Determinants

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

a, b, c in A.P $\rightarrow 2b = a + c \Rightarrow b - a = c - b$

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_2 \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1 \begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b-a \\ 1 & 1 & b-a \end{vmatrix} \Rightarrow 0$$

2. **Ans. (A)**

$$\begin{vmatrix} 1 & \log_x^y & \log_x^z \\ \log_y^x & 1 & \log_y^z \\ \log_z^x & \log_z^y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\log x}{\ln x} & \frac{\ln y}{\ln x} & \frac{\ln z}{\ln x} \\ \frac{\ln x}{\ln y} & \frac{\ln y}{\ln y} & \frac{\ln z}{\ln y} \\ \frac{\ln x}{\ln z} & \frac{\ln y}{\ln z} & \frac{\ln z}{\ln z} \end{vmatrix}$$

$$= \begin{vmatrix} \ln x & \ln y & \ln z \\ \ln x & \ln y & \ln z \\ \ln x & \ln y & \ln z \end{vmatrix} = 0$$

3. **Ans. (C)**

$$B = \begin{vmatrix} p+x & q+y & r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2 - R_3, R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_2$

$$B = 2 \begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = 2A = 12$$

4. Ans. (D)

$$S_r = \begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$$

$$\sum_{r=1}^{n} Sr = \begin{vmatrix} \Sigma 2r & x & n(n+1) \\ \Sigma(6r^2-1) & y & n^2(2n+3) \\ \Sigma(4r^3-2nr) & z & n^3(n+1) \end{vmatrix}$$

$$= \begin{vmatrix} n(n+1) & x & n(n+1) \\ n^2(2n+3) & y & n^2(2n+3) \\ n^3(n+1) & z & n^3(n+1) \end{vmatrix} = 0$$

5. Ans. (D)

$$\begin{vmatrix} \left(a^x+a^{-x}\right)^2 & \left(a^x-a^{-x}\right)^2 & 1 \\ \left(b^y+b^{-y}\right)^2 & \left(b^y-b^{-y}\right)^2 & 1 \\ \left(c^z+c^{-z}\right)^2 & \left(c^z-c^{-z}\right)^2 & 1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 4 & (a^x-a^{-x})^2 & 1 \\ 4 & (b^y-b^{-y})^2 & 1 \\ 4 & (c^z-c^{-z})^2 & 1 \end{vmatrix} = 4\begin{vmatrix} 1 & (a^x-a^{-x})^2 & 1 \\ 1 & (b^y-b^{-y})^2 & 1 \\ 1 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$$

C_1 & C_3 are identical so value of det. is 0

6. Ans. (B)

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$

$$\begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$= (a+b+c)\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix}$$

$= (a+b+c)\left((b-c)(a-b)-(c-a)^2\right)$

$= (a+b+c)\left(ab-b^2-ca+bc-c^2-a^2+2ac\right)$

$= -\dfrac{1}{2}(a+b+c)\left((a-b)^2+(b-c)^2+(c-a)^2\right) < 0$

7. Ans. (C)

$D = 0$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{vmatrix} = 0$$

$(3a - 25) - 2(a - 10) + 3\,(5 - 6) = 0$

$3a - 25 - 2a + 20 - 3 = 0$

$a = 8$

if $b = 15$

then

$$D_1 = \begin{vmatrix} 6 & 2 & 3 \\ 9 & 3 & 5 \\ 15 & 5 & 8 \end{vmatrix} = 3\begin{vmatrix} 2 & 2 & 3 \\ 3 & 3 & 5 \\ 5 & 5 & 8 \end{vmatrix} = 0$$

$\Rightarrow b = 15$

8. Ans. (C)

$C_1 \to C_1 + C_2 + C_3$

$$(2 + 4\sin 2x)\begin{vmatrix} 1 & \cos^2 x & 4\sin 2x \\ 1 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1\,;\; R_3 \to R_3 - R_1$

$$(2 + 4\sin 2x)\begin{vmatrix} 1 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$\Rightarrow (2 + 4\sin 2x)\,(1)$

$$\left(f(x)\right)_{max} = 2+4=6$$

9. Ans. (A)

$$D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$$

$$D_1 = \begin{vmatrix} a & b & a \\ c & d & c \\ a & b & a \end{vmatrix} + \begin{vmatrix} a & b & b \\ c & d & d \\ a & b & -b \end{vmatrix}$$

$R_1 \to R_1 - R_3$

$$\begin{vmatrix} 0 & 0 & 2b \\ c & d & d \\ 0 & b & -b \end{vmatrix} = 2b(bc - ad)$$

$$D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a & c & a \\ b & d & b \\ a & c & a \end{vmatrix} + \begin{vmatrix} a & c & c \\ b & d & d \\ a & c & b+c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a & c & 0 \\ b & d & 0 \\ a & c & b \end{vmatrix} + \begin{vmatrix} 0 & 0 & c \\ 0 & 0 & d \\ 0 & 0 & c \end{vmatrix}$$

$D_2 = -b(bc - ad)$

$\dfrac{D_1}{D_2} = -2$

10. Ans. (C)

$C_1 \to C_1 + C_2 + C_3$

$$(1 + x + x - (2)x) \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$f(x) \Rightarrow (1-x)^2$.

11. Ans. (B)

Apply

$R_1 \to R_1 + (sin\phi)R_2 + (cos\phi)R_3$

$$\begin{vmatrix} 0 & 0 & 2\cos^2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$$

$= 2cos^2\phi(sin^2\phi + cos^2\phi)$

$= 2cos^2\phi$ independent of (θ)

12. Ans. (A)

$a^2x - ay = 1 - a$

$bx + (3 - 2b)y = 3 + a$

$x = y = 1$ will satisfy both equation

$a^2 - a = 1 - a \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

I : $a = 1$	II : $a = -1$	
$b + 3 - 2b = 3 + a$	$a + b = 0$	$b = 1$ $b = -1$
$x - y = 0$	$x + y = 2$	$-x + 5y = 4$
	$x + y = 2$	

unique solution   coincident infinite solution

13. Ans. (D)

$$\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2 - R_3$

$$\begin{vmatrix} 0 & -2z & -2y \\ y & x+z & y \\ z & z & x+y \end{vmatrix}$$

$$R_2 \rightarrow R_2 + \frac{1}{2}R_1$$

$$R_3 \rightarrow R_3 + \frac{1}{2}R_1$$

$$\begin{vmatrix} 0 & -2z & -2y \\ y & x & 0 \\ z & 0 & x \end{vmatrix}$$

$0[x^2 - 0] + 2z\,[yx - 0] - 2y\,[0 - xz] + 2xyz + 2xyz$

$4xyz$

14. Ans. (B)

$x + ay + oz = 0$

$ox + y + az = 0$

$ax + oy + z = 0$

for unique solution.

$$D = \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$$

$= 1 - a(-a^2) = 0$

$\Rightarrow a^2 = 1$

$\Rightarrow a = \pm 1$

but if $a = 1 \Rightarrow x = y = z = 0$ is unique solution.

$a \neq -1$

$\Rightarrow \quad a \in R\{-1\}$

$$D = \begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix} = 0$$

$\sin\theta\,(\sin\theta\cos\theta + \lambda^2 + \lambda) + \cos\theta\,(\cos^2\theta + \lambda^2) + (\lambda + 1)\,(\cos\theta\,(\lambda + 1) - \lambda\sin\theta) = 0$

$\sin^2\theta\cos\theta + l(\lambda + 1)\sin\theta + \cos^2\theta + \lambda^2\cos\theta + (\lambda + 1)^2\cos\theta - l\,(\lambda + 1)\sin\theta = 0$

$\cos\theta + \cos\theta\,(\lambda^2 + (\lambda + 1)^2) = 0$

$\cos\theta\,(1 + (\lambda^2 + (\lambda + 1)^2) = 0$

$\Rightarrow \cos\theta = 0$ and $\lambda \in$ R

$\theta = (2n + 1)\dfrac{\pi}{g}, n \in I, \lambda \in R$

15. **Ans. (D)**

$D = 0$

$$D=\begin{vmatrix}1 & -c & -b\\ -c & 1 & -a\\ -b & -a & 1\end{vmatrix}$$

$D = 1[1 - a^2] + c[- c - ab] - b[ac + b]$

$D = 1 - a^2 - b^2 - c^2 - 2abc$

$D = 0$

so $a^2 + b^2 + c^2 + 2abc = 1$

16. **Ans. (A,B,C,D)**

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix}1+1+2\sin 4\theta & \cos^2 A & 2\sin 4\theta\\ 1+1+2\sin 4\theta & 1+\cos^2 A & 2\sin 4\theta\\ 1+1+2\sin 4\theta & \cos^2 A & 1+2\sin 4\theta\end{vmatrix}$$

$$2+2\sin 4\theta\begin{vmatrix}1 & \cos^2 A & 2\sin 4\theta\\ 1 & 1+\cos^2 A & 2\sin 4\theta\\ 1 & \cos^2 A & 1+2\sin 4\theta\end{vmatrix}$$

Apply $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Rightarrow 2(1+\sin 4\theta)\begin{vmatrix}0 & -1 & 0\\ 0 & 1 & -1\\ 1 & \cos^2 A & 1+2\sin 4\theta\end{vmatrix}=0$$

$$\Rightarrow 2(1+\sin 4\theta)\{1(1-0)\}=0$$

$$\Rightarrow \sin 4\theta=-1$$

$$\because\ \theta\in\left(\frac{-\pi}{4},\frac{\pi}{2}\right)$$

$$4\theta=\frac{-\pi}{2},\frac{3\pi}{2}$$

$$4\theta\in(-\pi,\ 2\pi)$$

$\boxed{\theta=\frac{-\pi}{8},\frac{3\pi}{8}}$ $\forall\ A \in R$ (Doesn't depends on A)

$\therefore$ All options are correct

17. **Ans. (B,D)**

$R_3 \to R_3 - (\alpha R_1 + R_2)$

$$\begin{vmatrix}a & b & 0\\ b & c & 0\\ a\alpha+b & b\alpha+c & -(\alpha(a\alpha+b)+b\alpha+c)\end{vmatrix}=0$$

Expand by R_3 :

$$-\underbrace{(a\alpha^2+2b\alpha+c)}_{\Downarrow\ 0}\underbrace{(b^2-ac)}_{\Downarrow\ 0}=0$$

If $a\alpha^2+2b\alpha+c=0$

$\Rightarrow$ $x=\alpha$ is root of QE $ax^2+2bx+c=0$

If $b^2-ac=0 \Rightarrow a,b,c$ are in G.P.

18. Ans. (A,B,D)

Let $y=\lambda\in R$

$2x + z = 1 - \lambda$ ...(1)

$x + z = 2 + 2\lambda$ ...(2)

(1) – (2)

$\Rightarrow x = -1 - 3\lambda$

Put in (1), we get $z = 3 + 5\lambda$

for option (B) replace λ by $(-\lambda)$

for option (D) put value of x, y, z in give equation

19. Ans. (B,C)

$C_1 \to C_1 + C_3,\ C_2 \to C_2 + C_3$

$$\Delta=\begin{vmatrix} a\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) & b\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) & 1 \\ b\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) & c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) & 1 \\ c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) & a\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) & 1 \end{vmatrix}=\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2\begin{vmatrix} a & b & 1 \\ b & c & 1 \\ c & a & 1\end{vmatrix}$$

$$=\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2\left[a^2+b^2+c^2-ab-bc-ca\right]=0$$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=0$

or $(a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow a = b = c$

20. Ans. (A,B,C,D)

$$D=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & p\end{vmatrix}=p-8$$

for infinitely many solutions $D = 0$

$\Rightarrow p = 8$

Now, $D_1=\begin{vmatrix} 6 & 1 & 1 \\ 14 & 2 & 3 \\ q & 5 & 8\end{vmatrix}=q-36=0$

$\Rightarrow q = 36$

and similarly, $D_2 = D_3 = 0$

for unique solutions it is sufficient to have $p \neq 8$

$\Rightarrow$ for atleast one solution, p may be any real number and $q = 36$.

21. Ans. (A,B,C)

$R_2 - \frac{R_1 + R_3}{2}$

$$\Delta = \begin{vmatrix} x+a & x^2+\log\alpha & k \\ 0 & 0 & 0 \\ x+c & x^2+\log\gamma & k \end{vmatrix} = 0$$

$\therefore$ Equation is an identity.

22. Ans. (A,B,D)

(A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is true

(B) Use C & D, $\frac{a_1+a_2}{a_1-a_2} = \frac{b_1+b_2}{b_1-b_2} = \frac{c_1+c_2}{c_1-c_2}$

(C) Both roots will be common

(D) Ratio of coefficients are equal i.e.

$$\frac{a_1^2 a_2}{a_1 a_2^2} = \frac{b_1^2 b_2}{b_1 b_2^2} = \frac{c_1^2 c_2}{c_2^2 c_1}$$

as $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, given system have infinite solutions.

23. Ans. (A)

$$D = \begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix}$$

$$= \begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$$

$= (xyz)(x-y)(y-z)(z-x)(xy+yz+zx) - (x-y)(y-z)(z-x)(x+y+z)$

$= (x-y)(y-z)(z-x)\{xyz(xy+yz+zx) - (x+y+z)\}$

$x \neq y \neq z$; $y^2 = xz$

$D = 0$

$\Rightarrow xyz(xy+yz+zx) = x+y+z$

$\Rightarrow y^3(xy+yz+y^2) = x+y+z$

$\Rightarrow (x+y+z)\left(y^4-1\right) = 0$

$y = 1 (y \in R^+)$

24. Ans. (B)

$$D=\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix}$$

$$=\begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix}-\begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

$$=xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}-\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$$

$$=(xyz)(x-y)(y-z)(z-x)(xy+yz+zx)-(x-y)(y-z)(z-x)(x+y+z)$$

$$=(x-y)(y-z)(z-x)\{xyz(xy+yz+zx)-(x+y+z)\}$$

$x + y + z = 21 \Rightarrow \text{AM}\ (x, y, z) = 7$

$xyz = 343 \Rightarrow \text{GM}\ (x, y, z) = 7$

$\because \text{AM} = \text{GM} \Rightarrow x = y = z$

$\Rightarrow x = y = z = 7$

$\Rightarrow D = 0$

25. Ans. (C)

$$D=\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix}$$

$$=\begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix}-\begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

$$=xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}-\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$$

$$=(xyz)(x-y)(y-z)(z-x)(xy+yz+zx)-(x-y)(y-z)(z-x)(x+y+z)$$

$$=(x-y)(y-z)(z-x)\{xyz(xy+yz+zx)-(x+y+z)\}$$

$x\neq y\neq z \qquad 2y = x + z$

$D=0$

$\Rightarrow xyz(xy+yz+zx)=x+y+z$

$\Rightarrow xyz\left(2y^2+zx\right)-3y=0$

$\Rightarrow y\left(2y^2xz+z^2x^2-3\right)=0$

$\Rightarrow 2y^2xz+z^2x^2=3$

26. **Ans. (A,C)**

The only even prime number is 2, hence $t = 2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ \alpha & \beta & \gamma \end{vmatrix} = 2$$

$\Rightarrow \alpha - 2\beta + \gamma = 2$ ......(i)

$\because$ α, β, γ are integral roots of $x^3 - 14x^2 + px - 36 = 0$

$\Rightarrow \alpha + \beta + \gamma = 14$ .....(ii)

$\alpha\beta\gamma = 36$ .....(iii)

on solving we gat $\alpha = 1, \beta = 4, \gamma = 9$

$\alpha\beta + \beta\gamma + \gamma\alpha = 49 \Rightarrow P = 49$

27. **Ans. (A,B)**

The only even prime number is 2, hence $t = 2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ \alpha & \beta & \gamma \end{vmatrix} = 2$$

$\Rightarrow \alpha - 2\beta + \gamma = 2$ ......(i)

$\because$ α, β, γ are integral roots of $x^3 - 14x^2 + px - 36 = 0$

$\Rightarrow \alpha + \beta + \gamma = 14$ ......(ii)

$\alpha\beta\gamma = 36$ .....(iii)

on solving we gat $\alpha = 1, \beta = 4, \gamma = 9$

The given expression is

$\alpha\beta(\alpha+\beta) + \beta\gamma(\beta+\gamma) + \gamma\alpha(\gamma+\alpha)$

$\alpha\beta(14-\gamma) + \beta\gamma(14-\alpha) + \gamma\alpha(14-\beta)$

$14(\alpha\beta + \beta\gamma + \gamma\alpha) - 3\alpha\beta\gamma$

$14.49 - 3.36 = 578$

28. **Ans. (A,B,C)**

The only even prime number is 2, hence $t = 2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ \alpha & \beta & \gamma \end{vmatrix} = 2$$

$\Rightarrow \alpha - 2\beta + \gamma = 2$ ......(i)

$\because$ α, β, γ are integral roots of $x^3 - 14x^2 + px - 36 = 0$

$\Rightarrow \alpha + \beta + \gamma = 14$ .....(ii)

$\alpha\beta\gamma = 36$ .....(iii)

on solving we gat $\alpha = 1, \beta = 4, \gamma = 9$

The value of t is 2 but $\alpha - \beta = -3$, $\beta - \gamma = -5$, $\gamma - \alpha = 8$

which does not divide 't'

29. Ans. (D)

$$\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix} \qquad \Delta_x=\begin{vmatrix}d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3\end{vmatrix}$$

$$\Delta_y=\begin{vmatrix}a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3\end{vmatrix} \qquad \Delta_z=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$$

(A) $$\Delta=\begin{vmatrix}k & b_1 & c_1\\ k & b_2 & c_2\\ k & b_3 & c_3\end{vmatrix}=k\begin{vmatrix}1 & b_1 & c_1\\ 1 & b_2 & c_2\\ 1 & b_3 & c_3\end{vmatrix}$$

$$\Delta_x=\begin{vmatrix}k^2 & b_1 & c_1\\ k^2 & b_2 & c_2\\ k^2 & b_3 & c_3\end{vmatrix}=k^2\begin{vmatrix}1 & b_1 & c_1\\ 1 & b_2 & c_2\\ 1 & b_3 & c_3\end{vmatrix}$$

$$\Delta_y=\begin{vmatrix}k & k^2 & c_1\\ k & k^2 & c_2\\ k & k^2 & c_3\end{vmatrix}=0$$ Similarly, $\Delta_z = 0$

$$\therefore\ x=\frac{\Delta_x}{\Delta} \qquad y=\frac{\Delta_y}{\Delta} \qquad z=\frac{\Delta_z}{\Delta}$$

$\alpha = k \qquad \beta = 0 \qquad \gamma = 0$

Given $\alpha + \beta + \gamma = 2$

$k + 0 + 0 = 2$

$k = 2$

(B) $$\Delta_x=\begin{vmatrix}k & b_1 & c_1\\ k & b_2 & c_2\\ k & b_3 & c_3\end{vmatrix}=\Delta$$

$\therefore x = 1 ; \alpha = 1$

$$\Delta_y=\begin{vmatrix}k & k & c_1\\ k & k & c_2\\ k & k & c_3\end{vmatrix}=0$$

$\Rightarrow \beta = 0$

Similarly $\Delta_z = 0$

$\Rightarrow \gamma = 0$

$\alpha + \beta + \gamma = 1$

(C,D) $$\Delta_x=\begin{vmatrix}k+1 & b_1 & c_1\\ k+1 & b_2 & c_2\\ k+1 & b_3 & c_3\end{vmatrix}$$

$$\Rightarrow\ x=\frac{\Delta_p}{\Delta}=\frac{k+1}{k}=\alpha$$

$$\Delta_y = \begin{vmatrix} k & k+1 & c_1 \\ k & k+1 & c_2 \\ k & k+1 & c_3 \end{vmatrix} = 0 \Rightarrow y = 0 = \beta$$

Similarly, $\Delta_z = 0 \Rightarrow z = 0 = r$

$$\alpha+\beta+\gamma = \frac{k+1}{k} = 1 + \frac{1}{k}$$

If $k > 0$, then $\alpha + \beta + \gamma > 1 \rightarrow$ Opt. Q, S

If $k < 0$, then $\alpha + \beta + \gamma > 1 \rightarrow$ Opt. R, T

30. Ans. (A → Q; B → P, R; C → P; D → R)

$\lambda x + y + z = 1$

$x + \lambda y + z = \lambda$

$x + y + \lambda z = \lambda^2$

$$\Delta = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix}$$

$$\Delta = (\lambda+2)\begin{vmatrix} 0 & 1-\lambda & 0 \\ 0 & \lambda-1 & 1-\lambda \\ 1 & 1 & \lambda \end{vmatrix}$$

$$\Delta = (\lambda+2)(\lambda-1)^2$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ \lambda & \lambda & 1 \\ \lambda^2 & 1 & \lambda \end{vmatrix} = (\lambda^2 - 1)(1 - \lambda)$$

$$\Delta_y = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & \lambda^2 & \lambda \end{vmatrix} = (\lambda - 1)^2$$

$$\Delta_z = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & \lambda \\ 1 & 1 & \lambda^2 \end{vmatrix} = (\lambda - 1)^2 (\lambda + 1)^2$$

(A) For $\lambda = 1$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ & all the three equations become identical so number of solutions is infinite.

(B) For $\lambda \neq 1$

but $\lambda = -2$ then $\Delta = 0$ & $\Delta_x, \Delta_y, \Delta_z \neq 0 \Rightarrow$ No solution

and if $\lambda \neq -2$ then $\Delta \neq 0 \Rightarrow$ unique solution

(C) for $\lambda \neq 1, \lambda \neq -2$

$\Delta \neq 0 \Rightarrow$ unique solution

(D) $\lambda = -2$

$\Delta = 0$ and $\Delta_x, \Delta_y, \Delta_z \neq 0 \Rightarrow$ No solution

EXERCISE - S

1. Ans. (80)

$x^3 - 5x^2 + 3x - 1 = 0$

$a + b + c = 5$

$ab + bc + ca = 3$

$$= \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

$= (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

$= 5(25 - 6 - 3) = 80$

2. Ans. (0)

$$\Delta(x) = \begin{vmatrix} 0 & 2x-2 & 2x+6 \\ x-1 & 4 & x^2+7 \\ 0 & 0 & x+4 \end{vmatrix}$$

$$f(x) = \sum_{j=1}^{3}\left(\sum_{i=1}^{3} a_{ij}\ c_{ij}\right)$$

$$= \sum_{j=1}^{3}\left(a_{1j}c_{1j} + a_{2j}c_{2j} + a_{3j}c_{3j}\right)$$

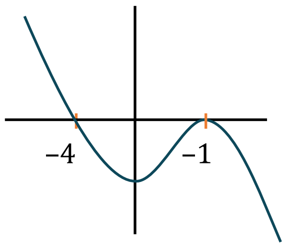
$$= -3\big((x-1)(x+4)(2x-2)\big)$$

$= -6(x - 1)\ (x + 4)\ (x - 1)$

$f(x) = -6\ (x - 1)^2(x + 4)$

$x \in [-3, 18]$

$f(x)_{max}$ is at $x = 1 = f_{max} = 0$



3. Ans. (2)

For non-trivial solution $\Delta = 0$

$$\therefore \begin{vmatrix} 1 & -a & -a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 - C_2$

$C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} 1+a & 0 & -a \\ 1+b & -(1+b) & b \\ 0 & (1+c) & -1 \end{vmatrix} = 0$$

$(1 + a)\ \{(1 + b) - b\ (1 + c)\} - (1 + b)\ a\ (1 + c) = 0$

Divide by $(1 + a)\ (1 + b)\ (1 + c)$

$$\therefore \frac{1}{1+c}-\frac{b}{1+b}-\frac{a}{1+a}=0$$

$$\Rightarrow \frac{1}{1+c}-\frac{b+1-1}{1+b}-\frac{a+1-1}{1+a}=0$$

$$\Rightarrow \frac{1}{1+c}-\left(1-\frac{1}{1+b}\right)-\left(1-\frac{1}{1+a}\right)=0$$

or $\boxed{\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=2}$

4. **Ans. (4)**

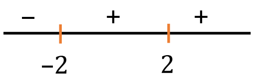
$x + ay = z$

$ax + 4y = 6$ ...(1)

$a(3 - ay) + 4y = 6$ ...(2)

$$y=\frac{3a-6}{a^2-4}>0$$

$$\frac{3(a-2)}{(a-2)(a+2)}>0$$



a ∈ (–2, 2) ∪ (2, ∞) ...(1.a)

(1) 4 – (2) a

$4x + 4ay = 12$

$a^2x + 4ay = 6a$

– – –

$(4-a^2)x = 6(2-a)$

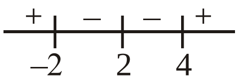
$$x=\frac{6(2-a)}{4-a^2}$$

$$\frac{6(a-2)}{a^2-4}>1$$

$$\frac{6a-12-a^2+4}{a^2-4}>0$$

$$\frac{a^2-6a+8}{a^2-4}<0$$

$$\frac{(a-2)(a-4)}{(a-2)(a+2)}<0$$



a ∈ (–2, 2)∪(2, 4) ....(2.a)

from (1.a) & (2.a)

$a = -1,0,1,3$

5. **Ans. (0)**

$$\Delta=\begin{vmatrix}0 & -\tan^{-1}(\tan 1) & -\tan^{-1}(\tan 2)\\ \tan^{-1}(\tan 1) & 0 & -\tan^{-1}(\tan 1)\\ \tan^{-1}(\tan 2) & \tan^{-1}(\tan 1) & 0\end{vmatrix}=0$$

because it is skew symmetric determinant.

6. **Ans. (1)**

$2x - y + 3z = 4$ ...(1)

$x + y - 3z = -1$ ...(2)

$5x - y + 3z = 7$ ...(3)

$$\Rightarrow \Delta = \begin{vmatrix}2 & -1 & 3\\ 1 & 1 & -3\\ 5 & -1 & 3\end{vmatrix}$$

Similarly, $\Delta_x = \Delta_y = \Delta_z = 0$

Let $z = k$ solving (i) & (ii) we get

$x = 1, y = 3k - 2, z = k$

$xyz \le 0$

$\Rightarrow k(3k - 2) \le 0$

$0 \le k \le 2/3$

$\Rightarrow k = 0$ is only integral value.

7. **Ans. (5)**

The given system of equation can be written as

$$a\left(\frac{\ell}{n}\right)+b\left(\frac{m}{n}\right)+c=0$$

$$b\left(\frac{\ell}{n}\right)+c\left(\frac{m}{n}\right)+a=0$$

$$c\left(\frac{\ell}{n}\right)+a\left(\frac{m}{n}\right)+b=0$$

Lines $ax + by + c = 0$

$bx + cy + a = 0$

$cx + ay + b = 0$

are concurrent.

$$\Rightarrow \begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=0$$

$\Rightarrow (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow a + b + c = 0$ {$\because$ a, b & c are distinct} $\Rightarrow f(1) = 5$

8. **Ans. (0)**

Put $\cos x = 0 \Rightarrow x = \frac{\pi}{2}$ is one of the solutions

$$\Rightarrow a_0 = \begin{vmatrix}0 & -1 & 2\\ 1 & 0 & -1\\ -2 & 1 & 0\end{vmatrix}=0$$

9. Ans. (9)

$$-\begin{vmatrix} z & y & 90+x \\ z & 90+y & x \\ 90+z & y & x \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$-\begin{vmatrix} z & y & 90+x \\ 0 & 90 & -90 \\ 90 & 0 & -90 \end{vmatrix}$$

expand + $8100z + y(8100) + (90 + x)(8100) + 8100(z + y + 90 + x)$

$x = 0, y = 1, z = 2$

$8100(93) = \lambda$

10. Ans. (4)

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} - \begin{vmatrix} b-c & b & b+a \\ a & -a-c & b-a \\ c+b & c-a & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} - \begin{vmatrix} b-c & a & b+c \\ b & -a-c & c-a \\ b+a & b-a & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} + \begin{vmatrix} a & b-c & b+c \\ -a-c & b & c-a \\ b-a & b+a & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2a & b-c & c+b \\ 0 & b & c-a \\ 0 & b+a & c \end{vmatrix} = 0$$

$\Rightarrow\ 2a[bc - (b + a)(c - a)] = 0$

$\Rightarrow\ 2a^2(a + b - c) = 0 \Rightarrow a + b = c$

$\because\ (a + b)^2 \geq 4ab \Rightarrow \dfrac{c^2}{ab} \geq 4$

EXERCISE - JEE (Main) PYQ

1. Ans. (2)

$$\Delta_1 = f(\theta) = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3$$

and $$\Delta_2 = f(2\theta) = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix} = -x^3$$

So $\Delta_1 + \Delta_2 = -2x^3$

2. **Ans. (2)**

$a + x = b + y = c + z + 1$

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} \quad C_3 \to C_3 - C_1$$

$$\begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

$$\begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \quad R_3 \to R_3 - R_1, R_2 \to R_2 - R_1$$

$$\begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$= (-y)[(y - x)(c - a) - (b - a)(z - x)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c - 1)]$

$= (-y)[(a - b)(c - a) + (a - b)(a - c) + b - a)$

$= -y(b - a) = y(a - b)$

3. **Ans. (4)**

For infinite many solutions

$D = D_1 = D_2 = D_3 = 0$

$$\text{Now } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$1.(2\lambda - 9) - 1.(\lambda - 3) + 1.(3 - 2) = 0$

$\therefore \lambda = 5$

$$\text{Now } D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0$$

$2(10 - 9) - 1(25 - 3\mu) + 1(15 - 2\mu) = 0$

$\mu = 8$

4. **Ans. (4)**

$$\text{Here } D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 2(-a-1) - 1(a-1) + 1 + 1 = 1 - 3a$$

$$D_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 2(-b-3) - 1(b-3) + 5(1+1) = 7 - 3b$$

for $a = \frac{1}{3}$, $b \neq \frac{7}{3}$, system has no solutions

5. Ans. (1)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28+[\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = -[\lambda] - 9$$

if $[\lambda] + 9 \neq 0$ then unique solution

if $[\lambda] + 9 = 0$ then $D_1 = D_2 = D_3 = 0$

so infinite solutions

Hence λ can be any real number.

6. Ans. (2)

$$\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$$

$R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$$

$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$

7. Ans. (6)

$$\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix} \begin{pmatrix} R_1 \to R_1 - R_2 \\ \& \ R_2 \to R_2 - R_3 \end{pmatrix}$$

$$-2(\cos^2 x) + 2(2 + 2\cos 2x + \sin^2 x)$$

$$4 + 4\cos 2x - 2(\cos^2 x - \sin^2 x)$$

$$f(x) = 4 + \underbrace{2\cos 2x}_{\max = 1}$$

$$f(x)_{\max} = 4 + 2 = 6$$

8. Ans. (2)

Case–I

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0 \ \ 0$$

or $4\sin3\theta = -2$

$\sin3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

9. **Ans. (2)**

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = 0$$

$\Rightarrow \delta = -3$

And $\begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ K & 4 & -3 \end{vmatrix} = 0 \Rightarrow K = 6$

$\Rightarrow \delta + K = 3$

Alternate

$2x + y - z = 7$ ...(1)

$x - 3y + 2z = 1$ ...(2)

$x + 4y + \delta z = k$ ...(3)

Equation (2) + (3)

We get $2x + y + (2+\delta)z = 1 + K$ ...(4)

For infinitely many solution

comparing equation (1) and (4)

$2 + \delta = -1 \Rightarrow \boxed{\delta = -3}$

$1 + k = 7 \Rightarrow \boxed{k = 6}$

$\delta + k = 3$

10. **Ans. (2)**

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0$$

$\Rightarrow |\lambda| = 7 \Rightarrow \lambda = \pm 7$ ...(1)

System :

$2x + 3y - z = -2$ ...(2)

$x + y + z = 4$ ...(3)

$x - y + |\lambda|z = 4\lambda - 4$ ...(4)

Eliminating y from equal (2) & (3) we get

$x + 4z = 14$ ...(5)

$(3) + (4) \Rightarrow x + \left(\frac{|\lambda| + 1}{2}\right)z = 2\lambda$ ...(6)

Clearly for $\lambda = -7$, system is inconsistent.

11. **Ans. (3)**

$$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$$

$3(-8a-9)+2(5a-18)+1(21)=0$

$\Rightarrow a=-3$

Also $\Delta_3 = \begin{vmatrix} 3 & -2 & b \\ 5 & -8 & 3 \\ 2 & 1 & -1 \end{vmatrix}$

If $b = \frac{1}{3}$

$\Delta_3 = 0$

So, *b* must be equal to

$-\frac{1}{3}$

12. **Ans. (2)**

$x+y+z=\alpha$

$\alpha x+2\alpha y+3z=-1$

$x+3\alpha y+5z=4$

Has inconsistent solution

$D = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix} = 0$

$\Rightarrow (\alpha-1)^2 = 0$

$\alpha = 1$

For $\alpha = 1$

$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 4 & 3 & 5 \end{vmatrix}$

$= (10-9) - (-5-12) + (-3-8)$

$= 1 + 17 - 11 \neq 0$

For $\alpha = 1$ the system of equation has Inconsistent solution

13. **Ans. (3)**

$\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$

$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(3 + \alpha)$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$\alpha \neq -3, x = 1, y = 1, z = 0,$

Now points $(\alpha, 1), (1, \alpha)$ & $(1, -1)$ are collinear

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \alpha(\alpha + 1) - 1(1 - 1) + 1(-1 - \alpha) = 0$

$\alpha^2 + \alpha - 1 - \alpha = 0$

$\alpha = \pm 1$

14. **Ans. (4)**

$$\Delta = \begin{vmatrix} P! & (P+1)! & (P+2)! \\ (P+1)! & (P+2)! & (P+3)! \\ (P+2)! & (P+3)! & (P+4)! \end{vmatrix}$$

$$\Delta = P!(P+1)!(P+2)! \begin{vmatrix} 1 & 1 & 1 \\ P+1 & P+2 & P+3 \\ (P+2)(P+1) & (P+3)(P+2) & (P+4)(P+3) \end{vmatrix}$$

$$\Delta = 2P!(P+1)!(P+2)!$$

Which is divisible by P^{α} & $(P+2)^{\beta}$

$\therefore \alpha = 3, \beta = 1$

15. **Ans. (3)**

$x + y + z = 6$ ...(1)

$2x + 5y + \alpha z = \beta$ ...(2)

$x + 2y + 3z = 14$ ...(3)

$x + y = 6 - z$

$x + 2y = 14 - 3z$

On solving

$x = z - 2 \Rightarrow y = 8 - 2z$ in (2)

$2(z - 2) + 5(8 - 2z) + \alpha z = \beta$

$(\alpha - 8) z = \beta - 36$ For having infinite solutions

$\alpha - 8 = 0$ & $\beta - 36 = 0$

$\alpha = 8, \beta = 36$ $(\alpha + \beta = 44)$

16. Ans. (1)

$C_1 \to C_1 + C_2 + C_3$

$$f(x)=\begin{vmatrix}2+\sin 2x & \cos^2 x & \sin 2x\\ 2+\sin 2x & 1+\cos^2 x & \sin 2x\\ 2+\sin 2x & \cos^2 x & 1+\sin 2x\end{vmatrix}$$

$$f(x)=(2+\sin 2x)\begin{vmatrix}1 & \cos^2 x & \sin 2x\\ 1 & 1+\cos^2 x & \sin 2x\\ 1 & \cos^2 x & 1+\sin 2x\end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$f(x)=(2+\sin 2x)\begin{vmatrix}1 & \cos^2 x & \sin 2x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$$

$= (2 + \sin 2x)\ (1) = 2 + \sin 2x$

$= \sin 2x \in \left[\frac{\sqrt{3}}{2}, 1\right]$

Hence $2+\sin 2x \in \left[2+\frac{\sqrt{3}}{2}, 3\right]$

17. Ans. (6)

$$D_k=\begin{vmatrix}1 & 2k & 2k-1\\ n & n^2+n+2 & n^2\\ n & n^2+n & n^2+n+2\end{vmatrix}$$

$$\sum_{k=1}^{n} D_k = 96$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow n(2n + 4) = 96 \Rightarrow n(n + 2) = 48 \Rightarrow n = 6$

18. Ans. (1)

$$\Delta=\begin{vmatrix}2 & 4 & 2a\\ 1 & 2 & 3\\ 2 & -5 & 2\end{vmatrix}=18(3-a)$$

$$\Delta_x=\begin{vmatrix}b & 4 & 2a\\ 4 & 2 & 3\\ 8 & -5 & 2\end{vmatrix}=(64+19b-72a)$$

For unique solution $\Delta \neq 0$

$\Rightarrow \boxed{a \neq 3}$ and $\boxed{b \in R}$

For Infinitely many solution

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$\Rightarrow a = 3 \quad \because \Delta = 0$

and $b = 8 \ \because \Delta_x = 0$

19. **Ans. (4)**

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$= a(15a^2+31a+36)=0 \Rightarrow a=0$

$\Delta \neq 0$ for all $a \in R-\{0\}$

Hence $S_1 = R-\{0\}$ $S_2 = \phi$

20. **Ans. (4)**

By equation 1 and 3 $y + 2z = 8$

$y = 8 - 2z$

and $x = -2 + z$

Now putting in equation 2

$\alpha(z-2)+\beta(-2z+8)+7z=3$

$\Rightarrow(\alpha-2\beta+7)z=2\alpha-8\beta+3$

So, equations have unique solution if

$\alpha-2\beta+7\neq 0$

And equations have no solution if

$\alpha-2\beta+7=0$ and $2\alpha-8\beta+3\neq 0$

And equations have infinite solution if

$\alpha-2\beta+7=0$ and $2\alpha-8\beta+3=0$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (B,C)**

$R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha$$

$R_3 \to R_3 - R_2$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix} = -648\alpha$$

$C_3 \to C_3 - C_2,\ C_2 \to C_2 - C_1$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(2+3\alpha) & \alpha(2+5\alpha) \\ 3+2\alpha & 2\alpha & 2\alpha \\ 2 & 0 & 0 \end{vmatrix} = -648\alpha$$

$\Rightarrow 2\alpha^2(2+3\alpha) - 2\alpha^2(2+5\alpha) = -324\alpha$

$\Rightarrow -4\alpha^3 = -324\alpha \Rightarrow \alpha = 0, \pm 9$

2. Ans. (2)

$$x.x^2 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 0 & 2 & 6x^3-1 \\ 0 & 6 & 24x^3-2 \end{vmatrix} = 10 \Rightarrow x^3(12x^3+2) = 10$$

$6x^6 + x^3 - 5 = 0 \Rightarrow 6x^6 + 6x^3 - 5x^3 - 5 = 0$

$(6x^3 - 5)(x^3 + 1) = 0 \Rightarrow x^3 = -1, x^3 = \frac{5}{6}$

$x = -1, x = \left(\frac{5}{6}\right)^{1/3}$ so, two solutions = 2

3. Ans. (B,C,D)

$ax + 2y = \lambda$

$3x - 2y = \mu$

(A) $a = -3$ gives

$\lambda = -\mu$

or $\lambda + \mu = 0$ not for all λ, μ

(B) $a \neq -3 \Rightarrow \Delta \neq 0$ where $\Delta = \begin{vmatrix} a & 2 \\ 3 & -2 \end{vmatrix} = -2a - 6$

$\therefore$ (B) is correct

(C) correct

(D) if $\lambda + \mu \neq 0$

$\Rightarrow -3x + 2y = \lambda$ ...(1)

& $3x - 2y = \mu$ ...(2)

Inconsistent $\Rightarrow$ (D) correct

4. Ans. (4)

$$det(P) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) \leq 6$$

value can be 6 only if $a_1 = 1, a_2 = -1, a_3 = 1, b_2c_3 = b_1c_3 = b_1c_2 = 1, b_3c_2 = b_3c_1 = b_2c_1 = -1$

$\Rightarrow (b_2c_3)(b_3c_1)(b_1c_2) = -1$

i.e. $b_1b_2b_3c_1c_2c_3 = 1$ and

hence not possible

Similar contradiction occurs when

$a_1 = 1, a_2 = 1, a_3 = 1, b_2c_2 = b_3c_1 = b_1c_2 = 1\ b_3c_2 = b_1c_3 = b_1c_2 = -1$

Now for value to be 5 one the terms must be zero but that will make 2 terms zero which means answer cannot be 5

Now $\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{vmatrix} = 4$ Hence max value = 4

5. Ans. (A,C)

$$f(\theta)=\frac{1}{2}\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}\sin\pi & \cos\left(\theta+\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right)\\ \sin\left(\theta-\frac{\pi}{4}\right) & -\cos\frac{\pi}{2} & \log_e\left(\frac{4}{\pi}\right)\\ \cot\left(\theta+\frac{\pi}{4}\right) & \log_e\frac{\pi}{4} & \tan\pi\end{vmatrix}$$

$$f(\theta)=\frac{1}{2}\begin{vmatrix}2 & \sin\theta & 1\\ 0 & 1 & \sin\theta\\ 0 & -\sin\theta & 1\end{vmatrix}+\begin{vmatrix}0 & -\sin\left(\theta-\frac{\pi}{4}\right) & \tan\left(\theta-\frac{\pi}{4}\right)\\ \sin\left(\theta-\frac{\pi}{4}\right) & 0 & \log_e\left(\frac{4}{\pi}\right)\\ -\tan\left(\theta-\frac{\pi}{4}\right) & -\log_e\left(\frac{4}{\pi}\right) & 0\end{vmatrix}$$

$f(\theta) = (1+\sin^2\theta) + 0$ (skew symmetric)

$g(\theta)=\sqrt{f(\theta)-1}+\sqrt{f\left(\frac{\pi}{2}-\theta\right)-1}$

$= |\sin\theta| + |\cos\theta|$ for $\theta\in\left[0,\frac{\pi}{2}\right]$

$g(\theta)\in\left[1,\sqrt{2}\right]$

Again let $P(x)=k\left(x-\sqrt{2}\right)(x-1)$

$2-\sqrt{2}=k\left(2-\sqrt{2}\right)(2-1)$

$\Rightarrow k = 1$ $\left(P(2)=2-\sqrt{2}\text{ given}\right)$

$\therefore\ P(x)=\left(x-\sqrt{2}\right)(x-1)$

for option (A) $P\left(\frac{3+\sqrt{2}}{4}\right)<0$ correct

option (B) $P\left(\frac{1+3\sqrt{2}}{4}\right)>0$ incorrect

option (C) $P\left(\frac{5\sqrt{2}-1}{4}\right)>0$ correct

option (D) $P\left(\frac{5-\sqrt{2}}{4}\right)<0$ incorrect

6. Ans. (B)

Let A is first of correct

If $\frac{q}{r} = 10 \Rightarrow A = D \Rightarrow D_x = D_y = D_z = 0$

So, there are infinitely many solutions

Look of infinitely many solutions can be given as

$x + y + z = 1$

& $10x + 100y + 1000z = 0 \Rightarrow x + 10y + 100z = 0$

Let $z = \lambda$

then $x + y = 1 - \lambda$

and $x + 10y = -100\lambda$

$\Rightarrow x = \frac{10}{9} + 10\lambda\ ;\ y = \frac{-1}{9} - 11\lambda$

i.e., $(x, y, z) \equiv \left(\frac{10}{9} + 10\lambda, \frac{-1}{9} - 11\lambda, \lambda\right)$

$Q\left(\frac{10}{9}, \frac{-1}{9}, 0\right)$ valid for $\lambda = 0$

$P\left(0, \frac{10}{9}, \frac{-1}{9}\right)$ valid for $\lambda = \frac{-1}{9}$

(I) $\rightarrow$ P,Q,R,T

(II) If $\frac{p}{r} \neq 100$, then $D_y \neq 0$

So, no solution

(II) $\rightarrow$ (S)

(III) If $\frac{p}{q} \neq 10$, then $D_z \neq 0$ so, no solution

(III) $\rightarrow$ (S)

(IV) If $\frac{p}{q} = 10 \Rightarrow D_z = 0 \Rightarrow D_x = D_y = 0$

so infinitely many solution

(IV) $\rightarrow$ Q,R,T

7. Ans. (A)

Given $x + 2y + z = 7$ .... (1)

$x + \alpha z = 11$ .... (2)

$2x - 3y + \beta z = \gamma$ .... (3)

Now, $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 7\alpha - 2\beta - 3$

$\therefore$ if $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \boxed{\Delta = 0}$

Now, $\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix}$

$= 21\alpha - 22\beta + 2\alpha\gamma - 33$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_x = 0$

$$\Delta_y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix}$$

$\Delta_y = 4\beta + 14\alpha - \alpha\gamma + \gamma - 22$

$\therefore$ if $\gamma = 28$

$\Rightarrow \Delta_y = 0$

Now, $\Delta_z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = 56 - 2\gamma$

If $\gamma = 28$

$\Rightarrow \boxed{\Delta_z = 0}$

$\therefore$ if $\gamma = 28$ and $\beta = \frac{1}{2}(7\alpha - 3)$

$\Rightarrow$ system has infinite solution

and if $\gamma \neq 28$

$\Rightarrow$ system has no solution

$\Rightarrow$ P $\to$ (3) ; Q $\to$ (2)

Now if $\beta \neq \frac{1}{2}(7\alpha - 3)$

$\Rightarrow \Delta \neq 0$

and for $\alpha = 1$ clearly

$y = -2$ is always be the solution

$\therefore$ if $\gamma \neq 28$

System has a unique solution

if $\gamma = 28$

$\Rightarrow x = 11, y = -2$ and $z = 0$ will be one of the solution

$\therefore$ R $\to$ 1 ; S $\to$ 4

$\therefore$ option 'A' is correct

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (2)**

Expanding $a(a^2) + b(b^2) = a^3 + b^3$

2. **Ans. (1)**

$$\Rightarrow \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$$

Now expanding we will get zero

$= -\sin B \tan A \cos C + \cos C \sin B \tan A = 0$

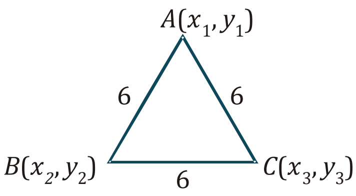
3. Ans. (1)

Put $x = 0$ on both sides.

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = a_0$$

$\therefore a_0 = -1$

4. Ans. (4)



$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$$

$= [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]^2 = 4 \times (\text{Area})^2$

$$4 \times \left(\frac{1}{2} \times 6 \times 3\sqrt{3}\right)^2 = 972$$

5. Ans. (3)

$c_1 \to c_1 + c_2 + c_3$

Column first becomes 0.

$\therefore \ \Delta = 0$

6. Ans. (4)

Applying $R_1 \to R_1 + R_2$ $\begin{vmatrix} 0 & 0 & 2c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

then expanding $= 2c(ab + ab) = 4\,abc$

7. Ans. (1)

Apply $R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

then taking $\sin(A-B)$, $\sin(B-C)$ common from determinant then solving this we will get 0.

8. Ans. (4)

$D_1 + D_2 + D_3 + D_4 + D_5$

$$= \begin{vmatrix} 1+2+3+4+5 & 15 & 8 \\ 1^2+2^2+3^2+4^2+5^2 & 35 & 9 \\ 1^3+2^3+3^3+4^3+5^3 & 25 & 10 \end{vmatrix} = \begin{vmatrix} 15 & 15 & 8 \\ 55 & 35 & 9 \\ 225 & 25 & 10 \end{vmatrix}$$

applying $C_1 \to C_1 - C_2$

$$= \begin{vmatrix} 0 & 15 & 8 \\ 20 & 35 & 9 \\ 200 & 25 & 10 \end{vmatrix} = 20 \times 5 \begin{vmatrix} 0 & 3 & 8 \\ 1 & 7 & 9 \\ 10 & 5 & 10 \end{vmatrix}$$

$= -100 \times 280$

9. **Ans. (4)**

$C_1 \rightarrow C_1 - C_2$

$$\begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^y - b^{-y})^2 & 1 \\ 4 & (c^z - c^{-z})^2 & 1 \end{vmatrix} = 0$$

10. **Ans. (4)**

Multiplying R_1, R_2, R_3 with c, a, b then applying

$c_1 \rightarrow c_1 - c_2$ & $c_2 \rightarrow c_2 - c_3$

$\alpha = 4$

11. **Ans. (3)**

$$D = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ ca^2b^2 & cab & c(a+b) \end{vmatrix}$$

$(R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3)$

$$= abc\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+cb \end{vmatrix}$$

$$= abc(ab+bc+ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} \qquad (C_3 \rightarrow C_3 + C_1)$$

$= 0$

12. **Ans. (1)**

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$

$C_3 \rightarrow C_3 - C_1 - C_2$

$$f(x) = \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x+1)(x-2) & 0 \end{vmatrix} \Rightarrow f(x) = 0$$

$\therefore f(100) = 0$

13. **Ans. (2)**

Obviously, each row can be split into 2 determinants so total = $2 \times 2 \times 2 = 8$

14. **Ans. (4)**

$\Delta = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & p & 2 \\ \mu & 4 & 1 \end{vmatrix} = 0$$

$$\Delta_x = \begin{vmatrix} 4 & 2 & 3 \\ 3 & p & 2 \\ 3 & 4 & 1 \end{vmatrix} = 0$$

$\Rightarrow 4(p-8) - 2(3-6) + 3(12-3p) = 0$

$\Rightarrow -5p + 10 = 0 \Rightarrow p = 2$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 3 & 2 \\ \mu & 3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 0 & 1 & 1 \\ 1 & 3 & 2 \\ \mu & 3 & 1 \end{vmatrix} = 0 \Rightarrow -1(1 - 2\mu) + 1(3 - 3\mu) = 0$$

$\Rightarrow \mu = 2$

15. Ans. (4)

$$\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & (1-x)^2 & 1-2x \\ 2x+1 & 3x & 3x-2 \\ x+1 & 2x & 2x-3 \end{vmatrix}$$

Since two columns are same in above determinants therefore we can add them along C_3.

$$\Rightarrow \begin{vmatrix} (1+x)^2 & (1-x)^2 & -(x+1)^2 \\ 2x+1 & 3x & -(1+2x) \\ x+1 & 2x & -(1+x) \end{vmatrix} = 0$$

$\Rightarrow 0 = 0 \Rightarrow$ infinite solution

16. Ans. (3)

for non-trivial solution $D = 0$

$$\begin{vmatrix} 1-a & 1 & 1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$$R_1 \to R_1 - R_3 \begin{vmatrix} -a & 0 & c \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix}$$

$\Rightarrow -a\{(1-b)(1-c)-1\} + c\{1-(1-b)\} = 0 \Rightarrow ab + ac + bc - abc = 0$

$\Rightarrow ab + ac + bc = abc$

17. Ans. (2)

$$\sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \sum_{r=1}^{n-1} r & \sum_{r=1}^{n-1} 2r-1 & \sum_{r=1}^{n-1} 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{1}{2}(n-1)(3n-4) \end{vmatrix} = \begin{vmatrix} \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \end{vmatrix}$$

R_1 and R_3 are identical so

$\sum_{r=1}^{n-1} \Delta_r = 0$ is independent of a, and n.

18. **Ans. (1)**

For nontrivial solution $\Delta = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1, -3$$

19. **Ans. (2)**

$$\frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$$

(1) = (2)

$\Rightarrow k^2 - 4k + 3 = 0$

$k = 1, 3$

for $k = 1$ (2) = (3)

for $k = 3$ (2) $\neq$ (3)

$k = 3$

20. **Ans. (1)**

For non-zero solution $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\Rightarrow k = 11$

Now equations

$x + 11y + 3z = 0$ ..... (1)

$3x + 11y - 2z = 0$ ..... (2)

$2x + 4y - 3z = 0$ ..... (3)

on equation (1) + (3) we get $3x + 15y = 0$

$\Rightarrow x = -5y$

Now put $x = -5y$ in equation (1)

we get $-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

$$\frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$$

SECTION–B

1. **Ans. (1)**

$$\begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$$

$C_3 \rightarrow C_3 - 2C_2$

$$\begin{vmatrix} 18 & 40 & 9 \\ 40 & 89 & 20 \\ 89 & 198 & 44 \end{vmatrix}$$

$C_2 \rightarrow C_2 - 2C_1$

$$\begin{vmatrix} 18 & 4 & 9 \\ 40 & 9 & 20 \\ 89 & 20 & 44 \end{vmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{vmatrix} 18 & 4 & 9 \\ 40 & 9 & 20 \\ 9 & 2 & 4 \end{vmatrix}$$

$R_1 \to R_1 - 2R_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ 40 & 9 & 20 \\ 9 & 2 & 4 \end{vmatrix}$$

= 1(80 – 81) EXPANDING ABOUT R_1

= –1

Hence, absolute value of determinant is 1.

2. Ans. (2)

$R_3 \to R_3 - (R_1 + R_2)$and $R_2 \to R_2 - 2R_1$

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ -1 & -x-2 & -2x-3 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

$R_1 \to R_1 + 2R_2$

$$\begin{vmatrix} x & -1 & -x-2 \\ -1 & -x-2 & -2x-3 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

Solving it we get $x = -1$ or $x = -2$

3. Ans. (6)

Let $\tan A = x, \tan B = y, \tan C = z$

$$\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

= $2 \tan A \tan B \tan C\ (\tan A + \tan B + \tan C)^3$

= $2(\tan A + \tan B + \tan C)^4 \geq\ 2 \times 729$

4. Ans. (8)

Given determinant is $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

= $(x-y)^2(y-z)^2(z-x)^2$

minimum value is 256

(take $x = 2, y = 0, z =-2$)

5. Ans. (1)

$2x - y + 3z = 4$ ...(i)

$x + y - 3z = -1$ ...(ii)

$5x - y + 3z = 7$ ...(iii)

$\Rightarrow \Delta = 0$

Similarly, $\Delta_x = \Delta_y = \Delta_z = 0$

Let $z = k$ solving (i) & (ii) we get

$x = 1, y = 3k - 2, z = k$

$xyz \leq 0$

$k(3k - 2) \leq 0$

$0 \leq k \leq 2/3$

$\Rightarrow k = 0$ is only integral value.

6. Ans. (0)

$$\Delta = \begin{vmatrix} \frac{\log xyz}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log xyz}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log xyz}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$$

Take $\log(xyz)$ common from C_1

Take $\log(y)$ common from C_2

Take $\log(z)$ common from C_3

$$\Delta = \log(xyz).\log y.\log z \begin{vmatrix} \frac{1}{\log x} & \frac{1}{\log x} & \frac{1}{\log x} \\ \frac{1}{\log y} & \frac{1}{\log y} & \frac{1}{\log y} \\ \frac{1}{\log z} & \frac{1}{\log z} & \frac{1}{\log z} \end{vmatrix} = 0$$

7. Ans. (1)

$x^3 - 3x + 2 = 0$

$\Rightarrow x = 1, 1, -2$

8. Ans. (0)

$a_1A_2 + b_1B_2 + c_1C_2 = 0$

Since sum of the product of elements of any raw with cofactors of corresponding elements of other row is always zero.

9. Ans. (33)

$3x + ky - 2z = 0$

$x + ky + 3z = 0$

$2x + 3y - 4x = 0$

$$D = \begin{vmatrix} 3 & k & -2 \\ 1 & k & 3 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$$\Rightarrow k = \frac{33}{2}$$

10. Ans. (9)

$$f(x)=x(x-1)^2(x-2)\begin{vmatrix}1 & x-1 & 1\\ 2 & x-2 & 1\\ 3 & x-3 & 1\end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$f(x) = x(x-1)^2(x-2)\begin{vmatrix}-1 & 1 & 0\\ -1 & 1 & 0\\ 3 & (x-3) & 1\end{vmatrix}=0$$

$f(x) = 0$

$f(50) = 0$

$$D_5=\begin{vmatrix}1 & 10 & 10\\ 70 & 17 & 16\\ 1 & 1 & 1\end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$D_5=\begin{vmatrix}1 & 0 & 10\\ 70 & 1 & 16\\ 1 & 0 & 1\end{vmatrix}$$

$D_5 = -9$

JEE (Advanced) Practice Paper

1. Ans. (C)

Applying $R_3 \to R_3 - R_2$

$R_2 \to R_2 - R_1$

$$\begin{vmatrix}1+\sin^2 x & \cos^2 x & 4\sin 2x\\ -1 & 1 & 0\\ 0 & -1 & 1\end{vmatrix}$$

$= (1+\sin^2 x)\ \cos^2 x\ (-1) + 4\sin 2x\,(1)$

$= 2 + 4\sin 2x$

maximum $\sin 2x = 1$ so $2 + 4 = 6$

2. Ans. (A)

$a, b, c \in R, a^2 + b^2 + c^2 = 1$

$$\begin{vmatrix}ax-by-c & bx+ay & cx+a\\ bx+ay & -ax+by-c & cy+b\\ cx+a & cy+b & -ax-by+c\end{vmatrix}$$

$$\frac{1}{abc}\begin{vmatrix}a^2x-aby-ca & b^2x+aby & c^2x+ac\\ abx+a^2y & -abx+b^2y-cb & c^2y+bc\\ acx+a^2 & cby+b^2 & -acx-bcy+c^2\end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$

$$\frac{1}{abc}\begin{vmatrix}(a^2+b^2+c^2)x & b^2x+aby & c^2x+ac\\(a^2+b^2+c^2)y & -abx+b^2y-cb & c^2y+bc\\(a^2+b^2+c^2) & cby+b^2 & -acx-bcy+c^2\end{vmatrix}$$

Taking $a^2+b^2+c^2$, b, c common from c_1 and c_2 , c_3

$$(a^2+b^2+c^2)\frac{1}{a}\begin{vmatrix}x & bx+ay & cx+a\\y & -ax+by-c & cy+b\\1 & cy+b & -ax-by+c\end{vmatrix}$$

$a^2+b^2+c^2=1$

$c_2 \to c_2 - bc_1$, $c_3 \to c_3 - cc_1$

$$\frac{1}{a}\begin{vmatrix}x & ay & a\\y & -ax-c & b\\1 & cy & -ax-by\end{vmatrix}$$

$R_3 \to R_3 + xR_1 + yR_2$

$$\frac{1}{a}\begin{vmatrix}x & ay & a\\y & -ax-c & b\\1+x^2+y^2 & 0 & 0\end{vmatrix}$$

$= 1(1+x^2+y^2)(ax+by+c)=0$

$ax+by+c=0$

3. **Ans. (C)**

$R_3 \to R_3 - (xR_1 + yR_2)$

$$\begin{vmatrix}p & q & px+qy\\q & r & qx+ry\\0 & 0 & -px^2-2qxy-ry^2\end{vmatrix}$$

$= -(pr-q^2)(px^2+2qxy+ry^2)$

$= -(+ve)(+ve) = -(ve)$

4. **Ans. (D)**

$$D=\begin{vmatrix}1 & -1 & 3\\2 & -1 & 1\\1 & -2 & \alpha\end{vmatrix}=0$$

$D_3 \neq 0$ so for $\alpha = 8$ no solution

for $\alpha \neq 8$ unique solution

5. **Ans. (C)**

$x+y=3$ ...(i)

$(1+K)x+(K+2)y=8$ ...(ii)

$x-(1+K)y=-K-2$ ...(iii)

If system is consistent then $\Delta = 0$

on solving we get

$K=1, \frac{-5}{3}$

6. **Ans. (C)**

For consistency we have

$$\begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0 \Rightarrow c = \frac{5b}{4b+3} < 1 \Rightarrow \frac{5b-4b-3}{4b+3} < 0 \Rightarrow \frac{b-3}{4b+3} < 0 \Rightarrow b \in \left(-\frac{3}{4}, 3\right)$$

7. **Ans. (B,C)**

$px^3 + x^2q - rx - r = 0$

$\therefore \alpha + \beta + \gamma = -q/p, \quad \alpha\beta\gamma = r/p, \ \sum \alpha\beta = -r/p$

$$\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix} = \alpha\beta\gamma\left(1 + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$$

$$= \alpha\beta\gamma\left(\frac{\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}\right)$$

$$= \frac{r}{p} - \frac{r}{p} = 0$$

8. **Ans. (A,B,C,D)**

$$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2 A & 2\sin 4\theta \\ 2 & 1+\cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 0 & -1 & 0 \\ 2 & 1+\cos^2 A & 2\sin 4\theta \\ 1 & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow 2(1 + 2\sin 4\theta) - 2\sin 4\theta = 0$

$\Rightarrow 1 + \sin 4\theta = 0 \Rightarrow \sin 4\theta = -1$

$4\theta = -\frac{\pi}{2}$ or $4\theta = \frac{3\pi}{2}$

$\theta = -\frac{\pi}{8}$ or $\theta = \frac{3\pi}{8}$ and $A \in R$

9. **Ans. (B, D)**

$$\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}=0$$

$c_3 \to c_3 - \alpha c_1 - c_2$

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha+b & b\alpha+c & -a\alpha^2-2b\alpha-c \end{vmatrix}=0$$

$\Rightarrow -(a\alpha^2+2b\alpha+c)\,(ac-b^2)=0$

$\Rightarrow a\alpha^2+2b\alpha+c=0$ or $b^2=ac$

$a, b, c,$ are in G.P.

10. **Ans. (B,C,D)**

$$\begin{vmatrix} 1 & 1 & 0 \\ \alpha+\beta & \gamma+\delta & 0 \\ \alpha\beta & \gamma\delta & 0 \end{vmatrix}\begin{vmatrix} 1 & \gamma+\delta & \gamma\delta \\ 1 & \alpha+\beta & \alpha\beta \\ 0 & 0 & 0 \end{vmatrix}$$

= 0.0= 0

11. **Ans. (B)**

Let $\frac{x^2}{a^2}=X, \frac{y^2}{b^2}=Y, \frac{z^2}{c^2}=z$

Then the system of equations are

$X + Y - Z = 1$

$X - Y + Z = 1$

$-X + Y + Z = 1$

So $\Delta=\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}=-4\neq 0$ it has unique solution.

12. **Ans. (A,C,D)**

A. As $D_1 = D_2 = D_3 = 0$ and

$$D=\begin{vmatrix} 1 & a & a \\ 1 & b & b \\ 1 & c & c \end{vmatrix}=0$$

$\therefore$ Non-trivial solution.

B. Homogeneous system is always consistent.

C. $x+ay+az=0$ ...(i)

$x+by+bz=0$ ...(ii)

$x+cy+cz=0$ ...(iii)

Now assume y = t.

equation (i) – equation (ii)

$\therefore y=-z$ and $x=0$

$\therefore x=0, y=t, z=-t$

D. $x + ay + az = 0$ ...(i)

$x + ay + az = 0$ ...(ii)

$x + ay + az = 0$ ...(iii)

Now assume $y = t_1,\ z = t_2$

On solving (i), (ii) & (iii)

$x = -a(t_1 + t_2)$

13. Ans. (1.00)

$A28, 3B9, 62C$ divisible by k

$A28 = 100A + 20 + 8 = \lambda_1 k$

$3B9 = 300 + 10B + 9 = \lambda_2 k$

$62C = 600 + 20 + C = \lambda_3 k$

$R_2 \to R_1 \times 100 + R_2 + 10R_3$

$$\begin{vmatrix} A & 3 & 6 \\ A28 & 3B9 & 62C \\ 2 & B & 2 \end{vmatrix} \Rightarrow \begin{vmatrix} A & 3 & 6 \\ \lambda_1 k & \lambda_2 k & \lambda_3 k \\ 2 & B & 2 \end{vmatrix}$$

$$= K \begin{vmatrix} A & 3 & 6 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ 2 & B & 2 \end{vmatrix} \therefore \text{ divisible by } K.$$

$\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$

$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1.$

14. Ans. (6.00)

Let $a = \beta + \gamma - \delta - \alpha$, $b = \gamma + \alpha - \beta - \delta$, $c = \alpha + \beta - \gamma - \delta$

$$\text{we get } \begin{vmatrix} a^4 & a^2 & 1 \\ b^4 & b^2 & 1 \\ c^4 & c^2 & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$= \begin{vmatrix} a^4 - b^4 & a^2 - b^2 & 0 \\ b^4 - c^4 & b^2 - c^2 & 0 \\ c^4 & c^2 & 1 \end{vmatrix} = (a^2 - b^2)(b^2 - c^2) \begin{vmatrix} a^2 + b^2 & 1 & 0 \\ b^2 + c^2 & 1 & 0 \\ c^4 & c^2 & 1 \end{vmatrix}$$

$= (a^2 - b^2)(b^2 - c^2)(a^2 - c^2) = (a - b)(a + b)(b - c)(b + c)(a + c)(a - c)$

$= -2^6 (\alpha - \beta)(\alpha - \gamma)(\beta - \gamma)(\alpha - \delta)(\beta - \delta)(\gamma - \delta)$

Compare with $-2^{\mu} (\alpha - \beta)(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)(\gamma - \delta)$

$\Rightarrow \mu = 6.$

15. Ans. (0.00)

Given determinant split into product of two determinants

$$= \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \begin{vmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

16. Ans. (1.00)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 3 \Rightarrow D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} = 3$$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 6 = 6$$

Similarly, $D_3 = 9$

So, consistent having $x = 1,\ y = 2,\ z = 3$

17. Ans. (1.00)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$\Rightarrow a = 1$

and at $a = 1$ $\quad D_1 = D_2 = D_3 = 0$

but $\quad$ at $a = 1$ and $b = 1$

First two equations are $x+y+z=1$
and third equation is $x+y+z=0$
$\Rightarrow$ There is no solution.

$\therefore\ b = \{1\} \Rightarrow$ it is a singleton set.

18. Ans. (7.00)

$D = 0 = D_1 = D_2$

So, for consistent solution

$D_3 = 0$

$$\begin{vmatrix} 2 & 3 & 0 \\ (c+2) & (c+4) & (c+6) \\ (c+2)^2 & (c+4)^2 & (c+6)^2 \end{vmatrix} = 0$$

$\Rightarrow c = -6, -1$

For $c = -6$ $\left.\begin{matrix} 2x+3y=0 \\ -4x-2y=0 \end{matrix}\right] \Rightarrow x = y = 0$

$c = -1$ $\left.\begin{matrix} 2x+3y=0 \\ x+3y=5 \end{matrix}\right] \Rightarrow y = \dfrac{10}{3},$

$= x = -5$