CHAPTER 19

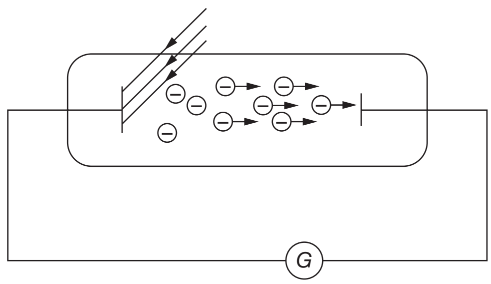
Modern Physics

Chapter Highlights

Dual nature of radiation. Photoelectric effect, Hertz and Lenard's observations; Einstein's photoelectric equation; particle nature of light. Matter waves-wave nature of particle, de Broglie relation. Davisson-Germer experiment. Alpha-particle scattering experiment; Rutherford's model of atom; Bohr model, energy levels, hydrogen spectrum. Composition and size of nucleus, atomic masses, isotopes, isobars; isotones. Radioactivity-alpha, beta and gamma particles/rays and their properties; radioactive decay law. Mass-energy relation, mass defect; binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

PHOTOELECTRIC EFFECT

When electromagnetic radiations of suitable wavelength are incident on a metallic surface then electrons are emitted, this phenomenon is called photo electric effect.



Photoelectron

The electron emitted in photoelectric effect is called photoelectron.

Photoelectric Current

If current passes through the circuit in photoelectric effect then the current is called photoelectric current.

Work Function

The minimum energy required to make an electron free from the metal is called work function. It is constant for a metal and denoted by ϕ or W. It is the minimum for Cesium. It is relatively less for alkali metals.

Work Functions of some Photosensitive Metals

Metal	Work function (eV)	Metal	Work function (eV)
Cesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

To produce photo electric effect only metal and light is necessary but for observing it the circuit is completed. Fig. 19.1 shows an arrangement used to study the photo-electric effect.

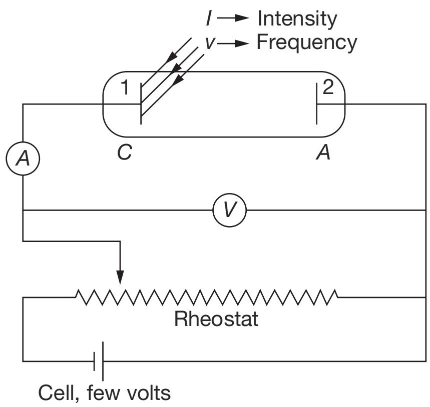


Fig. 19.1

Here the plate (1) is called emitter or cathode and other plate (2) is called collector or anode.

Saturation Current

When all the photo electrons emitted by cathode reach the anode then current flowing in the circuit at that instant is known as saturated current, this is the maximum value of photoelectric current.

Stopping Potential

Minimum magnitude of negative potential of anode with respect to cathode for which current is zero is called stopping potential. This is also known as cut-off voltage. This voltage is independent of intensity.

Retarding Potential

Negative potential of anode with respect to cathode which is less than stopping potential is called retarding potential.

OBSERVATIONS: (MADE BY EINSTEIN)

A graph between intensity of light and photoelectric current is found to be a straight line as shown in Fig. 19.2. Photoelectric current is directly proportional to the intensity of incident radiation. In this experiment the frequency and retarding potential are kept constant.

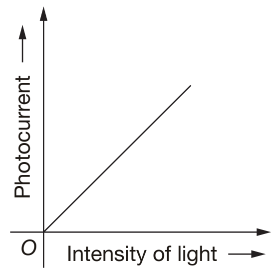


Fig. 19.2

A graph between photoelectric current and potential difference between cathode and anode is found as shown in Fig. 19.3.

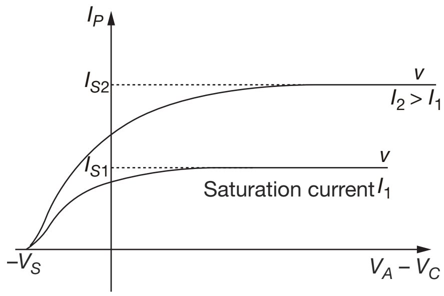


Fig. 19.3

In case of saturation current, rate of emission of photoelectrons = rate of flow of photoelectrons, here, $v_s \to$ stopping potential and it is a positive quantity

Electrons emitted from surface of metal have different energies.

Maximum kinetic energy of photoelectron on the cathode = eV_s

$$KE_{max} = eV_s$$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from

$$0 \text{ to } KE_{max} \quad \text{i.e.} \quad 0 \le KE_C \le eV_s$$

The energy distribution of photoelectron is shown in Fig. 19.4.

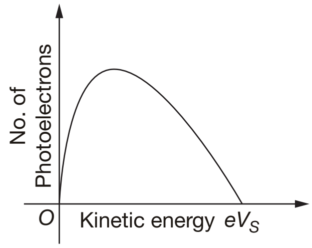


Fig. 19.4

If intensity is increased (keeping the frequency constant) then saturation current is increased by same factor by which intensity increases. Stopping potential is same, so maximum value of kinetic energy is not affected.

If light of different frequencies is used then obtained plots are shown in Fig. 19.5.

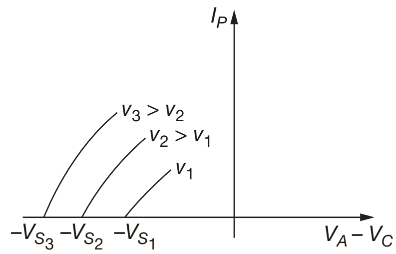


Fig. 19.5

It is clear from graph, as v increases, stopping potential increases, it means maximum value of kinetic energy increases.

Graphs between maximum kinetic energy of electrons ejected from different metals and frequency of light used are found to be straight lines of same slope as shown in Fig. 19.6

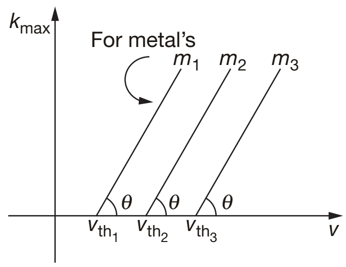


Fig. 19.6

Graph between K_{max} and ν

m_1, m_2, m_3: Three different metals.

It is clear from graph that there is a minimum frequency of electromagnetic radiation which can produce photoelectric effect, which is called threshold frequency.

ν_{th} = Threshold frequency

For photoelectric effect $\nu \geq \nu_{th}$

for no photoelectric effect $\nu < \nu_{th}$

Minimum frequency for photoelectric effect.

$$\nu_{min} = \nu_{th}$$

Threshold wavelength (λ_{th}) → The maximum wavelength of radiation which can produce photoelectric effect.

$$\lambda \leq \lambda_{th} \text{ for photo electric effect}$$

Maximum wavelength for photoelectric effect $\lambda_{max} = \lambda_{th}$.

Now, writing equation of straight line from graph.

We have $$K_{max} = A\nu + B$$

When $$\nu = \nu_{th},\ K_{max} = 0$$

and $$B = -A\nu_{th}$$

Hence $$[K_{max} = A(\nu - \nu_{th})]$$

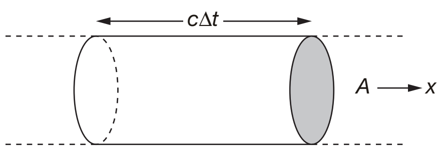
and A = tan $\theta = 6.63 \times 10^{-34}$ Js (from experimental data) later on A was found to be h.

It is also observed that photoelectric effect is an instantaneous process. When light falls on surface electrons, it starts ejecting without taking any time.

THREE MAJOR FEATURES OF THE PHOTOELECTRIC EFFECT CANNOT BE EXPLAINED IN TERMS OF THE CLASSICAL WAVE THEORY OF LIGHT

Intensity

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.



Consider a cylindrical volume with area of cross-section A and length $c\ \Delta t$ along the x-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained.

$$U = u_{av}(c \cdot \Delta t)A$$

The intensity is $$I = \frac{U}{A\Delta t} = u_{av}\, c.$$

In the terms of maximum electric field,

$$I = \frac{1}{2}\ \varepsilon_0\, E_0^{\ 2}\, c.$$

If we consider light as a wave then the intensity depends upon electric field.

If we take work function $W = I \,.\, A \,.\, t$,

then $$t = \frac{W}{IA}$$

so for photoelectric effect there should be time lag because the metal has work function.

But it is observed that photoelectric effect is an instantaneous process.

Hence, light is not of wave nature.

The Intensity Problem

Wave theory requires that the oscillating electric field vector E of the light wave increases in amplitude as the intensity of the light beam is increased. Since the force applied to the electron is eE, this suggests that the kinetic energy of the photoelectrons should also increase as the light beam is made more intense. However observation shows that maximum kinetic energy is independent of the light intensity.

The Frequency Problem

According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided only that the light is intense enough to supply the energy needed to eject the photoelectrons. However observations shows that there exists for each surface a characteristic cut-off frequency ν_{th}, for frequencies less than ν_{th}, the photoelectric effect does not occur, no matter how intense is light beam.

The Time Delay Problem

If the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, the "effective target area" for an electron in the metal is limited and probably not much more than that of a circle of diameter roughly equal to that of an atom. In the classical theory, the light energy is uniformly distributed over the wave front. Thus, if the light is feeble enough, there should be a measurable time lag, between the impinging of the light on the surface and the ejection of the photoelectron. During this interval the electron should be absorbing energy from the beam until it had accumulated enough to escape. However, no detectable time lag has ever been measured.

Now, quantum theory solves these problems in providing the correct interpretation of the photoelectric effect.

PLANCK'S QUANTUM THEORY

The light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence, energy $Q = NE$,

where $$E = h\nu$$

and N (number of photons) = 1,2,3,....

Here energy is quantized. $h\nu$ is the quantum of energy, it is a packet of energy called as photon.

$$E = h\nu = \frac{hc}{\lambda}$$

and $$hc \simeq 12400 \text{ eV Å}$$

EINSTEIN'S PHOTON THEORY

In 1905 Einstein made a remarkable assumption about the nature of light; namely, that, under some circumstances, it behaves as if its energy is concentrated into localized bundles, later called photons. The energy E of a single photon is given by

$$E = h\nu,$$

If we apply Einstein's photon concept to the photoelectric effect, we can write

$$h\nu = W + K_{max}, \quad \text{(energy conservation)}$$

Equation says that a single photon carries an energy $h\nu$ into the surface where it is absorbed by a single electron. Part of this energy W (called the work function of the emitting surface) is used in causing the electron to escape from the metal surface. The excess energy ($h\nu - W$) becomes the electron kinetic energy; if the electron does not lose energy by internal collisions as it escapes from the metal, it will still have this much kinetic energy after it emerges. Thus K_{max} represents the maximum kinetic energy that the photoelectron can have outside the surface. There is complete agreement of the photon theory with experiment.

Now $$IA = Nh\nu$$

$\Rightarrow N = \frac{IA}{h\nu}$ = number of photons incident per unit time on an area A when light of intensity I is incident normally.

If we double the light intensity, we double the number of photons and thus double the photoelectric current; we do not change the energy of the individual photons or the nature of the individual photoelectric processes.

The second objection (the frequency problem) is met if K_{max} equals zero, we have

$$h\nu_{th} = W,$$

Which asserts that the photon has just enough energy to eject the photoelectrons and none extra to appear as kinetic energy. If ν is reduced below ν_{th}, $h\nu$ will be smaller than W and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.

The third objection (the time delay problem) follows from the photon theory because the required energy is supplied in a concentrated bundle. It is not spread uniformly over the beam cross section as in the wave theory.

Hence Einstein's equation for photoelectric effect is given by

$$h\nu = h\nu_{th} + K_{max} \quad K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

SOLVED EXAMPLES

1. In an experiment on photo electric emission, following observations were made,

(i) Wavelength of the incident light = 1.98×10^{-7} m;

(ii) Stopping potential = 2.5 V

Find:

(A) Kinetic energy of photoelectrons with maximum speed.

(B) Work function and

(C) Threshold frequency.

Solution:

(A) Since $v_s = 2.5$ V, $K_{max} = eV_s$

so, $K_{max} = 2.5$ eV

(B) Energy of incident photon

$$E = \frac{12400}{1980} eV = 6.26 \text{ eV} \quad W = E - K_{max} = 3.76 \text{ eV}$$

(C) $h\nu_{th} = W = 3.76 \times 1.6 \times 10^{-19}$ J

$$\therefore \quad \nu_{th} = \frac{3.76 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \approx 9.1 \times 10^{14} \text{ Hz}.$$

2. A beam of light consists of four wavelength 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity 1.5×10^{-3} Wm^{-2}. The beam falls normally on an area 10^{-4} m^2 of a clean metallic surface of work function 1.9 eV. Assuming no loss of light energy (i.e. each capable photon emits one electron) calculate the number of photoelectrons liberated per second.

Solution:

$$E_1 = \frac{12400}{4000} = 3.1 \text{ eV}$$

$$E_2 = \frac{12400}{4800} = 2.58 \text{ eV}$$

$$E_3 = \frac{12400}{6000} = 2.06 \text{ eV}$$

and $$E_4 = \frac{12400}{7000} = 1.77 \text{ eV}$$

Therefore, light of wavelengths 4000 Å, 4800 Å and 6000 Å can only emit photoelectrons.

∴ Number of photoelectrons emitted per second = Number of photons incident per second)

$$= \frac{I_1A_1}{E_1} + \frac{I_2A_2}{E_2} + \frac{I_3A_3}{E_3}$$

$$= I\text{A}\left(\frac{1}{E_1}+\frac{1}{E_2}+\frac{1}{E_3}\right)$$

$$= \frac{(1.5\times10^{-3})(10^{-4})}{1.6\times10^{-19}}\left(\frac{1}{3.1}+\frac{1}{2.58}+\frac{1}{2.06}\right)$$

$$= 1.12 \times 10^{12}.$$

3. A metallic surface is irradiated with monochromatic light of variable wavelength. Above a wavelength of 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, stopping potential is 3 V. Find the unknown wavelength.

Solution:

using equation of photoelectric effect

$$K_{max} = E - W \ (K_{max} = eV_s)$$

$$\therefore \quad 3 \text{ eV} = \frac{12400}{\lambda} - \frac{12400}{5000}$$

$$= \frac{12400}{\lambda} - 2.48 \text{ eV}$$

or $$\lambda = 2262 \text{ Å}.$$

4. Illuminating the surface of a certain metal alternately with light of wavelengths $\lambda_1 = 0.35\ \mu$m and $\lambda_2 = 0.54\ \mu$m, it was found that the corresponding maximum velocities of photo electrons have a ratio $\eta = 2$. Find the work function of that metal.

Solution:

Using equation for two wavelengths

$$\frac{1}{2}mv_1^2 = \frac{hc}{\lambda_1} - W \qquad (1)$$

$$\frac{1}{2}mv_2^2 = \frac{hc}{\lambda_2} - W \qquad (2)$$

Dividing Equation (1) with Equation (2), with $v_1 = 2v_2$,

we have $$4 = \frac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W}$$

$$3W = 4\left(\frac{hc}{\lambda_2}\right) - \left(\frac{hc}{\lambda_1}\right)$$

$$= \frac{4\times12400}{5400} - \frac{12400}{3500} = 5.64 \text{ eV}.$$

5. Light described at a place by the equation $E = (100\text{ V/m})\ [\sin(5\times10^{15}\text{ s}^{-1})\,t + \sin(8\times10^{15}\text{ s}^{-1})\,t]$ falls on a metal surface having work function 2.0 eV. Calculate the maximum kinetic energy of the photoelectrons.

Solution:

The light contains two different frequencies. The one with larger frequency will cause photoelectrons with largest kinetic energy. This larger frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{8\times10^{15}\text{s}^{-1}}{2\pi}$$

The maximum kinetic energy of the photoelectrons is

$$K_{max} = h\upsilon - W$$

$$= (4.14 \times 10^{-15} \text{ eV-s}) \times \left(\frac{8\times10^{15}}{2\pi}\text{s}^{-1}\right) - 2.0 \text{ eV}$$

$$= 5.27 \text{ eV} - 2.0 \text{ eV} = 3.27 \text{ eV}.$$

6. Find the momentum of a 12.0 MeV photon.

Solution:

$$p = \frac{E}{c} = 12 \text{ MeV/c}.$$

7. Monochromatic light of wavelength 3000 Å is incident normally on a surface of area 4 cm^2. If the intensity of the light is 15×10^{-2} W/m^2, determine the rate at which photons strike the surface.

Solution:

Rate at which photons strike the surface

$$= \frac{IA}{hc/\lambda} = \frac{6\times10^{-5}\text{ J/s}}{6.63\times10^{-19}\text{ J/photon}}$$

$$= 9.05 \times 10^{13} \text{ photon/s}.$$

8. The kinetic energies of photoelectrons range from zero to 4.0×10^{-19} J when light of wavelength 3000 Å falls on a surface. What is the stopping potential for this light ?

Solution:

$$K_{max} = 4.0 \times 10^{-19} \text{ J} \times \frac{1\text{eV}}{1.6 \times 10^{-19}\text{J}} = 2.5 \text{ eV}.$$

Then, from $eV_s = K_{max}$, $V_s = 2.5$ V.

9. What is the threshold wavelength for the material in above problem?

Solution:

$$2.5 \text{ eV} = \frac{12.4 \times 10^3 \text{eV. Å}}{3000 \text{ Å}} - \frac{12.4 \times 10^3 \text{eV. Å}}{\lambda_{th}}$$

Solving, $\lambda_{th} = 7590$ Å.

FORCE DUE TO RADIATION (PHOTON)

Each photon has a definite energy and a definite linear momentum. All photons of light of a particular wavelength λ have the same energy $E = hc/\lambda$ and the same momentum $p = h/\lambda$.

When light of intensity I falls on a surface, it exerts force on that surface. Assume absorption and reflection coefficient of surface be a and r and assuming no transmission.

Assume light beam falls on surface of surface area A perpendicularly as shown in Fig. 19.7.

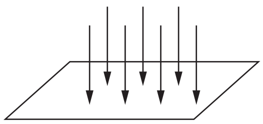


Fig. 19.7

For calculating the force exerted by beam on surface, we consider following cases.

Case: I

$$a = 1, r = 0$$

initial momentum of the photon $= \frac{h}{\lambda}$

final momentum of photon = 0

change in momentum of photon $= \frac{h}{\lambda}$ (upward)

$$\Delta P = \frac{h}{\lambda}$$

energy incident per unit time $= IA$

Number of photons incident per unit time $= \frac{IA}{h\nu} = \frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time $= n\,\Delta P$

$$= \frac{IA\lambda}{hc} \times \frac{h}{\lambda}$$

$$= \frac{IA}{c} \text{ (upward)}$$

force on photons = total change in momentum per unit time

$$= \frac{IA}{c} \text{ (upward)}$$

$\therefore$ force on plate due to photons$(F) = \frac{IA}{c}$ (downward)

$$\text{pressure} = \frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$$

Case: II

When $r = 1, a = 0$

initial momentum of the photon $= \frac{h}{\lambda}$ (downward)

final momentum of photon $= \frac{h}{\lambda}$ (upward)

change in momentum $= \frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda}$

$\therefore$ energy incident per unit time $= I$A

number of photons incident per unit time $= \frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time $= n \cdot \Delta P$

$$= \frac{IA\lambda}{hc} \cdot \frac{2h}{\lambda} = \frac{2IA}{C}$$

force = total change in momentum per unit time

$F = \frac{2IA}{c}$ (upward on photons and downward on the plate)

pressure $\quad P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

Case: III

When $\quad o < r < 1 \; a + r = 1$

change in momentum of photon when it is reflected $= \frac{2h}{\lambda}$ (upward)

change in momentum of photon when it is absorbed $= \frac{h}{\lambda}$ (upward)

Number of photons incident per unit time $= \frac{IA\lambda}{hc}$

Number of photons reflected per unit time $= \frac{IA\lambda}{hc} \,.\, r$

Number of photon absorbed per unit time $= \frac{IA\lambda}{hc}\,(1 - r)$

force due to absorbed photon $(F_a) = \dfrac{IA\lambda}{hc}(1-r)\,\dfrac{h}{\lambda}$.
$= \dfrac{IA}{c}(1-r)$ (downward)

Force due to reflected photon $(F_r) = \dfrac{IA\lambda}{hc}\cdot r\,\dfrac{2h}{\lambda} = \dfrac{2IA\lambda}{c}$ (downward)

Total force $= F_a + F_r$ (downward)

$$= \frac{IA}{c}(1-r) + \frac{2IAr}{c}$$

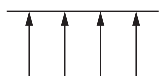
$$= \frac{IA}{c}(1+r)$$

Now pressure $P = (1+r)\times\dfrac{1}{A}$

$$= \frac{I}{c}(1+r).$$

SOLVED EXAMPLES

10. A plate of mass 10 gm is in equilibrium in air due to the force exerted by light beam on plate. Calculate power of beam. Assume plate is perfectly absorbing.



Solution:
Since plate is in air, so gravitational force will act on this

$$F_{gravitational} = mg \text{ (downward)}$$

$$= 10\times10^{-3}\times10$$

$$= 10^{-1}\text{ N}$$

for equilibrium force exerted by light beam should be equal to $F_{gravitational}$

$$F_{photon} = F_{gravitational}$$

Let power of light beam be P

$\therefore \quad F_{photon} = \dfrac{P}{c}$

$\Rightarrow \quad \dfrac{P}{c} = 10^{-1}$

$$P = 3.0\times10^{8}\times10^{-1}$$

$$P = 3\times10^{7}\text{ W}.$$

11. Calculate force exerted by light beam if light is incident on surface at an angle θ as shown in Fig. 19.8. Consider all cases.

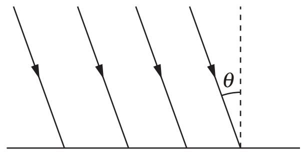
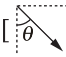
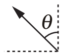


Fig. 19.8

Solution:
Case: I $a = 1, r = 0$
initial momentum of photon (in downward direction at an angle θ with vertical) $= \dfrac{h}{\lambda}$ []

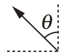
final momentum of photon = 0
change in momentum (in upward direction at an angle θ with vertical) $= \dfrac{h}{\lambda}$ []

energy incident per unit time $= IA\cos\theta$
Intensity = power per unit normal area

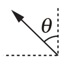
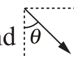
$$I = \frac{P}{A\cos\theta} \quad P = IA\cos\theta$$

Number of photons incident per unit time $= \dfrac{IA\cos\theta}{hc}.\lambda$

total change in momentum per unit time (in upward direction at an angle θ with vertical)

$$= \frac{IA\cos\theta\cdot\lambda}{hc}.\frac{h}{\lambda} = \frac{IA\cos\theta}{c}$$ []

Force (F) = total change in momentum per unit time

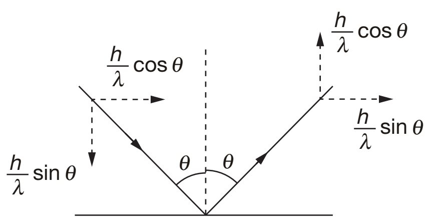
$F = \dfrac{IA\cos\theta}{c}$ (direction  on photon and  on the plate)

Pressure = normal force per unit Area

$$\text{Pressure} = \frac{F\cos\theta}{A} \quad P = \frac{IA\cos^2\theta}{cA} = \frac{I}{c}\cos^2\theta$$

Case: II When $r = 1, a = 0$
$\therefore$ change in momentum of one photon

$$= \frac{2h}{\lambda}\cos\theta \text{ (upward)}$$



Number of photons incident per unit time

$$= \frac{\text{Energy incident per unit time}}{h\nu}$$

$$= \frac{IA\cos\theta \cdot \lambda}{hc}$$

$\therefore$ total change in momentum per unit time

$$= \frac{IA\cos\theta \cdot \lambda}{hc} \times \frac{2h}{\lambda} \cos\theta = \frac{2IA\cos^2\theta}{c} \text{ (upward)}$$

$\therefore$ force on the plate $= \dfrac{2IA\cos^2\theta}{c}$ (downward)

Pressure $= \dfrac{2IA\cos^2\theta}{cA}$

$$P = \frac{2I\cos^2\theta}{c}$$

Case: III $0 < r < 1$, $a + r = 1$

change in momentum of photon when it is reflected = $\dfrac{2h}{\lambda}\cos\theta$ (downward)

change in momentum of photon when it is absorbed $= \dfrac{h}{\lambda}$ (in the opposite direction of incident beam)

energy incident per unit time $= IA\cos\theta$

Number of photons incident per unit time $= \dfrac{IA\cos\theta \cdot \lambda}{hc}$

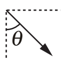
Number of reflected photon $(n_r) = \dfrac{IA\cos\theta \cdot \lambda r}{hc}$

Number of absorbed photon $(n_Q) = \dfrac{IA\cos\theta \cdot \lambda}{hc}(1 - r)$

Force on plate due to absorbed photons $F_a = n_a \cdot \Delta P_a$

$$= \frac{IA\cos\theta \cdot \lambda}{hc}(1 - r)\frac{h}{\lambda}$$

$$= \frac{IA\cos\theta}{c}(1 - r)$$

(at an angle θ with vertical 

)

Force on plate due to reflected photons $F_r = n_r \, \Delta P_r$

$$= \frac{IA\cos\theta \cdot \lambda}{hc} \times \frac{2h}{\lambda}\cos\theta \text{ (vertically downward)}$$

$$= \frac{IA\cos^2\theta}{c} \cdot 2r$$

Now resultant force is given by

$$F_R = \sqrt{F_r^2 + F_a^2 + 2F_aF_r\cos\theta}$$

$$= \frac{IA\cos\theta}{c}\sqrt{(1-r)^2 + (2r)^2\cos^2\theta + 4r(r-1)\cos^2\theta}$$

and, pressure

$$P = \frac{F_a\cos\theta + F_r}{A}$$

$$= \frac{IA\cos\theta(1-r)\cos\theta}{cA} + \frac{IA\cos^2\theta \cdot 2r}{cA}$$

$$= \frac{I\cos^2\theta}{c}(1 - r) + \frac{I\cos^2\theta}{c}2r$$

$$= \frac{I\cos^2\theta}{c}(1 + r).$$

DE-BROGLIE WAVELENGTH OF MATTER WAVE

A photon of frequency ν and wavelength λ has energy.

$$E = h\nu = \frac{hc}{\lambda}$$

By Einstein's energy mass relation, $E = mc^2$ the equivalent mass m of the photon is given by,

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$$

or $$\lambda = \frac{h}{\lambda c}$$

or $$\lambda = \frac{h}{p}$$

Here p is the momentum of photon. By analogy de-Broglie suggested that a particle of mass m moving with speed v behaves in some ways like waves of wavelength λ (called de-Broglie wavelength and the wave is called matter wave) given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad (19.1)$$

where p is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2Km}$$

and a charge q when accelerated by a potential difference V gains a kinetic energy $K = qV$. Combining all these relations Equation (19.2), can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}}$$

(de-Broglie wavelength) (19.2)

De-Broglie Wavelength for an Electron

If an electron (charge = e) is accelerated by a potential of V volts, it acquires a KE

$$K = \text{eV}$$

Substituting the values of h, m and q in Equation (19.2), we get a simple formula for calculating de-Broglie wavelength of an electron.

$$\lambda(\text{in Å}) = \sqrt{\frac{150}{V(\text{in volts})}} \quad (19.3)$$

De-Broglie Wavelength of a Gas Molecule

Let us consider a gas molecule at absolute temperature T. Kinetic energy of gas molecule is given by

$$KE = \frac{3}{2}kT;\ k = \text{Boltzman constant}$$

$$\therefore \quad \lambda_{\text{gas molecule}} = \frac{h}{\sqrt{3mkT}}$$

SOLVED EXAMPLES

12. An electron is accelerated by a potential difference of 50 V Find the de-Broglie wavelength associated with it.

Solution:

For an electron, de-Broglie wavelength is given by,

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{50}} = \sqrt{3} = 1.73\ \text{Å}.$$

13. Find the ratio of De-Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27ºC and 127ºC respectively.

Solution:

De-Broglie wavelength is given by

$$\therefore \quad \frac{\lambda_{H_2}}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_{H_2}T_{H_2}}} = \sqrt{\frac{4}{2}\cdot\frac{(127+273)}{(27+273)}} = \sqrt{\frac{8}{3}}.$$

14. Find the de Broglie wavelength of a 0.01 kg pellet having a velocity of 10 m/s.

Solution:

$$\frac{6.63\times10^{-34}\ \text{J.s}}{0.01\ \text{kg}\times10\ \text{m/s}} = 6.63\times10^{-23}\ \text{Å}.$$

15. Determine the accelerating potential necessary to give an electron a de Broglie wavelength of 1 Å, which is the size of the interatomic spacing of atoms in a crystal.

Solution:

$$V = \frac{h^2}{2m_0e\lambda^2} = 151\ \text{V}.$$

THOMSON'S ATOMIC MODEL

J.J. Thomson suggested that atoms are just positively charge lumps of matter with electrons embedded in them like raisins in a fruit cake. Thomson's model called the 'plum pudding' model is illustrated in Fig. 19.9.

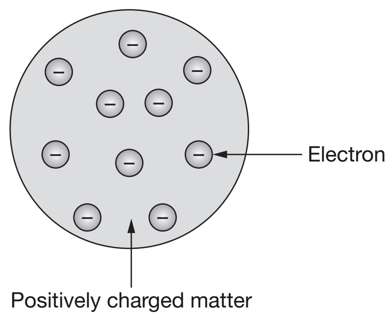


Fig. 19.9

Thomson played an important role in discovering the electron, through gas discharge tube by discovering cathode rays. His idea was taken seriously.

But the real atom turned out to be quite different.

RUTHERFORD'S NUCLEAR ATOM

Rutherford suggested that; "All the positive charge and nearly all the mass were concentrated in a very small volume of nucleus at the centre of the atom. The electrons were supposed to move in circular orbits round the nucleus (like planets round the sun). The electrostatic attraction between two opposite charges being the required centripetal force for such motion.

Hence $\quad \dfrac{mv^2}{r} = \dfrac{kZe^2}{r^2}$

and total energy = potential energy + kinetic energy = $\dfrac{-kZe^2}{2r}$

Rutherford's model of the atom, although strongly supported by evidence for the nucleus, is inconsistent with classical physics. This model suffers from two defects

Regarding Stability of Atom

An electron moving in a circular orbit round a nucleus is accelerating and according to electromagnetic theory it should therefore, emit radiation continuously and thereby lose energy. If total energy decreases then radius increases as given by above formula. If this happened the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of second. But atoms do not collapse. In 1913 an effort was made by Neils Bohr to overcome this paradox.

Regarding Explanation of Line Spectrum

In Rutherford's model, due to continuously changing radii of the circular orbits of electrons, the frequency of revolution of the electrons must be changing. As a result, electrons will radiate electromagnetic waves of all frequencies, i.e., the spectrum of these waves will be 'continuous' in nature. But experimentally the atomic spectra are not continuous. Instead they are line spectra.

THE BOHR'S ATOMIC MODEL

In 1913, Prof. Niel Bohr removed the difficulties of Rutherford's atomic model by the application of Planck's quantum theory. For this he proposed the following postulates

1. An electron moves only in certain circular orbits, called stationary orbits. In stationary orbits electron does not emit radiation, contrary to the predictions of classical electromagnetic theory.
2. According to Bohr, there is a definite energy associated with each stable orbit and atom radiates energy only when it makes a transition from one of these orbits to another. If the energy of electron in the higher orbit is E_2 and that in the lower orbit be E_1, then the frequency ν of the radiated waves is given by

   $$h\nu = E_2 - E_1$$

   or $$\nu = \frac{E_2 - E_1}{h}$$
3. Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be integral multiple of $\frac{h}{2\pi}$. The magnitude of the angular momentum is $L = mvr$ for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \; (n = 1, 2, 3....) \tag{19.4}$$

Each value of n corresponds to a permitted value of the orbit radius, which we will denote by r_n. The value of n for each orbit is called principal quantum number for the orbit. Thus,

$$mv_n r_n = mvr = \frac{nh}{2\pi} \tag{19.5}$$

According to Newton's second law a radially inward centripetal force of magnitude $F = \frac{mv^2}{r_n}$ is needed by the electron which is being provided by the electrical attraction between the positive proton and the negative electron.

Thus, $$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} \tag{19.6}$$

Solving Equation (19.5) and (19.6), we get

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \tag{19.7}$$

and $$v_n = \frac{e^2}{2\varepsilon_0 nh} \tag{19.8}$$

The smallest orbit radius corresponds to $n = 1$. We'll denote this minimum radius, called the Bohr radius as a_0. Thus,

$\bar{v}$

Substituting values of ε_0, h, p, m and e, we get

$$a_0 = 0.529 \times 10^{-10}, m = 0.529 \text{ Å} \tag{19.9}$$

Equation (19.7), in terms of a_0 can be written as,

$$r_n = n^2 a_0 \quad \text{or} \quad r_n \propto n^2 \tag{19.10}$$

Similarly, substituting values of e, ε_0 and h with $n = 1$ in Equation (19.8), we get

$$v_1 = 2.19 \times 10^6 \text{ m/s} \tag{9.11}$$

This is the greatest possible speed of the electron in the hydrogen atom, which is approximately equal to c/137, where c is the speed of light in vacuum.

Equation (19.9), in terms of v_1 can be written as,

$$v_n = \frac{v_1}{n} \quad \text{or} \quad v_n \propto \frac{1}{n}$$

Energy Levels

Kinetic and potential energies K_n and U_n in n^{th} orbit are given by

$$K_n = \frac{1}{2}\, m{v_n}^2 = \frac{me^4}{8{\varepsilon_0}^2 n^2 h^2}$$

and $$U_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{me^4}{4{\varepsilon_0}^2 n^2 h^2}$$

(assuming infinity as a zero potential energy level)

The total energy E_n is the sum of the kinetic and potential energies.

so, $$E_n = K_n + U_n = -\frac{me^4}{8{\varepsilon_0}^2 n^2 h^2}$$

Substituting values of m, e, ε_0 and h with $n = 1$, we get the least energy of the atom in first orbit, which is –13.6 eV. Hence,

$$E_1 = -13.6 \text{ eV} \quad (19.12)$$

and
$$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2} \text{ eV} \quad (19.13)$$

Substituting $n = 2, 3, 4, ...,$ etc., we get energies of atom in different orbits.

$$E_2 = -3.40 \text{ eV}, E_3 = -1.51 \text{ eV}, ... E_\infty = 0$$

Hydrogen Like Atoms

The Bohr model of hydrogen can be extended to hydrogen like atoms, i.e., one electron atoms, the nuclear charge is +ze, where z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 every where by ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under

$$r_n = \frac{\varepsilon_0 n^2 h^2}{nmze^2} = \frac{n^2}{z} a_0 \quad \text{or} \quad r_n \propto \frac{n^2}{z}$$

where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{ze^2}{2\varepsilon_0 nh} = \frac{z}{n} v_1 \quad \text{or} \quad v_n \propto \frac{z}{n}$$

where $v_1 = 2.19 \times 10^6$ m/s (speed of electron in first orbit of H)

$$E_n = -\frac{mz^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{z^2}{n^2} E_1 \quad \text{or} \quad E_n \propto \frac{z^2}{n^2}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)

Definitions Valid for Single Electron System

Ground State

Lowest energy state of any atom or ion is called ground state of the atom.

Ground state energy of H atom = –13.6 eV
Ground state energy of He^+ Ion = –54.4 eV
Ground state energy of Li^{++} Ion = –122.4 eV

Excited State

State of atom other than the ground state are called its excited states.

$n = 2$ first excited state
$n = 3$ second excited state
$n = 4$ third excited state
$n = n_0 + 1$ n_0^{th} excited state

Ionization Energy (I.E.)

Minimum energy required to move an electron from ground state to n = ∞ is called ionization energy of the atom or ion

Ionization energy of H atom = 13.6 eV
Ionization energy of He^+ Ion = 54.4 eV
Ionization energy of Li^{++} Ion = 122.4 eV

Ionization Potential (I.P.)

Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called ionization potential of the atom.

I.P of H atom = 13.6 V
I.P. of He^+ Ion = 54.4 V

Excitation Energy

Energy required to move an electron from ground state of the atom to any other exited state of the atom is called excitation energy of that state.

Energy in ground state of H atom = –13.6 eV
Energy in first excited state of H atom = –3.4 eV
Ist excitation energy = 10.2 eV.

Excitation Potential

Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.

Ist excitation energy = 10.2 eV.
Ist excitation potential = 10.2 V.

Binding Energy or Separation Energy

Energy required to move an electron from any state to $n = \infty$ is called binding energy of that state or energy released during formation of an H-like atom/ion from $n = \infty$ to some particular n is called binding energy of that state.

Binding energy of ground state of H atom = 13.6 eV

SOLVED EXAMPLE

16. First excitation potential of a hypothetical hydrogen like atom is 15 V Find third excitation potential of the atom.

Solution:

Let energy of ground state = E_0

$$E_0 = -13.6 \, Z^2 \text{ eV} \quad \text{and} \quad E_n = \frac{E_0}{n^2}$$

$$n = 2,\ E_2 = \frac{E_0}{4} \text{ given } \frac{E_0}{4} - E_0 = 15$$

$$-\frac{3E_0}{4} - = 15 \text{ for } n = 4,\ E_4 = \frac{E_0}{16}$$

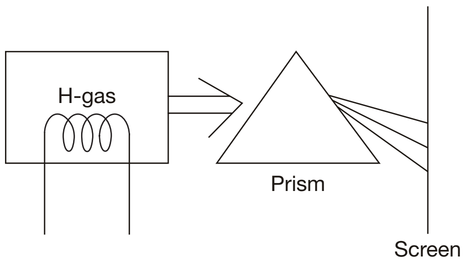
$$\text{third excitation energy} = \frac{E_0}{16} - E_0 = -\frac{15}{16}$$

$$E_0 = -\frac{15}{16}\cdot\left(\frac{-4\times 15}{3}\right) = \frac{75}{4} \text{ eV}$$

$\therefore$ third excitation potential is $\frac{75}{4}$ V.

Emission Spectrum of Hydrogen Atom

Under normal conditions the single electron in hydrogen atom stays in ground state (n = 1). It is excited to some higher energy state when it acquires some energy from external source. But it hardly stays there for more than 10^{-8} s.

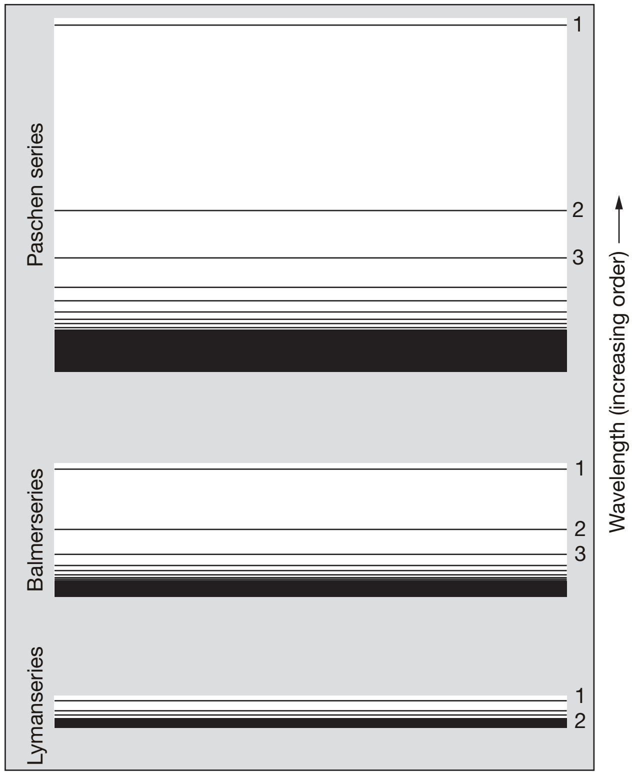


A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level.

Let n_i be the initial and n_f the final energy state, then depending on the final energy state following series are observed in the emission spectrum of hydrogen atom.

On Screen

A photograph of spectral lines of the Lymen, Balmer, Paschen series of atomic hydrogen.



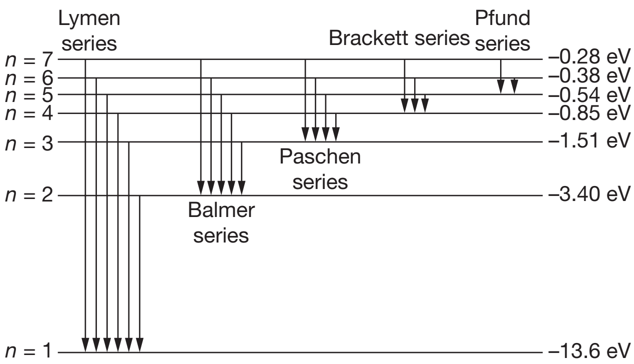
1, 2, 3..... represents the I, II and III line of Lymen, Balmer, Paschen series.

The hydrogen spectrum (some selected lines)

		Quantum Number			
Name of Series	**Number of Line**	**ni (Lower State)**	**nf (Upper State)**	**Wavelength (nm)**	**Energy**
Lymen	I	1	2	121.6	10.2 eV
	II	1	3	102.6	12.09 eV
	III	1	4	97	12.78 eV
	series limit	1	∞ (series limit)	91.2	13.6 eV
Balmer	I	2	3	656.3	1.89 eV
	II	2	4	486.1	2.55 eV
	III	2	5	434.1	2.86 eV
	series limit	2	∞ (series limit)	364.6	3.41 eV
Paschen	I	3	4	1875.1	0.66 eV
	II	3	5	1281.8	0.97 eV
	III	3	6	1093.8	1.13 eV
	series limit	3	∞ (series limit)	822	1.51 eV

Series Limit

Line of any group having maximum energy of photon and minimum wavelength of that group is called series limit.



For the Lymen series $n_f = 1$, for Balmer series $n_f = 2$ and so on.

Wavelength of Photon Emitted in De-excitation

According to Bohr when an atom makes a transition from higher energy level to a lower level it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $h\nu = \frac{hc}{\lambda}$, then conservation of energy gives,

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \text{ (energy of emitted photon)}$$

By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in the red and is called H_α, the next line, in the blue-green is called H_β and so on.

In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wave lengths of these lines. This is now called the Balmer series. The Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Here, $n = 3, 4, 5$ ...., etc.

R = Rydberg constant = $1.097 \times 10^7\ \text{m}^{-1}$

and λ is the wavelength of light/photon emitted during transition,

For $n = 3$, we obtain the wavelength of H_α line.

Similarly, for $n = 4$, we obtain the wavelength of H_β line. For $n = \infty$, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation, $E = \frac{hc}{\lambda}$ we can find the photon energies corresponding to the wavelength of the Balmer series.

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that,

$$E_n = -\frac{Rhc}{n^2},\ n = 1, 2, 3.....$$

The wavelengths corresponding to other spectral series (Lymen, Paschen, (etc.) can be represented by formula similar to Balmer's formula.

Lymen Series: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right),\ n = 2, 3, 4.....$

Paschen Series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right),\ n = 4, 5, 6.....$

Brackett Series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right),\ n = 5, 6, 7.....$

Pfund Series: $\frac{1}{m},\ n = 6, 7, 8$

The Lymen series is in the ultraviolet, and the Paschen. Brackett and Pfund series are in the infrared region.

SOLVED EXAMPLES

17. Calculate (A) the wavelength and (B) the frequency of the H_β line of the Balmer series for hydrogen.

Solution:

(A) H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. The corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= 0.2056 \times 10^7\ \lambda = 4.9 \times 10^{-7}\ \text{m}$$

(B) $\nu = r_0 = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14}$ Hz.

18. Find the largest and shortest wavelengths in the Lymen series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

Solution:

The transition equation for Lymen series is given by,

$$\frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right]\ n = 2, 3, ...$$

for largest wavelength, $n = 2$

$$\frac{1}{\lambda_{max}} = 1.097\times10^7\left(\frac{1}{1}-\frac{1}{4}\right) = 0.823\times10^7$$

$\therefore \quad \lambda_{max} = 1.2154\times10^{-7}$ m = 1215 Å

The shortest wavelength corresponds to $n = \infty$

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097\times10^7\left(\frac{1}{1}-\frac{1}{\infty}\right)$$

or $\quad \lambda_{min} = 0.911\times10^{-7}$ m = 911 Å

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

19. How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n?

Solution:

From the nth state, the atom may go to $(n-1)$th state, ..., 2nd state or 1st state. So there are $(n-1)$ possible transitions starting from the nth state. The atoms reaching $(n-1)$th state may make $(n-2)$ different transitions. Similarly for other lower states. The total number of possible transitions is

$$(n-1)+(n-2)+(n-3)+\ldots\ldots\ldots..2+1$$

$$= \frac{n(n-1)}{2} \text{ (Remember).}$$

20. (A) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

(B) How many spectral lines are observed in the emission spectrum of the above excited system?

Solution:

(A) The energy in the first orbit = $E_1 = Z^2 E_0$ where $E_0 = -13.6$ eV is the energy of a hydrogen atom in ground state thus for Li^{++},

$$E_1 = 9E_0 = 9\times(-13.6\text{ eV}) = -122.4\text{ eV}$$

The energy in the third orbit is

$$E_3 = \frac{E_1}{n^2} = \frac{E_1}{9} = -13.6\text{ eV}$$

Thus, $\quad E_3 - E_1 = 8\times13.6$ eV = 108.8 eV.

Energy required to excite Li^{++} from the first orbit to the third orbit is given by

$$E_3 - E_1 = 8\times13.6\text{ eV} = 108.8\text{ eV.}$$

The wavelength of radiation required to excite Li^{++} from the first orbit to the third orbit is given by

$$\frac{hc}{\lambda} = E_3 - E_1$$

or, $$\lambda = \frac{hc}{E_3 - E_1}$$

$$= \frac{1240\text{ eV}-\text{nm}}{108.8\text{ eV}} \approx 11.4\text{ nm}$$

(B) The spectral lines emitted are due to the transitions $n = 3 \rightarrow n = 2$, $n = 3 \rightarrow n = 1$

and $\quad n = 2 \rightarrow n = 1$.

Thus, there will be three spectral lines in the spectrum.

21. Find the kinetic energy potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Solution:

$$E_1 = -13.60\text{ eV } K_1 = -E_1 = 13.60\text{ eV}$$

$$U_1 = 2E_1 = -27.20\text{ eV}$$

$$E_2 = \frac{E_1}{(2)^2} = -3.40\text{ eV}$$

$$K_2 = 3.40\text{ eV}$$

and $\quad U_2 = -6.80$ eV

Now $U_1 = 0$, i.e., potential energy has been increased by 27.20 eV while kinetic energy will remain unchanged. So values of kinetic energy, potential energy and total energy in first orbit are 13.60 eV, 0, 13.60 respectively and for second orbit these values are 3.40 eV, 20.40 eV and 23.80 eV.

22. A lithium atom has three electrons, Assume the following simple picture of the atom. Two electrons move close to the nucleus making up a spherical cloud around it and the third moves outside this cloud in a circular orbit. Bohr's model can be used for the motion of this third electron but $n = 1$ states are not available to it. Calculate the ionization energy of lithium in ground state using the above picture.

Solution:

In this picture, the third electron moves in the field of a total charge $+3e - 2e = +e$. Thus, the energies are the same as that of hydrogen atoms. The lowest energy is:

$$E_2 = E_c = \frac{-13.6\text{ eV}}{4} = -3.4\text{ eV}$$

Thus, the ionization energy of the atom in this picture is 3.4 eV.

23. The energy levels of a hypothetical one electron atom are shown in the Fig. 19.10.

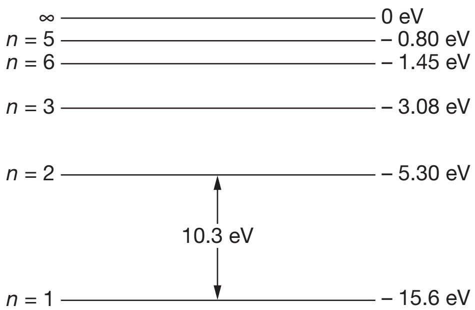


Fig. 19.10

(A) Find the ionization potential of this atom.
(B) Find the short wavelength limit of the series terminating at $n = 2$.
(C) Find the excitation potential for the state $n = 3$.
(D) Find wave number of the photon emitted for the transition $n = 3$ to $n = 1$.
(E) What is the minimum energy that an electron will have after interacting with this atom in the ground state if the initial kinetic energy of the electron is
(i) 6 eV (ii) 11 eV

Solution:

(A) Ionization potential = 15.6 V

(B) $\lambda_{\min} = \dfrac{12400}{5.3} = 2340$ Å

(C) $\Delta E_{31} = -3.08 - (-15.6) = 12.52$ eV
Therefore, excitation potential for state $n = 3$ is 12.52 volt.

(D) $$\frac{1}{\lambda_{31}} = \frac{\Delta E_{31}}{12400} \text{ Å}^{-1}$$
$$= \frac{12.52}{12400} \text{ Å}^{-1} 1.01 \times 10^7 \text{ m}^{-1}$$

(E) (i) $E_2 - E_1 = 10.3 \text{ eV} > 6 \text{ eV}$.
Hence electron cannot excite the atoms.
So, $K_{\min} = 6$ eV.
(ii) $E_2 - E_1 = 10.3 \text{ eV} < 11 \text{ eV}$.
Hence electron can excite the atoms.
So, $K_{\min} = (11 - 10.3) = 0.7$ eV.

24. A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

Solution:

The force at a distance r is,

$$F = -\frac{dU}{dr} = -2ar \qquad (1)$$

Suppose r be the radius of n^{th} orbit. The necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ar \qquad (2)$$

Further, the quantization of angular momentum gives,

$$mvr = \frac{nh}{2\pi}$$

Solving Equations (1) and (2) for r, we get

$$r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}.$$

25. An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge + 4e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to the system.
(A) Derive an expression for the radius of n^{th} Bohr orbit.
(B) Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to the second.

Solution:

(A) We have $$\frac{m_p v^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{ze^2}{r_n^2} \qquad (1)$$

The quantization of angular momentum gives,

$$m_p v r_n = \frac{nh}{2\pi} \qquad (2)$$

Solving Equations. (1) and (2), we get

$$r = \frac{n^2h^2\varepsilon_0}{z\pi m_p e^2}$$

Substituting $m_p = 100$ m
where m = mass of electron and $z = 4$

we get, $$r_n = \frac{n^2h^2\varepsilon_0}{400\,\pi m e^2}$$

(B) As we know,
Energy of hydrogen atom in ground state = – 13.60 eV

and $$E_n \propto \left(\frac{z^2}{n^2}\right) m$$

For the given particle,

$$E_4 = \frac{(-13.60)(4)^2}{4^2} \times 100 = 1360 \text{ eV}$$

$$= -1360 \text{ eV}$$

and $$E_2 = \frac{(-13.60)(4)^2}{(2)^2} \times 100 = -5440 \text{ eV}$$

$$E = E_4 - E_2 = 4080 \text{ eV}$$

$$\therefore \quad \lambda \text{ (in Å)} = \frac{12400}{4080} = 3.0 \text{ Å}.$$

26. A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelengths. All the wavelengths are equal or smaller than that of the absorbed photon.

(A) Determine the initial state of the gas atoms.
(B) Identify the gas atoms.
(C) Find the minimum wavelength of the emitted radiations.
(D) Find the ionization energy and the respective wavelength for the gas atoms.

Solution:

(A) $\frac{n(n-1)}{2} = 3$

$\therefore n = 3$

i.e., after excitation atom jumps to second excited state.

Hence $n_f = 3$. So n_i can be 1 or 2

If $n_i = 1$ then energy emitted is either equal to, greater than or less than the energy absorbed.

Hence the emitted wavelength is either equal to, less than or greater than the absorbed wavelength.

Hence $n_i \neq 1$.

If $n_i = 2$, then $E_e \geq E_a$.

Hence $\lambda_e \leq \lambda_0$

(B) $E_3 - E_2 = 68$ eV

$$\therefore (13.6)(Z^2)\left(\frac{1}{4} - \frac{1}{9}\right) = 68$$

$$\therefore \quad Z = 6$$

(C) $$\lambda_{min} = \frac{12400}{E_3 - E_1} = \frac{12400}{(13.6)(6)^2\left(1 - \frac{1}{9}\right)}$$

$$= \frac{12400}{435.2} = 28.49$$

(D) Ionization energy = $(13.6)(6)^2 = 489.6$ eV

$$\lambda = \frac{12400}{489.6} = 25.33 \text{ Å}.$$

27. An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantization of angular momentum holds good for this electron, find

(A) The allowed values of the radius r of the orbit.
(B) The kinetic energy of the electron in orbit.
(C) The potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.
(D) The total energy of the allowed energy levels.

Solution:

(A) Radius of circular path

$$r = \frac{mv}{Be} \tag{1}$$

$$mvr = \frac{nh}{2\pi} \tag{2}$$

Solving these two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}}$$

and $$v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

(B) $$K = \frac{1}{2} mv^2 = \frac{nhBe}{4\pi m}$$

(C) $$M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{evr}{2}$$

$$= \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}}$$

$$= \frac{\lambda hc}{4\pi m}$$

Now potential energy $U = -$ M . B

$$= \frac{\lambda heB}{4\pi m}$$

(D) $$E = U + K = \frac{nheB}{2\pi m}.$$

28. Determine the wavelength of the second line of the Paschen series for hydrogen.

Solution:

$$\frac{1}{\lambda} = (1.097 \times 10^{-3} \text{ Å}^{-1})\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

or $$\lambda = 12{,}820 \text{ Å}.$$

29. How many different photons can be emitted by hydrogen atoms that undergo transitions to the ground state from the $n = 5$ state?

Solution:

10 photons.

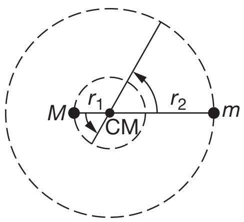
30. An electron rotates in a circle around a nucleus with positive charge Ze. How is the electrons' velocity related to the radius of its orbit?

Solution:

$$v = \sqrt{\frac{kZe^2}{mr}}.$$

EFFECT OF NUCLEUS MOTION ON ENERGY OF ATOM

Let both the nucleus of mass M, charge Ze and electron of mass m, and charge e revolve about their centre of mass (CM) with same angular velocity (ω) but different linear speeds. Let r_1 and r_2 be the distance of CM from nucleus and electron. Their angular velocity should be same then only their separation will remain unchanged in an energy level.



Let r be the distance between the nucleus and the electron. Then

$$Mr_1 = mr_2$$

$$r_1 + r_2 = r$$

$$\therefore \quad r_1 = \frac{mr}{M+m} \quad \text{and} \quad r_2 = \frac{Mr}{M+m}$$

Centripetal force to the electron is provided by the electrostatic force. So,

$$mr_2\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

or
$$m\left(\frac{Mr}{M+m}\right)\omega^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r^2}$$

or
$$\left(\frac{Mm}{M+m}\right) r^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$$

or
$$\mu r^3\omega^2 = \frac{e^2}{4\pi\varepsilon_0}$$

where
$$\frac{Mm}{M+m} = \mu$$

Moment of inertia of atom about CM,

$$I = Mr_1^2 + mr_2^2 = \left(\frac{Mm}{M+m}\right) r^2 = \mu r^2$$

According to Bohr's theory, $\frac{nh}{2\pi} = I\,\omega$

or
$$\mu r^2\omega = \frac{nh}{2\pi}$$

Solving above equations for r, we get

$$r = \frac{\varepsilon_0 n^2 h^2}{\pi\mu e^2 Z} \quad \text{and} \quad r = (0.529\text{ Å})\,\frac{n^2}{Z}\cdot\frac{m}{\mu}$$

Further electrical potential energy of the system,

$$U = \frac{-Ze^2}{4\pi\varepsilon_0 r} \qquad U = \frac{-Z^2e^4\mu}{4\varepsilon_0^2 n^2 h^2}$$

and kinetic energy,

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\mu r^2\omega^2 \text{ and } K = \frac{1}{2}\mu\text{N}^2$$

and
$$K = \frac{1}{2}\,\mu v^2$$

v-speed of electron with respect to nucleus. ($v = r\omega$)

here $\omega^2 = \dfrac{Ze^2}{4\pi\varepsilon_0\,\mu r^3}$

$$\therefore \quad K = \frac{Ze^2}{8\pi\varepsilon_0 r} = \frac{Z^2e^4\mu}{8\pi\varepsilon_0^2 n^2 h^2}$$

$\therefore$ Total energy of the system $E_n = K + U$

$$E_n = -\frac{\mu e^4}{8\varepsilon_0^2 n^2 h^2}$$

this expression can also be written as

$$E_n = -(13.6\text{ eV})\,\frac{Z^2}{n^2}\cdot\left(\frac{\mu}{m}\right)$$

The expression for E_n without considering the motion of proton is $E_n = -\dfrac{me^4}{8\varepsilon_0^2 n^2 h^2}$, i.e., m is replaced by μ while considering the motion of nucleus.

SOLVED EXAMPLES

31. A positronium 'atom' is a system that consists of a positron and an electron that orbit each other. Compare the wavelength of the spectral lines of positronium with those of ordinary hydrogen.

Solution:

Here the two particles have the same mass m, so the reduced mass is

$$\mu = \frac{mM}{m+M} = \frac{m^2}{2m} = \frac{m}{2}$$

where m is the electron mass. We know that $E_n \propto$ m

$$\therefore \frac{E'_n}{E_n} = \frac{\mu}{m} = \frac{1}{2} \text{ energy of each level is halved.}$$

$\therefore$ Their difference will also be halved.

Hence $\lambda'_n = 2\lambda_n$.

32. (A) Calculate the first three energy levels for positronium.

(B) Find the wavelength of the H_a line (3 $\rightarrow$ 2 transition) of positronium.

Solution:

(A) –6.8 eV, –1.7 eV , –0.76 eV ;

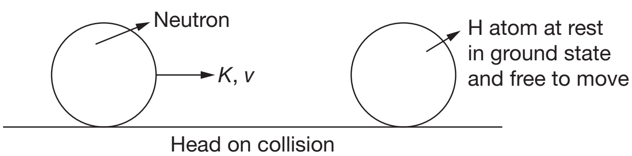
(B) 1313 Å .

ATOMIC COLLISION

In such collisions assume that the loss in the kinetic energy of system is possible only if it can excite or ionize.

SOLVED EXAMPLES

33. What will be the type of collision, if K = 14 eV, 20.4 eV, 22 eV, 24.18 eV



(elastic/inelastic/perfectly inelastic)

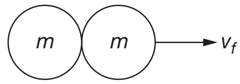
Solution:

Loss in energy (ΔE) during the collision will be used to excite the atom or electron from one level to another.

According to quantum Mechanics, for hydrogen atom.

$$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, ..., 13.6 \text{ eV})$$

According to Newtonian mechanics minimum loss = 0. (elastic collision) for maximum loss collision will be perfectly inelastic if neutron collides perfectly inelastic then, Applying momentum conservation



$$mv_0 = 2mv_f$$

$$v_f = \frac{v_0}{2}$$

$$\text{final } KE = \frac{1}{2} \times 2\text{ m} \times \frac{v_0^2}{4} = \frac{\frac{1}{2}mv_0^2}{2} = \frac{K}{2}$$

$$\text{maximum loss} = \frac{K}{2}$$

According to classical mechanics (ΔE) = [0, $\frac{K}{2}$]

(A) If K = 14 eV, According to quantum mechanics
(ΔE) = {0, 10.2 eV, 12.09 eV}
According to classical mechanics
ΔE = [0, 7 eV]
loss = 0, hence it is elastic collision speed of particle changes.

(B) If K = 20.4 eV
According to classical mechanics
loss = [0, 10.2 eV]
According to quantum mechanics
loss = {0, 10.2 eV, 12.09 eV,.........}
loss = 0 elastic collision
loss = 10.2 eV perfectly inelastic collision

(C) If K = 22 eV
Classical mechanics ΔE = [0, 11]
Quantum mechanics ΔE = {0, 10.2 eV, 12.09 eV, ...}
loss = 0 elastic collision
loss = 10.2 eV inelastic collision

(D) If K = 24.18 eV
According to classical mechanics
ΔE = [0, 12.09 eV]
According to quantum mechanics
ΔE = {0, 10.2 eV, 12.09 eV, ... 13.6 eV}
loss = 0 elastic collision
loss = 10.2 eV inelastic collision
loss = 12.09 eV perfectly inelastic collision.

34. A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the

He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particles.

Solution:

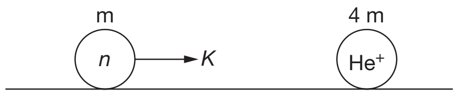
Here the loss during the collision can only be used to excite the atoms or electrons.

So according to quantum mechanics

loss = {0, 40.8 eV, 48.3 eV, ..., 54.4 eV} (1)

$$E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

Now according to newtonian mechanics



Minimum loss = 0

maximum loss will be for perfectly inelastic collision.

let v_0 be the initial speed of neutron and v_f be the final common speed.

so by momentum conservation $mv_0 = mv_f + 4mv_f$

$v_f = \frac{v_0}{5}$.

where m = mass of Neutron

$\therefore$ mass of He^+ ion = 4 m

so final kinetic energy of system

$$KE = \frac{1}{2} m\, v_f^2 + \frac{1}{2} 4\, m\, v_f^2$$

$$= \frac{1}{2} \cdot (5 \text{ m}) \cdot \frac{v_0^2}{25} = \frac{1}{5} . (\frac{1}{2} mv_0^2) = \frac{K}{5}$$

Maximum loss $= K - \frac{K}{5} = \frac{4K}{5}$

so loss will be $\left[0, \frac{4K}{5}\right]$ (2)

For inelastic collision there should be at least one common value other than zero in set (1) and (2)

$$\therefore \quad \frac{4K}{5} > 40.8 \text{ eV}$$

$$K > 51 \text{ eV}$$

Minimum value of K = 51 eV.

35. In previous question, find minimum value of K so that all types of collision is possible.

Solution:

$$K = \frac{4}{5} \times 12.09 \Rightarrow K = 60.45 \text{ eV}.$$

36. A H-atom in ground state is moving with initial kinetic energy K. It collides head on with a He^+ ion in ground state kept at rest but free to move. Find minimum value of K so that both the particles can excite to their first excited state.

Solution:

$$\frac{4K}{5} = 51 \text{ eV } K = \frac{51 \times 5}{4} \text{ eV } K = 63.75 \text{ eV}.$$

37. How many head-on, elastic collisions must a neutron have with deuterium nucleus to reduce its energy from 1 MeV to 0.025 eV.

Solution:

Let mass of neutron = m and mass of deuterium = 2 m

initial kinetic energy of neutron = K_0

Let after first collision kinetic energy of neutron and deuterium be K_1 and K_2.

Using C.O.L.M. along direction of motion

$$\sqrt{2mK_0} = \sqrt{2mK_1} + \sqrt{4mK_2}$$

velocity of separation = velocity of approach

$$\frac{\sqrt{4mK_2}}{2m} - \frac{\sqrt{2mK_1}}{m} = \frac{\sqrt{2mK_0}}{m}$$

Solving Equation on (1) and (2) we get

$$K_1 = \frac{K_0}{9}$$

Loss in kinetic energy after first collision

$$\Delta K_1 = K_0 - K_1$$

$$\Delta K_1 = \frac{8}{9} K_0 \quad (1)$$

After second collision

$$\Delta K_2 = \frac{8}{9} K_1 = \frac{8}{9} . \frac{K_0}{9}$$

$\therefore$ Total energy loss

$$\Delta K = \Delta K_1 + \Delta K_2 + ... + \Delta K_n$$

As, $$\Delta K = \frac{8}{9} K_0 + \frac{8}{9^2} K_0 + ... + \frac{8}{9^n} K_0$$

$$\Delta K = \frac{8}{9} \text{K}_0 \left(1 + \frac{1}{9} + ... + \frac{1}{9^{n-1}}\right)$$

$$\frac{\Delta K}{K_0} = \frac{8}{9} \left[\frac{1 - \frac{1}{9^n}}{1 - \frac{1}{9}} \right] = 1 - \frac{1}{9^n}$$

Here, $K_0 = 10^6$ eV, $\Delta K = (10^6 - 0.025)$ eV

$$\therefore \qquad \frac{1}{9^n} = \frac{K_0 - \Delta K}{K_0} = \frac{0.025}{10^6}$$

or $$9^n = 4 \times 10^7$$

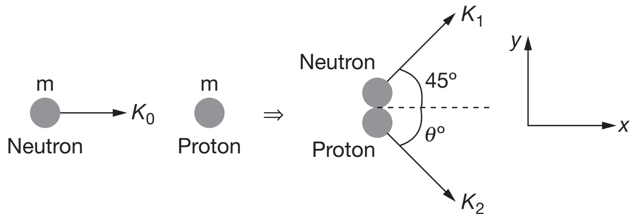
Taking log both sides and solving, we get

$$n = 8.$$

38. A neutron with energy of 4.6 MeV collides with protons and is retarded. Assuming that upon each collision neutron is deflected by 45º find the number of collisions which will reduce its energy to 0.23 eV.

Solution:

Mass of neutron mass of proton = m



From conservation of momentum in y-direction

$$\sqrt{2mK_1}\ \sin 45^\circ \ \sqrt{2mK_2} = \sin\theta \qquad (1)$$

In x-direction

$$\sqrt{2mK_0} - \sqrt{2mK_1}\ \cos 45^\circ = \sqrt{2mK_2}\ \cos\theta \quad (2)$$

Squaring and adding Equations (1) and (2), we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \qquad (3)$$

From conservation of energy

$$K_2 = K_0 - K_1 \qquad (4)$$

Solving Equations (3) and (4), we get

$$K_1 = \frac{K_0}{2}$$

i.e., after each collision energy remains half. Therefore, after n collisions,

$$K_n = K_0\left(\frac{1}{2}\right)^n$$

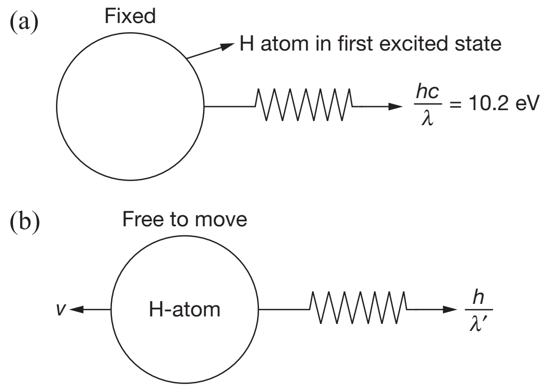
$$\therefore \qquad 0.23 = (4.6 \times 10^6)\left(\frac{1}{2}\right)^n \quad 2^n = \frac{4.6\times10^6}{0.23}$$

Taking log and solving, we get

$$n \approx 24.$$

Calculation of Recoil Speed of Atom on Emission of a Photon

momentum of photon = $mc = \dfrac{h}{\lambda}$



m - mass of atom

According to momentum conservation

$$mv = \frac{h}{\lambda'} \qquad (1)$$

According to energy conservation

$$\frac{1}{2}mv^2 + \frac{hc}{\lambda'} = 10.2 \text{ eV}$$

Since mass of atom is very large than photon

hence $\frac{1}{2}mv^2$ can be neglected

$$\frac{hc}{\lambda'} = 10.2 \text{ eV} \quad \frac{h}{\lambda} = \frac{10.2}{c}\text{ eV}$$

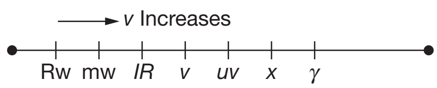
$$mv = \frac{10.2}{c}\text{eV} \quad v = \frac{10.2}{cm}$$

recoil speed of atom = $\dfrac{10.2}{cm}$

X-RAYS

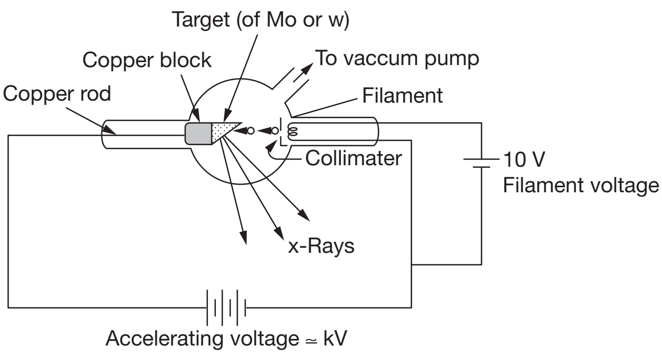
It was discovered by roentgen. The wavelength of x-rays is found between 0.1 Å to 10 Å. These rays are invisible to eye. They are electromagnetic waves and have speed $c = 3 \times 10^8$ m/s in vacuum.

Its photons have energy around 1000 times more than the visible light.



When fast moving electrons having energy of order of several KeV strike the metallic target then x-rays are produced.

Production of *x*-rays by Coolidge Tube



The melting point, specific heat capacity and atomic number of target should be high. When voltage is applied across the filament then filament on being heated emits electrons from it. Now for giving the beam shape of electrons, collimator is used. Now when electron strikes the target then *x*-rays are produced.

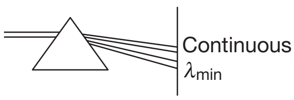
When electrons strike with the target, some part of energy is lost and converted into heat. Since, target should not melt or it can absorb heat so that the melting point, specific heat of target should be high.

Here copper rod is attached so that heat produced can go behind and it can absorb heat and target does not get heated very high.

For more energetic electron, accelerating voltage is increased.

For more number of photons voltage across filament is increased.

The *x*-ray were analysed by mostly taking their spectrum



Variation of Intensity of *x*-Rays with λ is Plotted as shown in Fig. 19.11.

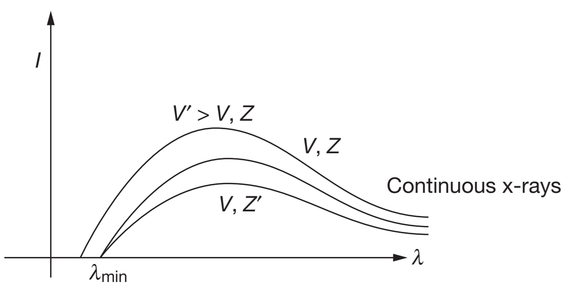


Fig. 19.11

The minimum wavelength corresponds to the maximum energy of the *x*-rays which in turn is equal to the maximum kinetic energy eV of the striking electrons thus

$$eV = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

$$\Rightarrow \qquad \lambda_{\min} = \frac{hc}{eV} = \frac{12400}{V(\text{involts})}\ \text{Å}.$$

We see that cut-off wavelength $\lambda_{\min}$ depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z')

SOLVED EXAMPLES

39. An *x*-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an *x*-ray photon at the first collision. Find the wavelength corresponding to this photon.

Solution:

Kinetic energy acquired by the electron is

$$K = \text{eV} = 20 \times 10^3 \text{ eV.}$$

The energy of the photon $= 0.05 \times 20 = 10^3$ eV
$= 10^3$ eV.

Thus, $\dfrac{hv}{\lambda} = 10^3\,\text{eV}$

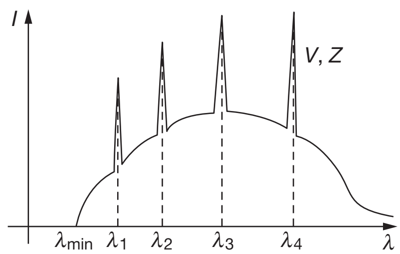
$$= \frac{(4.14 \times 10^{-15}\,\text{eV} - \text{s}) \times (3 \times 10^8\,\text{m/s})}{10^3\,\text{eV}}$$

$$= \frac{1242\,\text{eV} - \text{nm}}{10^3\,\text{eV}} = 1.24\,\text{nm}$$

Characteristic X-rays

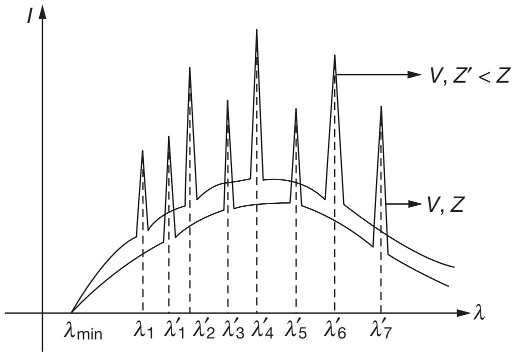
The sharp peaks obtained in graph are known as characteristic *x*-rays because they are characteristic of target material.

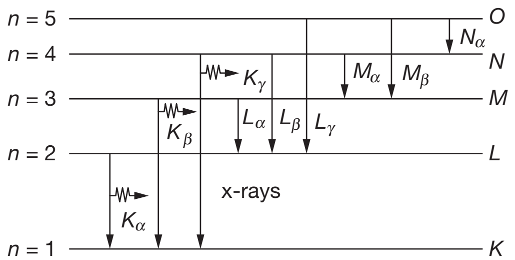
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$, ... = characteristic wavelength of material having atomic number Z are called characteristic *x*-rays and the spectrum obtained is called characteristic spectrum. If target of atomic number Z' is used then peaks are shifted.



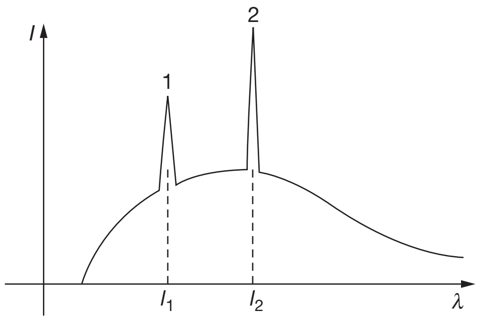
Characteristic *x*-ray emission occurs when an energetic electron collides with target and removes an inner shell electron from atom, the vacancy created in the shell is filled when an electron from higher level drops into it. Suppose vacancy created in innermost K-shell is filled by an electron dropping from next higher level L-shell then K_α characteristic x-ray is obtained. If vacancy in K-shell is filled by an electron from M-shell, K_β line is produced and so on

similarly L_α, L_β,.....M_α, M_β lines are produced.





40. Find which is K_α and K_β



Solution:

$\Delta E = \dfrac{hc}{\lambda}$, $\qquad \lambda = \dfrac{hc}{\Delta E}$

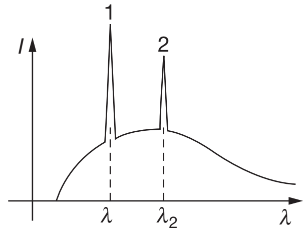
since energy difference of K_α is less than K_β

$$\Delta E_{k\alpha} < \Delta E_{k\beta}$$

$$\lambda_{k\beta} < \lambda_{k\alpha}$$

1 is K_β and 2 is K_α.

41. Find which is K_α and L_α



Solution:

$\therefore \qquad \Delta E_{K\alpha} > \Delta E_{L\alpha}$

1 is K_α and 2 is L_α.

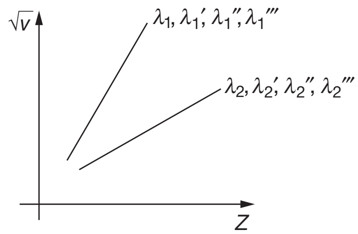
42. A TV tube operates with a 20 kV accelerating potential. What is the maximum–energy x-rays from the TV set?

Solution:

The electrons in the TV tube have energy of 20 keV, and if these electrons are brought to rest by a collision in which one x-ray photon is emitted, the photon energy is 20 keV.

MOSELEY'S LAW

Moseley measured the frequencies of characteristic x-rays for a large number of elements and plotted the square root of frequency against position number in periodic table. He discovered that plot is much closer to a straight line not passing through origin.



Z_1	l_1	l_2
Z_2	l_1'	l_2'
Z_3	l_1''	l_2''
Z_4	l_1'''	l_2'''

Wavelength of charactristic wavelengths

Moseley's observations can be mathematically expressed as

$$\sqrt{\nu} = a(Z - b)$$

a and b are positive constants for one type of x-rays and for all elements (independent of Z).

Moseley's Law can be derived on the basis of Bohr's theory of atom, frequency of x-rays is given by

$$\sqrt{\nu} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \cdot (Z - b)$$

by using the formula $\frac{1}{\lambda} = R z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ with modification for multi electron system.

$b \rightarrow$ known as screening constant or shielding effect, and $(Z - b)$ is effective nuclear charge.

for K_α line

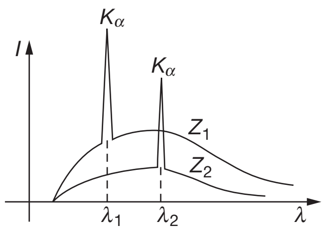
$$n_1 = 1, n_2 = 2$$

$$\therefore \quad \sqrt{\nu} = \sqrt{\frac{3RC}{4}}\,(Z - b) \quad \sqrt{\nu} = a(Z - b)$$

Here $a = \sqrt{\frac{3RC}{4}}$, [$b = 1$ for K_α lines].

SOLVED EXAMPLES

43. Find in Z_1 and Z_2 which one is greater.



Solution:

$$\because \sqrt{\nu} \equiv \sqrt{cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \cdot (Z - b)$$

If Z is greater then ν will be greater, λ will be less

$$\therefore \quad \lambda_1 < \lambda_2$$

$$\therefore \quad Z_1 > Z_2.$$

44. A cobalt target is bombarded with electrons and the wavelength of its characteristic spectrum is measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

Solution:

Using Moseley's law and putting c/λ for ν (and assuming $b = 1$), we obtain

$$\sqrt{\frac{c}{\lambda_{c_0}}} = aZ_{c_0} - a \text{ and } \sqrt{\frac{c}{\lambda_x}} = aZ_x - a$$

Dividing yields

$$\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$$

Substituting gives us

$$\sqrt{\frac{178.9\,\text{pm}}{143.5\,\text{pm}}} = \frac{Z_x - 1}{27 - 1}.$$

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

45. Find the constants a and b in Moseley's equation $\sqrt{\nu} = a(Z - b)$ from the following data.

Element	Z	Wavelength of K_α x-ray
Mo	42	71 pm
Co	27	178.5 pm

Solution:

Moseley's equation is

$$\sqrt{\nu} = a(Z - b)$$

Thus, $$\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b) \quad (1)$$

and $$\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b) \quad (2)$$

From (1) and (2) $$\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$$

or, $$a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$$

$$= \frac{(3 \times 10^8\,\text{m/s})^{1/2}}{42 - 27}\left[\frac{1}{(71 \times 10^{-12}\,\text{m})^{1/2}} - \frac{1}{(178.5 \times 10^{-12}\,\text{m})^{1/2}}\right]$$

$$= 5.0 \times 10^7\,(\text{Hz})^{1/2}$$

Dividing (1) by (2),

$$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$$

or, $\sqrt{\dfrac{178.5}{71}} = \dfrac{42-b}{27-b}$

or, $b = 1.37.$

46. In the Moseley relation, $\sqrt{v} = a(Z-b)$ which will have the greater value for the constant a for K_α or K_β transition ?

Solution:

A is larger for the K_β transitions than for the K_α transitions.

NUCLEAR PHYSICS

It is the branch of physics which deals with the study of nucleus.

Nucleus

1. ***Discoverer***: Rutherford
2. ***Constituents***: Neutrons (n) and Protons (p) [collectively known as nucleons]
   - **(a) Neutron:** It is a neutral particle. It was discovered by J. Chadwick.
     Mass of neutron, $m_n = 1.6749286 \times 10^{-27}$ kg.
   - **(b) Proton:** It has a charge equal to $+e$. It was discovered by Goldstein.
     Mass of proton, $m_p = 1.6726231 \times 10^{-27}$ kg

$$m_p \gtrsim m_n$$

3. ***Representation:***

$_Z X^A$ or $^A_Z X$

where $X \Rightarrow$ symbol of the atom

$Z \Rightarrow$ Atomic number = number of protons

$A \Rightarrow$ Atomic mass number = total number of nucleons.

= number of protons + number of neutrons.

Atomic Mass Number:

It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

1 a.m.u. = $\dfrac{1}{12}$ [mass of one atom of $_6C^{12}$ atom at rest and in ground state]

1.6603×10^{-27} kg; 931.478 MeV/c^2

mass of proton (m_p) = mass of neutron (m_n) = 1 a.m.u.

Some Definitions:

1. ***Isotopes***:
   The nuclei having the same number of protons but different number of neutrons are called isotopes.
2. ***Isotones:***
   Nuclei with the same neutron number N but different atomic number Z are called isotones.
3. ***Isobars***:
   The nuclei with the same mass number but different atomic number are called isobars.
4. ***Size of nucleus:*** Order of 10^{-15} m (fermi)
   Radius of nucleus; $R = R_0 A^{1/3}$
   where $R_0 = 1.1 \times 10^{-15}$ m (which is an empirical constant)
   A = Atomic mass number of atom.
5. ***Density:***

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

$$= \frac{3m_p}{4\pi R_0^{\ 3}}$$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.1 \times 10^{-15})^3} = 3 \times 10^{17} \text{ kg/m}^3$$

Nuclei of almost all atoms have almost same density as nuclear density is independent of the mass number (A) and atomic number (Z).

SOLVED EXAMPLES

47. Calculate the radius of ^{70}Ge.

Solution:

We have,

$$R = R_0 A^{1/3} = (1.1 \text{ fm})\,(70)^{1/3}$$

$$= (1.1 \text{ fm})\,(4.12) = 4.53 \text{ fm}.$$

48. Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of ^{12}C when they 'touch' each other at the surface

Solution:

The radius of a ^{12}C nucleus is

$$R = R_0 A^{1/3}$$

$$= (1.1 \text{ fm})\,(12)^{1/3} = 2.52 \text{ fm}.$$

The separation between the centres of the nuclei is 2R = 5.04 fm. The potential energy of the pair is

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

$$= (9 \times 10^9 \text{ N–m}^2/\text{C}^2)\ \frac{(6 \times 1.6 \times 10^{-19}\text{ C})^2}{5.04 \times 10^{-15}\text{ m}}$$

$$= 1.64 \times 10^{-12} \text{ J} = 10.2 \text{ MeV}.$$

Mass Defect

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{\text{expected}} = Z\,m_p + (A - Z)m_n$$

$$M_{\text{observed}} = M_{\text{atom}} - Zm_e$$

It is found that

$$M_{\text{observed}} < M_{\text{expected}}$$

Hence, mass defect is defined as

$$\text{Mass defect} = M_{\text{expected}} - M_{\text{observed}}$$

$$\Delta m = [Zm_p + (A - Z)m_n] - [M_{\text{atom}} - Zm_e]$$

Binding Energy

It is the minimum energy required to break the nucleus into its constituent particles.

or

Amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

Binding Energy (BE) = Δmc^2

$$\text{BE} = \Delta m \text{ (in amu)} \times 931.5 \text{ MeV/amu}$$

$$= \Delta m \times 931.5 \text{ MeV}$$

$$= \Delta m \times 931 \text{ MeV}$$

NOTE

If binding energy per nucleon is more for a nucleus then it is more stable.

For example

If $\left(\frac{\text{BE}_1}{A_1}\right) > \left(\frac{\text{BE}_2}{A_2}\right)$

then nucleus 1 would be more stable.

SOLVED EXAMPLES

49. Following data is available about 3 nuclei *P*, *Q* and *R*. Arrange them in decreasing order of stability

	P	Q	R
Atomic mass number (A)	10	5	6
Binding Energy (MeV)	100	60	66

Solution:

$$\left(\frac{\text{BE}}{A}\right)_P = \frac{100}{10} = 10$$

$$\left(\frac{\text{BE}}{A}\right)_Q = \frac{60}{5} = 12$$

$$\left(\frac{\text{BE}}{A}\right)_R = \frac{66}{6} = 11$$

$\therefore$ Stability order is $Q > R > P$.

50. A nucleus has binding energy of 100 MeV. It further releases 10 MeV energy. Find the new binding energy of the nucleus.

Solution:

After releasing 10 MeV, it will become more stable and its binding energy will increase.

New binding energy = 100 + 10 = 110 MeV.

51. A nuclear reaction is given as

$$A + B \rightarrow C + D$$

Binding energies of *A*, *B*, *C* and *D* are given as

$$B_1, B_2, B_3 \text{ and } B_4$$

Find the energy released in the reaction

Solution:

$$(B_3 + B_4) - (B_1 + B_2).$$

52. Calculate the binding energy of an alpha particle from the following data:

mass of ${}^1_1$H atom = 1.007826 u

mass of neutron = 1.008665 u

mass of ${}^4_2$He atom = 4.00260 u

Take 1 u = 931 MeV/c^2.

Solution:

The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$$B = (2 \times 1.007826 \text{ u} + 2 \times 1.008665 \text{ u} - 4.00260 \text{ u})c^2$$

$$= (0.03038 \text{ u})c^2$$

$$= 0.03038 \times 931 \text{ MeV} = 28.3 \text{ MeV}.$$

53. Find the binding energy of ${}^{56}_{26}$Fe. Atomic mass of ^{56}Fe is 55.9349 u and that of ^{1}H is 1.00783 u. Mass of neutron = 1.00867 u.

Solution:

The number of protons in ${}^{56}_{26}\text{Fe}$ = 26 and the number of neutrons = 56 – 26 = 30.

The binding energy of ${}^{56}_{26}\text{Fe}$ is

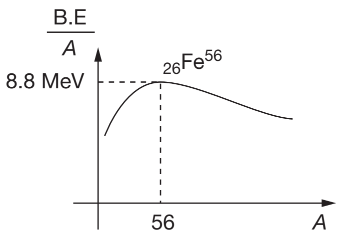
$= [26 \times 1.00783\text{ u} + 30 \times 1.00867\text{ u} - 55.9349\text{ u}]\, c^2$

$= (0.52878\text{ u})c^2$

$= (0.52878\text{ u})\,(931\text{ MeV/u}) = 492\text{ MeV}.$

Variation of Binding Energy Per Nucleon with Mass Number

The binding energy per nucleon first increases on an average and reaches a maximum of about 8.7 MeV for A ≃ 50 → 80. For still heavier nuclei, the binding energy per nucleon slowly decreases as A increases.



Binding energy per nucleon is maximum for $_{26}\text{Fe}^{56}$, which is equal to 8.8 MeV. Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.

The heavier nuclei being unstable have tendency to split into medium nuclei. This process is called Fission.

The Lighter nuclei being unstable have tendency to fuse into a medium nucleus. This process is called Fusion.

Radioactivity

It was discovered by Henry Becquerel.

Spontaneous emission of radiations (α, β, γ) from unstable nucleus is called radioactivity. Substances which shows radioactivity are known as radioactive substance.

Radioactivity was studied in detail by Rutherford.

In radioactive decay, an unstable nucleus emits α particle or β particle. After emission of α or β the remaining nucleus may emit γ particle, and converts into more stable nucleus.

α-particle

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

Mass of α-particle = Mass of $_2\text{He}^4$ atom – $2m_e = 4\, m_p$
Charge of α-particle = + 2 e

β-particle

1. **β^- (electron):**
   Mass = m_e; Charge = $-e$
2. **β^+ (positron):**
   Mass = m_e; Charge = $+e$
   positron is an antiparticle of electron.

Antiparticle

A particle is called antiparticle of other if on collision both can annihilate (destroy completely) and converts into energy.

For example:

1. electron ($-e$, m_e) and positron ($+e$, m_e) are anti-particles.
2. neutrino (ν) and antineutrino ($\bar{\nu}$) are anti-particles.

γ-particle: They are energetic photons of energy of the order of Mev and having rest mass zero.

Radioactive Decay (Displacement Law)

α-decay

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$$

Q value: It is defined as energy released during the decay process.

Q value = rest mass energy of reactants – rest mass energy of products.

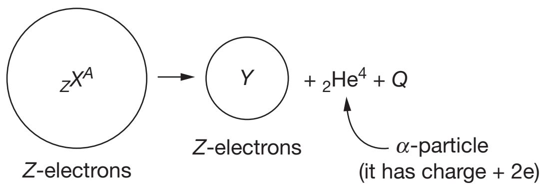
This energy is available in the form of increase in *KE* of the products.

Let, M_x = mass of atom $_ZX^A$
M_y = mass of atom $_{Z-2}Y^{A-4}$
M_{He} = mass of atom $_2\text{He}^4$.

$$Q \text{ value} = [(M_x - Zm_e) - \{(M_y - (Z-2)\, m_e) + (M_{He} - 2m_e)\}]\, c^2$$

$$= [M_x - M_y - M_{He}]\, c^2$$

Considering actual number of electrons in α-decay



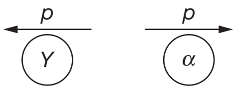
$$Q \text{ value} = [M_x - (M_y + 2m_e) - (M_{He} - 2m_e)]\, c^2$$

$$= [M_x - M_y - M_{He}]\, c^2$$

Calculation of *KE* of Final Products

As atom X was initially at rest and no external forces are acting, so final momentum also has to be zero. Hence both

Y and α-particle will have same momentum in magnitude but in opposite direction.



$$p_\alpha^2 = p_Y^2 \qquad 2m_\alpha T_\alpha = 2m_Y T_Y$$

(Here we are representing T for kinetic energy)

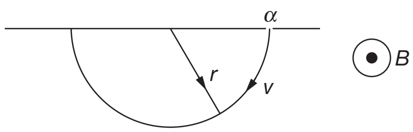
$$Q = T_y + T_\alpha \qquad m_\alpha T_\alpha = m_Y T_Y$$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y} Q \qquad T_Y = \frac{m_\alpha}{m_\alpha + m_Y} Q$$

$$T_\alpha = \frac{A-4}{A} Q \qquad T_Y = \frac{4}{A} Q$$

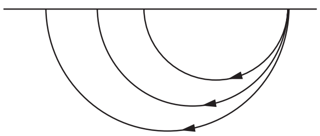
From the above calculation, one can see that all the α--particles emitted should have same kinetic energy. Hence, if they are passed through a region of uniform magnetic field having direction perpendicular to velocity, they should move in a circle of same radius.

$$r = \frac{mv}{qB} = \frac{mv}{2eB} = \frac{\sqrt{2Km}}{2eB}$$



Experimental Observation

Experimentally it has been observed that all the α-particles do not move in the circle of same radius, but they move in circles having different radii.



This shows that they have different kinetic energies. But it is also observed that they follow circular paths of some fixed values of radius i.e. yet the energy of emitted α-particles is not same but it is quantized. The reason behind this is that all the daughter nuclei produced are not in their ground state but some of the daughter nuclei may be produced in their excited states and they emits photon to acquire their ground state.

$X \longrightarrow Y^*$ (excited state) $+ \alpha + Q$

$\downarrow$

Y + photon (γ particle)

The only difference between Y and Y^* is that Y^* is in excited state and Y is in ground state.

Let, the energy of emitted γ-particles be E

$$\therefore \qquad Q = T_\alpha + T_Y + E$$

where $\qquad Q = [M_x - M_y - M_{\text{He}}]\, c^2$

$$T_\alpha + T_Y = Q - E$$

$$T_\alpha = \frac{m_Y}{m_\alpha + m_Y}(Q - E);$$

$$T_Y = \frac{m_\alpha}{m_\alpha + m_Y}(Q - E)$$

β decay

$$_Z X^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q$$

$_{-1}e^0$ can also be written as $_{-1}\beta^0$.

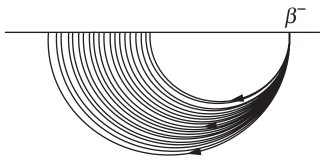
Here also one can see that by momentum and energy conversion, we will get

$$T_e = \frac{m_Y}{m_e + m_Y} Q;$$

$$T_Y = \frac{m_e}{m_e + m_Y} Q$$

as $m_e << m_Y$, we can consider that all the energy is taken away by the electron.

From the above results, we will find that all the β-particles emitted



will have same energy and hence they have same radius if passed through a region of perpendicular magnetic field. But, experimental observations were completely different. On passing through a region of uniform magnetic field perpendicular to the velocity, it was observed that β-particles take circular paths of different radius having a continuous spectrum.

To explain this, Paulling has introduced the extra particles called neutrino and antineutrino (antiparticle of neutrino).

$$\bar{\nu} \rightarrow \text{antineutrino}, \ \nu \rightarrow \text{neutrino}$$

Properties of Antineutrino ($\bar{\nu}$) and Neutrino (n)

1. They are like photons having rest mass = 0
   speed = c
   Energy, $E = mc^2$
2. They are charge less (neutral)

3. They have spin quantum number, $s = \pm \frac{1}{2}$

Considering the emission of antineutrino, the equation of β^- - decay can be written as

$$_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q + \bar{\nu}$$

Production of antineutrino along with the electron helps to explain the continuous spectrum because the energy is distributed randomly between electron $\bar{\nu}$ and it also helps to explain the spin quantum number balance (p, n and $\pm e$ each has spin quantum number $\pm$ 1/2).

During β^- decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$$n \rightarrow p + {}_{-1}e^0 + \bar{\nu}$$

Let, M_x = mass of atom $_ZX^A$
M_y = mass of atom $_{Z+1}Y^A$
m_e = mass of electron

Q value $= [(M_X - Zm_e) - \{(M_Y - (z+1)\ m_e) + m_e\}]\ c^2$

$= [M_X - M_Y]\ c^2$

Considering actual number of electrons.

Q value $= [M_X - \{(M_Y - m_e) + m_e\}]\ c^2 = [M_X - M_Y]\ c^2$

SOLVED EXAMPLE

54. Consider the beta decay

$$^{198}\text{Au} \rightarrow {}^{198}\text{Hg}^* + \beta^- +$$

where ^{198}Hg* represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass ^{198}Au is 197.968233 u and that of ^{198}Hg is 197.966760 u.

Solution:

If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = [m(^{198}\text{Au}) - m(^{198}\text{Hg})]c^2.$$

As ^{198}Hg* has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$$Q = [m(^{198}\text{Au}) - m(^{198}\text{Hg})]c^2 - 1.088 \text{ MeV}$$

$$= (197.968233 \text{ u} - 197.966760 \text{ u}) \left(931 \frac{\text{MeV}}{u}\right) - 1.088 \text{ MeV}$$

$$= 1.3686 \text{ MeV} - 1.088 \text{ MeV} = 0.2806 \text{ MeV}.$$

This is also the maximum possible kinetic energy of the electron emitted.

β^+ decay

$$_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu + Q$$

In β^+ decay, inside a nucleus a proton is converted into a neutron, positron and neutrino.

$$p \rightarrow n + {}_{+1}e^0 + \nu$$

As mass increases during conversion of proton to a neutron, hence it requires energy for β^+ decay to take place, $\therefore$ β^+ decay is rare process. It can take place in the nucleus where a proton can take energy from the nucleus itself.

$$Q \text{ value} = [(M_X - Zm_e) - \{(M_Y - (Z-1)\ m_e) + m_e\}]\ c^2$$

$$= [M_X - M_Y - 2m_e]\ c^2$$

Considering actual number of electrons.

$$Q \text{ value} = [M_X - \{(M_Y + m_e) + m_e\}]\ c^2$$

$$= [M_X - M_Y - 2m_e]\ c^2$$

SOLVED EXAMPLE

55. Calculate the Q-value in the following decays:

(A) $^{19}\text{O} \rightarrow {}^{19}\text{F} + e^- + \bar{\nu}$

(B) $^{25}\text{Al} \rightarrow {}^{25}\text{Mg} + e^+ + \nu$.

The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19.003576 u	18.998403 u	24.990432 u	24.985839 u

Solution:

(A) The Q-value of β^- decay is

$Q = [m(^{19}\text{O}) - m(^{19}\text{F})]\ c^2$

$= [19.003576 \text{ u} - 18.998403 \text{ u}]\ (931 \text{ MeV/u})$

$= 4.816$ MeV.

(B) The Q-value of β^+ decay is

$Q = [m(^{25}\text{Al}) - m(^{25}\text{Mg}) - 2m_e]\ c^2$

$= \left[24.99032 \text{ u} - 24.985839 \text{ u} - 2 \times 0.511 \frac{\text{MeV}}{c^2}\right] c^2$

$= (0.004593 \text{ u})\ (931 \text{ MeV/u}) - 1.022$ MeV

$= 4.276 \text{ MeV} - 1.022 \text{ MeV} = 3.254$ MeV.

K capture

It is a rare process which is found only in few nucleus. In this process the nucleus captures one of the atomic electrons from the K shell. A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is also emitted in the process and is emitted from the nucleus.

$$p + {}_{-1}e^0 \rightarrow n + \nu$$

If X and Y are atoms then reaction is written as:

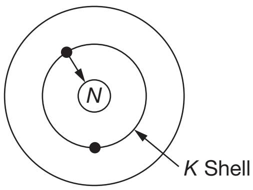
$${}_ZX^A \rightarrow {}_{Z-1}Y^A + \nu + Q + \text{characteristic x-rays of } Y.$$

If X and Y are taken as nucleus, then reaction is written as

$${}_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + \nu$$

NOTE

1. Nuclei having atomic numbers from $Z = 84$ to 112 shows radioactivity.
2. Nuclei having $Z = 1$ to 83 are stable (only few exceptions are there)
3. Whenever a neutron is produced, a neutrino is also produced.
4. Whenever a neutron is converted into a proton, an antineutrino is produced.



Nuclear Stability

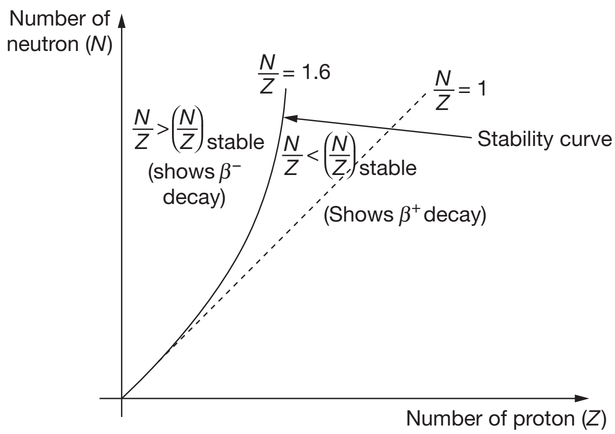


Fig. 19.12

Figure 19.12 a plot of neutron number N versus proton number Z for the nuclides found in nature. The solid line in the figure represents the stable nuclides. For light stable nuclides, the neutron number is equal to the proton number so that ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1.6 for the heaviest stable nuclides.

The points (Z, N) for stable nuclides fall in a rather well-defined narrow region. There are nuclides to the left of the stability belt as well as to the right of it. The nuclides to the left of the stability region have excess neutrons, whereas, those to the

right of the stability belt have excess protons. These nuclides are unstable and decay with time according to the laws of radioactive disintegration. Nuclides with excess neutrons (lying above stability belt) show β^- decay while nuclides with excess protons (lying below stability belt) show β^+ decay and K - capture.

Nuclear Force

1. Nuclear forces are basically attractive and are responsible for keeping the nucleons bound in a nucleus in spite of repulsion between the positively charge protons.
2. It is strongest force with in nuclear dimensions ($F_n \simeq 100\ F_e$)
3. It is short range force (acts only inside the nucleus)
4. It acts only between neutron-neutron, neutron-proton and proton-proton i.e. between nucleons.
5. It does not depend on the nature of nucleons.
6. An important property of nuclear force is that it is not a central force. The force between a pair of nucleons is not solely determined by the distance between the nucleons. For example, the nuclear force depends on the directions of the spins of the nucleons. The force is stronger if the spins of the nucleons are parallel (i.e., both nucleons have $m_s = +1/2$ or $-1/2$) and is weaker if the spins are antiparallel (i.e., one nucleon has $m_s = +1/2$ and the other has $m_s = -1/2$). Here m_s is spin quantum number.

Radiative Decay: Statistical Law

(Given by Rutherford and Soddy)

Rate of radioactive decay $\propto N$

where N = number of active nuclei

$$= \lambda N$$

where λ = decay constant of the radioactive substance.

Decay constant is different for different radioactive substances, but it does not depend on amount of substance and time.

SI unit of λ is s^{-1}

If $\lambda_1 > \lambda_2$ then first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ

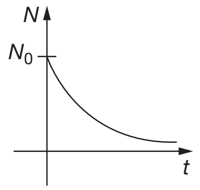
$$A \xrightarrow{\lambda} B$$

$t = 0$ $\quad N_0 \quad 0$ where N_0 = number of active nuclei of A at $t = 0$

$t = t$ $\quad N \quad N'$ where N = number of active nuclei of A at $t = t$

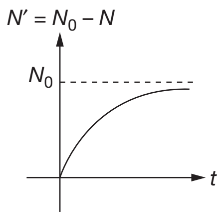
Rate of radioactive decay of $A = -\dfrac{dN}{dt} = \lambda N$

$$-\int_{N_0}^{N} \frac{dN}{N} = \int_0^t \lambda dt \Rightarrow N = N_0\, e^{-\lambda t} \text{ (it is exponential decay)}$$



Number of nuclei decayed (i.e. the number of nuclei of B formed)

$$N' = N_0 - N$$
$$= N_0 - N_0 e^{-\lambda t}$$
$$N' = N_0(1 - e^{-\lambda t})$$



Half-life (T1/2)

It is the time in which number of active nuclei becomes half.

$$N = N_0\, e^{-\lambda t}$$

After one half life, $N = \dfrac{N_0}{2}$

$$\frac{N_0}{2} = N_0\, e^{-\lambda t} \Rightarrow t = \frac{\ln 2}{\lambda} \Rightarrow \frac{0.693}{\lambda} = t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \text{ (to be remembered)}$$

Number of nuclei present after n half-lives i.e. after a time $t = n\, t_{1/2}$

$$N = N_0 e^{-\lambda t} = N_0\, e^{-\lambda nt1/2} = N_0\, e^{-\lambda n \frac{\ln 2}{\lambda}}$$

$$= N_0\, e^{\ln 2^{(-n)}} = N_0\,(2)^{-n} = N_0(1/2)^n = \frac{N_0}{2^n}$$

$\{n = \dfrac{t}{t_{1/2}}$. It may be a fraction, need not to be an integer$\}$

or $$N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3 \ldots\ldots\ldots\ldots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$$

SOLVED EXAMPLES

56. A radioactive sample has 6.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

Solution:

In one half-life the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$6.0 \times 10^{18} \times \left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right) = 1.5 \times 10^{18}.$$

57. The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5×10^{9} y. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U.

Solution:

Since the number of ^{206}Pb atoms equals the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is 4.5×10^9 y old.

Activity

Activity is defined as rate of radioactive decay of nuclei

It is denoted by A or R $A = \lambda N$

If a radioactive substance changes only due to decay then

$$A = -\frac{dN}{dt}$$

As in that case, $N = N_0\, e^{-\lambda t}$

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$
$$A = A_0\, e^{-\lambda t}$$

SI Unit of activity: Becquerel (Bq) which is same as 1 dps (disintegration per second)

The popular unit of activity is curie which is defined as

1 curie = 3.7×10^{10} dps (which is activity of 1 gm Radium)

SOLVED EXAMPLES

58. The decay constant for the radioactive nuclide ^{64}Cu is $1.516 \times 10^{-5}\ s^{-1}$. Find the activity of a sample containing 1 μg of ^{64}Cu. Atomic weight of copper = 63.5 g/mole. Neglect the mass difference between the given radioisotope and normal copper.

Solution:

63.5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 μg of *Cu* is

$$N = \frac{6\times10^{23}\times1\mu g}{63.5\,g} = 9.45 \times 10^{15}$$

The activity = λN

$$= (1.516 \times 10^{-5}\ s^{-1}) \times (9.45 \times 10^{15})$$

$$= 1.43 \times 10^{11} \text{ disintegrations/s}$$

$$= \frac{1.43\times10^{11}}{3.7\times10^{10}} Ci = 3.86\ Ci.$$

Activity after n half- lives: $\frac{A_0}{2^n}$.

59. The half-life of a radioactive nuclide is 20 hours. What fraction of original activity will remain after 40 hours?

Solution:

40 hours means 2 half-lives.

Thus, $A = \frac{A_0}{2^2} = \frac{A_0}{4}$ or, $\frac{A}{A_0} = \frac{1}{4}$.

So one fourth of the original activity will remain after 40 hours.

Specific activity: *The activity per unit mass is called specific activity.*

Average Life

$$T_{avg} = \frac{\text{sum of ages of all the nuclei}}{N_0}$$

$$= \frac{\int_0^\infty \lambda N_0 e^{-\lambda t}\ dt.t}{N_0} = \frac{1}{\lambda}$$

SOLVED EXAMPLES

60. The half-life of ^{198}Au is 2.7 days. Calculate (a) the decay constant, (b) the average-life and (c) the activity of 1.00 mg of ^{198}Au. Take atomic weight of ^{198}Au to be 198 g/mol.

Solution:

(A) The half-life and the decay constant are related as

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \text{ or, } \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7\,\text{days}}$$

$$= \frac{0.693}{2.7\times24\times3600\,s} = 2.9 \times 10^{-6}\ s^{-1}.$$

(B) The average-life is $t_{av} = \frac{1}{\lambda} = 3.9$ days.

(C) The activity is $A = \lambda N$. Now, 198 g of ^{198}Au has 6×10^{23} atoms. The number of atoms in 1.00 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1.0\,mg}{198\,g} = 3.03 \times 10^{18}.$$

Thus, $A = \lambda N = (2.9 \times 10^{-6}\ s^{-1})\ (3.03 \times 10^{18})$

$= 8.8 \times 10^{12}$ disintegrations/s

$= \frac{8.8\times10^{12}}{3.7\times10^{10}}$ Ci = 240 Ci.

61. Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time *t*. Find the condition for which the number of daughter nuclei becomes constant.

Solution:

The number of parent nuclei decaying in a short time interval t to t + dl is $\lambda_p N_p dt$. This is also the number of daughter nuclei decaying during the same time interval is $\lambda_d N_d\ dt$. The number of the daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt$$

or, $$\lambda_p N_p = \lambda_d N_d.$$

62. A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a resistance R. What should be the value of *R* so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?

Solution:

The activity of the sample at time t is given by

$$A = A_0\, e^{-\lambda t}$$

where λ is the decay constant and A_0 is the activity at time $t = 0$ when the capacitor plates are connected. The charge on the capacitor at time *t* is given by

$$Q = Q_0\, e^{-t/CR}$$

where Q_0 is the charge at $t = 0$ and $C = 100\ \mu$F is the capacitance. Thus,

$$\frac{Q}{A} = \frac{Q_0}{A_0}\frac{e^{-t/CR}}{e^{-\lambda t}}.$$

It is independent of t if $\lambda = \dfrac{1}{CR}$

or, $$R = \frac{1}{\lambda C} = \frac{t_{av}}{C} = \frac{20\times10^{-3}\,\text{s}}{100\times10^{-6}\,\text{F}} = 200\ \Omega.$$

63. A radioactive nucleus can decay by two different processes. The half-life for the first process is t_1 and that for the second process is t_2. Show that the effective half-life t of the nucleus is given by

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}.$$

Solution:

The decay constant for the first process is $\lambda_1 = \dfrac{\ln 2}{t_1}$ and for the second process it is $\lambda_2 = \dfrac{\ln 2}{t_1}$. The probability that an active nucleus decays by the first process in a time interval dt is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ, this probability is also equal to λdt. Thus.

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or, $$\lambda = \lambda_1 + \lambda_2$$

or, $$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}. \qquad \text{(To be remembered)}$$

64. A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find

(A) The number of nuclei of A and

(B) Number of nuclei of B, at any time t assuming the production of A starts at $t = 0$.

(C) Also find out the maximum number of nuclei of A present at any time during its formation.

Solution:

(A) Factory $\xrightarrow[\text{Constant rate}]{R} A \xrightarrow[\text{decay}]{\lambda} B$

Let N be the number of nuclei of A at any time t

$$\therefore \quad \frac{dN}{dt} = R - \lambda N \quad \int_0^N \frac{dN}{R-\lambda N} = \int_0^t dt$$

On solving we will get

$$N = R/\lambda(1 - e^{-\lambda t})$$

(B) Number of nuclei of B at any time t, $N_B = Rt - N_A = Rt - R/\lambda(1 - e^{-\lambda t}) = R/\lambda\,(\lambda t - 1 + e^{-\lambda t})$.

(C) Maximum number of nuclei of A present at any time during its formation $= R/\lambda$.

65. A radioactive substance A having N_0 active nuclei at $t = 0$, decays to another radioactive substance B with decay constant λ_1. B further decays to a stable substance C with decay constant λ_2.

(A) Find the number of nuclei of A, B and C after time t.

(B) What would be the answer of part (a) if $\lambda_1 >> \lambda_2$ and $\lambda_1 << \lambda_2$.

Solution:

The decay scheme is as shown

$$\text{A} \xrightarrow{\lambda_1} \text{B} \xrightarrow{\lambda_2} \text{C (stable)}$$

$t = 0$ N_0 0   0

t N_1 N_2 N_3

Here N_1, N_2 and N_3 represent the nuclei of A, B and C at any time t.

For A, we can write

$$N_1 = N_0\, e^{-}\lambda^{1t} \qquad (1)$$

For B, we can write

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad (2)$$

or, $$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1$$

This is a linear differential equation with integrating factor

I.F. $= e\lambda^{2t}$

$$e^{\lambda_2 t}\frac{dN_2}{dt} + e^{\lambda_2 t}\lambda_2 N_2 = \lambda_1 N_1\ e^{\lambda_2 t}$$

$$\int d(N_2 e^{\lambda_2 t}) = \int \lambda_1 N_1 e^{\lambda_2 t}\,dt$$

$$N_2\ e^{\lambda_2 t} = \lambda_1 N_0 \int e^{-\lambda_1 t}\, e^{\lambda_2 t}\,dt \qquad \text{using (1)}$$

$$N_2\ e^{\lambda_2 t} = \lambda_1 N_0 \frac{e^{(\lambda_2-\lambda_1)t}}{\lambda_2 - \lambda_1} + C \qquad (3)$$

At $t = 0$, $\quad N_2 = 0 \quad 0 = \dfrac{\lambda_1 N_0}{\lambda_2 - \lambda_1} + C$

Hence $$C = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2}$$

Using C in Equation (3), we get

$$N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

and $N_1 + N_2 + N_3 = N_0$

$\therefore$ $N_3 = N_0 - (N_1 + N_2)$

(B) For $\lambda_1 >> \lambda_2\ N_2 = \dfrac{\lambda_1 N_0}{-\lambda_1}\ (-e^{-\lambda_2 t}) = N_0\ e^{-}\lambda^{2t}$

For $\lambda_1 << \lambda_2\ N_2 = \dfrac{\lambda_1 N_0}{\lambda_2}\ (e^{-\lambda_1 t}) = 0$

Alternate Solution of (b) Part without use of Answer of Part (a)

If $\lambda_1 > \lambda_2$ that means A will decay very fast to B and B will then decay slowly. We can say that practically N_1 vanishes in very short time and B has initial number of atoms as N_0.

$\therefore$ Now $N_2 = N_0\, e^{-\lambda_2 t}$ and $N_1 = N_0\, e^{-\lambda_1 t}$

If $\lambda_1 << \lambda_2$ then B is highly unstable and it will soon decay into C.

So, it's rate of formation $\simeq$ its rate of decay.

$\therefore$ $\lambda_1 N_1 \simeq \lambda_2 N_2$

$\Rightarrow$ $N_2 = \dfrac{\lambda_1 N_1}{\lambda_2} = \dfrac{\lambda_1 N_0}{\lambda_2}\ (e^{-\lambda_1 t})$

SOLVED EXAMPLE

66. In the above problem find the time interval after which the activity of radionuclide A_2 reaches the maximum value.

Solution:

$$t_m = \frac{l\,n(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}.$$

Nuclear Fission

In nuclear fission heavy nuclei of A, above 200, break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use ${}_{92}U^{236}$ as the fission material. The technique is to hit a uranium sample by sample by slow-moving neutrons (kinetic energy ≈ 0.04 eV, also called thermal neutrons). A ${}_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming ${}_{92}U^{236}$ nucleus. These nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow X + Y + 2\,{}_0n^1,$$

or

$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow X' + Y' + 3\,{}_0n^1$$

and a number of other combinations.

On an average 2.5 neutrons are emitted in each fission event.

Mass lost per reaction ≃ 0.2 a.m.u.

In nuclear fission the total BE increases and excess energy is released.

In each fission event, about 200 MeV of energy is released a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV.

E.g., ${}^{235}_{92}U + {}_o n^1 \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_o n^1 +$ energy

$$Q \text{ value} = [(M_U - 92m_e + m_n) - \{(M_{Ba} - 56\, m_e) + (M_{Kr} - 36m_e) + 3m_n\}]\,c^2$$

$$= [(M_U + m_n) - (M_{Ba} + M_{Kr} + 3m_n)]\,c^2$$

A very important and interesting feature of neutron-induced fission is the chain reaction. For working of nuclear reactor refer your text book.

Nuclear Fusion (Thermo Nuclear Reaction)

Some unstable light nuclei of A below 20 fuse together, the BE per nucleon increases and hence the excess energy is released. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (D–D reaction) or fusion of a deuteron with a triton (D–T reaction).

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + {}_0n^1 + 3.3 \text{ MeV } (D\text{–}D)$$

$$Q \text{ value} = [2(M_D - m_e) - \{(M_{He3} - 2m_e) + m_n\}]\,c^2$$

$$= [2M_D - (M_{He3} + m_n)]\,c^2$$

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4.0 \text{ MeV } (D\text{–}D)$$

$$Q \text{ value} = [2(M_D - m_e) - \{(M_T - m_e) + (M_H - m_e)\}]\,c^2$$

$$= [2M_D - (M_T + M_H)]\,c^2$$

$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n + 17.6 \text{ MeV } (D - T)$$

$$Q \text{ value} = [\{(M_D - m_e) + (M_T - m_e)\} - \{(M_{He4} - 2m_e) + m_n\}]\,c^2$$

$$= [(M_D + M_T) - (M_{He4} + m_n)]\,c^2$$

NOTE

- *In case of fission and fusion,* $\Delta m = \Delta m_{atom} = \Delta m_{nucleus}$.
- These reactions take place at ultra-high temperature (≅ 10^7 to 10^9). At high pressure it can take place at low temperature also. For these reactions to take place nuclei should be brought upto 1 fermi distance which requires very high kinetic energy.
- Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.

SOLVED EXAMPLES

67. Calculate the energy released when three alpha particles combine to form a ^{12}C nucleus. The atomic mass of $^{4}_{2}He$ is 4.002603 u.

Solution:

The mass of a ^{12}C atom is exactly 12 u. The energy released in the reaction $3\left(^{4}_{2}He\right) \rightarrow ^{12}_{6}C$ is

$$[3\ m(^{4}_{2}He) - m(^{12}_{6}C)]\ c^2 = [3 \times 4.002603\ u - 12\ u]\ (931\ MeV/u)$$

$$= 7.27\ MeV.$$

68. Consider two deuterons moving towards each other with equal speeds in a deutron gas. What should be their kinetic energies (when they are widely separated) so that the closest separation between them becomes 2 fm? Assume that the nuclear force is not effective for separations greater than 2 fm. At what temperature will the deuterons have this kinetic energy on an average?

Solution:

As the deuterons move, the Coulomb repulsion will slow them down. The loss in kinetic energy will be equal to the gain in Coulomb potential energy. At the closest separation, the kinetic energy is zero and the potential energy is $\frac{e^2}{4\pi\varepsilon_0 r}$. If the initial kinetic energy of each deuteron is K and the closest separation is 2fm, we shall have

$$2\,K = \frac{e^2}{4\pi\varepsilon_0(2\,\text{fm})}$$

$$= \frac{(1.6\times10^{-19}\,C)^2 \times (9\times10^9\,N-m^2/C^2)}{2\times10^{-15}\,m}$$

or, $K = 5.7 \times 10^{-14}$ J.

If the temperature of the gas is T, the average kinetic energy of random motion of each nucleus will be 1.5 kT. The temperature needed for the deuterons to have the average kinetic energy of 5.7×10^{-14} J will be given by

$1.5\ kT = 5.7 \times 10^{-14}$ J where k = Botzmann constant

or, $$T = \frac{5.7\times10^{-14}\,J}{1.5\times1.38\times10^{-23}\,J/K} = 2.8 \times 10^9\ K.$$

BRAIN MAP

MODERN PHYSICS

1. Radius of allowed Bohrs orbit

$$= \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} = 0.53 \frac{n^2}{Z} \text{Å}$$

Energy of hydrogen atom in n^{th} energy

State $= -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2} = -13.6 \frac{Z^2}{n^2}$

Velocity of electron Bohr orbit

$$v = \frac{Ze^2}{2\varepsilon_0 hn} = 2.191.10^6 \frac{Z}{n}$$

2. The wave number of a spectral line is

given by $\bar{v} = RZ^2 \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$

$m = 1, n > 2$

Corresponds to Lyman series,

$m = 1, n > 2$

Corresponds to Balmer series,

$m = 3, n > 3$

Corresponds to Paschen series,

$m = 4, n > 4$

Corresponds to Bracket series,

$m = 5, n > 5$

Corresponds to Pfund series and so on.

(n is a natural number)

3. de-Broglie wavelength of a particle of mass m and moving with velocity v is

$$\lambda = \frac{h}{mv}$$

Einstein photoelectric equation is

$hv = W_0 + (\text{KE})_{max}$

Stopping potential

$$V_0 = \left(\frac{h}{e}\right)v - \frac{W_0}{e}$$

4. For continuous X-rays

$$\lambda_{min} = \frac{hc}{eV}$$

Moseley law for characteristic X-rays

$\sqrt{v} = a\,(Z - b)$

5. Einstein mass energy equivalence principle $\Delta E = (\Delta m)c^2$

Binding energy per nucleon

$$= \left[\frac{Z}{A}(m_P - m_n) + m_n - \frac{m}{A}\right]c^2$$

The statistical radioactive law

$N = N_0 e^{-\lambda t}$

Half life

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Mean life,

$$T_{av} = \frac{1}{\lambda}$$

Activity $A = A_0 e^{-\lambda t}$

EXERCISES

Single Options Correct Type

1. The energy that should be added to an electron, to reduce its de-Broglie wavelengths from 10^{-10} m to 0.5×10^{-10} m, will be
   (A) four times the initial energy.
   (B) thrice the initial energy.
   (C) equal to the initial energy.
   (D) twice the initial energy.

2. The radioactivity of an element becomes $\frac{1}{64}$th of its original value in 60 s. Then the half-life period of element is
   (A) 5 s (B) 10 s
   (C) 20 s (D) 30 s

3. The probability of a radioactive atom for not disintegrating till 3 times of its half-life is
   (A) $\frac{1}{3}$ (b) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{7}{8}$

4. The time by a photoelectron to come out after the photon strikes is approximately
   (A) 10^{-1}s (B) 10^{-4}s (C) 10^{-10}s (D) 10^{-16}s

5. The wavelength of a certain line in the x-ray spectrum for tungsten ($Z = 74$) is 200 Å. What would be the wavelength of the same line for platinum ($Z = 78$)? The screening constant a is unity.
   (A) 179.76 Å (B) 189.76 Å
   (C) 289.76 Å (D) 379.76 Å

6. When $_3Li^7$ nuclei are bombarded by protons, and the resultant nuclei are $_4Be^8$, the emitted particles will be
   (A) Neutrons. (B) Alpha particles.
   (C) Beta particles. (D) Gamma photons.

7. Two electrons of kinetic energy 2.5 eV fall on a metal plate, which has work function of 4.0 eV. Number of electrons ejected from the metal surface is
   (A) One (B) Two
   (C) Zero (D) More than two

8. The binding energies of the atoms of elements A and B are E_a and E_b respectively. Three atoms of the element B fuse to give one atom of element A. This fusion process is accompanied by release of energy e. Then E_a, E_b and e are related to each other as
   (A) $E_a + e = 3E_b$ (B) $E_a = 3E_b$
   (C) $E_a - e = 3E_b$ (D) $E_a + 3E_b + e = 0$

9. What is the ratio of the circumference of the first Bohr orbit for the electron in the hydrogen atom to the de-Broglie wavelength of electrons having the same velocity as the electron in the first Bohr orbit of the hydrogen atom?
   (A) 1 : 1 (B) 1 : 2 (C) 1 : 4 (D) 2 : 1

10. In the x-ray tube before striking the target we accelerate the electrons through a potential difference of V volt. For which of the following value of V, we will have x-rays of largest wavelength?
    (A) 10 kV (B) 20 kV (C) 30 kV (D) 40 kV

11. The electron emitted in beta radiation originates from
    (A) Inner orbits of atoms.
    (B) Free electrons existing in nuclei.
    (C) Decay of neutron in a nucleus.
    (D) Photon escaping from the nucleus.

12. Two radioactive substances X and Y initially contain equal number of nuclei. X has a half-life of 1 hour and Y has half-life of 2 hours. After two hours the ratio of the activity of X to the activity of Y will be
    (A) 1 : 4 (B) 1 : 2 (C) 1 : 1 (D) 2 : 1

13. The energy spectrum of a black body exhibits maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. The power radiated by the black body will now increase by a factor of
    (A) $\frac{256}{81}$ (B) $\frac{64}{27}$ (C) $\frac{16}{9}$ (D) $\frac{4}{3}$

14. The binding energy per nucleon of deuteron ($_1H^2$) and helium nucleus ($_2He^4$) are 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then energy released is
    (A) 13.9 MeV (B) 26.9 MeV
    (C) 23.6 MeV (D) 19.2 MeV

15. The electromagnetic waves that has highest wavelength is
    (A) X-rays.
    (B) Ultraviolet rays.
    (C) Infra-red rays.
    (D) Microwaves.

16. The ratio of de-Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C respectively will be

(A) $\sqrt{\frac{4}{3}}$ (B) $\sqrt{\frac{8}{3}}$ (C) $\sqrt{\frac{3}{8}}$ (D) $\sqrt{\frac{3}{4}}$

17. A proton with kinetic energy K describes a circle of radius r in a uniform magnetic field. An α-particle with kinetic energy K moving in the same magnetic field will describe a circle of radius

(A) $\frac{r}{2}$ (B) r (C) $2r$ (D) $4r$

18. An electron of mass m and charge e is accelerated by a potential difference V. It then enters a uniform magnetic field B applied perpendicular to its path. The radius of the circular path of the electron is

(A) $r=\left(\frac{2mV}{eB^2}\right)^{\frac{1}{2}}$ (B) $r=\left(\frac{2meV}{B^2}\right)^{\frac{1}{2}}$

(C) $r=\left(\frac{2mB}{eV^2}\right)^{\frac{1}{2}}$ (D) $r=\left(\frac{2B^2V}{em}\right)^{\frac{1}{2}}$

19. If the binding energy per nucleon in ${}^7_3\text{Li}$ and ${}^4_2\text{He}$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction ${}^1_1\text{H}+{}^7_3\text{Li}\rightarrow 2\,{}^4_2\text{He}$ energy of proton must be

(A) 39.2 MeV (B) 28.24 MeV
(C) 17.28 MeV (D) 1.46 MeV

20. A photosensitive metallic surface has work function $h\nu_0$. If photon of energy $2h\nu_0$ falls on this surface, the electrons come out with a maximum velocity of 4×10^6 m/s. When the photon energy is increased to $5h\nu_0$, then maximum velocity of photoelectrons will be:

(A) 2×10^6 m/s
(B) 2×10^7 m/s
(C) 8×10^7 m/s
(D) 8×10^6 m/s

21. In hydrogen like atoms the ratio of difference of energies $E_{4n}-E_{2n}$ and $E_{2n}-E_n$ varies with atomic number z and principle quantum number n as

(A) $\frac{z^2}{n^2}$ (B) $\frac{z^4}{n^4}$

(C) $\frac{z}{n}$ (D) None of these

22. A hydrogen atom is in an excited state of principle quantum number n. It emits a photon of wavelength λ when returns to the ground state. The value of n is (R = Rydberg constant)

(A) $\sqrt{\lambda R(\lambda R-1)}$ (B) $\sqrt{\frac{(\lambda R-1)}{\lambda R}}$

(C) $\sqrt{\frac{\lambda R}{\lambda R-1}}$ (D) $\sqrt{\lambda(R-1)}$

23. An x-ray tube is operating at 2 million V. What is the wavelength of shortest wave produced?

(A) 6×10^{-3} m (B) 6×10^{-5} m
(C) 6×10^{-1} m (D) None of these

24. The longest wavelength that a single ionized helium atom in its ground state will absorb is

(A) 912 Å (B) 304 Å
(C) 229 Å (D) 687 Å

25. A sample contains 16 g of a radioactive material, the half-life of which is 2 days. After 32 days the amount of radioactive material left in the sample is

(A) Less than 1 mg (B) (1/4) g
(C) (1/2) g (D) 1 g

26. A freshly prepared radioactive source half-life 2hr emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

(A) 6 hr (B) 12 hr (C) 42 hr (D) 128 hr

27. If the deBroglie wavelength of a proton is 1.0×10^{-13} m, the electric potential through which it must have been accelerated is

(A) 4.07×10^4 V (B) 8.2×10^4 V
(C) 8.2×10^3 V (D) 4.07×10^5 V

28. If proton and α -particles are accelerated by the same potential difference, then their De-Broglie wavelength will be in the ratio of

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4

29. If the maximum kinetic energy of emitted photo electrons from a metal surface of work function 2.5 eV, is 1.7 eV. If wavelength of incident radiation is halved, then stopping potential will be

(A) 2.5 V (B) 6.7 V (C) 5 V (D) 1.1 V

30. If photons of energy 12.75 eV are passing through hydrogen gas in ground state then number of lines in emission spectrum will be

(A) 6 (B) 4 (C) 3 (D) 2

31. An x-ray tube operating at 30 kV, will emit x-ray of minimum wavelength

(A) 2840 Å (B) 0.414 Å
(C) 2.14 Å (D) 1.78 Å

32. The half-life of a radioactive element $^{222}R_u$ is 3.8 hrs. Mass of this element which has activity equal to 10^{16} Rutherford is

(A) 0.37 kg (B) 0.37 g
(C) 0.073 g (D) 0.07g

33. If the shortest wavelengths of the continuous spectrum coming out of a Coolidge tube is 0.1Å, then the de Broglie wavelength of the electron reaching the target metal in the Coolidge tube is approximately (hc = 12400 eVÅ, $h = 6.63 \times 10^{-34}$ in MKS, mass of electron = 9.1×10^{-31} kg)

(A) 0.35 Å (B) 0.035 Å
(C) 35 Å (D) 1 Å

34. An electron collides with a fixed hydrogen atom in its ground state. Hydrogen atom gets excited and the colliding electron loses all its kinetic energy. Consequently the hydrogen atom may emit a photon corresponding to the largest wavelength of the Balmer series. The minimum kinetic energy of colliding electron is

(A) 10.2 eV (B) 1.9 eV
(C) 12.09 eV (D) 13.6 eV

35. A radiation of energy E falls normally on a perfectly absorbing surface. The momentum transferred to the surface is

(A) $\frac{E}{c}$ (B) $\frac{2E}{c}$ (C) Ec (D) $\frac{E}{c^2}$

36. A lead ball moving with velocity v strikes a wall and stops. If 50% of its energy is converted into heat, then what will be the increase in temperature? (Specific heat of lead is s)

(A) $\frac{2v^2}{\text{Js}}$ (B) $\frac{v^2}{4\text{Js}}$ (C) $\frac{v^2 s}{\text{J}}$ (D) $\frac{v^2 s}{2\text{J}}$

37. If doubly ionized lithium atom is hydrogen like with atomic number 3, the wavelength of radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit and the number of different spectral lines observed in the emission spectrum of the above excited system are

(A) 296 Å, 6 (B) 114 Å, 3
(C) 1026 Å, 6 (D) 8208 Å, 3

38. Uranium ores contain one radium-226 atom for every 2.8×10^6 Uranium-238 atoms. Calculate the half-life of $_{92}U^{238}$ given that the half-life of $_{88}Ra^{226}$ is 1600 years and $_{88}Ra^{226}$ is a decay product of $_{92}U^{238}$.

(A) 1.75×10^3 years (B) $1600 \times \frac{238}{92}$ years
(C) 4.5×10^9 years (D) $1600 \times \frac{92}{238}$ years

39. The ratio of de Broglie wavelength of α-particle to that of a proton being subjected to the same magnetic field so that the radii of their paths are equal to each other assuming the field induction vector $\vec{B}$ is perpendicular to the velocity vectors of the α-particle and the proton is

(A) 1 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 2

40. The ratio (in SI units) of magnetic dipole moment to that of the angular momentum of electron of mass m kg and charge e coulomb in Bohr's orbit of hydrogen atom is

(A) $\frac{e}{22m}$ (B) $\frac{e}{6m}$
(C) $\frac{12e}{m}$ (D) None of these

41. The speed of an electron having a wavelength of the order of 1 Å will be

(A) 7.25×10^6 m/s (B) 6.26×10^6 m/s
(C) 5.25×10^6 m/s (D) 4.24×10^6 m/s

42. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$; $B = B_0 \sin(kx - \omega t)$. Which of the following equation is true?

(A) $E_0 k = B_0 \omega$ (B) $E_0 \omega = B_0 k$
(C) $E_0 k_0 = \omega k$ (D) None of these

43. The number densities of electrons and holes in a pure germanium at room temperature are equal and its value is 3×10^{16} per m^3. On doping with aluminium, the whole density increases to 4.5×10^{22} per m^3. Then the electron density in doped germanium is

(A) 2.5×10^{10} m^{-3} (B) 2×10^{10} m^{-3}
(C) 4.5×10^9 m^{-3} (D) 3×10^9 m^{-3}

44. The energy that should be added to an electron, to reduce its de-Broglie wavelengths from 10^{-10} m to 0.5×10^{-10} m, will be

(A) four times the initial energy.
(B) thrice the initial energy.
(C) equal to the initial energy.
(D) twice the initial energy.

45. A proton of mass m and charge +e is moving in a circular orbit in a magnetic field with energy 1 MeV. What should be the energy of α-particle (mass = 4m and charge = +2e), so that it can revolve in the path of same radius

(A) 1 MeV (B) 4 MeV
(C) 2 MeV (D) 0.5 MeV

46. In an electromagnetic wave, the electric and magnetizing fields are 100 Vm^{-1} and 0.265 Am^{-1}. The maximum energy flow is

(A) 26.5 W/m^2 (B) 36.5 W/m^2
(C) 46.7 W/m^2 (D) 765 W/m^2

47. The speed of an electron having a wavelength of the order of 1Å will be
(A) 7.25×10^6 m/s (B) 6.26×10^6 m/s
(C) 5.25×10^6 m/s (D) 4.24×10^6 m/s

48. In a photoelectric experiment, the wavelength of incident radiation is reduced from 6000 Å to 4000 Å then
(A) Stopping potential will decrease.
(B) Stopping potential will increase.
(C) Kinetic energy of emitted electrons will decrease.
(D) The value of work function will decrease.

49. Figure 19.13 represents the graph of kinetic energy (K) of photoelectrons (in eV) and frequency (v) for a metal used as cathode in photoelectric experiment. The work function of metal is

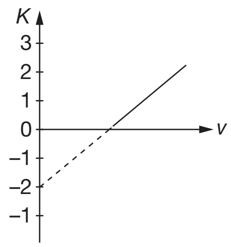


Fig. 19.13

(A) 1 eV (B) 1.5 eV (C) 2 eV (D) 3 eV

50. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized *Li* and ($Z = 3$) is
(A) 1.51 (B) 13.6 (C) 40.8 (D) 122.4

51. In Bohr's model, the atomic radius of the first orbit is r_0, then the radius of the third orbit is
(A) $\frac{r_0}{9}$ (B) r_0 (C) $9r_0$ (D) $3r_0$

52. A particle of mass $3m$ at rest decays into two particles of masses m and $2m$ having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles (λ_1/λ_2) is
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$
(C) 2 (D) None of these

53. In a photoelectric effect experiment
(A) on increasing intensity and keeping frequency fixed the saturation current decreases.
(B) on increasing intensity and keeping frequency fixed the saturation current remains constant.
(C) on increasing intensity, saturation current may increase.
(D) on increasing frequency saturation current may increase.

54. The wavelength of the K_α line for the uranium atom ($Z = 92$) is ($R = 10^7$ m^{-1})
(A) 1.6Å (B) 0.16Å (C) 0.5Å (D) 2.0Å

55. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li and ($Z = 3$) is
(A) 1.51 (B) 13.6 (C) 40.8 (D) 122.4

56. The de-Broglie wavelength of a particle moving with a velocity 2.25×10^8 m/s is equal to the wavelength of photon. The ratio of kinetic energy of the particle to the energy of the photon is (velocity of light is 3×10^8 m/s)
(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{7}{8}$

57. When a certain metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$. When the same surface is illuminated with the light of wavelength 2λ, the stopping potential is V_0. The threshold wavelength for the surface for photoelectric effect is
(A) $\frac{4\lambda}{3}$ (B) 4λ (C) 6λ (D) 8λ

58. In the hydrogen atom spectrum, λ_{3-1} and λ_{2-1} represent wavelengths emitted due to transition from second and first excited states to the ground state respectively. The value of $\frac{\lambda_{3-1}}{\lambda_{2-1}}$ is
(A) $\frac{27}{32}$ (B) $\frac{32}{27}$ (C) $\frac{4}{9}$ (D) $\frac{9}{4}$

59. The work function of aluminium is 4.2 eV. If two photons, each of energy 3.5 eV strike an electron of aluminium, then emission of electrons will be
(A) Possible.
(B) Not possible.
(C) Data is incomplete.
(D) Depend upon the density of the surface.

60. An electron and a proton are separated by a large distance. The electron starts approaching the proton with energy 2eV. The proton captures the electron and forms a hydrogen atom in first excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4600Å. The maximum *KE* of the emitted photoelectron is (Take hc = 12420 eV Å)
(A) 2.4 eV (B) 2.7 eV
(C) 2.9 eV (D) 5.4 eV

61. Two radioactive elements *R* and *S* disintegrate as
$R \longrightarrow P + \alpha;\ \lambda_R = 4.5 \times 10^{-3}$ years $^{-1}$
$S \longrightarrow Q + \beta;\ \lambda_S = 3 \times 10^{-3}$ years $^{-1}$

Starting with number of atoms of R and S in the ratio of 2 : 1, this ratio after the lapse of three half-lives of R will be

(A) 3: 2 (B) 1: 3
(C) 1 : 1 (D) 2 : 1

62. An electron in hypothetical hydrogen atom is in its 3rd excited state and makes transition from 3rd to 2nd excited, then to 1st excited state and then to ground state. If the amount of time spent by the electron in any state of quantum number n, is proportional to $\left(\frac{1}{n-1}\right)$, then the ratio of number of revolutions completed by the electron in 1st excited state to that in the 2nd excited state will be

(A) 2 (B) $\frac{27}{8}$ (C) $\frac{27}{4}$ (D) $\frac{27}{6}$

63. In an x-ray set up accelerating potential difference is set at V volt and first a target metal having atomic number Z_1 is used then target metal having Z_2 is used (λ_{α_1} is wavelength of K_α x-ray from Z_1 target and λ_{α_2} is wavelength of K_α, x-ray with Z_2 target)

(A) If $Z_1 > Z_2$ then $\left(\lambda_{\alpha_1} - \lambda_{\min}\right) > \left(\lambda_{\alpha_2} - \lambda_{\min}\right)$

(B) If $Z_1 < Z_2$ then $\left(\lambda_{\alpha_1} - \lambda_{\min}\right) > \left(\lambda_{\alpha_2} - \lambda_{\min}\right)$

(C) The difference $\left(\lambda_{\alpha_1} - \lambda_{\min}\right)$ and $\left(\lambda_{\alpha_{12}} - \lambda_{\min}\right)$ will be same for both the target metals.

(D) If $Z_1 \le Z_2$ then $\left(\lambda_{\alpha_1} - \lambda_{\min}\right) \le \left(\lambda_{\alpha_2} - \lambda_{\min}\right)$

64. The minimum KE of proton incident on ${}^{13}_{6}C$ nuclei at rest that will produce the reaction ${}^{13}C(p,n)^{13}N$ is (Mass of nitrogen = 13.005738 amu, Mass of neutron = 1.008665 amu, Mass of carbon = 13.005738 amu Mass of proton = 1.007825 amu)

(A) 0.02 MeV (B) 0.10 MeV
(C) 0.03 MeV (D) 0.05 MeV

65. The wavelength corresponding to maximum spectral radiance of a black body A is λ_A = 5000 Å. Consider another black body B, whose surface area is twice that of A and total radiant energy by B is 16 times that emitted by A. The wavelength corresponding to maximum spectrum radiance for B will be

(A) $5000\ (2)^{3/4}$ Å (B) 2500 Å
(C) 10,000 Å (D) $5000\ (2)^{-3/4}$ Å

66. An electron collides with a fixed hydrogen atom in its ground state. Hydrogen atom gets excited and the colliding electron loses all its KE. Consequently the hydrogen atom may emit a photon corresponding to the largest wavelength of the Balmer series. The minimum KE of colliding electron is

(A) 10.2 eV (B) 1.9 eV
(C) 12.09 eV (D) 13.6 eV

67. Electromagnetic radiation whose electric component varies with time as $E = C_1(C_2 + C_3 \cos \omega t)\cos \omega_0 t$, here C_1, C_2 and C_3 are constants, is incident on lithium and liberates photoelectrons. Let the kinetic energy of most energetic electrons be 0.592×10^{-19} J. Given that $\omega_0 = 3.6 \times 10^{15}$ rad/s and $\omega = 6 \times 10^{14}$ rad/sec. The work function of lithium is (take planks constant $h = 6.6 \times 10^{-34}$ MKS).

(A) 1.2 eV (B) 1.5 eV
(C) 2.1 eV (D) 2.39 eV

68. In hydrogen atom, if potential energy of electron in ground state is assumed to be zero, then its energy in first excited state is equal to

(A) – 3.4 eV (B) 23.8 eV
(C) 17.2 eV (D) – 6.8 eV

69. The wavelength of characteristic K_α-line emitted by a hydrogen like element is 0.32 Å. The wavelength of the K_β-line emitted by the same element will be

(A) 0.25 Å (B) 0.27 Å
(C) 0.30 Å (D) 0.35 Å

70. The mean lives of a radioactive sample are 30 years and 60 years for α-emission and β-emission respectively. If the sample decays both by α-emission and β-emission simultaneously, the time after which, only one-fourth of the sample remain is

(A) 10 years (B) 20 years
(C) 40 years (D) 45 years

71. An electron and a proton are separated by a large distance. The electron starts approaching the proton with energy 2 eV. The proton captures the electron and forms a hydrogen atom in first excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4600 Å. The maximum KE of the emitted photoelectron is (hc = 12420 eV Å)

(A) 2.4 eV (B) 2.7 eV
(C) 2.9 eV (D) 5.4 eV

72. If the shortest wavelengths of the continuous spectrum coming out of a Coolidge tube is 0.1Å, then the de Broglie wavelength of the electron reaching the target metal in the Coolidge tube is approximately

($hc = 12400$ eVÅ, $h = 6.63 \times 10^{-34}$ in MKS, mass of electron = 9.1×10^{-31} kg)

(A) 0.35 Å (B) 0.035 Å
(C) 35 Å (D) 1 Å

73. A radioactive nucleus decays according to following series

$$X \xrightarrow{\alpha} X_1 \xrightarrow{\beta} X_2 \xrightarrow{\alpha} X_3 \xrightarrow{\gamma} X_4$$

If the atomic number and atomic weight of the parent element X are 72 and 180 respectively, then the atomic number and atomic mass of X_4 are respectively

(A) 70, 172
(B) 69, 171
(C) 69, 172
(D) 68, 172

74. The 'rad' is the correct unit used to report the measurement of

(A) the rate of decay of radioactive source.
(B) the ability of a beam of gamma ray photons to produce ions in a target.
(C) the energy delivered by radiation to a target.
(D) the biological effect of radiation.

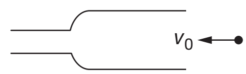
75. In certain element the K-shell electron energy is –18.525 keV and the L-shell electron energy is –3 keV. When an electron jumps from the L-shell to K shell, an x-ray photon is emitted. The wavelength of the emitted x-rays is

(A) 0.8 Å (B) 1 Å
(C) 0.6 Å (D) 1.2 Å

76. In an ore containing uranium, the ratio of U^{238} to Pb^{206} nuclei is 3. The age of the ore, assuming that all the lead present in the ore is the final stable product of U^{238} is (Take the half-life of U^{238} to be 4.5×10^9 years, ln 2 = 0.7, ln 3 = 1.1)

(A) 1.95×10^9 years
(B) 1.95×10^{10} years
(C) 1.95×10^8 years
(D) 1.95×10^7 years

77. An electron moves along a metal tube with variable section. The velocity of the electron when it approaches the neck of tube, is



(A) greater than v_0 (B) equal to v_0
(C) less than v_0 (D) not defined

78. In a hydrogen atom, an electron of mass m and charge e is in an orbit of radius r making n revolutions per second. If the mass of the hydrogen nucleus is M, the magnetic moment associated with the orbital motion of the electron is

(A) $\dfrac{\pi ner^2 m}{M}$ (B) $\dfrac{\pi ner^2 M}{m}$

(C) $\dfrac{\pi ner^2 m}{(M+m)}$ (D) πner^2

More than One Option Correct Type

79. An electron in H-atom jumps from second excited state to first excited state and then from first excited to ground state. Let the ratio of wavelength, momentum and energy of photons emitted in these two cases be a, b and c respectively. Then

(A) $c = \dfrac{1}{a}$ (B) $a = \dfrac{9}{4}$

(C) $b = \dfrac{5}{27}$ (D) $c = \dfrac{5}{27}$

80. If the potential difference of Coolidge tube producing x-ray is increased, then choose the correct option(s).

(A) the interval between $\lambda_{k\alpha}$ and $\lambda_{k\beta}$ increases
(B) the interval between $\lambda_{k\alpha}$ and λ_0 increases
(C) the interval between $\lambda_{k\beta}$ and λ_0 increases
(D) λ_0 does not change

81. Pick the correct statements

(A) gravitational force between two protons may be greater than the electrostatic force between the protons.
(B) electromagnetic force is greater than the gravitational force between two protons.
(C) electrostatic force is a fundamental force.
(D) nuclear force is attractive or repulsive as per the nature of charges.

82. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr model to be valid. If the time period of the electron in the initial state is eight times that in the final state then the possible values of n_1 and n_2 are

(A) $n_1 = 4, n_2 = 2$ (B) $n_1 = 8, n_2 = 2$
(C) $n_1 = 8, n_2 = 1$ (D) $n_1 = 6, n_2 = 3$

83. Which of the following is a correct statement?
(A) Beta rays are same as cathode rays.
(B) Gamma rays are high energy neutrons.
(C) Alpha particles are double ionized helium atoms.
(D) Protons and neutrons have exactly the same mass.

84. The energy, the magnitude of linear momentum and orbital radius of an electron in a hydrogen atom corresponding to the quantum number n are E, P and r respectively. Then according to Bohr's theory for hydrogen atom
(A) EPr is proportional to $\frac{1}{n}$.
(B) $\frac{P}{E}$ is proportional to n.
(C) Er is constant for all orbits.
(D) Pr is proportional to n.

85. Which of the following statement about x-rays is/are true?
(A) $E(K_\alpha)+E(L_\beta)=E(K_\beta)+E(M_\alpha)=E(K_\gamma)$, where E is the energy of respective x-rays.
(B) For the harder x-rays, the intensity is higher than soft x-rays.
(C) The continuous and the characteristic x-rays differ only in the method of creation.
(D) The cut-off wavelength λ_{min} depends only on the accelerating voltage applied between the target and the filament.

86. Due to annihilation of electron-positron of same kinetic energy 0.95 MeV, a photon is produced which can also be produced by a photo-electron of energy E, the possible value(s) of E is/are (mass of electron $= 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ Coulomb)

(A) 1.02 MeV (B) 2.42 MeV
(C) 4.03 MeV (D) 2.93 MeV

87. An electron in hydrogen atom first jumps from second excited state to first excited state and then from first excited state to ground state. Let the ratio of wavelength, momentum and energy of photons emitted in these two cases be a, b and c respectively, then

(A) $c = \frac{1}{a}$ (B) $a = \frac{9}{4}$
(C) $b = \frac{5}{27}$ (D) $c = \frac{5}{27}$

88. The bodies A and B have thermal emissivity's of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are equal. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiance in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiance in the radiation from A, by 1.00 μm. If the temperature of A is 5802 K.
(A) the temperature of B is 1934 K.
(B) $\lambda_B = 1.5$ μm.
(C) the temperature of B is 1160 K.
(D) the temperature of B is 2901 K.

Passage Based Questions

Passage I

An ideal gas is isothermally expanded at temperature T to double its volume. Then, it is expanded adiabatically to further increase the volume 4 times. Then, it is taken back to starting state by a thermodynamic process which can be shown by a straight line in a PV-diagram of the gas. (Number of moles $n = 1$, $\gamma = 1.5$ and $R =$ gas constant)

89. Work done by the gas in third process

(A) $-\frac{129}{32}RT$ (B) $-\frac{219}{32}RT$
(C) $-\frac{119}{32}RT$ (D) $-\frac{139}{32}RT$

90. Work done by the gas in adiabatic process

(A) $2RT$ (B) RT (C) $\frac{1}{2}RT$ (D) $\frac{3}{2}RT$

91. Heat given in third process

(A) $-\frac{87}{32}RT$ (B) $-\frac{85}{32}RT$
(C) $-\frac{29}{32}RT$ (D) $-\frac{1}{32}RT$

Passage 2

In hydrogen like atoms, in which every atom is in a particular excited state. Now a stream of photons of energy $\frac{64}{225}E_0$ bombarded into it and is absorbed by the hydrogen like atoms and subsequently its emission spectrum shows 10 different lines in which some lines have energy less than $\frac{64}{225}E_0$ and some lines have energy more than $\frac{64}{225}E_0$ and some lines have energy equal to $\frac{64}{225}E_0$. (Where E_0 is ionization energy of hydrogen atom in ground state).

92. The initial quantum number of electron in the atom is
(A) 2 (B) 3 (C) 4 (D) 5

93. The final quantum number of electron in the atom is
(A) 2 (B) 3 (C) 4 (D) 5

94. Atomic number of the atom is
(A) 2 (B) 3 (C) 4 (D) 5

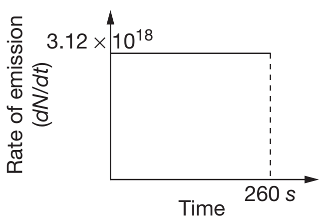
Passage 3

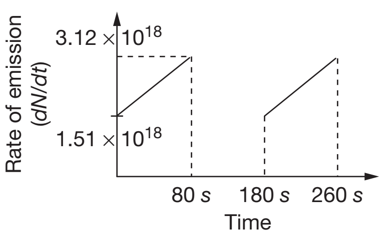
A light of wavelength λ is incident on a metal sheet of work function $\phi = 2$ eV. The wavelength λ varies with time as $\lambda = 3000 + 40t$, where λ is in Å and t is in second. The power incident on metal sheet is constant at 100 W. This signal is switched on and off for time intervals of 2 minutes and 1 minute respectively. Each time the signal is switched on, when the λ start from initial value of 3000 Å. The metal plate is grounded and electron clouding is negligible. The efficiency of photoemission is 1% (hc = 12400 eVÅ).

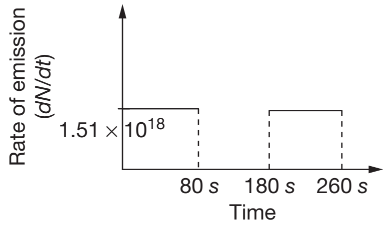
95. The time after which photoemission will stop is
(A) 79 s (B) 80 s
(C) 81 s (D) 78 s

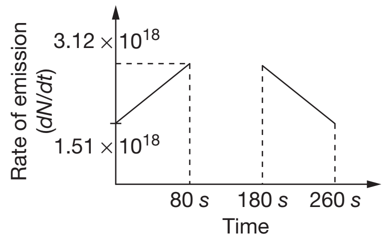
96. The total number of photoelectrons ejected in one hour is
(A) 3.71×10^{20} (B) 3.71×10^{21}
(C) 3.71×10^{22} (D) 3.71×10^{23}

97. The variation of rate of emission vs time is

(A)


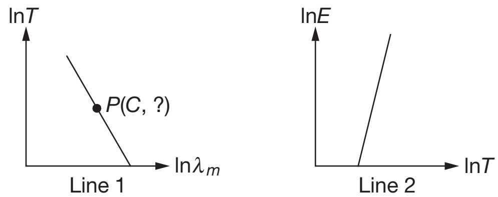
(B)


(C)


(D)


Passage 4

The variation of the ln$T$ versus lnλ_m and ln$E$ versus ln$T$ are given as shown. T is the temperature of the body in Kelvins, λ_m is the wave length corresponding to which maximum number of photons is emitted and E is the energy emitted by the body per second. The intercept made by the line 1 on the Y-axis has length A.



98. What is the slope of the line 1?
(A) $\frac{-1}{2}$ (B) –1 (C) –2 (D) –4

99. What is the slope of the line 2?
(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4

100. What is the value of Wein's displacement constant?
(A) e^A (B) $1/e^A$
(C) ln A (D) 1/InA

Passage 5

A gas of identical hydrogen like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. If all the excited atoms of the gas make transition to higher energy level by absorbing monochromatic light of photon 10.2 eV, subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 10.2 eV, some have energy more and some have less than 10.2 eV.

101. Find the principal quantum number of the initially excited level B.

(A) 1 (B) 2 (C) 3 (D) 4

102. Find the ionization energy for the gas atoms.

(A) 54.4 eV (B) 13.6 eV
(C) 122.4 eV (D) 217.6 eV

103. Find the minimum energies of the emitted photons.

(A) 1.5 eV (B) 2.1 eV
(C) 2.64 eV (D) 3.4 eV

104. Find the maximum energies of the emitted photons.

(A) 51 eV (B) 40.8 eV
(C) 48.4 eV (D) 10.2 eV

Match the Column Type

105. Match the column-I with column-II.

Column-I	Column-II
(A) Mass of products formed is less than the original mass of the system in	(1) α-decay
(B) Binding energy per nucleon increases in	(2) β-decay
(C) Mass number is conserved in	(3) Nuclear fission
(D) Charge number is conserved in	(4) Nuclear fusion

106. In a photoelectric effect experiment, if f is the frequency of radiations incident on the metal surface and I is the intensity of the incident radiations, then match the following.

Column-I	Column-II
(A) If f is increased keeping I and work function constant.	(1) Stopping potential increases
(B) If distance between cathode and anode is increased	(2) Saturation current increases
(C) If I is increased keeping f and work function constant	(3) Maximum kinetic energy of photoelectron increases
(D) Work function is decreased keeping f and I constant.	(4) Stopping potential remains same

107. Column-I represent the physical parameters being changed in the experiment of photo electric effect and in Column-II is its effect.

Column-I	Column-II
(A) Intensity	(1) photo electric current
(B) Frequency	(2) stopping potential
(C) Potential difference between anode and cathode	(3) work function
(D) Metal	(4) maximum kinetic energy

108. In a hydrogen like atoms in which electron in every atom has in a particular excited state. Now a stream of photons of energy $\frac{64}{225}E_0$ bombarded into it and is absorbed by the hydrogen like atoms and subsequently its emission spectrum shows 10 different lines in which some lines have energy less than $\frac{64}{225}E_0$ and some lines have energy more than $\frac{64}{225}E_0$ and some lines have energy equal to $\frac{64}{225}E_0$. (Where E_0 is ionization energy of hydrogen atom in ground state).

Column-I	Column-II
(A) The initial quantum number of electron in the atom is	(1) 2
(B) The final quantum number of electron in the atom is	(2) 3
(C) Atomic number of the H-like atom is	(3) 4
	(4) 5

109. Column-II represents the possible effects of the processes performed in column-I.

Column-I	Column-II
(A) The voltage applied to x-ray tube is increased.	(1) Average KE of the electrons decreases
(B) In photo-electric effect experiment, work function of the photosensitive metal is increased.	(2) Average KE of the electrons increases
(C) Magnitude of stopping potential decreases	(3) Cut off wavelength decreases
(D) Atomic number of target material decreases	(4) Wavelength of K_α X-ray increases

Assertion-Reason Type

110. Assertion: The nuclear energy can be obtained by the nuclear fission of heavier nuclei as well as nuclear fusion of lighter nuclei.

Reason: The binding energy per nucleon with increase in atomic number first increases and then decreases.

(A) A (B) B (C) C (D) D

111. Assertion: Heavy nuclides tend to have more number of neutrons than protons.

Reason: In heavy nuclei, there is an excess of neutrons due to Coulomb repulsion between protons.

(A) A (B) B (C) C (D) D

112. Assertion: In a hydrogen atom energy of emitted photon corresponding to transition from $n = 2$ to $n = 1$ is much greater as compared to transition from $n = \infty$ to $n = 2$.

Reason: Wavelength of photon is directly proportional to the energy of emitted photon.

(A) A (B) B (C) C (D) D

113. Assertion: Time required for 75% radioactive disintegration $(t_{3/4}) = 2 \times t_{1/2}$.

Reason: Half-life $(t_{1/2})$ of the radioactive disintegration is independent of temperature.

(A) A (B) B (C) C (D) D

114. Assertion: x-rays are not deflected by electric and magnetic field.

Reason: x-rays travel with velocity equal to that of light

(A) A (B) B (C) C (D) D

115. Assertion: Work function of aluminium is 4.2 eV. If two photons of each of energy 2.5 eV strike on an electron of aluminium, the electron is not emitted.

Reason: In photoelectric effect a single photon interacts with a single electron and electron is emitted only if energy of each of incident photon is greater than the work function.

(A) A (B) B (C) C (D) D

116. Assertion: ${}_ZX^A$ undergoes 2α-decays, 2β-decays and 2γ-decays and the daughter product is ${}_{Z-2}X^{A-8}$.

Reason: In α-decay the mass number decreases by 4 units and atomic number decreases by 2 units. In β-decay the mass number remains unchanged, but atomic number increases by 1 unit only. In γ-decay, mass number and atomic number remain unchanged.

(A) A (B) B (C) C (D) D

117. Assertion: In an isothermal process, heat supplied to an ideal gas is completely used.

Reason: In isothermal process, internal energy remains unchanged.

(A) A (B) B (C) C (D) D

118. Assertion: Adiabatic compressibility of an ideal gas is greater than its isothermal compressibility at same pressure.

Reason: Slope of adiabatic P-V graph has greater magnitude than the slope of isothermal P-V graph at same pressure.

(A) A (B) B (C) C (D) D

119. Assertion: Photoelectric effect takes place only with bound electrons.

Reason: The photon cannot transfer all of its energy and momentum to a free electron.

(A) A (B) B (C) C (D) D

Integer Type

120. In a certain hypothetical radioactive decay process, species A decays into species B and species B decays into species C according to the reactions

$$A \rightarrow 2B + \text{particles} + \text{energy}$$
$$B \rightarrow 3C + \text{particles} + \text{energy}$$

The decay constant for species A is $\lambda_1 = 1\,\text{sec}^{-1}$ and that for species B is $\lambda_2 = 100\ \text{s}^{-1}$. Initially 10^4 moles of species of A were present while there was none of B and C. It was found that species B reaches its maximum number at a time $t_0 = 2\ln(10)$ sec. Calculate the value of maximum number of moles of B.

121. When photons of energy 5 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy K_A eV and de Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 5.30 eV is $K_B = (K_A - 1.5)$. eV. If the de Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then find K_A and K_B.

122. The work function W_A for photoelectric material A is 2 eV and W_B for another photoelectric material B is 4 eV. If photons of energy E_A strike the surface of A, the ejected photoelectrons have a minimum de Broglie wavelength and photons of energy E_B strike the surface B, the ejected photoelectrons also have a minimum de Broglie's wavelength. If $E_B - E_A = 0.5$ eV and V_A and V_B are the respective stopping potentials, find $V_A - V_B$.

123. In hydrogen like atom an electron is orbiting in an orbit having quantum number n. Its frequency of revolution is found to be 13.2×10^{15} Hz. Energy required to free the electron from the atom from the above orbit is 54.4 eV. In time 7 nano second the electron jumps back to orbit having quantum number $\frac{n}{2}$. τ be the average torque acted on the electron during the above process, then find $\tau \times 10^{27}$ in Nm. (given : $\frac{h}{\pi} = 2.1 \times 10^{-34}$ J-s, frequency of revolution of electron in the ground state of H-atom $v_0 = 6.6 \times 10^{15}$ Hz and ionization energy of H-atom, $E_0 = 13.6$ eV).

124. When the voltage applied to an x-ray tube is increased from 10 kV to 20 kV the wavelength interval between the K_α line and the short wave cut off of the continuous x-ray spectrum increases by a factor 3. Find the atomic number of element of which the tube anti-cathode is made. (Rydberg's constant = $10^7\ \text{m}^{-1}$)

125. Suppose potential energy between electron and proton at separation r is given by $U = k \ln r$, where k is constant. For such hypothetical hydrogen atom, the ratio of energy difference between energy levels ($n = 1$ and $n = 2$) and ($n = 2$ and $n = 4$) is

126. There are two radioactive nuclei A and B. A is an alpha emitter and B is a beta emitter. If their disintegration constants are in the ratio 1: 2, then the ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant

127. The radius of hydrogen atom in its ground state is 5.3×10^{-11} m. After collision with an electron it is found to have a radius of 21.2×10^{-11} m. What is the principal quantum number n of the final state of the atom?

Previous Years' Questions

128. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is **[2002]**

(A) 10.2 eV (B) 0 eV
(C) 3.4 eV (D) 6.8 eV

129. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit: **[2002]**

(i) Electrons (ii) Protons
(iii) He^{2+} (iv) Neutrons

The emission at instant can be

(A) (i), (ii), (iii) (B) (i), (ii), (iii), (iv)
(C) (iv) (D) (ii), (iii)

130. Sodium and copper have work done functions 2.3 eV and 4.5 eV respectively. Ten the ratio of the wavelengths is nearest to **[2002]**

(A) 1 : 2 (B) 4 : 1 (C) 2 : 1 (D) 1 : 4

131. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ years, then the amount of substance left after 15 years is **[2002]**

(A) $N_0/8$ (B) $N_0/16$
(C) $N_0/2$ (D) $N_0/4$

132. A strip of copper and another of germanium are cooled from room temperature to 80 K. The resistance of **[2003]**

(A) each of these decreases.
(B) copper strip increases and that of germanium decreases.
(C) copper strip decreases and that of germanium increases.
(D) each of these increases.

133. Which of the following radiations has the least wavelength? **[2003]**

(A) γ-rays (B) β-rays
(C) α-rays (D) X-rays

134. When a U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is **[2003]**

(A) $\frac{4u}{238}$ (B) $-\frac{4u}{234}$
(C) $\frac{4u}{234}$ (D) $-\frac{4u}{238}$

135. A radioactive sample at any instant has its disintegration rate 5000 disintegration per minute. After 5 minute, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is **[2003]**

(A) 0.4 ln 2 (B) 0.2 ln 2
(C) 0.1 ln 2 (D) 0.8 ln 2

136. A nucleus with $Z = 92$ emits the following in a sequence:

$\alpha, \beta^-, \beta^-\alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$

Then z of the resulting nucleus is **[2003]**

(A) 76 (B) 78 (C) 82 (D) 74

137. Which of the following cannot be emitted by radioactive substances during their decay? **[2003]**

(A) Protons (B) Neutrino's
(C) Helium nuclei (D) Electrons

138. In the nuclear nuclei

$$^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + n$$

given that the repulsive potential energy between the two nuclei is $\sim 7.7 \times 10^{-14}$ J , the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K] **[2003]**

(A) 10^7 K (B) 10^5 K (C) 10^3 K (D) 10^9 K

139. Which of the following atoms has the lowest ionization potential? **[2003]**

(A) $^{14}_{7}N$ (B) $^{133}_{55}Cs$
(C) $^{40}_{18}Ar$ (D) $^{16}_{8}O$

140. The wavelength involved in the spectrum of deuterium $\left(^2_1D\right)$ are slightly different from that of hydrogen spectrum because **[2003]**

(A) the size of two nuclei are different.
(B) the nuclear forces are different in the two cases.
(C) the masses of the two nuclei are different.
(D) the attraction between the electron and the nuclei is different in the two cases.

141. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is **[2003]**

(A) 30.6 eV (B) 13.6 eV
(C) 3.4 eV (D) 122.4 eV

142. A nucleus disintegrated into two nuclear parts which have their velocities in the ratio 2: 1. The ratio of their nuclear sizes will be **[2004]**

(A) $3^{1/2}:1$ (B) $1:2^{1/3}$
(C) $2^{1/3}:1$ (D) $1:3^{1/2}$

143. The binding energy per nucleon of deuteron $\left(^2_1\text{H}\right)$ and helium nucleus $\left(^4_2\text{He}\right)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is **[2004]**

(A) 23.6 MeV (B) 26.9 MeV
(C) 13.9 MeV (D) 19.2 MeV

144. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closet approach is of the order of **[2004]**

(A) 10^{-12} cm (B) 10^{-10} cm
(C) 1 A (D) 10^{-15} cm

145. If radius of the $^{27}_{13}\text{Al}$ nucleus is estimated to be 3.6 fermi then the radius of $^{125}_{52}\text{Te}$ nucleus be nearly **[2005]**

(A) 8 fermi (B) 6 fermi
(C) 5 fermi (D) 4 fermi

146. Starting with a sample of pure ^{66}Cu, $\frac{7}{8}$ of it decays into Zn in 15 minutes. The corresponding half-life is **[2005]**

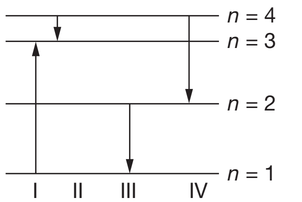
(A) 15 min (B) 10 min

(C) $7\frac{1}{2}$ min (D) 5 min

147. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $\frac{I}{8}$. The thickness of lead which will reduce the intensity to $\frac{I}{2}$ will be **[2005]**

(A) 9 mm (B) 6 mm
(C) 12 mm (D) 18 mm

148. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy? **[2005]**



(A) IV (B) III (C) II (D) I

149. A nuclear transformation is denoted by $X(n, \alpha)^7_3 Li$. Which of the following is the nucleus of element x? **[2005]**

(A) $^{10}_{5}Be$ (B) $^{12}C_6$ (C) $^{11}_{4}Be$ (D) $^{9}_{5}B$

150. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to **[2006]**

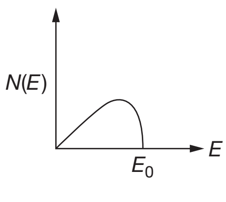
(A) v^2 (B) $\frac{1}{m}$ (C) $\frac{1}{v^2}$ (D) $\frac{1}{Ze}$

151. When ${}_3Li^7$ nuclei are bombarded by protons, and the resultant nuclei are ${}_4Be^8$, the emitted particles will be **[2006]**

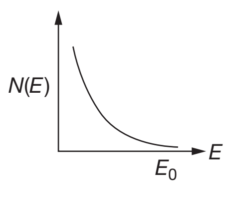
(A) Alpha particles (B) Beta particles
(C) Gamma photons (D) Neutrons

152. The energy spectrum of β-particles. [number $N(E)$ as a function of β energy E] emitted from a radioactive source is **[2006]**

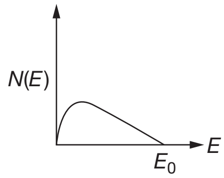
(A)



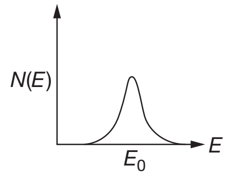
(B)



(C)



(D)



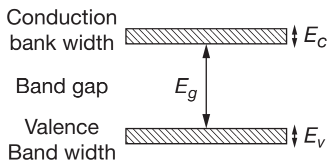
153. If the binding energy per nucleon in $^7_3$Li and $^4_2$He nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction $p + ^7_3 \text{Li} \rightarrow 2^4_2\text{He}$ energy of proton must be **[2006]**

(A) 28.24 MeV (B) 17.28 MeV
(C) 1.46 MeV (D) 39.2 MeV

154. The 'rad' is the correct unit used to report the measurement of **[2006]**

(A) the ability of a beam of gamma - ray photons to produce ions in a target.
(B) the energy delivered by radiation to a target.
(C) the biological effect of radiation.
(D) the rate of decay of a radioactive source.

155. If the lattice constant of this semiconductor is decreased, then which of the following is correct? **[2006]**



(A) All E_c, E_g, E_v increase
(B) E_c and E_v increase, but E_g decreases
(C) E_c and E_v decrease, but E_g increases
(D) All E_c, E_g, E_v decrease

156. If M_O is the mass of an oxygen isotope ${}_8O^{17}, M_P$ and are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is **[2007]**

(A) $(M_O - 17M_N)c^2$
(B) $(M_O - 8M_P)c^2$
(C) $(M_O - 8M_P - 9M_N)c^2$
(D) $M_O c^2$

157. In gamma ray emission from a nucleus **[2007]**

(A) only the proton number changes.
(B) both the neutron number and the proton number change.
(C) there is no change in the proton number and the neutron number.
(D) only the neutron number changes.

158. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then **[2007]**

(A) X and Y decay at same rate always
(B) X will decay faster than Y
(C) Y will decay faster than X
(D) X and Y have same decay rate initially

159. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate? **[2007]**

(A) The number of free electrons for conduction is significant only in Si and Ge but small in C
(B) The number of free conduction electrons is significant in C but small in Si and Ge
(C) The number of free conduction electrons is negligibly small in all the three
(D) The number of free electrons for conduction is significant in all the three.

160. Which of the following transitions in hydrogen atoms emit photons of highest frequency? **[2007]**

(A) $n = 1$ to $n = 2$ (B) $n = 2$ to $n = 6$
(C) $n = 6$ to $n = 2$ (D) $n = 2$ to $n = 1$

161. This question contains Statement 1 and statement 2. Of the four choices given after the statements, choose the one that best describes the two statements. **[2008]**

Statement 1: Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion and
Statement 2: For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

(A) Statement 1 is true, Statement 2 is false
(B) Statement 1 is false, Statement 2 is true
(C) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for statement 1
(D) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1

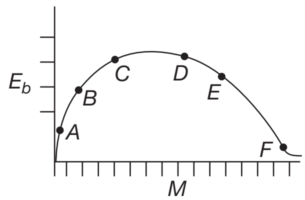
162. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where k a constant and r is is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then which of the following is true? **[2008]**

(A) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$ (B) $T_n \propto \frac{1}{n}, r_n \propto n^2$
(C) T_n independent of n, $r_n \propto n$ (D) $T_n \propto \frac{1}{n}, r_n \propto n$

163. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from: **[2009]**

(A) $3 \to 2$ (B) $4 \to 2$
(C) $5 \to 4$ (D) $2 \to 1$

164. The above is a plot of binding energy per nucleon E_b, against the nuclear mass M ; A, B, C, D, E, F correspond to different nuclei. Consider four reactions: **[2009]**



(i) $A + B \to C + \varepsilon$ (ii) $C \to A + B + \varepsilon$
(iii) $D + E \to F + \varepsilon$ and (iv) $F \to D + E + \varepsilon$

where ε is the energy released ? In which reactions is ε positive?

(A) (i) and (iii) (B) (ii) and (iv)
(C) (ii) and (iii) (D) (i) and (iv)

165. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **[2010]**

(A) $\frac{A - Z - 8}{Z - 4}$ (B) $\frac{A - Z - 4}{Z - 8}$
(C) $\frac{A - Z - 12}{Z - 4}$ (D) $\frac{A - Z - 4}{Z - 2}$

166. The speed of daughter nuclei is

(A) $c\sqrt{\frac{\Delta m}{M + \Delta m}}$ (B) $c\frac{\Delta m}{M + \Delta m}$
(C) $c\sqrt{\frac{2\Delta m}{M}}$ (D) $c\sqrt{\frac{\Delta m}{M}}$

167. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(A) $E_1 = 2E_2$ (B) $E_2 = 2E_1$
(C) $E_1 > E_2$ (D) $E_2 > E_1$

168. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(A) 12.1 eV (B) 36.3 eV
(C) 108.8 eV (D) 122.4 eV

169. The half-life of a radioactive substance is 20 min. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is

(A) 7 min (B) 14 min
(C) 20 min (D) 28 min

170. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be: **[2012]**

(A) 2 (B) 3 (C) 5 (D) 6

171. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by: (n is integer) **[2012]**

(A) $\frac{(m_1+m_2)^2 n^2h^2}{2m_1^2\ m_2^2r^2}$ (B) $\frac{n^2h^2}{2(m_1+m_2)r^2}$

(C) $\frac{2n^2h^2}{(m_1+m_2)r^2}$ (D) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$

172. Assume that a neutron breaks into a proton and an electron. The energy released during this process is **[2012]**

Mass of neutron $= 1.6725 \times 10^{-27}$ kg

Mass of proton $= 1.6725 \times 10^{-27}$ kg

Mass of electron $= 9 \times 10^{-31}$ kg

(A) 0.73 MeV (B) 7.10 MeV
(C) 6.30 MeV (D) 5.4 MeV

173. In hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n - 1)$. If $n >> 1$, the frequency of radiation emitted is proportional to: **[2013]**

(A) $\frac{1}{n^2}$ (B) $\frac{1}{n^{3/2}}$ (C) $\frac{1}{n^3}$ (D) $\frac{1}{n}$

174. The current voltage relation of diode is given by $I = (e^{1000\,V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **[2014]**

(A) 0.2 mA (B) 0.252 mA
(C) 0.5 mA (D) 0.05 mA

175. Hydrogen $({}_1H^1)$, Deuterium $({}_1H^2)$, singly ionised Helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct? **[2014]**

(A) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(B) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(C) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(D) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = 4\lambda_4$

176. The radiation corresponding to $3 \to 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to **[2014]**

(A) 1.8 eV (B) 1.1 eV
(C) 0.8 eV (D) 1.6 eV

177. As an electron makes a transition from an excited state to the ground state of hydrogen - like atom/ion **[2015]**

(A) *KE*, potential energy and total energy decrease.
(B) *KE* decreases, potential energy increases but total energy remains same.
(C) *KE* and total energy decrease but potential energy increases.
(D) Its kinetic energy increases but potential energy and total energy decrease.

178. Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list **[2015]**

List-I	List-II
(a) Franck-Hertz Experiment	(i) Particle Nature of Light
(b) Photo-Electric Experiment	(ii) Discrete Energy Levels of Atom
(c) Davison-Germer Experiment	(iii) Wave nature of Electron
	(iv) Structure of Atom

(A) (a) - (ii), (b) - (iv), (c) - (iii)
(B) (a) - (ii), (b) - (i), (c) - (iii)
(C) (a) - (iv), (b) - (iii), (c) - (ii)
(D) (a) - (i), (b) - (iv), (c) - (iii)

179. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be **[2016]**

(A) 4: 1 (B) 1: 4
(C) 5: 4 (D) 1: 16

180. Radiation wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be: **[2016]**

(A) $< v\left(\frac{4}{3}\right)^{1/2}$ (B) $= v\left(\frac{4}{3}\right)^{1/2}$

(C) $= v\left(\frac{3}{4}\right)^{1/2}$ (D) $> v\left(\frac{4}{3}\right)^{1/2}$

ANSWER KEYS

Single Option Correct Type

1. (B)	**2.** (B)	**3.** (C)	**4.** (C)	**5.** (A)	**6.** (D)	**7.** (C)	**8.** (C)	**9.** (A)	**10.** (A)
11. (C)	**12.** (C)	**13.** (A)	**14.** (C)	**15.** (D)	**16.** (B)	**17.** (B)	**18.** (A)	**19.** (C)	**20.** (D)
21. (D)	**22.** (C)	**23.** (D)	**24.** (B)	**25.** (A)	**26.** (B)	**27.** (B)	**28.** (C)	**29.** (B)	**30.** (A)
31. (B)	**32.** (C)	**33.** (C)	**34.** (C)	**35.** (A)	**36.** (B)	**37.** (B)	**38.** (C)	**39.** (C)	**40.** (D)
41. (A)	**42.** (A)	**43.** (B)	**44.** (B)	**45.** (A)	**46.** (A)	**47.** (A)	**48.** (B)	**49.** (C)	**50.** (D)
51. (C)	**52.** (D)	**53.** (C)	**54.** (B)	**55.** (D)	**56.** (B)	**57.** (B)	**58.** (A)	**59.** (B)	**60.** (B)
61. (C)	**62.** (C)	**63.** (B)	**64.** (D)	**65.** (D)	**66.** (C)	**67.** (D)	**68.** (B)	**69.** (B)	**70.** (C)
71. (B)	**72.** (C)	**73.** (C)	**74.** (D)	**75.** (A)	**76.** (A)	**77.** (A)	**78.** (D)		

More than One Option Correct Type

79. (A), (C) and (D) **80.** (B) and (C) **81.** (B) and (C) **82.** (A) and (D) **83.** (A) and (C)
84. (A), (B), (C) and (D) **85.** (A), (C) and (D) **86.** (C) and (D) **87.** (A), (C) and (D)
88. (A) and (B)

Passage Based Questions

Passage 1

89. (C) **90.** (B) **91.** (A)

Passage 2

92. (D) **93.** (B) **94.** (A)

Passage 3

95. (B) **96.** (B) **97.** (B)

Passage 4

98. (B) **99.** (D) **100.** (A)

Passage 5

101. (B) **102.** (A) **103.** (C) **104.** (A)

Match the Column Type

105. (A) → 1, 2, 3, 4; (B) → 1, 2, 3, 4; (C) → 1, 2, 3, 4; (D) → 1, 2, 3, 4
106. (A) → 1, 3; (B) → 4; (C) → 2, 4; (D) → 1, 3
107. (A) → 1; (B) → 2, 4; (C) → 1; (D) → 2, 3, 4
108. (A) → 2; (B) → 4; (C) → 1
109. (A) → 2, 3; (B) → 1, 3; (C) → 1; (D) → 4

Assertion-Reason Type

110. (A) **111.** (A) **112.** (C) **113.** (B) **114.** (B) **115.** (A) **116.** (A) **117.** (A) **118.** (D) **119.** (A)

Integer Type

120. 2
121. $K_A = 2$ eV
122. 1.5 V
123. 15
124. $Z = 30$
125. 1
126. 2
127. $n = 2$

Previous Years' Questions

128. (C)	**129.** (A)	**130.** (C)	**131.** (A)	**132.** (C)	**133.** (A)	**134.** (B)	**135.** (A)	**136.** (B)	**137.** (A)
138. (D)	**139.** (B)	**140.** (C)	**141.** (A)	**142.** (B)	**143.** (A)	**144.** (A)	**145.** (B)	**146.** (D)	**147.** (C)
148. (B)	**149.** (A)	**150.** (B)	**151.** (C)	**152.** (C)	**153.** (B)	**154.** (C)	**155.** (C)	**156.** (C)	**157.** (C)
158. (B)	**159.** (A)	**160.** (D)	**161.** (A)	**162.** (C)	**163.** (C)	**164.** (D)	**165.** (B)	**166.** (C)	**167.** (D)
168. (C)	**169.** (C)	**170.** (D)	**171.** (D)	**172.** (A)	**173.** (C)	**174.** (A)	**175.** (C)	**176.** (B)	**177.** (D)
178. (B)	**179.** (C)	**180.** (D)							

HINTS AND SOLUTIONS

Single Option Correct Type

1. $\lambda = \dfrac{h}{\sqrt{2mE}}$

$\Rightarrow \lambda \propto \dfrac{1}{\sqrt{E}}$

$\Rightarrow \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{E_2}{E_1}}$

$\Rightarrow \dfrac{10^{-10}}{0.5 \times 10^{-10}} = \sqrt{\dfrac{E_2}{E_1}}$

$\Rightarrow E_2 = 4E_1$

Hence added energy $= E_2 - E_1 = 3E_1$

The correct option is (B)

2. $2^n = 64$

$\Rightarrow n = 6$

$\therefore T_{1/2} = \dfrac{60}{6} = 10\,\text{s}$

The correct option is (B)

3. Probability = (not disintegrate for 1st half) × (not disintegrate in 2nd half) × (not disintegrated in 3rd half = ½ × ½ × ½ = $\dfrac{1}{8}$

The correct option is (C)

4. 10^{-10} s

The correct option is (C)

5. Using Moseley's law, we get

$\dfrac{\lambda_2}{\lambda_1} = \dfrac{(Z_1 - a)^2}{(Z_2 - a)^2}$

$\lambda_2 = \dfrac{200 \times (74-1)^2}{(78-1)^2} = 179.76$ Å

The correct option is (A)

6. Gamma-photon

The correct option is (D)

7. Energy of photon is less than work function of material.

The correct option is (C)

8. $E_a - 3E_b = e \Rightarrow E_a - e = 3E_b$

The correct option is (C)

9. For first Bohr orbit of hydrogen atom,

$mvr = \dfrac{h}{2\pi}, \Rightarrow 2\pi r = \dfrac{h}{mv}$

de-Broglie wavelength of electron having same velocity,

$\lambda = \dfrac{h}{mv} \quad \therefore \dfrac{2\pi r}{\lambda} = 1$

The correct option is (A)

10. $\lambda \propto \dfrac{1}{\sqrt{V}}$

The correct option is (A)

11. In β-decay, a neutron decays into a proton and an electron is emitted.

The correct option is (C)

12. The correct option is (C)

13. $\lambda_m T = C$

$E \propto T^4$

$\dfrac{E_2}{E_1} = \left(\dfrac{T_2}{T_1}\right)^4 = \left(\dfrac{4}{3}\right)^4 = \dfrac{256}{81}$

The correct option is (A)

14. The correct option is (C)

15. The correct option is (D)

16. de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{3mkT}} \qquad \therefore \frac{\lambda_{H_2}}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_{H_2}T_{H_2}}} = \sqrt{\frac{8}{3}}$$

The correct option is (B)

17. $r = \frac{\sqrt{2mK}}{qB} \qquad (m_\alpha = 4\, m_P,\ q_\alpha = 2q_p)$

The correct option is (B)

18. $K = eV,\ r = \frac{\sqrt{2mK}}{eB} = \sqrt{\frac{2meV}{e^2B^2}} = \sqrt{\frac{2mV}{eB^2}}$

The correct option is (A)

19. $E_P = (8 \times 7.06 - 7 \times 5.60)$ MeV = 17.28 MeV

The correct option is (C)

20. $K_1 = E_1 - W = 2h\nu_0 - h\nu_0 = h\nu_0$

$K_2 = E_2 - W = 5h\nu_0 - h\nu_0 = 4h\nu_0$

The correct option is (D)

21. $$\frac{E_{4n} - E_{2n}}{E_{2n} - E_n} = \frac{\frac{E_1}{16n^2} - \frac{E_1}{4n^2}}{\frac{E_1}{4n^2} - \frac{E_1}{n^2}} = \frac{1}{4} = \text{constant}$$

The correct option is (D)

22. $\frac{hc}{\lambda} = Rhc\left(1 - \frac{1}{n^2}\right)$

$n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$

The correct option is (C)

23. $$\lambda_{min} = \frac{hc}{eV} = \frac{6.64\times10^{-34} \times 3\times10^8}{20\times10^5 \times 1.6\times10^{-19}}$$

$$= \frac{6.64\times10^{-12} \times 3}{32} \approx 6 \times10^{-13} \text{ m}$$

The correct option is (D)

24. $\frac{hC}{\lambda_{max}} = \Delta E_{12}$

$$\Delta E = (13.6)\ (4) \left[\frac{1}{1^2} - \frac{1}{2^2}\right] \times 1.6 \times10^{-19} \text{ J}$$

$$= (13.6)\ (4) \left(\frac{3}{4}\right) \times 1.6\times10^{-19}$$

$$= \frac{19.8}{(13.6)(4.8)} \times10^{-7} = 0.304 \times 10^{-7} \text{ m} = 304 \text{ Å}$$

The correct option is (B)

25. We know that $N = N_0\ (1/2)^{t/T}$

where $N_0 = 16$ g, $t = 32$ and $T = 2$

$$N = 16 \left(\frac{1}{2}\right)^{32/2} = 16\left(\frac{1}{2}\right)^{16} \text{ g},\ N < 1 \text{ mg}$$

The correct option is (A)

26. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

or $\frac{1}{64} = \left(\frac{1}{2}\right)^{t/T}$

or $\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{t/T}$

or $\frac{t}{T} = 6$

$\therefore \quad t = 6\,T$ or $t = 6 \times 2 = 12$

The correct option is (B)

27. $V = \frac{h^2}{2me\lambda^2} = 8.2 \times 10^4$ V

The correct option is (B)

28. $\lambda = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2V_2q_2}{m_1V_2q_2}} = \sqrt{\frac{(4m)(2e)}{m\ e}} = 2\sqrt{2}$$

The correct option is (C)

29. $\frac{hc}{\lambda} = h\nu = 2.5 + 1.7 = 4.2eV$

$\nu\lambda = C$

and $\nu' \frac{\lambda}{2} = C \Rightarrow \nu' = 2\nu$

$eV_0 = h(2\nu) - \phi = 2 \times 4.2 - 1.7$

$V_0 = 6.7$ V

The correct option is (B)

30. $12.75 = E_0 - \frac{E_0}{n^2} = -13.6 - \frac{(-13.6)}{n_1^2}$

$\Rightarrow \quad n = 4$

Number of lines $= \frac{n(n-1)}{2} = 6$

The correct option is (A)

31. $$eV = \frac{hc}{\lambda_{min}} \Rightarrow \lambda_{min} = \frac{h\nu}{eV} = \frac{6.67\times10^{-34} \times 3\times10^8}{1.6\times10^{-19} \times 30\times10^3}$$

$$= \frac{12.51}{30} \text{Å} = 0.414 \text{ Å}$$

The correct option is (B)

32. Activity = λN

$$10^{16} = \frac{0.693}{3.8\times60\times60} \times \frac{m\ 6\times10^{23}}{222}$$

$m = 0.073$ g

The correct option is (C)

33. Kinetic energy of the electron reaching the target metal = maximum kinetic energy of x-ray photon

$$= \frac{12400}{0.1} \text{ eV} = 124 \times 10^3 \text{eV} = 124 \times 1.6 \times 10^{-16} \text{ J}$$

de Broglie wavelength of the electron

$$= \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times124\times1.6\times10^{-16}}} = 0.035\ \text{Å}$$

The correct option is (C)

34. For emission of photon corresponding to the target wavelength the transition of electron will be from $n = 3$ to $n = 2$.

Hence after collision of electron with the hydrogen atom, the hydrogen atom will have excited to the state whose quantum number n is at least equal to 3.

Minimum energy of colliding electron $= 13.6\left(\frac{1}{I^2} - \frac{1}{3^2}\right)$ = 12.09 eV

The correct option is (C)

35. The correct option is (A)

36. $(ms\Delta\theta)\text{J} = \frac{1}{2}\left(\frac{1}{2}mv^2\right),$

$$\Delta\theta = \frac{v^2}{4Js}$$

The correct option is (B)

37. $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

The correct option is (B)

38. The correct option is (C)

39. $\frac{\lambda_\alpha}{\lambda_p} = \frac{P_p}{P_\alpha} = \frac{q_p}{q_\alpha}$

The correct option is (C)

40. The correct option is (D)

41. $v = \frac{p}{m} = \frac{h}{m\lambda} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times10^{-10}} = 7.25 \times 10^6$ m/s

The correct option is (A)

42. $\frac{E_0}{B_0} = c$ and $\frac{\omega}{k} = c$ $\quad\therefore\quad \frac{E_0}{B_0} = \frac{\omega}{k}$

$\Rightarrow\ E_0k = B_0\omega$

The correct option is (A)

43. The electron density in doped semiconductor,

$$\eta_e = \frac{n_i^2}{n_h} = \frac{(3\times10^{16})^2}{4.5\times10^{22}} = 2\times10^{10}\ /\text{m}^3$$

The correct option is (B)

44. $\lambda = \frac{h}{\sqrt{2mE}}$

$$\Rightarrow\ \lambda \propto \frac{1}{\sqrt{E}}$$

$$\Rightarrow\ \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow\ \frac{10^{-10}}{0.5\times10^{-10}} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow\ E_2 = 4E_1$$

Hence added energy $= E_2 - E_1 = 3E_1$

The correct option is (B)

45. $r = \frac{\sqrt{2mK}}{qB}$

$$\Rightarrow\ K \propto \frac{q^2}{m}$$

$$\Rightarrow\ \frac{K_p}{K_\alpha} = \left(\frac{q_p}{q_\alpha}\right)^2 \times \frac{m_\alpha}{m_p}$$

$$\Rightarrow\ \frac{K_p}{K_\alpha} = \left(\frac{q_p}{2q_p}\right)^2 \times \frac{4m_p}{m_p} = 1$$

$$\Rightarrow\ K_\alpha = 1\ \text{MeV}$$

The correct option is (A)

46. Here $E_0 = 100$ V/m, $B_0 = 0.265$ A/m

$\therefore$ Maximum rate of energy flow

$$S = E_0 \times \frac{B_0}{\mu_0} = 100 \times 0.265 = 26.5\frac{\text{W}}{\text{m}^2}$$

The correct option is (A)

47. $v = \frac{p}{m} = \frac{h}{m\lambda} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times10^{-10}} = 7.25 \times 10^6$ m/s

The correct option is (A)

48. Stopping potential $V_0 = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$. As λ decreases so V_0 increases.

The correct option is (B)

49. Work function is the intercept on KE axis i.e. 2 eV.

The correct option is (C)

50. $E = -Z^2\times13.6\,\text{eV} = -9\times13.6\,\text{eV} = -122.4\,\text{eV}$

So, ionization energy = +122.4 eV

The correct option is (D)

51. $r_n \propto n^2$

The correct option is (C)

52. From conservation of linear momentum both the particles will have equal and opposite momentum. The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \qquad \therefore\ \frac{\lambda_1}{\lambda_2} = 1$$

The correct option is (D)

53. The correct option is (C)

54. $\frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$

$\lambda = 0.16$ Å

The correct option is (B)

55. $E = -Z^2\times13.6\,\text{eV} = -9\times13.6\,\text{eV} = -122.4\,\text{eV}$

So, ionization energy = +122.4 eV

The correct option is (D)

56. $K_{particle} = \frac{1}{2}mv^2$ also $\lambda = \frac{h}{mv}$

$$\Rightarrow K_{particle} = \frac{1}{2}\left(\frac{h}{\lambda v}\right)\cdot v^2 = \frac{vh}{2\lambda} \quad (1)$$

$$K_{photon} = \frac{hc}{\lambda} \quad (2)$$

$$\therefore \frac{K_{particle}}{K_{photon}} = \frac{v}{2c} = \frac{2.25\times 10^8}{2\times 3\times 10^8} = \frac{3}{8}$$

The correct option is (B)

57. $3eV_0 = \frac{hc}{\lambda} - W_0$

$$eV_0 = \frac{hc}{2\lambda} - W_0$$

$$W_0 = \frac{hc}{4\lambda}$$

The correct option is (B)

58. $\frac{1}{\lambda_{3-1}} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8R}{9}$

$$\frac{1}{\lambda_{2-1}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\Rightarrow \frac{\lambda_{3-1}}{\lambda_{3-1}} = \frac{27}{32}$$

The correct option is (A)

59. For emission of electrons incident energy of each photon must be greater than work function (threshold energy).

The correct option is (B)

60. Energy of the H atom in first excited state = –3.4 eV

Initial energy of the electron = 2 eV

Energy released = 2 – (–3.4) eV = 5.4 eV

Work function of the metal = $\frac{12420}{4600} = 2.7$ eV

$K_{max} = 5.4 - 2.7 = 2.7$ eV

The correct option is (B)

61. $\frac{\lambda_R}{\lambda_S} = 1.5$

So, the rate of disintegration of R will be 1.5 times that of S. Thus, the half-life of S will be 1.5 times that of R. So, two half-lives of S will be equal to the three half –lives of R.

$$\frac{N_R}{N_S} = \frac{0.25}{0.25} = 1$$

The correct option is (C)

62. Time period $\propto n^3$

Total time $\propto \frac{1}{n-1}$

Number of revolution $\propto \frac{1}{n^3(n-1)}$

$$\frac{n_1}{n_2} = \frac{27}{4}$$

The correct option is (C)

63. $\frac{1}{\lambda} = R(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For K_α transition $\frac{1}{\lambda_\alpha} = R(Z-1)^2\left[1 - \frac{1}{4}\right]$

$\lambda_\alpha - \lambda_{min}$ depends on $\frac{1}{3R(Z-1)^2}$

The correct option is (B)

64. $E_{Th} = \frac{m}{m+M}|Q|$,

$$Q = \left(M_C + M_H - M_N + M_n\right)931 \text{ MeV} = 0.782040$$

$$E_{Th} = 0.05 \text{ MeV}$$

The correct option is (D)

65. $P = \sigma A T^4$

$$\Rightarrow \frac{P_A}{P_B} = \frac{A_A}{A_B}\left(\frac{T_A}{T_B}\right)^4 \Rightarrow T_B = T_A 2^{3/4}$$

as $\lambda_m T$ = constant

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} \Rightarrow \lambda_B = 5000(2)^{-3/4} \text{ Å}.$$

The correct option is (D)

66. For emission of photon corresponding to the target wavelength the transition of electron will be from $n = 3$ to $n = 2$.

Hence after collision of electron with the hydrogen atom, the hydrogen atom will have excited to the state whose quantum number n is at least equal to 3.

Minimum energy of colliding electron

$$= 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.09 \text{eV}$$

The correct option is (C)

67. $E = C_1C_2\cos\omega_0 t + \frac{C_1C_3}{2}\cos(\omega_0+\omega)t + \frac{C_1C_3}{2}\cos(\omega_0-\omega)t$

Of the three components, the highest frequency component will liberate the electrons with maximum kinetic energy

$$(\text{KE})_{max} + \phi = \frac{h}{2\pi}(\omega_0+\omega) \Rightarrow \phi = 2.39 \text{ eV}.$$

The correct option is (D)

68. In the reference frame of infinity, $U = 0$

$E_1 = -13.6$ eV, $K_1 = 13.6$ eV, $U_1 = -27.2$ eV

$E_2 = -3.4$ eV, $K_2 = 3.4$ eV, $U_2 = -6.8$ eV

Now for U_1 to be zero, we have to add 27.2 eV to U_1.

Hence $E_2 = -3.4 + 27.2 = 23.8$ eV

The correct option is (B)

69. $\frac{1}{\lambda_{k\alpha}} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ and $\frac{1}{\lambda_{k_\beta}} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$;

dividing we get,

$\lambda_{k_\beta} = 0.27$ Å

The correct option is (B)

70. $\lambda_{\text{sample}} = \lambda_\alpha + \lambda_\beta = 0.6932\left[\frac{1}{30} + \frac{1}{60}\right] \text{year}^{-1}$

$$N = N_0 e^{-(\lambda_{\text{sample}})t}$$

$$\frac{N_0}{4} = N_0 e^{-(\lambda_{\text{sample}})t}$$

$$\ln 4 = (\lambda_{\text{sample}})t$$

$$2\ln 2 = 0.6932\left[\frac{1}{20}\right]t$$

$$t = 40 \text{ years}$$

The correct option is (C)

71. Energy of the H-atom in first excited state = –3.4 eV

Initial energy of the electron = 2 eV

Energy released = 2 – (–3.4) eV = 5.4 eV

Work function of the metal = $\frac{12420}{4600} = 2.7$ eV

$K_{\text{max}} = 5.4 - 2.7 = 2.7$ eV

The correct option is (B)

72. Kinetic energy of the electron reaching the target metal = maximum kinetic energy of x-ray photon = $\frac{12400}{0.1}$ eV = 124 × 10^3eV = 124 × 1.6 × 10^{-16} J

de Broglie wavelength of the electron

$$= \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times124\times1.6\times10^{-16}}} = 0.035 \text{ Å}$$

The correct option is (C)

73. ${}^{180}_{72}\text{X} \xrightarrow{\alpha} {}^{176}_{70}\text{X}_1 \xrightarrow{\beta} {}^{176}_{71}\text{X}_2 \xrightarrow{\alpha} {}^{172}_{69}\text{X}_3 \xrightarrow{\gamma} {}^{172}_{69}\text{X}_4$

The correct option is (C)

74. The correct option is (D)

75. $E_L - E_K = -3 - (-18.525) = 15.525$ keV

$$\lambda = \frac{hc}{15.525 \text{ keV}} = \frac{6.63\times10^{-34}\times3\times10^8}{15.525\times10^3\times1.6\times10^{-19}} = 0.8 \text{ Å}$$

The correct option is (A)

76. Given that $U^{238} : Pb^{206} = 3 : 1$

$$\lambda t = \ln\left(\frac{4}{3}\right)$$

$$\Rightarrow t = \frac{1}{\lambda}\times\ln\left(\frac{4}{3}\right) = 1.95\times10^9 \text{ years}$$

The correct option is (A)

77. When electron reaches near the curved surface, the force due to induce charges accelerate the electron.

The correct option is (A)

78. Equivalent current, $I = ne$

Area = πr^2

Magnetic moment = $\pi n e r^2$

The correct option is (D)

More than One Option Correct Type

79. As $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and $P = \frac{h}{\lambda}$

Also, $E = \frac{hc}{\lambda}$

The correct option is (A), (C) and (D)

80. Here λ_0 is cut-off wavelength and $\lambda_{k\alpha}$ and λ_{kB} are wavelength of k_α and k_B characteristic x-rays.

As $\lambda_0 = \frac{hc}{eV}$ and as V increases. λ_0 decreases but characteristic wavelengths do not change so interval between $\lambda_{k\alpha}$ and λ_0 increases and the same for the interval between $\lambda_{k\beta}$ and λ_0

The correct option is (B) and (C)

81. The correct option is (B) and (C)

82. For a Bohr's model. $T \propto n^3$, $\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$

If $\frac{T_1}{T_2} = 8$ then $\frac{n_1}{n_2} = 2$

The correct option is (A) and (D)

83. The correct option is (A) and (C)

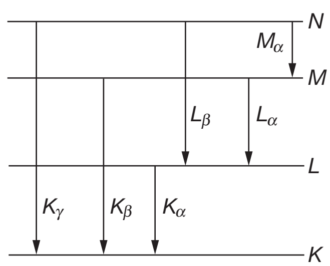
84. $E \propto \frac{1}{n^2}, r \propto n^2, P \propto \frac{1}{n}$

Hence all the four option are correct.

The correct option is (A), (B), (C) and (D)

85. $E(K_\alpha) + E(L_\beta) = E(K_\beta) + E(M_\alpha) = E(K_\gamma)$

Harder x-rays have higher frequency than the softer x-ray. Continuous and characteristic x-rays differ only in the method of creation.



The correct option is (A), (C) and (D)

86. $E_{min} = 2\left[\frac{9.1\times10^{-31}\times(3\times10^8)^2}{1.6\times10^{-13}}\text{MeV} + 0.95\text{ MeV}\right]$

$= 2.924$ MeV

The correct option is (C) and (D)

87. $E \propto \frac{1}{\lambda}$ and $p \propto E$

Further $E \propto \frac{1}{n^2}$

$\therefore b = c$ and $a = \frac{1}{b}$ and $c = \frac{5}{27}$

$\therefore b = c = 5/27$ and $a = \frac{27}{5}$

The correct option is (A), (C) and (D)

88. $P = e\sigma AT^4$

$P_A = P_B$ and $A_A = A_B$

$\therefore e_A T_A^4 = e_B T_B^4$

$$T_B = \left(\frac{e_A}{e_B}\right)^{1/4} T_A = \left(\frac{0.01}{0.81}\right)^{1/4} \times 5802 = 1934 \text{ K}$$

Also, $\lambda_A T_A = \lambda_B T_B$

$$\lambda_B = \frac{\lambda_A \times 5802}{1934}$$

$\lambda_B = 3\lambda_A$

$\lambda_B - \lambda_A = 1\ \mu$m

$\lambda_B = \frac{3}{2}\mu\text{m} = 1.5\ \mu$m

The correct option is (A) and (B)

Passage Based Questions

Passage 1

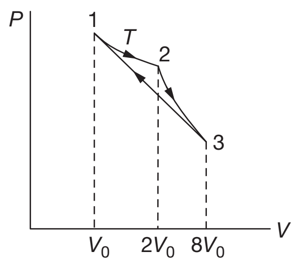
89. Let initial volume be V_0

for 1 – 2, $P_1V_0 = P_2(2V_0) \Rightarrow \frac{P_1}{P_2} = 2$ (1)

for 2 – 3, $P_2(2V_0)^{1.5} = P_3(8V_0)^{1.5} \Rightarrow \frac{P_2}{P_3} = 8$ (2)

from (1) and (2) $\frac{P_1}{P_3} = 16$

Now, $w_{3-1} = -\frac{1}{2}(P_1 + P_3)(7V_0) = -\frac{119}{32}RT$



The correct option is (C)

90. $w_{2-3} = -\frac{nR(T_3 - T_2)}{\gamma - 1} = RT$

The correct option is (B)

91. For process

3 – 1, $\Delta Q_{3-1} = \Delta U_{3-1} + w_{3-1}$

$$= \frac{nR(T_1 - T_3)}{\gamma - 1} + w_{31} = -\frac{87}{32}RT$$

The correct option is (A)

Passage 2

92. As emission spectrum of the atom shows 10 different lines, so the highest excited state of the electron is $n = 5$.

The correct option is (D)

93. As some lines have energy less then $\frac{64}{225}E_0$ and some have more than $\frac{64}{225}E_0$ and some have equal to it. So initial state of electron can be $n = 2$ or $n = 3$. Again if initial level is $n = 2$, then

$$Z^2\left(-\frac{E_0}{25}\right) - Z^2\left(-\frac{E_0}{4}\right) = \frac{64}{225}E_0,$$

Z is in fraction (not possible)

The correct option is (B)

94. If $n = 3$, then $Z^2\left(-\frac{E_0}{25}\right) - Z^2\left(\frac{-E_0}{9}\right) = \frac{64}{225}E_0, Z = 2$

The correct option is (A)

Passage 3

95. Photo emission will stop if $\lambda = \lambda_0 = \frac{hc}{\phi}$

$3000 + 40t_0 = \frac{12400}{2}$, $t_0 = 80$ s

The correct option is (B)

96. Rate of photo emission at time t

$$\frac{dN}{dt} = \frac{1}{100} \times \frac{100}{19840\times10^{-19}}[3000 + 40t]$$

$$N = \frac{1}{19840\times10^{-19}}[3000t + 20t^2]_0^{80}$$

$N = 18.55\times10^{19} = 1.855\times10^{20}$

This is the number of photoelectron ejected in 1 blinking.

In 1 hr, total number of photoelectron ejected is

$= 1.855 \times 10^{20} \times 20 = 3.71 \times 10^{21}$

The correct option is (B)

97. The correct option is (B)

Passage 4

98. $T\lambda_m = b \Rightarrow \ln T = \ln b - \ln \lambda_m$

$\therefore$ slope = –1

The correct option is (B)

99. $E = e\sigma AT^4 \Rightarrow \ln E = \ln(\rho\sigma A) + 4\ln T$

$\therefore$ slope = 4

The correct option is (D)

100. $\ln T = \ln b - \ln \lambda_m$

when $\ln \lambda_m = 0$, $\ln T = A$

$\Rightarrow \ln b = A \Rightarrow b = e^A$

The correct option is (A)

Passage 5

Emitting only six different radiation means the transition was made from charge level n such that

$$\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$$

so final energy level after absorbing the photon is $n = 4$

Since some emitted radiation have more, some have less and some have equal energy to 10.2 eV energy of the absorbed photon, so initial state must be $n = 2$

So, $10.2 = 13.6.\ 2^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \Rightarrow Z = 2$

$\therefore$ ionization energy $= 13.6 \times 2^2 = 24.4$ eV

Minimum energy would be for $n = 4 \Rightarrow$ and more for $n = 4 \Rightarrow n = 1$

$$E_{\min} = 13.6 \times 2^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 2.64 \text{ eV}$$

$$E_{\max} = 13.6 \times 2^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 51 \text{ eV}$$

101. The correct option is (B)

102. The correct option is (A)

103. The correct option is (C)

104. The correct option is (A)

Match the Column Type

105. (A) → (1), (2), (3) and (4); (B) → (1), (2), (3) and (4); (C) → (1), (2), (3) and (4); (D) → (1), (2), (3) and (4)

106. (A) → (1)and (3); (B) → (4); (C) → (2) and (4); (D) → (1) and (3)

107. (A) → (1); (B) → (2) and(4); (C) → (1); (D) → (2), (3) and (4)

108. As emission spectrum of the atom shows 10 different lines, so the highest excited state of the electron is $n = 5$. Again as some lines have energy less then $\frac{64}{225}E_0$ and some have more than $\frac{64}{225}E_0$ and some have equal to it. So initial state of electron cannot have $n = 2$. Again if initial level is $n = 2$, then

$$Z^2\left(-\frac{E_0}{25}\right) - Z^2\left(-\frac{E_0}{4}\right) = \frac{64}{225}E_0,$$

Z is in fraction (not possible)

If $n = 3$, then $Z^2\left(-\frac{E_0}{25}\right) - Z^2\left(\frac{-E_0}{9}\right) = \frac{64}{225}E_0$, $Z = 2$

$\therefore$ (A) → (2); (B) → (4); (C) → (1)

109. (A) → (2) and (3); (B) → (1) and (3); (C) → (1); (D) → (4)

Integer Type

120. $\frac{dN_A}{dt} = -\lambda_1 N_A$, $\frac{dN_B}{dt} = 2\lambda_1 N_A - \lambda_2 N_B$,

N_B = maximum $\Rightarrow \frac{dN_B}{dt} = 0$

$\Rightarrow 2\lambda_1 N_A = \lambda_2 N_{B_{\max}}$

$\Rightarrow N_{B_{\max}} = \frac{2\lambda_1}{\lambda_2} N_A$

$\Rightarrow N_{B_{\max}} = \frac{2\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t} = 2.$

121. $K_A = \frac{hc}{\lambda} - W_A = 5 - W_A$ (1)

$K_B = 5.3 - W_B$ (2)

It is given $K_B = K_A - 1.5$ (3)

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}}$$

$\therefore \lambda_B^2 = 4\lambda_A^2$

$\frac{h^2}{2m_e K_B} = \frac{4h^2}{2m_e K_A} \Rightarrow K_A = 4K_B$ (4)

$\therefore \quad K_B = 4K_B - 1.5$

$3K_B = 1.5$

$\therefore \quad K_B = 0.5$ eV

$\therefore \quad K_A = 2$ eV

122. For the material A,

$$E_A - W_A = h\nu_A \quad (1)$$

For the material B,

$$E_B - W_B = h\nu_B \quad (2)$$

Subtracting (2) from (1), we have

$$h(\nu_A - \nu_B) = (1.5 \times e) \text{ J} \quad (3)$$

Now, $eV_A = h\nu_A$ and $eV_B = h\nu_B$

$\therefore \quad e(V_A - V_B) = h(\nu_A - \nu_B)$ or, $e[V_A - V_B] = 1.5 \times e$

$\Rightarrow \quad V_A - V_B = 1.5$ V

123. $v = v_0 \cdot \dfrac{Z^2}{n^3} \Rightarrow \dfrac{Z^2}{n^3} = 2 \quad (1)$

$E = E_0 \cdot \dfrac{Z^2}{n^2} \Rightarrow \dfrac{Z^2}{n^2} = 4 \quad (2)$

Solving (1) and (2) $n = 2, Z = 4$

$$L = mvr = \frac{nh}{2\pi},\ \Delta L = \tau\Delta t = \Delta n \cdot \frac{h}{2\pi}$$

$$\tau = \frac{\Delta n}{\Delta t} \cdot \frac{h}{2\pi} = \frac{1}{7\times10^{-9}} \times \frac{1}{2} \times 2.1\times10^{-34} = \frac{2.1}{14}\times10^{-25}$$

$$\Rightarrow \quad \tau\times10^{27} = \frac{2.1}{14}\times10^{-25}\times10^{27} = 15$$

124. $\lambda_{C_1} = \dfrac{hc}{eV} = \dfrac{12375}{10\times10^3}$ Å $= 1.2375$ Å $\quad (1)$

$\lambda_{C_2} = 0.61875$ Å $\quad (2)$

$$\frac{1}{\lambda_{K_\alpha}} = (Z-1)^2 \left\{\frac{1}{1} - \frac{1}{4}\right\}\times10^7 \quad (3)$$

It is given

$3 \times (\lambda_k\alpha - 1.2375) = \lambda_k\alpha - 0.61875$

$\therefore \quad \lambda_k\alpha = 1.54338$ Å $\quad (4)$

Putting this value in (3)

$Z - 1 = 29$

$Z = 30$

125. $F = -\dfrac{dU}{dr} = -\dfrac{k}{r}, \dfrac{k}{r} = \dfrac{mv^2}{r} \quad (1)$

$$E_n = \frac{1}{2}mv^2 + k\ln r \quad (2)$$

$$mvr = \frac{nh}{2\pi} \quad 3)$$

Solving these $E_n = \dfrac{k}{2}\left(1 + \ln\left(\dfrac{n^2h^2}{4\pi^2 mk}\right)\right)$

Required ratio $= \dfrac{E_2 - E_1}{E_4 - E_2} = 1.$

126. $\dfrac{\lambda_A}{\lambda_B} = \dfrac{1}{2}$

Probabilities of getting α and β particles are same. Thus rate of disintegration are equal

$\therefore \quad \lambda_A N_A = \lambda_B N_B$ or $\dfrac{N_A}{N_B} = \dfrac{\lambda_B}{\lambda_A} = 2$

127. $n^2 \times 5.3 \times 10^{-11} = 21.2 \times 10^{-11}$

$n^2 = 4$

$n = 2$

Previous Years' Questions

128. $E_n = \dfrac{13.6}{n^2} \Rightarrow E_2 = \dfrac{13.6}{(2)^2} = 3.4$ eV

The correct option is (C)

129. Neutrons are electrically neutral. They are not deflected by magnetic field.

The correct option is (A)

130. The correct option is (C)

131. $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{t/T} = \left(\dfrac{1}{2}\right)^{15/5} = \left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8}$

$\therefore \quad N = N_0 / 8$

The correct option is (A)

132. Copper is conductor and germanium is semiconductor, when cooled, the resistance of copper strip decreases and that of germanium increases.

The correct option is (C)

133. Gamma rays have the least wavelength.

The correct option is (A)

134. α-particle $= {}_2^4$He

$U^{238} \to X^{234} + He^4$

$\therefore \quad (238\times0) = (238\times v) + 4u$

or $v = -\dfrac{4u}{234}$

The correct option is (B)

135. Let decay constant per minute $= \lambda$

Disintegration rate, initially = 5000

$\therefore \quad N_0\lambda = 5000 \quad (1)$

Disintegration rate, finally = 1250

$\therefore \quad N\lambda = 1250 \quad (2)$

or $\dfrac{N}{N_0} = \dfrac{1}{4} \Rightarrow \dfrac{N_0 e^{-5\lambda}}{N_0} = \dfrac{1}{4} \Rightarrow e^{-5\lambda} = (4)^{-1}$

$\therefore \quad 5\lambda = \ln 4 = 2\ln 2$

$\therefore \quad \lambda = \frac{2}{5}\ln = 0.4\ln 2$

The correct option is (A)

136. $\therefore$ Decrease in $Z = 8 \times 2 = 16$ (1)

Four β^- particles are emitted i.e. $4({}_{-1}\beta^0)$,

$\therefore$ Increase in $Z = 4 \times 1 = 4$ (2)

2 positrons are emitted i.e. $2({}_1\beta^0)$,

$\therefore$ Decrease in $Z = 2 \times 1 = 2$ (3)

$\therefore$ Z of resultant nucleus $= 92 - 16 + 4 - 2 = 78$

The correct option is (B)

137. Protons are not emitted during radioactive decay

The correct option is (A)

138. At temperature T, molecules of a gas acquire a kinetic energy $= \frac{3}{2}$kT where k = Boltzmann's constant

$\therefore$ To initiate the fusion reaction

$$\frac{3}{2}\text{kT} = 7.7 \times 10^{-14}\ \text{J}$$

$$\therefore \quad T = \frac{7.7 \times 10^{-14} \times 2}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9\ \text{K}$$

The correct option is (D)

139. ${}^{133}_{55}$Cs has the largest size. Electrons in the outer most orbit are at large distance from nucleus in a large-size atom. Hence the ionization potential is the least.

The correct option is (B)

140. Masses of ${}_1\text{H}^1$ and ${}_1\text{D}^2$ are different, hence the corresponding wavelengths are different

The correct option is (C)

141. Energy $E_2 = \frac{-Z^2E_0}{n^2} = \frac{-(3)^2 \times 13.6}{(2)^2} = -30.6\,\text{eV}$

$\therefore$ energy required = 30.6 eV

The correct option is (A)

142. Momentum is conserved during disintegration

$\therefore \quad m_1v_1 = m_2v_2$ (1)

For an atom, $R = R_0A^{1/3}$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$= \left(\frac{m_1}{m_2}\right)^{1/3} = \left(\frac{m_2v_2}{m_2v_1}\right)^{1/3}, \text{ from (1)}$$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{1}{2}\right)^{1/3} = \frac{1}{2^{1/3}}$$

The correct option is (B)

143. Total binding energy for (each deuteron)

$= 2 \times 1.1 = 2.2$ MeV

Total binding energy for helium $= 4 \times 7 = 28$ MeV

$\therefore$ Energy released $= 28 - 2(2 \times 2.2)$

$= 28 - 4.4 = 23.6$ MeV

The correct option is (A)

144. Kinetic energy is converted into potential energy at closest approach

$\therefore$ KE = PE

$$\therefore \quad 5\,\text{MeV} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$\text{or} \quad 5 \times 10^6 \times e = \frac{(9 \times 10^9) \times (92e)(2e)}{r}$$

$$\text{or} \quad r = \frac{9 \times 10^9 \times 92 \times 2 \times e}{r}$$

$$= \frac{9 \times 10^9 \times 92 \times 2 \times (1.6 \times 10^{-19})}{5 \times 10^6}$$

$$\therefore r = 5.3 \times 10^{-14}\ \text{m} = 5.3 \times 10^{-12}\ \text{cm}$$

The correct option is (A)

145. R is proportional to $\text{A}^{1/3}$ where A is mass number

$3.6 = R_0(27)^{1/3} = 3R_0$, for ${}^{27}_{13}$Al

Again $R = R_0(125)^{1/3}$, for ${}^{125}_{52}$Al

$$\therefore \quad R = \frac{(3.6)}{3} \times 5 = 6\ \text{fermi}$$

The correct option is (B)

146. $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

$$\therefore \quad \frac{1}{8} = \left(\frac{1}{2}\right)^{15/T} \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{15/T}$$

$$\therefore \quad \frac{15}{T} = 3 \Rightarrow T = 5\,\text{min}$$

The correct option is (D)

147. $\because \quad I = I_0e^{-kx} \Rightarrow \frac{I}{I_0} = e^{-kx}$

$$\therefore \ln\left(\frac{I}{I_0}\right) = -kx$$

In first case

$$\ln\left(\frac{1}{8}\right) = -k \times 36$$

$$\ln(2^{-3}) = -k \times 36$$

or $3\ln 2 = k \times 36$

In second case, $\ln\left(\frac{1}{2}\right) = -k \times x$

or $\ln\left(2^{-1}\right) = -kx$

or $\ln 2 = kx$

From (1) and (2)

$3 \times (kx) = k \times 36$ or $x = 12$ mm

The correct option is (C)

148. I is showing absorption photon.

From rest of three, III having maximum energy from

$$\Delta E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The correct option is (B)

149. The nuclear transformation is given by

$$^{A}_{Z}X + ^{1}_{0}n \rightarrow ^{4}_{2}He + ^{7}_{3}Li$$

According to conservation of mass number

$$A + 1 = 4 + 7$$

or $A = 10$

According to conservation of charge number

$$Z + 0 \rightarrow 2 + 3$$

or $Z = 5$

So the nucleus of the element be $^{10}_{5}B$

The correct option is (A)

150. For closest approach, kinetic energy is converted into potential energy.

$$\therefore \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_0}$$

or $r_0 = \dfrac{4Ze^2}{4\pi\varepsilon_0 mv^2} = \dfrac{Ze^2}{\pi\varepsilon_0 v^2}\left(\dfrac{1}{m}\right)$

or r_0 is proportional to $\left(\dfrac{1}{m}\right)$

The correct option is (B)

151. $_3Li^7 + _1H^1 \rightarrow _4Be^8 + _ZX^A$

Z for the unknown X nucleus $= (3+1) - 4 = 0$

A for the unknown X nucleus $= (7+1) - 8 = 0$

Hence particle emitted has zero Z and zero A

It is a gamma photon.

The correct option is (C)

152. Graph (C) represents the variation.

The correct option is (C)

153. Binding energy of

$^{7}_{3}Li = 7 \times 5.60 = 39.2$ MeV

Binding energy of $^{4}_{2}He = 4 \times 7.06 = 28.24$ MeV

$\therefore$ Energy of proton $=$ Energy of $\left[2\left(^{4}_{2}He\right) - ^{7}_{3}Li\right]$

$$= 2 \times 28.24 - 39.2 = 17.28 \text{ MeV}$$

The correct option is (B)

154. The 'rad' the biological effect of radiation.

The correct option is (C)

155. *Ec* and *Ev* decrease but *Eg* increases if the lattice constant of the semiconductor is decreased.

The correct option is (C)

156. Binding energy $= [ZM_P + (A - Z)M_N - M]c^2$

$$= [8M_P + (17-8)M_N - M_0]c^2$$

$$= (8M_P + 9M_N - M_0)c^2$$

[But the option given is negative of this]

The correct option is (C)

157. γ-ray emission takes place due to de-excitation of the nucleus. Therefore during γ-ray emission, there is no change in the proton and neutron number.

The correct option is (C)

158. $T_{1/2}$, half-life of $X = \tau_y$, mean life of Y

$$\frac{\ln 2}{\lambda_x} = \frac{1}{\lambda_y} \Rightarrow \lambda_x = \lambda_y \ln 2$$

$$\lambda_x > \lambda_y$$

$$\therefore A_x = A_0 e^{-\lambda x'};\ A_y = A_0 e^{-\lambda y'}$$

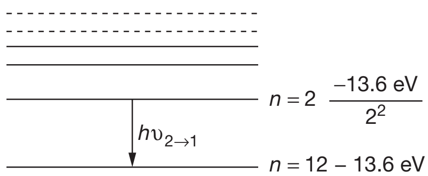
X will decay faster than Y.

The correct option is (B)

159. C, Si and Ge have the same lattice structure and their valence electrons are 4. For *C*, these electrons are in the second orbit, for Si it is third and germanium it is the fourth orbit. In solid state, higher the orbit, greater the possibility of overlapping of energy bands. Ionization energies are also lell therefore Ge has more conductivity compared to Si. Both are semiconductors. Carbon is an insulator.

The correct option is (A)

160.



$$h\upsilon_{2\to1} = -13.6\left(\frac{1}{2^2} - \frac{1}{2^2}\right)eV$$

$$= +13.6 \times \frac{3}{4}eV = 10.2\,eV$$

Emission is $n = 2 \rightarrow n = 1$ i.e., higher *n* to lower *n*.

Transition from lower to higher levels is absorption lines.

$$= -13.6\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = +13.6 \times \frac{2}{9}$$

This is $< E_{n=2} \rightarrow E_{n=1}$

The correct option is (D)

161. Statement 1 states that energy is released when heavy nuclei undergo fission and light nuclei undergo fusion is correct. Statement 2 is wrong.

The binding energy per nucleon, B/A, starts at a small value, rises to a maximum at ^{62}Ni, then decreases to 7.5 meV for the heavy nuclei.

The correct option is (A)

162. Supposing that the force of attraction is Bohr's atom does not follow inverse square law but inversely proportional to r,

$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ would have been $=\frac{mv^2}{r}$

$$\therefore\ mv^2=\frac{e^2}{4\pi\varepsilon_0}=k\Rightarrow\frac{1}{2}mv^2=\frac{1}{2}k$$

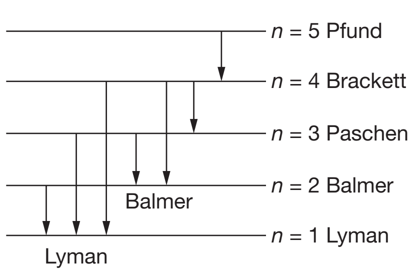
This is independent of n.

From $mvr_n=\frac{nh}{2\pi}$,

as mv is independent of $n, r_n \propto n$.

The correct option is (C)

163.



Transition $4\rightarrow 3$ is in Paschen series. This is not in the ultraviolet region but this is in infrared region. Transition $5\rightarrow 4$ will also be in infrared region (Brackett).

The correct option is (C)

164. When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements.

Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A+B\rightarrow C+\varepsilon$ is correct. In the possibility of fission is only for F and not C. Therefore.

$F\rightarrow D+E+\varepsilon$ is the correct choice.

The correct option is (D)

165. When a radioactive nucleus emits an alpha particle, its mass number decreases by 4 while the atomic number decreases by 2.

When a radioactive nucleus, emits a β^+ particle (or positron (e^+)) its mass number remains unchanged while the atomic number decreases by 1.

$$\therefore\ {}_Z^AX\xrightarrow{3\alpha}{}_{Z-6}^{A-12}Y\xrightarrow{2e^-}{}_{Z-8}^{A-12}W$$

In the final nucleus,

Number of protons, $N_p=Z-8$

Number of neutrons, $N_n=A-12-(Z-8)$

$=A-Z-4$

$$\therefore\ \frac{N_n}{N_p}=\frac{A-Z-4}{Z-8}$$

Direction for Questions 10–12 are based on the following paragraph.

A nucleus of mass $M+\Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

166. Mass defect, $\Delta M=\left[(M+\Delta m)-\left(\frac{M}{2}+\frac{M}{2}\right)\right]$

$=[M+\Delta m-M]=\Delta m$

Energy released, $Q=\Delta Mc^2=\Delta Mc^2$ (1)

According to law of conservation of momentum, we get

$$(M+\Delta m)\times 0=\frac{M}{2}\times v_1-\frac{M}{2}\times v_2\ \text{or}\ v_1=v_2$$

Also, $Q=\frac{1}{2}\left(\frac{M}{2}\right)v_1^2+\frac{1}{2}\left(\frac{M}{2}\right)v_2^2-\frac{1}{2}(M+\Delta m)\times(0)^2$

$$=\frac{M}{2}v_1^2\ (\because v_1=v_2)\quad (2)$$

Equating Equations (1) and (2), we get

$$\left(\frac{M}{2}\right)v_1^2=\Delta mc^2$$

$$v_1^2=\frac{2\Delta mc^2}{M}$$

$$v_1=c\sqrt{\frac{2\Delta m}{M}}$$

The correct option is (C)

167. After decay, the daughter nuclei will be more stable, hence binding energy per nucleon of daughter nuclei is more than that of their parent nucleus.

Hence, $E_2>E_1$

The correct option is (D)

168. Using, $E_n=-\frac{13.6Z^2}{n^2}\text{eV}$

Here, $Z=3(\text{For Li}^{++})$

$$\therefore\ E_1=-\frac{13.6(3)^2}{(1)^2}\text{eV}$$

$$E_1=-122.4\,\text{eV}$$

and $E_3=\frac{-13.6\times(3)^2}{(3)^2}=-13.6\,\text{eV}$

$$\Delta E-E_3-E_1=-13.6+122.4=108.8\ \text{eV}$$

The correct option is (C)

169. Number of not decayed atoms after time t_2,

$$\frac{N_0}{3}=N_0e^{-\lambda t_2}\quad (1)$$

number of not decayed atoms after time t_1,

$$\frac{2}{3}N_0 = N_0 e^{-\lambda t_1} \qquad (2)$$

Dividing (2) by (1), we get

$$2 = e^{\lambda(t_2 - t_1)} \text{ or In } 2 = \lambda(t_2 - t_1)$$

or $(t_2 - t_1) = \dfrac{\ln 2}{\lambda}$

As per question, $t_{1/2}$ = half life time = 20 min

$\therefore t_2 - t_1 = 20$ min $\left[\because t_{1/2} = \dfrac{\ln 2}{\lambda}\right]$

The correct option is (C)

170. Number of spectral line in emission spectra = $[n(n-1)/2] = 6$

The correct option is (D)

171. $E = (1/2)\mu r^2 \cdot \omega^2 \qquad (1)$

and $L = \mu\omega^2 r^2 = nh \qquad (2)$

where $\mu = [m_1 m_2 / (m_1 + m_2)]$

using (1) and (2) $E = \dfrac{(m_1 + m_2)n^2 h^2}{2m_1 m_2 r^2}$ where h stands for $(h/2\pi)$

The correct option is (D)

172. $\Delta m = (m_p + m_e) - m_n = 9 \times 10^{-31}$ kg

Energy Released = $(9 \times 10^{-31}$ kg$)$ c^2 J

$$= \frac{9\times10^{-31}\times(3\times10^8)^2}{1.6\times10^{-13}} \text{MeV}$$

$= 0.73$ MeV

The correct option is (A)

173. $\Delta E = 13.6\, z^2\left(\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2}\right)$

$$hf = \frac{13.6z^2(n^2 - n^2 - 1 + 2n)}{n^2(n-1)^2} \approx \frac{13.6z^2}{n^3} \quad (n >> 1)$$

$\therefore \quad f \propto \dfrac{1}{n^3}$

The correct option is (C)

174. $I = \left(e^{\frac{1000V}{T} - 1}\right) mA$

$$dI = \frac{1000}{T} e^{\frac{1000V}{T}} dV$$

$$dI = \frac{1000}{T}[I+1]dV = \frac{1000}{300}[5+1]\times 0.01 = 0.2\,\text{mA}$$

The correct option is (A)

175. $\dfrac{hc}{\lambda} = 13.6Z^2\left(\dfrac{1}{1} - \dfrac{1}{4}\right)$

$\Rightarrow \lambda Z^2 = \text{constant} \Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

The correct option is (C)

176. $E_{\text{photon}} = 13.6\left(\dfrac{1}{4} - \dfrac{1}{9}\right) = \dfrac{13.6\times5}{36} = 1.89$ eV

$$r_{\max} = \frac{mv_{\max}}{eB} \Rightarrow mv_{\max} = er_{\max}B$$

Now,

$$K_{\max} = \frac{(mv_{\max})^2}{2m} = \frac{\left(10\times10^{-3}\times1.6\times10^{-19}\times3\times10^{-4}\right)^2}{2\times9.1\times10^{-31}}$$

$= 0.791$ eV

$\therefore \ \phi = E - K = 1.89 - 0.79 = 1.1$ eV

The correct option is (B)

177. Total energy = $-13.6\dfrac{z^2}{n^2}$ and $\left[\text{TE} = -\text{KE} = \dfrac{\text{PE}}{2}\right]$

As n decreases, Total energy decreases.

Potential energy $= -\dfrac{Kze^2}{r_n}$

As r_n decreases, PE also decreases.

$$\text{KE} = \frac{1}{2}mv^2 = \frac{Kze^2}{r_n}$$

As r_n decreases, KE increases.

The correct option is (D)

178. (A) Frank hertz experiment — Discrete energy levels of atom

(B) Photo-electric experiment — Particle nature of light

(C) Davison-germer experiment — wave nature of electron

The correct option is (B)

179.

A	B
N_0	N_0

$$\frac{N_A}{N_B} = \frac{1 - e^{-\frac{\ell n2}{20}\times 80}}{1 - e^{-\frac{\ell n2}{40}\times 80}} = 5/4$$

The correct option is (C)

180. $K_{\max} = \dfrac{hc}{\lambda} - \phi$

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$$

Now on changing wavelength of incident light to $\dfrac{3\lambda}{4}$, if v_1 is the new speed of fastest emitted electrons

then $\dfrac{1}{2}mv_1^2 = \dfrac{hc}{\left(\dfrac{3\lambda}{4}\right)} - \phi$

$$v_1 = \sqrt{\left(\frac{4}{3}\right)v^2 + \left(\frac{2\phi}{3m}\right)}$$

$$v_1 > \left(\frac{4}{3}\right)^{1/2} v$$

The correct option is (D)