFN071

26. Range of $g(f(x))$ is -

(A) $(-\infty, \infty)$ (B) $[1,3) \cup (3, \infty)$

(C) $[1, \infty)$ (D) $[0, \infty)$

MFN072

Paragraph for Question No. 27 to 28

Let $f:\mathbb{R}\to\mathbb{R}$ is a function satisfying $f(2-x) = f(2+x)$ and $f(20-x) = f(x), \forall x \in \mathbb{R}$.

On the basis of above information, answer the following questions :

27. If $f(0) = 5$, then minimum possible number of values of x satisfying $f(x) = 5$, for $x \in [0, 170]$ is-

(A) 22 (B) 23

(C) 24 (D) 25

MFN073

28. Graph of $y = f(x)$ is symmetrical about $x = k$. Find k –

(A) 15 (B) 16

(C) 17 (D) 18

MFN074

MATCHING LIST TYPE QUESTION

29. **List-I** $f(x)$ — **List-II** Range

	List-I $f(x)$		List-II Range
(I)	$\dfrac{\cos^2 x+\cos x+2}{\cos^2 x+\cos x+1}$	(P)	$\left(0,\dfrac{7}{3}\right]$
(II)	$\left\|\dfrac{\left(\sqrt{\cos x}-\sqrt{\sin x}\right)\left(\sqrt{\cos x}+\sqrt{\sin x}\right)}{3(\cos x+\sin x)}\right\|$	(Q)	$\left[\dfrac{4}{3},\dfrac{7}{3}\right]$
(III)	$\dfrac{7}{3(x^6+2x^4+3x^2+1)}$	(R)	$\left[0,\dfrac{1}{3}\right]$
(IV)	$\log_8(x^2+2x+2)$	(S)	$[0,\infty)$

Which of the following is correct ?

(A) I $\to$ Q; II $\to$ S; III $\to$ P; IV $\to$ S (B) I $\to$ Q; II $\to$ R; III $\to$ S; IV $\to$ P

(C) I $\to$ P; II $\to$ S; III $\to$ Q; IV $\to$ R (D) I $\to$ P; II $\to$ S; III $\to$ R; IV $\to$ Q

MFN077

MATRIX MATCH TYPE QUESTION

30.

	Column-I Number of integers in		Column-II
(A)	Domain of $f(x)=\ell n\{x\}$	(P)	0
(B)	Domain of $f(x)=\sqrt{\sec(\sin x)}+\sqrt{\left[x+\dfrac{1}{x}\right]}+\sqrt{10-[x]^2}$	(Q)	2
(C)	Range of $f(x)=x^2-2x+2, x\in[0,2]$	(R)	3
(D)	Range of $f(x)=\sqrt{25-[x]^2}$	(S)	less than 3
		(T)	more than 3

(where [.] and {.} denote greatest integer function and fractional part function respectively)

MFN078

EXERCISE - S

1. The sum of integral values of the elements in the domain of $f(x)=\sqrt{\log_{\frac{1}{2}}|3-x|}$ is

MFN021

2. Let $f(x)=\frac{9^x}{9^x+3}$ then find the value of the sum $f\left(\frac{1}{2006}\right)+f\left(\frac{2}{2006}\right)+f\left(\frac{3}{2006}\right)+.......f\left(\frac{2005}{2006}\right)$

MFN022

3. The number of integral values of x satisfying the inequality $[x-5][x-3]+2<[x-5]+2[x-3]$ (where [.] represents greatest integer function) is –

MFN023

4. Suppose, $f(x,n)=\sum_{k=1}^{n}\log_x\left(\frac{k}{x}\right)$, then the value of x satisfying the equation $f(x,10)=f(x,11)$, is

MFN024

5. If $f(x)=a\log\left(\frac{1+x}{1-x}\right)+bx^3+c\sin x+5$ and $f(\log_3 2)=4$, then $f\left(\log_3\left(\frac{1}{2}\right)\right)$ is equal to

MFN025

6. If $f(x)=px+q$ and $f(f(f(x)))=8x+21$, where p and q are real numbers, then $p+q$ equals-

MFN026

7. Let $f(x)=\frac{2x-1}{x+3}$. If $f^{-1}=\frac{ax+b}{c-x}$, then $a+b+c$ is

MFN027

8. A function $'f'$ is defined for all real numbers and satisfies $f(2+x)=f(2-x)$ and $f(7+x)=f(7-x)$ for all real x. If $x=0$ is a root of $f(x)=0$, then find least number of roots of $f(x)=0$ for $x\in[-1000,1000]$.

MFN028

9. If $f(x,y)=\max(x,y)+\min(x,y)$ and $g(x,y)=\max(x,y)-\min(x,y)$, then the value of $f\left(g\left(-\frac{2}{3},-\frac{3}{2}\right),g(-3,-4)\right)$ is equal to $\frac{k}{6}$ then k ($k\in\mathbb{N}$) is -

MFN029

10. Let $'f'$ be an even periodic function with period $'4'$ such that $f(x)=2^x-1, 0\le x\le 2$. The number of solutions of the equation $f(x)=1$ in $[-10,20]$ are

MFN030

EXERCISE - JEE (Main) PYQ

1. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then $J(x)$ is equal to :- **[JEE (Main) 2019]**

(1) $f_3(x)$ (2) $f_1(x)$ (3) $f_2(x)$ (4) $\frac{1}{x} f_3(x)$

MFN031

2. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function f satisfies $f(x+y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. then the natural number 'a' is **[JEE (Main) 2019]**

(1) 4 (2) 3 (3) 16 (4) 2

MFN036

3. The domain of the definition of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :- **[JEE (Main) 2019]**

(1) $(1, 2) \cup (2, \infty)$ (2) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

MFN038

4. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to **[JEE (Main) 2020]**

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$ (3) $-\frac{3}{2}$ (4) $\frac{1}{2}$

MFN039

5. Let $f : (1,3) \rightarrow R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is **[JEE (Main) 2020]**

(1) $\left(\frac{3}{5}, \frac{4}{5}\right)$ (2) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$ (3) $\left(\frac{2}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

MFN040

6. Let $f : R \rightarrow R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is **[JEE (Main) 2020]**

(1) 0 (2) 3 (3) 2 (4) 4

MFN041

7. The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1,1)$, is **[JEE (Main) 2020]**

(1) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$ (2) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(3) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$ (4) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

MFN042

8. Let $f : R \rightarrow R$ be a function which satisfies $f(x+y) = f(x) + f(y) \forall x, y \in R$. If $f(1) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in \mathrm{N}$ then the value of n, for which $g(n) = 20$, is : **[JEE (Main) 2020]**

(1) 5 (2) 9 (3) 20 (4) 4

MFN043

9. For a suitably chosen real constant a, let a function, $f: R - \{-a\} \to R$ be defined by $f(x)=\frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to : **[JEE (Main) 2020]**

(1) $\frac{1}{3}$ (2) 3 (3) –3 (4) $-\frac{1}{3}$

MFN045

10. Suppose that a function $f: R \to R$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i)=363$, then n is equal to. **[JEE (Main) 2020]**

MFN046

11. The range of the function, $f(x)=\log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$ is : **[JEE (Main) 2021]**

(1) $\left(0,\sqrt{5}\right)$ (2) $[-2, 2]$ (3) $\left[\frac{1}{\sqrt{5}},\sqrt{5}\right]$ (4) $[0, 2]$

MFN047

12. Let $f(x)$ be a polynomial of degree 3 such that $f(k)=-\frac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10\,f(10)$ is equal to : **[JEE (Main) 2021]**

MFN048

13. The inverse of $y = 5^{\log x}$ is : **[JEE (Main) 2021]**

(1) $x = 5^{\log y}$ (2) $x = y^{\log 5}$ (3) $x=y^{\frac{1}{\log 5}}$ (4) $x=5^{\frac{1}{\log y}}$

MFN049

14. Let $f: R - \{3\} \to R - \{1\}$ be defined by $f(x)=\frac{x-2}{x-3}$. Let $g: R \to R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to **[JEE (Main) 2021]**

(1) 7 (2) 2 (3) 5 (4) 3

MFN050

15. Let $[x]$ denote the greatest integer $\leq x$, where $x \in \boldsymbol{R}$. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty,a)\cup[b,c)\cup[4,\infty), a<b<c$, then the value of $a + b + c$ is: **[JEE (Main) 2021]**

(1) 8 (2)1 (3) –2 (4) –3

MFN051

16. The number of bijective functions $f:\{1,3,5,7,\ldots\ldots.99\}\to\{2,4,6,8,\ldots\ldots.,100\}$, such that $f(3)\geq f(9)\geq f(15)\geq f(21)\geq\ldots\geq f(99)$, is. **[JEE (Main) 2022]**

(1) ${}^{50}P_{17}$ (2) ${}^{50}P_{33}$ (3) $33! \times 17!$ (4) $\frac{50!}{2}$

MFN052

17. Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0) = p$, $p \neq 0$ and $f(1) = \frac{1}{3}$. If the equation $f(x) = 0$ and $fofofof(x) = 0$ have a common real root, then $f(-3)$ is equal to. **[JEE (Main) 2022]**

MFN053

18. The total number of functions, $f: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4, 5, 6\}$ such that $f(1) + f(2) = f(3)$, is equal to : **[JEE (Main) 2022]**

(1) 60 (2) 90 (3) 108 (4) 126

MFN054

19. Let $f: R \rightarrow R$ be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to : **[JEE (Main) 2022]**

(1) 4 (2) 10 (3) 11 (4) 16

MFN055

20. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x - 1$ and $g: \mathbb{R} - \{1, -1\} \rightarrow \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$.

Then the function fog is : **[JEE (Main) 2022]**

(1) one-one but not onto function (2) onto but not one-one function
(3) both one-one and onto function (4) neither one-one nor onto function

MFN056

21. The number of one-one function $f: \{a, b, c, d\} \rightarrow \{0, 1, 2, \ldots, 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is. **[JEE (Main) 2022]**

MFN057

22. Let $f: R \rightarrow R$ be a function defined by $f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)\left(2 + x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x) = f(f(f(x))) + f(f(x))$, the greatest integer less than or equal to $g(1)$ is **[JEE (Main) 2022]**

MFN059

23. If domain of the function $\log_e\left(\frac{6x^2 + 5x + 1}{2x - 1}\right) + \cos^{-1}\left(\frac{2x^2 - 3x + 4}{3x - 5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then $18(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to. **[JEE (Main) 2023]**

MFN088

24. Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Total number of onto function $f : R \rightarrow S$ such that $f(a) \neq 1$, is equal to ____. **[JEE (Main) 2023]**

MFN089

25. Consider a function $f : \mathbb{N} \rightarrow \mathbb{R}$, satisfying $f(1) + 2f(2) + 3f(3) + \ldots + xf(x) = x(x + 1) f(x)$; $x \geq 2$ with $f(1)=1$. Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to **[JEE (Main) 2023]**

(1) 8200 (2) 8000 (3) 8400 (4) 8100

MFN091

EXERCISE - JEE (Advanced) PYQ

1. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is - **[JEE 2011]**

(A) $\pm\sqrt{n\pi}, n \in \{0,1,2,....\}$ (B) $\pm\sqrt{n\pi}, n \in \{1,2,....\}$

(C) $\frac{\pi}{2}+2n\pi, n \in \{......,-2,-1,0,1,2,....\}$ (D) $2n\pi, n \in \{......,-2,-1,0,1,2,....\}$

MFN080

2. The function $f:[0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is : **[JEE 2012]**

(A) one-one and onto (B) onto but not one-one

(C) one-one but not onto (D) neither one-one nor onto

MFN081

3. Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$. Then:- **[JEE (Advanced) 2014]**

(A) $f(x)$ is an odd function (B) $f(x)$ is a one-one function

(C) $f(x)$ is an onto function (D) $f(x)$ is an even function

MFN083

4. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta-\alpha)$ is_____ **[JEE (Advanced) 2018]**

MFN084

5. If the function $f:\mathbb{R} \to \mathbb{R}$ is defined by $f(x) = |x|(x - \sin x)$, then which of the following statements is TRUE? **[JEE (Advanced) 2020]**

(A) f is one-one, but NOT onto (B) f is onto, but NOT one-one

(C) f is BOTH one-one and onto (D) f is NEITHER one-one NOR onto

MFN085

6. Let the function $f:[0,1] \to \mathbb{R}$ be defined by $f(x)=\frac{4^x}{4^x+2}$

Then the value of $f\left(\frac{1}{40}\right)+f\left(\frac{2}{40}\right)+f\left(\frac{3}{40}\right)+.........+f\left(\frac{39}{40}\right)-f\left(\frac{1}{2}\right)$ is_____

[JEE (Advanced) 2020]

MFN086

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0, if none of the bubbles is darkened.
  Negative Marks : –1 in all other cases.

---

1. Let f be a function satisfying $f(xy)=\frac{f(x)}{y}$ for all positive real numbers x and y. If $f(30) = 20$, then the value of $f(40)$ is-

(1) 15 (2) 20 (3) 40 (4) 60

MFN092

2. Let $f: A \to B$ be an onto function such that $f(x)=\sqrt{x-2-2\sqrt{x-3}}-\sqrt{x-2+2\sqrt{x-3}}$, then set 'B' is-

(1) [–2,0] (2) [0,2] (3) [–3,0] (4) [–1,0]

MFN093

3. Let $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = x^3 + ax^2 + bx - 8$. If $f(x) = 0$ has three real roots & $f(x)$ is a bijective function, then $(a + b)$ is equal to

(1) 0 (2) 6 (3) –6 (4) 12

MFN094

4. $f(x)$ and $g(x)$ are linear function such that for all x, $f(g(x))$ and $g(f(x))$ are Identity functions. If $f(0) = 4$ and $g(5) = 17$, compute $f(2006)$.

(1) 122 (2) 102 (3) 96 (4) 205

MFN095

5. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = ln(x + \sqrt{x^2+1})$, then number of solutions of $|f^{-1}(x)| = e^{-|x|}$ is :-

(1) 1 (2) 2 (3) 3 (4) Infinite

MFN096

6. The domain of the function $f(x) = log_{1/2}\left(-\log_2\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is:

(1) $0 < x < 1$ (2) $0 < x \le 1$ (3) $x \ge 1$ (4) null set

MFN097

7. If $q^2 - 4pr = 0, p > 0$, then the domain of the function $f(x) = log(p\,x^3 + (p+q)x^2 + (q+r)\,x + r)$ is:

(1) $R - \left\{-\frac{q}{2p}\right\}$ (2) $R - \left[(-\infty, -1] \cup \left\{-\frac{q}{2p}\right\}\right]$

(3) $R - \left[(-\infty, -1) \cap \left\{-\frac{q}{2p}\right\}\right]$ (4) R

MFN098

8. Let $f(x) = \dfrac{x-[x]}{1+x-[x]}$, $R \to A$ is onto then find set A. (where {.} and [.] represent fractional part and greatest integer part functions respectively)

(1) $\left(0, \frac{1}{2}\right]$ (2) $\left[0, \frac{1}{2}\right]$ (3) $\left[0, \frac{1}{2}\right)$ (4) $\left(0, \frac{1}{2}\right)$

MFN099

9. The range of the function $f(x) = \log_{\sqrt{2}}\left(2 - \log_2\left(16\sin^2 x + 1\right)\right)$ is

(1) $(-\infty, 1)$ (2) $(-\infty, 2)$ (3) $(-\infty, 1]$ (4) $(-\infty, 2]$

MFN100

10. If domain of $f(x)$ is $(-\infty, 0]$, then domain of $f(6\{x\}^2 - 5\{x\} + 1)$ is (where $\{\cdot\}$ represents fractional part function).

(1) $\bigcup_{n\in I}\left[n+\frac{1}{3}, n+\frac{1}{2}\right]$ (2) $(-\infty, 0)$

(3) $\bigcup_{n\in I}\left[n+\frac{1}{6}, n+1\right]$ (4) $\bigcup_{n\in I}\left[n-\frac{1}{2}, n-\frac{1}{3}\right]$

MFN101

11. Let $f: (e, \infty) \to R$ be defined by $f(x) = \ell n(\ell n(\ell n\, x))$, then

(1) f is one one but not onto (2) f is onto but not one - one

(3) f is one-one and onto (4) f is neither one-one nor onto

MFN102

12. If $f(x) = 2[x] + cosx$, then $f: R \to R$ is: (where [.] denotes greatest integer function)

(1) one–one and onto (2) one–one and into

(3) many–one and into (4) many–one and onto

MFN103

13. Let $f: (2, 4) \to (1, 3)$ be a function defined by $f(x) = x - \left[\frac{x}{2}\right]$ (where [.] denotes the greatest integer function), then $f^{-1}(x)$ is equal to :

(1) $2x$ (2) $x + \left[\frac{x}{2}\right]$ (3) $x + 1$ (4) $x - 1$

MFN104

14. If the function $f: [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$ then f^{-1} is

(1) $\left(\frac{1}{2}\right)^{x(x-1)}$ (2) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(3) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (4) Not defined

MFN105

15. Find the domain of definitions of the functions, $f(x) = \sqrt{1-\sqrt{1-x^2}}$

(1) $[0, 3]$ (2) $[-1, 1]$ (3) R (4) ϕ

MFN106

16. Let $f : [-\sqrt{2}+1, \sqrt{2}+1] \to \left[\frac{-\sqrt{2}+1}{2}, \frac{\sqrt{2}+1}{2}\right]$ be a function defined by $f(x) = \frac{1-x}{1+x^2}$.

If $f^{-1}(x) = \begin{cases} \frac{-1+\lambda\left(\sqrt{4x-4x^2+1}\right)}{2x}, & x \neq 0 \\ \mu \quad , & x = 0 \end{cases}$, then $\lambda + \mu$ is.

(1) 1 (2) 2 (3) 3 (4) 4

MFN107

17. The number of real solutions of the equation $x^3 + 1 = 2\sqrt[3]{2x-1}$, is :

(1) 1 (2) 2 (3) 3 (4) 4

MFN108

18. Which one of the following pair of functions are identical ?

(1) $e^{(\ell nx)/2}$ and $\sqrt{x}$

(2) tan (tan$x$) and cot (cot$x$)

(3) $cos^2x + sin^4x$ and $sin^2x + cos^4x$

(4) $\frac{|x|}{x}$ and sgn (x), where sgn(x) stands for signum function.

MFN109

19. Find range of the function $f(x) = \log_2\left[3x - \left[x + \left[x + [x]\right]\right]\right]$

(where [·] is greatest integer function)

(1) {0, 1} (2) {–1, 4} (3) {3, 7} (4) {4, 9}

MFN110

20. The function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is

(1) an odd function (2) an even function

(3) neither an odd nor an even function (4) a periodic function

MFN111

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4, if only correct answer is given.

Zero Marks : 0, if no answer is given.

Negative Marks : -1 for incorrect answer

1. The number of even integral value(s) in the range of the function $f(x) = \frac{\tan^2 x + 8\tan x + 15}{1 + \tan^2 x}$ is

MFN112

2. Number of integers in the range of the function $f(x) = \dfrac{x^3+2x^2+3x+2}{x^3+2x^2+2x+1}$; $x \in \mathbb{R}^*$ is :

MFN113

3. If the range of the function $f(x) = \left\{\dfrac{x}{4}\right\} + \cos\pi\left(\dfrac{(1-2[x])}{2}\right) + \sin\left(\dfrac{\pi[x]}{2}\right)$ is $\left[\dfrac{\alpha}{4},\dfrac{\beta}{4}\right) \cup \left[\dfrac{\gamma}{4},\dfrac{\delta}{4}\right) \cup \left[\dfrac{2\gamma+1}{4},\dfrac{\delta}{2}\right)$, (where {.} and [.] represent fractional part and greatest integer part functions respectively), then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is

MFN114

4. The function $f(x)$ has the property that for each real number x in its domain, $1/x$ is also in its domain and $f(x) + f(1/x) = x$. The largest set of real numbers that can be in the domain of $f(x)$ contains k elements then k is equal to ?

MFN115

5. If f and g are two distinct linear functions defined on R such that they map$[-1, 1]$ onto $[0, 2]$ and $h: R - \{-1, 0, 1\} \to R$ defined by $h(x) = \dfrac{f(x)}{g(x)}$, then $|h(h(x)) + h(h(1/x))| > n$. Then maximum integral value of n is :

MFN116

6. Let $f : R-\left\{\dfrac{1}{2}\right\} \to R-\left\{\dfrac{1}{2}\right\}, f(x)=\dfrac{x-2}{2x-1}$ be a function such that $x = m$ is the solution of $f(x) + 2f^{-1}(x) + 2 = f(f(x))$, then m is equal to

MFN117

7. Period of $f(x)=\sec^2\left(\dfrac{\pi x}{6}\right)+\cot^2\left(\dfrac{\pi x}{6}\right)$ is

MFN118

8. Number of integral points in the domain of function $f(x)=\dfrac{1}{1-\sqrt{-x^2+4x+11}}$, is -

MFN119

9. The value of $\left[\sqrt[3]{1}\right]+\left[\sqrt[3]{2}\right]+....+\left[\sqrt[3]{124}\right]$ is (where [.] denotes greatest integer function)

MFN120

10. If $f(x)=\sqrt{\ell n\left(\sin\dfrac{\pi}{4}\sqrt{x}\right)}+\sqrt{16-x^2}$, then number of integers in the domain of $f(x)$ is

MFN121

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-I

- This section contains **TEN** questions.
- Each question has **FOUR** options for correct answer(*s*). **ONE OR MORE THAN ONE** of these four option(*s*) is (are) correct option(*s*).
- For each question, choose the correct option(*s*) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 if only (all) the correct option(*s*) is (are) chosen.
Partial Marks : +3 if all the four options are correct but ONLY three options are chosen.
Partial Marks : +2 if three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks : +1 if two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered).
Negative Marks : –2 in all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(*s*) (second option in this case), with or without selection of any correct option(*s*) will result in –2 marks.

---

1. If $f(x) = \sin \ell n\left(\frac{\sqrt{4-x^2}}{1-x}\right)$, then

(A) domain of $f(x)$ is $(-2, 1)$
(B) domain of $f(x)$ is $[-1, 1]$
(C) range of $f(x)$ is $[-1, 1]$
(D) range of $f(x)$ is $[-1, 1)$

MFN122

2. D is domain and R is range of $f(x) = \sqrt{x-1} + 2\sqrt{3-x}$, then

(A) $D : [1, 3]$;
(B) $D : (-\infty, 1] \cup [3, \infty)$,
(C) $R : [1, \sqrt{3}]$
(D) $R : [\sqrt{2}, \sqrt{10}]$

MFN123

3. Let $D \equiv [-1, 1]$ is the domain of the following functions, state which of them are injective.

(A) $f(x) = \begin{cases} \tan^{-1}\frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$
(B) $g(x) = x^3$
(C) $h(x) = \sin 2x$
(D) $k(x) = \sin(\pi x/2)$

MFN124

4. Let $f(x)=x^{135}+x^{125}-x^{115}+x^5+1$. If f(x) divided by x^3-x, then the remainder is some function of x say $g(x)$. Then $g(x)$

(A) one-one function
(B) many one function
(C) $g(0)=0$
(D) $g(10)=21$

MFN125

5. If $f(x)=\begin{cases} x^2 & x\le 1 \\ 1-x & x>1 \end{cases}$ & composite function $h(x)=|f(x)|+f(x+2)$, then

(A) $h(x)=2x^2+4x+4 \quad \forall\ x\le -1$
(B) $h(x)=x^2+x+1 \quad \forall\ -1<x\le 1$
(C) $h(x)=x^2-x-1 \quad \forall\ -1<x\le 1$
(D) $h(x)=-2 \quad \forall\ x>1$

MFN126

6. Let $f(x)=\begin{cases} 0 & \text{for } x=0 \\ x^2\sin\left(\frac{\pi}{x}\right) & \text{for } -1<x<1\ (x\neq 0) \\ x\ |x| & \text{for } x>1 \text{ or } x<-1 \end{cases}$, then:

(A) $f(x)$ is an odd function
(B) $f(x)$ is an even function
(C) $f(x)$ is neither odd nor even
(D) $f'(x)$ is an even function

MFN127

7. If $f:[-2,2]\to R$ where $f(x)=x^3+\tan x+\left[\frac{x^2+1}{P}\right]$ is a odd function, then the value of parametric P, where [.] denotes the greatest integer function, can be

(A) $5<P<10$
(B) $P<5$
(C) $P>5$
(D) $P=15$

MFN128

8. If $f(x)=\dfrac{2x\ (\sin x+\tan x)}{2\left[\frac{x+2\pi}{\pi}\right]-3}$ then it is, (where [.] denotes the greatest integer function)

(A) odd
(B) Even
(C) many one
(D) one-one

MFN129

9. If $F(x)=\dfrac{\sin\pi[x]}{\{x\}}$, then $F(x)$ is: (where { . } denotes fractional part function and [.] denotes greatest integer function and sgn (x) is a signum function)

(A) periodic with fundamental period 1
(B) even
(C) range is singleton
(D) identical to sgn $\left(\text{sgn}\dfrac{\{x\}}{\sqrt{\{x\}}}\right)-1$

MFN130

10. Let $f:R\to R$ and $g:R\to R$ be two one-one and onto functions such that they are mirror images of each other about the line $y=a$. If $h(x)=f(x)+g(x)$, then $h(x)$ is

(A) one-one
(B) into
(C) onto
(D) many-one

MFN131

SECTION–II

- This section contains **ONE** paragraph.
- Based on each paragraph, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 in all other cases.

Comprehension # 1 (Q. No. 11 to 13)

Given a function $f : A \to B$; where $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7, 8\}$

11. Find number of all such functions $y = f(x)$ which are one-one?

(A) 0 (B) 3^5 (C) ${}^5P_3$ (D) 5^3

MFN132

12. Find number of all such functions $y = f(x)$ which are onto

(A) 243 (B) 93 (C) 150 (D) none of these

MFN133

13. The number of mappings of $g(x) : B \to A$ such that $g(i) \le g(j)$ whenever $i < j$ is

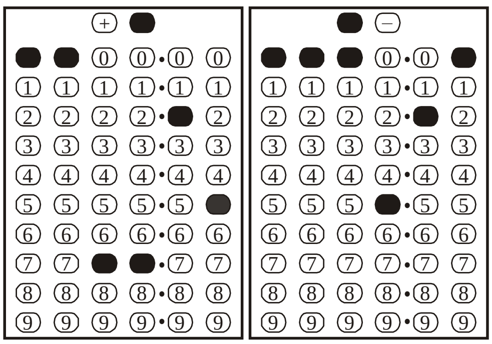
(A) 60 (B) 140 (C) 10 (D) 35

MFN134

SECTION-III

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.

  For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the correct numerical value is entered as answer.

  Zero Marks : 0 In all other cases.

14. The domain of the function $y = \sqrt{\sin x + \cos x} + \sqrt{7x - x^2 - 6}$ is $\left[p, \frac{q\pi}{4}\right] \cup \left[\frac{r\pi}{4}, s\right]$ then value of $p + q + r + s$ is

MFN135

15. The domain of $f(x)$ such that the $f(x) = \dfrac{\left[x+\frac{1}{2}\right]}{\left[x-\frac{1}{2}\right]}$ is prime is $[x_1, x_2)$, then the value of $2\left(x_1^2 + x_2^2\right)$.

[Where [.] denotes greatest integer function less than or equal to x]

MFN136

16. Range of the function $f(x) = |\sin x|\cos x| + \cos x|\sin x||$ is $[a, b]$ then $(a + b)$ is equal to

MFN137

17. If $f(x) = ax^7 + bx^3 + cx - 5$; a, b, c are real constants and $f(-7) = 7$ then maximum value of $|f(7) + 17\cos x|$ is

MFN138

18. If $f(x) = \dfrac{4a-7}{3}x^3 + (a-3)x^2 + x + 5$ is a one-one function, then number of possible integral values of a is

MFN139

19. Let f be a one–one function with domain {21, 22, 23} and range $\{x, y, z\}$. It is given that exactly one of the following statements is true and the remaining two are false. $f(21) = x$; $f(22) \neq x$; $f(23) \neq y$. Then $f^{-1}(x)$ is :

MFN140

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	B	B	D	A	C	D	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B	D	C	C	A	B	A,C	A,B,C	B,C,D	C,D
Que.	21	22	23	24	25	26	27	28	29	
Ans.	A,B,C	A,D	B,C	A,B,C	A	C	A	D	A	
Que.	30									
Ans.	$A \to P,S$; $B \to R$; $C \to Q,S$; $D \to T$									

EXERCISE - S

1. 6 **2.** 1002.50 **3.** 2 **4.** 11 **5.** 6
6. 5 **7.** 6 **8.** 401 **9.** 11 **10.** 15

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	2	3	2	4	2	3	1	2	5.00
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	26	3	3	3	2	25	2	2	4
Que.	21	22	23	24	25					
Ans.	31	2	20	180	4					

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6
Ans.	A	B	A,B,C	119	C	19.00

JEE (Main) Practice Paper

		1	2	3	4	5	6	7	8	9	10
Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	1	1	2	1	2	4	2	3	4	1
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	3	3	3	2	2	2	3	3	1	2
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	9	0	15	2	2	2	3	7	400	1

JEE (Advanced) Practice Paper

Section-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	A.	A,C	A,D	B,D	A,D	A,C,D	A,D	A,C,D	A,C
	Q.	**9**	**10**						
	A.	A,B,C,D	B,D						
Section-II	**Q.**	**11**	**12**	**13**					
	A.	A	C	D					
Section-III	**Q.**	**14**	**15**	**16**	**17**	**18**	**19**		
	A.	17	17	1	34	7	22		