EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

1. For the function $f(x)=\begin{cases}\dfrac{1}{x+2^{\left(\frac{1}{x-2}\right)}}, & x\neq 2\\ k, & x=2\end{cases}$ which of the following holds ?

(A) $k = 1/2$ and f is continuous at $x = 2$ (B) $k \neq 0, 1/2$ and f is continuous at $x = 2$

(C) f can not be continuous at $x = 2$ (D) $k = 0$ and f is continuous at $x = 2$.

MCT002

2. If $f(x)=\dfrac{x-e^x+\cos 2x}{x^2}, x\neq 0$ is continuous at $x = 0$, then -

(A) $f(0)=\dfrac{5}{2}$ (B) $[f(0)] = -2$ (C) $\{f(0)\} = -0.5$ (D) $[f(0)].\{f(0)\} = -1.5$

where [.] and {.} denotes greatest integer and fractional part function

MCT004

3. Number of points of discontinuity of $f(x) = [2x^3 - 5]$ in [1,2), is equal to-
(where $[x]$ denotes greatest integer less than or equal to x)

(A) 14 (B) 13 (C) 10 (D) 8

MCT007

4. f is a continuous function on the real line. Given that $x^2 + (f(x) - 2)x - \sqrt{3}\cdot f(x) + 2\sqrt{3} - 3 = 0$. Then the value of $f(\sqrt{3})$

(A) can not be determined (B) is $2(1 - \sqrt{3})$

(C) is zero (D) is $\dfrac{2(\sqrt{3}-2)}{\sqrt{3}}$

MCT010

5. The function $f(x) = [x]^2 - [x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at :

(A) all integers (B) all integers except 0 & 1

(C) all integers except 0 (D) all integers except 1

MCT011

6. Consider $f(x)=\begin{cases}\dfrac{\sin(2\pi\sec x)}{e^x-1-x} & x\neq 0 \text{ or } (2n+1)\dfrac{\pi}{2}, n\in I\\ k & x=0\end{cases}$

Value of k for which $f(x)$ is continuous at $x = 0$, is -

(A) $\dfrac{\pi}{2}$ (B) π (C) 2π (D) 4π

MCT016

7. The number of values of $x, x \in [-2,3]$ where $f(x) = [x^2]\sin(\pi x)$ is discontinuous is
(where [.] denotes greatest integer function)-

(A) 19 (B) 8 (C) 6 (D) 0

MCT020

MULTIPLE CORRECT TYPE QUESTIONS

8. If $f(x)=\begin{cases}\sin^{-1}\left(\dfrac{a}{\ell nx}-\dfrac{b}{x-1}\right) & x\neq 1\\ \dfrac{\pi}{3} & x=1\end{cases}$ is continuous at $x=1$, then-

(A) $a=b$ (B) $a=\sqrt{3}$ (C) $b=\sqrt{3}$ (D) $b=-\sqrt{3}$

MCT036

9. Let $f(x)=\begin{cases}\dfrac{\ell n(1+x)-x+kx^2}{x^3} & ;\ x>0\\ \ell & ;\ x=0\\ \dfrac{\tan\frac{x}{3}}{x} & ;\ x<0\end{cases}$, then-

(A) $f(x)$ is continuous at $x=0$ if $k=\dfrac{1}{2},\ell=\dfrac{1}{3}$

(B) $f(x)$ is discontinuous at $x=0$ if $\ell\neq\dfrac{1}{3}$

(C) $f(x)$ has irremovable discontinuity at $x=0$ if $k\neq\dfrac{1}{2}$

(D) $f(x)$ has removable discontinuity at $x=0$ if $k=\dfrac{1}{2}$, $\ell\neq\dfrac{1}{3}$

MCT037

10. A function $f(x)$ is defined as $f(x)=\dfrac{A\sin x+\sin 2x}{x^3},(x\neq 0)$. If the function is continuous at $x=0$, then -

(A) $A=-2$ (B) $f(0)=-1$ (C) $A=1$ (D) $f(0)=1$

MCT039

11. Which of the following function(s) can't be defined at $x=0$ to make it continuous at $x=0$?

(A) $f(x)=\dfrac{1}{1+2^{\frac{1}{x}}}$ (B) $f(x)=\tan^{-1}\dfrac{1}{x}$ (C) $f(x)=\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$ (D) $f(x)=\dfrac{1}{\ell n|x|}$

MCT040

12. If f is defined on an interval $[a,b]$. Which of the following statement(s) is/are INCORRECT ?

(A) If $f(a)$ and $f(b)$, have opposite sign, then there must be a point $c\in(a,b)$ such that $f(c)=0$.

(B) If f is continuous on $[a,b]$, $f(a)<0$ and $f(b)>0$, then there must be a point $c\in(a,b)$ such that $f(c)=0$.

(C) If f is continuous on $[a,b]$ and there is a point c in (a,b) such that $f(c)=0$, then $f(a)$ and $f(b)$ have opposite sign.

(D) If f has no zeroes on $[a,b]$, then $f(a)$ and $f(b)$ have the same sign.

MCT042

13. Which of the following functions can be defined at indicated point so that resulting function is continuous -

(A) $f(x)=\dfrac{x^2-2x-8}{x+2}$ at $x=-2$ (B) $f(x)=\dfrac{x-7}{|x-7|}$ at $x=7$

(C) $f(x)=\dfrac{x^3+64}{x+4}$ at $x=-4$ (D) $f(x)=\dfrac{3-\sqrt{x}}{9-x}$ at $x=9$

MCT043

14. In which of the following cases the given equations has atleast one root in the indicated interval ?

(A) $x-\cos x=0$ in $(0,\pi/2)$

(B) $x+\sin x=1$ in $(0,\pi/6)$

(C) $\dfrac{a}{x-1}+\dfrac{b}{x-3}=0, a,b>0$ in $(1,3)$

(D) $f(x)-g(x)=0$ in (a,b) where f and g are continuous on $[a,b]$ and $f(a)>g(a)$ and $f(b)<g(b)$.

MCT044

15. Let $f(x)=x^3-x^2-3x-1$ and $h(x)=\dfrac{f(x)}{g(x)}$, where h is a rational function such that

(a) Domain of $h(x)$ is $\mathbb{R}-\{-1\}$

(b) $\lim\limits_{x\to\infty} h(x)=\infty$ and

(c) $\lim\limits_{x\to -1} h(x)=\dfrac{1}{2}$.

Which of the following is/are True ?

(A) $\lim\limits_{x\to 0}(3h(x)+f(x)-2g(x))=-\dfrac{39}{4}$ (B) $g(x)=4(x+1)$

(C) $\lim\limits_{x\to 0}(3h(x)+f(x)-2g(x))=\dfrac{39}{4}$ (D) $g(x)=2(x+1)$

MCT045

16. If the function $f(x)=\dfrac{3x^2+ax+a+3}{x^2+x-2}$, $(x\neq -2)$ is continuous at $x=-2$, then

(A) $a=15$ (B) $a=-15$ (C) $f(-2)=1$ (D) $f(-2)=-1$

MCT046

17. Let $f(x)=\begin{cases}\dfrac{\ell n\cos x}{\sqrt[4]{1+x^2}-1} & \text{if } x>0\\ \dfrac{e^{\sin 4x}-1}{\ell n(1+\tan 2x)} & \text{if } x<0\end{cases}$

(A) $f(0^+)=-2$ (B) $f(0^-)=2$

(C) $f(0^+)=2$ (D) $f(0)$ can not be defined to remove discontinuity

MCT048

18. Let $f(x) = \dfrac{\sin^{-1}(1-\{x\}) \ . \ \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\ (1-\{x\})}$ then

(where {.} denotes the fractional part function)

(A) $\lim\limits_{x\to 0^+} f(x) = \dfrac{\pi}{2}$ (B) f is discontinuous at $x = 0$

(C) $\lim\limits_{x\to 0^-} f(x) = \dfrac{\pi}{2\sqrt{2}}$ (D) f is continuous at $x = 0$

MCT049

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 19 to 21

Let $f(x) = \lim\limits_{n\to\infty} \dfrac{\ell nx + x^n}{(x-1)+x^n}$ $(x > 0)$.

On the basis of above information, answer the following questions :

19. $\lim\limits_{x\to 1^-} \dfrac{\sin(f(x)-1)}{(x-1)}$ is equal to -

(A) 0 (B) $\dfrac{1}{2}$ (C) $-\dfrac{1}{2}$ (D) 1

MCT053

20. For $f(x)$ which of the following statement is true -

(A) $f(x)$ is discontinuous at $x = 1$

(B) non-negative difference between LHL & RHL at $x = 1$ is equal to 2.

(C) $f(x)$ is many-one function

(D) $f(x)$ is one-one function

MCT054

21. The number of solutions of the equation $f(x) = \dfrac{1}{(x-1)^2}$ is -

(A) 0 (B) 1 (C) 2 (D) 3

MCT055

Paragraph for Question No. 22 to 23

Consider $f(x) = \dfrac{1}{\ell n|x|}$ and $g(x) = \tan^{-1}|x|$.

On the basis of above information, answer the following questions :

22. Sum of solutions of the equation $\dfrac{g(x)}{f(x)} = \dfrac{1}{2}$ will be less than or equal to :

(A) 0 (B) 1 (C) 2 (D) 3

MCT062

23. The value(s) of x at which $h(x) = f(x) + g(x)$ will be discontinuous is/are :

(A) 0 (B) 1 (C) –1 (D) 2

MCT063

MATCHING LIST TYPE QUESTION

24.

	Column-I		**Column-II**
(A)	$\lim\limits_{x\to1}\dfrac{x^3-1}{\ell nx}$ is	(P)	2
(B)	$\lim\limits_{x\to0}\dfrac{x(\cos x-\cos 2x)}{2\sin x-\sin 2x}$ is	(Q)	3
(C)	$\lim\limits_{x\to0}\dfrac{\tan x\sqrt{\tan x}-\sin x\sqrt{\sin x}}{x^3.\sqrt{x}}$ is	(R)	$\frac{3}{2}$
(D)	If $f(x)=\cos(x\cos\frac{1}{x})$ and $g(x)=\dfrac{\ell n(\sec^2 x)}{x\sin x}$ are both continuous at $x=0$ then $f(0)+g(0)$ equals	(S)	$\frac{3}{4}$

MCT064

25. Match the function in column-I with its behaviour at $x = 0$ in column-II, where [.] denotes greatest integer function & sgn(x) denotes signum function.

	Column-I		**Column-II**
(A)	$f(x)=[x][1+x]$	(P)	LHL exist at $x = 0$
(B)	$f(x)=[-x][1+x]$	(Q)	RHL exist at $x = 0$
(C)	$f(x)=(\text{sgn}(x))[2-x][1+\lvert x\rvert]$	(R)	Continuous at $x = 0$
(D)	$f(x)=[\cos x]$	(S)	$\lim\limits_{x\to0} f(x)$ exists but function is discontinuous at $x = 0$
		(T)	$\lim\limits_{x\to0} f(x)$ does not exist

MCT065

EXERCISE - S

1. The function $f(x) = \begin{cases} \left(\frac{6}{5}\right)^{\frac{\tan 6x}{\tan 5x}} & \text{if} \quad 0 < x < \frac{\pi}{2} \\ b+2 & \text{if} \quad x = \frac{\pi}{2} \\ (1+|\cos x|)^{\left(\frac{a|\tan x|}{b}\right)} & \text{if} \quad \frac{\pi}{2} < x < \pi \end{cases}$

If $f(x)$ is continuous at $x = \pi/2$, then the value of $(a - b)$ is

MCT066

2. Let $f(x) = \begin{cases} \frac{1-\sin \pi x}{1+\cos 2\pi x}, & x < \frac{1}{2} \\ p, & x = \frac{1}{2} \\ \frac{\sqrt{2x-1}}{\sqrt{4+\sqrt{2x-1}}-2}, & x > \frac{1}{2} \end{cases}$. The number of all possible values of 'p' so that the function $f(x)$ is continuous at $x = 1/2$, is –

MCT067

3. Let $f(x) = \begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$ and $g(x) = f[f(x)]$ & hence the number of points of discontinuity of $g(x)$ is

MCT068

4. If $f(x)$ is continuous at $x = \frac{\pi}{2}$ where $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & \text{if} \quad x < \frac{\pi}{2} \\ a & \text{if} \quad x = \frac{\pi}{2}, \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & \text{if} \quad x > \frac{\pi}{2} \end{cases}$

then the value of $(4a + 3b)$ is

MCT069

5. If $f(x) = \frac{\sin 3x + A\sin 2x + B\sin x}{x^5}$ $(x \neq 0)$ is continuous at $x = 0$, then the value of $(A + B + f(0))$ is

MCT071

6. If $f(x) = x + \{-x\} + [x]$, where $[x]$ is the integral part & $\{x\}$ is the fractional part of x, then the number of points of discontinuity of $f(x)$ in $[-2, 2]$ is

MCT072

7. Let $f(x)=\begin{cases}(\sin x+\cos x)^{\operatorname{cosec} x} & ; \quad \frac{-\pi}{2}<x<0 \\ a & ; \quad x=0 \\ \frac{e^{1/x}+e^{2/x}+e^{3/|x|}}{ae^{2/x}+be^{3/|x|}} & ; \quad 0<x<\frac{\pi}{2}\end{cases}$

If $f(x)$ is continuous at $x=0$, then the value of $(a^2 \cdot b^2)$ is

MCT073

8. Let f be a real valued continuous function on $\mathbb{R}$ and satisfying $f(-x)-f(x)=0 \ \forall \in \mathbb{R}$. If $f(-5)=5$, $f(-2)=4$, $f(3)=-2$ and $f(0)=0$ then find the minimum number of zero's of the equation $f(x)=0$.

MCT074

EXERCISE - JEE (Main) PYQ

1. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by $f(x)=\begin{cases}\frac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\frac{\pi}{4}\\ k, & x=\frac{\pi}{4}\end{cases}$ is continuous, then k is equal to

[JEE (Main) 2019]

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) 2

MCT026

2. If the function $f(x)=\begin{cases}a|\pi-x|+1, x\leq 5\\ b|x-\pi|+3, x>5\end{cases}$ is continuous at $x=5$, then the value of $a-b$ is :-

[JEE (Main) 2019]

(1) $\frac{2}{5-\pi}$ (2) $\frac{2}{\pi-5}$ (3) $\frac{2}{\pi+5}$ (4) $\frac{-2}{\pi+5}$

MCT027

3. Let [t] denote the greatest integer ≤ t and $\lim\limits_{x\to 0} x\left[\frac{4}{x}\right]=A$. Then the function, $f(x)=[x^2]\sin(\pi x)$ is discontinuous, when x is equal to : **[JEE (Main) 2020]**

(1) $\sqrt{A+5}$ (2) $\sqrt{A+1}$ (3) $\sqrt{A}$ (4) $\sqrt{A+21}$

MCT028

4. Let $f(x)=x\cdot\left[\frac{x}{2}\right]$, for $-10<x<10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to. **[JEE (Main) 2020]**

MCT029

5. Let $f: R\to R$ and $g: R\to R$ be defined as

$f(x)=\begin{cases}x+a, & x<0\\ |x-1|, & x\geq 0\end{cases}$ and $g(x)=\begin{cases}x+1, & x<0\\ (x-1)^2+b, & x\geq 0\end{cases}$

where a, b are non-negative real numbers. If $(gof)(x)$ is continuous for all $x\in R$, then $a+b$ is equal to. **[JEE (Main) 2021]**

MCT030

6. If the function $f(x)=\frac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0)=\frac{1}{k}$, then k is. **[JEE (Main) 2021]**

MCT031

7. Let a function $f: \boldsymbol{R}\to\boldsymbol{R}$ be defined as

$$f(x)=\begin{cases}\sin x-e^x & \text{if } x\leq 0\\ a+[-x] & \text{if } 0<x<1\\ 2x-b & \text{if } x\geq 1\end{cases}$$

Where $[x]$ is the greatest integer less than or equal to x. If f is continuous on **R**, then $(a+b)$ is equal to: **[JEE (Main) 2021]**

(1) 4 (2) 3 (3) 2 (4) $5d$

MCT032

8. The number of points where the function

$$f(x)=\begin{cases}|2x^2-3x-7| & \text{if} \quad x\le -1\\ [4x^2-1] & \text{if} \quad -1<x<1\\ |x+1|+|x-2| & \text{if} \quad x\ge 1\end{cases}$$

$[t]$ denotes the greatest integer $\le t$, is discontinuous is. **[JEE (Main) 2022]**

MCT033

9. Let $f:\mathbb{R}\to\mathbb{R}$ be defined as

$$f(x)=\begin{cases}[e^x], & x<0\\ ae^x+[x-1], & 0\le x<1\\ b+[\sin(\pi x)], & 1\le x<2\\ [e^{-x}]-c, & x\ge 2\end{cases}$$

where $a,b,c\in\mathbb{R}$ and $[t]$ denotes greatest integer less than or equal to t. Then, which of the following statements is true ? **[JEE (Main) 2022]**

(1) There exists $a, b, c \in \mathbb{R}$ such that f is continuous of $\mathbb{R}$.
(2) If f is discontinuous at exactly one point, then $a + b + c = 1$.
(3) If f is discontinuous at exactly one point, then $a + b + c \neq 1$.
(4) f is discontinuous at atleast two points, for any values of a, b and c.

MCT035

10. If the function $f(x)=\begin{cases}\dfrac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\dfrac{-\pi}{2},\dfrac{\pi}{2}\right)-\{0\}\\ k, & x=0\end{cases}$

is continuous at $x = 0$, then k is equal to : **[JEE (Main) 2022]**

(1) 1 (2) –1 (3) e (4) 0

MCT076

11. Let $f, g:\mathbf{R}\to\mathbf{R}$ be functions defined by

$$f(x)=\begin{cases}[x] & , \quad x<0\\ |1-x| & , \quad x\ge 0\end{cases} \text{ and } g(x)=\begin{cases}e^x-x & , \quad x<0\\ (x-1)^2-1 & , \quad x\ge 0\end{cases}$$

where $[x]$ denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly : **[JEE (Main) 2022]**

(1) one point (2) two points (3) three points (4) four points

MCT077

12. Let $f(x)=[x^2-x]+|-x+[x]|$, where $x \in \mathbb{R}$ and $[t]$ denotes the greatest integer less than or equal to t. Then, f is **[JEE (Main) 2023]**

(1) continuous at $x = 0$, but not continuous at $x = 1$
(2) continuous at $x = 0$ and $x = 1$
(3) not continuous at $x = 0$ and $x = 1$
(4) continuous at $x = 1$, but not continuous at $x = 0$

MCT060

13. Let $[x]$ be the greatest integer $\le x$. Then the number of points in the interval (–2,1), where the function $f(x) = |[x]| + \sqrt{x-[x]}$ is discontinuous is. **[JEE (Main) 2023]**

MCT061

EXERCISE - JEE (Advanced) PYQ

1. For every integer n, let a_n and b_n be real numbers. Let function $f:\mathbb{R}\to\mathbb{R}$ be given by

$$f(x)=\begin{cases}a_n+\sin\pi x, & \text{for } x\in[2n,2n+1]\\ b_n+\cos\pi x, & \text{for } x\in(2n-1,2n)\end{cases}$$, for all integers n.

If f is continuous, then which of the following holds(s) for all n ? **[JEE (Advanced) 2012]**

(A) $a_{n-1}-b_{n-1}=0$ (B) $a_n-b_n=1$

(C) $a_n-b_{n+1}=1$ (D) $a_{n-1}-b_n=-1$

MCT057

2. For every pair of continuous function $f,g:[0,1]\to\mathbb{R}$ such that

$\max\{f(x):x\in[0,1]\}=\max\{g(x):x\in[0,1]\}$,

the correct statement(s) is(are) : **[JEE (Advanced) 2014]**

(A) $(f(c))^2+3f(c)=(g(c))^2+3g(c)$ for some $c\in[0,1]$

(B) $(f(c))^2+f(c)=(g(c))^2+3g(c)$ for some $c\in[0,1]$

(C) $(f(c))^2+3f(c)=(g(c))^2+g(c)$ for some $c\in[0,1]$

(D) $(f(c))^2=(g(c))^2$ for some $c\in[0,1]$

MCT058

3. Let $[x]$ be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x)=x\cos(\pi(x+[x]))$ is discontinuous ? **[JEE (Advanced) 2017]**

(A) $x=-1$ (B) $x=0$

(C) $x=2$ (D) $x=1$

MCT059

JEE (Main) Practice Paper

SECTION–A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1 in all other cases.

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1. Let $f(x)=\begin{cases} ax+1 & \text{if} \quad x<1 \\ 3 & \text{if} \quad x=1 \\ bx^2+1 & \text{if} \quad x>1 \end{cases}$. If $f(x)$ is continuous at $x = 1$ then $(a - b)$ is equal to-

(1) 0 (2) 1 (3) 2 (4) 4

MCT001

2. If $f(x)=\dfrac{x^2-bx+25}{x^2-7x+10}$ for $x \neq 5$ and f is continuous at $x = 5$, then $f(5)$ has the value equal to-

(1) 0 (2) 5 (3) 10 (4) 25

MCT003

3. $y = f(x)$ is a continuous function such that its graph passes through $(a, 0)$. Then $\lim\limits_{x\to a}\dfrac{\log_e(1+3f(x))}{2f(x)}$ is-

(1) 1 (2) 0 (3) $\frac{3}{2}$ (4) $\frac{2}{3}$

MCT005

4. In [1,3], the function $[x^2 + 1]$,[.] denoting the greatest integer function, is continuous -

(1) for all x (2) for all x except at nine points
(3) for all x except at seven points (4) for all x except at eight points

MCT006

5. Given $f(x)=\begin{cases} |x+1| & \text{if} \quad x<-2 \\ 2x+3 & \text{if} \quad -2\le x<0 \\ x^2+3 & \text{if} \quad 0\le x<3 \\ x^3-15 & \text{if} \quad x\ge 3 \end{cases}$. Then number of point(s) of discontinuity of $f(x)$ is-

(1) 0 (2) 1 (3) 2 (4) 3

MCT008

6. If $f(x)$ is continuous and $f(9/2) = 2/9$, then $\lim\limits_{x\to 0} f\left(\dfrac{1-\cos 3x}{x^2}\right)$ is equal to :

(1) 9/2 (2) 0 (3) 2/9 (4) 8/9

MCT021

7. The function $f(x) = [x].\cos\frac{2x-1}{2}\pi$, where [·] denotes the greatest integer function, is discontinuous at :-

(1) all x (2) all integer points (3) no x (4) x which is not an integer

MCT013

8. Let $f : R \to Q$ be a continuous function such that $f(2) = 7$, then-

(1) $f(x)$ is always an even function

(2) $f(x)$ is always an odd function

(3) nothing can said about $f(x)$ being even or odd

(4) $f(x)$ is an increasing function

MCT015

9. Let $f(x)=\begin{cases}\frac{\tan(bx^3)}{x^3}, & x\neq 0\\ \frac{5}{4}+\frac{3}{4b}, & x=0\end{cases}$, be continuous at $x = 0$, then

(1) $b=\frac{5+\sqrt{73}}{8}$ (2) $b=\frac{3+\sqrt{73}}{8}$ (3) $b=\frac{-5+\sqrt{73}}{8}$ (4) $b=\frac{-3+\sqrt{73}}{8}$

MCT019

10. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function $\forall\ x \in \mathbb{R}$ and $f(x) = 5\ \forall\ x \in$ irrational. Then the value of $f(3)$ is-

(1) 1 (2) 2 (3) 5 (4) cannot determine

MCT012

11. A function $f(x)$ is defined as below $f(x) = \frac{\cos(\sin x)-\cos x}{x^2}$, $x \neq 0$ and $f(0) = a$

(1) 0 (2) 4 (3) 5 (4) 6

MCT078

12. Let $f(x) = \left|\left(x+\frac{1}{2}\right)[x]\right|$, when $-2 \le x \le 2$. where [.] represents greatest integer function. Then

(1) $f(x)$ is continuous at $x = 2$ (2) $f(x)$ is continuous at $x = 1$

(3) $f(x)$ is continuous at $x = -1$ (4) $f(x)$ is discontinuous at $x = 0$

MCT079

13. The function $f(x)$ is defined by $f(x) = \begin{cases}\log_{(4x-3)}(x^2-2x+5), & \text{if } \frac{3}{4} < x < 1 \text{ or } x > 1\\ 4, & \text{if } x = 1\end{cases}$

(1) is continuous at $x = 1$

(2) is discontinuous at $x = 1$ since f(1^+) does not exist though f(1^-) exists

(3) is discontinuous at $x = 1$ since f(1^-) does not exist though f(1^+) exists

(4) is discontinuous since neither f(1^-) nor f(1^+) exists.

MCT080

14. If $f(x) = x\sin\left(\frac{\pi}{2}(x+2[x])\right)$, then $f(x)$ is {where [.] denotes GIF}

(1) Discontinuous at $x = 2$ (2) Discontinuous at $x = 1$

(3) Continuous at $x = 1$ (4) Continuous at $x = 3$

MCT081

15. $f(x)=\begin{cases}\dfrac{\sqrt{(1+px)}-\sqrt{(1-px)}}{x}, & -1\le x<0\\ \dfrac{2x+1}{x-2}, & 0\le x\le 1\end{cases}$ is continuous in the interval $[-1, 1]$, then 'p' is equal to:

(1) -1 (2) $-1/2$ (3) $1/2$ (4) 1

MCT082

16. Let $f(x) = \text{Sgn}(x)$ and $g(x) = x(x^2-5x+6)$. The function $f(g(x))$ is discontinuous at

(1) infinitely many points (2) exactly one point

(3) exactly three points (4) no point

MCT083

17. If $y = \dfrac{1}{t^2+t-2}$ where $t = \dfrac{1}{x-1}$, then the number of points of discontinuities of $y = f(x)$, $x \in R$ is

(1) 1 (2) 2 (3) 3 (4) infinite

MCT084

18. The equation $2\tan x + 5x - 2 = 0$ has

(1) no solution in $[0, \pi/4]$ (2) at least one real solution in $[0, \pi/4]$

(3) two real solution in $[0, \pi/4]$ (4) three real solution in $[0, \pi/4]$

MCT085

19. Let $[x]$ denote the integral part of $x \in R$ and $g(x) = x - [x]$. Let $f(x)$ be any continuous function with $f(0) = f(1)$, then the function $h(x) = f\big(g(x)\big)$:

(1) has finitely many discontinuities (2) is continuous on R

(3) is discontinuous at some $x = c$ (4) is a constant function.

MCT086

20. Let $f(x)=\begin{cases}x^2 & \text{if } x \text{ is irrational}\\ 1 & \text{if } x \text{ is irrational}\end{cases}$, then:

(1) $f(x)$ is discontinuous for all x

(2) discontinuous for all x except at $x = 0$

(3) discontinuous for all x except at $x = 1$ or -1

(4) continuous for all x except at $x = 1$ or -1

MCT087

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4, if only correct answer is given.

  Zero Marks : 0, if no answer is given.

  Negative Marks : -1 for incorrect answer

1. Let $f(x) = \begin{cases} \dfrac{a(1-x\sin x)+b\cos x+5}{x^2} & x<0 \\ 3 & x=0 \\ \left(1+\left(\dfrac{cx+dx^3}{x^2}\right)\right)^{1/x} & x>0 \end{cases}$

If $f(x)$ is continuous at $x = 0$ then find $\left(a-b-c+e^d\right)$

MCT088

2. A point (x, y), where function $f(x) = [\sin[x]]$ in $(0, 2\pi)$ is not continuous, is ([.] denotes greatest integer $\le x$) then the value of $x + y$ is

MCT089

3. The number of points of discontinuity of $f(x) = \begin{cases} |4x-5|\ [x] & \text{for } x > 1 \\ [\cos \pi x] & \text{for } x \le 1 \end{cases}$

(where $[x]$ is the greatest integer not greater than x) in $[0, 2]$ is

MCT090

4. Let f be a continuous function on R such that $f\left(\dfrac{1}{4x}\right) = \left(\sin e^x\right)e^{-x^2} + \dfrac{x^2}{x^2+1}$, then find the value of $f(0)$.

MCT091

5. $g(x) = \lim\limits_{n\to\infty} \dfrac{x^n f(x)+h(x)+1}{2x^n+3x+3}$, $x \ne 1$

$g(1) = \lim\limits_{x\to 1} \dfrac{\sin^2(\pi 2^x)}{\log_e \sec(\pi 2^x)}$ be a continuous function at $x = 1$, then find the value of

$4g(1)+2f(1)-h(1)$, assume that $f(x)$ and $h(x)$ are continuous at $x = 1$

MCT092

6. If $f(x)=x^2-2|x|$, and $g(x) = \begin{cases} \min.\{f(t);\ -2\le t\le x\} & ,\ -2\le x<0 \\ \max.\ \{f(t);\ 0\le t\le x\} & ,\ 0\le x\le 3 \end{cases}$ then number of discontinuous point of $g(x)$ in the interval $[-2, 3]$, is

MCT093

7. If $f(x) = \begin{cases} \dfrac{x}{1+|x|} & ;\ |x|\ge 1 \\ \dfrac{x}{1-|x|} & ;\ |x|<1 \end{cases}$ then number of discontinuous point is-

MCT094

8. If $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & ,\ x < \pi/2 \\ a & ,\ x = \pi/2 \\ \dfrac{b\ (1-\sin x)}{(\pi-2x)^2} & ,\ x > \pi/2 \end{cases}$ is continuous at $x=\dfrac{\pi}{2}$. Then the value of $2a + b$ is

MCT095

9. If $g(x)=\left(|x-1|+|4x-11|\right)\left[x^2-2x-2\right]$, then find the number of point of discontinuity of $g(x)$ in $\left(\frac{1}{2}, \frac{5}{2}\right)$. {where [.] denotes GIF}

MCT096

10. Let $f(x)=\begin{bmatrix}1+x & , & 0 \le x \le 2\\ 3-x & , & 2 < x \le 3\end{bmatrix}$. Then find the number of points of discontinuity of $g(x)=f\left(f(x)\right)$

MCT097

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION–I

- This section contains **SIX** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : –1, in all other cases

1. If $f(x)=\begin{cases}\dfrac{x^2+ax+b}{x^2-3x+2} & x\neq 1\\ c & x=1\end{cases}$ is continuous at $x=1$, then $(2a+b+c)$ is-

(A) 0 (B) 1 (C) 3 (D) –3

MCT098

2. Number of points where $f(x)=|x-sgn(x)|$ is non differentiable, is (sgn(.) denotes signum function)

(A) 0 (B) 1 (C) 2 (D) 3

MCT099

3. Let $f(x)=\begin{cases}\dfrac{\ell n\cos x}{ax} & , \quad x>0\\ 0 & , \quad x=0\\ \dfrac{e^{x^2}-1}{bx} & , \quad x<0\end{cases}$. If $f'(0)=\dfrac{1}{4}$, then

(A) $a+b=2$ (B) $b-a=2$ (C) $a+b=-2$ (D) $b-a=4$

MCT100

4. Let $f(x)=\begin{cases}\dfrac{e^x-e^{\sin x}}{ax^3} & ; \quad x<0\\ b & ; \quad x=0\\ \dfrac{x}{\ell n(1+4x)} & ; \quad x>0\end{cases}$. If f is continuous at $x=0$, then $(3a+4b)$ is equal to -

(A) –3 (B) 0 (C) 3 (D) 4

MCT101

5. If $f(x)=\begin{cases}-4\sin x+\cos x & \text{for} \quad x\leq -\dfrac{\pi}{2}\\ a\sin x+b & \text{for} \quad -\dfrac{\pi}{2}<x<\dfrac{\pi}{2}\\ \cos x+2 & \text{for} \quad x\geq \dfrac{\pi}{2}\end{cases}$ is continuous then :

(A) $a=-1, b=3$ (B) $a=1, b=-3$ (C) $a=1, b=3$ (D) $a=-1, b=-3$

MCT102

6. The function $f(x)=\begin{cases}\frac{1}{4}(3x^2+1) & -\infty<x\le 1\\ 5-4x & 1<x<4\\ 4-x & 4\le x<\infty\end{cases}$ is -

(A) continuous at $x=1$ & $x=4$
(B) continuous at $x=1$, discontinuous at $x=4$
(C) continuous at $x=4$, discontinuous at $x=1$
(D) discontinuous at $x=1$ & $x=4$

MCT103

SECTION-II

- This section contains **SIX** questions.
- Each question has **FOUR** options for correct answer(*s*). **ONE OR MORE THAN ONE** of these four option(*s*) is (are) correct option(*s*).
- For each question, choose the correct option(*s*) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	if only (all) the correct option(*s*) is (are) chosen.
Partial Marks	:	+3	if all the four options are correct but ONLY three options are chosen.
Partial Marks	:	+2	if three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	:	+1	if two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0	if none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	–2	in all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(*s*) (second option in this case), with or without selection of any correct option(*s*) will result in –2 marks.

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7. A function $f:\mathbb{R}\to\mathbb{R}$ is defined as $f(x)=\lim\limits_{n\to\infty}\frac{ax^2+bx+c+e^{nx}}{1+c\cdot e^{nx}}$ where f is continuous on $\mathbb{R}$, then

(A) $a\in\mathbb{R}$ (B) $b\in\mathbb{R}$ (C) $c=-1$ (D) $c=1$

MCT051

8. Suppose that $f(x)=x^3-3x^2-4x+12$ and $h(x)=\begin{cases}\frac{f(x)}{x-3} & , \quad x\neq 3\\ K & , \quad x=3\end{cases}$ then

(A) $f(x)=0$ has 3 real roots
(B) If $K=5$ then $h(x)$ is continuous at $x=3$
(C) for $K=5$, $h(x)$ is an even function
(D) for $K=5$, $h(x)$ is an odd function

MCT052

9. If $f(x) = sgn(x^5)$, then which of the following is/are **false** (where sgn denotes signum function) -

(A) $f'(0^+) = 1$

(B) $f'(0^-) = -1$

(C) f is continuous but not differentiable at $x = 0$

(D) f is discontinuous at $x = 0$

MCT104

10. Given $f(x) = \sum_{r=1}^{n} \tan\left(\frac{x}{2^r}\right)\sec\left(\frac{x}{2^{r-1}}\right); r, n \in N$

$$g(x) = \lim_{n\to\infty} \frac{\ell n\left(f(x)+\tan\frac{x}{2^n}\right) - \left(f(x)+\tan\frac{x}{2^n}\right)^n \cdot \left[\sin\left(\tan\frac{x}{2}\right)\right]}{1+\left(f(x)+\tan\frac{x}{2^n}\right)^n} = k$$

for $x = \frac{\pi}{4}$ and the domain of $g(x)$ is $(0, \pi/2)$.

where [.] denotes the greatest integer function.

If $g(x)$ is continuous at $x = \pi/4$, then

(A) $k = 0$

(B) $k = 1$

(C) $g(x) = \begin{cases} \ell n(\tan x) & \text{if } 0 < x < \frac{\pi}{4} \\ 0 & \text{if } \frac{\pi}{4} \le x < \frac{\pi}{2} \end{cases}$

(D) $g(x)$ is continuous everywhere

MCT105

11. Which of the following function(s) can be defined continuously at $x = 0$?

(A) $f(x) = \frac{1}{1+2^{\cot x}}$

(B) $f(x) = \cos\left(\frac{|\sin x|}{x}\right)$

(C) $f(x) = x \sin\frac{\pi}{x}$

(D) $f(x) = \frac{1}{\ell n|x|}$

MCT106

12. Let $f(x) = \begin{cases} \frac{\left(\frac{\pi}{2} - \sin^{-1}\left(1-\{x\}^2\right)\right)\sin^{-1}(1-\{x\})}{\sqrt{2}\left(\{x\}-\{x\}^3\right)} & \text{for } x \neq 0 \\ \frac{\pi}{2} & \text{for } x = 0 \end{cases}$ where $\{x\}$ is the fractional part of x.

Consider another function $g(x)$; such that

$$g(x) = \begin{cases} f(x) & \text{for } x \ge 0 \\ 2\sqrt{2}f(x) & \text{for } x < 0 \end{cases}$$

(A) $f(0^-) = \frac{\pi}{4\sqrt{2}}$

(B) f is discontinuous at $x = 0$

(C) $g(0^+) = \pi/2$

(D) g is discontinuous at $x = 0$

MCT107

SECTION–III

- This section contains **ONE** paragraph.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+3	if only the bubble corresponding to the correct answer is darkened.
Zero Marks	:	0	in all other cases.

Comprehension # 1 (Q. No. 13 - 14)

Let $f : R \to R$ be a function defined as,

$f(x) = \begin{cases} 1-|x| & , \ |x| \le 1 \\ 0 & , \ |x| > 1 \end{cases}$ and $g(x) = f(x-1) + f(x+1), \forall\, x \in R$. Then

13. The value of $g(x)$ is :

(A) $g(x) = \begin{cases} 0 & , \ x \le -3 \\ 2+x & , \ -3 \le x \le -1 \\ -x & , \ -1 < x \le 0 \\ x & , \ 0 < x \le 1 \\ 2-x & , \ 1 < x \le 3 \\ 0 & , \ x > 3 \end{cases}$

(B) $g(x) = \begin{cases} 0 & , \ x \le -2 \\ 2+x & , \ -2 \le x \le -1 \\ -x & , \ -1 < x \le 0 \\ x & , \ 0 < x \le 1 \\ 2-x & , \ 1 < x \le 2 \\ 0 & , \ x > 2 \end{cases}$

(C) $g(x) = \begin{cases} 0 & , \ x \le 0 \\ 2+x & , \ 0 < x < 1 \\ -x & , \ 1 \le x \le 2 \\ x & , \ 2 < x < 3 \\ 2-x & , \ 3 \le x < 4 \\ 0 & , \ 4 \le x \end{cases}$

(D) $g(x) = \begin{cases} 0 & , \ x \le -3 \\ 1+x & , \ -3 \le x \le -1 \\ -x & , \ -1 < x \le 0 \\ x & , \ 0 < x \le 1 \\ 1-x & , \ 1 < x \le 3 \\ 0 & , \ x > 3 \end{cases}$

MCT108

14. The function $g(x)$ is continuous for, $x \in$

(A) $R - \{0, 1, 2, 3, 4\}$ (B) $R - \{-2, -1, 0, 1, 2\}$

(C) R (D) $R - \{0, 1, 2, 3\}$

MCT109

SECTION–IV

- This section contains **ONE** question.
- Each question contains two columns, Column-I and Column-II.
- Column-I has four entries (P), (Q), (R) and (S).
- Column-II has four entries (1), (2), (3), (4).
- Match the entries in Column-I with the entries in Column-II.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only correct answer is given.
Zero Marks	: 0	If no answer is given.
Negative Marks	: –1	For incorrect answer

15. Match column-I with column-II

	Column I		Column II
(P)	The number of points of discontinuity of $f(x) = ([x] + [-x])x^2$ in (–5, 5) is equal to, (where [.] is greatest integer function)	(1)	0
(Q)	The number of elements in the range of $f(x)=\sqrt{\cos(\cos x)} + \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is equal to	(2)	2
(R)	The number of integral values of x for which $\|3x-1\| < \|2x-1\| + \|x\|$	(3)	8
(S)	The number of points where $f(x) = sgn(sinx) + sgn(x^2 - \pi x)$ is discontinuous in $(0, 2\pi)$ is equal to (where sgn indicates signum function)	(4)	1

(A) P → 3; Q → 2; R → 4; S → 4 (B) P → 4; Q → 2; R → 1; S → 4

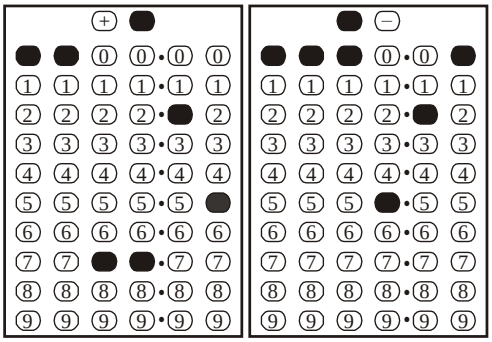
(C) P → 3; Q → 2; R → 1; S → 1 (D) P → 3; Q → 1; R → 1; S → 2

MCT110

SECTION-V

- This section contains **THREE** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, –0.33, –.30, 30.27, –127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darken the corresponding bubbles in the ORS.

For Example : If answer is –77.25, 5.2 then fill the bubbles as follows.



- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

Zero Marks : 0 In all other cases.

16. If $f(x) = \begin{cases} 2x^2+12x+16, & -4 \le x \le -2 \\ 2-\|x\|, & -2 < x \le 1 \\ 4x-x^2-2, & 1 < x \le 13 \end{cases}$, then the maximum length of interval for which $f(|x|)$ is continuous is

MCT111

17. Let $f(x)=\dfrac{1-\sin x}{(\pi-2x)^2}\cdot\dfrac{\ell n(\sin x)}{\ell n\left(1+\pi^2-4\pi x+4x^2\right)}$, $x\neq\dfrac{\pi}{2}$. The value of $f\left(\dfrac{\pi}{2}\right)$ so that the function is continuous at $x=\dfrac{\pi}{2}$ is λ and $|\lambda|\alpha^{\beta}=1$ where $\alpha,\beta\in N$ then find product of all possible values of β

MCT112

18. If the function $f(x)$ defined as $f(x)=\begin{cases}(\sin x+\cos x)^{\text{cosec}x} & , \quad -\dfrac{\pi}{2}<x<0\\ a & , \quad x=0\\ \dfrac{e^{\frac{1}{x}}+e^{\frac{2}{x}}+e^{\frac{3}{x}}}{ae^{-2+\frac{1}{x}}+be^{-1+\frac{3}{x}}} & , \quad 0<x<\dfrac{\pi}{2}\end{cases}$

is continuous at $x=0$, then the value of $\log_{e^{1/7}}a+9b$ is :

MCT113

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	D	B	B	D	C	B	A,B,C	A,B,C,D	A,B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,B,C	A,C,D	A,C,D	A,B,C,D	A,B	A,D	A,B,D	A,B,C	C	C
Que.	21	22	23	24						
Ans.	C	A,B,C,D	A,B,C	(A→Q; B→R; C→S; D→P)						
Que.	25									
Ans.	(A→P,Q,R; B→P,Q,T; C→P,Q,T; D→P,Q,S)									

EXERCISE - S

1. 1 **2.** 0 **3.** 2 **4.** 14 **5.** 2.00

6. 5.00 **7.** 1.00 **8.** 5.00

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	2	8	1	6	2	7	3	1
Que.	11	12	13							
Ans.	2	4	2							

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3
Ans.	B,D	A,D	A,C,D

JEE (Main) Practice Paper

	Q.	1	2	3	4	5	6	7	8	9	10
Section-A	A.	1	1	3	4	2	3	3	1	1	3
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	1	4	4	2	2	3	3	2	2	3
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	6	3	4	1	5	1	2	5	2	2

JEE (Advanced) Practice Paper

Section-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**
	A.	D	D	A	C	A	B
Section-II	**Q.**	**7**	**8**	**9**	**10**	**11**	**12**
	A.	A,B,D	A,B,C	A,B,C	A,C,D	B,C,D	A,B,C
Section-III	**Q.**	**13**	**14**				
	A.	B	C				
Section-IV	**Q.**	**15**					
	A.	C					
Section-V	**Q.**	**16**	**17**	**18**			
	A.	26	36	16			