Trigonometric Equations

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

$2\tan^2\theta = \sec^2\theta$

$2(\sec^2\theta - 1) = \sec^2\theta$

$\Rightarrow\ 2\sec^2\theta - 2 = \sec^2\theta \Rightarrow\ \sec^2\theta = 2$

$\Rightarrow\ \cos^2\theta = \frac{1}{2} = \cos^2\frac{\pi}{4}$

$\theta = n\pi \pm \frac{\pi}{4}, n \in I$ Ans. option (C) is correct.

2. **Ans. (D)**

$\frac{1-\cos 2\theta}{1+\cos 2\theta} = 3$

$\Rightarrow\ 1 - \cos 2\theta = 3 + 3\cos 2\theta \Rightarrow\ 4\cos 2\theta = -2$

$\Rightarrow\ \cos 2\theta = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right)\ \Rightarrow\ 2\theta = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow\ \theta = n\pi \pm \frac{\pi}{3}; n \in I$ option (D) is correct.

3. **Ans. (C)**

Given

$A = \{\theta : \sin\theta = \tan\theta\}$

$B = \{\theta : \cos\theta = 1\}$

$\sin\theta = \tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow\ \sin\theta\,(\cos\theta - 1) = 0$

$\sin\theta = 0$ or $\cos\theta = 1$

$A = \{\theta : \theta = n\pi,\ \theta = 2m\pi; \forall m, n \in I\} = \{\theta : \theta = n\pi\ \forall\ n \in I\}$

$B = \{\theta : \theta = 2n\pi; \forall n \in I\}$

Option (C) is correct.

4. **Ans. (C)**

$(5 + 4\cos\theta)(2\cos\theta + 1) = 0$

$\cos\theta = -\frac{5}{4}$ Not possible as $\cos\theta \in [-1,1]$	$\cos\theta = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right)$ $\theta = 2n\pi \pm \frac{2\pi}{3}; n \in I$ $\left\{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ Ans. (C)

5. **Ans. (B)**

$4\cos^2 x + 6\sin^2 x = 5$

$6\sin^2 x - 4\sin^2 x + 4 = 5$

$\Rightarrow\ 2\sin^2 x = 1 \Rightarrow\ \sin^2 x = \frac{1}{2}$

$\sin^2 x = \sin^2 \frac{\pi}{4}$

$x = n\pi \pm \frac{\pi}{4}\ (n \in I)$ option (B) is correct.

6. **Ans. (A)**

$\tan^2\theta - (1 + \sqrt{3})\tan\theta + \sqrt{3} = 0$

$\tan\theta(\tan\theta - 1) - \sqrt{3}(\tan\theta - 1) = 0$

$\Rightarrow (\tan\theta - \sqrt{3})(\tan\theta - 1) = 0$

$\tan\theta = \sqrt{3}, \tan\theta = 1$

$\theta = n\pi + \frac{\pi}{3}$ or $\tan\theta = n\pi + \frac{\pi}{4}$ option (A) is correct.

7. **Ans. (C)**

$\sin 2x - 2\cos x + 4\sin x = 4$

$2\cos x(\sin x - 1) + 4(\sin x - 1) = 0$

$(\sin x - 1)(2\cos x + 4) = 0$

$\sin x = 1$	or $2\cos x + 4 = 0$
$x = (4n+1)\frac{\pi}{2}, n \in I$	$\Rightarrow \cos x = -2$ (Not possible as $\cos x \in [-1,1]$)

$\therefore In[0, 5\pi] \Rightarrow \left\{x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}\right\}$ option (C) is correct.

8. **Ans. (B)**

$4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$

$\cos\theta = \frac{2\sqrt{2} \pm \sqrt{8+16}}{2\times 4} = \frac{\sqrt{2} \pm \sqrt{6}}{4}$

$\cos\theta = \frac{1+\sqrt{3}}{2\sqrt{2}}, \frac{1-\sqrt{3}}{2\sqrt{2}}$

$\cos\theta = \frac{1+\sqrt{3}}{2\sqrt{2}}$	$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}}$
$\cos\theta = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$	$\cos\theta = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$
$\theta = 2n\pi \pm \frac{\pi}{12}; n \in I$	$\theta = 2n\pi \pm \frac{7\pi}{12}; n \in I$
for $n = 0, 1$	For $n = 0, 1$
$\theta = \frac{\pi}{12}, \frac{23\pi}{12}$	$\theta = \frac{7\pi}{12}, \frac{17\pi}{12}$

$\theta = \left\{\frac{\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}\right\}$ option (B) is correct.

9. Ans. (C)

$\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \cdot \tan 4\theta \cdot \tan 7\theta$

$$\tan(\theta+4\theta+7\theta)=\frac{\tan\theta+\tan 4\theta+\tan 7\theta-\tan\theta\cdot\tan 4\theta\cdot\tan 7\theta}{1-\tan\theta\cdot\tan 4\theta-\tan 4\theta\cdot\tan 7\theta-\tan 7\theta\cdot\tan\theta}$$

$\Rightarrow \tan(12\,\theta) = 0$

$\Rightarrow 12\theta = n\pi$

$\Rightarrow \theta=\dfrac{n\pi}{12}$

But $\tan\theta$, $\tan 4\theta$ and $\tan 7\theta$ won't be defined for

$\theta=(2\pi+1)\dfrac{\pi}{2}$, $(2\theta+1)\dfrac{\pi}{8}$ & $(2r+1)\dfrac{\pi}{14}$

$\therefore\ \theta=\dfrac{n\pi}{12}$ where n can't be an odd multiple of 6

$\therefore\ \theta=\dfrac{n\pi}{12}, \forall\, n \in I$

Such that $n \neq 12m - 6, m \in I$

Ans. Option (C) is correct.

10. Ans. (B)

$\tan 3\theta=0$

$3\theta = n\pi$

$\theta=\dfrac{n\pi}{3}, n \in I.$

11. Ans. (D)

$\tan\theta - \sqrt{2}\sec\theta = \sqrt{3}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta}-\dfrac{\sqrt{2}}{\cos\theta}=\sqrt{3} \Rightarrow \sin\theta - \sqrt{2} = \sqrt{3}\cos\theta$

$\Rightarrow \sin\theta - \sqrt{3}\cos\theta = \sqrt{2}$

divide both sides by 2

$\Rightarrow \dfrac{1}{2}\cdot\sin\theta-\dfrac{\sqrt{3}}{2}\cdot\cos\theta=\dfrac{1}{\sqrt{2}} \Rightarrow \sin\left(\theta-\dfrac{\pi}{3}\right)=\dfrac{1}{\sqrt{2}}$

$\theta-\dfrac{\pi}{3}=n\pi+(-1)^n\dfrac{\pi}{4}$

$\theta=n\pi+(-1)^n\dfrac{\pi}{4}+\dfrac{\pi}{3}; n \in I$

Option (D) is correct.

12. Ans. (C)

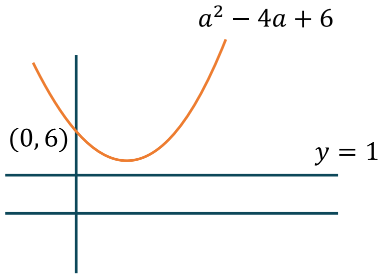
$\sin x + \cos x = \min\limits_{a\in R}\{1, a^2 - 4a + 6\}$

$\min(1, a^2 - 4a + 6) = 1$

$\sin x + \cos x = 1$

divide by $\sqrt{2}$ both side

$\dfrac{1}{\sqrt{2}}\sin x+\dfrac{1}{\sqrt{2}}\cos x=\dfrac{1}{\sqrt{2}}$



$$\Rightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}} \Rightarrow x+\frac{\pi}{4}=n\pi+(-1)^n\frac{\pi}{4}$$

$$x=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}\,;\, n\in I$$

Option (C) is correct.

13. Ans. (D)

$\sin^4 x-(k+2)\sin^2 x-(k+3)=0$

$$\sin^2 x=\frac{(k+2)\pm\sqrt{(k+2)^2+4(k+3)}}{2}=\frac{k+2\pm\sqrt{(k+4)^2}}{2}$$

$\Rightarrow \sin^2 x=\dfrac{2k+6}{2},-1$ $\{\because \sin^2 x=-1 \text{ not possible}\}$

$\sin^2 x=k+3$

$1\geq k+3\geq 0$

$\therefore \left\{\begin{matrix} k\geq -3 \\ k\leq -2\end{matrix}\right\} \therefore k\in[-3,-2]$

Option (D) is correct.

14. Ans. (B)

$\log_2(\sin x)+\log_{1/2}(-\cos x)=0$

$\log_2(\sin x)-\log_2(-\cos x)=0$

$\Rightarrow \sin x>0$ and $\cos x<0$

$\therefore$ x must be lie in $\left(\dfrac{\pi}{2},\pi\right)\Rightarrow \log_2\left(-\dfrac{\sin x}{\cos x}\right)=0$

$\tan x=-1$

in the interval $[-\pi,\pi]$ and $\left(\dfrac{\pi}{2},\pi\right)$

$x=\dfrac{3\pi}{4}$ Ans.

$\therefore$ Option (B) is correct.

15. Ans. (B)

$\because \sin^4 x\geq 0$

$\therefore$ L.H.S. = $1+\sin^4 x\geq 1$

And R.H.S. = $\cos^2 3x\leq 1$

$\therefore$ L.H.S. = R.H.S. is possible only if

$1+\sin^4 x=\cos^2 3x=1$

$\Rightarrow \sin^4 x=0$ and $1-\cos^2 3x=0$

$\Rightarrow \sin^4 x=0$ and $\sin^2 3x=0$

$\therefore I\in\left[-\dfrac{5\pi}{2},\dfrac{5\pi}{2}\right]$

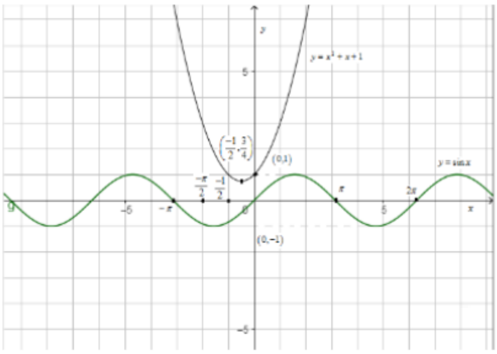
$x=-2\pi,-\pi,0,\pi,2\pi$

are the solution of $\sin^4 x=0$ which also satisfy $\sin^2 3x=0$

$\therefore$ The greatest positive solution = 2π

Ans. (B) is correct.

16. **Ans. (A)**



So, there is no solution

option (A) is correct.

17. **Ans. (A)**

$$2\cos\left(\frac{x}{2}\right) = 3^x + 3^{-x}$$

Range of $2\cos\left(\frac{x}{2}\right) \in [-2, 2]$

Range of $3^x + 3^{-x} \in [2, \infty)$

$AM \geq GM$

$$\frac{3^x + 3^{-x}}{2} \geq \sqrt{3^x \cdot 3^{-x}}$$

$3^x + 3^{-x} \geq 2$

$x = 0$ is a solution

Option (A) is correct.

18. **Ans. (C)**

$$2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + x^{-2}$$

L.H.S = $2\cos^2\left(\frac{x}{2}\right)\sin^2 x < 2$

R.H.S = $x^2 + x^{-2} \in [2, \infty)$

So there is no solution

Option (C) is correct.

19. **Ans. (B)**

$$a^2 + 2a + \operatorname{cosec}^2\left[\frac{\pi}{2}(a + x)\right] = 0$$

$$(a+1)^2 + \operatorname{cosec}^2\left[\frac{\pi}{2}(a + x)\right] = 1$$

Since $\operatorname{cosec}^2\left[\frac{\pi}{2}(a + x)\right] \geq 1$

$\therefore (a + 1) = 0$

$a = -1$ & $\frac{x}{2} \in I$ satisfies this equation

Option (B) is correct.

20. Ans. (A)

$\tan^2 x - 2\sqrt{2}\tan x + 1 \le 0$

$\tan x = \dfrac{2\sqrt{2} \pm \sqrt{4}}{2}$

$\tan x = (\sqrt{2}+1)$ or $(\sqrt{2}-1)$

$\left(\tan x - (\sqrt{2}+1)\right)\left(\tan x - (\sqrt{2}-1)\right) \le 0$

$\sqrt{2} - 1 \le \tan x \le \sqrt{2} + 1$

$n\pi + \dfrac{\pi}{8} \le x \le n\pi + \dfrac{3\pi}{8}$; $n \in I$

Option (A) is correct.

21. Ans. (A,B,C,D)

$2\sin\dfrac{x}{2}.(\cos^2x - \sin^2x) + (\sin^2x - \cos^2x) = 0$

$\Rightarrow 2\sin\dfrac{x}{2}(\cos^2x - \sin^2x) + (\sin^2x - \cos^2x) = 0 \Rightarrow \left(2\sin\dfrac{x}{2} - 1\right)(\cos^2x - \sin^2x) = 0$

Either $\sin\dfrac{x}{2} = \dfrac{1}{2}$ or $\cos^2x - \sin^2x = 0$

$\Rightarrow \dfrac{x}{2} = n\pi + (-1)^n\dfrac{\pi}{6}$ or $\cos 2x = 0$

$\Rightarrow x = 2n\pi + (-1)^n\dfrac{\pi}{3}$ or $2x = (2m+1)\dfrac{\pi}{2}$ $(n, m \in I)$

or $x = (2m+1)\dfrac{\pi}{4}$

$\therefore \sin 2x = \sin(2m+1)\dfrac{\pi}{2} = 1$ or -1

$\cos x = \cos\left(2n\pi + (-1)^n\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

$\cos 2x = \cos\left(4m\pi + (-1)^n\dfrac{2\pi}{3}\right) = \dfrac{-1}{2}$

22. Ans. (B,C)

Let $2\sin^2x - 3\sin x = t$ $\therefore$ Equation becomes,

$2^{t+1} + 2^{2-t} = 9 \Rightarrow 2^t . 2^1 + \dfrac{2^2}{2^t} = 9$

Let $2^t = x$

$\therefore 2x + \dfrac{4}{x} = 9 \Rightarrow 2x^2 + 4 = 9x$

$\Rightarrow 2x^2 - 9x + 4 = 0$

$\Rightarrow 2x^2 - 8x - x + 4 = 0$

$\Rightarrow (2x - 1)(x - 4) = 0$

$\Rightarrow x = 2^{-1}$ or $x = 2^2$ $\therefore t = -1$ or $t = 2$

$\Rightarrow 2\sin^2x - 3\sin x = -1$ or $2\sin^2x - 3\sin x = 2$

$\Rightarrow 2\sin^2x - 3\sin x + 1 = 0$ or $2\sin^2x - 3\sin x - 2 = 0$

$\Rightarrow \sin x = 1$ or $\sin x = \dfrac{1}{2}$ or $\sin x = 2$ or $\sin x = \dfrac{-1}{2}$

$\because \sin x \neq 2, \sin x \in [-1, 1]$

$\therefore \sin x = 1$ or $\sin x = \pm\frac{1}{2}$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $x = m\pi \pm \frac{\pi}{6}$

Where $m \in I$ and $n \in I$

23. Ans. (A,B,C,D)

$\cot x + \operatorname{cosec} x + \sec x = \tan x$

$\Rightarrow \frac{\cos x}{\sin x} + \frac{1}{\sin x} + \frac{1}{\cos x} = \frac{\sin x}{\cos x} \Rightarrow \cos^2 x + \cos x + \sin x = \sin^2 x$

$\Rightarrow \cos^2 x - \sin^2 x + \cos x + \sin x = 0 \Rightarrow (\cos x - \sin x)(\cos x + \sin x) + (\cos x + \sin x) = 0$

$\Rightarrow (\cos x + \sin x)(\cos x - \sin x + 1) = 0 \Rightarrow \cos x + \sin x = 0$ or $\cos x - \sin x = -1$

$\Rightarrow \cos x = -\sin x$ or $\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x = \frac{-1}{\sqrt{2}} \Rightarrow \tan x = -1$ or $\cos\left(x + \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

$\Rightarrow x = n\pi - \frac{\pi}{4}; n \in I$ or $x + \frac{\pi}{4} = 2m\pi \pm \frac{3\pi}{4}; m \in I$

In $[0, 2\pi]$

$x = \left\{\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}\right\}$ (By putting values of n, m)

or $x = \left\{\frac{3\pi}{4} - \frac{\pi}{4}, 2\pi - \frac{3\pi}{4} - \frac{\pi}{4}\right\} = \left\{\frac{\pi}{2}, \pi\right\}$

Both rejected due to $\operatorname{cosec} x$, $\sec x$ & $\tan x$

$\therefore$ Sum $= \pi - \frac{\pi}{4} + 2\pi - \frac{\pi}{4} = 3\pi - \frac{\pi}{2} = \frac{5\pi}{2} = \frac{k\pi}{2}$

$\Rightarrow k = 5$

24. Ans. (B,C)

$\sin x + \cos(k + x) + \cos(k - x) = 2$

$\Rightarrow \sin x + 2\cos k.\cos x = 2$

we have solution when,

$2 \leq \sqrt{1^2 + (2\cos k)^2} \Rightarrow 1 + 4\cos^2 k \geq 4$

$\Rightarrow \cos^2 k \geq \frac{3}{4} \Rightarrow 1 - \sin^2 k \geq \frac{3}{4}$

$\Rightarrow 1 - \frac{3}{4} \geq \sin^2 k \Rightarrow \frac{1}{4} \geq \sin^2 k$

$\Rightarrow \frac{1}{4} \geq \sin^2 k \geq 0 \Rightarrow \frac{-1}{2} \leq \sin k \leq \frac{1}{2}$

For $a\sin x + b\cos x = c$, have solutions when

As, $|a\sin x + b\cos x| \leq \sqrt{a^2 + b^2}$

$\Rightarrow |c| \leq \sqrt{a^2 + b^2}$.

25. Ans. (A,C)

$m > 0; n < 5; 0 < m + n < 10$

$\frac{x^2 + 5}{2} = x - 2\cos(m + nx) \Rightarrow x^2 - 2x + 5 = -4\cos(m + nx)$

$\Rightarrow (x - 1)^2 + 4 = -4\cos(m + nx)$

Range of $(x - 1)^2 + 4 = [4, \infty]$ and range of

$-4\cos(m+nx) = [-4,4]$
Equality 4 = 4 when
$(x-1)^2 = 0$ & $\cos(m+nx) = -1$
$\Rightarrow x = 1$ & $\cos(m+n(1)) = -1 \Rightarrow m+n = (2k+1)\pi$
$k = 0, k = 1$
$m+n = \{\pi, 3\pi\}$

26. **Ans. (B,C)**

$x^4 - 2x^2 + 4 + 3\cos(ax^2+bx+c) = 0$
$\Rightarrow (x^2-1)^2 + 3 = -3\cos(ax^2+bx+c)$
Range of $(x^2-1)^2 + 3 = [3,\infty)$ and range of
$-3\cos(ax^2+bx+c) = [-3,3]$
Equality holds when $x^2-1 = 0 \Rightarrow x = \pm 1$
& $\cos(ax^2+bx+c) = -1$
when; $x = 1$;
$\cos(a+b+c) = -1 \Rightarrow a+b+c = \pi, 3\pi$
when; $x = -1$;
$\cos(a-b+c) = -1 \Rightarrow a-b+c = \pi, 3\pi$
as $a, b, c \in (2,5)$
$\therefore 6 < a+b+c < 15$
So, $a+b+c = 3\pi$ only (C)
and $4 < a+c < 10$ & $-5 < -b < -2$
$\Rightarrow -1 < a+c-b < 8$
$\therefore a+c-b = \pi$ only (B)

27. **Ans. (B,D)**

$\sin^6 x + \cos^6 x = a^2 \Rightarrow (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x) = a^2$
$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x = a^2 \Rightarrow 1 - 3\sin^2 x\cos^2 x = a^2$
$\Rightarrow 1 - \frac{3}{4}\sin^2 2x = a^2 \Rightarrow \frac{4(1-a^2)}{3} = \sin^2 2x \Rightarrow 0 \le \frac{4}{3}(1-a^2) \le 1$
$1 - a^2 \ge 0$ and $4 - 4a^2 \le 3$
$a^2 \le 1$ and $\frac{1}{4} \le a^2$
$-1 \le a \le 1$ and $a \ge \frac{1}{2}$ or $a \le -\frac{1}{2}$
$a \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

28. **Ans. (A,B,C)**

$\cos 4x\cos 8x - \cos 5x\cos 9x = 0 \Rightarrow 2\cos 4x\cos 8x = 2\cos 5x\cos 9x$
$\Rightarrow \cos 12x + \cos 4x = \cos 14x + \cos 4x \Rightarrow 14x = 2n\pi \pm (12x)$
$\Rightarrow 2x = 2n\pi$ or $26x = 2n\pi \Rightarrow x = n\pi$ or $\frac{n\pi}{13}$ $\therefore \sin x = 0$ or $\sin 13x = 0$.

29. **Ans. (C)**

$f(x) = g(x) \Rightarrow \cos x + \sin x - 1 = \sin 2x - 2$
$\Rightarrow \cos x + \sin x - 1 = \sin 2x + 1 - 3$
Let $\cos x + \sin x = t \Rightarrow (\cos x + \sin x)^2 = t^2$
$\Rightarrow \cos^2 x + \sin^2 x + 2\sin x\cos x = t^2 \Rightarrow 1 + \sin 2x = t^2$
$\therefore$ Equation becomes,
$t - 1 = t^2 - 3 \Rightarrow t^2 - t - 2 = 0$

$\Rightarrow t = -1$ or $t = 2$

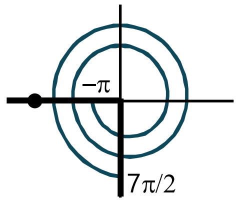
$\Rightarrow \cos x + \sin x = -1$

$\cos x + \sin x = 2$

But range of $\cos x + \sin x$ is $\left[-\sqrt{2}, \sqrt{2}\right]$

$\therefore$ $\cos x + \sin x = 2$ has no solution

$\Rightarrow \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{-1}{\sqrt{2}} \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$



$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$; $n \in I \Rightarrow x = 2n\pi + \pi$ or $x = 2n\pi - \frac{\pi}{2}$; $n \in I$

$\therefore x \in \left[-\pi, \frac{7\pi}{2}\right]$, $x = -\pi, \pi, 3\pi, -\pi/2, 3\pi/2, 7\pi/2$

$\therefore$ Number of solutions in $\left[-\pi, \frac{7\pi}{2}\right]$ is 6.

30. **Ans. (C)**

$f(x) = k$

$\Rightarrow \cos x + \sin x - 1 = k \Rightarrow \cos x + \sin x = k + 1$

will have solution when

$-\sqrt{2} \le k + 1 \le \sqrt{2} \Rightarrow -\sqrt{2} - 1 \le k \le \sqrt{2} - 1$

Integral values of $k = -2, -1, 0$

$\therefore$ Number of integral values of $k = 3$

31. **Ans. (A)**

$f(x) = 0 \Rightarrow \sin x = a - 3$ or $\sin x = 2$ (No sol.)

$\therefore \sin x = (a - 3)$ will have exactly one real not in $[0, \pi]$

$\therefore a - 3 = 1 \Rightarrow a = 4$

$\because a \in (3, 5)$

32. **Ans. (B)**

$f(x) = 0$ will have exactly two real roots in $(0, \pi)$ only if

$a - 3 \in (0, 1) \Rightarrow a \in (3, 4)$

33. **Ans. (A)**

(i) $2\cos^2 x - 3\cos x + 1 = 0; x \in (-\pi, 3\pi]$

$\Rightarrow \cos x = 1$ or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0, 2\pi$ or $x = \frac{-\pi}{3}, \frac{\pi}{3}, 2\pi + \frac{\pi}{3}$

Also, $\tan\frac{3x}{4} + 1 = 0$

$\therefore \frac{3x}{4} = \frac{-\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\therefore x = \frac{-\pi}{3}, \pi, \frac{5\pi}{3}, \frac{7\pi}{3}$

only $x = \frac{-\pi}{3}$ and $x = 2\pi + \frac{\pi}{3}$

satisfy both the equations

(ii) $\sin^2 x + 2\sin x - 4\cos x - \sin 2x = 0$

$\Rightarrow \sin x(\sin x + 2) - 4\cos x - 2\sin x\cos x = 0$

$\Rightarrow (\sin x - 2\cos x)(\sin x + 2) = 0$

$\Rightarrow \sin x = 2\cos x$ or $\sin x = -2$ (Not possible)

$\Rightarrow \tan x = 2$

Two solutions in $(0, 2\pi)$

(iii) $2\cos x\cdot\operatorname{cosec} x-4\cos x-\operatorname{cosec} x=-2$

$\Rightarrow 2\cos x\cdot\operatorname{cosec} x-4\cos x-\operatorname{cosec} x+2=0$

$\Rightarrow 2\cos x(\operatorname{cosec} x-2)-1(\operatorname{cosec} x-2)=0$

$\Rightarrow (\operatorname{cosec} x-2)(2\cos x-1)=0$

$\Rightarrow \cos x=\frac{1}{2}$ or $\sin x=\frac{1}{2}$; $x\in(0,2\pi)$

$\Rightarrow x=\frac{\pi}{3}, 2\pi-\frac{\pi}{3}$ or $x=\frac{\pi}{6}$ or $\pi-\frac{\pi}{6}$

$\therefore$ Sum of all solution = $3\pi=\frac{k\pi}{4}\Rightarrow k=12$

(iv) $x-y=\frac{1}{4}$; $\cos\pi x.\cos\pi y=\frac{1}{2\sqrt{2}}$

$\Rightarrow 2\cos\pi x.\cos\pi y=\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\pi(x+y)+\cos\pi(x-y)=\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\pi(x+y)+\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\pi(x+y)=0$

$\Rightarrow \pi(x+y)=(2n+1)\frac{\pi}{2}$

$\Rightarrow x+y=\frac{2n+1}{2}\ n\in I$

$\therefore 2x=\frac{2n+1}{2}+\frac{1}{4}$

$\Rightarrow x=\frac{4n+3}{8}$ as $0<\frac{4n+3}{8}<2$

$\Rightarrow 0<4n+3<16\Rightarrow -3<4n<13$

$\Rightarrow \frac{-3}{4}<n<\frac{13}{4}\Rightarrow n=0,1,2,3$

$\Rightarrow x=\frac{3}{8},\frac{7}{8},\frac{11}{8},\frac{15}{8}$

So, $y=\frac{1}{8},\frac{5}{8},\frac{9}{8},\frac{13}{8}$

$\therefore$ there will be four ordered pairs of (x,y)

34. Ans. (A $\rightarrow$ S; B $\rightarrow$ P; C $\rightarrow$ R; D $\rightarrow$ Q)

(A) $\because \sin^4x\le\sin^2x$ and $\cos^7x\le\cos^2x$

$\therefore \sin^4x+\cos^7x\le\sin^2x+\cos^2x$

$\Rightarrow \sin^4x+\cos^7x\le 1$

Given that

$\sin^4x+\cos^7x=1$

$\therefore \sin^4x=\sin^2x$ and $\cos^7x=\cos^2x$

$\therefore$ in $(-\pi,\pi)$

$x\in\left\{-\frac{\pi}{2},0,\frac{\pi}{2}\right\}$

(B) $|\cot x| = \cot x + \dfrac{1}{\sin x}$; $0 < x < \pi$

when $0 < x < \dfrac{\pi}{2}$; $\cot x = \cot x + \dfrac{1}{\sin x} \Rightarrow \text{cosec}x = 0$

$\Rightarrow$ No solution

When $\dfrac{\pi}{2} \le x < \pi$; $-\cot x = \cot x + \dfrac{1}{\sin x}$

$\Rightarrow -2\cot x = \dfrac{1}{\sin x} \Rightarrow \cos x = \dfrac{-1}{2}$; {as $\sin x \neq 0$}

$\Rightarrow x = \dfrac{2\pi}{3}$

(C) $\because$ L.H.S. ≥ 3 and R.H.S. ≤ -2

$\therefore$ there will be no real solution of given equation.

(D) $\text{cosec}\, x = 1 + \cot x$

$\Rightarrow \dfrac{1}{\sin x} = 1 + \dfrac{\cos x}{\sin x} \Rightarrow \sin x + \cos x = 1$; $(\sin x \neq 0)$

$\Rightarrow \dfrac{1}{\sqrt{2}}\cos x + \dfrac{1}{\sqrt{2}}\sin x = \dfrac{1}{\sqrt{2}} \Rightarrow \cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}} = \cos\dfrac{\pi}{4}$

$\Rightarrow x - \dfrac{\pi}{4} = 2n\pi \pm \dfrac{\pi}{4}$; $n \in I \Rightarrow x = 2n\pi + \dfrac{\pi}{4} + \dfrac{\pi}{4}$ or $x = 2n\pi - \dfrac{\pi}{4} + \dfrac{\pi}{4}$; $n \in I$

$\Rightarrow x = 2n\pi + \dfrac{\pi}{2}$ or $x = 2n\pi$ (reject) as $\sin x \neq 0$

$\therefore$ In $[-2\pi, 2\pi]$

x can be $-\dfrac{3\pi}{2}$ *and* $\dfrac{\pi}{2}$

$\therefore$ In $[-2\pi, 2\pi]$ the number of solutions = 2.

EXERCISE - S

1. **Ans. (3)**

$4 \cdot 16^{\sin^2 x} = 2^{6\sin x}$

$\Rightarrow 2^{2+4\sin^2 x} = 2^{6\sin x} \Rightarrow 2 + 4\sin^2 x = 6\sin x$

$\Rightarrow 4\sin^2 x - 6\sin x + 2 = 0 \Rightarrow 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \dfrac{1}{2}$ or $\sin x = 1$

$x = n\pi + (-1)^n \dfrac{\pi}{6}$	Or	$x = \dfrac{\pi}{2}$
$= \dfrac{\pi}{6}, -\dfrac{\pi}{6}, \dfrac{11\pi}{6}$		

In $[0, 2\pi)$ there are 3 solutions $\Rightarrow x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{11\pi}{6}$

2. **Ans. (3)**

$\tan^2 x - \sec^{10} x + 1 = 0$

$\sec^2 x - 1 - \sec^{10} x + 1 = 0$

$\sec^2 x\,(1 - \sec^8 x) = 0$

$\sec^2 x = 0$	$\sec^8 x = 1$
No solution	$\cos^8 x = 1$
	$x = n\pi$

$x = \pi, 2\pi, 3\pi$ Ans.

3. **Ans. (6)**

Case-I : When $\cos x = -1$ and $\sin y = -1$

$$x = \pi, 3\pi, \text{ and } y = \frac{3\pi}{2}$$

$\therefore$ Number of ordered pairs (x, y) in this case = 2

Case-II : When $\cos x = 1$ and $\sin y = 1$

$$x = 0, 2\pi, \text{ and } y = \frac{\pi}{2}, \frac{5\pi}{2}$$

$\therefore$ Number of ordered pairs (x, y) in this case = 2 × 2 = 4

$\therefore$ Total number of ordered pairs $(x, y) = 6$

4. **Ans. (0)**

$\cot^4 x - 2\operatorname{cosec}^2 x + a^2 = 0$

$\cot^4 x - 2(\cot^2 x + 1) + a^2 = 0$

$\Rightarrow \cot^4 x - 2\cot^2 x - 2 + a^2 = 0 \Rightarrow (\cot^2 x - 1)^2 = 3 - a^2$

$\Rightarrow (\cot^2 x - 1)^2 = \left(\sqrt{3} - a\right)\left(\sqrt{3} + a\right)$

$\because$ L.H.S. ≥ 0 $\qquad \therefore$ R.H.S. $= 3 - a^2 \geq 0$

$\therefore a \in [-\sqrt{3}, \sqrt{3}]$ and $a \in I$

$\therefore a \in \{-1, 0, 1\}$

$\therefore$ sum of integral values of $a = 0$

5. **Ans. (5)**

$2\sin^2 x - 5\sin x + 2 > 0 \,;\, x \in [0, 2\pi]$

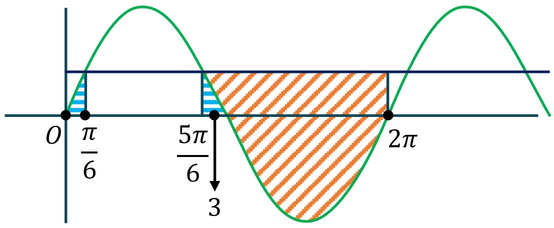
$2\sin x - 4\sin x - \sin x + 2 > 0$

$(2\sin x - 1)(\sin x - 2) > 0$

$\sin x < \frac{1}{2}$ or $\sin x > 2$

integral value of x are

$x = 0, 3, 4, 5, 6$



6. **Ans. (5025)**

$\sin \pi x + \cos \pi x = 0$

or $\sin\left(\frac{\pi}{4} + \pi x\right) = 0$ $\qquad$ or $\frac{\pi}{4} + \pi x = n\pi$

or $x = \left(n - \frac{1}{4}\right)\frac{\pi}{\pi} = n - \frac{1}{4}$ where $n \in I$

sum = $\sum_{n=0}^{100}\left(n - \frac{1}{4}\right) = \frac{n(n+1)}{2} - \frac{n}{4} = \frac{100 \times 101}{2} - \frac{100}{4}$

$= 5050 - \frac{100}{4} = 5025$

7. Ans. (24)

$5 - 12\tan\theta = 11\sec\theta \Rightarrow 25 + 144\tan^2\theta - 120\tan\theta = 121 + 121\tan^2\theta$

$23\tan^2\theta - 120\tan\theta - 96 = 0 \Rightarrow \tan\alpha + \tan\beta = \frac{120}{23} \Rightarrow \tan\alpha\tan\beta = -\frac{96}{23}$

$\tan(\alpha+\beta) = \frac{\frac{120}{23}}{1+\frac{96}{23}} = \frac{120}{119} \Rightarrow \sin(\alpha+\beta) = -\frac{120}{169}.$

8. Ans. (2)

$4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sqrt{4\sin^4 x+4\sin^2 x\cos^2 x} = 4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right) + |2\sin x| = 4\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right) - 2\sin x$

$= 2\left(1+\cos\left(\frac{\pi}{2}-x\right)\right) - 2\sin x = 2$

9. Ans. (50)

$4\cos^3 x - 4\cos^2 x + \cos x - 1 = 0$

$\Rightarrow (4\cos^2 x + 1)(\cos x - 1) = 0 \Rightarrow \cos x = 1$

$x = 2n\pi$

solutions in the interval [0, 315] are $0, 2\pi, 4\pi, \ldots\ldots, 100\pi$

$\therefore$ arithmetic mean $= \frac{0+2\pi+4\pi+\ldots.+100\pi}{51} = 50\pi$

10. Ans. (17)

$\tan\theta + \tan 2\theta + \tan 3\theta - \tan\theta\tan 3\theta\tan 2\theta = 0$ ...(1)

$\Rightarrow \tan(\theta + 2\theta + 3\theta) = 0 \Rightarrow \tan 6\theta = 0 \Rightarrow 6\theta = n\pi \Rightarrow \theta = \frac{n\pi}{6}$

11. Ans. (07)

$\cos 6x\,(1+\tan^2 x) = 1 - \tan^2 x$ or $\cos 6x = \frac{1-\tan^2 x}{1+\tan^2 x}$

or $\cos 6x = \cos 2x$ or $6x = 2n\pi \pm 2x \Rightarrow x = \frac{n\pi}{2}, \frac{n\pi}{4}$

$\Rightarrow x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$ (At, $x = \frac{\pi}{2}, \frac{3\pi}{2}$, $\tan x$ does not exist)

12. Ans. (0)

$\sin\theta + 2\sin 2\theta + 3\sin 3\theta + 4\sin 4\theta = 10 \Rightarrow \sin\theta = \sin 2\theta = \sin 3\theta = \sin 4\theta = 1$ is $(0, \pi)$

which is not possible

EXERCISE - JEE (Main) PYQ

1. **Ans. (1)**

$\sin\theta + \sin 4\theta + \sin 7\theta = 0$

$2\sin\left(\frac{\theta+7\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right) + \sin 4\theta = 0$

$\Rightarrow \sin 4\theta[2\cos 3\theta + 1] = 0 \Rightarrow \sin 4\theta = 0 \Rightarrow 4\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

$\Rightarrow \theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

$\therefore\ \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ in $(0, \pi)$

and $2\cos 3\theta + 1 = 0 \Rightarrow \cos 3\theta = \frac{-1}{2}$

$\Rightarrow 3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$ $\{\because \theta \in (0, \pi) \therefore 3\theta \in (0, 3\pi)\}$

$\Rightarrow \theta = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

So, $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

2. **Ans. (3)**

$3\sin P + 4\cos Q = 6$ ...(1)

$4\sin Q + 3\cos P = 1$ ...(2)

Squaring (1) and (2). Then adding

$9 + 16 + 24\sin(P + Q) = 37$

$\Rightarrow 24\sin(P + Q) = 12$

$\Rightarrow \sin(P + Q) = \frac{1}{2} \Rightarrow \sin R = \frac{1}{2}$

$\therefore R = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

If $R = \frac{5\pi}{6}$

then $P + Q = \frac{\pi}{6}$ $\therefore\ 0 < P, Q < \frac{\pi}{6}$

$\therefore 0 < 3\sin P < 3\sin\frac{\pi}{6} = \frac{3}{2}$

and $4\cos\frac{\pi}{6} < 4\cos Q < 4$

$\Rightarrow 2\sqrt{3} < 3\sin P + 4\cos Q < 5.5$

but given that $3\sin P + 4\cos Q = 6$

$\therefore\ R = \frac{5\pi}{6}$ can't be true.

$\therefore\ R = \frac{\pi}{6}$.

3. Ans. (4)

$2\cos 2x \cos x + 2\cos 3x \cos x = 0$

$\Rightarrow 2\cos x\,(\cos 2x + \cos 3x) = 0$

$2\cos x\, 2\cos 5x/2 \cos x/2 = 0$

$\Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos\frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$

$\Rightarrow x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{\pi}{2}, \frac{7\pi}{5}, \frac{9\pi}{5}$

& $\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

7 Solutions

4. Ans. (1)

$8\cos x\left(\cos^2\frac{\pi}{6} - \sin^2 x - \frac{1}{2}\right) = 1$

$\Rightarrow 8\cos x\left(\frac{1}{4} - (1 - \cos^2 x)\right) = 1 \Rightarrow 8\cos x\left(\cos^2 x - \frac{3}{4}\right) = 1 \Rightarrow 2\cos 3x = 1 \Rightarrow \cos 3x = \frac{1}{2}$

$\therefore\ 3x = 2n\pi \pm \frac{\pi}{3},\ n \in I \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

In $x \in [0, \pi]$: $x = \frac{\pi}{9}, \frac{2\pi}{3} + \frac{\pi}{9}, \frac{2\pi}{3} - \frac{\pi}{9}$ only

sum $= \frac{13\pi}{9}$

5. Ans. (1)

$1 + \sin^4 x = \cos^2 3x$

$\sin x = 0$ & $\cos 3x = 1$

$0, 2\pi, -2\pi, -\pi, \pi$

$\therefore$ Number of solutions = 5

6. Ans. (1)

$\cos 2x + \alpha \sin x = 2\alpha - 7$

$\Rightarrow 2\sin^2 x - \alpha \sin x + 2\alpha - 8 = 0$

$\sin^2 x - \frac{\alpha}{2}\sin x + \alpha - 4 = 0$

$\Rightarrow \sin x = 2$ (rejected) or $\sin x = \frac{\alpha - 4}{2}$

$\Rightarrow \left|\frac{\alpha - 4}{2}\right| \le 1$

$\Rightarrow \alpha \in [2, 6]$

7. **Ans. (2)**

AM $\geq GM$

$$\frac{\sin^4\alpha+4\cos^4\beta+1+1}{4}\geq\left(4\sin^4\alpha\cos^4\beta\right)^{\frac{1}{4}}$$

$$\sin^4\alpha+4\cos^4\beta+2\geq4\sqrt{2}\sin\alpha\cos\beta$$

To hold equality

$$\sin^4\alpha=4\cos^4\beta=1$$

$$\alpha=\frac{\pi}{2},\ \beta=\frac{\pi}{4}$$

$\therefore$ cos $(\alpha+\beta)$ – cos $(\alpha-\beta)$ = –2 sin α sin β = $-\sqrt{2}$

8. **Ans. (4)**

$$\lambda=-(\sin^4\theta+\cos^4\theta)$$

$$\lambda=-(\sin^2\theta+\cos^2\theta)^2+2\sin^2\theta\cos^2\theta$$

$$\lambda=\frac{\sin^2 2\theta}{2}-1$$

$$\frac{\sin^2 2\theta}{2}\in\left[0,\ \frac{1}{2}\right]$$

$$\lambda\in\left[-1,\ -\frac{1}{2}\right]$$

9. **Ans. (4)**

sin 2θ + tan 2θ > 0

$$\Rightarrow\ \sin2\theta+\frac{\sin2\theta}{\cos2\theta}>0$$

$$\Rightarrow\ \sin2\theta\frac{(\cos2\theta+1)}{\cos2\theta}>0\Rightarrow\tan2\theta(2\cos^2\theta)>0$$

Note : $\cos2\theta\neq0$

$$\Rightarrow 1-2\sin^2\theta\neq0\Rightarrow\sin\theta\neq\pm\frac{1}{\sqrt{2}}$$

Now, tan 2θ (1 + cos 2θ) > 0

$\Rightarrow$ tan 2θ > 0 (as cos 2θ + 1 > 0)

$$\Rightarrow2\theta\in\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)\cup\left(2\pi,\frac{5\pi}{2}\right)\cup\left(3\pi,\frac{7\pi}{2}\right)$$

$$\Rightarrow\theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$$

As $\sin\theta\neq\pm\frac{1}{\sqrt{2}}$; which has been already considered

10. Ans. (1)

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$$

$$2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1$$

$$2\cos x\left(4\left(\frac{1}{2}-\sin^2 x\right)-1\right)=1$$

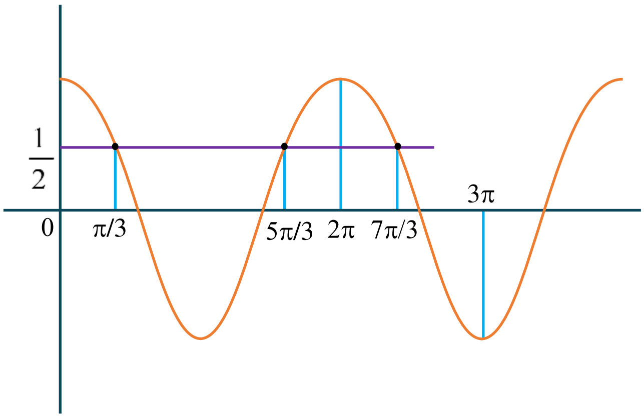
$$2\cos x\left(2-4\sin^2 x-1\right)=1$$

$$2\cos x\left(1-4\sin^2 x\right)=1$$

$$2\cos x\left(4\cos^2 x-3\right)=1$$

$$4\cos^3 x-3\cos x=\frac{1}{2}$$

$$\cos 3x=\frac{1}{2}$$



$x \in [0,\pi]$ $\therefore$ $3x \in [0,3\pi]$

$$\therefore\ 3x=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3}$$

$$\therefore\ x=\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9}$$

$$\therefore\ n = 3 \text{ and } S=\frac{13\pi}{9}$$

11. Ans. (2)

$$(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$$

$$(81)^{\sin^2 x}+\frac{(81)^1}{(18)^{\sin^2 x}}=30$$

$$(81)^{\sin^2 x}=t$$

$$t+\frac{81}{t}=30$$

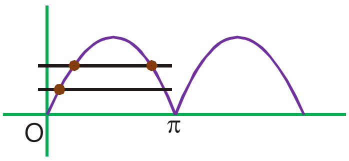
$$t^2-30t+81=0$$

$$(t-3)(t-27)=0$$

$(81)^{\sin^2 x}=3^1$ or $(81)^{\sin^2 x}=3^3$

$3^{4\sin^2 x}=3^1$ or $3^{4\sin^2 x}=3^3$

$\sin^2 x=\frac{1}{4}$ or $\sin^2 x=\frac{3}{4}$



Total sol. = 4

12. Ans. (4)

$$\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$$

$x\in[-3\pi,3\pi]$

$$4\left(\cos^2\left(\frac{\pi}{3}\right)-\sin^2 x\right)=\cos^2 2x$$

$$4\left(\frac{1}{4}-\sin^2 x\right)=\cos^2 2x$$

$1-4\sin^2 x=\cos^2 2x$

$1-2(1-\cos 2x)=\cos^2 2x$

let $\cos 2x=t$

$-1+2\cos 2x=\cos^2 2x$

$t^2-2t+1=0$

$(t-1)^2=0$

$\boxed{t=1}$ $\boxed{\cos 2x=1}$

$2x=2n\pi$

$\boxed{x=n\pi}$

$n=-3,-2,-1,0,1,2,3$

(4) option is correct.

13. Ans. (2)

$\sin\theta\tan\theta+\tan\theta=\sin 2\theta$

$$\tan\theta(\sin\theta+1)=\frac{2\tan\theta}{1+\tan^2\theta}$$

$\tan\theta=0 \Rightarrow \theta=-\pi,0,\pi$

$(\sin\theta+1)=2.\cos^2\theta = 2(1+\sin\theta)(1-\sin\theta)$

$\sin\theta=-1$ which is not possible

$\sin\theta=\frac{1}{2}$ $\quad\theta=\frac{\pi}{6},\frac{5\pi}{6}$

$n(s)=5$

$$T=\cos 0+\cos 2\pi+\cos 2\pi+\cos\frac{\pi}{3}+\cos\frac{5\pi}{3}$$

$T=4$

$T+n(s)=9$

14. Ans. (3)

Let $\alpha=\theta+(m-1)\frac{\pi}{6}$ & $\beta=\theta+m\frac{\pi}{6}$

So, $\beta-\alpha=\frac{\pi}{6}$

Here, $\sum_{m=1}^{9}\sec\alpha\cdot\sec\beta=\sum_{m=1}^{9}\frac{1}{\cos\alpha\cdot\cos\beta}$

$$=2\sum_{m=1}^{9}\frac{\sin(\beta-\alpha)}{\cos\alpha\cdot\cos\beta}=2\sum_{m=1}^{9}(\tan\beta-\tan\alpha)=2\sum_{m=1}^{9}\left(\tan\left(\theta+m\frac{\pi}{6}\right)-\tan\left(\theta+(m-1)\frac{\pi}{6}\right)\right)$$

$$=2\left(\tan\left(\theta+\frac{9\pi}{6}\right)-\tan\theta\right)=2(-\cot\theta-\tan\theta)=-\frac{8}{\sqrt{3}}\ \text{(Given)}$$

$$\therefore\ \tan\theta+\cot\theta=\frac{4}{\sqrt{3}}$$

$$\Rightarrow\ \tan\theta=\frac{1}{\sqrt{3}}\ \text{or}\ \sqrt{3}$$

So, $S=\left\{\frac{\pi}{6},\frac{\pi}{3}\right\}$

$$\sum_{\theta\in S}\theta=\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi}{2}$$

15. Ans. (1)

$$2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}$$

L.H.S $\le$ 2. & R.H.S. $\ge$ 2

Hence L.H.S = 2 & R.H.S = 2

$$2cos\left(\frac{x^2+x}{6}\right)=2\quad 4^x+4^{-x}=2$$

only $x=0$ is possible hence exactly one solution.

16. Ans. (3)

$$3\cos^4\theta-5\cos^2\theta-2\sin^6\theta+2=0$$

$$\Rightarrow 3\cos^4\theta-3\cos^2\theta-2\cos^2\theta-2\sin^6\theta+2=0\Rightarrow 3\cos^4\theta-3\cos^2\theta+2\sin^2\theta-2\sin^6\theta=0$$

$$\Rightarrow 3\cos^2\theta(\cos^2\theta-1)-2\sin^2\theta(\sin^4\theta-1)=0\Rightarrow -3\cos^2\theta\sin^2\theta+2\sin^2\theta(1+\sin^2\theta)\cos^2\theta=0$$

$$\Rightarrow \sin^2\theta\cos^2\theta(2+2\sin^2\theta-3)=0\ \Rightarrow \sin^2\theta\cos^2\theta(2\sin^2\theta-1)=0$$

(C1) $\sin^2\theta=0\Rightarrow$ 3 solutions ; $\theta=0,\pi,2\pi$

(C2) $\cos^2\theta=0\ \Rightarrow$ 2 solutions ; $\theta=\frac{\pi}{2},\frac{3\pi}{2}$

(C3) $\sin^2\theta=\frac{1}{2}\ \Rightarrow$ 4 solutions ; $\theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

No. of solutions = 9

17. Ans. 25

$$\cos2\theta\cdot\cos\frac{\theta}{2}=\cos3\theta\cdot\cos\frac{9\theta}{2}$$

$$\Rightarrow 2\cos2\theta\cdot\cos\frac{\theta}{2}=2\cos\frac{9\theta}{2}\cdot\cos3\theta\Rightarrow\cos\frac{5\theta}{2}+\cos\frac{3\theta}{2}=\cos\frac{15\theta}{2}+\cos\frac{3\theta}{2}$$

$$\Rightarrow\cos\frac{15\theta}{2}=\cos\frac{5\theta}{2}\Rightarrow\frac{15\theta}{2}=2k\pi\pm\frac{5\theta}{2}$$

$$5\theta=2k\pi\ \text{or}\ 10\theta=2k\pi$$

$\theta = \frac{2k\pi}{5}$ or $\theta = \frac{k\pi}{5}$

$\therefore \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$

$m = 5, n = 5$

$\therefore m.n = 25$

18. Ans. (4)

$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1$

$\Rightarrow \frac{\ln\cos x - \ln\sin x}{\ln\cos x} + 4\frac{\ln\sin x - \ln\cos x}{\ln\sin x} = 1$

$\Rightarrow (\ln\sin x)^2 - 4(\ln\sin x)(\ln\cos x) + 4(\ln\cos x)^2 = 0$

$\Rightarrow (\ln\sin x - 2\ln\cos x)^2 = 0$

$\Rightarrow \ln\sin x = 2\ln\cos x$

$\Rightarrow \sin x = \cos^2 x > 0$

$\Rightarrow \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$

$\therefore \alpha + \beta = 4$

19. Ans. (1)

Let $9^{\tan^2 x} = P$

$\frac{9}{P} + P = 10$

$P^2 - 10P + 9 = 0$

$(P - 9)(P - 1) = 0$

$P = 1, 9$

$9^{\tan^2 x} = 1$, $9^{\tan^2 x} = 9$

$\tan^2 x = 0, \tan^2 x = 1$

$x = 0, \pm\frac{\pi}{4}$ $\qquad \therefore x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\beta = \tan^2(0) + \tan^2\left(+\frac{\pi}{12}\right) + \tan^2\left(-\frac{\pi}{12}\right)$

$= 0 + 2(\tan 15°)^2 = 2(2-\sqrt{3})^2 = 2(7-4\sqrt{3})$

$\therefore \frac{1}{6}(\beta - 14)^2$

$= \frac{1}{6}(14 - 8\sqrt{3} - 14)^2 = 32$

20. Ans. (2)

$\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$

$\tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\pi\cos\theta = n\pi - \pi\sin\theta$

$\sin\theta + \cos\theta = n$ where $n \in I$

possible values are $n = 0$, 1 and –1 because

$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$

Now it gives $\theta \in \left\{0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{2}, \pi\right\}$

So $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = 2(0) + 4\left(\frac{1}{2}\right) = 2$

EXERCISE - JEE (Advanced) PYQ

1. Ans. (D)

$\sin x - \sin 3x + 2\sin 2x = 3$

$-2\sin x\cos 2x + 4\sin x\cos x = 3$

$2\sin x\{-\cos 2x + 2\cos x\} = 3$

$2\sin x\{-(2\cos^2 x - 1) + 2\cos x\} = 3$

$2\sin x\{-2\cos^2 x + 2\cos x + 1\} = 3$

$$\underbrace{2\sin x}_{\le 2}\underbrace{\left\{\frac{3}{2} - 2\left(\cos x - \frac{1}{2}\right)^2\right\}}_{\le \frac{3}{2}} = 3$$

for equality to hold

$\sin x = 1$ & $\cos x = \frac{1}{2}$

which is not possible

hence no solution.

2. Ans. (8)

Given equation

$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

$$\frac{5}{4}\cos^2 2x + 1 - 2\sin^2 x\cos^2 x + 1 - 3\sin^2 x\cos^2 x = 2$$

$$\frac{5}{4}\cos^2 2x - \frac{1}{2}(1 - \cos^2 2x) - \frac{3}{4}(1 - \cos^2 2x) = 0$$

$$\frac{5}{2}\cos^2 2x = \frac{5}{4} \Rightarrow \cos^2 2x = \frac{1}{2} \Rightarrow \cos 4x = 0 \Rightarrow 4x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \text{ and } \pm\frac{7\pi}{2}$$

Clearly having 8 solutions in $[0, 2\pi]$

3. Ans. (C)

$\sqrt{3}\sin x + \cos x = 2\cos 2x$

$\Rightarrow \cos 2x = \cos\left(x - \frac{\pi}{3}\right) \Rightarrow 2x = 2n\pi \pm \left(x - \frac{\pi}{3}\right)$

$x = (6n - 1)\frac{\pi}{3}$ or $(6n + 1)\frac{\pi}{9}$

$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}$ and $-\frac{5\pi}{9}$ in $(-\pi, \pi)$

$\Rightarrow$ sum = 0.

4. Ans. (0.50)

$$\sqrt{3}\cos x+\frac{2b}{a}\sin x=\frac{c}{a}$$

Now, $\sqrt{3}\cos\alpha+\frac{2b}{a}\sin\alpha=\frac{c}{a}$ ...(1)

$\sqrt{3}\cos\beta+\frac{2b}{a}\sin\beta=\frac{c}{a}$ ...(2)

$$\sqrt{3}[\cos\alpha-\cos\beta]+\frac{2b}{a}(\sin\alpha-\sin\beta)=0$$

$$\sqrt{3}\left[-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]+\frac{2b}{a}\left[2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)\right]=0$$

$$-\sqrt{3}+2\sqrt{3}.\frac{b}{a}=0$$

$$\frac{b}{a}=\frac{1}{2}=0.5$$

5. Ans. (C)

6. Ans. (B)

Solution (5 or 6)

$f(x)=\sin(\pi\cos x)$

$X:\{x:f(x)=0\}$

$f(x)=0\Rightarrow\sin(\pi\cos x)=0\Rightarrow\cos x=\text{n}\Rightarrow\cos x=1,-1,0\Rightarrow x=\frac{n\pi}{2}$

$$X=\left\{\frac{n\pi}{2}:n\in N\right\}=\left\{\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi,....\right\}$$

$g(x)=\cos(2\pi\sin x)$

$Z=\{x:g(x)=0\}$

$\cos(2\pi\sin x)=0\Rightarrow 2\pi\sin x=(2n+1)\frac{\pi}{2}\Rightarrow\sin x=\frac{(2n+1)}{4}$

$$\sin x=-\frac{1}{4},\frac{1}{4},\frac{-3}{4},\frac{3}{4}$$

$$Z=\left\{n\pi\pm\sin^{-1}\left(\frac{1}{4}\right),n\pi\pm\sin^{-1}\left(\frac{3}{4}\right),n\in I\right\}$$

$Y=\{x:f'(x)=0\}$

$f(x)=\sin(\pi\cos x)\Rightarrow f'(x)=\cos(\pi\cos x).(-\pi\sin x)=0$

$\sin x=0\Rightarrow x=n\pi.$

$\cos(\pi\cos x)=0\Rightarrow\pi\cos x=(2n+1)\frac{\pi}{2}\Rightarrow\cos x=\frac{(2n+1)}{2}\Rightarrow\cos x=-\frac{1}{2},\frac{1}{2}$

$$Y=\left\{n\pi,n\pi\pm\frac{\pi}{3}\right\}=\left\{\frac{\pi}{3},\frac{2\pi}{3},\pi,\frac{4\pi}{3},\frac{5\pi}{3},2\pi,......\right\}$$

$W=\{x:g'(x)=0\}$

$g(x)=\cos(2\pi\sin x)\Rightarrow g'(x)=-\sin(2\pi\sin x).(2\pi\cos x)=0$

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

$\sin(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = n\pi \Rightarrow \sin x = \frac{n}{2} = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1$

$W = \left\{\frac{n\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in I\right\} = \left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, ...\right\}$

Now check the options.

7. Ans. (1.00)

Let $\pi x - \frac{\pi}{4} = \theta \in \left[\frac{-\pi}{4}, \frac{7\pi}{4}\right]$

So, $\left(3 - \sin\left(\frac{\pi}{2} + 2\theta\right)\right)\sin\theta \geq \sin(\pi + 3\theta)$

$\Rightarrow (3 - \cos 2\theta)\sin\theta \geq -\sin 3\theta \Rightarrow \sin\theta[3 - 4\sin^2\theta + 3 - \cos 2\theta] \geq 0$

$\Rightarrow \sin\theta(6 - 2(1 - \cos 2\theta) - \cos 2\theta) \geq 0 \Rightarrow \sin\theta(4 + \cos 2\theta) \geq 0$

$\Rightarrow \sin\theta \geq 0 \Rightarrow \theta \in [0, \pi] \Rightarrow 0 \leq \pi x - \frac{\pi}{4} \leq \pi \Rightarrow x \in \left[\frac{1}{4}, \frac{5}{4}\right]$

$\Rightarrow \beta - \alpha = 1.$

8. Ans. (B)

(I) $\left\{x \in \left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$

$\cos x + \sin x = 1$

$\Rightarrow \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \frac{1}{\sqrt{2}} \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$; $n \in Z \Rightarrow x = 2n\pi$; $x = 2n\pi + \frac{\pi}{2}$; $n \in Z$

$\Rightarrow x \in \left\{0, \frac{\pi}{2}\right\}$ in given range has two solutions

(II) $\left\{x \in \left[\frac{-5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$

$\sqrt{3}\tan 3x = 1 \Rightarrow \tan 3x = \frac{1}{\sqrt{3}} \Rightarrow 3x = n\pi + \frac{\pi}{6} \Rightarrow x = (6n+1)\frac{\pi}{18}$; $n \in Z$

$\Rightarrow x \in \left\{\frac{\pi}{18}, \frac{-5\pi}{18}\right\}$ in given range has two solutions

(III) $\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$

$2\cos 2x = \sqrt{3}$

$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow 2x = 2n\pi \pm \frac{\pi}{6}$; $n \in Z$

$\Rightarrow x = n\pi \pm \frac{\pi}{12}$; $n \in Z$

$x \in \left\{\pm\frac{\pi}{12}, \pi \pm \frac{\pi}{12}, -\pi \pm \frac{\pi}{12}\right\}$

Six solutions in given range

(IV) $\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$

$\cos x - \sin x = -1$

$\Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4} \Rightarrow x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$; $n \in Z$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi - \pi$; $n \in Z$

$\Rightarrow x \in \left\{\frac{\pi}{2}, \frac{-3\pi}{2}, \pi, -\pi\right\}$ four solutions in given range.

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (3)**

$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3 \Rightarrow 3\tan 3x = 3$

$\Rightarrow \tan 3x = 1 \Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{12}, n \in I$

2. **Ans. (1)**

$4^{\tan x} - 3.2^{\tan x} + 2 \le 0 \Rightarrow t^2 - 3t + 2 \le 0,\ t = 2^{\tan x}$

$(t-1)(t-2) \le 0 \Rightarrow t \in [1, 2]$

$\Rightarrow 1 \le 2^{\tan x} \le 2$

$\Rightarrow 2^{\circ} \le 2^{\tan x} \le 2^1 \Rightarrow 0 \le \tan x \le 1 \Rightarrow x \in \left[0, \frac{\pi}{4}\right]$

$\because x \in \left[n\pi, n\pi + \frac{\pi}{4}\right], n \in I$

3. **Ans. (3)**

$\sin^2 x - \cos 2x = 2 - \sin 2x$

$\Rightarrow \sin^2 x - (1 - 2\sin^2 x) = 2 - 2\sin x \cos x$

$\Rightarrow 3\sin^2 x + 2\sin x \cos x = 3$

case-I : $\cos x \neq 0$

$\therefore 3\tan^2 x + 2\tan x = 3(1 + \tan^2 x)$

$\Rightarrow \tan x = \frac{3}{2}$

case-II : $\cos x = 0$

$\therefore 3(1) + 2(\pm 1)(0) = 3$

which is true $\therefore x = (2n+1)\frac{\pi}{2}$

4. **Ans. (1)**

Given $\cos x + \frac{1}{\cos x} = 2$

$\Rightarrow \cos^2 x - 2\cos x + 1 = 0 \Rightarrow \cos x = 1$

$\Rightarrow x = 2n\pi, n \in I$

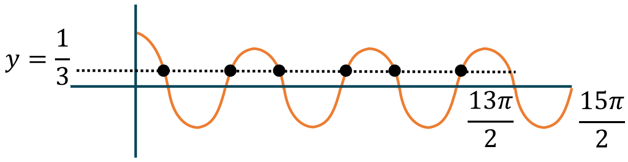
5. Ans. (4)

$2\tan^2 x - 5\sec x - 1 = 0$

$\Rightarrow 2(\sec^2 x - 1) - 5\sec x - 1 = 0 \Rightarrow 2\sec^2 x - 5\sec x - 3 = 0$

$\Rightarrow \sec x = \frac{6}{2}, \frac{-1}{2} = 3, \frac{-1}{2}$

$\Rightarrow \sec x = 3 \qquad \left(\sec x \neq \frac{-1}{2}\right)$



$\Rightarrow \cos x = \frac{1}{3} \Rightarrow$ 7 solutions in $\left[0, \frac{15\pi}{2}\right]$

$\therefore n = 15$

6. Ans. (3)

$16^{\cos^2 x} + 16^{1-\cos^2 x} = 10$

Let $16^{\cos^2 x} = y$, $y + \frac{16}{y} = 10$, $y^2 - 10y + 16 = 0$

$\Rightarrow y = 2, 8 \Rightarrow 16^{\cos^2 x} = 2, 8 \Rightarrow \cos^2 x = \frac{1}{4}, \frac{3}{4}$

$$\left.\begin{aligned} \cos^2 x = \frac{1}{4} &\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3} \\ \cos^2 x = \frac{3}{4} &\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \end{aligned}\right\} \text{12 solutions}$$

7. Ans. (1)

$\sqrt{3}\sin\theta - \cos\theta = \sqrt{2} \Rightarrow 2\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right) = \sqrt{2}$

$\Rightarrow 2\sin\left(\theta - \frac{\pi}{6}\right) = \sqrt{2} \Rightarrow \sin\left(\theta - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\Rightarrow \theta - \frac{\pi}{6} = n\pi + (-1)^n \frac{\pi}{4} \Rightarrow \theta = n\pi + \frac{\pi}{6} + (-1)^n \frac{\pi}{4}, n \in I$

8. Ans. (3)

$5\sin\theta + 2\cos\theta = 5$

$\Rightarrow \frac{5}{\sqrt{29}}\sin\theta + \frac{2}{\sqrt{29}}\cos\theta = \frac{5}{\sqrt{29}} \Rightarrow \sin\phi\sin\theta + \cos\phi\cos\theta = \frac{5}{\sqrt{29}}$

$\Rightarrow \cos(\theta - \phi) = \sin\phi = \cos\left(\frac{\pi}{2} - \phi\right) \Rightarrow \theta - \phi = 2n\pi \pm \left(\frac{\pi}{2} - \phi\right)$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{2} \mp \phi + \phi$

$\theta = 2n\pi + \frac{\pi}{2}, 2n\pi - \frac{\pi}{2} + 2\phi \Rightarrow \theta = 2n\pi + \frac{\pi}{2}, 2n\pi - \frac{\pi}{2} + 2\phi$

For $\theta = 2n\pi - \frac{\pi}{2} + 2\phi$, We have $\theta = 2n\pi + 2\left(\phi - \frac{\pi}{4}\right) = 2n\pi + 2\left(\tan^{-1}\frac{5}{2} - \tan^{-1}1\right)$

$= 2n\pi + 2\tan^{-1}\left(\frac{\frac{5}{2}-1}{1+\frac{5}{2}}\right) = 2n\pi + 2\tan^{-1}\left(\frac{3}{7}\right)$

$\therefore\ \theta = 2n\pi + \frac{\pi}{2}$ or $2n\pi + 2\alpha$ where $\tan^{-1}\frac{3}{7} = \alpha$

9. **Ans. (3)**

$\tan\theta = -1 \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi]$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\theta = \frac{\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi]$

common value is $x = \frac{7\pi}{4}$

general solution is $2n\pi + \frac{7\pi}{4},\ n \in I$.

10. **Ans. (3)**

Let $\tan\theta = t \Rightarrow \theta \in [0, \pi/2] \cup \{\pi, 2\pi\}$

$\Rightarrow \sqrt{t} = 2\sin\theta$

squaring

$\left(1+\frac{1}{t^2}\right)t = 4$

$t + \frac{1}{t} = 4 \Rightarrow t = 2 + \sqrt{3}, 2 - \sqrt{3}$

$\Rightarrow \theta = 0, \frac{\pi}{12}, \frac{5\pi}{12}, \pi, 2\pi$

11. **Ans. (1)**

$\log_2\left(\frac{\sin x + \cos x}{\cos x}\right) = -1 \Rightarrow \log_2(\tan x + 1) = -1 \Rightarrow \tan x + 1 = \frac{1}{2} \Rightarrow \tan x = -\frac{1}{2}$

$\Rightarrow x = \tan^{-1}\left(-\frac{1}{2}\right)$

12. **Ans. (2)**

$\sin x + \sin 2x + \sin 3x = 0$

$\Rightarrow 2\sin 2x\cos x + \sin 2x = 0 \Rightarrow \sin 2x = 0$ or $\cos x = \frac{-1}{2}$

13. **Ans. (4)**

$4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$

$2\cos\theta\,(2\sin\theta - 1) - \sqrt{3}\,(2\sin\theta - 1) = 0$

$(2\sin\theta - 1)(2\cos\theta - \sqrt{3}) = 0$

$\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

14. Ans. (3)

As $\sin^{50}x$ and $\cos^{50}x$ are both non-negative

clearly $\sin^{50}x = 1$ and $\cos^{50}x = 0$

$\Rightarrow \sin x = \pm 1, \cos x = 0 \Rightarrow x = n\pi + \frac{\pi}{2}$

15. Ans. (4)

Dividing by $\cos^2 x$, we get

$7 + \tan x - 3\sec^2 x = 0 \Rightarrow 7 + \tan x - 3 - 3\tan^2 x = 0$

this gives $\tan x = -1, \frac{4}{3}$

$\tan x = -1 \Rightarrow x = n\pi - \frac{\pi}{4}, n \in I$

$\tan x = \frac{4}{3} \Rightarrow x = m\pi + \tan^{-1}\left(\frac{4}{3}\right), m \in I$

16. Ans. (2)

$\cos 2x \le \cos x \Rightarrow 2\cos^2 x - \cos x - 1 \le 0$

$\Rightarrow 2\cos^2 x - 2\cos x + \cos x - 1 \le 0$

$\Rightarrow 2\cos x(\cos x - 1) + 1(\cos x - 1) \le 0$

$\Rightarrow (\cos x - 1)(2\cos x + 1) \le 0$

$\Rightarrow \cos x \in \left[-\frac{1}{2}, 1\right] \quad \therefore x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$

General solution is $x \in \left[2n\pi - \frac{2\pi}{3}, 2n\pi + \frac{2\pi}{3}\right], n \in I$

17. Ans. (1)

$\tan^2 x - \tan x - \sqrt{3}\tan x + \sqrt{3} < 0$

$\tan x(\tan x - 1) - \sqrt{3}(\tan x - 1) < 0$

$\Rightarrow (\tan x - 1)(\tan x - \sqrt{3}) < 0$

$\Rightarrow \tan x \in (1, \sqrt{3})$

$\Rightarrow x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$\therefore$ General solution is $x \in \left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}\right), n \in I$

18. Ans. (3)

$\tan^2\alpha + 2\sqrt{3}\tan\alpha - 1 = 0 \Rightarrow \tan\alpha = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = \frac{-2\sqrt{3} \pm 4}{2} = -\sqrt{3} \pm 2$

$= 2 - \sqrt{3}, -(2+\sqrt{3}) = \tan 15°, -\cot 15° = \tan\frac{\pi}{12}, \tan\left(\frac{-5\pi}{12}\right) \Rightarrow \alpha = n\pi + \frac{\pi}{12}, n\pi - \frac{5\pi}{12}$

or $n\pi + \frac{\pi}{12}, (2n-1)\frac{\pi}{2} + \frac{\pi}{12} \Rightarrow \alpha = \frac{2n\pi}{2} + \frac{\pi}{12}, (2n-1)\frac{\pi}{2} + \frac{\pi}{12} \Rightarrow \alpha = \frac{n\pi}{2} + \frac{\pi}{12}$.

19. **Ans. (1)**

$1 + 2\text{ cosec } x = \dfrac{-\sec^2\left(\dfrac{x}{2}\right)}{2} \Rightarrow 1 + \dfrac{2}{\sin x} = \dfrac{-1}{1+\cos x}$

$\Rightarrow (2 + \sin x)(1 + \cos x) = -\sin x$

$\Rightarrow 2 + 2\cos x + \sin x + \sin x \cos x = -\sin x$

$\Rightarrow 2(\sin x + \cos x) + \sin x \cos x + 2 = 0$

Put $\sin x + \cos x = t \Rightarrow 1 + 2\sin x \cos x = t^2$

$\therefore\ 2t + \dfrac{t^2-1}{2} + 2 = 0 \Rightarrow t^2 + 4t + 3 = 0$

$\Rightarrow t = -1, -3 \Rightarrow \sin x + \cos x = -1 \Rightarrow \cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{-1}{\sqrt{2}} = \cos\dfrac{3\pi}{4}$

$\Rightarrow x - \dfrac{\pi}{4} = 2n\pi \pm \dfrac{3\pi}{4} \Rightarrow x = 2n\pi + \pi, 2n\pi - \dfrac{\pi}{2} \Rightarrow x = 2n\pi + \pi$ at which cosec x is not defined

$\therefore\ x = 2n\pi - \dfrac{\pi}{2}$.

20. **Ans. (4)**

$|\cos x| = \cos x - 2\sin x$

Case-I : $\cos x \geq 0 \Rightarrow \cos x = \cos x - 2\sin x \Rightarrow \sin x = 0 \Rightarrow x = n\pi = 2m\pi$ or $(2m + 1)\pi$

As $\cos x \geq 0$ $\quad\quad \therefore\ x = 2m\pi$

Case-II : $\cos x < 0 \Rightarrow -\cos x = \cos x - 2\sin x \Rightarrow \tan x = 1 \Rightarrow x = n\pi + \dfrac{\pi}{4}$

$\Rightarrow x = 2m\pi + \dfrac{\pi}{4}$ or $(2m + 1)\pi + \dfrac{\pi}{4}$ As $\cos x < 0$

$\therefore\ x = (2m + 1)\pi + \dfrac{\pi}{4}$.

SECTION–B

1. **Ans. (2)**

$\sec\theta + \tan\theta = \sqrt{3}$ ...(i)

Also we have $\sec^2\theta - \tan^2\theta = 1$ ...(ii)

$\Rightarrow\ \sec\theta - \tan\theta = \dfrac{1}{\sqrt{3}}$ ...(iii)

Now (i) and (iii) gives,

$\tan\theta = \dfrac{1}{2}\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right) = \dfrac{1}{\sqrt{3}} = \tan\left(\dfrac{\pi}{6}\right)$

$\Rightarrow\ \theta = n\pi + \dfrac{\pi}{6}$.

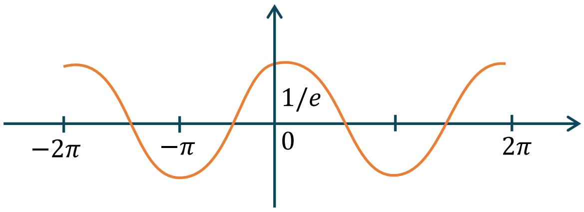
$\therefore$ Solutions for $0 \leq \theta \leq 2\pi$ are $\dfrac{\pi}{6}$ and $\dfrac{7\pi}{6}$.

Hence there are two solutions.

2. Ans. (4)

$\alpha - \beta = 0, \ -2\pi$ or 2π

$\alpha - \beta = 0 \Rightarrow \alpha = \beta \Rightarrow \cos 2\beta = \frac{1}{e}$



This is true for '4' value of 'α', 'β'

If $\alpha - \beta = -2\pi \Rightarrow \alpha = -\pi$ and $\beta = \pi$

and $\cos(\alpha + \beta) = 1 \Rightarrow$ (No solution)

similarly if $\alpha - \beta = 2\pi \Rightarrow \alpha = \pi$ and $\beta = -\pi$ again no solution results

3. Ans. (2)

$\sec^2\theta.\operatorname{cosec}^2\theta + 2\operatorname{cosec}^2\theta = 8 \Rightarrow \frac{1}{\sin^2\theta\cos^2\theta} + \frac{2}{\sin^2\theta} = 8$

$\Rightarrow 1 + 2\cos^2\theta = 8\sin^2\theta\cos^2\theta \Rightarrow 8\sin^2\theta\cos^2\theta - 2\cos^2\theta - 1 = 0$

$\Rightarrow 8(1 - \cos^2\theta)\cos^2\theta - 2\cos^2\theta - 1 = 0 \Rightarrow 8\cos^4\theta - 6\cos^2\theta + 1 = 0$

$\Rightarrow \cos^2\theta = \frac{4}{8}, \frac{2}{8} = \frac{1}{2}, \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{2} \Rightarrow \theta = 45°, 60°$ in $(0°, 90°)$

4. Ans. (17)

$\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0 \Rightarrow 2\cos 4\theta\cos 2\theta + (1 + \cos 4\theta) = 0$

$\Rightarrow 2\cos 4\theta\cos 2\theta + 2\cos^2 2\theta = 0 \Rightarrow 2\cos 2\theta(\cos 4\theta + \cos 2\theta) = 0$

$\Rightarrow \cos 2\theta(2\cos 3\theta\cos\theta) = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, (2n+1)\frac{\pi}{4}, (2n+1)\frac{\pi}{6}$

5. Ans. (6)

$3\sin^2 x - 7\sin x + 2 = 0$

$\Rightarrow 3\sin^2 x - 6\sin x - \sin x + 2 = 0 \Rightarrow 3\sin(\sin x - 2) - (\sin x - 2) = 0$

$\Rightarrow (3\sin x - 1)(\sin x - 2) = 0 \Rightarrow \sin x = \frac{1}{3}$ or $2 \Rightarrow \sin x = \frac{1}{3}, (\because \sin x \neq 2)$

Let $\sin^{-1}\frac{1}{3} = \alpha, 0 < \alpha < \frac{\pi}{2}$ are the solutions in $[0, 5\pi]$. Then $\alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha, 5\pi - \alpha$ are the solutions in $[0, 5\pi]$.

$\therefore$ Required number of solutions = 6.

6. Ans. (6)

$\cos x.\sin y = 1 \Rightarrow$ Either $\cos x = 1$ and $\sin y = 1$

or $\cos x = -1$ and $\sin y = -1 \Rightarrow (x, y) = \left(0, \frac{\pi}{2}\right), \left(0, \frac{5\pi}{2}\right), \left(2\pi, \frac{\pi}{2}\right), \left(2\pi, \frac{5\pi}{2}\right)$

or $(x, y) = \left(\pi, \frac{3\pi}{2}\right), \left(3\pi, \frac{3\pi}{2}\right)$ $\therefore$ Number of pairs = 6

7. Ans. (15)

$\sin 3\theta = 4\sin\theta\sin 2\theta\sin 4\theta \Rightarrow \sin 3\theta = (2\sin\theta)(2\sin 2\theta\sin 4\theta)$

$\Rightarrow 3\sin\theta - 4\sin^3\theta = 2\sin\theta(\cos 2\theta - \cos 6\theta) \Rightarrow 3 - 4\sin^2\theta = 2(\cos 2\theta - \cos 6\theta)$ or $\sin\theta = 0$

$\Rightarrow 3 - 2(1 - \cos 2\theta) = 2\cos 2\theta - 2\cos 6\theta$ or $\sin\theta = 0$

$\Rightarrow 1 = -2\cos 6\theta \Rightarrow \cos 6\theta = \frac{-1}{2}$ or $\sin\theta = 0$ $\therefore \sin\theta = 0$ or $\cos 6\theta = \frac{-1}{2}$

$\Rightarrow \theta = n\pi$ or $\theta = \frac{2n\pi \pm \left(\frac{2\pi}{3}\right)}{6} = \frac{n\pi}{3} \mp \frac{\pi}{9} \Rightarrow \theta = 0, \pi, \frac{\pi}{9}, \frac{\pi}{3} \mp \frac{\pi}{9}, \frac{2\pi}{3} \mp \frac{\pi}{9}, \pi - \frac{\pi}{9}$

So eight solutions.

8. Ans. (6)

The first equation can be written as

$$2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)=2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x+y)$$

$\therefore$ Either $\sin\frac{1}{2}(x+y)=0$ or $\sin\frac{1}{2}x=0$ or $\sin\frac{1}{2}y=0$

Thus $x+y=-1, x-y=-1$.

When $x+y=0$, we have to reject $x+y=1$ and check with the options or $x+y=-1$ and solve it with $x-y=1$ or $x-y=-1$ which gives $\left(\frac{1}{2},-\frac{1}{2}\right)$ or $\left(-\frac{1}{2},\frac{1}{2}\right)$ as the possible solution. Again solving with $x=0$, we get $(0, \pm 1)$ and solving with $y=0$, we get $(\pm 1, 0)$ as the other solution. Thus we have six pairs of solution for x and y.

9. Ans. (6)

$\cot x - 2\sin 2x = 1 \Rightarrow \frac{1}{\tan x} - \frac{4\tan x}{1+\tan^2 x} = 1 \Rightarrow 1 + t^2 - 4t^2 = t + t^3$ where $t = \tan x$

$\Rightarrow t^3 + 3t^2 + t - 1 = 0 \Rightarrow (t+1)(t^2 + 2t - 1) = 0 \Rightarrow t = -1, -1 \pm \sqrt{2} = -1, -1-\sqrt{2}, -1+\sqrt{2}$

$\Rightarrow \tan x = \tan\left(-\frac{\pi}{4}\right), -\tan\left(\frac{\pi}{2}-\frac{\pi}{8}\right), \tan\frac{\pi}{8} \Rightarrow x = n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{8}, n\pi + \frac{\pi}{8} - \frac{\pi}{2}$

$\Rightarrow x = n\pi - \frac{\pi}{4}, \frac{2n\pi}{2} + \frac{\pi}{8}, (2n-1)\frac{\pi}{2} + \frac{\pi}{8} = n\pi - \frac{\pi}{4}, \frac{n\pi}{2} + \frac{\pi}{8}$.

10. Ans. (6)

$2\sin^2 x + 2\sin^2 2x = 1 + 2\cos^2 3x$

$1 - \cos 2x + 1 - \cos 4x = 1 + 1 + \cos 6x$

$\cos 2x + \cos 4x + \cos 6x = 0$

$(\cos 4x)(2\cos 2x + 1) = 0$

$\cos 4x = 0$	$\cos 2x = -\frac{1}{2}$
$4x=(2n+1)\frac{\pi}{2}$	$2x=2n\pi\pm\frac{2\pi}{3}$
$x=(2n+1)\frac{\pi}{8}$	$x=n\pi\pm\frac{\pi}{3}$
$\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8}$	$\frac{\pi}{3},\frac{2\pi}{3}$

Total 6 solutions.

JEE (Advanced) Practice Paper

1. Ans. (A,B,C,D)

$2\sin 2x = \sin x + \sin 3x \Rightarrow 2\sin 2x = 2\sin x 2x \cos x \Rightarrow \sin 2x = 0$ or $\cos x = 1$

$\Rightarrow 2x = n\pi$ or $x = 2m\pi \Rightarrow x = \frac{n\pi}{2}$, $2m\pi$. options (A), (B), (C), (D) are all a part of $x = \frac{n\pi}{2}$.

2. Ans. (B,D)

$\sin x + \sin 2x + \sin 3x = 0 \Rightarrow 2\sin 2x \cos x + \sin 2x = 0 \Rightarrow \sin 2x = 0$ or $\cos x = \frac{-1}{2}$.

3. Ans. (A,D)

$\cos^2\theta = \frac{1}{8}$

$\cos\theta = \pm\frac{1}{2\sqrt{2}}$

Solutions are

$\theta, \pi-\theta, \pi+\theta, 2\pi-\theta, 2\pi+\theta, 3\pi-\theta,$

$3\pi+\theta, 4\pi-\theta$

common difference = $-\pi$ or π

4. Ans. (A,D)

Let $E = \sin x - \cos^2 x - 1 \Rightarrow E = \sin x - 1 + \sin^2 x - 1 = \sin^2 x + \sin x - 2$

$= \left(\sin x + \frac{1}{2}\right)^2 - \frac{9}{4}$ assumes least value when $\sin x = -\frac{1}{2} \Rightarrow x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$.

5. Ans. (A,C)

$\cos x.\cos 6x = -1 \Rightarrow$ Either $\cos x = 1$ and $\cos 6x = -1$ or $\cos x = -1$ and $\cos 6x = 1$

$\Rightarrow x = 2n\pi$ and $\cos 6x = -1$ or $x = (2n+1)\pi$ and $\cos 6x = 1$

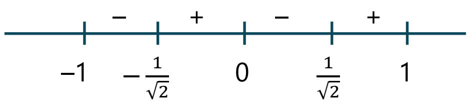
If $x = 2n\pi$ then $\cos 6x$ cannot be -1 However if $x = (2n+1)\pi$ then $\cos 6x = 1$ is possible

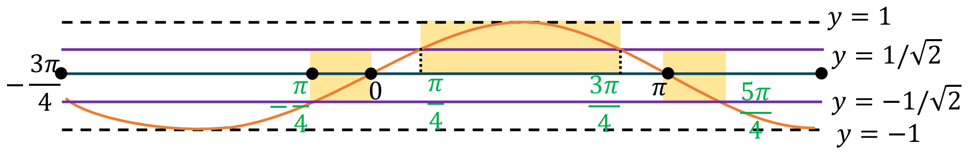
$\therefore x = (2n+1)\pi$ & $x = (2n-1)\pi$ are general solutions where $n \in I$.

6. Ans. (A,C,D)

$\sin 3x < \sin x \Rightarrow 3\sin x - 4\sin^3 x - \sin x < 0 \Rightarrow 2\sin x(1 - 2\sin^2 x) < 0$

$\Rightarrow \sin x\left(\sin x + \frac{1}{\sqrt{2}}\right)\left(\sin x - \frac{1}{\sqrt{2}}\right) > 0$





$\Rightarrow \sin x \in \left(\frac{-1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right) \Rightarrow x \in \left(\frac{-\pi}{4}, 0\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right)$

$\therefore$ General solution $x \in \bigcup_{n\in I}\left(2n\pi - \frac{\pi}{4}, 2n\pi\right) \cup \left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right) \cup \left(2n\pi + \pi, 2n\pi + \frac{5\pi}{4}\right)$

7. **Ans. (A,B,C,D)**

$2\sin\frac{x}{2}.\cos^2 x + \sin^2 x = 2\sin\frac{x}{2}.\sin^2 x + \cos^2 x \Rightarrow 2\sin\frac{x}{2}(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$

$\Rightarrow \cos 2x = 0$ or $2\sin\frac{x}{2} = 1$

$\Rightarrow \sin 2x = 1$ or -1 and $\cos x = 1 - 2\sin^2\frac{x}{2} = 1 - 2\left(\frac{1}{4}\right) = \frac{1}{2}$

$\cos 2x = 2\cos^2 x - 1 = 2 \times \frac{1}{4} - 1 = -\frac{1}{2}$

8. **Ans. (A,D)**

$\sec x + \operatorname{cosec} x = \sin x + \cos x$

$\Rightarrow \frac{\sin x + \cos x}{\sin x \cos} = \sin x + \cos x \Rightarrow (\sin x + \cos x)(1 - \sin x\cos x) = 0$

$\Rightarrow \sin x = -\cos x$ or $1 = \sin x\cos x \Rightarrow \tan x = -1$ or $\sin 2x = 2$ not possible

$\Rightarrow x = n\pi - \frac{\pi}{4}, x \in I \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

9. **Ans. (A,C)**

$5\sin^2 x + \sqrt{3}\sin x\cos x + 6\cos^2 x = 5$

Case-I: $\cos x = 0 \Rightarrow 5 + 0 + 0 = 5$

$\therefore \quad x = n\pi + \frac{\pi}{2}$

Case -II: $\cos x \neq 0$

$\therefore 5\tan^2 x + \sqrt{3}\tan x + 6 = 5(1 + \tan^2 x) \Rightarrow \tan x = -\frac{1}{\sqrt{3}}$

10. **Ans. (C,D)**

$\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$

case-I : $\cos x \neq 0$

$\therefore \tan^2 x + 2\tan x - 3 = 0$

$\Rightarrow \tan x = 3, 1$

$\Rightarrow x = n\pi + \tan^{-1}(-3), n\pi + \frac{\pi}{4}$

case-II: $\cos x = 0 \Rightarrow 1 + 0 - 0 = 0$ not true.

11. **Ans. (D)**

12. **Ans. (D)**

Solution (11, 12)

$[\sin(\alpha - \beta) + \cos(\alpha + 2\beta)\sin\beta]^2 = 4\cos\alpha\sin\beta\sin(\alpha + \beta)$

Use : $\sin(A - B), \sin(A + B)$

$\cos(A + B)$

$(\sin\alpha\cos\beta - \cos\alpha\sin\beta(1 - \cos 2\beta) - 2\sin\alpha\sin^2\beta\cos\beta)^2 = 4\cos\alpha\sin\beta\sin\alpha\cos\beta + 4\cos^2\alpha\sin^2\beta$

divide by $\cos^2\alpha\sin^2\beta$, $\sin\beta \neq 0$

we get $(3 - 8\sin^2\beta)^2 = 16$

$\sin^2\beta = -1$ or $\frac{7}{8}$

on check $\sin\beta = 0$ also satisfies system of equation

$\therefore\ \sin\beta = \sqrt{\frac{7}{8}}, -\sqrt{\frac{7}{8}}, 0$

13. Ans. (C)

$f(x) = \frac{1+\sin x}{1-\sin x}$ also $f(x) = \tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$\frac{1+\sin x}{1-\sin x} = \frac{4}{1+\tan^2 x}$

$\frac{\cos^2 x}{(1-\sin x)^2} = 4\cos^2 x$

$\cos^2 x = 0$ $(1-\sin x)^2 = \frac{1}{4}$

$\cos^2 x = 0$, $\sin x = \frac{1}{2}$, $\sin x = \frac{3}{2}$ (rejected)

$\sin x = \frac{1}{2}, \cos^2 x = 0(\text{rejected})$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

14. Ans. (B)

$9\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right) + 16\tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right) \geq 24$

15. Ans. (8)

Given equation is $\sin^2 3x - \sin^2 x$

$= \sin 3x \cos x - \cos 3x . \sin x$

$\Rightarrow \sin 2x(\sin 4x - 1) = 0$

$\Rightarrow \sin 2x = 0 \Rightarrow x = \frac{n\pi}{2}$

$\Rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$\sin 4x = 1$

$4x = (4m+1)\frac{\pi}{2}$ $\qquad \Rightarrow x = (4m+1)\frac{\pi}{8}$

$\Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

$\therefore\ \Sigma x_i = 5\pi + \frac{28\pi}{8} = 5\pi + \frac{7\pi}{2} = \frac{17\pi}{2} = \frac{a\pi}{b}$

$\Rightarrow \left[\frac{a}{b}\right] = \left[\frac{17}{2}\right] = 8$

16. Ans. (2)

Let $\sin x(\sin x + \cos x) = n \Rightarrow \sin^2 x + \sin x \cos x = n \Rightarrow 1 - \cos 2x + \sin 2x = 2n$

$\Rightarrow \sin 2x - \cos 2x = 2n - 1 \Rightarrow -\sqrt{2} \le 2n - 1 \le \sqrt{2} \Rightarrow \dfrac{1-\sqrt{2}}{2} \le n \le \dfrac{1+\sqrt{2}}{2} \Rightarrow n = 0, 1$

So number of integral values of $n = 2$

17. Ans. (0)

$\sin x \cos x - 3 \cos x + 4 \sin x - 12 - 1 > 0$

$\Rightarrow (\sin x - 3)(\cos x + 4) - 1 > 0$

$\Rightarrow$ (–ve) (+ve) $-1 > 0$ (which is not posible)

$\Rightarrow x \in \phi$

18. Ans. (6)

$5\cos\theta - \cos 3\theta = 4$

$\Rightarrow \cos^3\theta - 2\cos\theta + 1 = 0$

$\Rightarrow (\cos\theta - 1)(\cos^2\theta + \cos\theta - 1) = 0$

valid values of $\cos\theta = 1, \dfrac{\sqrt{5}-1}{2}$

$\therefore$ 6 solutions in $\left[-\dfrac{\pi}{2}, \dfrac{5\pi}{2}\right]$