Point and Straight Lines

SOLUTIONS

EXERCISE - O

1. **Ans. (B)**

x-coordinate $=1+\frac{1}{4}+\frac{1}{16}+......=\frac{1}{1-(1/4)}=\frac{4}{3}$

y-coordinate $=\frac{1}{2}-\frac{1}{8}+\frac{1}{16}.......=\frac{1/2}{1+(1/4)}=\frac{2}{5}$

2. **Ans. (D)**

$C<\frac{\pi}{2} \Rightarrow A+B>\frac{\pi}{2}$

$\Rightarrow \sin A > \cos B$ & $\cos A < \sin B$

$\Rightarrow \cos B - \sin A < 0$ and $\sin B - \cos A > 0$

3. **Ans. (A)**

Let it be (a, b)

$\frac{2a+1}{3}=\frac{-\frac{2}{3}+\frac{11}{3}}{3} \Rightarrow a = 1$

$\frac{2b+10}{3}=\frac{\frac{4}{3}+\frac{4}{3}}{3} \Rightarrow b=-\frac{11}{3}$

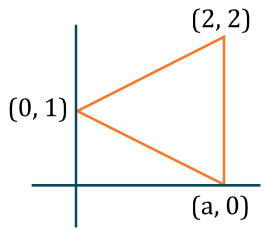
4. **Ans. (C)**

It is a right angled triangle

$\Rightarrow$ Orthocentre will be vertex containing $\perp$ sides

5. **Ans. (A)**

$\frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ a & 0 & 1 \\ 2 & 2 & 1 \end{vmatrix}=\pm 2$



$(2-a)+(2a)=\pm 4$

$2+a=\pm 4$

$a=-6, 2$

Sum = – 4

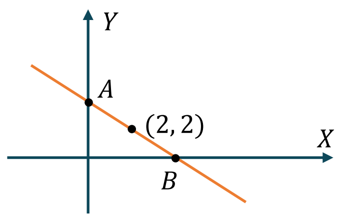
6. **Ans. (A)**

Let the line be $y-2=m(x-2)$

$\Rightarrow A \equiv (0, -2m+2)$ and $B\equiv\left(\frac{2m-2}{m}, 0\right)$

Since A lie on +ve y-axis and B lies on +ve x-axis therefore

$-2m+2>0$ and $\frac{2m-2}{m}>0$



$\Rightarrow m<1$ and $m \in (-\infty, 0) \cup (1, \infty)$

$\Rightarrow m<0$

Area $=\frac{1}{2}\times(-2m+2)\frac{(2m-2)}{m}=9$

$\Rightarrow -2(m-1)^2 = 9m \;\; \Rightarrow 2(m-1)^2 + 9m = 0$

$\Rightarrow 2m^2 + 5m + 2 = 0$

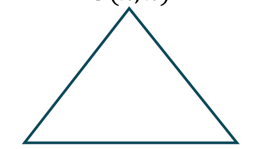
Since $D > 0$, product > 0, sum < 0

$\Rightarrow$ both root negative

$\therefore$ Sum of possible value of $m = -2.5$.

7. Ans. (A)

Centroid $\equiv\left(\frac{h}{3},\frac{k-2}{3}\right)$



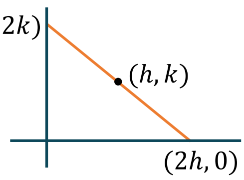
Lies on line $2x + 3y = 1$

So, $\frac{2h}{3}+\frac{3(k-2)}{3}=1$

$\Rightarrow 2h + 3k = 9$

$\Rightarrow 2x + 3y = 9$

8. Ans. (B)



$(2h)^2 + (2k)^2 = 10^2$

$\Rightarrow h^2 + k^2 = 25$

$\Rightarrow x^2 + y^2 = 25$

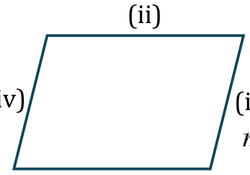
9. Ans. (D)

$3x + 4y = 7a \Rightarrow y=\frac{-3}{4}x+\frac{7a}{4}$ ...(i)

$3x + 4y = 7b \Rightarrow y=\frac{-3}{4}x+\frac{7b}{4}$ ...(ii)

$4x + 3y = 7c \Rightarrow y=\frac{-4}{3}x+\frac{7c}{3}$ ...(iii)

$4x + 3y = 7d \Rightarrow y=\frac{-4}{3}x+\frac{7d}{3}$ ...(iv)



$c_1=\frac{7a}{4},\; c_2=\frac{7b}{4}$

$d_1=\frac{7c}{3},\; d_2=\frac{7d}{3}$

Area of parallelogram = $\left|\frac{(c_1-c_2)(d_1-d_2)}{(m_1-m_2)}\right|$

$=\left|\frac{\left(\frac{7a}{4}-\frac{7b}{4}\right)\left(\frac{7c}{3}-\frac{7d}{3}\right)}{\left(\frac{-3}{4}+\frac{4}{3}\right)}\right|$

$= 7|(a-b)(c-d)|$

Hence options (D)

10. **Ans. (B)**

$$\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix} = 0$$

$\Rightarrow \lambda = \cos 2\alpha + \sin 2\alpha$

$\Rightarrow \lambda \in \left[-\sqrt{2}, \sqrt{2}\right]$

11. **Ans. (D)**

$$\begin{vmatrix} a & am & 1 \\ b & mb+b & 1 \\ c & mc+2c & 1 \end{vmatrix} = 0$$

$C_2 \to C_2 - mC_1$

$$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ b & b & 1 \\ c & 2c & 1 \end{vmatrix} = 0$$

$a(b - 2c) + 1(2bc - bc) = 0$

$ab + bc = 2ac$

$\Rightarrow \dfrac{2}{b} = \dfrac{1}{a} + \dfrac{1}{c}$

12. **Ans. (D)**

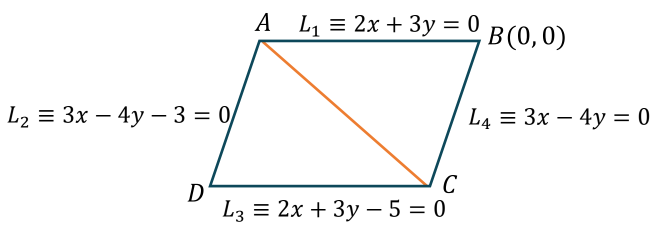
We need to find equation of AC.

Diagonal belongs to two families of lines

$L_4 + \lambda L_3 = 0$ and $L_2 + \mu L_1 = 0$

$\Rightarrow (3 + 2\lambda)x + y(3\lambda - 4) - 5\lambda = 0$ & $(3 + 2\mu)x + y(3\mu - 4) = 3$

$\Rightarrow \dfrac{3+2\lambda}{3+2\mu} = \dfrac{3\lambda-4}{3\mu-4} = \dfrac{-5\lambda}{-3} \Rightarrow \lambda = \dfrac{3}{5}$



Equation of AC :

$\dfrac{21}{5}x - \dfrac{11y}{5} - \dfrac{15}{5} = 0$

$\Rightarrow 21x - 11y - 15 = 0$

13. **Ans. (B)**

$\because \Delta = 0$, It represent pair of straight lines

Let $x^2 + 2xy + y^2 - 3x - 3y + 2 = (x + y + k_1)(x + y + k_2)$

On comparing coeff.

$k_1 + k_2 = -3$ and $k_1 k_2 = 2$

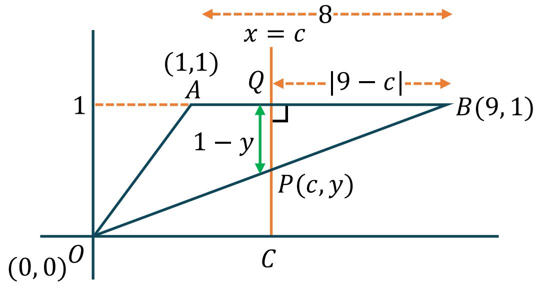
On solving $k_1 = -1$; $k_2 = -2$ or $k_1 = -2$; $k_1 = -1$

$\Rightarrow k_1 \neq k_2$

Hence, lines are parallel – (B)

14. Ans. (B)

$\because$ O, P, B are collinear



$\therefore m_{OP} = m_{OB} \Rightarrow \frac{y}{c} = \frac{1}{9} \Rightarrow \boxed{y = \frac{c}{9}}$

Area of $\Delta OAB = \frac{1}{2} \cdot 1 \cdot 8 = 4$ sq. unit

$\therefore$ Area of $\Delta BPQ = \frac{1}{2}$ Area of $\Delta OAB = 2$ sq. units

$\Rightarrow \left|\frac{1}{2} \cdot PQ \cdot BQ\right| = 2 \Rightarrow \frac{1}{2}(1 - y)(9 - c) = \pm 2$

$\Rightarrow \frac{1}{2}\left(1 - \frac{c}{9}\right)(9 - c) = \pm 2$

$\Rightarrow c = 3$ or 15 (reject) ; $\{\because 0 < c < 9\}$

$\Rightarrow c = 3$

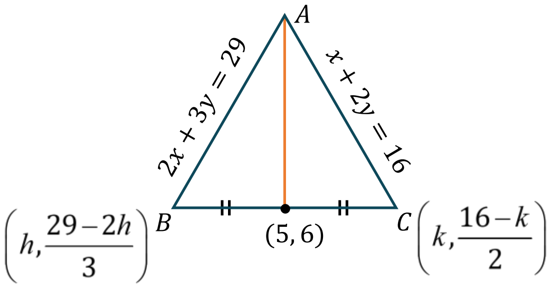
15. Ans. (B)

$y = mx + b$ cuts x-axis at $\left(-\frac{b}{m}, 0\right)$

Since $-\frac{b}{m} < 0$; (2009, 0) won't lie on the line.

16. Ans. (C)

Let $B \equiv \left(h, \frac{29-2h}{3}\right)$ & $C \equiv \left(k, \frac{16-k}{2}\right)$



$h + k = 10$ & $\frac{29-2h}{3} + \frac{16-k}{2} = 12$

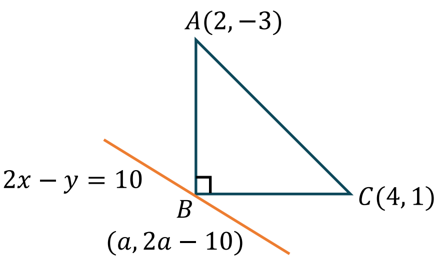
$\Rightarrow h = 4$ & $k = 6$

$B \equiv (4, 7)$ & $C \equiv (6, 5)$

Equation of $BC \Rightarrow y - 5 = -1(x - 6)$

$\Rightarrow x + y = 11$

17. Ans. (B)



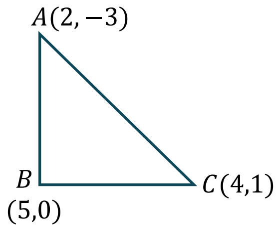
$\because m_{AB} \times m_{BC} = -1$

$\Rightarrow \left(\frac{2a-10+3}{a-2}\right)\left(\frac{2a-10-1}{a-4}\right) = -1$

$\Rightarrow a = 5$ or $\frac{19}{5}$ (reject)

$\therefore$ Area of $\Delta ABC = \frac{1}{2}.AB.AC$

$= \frac{1}{2}\cdot\sqrt{18}\cdot\sqrt{2} = 3$ sq. unit

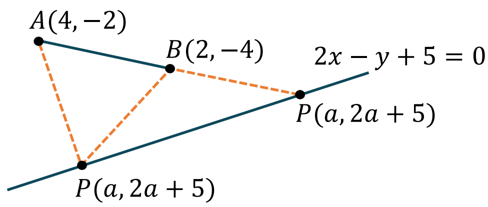


18. Ans. (B)

$\because$ Point $A(4, -2)$ & $B(2, -4)$ lie same side of the given line $2x - y + 5 = 0$

Let point 'P' lie on the line $2x - y + 5 = 0$

$\therefore P(a, 2a + 5)$



For ΔPAB,

$|PA - PB| < AB$

{In a triangle, difference b/w two sides always less than the third side}

So, $|PA - PB|_{\min} = AB$

{But in this case all three points A, B, P must be collinear}

$\therefore m_{AP} = m_{AB}$

$\Rightarrow \frac{2a+5+2}{a-4} = \frac{-4+2}{2-4}$

$\Rightarrow a = -11$

So, point 'P' is $P(-11, -17)$

19. Ans. (A)

$(k^2 + 1)(x - 2) + k(y + 2x) = 0$

Point is $(2, -4)$

$y + 4 = 2(x - 2) \Rightarrow y = 2x - 8$

20. Ans. (B)

$5x^2 + 12xy - 6y^2 + 4x(x + ky) - 2y(x + ky) + 3(x + ky)^2 = 0$

$\Rightarrow x^2(12) + xy(12 + 4k - 2 + 6k) + y^2(-6 - 2k + 3k^2) = 0$

$\Rightarrow 12x^2 + (10k + 10)xy + (3k^2 - 2k - 6)y^2 = 0$

For equally inclined lines,

$10k + 10 = 0 \Rightarrow k = -1$

21. Ans. (A)

$$\left|\sqrt{x^2+(y-4)^2} - \sqrt{x^2+(y+4)^2}\right| = 6$$

$$2x^2+2y^2+32+2\sqrt{\left(x^2+(y-4)^2\right)\left(x^2+(y+4)^2\right)} = 36$$

$\Rightarrow x^4 + x^2[2y^2 + 32] + (y^2 - 16)^2 = [2 - x^2 - y^2]^2$

$\Rightarrow x^4 + y^4 + 2x^2y^2 + 32x^2 - 32y^2 + 256$

$= 4 + x^4 + y^4 + 2x^2y^2 - 4x^2 - 4y^2$

$36x^2 - 28y^2 + 252 = 0$

$9x^2 - 7y^2 + 63 = 0$

22. Ans. (A)

$x_1 + y_1 = -8, x_2 + y_2 = -8, x_3 + y_3 = -8$

Now Area of Δ formed by points

$(x_1, y_1), (x_2, y_2), (x_3, y_3)$

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x_1+y_1 & y_1 & 1 \\ x_2+y_2 & y_2 & 1 \\ x_3+y_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}(-8)\begin{vmatrix} 1 & y_1 & 1 \\ 1 & y_2 & 1 \\ 1 & y_3 & 1 \end{vmatrix} = 0$$

$\therefore$ Points are collinear $\Rightarrow$ hence option (A)

23. Ans. (B)

Let slope of initial line is m.

$y - 4 = m(x - 3)$

$y = 4 - 3m$

Line reflected off y-axis is

$y - (4 - 3m) = -m(x) \Rightarrow x = \frac{4-3m}{m}$

Line reflected off x-axis is $y - 2 = m(x - 8)$

$(x, 0)$ lies on this line

$\Rightarrow -2 = m\left(\frac{4-3m}{m} - 8\right)$

$\Rightarrow m = \frac{6}{11} \Rightarrow x = \frac{13}{3}$

24. Ans. (B)

$\cos\theta\,(x + y - 9) + \sin\theta(x - y - 3) = 0$

$\Rightarrow M \equiv (6, 3)$

Reflection of M in $y = x \equiv (3, 6)$

25. Ans. (B)

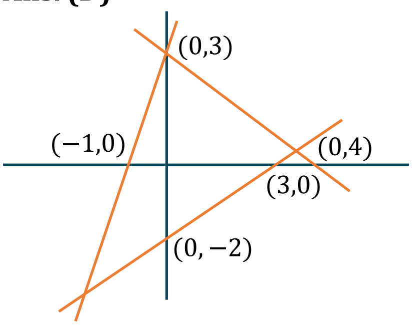
$m + m^2 = 6, m^3 = a$

$\Rightarrow m^3 + m^6 + 3m^3(m + m^2) = 216$

$\Rightarrow a^2 + a + 3a(6) = 216 \Rightarrow a^2 + 19a - 216 = 0$

sum = –19

26. Ans. (D)



$\alpha \in [-1, 3]$

$\beta \in [-2, 3]$

27. **Ans. (A,B)**

* $x^2 - 3|x| + 2 = 0$
* $y^2 - 3y + 2 = 0$

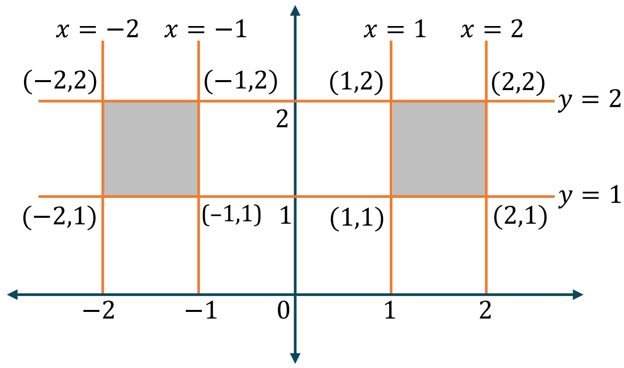
$\Rightarrow |x|^2 - 3|x| + 2 = 0 \Rightarrow y = 1$ or 2

$\Rightarrow |x| = 1$ or 2

$\Rightarrow x = \pm 1$ or ± 2

so, possible verties of square(s)

* $\boxed{(1,1),(2,1),(2,2),(1,2)}$ option (A)
* $\boxed{(-1,1),(-2,1),(-2,2),(-1,2)}$ option (B)



28. **Ans. (B,C)**

Altitude from $B \to x = 0$

Altitude from $A \to y = \frac{a}{b}(x + a')$

Solving, $\left(0, \frac{aa'}{b}\right)$

Also, as they are concyclic,

$aa' = bb'$

$\Rightarrow \left(0, \frac{aa'}{b}\right) \equiv (0, b')$

29. **Ans. (C,D)**

$a^2 - 6ab + 9b^2 = 4c^2$

$\Rightarrow (a - 3b)^2 = (2c)^2$

$\Rightarrow a - 3b - 2c = 0$ or $a - 3b + 2c = 0$

$\Rightarrow \left(-\frac{1}{2}, \frac{3}{2}\right)$ & $\left(\frac{1}{2}, -\frac{3}{2}\right)$

30. **Ans. (B,C,D)**

$(3x^2 + 10xy + 8y^2 + 14x + 22y + 15)$

$\equiv (3x + 4y + c_1)(x + 2y + c_2)$

$\Rightarrow c_1 + 3c_2 = 14$ & $2c_1 + 4c_2 = 22$

$\Rightarrow c_1 = 5, c_2 = 3$

$\Rightarrow m_1 + m_2 = -\frac{3}{4} - \frac{1}{2} = -\frac{5}{4}$

$m_1 m_2 = \left(-\frac{3}{4}\right)\left(-\frac{1}{2}\right) = \frac{3}{8}$

$\tan\theta = \left|\dfrac{-\frac{3}{4} + \frac{1}{2}}{1 + \frac{3}{8}}\right| = \frac{2}{11}$

$\sin\theta = \frac{2}{5\sqrt{5}}$

$P \equiv (1, -2) \Rightarrow 1 + (-2) = -1$

31. **Ans. (B,D)**

$m + m^2 = 6$ $\quad\quad$ $m^3 = a$

$(m + m^2)^3 = 216$

$m^3 + m^6 + 3m^3(m + m^2) = 216$

$a + a^2 + 18a = 216$

$a^2 + 19a - 216 = 0$

$(a + 27)(a - 8) = 0$

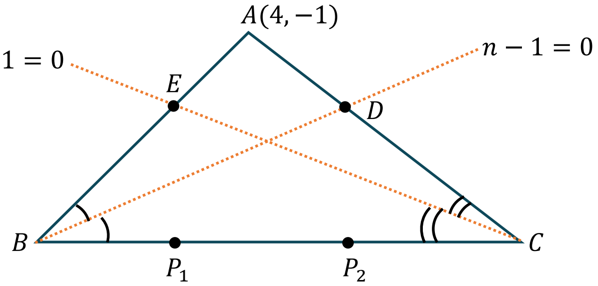
$a = 8$ $\quad\quad$ $(\because a > 0)$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} = \frac{2\sqrt{9-8\times 1}}{8+1} = \frac{2}{9}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

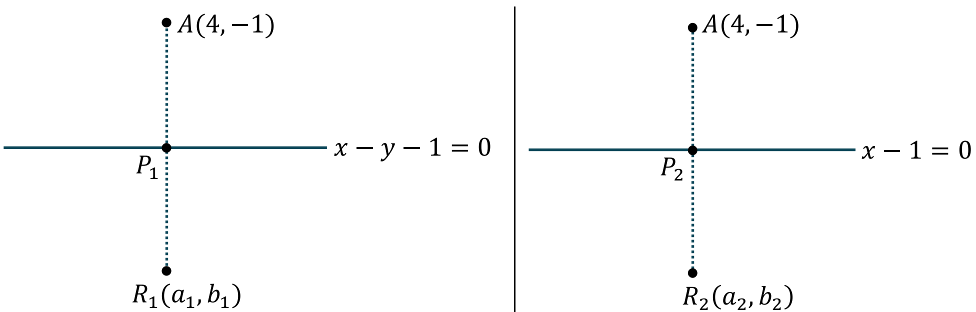
$$= \frac{2\times\frac{2}{9}}{1-\frac{4}{81}} = \frac{36}{77}$$

32. **Ans. (A,C,D)**



$\because$ BD is the interval $\angle$ bisector ofᵣ $\angle ABC$

$\therefore$ image of point A through line BD must Lie on side BC.



$P_1 = (0, 3)$ $\quad\quad$ $P_2 = (-2, -1)$

$\therefore$ equation of BC

$$y - 3 = (x - 0)\left(\frac{3+1}{0+2}\right)$$

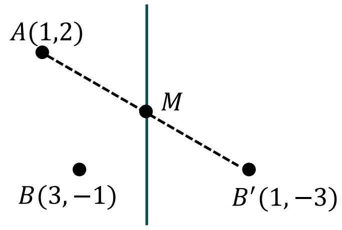
$\therefore$ $2x - y + 3 = 0$

$B = (1, 5)$ $\quad$ $C = (-4, -5)$

equation of line AB $\quad\quad$ equation of line

$2x + y - 7 = 0$ $x - 2y - 6 = 0$

33. **Ans.(B)**



B' is reflection of B in $x+y=0$

$AB' \equiv x = 1$

$M \equiv (1,-1)$

Reflection of M in $x = y \equiv (-1,1)$

34. **Ans.(D)**

In ΔABM,

$AB + BM > AM \Rightarrow AB > |AM - BM|$

$\Rightarrow$ Maximum value of $|AM - BM| = AB$

$\Rightarrow M$ is point of intersection of lines AB & $x + y = 0$

$\Rightarrow M \equiv (7,-7) \Rightarrow MN = 10$

35. **Ans. (A)**

$|AM - BM|$ is minimum when $AM = BM$.

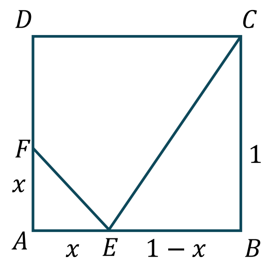
Let $M \equiv (h,-h)$

$(h-1)^2 + (h+2)^2 = (h-3)^2 + (h-1)^2$

$\Rightarrow h = \frac{1}{2} \Rightarrow M \equiv \left(\frac{1}{2}, -\frac{1}{2}\right)$

Area of triangle AMB $= \left|\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{vmatrix}\right| = \frac{13}{4}$

36. **Ans. (C)**



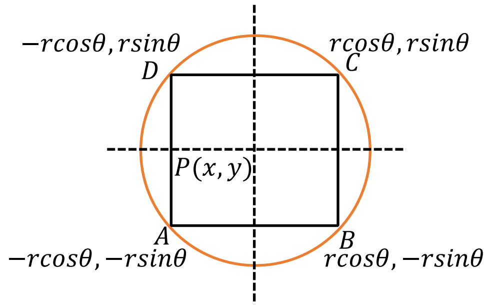
$Ar(CDFE) = 1 - \left(\frac{x^2}{2}\right) - \left(\frac{(1-x)}{2}\right)$

$= \frac{2 - x^2 - 1 + x}{2} = \frac{1 + x - x^2}{2} = \frac{1 + 1/4 - (x - 1/2)^2}{2}$

Maximum at $x = \frac{1}{2}$

Max. $Ar(CDFE) = \frac{5}{8}$

37. **Ans. (A,B,C)**



$(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2$

$(x + r\cos\theta)^2 + (y + r\cos\theta)^2 - (x - r\cos\theta)^2$

$-(y + r\sin\theta)^2 + (x - r\cos\theta)^2 - (y + r\sin\theta)^2$

$-(x + r\cos\theta)^2 - (y - r\sin\theta)^2 = 0$

38. Ans. (C)

(A) Triangle formed by coordinate axes and line $3x + 4y = 2$ will have vertices (0, 0), (4, 0) and (0, 3)
Therefore coordinates of the incentre will be given by
$$\left(\frac{5.0+4.0+3.4}{5+4+3}, \frac{5.0+4.3+3.0}{5+4+3}\right) \equiv (a, b)$$
$\therefore a = 1, b = 1 \Rightarrow a + b = 2$

(B) Perpendicular distance from origin to the line $2x - 3y = 6$ is $p = \frac{6}{\sqrt{13}}$.

So, equation of line in new position is
$2x - 3y = \lambda$

Perpendicular distance from (0, 0) to this line is $\left|\frac{\lambda}{\sqrt{13}}\right| = \frac{6}{\sqrt{13}} - 1 \Rightarrow |\lambda| = 6 - \sqrt{13}$

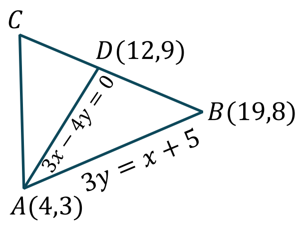
So equation of the line is $2x - 3y = 6 - \sqrt{13}$
$\Rightarrow k = -1$ or $|k| = 1$

(C) Let the point be $(a, a - 3)$
$$\Rightarrow \left|\frac{4a - 3(a-3) - 12}{5}\right| = 1 \Rightarrow \left|\frac{a-3}{5}\right| = 1$$
$\therefore A = 8, -2 \Rightarrow$ their sum = 6.

(D) Put $x = 2$ in $x + 2y = 4$ and $x + 2y = 5$, then
$y = 1$ and $y = \frac{3}{2}$
$\therefore \quad 1 < a < \frac{3}{2} \Rightarrow$ Integral value of $a = 1$

39. Ans. (A→R; B→P; C→Q; D→T)

(A) If B is (h, k)



$$\frac{h-13}{3} = \frac{k-16}{-4} = \frac{-2(39-64)}{25}$$
$$\frac{h-13}{3} = \frac{k-16}{-4} = +2$$
$\Rightarrow h = 19, k = 8 \Rightarrow$ line AB is $3y = x + 5$

(B) $\because$ Point (13, 16) lies on side AC
$$\Rightarrow \quad m_{AC} = \frac{16-3}{13-4} = \frac{13}{9}$$

(C) Other bisector is
$4x + 3y = \lambda$
passes through (4,3) $\Rightarrow \lambda = 25$
x-intercept = $\frac{25}{4}$

(D) Line AC is $(y-3) = \frac{13}{9}(x-4)$

& line BD is $(y - 8) = -\frac{1}{7}(x-19)$

$\Rightarrow \quad$ Solve to get vertex $C\left(\frac{17}{2}, \frac{19}{2}\right)$

EXERCISE - S

1. Ans. (5)

If P lies on angle bisector of $\angle BOA$ or $\angle BAD$

then it equidistant from sides OB and OA or AB and OA. Point P must lie in shaded region required region is shaded region.

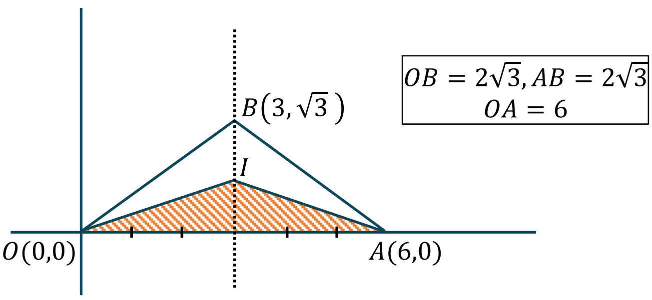
I is incentre

$$\left(\frac{6(3)+3\sqrt{2}(0)+2\sqrt{3}(6)}{6+2\sqrt{3}+2\sqrt{3}}, \frac{6(\sqrt{3})+0+0}{6+2\sqrt{3}+2\sqrt{3}}\right)$$

$$I\left(\frac{6(3+2\sqrt{3})}{6+4\sqrt{3}}, \frac{6\sqrt{3}}{6+4\sqrt{3}}\right)$$

$$=\frac{1}{2}\times 6\times\frac{6\sqrt{3}}{6+4\sqrt{3}}=\frac{9\sqrt{3}}{3+2\sqrt{3}}\times\frac{3-2\sqrt{3}}{3-2\sqrt{3}}$$

$$=\frac{9\sqrt{3}(3-2\sqrt{3})}{-3}=\frac{9\times 3(\sqrt{3}-2)}{-3}=9(2-\sqrt{3})$$



$a = 2$ and $b = 3$

2. Ans. (3)

$24^2 + 18^2 = 30^2$

$\Rightarrow$ Right angle Δ

Circumcentre: Mid point of hypotenuse

0 (9, 12)

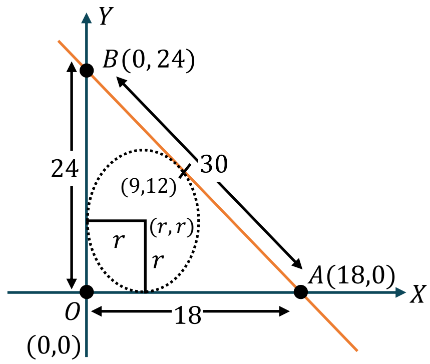
In centre :-

$$r = \frac{\Delta}{S} = \frac{\frac{1}{2}(18)(24)}{\frac{18+30+24}{2}} \quad \boxed{r=6}$$

then $I(6, 6)$ **Ans**

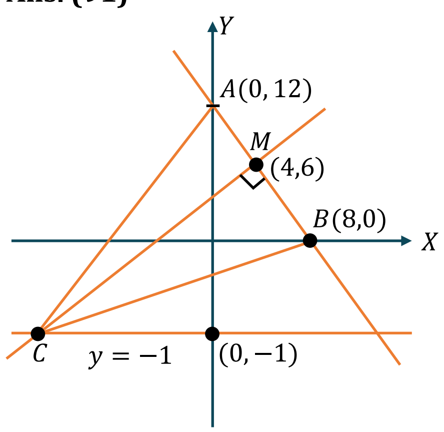
centroid :- $\left(\frac{0+18+0}{3}, \frac{0+0+24}{3}\right)$

$G(6, 8)$



$$\text{Now the area of } \Delta = \frac{1}{2}\begin{vmatrix} 9 & 12 & 1 \\ 6 & 6 & 1 \\ 6 & 8 & 1 \end{vmatrix} = 3$$

3. Ans. (91)



The perpendicular bisector of AB is $2x - 3y + 10 = 0$

Point C is intersection point of $2x - 3y + 10 = 0$

and $y = -1$, which is $\left(-\frac{13}{2}, -1\right)$

Now Area of triangle ABC where $A(0, 12)$ $B(8, 0)$ and $C\left(-\frac{13}{2}, -1\right)$ = 91 sq. unit

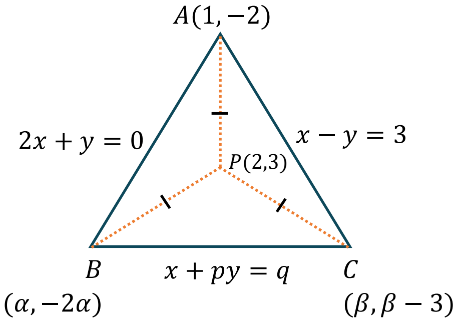
4. **Ans. (47)**

$PA^2 = 26 = PB^2 = PC^2$

$\Rightarrow (\alpha - 2)^2 + (3 + 2\alpha)^2 = (\beta - 2)^2 + (\beta - 6)^2 = 26$

$\Rightarrow \alpha = \frac{-13}{5}$ or 1

$\alpha = 1$ (rejected because vertices A and C coincide)



Hence $\alpha = \frac{-13}{5}$ and $\beta = 7$

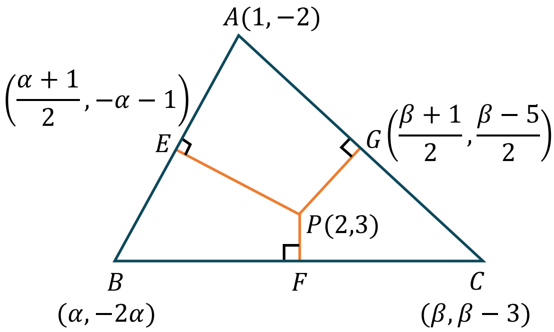
$B = \left(\frac{-13}{5}, \frac{26}{5}\right)$, $C = (7, 4)$

The equation of BC is $x + 8y = 39$

$\Rightarrow (p + q) = 47$ **Ans**

Alternatively:

Slope of line $EP = \frac{1}{2}$



$\frac{1}{2} = \frac{3 + \alpha + 1}{2 - \frac{\alpha + 1}{2}}$; $\frac{1}{2} = 2\left(\frac{4 + \alpha}{3 - \alpha}\right)$

$3 - \alpha = 16 + 4\alpha \Rightarrow 5\alpha = -13 \Rightarrow \alpha = \frac{-13}{5}$

Slope of line $PG = -1$

$$-1 = \frac{\frac{\beta - 5}{2} - 3}{\frac{\beta + 1}{2} - 2} = \frac{\beta - 11}{\beta - 3};$$

$-\beta + 3 = \beta - 11$; $2\beta = 14$; $\beta = 7$

$B = \left(\frac{-13}{5}, \frac{26}{5}\right)$, $C = (7, 4)$

The equation of BC is $x + 8y = 39$

$\Rightarrow (p + q) = 47$

5. **Ans. (6)**

$y^2 - 4y + 3 = 0$

$\Rightarrow (y - 3)(y - 1) = 0 \Rightarrow y = 3, y = 1$

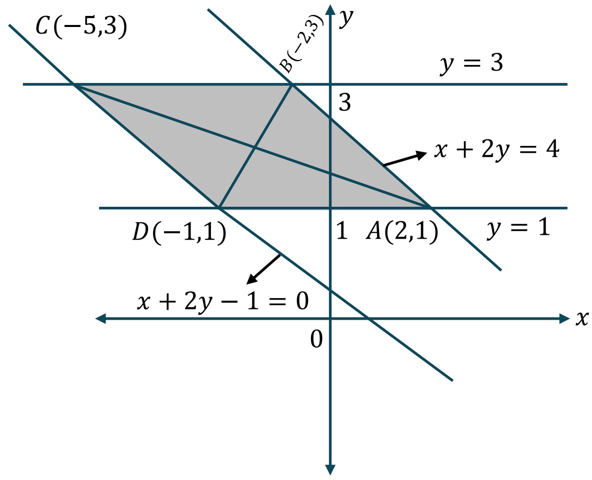
$x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$

$\Rightarrow x + 2y - 4 = 0$ & $x + 2y - 1 = 0$

Now

$x^2 + 4xy + 4y^2 - 5x - 10y + 4$

$= (x + 2y + c_1)(x + 2y + c_2) = x^2 + 4xy + 4y^2 + x(c_1 + c_2) + y\,(2c_2 + 2c_1) + c_1c_2$



compare $\Rightarrow c_1 + c_2 = -5$

$c_1c_2 = 4 \Rightarrow c_2 = \dfrac{4}{c_1}$

$c_1 + \dfrac{4}{c_1} = -5 \Rightarrow c_1^2 + 5c_1 + 4 = 0$

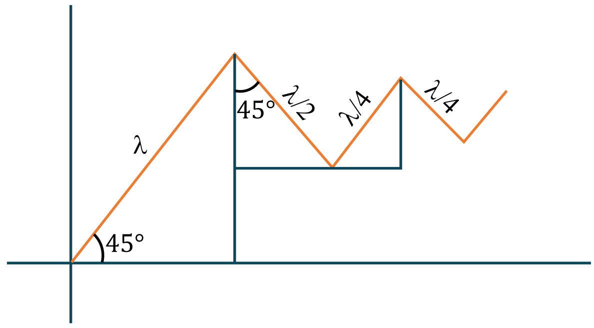
$(c_1 + 4)(c_1 + 1) = 0$

$c_1 = -4 \quad c_1 = -1$

$c_2 = -1 \quad c_2 = -4$

Area of polygon = 3.2 = 6 **Ans.**

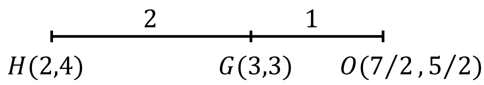
6. Ans. (8)



$$\alpha = \frac{\lambda}{\sqrt{2}} + \frac{\lambda}{2\sqrt{2}} + \frac{\lambda}{4\sqrt{2}} + \frac{\lambda}{8\sqrt{2}} + \ldots$$

$$= \frac{\frac{\lambda}{\sqrt{2}}}{1 - \frac{1}{2}} = \lambda\sqrt{2} = 6$$

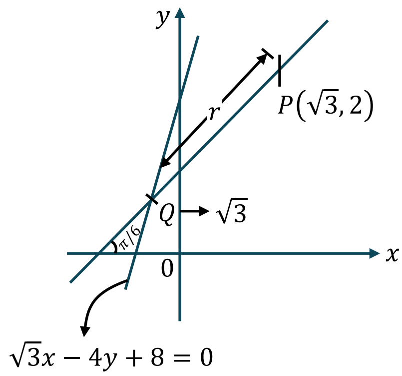
$$\beta = \frac{\lambda}{\sqrt{2}} - \frac{\lambda}{2\sqrt{2}} + \frac{\lambda}{4\sqrt{2}} - \frac{\lambda}{8\sqrt{2}} \ldots\ldots = \frac{\lambda/\sqrt{2}}{1 + \frac{1}{2}} = \frac{\lambda\sqrt{2}}{3}$$



$\alpha = 6, \beta = 2$

7. Ans. (6)

point Q is on $\sqrt{3}x - 4y + 8 = 0$ so it will satisfy this equation



$$\sqrt{3}\left(\sqrt{3} - \frac{\sqrt{3}r}{2}\right) - 4\left(2 - \frac{r}{2}\right) + 8 = 0$$

$$3 - \frac{3}{2}r - 8 + 2r + 8 = 0$$

$$r\left(2 - \frac{3}{2}\right) = -3$$

$r = -6$

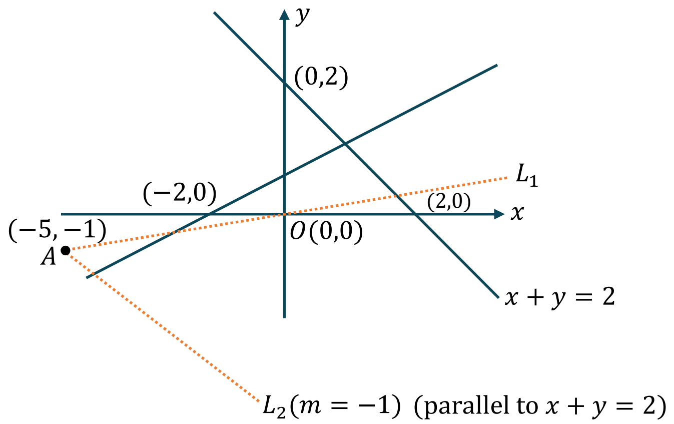
So, $PQ = 6$ **Ans.**

8. Ans. (24)

Equation of pair of straight line is.

$x^2 - 3y^2 - 2xy + 8y - 4 = 0$

$\Rightarrow x^2 + (-2y)x + (-3y^2 + 8y - 4) = 0$



$x = \dfrac{2y \pm \sqrt{(-2y)^2 - 4(-3y^2 + 8y - 4)}}{2}$

$= \dfrac{2y \pm \sqrt{4y^2 + 12y^2 - 32y + 16}}{2}$

$= \dfrac{2y \pm \sqrt{16y^2 - 32y + 16}}{2}$

$= \dfrac{2y \pm 4\sqrt{y^2 - 2y + 1}}{2} = y \pm 2\sqrt{(y-1)^2}$

$\Rightarrow x = y \pm 2(y-1) \Rightarrow x = y + 2(y-1)$ or $x = y - 2(y-1)$

$\Rightarrow x = 3y - 2$ or $x = -y + 2$

Let the equation of straight line passes through point $(-5, -1)$ & of slope m is

$y + 1 = m(x + 5)$

$\Rightarrow y + 1 = mx + 5m$

Lines passes through point A such that origin is interior part of Δ. We have to form a triangle using sides $x + y = 2, x = 3y - 2$ & $y + 1 = mx + 5x$ such that origin is the interior. It is possible only when line passes through point $(-5, -1)$ lies between line L_1 & L_2

Slope of line $L_1 = \dfrac{-1-0}{-5-0} = \dfrac{1}{5}$

Slope of line $L_2 = -1$

So, slope lies between $(-1, 1/5)$

Therefore $a = -1$ and $b = 1/5$

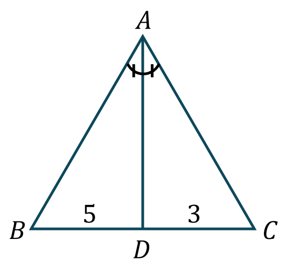
$\therefore a + \dfrac{1}{b^2} = -1 + 5^2$

$= -1 + 25 = 24$

9. Ans. (89)

$AB = \sqrt{7^2 + 24^2} = 25$

$AC = \sqrt{9^2 + 12^2} = 15$



AD is internal angle bisector

$\Rightarrow \dfrac{BD}{CD} = \dfrac{AB}{AC} \Rightarrow \dfrac{BD}{CD} = \dfrac{25}{15} = \dfrac{5}{3}$

$D \equiv \left(\dfrac{5-45}{8}, \dfrac{-35-57}{8}\right) \equiv \left(-5, \dfrac{-23}{2}\right)$

$m_{AD} = \dfrac{\frac{-23}{2} - 5}{-5+8} = -\dfrac{33}{2\times 3} = -\dfrac{11}{2}$

Equation of AD:

$y - 5 = \dfrac{-11}{2}(x + 8) \Rightarrow 11x + 2y + 78 = 0$

So, $a = 11$ & $c = 78$

$\therefore a + c = 89$

10. Ans. (2)

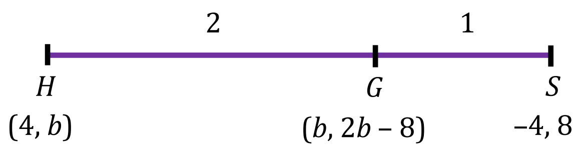
Fixed point is arithmetic mean of coordinates of vertices

$$\Rightarrow p = \frac{-2+0+4+3+2-1}{6} = 1$$

$$\Rightarrow q = \frac{-1-2+0+2+3+4}{6} = 1$$

$\therefore p + q = 2$

11. Ans. (0)



As H, G and S are collinear

$$\therefore \begin{vmatrix} 4 & b & 1 \\ b & 2b-8 & 1 \\ -4 & 8 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 4 & b & 1 \\ b-4 & b-8 & 0 \\ -(b+4) & 16-2b & 0 \end{vmatrix} = 0$$

$$\Rightarrow (b-4)(16-2b)+(b+4)(b-8)=0 \Rightarrow 2(b-4)(8-b)+(b+4)(b-8)=0$$

$$\Rightarrow (8-b)[2b-8-(b+4)]=0 \Rightarrow (8-b)(b-12)=0$$

Also

$$\therefore \frac{-8+4}{3} = b \Rightarrow b = \frac{-4}{3}$$

And $\frac{16+b}{3} = 2b-8 \Rightarrow b = 8$

But no common value of 'b' is possible.

12. Ans. (7)

Using section formula $A\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$

Area of triangle ABC is 2 sq. units

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 1 & 5 & 1 \\ 7 & -2 & 1 \\ \frac{3k-5}{k+1} & \frac{5k+1}{k+1} & 1 \end{vmatrix} = \pm 2$$

Operating $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 5 & 1 \\ 6 & -7 & 0 \\ \frac{3k-5}{k+1} & \frac{5k+1}{k+1}-5 & 0 \end{vmatrix} = \pm 4$$

$$\Rightarrow 6\left(\frac{5k+1-5k-5}{k+1}\right)+7\left(\frac{3k-5-k-1}{k+1}\right)=\pm 4$$

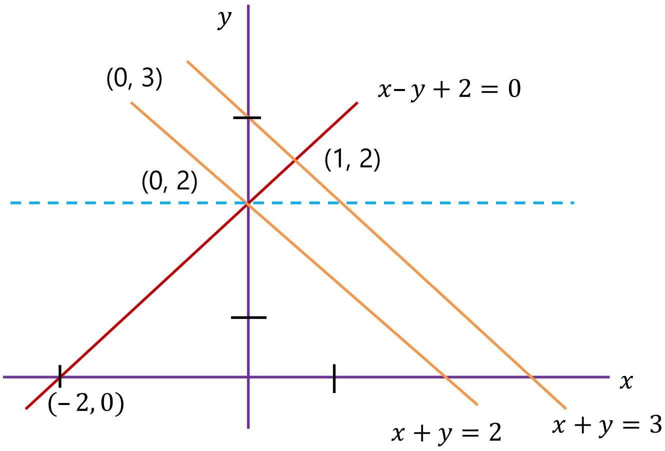
$$\Rightarrow -24+7(2k-6)=\pm 4(k+1) \Rightarrow k=7 \text{ or } k=\frac{31}{9}$$

13. **Ans. (8)**

Given pair of lines $x^2-(y^2-4y+4)=0$

$\Rightarrow x^2-(y-2)^2=0$

$\Rightarrow (x+y-2)(x-y+2)=0$



Required area is $A=\frac{1.1}{2}=\frac{1}{2}$

14. **Ans. (2)**

Lines $(2a+b)x+(a+3b)y+(b-3a)=0$ or $a(2x+y-3)+b(x+3y+1)=0$ are concurrent at point of intersection of lines $2x+y-3=0$ and $x+3y+1=0$ which is $(2,-1)$.

Now line $mx+2y+6=0$ must pass through this point

$\Rightarrow 2m-2+6=0$ or $m=-2$.

15. **Ans. (5)**

P is orthocentre

$\Rightarrow AP \perp BC$

$$\Rightarrow \left(-\frac{1}{p}\right)\left(\frac{3+2}{2-1}\right)=-1$$

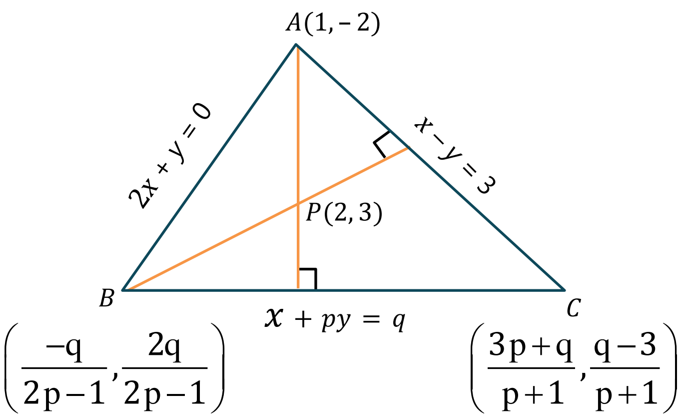
$$\Rightarrow \frac{5}{p} \Rightarrow p=5$$

$\because BP \perp AC$

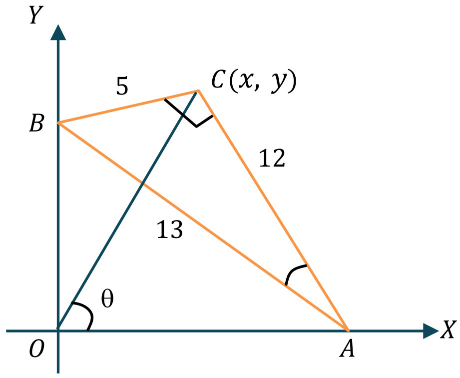
$$\Rightarrow \frac{27-2q}{18+q}=-1 \quad \Rightarrow q=27+18$$

$\Rightarrow q=45$

$\therefore\ p+q=5+45=50$



16. Ans. (5)



Since $\angle BCA = 90°$

Points A, O, B, C are concyclic

Let $\angle AOC = \theta$

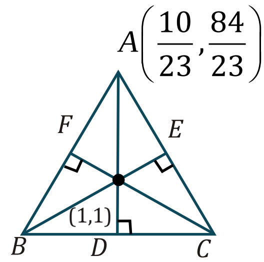
$\angle BOC = \angle BAC$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{5}{12}$$

$$\frac{x}{y} = \frac{5}{12} \Rightarrow 12x - 5y = 0$$

EXERCISE - JEE (Main) PYQ

1. Ans. (4)



Consider equation of AB is $3x - 2y + 6 = 0$ ...(i)

and equation of AC is $4x + 5y - 20 = 0$ ...(ii)

$$m_{BE} = -\frac{1}{m_{AC}} = \frac{5}{4}$$

Equation of BE : $5x - 4y - 1 - 0$ ...(iii)

Solve (i) & (iii) to get point B

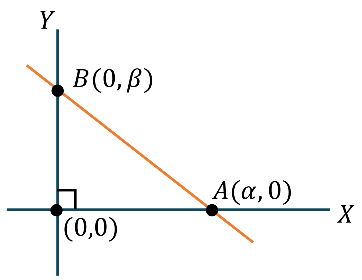
$$B \equiv \left(-13, -\frac{33}{2}\right)$$

$$\because m_{AD} = -\frac{61}{13} \Rightarrow m_{BC} == -\frac{1}{m_{AD}} = \frac{13}{61}$$

Equation of BC : $y + \frac{33}{2} = (x + 13)\frac{13}{61}$

$\Rightarrow$ $\boxed{26x - 122y - 1675 = 0}$

2. Ans. (4)



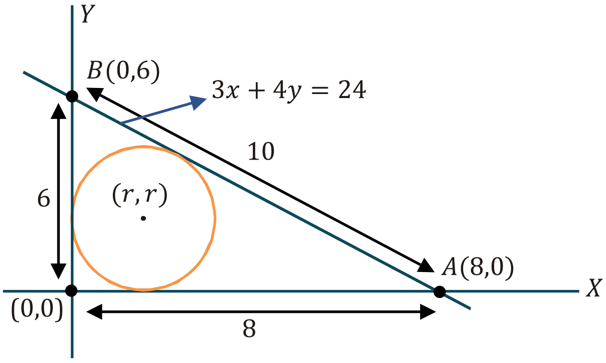
Let $A(\alpha,0)$ and $B(0,\beta)$ be the vertices of the given triangle AOB

$\Rightarrow |\alpha\beta| = 100$

$\Rightarrow$ Number of triangles = 4 × (number of divisors of 100)

= 4 × 9 = 36

3. Ans. (2)



$$I(r,r)=\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)=\left(\frac{6(8)+8(0)+10(0)}{6+8+10},\frac{6(0)+8(6)+10(0)}{6+8+10}\right)$$

$$=\left(\frac{48}{24},\frac{48}{14}\right)=(2,2)$$

4. Ans. (2)

$$m_{BD}\times m_{AD}=-1\Rightarrow\left(\frac{3-2}{4-0}\right)\times\left(\frac{b-0}{a-0}\right)=-1$$

$\Rightarrow b+4a=0$ ...(i)

$$m_{AB}\times m_{CF}=-1\Rightarrow\left(\frac{(b-2)}{a-0}\right)\times\left(\frac{3}{4}\right)=-1$$

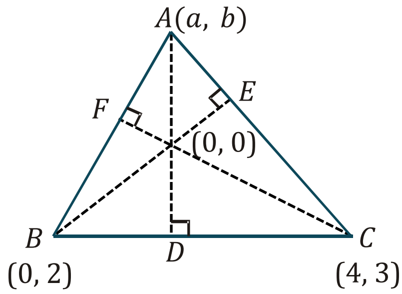
$\Rightarrow 3b-6=-4a\Rightarrow 4a+3b=6$ ...(ii)

From (i) and (ii)

$a=\frac{-3}{4}, b=3$

$\therefore$ IInd quadrant.

Option (2)



5. Ans. (4)

Solving $x+y=3$ and $x-y=-3$ are $A(0,3)$

$\frac{x_1+0}{2}=2;\ x_1=4$ similarly $y_1=5$

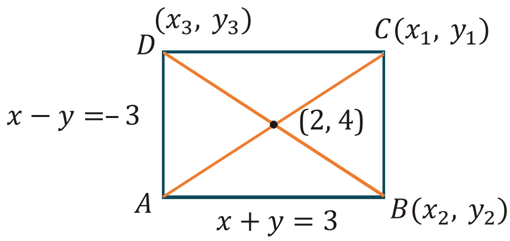
$\Rightarrow C\equiv(4,5)$

Now equation of BC is $x-y=-1$

and equation of CD is $x+y=9$

Solving $x+y=9$ and $x-y=-3$

Point D is $(3,6)$



6. Ans. (3)

Slope of $AB=\frac{-h}{k}$

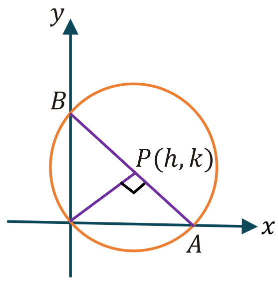
Equation of AB is $hx+ky=h^2+k^2$

$A\left(\frac{h^2+k^2}{h},0\right)$, $B\left(0,\frac{h^2+k^2}{k}\right)$

$AB=2R$

$\Rightarrow(h^2+k^2)^3=4R^2h^2k^2$

$\Rightarrow(x^2+y^2)^3=4R^2x^2y^2$



7. Ans. (1)

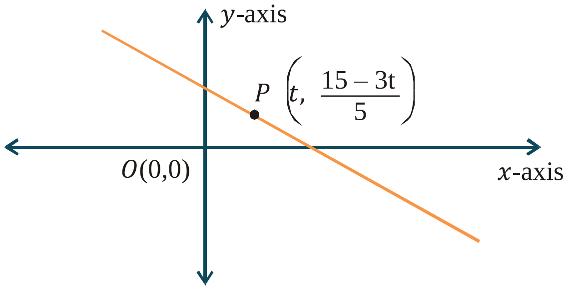
Now, $\left|\frac{15-3t}{5}\right| = |t|$

$\Rightarrow \frac{15-3t}{5} = t$ or $\frac{15-3t}{5} = -t$

$\therefore\ t = \frac{15}{8}$ or $t = \frac{-15}{2}$

So, $P\left(\frac{15}{8}, \frac{15}{8}\right) \in I^{st}$ quadrant

or $P\left(\frac{-15}{2}, \frac{15}{2}\right) \in II^{nd}$ quadrant



8. Ans. (3)

Equation of L_1 is $y = -\frac{1}{2}x + \frac{5}{2}$ ...(i)

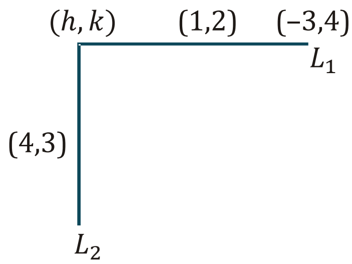
Equation of L_2 is $y = 2x - 5$ ...(ii)

By (i) and (ii)

$x = 3$ & $y = 1$

$\Rightarrow h = 3, k = 1$

$\Rightarrow \frac{k}{h} = \frac{1}{3}$



9. Ans. (3)

$x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

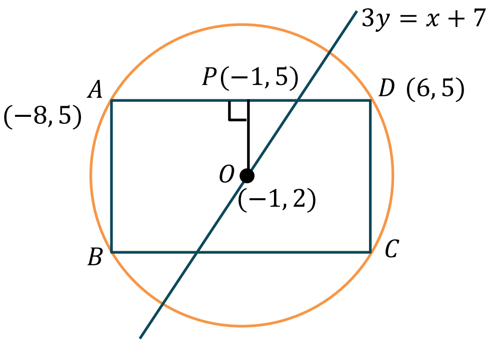
$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7 \Rightarrow r(\cos\theta + \sin\theta) = 2$

$\Rightarrow \sin\theta + \cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2} \Rightarrow 1 + \sin 2\theta = \frac{1}{4} \Rightarrow \sin 2\theta = -\frac{3}{4}$

$\Rightarrow \frac{2m}{1+m^2} = -\frac{3}{4} \Rightarrow 3m^2 + 8m + 3 = 0 \Rightarrow m = \frac{-4\pm\sqrt{7}}{3}$

$\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{\left(1-\sqrt{7}\right)^2}{1-7} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$

10. Ans. (2)



$AD = 14$

$P\left(\frac{-8+6}{2}, \frac{5+5}{2}\right) \Rightarrow P(-1,5)$

Since $OP \perp AD \Rightarrow O(-1, a)$

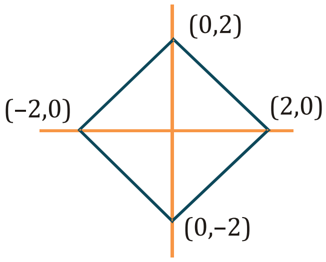
Lies on line $3y = x + 1 \Rightarrow a = 2$

$\therefore OP = 3 \Rightarrow AB = 6$

Area $(A) = 6 \times 14$

$\Rightarrow A = 84$

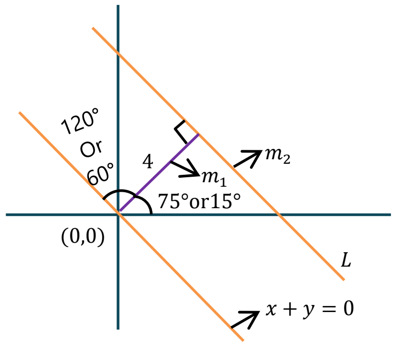
11. **Ans. (1)**



$|x-y| \leq 2$ and $|x+y| \leq 2$

Square whose side is $2\sqrt{2}$

12. **Ans. (1) or (2)**



$m_1 = \tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

or $m = \tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$m_2 = \frac{-1}{m_1} = \frac{-\left(\sqrt{3}-1\right)}{\sqrt{3}+1}$

or $m_2 = \frac{-1}{m_1} = \frac{-\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$

$\Rightarrow y = m_2 x + C$

$\Rightarrow y = \frac{-\left(\sqrt{3}-1\right)x}{\sqrt{3}+1} + C \Rightarrow L$ or $y = \frac{-\left(\sqrt{3}+1\right)x}{\sqrt{3}-1} + C \Rightarrow L$

Distance from origin = 4

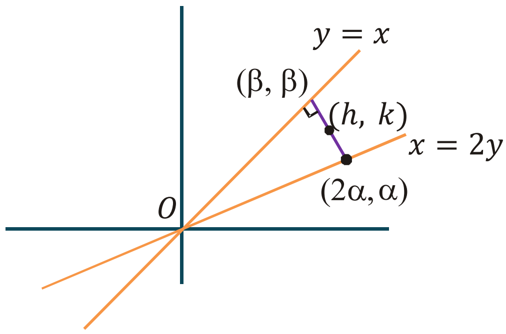
$\therefore \left|\frac{C}{\sqrt{1+\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)^2}}}\right| = 4$ or $\left|\frac{C}{\sqrt{1+\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)^2}}}\right| = 4$

$\Rightarrow C = \frac{8\sqrt{2}}{\left(\sqrt{3}+1\right)}$ or $C = \frac{8\sqrt{2}}{\left(\sqrt{3}-1\right)}$

$\Rightarrow \left(\sqrt{3}-1\right)y + \left(\sqrt{3}+1\right)x = 8\sqrt{2}$

or $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

13. Ans. (3)



$$\frac{\alpha-\beta}{2\alpha-\beta}=-1$$

$3\alpha = 2\beta$

$$h = \frac{2\alpha+\beta}{2}$$

$$2h = \frac{7\alpha}{2}$$

$$k = \frac{\alpha+\beta}{2}$$

$$2k = \frac{5\alpha}{2}$$

$$\frac{h}{k}=\frac{7}{5}$$

$5x = 7y$

14. Ans. (5)

P is centroid of the triangle ABC

$$\Rightarrow P \equiv \left(\frac{17}{6},\frac{8}{3}\right)$$

$\Rightarrow PQ = 5$

15. Ans. (3)

$\overleftrightarrow{AB} : 3x + y - 2 = 0$

Also, $\frac{1}{2}\times\sqrt{10}\times h = 5$

$\Rightarrow h = \sqrt{10}$

$$\Rightarrow \frac{|4\lambda-2|}{\sqrt{10}}=\sqrt{10} \Rightarrow \lambda = 3,-2$$

16. Ans. (2)

Centroid of Δ = (2, 2)

line passing through intersection of

$x + 3y - 1 = 0$ and

$3x - y + 1 = 0$, be given by

$(x + 3y - 1) + \lambda(3x - y + 1) = 0$

$\because$ It passes through (2, 2)

$$\Rightarrow 7 + 5\lambda = 0 \Rightarrow \lambda = -\frac{7}{5}$$

$\therefore$ Required line is $8x - 11y + 6 = 0$

$\because$ (–9, –6) satisfies this equation.

17. Ans. (4)

Given that both points (1, 2) & ($\sin\theta$, $\cos\theta$) lie on same side of the line $x + y - 1 = 0$

So, (Put (1,2) in given line) (Put ($\sin\theta, \cos\theta$) in given line) > 0

$\Rightarrow (1 + 2 - 1)(\sin\theta + \cos\theta - 1) > 0$

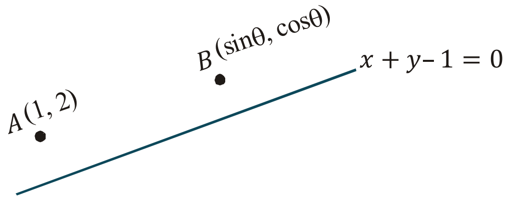
$\Rightarrow \sin\theta + \cos\theta > 1 \left\{\div by \sqrt{2}\right\}$

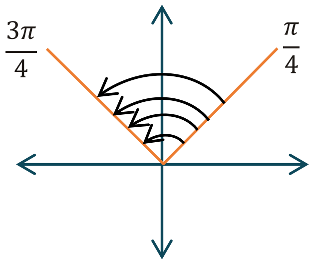
$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{3\pi}{4}$$

$$\Rightarrow \boxed{0 < \theta < \frac{\pi}{2}}$$





18. Ans. (4)

Slope = $m = \frac{1}{1-k}$

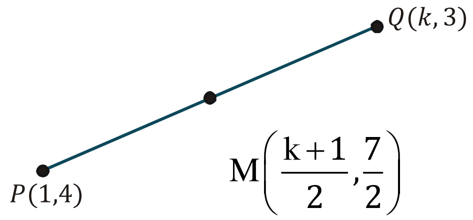
Equation of $\perp^r$ bisector is

$y + 4 = (k - 1)(x - 0)$

$\Rightarrow y + 4 = x(k - 1)$

$$\Rightarrow \frac{7}{2} + 4 = \frac{k+1}{2}(k-1)$$

$$\Rightarrow \frac{15}{2} = \frac{k^2-1}{2} \Rightarrow k^2 = 16 \Rightarrow k = 4, -4$$



19. Ans. (30)

Apply distance between parallel line formula

$4x - 2y + \alpha = 0$

$4x - 2y + 6 = 0$

$$\left|\frac{\alpha - 6}{2\sqrt{5}}\right| = \frac{1}{\sqrt{5}}$$

$|\alpha - 6| = 2 \Rightarrow \alpha = 8, 4$

sum = 12

again

$6x - 3y + \beta = 0$

$6x - 3y + 9 = 0$

$$\left|\frac{\beta - 9}{3\sqrt{5}}\right| = \frac{2}{\sqrt{5}}$$

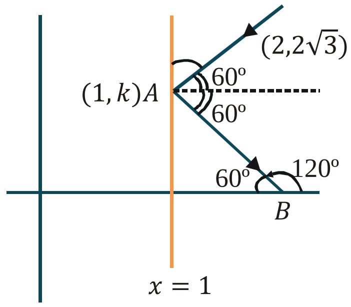
$|\beta - 9| = 6 \Rightarrow \beta = 15, 3$

sum = 18

sum of all values of α and β is = 30

20. Ans. (2)

For point A



$$\tan 60° = \frac{2\sqrt{3}-k}{2-1}$$

$$\sqrt{3} = 2\sqrt{3} - k$$

$$\therefore\ k = \sqrt{3}$$

so point $A\left(1, \sqrt{3}\right)$

Now slope of line AB is m_{AB} = tan120º

$$m_{AB} = -\sqrt{3}$$

Now equation of line AB is

$$y - \sqrt{3} = -\sqrt{3}(x-1)$$

$$\sqrt{3}x + y = 2\sqrt{3}$$

Now satisfy options

21. Ans. (3)

$$\frac{x^2-y^2}{1-(-5)} = \frac{xy}{-2}$$

$$\frac{x^2-y^2}{6} = \frac{xy}{-2}$$

$$\Rightarrow x^2 - y^2 = -3xy$$

$$\Rightarrow x^2 + 3xy - y^2 = 0$$

22. Ans. (1)

First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$ ...(i)

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$ ...(ii)

Hence $4p^2 + q^2 = k^2$ (From (i) & (ii))

23. Ans. (3)

$$\frac{1}{2}\begin{vmatrix} \alpha & 0 & 1 \\ 1 & \alpha & 1 \\ 0 & \alpha & 1 \end{vmatrix} = \pm 4$$

$\alpha = \pm 8$

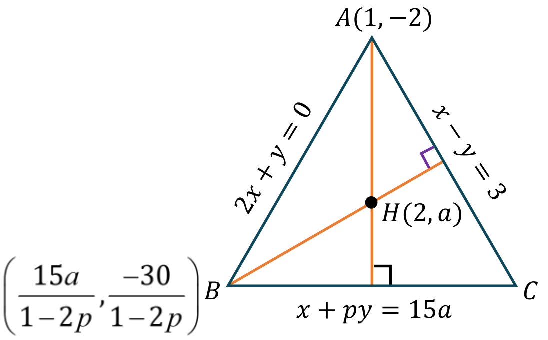
Now given points (8, –8), (–8, 8), (64, β)

OR (–8, 8), (8, –8), (64, β)

are collinear $\Rightarrow$ Slope = –1.

$\boxed{\beta = -64}$ Ans. (C)

24. **Ans. (3)**



Coordinates of $A(1, -2)$, $B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and orthocentre $H(2, a)$

Slope of $AH = p$

$a + 2 = p$ ...(1)

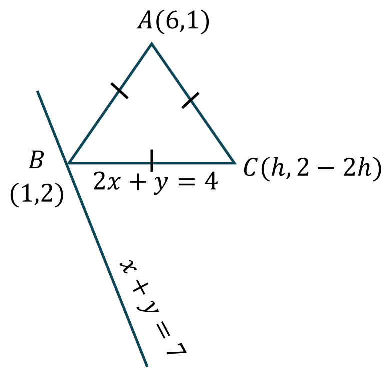
Slope of $BH = -1$

$31a - 2ap = 15a + 4p - 2$ ...(2)

From (1) and (2)

$a = 1$ & $p = 3$

25. **Ans. (3)**



Point $B(1, 2)$

Now let C be $(h, 4 - 2h)$

(As C lies on $2x + y = 4$)

$\because \Delta$ is isosceles with base BC

$\therefore AB = AC$

$\sqrt{25+1} = \sqrt{(6-h)^2 + (2h-3)^2}$

$\sqrt{26} = \sqrt{36 + h^2 - 12h + 4h^2 + 9 - 12h}$

$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$

$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$

$h = \frac{19}{5}$ or $h = 1$

Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

Centroid $\left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right)$

$\left(\frac{35+19}{15}, \frac{15-18}{15}\right)$

$\left(\frac{54}{15}, \frac{-3}{15}\right)$

$\alpha = \frac{54}{15}; \beta = \frac{-3}{15}$

$15(\alpha + \beta) = 51$

26. Ans. (4)

Perpendicular bisector of AB

$x + y = 5$

Take image of A

$$\frac{x-1}{1}=\frac{y+2}{1}=\frac{-2(-6)}{2}=6$$

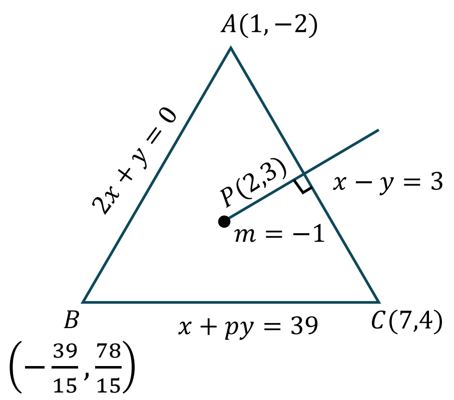
$(7, 4)$

$7 + 4p = 39$

$p = 8$

solving $x + 8y = 39$ and $y = -2x$

$x = \frac{-39}{15}$ $y = \frac{78}{15}$



$AC^2 = 72 = 9p$

$AC^2 + p^2 = 72 + 64 = 136$

$$\Delta ABC=\frac{1}{2}\begin{vmatrix}1 & -2 & 1\\ 7 & 4 & 1\\ \frac{-39}{15} & \frac{78}{15} & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[4-\frac{78}{15}+2\left(7+\frac{39}{15}\right)+7\left(\frac{78}{15}\right)+\frac{4\times 39}{15}\right]=\frac{1}{2}\left[18+18\times\frac{13}{5}\right]$$

$$=9\left[\frac{18}{5}\right]=\frac{162}{5}=32.4$$

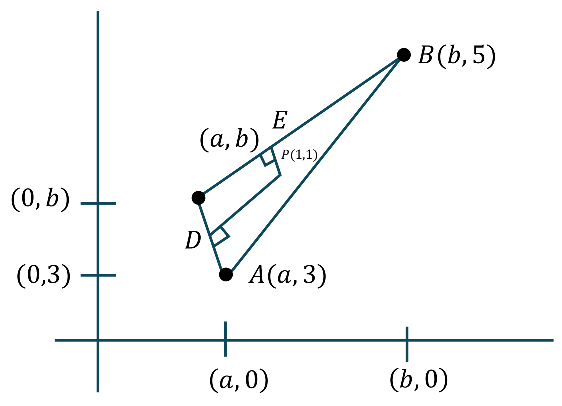
27. Ans. (2)

$m_{AC} \longrightarrow \infty$

$m_{PD} = 0$

$$D\left(\frac{a+a}{2},\frac{b+3}{2}\right)$$

$$D\left(a,\frac{b+3}{2}\right)$$



$m_{PD} = 0$

$$\frac{b+3}{2}-1=0$$

$b + 3 - 2 = 0$

$\boxed{b=-1}$

$$E\left(\frac{b+a}{2},\frac{5+b}{2}\right)=\left(\frac{af}{2},2\right)$$

$m_{CB} . m_{EP} = -1$

$$\left(\frac{5-b}{b-a}\right)=\left(\frac{2-1}{\frac{a-1}{2}-1}\right)=-1$$

$$\left(\frac{6}{-1-a}\right)=\left(\frac{2}{a-3}\right)=-1$$

$12 = (1 + a)\ (a - 3)$

$12 = a^2 - 3a + a - 3$

$\Rightarrow a^2 - 2a - 15 = 0$

$(a - 5)\ (a + 3) = 0$

$a = 5\ or\ a = -3$

Given $ab > 0$

$a(-1) > 0$

$-a > 0$

$a < 0$

$\boxed{a=-3}$ Accept

AP line A (−3, 3) $P(1, 1)$

$y - 1 = \left(\frac{3-1}{-3-1}\right)(x-1)\ \ -2y + 2 = x - 1$

$\Rightarrow \boxed{x+2y=3}$ Appling .....(1)

Line BC $B(-1, 5)$

$C(-3, -1)$

$(y-5)=\frac{6}{2}(x+1)$

$y - 5 = 3x + 3$

$\boxed{y=3x+8}$ .....(2)

Solving (1) & (2)

$x + 2(3x + 8) = 3$

$\Rightarrow 7x + 16 = 3$

$7x = -13$

$x=-\frac{13}{7}$

$y=3\left(-\frac{13}{7}\right)+8\ =\frac{-39+56}{7}$

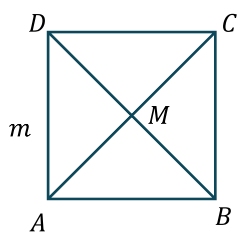
$y=\frac{17}{7}$

$x+y=\frac{-13+17}{7}=\frac{4}{7}$

28. **Ans. (2)**

$m_1 m_2 = -1$

$a^2 + 11a + 3\left(m_1^2+\frac{1}{m_1^2}\right)=220$



Eq. of AC

AC = (cosα – sinα) + (sinα + cosα) y = 10

BD = (sinα – cosα) x + (sinα – cosα) y = 0

(10 (cosα – sinα), 10 (sinα – cosα))

Slope of $AC = \left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)=\tan\theta=M$

Eq. of line making an angle π_4 with AC

$$m_1, m_2 = \frac{m\pm\tan\frac{\pi}{4}}{1\pm m\tan\frac{\pi}{4}} = \frac{m+1}{1-m} or \frac{m-1}{1+m}$$

$$\frac{\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}+1}{1-\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)}, \frac{\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}-1}{1+\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}}$$

$m_1, m_2 = \tan\alpha, \cot\alpha$

mid point of AC & BD

$= M(5(\cos\alpha-\sin\alpha), 5(\cos\alpha+\sin\alpha))$

$B(10(\cos\alpha-\sin\alpha), 10(\cos\alpha+\sin\alpha))$

$a = AB = \sqrt{2}\,BM = \sqrt{2}\left(5\sqrt{2}\right) = 10$

$a = 10$

$\because a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1 2}\right) = 220$

$100 + 110 + 3(\tan^2\alpha + \cot^2\alpha) = 220$

Hence $\tan^2\alpha = 3$, $\tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3} or \frac{\pi}{6}$

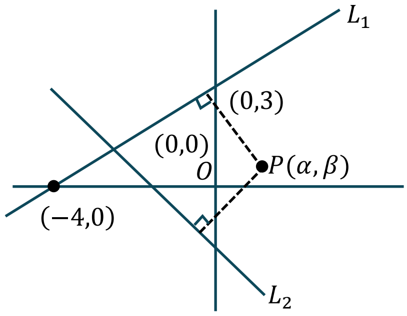
Now $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$

$= 72\left(\frac{9}{16}+\frac{1}{16}\right)+100-30+13$

$= 72\left(\frac{5}{8}\right)+83 = 45+83 = 128$

29. Ans. (4)

By observing origin and P lies in same region.



$L_1(0,0) > 0$; $L_1(\alpha,\beta) > 0 \Rightarrow 3\alpha - 4\beta + 12 > 0$

$1 = \left|\frac{3\alpha-4\beta+12}{5}\right|$

$3\alpha - 4\beta + 12 = 5$ ....(1)

Similarly for L_2

$L_2(0,0) > 0$; $L_2(\alpha,\beta) > 0$

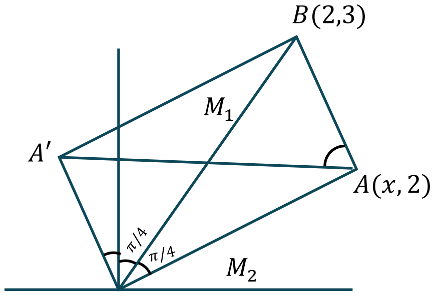
$1 = \left|\frac{8\alpha+6\beta+11}{10}\right| \Rightarrow 8\alpha+6\beta+11 = 10$ .....(2)

Solving (1) and (2)

$\alpha = -\frac{23}{25}; \beta = \frac{106}{100}$

$100(\alpha+\beta) = 100\left(\frac{-92}{100}+\frac{106}{100}\right) = 14$

30. Ans. (3)



$M_1 = 3/2 \qquad\qquad M_2 = 2/x$

$$\tan \pi/4 = \left|\frac{3/2 - 2/x}{1 + 6/2x}\right| = 1$$

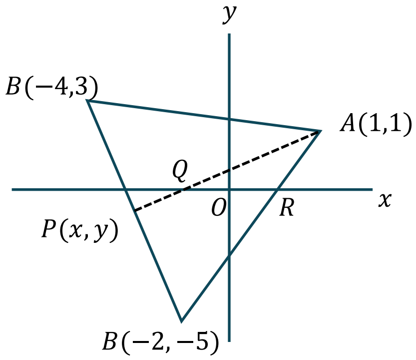
$\Rightarrow x_1 = 10, \quad x_2 = -2/5$

$\Rightarrow AA^1 = 52/5$

31. Ans. (3)

Given $\Delta_1 = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix}$ & $\Delta_2 = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -4 & 3 & 1 \\ -2 & -5 & 1 \end{vmatrix}$

Given $\frac{\Delta_1}{\Delta_2} = \frac{4}{7} \Rightarrow \frac{-2x - 5y + 7}{36} = \frac{4}{7}$



$\Rightarrow 14x + 35y = -95$ ...(1)

Equation of BC is $4x + y = -13$ ...(2)

Solve equation (1) & (2)

Point $P\left(\frac{-20}{7}, \frac{-11}{7}\right)$

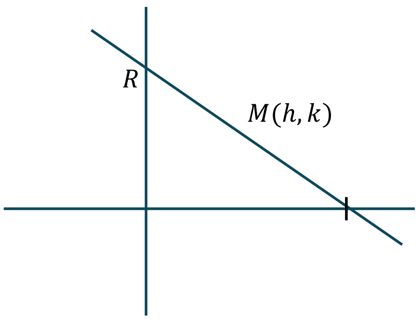
Here point $Q\left(\frac{-1}{2}, 0\right)$ & $R\left(\frac{1}{2}, 0\right)$

So Area of triangle $AQR = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

32. Ans. (1)

$pt(\alpha, \frac{7\sqrt{3}}{3})$

$x\cos\theta + y\sin\theta = 7$



x - intercept $= \frac{7}{\cos\theta}$

y - intercept $= \frac{7}{\sin\theta}$

$A : \left(\frac{7}{\cos\theta}, 0\right) \quad B : \left(0, \frac{7}{\sin\theta}\right)$

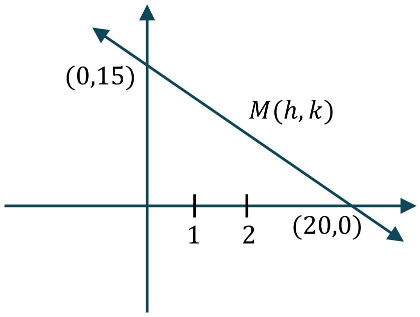
Locus of mid pt $M : (h, k)$

$h=\frac{7}{2\cos\theta}, k=\frac{7}{2\sin\theta}$

$\frac{7}{2\sin\theta}=\frac{7\sqrt{3}}{3} \Rightarrow \sin\theta=\frac{\sqrt{3}}{2} \Rightarrow \theta=\frac{\pi}{3}$

$\alpha=\frac{7}{2\cos\theta}=7$

33. Ans. (31)



If $x = 1, y = \frac{57}{4}=14.25$

(1, 1) (1, 2) – (1, 14) $\Rightarrow$ 14 pts.

If $x = 2,\ y=\frac{27}{2}=13.5$

(2, 2) (2, 4) .... (2, 12) $\Rightarrow$ 6 pts.

If $x = 3,\ y=\frac{51}{4}=12.75$

(3, 3) (3, 6) – (3, 12) $\Rightarrow$ 4 pts.

If $x = 4, y = 12$

(4, 4) (4, 8) $\Rightarrow$ 2 pts.

If $x = 5.\ y=\frac{45}{4}=11.25$

(5, 5), (5, 10) $\Rightarrow$ 2 pts.

If $x = 6,\ y=\frac{21}{2}=10.5$

(6, 6) $\Rightarrow$ 1 pt.

If $x = 7,\ y=\frac{39}{4}=9.75$

(7, 7) $\Rightarrow$ 1 pt.

If $x = 8, y = 9$

(8, 8) $\Rightarrow$ 1 pt.

If $x = 9,\ y=\frac{33}{4}=8.25 \Rightarrow$ no pt.

Total = 31 pts.

34. Ans. (2)

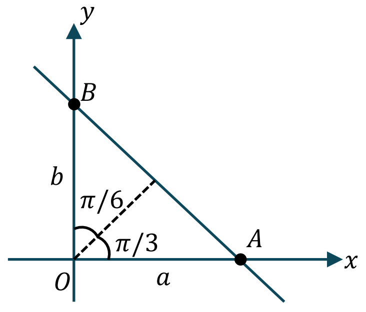
Slope of reflected ray = $\tan 60°=\sqrt{3}$

Line $y=\frac{x}{\sqrt{3}}$ intersect $y+x=1$ at $\left(\frac{\sqrt{3}}{\sqrt{3}+1},\frac{1}{\sqrt{3}+1}\right)$

Equation of reflected ray is $y-\frac{1}{\sqrt{3}+1}=\sqrt{3}\left(x-\frac{\sqrt{3}}{\sqrt{3}+1}\right)$

Put $y = 0 \Rightarrow x=\frac{2}{3+\sqrt{3}}$

35. Ans. (1)



Equation of straight line : $\frac{x}{a}+\frac{y}{b}=1$

Or $x\cos\frac{\pi}{3}+y\sin\frac{\pi}{3}=p$

$\frac{x}{2}+\frac{y\sqrt{3}}{2}=p$

$\frac{x}{3p}+\frac{y}{2p}=1$

Comparing both : $a=2p, b=\frac{2p}{\sqrt{3}}$

Now area of $\Delta OAB=\frac{1}{2}\cdot ab=\frac{98}{3}\cdot\sqrt{3}$

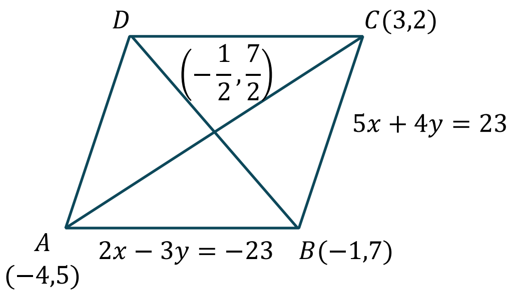
$\frac{1}{2}\cdot 2p\cdot\frac{2p}{\sqrt{3}}=\frac{98}{3}\cdot\sqrt{3}$

$p^2=49$

$a^2-b^2=4p^2-\frac{4p^2}{3}=\frac{2}{3}4p^2$

$=\frac{8}{3}\cdot 49=\frac{392}{3}$

36. Ans. (529)



A & C point will be (–4, 5) & (3, 2)

mid point of AC will be $\left(-\frac{1}{2},\frac{7}{2}\right)$

equation of diagonal BD is

$y-\frac{7}{2}=\frac{\frac{7}{2}}{-\frac{1}{2}}\left(x+\frac{1}{2}\right)$

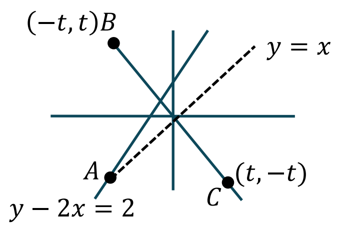
$\Rightarrow 7x+y=0$

Distance of A from diagonal BD

$=d=\frac{23}{\sqrt{50}}\Rightarrow 50d^2=(23)^2$

$50d^2=529$

37. Ans. (3)



At A $x=y$

$y-2x=2$

(–2, –2)

Height from line $x+y=0$

$h=\frac{4}{\sqrt{2}}$

Area of $\Delta=\frac{\sqrt{3}}{4}\;\frac{h^2}{\sin^2 60}=\frac{8}{\sqrt{3}}$

EXERCISE - JEE (Advanced) PYQ

1. Ans. (A→S; B→P,Q; C→R; D→ P,Q,S)

$x+3y-5=0,\ 3x-xy-1=0,\ 5x+2y-12=0$

(A) For concurrency

$$\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix}=0$$

$\Rightarrow (12k+2)-3(-36+5)-5(6+5k)=0$

$\therefore k=5$

(B) For parallel

either $-\frac{1}{3}=\frac{3}{k}$ $\qquad\therefore k=-9$

or $-\frac{5}{2}=\frac{3}{k}$ $\qquad\therefore k=\frac{-6}{5}$

(C) They will form triangle

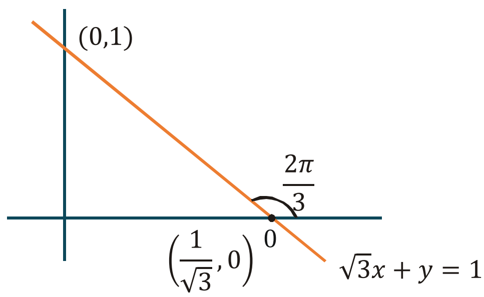
when $k\neq 5,-9,\frac{-6}{5}$

(D) They will not form triangle

when $k=5,-9,\frac{-6}{5}$

2. Ans. (B)

Line L has two possible slopes with inclination; $\theta=\frac{\pi}{3}$, $\theta=0$



$\therefore$ equation of line L when $\theta=\frac{\pi}{3}$, $y+2=\sqrt{3}(x-3)$

$\Rightarrow y-\sqrt{3}x+2+3\sqrt{3}=0$

equation of line L when $\theta=0$, $y=-2$ (rejected)

$\therefore$ required line L is $y-\sqrt{3}x+2+3\sqrt{3}=0$

3. Ans. (A,C)

Point of intersection of both lines is

$\left(-\frac{c}{(a+b)},-\frac{c}{(a+b)}\right)$

Distance between $\left(-\frac{c}{(a+b)},-\frac{c}{(a+b)}\right)$ & (1,1) is

Distance $=\sqrt{\frac{(a+b+c)^2}{(a+b)^2}\times 2}<2\sqrt{2}$

$(a+b+c)^2<2^2(a+b)^2$

$(a+b+c)^2-\left(2(a+b)\right)^2<0$

$(a+b+c+2(a+b))(a+b+c-2(a+b)) < 0$

$(3a+3b+c)(c-a-b) < 0$

a, b, c are positive

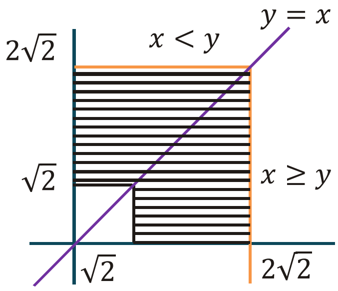
hence $c-a-b < 0$ or $a+b-c > 0$

According to given condition option (A,C) are correct.

4. **Ans. (6)**

Let $P(x, y)$ is the point in I quad.

Now $2 \le \left|\frac{x-y}{\sqrt{2}}\right| + \left|\frac{x+y}{\sqrt{2}}\right| \le 4$



$2\sqrt{2} \le |x-y| + |x+y| \le 4\sqrt{2}$

Case-I : $x \ge y$

$2\sqrt{2} \le (x-y) + (x+y) \le 4\sqrt{2}$

$\Rightarrow x \in [\sqrt{2}, 2\sqrt{2}]c$

Case-II : $x < y$

$2\sqrt{2} \le y - x + (x+y) \le 4\sqrt{2}$

$y \in [\sqrt{2}, 2\sqrt{2}]$

$A = (2\sqrt{2})^2 - (\sqrt{2})^2 = 6$

5. **Ans. (9.00)**

$P(x,y) \quad \left|\frac{\sqrt{2}x+y-1}{\sqrt{3}}\right|\left|\frac{\sqrt{2}x-y+1}{\sqrt{3}}\right| = \lambda^2$

$\left|\frac{2x^2-(y-1)^2}{3}\right| = \lambda^2$, $C: \left|2x^2-(y-1)^2\right| = 3\lambda^2$

line $y = 2x+1$, $RS = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$, $R(x_1, y_1)$ and $S(x_2, y_2)$

$y_1 = 2x_1 + 1$ and $y_2 = 2x_2 + 1 \Rightarrow (y_1 - y_2) = 2(x_1 - x_2)$

$RS = \sqrt{5(x_1-x_2)^2} = \sqrt{5}|x_1 - x_2|$

solve curve C and line $y = 2x + 1$ we get

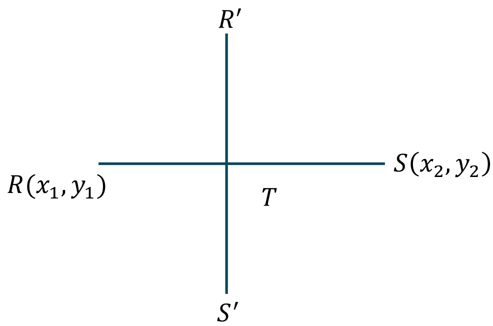
$\left|2x^2 - (2x)^2\right| = 3\lambda^2 \Rightarrow x^2 = \frac{3\lambda^2}{2}$

$RS = \sqrt{5}\left|\frac{2\sqrt{3}\lambda}{\sqrt{2}}\right| = \sqrt{30}\lambda = \sqrt{270} \Rightarrow 30\lambda^2 = 270 \Rightarrow \lambda^2 = 9$

6. **Ans. (77.14)**

$\perp$ bisector of RS

$T \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



Here $x_1 + x_2 = 0$

$T = (0,1)$

Equation of

$R'S' : (y-1) = -\frac{1}{2}(x-0) \Rightarrow x + 2y = 2$

$R'(a_1, b_1)S'(a_2, b_2)$

$D = (a_1 - a_2)^2 + (b_1 - b_2)^2 = 5(b_1 - b_2)^2$

solve $x + 2y = 2$ and $\left|2x^2 - (y-1)^2\right| = 3\lambda^2$

$$\left|8(y-1)^2 - (y-1)^2\right| = 3\lambda^2 \Rightarrow (y-1)^2 = \left(\frac{\sqrt{3}\lambda}{\sqrt{7}}\right)^2$$

$$y - 1 = \pm\frac{\sqrt{3}\lambda}{\sqrt{7}} \Rightarrow b_1 = 1 + \frac{\sqrt{3}\lambda}{\sqrt{7}}, b_2 = 1 - \frac{\sqrt{3}\lambda}{\sqrt{17}}$$

$$D = 5\left(\frac{2\sqrt{3}\lambda}{\sqrt{7}}\right)^2 = \frac{5\times4\times3\lambda^2}{7} = \frac{5\times4\times27}{7} = 77.14$$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (4)

Lines are ||

$$\frac{-p(p^2+1)}{-1} = \frac{-(p^2+1)^2}{p^2+1}$$

$p(p^2 + 1) = -(p^2 + 1)$

$p = -1$

2. Ans. (3)

Case–I

Let $a > 0$

$x + y = a$ and $ax - y = 1$

$\therefore x = 1$ $\qquad$ $y = a - 1$

$\therefore a - 1 > 0$

$\Rightarrow a > 1$

Case–II

Let $a < 0$

$x + y = -a$ and $-a - y = 1$

But line $x + y = -a$ does not passes through the Ist quadrant.

Case–III

$a = 0$

$x + y = 0$ $\qquad$ ...(I)

$y = -1$ $\qquad$ ...(II)

line (I) and line (II) do not pass through Ist quadrant.

Hence $a \in (1, \infty)$

3. Ans. (3)

Let equation of any tangent to $y^2 = 16\sqrt{3}\,x$

be $y = mx + \frac{4\sqrt{3}}{m}$ $\qquad$ ...(i)

and equation of any tangent to $2x^2 + y^2 = 4$

be $y = mx + \sqrt{2m^2+4}$ ...(ii)

but (i) and (ii) are same lines

$\therefore \ \frac{4\sqrt{3}}{m} = \sqrt{2m^2+4}$

$\Rightarrow m^4 + 2\,m^2 - 24 = 0$

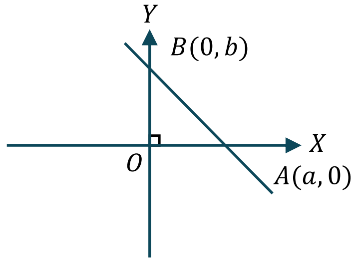
$\Rightarrow m^2 = -6, 4$

$\therefore m = \pm\, 2$

4. Ans. (4)

Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$

It passes through (1, 2)



$\therefore \ \frac{1}{a}+\frac{2}{b}=1 \Rightarrow b = \frac{2a}{a-1}$

Area of $\Delta = \frac{1}{2}ab \Rightarrow \Delta = \frac{a^2}{a-1}$

$\frac{d\Delta}{da} = \frac{a^2-2a}{(a-1)^2} = 0 \Rightarrow \frac{a(a-2)}{(a-1)^2} = 0 \ \Rightarrow a = 0, 2$

but at $a = 0$, Δ not possible

$\therefore a = 2$

slope of line $-\frac{b}{2} = 2.$

5. Ans. (3)

Put $y = -x$ [because equidistant points in forth quadrant lies on $y = -x$]

$4ax + 2ay + c = 0$

$\Rightarrow 4ax - 2ax + c = 0$

$\Rightarrow x = -\frac{c}{2a}$ ...(1)

$5bx + 2by + d = 0$

$\Rightarrow 5bx - 2bx + d = 0 \Rightarrow 3bx + d = 0$

$\Rightarrow x = -\frac{d}{3b}$ ...(2)

from (1) & (2)

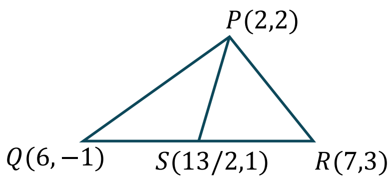
$-\frac{c}{2a} = -\frac{d}{3b}$

$\Rightarrow 3bc - 2ad = 0.$

6. Ans. (2)

Slope of PS

$M_{PS} = \frac{2-1}{2-\frac{13}{2}}$



$M_{PS} = -\frac{2}{9}$

Equation of the line passing through (1, – 1) and parallel to PS

$$y + 1 = -\frac{2}{9}(x - 1)$$

$\Rightarrow 9y + 9 = -2x + 2$

$\Rightarrow 2x + 9y + 7 = 0$

7. **Ans. (4)**

equation of $RQ \equiv x - 2y = 2$

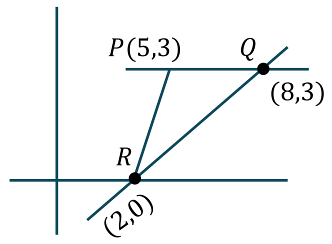
$\Rightarrow R(2, 0)$

equation of $PQ = y = 3$

point of intersection of PQ and RQ

$x - 2(3) = 2$

$x = 8 \Rightarrow R(8, 3)$



Centroid $\left(\frac{2+8+5}{3}, \frac{0+3+3}{3}\right)$

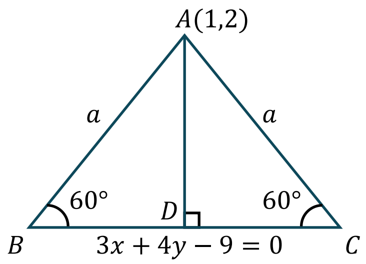
$\equiv (5, 2)$ as is simplified by $2x - 5y = 0$

8. **Ans. (1)**

Let BC is base of equilater triangle ABC with side a and A (1, 2)

$AD = a \sin 60°$

AD is perpendicular distance of PtA from line $3x + 4y - 9 = 0$



$$AD = \left|\frac{3\times1+4\times2-9}{\sqrt{3^2+4^2}}\right|$$

$$a \sin 60° = \frac{2}{5}$$

$$a = \frac{4}{5\sqrt{3}} = \frac{4\sqrt{3}}{15}.$$

9. **Ans. (2)**

$x + a(2y + 1) = 0$

$x + b(3y + 1) = 0$

$x + a(4y + 1) = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4a & a \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 2a & 0 \end{vmatrix} = 0$$

$\Rightarrow 2a(b - a) = 0$

$2a = 0$ or $b = a$

Locus of $(a, b) \Rightarrow x = 0$ or $y = x$

10. Ans. (2)

Equation = of line L is

$x + 5y + c = 0$

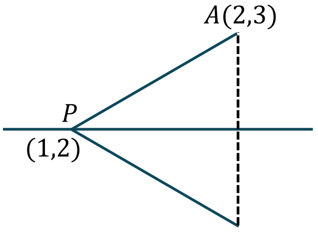
$\frac{c^2}{2|1\times 5|} = 5 \Rightarrow c = \pm 5\sqrt{2}$

$\therefore$ Eq of line L is

$x + 5y \pm 5\sqrt{2} = 0$

Its distance from $x + 5y = 0$ is $\frac{5}{\sqrt{13}}$.

11. Ans. (1)



P is the fixed point for given family of line

$PB = PA \Rightarrow (\alpha - 1)^2 + (\beta - 2)^2 = 1 + 1$

$(x-1)^2 + (y-2)^2 = \left(\sqrt{2}\right)^2$

$\therefore$ locus is a circle of radius $\sqrt{2}$.

12. Ans. (3)

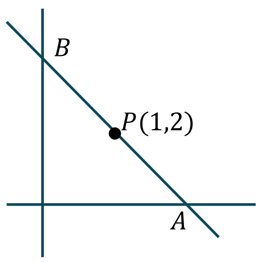
Equation of line $L \neq (y-2) = m(x-1)$

co-ordinates of $A = \left(\frac{m-2}{m}, 0\right)$

of $B = (0, 2-m)$

P is mid point of AB

hence $\frac{m-2}{2m} = 1$ $\qquad m = -2$



Hence line $L \equiv y + 2x = 4$

equation of line $L_1 \equiv 2y - x = k$

passes through $(-2, 1) \equiv k = 4$

$L_1 \equiv 2y - x = 4$

intersection of L & L_1

$x = \frac{4}{5}$ & $y = \frac{12}{5}$.

13. Ans. (2)

$\pm \tan 60° = \frac{-\sqrt{3} - m}{1 - \sqrt{3}m} \Rightarrow m = 0, \sqrt{3}$

$\Rightarrow$ required line is $y + 2 = \sqrt{3}(x-3)$

$y - \sqrt{3}x + \left(3\sqrt{3} + 2\right) = 0$

14. Ans. (4)

Equation of angle bisector of the lines $x - y + 1 = 0$ and $7x - y - 5 = 0$ is given by

$\frac{x-y+1}{\sqrt{2}} = \pm\frac{7x-y-5}{5\sqrt{2}} \Rightarrow 5(x - y + 1) = 7x - y - 5$

and $5(x - y + 1) = -7x + y + 5$

$\therefore\ 2x + 4y - 10 = 0 \Rightarrow x + 2y - 5 = 0$ and

$12x - 6y = 0 \Rightarrow 2x - y = 0$

Now equation of diagonals are

$(x + 1) + 2(y + 2) = 0 \Rightarrow x + 2y + 5 = 0$ ...(1) and

$2(x + 1) - (y + 2) = 0 \Rightarrow 2x - y = 0$ ...(2)

Clearly $\left(\frac{1}{3}, -\frac{8}{3}\right)$ lies on (1)

15. Ans. (2)

$L: x - y = 4$ $\quad\tan\theta = 1$ $\quad\theta = 45°$

Point which is at $2\sqrt{3}$ units from (2, 1)

in III Quad. along line $x - y = 4$ is given by

$\frac{x-2}{\cos 45°} = \frac{y-1}{\sin 45°} = -2\sqrt{3}$

$x = 2 - \frac{2\sqrt{3}}{\sqrt{2}}\ y = 1 - \frac{2\sqrt{3}}{\sqrt{2}}$

line $\perp$ to 'L' has slope -1

$y - \left(1 - \frac{2\sqrt{3}}{\sqrt{2}}\right) = -1\left(x - \left(2 - \frac{2\sqrt{3}}{2}\right)\right)$

$x + y = 3 - 2\sqrt{6}$.

16. Ans. (3)

$L_1: \frac{x}{a} + \frac{y}{b} = 1$ and $L_2: \frac{x}{b} + \frac{y}{a} = 1$

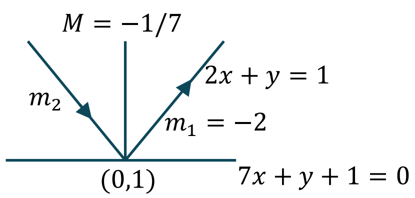
Then locus of mid point of line passing through intersection of L_1 and L_2 is

$2xy(a + b) = ab(x + y)$

$\therefore\ 2xy(7) = 12(x + y)$

$7xy = 6(x + y)$

17. Ans. (4)



$\frac{m_2 - m}{1 + mm_2} = \frac{m - m_1}{1 + mm_1}$

$\frac{m_2 + \frac{1}{7}}{1 - \frac{1}{7}m_2} = \frac{-\frac{1}{7} + 2}{1 + (-2)\left(-\frac{1}{7}\right)} = \frac{\frac{13}{7}}{1 + \frac{2}{7}}$

$\frac{7m_2 + 1}{7 - m_2} = \frac{\frac{13}{7}}{\frac{9}{7}} = \frac{13}{9}$

$7m_2 \times 9 + 9 = 13.7 - 13m_2$

$(63 + 13)m_2 = 91 - 9$

$76m_2 = 82$

$m_2 = \frac{41}{38}$

$y - 1 = \frac{41}{38}x$

$38y - 38 = 41x$

$41x - 38y + 38 = 0$

18. Ans. (1)

$y = mx$

$3y = 10 - 4x$

$(3m + 4)x = 10$

$A = \left(\frac{10}{3m+4}, \frac{10m}{3m+4}\right)$

$B = \left(\frac{-5}{6m+8}, \frac{-5m}{6m+8}\right)$

$$\frac{-\frac{5\lambda}{6m+8} + \frac{10}{3m+4}}{\lambda+1} = 0$$

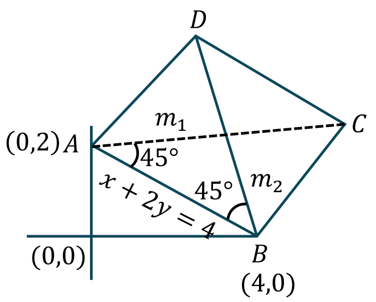
$15\lambda m - 20\lambda + 60m + 80 = 0$

$\frac{-5m\lambda}{6m+8} + \frac{10m}{3m+4}$

$\lambda : 1 \equiv 4 : 1.$

19. Ans. (3)

$\tan 45° = \left|\frac{m + \frac{1}{2}}{1 - \frac{m}{2}}\right| \Rightarrow \pm 1 = \frac{2m+1}{2-m} \Rightarrow m = \frac{1}{3}, -3$



$\therefore$ Equation of AC

$y - 2 = \frac{1}{3}(x) \Rightarrow x - 3y + 6 = 0$ ...(i)

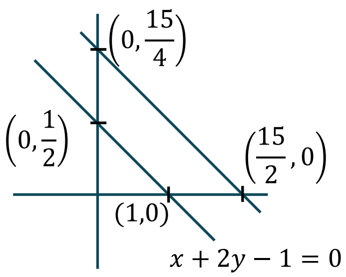
Equation of BD

$y = -3(x - 4) \Rightarrow 3x + y - 12 = 0$ ...(ii)

(i) & (ii) $x = 3$ & $y = 3$

20. Ans. (1)

Point $P\left(1 + \frac{t}{\sqrt{2}}, 2 + \frac{t}{\sqrt{2}}\right)$ lies between given line



Hence $L_1(P) = \left(1 + \frac{t}{\sqrt{2}}\right) + 2\left(2 + \frac{t}{\sqrt{2}}\right) - 1 = 0$

$5 + \frac{3t}{\sqrt{2}} - 1 = 0 \Rightarrow t = -\frac{4\sqrt{2}}{3}$

Now, $L_2(P) = 2\left(1+\frac{t}{\sqrt{2}}\right) + 4\left(2+\frac{t}{\sqrt{2}}\right) - 15 = 0 \Rightarrow 10 + \frac{6t}{\sqrt{2}} - 15 = 0 \Rightarrow t = \frac{5\sqrt{2}}{6}$

and $L_1(P) \times L_2(P) < 0$

Hence $t \in \left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}\right)$.

SECTION–B

1. **Ans. (2)**

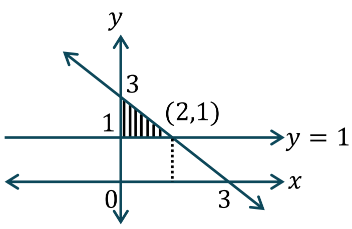
$x^2 - y^2 + 2y = 1 \Rightarrow x = \pm(y-1)$

Bisector of above lines are $x = 0, y = 1$

so Area between $x = 0, y = 1$ and $x + y = 3$

$x = 0, y = 1$

$x + y = 3 = \frac{1}{2} \times 2 \times 2 = 2$ squ. units



2 **Ans. (2)**

Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$

It passes through (1, 2)

$\therefore \frac{1}{a}+\frac{2}{b}=1 \Rightarrow b = \frac{2a}{a-1}$

Area of $\Delta = \frac{1}{2}ab \Rightarrow \Delta = \frac{a^2}{a-1}$

$\frac{d\Delta}{da}=\frac{a^2-2a}{(a-1)^2}=0$

$\Rightarrow \frac{a(a-2)}{(a-1)^2}=0 \Rightarrow$ a = 0, 2

but at $a = 0$, Δ not possible

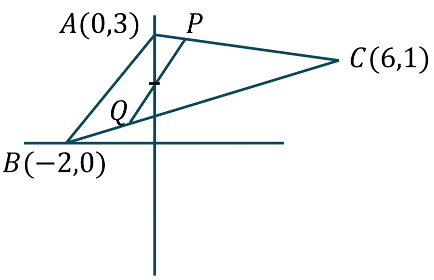
$\therefore a = 2$

slope of line $-\frac{b}{2} = -2$

3. **Ans. (2)**

Since $(\lambda, \lambda+1)$ lies on $y = x + 1$

equation of $AB: 3x - 2y + 6 = 0$; $BC: x - 8y + 2 = 0$; $AC: x + 3y - 9 = 0$

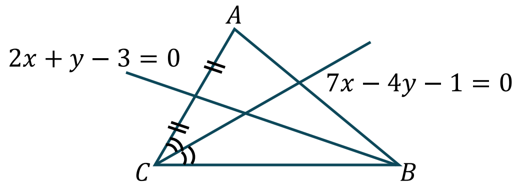


Line $y = x + 1$ cuts AC at $P\left(\frac{3}{2},\frac{5}{2}\right)$ cut BC at $Q\left(\frac{-6}{7},\frac{1}{7}\right)$. Hence $\lambda \in \left(\frac{-6}{7},\frac{3}{2}\right)$

4. **Ans. (18)**

Since C lies on $7x-4y-1=0$, therefore let us choose its coordinates as $\left(h,\frac{7h-1}{4}\right)$.

The mid point of AC, i.e. $\left(\frac{h-3}{2},\frac{7h+3}{8}\right)$ lies on



$2x+y-3=0$,

therefore we have $\left(\frac{h-3}{2}\right)+\left(\frac{7h+3}{8}\right)-3=0$ gives $h=3$

Hence, coordinates of C are (3, 5) and equation of AC is

$y-5=\frac{5-1}{3+3}(x-3)$ i.e., $2x-3y+9=0$ ...(1)

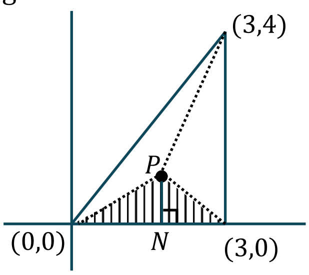
Let slope of $BC=m$. Since lines BC and AC $\left(slope=\frac{2}{3}\right)$ are equally inclined to the line

$7x-4y-1=0$ $\left(slope=\frac{7}{4}\right)$, therefore we have i.e., $\frac{m-\frac{7}{4}}{1+\frac{7m}{4}}=\frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{7}{6}}$ (see figure)

i.e., $\frac{4m-7}{7m+4}=\frac{1}{2}$ gives $m=18$.

5. **Ans. (1)**

Here BP and CP are angular bisectors. Maximum of $d(P,BC)$ occurs, when P is incentre of ΔABC.



$\therefore$ Maximum of $d(P,BC)=PN=$ ordinate of incentre = 1.

$\therefore d(P,BC)=PN==1$

6. **Ans. (5)**

$p=\left|\frac{0+0-\sqrt{5}}{\sqrt{5}}\right|=\frac{a}{\sqrt{5}}=\sqrt{5}$

$\tan 45° = \frac{p}{x}\Rightarrow p=x$

Hence area = $\frac{1}{2}(2x)(p)=px=5$

7. **Ans. (52)**

Point be (x,y) but it lies on $y=x+2$ So, $(x,x+2)$

$F(x)=\left[\frac{3x-4(x+2)+8}{\sqrt{3^2+4^2}}\right]^2+\left[\frac{3x-(x+2)-1}{\sqrt{3^2+1^2}}\right]^2=\frac{2x^2+5[4x^2-12x+9]}{50}=\frac{22\left[\left(x-\frac{30}{22}\right)^2-\frac{900}{484}\right]+45}{50}$

$F(x)$ is minimum at $x=\frac{15}{11}$. So point is $\left(\frac{15}{11},\frac{37}{11}\right)=(a,b)$

$11(a+b)=52$.

8. Ans. (144)

Since orthocentre and circumcentre both lies on y –axis

$\Rightarrow$ Centroid also lies on y-axis

$\Rightarrow \Sigma\cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$\therefore \dfrac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\,\cos\gamma}$

$= \dfrac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\,\cos\gamma} = 12$

9. Ans. (4)

Let $\angle OAB = \theta$

and $AB = AC = b$ cosec θ.

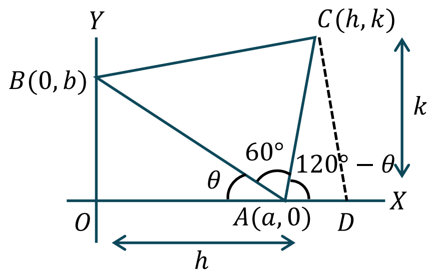
Since $h = OA + AD = OA + AC\cos(120^0 - \theta)$

$\Rightarrow h = a + b\,\text{cosec}\theta\cos(120^0 - \theta)$

$\Rightarrow h = b\cot\theta + b\,\text{cosec}\theta\cos(120^0 - \theta)$.

(since in ΔOAB, $a = b\cot\theta$)

$\Rightarrow h = \dfrac{b}{2}\left(\cot\theta + \sqrt{3}\right)$. ...(1)



Also $k = AD\sin(120^0 - \theta)$

$\Rightarrow k = b\,\text{cosec}\theta\cdot\sin\left(120^0 - \theta\right) = \dfrac{b}{2}\left(\sqrt{3}\cot\theta + 1\right)$ ...(2)

from (1) and (2)

$\dfrac{2h}{b} - \sqrt{3} = \left(\dfrac{2k}{b} - 1\right)\dfrac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h - k = b$

Hence locus of C is $\sqrt{3}x - y = b$

10. Ans. (1)

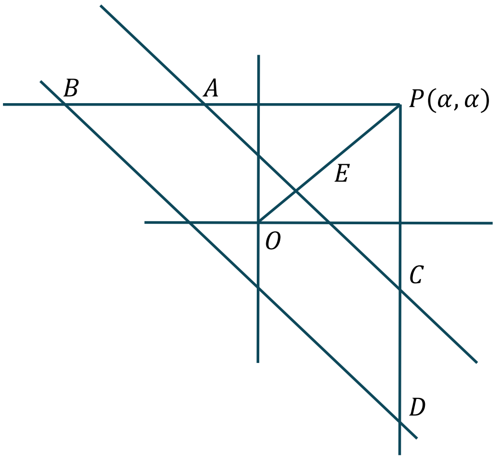
Distance between AC and BD is $\dfrac{2|f(\alpha)|}{\sqrt{2}} = \sqrt{2}|f(\alpha)|$

$AC = 2(OP - OE) = 2\left(\sqrt{2}\alpha - \dfrac{|f(\alpha)|}{\sqrt{2}}\right)$

And $BD = 2\left(\sqrt{2}\alpha + \dfrac{|f(\alpha)|}{\sqrt{2}}\right)$

So $AC + BD = 4\sqrt{2}\alpha$

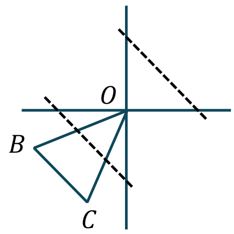
Area of $ABDC = \dfrac{1}{2}\sqrt{2}\,f(\alpha)\,.\,4\sqrt{2}\,\alpha = 4\alpha\,f(\alpha)$.



If area is independent of α, then $f(\alpha) = \frac{k}{\alpha}$.

JEE (Advanced) Practice Paper

1. Ans. (A)



Slope of BC is $= \frac{-3-(-1)}{-1-(-3)} = \frac{-2}{2} = -1$

$\therefore$ Equation of a line parallel to BC is

$y = -x + c$ i.e. $x + y - c = 0$

its distnace from the origin is $\left|\frac{c}{\sqrt{2}}\right| = \frac{1}{2}$ $\quad \therefore c = \pm\frac{1}{\sqrt{2}}$

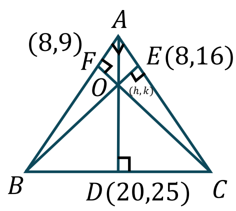
$\therefore$ Equations of the lines are $x + y \pm \frac{1}{\sqrt{2}} = 0$

Since the required line intersects OB and OC, therefore, it is the line whose y intercept is negative.

Hence the required line is $x + y + \frac{1}{\sqrt{2}} = 0$.

2. Ans. (C)

Orthocentre O of the ΔABC is the incentre of the pedal ΔDEF.



$ED = \sqrt{(20-8)^2 + (25-16)^2} = 15$

$FD = 20, EF = 7$

$H = \frac{7\times20+20\times8+15\times8}{7+20+15} = 10$

$K = \frac{7\times25+20\times16+15\times9}{7+20+15} = 15$

$0(10, 15)$

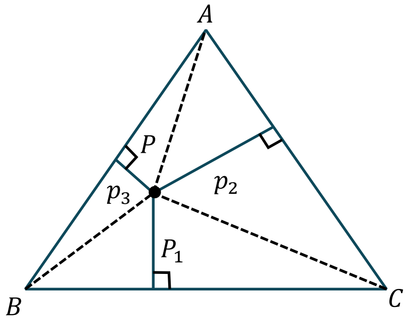
$AC \equiv y - 2x = 0$

$AB \equiv 3y + x - 35 = 0$

$BC \equiv x + y - 45 = 0 \Rightarrow A(5, 10), B(50, -5) C(15, 30)$

3. **Ans. (D)**

Area ΔABC = Area ΔBCP + Area ΔCPA + Area ΔAPB



$$12 = 3p_1 + \frac{5}{2}p_2 + \frac{5}{2}p_3$$

Applying $AM \geq G.M$

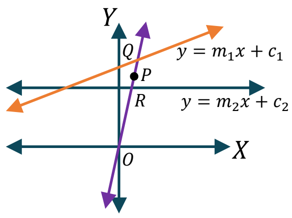
$$\frac{3p_1 + \frac{5}{2}p_2 + \frac{5}{3}p_3}{3} \geq \left((3p_1)\left(\frac{5}{2}p_2\right)\left(\frac{5}{2}p_3\right)\right)^{1/3}$$

$$\frac{12}{3} \geq \left(\frac{75}{4}p_1p_2p_3\right)^{1/3}$$

$$p_1p_2p_3 \leq \frac{256}{75}.$$

4. **Ans. (B)**

Let the line (L) through the origin is



$x = r\cos\theta$; $y = r\sin\theta$

line $y = 2x + 4, m_1 = 2, C_1 = 4$

Line $y = 5x + 3, m_2 = 5, C_2 = 3$

as L intersects L_1 at Q and $OQ = r_1$

$\therefore r_1\sin\theta = m_1r_1\cos\theta + c_1$ ...(1)

similarly, L intersects L_2 at R and $OR = r_2$

$r_2\sin\theta = m_2r_2\cos\theta + c_2$ ...(2)

Let $P \equiv (h, k)$ & $OP = r$

$\therefore r^2 = r_1 r_2$ ...(3)

& $h = r\cos\theta$ ...(4)

$k = r\sin\theta$ ...(5)

putting the values of r_1 and r_2 from (1) and (2) in (3)

$$\therefore r^2 = \frac{c_1}{(\sin\theta - m_1\cos\theta)} \ \frac{c_2}{(\sin\theta - m_2\cos\theta)}. \quad ...(6)$$

putting the value of $\cos\theta$ and $\sin\theta$ from (4) and (5) in (6), we get

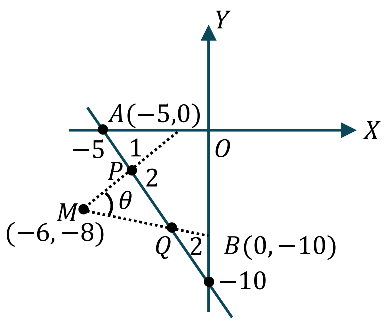
$$\Rightarrow r^2 = \frac{c_1c_2}{\left(\frac{k}{r} - m_1\frac{h}{r}\right)\left(\frac{k}{r} - m_2\frac{h}{r}\right)} \Rightarrow (k - m_1h)(k - m_2h) = c_1c_2$$

replacing (h, k) by (x, y) we get the desired locus as

$(y - m_1x)(y - m_2x) = c_1c_2 \Rightarrow (y - 2x)(y - 5x) = 12$

5. **Ans. (A)**

$\therefore P \equiv (-4, -2)$ and $Q \equiv (-2, -6)$



$\therefore$ Let slopes of PM and QM be m_1 and m_2 respectively.

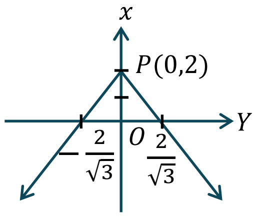
$$\therefore m_1 = 3 \text{ and } m_2 = \frac{1}{2}.$$

Let 'θ' be the acute angle between PM and QM

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}.$$

6. **Ans. (B)**

$\because$ point of intersection of the two ray is $P(0, 2)$



$\therefore$ Point A is $\left(\frac{2}{\sqrt{3}}, 0\right)$ or $\left(-\frac{2}{\sqrt{3}}, 0\right)$

and PO is bisector of the angle between two rays

$\therefore$ required point is (0, 0)

7. **Ans. (B,C,D)**

Equation of line

(A) $y - 2 = m(x - 8)$ and $m < 0 \Rightarrow P\left(8 - \frac{2}{m}, 0\right)$ and $Q(0, 2 - 8m)$

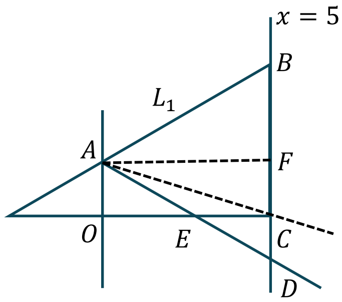
$OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m| = 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18.$

Area of ΔOPQ is minimum when (8,2) is midpoint of line.

So, $P(16, 0)$, $Q(0,4)$ $\Delta OPQ = \frac{1}{2}$ (16) (4) = 32.

8. **Ans. (A,C)**

$L_1 \equiv 3y - 2x - 6 = 0$



Point about which line rotated is $A \equiv (0, 2)$

Let equation of L_2 be $y = mx + 2$

As lying L_2 will be cutting line $x = 5$ below x–axis.

$A \equiv (0, 2)$, $b \equiv \left(5, \frac{16}{3}\right)$, $C \equiv (5, 0)$, $D \equiv (5, 5m + 2)$,

$E \equiv \left(\frac{-2}{m}, 0\right)$

Area $AECB$ = area ADB – area ECD

$\frac{49}{3} = \frac{1}{2} \times BD \times AF - \frac{1}{2} \times EC \times CD$

$= \frac{1}{2} \times \left[\frac{16}{3} - (5m + 2)\right] \times 5 - \frac{1}{2} \times \left(5 + \frac{2}{m}\right) [-(5m + 2)]$

$\Rightarrow \frac{49}{3} = \frac{110m + 12}{6m} \Rightarrow m = -1$

$\Rightarrow$ Equation of L_2 is $y + x = 2$

9. **Ans. (A,D)**

$k_1 u - k_2 v = 0$ ...(i)

$k_1 u + k_2 v = 0$ ...(ii)

$\because$ equations of bisectors of the angles formed by lines (i) and (ii) are

$$\frac{k_1 u - k_2 v}{\sqrt{(ak_1 - bk_2)^2 + (k_1 b + ak_2)^2}} = \frac{\pm(k_1 u + k_2 v)}{\sqrt{(k_1 a + bk_2)^2 + (k_1 b - ak_2)^2}}$$

$\Rightarrow k_1 u - k_2 v = \pm(k_1 u + k_2 v)$ ...(iii)

(i) by taking positive sign in (iii), we get $k_1 u - k_2 v = k_1 u + k_2 v \Rightarrow 2k_2 v = 0 \Rightarrow v = 0$

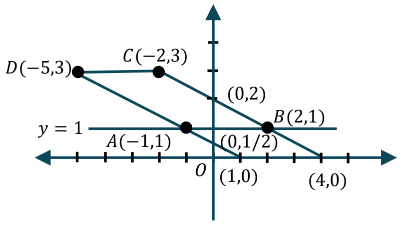
(ii) by taking negative sing in (iii), we get $u = 0$

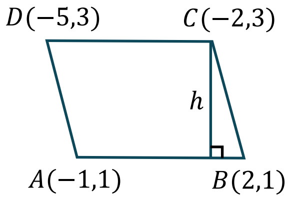
10. **Ans. (A,B,D)**

$y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$

$(y-3)(y-1)$ $\quad\quad$ $(x+2y-1)(x+2y-4) = 0$

$y = 1, y = 3$





$\ell(AB) = 3$ and $h = 2$

Area of parallelogram = 3 × 2 = 6

$\therefore AC = \sqrt{1^2+2^2} = \sqrt{5}$, $BD = \sqrt{7^2+2^2} = \sqrt{53}$

11. **Ans. (A)**

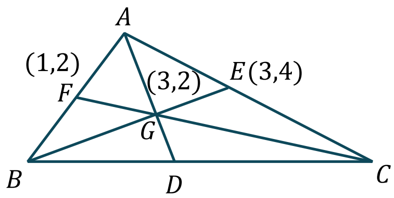
12. **Ans. (B)**

13. **Ans.(C)**

Sol. (11, 12, 13)

Let the co-ordinates of $D(\alpha, \beta)$ then $\frac{\alpha+1+3}{3} = 3 \Rightarrow \alpha = 5$

and $\frac{\beta+2+4}{3} = 2 \Rightarrow \beta = 0$



$\therefore D(5, 0)$

Taking $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

then by $\frac{x_1+x_2}{2} = 1$, $\frac{x_2+x_3}{2} = 5$, $\frac{x_3+x_1}{2} = 3$ and $\frac{y_1+y_2}{2} = 2$,

$\frac{y_2+y_3}{2} = 0$, $\frac{y_1+y_3}{2} = 4$

we get $A(-1, 6)$, $B(3, -2)$, $C(7, 2)$ equation of AB is $2x + y = 4$

Height of altitude from A is = $\frac{2\times area(\Delta ABC)}{BC} = 6\sqrt{2}$

14. **Ans. (A → p;B → q; C → r; D → s)**

(A) $P = (h, h)$; $Q \equiv (k, 2k) \Rightarrow (PQ)^2 = 16 = (h-k)^2 + (h-2k)^2$

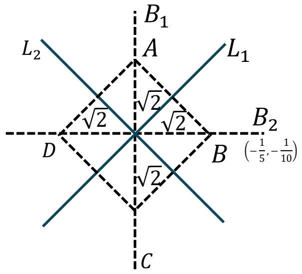
Mid point of $PQ \equiv M \equiv \left(\frac{h+k}{2}, \frac{h+2k}{2}\right) = (x, y)$

$h + k = 2x$; $h + 2k = 2y$ $\Rightarrow k = 2(y-x)$; $h = 4x - 2y \Rightarrow 25x^2 + 13y^2 - 36xy = 4$

(B) $(k^2 + 2k + 1)h + ky - 2k^2 - 2 = 0 \Rightarrow k^2(x-2) + k(2x+y) + x - 2 = 0 \Rightarrow x = 2$ and $y = -4$

(C) $B_1 = 2x + 14y - 1 = 0$; $B_2 = 14x - 2y + 3 = 0$

$\tan\theta_1 = \frac{-1}{7}$ and $\tan\theta_2 = 7$



sum of all possible value of α are $= 0 + \frac{6}{5} - \frac{8}{5} - \frac{2}{5} = \frac{-4}{5}$.

(D) Lines represented by given equation are $x + y + a = 0$ and $x + y - 9a = 0$

$\therefore$ distance between these parallel lines is $= \left|\frac{10a}{\sqrt{2}}\right| = 25\sqrt{2}$

15. Ans. (0)

Let equation of line is $\ell x + my + n = 0$ ...(i)

given $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear

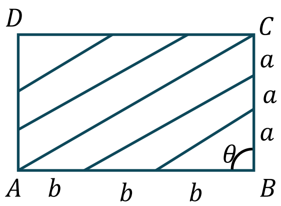
$\left(\frac{t^3}{t-1}, \frac{t^2-3}{t-1}\right)$ is general point which satisfies line (i)

$\ell\left(\frac{t^3}{t-1}\right) + m\left(\frac{t^2-3}{t-1}\right) + n = 0 \Rightarrow \ell t^3 + mt^2 + nt - (3m + n) = 0$

$a + b + c = -\frac{m}{\ell} \Rightarrow ab + bc + ac = \frac{n}{\ell} \Rightarrow abc = \frac{3m+n}{\ell}$

Now LHS $= abc - (ab + bc + ac) + 3(a + b + c) = \frac{(3m+n)}{\ell} - \frac{n}{\ell} + 3\left(\frac{-m}{\ell}\right) = 0$

16. Ans. (4)



$\frac{\frac{1}{2}ab \times \sin\theta}{na \times nb \times \sin\theta} = \frac{1}{32} \Rightarrow n^2 = 16 \Rightarrow n = 4$

17. Ans. (2)

Let $\angle OPQ = \theta$

and $PQ = PR = b$ cosec θ.

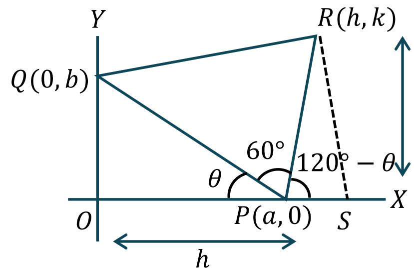
Since $h = OP + Ps = OP + PR \cos(120^0 - \theta)$

$\Rightarrow h = a + b \operatorname{cosec}\theta \cos(120^0 - \theta)$

$\Rightarrow h = b \cot\theta + b \operatorname{cosec}\theta \cos(120^0 - \theta)$.

(since in ΔOPQ, $a = b \cot\theta$)

$\Rightarrow h = \frac{b}{2}\left(\cot\theta + \sqrt{3}\right)$. ...(1)



Also $k = PS \sin(120^0 - \theta)$

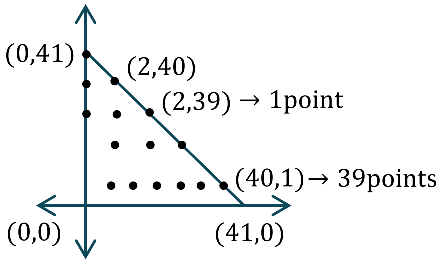
$\Rightarrow k = b\operatorname{cosec}\theta \cdot \sin\left(120^0 - \theta\right) = \frac{b}{2}\left(\sqrt{3}\cot\theta + 1\right)$ ... (2)

from (1) and (2)

$\frac{2h}{b} - \sqrt{3} = \left(\frac{2k}{b} - 1\right)\frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h - k = b$

Hence locus of R is $\sqrt{3}x - y = b$

18 Ans. (780)



$1 + 2 + \ldots\ldots 39 = \frac{39}{2}(39 + 1) = 780$