

EXERCISE - O

SINGLE CORRECT TYPE QUESTIONS

- Let f be differentiable at $x = 0$ and $f'(0) = 1$. Then $\lim_{h \rightarrow 0} \frac{f(h) - f(-2h)}{h} =$
 (A) 3 (B) 2 (C) 1 (D) -1
MDY002
- If $f(x) = (x^5 + 1)|x^2 - 4x - 5| + \sin|x| + \cos(|x - 1|)$, then $f(x)$ is not differentiable at -
 (A) 2 points (B) 3 points (C) 4 points (D) zero points
MDY006
- Let $f(x) = \begin{cases} x^3 + 2x^2 & x \in \mathbb{Q} \\ -x^3 + 2x^2 + ax & x \notin \mathbb{Q} \end{cases}$, then the integral value of 'a' so that $f(x)$ is differentiable at $x = 1$, is
 (A) 2 (B) 6 (C) 7 (D) not possible
MDY007
- Let $\mathbb{R}$ be the set of real numbers and $f : \mathbb{R} \rightarrow \mathbb{R}$, be a differentiable function such that $|f(x) - f(y)| \leq |x - y|^3 \forall x, y \in \mathbb{R}$. If $f(10) = 100$, then the value of $f(20)$ is equal to -
 (A) 0 (B) 10 (C) 20 (D) 100
MDY008
- The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at :
 (A) -1 (B) 0 (C) 1 (D) 2
MDY009
- If $f(x) = \begin{cases} e^{x^2} + x^3 + 1 & ; x > 0 \\ ax^2 + bx + 2 & ; x \leq 0 \end{cases}$ is differentiable at $x = 0$, then -
 (A) $a \in \mathbb{R} \& b = 0$ (B) $a \in \mathbb{R} \& b = 1$ (C) $a \in \mathbb{R} \& b = 2$ (D) $a \in \mathbb{R} \& b = 3$
MDY010
- If $f(x) = [\sin x] + \sqrt{\sin x - [\sin x]}$, where $[.]$ denotes greatest integer function, then $f(x)$ is -
 (A) differentiable at $x = \frac{\pi}{2}$ & $x = \pi$ both (B) differentiable at $x = \frac{\pi}{2}$ but not at $x = \pi$
 (C) differentiable at $x = \pi$ but not at $x = \frac{\pi}{2}$ (D) neither differentiable at $x = \pi$ nor at $x = \frac{\pi}{2}$
MDY011
- If $f(x) = \begin{cases} \sin^{-1} x + a \cos \pi x & -1 < x < 0 \\ b \cos^{-1} x + \sin \pi x & 0 \leq x < 1 \end{cases}$ is differentiable in $(-1, 1)$, then $(2a + b)$ is equal to -
 (A) π^2 (B) $1 + \pi^2$ (C) $1 - \pi^2$ (D) $\pi^2 - 1$
MDY012
- Number of points where $f(x) = |x - \operatorname{sgn}(x)|$ is non-differentiable, is ($\operatorname{sgn}(\cdot)$ denotes signum function)
 (A) 0 (B) 1 (C) 2 (D) 3
MCT014

10. Let $f(x) = \begin{cases} \frac{\ln \cos x}{ax} & , x > 0 \\ 0 & , x = 0 \\ \frac{e^{x^2} - 1}{bx} & , x < 0 \end{cases}$. If $f'(0) = \frac{1}{4}$, then

- (A) $a + b = 2$ (B) $b - a = 2$ (C) $a + b = -2$ (D) $b - a = 4$

MCT017

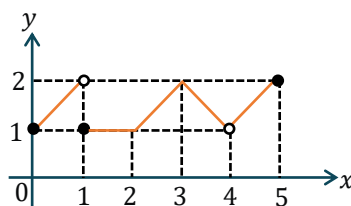
MULTIPLE CORRECT TYPE QUESTIONS

11. Select the correct statements -

- (A) The function f defined by $f(x) = \begin{cases} 2x^2 + 3 & \text{for } x \leq 1 \\ 3x + 2 & \text{for } x > 1 \end{cases}$ is neither differentiable nor continuous at $x = 1$.
 (B) The function $f(x) = x^2 |x|$ is twice differentiable at $x = 0$.
 (C) If f is continuous at $x = 5$ and $f(5) = 2$ then $\lim_{x \rightarrow 2} f(4x^2 - 11)$ exists
 (D) If $\lim_{x \rightarrow a} (f(x) + g(x)) = 2$ and $\lim_{x \rightarrow a} (f(x) - g(x)) = 1$ then $\lim_{x \rightarrow a} f(x) \cdot g(x)$ need not exist.

MDY043

12. Graph of $f(x)$ is shown in adjacent figure, then



- (A) $f(x)$ has non-removable discontinuity at two points
 (B) $f(x)$ is non-differentiable at three points in its domain
 (C) $\lim_{x \rightarrow 1} f(f(x)) = 1$
 (D) Number of points of discontinuity = number of points of non-differentiability

MDY045

13. Let S denotes the set of all points where $\sqrt[3]{x^2|x|^3} - \sqrt[3]{x^2|x|} - 1$ is not differentiable then S is a subset of -

- (A) $\{0, 1\}$ (B) $\{0, 1, -1\}$ (C) $\{0, 1\}$ (D) $\{0\}$

MDY046

14. Which of the following statements is/are correct ?

- (A) There exists a function $f : [0, 1] \rightarrow \mathbb{R}$ which is discontinuous at every point in $[0, 1]$ & $|f(x)|$ is continuous at every point in $[0, 1]$
 (B) Let $F(x) = f(x) \cdot g(x)$. If $f(x)$ is differentiable at $x = a$, $f(a) = 0$ and $g(x)$ is continuous at $x = a$ then $F(x)$ is always differentiable at $x = a$.
 (C) If $Rf'(a) = 2$ & $Lf'(a) = 3$, then $f(x)$ is non-differentiable at $x = a$ but will be always continuous at $x = a$
 (D) If $f(a)$ and $f(b)$ possess opposite signs then there must exist at least one solution of the equation $f(x) = 0$ in (a, b) provided f is continuous on $[a, b]$

MDY047

15. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function. Define $g : \mathbb{R} \rightarrow \mathbb{R}$ by $g(x) = |f(x)|$ for all x . Then which of the following is/are not always true-
 (A) If f is continuous then g is also continuous
 (B) If f is one-one then g is also one-one
 (C) If f is onto then g is also onto
 (D) If f is differentiable then g is also differentiable
MDY048
16. The function $\phi(x) = [x] - \sin|x|$ (where $[.]$ denotes greater integer function) is -
 (A) derivable at $x = 0$ (B) continuous at $x = 0$
 (C) $\lim_{x \rightarrow 0} \phi(x)$ does not exists (D) continuous and derivable at $x = 0$
MDY049
17. Let $f(x+y) + f(x-y) = 2f(x) \quad \forall x, y \in \mathbb{R}$, then which of the following is/are be correct
 ($f(0) = 0$ & f is differentiable $\forall x \in \mathbb{R}$)
 (A) f must be odd (B) f must be even (C) $f'(1) = f'(2)$ (D) $f'(2) = f'(3) = f'(4)$
MDY052
18. Let $f(x) = \begin{cases} 1-x & , (0 \leq x \leq 1) \\ x+2 & , (1 < x < 2) \\ 4-x & , (2 \leq x \leq 4) \end{cases}$. Then in $0 \leq x \leq 4$ function $y = f(f(x))$ is
 (A) continuous at $x = 1$
 (B) not differentiable at $x = 1$
 (C) continuous and differentiable at $x = 2$
 (D) continuous but not differentiable at $x = 3$
MDY053
19. If $\lim_{x \rightarrow 0} \frac{1 - \cos\left(1 - \cos \frac{x}{2}\right)}{2^m x^n}$ is equal to the left hand derivative of $e^{-|x|}$ at $x = 0$, then the value of $|n + 2m|$ is divisible by
 (A) 2 (B) 3 (C) 5 (D) 7
MDY054

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 20 to 21

A new function tignum (x) (i.e. $\text{tgn}(x)$) is defined such that $\text{tgn}(x) = \begin{cases} 1 & \text{if } [x] = \text{even} \\ -1 & \text{if } [x] = \text{odd} \end{cases}$, where $[.]$

denotes greatest integer function. Let $f(x) = \text{tgn}(x) \sin(x) |x|$.

On the basis of above information, answer the following questions :

20. Number of points in $[0, 10]$ where $f(x)$ is discontinuous is -
 (A) 8 (B) 9 (C) 10 (D) 11
MDY055
21. Number of points in $[-10, 10]$ where $f(x)$ is non- derivable is -
 (A) 18 (B) 19 (C) 20 (D) 21
MDY056

Paragraph for Question No. 22 to 23

Consider the function $f(x) = \begin{cases} x^2 - 5x + 6 & ; -2 \leq x \leq 4 \\ k - \tan\left(\frac{\pi x}{4}\right) & ; 4 < x \leq 5 \\ \log_{10}(\alpha x) & ; 5 < x \leq 10 \end{cases}$. Let $f_1(x) = |f(|x|)|$.

On the basis of above information, answer the following questions :

22. If $f(x)$ is continuous in $[-2, 10]$ then -

- (A) $\alpha = 2$ (B) $\alpha = 4$ (C) $k = 2$ (D) $k = 4$

MDY057

23. If $f(x)$ is continuous in $[-2, 10]$, then the number of points, where $f_1(x)$ is NOT differentiable, is -

- (A) 9 (B) 8 (C) 7 (D) 5

MDY058

MATCHING LIST TYPE QUESTION

24.

List-I

List-II

(P) If $f(x)$ is derivable at $x = 3$ & $f'(3) = 2$,

(1) 0

then $\lim_{h \rightarrow 0} \frac{f(3+h^2) - f(3-h^2)}{2h^2}$ equals

(Q) Let $f(x)$ be a function satisfying the condition $f(-x) = f(x)$ for all real x . If $f'(0)$ exists, then its value is equal to

(2) 1

(R) For the function $f(x) = \begin{cases} \frac{x}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

(3) 2

the derivative from the left $Lf'(0)$ equals

(S) The number of points at which the function $f(x) = \max\{a - x, a + x, b\}$, $-\infty < x < \infty$, $0 < a < b$ cannot be differentiable is

(4) 3

(A) (P) $\rightarrow$ 3, (Q) $\rightarrow$ 2, (R) $\rightarrow$ 1, (S) $\rightarrow$ 3

(B) (P) $\rightarrow$ 4, (Q) $\rightarrow$ 1, (R) $\rightarrow$ 2, (S) $\rightarrow$ 4

(C) (P) $\rightarrow$ 3, (Q) $\rightarrow$ 1, (R) $\rightarrow$ 2, (S) $\rightarrow$ 3

(D) (P) $\rightarrow$ 2, (Q) $\rightarrow$ 2, (R) $\rightarrow$ 1, (S) $\rightarrow$ 3

MDY059

EXERCISE - S

1. If the function $f(x)$ defined as $f(x) = \begin{cases} -\frac{x^2}{2} & \text{for } x \leq 0 \\ x^n \sin \frac{1}{x} & \text{for } x > 0 \end{cases}$ is continuous but not derivable at $x = 0$ then sum of all non-negative integral value(s) of 'n' is
MDY013
2. If $f(x) = |x - 1| \cdot ([x] - [-x])$, where $[x]$ denotes greatest integer function, then the value of $(Rf'(1) + Lf'(1))$ is
MDY014
3. For any real number x , let $[x]$ denote the largest integer less than or equal to x . Let f be a real valued function defined on the interval $[-3, 3]$ by $f(x) = \begin{cases} -x - [-x] & \text{if } [x] \text{ is even} \\ x - [x] & \text{if } [x] \text{ is odd} \end{cases}$
If L denotes the number of point of discontinuity and M denotes the number of points of non-derivability of $f(x)$, then find $(L + M)$.
MDY015
4. A derivable function $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ satisfies the condition $f(x) - f(y) \geq \ell n(x/y) + x - y$ for every $x, y \in \mathbb{R}^+$. If g denotes the derivative of f then compute the value of the sum $\sum_{n=1}^{100} g\left(\frac{1}{n}\right)$.
MDY016
5. Let $f(x) = [[x] + \{x^2\}] + \{[x^2] + \{x\}\}$, then number of points where $|f(x)|$ is non-derivable in $[-3, 3]$ is equal to (where $[.]$ denotes greatest integer function & $\{.\}$ denotes fractional part function)
MDY017
6. If f is a differentiable function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x) + f(y) + f(0)}{3}$, $\forall x, y \in \mathbb{R}$ and $f'(0) = 2$, then the slope of tangent to $f(x)$ at $x = 7$ is
MDY018
7. Let $f(x)$ be a differentiable function such that $\lim_{x \rightarrow 0} \frac{f(3 - \sin x) - f(3 + x)}{x} = 8$, then $|f'(3)|$ is
MDY019
8. Let $f(x)$ be a differentiable function such that $2f(x + y) + f(x - y) = 3f(x) + 3f(y) + 2xy$ $\forall x, y \in \mathbb{R}$ & $f'(0) = 0$, then $f(10) + f'(10)$ is equal to
MDY020
9. Let $f(x) = \max\{2\sin x, 1 - \cos x\} \forall x \in (0, \pi)$. If $f(x)$ not differentiable at $x = \pi - \cos^{-1} \frac{k}{5}$ then the value of k is
MDY078
10. $f(x) = \begin{cases} |1 - 4x^2| & , 0 \leq x < 1 \\ [x^2 - 2x] & , 1 \leq x < 2 \end{cases}$ then number of points where $f(x)$ is non-derivable in $x \in (0, 2)$ is equal to
MDY079

EXERCISE - JEE (Main) PYQ

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function satisfying $f'(3) + f'(2) = 0$.
Then $\lim_{x \rightarrow 0} \left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)} \right)^{\frac{1}{x}}$ is equal to [JEE (Main) 2019]
(1) e^2 (2) e (3) e^{-1} (4) 1 MDY021
2. Let f be a differentiable function from $\mathbb{R}$ to $\mathbb{R}$ such that $|f(x) - f(y)| \leq 2|x - y|^{\frac{3}{2}}$, for all $x, y \in \mathbb{R}$. If $f(0) = 1$ then $\int_0^1 f^2(x) dx$ is equal to [JEE (Main) 2019]
(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1 MDY022
3. Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \leq 2 \\ 8 - 2|x|, & 2 < |x| \leq 4 \end{cases}$. Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then S : [JEE (Main) 2019]
(1) is an empty set (2) equals $\{-2, -1, 1, 2\}$
(3) equals $\{-2, -1, 0, 1, 2\}$ (4) equals $\{-2, 2\}$ MDY023
4. Let $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 \leq x \leq 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2)$, g is:- [JEE (Main) 2019]
(1) differentiable at all points (2) not differentiable at two points
(3) not continuous (4) not differentiable at one point MDY027
5. Let S be the set of points where the function, $f(x) = |2 - |x - 3||$, $x \in \mathbb{R}$, is not differentiable. Then $\sum_{x \in S} f(f(x))$ is equal to _____. [JEE (Main) 2020] MDY030
6. Suppose a differentiable function $f(x)$ satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y . If $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$, then $f'(3)$ is equal to. [JEE (Main) 2020] MDY031
7. The function $f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \geq 1 \\ \frac{1}{2}(|x| - 1), & |x| < 1 \end{cases}$ is : [JEE (Main) 2020]
(1) continuous on $\mathbb{R} - \{1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$.
(2) both continuous and differentiable on $\mathbb{R} - \{-1\}$.
(3) continuous on $\mathbb{R} - \{-1\}$ and differentiable on $\mathbb{R} - \{-1, 1\}$.
(4) both continuous and differentiable on $\mathbb{R} - \{1\}$ MDY032
8. If the function $f(x) = \begin{cases} k_1(x - \pi)^2 - 1, & x \leq \pi \\ k_2 \cos x, & x > \pi \end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to : [JEE (Main) 2020]
(1) $\left(\frac{1}{2}, 1\right)$ (2) $(1, 1)$ (3) $\left(\frac{1}{2}, -1\right)$ (4) $(1, 0)$ MDY033

9. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2 & , x < 0 \\ 0 & , x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2 & , x > 0 \end{cases}$

The value of λ for which $f''(0)$ exists, is.

[JEE (Main) 2020]

MDY034

10. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in $\mathbb{R}$, where f is not differentiable. Then:

[JEE (Main) 2020]

- (1) $\{0, 1\}$ (2) $\{0\}$ (3) $\emptyset$ (an empty set) (4) $\{1\}$

MDY035

11. Let a function $g : [0, 4] \rightarrow \mathbb{R}$ be defined as $g(x) = \begin{cases} \max\{t^3 - 6t^2 + 9t - 3\}, & 0 \leq x \leq 3 \\ 4 - x & , 3 < x \leq 4 \end{cases}$

then the number of points in the interval $(0, 4)$ where $g(x)$ is NOT differentiable, is _____.

[JEE (Main) 2021]

MDY036

12. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } |x| \leq 2 \\ 0 & \text{if } |x| > 2 \end{cases}$$

Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in $\mathbb{R}$ where g is not continuous and not differentiable, respectively, then $n + m$ is equal to _____.

[JEE (Main) 2021]

MDY037

13. The number of points, at which the function $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in \mathbb{R}$ is not differentiable, is _____.

[JEE (Main) 2021]

MDY038

14. Let $[t]$ denote the greatest integer less than or equal to t . Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$, $x \in [-2, 2]$. Then h is :

[JEE (Main) 2021]

- (1) continuous in $[-2, 2]$ but not differentiable at more than four points in $(-2, 2)$
 (2) not continuous at exactly three points in $[-2, 2]$
 (3) continuous in $[-2, 2]$ but not differentiable at exactly three points in $(-2, 2)$
 (4) not continuous at exactly four points in $[-2, 2]$

MDY039

15. The function $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2 - 12x + 4|}$ is not differentiable at exactly :

[JEE (Main) 2021]

- (1) four points (2) three points (3) two points (4) one point

MDY040

16. If $[t]$ denotes the greatest integer $\leq t$, then number of points, at which the function

$$f(x) = 4|2x + 3| + 9\left[x + \frac{1}{2}\right] - 12[x + 20]$$

is not differentiable in the open interval $(-20, 20)$, is.

[JEE (Main) 2022]

MDY073

17. Let $f(x) = \min \{1, 1 + x \sin x\}$, $0 \leq x \leq 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to

[JEE (Main) 2022]

- (1) (2, 0) (2) (1, 0) (3) (1, 1) (4) (2, 1)

MDY074

18. Let $f: (-2, 2) \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x[x] & , -2 < x < 0 \\ (x-1)[x] & , 0 \leq x < 2 \end{cases}$$

Where $[x]$ denotes the greatest integer function. If m and n respectively are the number of points in $(-2, 2)$ at which $y = |f(x)|$ is not continuous and not differentiable, then $m + n$ is equal to ____.

[JEE (Main) 2023]

MDY075

19. Let $[x]$ denote the greatest integer function and $f(x) = \max \{1 + x + [x], 2 + x, x + 2[x]\}$, $0 \leq x \leq 2$. Let m be the number of points in $[0, 2]$, where f is not continuous and n be the number of points in $(0, 2)$, where f is not differentiable. Then $(m + n)^2 + 2$ is equal to:

[JEE (Main) 2023]

- (1) 11 (2) 2 (3) 6 (4) 3

MDY076

20. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$; Then at $x = 0$

[JEE (Main) 2023]

- (1) f is continuous but not differentiable
(2) f is continuous but f' is not continuous
(3) f and f' both are continuous
(4) f' is continuous but not differentiable

MDY077

EXERCISE - JEE (Advanced) PYQ

1. Let $f(x) = \begin{cases} x^2 \cos \frac{\pi}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, $x \in \mathbb{R}$, then f is - [JEE (Advanced) 2012]

- (A) differentiable both at $x = 0$ and at $x = 2$
 (B) differentiable at $x = 0$ but not differentiable at $x = 2$
 (C) not differentiable at $x = 0$ but differentiable at $x = 2$
 (D) differentiable neither at $x = 0$ nor at $x = 2$

MDY061

2. Let $f_1 : \mathbb{R} \rightarrow \mathbb{R}$, $f_2 : [0, \infty) \rightarrow \mathbb{R}$, $f_3 : \mathbb{R} \rightarrow \mathbb{R}$ and $f_4 : \mathbb{R} \rightarrow [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0, \\ e^x & \text{if } x \geq 0; \end{cases}$$

$$f_2(x) = x^2; f_3(x) = \begin{cases} \sin x & \text{if } x < 0, \\ x & \text{if } x \geq 0 \end{cases}$$

$$\text{and } f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0, \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0. \end{cases}$$

- | List-I | | List-II | |
|--------|--------------------|---------|--------------------------------|
| P. | f_4 is | 1. | onto but not one-one |
| Q. | f_3 is | 2. | neither continuous nor one-one |
| R. | $f_2 \circ f_1$ is | 3. | differentiable but not one-one |
| S. | f_2 is | 4. | continuous and one-one |

Codes :

- | | P | Q | R | S |
|-----|---|---|---|---|
| (A) | 3 | 1 | 4 | 2 |
| (B) | 1 | 3 | 4 | 2 |
| (C) | 3 | 1 | 2 | 4 |
| (D) | 1 | 3 | 2 | 4 |

[JEE (Advanced) 2014]

MDY062

3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define

$$h : \mathbb{R} \rightarrow \mathbb{R} \text{ by } h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \leq 0, \\ \min\{f(x), g(x)\} & \text{if } x > 0. \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

[JEE (Advanced) 2014]

MDY063

4. Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$. Then f is -

- (A) differentiable at $x = 0$ if $a = 0$ and $b = 1$
 (B) differentiable at $x = 1$ if $a = 1$ and $b = 0$
 (C) **NOT** differentiable at $x = 0$ if $a = 1$ and $b = 0$
 (D) **NOT** differentiable at $x = 1$ if $a = 1$ and $b = 1$

[JEE (Advanced) 2016]

MDY064

5. Let $f: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ and $g: \left[-\frac{1}{2}, 2\right] \rightarrow \mathbb{R}$ be function defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + |4x - 7|f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

- (A) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
 (B) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
 (C) g is NOT differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$
 (D) g is NOT differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

[JEE (Advanced) 2016]

MDY065

6. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x + y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in \mathbb{R}$. Then, then value of $\log_e(f(4))$ is ____.

[JEE (Advanced) 2018]

MDY066

7. Let $f_1: \mathbb{R} \rightarrow \mathbb{R}$, $f_2: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$, $f_3: \left(-1, e^{\frac{\pi}{2}} - 2\right) \rightarrow \mathbb{R}$ and $f_4: \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin(\sqrt{1 - e^{-x^2}})$

(ii) $f_2(x) = \begin{cases} |\sin x| & \text{if } x \neq 0 \\ \tan^{-1} x & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x + 2))]$, where for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t ,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

List-I

- P. The function f_1 is
 Q. The function f_2 is
 R. The function f_3 is
 S. The function f_4 is

List-II

1. NOT continuous at $x = 0$
 2. continuous at $x = 0$ and NOT differentiable at $x = 0$
 3. differentiable at $x = 0$ and its derivative is NOT continuous at $x = 0$
 4. differentiable at $x = 0$ and its derivative is continuous at $x = 0$

[JEE (Advanced) 2018]

The correct option is :

- (A) $P \rightarrow 2$; $Q \rightarrow 3$; $R \rightarrow 1$; $S \rightarrow 4$
 (B) $P \rightarrow 4$; $Q \rightarrow 1$; $R \rightarrow 2$; $S \rightarrow 3$
 (C) $P \rightarrow 4$; $Q \rightarrow 2$; $R \rightarrow 1$; $S \rightarrow 3$
 (D) $P \rightarrow 2$; $Q \rightarrow 1$; $R \rightarrow 4$; $S \rightarrow 3$

MDY067

8. Let the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^3 - x^2 + (x-1)\sin x$ and let $g : \mathbb{R} \rightarrow \mathbb{R}$ be an arbitrary function. Let $fg : \mathbb{R} \rightarrow \mathbb{R}$ be the product function defined by $(fg)(x) = f(x)g(x)$. Then which of the following statements is/are TRUE ? [JEE (Advanced) 2020]

- (A) If g is continuous at $x = 1$, then fg is differentiable at $x = 1$
 (B) If fg is differentiable at $x = 1$, then g is continuous at $x = 1$
 (C) If g is differentiable at $x = 1$, then fg is differentiable at $x = 1$
 (D) If fg is differentiable at $x = 1$, then g is differentiable at $x = 1$

MDY068

9. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be functions satisfying $f(x+y) = f(x) + f(y) + f(x)f(y)$ and $f(x) = xg(x)$ for all $x, y \in \mathbb{R}$. If $\lim_{x \rightarrow 0} g(x) = 1$, then which of the following statements is/are TRUE? [JEE (Advanced) 2020]

- (A) f is differentiable at every $x \in \mathbb{R}$
 (B) If $g(0) = 1$, then g is differentiable at every $x \in \mathbb{R}$
 (C) The derivative $f'(1)$ is equal to 1
 (D) The derivative $f'(0)$ is equal to 1

MDY069

10. Let the functions $f : (-1, 1) \rightarrow \mathbb{R}$ and $g : (-1, 1) \rightarrow (-1, 1)$ be defined by $f(x) = |2x - 1| + |2x + 1|$ and $g(x) = x - [x]$, where $[x]$ denotes the greatest integer less than or equal to x . Let $fog : (-1, 1) \rightarrow \mathbb{R}$ be the composite function defined by $(fog)(x) = f(g(x))$. Suppose c is the number of points in the interval $(-1, 1)$ at which fog is **NOT** continuous, and suppose d is the number of points in the interval $(-1, 1)$ at which fog is **NOT** differentiable. Then the value of $c + d$ is ____ [JEE (Advanced) 2020]

MDY070

11. Let $f : (0, 1) \rightarrow \mathbb{R}$ be the function defined as $f(x) = [4x] \left(x - \frac{1}{4} \right)^2 \left(x - \frac{1}{2} \right)$, where $[x]$ denotes the greatest integer less than or equal to x . Then which of the following statements is(are) true? [JEE (Advanced) 2023]

- (A) The function f is discontinuous exactly at one point in $(0, 1)$
 (B) There is exactly one point in $(0, 1)$ at which the function f is continuous but **NOT** differentiable
 (C) The function f is **NOT** differentiable at more than three points in $(0, 1)$
 (D) The minimum value of the function f is $-\frac{1}{512}$

MDY071

12. Let $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$. Then which of the following statements is(are) true ? [JEE (Advanced) 2023]

- (A) There are infinitely many functions from S to T
 (B) There are infinitely many strictly increasing functions from S to T
 (C) The number of continuous functions from S to T is at most 120
 (D) Every continuous function from S to T is differentiable

MDY072

JEE (Main) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-A

- This section contains **TWENTY** questions.
- Each question has **FOUR** options (1), (2), (3) and (4). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
Full Marks : +4, if only the bubble corresponding to the correct option is darkened.
Zero Marks : 0, if none of the bubbles is darkened.
Negative Marks : -1 in all other cases.

1. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then indicate the correct alternative(s):

- (1) $f(x)$ is continuous but not differentiable at $x = 0$
 (2) $f(x)$ is differentiable at $x = 0$
 (3) $f(x)$ is not differentiable at $x = 0$
 (4) none

MDY080

2. If $f(x) = \frac{x}{\sqrt{x+1} - \sqrt{x}}$ be a real valued function, then

- (1) $f(x)$ is continuous, but $f'(0)$ does not exist
 (2) $f(x)$ is differentiable at $x = 0$
 (3) $f(x)$ is not continuous at $x = 0$
 (4) $f(x)$ is not differentiable at $x = 0$

MDY081

3. The function $f(x) = \sin^{-1}(\cos x)$ is :

- (1) discontinuous at $x = 0$ (2) continuous at $x = 0$
 (3) differentiable at $x = 0$ (4) none of these

MDY082

4. If $f(x) = \begin{cases} x + \{x\} + x \sin\{x\} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$, where $\{ \cdot \}$ denotes the fractional part function, then:

- (1) f is continuous & differentiable at $x = 0$
 (2) f is continuous but not differentiable at $x = 0$
 (3) f is continuous & differentiable at $x = 2$
 (4) none of these.

MDY083

5. The set of all points where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable is:

- (1) $(-\infty, \infty)$ (2) $[0, \infty)$ (3) $(-\infty, 0) \cup (0, \infty)$ (4) $(0, \infty)$

MDY084

6. If $f(x)$ is differentiable everywhere, then :
- (1) $|f|$ is differentiable everywhere
 - (2) $|f|^2$ is differentiable everywhere
 - (3) $f|f|$ is not differentiable at some point
 - (4) $f + |f|$ is differentiable everywhere

MDY085

7. Let $f(x)$ be defined in $[-2, 2]$ by
- $$f(x) = \begin{cases} \max(\sqrt{4-x^2}, \sqrt{1+x^2}) & , -2 \leq x \leq 0 \\ \min(\sqrt{4-x^2}, \sqrt{1+x^2}) & , 0 < x \leq 2 \end{cases} \text{ , then } f(x) :$$

- (1) is continuous at all points
- (2) is not continuous at more than one point .
- (3) is not differentiable only at one point
- (4) is not differentiable at more than one point

MDY086

8. The number of points at which the function $f(x) = \max. \{a-x, a+x, b\}$, $-\infty < x < \infty$, $0 < a < b$ cannot be differentiable is:
- (1) 1
 - (2) 2
 - (3) 3
 - (4) none of these

MDY087

9. Let $f(x) = x - x^2$ and $g(x) = \begin{cases} \max f(t), 0 \leq t \leq x, 0 \leq x \leq 1 \\ \sin \pi x, x > 1 \end{cases}$, then in the interval $[0, \infty)$
- (1) $g(x)$ is everywhere continuous except at two points
 - (2) $g(x)$ is everywhere differentiable except at two points
 - (3) $g(x)$ is everywhere differentiable except at $x = 1$
 - (4) none of these

MDY088

10. Consider the following statements :

S_1 : Number of points where $f(x) = |x \operatorname{sgn}(1-x^2)|$ is non-differentiable is 3.

S_2 : Defined $f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1) & , x \leq 0 \\ \frac{\tan x - \sin x}{x^3} & , x > 0 \end{cases}$, In order that $f(x)$ be continuous at $x = 0$, 'a' should be equal to $\frac{1}{2}$

S_3 : The set of all points, where the function $\sqrt[3]{x^2|x|}$ is differentiable is $(-\infty, 0) \cup (0, \infty)$

S_4 : Number of points where $f(x) = \frac{1}{\sin^{-1}(\sin x)}$ is non-differentiable in the interval $(0, 3\pi)$ is 3.

State, in order, whether S_1, S_2, S_3, S_4 are true or false

- (1) TTTF
- (2) TTTT
- (3) FTTF
- (4) TFTT

MDY089

11. Consider the following statements :

S_1 : Let $f(x) = \frac{\sin(\pi [x - \pi])}{1 + [x]^2}$, where $[.]$ stands for the greatest integer function. Then $f(x)$ is

discontinuous at $x = n + \pi, n \in \mathbb{I}$

S_2 : The function $f(x) = p[x + 1] + q[x - 1]$, (where $[.]$ denotes the greatest integer function) is continuous at $x = 1$ if $p + q = 0$

S_3 : Let $f(x) = |[x]x|$ for $-1 \leq x \leq 2$, where $[.]$ is greatest integer function, then f is not differentiable at $x = 2$.

S_4 : If $f(x)$ takes only rational values for all real x and is continuous, then $f'(10) = 10$.

State, in order, whether S_1, S_2, S_3, S_4 are true or false

- (1) FTTT (2) TTTF (3) FTTF (4) FFTF

MDY090

12. For what triplets of real numbers (a, b, c) with $a \neq 0$ the function

$f(x) = \begin{cases} x, & x \leq 1 \\ ax^2 + bx + c, & \text{otherwise} \end{cases}$ is differentiable for all real x ?

- (1) $\{(a, 1-2a, a) \mid a \in \mathbb{R}, a \neq 0\}$ (2) $\{(a, 1-2a, c) \mid a, c \in \mathbb{R}, a \neq 0\}$
(3) $\{(a, b, c) \mid a, b, c \in \mathbb{R}, a + b + c = 1\}$ (4) $\{(a, 1-2a, 0) \mid a \in \mathbb{R}, a \neq 0\}$

MDY091

13. Given that $f'(2) = 6$ and $f'(1) = 4$, then $\lim_{h \rightarrow 0} \frac{f(2h+2+h^2) - f(2)}{f(h-h^2+1) - f(1)} =$

- (1) does not exist (2) is equal to $-3/2$ (3) is equal to $3/2$ (4) is equal to 3

MDY092

14. If $f(x + y) = f(x) \cdot f(y)$, $\forall x \& y \in \mathbb{N}$ and $f(1) = 2$, then the value of $\sum_{n=1}^{10} f(n)$ is

- (1) 2036 (2) 2046 (3) 2056 (4) 2066

MDY093

15. If $f(1) = 1$ and $f(n + 1) = 2f(n) + 1$ if $n \geq 1$, then $f(n)$ is equal to

- (1) $2^n + 1$ (2) 2^n (3) $2^n - 1$ (4) $2^{n-1} - 1$

MDY094

16. If $y = f(x)$ satisfies the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ ($x \neq 0$), then $f(x)$ is equal to

- (1) $-x^2 + 2$ (2) $-x^2 - 2$
(3) $x^2 - 2, x \in \mathbb{R} - \{0\}$ (4) $x^2 - 2, |x| \in [2, \infty)$

MDY095

17. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies the condition $x^2 f(x) + f(1 - x) = 2x - x^4$. Then $f(x)$ is:

- (1) $-x^2 - 1$ (2) $-x^2 + 1$ (3) $x^2 - 1$ (4) $-x^4 + 1$

MDY096

18. If $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function, such that $f(x + 2y) = f(x) + f(2y) + 4xy \forall x, y \in \mathbb{R}$. then

- (1) $f'(1) = f'(0) + 1$ (2) $f'(1) = f'(0) - 1$ (3) $f'(0) = f'(1) + 2$ (4) $f'(0) = f'(1) - 2$

MDY097

19. If $f(x) = \begin{cases} \sin^{-1} x + \frac{\pi}{2} & -1 \leq x \leq 0 \\ \log |x+1| - \pi & 0 < x < \pi \\ |\sin x| & \pi \leq x \leq 4\pi \end{cases}$ and number of points, where $f(x)$ is not differentiable is m and

number of points where $f(x)$ is discontinuous is n , $\forall x \in [-1, 4\pi]$, then $(m + n)$ is

- (1) 3 (2) 7 (3) 4 (4) 2

MDY098

20. Given $f(x) = (x-1)\sqrt{\ell nx}$, then at $x = 1$, $f(x)$ is-

- (1) discontinuous
(2) continuous but not differentiable
(3) differentiable but not continuous
(4) both continuous and differentiable

MDY099

SECTION-B

- This section will have **TEN** questions. Candidate can choose to attempt any 5 question out of these 10 questions. In case if candidate attempts more than 5 questions, first 5 attempted questions will be considered for marking.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (Answer should be rounded off to the nearest integer).
- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +4, if only correct answer is given.
Zero Marks : 0, if no answer is given.
Negative Marks : -1 for incorrect answer

1. Let $f : R \rightarrow R$ is a function satisfying $f(10 - x) = f(x)$ and $f(2 - x) = f(2 + x)$, $\forall x \in R$. If $f(0) = 101$, then the minimum possible number of values of x satisfying $f(x) = 101$ for $x \in [0, 30]$ is

MDY100

2. If $f(x) = \begin{cases} ax^2 - b & \text{if } |x| < 1 \\ -\frac{1}{|x|} & \text{if } |x| \geq 1 \end{cases}$ is derivable at $x = 1$, then value of $a + b$.

MDY101

3. Let $f(x)$ be a polynomial function satisfying the relation $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \forall x \in R - \{0\}$ and $f(3) = -26$, then $f(-2)$ is

MDY102

4. Let $f(x) = |x|^2 - 9|x| + 20$ and number of points where $f(x)$ is not differentiable is ' n ', then the value of ' n ' is

MDY103

5. Let on the graph of $y = f(x)$ where $f(x) = \begin{cases} 4x\sqrt{3} + a, & x \leq 0 \\ (7+b)x + 5\sqrt{2}, & x > 0 \end{cases}$

the points with abscissae $-1, 0, 1, 10$ are collinear then the value of $[a + b]$ where $[\cdot]$ denotes the G.I.F., is

MDY104

6. Let $f(xy) + f(x - y) + f(x + y + 1) = xy + 2x + 1$ and $P(x) = \frac{2016^{f(x)}}{2016^{f(x)} + \sqrt{2016}}$.

Find $P\left(\frac{1001}{2016}\right) + P\left(\frac{1015}{2016}\right)$

MDY105

7. $f(x) = \prod_{k=1}^{\infty} \left(\frac{1 + 2\cos\left(\frac{2x}{3^k}\right)}{3} \right)$, then number of points where $[x f(x)] + |x f(x)| + (x-1)|x^2 - 3x + 2|$ is

non-differentiable in $x \in (0, 3\pi)$ is equal to (where $[\cdot]$ denotes greatest integer function)

MDY106

8. If $f(x) = [2 + 3n\sin x]$ has exactly 11 points of discontinuity in $\left(0, \frac{\pi}{2}\right)$, then the number of integers in the range of n is (where $[\cdot]$ denotes greatest integer function)

MDY107

9. Let $f(x) = x^2 - x + k - 2, k \in R$. If the complete set of values of k for which $y = |f(|x|)|$ is non derivable at 5 distinct points is (a, b) then the value of $8(b - a)$ is

MDY108

10. Let $f(x) = (x^2 + \lambda x + 12), g(x) = x^2 + \mu x + 15, r(x) = x^2 + (\lambda + \mu)x + 36$ and $h(x) = f(x)|g(x)| + r(x)|f(x)| + g(x)|r(x)|$. If equations $f(x) = 0, g(x) = 0, r(x) = 0$ has real roots and $h(x)$ is non differentiable at exactly 3 points, then number of ordered pairs (λ, μ) , where $\lambda, \mu \in I$, is

MDY109

JEE (Advanced) Practice Paper

This paper is for yourself practice and assessment the discussion of this paper is optional though you can see PDF solutions or video solutions or solutions in hardcopy whichever is provided.

SECTION-I

- This section contains **FIVE** questions.
 - Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
 - For each question, darken the bubble corresponding to the correct option in the ORS.
 - For each question, marks will be awarded in one of the following categories:
 Full Marks : +3, if only the bubble corresponding to the correct option is darkened.
 Zero Marks : 0, if none of the bubbles is darkened.
 Negative Marks : -1, in all other cases
-
1. The functions defined by $f(x) = \max \{x^2, (x-1)^2, 2x(1-x)\}$, $0 \leq x \leq 1$
 (A) is differentiable for all x
 (B) is differentiable for all x except at one point
 (C) is differentiable for all x except at two points
 (D) is not differentiable at more than two points. **MDY110**
2. Let $f(x) = [n + p \sin x]$, $x \in (0, \pi)$, $n \in \mathbb{Z}$, p is a prime number and $[x]$ is greatest integer less than or equal to x . The number of points at which $f(x)$ is not differentiable is
 (A) p (B) $p-1$ (C) $2p+1$ (D) $2p-1$ **MDY111**
3. Let $f(x) = x^3 - x^2 + x + 1$ and $g(x) = \begin{cases} \max \{f(t) \text{ for } 0 \leq t \leq x\} & \text{for } 0 \leq x \leq 1 \\ 3 - x + x^2 & \text{for } 1 < x \leq 2 \end{cases}$ then:
 (A) $g(x)$ is continuous & derivable at $x = 1$
 (B) $g(x)$ is continuous but not derivable at $x = 1$
 (C) $g(x)$ is neither continuous nor derivable at $x = 1$
 (D) $g(x)$ is derivable but not continuous at $x = 1$ **MDY112**
4. If a differentiable function f satisfies $f\left(\frac{x+y}{3}\right) = \frac{4-2(f(x)+f(y))}{3} \forall x, y \in \mathbb{R}$, then $f(x)$ is equal to
 (A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{8}{7}$ (D) $\frac{4}{7}$ **MDY113**
5. If $f(x) = \begin{cases} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is
 (A) continuous as well differentiable at $x = 0$
 (B) continuous but not differentiable at $x = 0$
 (C) neither differentiable at $x = 0$ nor continuous at $x = 0$
 (D) none of these **MDY114**

SECTION-II

- This section contains **FOUR** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 if only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 if all the four options are correct but **ONLY** three options are chosen.

Partial Marks : +2 if three or more options are correct but **ONLY** two options are chosen, both of which are correct options.

Partial Marks : +1 if two or more options are correct but **ONLY** one option is chosen and it is a correct option.

Zero Marks : 0 if none of the options is chosen (i.e. the question is unanswered).

Negative Marks : -2 in all other cases.

For Example : If first, third and fourth are the **ONLY** three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

6. If $f(x) = \begin{cases} \cos^{-1}(\cos x) & |x| < \pi \\ \pi \log_{\pi} |x| & |x| \geq \pi \end{cases}$, then correct statement(s) is/are -

(A) $f(x)$ is not differentiable at three points.

(B) $f'(\pi^+) + f'(-\pi^-) = 0$

(C) $f'(\pi^+) = f'(-\pi^-)$

(D) $f'(0^+) + f'(0^-) = 0$

MDY115

7. The points at which the function, $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval $(0, 2)$ are:

(A) 1

(B) $\pi/2$

(C) $\pi/4$

(D) $1/2$

MDY116

8. $f(x) = (\sin^{-1} x)^2 \cdot \cos(1/x)$ if $x \neq 0$; $f(0) = 0$, $f(x)$ is:

(A) continuous no where in $-1 \leq x \leq 1$

(B) continuous everywhere in $-1 \leq x \leq 1$

(C) differentiable no where in $-1 \leq x \leq 1$

(D) differentiable everywhere in $-1 < x < 1$

MDY117

9. If $f(x) = a_0 + \sum_{k=1}^n a_k |x|^k$, where a_i 's are real constants, then $f(x)$ is

(A) continuous at $x = 0$ for all a_i

(B) differentiable at $x = 0$ for all $a_i \in R$

(C) differentiable at $x = 0$ for all $a_{2k-1} = 0$

(D) none of these

MDY118

SECTION-III

- This section contains **ONE** paragraph.
- Based on each paragraph, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D) **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :
Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 in all other cases.

Comprehension # 1 (Q. No. 10 - 12)

Let $f(x) = \begin{cases} x g(x) & , x \leq 0 \\ x + ax^2 - x^3 & , x > 0 \end{cases}$, where $g(t) = \lim_{x \rightarrow 0} (1 + a \tan x)^{t/x}$, a is positive constant, then

10. If a is even prime number, then $g(2) =$
 (A) e^2 (B) e^3 (C) e^4 (D) none of these MDY119
11. Set of all values of a for which function $f(x)$ is continuous at $x = 0$
 (A) $(-1, 10)$ (B) $(-\infty, \infty)$ (C) $(0, \infty)$ (D) none of these MDY120
12. If $f(x)$ is differentiable at $x = 0$, then $a \in$
 (A) $(-5, -1)$ (B) $(-10, 3)$ (C) $(0, \infty)$ (D) none of these MDY121

SECTION-IV

- This section contains **SIX** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, -0.33, -30, 30.27, -127.30, if answer is 11.36777..... then both 11.36 and 11.37 will be correct) by darkening the corresponding bubbles in the ORS.
For Example : If answer is -77.25, 5.2 then fill the bubbles as follows.

+ ● <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">0</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">2</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">●</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">7</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> </tr> </table>	●	●	0	0	0	0	1	1	1	1	1	1	2	2	2	2	●	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	●	6	6	6	6	6	6	7	7	●	●	7	7	8	8	8	8	8	8	9	9	9	9	9	9	● - <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">●</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">1</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">2</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> <td style="border: 1px solid black; padding: 2px;">3</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> <td style="border: 1px solid black; padding: 2px;">4</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">●</td> <td style="border: 1px solid black; padding: 2px;">5</td> <td style="border: 1px solid black; padding: 2px;">5</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> <td style="border: 1px solid black; padding: 2px;">6</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">7</td> <td style="border: 1px solid black; padding: 2px;">7</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> <td style="border: 1px solid black; padding: 2px;">8</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> <td style="border: 1px solid black; padding: 2px;">9</td> </tr> </table>	●	●	0	0	●	●	1	1	1	1	1	1	2	2	2	2	●	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	●	5	5	6	6	6	6	6	6	7	7	7	7	7	7	8	8	8	8	8	8	9	9	9	9	9	9
●	●	0	0	0	0																																																																																																																				
1	1	1	1	1	1																																																																																																																				
2	2	2	2	●	2																																																																																																																				
3	3	3	3	3	3																																																																																																																				
4	4	4	4	4	4																																																																																																																				
5	5	5	5	5	●																																																																																																																				
6	6	6	6	6	6																																																																																																																				
7	7	●	●	7	7																																																																																																																				
8	8	8	8	8	8																																																																																																																				
9	9	9	9	9	9																																																																																																																				
●	●	0	0	●	●																																																																																																																				
1	1	1	1	1	1																																																																																																																				
2	2	2	2	●	2																																																																																																																				
3	3	3	3	3	3																																																																																																																				
4	4	4	4	4	4																																																																																																																				
5	5	5	●	5	5																																																																																																																				
6	6	6	6	6	6																																																																																																																				
7	7	7	7	7	7																																																																																																																				
8	8	8	8	8	8																																																																																																																				
9	9	9	9	9	9																																																																																																																				

- Answer to each question will be evaluated according to the following marking scheme:
Full Marks : +3 If **ONLY** the correct numerical value is entered as answer.
Zero Marks : 0 In all other cases.

13. The number of points of non differentiability of the function $f(x) = |\sin x| + \sin |x|$ in $[-4\pi, 4\pi]$ is MDY122

14. If $f(x) = \begin{cases} \frac{\sin [x^2] \pi}{x^2 - 3x + 8} + ax^3 + b & , 0 \leq x \leq 1 \\ 2\cos \pi x + \tan^{-1} x & , 1 < x \leq 2 \end{cases}$ is differentiable in $[0, 2]$, then the value of $[a + b + 6]$

is

(Here $[\cdot]$ stands for the greatest integer function)

MDY123

15. If $f(x) = \begin{cases} x^2 e^{2(x-1)} & \text{for } 0 \leq x \leq 1 \\ a \operatorname{sgn}(x+1) \cos(2x-2) + bx^2 & \text{for } 1 < x \leq 2 \end{cases}$ is differentiable at $x = 1$ then $a^3 + b^3 =$

MDY124

16. Find number of points of non-differentiability of $f(x) = \lim_{n \rightarrow \infty} \frac{\{e^x\}^n - 1}{\{e^x\}^n + 1}$ in interval $[0, 1]$ where $\{ \cdot \}$ represents fractional part function

MDY125

17. Let $[x]$ denote the greatest integer less than or equal to x . The number of integral points in $[-1, 1]$ where $f(x) = [x \sin \pi x]$ is differentiable are

MDY126

18. If $g(x) = (|x-1| + |4x-11|) [x^2 - 2x - 2]$, then find the number of point of discontinuity of $g(x)$ in $\left(\frac{1}{2}, \frac{5}{2}\right)$
{where $[\cdot]$ denotes GIF}

MDY127

ANSWER KEY

EXERCISE - O

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	D	D	D	A	B	D	D	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	B,C	B,C	A,B,C,D	A,B,C,D	B,C,D	A,B,D	A,C,D	A,B	A,C	C
Que.	21	22	23	24						
Ans.	B	A,C	A	C						

EXERCISE - S

1.	1	2.	2	3.	8	4.	5150	5.	1
6.	2	7.	4	8.	120	9.	3	10.	2

EXERCISE - JEE (Main) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	3	4	3	10	1	1	5	1
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	1	4	2	1	3	79	2	4	4	2

EXERCISE - JEE (Advanced) PYQ

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	D	3	A,B	B,C	2	D	A,C	A,B,D	4
Que.	11	12								
Ans.	A,B	A,C,D								

JEE (Main) Practice Paper

Section-A	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	2	2	4	1	2	4	2	3	1
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	3	1	4	2	3	4	2	4	2	4
Section-B	Q.	1	2	3	4	5	6	7	8	9	10
	A.	11	2	9	5	6	1	5	1	2	2

JEE (Advanced) Practice Paper

Section-I	Q.	1	2	3	4	5		
	A.	C	D	C	D	B		
Section-II	Q.	6	7	8	9			
	A.	A,B,D	A,B,D	B,D	A,C			
Section-III	Q.	10	11	12				
	A.	C	C	C				
Section-IV	Q.	13	14	15	16	17	18	
	A.	7	4	7	0	3	2	

Important Notes