Continuity

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

Let us check continuity at $x = 2$

if continuous at $x = 2$ then $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^-} f(x) = f(2)$

$$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{1}{x + 2^{\frac{1}{x-2}}} = \lim_{x\to 2^-} \frac{1}{2^- + 2^{\frac{1}{2^- - 2}}} = \lim_{x\to 2^-} \frac{1}{2 + 2^{-\infty}} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{1}{x + 2^{\frac{1}{x-2}}} = \frac{1}{2^+ + 2^{\frac{1}{2^+ - 2}}} = \frac{1}{2 + 2^{\infty}} = 0$$

LHL $\neq$ RHL

So, cannot be continuous at $x = 2$.

2. **Ans. (D)**

$$f(x) = \frac{x - e^x + \cos 2x}{x^2},\ x \neq 0$$

$f(x)$ is continuous at $x = 0$

So $\lim\limits_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0} \frac{x - e^x + \cos 2x}{x^2} = \lim_{x\to 0} \frac{x - \left(1 + x + \frac{x^2}{2!} + \ldots\ldots\right) + \left(1 - \frac{(2x)^2}{2!} + \ldots\ldots\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{x - 1 - x - \frac{x^2}{2} + \ldots\ldots + 1 - \frac{4x^2}{2!} + \ldots\ldots}{x^2} = \lim_{x\to 0} \frac{-\frac{x^2}{2} - 2x^2}{x^2} = \lim_{x\to 0} \frac{-\frac{5}{2}x^2}{x^2} = \frac{-5}{2}$$

$$[f(0)] = \left[\frac{-5}{2}\right] = -3$$

$$\{f(0)\} = \left\{-\frac{5}{2}\right\} = \frac{1}{2}$$

$$[f(0)] \cdot \{f(0)\} = -3 \times \frac{1}{2} = -1.5$$

3. **Ans. (B)**

$f(x) = [2x^3 - 5]$ in $[1, 2)$

$-3 \leq 2x^3 - 5 < 11$ in $[1, 2)$

$f(x)$ is discontinuous where $2x^3 - 5$ become integer

at $x = 1$

$$\text{LHL} = \lim_{x\to 1^-} \left[2x^3 - 5\right] = \lim_{x\to 1^-} \left[2^- - 5\right] = \left[-3^+\right] = -4$$

$$f(1) = [2 - 5] = -3$$

$f(x)$ is discontinuous where

$2x^3 - 5 \Rightarrow -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

4. **Ans. (B)**

$x^2 + (f(x) - 2)x - \sqrt{3}f(x) + 2\sqrt{3} - 3 = 0$ ...(1)

For

$$x = \frac{-f(x)+2 \pm \sqrt{(f(x)-2)^2 + 4\sqrt{3}f(x) - 8\sqrt{3} + 12}}{2}$$

$$x = \frac{-f(x)+2 \pm \sqrt{(f(x))^2 + 4(\sqrt{3}-1)f(x) + 16 - 8\sqrt{3}}}{2}$$

$$x = \frac{-f(x)+2 \pm \sqrt{\left(f(x)+2(\sqrt{3}-1)\right)^2}}{2}$$

if consider + sign then $f(x)$ will cancel out.

$$x = \frac{-f(x)+2-f(x)-2(\sqrt{3}-1)}{2}$$

$2x = -2f(x) + 4 - 2\sqrt{3}$

Put $x = \sqrt{3}$

$2\sqrt{3} = -2f(\sqrt{3}) + 4 - 2\sqrt{3}$

$2f(\sqrt{3}) = 4 - 4\sqrt{3}$

$f(\sqrt{3}) = 2 - 2\sqrt{3} = 2(1 - \sqrt{3})$.

5. **Ans. (D)**

$f(x) = [x]^2 - [x^2]$

We know that greatest integer is discontinuous at integers but here we have to check at integers.

First so we will check at $x = 0$ and $x = 1$ at $x = 0$

$\text{LHL} = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} [x]^2 - [x^2] = (-1)^2 - 0 = 1$

$\text{RHL} = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} [x]^2 - [x^2] = 0 - 0 = 0$

LHL $\neq$ RHL discontinuous at $x = 0$

Now at $x = 1$

$\text{LHL} = \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} [x]^2 - [x^2] = 0^2 - [1^-] = 0$

$\text{RHL} = \lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} [x]^2 - [x^2] = \left[1^+\right]^2 - \left[(1^+)^2\right] = 1 - 1 = 0$

$f(1) = 0$

LHL = RHL = $f(1)$ so continuous at $x = 1$ at all other integers it will discontinuous.

6. **Ans. (C)**

If $f(x)$ is continuous at $x = 0$ then

$\lim\limits_{x \to 0} f(x) = f(0) = k$

$$\Rightarrow k = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin(2\pi \sec x)}{e^x - 1 - x} \qquad \left\{\frac{0}{0} \text{ form}\right\}$$

$$= \lim_{x \to 0} \frac{-\sin(2\pi - 2\pi \sec x)}{e^x - 1 - x} = \lim_{x \to 0} \frac{-\sin(2\pi - 2\pi \sec x)}{(2\pi - 2\pi \sec x)} \times \frac{(2\pi - 2\pi \sec x)}{e^x - 1 - x}$$

$$= \lim_{x\to 0}\frac{-2\pi(1-\sec x)}{e^x-1-x} = \lim_{x\to 0}\frac{2\pi(1-\cos x)}{\cos x\left(e^x-1-x\right)}$$

$$= \lim_{x\to 0}\frac{2\pi(1-\cos x)}{\cos x.x^2}\times\frac{x^2}{e^x-1-x} = \lim_{x\to 0}\left[2\pi\times\frac{1}{2}\times\frac{x^2}{e^x-1-x}\right]$$

$$= \lim_{x\to 0}\frac{\pi \quad x^2}{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....\right)-1-x} = \lim_{x\to 0}\frac{\pi \quad x^2}{x^2\left[\frac{1}{2!}+\frac{x}{3!}+......\right]} = \frac{\pi}{\left(\frac{1}{2}\right)} = 2\pi$$

7. **Ans. (B)**

$f(x)=\left[x^2\right]\sin(\pi x)$

$f(x)=g(x).h(x)$

For this kind of functions, if $g(x)$ is discontinuous and $h(x)$ is not zero at those points then overall function will be discontinuous. So possible of discontinuity will be where $[x^2]$ is discontinuous.

Possible points of discontinuity in $[-2,3]$

$x=-2,-\sqrt{3},-\sqrt{2},-1,0,1,\sqrt{2},\sqrt{3},2,\sqrt{5},\sqrt{6},\sqrt{7},\sqrt{8},3$

But $\sin(\pi x)$ will be zero on integral values.

i.e. $x = -2, -1, 0, 1, 2, 3$

So, $f(x)$ will not be discontinuous at these points.

So, remaining points where $f(x)$ will be discontinuous are

$x=-\sqrt{3},-\sqrt{2},\sqrt{2},\sqrt{3},\sqrt{5},\sqrt{6},\sqrt{7},\sqrt{8}$

Total number of points = 8

8. **Ans. (A,B,C)**

$$f(x)=\begin{cases}\sin^{-1}\left(\frac{a}{\ell nx}-\frac{b}{x-1}\right), & x\neq 1\\ \frac{\pi}{3}, & x=1\end{cases}$$

For continuity at $x=1$

$$\Rightarrow f\left(1^+\right)=f\left(1^-\right)=f(1)$$

$$\Rightarrow \lim_{x\to 1^+}\sin^{-1}\left(\frac{a}{\ell nx}-\frac{b}{x-1}\right)=\frac{\pi}{3}$$

Put $x=1+h$

$$\Rightarrow \lim_{h\to 0}\sin^{-1}\left(\frac{a}{\ell n(1+h)}-\frac{b}{h}\right)=\frac{\pi}{3}$$

$$\Rightarrow \lim_{h\to 0}\frac{a}{\ell n(1+h)}-\frac{b}{h}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \lim_{h\to 0}\frac{ah-b\ell n(1+h)}{h.\ell n(1+h)}$$

$$\Rightarrow \lim_{h\to 0}\frac{ah-b\left(h-\frac{h^2}{2}+\frac{h^3}{3}-\frac{h^4}{4}+...\right)}{h^2}\times\left(\frac{h}{\ell n(1+h)}\right)$$

$$\Rightarrow \lim_{h\to 0}\frac{\left[(a-b)h+\frac{b}{2}h^2-\frac{bh^3}{3}+\frac{bh^4}{4}-....\right]}{h^2}$$

For limit to exist $a-b=0$

$\Rightarrow a=b$

$$\Rightarrow \lim_{h\to 0}\frac{\left(\frac{b}{2}h^2-\frac{b}{3}h^3+\frac{bh^4}{4}-...\right)}{h^2}$$

$$\Rightarrow \frac{b}{2}=\frac{\sqrt{3}}{2}$$

$\Rightarrow b=\sqrt{3}, a=\sqrt{3}$

9. Ans. (A,B,C,D)

$$f(x)=\begin{cases}\dfrac{\ell n(1+x)-x+kx^2}{x^3} & , \quad x>0\\ \ell & , \quad x=0\\ \dfrac{\tan\left(\frac{x}{3}\right)}{x} & , \quad x<0\end{cases}$$

For continuity at $x=0$

$$f(0^+)=f(0^-)=f(0)$$

$$\text{RHL } \lim_{h\to 0}\frac{\ell n(1+h)-h+kh^2}{h^3}$$

$$\Rightarrow \lim_{h\to 0}\frac{\left(h-\frac{h^2}{2}+\frac{h^3}{3}-\frac{h^4}{4}.....\right)-h+kh^2}{h^3}$$

$$\Rightarrow \lim_{h\to 0}\frac{\left(k-\frac{1}{2}\right)h^2+\frac{h^3}{3}-\frac{h^4}{4}+...}{h^3}$$

For limit to exist $k-\frac{1}{2}=0$

$$\Rightarrow k=\frac{1}{2}$$

$$\Rightarrow \lim_{h\to 0}\frac{\frac{h^3}{3}-\frac{h^4}{4}+\frac{h^5}{5}.....}{h^3}$$

$$\Rightarrow \frac{1}{3}$$

$$\text{LHL } \lim_{h\to 0}\frac{\tan\left(-\frac{h}{3}\right)}{-h}$$

$$\Rightarrow \lim_{h\to 0}\frac{\tan\left(-\frac{h}{3}\right)}{\left(-\frac{h}{3}\right)\times 3}$$

$$\Rightarrow \frac{1}{3}$$

For continuity at $x = 0$, $\ell = \frac{1}{3}$, $k = \frac{1}{2}$

If $k \neq \frac{1}{2}$, then R.H.L. does not exist so, we have non-removable discontinuity.

10. **Ans. (A,B)**

For continuity at $x = 0$

$$f(0^+) = f(0^-) = f(0)$$

$$\Rightarrow \lim_{h\to 0}\frac{A\sin h + \sin(2h)}{h^3}$$

$$\Rightarrow \lim_{h\to 0}\frac{A\left(h - \frac{h^3}{3!} + \frac{h^5}{5!}...\right) + \left(2h - \frac{8h^3}{3!} + \frac{32h^5}{5!}...\right)}{h^3}$$

$$\Rightarrow \lim_{h\to 0}\frac{(A+2)h - \left(\frac{A}{6}+\frac{8}{6}\right)h^3 + ...}{h^3}$$

For limit to exist

$A + 2 = 0 \Rightarrow A = -2$

$$\Rightarrow \lim_{h\to 0}\frac{-\left(\frac{A}{6}+\frac{8}{6}\right)h^3 + ...}{h^3}$$

$$= -\left(\frac{A}{6}+\frac{8}{6}\right)$$

$$= -\left(\frac{-2}{6}+\frac{8}{6}\right) = -1$$

For continuity at $x = 0$, $f(0) = -1$

11. **Ans. (A,B,C)**

We need non-removable discontinuity

(A) R.H.L. $\lim_{h\to 0}\frac{1}{1+2^{\frac{1}{h}}} = 0$

L.H.L. $\lim_{h\to 0}\frac{1}{1+2^{-\frac{1}{h}}} = 1$

L.H.L. $\neq$ R.H.L.

So, non-removable discontinuity

(B) R.H.L. $\lim_{h\to 0}\tan^{-1}\left(\frac{1}{h}\right)=\frac{\pi}{2}$

L.H.L. $\lim_{h\to 0}\tan^{-1}\left(\frac{1}{-h}\right)=\frac{-\pi}{2}$

L.H.L. $\neq$ R.H.L.

(C) R.H.L. $\lim_{h\to 0}\frac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1}$

$\Rightarrow \lim_{h\to 0}\frac{1-e^{-\frac{1}{h}}}{1+e^{-\frac{1}{h}}}=1$

L.H.L. $\lim_{h\to 0}\frac{e^{-\frac{1}{h}}-1}{e^{-\frac{1}{h}}+1}=-1$

L.H.L. $\neq$ R.H.L.

(D) R.H.L. $\lim_{h\to 0}\frac{1}{\ell nh}=0$

L.H.L. $\lim_{h\to 0}\frac{1}{\ell n|0-h|}=0$

$=\lim_{h\to 0}\frac{1}{\ell nh}=0$

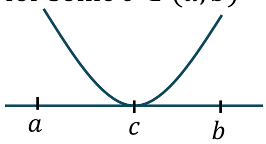
So, $f(x)$ will have removable type discontinuity.

12. Ans. (A,C,D)

(A) Given statement is true only if we are given that $f(x)$ is continuous in $[a,b]$.

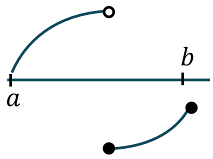
(B) If $f(x)$ is continuous in $[a,b]$ and $f(a).f(b)<0$, by intermediate value theorem, $f(x)=0$, for some $c\in(a,b)$

(C)



It is not necessary that $f(a)$ and $f(b)$ are of opposite sign.

(D)



$f(x)$ does not have any roots in $[a,b]$.

Also $f(a)$ and $f(b)$ are of opposite signs.

13. Ans. (A,C,D)

We need removable type discontinuity in this question.

(A) $\lim_{x\to -2}\frac{x^2-2x-8}{x+2}$

$\Rightarrow \lim_{x\to -2}\frac{(x-4)(x+2)}{(x+2)}$

$\Rightarrow \lim_{x\to -2}(x-4)=-6$

L.H.L. = R.H.L. = -6

(B) $f(x)=\frac{x-7}{|x-7|}$ at $x = 7$

R.H.L. $\lim_{h\to 0}\frac{(7+h)-7}{|7+h-7|}=1$

L.H.L. $\lim_{h\to 0}\frac{(7-h)-7}{|7-h-7|}$

$\Rightarrow \lim_{h\to 0}\frac{-h}{h}=-1$

L.H.L. $\neq$ R.H.L.

(C) $f(x)=\frac{x^3+64}{x+4}$ at $x = -4$

$\lim_{x\to -4}\frac{x^3+64}{x+4}$

$\Rightarrow \lim_{x\to -4}\frac{(x+4)(x^2+16-4x)}{(x+4)}$

$\Rightarrow \lim_{x\to -4}(x^2+16-4x)$

$= (-4)^2+16-4(-4)=48$

$\Rightarrow$ L.H.L. = R.H.L. = 48

(D) $f(x)=\frac{3-\sqrt{x}}{9-x}$ at $x = 9$

$\lim_{x\to 9}\frac{3-\sqrt{x}}{9-x}\Rightarrow \lim_{x\to 9}\frac{(3-\sqrt{x})}{(3-\sqrt{x})(3+\sqrt{x})}$

$\Rightarrow \lim_{x\to 9}\frac{1}{(3+\sqrt{x})}=\frac{1}{6}$

L.H.L. = R.H.L. = $\frac{1}{6}$

14. **Ans. (A,B,C,D)**

(A) $f(x)=x-\cos x$

$f(x)$ is continuous in $\left[0,\frac{\pi}{2}\right]$

$f(0)=-1, f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$

So, $f(x) = 0$ has atleast one root in $\left(0,\frac{\pi}{2}\right)$

(B) $f(x)=x+\sin x-1$

$f(x)$ continuous in $\left[0,\frac{\pi}{6}\right]$

$f(0) = -1$

$$f\left(\frac{\pi}{6}\right)=\frac{\pi}{6}+\frac{1}{2}-1$$

$$=\frac{\pi-3}{6}>0$$

$$f(0).f\left(\frac{\pi}{6}\right)<0$$

So, $f(x) = 0$ has atleast on root in $\left(0,\frac{\pi}{6}\right)$

(C) $f(x)=\frac{a}{x-1}+\frac{b}{x-3}$

$y = f(x)$ is continuous in $(1,3)$

$$f(1^{+})=\frac{a}{x-1}+\frac{b}{x-3}\to\infty$$

$$f(3^{-})=\frac{a}{x-1}+\frac{b}{x-3}\to-\infty$$

So, $f(x) = 0$ has atleast one root in $(1,3)$

(D) Let $p(x) = f(x) - g(x)$

$p(a) = f(a) - g(a) > 0$

$P(b) = f(b) - g(b) < 0$

$p(x)$ is continuous in $[a, b]$

So, $p(x) = 0$ has atleast on roots in (a, b).

15. **Ans. (A,B)**

$g(x)$ must have a factor $(x + 1)$

$f(x) = (x + 1)(x^2 - 2x - 1)$

Let $g(x) = A(x + 1)$

$$h(x)=\frac{(x+1)(x^2-2x-1)}{A(x+1)}$$

$$\lim_{x\to-1}h(x)\Rightarrow\lim_{x\to-1}\frac{x^2-2x-1}{A}$$

$$\Rightarrow\frac{2}{A}=\frac{1}{2}\Rightarrow A=4$$

$\Rightarrow g(x) = 4(x + 1)$

$$\Rightarrow h(x)=\frac{(x+1)(x^2-2x-1)}{4(x+1)}$$

$3h(0) + f(0) - 2g(0)$

$$\Rightarrow\frac{-3}{4}-1-8=\frac{-39}{4}$$

16. **Ans. (A,D)**

$$f(x)=\frac{3x^2+ax+a+3}{x^2+x-2},x\neq-2$$

For function to be continuous at $x = -2$

R.H.L. = L.H.L. = $f(-2)$

$$\Rightarrow\lim_{x\to-2}\frac{3x^2+ax+a+3}{x^2+x-2}\text{ exists}$$

$\Rightarrow 3(-2)^2 + a(-2) + a + 3 = 0$

$\Rightarrow 12 - a + 3 = 0$

$\Rightarrow a = 15$

$$\Rightarrow f(x) = \frac{3x^2+15x+18}{x^2+x-2} = \frac{3(x^2+5x+6)}{(x+2)(x-1)}$$

$$\Rightarrow \lim_{x\to -2} \frac{3(x^2+5x+6)}{(x+2)(x-1)}$$

$$\Rightarrow \lim_{x\to -2} \frac{3(x+2)(x+3)}{(x+2)(x-1)} = \frac{3}{-3} = -1$$

$\Rightarrow f(-2) = -1$

17. **Ans. (A,B,D)**

LHL $\lim_{x\to 0^-} \frac{e^{\sin 4x}-1}{\ell n(1+\tan 2x)}$

Apply LH rule

$$\lim_{x\to 0^-} \frac{4e^{\sin 4x}(1+\tan 2x)\cos 4x}{2\sec^2 2x} = 2$$

RHL

$$\lim_{x\to 0^+} \frac{\ell n \cos x}{(1+x^2)^{\frac{1}{4}}-1}$$

Apply LH rule

$$\lim_{x\to 0^+} \frac{\frac{-\sin x}{\cos x}}{\frac{1}{4}(1+x^2)^{\frac{-3}{4}} \times 2x} = \lim_{x\to 0^+} \frac{-\tan x \times 2}{(1+x^2)^{\frac{-3}{4}} \times x} = -2$$

$f(0)$ cannot be defined

18. **Ans. (A,B,C)**

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sin^{-1}(1-\{x\})\ .\ \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\ \ (1-\{x\})} = \lim_{h\to 0} \frac{\sin^{-1}(1-h)\ .\ \cos^{-1}(1-h)}{\sqrt{2h}\ \ (1-h)}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(1-h)\ \sin^{-1}\sqrt{h\ (2-h)}}{(1-h)\ \ \sqrt{2h}} = \lim_{h\to 0} \frac{\frac{\pi}{2}}{1}\ \frac{\sin^{-1}\sqrt{h(2-h)}}{\sqrt{2h-h^2}\ \ \sqrt{2h}}\ \sqrt{2h-h^2}$$

$$= \frac{\pi}{2}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin^{-1}(1-\{x\})\ \cos^{-1}(1-\{x\})}{\sqrt{2\ \{x\}}\ \ (1-\{x\})} = \lim_{h\to 0} \frac{\sin^{-1}h\ .\ \cos^{-1}h}{\sqrt{2(1-h)}\ \ h} = \frac{\frac{\pi}{2}}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

$\because\ f(x)$ is discontinuous at $x = 0$

19. Ans. (C)

$$f(x)=\lim_{n\to\infty}\frac{\ell nx+x^n}{(x-1)+x^n}\ (x>0)$$

(i) $0<x<1$

$x^n\to 0$ when $n\to\infty$

$$f(x)=\frac{\ell nx}{x-1}$$

(ii) $x=1$

$f(x)=1$

(iii) $x>1$

$x^n\to\infty$ when $n\to\infty$

$$f(x)=\lim_{n\to\infty}\frac{x^n\left(\frac{\ell nx}{x^n}+1\right)}{x^n\left(\frac{x-1}{x^n}+1\right)}=1$$

Now, $\displaystyle\lim_{x\to1^-}\frac{\sin(f(x)-1)}{(f(x)-1)}\times\frac{(f(x)-1)}{x-1}$

$$\Rightarrow \lim_{x\to1^-}\frac{f(x)-1}{x-1}=\lim_{x\to1^-}\frac{\frac{\ell nx}{x-1}-1}{x-1}$$

$$\Rightarrow \lim_{x\to1^-}\frac{\ell nx-x+1}{(x-1)^2}$$ Apply LH Rule

$$\Rightarrow \lim_{x\to1^-}\frac{\frac{1}{x}-1}{2(x-1)}$$ Again apply LH rule

$$\Rightarrow \lim_{x\to1^-}-\frac{1}{x^2\times2}=-\frac{1}{2}$$

20. Ans. (C)

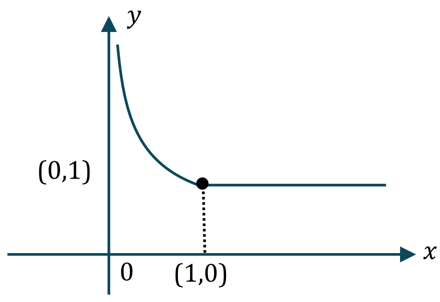
LHL $\displaystyle\lim_{x\to1^-}f(x)=\lim_{x\to1^-}\frac{\ell nx}{x-1}$ apply LH rule

LHL $\displaystyle\lim_{x\to1^-}\frac{\frac{1}{x}}{1}=1$

RHL $\displaystyle\lim_{x\to1^+}f(x)=1$

$f(1)=1$

$f(x)$ is continuous at $x=1$



$$f(x)\begin{cases}\frac{\ell nx}{x-1} & 0<x<1\\ 1 & x=1\\ 1 & x>1\end{cases}$$

$\because f(x)$ is many one function

21. Ans. (C)

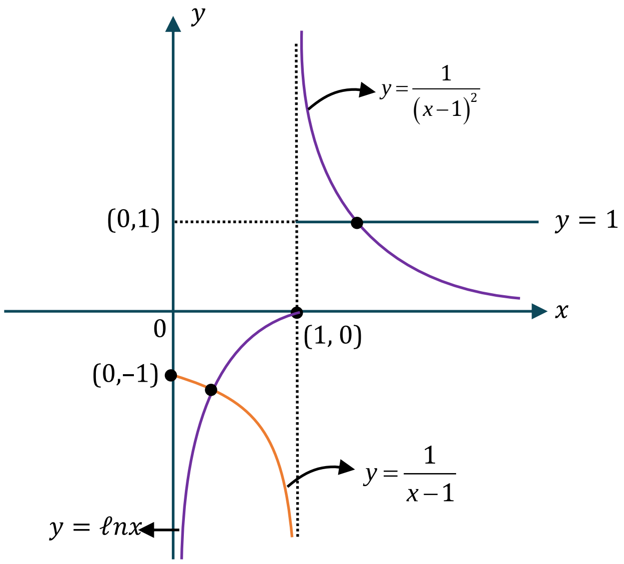
$$f(x)=\frac{1}{(x-1)^2}$$

$$f(x)\begin{cases}\frac{\ell nx}{x-1} & 0<x<1\\ 1 & x=1\\ 1 & x>1\end{cases}$$

For (i) $0 < x < 1$ (ii) for $x \geq 1$

$$\frac{\ell nx}{x-1}=\frac{1}{(x-1)^2} \qquad 1=\frac{1}{(x-1)^2}$$

$$\ell nx=\frac{1}{x-1}$$

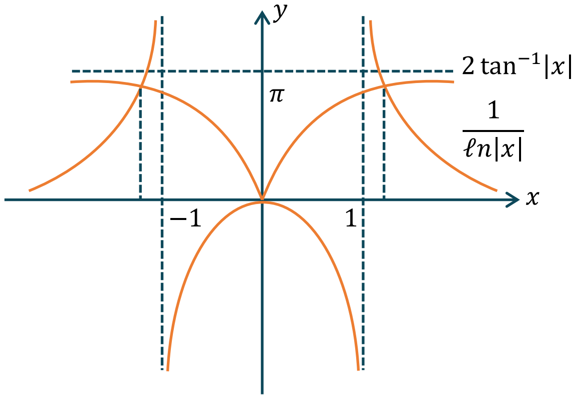


Now will make graph of LHS & RHS

No. of solutions = No. of point of intersections = 2

22. Ans. (A,B,C,D)

$$\frac{\tan^{-1}|x|}{\frac{1}{\ell n|x|}}=\frac{1}{2}\Rightarrow 2\tan^{-1}|x|=\frac{1}{\ell n|x|}$$



2 points of intersection which are symmetrical about origin

$\therefore$ sum = 0

23. Ans. (A,B,C)

$f(x)$ is discontinuous at $x = -1$, 0 and 1

$g(x)$ is continuous $\forall\ x \in R$.

$\therefore$ 3 points.

24. Ans. (A→Q; B→R; C→S; D→P)

(A) $$\lim_{x\to1}\frac{x^3-1}{\ell n\,x}=\lim_{x\to1}\frac{\frac{x^3-1}{x-1}}{\frac{\ell n(1+(x-1))}{(x-1)}}=\frac{3}{1}=3$$

(B) $\lim\limits_{x\to 0}\dfrac{x(\cos x-\cos 2x)}{2\sin x-\sin 2x}=\lim\limits_{x\to 0}\dfrac{x.2\sin\frac{3x}{2}.\sin\frac{x}{2}}{2\sin x(1-\cos x)}$

$$=\lim_{x\to 0}\frac{\frac{\sin\frac{3x}{2}}{x}\;\frac{\sin\frac{x}{2}}{x}}{\frac{\sin x}{x}\left(\frac{1-\cos x}{x^2}\right)}=\frac{\frac{3}{2}\times\frac{1}{2}}{\frac{1}{2}}=\frac{3}{2}$$

(C) $\lim\limits_{x\to 0}\dfrac{(\tan x)^{3/2}-(\sin x)^{3/2}}{x^3\sqrt{x}}=\lim\limits_{x\to 0}\dfrac{\tan^3 x-\sin^3 x}{x^3\sqrt{x}\left((\tan x)^{3/2}+(\sin x)^{3/2}\right)}$

$$=\lim_{x\to 0}\frac{\left(\frac{\tan x-\sin x}{x^3}\right)\left(\frac{\tan^2 x}{x^2}+\frac{\tan x\sin x}{x^2}+\frac{\sin^2 x}{x^2}\right)}{\frac{(\tan x)^{3/2}}{x^{3/2}}+\frac{(\sin x)^{3/2}}{x^{3/2}}}$$

$$=\frac{\frac{1}{2}\times 3}{2}=\frac{3}{4}$$

(D) $f(0)=\lim\limits_{x\to 0}\cos\left(x\cos\frac{1}{x}\right)=\cos(0)=1$

$$g(0)=\lim_{x\to 0}\frac{\ell n(1+\tan^2 x)}{\tan^2 x}\times\frac{\frac{\tan^2 x}{x^2}}{\frac{\sin x}{x}}=1$$

$\therefore f(0)+g(0)=2$

25. **Ans. (A→P,Q,R; B→P,Q,T; C→P,Q,T; D→P,Q,S)**

(A) $f(x)=[x]\,[1+x]$

$f(0^-)=(-1)\times 0=0$

$f(0^+)=0\times 1=0$

$f(0)=0$

$\therefore$ continuous at $x=0$

(B) $f(x)=[-x]\,[1+x]$

$f(0^-)=0\times 0=0$

$f(0^+)=-1\times 1=-1$

$\lim\limits_{x\to 0} f(x)$ does not exist

(C) $f(x)=\mathrm{sgn}(x)\,[2-x]\,[1+|x|]$

$f(0^-)=-1\times 2\times 1=-2$

$f(0^+)=1\times 1\times 1=1$

$\lim\limits_{x\to 0} f(x)$ does not exist

(D) $f(x)=[\cos x]$

$f(0^-)=0$

$f(0^+)=0$

$f(0)=1$

$\lim\limits_{x\to 0} f(x)$ exists but $f(x)$ is discontinuous at $x=0$

EXERCISE - S

1. **Ans. (1)**

$$f\left(\frac{\pi}{2}\right)=b+2 \qquad \text{...(i)}$$

LHL at $x=\frac{\pi}{2}$

$$f\left(\frac{\pi}{2}^{-}\right)=\lim_{x\to\frac{\pi}{2}^{-}}\left(\frac{6}{5}\right)^{\frac{\tan 6x}{\tan 5x}}$$

replace $x=\frac{\pi}{2}-h$

$$=\lim_{h\to 0}\left(\frac{6}{5}\right)^{\frac{\tan(3\pi-6h)}{\tan\left(\frac{5\pi}{2}-5h\right)}}=\lim_{h\to 0}\left(\frac{6}{5}\right)^{\frac{\tan 6h}{\cot 5h}}=\left(\frac{6}{5}\right)^{0}=1$$

$$f\left(\frac{\pi}{2}^{-}\right)=1 \qquad \text{...(ii)}$$

RHL at $x=\frac{\pi}{2}$

$$f\left(\frac{\pi}{2}^{+}\right)=\lim_{x\to\frac{\pi}{2}^{+}} f(x)=\lim_{x\to\frac{\pi}{2}^{+}}(1+|\cos x|)^{\frac{a|\tan x|}{b}}$$

replace $x=\frac{\pi}{2}+h$

$$=\lim_{h\to 0}(1+|\sin h|)^{\frac{a|\cot h|}{b}} \quad (1^{\infty}\text{ form})$$

$$=e^{\lim_{h\to 0}\frac{a}{b|\tan h|}(1+|\sin h|-1)}=e^{\lim_{h\to 0}\frac{a|\cos h|}{b}}=e^{\frac{a}{b}}$$

$$f\left(\frac{\pi}{2}^{+}\right)=e^{a/b} \qquad \text{...(iii)}$$

$\because f(x)$ is continuous at

So $f\left(\frac{\pi}{2}\right)=f\left(\frac{\pi}{2}^{-}\right)=f\left(\frac{\pi}{2}^{+}\right)$

$b+2=1=e^{\frac{a}{b}}$

$b=-1 \qquad a=0$

2. **Ans. (0)**

$$f\left(\frac{1}{2}\right)=p \qquad \text{...(i)}$$

LHL at $x=\frac{1}{2}$

$$f\left(\frac{1}{2}^{-}\right)=\lim_{x\to\frac{1}{2}^{-}}\frac{1-\sin\pi x}{1+\cos 2\pi x}$$

Put $x=\frac{1}{2}-h$

$$= \lim_{h\to 0}\frac{1-\sin\left(\frac{\pi}{2}-\pi h\right)}{1+\cos(\pi-2\pi h)}$$

$$= \lim_{h\to 0}\frac{(1-\cos\pi h)}{(1-\cos 2\pi h)}$$

$$= \lim_{h\to 0}\frac{2\sin^2\left(\frac{\pi h}{2}\right)}{2\sin^2(\pi h)} = \frac{1}{4} \quad ...(ii)$$

RHL at $x = \frac{1}{2}$

$$f\left(\frac{1}{2}^+\right) = \lim_{x\to \frac{1}{2}^+}\frac{\sqrt{2x-1}}{\sqrt{4+\sqrt{2x-1}}-2}$$

$$= \lim_{x\to \frac{1}{2}^+}\frac{\left(\sqrt{2x-1}\right)\left(\sqrt{4+\sqrt{2x-1}}+2\right)}{\left(\sqrt{2x-1}\right)}$$

$$= \lim_{x\to \frac{1}{2}^+}\left(\sqrt{4+\sqrt{2x-1}}+2\right) = 4 \quad ...(iii)$$

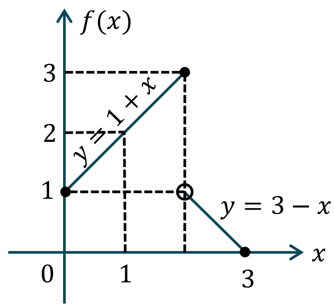
$\because$ RHL $\neq$ LHL at $x = \frac{1}{2}$

So $f(x)$ can not be continuous

$\Rightarrow$ No value of p exists

3. **Ans. (2)**

$$g(x) = f(f(x)) = \begin{cases} 1+f(x), & 0\le f(x)\le 2 \\ 3-f(x), & 2< f(x)\le 3\end{cases}$$



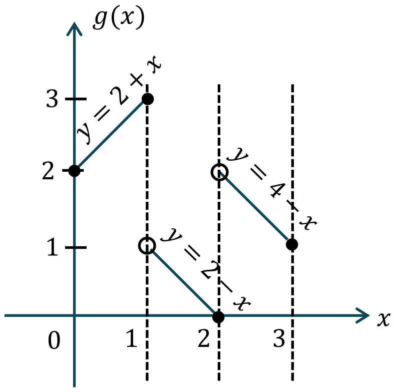
$\Rightarrow 0 \le f(x) < 1 \Rightarrow f(x) = 3 - x \,;\, 2 < x \le 3$

$\Rightarrow 1 \le f(x) \le 2 \Rightarrow f(x) = 1 + x \,;\, 0 \le x \le 1$

$\Rightarrow 2 < f(x) \le 3 \Rightarrow f(x) = 1 + x \,;\, 1 \le x \le 2$

$$\therefore\ g(x) = f(f(x)) = \begin{cases} 1+(3-x) & ; \quad 2< x\le 3 \\ 1+(1+x) & ; \quad 0\le x\le 1 \\ 3-(1+x) & ; \quad 1< x\le 2\end{cases}$$

$$\therefore\ g(x) = \begin{cases} 2+x & ; \quad 0\le x\le 1 \\ 2-x & ; \quad 1< x\le 2 \\ 4-x & ; \quad 2< x\le 3\end{cases}$$



$\therefore\ g(x)$ is discontinuous at $x = 1$ and at $x = 2$.

4. Ans. (14)

$$f\left(\frac{\pi}{2}\right)=a \qquad \text{... (1)}$$

$$f\left(\frac{\pi^+}{2}\right)=\lim_{x\to\frac{\pi^+}{2}}\frac{b(1-\sin x)}{(\pi-2x)^2}$$

$$x=\frac{\pi}{2}+h$$

$$=\lim_{h\to 0}\frac{b.(1-\cos h)}{(-2h)^2}$$

$$=\lim_{h\to 0}\frac{b}{4}\frac{1-\cos h}{h^2}=\frac{b}{8}$$

$$f\left(\frac{\pi^+}{2}\right)=\frac{b}{8} \qquad \text{... (2)}$$

$$f\left(\frac{\pi^-}{2}\right)=\lim_{x\to\frac{\pi^-}{2}}\frac{1-\sin^3 x}{3\cos^2 x}$$

$$=\lim_{x\to\frac{\pi^-}{2}}\frac{(1-\sin x)(1+\sin^2 x+\sin x)}{3(1-\sin x)(1+\sin x)}$$

$$f\left(\frac{\pi^-}{2}\right)=\frac{1}{2} \qquad \text{... (3)}$$

$\because f(x)$ is continuous at $x=\frac{\pi}{2}$

So $f\left(\frac{\pi^-}{2}\right)=f\left(\frac{\pi^+}{2}\right)=f\left(\frac{\pi}{2}\right)$

$$a=\frac{1}{2}=\frac{b}{8}$$

$$\boxed{a=\frac{1}{2}}\ \boxed{b=4}$$

5. Ans. (2.00)

$$f(0)=\lim_{x\to 0}\frac{A\sin 2x+B\sin x+\sin 3x}{x^5}$$

$$f(0)=\lim_{x\to 0}\frac{A\left(2x-\frac{(2x)^3}{6}+\frac{(2x)^5}{120}\right)+B\left(x-\frac{x^3}{6}+\frac{x^5}{120}\right)+\left(3x-\frac{(3x)^3}{6}+\frac{(3x)^5}{120}\right)}{x^5}$$

$$f(0)=\lim_{x\to 0}\frac{(2A+B+3)+\frac{x^3}{6}(-8A-B-27)+\frac{x^5}{120}(32A+B+243)}{x^5}$$

$2A+B+3=0,\ 8A+B+27=0$

$(A,B)\equiv(-4,5)$

$$f(0)=\frac{32A+B+243}{120}=1 \text{ so}$$

$A+B+f(0)=-4+5+1=2$

6. **Ans. (5.00)**

$f(x) = x + [x] + \{-x\}$

Case-1: $x \in I, f(x) = 2x$

Case-2: $x \notin I, f(x) = x + [x] + 1 - \{x\}$

$f(x) = 2[x] + 1$

$f(n) = 2n, f(n^+) = 2n + 1, f(n^-) = 2n - 1$

It is discontinuous at every integer in [–2, 2]

Total = 5 integral points.

7. **Ans. (1.00)**

RHL

$= f(0^+) = \lim\limits_{x\to 0}\dfrac{e^{1/x}+e^{2/x}+e^{3/x}}{ae^{2/x}+be^{3/x}}$ $\qquad f(0) = a$

$= f(0^+) = \lim\limits_{x\to 0}\dfrac{e^{3/x}}{be^{3/x}} = \dfrac{1}{b}$

LHL

$= f(0^-) = e^{\lim\limits_{x\to 0^-}\operatorname{cosec}x(\sin x+\cos x-1)}$ $\qquad (1^\infty \text{ form})$

$= e^{\lim\limits_{x\to 0}\left(\frac{\sin x+\cos x-1}{\sin x}\right)} = e^{\lim\limits_{x\to 0}\left(1-\left(\frac{1-\cos x}{\sin x}\right)\right)}$

$= e^{1-0} = e^1$

$\therefore f(0) = 6(0^+) = 6(0^-)$

$\Rightarrow a = \dfrac{1}{b} = e$

$\Rightarrow (a, b) = \left(e, \dfrac{1}{e}\right)$

$\therefore a^2 \cdot b^2 = e^2 \cdot \dfrac{1}{e^2} = 1$

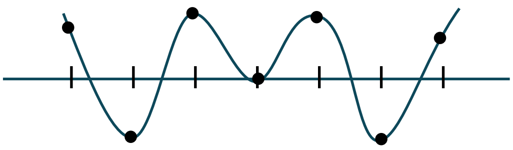
8. **Ans. (5.00)**

$f(x) = f(-x)$

$f(-5) = f(5) = 5, f(-2) = f(2) = 4$

$f(3) = f(-3) = -2, f(0) = 0$

Now plot the graph



min 5 roots

EXERCISE - JEE (Main) PYQ

1. **Ans. (1)**

$\therefore$ function should be continuous at $x=\frac{\pi}{4}$

$$\therefore \lim_{x\to\frac{\pi}{4}} f(x)=f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}=k$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\left(\sqrt{2}\cos x-1\right)\left(\sqrt{2}\cos x+1\right)\left(\cot x+1\right)}{\left(\cot x-1\right)\left(\cot x+1\right)\left(\sqrt{2}\cos x+1\right)}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\left(2\cos^2 x-1\right)\left(\cot x+1\right)}{\left(\cot^2 x-1\right)\left(\sqrt{2}\cos x+1\right)}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\cos 2x\left(\cot x+1\right)}{\frac{\left(\cos^2 x-\sin^2 x\right)}{\sin^2 x}\left(\sqrt{2}\cos x+1\right)}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\cos 2x\left(\cot x+1\right)\sin^2 x}{\cos 2x\left(\sqrt{2}\cos x+1\right)}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\left(\cot x+1\right)\sin^2 x}{\left(\sqrt{2}\cos x+1\right)}$$

$$\Rightarrow \frac{(1+1)\times\frac{1}{2}}{\sqrt{2}\times 0+1}=1$$

2. **Ans. (1)**

$$f(x)=\begin{cases} a|\pi-x|+1; & x\le 5 \\ b|\pi-x|+3; & x>5 \end{cases}$$

And it is also given that $f(x)$ is continuous at $x = 5$

$a|\pi - 5| + 1 = b|5 - \pi| + 3$

$a(5 - \pi) + 1 = b(5 - \pi) + 3$

$(a - b)\ (5 - \pi) = 2$

$$a-b=\frac{2}{5-\pi}$$

3. **Ans. (2)**

$$A=\lim_{x\to 0} x\left[\frac{4}{x}\right]=\lim_{x\to 0} x\left(\frac{4}{x}\right)-x\left\{\frac{4}{x}\right\}=4$$

$f(x) = [x^2]\sin(\pi x)$ will be discontinuous at non integers

$\therefore\ x=\sqrt{A+1}$ *i.e.* $\sqrt{5}$

But for $\sqrt{A+1}=3, \sqrt{A}=2, \sqrt{A+21}=5$

All are integers so, be continuous at those points because sin will be zero.

4. Ans. (8)

$x \in (-10, 10)$

$\frac{x}{2} \in (-5,\ 5) \rightarrow 9$ integers

check continuity at $x = 0$

$$\left.\begin{array}{l} f(0)=0 \\ f(0^+)=0 \\ f(0^-)=0 \end{array}\right\} \text{continuous at } x = 0$$

function will be discontinuous when $\frac{x}{2} = \pm 4,\ \pm 3,\ \pm 2,\ \pm 1$

8 points of discontinuity

5. Ans. (1)

$$g[f(x)] = \begin{cases} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x)\ge 0 \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x+a<0\ \&\ x<0 \\ |x-1|+1 & |x-1|<0\ \&\ x\ge 0 \\ (x+a-1)^2+b & x+a\ge 0\ \&\ x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0\ \&\ x\ge 0 \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a)\ \&\ x\in(-\infty,0) \\ |x-1|+1 & x\in\phi \\ (x+a-1)^2+b & x\in[-a,\infty)\ \&\ x\in(-\infty,0) \\ (|x-1|-1)^2+b & x\in R\ \&\ x\in[0,\infty) \end{cases}$$

$$g[f(x)] = \begin{cases} x+a+1 & x\in(-\infty,-a) \\ (x+a-1)^2+b & x\in[-a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty) \end{cases}$$

$g(f(x))$ is continuous

at $x = -a$	&	at $x = 0$
$1 = b + 1$	&	$(a-1)^2 + b = b$
$b = 0$	&	$a = 1$

$\Rightarrow a + b = 1$

6. Ans. (6)

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4} = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4} = \frac{1}{K} \Rightarrow \lim_{x\to 0} 2\left(\frac{\sin x+x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right) = \frac{1}{K}$$

$$\Rightarrow 2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6} = \frac{1}{K} \Rightarrow K = 6$$

7. **Ans. (2)**

Continuous at $x = 0$

$f(0^+) = f(0^-) \Rightarrow a - 1 = 0 - e^0$

$\Rightarrow a = 0$

Continuous at $x = 1$

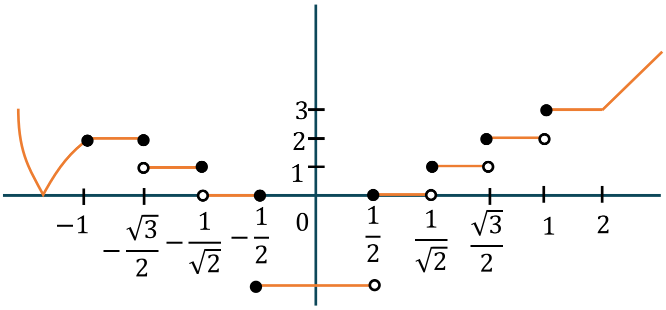
$f(1^+) = f(1^-)$

$\Rightarrow 2(1) - b = a + (-1)$

$\Rightarrow b = 2 - a + 1 \Rightarrow b = 3$

$\therefore a + b = 3$

8. **Ans. (7)**



9. **Ans. (3)**

$f(x)$ is discontinuous at $x = 1$

For continuous at $x = 0$; $a = 1$

For continuous at $x = 2$; $b + c = 1$

$a + b + c = 2$

10. **Ans. (1)**

$$\lim_{x\to 0} \frac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}$$

$$\lim_{x\to 0} \frac{\left(\frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)x^2\left(1+x^2\right)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$$

$\therefore k = 1$

11. **Ans. (2)**

Check continuity at $x = 0$ and also check continuity at those x where $g(x) = 0$

$g(x) = 0$ at $x = 0, 2$

$\text{fog}(0^+) = -1$

$\text{fog}(0) = 0$

Hence, discontinuous at $x = 0$

$\text{fog}(2^+) = 1$

$\text{fog}(2^-) = -1$

Hence, discontinuous at $x = 2$

12. **Ans. (4)**

Here $f(x)=[x(x-1)]+\{x\}$

$f(o^{+})=-1+0=-1$ $f(o)=0$	$f(1^{+})=0+0=0$ $f(1)=0$ $f(1^{-})=-1+1=0$

$\therefore f(x)$ is continuous at $x=1$, discontinuous at $x=0$

13. **Ans. (2)**

Need to check at doubtful points

discont at $x \in$ I only

at $x = -1 \Rightarrow f(-1^{+}) = 1 + 0 = 1$

$\Rightarrow f(-1^{-}) = 2 + 1 = 3$

at $x = 0 \Rightarrow f(0^{+}) = 0 + 0 = 0$

$\Rightarrow f(0^{-}) = 1 + 1 = 2$

at $x = 1 \Rightarrow f(1^{+}) = 1 + 0 = 1$

$\Rightarrow f(1^{-}) = 0 + 1 = 1$

discont. at two points

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (B,D)**

For continuity at $x=2n$,

$\lim\limits_{x\to 2n^{+}} f(x) \Rightarrow \lim\limits_{x\to 2n^{+}} a_n+\sin(\pi x)$

$a_n+0=a_n$

$\lim\limits_{x\to 2n^{-}} b_n+\cos(\pi x)$

$\Rightarrow b_n+1$

$f(2n)=a_n+\sin(2n\pi)=a_n$

$\Rightarrow \boxed{a_n=b_n+1}$

for continuity at $x=2n+1$, $\lim\limits_{x\to(2n+1)^{+}} b_{n+1}+\cos(\pi(n+1))$

$=b_{n+1}-1$

$\lim\limits_{x\to(2n+1)^{-}} a_n+\sin(\pi x)=a_n$

$\Rightarrow \boxed{a_n=b_{n+1}-1}$

2. **Ans. (A,D)**

$\max\{f(x): x\in[0,1]\}=\max\{g(x): x\in[0,1]\}$

$\Rightarrow y=f(x)$ and $y=g(x)$ intersect atleast once in $[0,1]$

Let graphs intersect at $x=c$

$\Rightarrow f(c)=g(c)$

our answer should be the option in which atleast one of the factors is $f(c)-g(c)$.

3. Ans. (A,C,D)

$f(x) = x \cdot \cos(\pi(x + [x]))$

$= x \cdot \cos(\pi x + \pi[x])$

x is continuous everywhere.

$\cos(\pi x + \pi[x]) = \cos(\pi x)$ when $[x]$ is even,

$\cos(\pi x + \pi[x]) = -\cos(\pi x)$ when $[x]$ is odd

At any $x = n$, where n is integer then $[x]$ has different parity in $(n-1, n)$ and $(n, n+1)$.

JEE (Main) Practice Paper

SECTION–A

1. Ans. (1)

If $f(x)$ is continuous at $x = 1$

Then $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^-} f(x) = f(1)$

$\text{LHL} = \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}(ax+1) = a+1$

$\text{RHL} = \lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} bx^2+1 = b+1$

LHL = RHL = $f(1)$

$a + 1 = b + 1 = 3$

$a + 1 = 3, b + 1 = 3$

$a = 2, b = 2$

$a - b = 0$

2. Ans. (1)

$f(x) = \dfrac{x^2 - bx + 25}{x^2 - 7x + 10}$ for $x \neq 5$

$f(x)$ is continuous at $x = 5$

So, $\lim\limits_{x\to 5} f(x) = f(5)$

$\lim\limits_{x\to 5} f(x) = \lim\limits_{x\to 5} \dfrac{x^2 - bx + 25}{x^2 - 7x + 10} = \lim\limits_{x\to 5} \dfrac{x^2 - bx + 25}{(x-2)(x-5)}$

If limit exist then in numerator there is factor of $(x - 5)$

So $x = 5$ will satisfy $x^2 - bx + 25$

$25 - 5b + 25 = 0 \Rightarrow b = 10$

$\lim\limits_{x\to 5} \dfrac{x^2 - 10x + 25}{(x-2)(x-5)} = \lim\limits_{x\to 5} \dfrac{(x-5)(x-5)}{(x-2)(x-5)} = \lim\limits_{x\to 5} \dfrac{x-5}{x-2} = 0$

3. Ans. (3)

$f(x)$ passes through $(a, 0)$

So $f(a) = 0$

$= \lim\limits_{x\to a} \dfrac{\log(1+3f(x))}{2f(x)} \quad \left(\dfrac{0}{0}\right)$

$= \lim\limits_{x\to a} \dfrac{\log(1+3f(x))}{3f(x)} \dfrac{3f(x)}{2f(x)} = \lim\limits_{x\to a} \dfrac{3f(x)}{2f(x)} = \dfrac{3}{2} \qquad \left(\because \lim\limits_{(x)\to 0} \dfrac{\log(1+(x))}{(x)} = 1\right)$

4. Ans. (4)

function $f(x) = [x^2 + 1]$

it will be discontinuous where $x^2 + 1$ become integer

$x^2 + 1$ = integer in [1, 3]

Now $x^2 + 1 = 2, 3, 4, 5, 6, 7, 8, 9, 10$

$x^2 + 1 = 2$

When $x = 1$

$\text{RHL} = \lim_{x\to1^+}\left[x^2+1\right] = 2$

$f(1) = 2$

$x^2 + 1 = 10 \Rightarrow x = 3$

$\text{LHL} = \lim_{x\to3^-}\left[x^2+1\right] = 9$

$f(3) = 10$

$\text{LHL} \neq f(3)$

So discontinuous points where $x^2 + 1 = 3, 4, 5, 6, 7, 8, 9, 10$ eight points.

5. Ans. (2)

We will check where definition of function changes we will check at $x = -2, 0, 3$

At $x = -2$

$\text{LHL} = \lim_{x\to-2^-}|x+1| = 1$ $\quad$ $\text{RHL} = \lim_{x\to-2^+} 2x+3 = -1$

$\text{LHL} \neq \text{RHL}$ $\quad$ So, discontinuous

At $x = 0$

$\text{LHL} = \lim_{x\to0^-} 2x+3 = 3$ $\quad$ $\text{RHL} = \lim_{x\to0^+} x^2+3 = 3$

$f(0) = 3$ $\quad$ So, continuous at $x = 0$

At $x = 3$

$\text{LHL} = \lim_{x\to3^-} x^2+3 = 9 + 3 = 12$ $\quad$ $\text{RHL} = \lim_{x\to3^+} x^3-15 = 27 - 15 = 12$

$f(3) = 12$ $\quad$ So, LHL = RHL = $f(3)$ so continuous at $x = 3$

Only discontinuous at $x = -2$.

6. Ans. (3)

$$\lim_{x\to0} f\left(\frac{1-\cos 3x}{x^2}\right) = \lim_{x\to0} f\left(\frac{2\sin^2\frac{3}{2}x}{x^2}\right) = \lim_{x\to0} f\left(\frac{2\left(\sin\frac{3}{2}x\right)^2}{\left(\frac{3}{2}x\right)^2}\times\frac{9}{4}\right) = f\left(\frac{9}{2}\right) = \frac{2}{9}$$

7. Ans. (3)

$$f(x) = [x]\cdot\cos\left(\frac{2x-1}{2}\right)\pi$$

here greatest integer is involve so we will check on integers

we will check at $x = I$

$$\text{LHL} = \lim_{x\to I^-}[x]\cos\left(\frac{2x-1}{2}\right)\pi = [I^-]\cos\left(\frac{2(I^-)-1}{2}\right)\pi = (I-1)\cos\left(\text{odd multiple of }\frac{\pi}{2}\right) = 0$$

$$\text{RHL} = \lim_{x\to I^+}[x]\cos\left(\frac{2x-1}{2}\right)\pi = [I^+]\cos\left(\frac{2(I^+)-1}{2}\right)\pi = I\cos\left(\text{odd multiple of }\frac{\pi}{2}\right) = 0$$

$f(I) = [I]\cos\left(\frac{2I-1}{2}\right)\pi = I\cos\left(\text{odd multiple of } \frac{\pi}{2}\right) = 0$

LHL = RHL = $f(I)$

So, no value of x where $f(x)$ is discontinuous.

8. Ans. (1)

f is continuous function and its co-domain is rational numbers then it must be a constant function as $f(2) = 7$

So, $f(x) = 7$ will be function $f(x)$ and constant functions are always even function.

9. Ans. (1)

If $f(x)$ is continuous at $x = 0$ then

$\Rightarrow \lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x\to 0} \frac{\tan(bx^3)}{x^3} = \frac{5}{4} + \frac{3}{4b}$$

$$\Rightarrow \lim_{x\to 0} \frac{\tan(bx^3)}{(bx^3)} \times b = \frac{5b+3}{4b}$$

$$\Rightarrow b = \frac{5b+3}{4b} \Rightarrow 4b^2 = 5b+3 \Rightarrow 4b^2 - 5b - 3 = 0$$

$$\Rightarrow b = \frac{5 \pm \sqrt{25+48}}{8} \Rightarrow b = \frac{5 \pm \sqrt{73}}{8}$$

$$\Rightarrow b = \frac{5+\sqrt{73}}{8} \text{ or } \frac{5-\sqrt{73}}{8}$$

10. Ans. (3)

$f: R \to R$ continuous function $\forall x \in R$

and $f(x) = 5, \forall\, x \in$ irrational

$f(x)$ is continuous so

if $f(x) = 5, \forall\, x \in$ irrational

then $\forall\, x \in$ Rational $f(x) = 5$ Also

$f(3) = 5$.

11. Ans. (1)

$\lim_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^2} = a \Rightarrow \frac{2}{x^2}\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x - \sin x}{2}\right) = a$$

$$\Rightarrow a = \lim_{x\to 0}\ 2.\frac{\sin\left(\frac{\sin x + x}{2}\right)}{\frac{\sin x + x}{2}}.\frac{\sin\left(\frac{x - \sin x}{2}\right)}{\frac{x - \sin x}{2}}.\frac{1}{4}\left(\frac{\sin x + x}{x}\right)\left(\frac{x - \sin x}{x}\right) = 2.1.1\frac{1}{4}(1+1)(1-1) = 0$$

12. Ans. (4)

$$f(x) = \left|\left(x+\frac{1}{2}\right)[x]\right| = \begin{cases} -(2x+1) & , \quad -2 \le x < -1 \\ \left|x+\frac{1}{2}\right| & , \quad -1 \le x < 0 \\ 0 & , \quad 0 \le x < 1 \\ x+\frac{1}{2} & , \quad 1 \le x < 2 \\ 5 & , \quad x = 2 \end{cases}$$

$f(x)$ is discontinuous at $x = -1, 0, \frac{1}{2}, 1, 2$

13. Ans. (4)

$$\lim_{x\to1^-} f(x) = \lim_{h\to0} f(1-h) = \lim_{h\to0} \frac{\ell n\ [(1-h)^2 - 2(1-h)+5]}{\ell n\ [1-4h]}$$

does not exist as denominater is tending to zero.

similarly $\lim_{n\to1^+} f(x)$ also does not exist.

14. Ans. (2)

$$\lim_{x\to1^+} x\sin\left(\frac{\pi}{2}(x+2)\right) = \lim_{x\to1^+}(-x)\sin\left(\frac{\pi x}{2}\right) = -1 \text{ and } \lim_{x\to1^-} x\sin\left(\frac{\pi}{2}(x+0)\right) = 1$$

15. Ans. (2)

LHL $(x = 0) = f(0) =$ RHL $(x = 0)$; LHL $= \underset{x\to0^-}{\text{Lim}} \frac{\sqrt{1+px}-\sqrt{1-px}}{x} = \frac{2p}{2} = p$

$f(0) = -\frac{1}{2} =$ RHL

16. Ans. (3)

$$f(g(x)) = \text{sgn}(g(x)) = \text{sgn}\left(x\left(x^2-5x+6\right)\right) = \text{sgn}\left(x(x-2)(x-3)\right) = \begin{bmatrix} 1 & ; & x>3,\ 1<x<2 \\ 0 & ; & x=1,\ 2,\ 3 \\ -1 & ; & 2<x<3\ ;\ x<1 \end{bmatrix}$$

$f(g(x))$ is discontinuous at 3 points (0, 2 and 3)

17. Ans. (3)

$y = \frac{1}{t^2+t-2}$, where $t = \frac{1}{x-1}$, $y = f(x)$ is discontinuous at $x = 1$, where t is discontinuous and $y = \frac{1}{(t+2)(t-1)}$ at $t = -2$ and $t = 1$

$\Rightarrow \frac{1}{x-1} = -2 \quad \Rightarrow -2x + 2 = 1, x = \frac{1}{2}$

and $\frac{1}{x-1} = 1 \quad \Rightarrow x = 2$

$f(g(x))$ is discontinuous at $x = \frac{1}{2}$, 2,1

18. **Ans. (2)**

Let $f(x) = 2\tan x + 5x - 2$

$$f(0) = -2\; f\left(\frac{\pi}{4}\right) = 2\tan\frac{\pi}{4} + \frac{5\pi}{4} - 2 = \frac{5\pi}{4}$$

Now $x \in \left[-2, \frac{5\pi}{4}\right]$ and $f(x)$ is continuous on $\left[0, \frac{\pi}{4}\right]$

$\therefore$ By intermediate value theorem $c \in \left[0, \frac{\pi}{4}\right]$ for which $f(c) = 0$

$\therefore$ (b) is correct.

19. **Ans. (2)**

$g(x) = x - [x] = \{x\} \in [0, 1)$

$g(x)$ is discontinuous only at $x \in I$

Now $h(x) = fog(x)$

$h(x)$ is continuous $\forall\ x \in R - I$

Let $x \in I$, consider $x = n$

$h(n) = f\,[g(n)] = f(0)$

$$\lim_{x\to n^-} h(x) = \lim_{x\to n^-} f(\{x\}) = f(1) = f(0)$$

$\lim_{x\to n^+} h(x) = \lim_{x\to n^+} f(\{x\}) = f(0) \Rightarrow h(x)$ is continuous $x \in I$

$h(x)$ is continuous $\forall\ x \in R$

20. **Ans. (3)**

$$f(x) = \begin{bmatrix} x^2 & x \text{ is irrational} \\ 1 & x \text{ is rational} \end{bmatrix}$$

It is continuous $x^2 = 1$ $\qquad$ $x = \pm 1$

Discontinuous at all x except $x = 1, -1$

SECTION–B

1. **Ans. (6)**

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{(a+b+5) + \left(-a - \frac{b}{2}\right)x^2 + ...}{x^2} = 3$$

$\Rightarrow a + b + 5 = 0$ and $-a - \frac{b}{2} = 3 \Rightarrow a = -1, b = -4$

Because $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(1 + \left(\frac{cx + dx^3}{x^2}\right)\right)^{1/x}$ exists so $\lim_{x\to 0^+} \left(\frac{(cx+dx)^3}{x^2}\right) = 0 \Rightarrow c = 0$

Now $\lim_{x\to 0^+} (1 + dx)^{1/x} = e^d = 3$

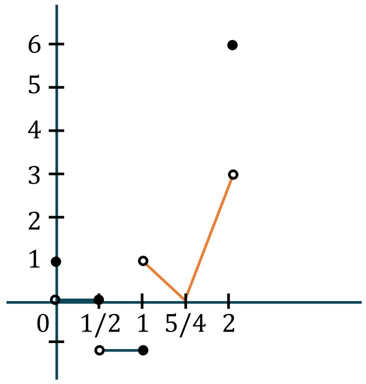
2. Ans. (3)

$$f(x) = [\sin[x]] = \begin{cases} [\sin 0] = 0 & 0 \le x < 1 \\ [\sin 1] = 0 & 1 \le x < 2 \\ [\sin 2] = 0 & 2 \le x < 3 \\ [\sin 3] = 0 & 3 \le x < 4 \\ [\sin 4] = -1 & 4 \le x < 5 \\ [\sin 5] = -1 & 5 \le x < 6 \\ [\sin 6] = -1 & 6 \le x < 2\pi \end{cases}$$

$f(x)$ is discontinuous at $(4, -1)$

3. Ans. (4)

$$f(x) = \begin{cases} 1 & , \quad x = 0 \\ 0 & , \quad 0 < x \le 1/2 \\ -1 & , \quad 1/2 < x \le 1 \\ 5 - 4x & , \quad 1 < x < 5/4 \\ 4x - 5 & , \quad 5/4 \le x < 2 \\ 6 & , \quad x = 2 \end{cases}$$



$f(x)$ is discontinous at $x = 0, \frac{1}{2}, 1, 2$ in $[0, 2]$

4. Ans. (1)

As f is continuous on R, so $f(0) = \lim\limits_{x \to 0} f(x)$

$$\text{Thus } f(0) = \lim_{n \to \infty} f\left(\frac{1}{4n}\right) = \lim_{n \to \infty} \left((\sin e^{n}) e^{-n^2} + \frac{1}{1 + \frac{1}{n^2}} \right) = 0 + 1 = 1$$

5. Ans. (5)

$$g(x) = \begin{cases} \dfrac{f(x)}{2} & x > 1 \\ \dfrac{h(x) + 1}{6} & x < 1 \end{cases}$$

$$g(1) = \lim_{x \to 1} \frac{\sin^2 \pi \ . \ 2^x}{\log_e \sec \pi \ . \ 2^x} = \lim_{x \to 1} \frac{\sin^2(2\pi - \pi \ . \ 2^x)}{\log_e \sec(2\pi - \pi \ . \ 2^x)} = 2$$

Now $g(x)$ is continuous at $x = 1$

$g(1^+) = g(1^-) = g(1)$

$\frac{f(1)}{2} = \frac{h(1)+1}{6} = 2$

$f(1) = 4 \qquad h(1) = 11$

$g(1) = 2$

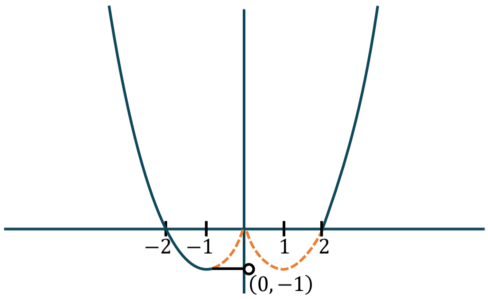
$4g(1)+2f(1)-h(1)=8+8-11=5$

6. **Ans. (1)**

$\because f(x) = \begin{cases} x^2+2x & , x<0 \\ x^2-2x & , x \ge 0 \end{cases}$

by definition of $g(x)$

$g(x) = \begin{cases} x^2+2x & , -2 \le x < -1 \\ -1 & , -1 \le x < 0 \\ 0 & , 0 \le x < 2 \\ x^2-2x & , 2 \le x < 3 \end{cases}$



clearly $g(x)$ is discontinuous at $x = 0$.

7. **Ans. (2)**

$\because f(x) = \begin{cases} \frac{x}{1-x} & , x \le -1 \\ \frac{x}{1+x} & , -1 < x < 0 \\ \frac{x}{1-x} & , 0 \le x < 1 \\ \frac{x}{1+x} & , x \ge 1 \end{cases}$

$\because \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{1-x} = 0$ and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{1+x} = 0$

and $f(0) = 0 \therefore f(x)$ is continuous at $x = 0$

$\because \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x}{1+x} = \frac{1}{2}$ and $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x}{1-x} \to \infty$

$\therefore f(x)$ is discontinuous for $x = 1$

Similarly, we can check that $f(x)$ is discontinuous at $x = -1$

8. **Ans. (5)**

$\lim_{x\to \frac{\pi}{2}^-} f(x) = \lim_{x\to \frac{\pi}{2}^-} \frac{1-\sin^3 x}{3\cos^2 x} = \lim_{h\to 0} \frac{1-\cos^3 h}{3\sin^2 h} \qquad \left[\text{put } x = \frac{\pi}{2} - h\right]$

$= \lim_{h\to 0} \frac{1-\left(1-\frac{h^2}{2\,!}+\frac{h^4}{4\,!}-....\right)^3}{3h^2} = \frac{1}{2}$

and $\lim_{x\to \frac{\pi}{2}^+} f(x) = \lim_{x\to \frac{\pi}{2}^+} \frac{b(1-\sin x)}{(\pi-2x)^2} = \lim_{h\to 0} \frac{b(1-\cosh)}{4h^2} = \frac{b}{8}$

so, $\frac{1}{2} = \frac{b}{8} = a$

$\Rightarrow 2a + b = 5$

9. Ans. (2)

$$g(x)=(|x-1|+|4x-11|)\left[(x-1)^2-3\right]=(|x-1|+|4x-11|)\left(\left[(x-1)^2\right]-3\right)$$

Now $|x-1|+|4x-11|$ is continuous every where

& $\left[(x-1)^2-3\right]$ is discontinuous at $x=1, 2\,;\ \sqrt{2}+1$

At $x=1, g(x)$ is continuous

At $x=2, g(x)$ is discontinuous

At $x=\left(\sqrt{2}+1\right), g(x)$ is discontinuous

10. Ans. (2)

$$f(x)=\begin{bmatrix} 1+x & , 0\le x\le 2 \\ 3-x & , 2< x\le 3\end{bmatrix}$$

$$y(x)=fof(x)=\begin{bmatrix} 1+f(x) & \text{when} & 0\le f(x)\le 2 \\ 3-f(x) & \text{when} & 2< f(x)\le 3\end{bmatrix}=\begin{cases} 1+(1+x) & \text{when}\quad 0\le x\le 1 \\ 1+(3-x) & \text{when}\quad 2\le x\le 3 \\ 3-(1+x) & \text{when}\quad 1\le x\le 2\end{cases}$$

$g\left(1^-\right)=g(1)=3\ne g\left(1^+\right)=1$ $\qquad$ $g\left(2^-\right)=g(2)=0\ne g\left(2^+\right)=2$

JEE (Advanced) Practice Paper

1. Ans. (D)

$$f(1)=\lim_{x\to 1} f(1)$$

$$c=\lim_{x\to 1}\frac{x^2+ax+b}{(x-1)(x-2)}$$

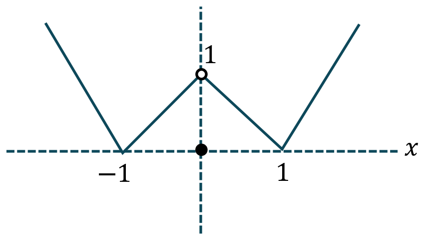
$$c=\lim_{x\to 1}\frac{x^2+ax-1-a}{(x-1)(x-2)} \qquad \{\because\ 1+a+b=0\}$$

$$c=\lim_{x\to 1}\frac{(x-1)(x+1+a)}{(x-1)(x-2)}$$

$$c=\frac{(a+2)}{-1}\Rightarrow a+c=-2 \ \&\ a+b=-1$$

2. Ans. (D)

$$f(x)=\begin{cases} |x-1| & x>0 \\ 0 & x=0 \\ |x+1| & x<0\end{cases}$$



3. Ans. (A)

$$f(x)=\begin{cases}\dfrac{\ell n\cos x}{ax} & , \quad x>0\\ 0 & , \quad x=0 \text{ ; if } f'(0)=\dfrac{1}{4}\\ \dfrac{e^{x^2}-1}{bx} & , \quad x<0\end{cases}$$

$$\Rightarrow f'(0^+)=\lim_{h\to 0}\frac{\frac{\ell n\cos h}{ah}-0}{h}=\lim_{h\to 0}\frac{\ell n\cos h}{ah^2}=\lim_{h\to 0}\frac{\cos h-1}{ah^2}=-\frac{1}{2a}=\frac{1}{4}$$

$\Rightarrow a = -2$

$$\text{and } f'(0^-)=\lim_{h\to 0}\frac{\frac{e^{h^2}-1}{bh}-0}{h}$$

$$=\lim_{h\to 0}\frac{e^{h^2}-1}{bh^2}=\frac{1}{b}=\frac{1}{4}$$

$\Rightarrow b = 4 \Rightarrow a + b = 2$

4. Ans. (C)

$f(0^-) = f(0) = f(0^+)$ $\quad \{\because f' \text{ is continuous}\}$

$$\lim_{x\to 0^-}\frac{e^x-e^{\sin x}}{ax^3}=b$$

$$\therefore \lim_{x\to 0^-}\frac{e^{\sin x}}{a}\left(\frac{e^{x-\sin x}-1}{x-\sin x}\right)\times\left(\frac{x-\sin x}{x^3}\right)=b$$

$$\therefore \frac{1}{6a}=b \qquad \text{...(1)}$$

$$\lim_{x\to 0^+}\frac{x}{\ell n(1+4x)}=b$$

$$\Rightarrow b=\frac{1}{4} \qquad \text{....(2)}$$

from (1) and (2)

$$\Rightarrow \frac{1}{6a}=\frac{1}{4}\Rightarrow a=\frac{2}{3}$$

$\therefore 3a + 4b = 3$

5. Ans. (A)

$f(x)$ is continuous

at $x=\dfrac{-\pi}{2}$

$$\lim_{x\to\frac{-\pi}{2}^+}f(x)=\lim_{x\to\frac{-\pi}{2}^-}f(x)$$

$$\Rightarrow \lim_{x\to\frac{-\pi}{2}^+}(a\sin x+b)=4$$

$$\Rightarrow -a+b=4 \qquad \text{...(i)}$$

at $x=\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi}{2}^+} f(x)=\lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$2=\lim_{x\to\frac{\pi}{2}^-}(a\sin+\mathrm{b})$$

$\Rightarrow a+b=2$ ...(ii)

From (i) and (ii)

$a=-1, b=3$

6. Ans. (B)

At $x=1$

$$\rightarrow \lim_{x\to1^+} f(x)=\lim_{x\to1^+}(5-4x)=1$$

$$f(1)=\frac{1}{4}(3\times1^2+1)=1$$

$$\Rightarrow \lim_{x\to1^+} f(x)=f(1)$$

so $f(x)$ is continuous at $x=1$.

At $x=4$

$f(4) = 0.$

$$\rightarrow \lim_{x\to4^-} f(x)=\lim_{x\to4^-} 5-4x=-11$$

$$\Rightarrow \lim_{x\to4^-} f(x)\neq f(4)$$

so $f(x)$ is not continuous at $x=4$

7. Ans. (A,B,D)

$$f(x)=\lim_{n\to\infty}\frac{ax^2+bx+c+(e^x)^n}{1+c(e^x)^n}$$

For $x<0$ $(e^x)^n\rightarrow 0$

$$f(x)=ax^2+bx+c$$

For $x>0$ $(e^x)^n\rightarrow\infty$

$$f(x)=\lim_{n\to\infty}\frac{\frac{ax^2+bx+c}{(e^x)^n}+1}{\frac{1}{(e^x)^n}+c}=\frac{1}{c}$$

$$f(0)=\frac{c+1}{c+1}=1$$

$f(x)$ is continuous on R so it is continuous at $x=0$

$$f(0^+)=f(0^-)=f(0)$$

$$\frac{1}{c}=c=1$$

$c=1,\quad a,b\in R$

8. Ans. (A,B,C)

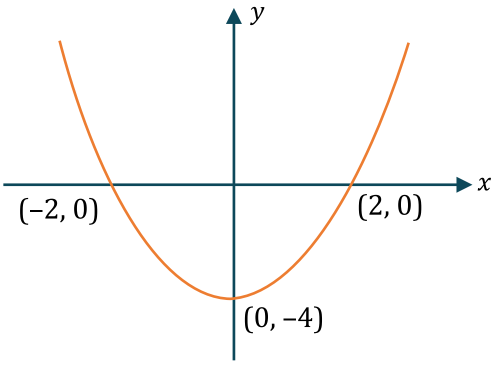
$f(x)=x^3-3x^2-4x+12$

$f(x)=x^2(x-3)-4(x-3)=(x-3)(x^2-4)$

$f(x)=(x-3)(x-2)(x+2)$

$f(x)=0$

$x=2,3,-2$ $\qquad$ $f(x)$ has 3 real roots



$\lim_{x\to3}\frac{f(x)}{x-3}=\lim_{x\to3}\frac{(x-2)(x-3)(x+2)}{(x-3)}=5$

$f(x)$ is continuous at $x = 3$

So, $f(3)=\lim_{x\to3}\frac{f(x)}{x-3}$

$K = 5$

$h(x) = \begin{bmatrix} x^2-4 & x\neq3 \\ 5 & x=3 \end{bmatrix}$

$\because h(x)$ is an even function.

9. Ans. (A,B,C)

$f(x) = sgn(x^5) = \begin{cases} 1 & if \quad x>0 \\ 0 & if \quad x=0 \\ -1 & if \quad x<0 \end{cases}$

$f'(0^+)=\lim_{h\to0}\frac{1-0}{h}$ $\qquad$ = DNE

$f'(0^-)=\lim_{h\to0}\frac{-1-0}{-h}=D.N.E.$

Also, $f(0^+) = 1$ & $f(0^-) = -1$

$\Rightarrow f$ is neither continuous nor differentiable at $x = 0$

10. Ans. (A,C,D)

Let $\theta=\frac{x}{2^{r-1}}$, $\tan\frac{\theta}{2}.\sec\theta=\frac{\sin\frac{\theta}{2}}{\cos\theta\cos\frac{\theta}{2}}$

$\frac{\sin\left(\theta-\frac{\theta}{2}\right)}{\cos\theta.\cos\frac{\theta}{2}}=\tan\theta-\tan\frac{\theta}{2}$

$\therefore\ f(x)=\sum_{r=1}^{n}\left(\tan\frac{x}{2^{r-1}}-\tan\frac{x}{2^r}\right)$ on expanding we get,

$\Rightarrow f(x) = \tan x - \tan\frac{x}{2^n} \Rightarrow f(x) + \tan\frac{x}{2^n} = \tan x$

$g(x)=\lim_{n\to\infty}\frac{\log_e(\tan x)-(\tan x)^n.\left[\sin\left(\tan\frac{x}{2}\right)\right]}{1+(\tan x)^n}$

$$g(x)=\begin{cases} \log_e \tan x & ; \quad x\in\left(0,\frac{\pi}{4}\right) \\ k & ; \quad x=\frac{\pi}{4} \\ -\left[\sin\tan\frac{x}{2}\right] & ; \quad x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right) \end{cases}$$

Since, $g(x)$ is continuous at $x=\frac{\pi}{4}$

then $g\left(\frac{\pi}{4}^{-}\right)=g\left(\frac{\pi}{4}\right)=g\left(\frac{\pi}{4}\right)$

$\Rightarrow \ell n1 = k = \sin 0$

$\therefore k = 0$

11. **Ans. (B,C,D)**

(A) $f(0^-)=\lim\limits_{x\to 0^-}\frac{1}{1+2^{\cot x}}=1$

$f(0^+)=\lim\limits_{x\to 0^+}\frac{1}{1+2^{\cot x}}=0$

$\because$ Limit does not exist

$\therefore$ can't be defined continuously at $x = 0$

(B) $f(0^-)=\lim\limits_{x\to 0^-}\cos\left(\frac{-\sin x}{x}\right)=\cos(-1)=\cos(1)$

$f(0^+)=\lim\limits_{x\to 0^+}\cos\left(\frac{\sin x}{x}\right)=\cos(1)$

$\therefore$ for $f(x)$ to be continuous at $x = 0$ $f(0) = cos1$

(C) $f(0^-)=f(0^+)=\lim\limits_{x\to 0}x\sin\frac{\pi}{x}=0$

$\therefore$ for $f(x)$ to be continuous at $x = 0$, $f(0) = 0$

(D) $f(0^-)=f(0^+)=\lim\limits_{x\to 0}\frac{1}{\ell n|x|}=0$

$\therefore$ for $f(x)$ to be continuous at $x = 0$, $f(0) = 0$

12. **Ans. (A,B,C)**

Let $\cos^{-1}(1 - x^2) = \theta$

$\cos\theta = 1 - x^2$

$\sin\theta=\sqrt{1-\cos^2\theta}$

$=\sqrt{1-(1-x^2)^2}$

$=\sqrt{2x^2-x^4}$

$\therefore \quad \theta=\sin^{-1}\left(\sqrt{2x^2-x^4}\right)$

$x \to 0^+$, $\{x\} = x$

$$\Rightarrow \lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{\cos^{-1}(1-x^2).\sin^{-1}(1-x)}{\sqrt{2}\,x(1-x^2)}=f(0^+)$$

$$=\frac{\pi}{2}\lim_{x\to0^+}\frac{\sin^{-1}\sqrt{2x^2-x^4}}{\sqrt{2}\,x}=\frac{\pi}{2}\lim_{x\to0^+}\frac{x\sqrt{2-x^2}}{\sqrt{2}\,x}=\frac{\pi}{2}$$

$x \to 0^{--}$, $\{x\} = x + 1$

$$f(0^-)=\lim_{x\to0^-}\frac{\cos^{-1}\left(1-(x+1)^2\right).\sin^{-1}(-x)}{\sqrt{2}(x+1)\left(1-(x+1)^2\right)}$$

$$f(0^-)=\lim_{x\to0^-}\frac{\cos^{-1}(-x^2-2x).\sin^{-1}x}{\sqrt{2}(x+1)(-x^2-2x)}=\lim_{x\to0^-}\left(\frac{-\left(\frac{\pi}{2}\right)\sin^{-1}x}{\sqrt{2}(x+1)x(-x-2)}\right)$$

$$f(0^-)=\frac{\pi}{2\sqrt{2}}\lim_{x\to0^-}\frac{\sin^{-1}x}{2x}=\frac{\pi}{4\sqrt{2}}$$

$$g(x)=\begin{cases} f(x) & ; \quad x\ge0 \\ 2\sqrt{2}\,f(x) & ; \quad x<0\end{cases}$$

$$g(0^+)=\lim_{x\to0^+} f(x)=\frac{\pi}{2}$$

$$g(0^-)=\lim_{x\to0^-} 2\sqrt{2}\,f(x)=\frac{\pi}{2}$$

$g(x)$ is cont at $x=\frac{\pi}{2}$

13. Ans. (B)

14. Ans. (C)

Solution (Q. No. 13 - 14)

Given function $f(x)$ can be rewritten as, $f(x) = \begin{cases} 0 & , \quad x<-1 \\ 1+x & , \quad -1\le x\le 0 \\ 1-x & , \quad 0<x\le1 \\ 0 & , \quad x>1\end{cases}$

$\Rightarrow f(x-1) = \begin{cases} 0 & , \quad x-1<-1 \\ 1+(x-1) & , \quad -1\le x-1\le 0 \\ 1-(x-1) & , \quad 0<x-1\le1 \\ 0 & , \quad x-1>1\end{cases}$ or $f(x-1) = \begin{cases} 0 & , \quad x<0 \\ x & , \quad 0\le x\le 1 \\ 2-x & , \quad 1<x\le2 \\ 0 & , \quad x>2\end{cases}$

also, $f(x+1) = \begin{cases} 0 & , \quad x+1<-1 \\ 1+(x+1) & , \quad -1\le x+1\le 0 \\ 1-(x+1) & , \quad 0<x+1\le1 \\ 0 & , \quad x+1>1\end{cases}$ or $f(x+1) = \begin{cases} 0 & , \quad x<-2 \\ 2+x & , \quad -2\le x\le -1 \\ -x & , \quad -1<x\le0 \\ 0 & , \quad x>0\end{cases}$

Now, $g(x) = f(x-1) + f(x+1) = \begin{cases} 0 & , \quad x<-2 \\ 2+x & , \quad -2<x\le -1 \\ -x & , \quad -1<x\le 0 \\ x & , \quad 0<x\le1 \\ 2-x & , \quad 1<x\le2 \\ 0 & , \quad x>2\end{cases}$

It is easy to check that $g(x)$ is continuous for all $x \in R$.

15. Ans. (C)

(P) $f(x)$ is continuous at $x = 0$ and discontinuous at all other integral points

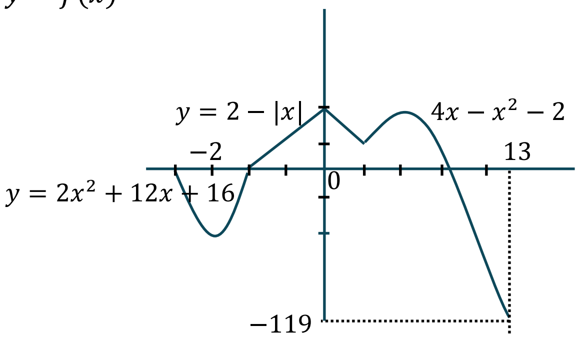
(Q) domain contains only two elements {–1, 1}

(R) $(2x - 1)\ x < 0$

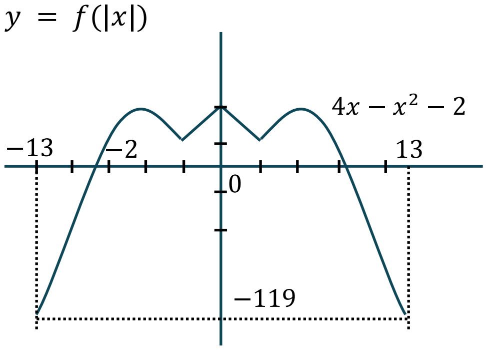
(S) $f(x)$ is continuous everywhere

16. Ans. (26)

$y = f(x)$



$y = f(|x|)$



17. Ans. (36)

$$f\left(\frac{\pi}{2}\right)=\lim_{h\to 0}\frac{1-\cos h}{4h^2}\cdot\frac{\ln(1+\cos h-1)}{\ln(1+4h^2)}$$

$$=\lim_{h\to 0}\frac{1}{8}\times\frac{\cos h-1}{4h^2}$$

$$=\frac{1}{8}\times\left(-\frac{1}{8}\right)=-\frac{1}{64}$$

$\Rightarrow \alpha^\beta = 64 = 2^6,\ 4^3, 8^2,\ 64^1$

18. Ans. (16)

We have

$$\lim_{x\to 0^-} f(x) = {}^*\ (\sin(-h) + \cos(-h))^{\text{cosec}(-h)} = \lim_{h\to 0^+} (\cos h - \sin h)^{-\text{cosech}}$$

$$= \lim_{h\to 0^+} (1+(\cos h-\sin h-1))^{\frac{1}{(\cos h-\sin h-1)}\cdot\frac{(\cos h-\sin h-1)}{(-\sin h)}} = \lim_{h\to 0^+} e^{\frac{\cos h-\sin h-1}{-\sin h}} = e$$

Now we have

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0^+}\frac{e^{\frac{1}{h}}+e^{2/h}+e^{3/h}}{ae^{-2+1/h}+be^{-1+3/h}} = \lim_{h\to 0^+}\frac{e^{-\frac{2}{h}}+e^{\frac{-1}{h}}+1}{(ae^{-2})e^{-2/h}+(be^{-1})} = \frac{e}{b}$$

If 'f' is continuous at $x = 0$, then

$e = a = \frac{e}{b}$ gives $a = e$ and $b = 1$