CHAPTER 18 Ray Optics and Wave Optics

Chapter Highlights

Reflection and refraction of light at plane and spherical surfaces, mirror formula, Total internal reflection and its applications, Deviation and Dispersion of light by a prism, Lens Formula, Magnification, Power of a Lens, Combination of thin lenses in contact, Microscope and Astronomical Telescope (reflecting and refracting) and their magnifying powers.
Wave optics: wavefront and Huygens' principle, Laws of reflection and refraction using Huygen's principle. Interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light. Diffraction due to a single slit, width of central maximum. Resolving power of microscopes and astronomical telescopes, Polarisation, plane polarized light; Brewster's law, uses of plane polarized light and Polaroids.

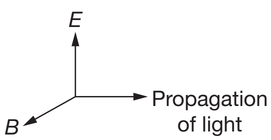
GEOMETRICAL OPTICS

Introduction

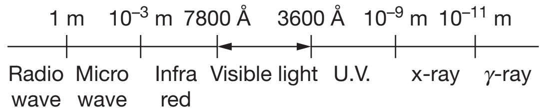
Blue lakes, golden deserts, green forest, and multicoloured rainbows can be enjoyed by anyone who has eyes with which to see them. But by studying the branch of physics called optics, which deals with the behaviour of light and other electromagnetic waves, we can reach a deeper appreciation of the visible world. A knowledge of the properties of light allows us to understand the blue colour of the sky and the design of optical devices such as telescopes, microscopes, cameras, eyeglasses, and the human eye. The same basic principles of optics also lie at the heart of modern developments such as the laser, optical fibres, holograms, optical computers, and new techniques in medical imaging.

Properties of Light

1. Speed of light in vacuum, denoted by c, is equal to 3×10^8 m/s approximately.
2. Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.

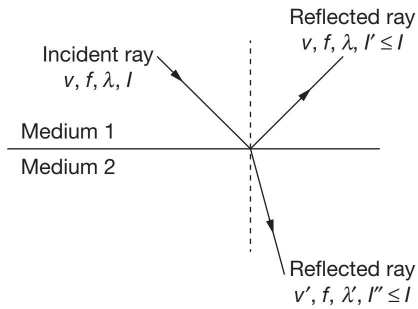


3. Light carries energy and momentum.
4. The formula $v = f\lambda$ is applicable to light.



Electromagnetic spectrum

5. When light gets reflected in same medium, it suffers no change in frequency, speed, and wavelength.
6. When light gets reflected in same medium, it suffers no change in frequency, speed, and wavelength.
7. Frequency of light remains unchanged when it gets reflected or refracted.



Some Interesting Facts about Light

Until the time of Isaac Newton (1642–1727), most scientists thought that light consisted of streams of particles (called corpuscles) emitted by light source. Galileo and others tried (unsuccessfully) to measure the speed of light. Around 1665, evidence of wave properties of light began to be discovered. By the early nineteenth century, evidence that light is a wave had grown very persuasive.

In 1873, James Clerk Maxwell predicted the existence of electromagnetic wave and calculated their speed of propagation. The development, along with the experimental work of Heinrich Hertz starting in 1887, showed conclusively that light is indeed an electromagnetic wave.

The wave picture of light is not the whole story; however, several effects associated with emission and absorption of light reveal a particle aspect, in that the energy carried by light waves is packaged in discrete bundles called photons or quanta. These apparently contradictory wave and particle properties have been reconciled since 1930 with the development of quantum electrodynamics, a comprehensive theory that includes both wave and particle properties. The propagation of light is best described by a wave model, but understanding emission and absorption requires a particle approach.

The speed of light in a vacuum is exactly 299,792,458 m/s (about 186,282.397 miles per second). The speed of light depends upon the medium in which it is traveling, and the speed will be lower in a transparent medium.

Different physicists have attempted to measure the speed of light throughout history. *Galileo* attempted to measure the speed of light in the seventeenth century. A good early experiment to measure the speed of light was conducted by *Ole Rømer*, a Danish physicist, in 1676. Using a telescope, Ole observed the motions of *Jupiter* and one of its *moons*, Io. Noting discrepancies in the apparent period of Io's orbit, Rømer calculated that light takes about 18 minutes to traverse the diameter of Earth's orbit. Unfortunately, this was not a value that was known at that time. If Ole had known the diameter of the earth's orbit, he would have calculated a speed of 227,000,000 m/s.

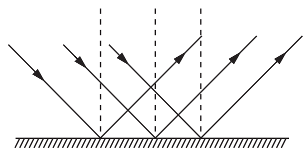
Another, more accurate, measurement of the speed of light was performed in Europe by *Hippolyte Fizeau* in 1849. Fizeau directed a beam of light at a mirror several kilometers away. A rotating cog wheel was placed in the path of the light beam as it traveled from the source to the mirror and then returned to its origin. Fizeau found that at a certain rate of rotation, the beam would pass through one gap in the wheel on the way out and the next gap on the way back. Knowing the distance to the mirror, the number of teeth on the wheel, and the rate of rotation, Fizeau was able to calculate the speed of light as 313,000,000 m/s.

Léon Foucault used an experiment which used rotating mirrors to obtain a value of 298,000,000 m/s in 1862. *Albert A. Michelson* conducted experiments on the speed of light from 1877 until his death in 1931. He refined Foucault's methods in 1926 using improved rotating mirrors to measure the time it took light to make a round trip from Mt. Wilson to Mt. San Antonio in *California*. The precise measurements yielded a speed of 299,796,000 m/s.

Reflection of Light

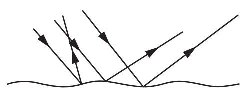
When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called *Reflection of Light.*

1. **Regular Reflection:** When the reflection takes place from a perfect plane surface it is called regular reflection. In this case, the reflected light has large intensity in one direction and negligibly small intensity in other directions.



Regular reflection

2. **Diffused Reflection:** When the surface is rough, we do not get a regular behaviour of light. Although at each point, light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called diffused reflection.

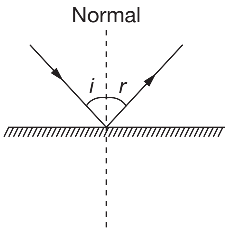


Diffused reflection

For example, reflection from a wall, from a newspaper, etc. This is why you can not see your face in news paper and in the wall.

Laws of Reflection

1. The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the plane of incidence (or plane of reflection). This condition can be expressed mathematically as $\vec{R} \cdot (\vec{I} \times \vec{N}) = \vec{N} \cdot (\vec{I} \times \vec{R}) = \vec{I} \cdot (\vec{N} \times \vec{R}) = 0$, where $\vec{I}$, $\vec{N}$, and $\vec{R}$ are vectors of any magnitude along incident ray, the normal, and the reflected ray, respectively.
2. The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e. $\angle i = \angle r$



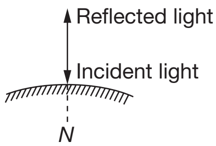
Special Cases

Normal Incidence

In case light is incident normally,

$$i = r = 0$$

$$\delta = 180°$$



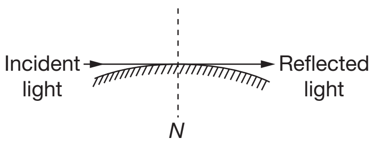
> **NOTE**
>
> We say that the ray has retraced its path.

Grazing Incidence

In case light strikes the reflecting surface tangentially,

$$i = r = 90$$

$$\delta = 0° \quad \text{or} \quad 360°$$



> **NOTE**
>
> In case of reflection, speed (magnitude of velocity) of light remains unchanged, but in Grazing incidence, velocity remains unchanged.

SOLVED EXAMPLE

1. Show that for a light ray incident at an angle i on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.

Solution:

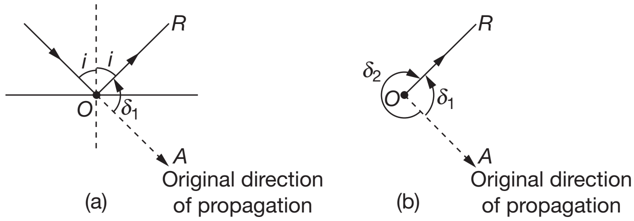


Fig. 18.1

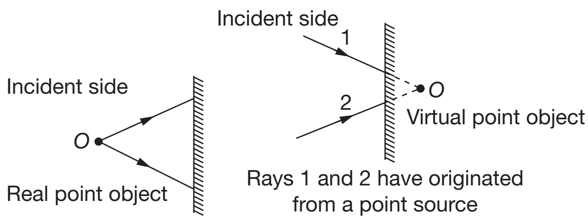
From Fig. 18.1(b) it is clear that light ray bends either by δ_1 anti-clockwise or by δ_2 $(= 2\pi - \delta_1)$ clockwise. From Fig. 18.1(a),

$$\delta_1 = \pi - 2i.$$

$$\therefore \qquad \delta_2 = \pi + 2i.$$

Object and Image

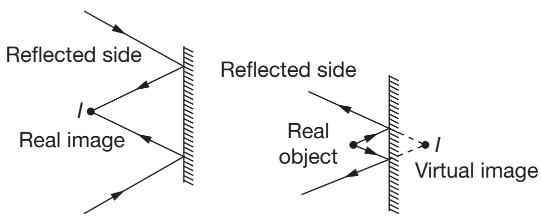
1. **Object (O):** Object is defined as point of intersection of incident rays.



Let us call the side in which incident rays are present as incident side and the side in which reflected (refracted) rays are present, as reflected (refracted) side.

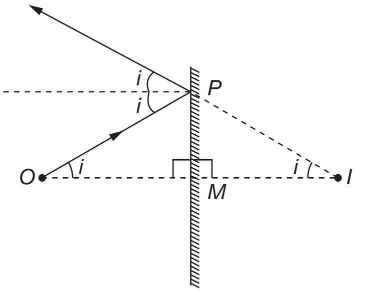
> **NOTE**
>
> An object is called real if it lies on incident side otherwise it is called virtual.

2. **Image (I):** Image is defined as point of intersection of reflected rays (in case of reflection) or refracted rays (in case of refraction).



> **NOTE**
>
> An image is called real if it lies on reflected or refracted side otherwise it is called virtual.

Formation of Image of a Point Object

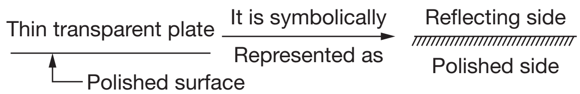


Δ OMP and Δ IMP are congruent so we can say that OM = MI

> **NOTE**
>
> Plane mirror is the perpendicular bisector of OI.

Plane Mirror

Plane mirror is formed by polishing one surface of a plane thin glass plate. It is also said to be silvered on one side.



PLANE MIRROR

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

SOLVED EXAMPLE

2. For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.

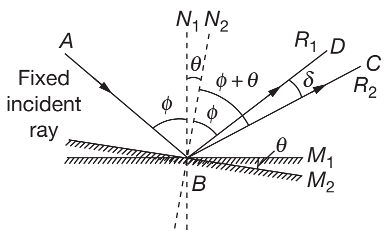


Fig. 18.2

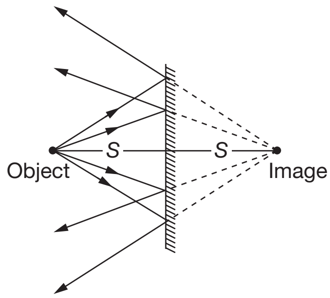
Solution:

See Fig. 18.2, M_1, N_1, and R_1 indicate the initial position of mirror, initial normal, and initial direction of reflected light ray, respectively. M_2, N_2, and R_2 indicate the final position of mirror, final normal, and final direction of reflected light ray, respectively. From Fig. 18.2, it is clear that $\angle ABC = 2\phi + \delta = 2(\phi + \theta)$ or $\delta = 2\theta$.

> **NOTE**
>
> Keeping the mirror fixed, if the incident ray is rotated by angle θ then reflected ray rotates by same angle in the opposite direction of rotation

Point Object

Characteristics of image due to reflection by a plane mirror:



1. Distance of object from mirror = Distance of image from the mirror.
2. All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called image.
3. The line joining a point object and its image is normal to the reflecting surface.
4. For a real object, the image is virtual and for a virtual object, the image is real.
5. The region in which observer's eye must be present in order to view the image is called field of view.

SOLVED EXAMPLES

3. Figure 18.3 shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called field of view.)

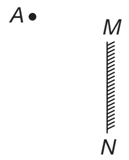


Fig. 18.3

Solution:

See Fig. 18.4, consider any two rays emanating from the object. N_1 and N_2 are normals;

$$i_1 = r_1 \quad \text{and} \quad i_2 = r_2$$

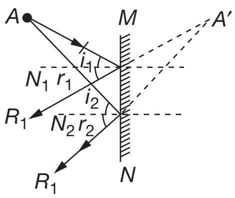


Fig. 18.4

The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror such that their meeting point is A'. To obtain the region in which reflected rays are present, join A' with the ends of mirror and extend. The following Fig. 18.5 shows this region as shaded. In Fig. 18.5, there are no reflected rays beyond rays 1 and 2; therefore, the observers P and Q cannot see the image because they do not receive any reflected ray.

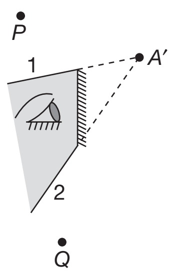
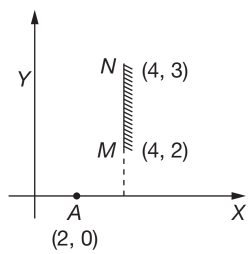


Fig. 18.5

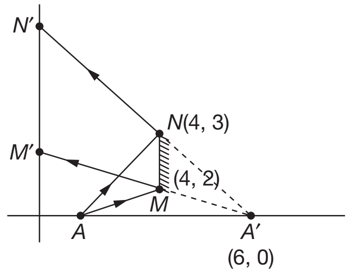
4. Find the region on y-axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown.



Solution:

The image of point A, in the mirror is at A' (6, 0).

Join $A'M$ and extend to cut y-axis at M' (Ray originating from A which strikes the mirror at M gets reflected as the ray MM' appears to come from A'). Join $A'N$ and extend to cut y-axis at N' (Ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A').



From geometry,

$$M' \equiv (0, 6)$$

$N' \equiv (0, 9)$. $M'N'$ is the region on y-axis in which reflected rays are present.

Extended Object

An extended object like AB shown in Fig. 18.6 is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A' will be image of A, C' will be image of C, B' will be image of B, and so on. All point images together form extended image. Thus, extended image is formed of an extended object.

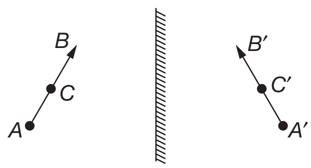
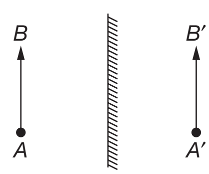


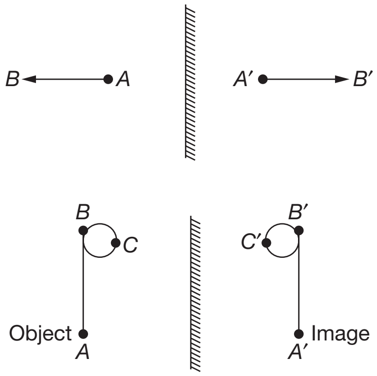
Fig. 18.6

Properties of Image of an Extended Object, Formed by a Plane Mirror

1. Size of extended object = size of extended image.
2. The image is erect, if the extended object is placed parallel to the mirror.



3. The image is inverted if the extended object lies perpendicular to the plane mirror.



4. If an extended horizontal object is placed in front of a mirror inclined 45° with the horizontal, the image formed will be vertical. See Fig. 18.7.

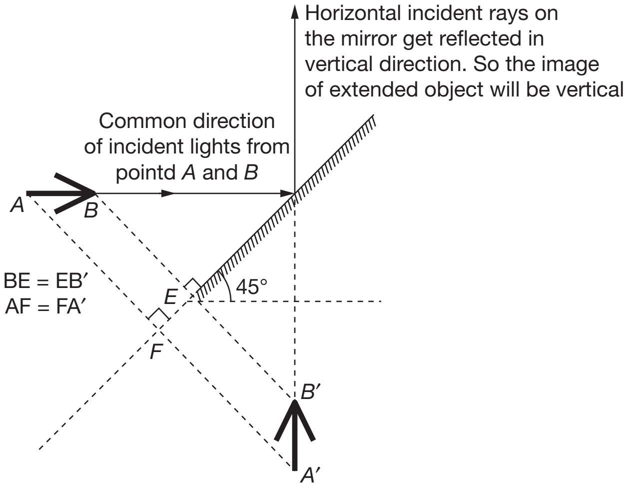


Fig. 18.7

SOLVED EXAMPLE

5. Show that the minimum size of a plane mirror, required to see the full image of an observer is half the size of that observer.

Solution:

See the following Fig. 18.8. It is self-explanatory if you consider lengths x and y as shown in it.

Aliter:

$\Delta E\ M_1$, M_2, and $\Delta E\ H'F'$ are similar

$$\therefore \qquad \frac{M_1M_2}{H'F'} = \frac{z}{2z}$$

or $\qquad M_1\ M_2 = H'\ F'/2 = HF/2.$

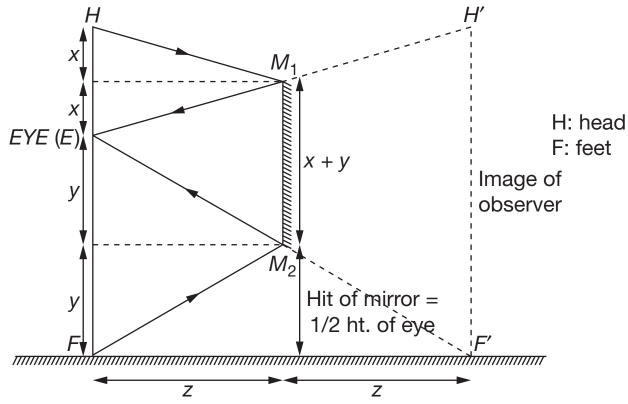


Fig. 18.8

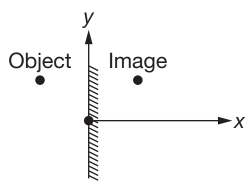
NOTE

- The requirement of the mirror is independent of its distance from the person.
- The image can be seen only if the mirror is placed in a proper position. (The upper end of the mirror should be just between the eyes and head of the person.)

Relation between Velocity of Object and Image (Take Origin on the Mirror)

From mirror property: $x_{im} = -x_{om}$, $y_{im} = y_{om}$, and $z_{im} = z_{om}$

Here x_{im} means x coordinate of image with respect to mirror. Similarly, others have meaning.



Differentiating with respect to time, we get

$$v_{(im)x} = -v_{(om)x};\ v_{(im)y} = v_{(om)y};\ v_{(im)z} = v_{(om)z},$$

$\Rightarrow$ for x-axis

$$v_{iG} - v_{mG} = -(v_{oG} - v_{mG})$$

but for y-axis and z-axis

$$v_{iG} - v_{mG} = (v_{oG} - v_{mG})$$

or

$$v_{iG} = v_{oG}.$$

here v_{iG} = velocity of image with respect to ground.

NOTE

- For perpendicular direction: Velocity of object and image are different; we can also say that
  $\vec{V}_{OM} = -\vec{V}_{IM}$ (in perpendicular direction)
- For parallel direction: Velocity of object and image are same.
  $\vec{V}_{OM} = \vec{V}_{IM}$ (in parallel direction)

SOLVED EXAMPLES

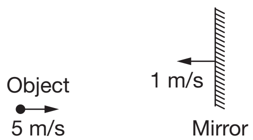
6. An object moves with 5 m/s towards right while the mirror moves with 1 m/s towards the left as shown. Find the velocity of image.

Solution:
Take $\rightarrow$ as + direction.

$$v_i - v_m = v_m - v_0$$

$$v_i - (-1) = (-1) - 5$$

$$\therefore \quad v_i = -7 \text{ m/s.}$$



$\Rightarrow$ 7 m/s and direction towards left.

7. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object, find the distance which the image will move.

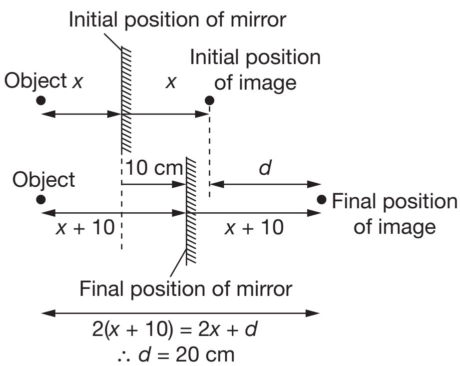
Solution:
We know that

$$x_{im} = -x_{om} \text{ or } x_i - x_m = x_m - x_o$$

or $$\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o.$$

In this Q. $\Delta x_o = 0$; $\Delta x_m = 10$ cm.

Therefore $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.



8. In the situation shown in Fig. 18.9, find the velocity of image.

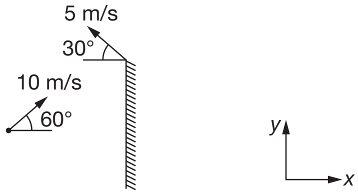


Fig. 18.9

Solution:
Along x direction, applying $v_i - v_m = -(v_0 - v_m)$

$$v_i - (-5\cos 30°) = -(10 \cos 60° - (-5 \cos 30°))$$

$$\therefore \quad v_i = -5\,(1+\sqrt{3}\,) \text{ m/s}$$

Along y direction $v_0 = v_i$

$$\therefore \quad v_i = 10 \sin 60° = 5\sqrt{3} \text{ m/s}$$

$\therefore$ Velocity of the image $= -5\,(1+\sqrt{3})\hat{i} + 5\sqrt{3}\hat{j}$ m/s.

Images formed by Two Plane Mirrors

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

SOLVED EXAMPLES

9. Figure 18.10 shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.

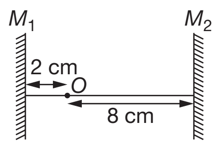
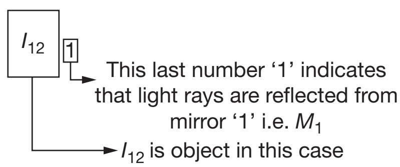


Fig. 18.10

Solution:
To understand how images are formed, see the following Fig. 18.11 and table. You will require to know what symbols like I_{121} stands for. See the following diagram.



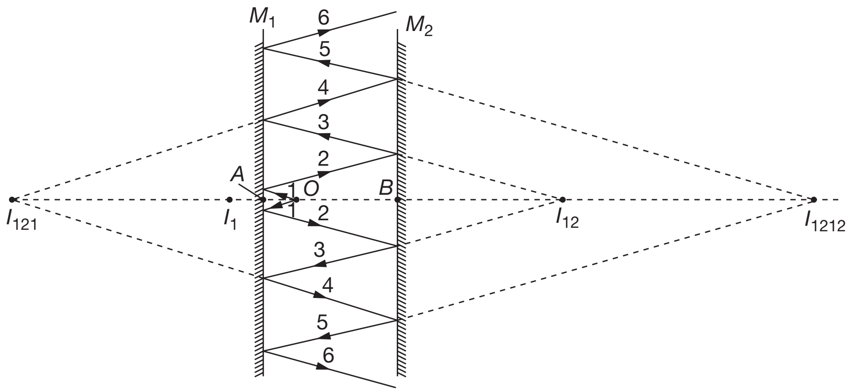


Fig. 18.11

Incident rays	Reflected by	Reflected rays	Object	Image	Object distance	Image distance
Rays 1	M_1	Rays 2	O	I_1	$AO = 2$ cm	$AI_1 = 2$ cm
Rays 2	M_2	Rays 3	I_1	I_{12}	$BI_1 = 12$ cm	$BI_{12} = 12$ cm
Rays 3	M_1	Rays 4	I_{12}	I_{121}	$AI_{12} = 22$ cm	$AI_{121} = 22$ cm
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	$BI_{121} = 32$ cm	$BI_{1212} = 32$ cm

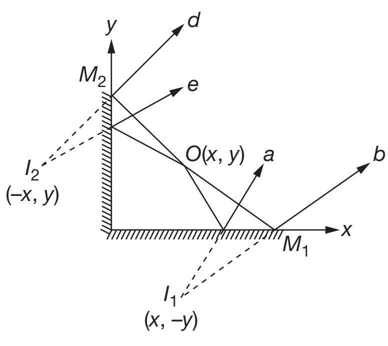
And so on …

Similarly, images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

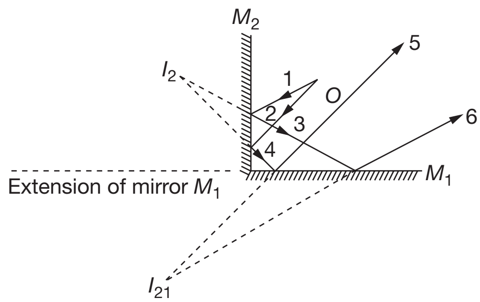
10. Consider two perpendicular mirrors. M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinate of object as (x, y), find the position and number of images.

Solution:

Rays a and b strike mirror M_1 only and these rays will form image I_1 at $(x, -y)$, such that O and I_1 are equidistant from mirror M_1. These rays do not form further image because they do not strike any mirror again. Similarly, rays d and e strike mirror M_2 only and these rays will form image I_2 at $(-x, y)$, such that O and I_2 are equidistant from mirror M_2.



Now consider those rays which strike mirror M_2 first and then the mirror M_1.



For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.

Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

I_2 and I_{21}, are equidistant from M_1. To summarize, see the following Fig. 18.12.

Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

For rays reflecting first from M_1 and then from M_2, first image I_1 (at $(x, -y)$) will be formed and this will function as object for mirror M_2 and then its image I_{12} (at $(-x, -y)$) will be formed.

$$I_{12} \text{ and } I_{21} \text{ coincide.}$$

∴ three images are formed.

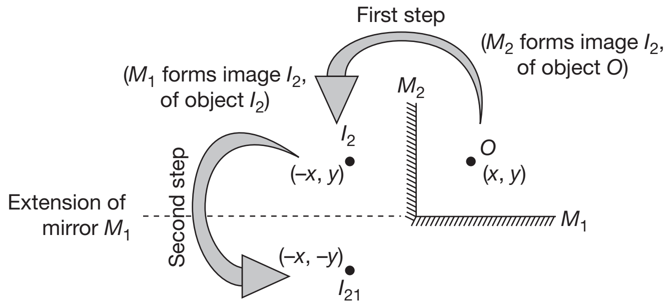


Fig. 18.12

Locating all the Images Formed by Two Plane Mirrors

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in Fig. 18.13.

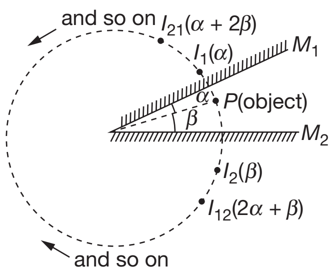


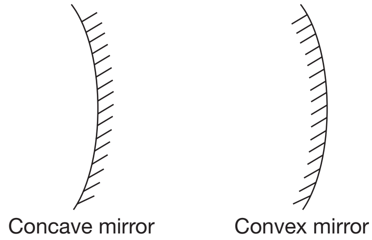
Fig. 18.13

Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the Fig. 18.13 besides I_1. Similarly, image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the Fig. 18.13 besides I_2.

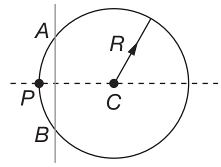
Now I_2 will act as an object for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at an angle $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when this will process stops. Hint: The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

SPHERICAL MIRRORS

Spherical mirror is formed by polishing one surface of a part of sphere. Depending upon which part is shining, the spherical mirror is classified as (a) Concave mirror, if the side towards centre of curvature is shining and (b) Convex mirror if the side away from the centre of curvature is shining.



1. Important terms related to spherical mirrors:
   - **(a) Centre of Curvature (C):** The centre of the sphere from which the spherical mirror is formed is called the centre of curvature of the mirror. It is represented by C and is indicated in Fig. 18.14.



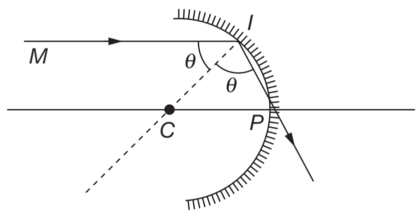
A spherical shell with the center of curvature, pole aperture and radius of curvature identified

Fig. 18.14

   - **(b) Pole (P):** The centre of the mirror is called the pole. It is represented by the point P on the mirror APB in Fig. 18.14.
   - **(c) Principal Axis:** Principal axis is a line which is perpendicular to the plane of the mirror and passes through the pole. The principal axis can also be defined as the line which joins the pole to the centre of curvature of the mirror.
   - **(d) Aperture (A):** The aperture is the segment or area of the mirror which is available for reflecting light. In Fig. 18.14, APB is the aperture of the mirror.

SOLVED EXAMPLES

11. Find the angle of incidence of ray for which it passes through the pole, given that $MI \parallel CP$.



Solution:

$$\angle MIC = \angle CIP = \theta$$

$$MI \parallel CP \ \angle MI\theta = \angle ICP = \theta$$

$$CI = CP$$

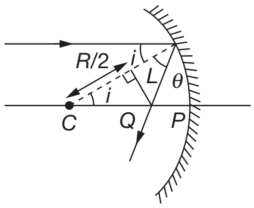
$$\angle CIP = \angle CPI = \theta$$

$\therefore$ In ΔCIP, all angles are equal

$$3\theta = 180°$$

$$\Rightarrow \quad \theta = 60°.$$

12. Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also find the distance CQ if $i \to 0$.

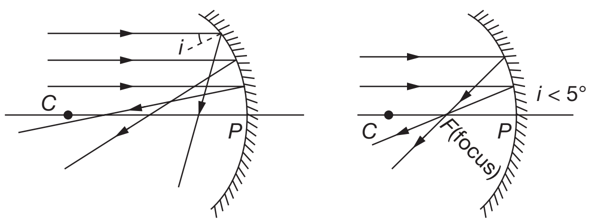


Solution:

$$\cos i = \frac{R}{2CQ}$$

$$\Rightarrow \quad CQ = \frac{R}{2\cos i}$$

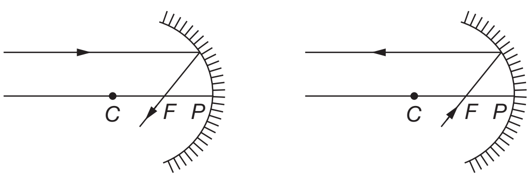
As i increases, $\cos i$ decreases.
Hence, CQ increases
If i is a small angle, $\cos i \approx 1$
$\therefore \quad CQ = R/2$



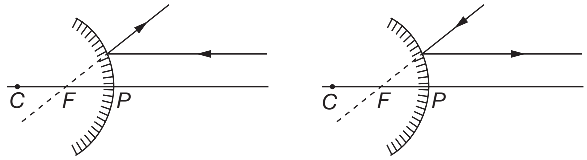
So, paraxial rays meet at a distance equal to $R/2$ from centre of curvature, which is called focus.

Principal Focus (F)

It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In concave mirror, it is real, and in the convex mirror, it is virtual. The distance from pole to focus is called focal length.



Concave mirror

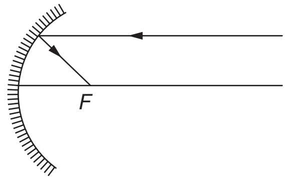


Convex mirror

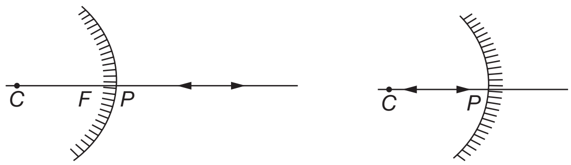
Ray Tracing

Following facts are useful in ray tracing.

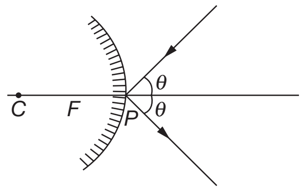
1. If the incident ray is parallel to the principal axis, the reflected ray passes through the focus.



2. If the incident ray passes through the focus, then the reflected ray is parallel to the principal axis.
3. Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is a normally incident ray).



4. It is easy to make the ray tracing of a ray incident at the pole as shown below.



Sign Convention

We are using co-ordinate sign convention.

1. Take origin at pole (in case of mirror) or at optical centre (in case of lens).
   Take x-axis along the principal axis, taking positive direction along the incident light.
   u, v, r, and f indicate the x-coordinate of object, image, centre of curvature, and focus, respectively.
2. y-coordinates are taken positive above principal axis and negative below principal axis.
   h_1 and h_2 denote the y-coordinate of object and image, respectively.

NOTE

This sign convention is used for reflection from mirror, reflection through flat or curved surfaces or lens.

Formulae for Reflection from Spherical Mirrors

Mirror Formula

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{R}=\frac{1}{f}$$

x-coordinate of centre of curvature and focus of concave mirror are negative and those for convex mirror are positive. In case of mirrors, since light rays reflect back in x-direction, –ve sign of v indicates real image and +ve sign of v indicates virtual image**.**

SOLVED EXAMPLES

13. Figure 8.15 shows a spherical concave mirror with its pole at (0, 0) and principal axis along x-axis. There is a point object at (–40 cm, 1 cm), find the position of image.

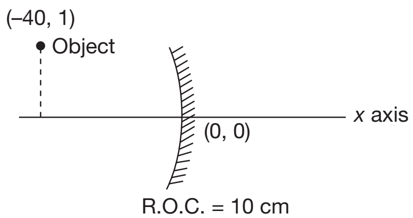


Fig. 18.15

Solution:

According to sign convention,

$$u = -40 \text{ cm}$$

$$h_1 = +1 \text{ cm}$$

$$f = -5 \text{ cm}$$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{-40}=\frac{1}{-5};$$

$$v=\frac{-40}{7}\text{ cm}; \quad \frac{h_2}{h_1}=\frac{-v}{u}$$

$$\Rightarrow \quad h_2=-\left(\frac{-v}{u}\times h_1\right)=\frac{-\left(-\frac{40}{7}\right)\times 1}{-40}$$

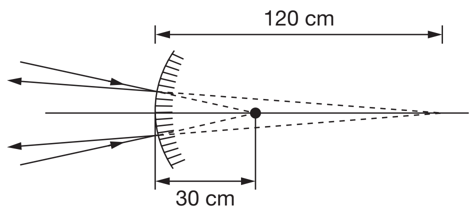
$$=-\frac{1}{7}\text{ cm.}$$

$\therefore$ The position of image is $\left(\frac{-40}{7}\text{cm}, -\frac{1}{7}\text{cm}\right)$

14. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

Solution:

In this case,



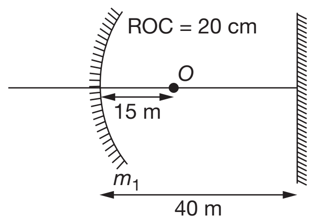
$$u = +30$$

$$\Rightarrow \quad v = +120$$

$$\therefore \quad \frac{1}{f}=\frac{1}{v}+\frac{1}{u}=\frac{1}{120}+\frac{1}{30}$$

$$f = 24 \text{ cm}$$

15. Find the position of final image after three successive reflections taking first reflection on m_1.



Solution:

I reflection:

$$\text{Focus of mirror} = -10 \text{ cm}$$

$$\Rightarrow \quad u = -15 \text{ cm}$$

Applying mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$\Rightarrow$ $v = -30$ cm.

For II reflection on plane mirror:

$u = -10$ cm

$\therefore$ $v = 10$ cm

For III reflection on curved mirror:

$u = -50$ cm

$f = -10$ cm

Applying mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$v = -12.5$ cm.

1. Lateral magnification (or transverse magnification) denoted by m is defined as $m = \frac{h_2}{h_1}$ and is related as $m = -\frac{v}{u}$. From the definition of m, positive sign of m indicates erect image and negative sign indicates inverted image.

2. In case of successive reflection from mirrors, the overall lateral magnification is given by $m_1 \times m_2 \times m_3 \ldots$, where m_1, m_2, etc. are lateral magnifications produced by individual mirrors.

h_1 and h_2 denote the y-coordinate of object and image, respectively.

NOTE

- Using (1) and (2) the following conclusions can be made (check yourself).
- Derivation to be studied by student from HCV/NCERT.

Nature of Object	Nature of Image	Inverted or erect
Real	Real	Inverted
Real	Virtual	Erect
Virtual	Real	Erect
Virtual	Virtual	Inverted

From (1) and (2), we get

$$m = \frac{f}{f-u} = \frac{f-v}{f} \quad \text{(just a time-saving formula)}$$

SOLVED EXAMPLES

16. An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from pole. Find the lateral magnification produced.

Solution:

$u = -15$ cm

$f = -10$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

we get, $v = -30$ cm

$\therefore$ $m = -\frac{v}{u} = -2.$

Aliter:

$$m = \frac{f}{f-u} = \frac{-10}{-10-(-15)} = -2$$

17. A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance 20 cm, then find the nature of radius of curvature of the mirror.

Solution:

Person will see his face only when the image is virtual. Virtual image of real object is erect.

Hence, $m = 2$

$\therefore$ $\frac{-v}{u} = 2$

$\Rightarrow$ $v = 40$ cm

Applying $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$;

$f = -40$ cm

or $R = 80$ cm.

Aliter:

$$m = \frac{f}{f-u}$$

$\Rightarrow$ $2 = \frac{f}{f-(-20)}$

$\Rightarrow$ $f = -40$ cm

or $R = 80$ cm.

18. An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance 5 cm, then the image become triple its size. Find the nature and *ROC* of the mirror.

Solution:

Since the images formed on screen is real. Real object and real image implies concave mirror.

Applying $$m = \frac{f}{f-u}$$

or $$-2 = \frac{f}{f-(u)} \quad (1)$$

After shifting $$-3 = \frac{f}{f-(u+5)} \quad (2)$$

[Why $u + 5$?, why not $u - 5$: In a concave mirror, size of real image will increase, only when the real object is brought closer to the mirror. In doing so, its x-coordinate will increase]

From (1) and (2) we get,

$$f = -30 \text{ cm} \quad \text{or} \quad R = 60 \text{ cm}$$

VELOCITY OF IMAGE

Object Moving Perpendicular to Principal Axis

From the relation in 5.3.2, we have

$$\frac{h_2}{h_1} = -\frac{v}{u} \quad \text{or} \quad h_2 = -\frac{v}{u} \cdot h_1$$

If a point object moves perpendicular to the principal axis, x coordinate of both the object and the image become constant. On differentiating the above relation with respect to time, we get,

$$\frac{dh_2}{dt} = -\frac{v}{u}\frac{dh_1}{dt}$$

Here, $\frac{dh_1}{dt}$ denotes velocity of object perpendicular to the principal axis and $\frac{dh_2}{dt}$ denotes velocity of image perpendicular to the principal axis.

Object Moving along Principal Axis

On differentiating the mirror formula with respect to time, we get $\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$, where $\frac{dv}{dt}$ is the velocity of image along principal axis and $\frac{du}{dt}$ is the velocity of object along principal axis. Negative sign implies that the image, in case of mirror, always moves in the direction opposite to that of object. This discussion is for velocity with respect to mirror and along the x-axis.

Object Moving at an Angle with the Principal Axis

Resolve the velocity of object along and perpendicular to the principal axis and find the velocities of image in these directions separately and then find the resultant.

Optical Power of a Mirror (in Diopters) = $\frac{1}{f}$

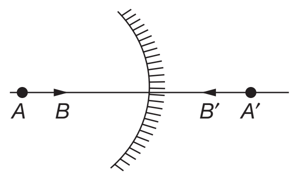
f = focal length with sign and in meters.

If object lying along the principal axis is not of very small size, the longitudinal magnification = $\frac{v_2 - v_1}{u_2 - u_1}$ (it will always be inverted)

If the size object is very small as compared to its distance from pole then,

On differentiating the mirror formula, we get $\frac{dv}{du} = -\frac{v^2}{u^2}$:

Mathematically, du implies small change in position of object and dv implies corresponding small change in position of image. If a small object lies along principal axis, du may indicate the size of object and dv the size of its image along principal axis (Note that the focus should not lie in between the initial and final points of object). In this case, $\frac{dv}{du}$ is called longitudinal magnification. Negative sign indicates inversion of image irrespective of nature of image and nature of mirror.



SOLVED EXAMPLE

19. A pointed object is placed 60 cm from pole of a concave mirror of focal length 10 cm on the principal axis. Find

(A) The position of image

(B) If object is shifted 1 mm towards the mirror along principal axis, find the shift in image. Explain the result.

Solution:

(A) $u = -60$ cm

$f = -10$ cm

$$v = \frac{fu}{u-f} = \frac{-10(-60)}{-60-(-10)} = \frac{600}{-50} = -12 \text{ cm.}$$

(B) $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Differentiating, we get

$$dv=-\frac{v^2}{u^2}\,du=-\left(\frac{-12}{-60}\right)^2[1\text{ mm }]=-\frac{1}{25}\text{ mm}$$

[∵ $du = 1$ mm; sign of du is + because it is shifted in +ve direction defined by sign convention.]

(A) –ve sign of dv indicates that the image will shift towards negative direction.

(B) The sign of v is negative. Which implies the image is formed on negative side of pole. (A) and (B) together imply that the image will shift away from pole.

Note that differentials dv and du denote small changes only.

Newton's formula: $XY = f^2$

X and Y are the distances (along the principal axis) of the object and image, respectively, from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.

Refraction of Light

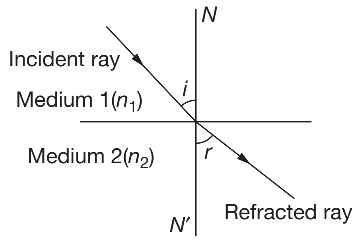
When the light changes its medium some changes occurs in its properties; the phenomenon is known as refraction.

- If the light is incident at an angle ($0 < i < 90$) then it deviates from its actual path. It is due to change in speed of light as light passes from one medium to another medium.
- If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.
- Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum $\mu=\frac{c}{v}=\frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$.

More (less) refractive index implies less (more) speed of light in that medium, which, therefore, is called denser (rarer) medium.

Laws of Refraction

1. The incident ray, the normal to any refracting surface at the point of incidence, and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.
2. $\frac{\sin i}{\sin r}$ = Constant for any pair of media and for light of a given wave length. This is known as *Snell's* law.



Also, $\frac{\sin i}{\sin r}=\frac{n_2}{n_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}$

For applying in problems remember,

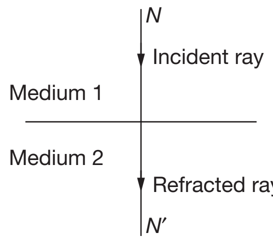
$$n_1\sin i = n_2\sin r$$

$=\frac{n_2}{n_1}\ {}_1n_2$ = Refractive index of the second medium with respect to the first medium.

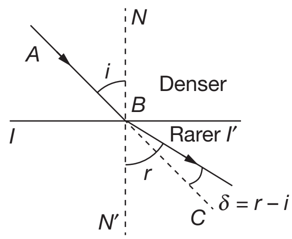
C = speed of light in air (or vacuum) = 3×10^8 m/s.

Special Cases

- **Normal incidence:** $i = 0$ from Snell's law: $r = 0$

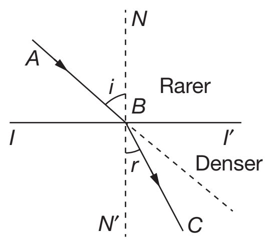


- When light moves from denser to rarer medium, it bends away from normal.



$r > i$

- When light moves from rarer to denser medium, it bends towards the normal.



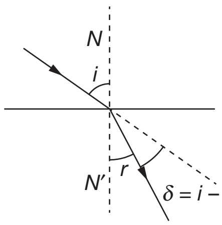
$r > i$

NOTE

- Higher the value of RI, denser (optically) is the medium.
- Frequency of light does not change during refraction.
- Refractive index of the medium relative to vacuum $= \sqrt{\mu_r \in_r}$
  $n_{vacuum} = 1$; $n_{air} \gtrsim 1$; n_{water} (average value) = 4/3; n_{glass} (average value)= 3/2

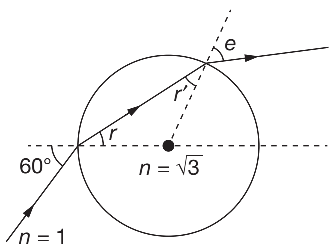
Deviation of a Ray Due to Refraction

Deviation (δ) of ray incident at $\angle i$ and refracted at $\angle r$ is given by $\delta = |i - r|$.



SOLVED EXAMPLES

20. A light ray is incident on a glass sphere at an angle of incidence 60° as shown. Find the angles *r*, *r*, e, and the total deviation after two refractions.



Solution:

Applying Snell's law,

$$1 \sin 60° = \sqrt{3} \ \sin r$$

$\Rightarrow$ $$r = 30°$$

From symmetry $r' = r = 30°$.

Again applying Snell's law at second surface, 1 sin $e = \sqrt{3} \sin r$

$\Rightarrow$ $$e = 60°$$

Deviation at first surface $= i - r = 60° - 30° = 30°$
Deviation at second surface $= e - r' = 60° - 30° = 30°$
Therefore, total deviation = 60°.

21. Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in Fig. 8.16.

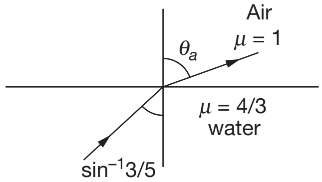


Fig. 18.16

Solution:

Snell's law

$$\mu_W \sin \theta_W = \mu_a \sin \theta_a$$

$$\frac{4}{3} \times \frac{3}{5} = 1 \sin \theta_a$$

$$\sin \theta_a = \frac{4}{5}$$

$$\theta_a = \sin^{-1} \frac{4}{5}.$$

22. Find the speed of light in medium *a* if speed of light in medium *b* is $\frac{c}{3}$, where c = speed of light in vacuum and light refracts from medium *a* to medium *b* making 45° and 60°, respectively, with the normal.

Solution:

Snell's law

$$\mu_a \sin \theta_a = \mu_b \sin \theta_b.$$

$$\frac{c}{v_a} \sin \theta_a = \frac{c}{v_b} \sin \theta_b.$$

$$\frac{c}{v_a} \sin 45° = \frac{c}{c/3} \sin 60°.$$

$\Rightarrow$ $$v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$$

Principle of Reversibility of Light Rays

1. A ray travelling along the path of the reflected ray is reflected along the path of the incident ray.
2. A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus, the incident and refracted rays are mutually reversible.

Refraction through a Parallel Slab

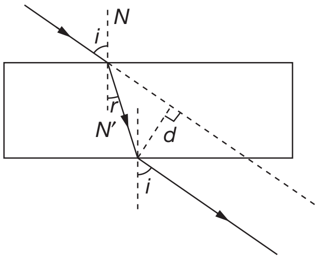
When light passes through a parallel slab, having same medium on both sides, then

1. Emergent ray is parallel to the incident ray.

NOTE

Emergent ray will not be parallel to the incident ray if the medium on both the sides of slab are different.

2. Light is shifted laterally, given by (Students should be able to derive it.)



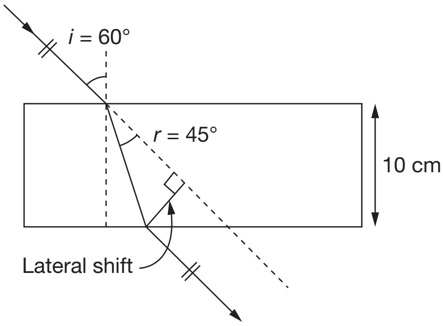
$$d = \frac{t\ \sin(i - r)}{\cos r}$$

t = thickness of slab

SOLVED EXAMPLE

23. Find the lateral shift of light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.

Solution:



$$d = \frac{t \sin\ (i - r)}{\cos r}$$

$$= \frac{10 \sin\ (60° - 45°)}{\cos 45°}$$

$$= \frac{10 \sin 15°}{\cos 45°}$$

$$= 10\ \sqrt{5}\ \sin 15°.$$

Apparent Depth and Shift of Submerged Object

At near normal incidence (small angle of incidence i), apparent depth (d') is given by:

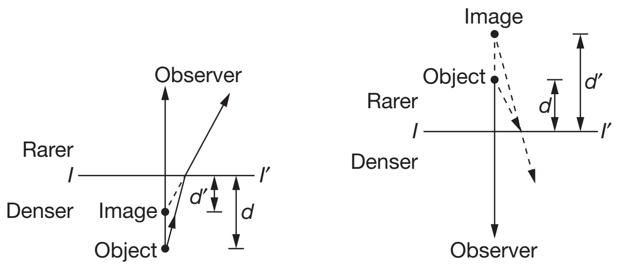
$$d' = \frac{d}{n_{\text{relative}}}$$

and $$v' = \frac{v}{n_{\text{relative}}}$$

where $$n_{\text{relative}} = \frac{n_i \text{ (R.I. of medium of incidence)}}{n_r \text{ (R.I. of medium of refraction)}}$$

d = distance of object from the interface = real depth
d' = distance of image from the interface = apparent depth
v = velocity of object perpendicular to interface relative to surface
v' = velocity of image perpendicular to interface relative to surface

This formula can be easily derived using Snell's law and applying the condition of nearly normal incidence.... (try it or see in text book).



$$\text{Apparent shift} = d\left(1 - \frac{1}{n_{\text{relative}}}\right)$$

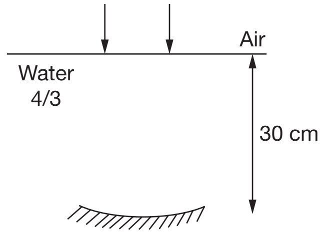
SOLVED EXAMPLES

24. An object lies 100 cm inside water. It is viewed from air nearly normally. Find the apparent depth of the object.

Solution:

$$d' = \frac{d}{n_{\text{relative}}} = \frac{100}{\frac{4/3}{1}} = 75 \text{ cm}$$

25. A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Solution:

The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞. Its image A (in Fig. 18.17) will be formed at focus which is 20 cm from the mirror.

Now for the interface between water and air, $d = 10$ cm.

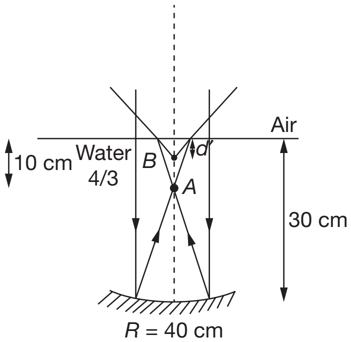


Fig. 18.17

$$\therefore \quad d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5 \text{ cm.}$$

26. See the Fig. 18.18

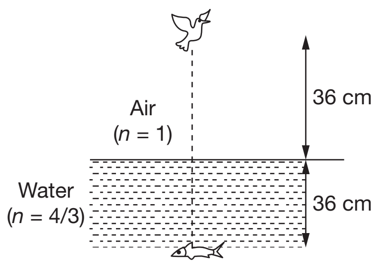


Fig. 18.18

(A) Find apparent height of the bird
(B) Find apparent depth of fish
(C) At what distance will the bird appear to the fish.
(D) At what distance will the fish appear to the bird
(E) If the velocity of bird is 12 cm/s downward and the fish is 12 cm/s in upward direction, then find out their relative velocities with respect to each other.

Solution:

(A) $d'_B = \dfrac{36}{\dfrac{1}{\left(\frac{4}{3}\right)}} = \dfrac{36}{3/4} = 48$ cm

(B) $d'_F = \dfrac{36}{4/3} = 27$ cm

(C) For fish: $d_B = 36 + 48 = 84$ cm
$d_B = 36 + 48 = 84$ cm

(D) For bird: $d_F = 27 + 36 = 63$ cm
$d_F = 27 + 36 = 63$ cm.

(E) Velocity of fish with respect to bird

$$= 12 + \left(\frac{12}{4/3 \diagup 1/1}\right) = 21 \text{ cm/s}$$

Velocity of bird with respect to fish

$$= 12 + \left(\frac{12}{3/4 \diagup 1/1}\right) = 28 \text{ cm/s}$$

27. See the Fig 18.19. Find the distance of final image formed by mirror

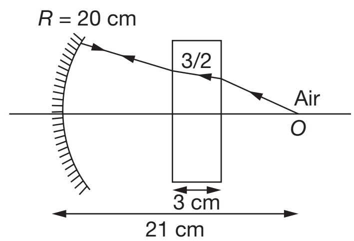


Fig. 18.19

Solution:

$$\text{Shift} = 3\left(1 - \frac{1}{3/2}\right) = 3\left(1 - \frac{1}{3/2}\right)$$

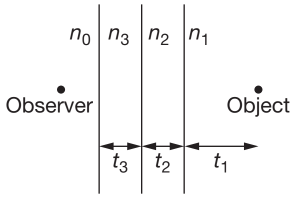
For mirror object is at a distance $= 21 - 3\left(1 - \dfrac{1}{3/2}\right)$
$= 20$ cm

∴ object is at the centre of curvature of mirror. Hence, the light rays will retrace and image will formed on the object itself.

Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of $RI\ n_0$)

Apparent Depth (Distance of Final Image from Final Surface)

$$= \frac{t_1}{n_{1\text{ relative}}} + \frac{t_2}{n_{2\text{ relative}}} + \frac{t_3}{n_{3\text{ relative}}} + \ldots + \frac{t_n}{n_{n\text{ relative}}}$$



Apparent Shift

$$= t_1\left[1 - \frac{1}{n_{1\text{ relative}}}\right] + t_2\left[1 - \frac{1}{n_{2\text{ relative}}}\right] + \ldots + \left[1 - \frac{n}{n_{n\text{ relative}}}\right]t_n$$

Where t represents thickness and n represents the *RI* of the respective media, relative to the medium of observer. (i.e., $n_{1\text{relative}} = n_1/n_0$, $n_{2\text{ relative}} = n_2/n_0$, etc.)

SOLVED EXAMPLE

28. See Fig. 18.20. Find the apparent depth of object seen below surface AB.

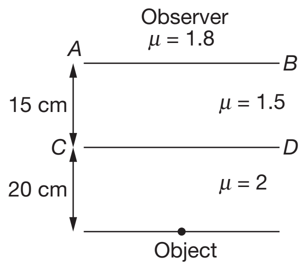


Fig. 18.20

Solution:

$$D_{\text{app}} = \sum \frac{d}{\mu} = \frac{20}{\left(\frac{2}{1.8}\right)} + \frac{15}{\left(\frac{1.5}{1.8}\right)} = 18 + 18 = 36 \text{ cm.}$$

Critical Angle and Total Internal Reflection (TIR)

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90°. When angle in denser medium is more, then critical angle the light ray reflects back in denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror.

In the Fig. 18.21

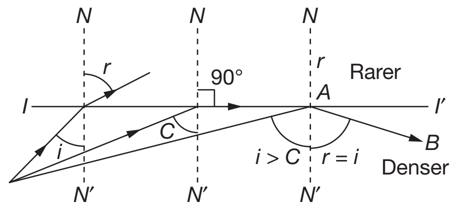


Fig. 18.21

O = Object
NN' = normal to the interface
II' = interface
C = critical angle
AB = reflected ray due to TIR
When $i = C$ then $r = 90^\circ$

$$\therefore \qquad C = \sin^{-1}\frac{n_r}{n_d}$$

Conditions of TIR

1. Light is incident on the interface from denser medium.
2. Angle of incidence should be greater than the critical angle ($i > c$). Fig. 18.22 shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r and d stand for rarer and denser medium, respectively.

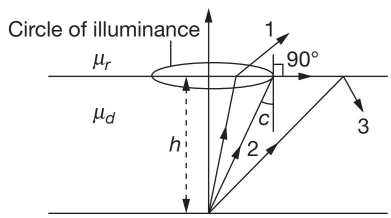


Fig. 18.22

In the Fig. 18.22, ray 1 strikes the surface at an angle less than critical angle C and gets refracted in rarer medium. Ray 2 strikes the surface at a critical angle and grazes the interface. Ray 3 strikes the surface making an angle more than critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called circle of illuminance. All light rays striking inside the circle of illuminance get refracted in rarer medium. If an observer is in rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in rarer medium and then the object cannot be seen from the rarer medium. Radius of COI can be easily found.

SOLVED EXAMPLES

29. Find the max. angle that can be made in glass medium ($\mu = 1.5$) if a light ray is refracted from glass to vacuum.

Solution:

$$1.5 \sin C = 1 \sin 90°,$$

where C = critical angle.

$$\sin C = 2/3$$

$$C = \sin^{-1} 2/3.$$

30. Find the angle of refraction in a medium ($m = 2$) if light is incident in vacuum, making angle equal to twice the critical angle.

Solution:

Since the incident light is in rarer medium. Total internal reflection cannot take place.

$$C = \sin^{-1}\frac{1}{\mu} = 30°$$

$\therefore$ $$i = 2C = 60°$$

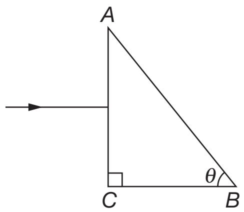
Applying Snell's law,

$$1 \sin 60° = 2 \sin r$$

$$\sin r = \frac{\sqrt{3}}{4}$$

$\Rightarrow$ $$r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right).$$

31. What should be the value of angle θ so that light entering normally through the surface AC of a prism ($n = 3/2$) does not cross the second refracting surface AB.



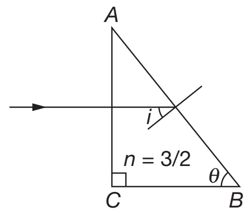
Solution:

Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.

$$i = 90° - \theta$$

For the required condition

$$90° - \theta > C$$



or $$\sin(90° - \theta) > \sin C$$

or $$\cos\theta > \sin C = \frac{1}{3/2} = \frac{2}{3}$$

or $$\theta < \cos^{-1}\frac{2}{3}.$$

32. What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?

Solution:

It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle (A)

Angle r' is minimum when r is maximum, i.e., C (why?). Therefore, the minimum value of r' is $90° - C$.

From condition (A):

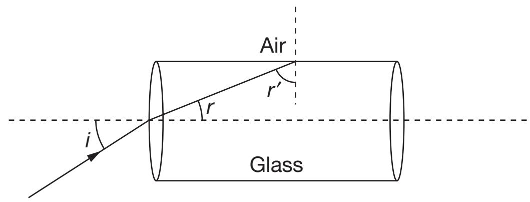
$$90° - C > C$$

or $$C < 45°$$

$$\sin C < \sin 45°;$$

$$\frac{1}{n} < \frac{1}{\sqrt{2}}$$

or $$n > \sqrt{2}.$$

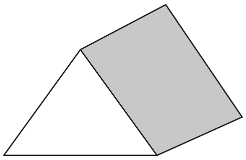


Characteristics of a Prism

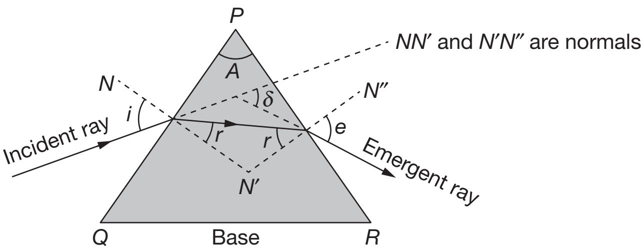
1. A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a prism

3D view

Refraction through a prism



View from one side



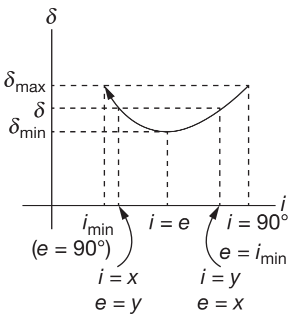
2. *PQ* and *PR* are refracting surfaces.
3. $\angle QPR = A$ is called refracting angle or the angle of prism (also called apex angle).
4. δ = angle of deviation
5. For refraction of a monochromatic (single wave length) ray of light through a prism

$$\delta = (i + e) - (r_1 + r_2)$$

and $\quad r_1 + r_2 = A$

$\therefore \quad \delta = i + e - A.$

6. Variation of δ versus *i* (shown in diagram).



For one δ (except δ min) there are two values of angle of incidence. If *i* and *e* are interchanged, then we get the same value of δ because of reversibility principle of light.

NOTE

- For application of above result, medium on both sides of prism must be same.
- Based on above graph, we can also derive following result, which says that *i* and *e* can be interchanged for a particular deviation, in other words, there are two angles of incidence for a given deviation (except minimum deviation).

i	r_1	r_2	e	δ
θ_1	θ_2	θ_4	θ_4	θ_5
θ_4	θ_3	θ_2	θ_1	θ_5

7. There is one and only one angle of incidence for which the angle of deviation is minimum.
8. When $\delta = \delta_{min}$, the angle of minimum deviation, then $i = e$ and $r_1 = r_2$, the ray passes symmetrically with respect to the refracting surfaces. We can show by simple calculation that

$$\delta_{min} = 2i_{min} - A$$

where i_{min} = angle of incidence for minimum deviation, and $r = A/2$.

$$\therefore \qquad n_{rel} = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]},$$

where $\quad n_{rel} = \dfrac{n_{prism}}{n_{surroundings}}$

Also

$$\delta_{min} = (n - 1)A \quad \text{(for small values of } \angle A)$$

9. For a thin prism ($A \le 10°$) and for small value of *i*, all values of

$$\delta = (n_{rel} - 1)A,$$

where $\quad n_{rel} = \dfrac{n_{prism}}{n_{surrounding}}$

SOLVED EXAMPLES

33. Refracting angle of a prism $A = 60°$ and its refractive index is, $n = 3/2$, what is the angle of incidence *i* to get minimum deviation. Also find the minimum deviation. Assume the surrounding medium to be air ($n = 1$).

Solution:

For minimum deviation,

$$r_1 = r_2 = \frac{A}{2} = 30°.$$

applying Snell's law at *I* surface

$$1 \times \sin i = \frac{3}{2}\sin 30°$$

$$\Rightarrow \quad i = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \delta_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}.$$

34. See the Fig. 18.23, Find the deviation caused by a prism having refracting angle 4° and refractive index $\frac{3}{2}$.

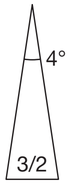


Fig. 18.23

Solution:

$$\delta = \left(\frac{3}{2} - 1\right) \times 4° = 2°.$$

35. For a prism, $A = 60°$, $n = \sqrt{\frac{7}{3}}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also find δ_{max}.

Solution:

In minimum incidence case, the angles will be as shown in Fig. 18.24.

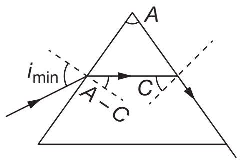


Fig. 18.24

Applying Snell's law:

$$1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin (A - C)$$

$$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C)$$

$$= \sqrt{\frac{7}{3}} \left(\sin 60 \sqrt{1 - \frac{3}{7}} - \cos 60 \sqrt{\frac{3}{7}}\right) = \frac{1}{2}$$

$$\therefore \quad i_{min} = 30°$$

$$\therefore \quad \delta_{max} = i_{min} + 90° - \text{A}$$

$$= \quad 30° + 90° - 60° = 60°.$$

36. Show that if $A > A_{max}$ (=2 C), then total internal reflection occurs at second refracting surface PR for any value of i.

Solution:

For TIR at second surface,

$$r' > C$$

$$\Rightarrow \quad (A - r) > C$$

$$\text{or} \quad A > (C + r)$$

The above relation will be fulfilled if

$$\text{or} \quad A > C + r_{max}$$

$$\text{or} \quad A > C + C$$

$$\text{or} \quad A > 2C$$

1. On the basis of above example and similar reasoning, it can be shown that (you should try the following cases (ii) and (iii) yourself.)
   (a) If $A > 2C$, all rays are reflected back from the second surface.
   (b) If $A \leq C$, no rays are reflected back from the second surface, i.e., all rays are refracted from second surface.
   (c) If $2C \geq A > C$, some rays are reflected back from the second surface and some rays are refracted from second surface, depending on the angle of incidence.
2. δ is maximum for two values of i
   $\Rightarrow$ i_{min} (corresponding to $e = 90°$) and $i = 90°$ (corresponding to e_{min}).
   For i_{min}: $n_s \sin i_{min} = n_p \sin(A - C)$
   If $i < i_{min}$, then T.I.R takes place at second refracting surface PR.

Dispersion

You see a rainbow when you are facing falling rain with the sun behind you. The arc is formed by rays of sunlight that change direction three times: the rays refract, or bend, when they enter a raindrop; then reflect off the back of the raindrop and finally refract again as they exit the raindrop. You see colours because different wavelengths refract through different angles, a phenomenon called dispersion.

Dispersion of Light

The angular splitting of a ray of white light into a number of components and spreading in different directions is called *dispersion of light.* [It is for whole electromagnetic wave in totality.] This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula $n(\lambda) = a + \dfrac{b}{\lambda^2}$, where a and b are positive constants of a medium.

NOTE

- Such phenomenon is not exhibited by sound waves.

  Angle between the rays of the extreme colours in the refracted (dispersed) light is called angle of dispersion.

$$\theta = \delta_v - \delta_r \qquad \text{(Fig. 18.25 (a))}$$

Fig. 18.25(a) and (c) represents dispersion, whereas in Fig. 18.25(b), there is no dispersion.

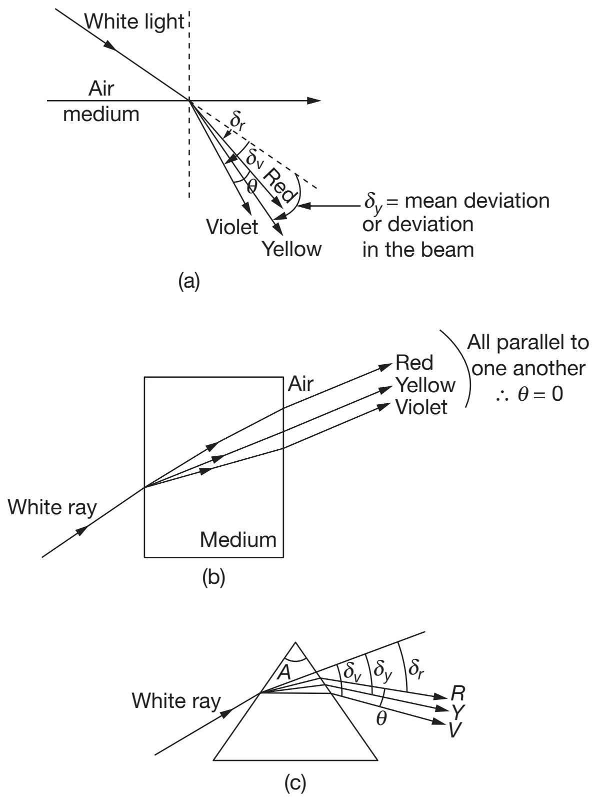


Fig. 18.25

For prism of small A with small i:

$$\theta = \delta_v - \delta_r = (n_v - n_r)A$$

SOLVED EXAMPLES

37. The refractive indices of flint glass for red and violet light are 1.613 and 1.632, respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

Solution:

Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

$$\text{The dispersion} = \delta_v - \delta_r = (\mu_v - \mu_r)A$$

$$= (1.632 - 1.613) \times 5° = 0.095°.$$

Deviation of beam (also called mean deviation)

$$\delta = \delta_y = (n_y - 1)A$$

n_v, n_r, and n_y are *RI* of material for violet, red, and yellow colours, respectively.

NOTE

- Numerical data reveals that if the average value of μ is small, $\mu_v - \mu_r$ is also small and if the average value of μ is large, $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

  Dispersive power (ω) of the medium of the material of prism is given by:

$$\omega = \frac{n_v - n_r}{n_y - 1}$$

- ω is the property of a medium.

  For small angled prism ($A \le 10°$) with light incident at small angle i:

$$\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y} = \frac{\text{Angular dispersion}}{\text{Deviation of mean ray (yellow)}}$$

  [$n_y = \dfrac{n_v + n_r}{2}$ if n_y is not given in the problem.]

- $n - 1$ = refractivity of the medium for the corresponding colour.

38. Refractive index of glass for red and violet colours are 1.50 and 1.60, respectively. Find

(A) The refractive index for yellow colour approximately

(B) Dispersive power of the medium

Solution:

(A) $\mu_r \simeq \dfrac{\mu_v + \mu_R}{2} = \dfrac{1.50 + 1.60}{2} = 1.55$

(B) $\omega = \dfrac{\mu_v - \mu_R}{\mu_r - 1} = \dfrac{1.60 - 1.50}{1.55 - 1} = 0.18.$

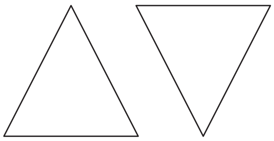
Dispersion without Deviation (Direct Vision Combination)

The condition for direct vision combination is:

$$\left[n_y - 1\right] \ A = \left[n_y - 1\right] \ A'$$

$$\Leftrightarrow \qquad \left[\frac{n_v + n_r}{2} - 1\right] A = \left[\frac{n'_v + n'_r}{2} - 1\right] A'$$

Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.



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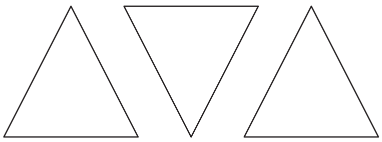

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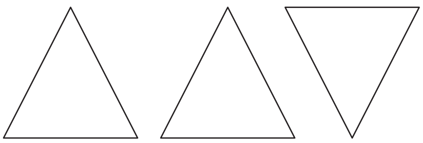
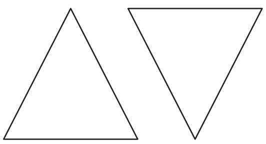


Deviation without Dispersion (Achromatic Combination)

Condition for achromatic combination is: $(n_v - n_r)\ A = (n'_v - n'_r)\ A'$



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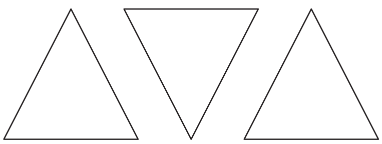

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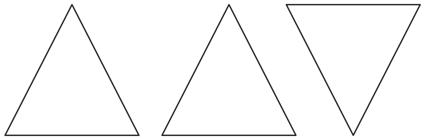


SOLVED EXAMPLES

39. If two prisms are combined, as shown in Fig. 18.26, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.

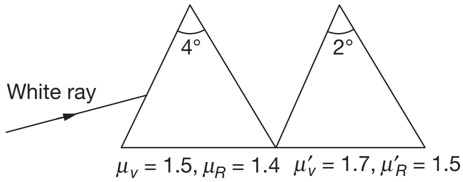


Fig. 18.26

Solution:

Both prisms will turn the light rays towards their bases and also in same direction. Therefore, turnings caused by both prisms are additive.

Total angular dispersion

$$= \theta + \theta' = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)\ A'$$

$$= (1.5 - 1.4)\ 4° + (1.7 - 1.5)2° = 0.8°$$

Total deviation

$$= \delta + \delta'$$

$$= \left(\frac{\mu_V + \mu_R}{2} - 1\right) A + \left(\frac{\mu'_V + \mu'_R}{2} - 1\right) A'$$

$$= \left(\frac{1.5 + 1.4}{2} - 1\right) 0.4° + \left(\frac{1.7 + 1.5}{2} - 1\right) 0.2°$$

$$= (1.45 - 1)\ 0.4° + (1.6 - 1)\ 0.2°$$

$$= 0.45 \times 0.4° + 0.6 \times 0.2°$$

$$= 1.80 + 1.2 = 3.0°$$

40. Two thin prisms are combined to form an achromatic combination. For I prism, A = 4°, μ_R = 1.35, μ_Y = 1.40, μ_v = 1.42. For II prism, μ'_R= 1.7, μ'_Y = 1.8 and μ'_R = 1.9, find the prism angle of II prism and the net mean deviation.

Solution:

Condition for achromatic combination.

$$\theta = \theta'$$

$$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$$

$$\therefore \quad A' = \frac{(1.42 - 1.35)4°}{1.9 - 1.7} = 1.4°$$

$$\delta_{Net} = \delta \sim \delta' = (\mu_Y - 1)A \sim (\mu'_Y - 1)A'$$

$$= (1.40 - 1)\ 4° \sim (1.8 - 1)\ 1.4°$$

$$= 0.48°.$$

41. A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find

(A) the angle of the flint glass prism needed and

(B) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow, and violet light are 1.5, 1.6, and 1.7, respectively, for crown glass and 1.8, 2.0, and 2.2 for flint glass.

Solution:

The deviation produced by the crown prism is

$$\delta = (\mu - 1)A$$

and by the flint prism is

$$\delta' = (\mu' - 1)A'.$$

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'. \quad (1)$$

(A) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'.$$

or, $$A' = \frac{(\mu - 1)}{(\mu' - 1)}A = \frac{1.6 - 1}{2.0 - 1} \times 5^0 = 3°$$

(B) The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is,

$$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$$

The net angular dispersion is,

$$(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A$$

$$= (1.7 - 1.5) \times 5° - (2.2 - 1.8) \times 3°$$

$$= -0.2°.$$

The angular dispersion has magnitude 0.2°.

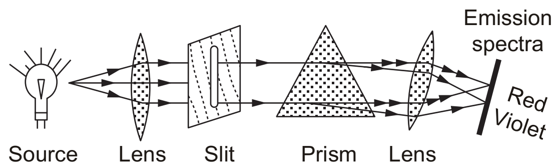
Spectrum (Only for your knowledge and not of much use for JEE)

Types of Spectra

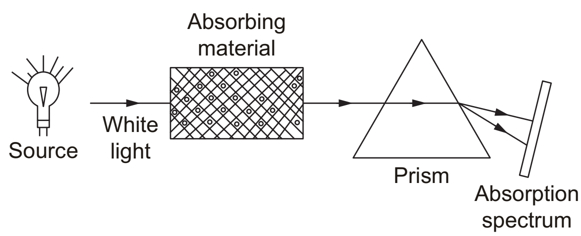
In general, a spectrum is defined as orderly arrangement of something. The something may be momentum, energy, wavelength, mass, etc., and the corresponding spectra are known as momentum spectrum, energy spectrum, wavelength spectrum, mass spectrum, etc. Here we are concerned with the light spectrum. When a beam of composite light, say white light is passed through a prism, the light splits into its component colours. The regular arrangement of colours is known as spectrum. The spectra may be divided into two principal classes.

1. Emission spectra and
2. Absorption spectra.

The emission spectra are obtained when light coming directly from the source is examined with a spectroscope while the absorption spectra are obtained when the light from the source which gives continuous spectrum is passed through an absorbing material, and the transmitted light is examined with a spectroscope.



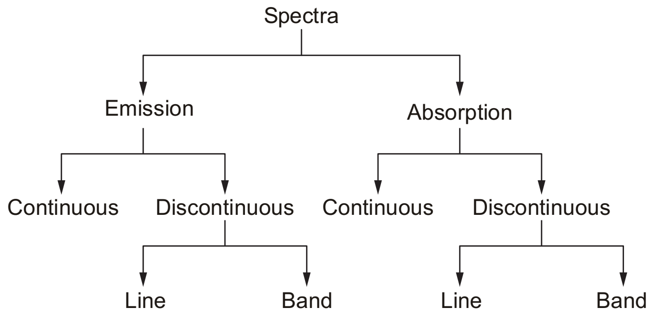
In absorption spectrum, the absorbing material absorbs all those wavelengths which it emits in emission spectra. The dark lines in the absorption spectra correspond to the lines which absorbing medium would have emitted on excitation. The question is that what is the utility of considering the absorption spectra? The answer is that the substances which cannot be conveniently excited to give emission spectra are easily studied with the help of absorption spectra.



Both, the emission and absorption spectra are subdivided into three classes:

1. Continuous spectra
2. Band spectra
3. Line spectra

The classification is shown below



We shall consider these spectra one by one.

Emission Spectra

1. **Continuous Emission Spectrum:** A continuous emission spectrum is one which contains all the wavelengths from one end to the other of the spectrum. Spectrum of this type appears to be unbroken and luminous. In visible region, it appears in the form of multicoloured strips in which no line of demarcation can be drawn between two colours. In the spectrum of this type, the intensity depends upon temperature and the body itself. This spectrum is obtained when a matter in bulk is heated. When light from a hot solid such as tungsten filament of an electric lamp, hot iron, charcoal, etc., is sent through prism, spectrum of this type is produced. Since the continuous spectra produced by different substances have all possible wavelengths, such spectra are not the characteristic of a substance.
2. **Band Emission Spectrum:** If the emission spectrum has coloured bands separated by dark spaces on the two sides, the spectrum is known as band emission spectrum. These bands are sharply defined at one end and shade off gradually at the other end. The sharply defined edge is called the head of the band and the other tail. This spectrum is produced when matter in molecular state is excited. Due to this fact, it is also known as molecular spectra. Band spectra are different for different types of molecules.
3. **Line Emission Spectrum:** When the emission spectrum contains a series of fine sharp lines having dark spaces in between, it is called line emission spectrum. In visible region, these lines are coloured. Such a spectrum is produced when the matter in atomic state is excited; therefore, it is also called as atomic spectra. It is different for different types of atoms.

   The continuous, band and line spectra are shown in Fig. 18.27.

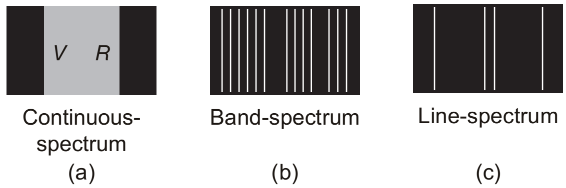


Fig. 18.27

Absorption Spectra

1. **Continuous Absorption Spectrum:** Continuous absorption spectrum is produced when the absorbing material absorbs a continuous range of wavelengths without any gap. For example, when light from a filament of electric lamp is passed through a violet piece of glass, it absorbs all visible light except violet, and the spectrum thus obtained will be continuous absorption type. Continuous absorption spectrum is a characteristic of the absorbing medium.
2. **Band Absorption Spectrum:** Band absorption spectrum is produced when light from a source of continuous wavelength passes through a medium in molecular state. At certain places, the dark bands appear with coloured strips on their two sides. For example, when the absorbing material is an aqueous solution of $KMnO_4$, it gives five bands depending upon absorbing medium while the widths of the dark bands depend upon the thickness of absorbing material.
3. **Line Absorption Spectrum:** Line absorption spectrum is produced when light passes through the absorbing medium in atomic state. The characteristic dark line appears in absorption at the places where the coloured lines would have been present when the absorbent is excited in atomic state, for example, when light is passed through sodium vapour in atomic state, two dark lines appear exactly at the places of D_1 and D_2 lines in emission spectra. Similarly, solar spectrum is an example of line absorption spectrum which consists of Fraunhofer absorption lines which correspond to vapours of different elements present in solar atmosphere. The band and line absorption spectra are shown in the Fig. 18.28.

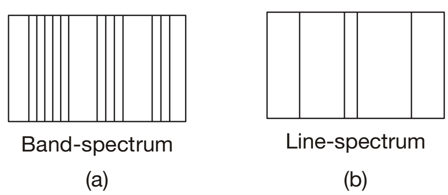


Fig. 18.28

Few Definitions

1. **Visible and Invisible Spectrum:** The spectrum beyond violet end of visible region known as ultraviolet spectrum. It ranges from 4000 Å to 100 Å. The spectrum beyond red end of visible region is known as infra-red spectrum. It range from 8000 Å to one mm. Ultraviolet and infrared constitute the invisible spectrum.
2. **Fraunhofer Lines:** When sun's light is allowed to fall on the slit of a spectrometer, its spectrum is found to consist of many dark lines. These lines are called Fraunhofer lines. The number of these lines is approximately 700. These lines are due to the absorption of certain wavelengths in sunlight by the gases in the sun's outer sphere. It is an example of line absorption spectrum.
3. **Fluorescence:** The phenomenon of absorption of light of one wavelength by a substance and then re-emission of light of greater wavelength is known as fluorescence. Such substances are fluorspar (calcium fluoride), paraffin oil, Quinine sulphate, uranium oxide, etc.
4. **Phosphorescence:** The phenomenon of re-emission of visible light, even after the incident light is cut off, is known as phosphorescence. Examples are cadmium sulphide, barium sulphide, strontium sulphide, zinc sulphide, etc.

REFRACTION AT SPHERICAL SURFACES

For paraxial rays incident on a spherical surface separating two media

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}. \quad (1)$$

where light moves from the medium of refractive index n_1 to the medium of refractive index n_2.

Transverse magnification (m) (of dimension perpendicular to principal axis) due to refraction at spherical surface is given by $m = \frac{v-R}{u-R} = \left(\frac{v/n_2}{u/n_1}\right)$

SOLVED EXAMPLE

42. Find the position, size, and nature of image, for the situation shown in Fig. 18.29. Draw ray diagram.

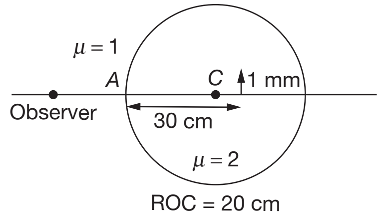
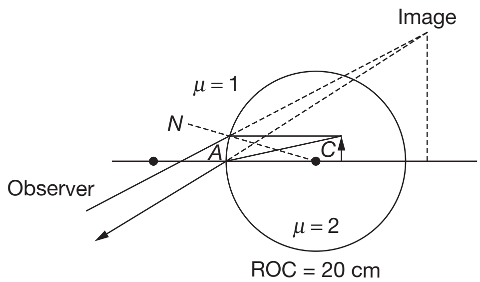


Fig. 18.29

Solution:



For refraction near point A,

$$u = -30;\ R = -20;\ n_1 = 2;\ n_2 = 1.$$

Applying refraction formula,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{v} - \frac{2}{-30} = \frac{1-2}{-20}$$

$$v = -60 \text{ cm}$$

$$m = \frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} = \frac{2(-60)}{1(-30)} = 4$$

$\therefore$ $h_2 = 4$ mm.

Special Case

Refraction at Plane Surfaces

Putting $R = \infty$ in the formula $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, we get

$$v = \frac{n_2 u}{n_1}$$

The same sign of v and u implies that the object and the image are always on the same side of the interface separating the two media. If we write the above formula as

$$v = \frac{u}{n_{rel}},$$

it gives the relation between the apparent depth and real depth, as we have seen before.

SOLVED EXAMPLE

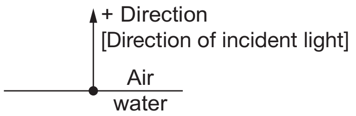
43. Using formula of spherical surface or otherwise, find the apparent depth of an object placed 10 cm below the water surface, if seen near normally from air.

Solution:

Put $R = \infty$ in the formula of the refraction at spherical surfaces to get,

$$v = \frac{un_2}{n_1}$$

$$u = -10 \text{ cm}$$



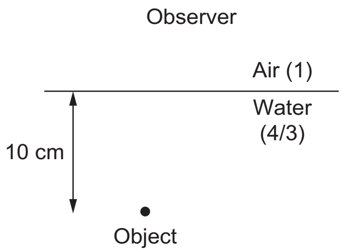
$$n_1 = \frac{4}{3}$$

$$n_2 = 1$$

$$v = -\frac{10 \times 1}{4/3} = -7.5 \text{ cm}$$

negative sign implies that the image is formed in water.

Aliter:



$$d_{app} = \frac{d_{real}}{\mu_{rel}}$$

$$= \frac{10}{4/3} = \frac{30}{4} = 7.5 \text{ cm.}$$

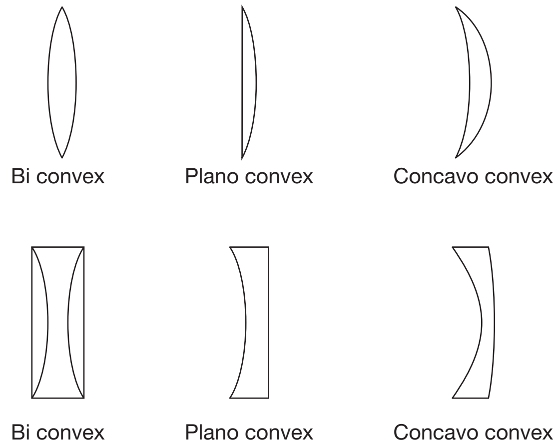
Thin Lens

A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends.

One surface of a convex lens is always convex. Depending on the other surface, a convex lens is categorized as

1. Biconvex or convexo convex, if the other surface is also convex,
2. Plano convex if the other surface is plane and
3. Concavo convex if the other surface is concave.

Similarly, concave lens is categorized as concavo-concave or biconcave, plano-concave and convexo-concave.



For a spherical, thin lens having the same medium on both sides

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (1),$$

where $n_{rel} = \frac{n_{lens}}{n_{medium}}$ and R_1 and R_2 are x coordinates of the centre of curvature of the 1st surface and 2nd surface, respectively.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \rightarrow \text{Lens Maker's Formula} \quad (2)$$

Lens has Two Focii

If $\quad u = \infty,$

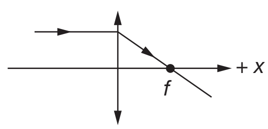
then $\quad \frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$

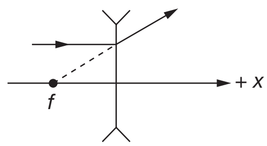
$\Rightarrow \quad v = f$

If incident rays are parallel to principal axis, then its refracted ray will cut the principal axis at 'f'.

It is called 2nd focus.

In case of converging lens, it is positive and in case of diverging lens, it is negative.



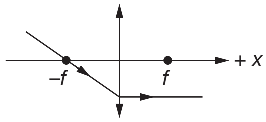


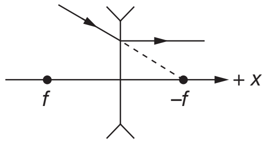
If $v = \infty$ that means

$$\frac{1}{\infty} - \frac{1}{u} = \frac{1}{f}$$

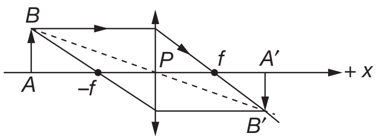
$$\Rightarrow \qquad u = -f$$

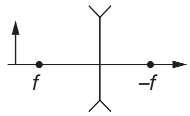
If incident rays cuts principal axis at $-f$, then its refracted ray will become parallel to the principal axis. It is called 1st focus. In case of converging lens, it is negative ($\because f$ is positive) and in the case of diverging lens, it is positive ($\because f$ is negative) use of $-f$ and $+f$ is in drawing the ray diagrams.





Notice that the point B, its image B', and the pole P of the lens are collinear. It is due to parallel slab nature of the lens at the middle. This ray goes straight. (Remember this.)





From the relation $\frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ it can be seen that the second focal length depends on two factors.

1. The factor $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ is
   (a) Positive for all types of convex lenses
   (b) Negative for all types of concave lenses
2. The factor $(n_{\text{rel}} - 1)$ is
   (a) Positive when surrounding medium is rarer than the medium of lens.
   (b) Negative when surrounding medium is denser than the medium of lens.
3. So a lens is converging if f is positive which happens when both the factors (A) and (B) are of same sign.
4. And a lens is diverging if f is negative which happens when the factors (A) and (B) are of opposite signs.
5. Focal length of the lens depends on medium of lens as well as surrounding.
6. It also depends on wavelength of incident light. Incapability of lens to focus light rays of various wavelengths at single point is known as chromatic aberration.

SOLVED EXAMPLES

44. Find the behaviour of a concave lens placed in a rarer medium.

Solution:

Factor (A) is negative, because the lens is concave.

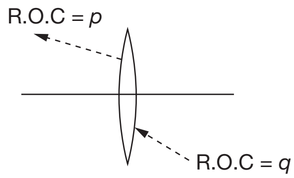
Factor (B) is positive, because the lens is placed in a rarer medium.

Therefore, the focal length of the lens, which depends on the product of these factors, is negative and hence the lens will behave as diverging lens.

45. Show that the factor $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (and therefore focal length) does not depend on which surface of the lens light strike first.

Solution:

Consider a convex lens of radii of curvature p and q as shown.



Case I: Suppose light is incident from left side and strikes the surface with radius of curvature p, first.

Then $R_1 = +p$; $R_2 = -q$ and $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1}{p} - \frac{1}{-q}\right)$

Case II: Suppose light is incident from right side and strikes the surface with radius of curvature q, first.

Then $R_1 = +q$; $R_2 = -p$ and $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1}{q} - \frac{1}{-p}\right)$

Though we have shown the result for biconvex lens, it is true for every lens.

SOLVED EXAMPLES

46. Find the focal length of the lens shown in the Fig. 18.30.

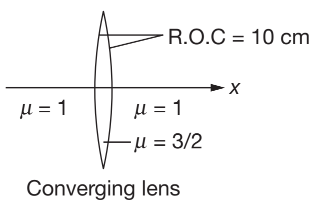


Fig. 18.30

Solution:

$$\because \quad \frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (3/2 - 1)\left(\frac{1}{10} - \frac{1}{(-10)}\right)$$

$$\frac{1}{f} = \frac{1}{2} \times \frac{2}{10}$$

$$\Rightarrow \quad f = +10 \text{ cm.}$$

47. Find the focal length of the lens shown in Fig. 18.31

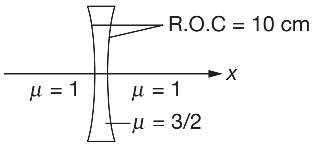


Fig. 18.31

Solution:

$$\frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{10}\right)$$

$f = -10$ cm.

48. Find the focal length of the lens shown in Fig. 18.32

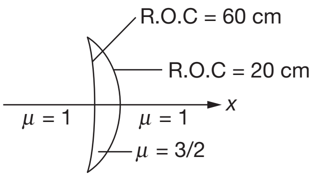


Fig. 18.32

(A) If the light is incident from left side.
(B) If the light is incident from right side.

Solution:

(A) $$\frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-60} - \frac{1}{-20}\right)$$

$f = 60$ cm

(B) $$\frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{20} - \frac{1}{60}\right)$$

$f = 60$ cm.

49. Point object is placed on the principal axis of a thin lens with parallel curved boundaries, i.e., having same radii of curvature. Discuss about the position of the image formed.

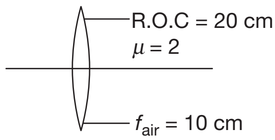
Solution:

$$\frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0 \qquad [\because R_1 = R_2]$$

$$\frac{1}{v} - \frac{1}{u} = 0 \quad \text{or} \quad v = u,$$

i.e., rays pass without appreciable bending.

50. Focal length of a thin lens in air is 10 cm. Now medium on one side of the lens is replaced by a medium of refractive index $\mu = 2$. The radius of curvature of surface of lens, in contact with the medium, is 20 cm. Find the new focal length.



Solution:

Let radius of I surface be R_1 and refractive index of lens be μ. Let parallel rays be incident on the lens. Applying refraction formula at first surface,

$$\frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \tag{1}$$

At II$^{\text{nd}}$ surface,

$$\frac{2}{V} - \frac{\mu}{V_1} = \frac{2 - \mu}{-20} \tag{2}$$

Adding (1) and (2),

$$\frac{\mu}{V_1} - \frac{1}{\infty} + \frac{2}{V} - \frac{\mu}{V_1} = \frac{\mu - 1}{R_1} + \frac{2 - \mu}{-20}$$

$$= (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{\mu - 1}{20} - \frac{2 - \mu}{20}$$

$$= \frac{1}{f}\text{(in air)} + \frac{1}{20} - \frac{2}{20}$$

$$\Rightarrow \quad v = 40 \text{ cm}$$

$$\Rightarrow \quad f = 40 \text{ cm.}$$

51. Figure 18.33 shown a point object and a converging lens. Find the final image formed.

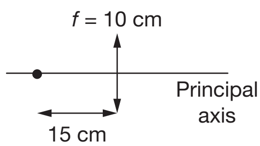
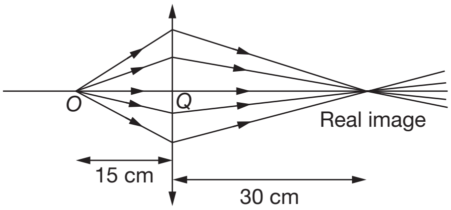


Fig. 18.33

Solution:



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$$

$$v = +30 \text{ cm.}$$

52. See the Fig. 18.34. Find the position of final image formed.

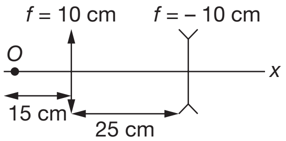


Fig. 18.34

Solution:

For converging lens

$$u = -15 \text{ cm},$$

$$f = 10 \text{ cm}$$

$$v = \frac{fu}{f+u} = 30 \text{ cm}$$

For diverging lens

$$u = 5 \text{ cm}$$

$$f = -10 \text{ cm}$$

$$v = \frac{fu}{f+u} = 10 \text{ cm.}$$

53. Figure 18.35 shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel.

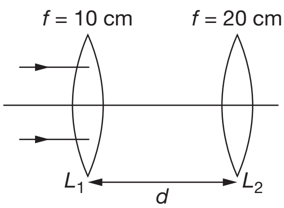


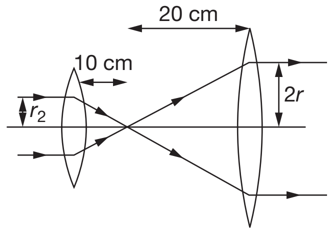
Fig. 18.35

Solution:

Final rays should be parallel. For this the IInd focus of L_1 must coincide with I focus of L_2.

$$d = 10 + 20$$

$$= 30 \text{ cm.}$$



Here the diameter of ray beam becomes wider.

54. See the Fig. 18.36. Find the position of final image formed.

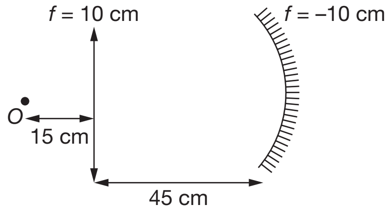


Fig. 18.36

Solution:

For lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

$\Rightarrow \quad v = +30$ cm.

Hence, it is object for mirror

$$u = -15 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10}$$

$\Rightarrow \quad v = -30$ cm.

Now for second time, it again passes through lens

$$u = -15 \text{ cm}$$

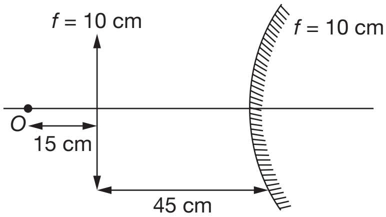
$$v = ?; \quad f = 10 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

$\Rightarrow \quad v = +30.$

Hence, final image will form at a distance 30 cm from the lens towards left.

55. What should be the value of d so that image is formed on the object itself.



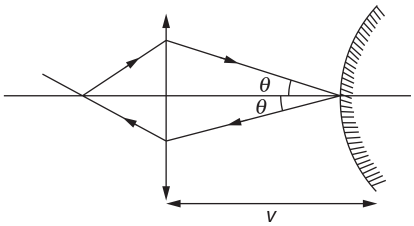
Solution:

For lens:

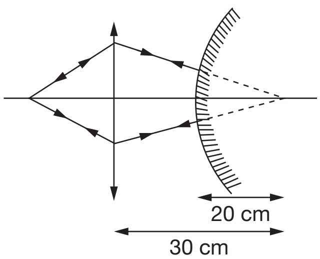
$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

$$v = +30 \text{ cm.}$$

Case I: If $d = 30$, the object for mirror will be at pole and its image will be formed there itself.



Case II: If the rays strike the mirror normally, they will retrace and the image will be formed on the object itself



$\therefore \qquad d = 30 - 20 = 10$ cm.

Transverse Magnification (*m*)

Transverse magnification (m) of (of dimension perpendicular to principal axis) is given by

$$m = \frac{v}{u}$$

If the lens is thick or/and the medium on both sides is different, then we have to apply the formula given for refraction at spherical surfaces step by step.

SOLVED EXAMPLE

56. An extended real object of size 2 cm is placed perpendicular to the principal axis of a converging lens of focal length 20 cm. The distance between the object and the lens is 30 cm.

(A) Find the lateral magnification produced by the lens.
(B) Find the height of the image.
(C) Find the change in lateral magnification, if the object is brought closer to the lens by 1 mm along the principal axis.

Solution:

(A) Using $\qquad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

and $\qquad m = \frac{v}{u}$

we get $\qquad m = \frac{f}{f+u} \qquad (1)$

$\therefore \qquad m = \frac{+20}{+20+(-30)}$

$$= \frac{+20}{-10} = -2$$

–ve sign implies that the image is inverted.

(B) $\frac{h_2}{h_1} = m$

$\therefore \quad h_2 = mh_1 = (-2)(2) = -4$ cm

(C) Differentiating (1) we get

$$dm = \frac{-f}{(f+u)^2}\,du = \frac{-(20)}{(-10)^2}\,(0.1) = \frac{-2}{100}$$

$= -02$

Note that the method of differential is valid only when changes are small.

Aliter: u (after displacing the object)

$$= -(30 + 0.1) = -29.9 \text{ cm}$$

Applying the formula,

$$m = \frac{f}{f+u}$$

$$m = \frac{20}{20+(-29.9)} = -2.02$$

$\therefore \quad$ change in $m = -0.02$.

Since in this method differential is not used, this method can be used for any changes, small or large.

Displacement Method to find Focal Length of Converging Lens

Fix an object of small height H and a screen at a distance D from object (as shown in Fig. 18.37). Move a converging lens from the object towards the screen. Let a sharp image form on the screen when the distance between the object and the lens is a. From lens formula, we have

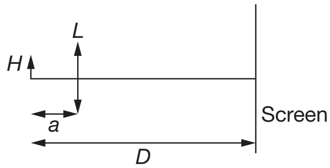


Fig. 18.37

$$\frac{1}{D-a} - \frac{1}{-a} = \frac{1}{f}$$

or $$a^2 - D\,a + f\,D = 0 \qquad (1)$$

This is quadratic equation and hence two values of a are possible. Call them a_1 and a_2. Thus, a, and a_2 are the roots of the equation. From the properties of roots of a quadratic equation,

$\therefore \quad a_1 + a_2 = D$

$\Rightarrow \quad a_1 a_2 = f\,D$

Also $$(a_1 - a_2) = \sqrt{(a_1 + a_2)^2 - 4a_1 a_2}$$

$$= \sqrt{D^2 - 4fD} = d \quad \text{(suppose).}$$

d physically means the separation between the two position of lens.

COMBINATION OF LENSES

The equivalent focal length of thin lenses in contact is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \ldots$$

where f_1, f_2, f_3 are focal lengths of individual lenses.

If two converging lenses are separated by a distance d and the incident light rays are parallel to the common principal axis, then the combination behaves like a single lens of focal length given by the relation

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

and the position of equivalent lens is $\frac{-d\ F}{f_1}$ with respect to IInd lens

SOLVED EXAMPLES

57. Find the lateral magnification produced by the combination of lenses as shown in the Fig. 18.38.

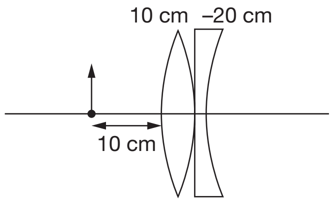


Fig. 18.38

Solution:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$$

$\Rightarrow \quad f = +20$

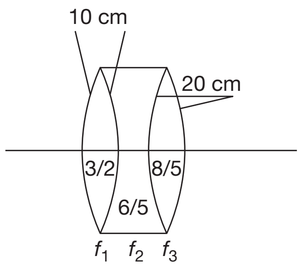
$\therefore \quad \frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$

$\Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{10}$

$$= \frac{-1}{20} - 20 \text{ cm}$$

$\therefore \quad m = \frac{-20}{-10} = 2.$

58. Find the focal length of equivalent system.



Solution:

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right)$$

$$= \frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$$

$$\frac{1}{f_2} = \left(\frac{6}{5} - 1\right)\left(\frac{-1}{10} - \frac{1}{20}\right)$$

$$= \frac{1}{5} \times \left(\frac{-30}{10 \times 20}\right) = \frac{-3}{100}$$

$$\frac{1}{f_3} = \left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{3}{50}$$

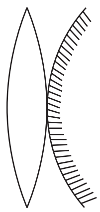
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{10} + \frac{-3}{100} + \frac{3}{50}$$

$$\Rightarrow \quad f = \frac{100}{13}$$

Combination of Lens and Mirror

The combination of lens and mirror behaves like a mirror of focal length f given by



$$\frac{1}{f} = \frac{1}{F_m} - \frac{2}{F_\ell}.$$

If lenses are more then one, f is given by

$$\frac{1}{f} = \frac{1}{F_m} - 2\left(\sum \frac{1}{f_\ell}\right)$$

For the following Fig. 18.39,

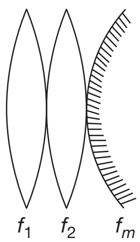


Fig. 18.39

f is given by

$$\frac{1}{f} = \frac{1}{F_m} - 2\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$$

SOLVED EXAMPLES

59. Find the position of final image formed. (The gap shown in Fig. 18.40 is of negligible width.)

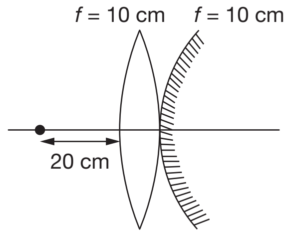


Fig. 18.40

Solution:

$$\frac{1}{f_{eq}} = \frac{1}{10} - \frac{2}{10} = \frac{-1}{10}$$

$$\Rightarrow \quad f_{eq} = -10 \text{ cm}$$

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-10}$$

$$\Rightarrow \quad v = -20 \text{ cm.}$$

Hence, image will be formed on the object itself

60. Two mirrors are inclined by an angle 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using

(A) Direct formula and

(B) Counting the images

Solution:

Figure 18.41 is self-explanatory.

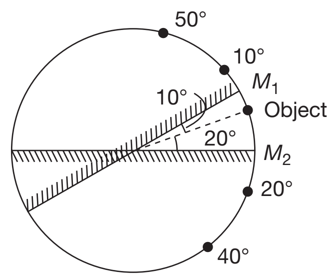


Fig. 18.41

Number of images

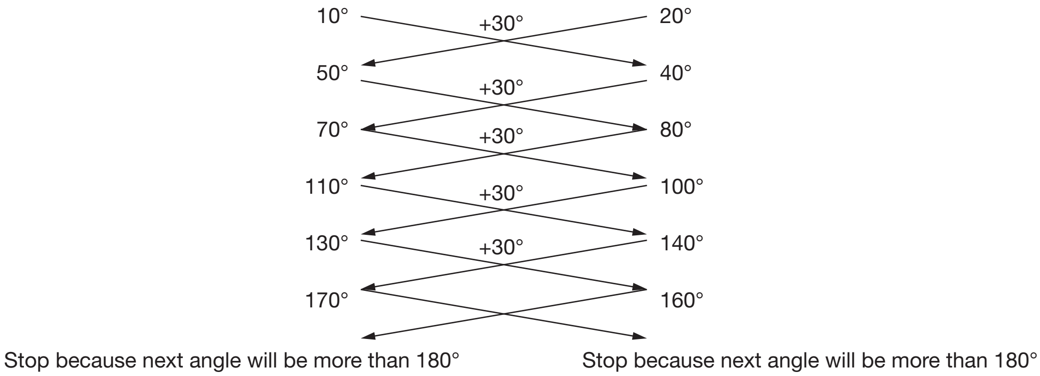
(A) Using direct formula

$$\frac{360°}{30°} = 12 \text{ (even number)}$$

$\therefore$ number of images = 12 – 1 = 11

(B) By counting, see the following table

Image formed by mirror M_1 (angles are measured from the mirror M_1)	Image formed by mirror M_2 (angles are measured from the mirror M_2)

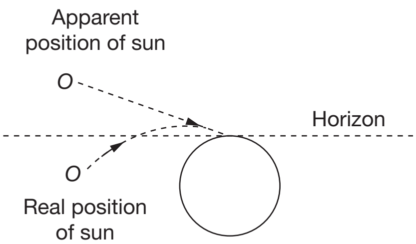


To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images fromed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors = 160° + 170° + 30° = 360°. Therefore in this case the last images coincide. Therfore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 – 1 (as the last images coincide) = 6 + 6 – 1 = 11.

SOME INTERESTING FACTS ABOUT LIGHT

The Sun Rises before it Actually Rises and Sets After it Actually Sets

The atmosphere is less and less dense as its height increase, and it is also known that the index of refraction decrease with a decrease in density. So, there is a decrease of the index of refraction with height. As a result, the light rays bend as they move in the earth's atmosphere.



The Sun is Oval Shaped at the Time of its Rise and Set

The rays diverging from the lower edge of the sun have to cover a greater thickness of air than the rays from the upper edge. Hence, the former are refracted more than the latter and so the vertical diameter of the sun appears to be a little shorter than the horizontal diameter which remains unchanged.

The Stars Twinkle but not the Planets

The refractive index of atmosphere fluctuates by a small amount due to various reasons. This causes slight variation in bending of light due to which the apparent position of star also changes, producing the effect of twinkling.

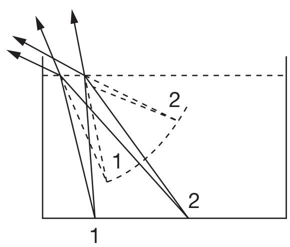
Glass is Transparent, but its Powder is White

When powdered, light is reflected from the surface of innumerable small pieces of glass and so the powder appears white. Glass transmits most of the incident light and reflects very little hence it appears transparent.

Greased or Oiled Paper is Transparent, but Paper is White

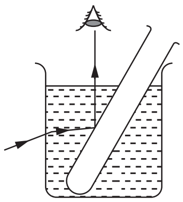
The rough surface of paper diffusely reflects incident light and so it appears white. When oiled or greased, very little reflection takes place and most of the light is allowed to pass and hence it appears transparent.

An Extended Water Tank Appears Shallow at the Far End



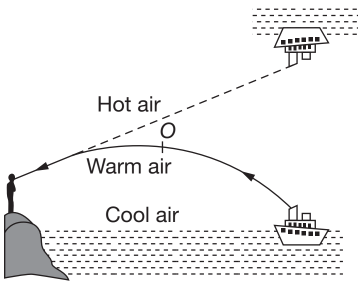
A Test Tube or a Smoked Ball Immersed in Water Appears Silvery white when Viewed from the Top

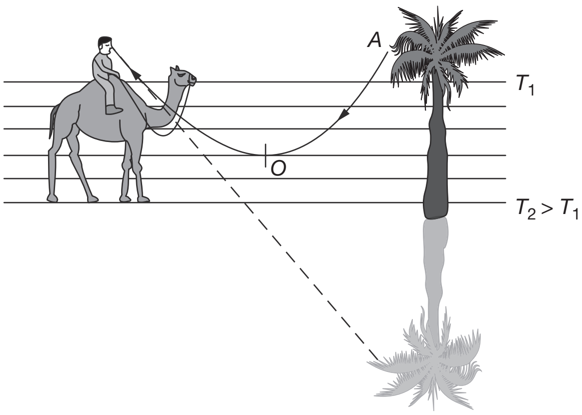
This is due to total internal reflection.



Ships Hang Inverted in the Air in Cold Countries and Trees Hang Inverted Underground in Deserts

This is due to total internal reflection.





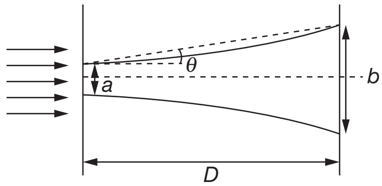
Condition for Rectilinear Propagation of Light (Only for information not in JEE syllabus)

Some part of the optics can be understood if we assume that light travels in a straight line and it bends abruptly when it suffers reflection or refraction.

The assumption that the light travels in a straight line is correct if

1. the medium is isotropic, i.e., its behaviour is same in all directions and
2. the obstacle past which the light moves or the opening through which the light moves is not very small.

Consider a slit of width a through which monochromatic light rays pass and strike a screen, placed at a distance D as shown.



It is found that the light strikes in a band of width b more than a. This bending is called diffraction.

Light bends by $(b–a)/2$ on each side of the central line. It can be shown by wave theory of light that

$$\sin\theta = \frac{\lambda}{a} \quad (1),$$

where θ is shown in figure.

This formula indicates that the bending is considerable only when $a \sim \lambda$. Diffraction is more pronounced in sound because its wavelength is much more than that of light and it is of the order of the size of obstacles or apertures. Formula (A) gives $\frac{b-a}{2D} \approx \frac{\lambda}{a}$.

It is clear that the bending is negligible if $\frac{D\lambda}{a} << a$ or $a >> \sqrt{D\lambda}$. If this condition is fulfilled, light is said to move rectilinearly. In most of the situations including geometrical optics, the conditions are such that we can safely assume that light moves in straight line and bends only when it gets reflected or refracted.

Thus, geometrical optics is an approximate treatment in which the light waves can be represented by straight lines which are called rays. A ray of light is the straight line path of transfer of light energy. Arrow represents the direction of propagation of light.

Figure 18.42 shows a ray which indicates light is moving from A to B.

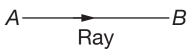
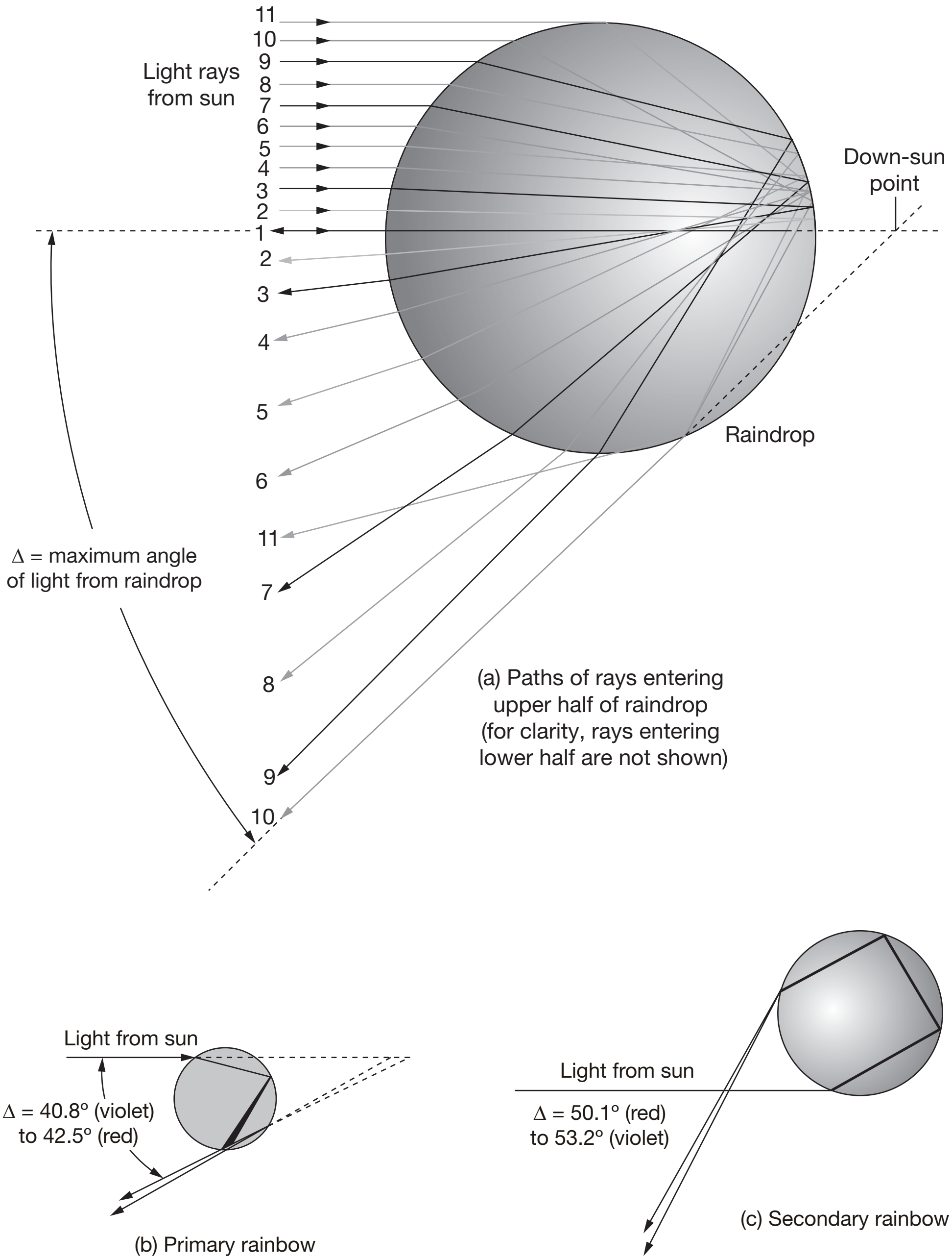


Fig. 18.42

Formation of Rainbow



Rainbows are formed by refraction, reflection, and dispersion in water drops.

1. The paths of light rays entering the upper half of rainbow. (The pattern of rays entering the lower half is the same, but flipped upside down.) All of the rays emerge within an angle Δ from the down-sun point. The angle Δ is the angular radius of the rainbow.
2. The colours of the rainbow are due to dispersion. Red appears on the outside of the primary rainbow.
3. Red appears on the inside of the fainter secondary rainbow.

OPTICAL INSTRUMENT

Compound Microscope

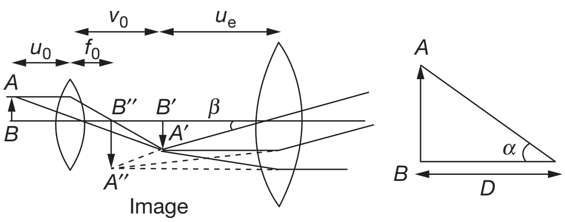
When a greater magnification than is obtained from a simple magnifier is needed, we usually use compound microscope.

Construction

A compound microscope consists of two convex lenses co-axially separated by some distance. The one facing the object is called the objective and the one close to the eye is called the eye piece or ocular. The objective has a smaller aperture and smaller focal length than those of the eye piece.

Formation of Image

The object AB to be viewed is placed slightly beyond the principal focus F_0 of the objective. The objective forms an enlarged, real inverted image $A'B'$. This image works as the object for the eyepiece. A very much enlarged virtual image $A''B''$ is formed by the eyepiece.



Angular Magnification or Magnifying Power

Angular magnification of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when it is placed at the least distance of distinct vision (D)

$$\text{Angular magnification } m = \frac{\beta}{\alpha} = \frac{A'B'}{u_e} \times \frac{D}{AB}$$

$$= \left(\frac{A'B'}{AB}\right) \times \left(\frac{D}{u_e}\right)$$

Also from figure

$$\frac{A'B'}{AB} = \frac{-v_0}{u_0}$$

Now $\frac{D}{u_e}$ is the magnifying power of the eye piece treated as a simple microscope. This is equal to $\frac{D}{f_e}$ in normal adjustment (image at infinity) and $1 + \frac{D}{f_e}$ for the adjustment when the image is formed at the least distance for clear vision i.e. at D.

Thus, the magnifying power of the compound microscope is

$$m = \frac{v}{u}\left(\frac{D}{f_e}\right) \text{ for normal adjustment and}$$

$m = \frac{v}{u}\left(1 + \frac{D}{f_e}\right)$ for the adjustment when the final image is formed at the least distance of clear vision. Since the object is placed very close to the principal focus of the objective, therefore u_0 is nearly equal to f_0. Moreover, the focal length of the eyepiece is small. So, the image $A'B'$ is formed very close to the eye piece. So v_0 is nearly equal to the length L of the microscope tube (separation between the two lenses)

$$m = -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \quad \text{for image at } D \text{ and}$$

$$m = -\frac{L}{f_0} \times \frac{D}{f_e} \quad \text{for normal adjustment}$$

Telescope

To look at distant objects such as a star, a planet or a distant tree etc. we use another instrument called a telescope.

Astronomical Refracting Telescope

It consists of an objective lens system and an eyepiece. The one facing the distant object is called the objective and has a large aperture and a large focal length. The other is called the eyepiece, as the eye is placed close to it. It has a smaller aperture and a smaller focal length. The lenses are fixed in tube.

Light from the distant object enters the objective and a real image is formed within the tube. The eye piece used again as a simple magnifier, leaves the final image inverted.

1. **When the final image is formed at infinity (normal adjustment)**
   From the figure, we see that the objective forms image $A'B'$ of an object AB placed at a distant place. If the point P is on the principal axis, the image point P is at the second focus of the objective. For normal adjustment image is formed at infinity.
   Hence $u_e = f_e$
   Angular magnification
   $$m = \frac{\beta}{\alpha} = -\frac{A'B'/u_e}{A'B'/f_0} = -\frac{f_0}{u_e} = -\frac{f_0}{f_e}$$
   Where f_0 = focal length of objective
   f_e = focal length of eye piece
2. **When the final image is formed at the least distance of distinct vision. (near point)**
   If the telescope is adjusted, so that the final image is formed at the near point of the eye, the angular magnification is further increased.

Angular magnification $\Rightarrow -\dfrac{f_0}{v_e} = m$

But, for near point adjustment $v_e = -D$

So by lens formula

$$-\frac{1}{D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{f_e + D}{f_e D}$$

$$m = -\frac{f_0\left(f_e + D\right)}{f_e\ D} = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Length of Telescope

From the given Fig. 18.43, we see that the length of the telescope is $L = f_0 + u_e$

For normal adjustment $L = f_0 + f_e$

For near point adjustment $L = f_0 + u_e = f_0 + \dfrac{f_e D}{f_e + D}$

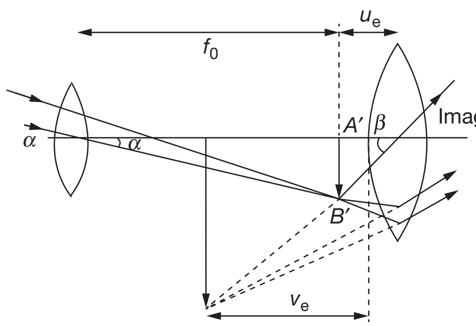
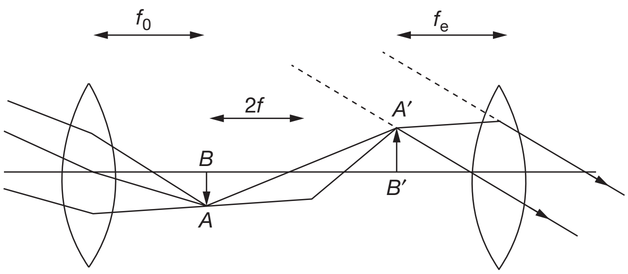


Fig. 18.43

Terrestrial Telescope

The astronomical telescope produces an inverted image of an object viewed through it. So it is unsuitable for terrestrial use. The astronomical telescope can be converted into terrestrial telescope by introducing a converging lens between the objective and the eye piece. This additional lens is called an erecting lens.



The erecting lens of focal length f is introduced in such a manner that the focal plane of the objective is a distance $2f$ away from this lens. An erect image $A'\ B'$ of same size is made by the new lens introduced. The length of the tube for normal adjustment is

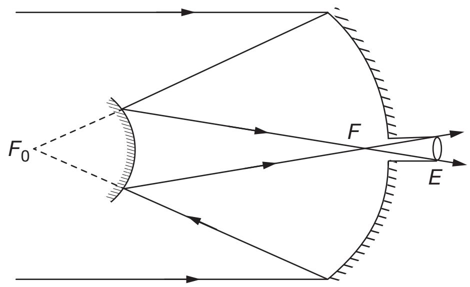
$$L = f_0 + 4f + f_e$$

Magnifying power $m = \dfrac{f_0}{f_e}$ for normal adjustment

$$= \frac{f_0}{f_e}\left(1 + \frac{v_e}{D}\right)$$

Reflecting Telescope (Cassegrain)

Telescope with mirror objectives are called reflecting telescopes. The objective O of the telescope is a large parabolic mirror with a hole (small circular aperture) in the centre. A small convex parabolic mirror is placed in front of the objective. A convex lens acts as the eye piece. The eyepiece has a large focal length in a short telescope.



A beam of light parallel to the axis of the telescope from some distant light source is incident on the objective. The objective converges this light towards its principle focus F_0. The reflected beam is intercepted by the convex mirror. The convex mirror forms an inverted image at F. This inverted image is seen through the eye piece E.

Magnification of reflecting telescope $= \dfrac{f_0}{f_e}$

Where f_0 = focal length of objective mirrors
f_e = focal length of objective eye lens.

Advantages of Reflecting Telescope

1. There is no chromatic aberration, because it is based on reflection.
2. Spherical aberration can be removed by choosing parabolic reflector.
3. Mechanical support required is much less of a problem since the mirror weighs much less than a lens of equivalent optical quality and can be supported over its entire back surface.

SOLVED EXAMPLES

61. An astronomical telescope consists of two thin lenses set 36 cm. apart and has a magnifying power of 8 in normal adjustment. Calculate the focal lengths of lenses.

Solution:

In the normal adjustment, the final image is formed at infinity.

$\therefore \quad f_0 + f_e = 36 \quad (1)$

and $\quad -\dfrac{f_0}{f_e} = -8$

$\therefore \quad f_0 = 8f_e \quad (2)$

$\therefore$ From equation (1) and (2), we have

$$8f_0 + f_e = 36$$

$\therefore \quad 9f_e = 36$

$\therefore \quad f_e = 4$ cm

and $\quad f_0 = 8 \times 4 = 32$ cm

62. A compound microscope has a magnification of 30. The focal length of its eye piece is 5 cm. Assuming the final image is to be formed at the least distance of distinct vision 25 cm, calculate the magnification produced by the objective.

Solution:

Given, $\quad M = 30, \quad f_e = 5$ cm, $\quad D = 25$ cm

From $M = M_0 \times m_e \times \left(\dfrac{D}{f_e} + 1\right) = m_0\left(1 + \dfrac{25}{5}\right)$

$\therefore \quad 30 = 60\, m_0$

$\therefore \quad m_0 = \dfrac{30}{6} = 5$

63. A reflecting type telescope has a large mirror with a radius of curvature equal to 80 cm. What is the magnifying power of telescope if the eye piece used has a focal length of 1.6 cm?

Solution:

Given, $\quad R = 80$ cm

$\therefore \quad f_0 = \dfrac{80}{2} = 40$ cm

$f_e = 1.6$ cm

$\therefore \quad M = \dfrac{f_0}{f_e} = \dfrac{40}{1.6} = 25$

WAVE OPTICS

Huygen's Wave Theory

According to this theory light is a wave. This theory is capable of explaining rectilinear propagation of light, reflection, refraction, interference, diffraction and polarisation.

Wave Front

It is the locus of all the particles of the medium which vibrate in same phase and where disturbances reach at the same instant of time. A point source gives rise to a spherical wave-front while along line source produces a cylindrical wave-front.

Huygen's Principle

It has following two basic postulates:

1. Consider all the points on a primary wave-front to be the sources of light, which emit disturbances known as secondary disturbances.
2. Tangent envelope to all secondary wavelets gives the position of new wave-front.

INTERFERENCE

Interference of light is due to the superposition of two or more wave trains act simultaneously at the same position in a medium. In this phenomenon, there is a modification of the distribution of intensity of light due to superposition of more than one wave from coherent sources.

Coherent Sources

Two sources are said to be coherent if their frequencies are equal and they have a constant phase difference. Two independent sources of light cannot be coherent. Therefore, to obtain steady interference pattern, two interfering waves are derived from same source. For example, two sources are obtained from a single source by reflection in Lloyd's single mirror, by refraction in Fresnel's biprism, by diffraction in Young's double slit experiment etc.

Constructive and Destructive Interference

If two wave trains interfere in the same phase, then the interference is called constructive, while if they interfere in the opposite phases then the interference is called destructive. The constructive interference occurs when

- The phase difference between the two interfering waves at any point is $2n\pi$ where $n = 0, 1, 2, 3, \ldots\ldots\ldots$
- The path difference between the two interfering waves at any point is nl where $n = 0, 1, 2, 3$.

  The amplitude of the resultant is the sum of the individual amplitudes and thus intensity is maximum.

$$A_{max} = A_1 + A_2$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

The destructive interference occurs when

- The phase difference of the two interfering waves at any point is $(2n - 1)\pi$ where $n = 1, 2, 3, ...$
- The path difference between two interfering waves at any point is $(2n-1)\frac{\lambda}{2}$ where $n = 1, 2, 3.$

The amplitude of the resultant is the difference of the individual amplitudes and thus intensity is the minimum.

$$A_{min} = A_1 - A_2$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

Thus in the interference pattern of the light we get bright and dark bands called fringe.

Intensity Variation in Interference

When two coherent waves of intensities I_1 and I_2 are superimposed with a constant phase difference ϕ, then intensity of the resultant wave obtained is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

At the position of maxima or bright fringe,

$$\phi = 2n\pi \qquad n = 0, 1, 2, 3, ...$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

or $$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

At the position of minima or dark fringe,

$$\phi = (2n-1)\pi \qquad n = 0, 1, 2, 3, ...$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

or $$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

NOTE

The phenomenon of interference is not in violation of energy conservation. The initial average total energy of the waves is

$$U_{initial} = I_1 + I_2$$

and the final average total energy on the screen after the interference is

$$U_{final} = \frac{I_{max} + I_{min}}{2}$$

or $$U_{final} = I_1 + I_2$$

i.e. $$U_{final} = U_{initial}$$

SOLVED EXAMPLE

64. Light from two sources, each of same frequency and travelling in same direction, but with intensity in the ratio 4 : 1 interfere. Find ratio of maximum to minimum intensity.

Solution:

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right)^2$$

$$= \left(\frac{2+1}{2-1}\right)^2 = 9 : 1.$$

Young's Double Slit Experiment

The Fig. 18.44 shows a schematic representation of the original experiment performed by Young. In the experiment, sunlight is first allowed to pass through a pinhole S and then, at a considerable distance away, through two pinholes S_1 and S_2. The two set of spherical waves emerging from the two holes interfere with each other in such a way as to form a symmetrical pattern (circular shape) of varying intensity on the screen.

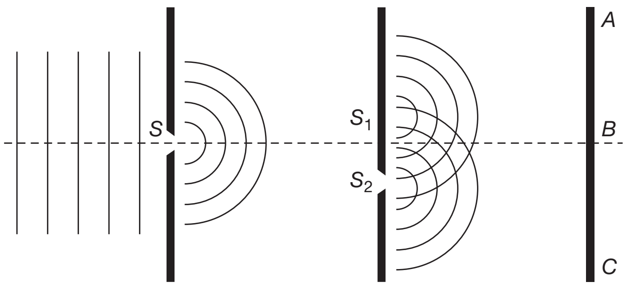


Fig. 18.44

Location of Fringes on the Screen

The Fig. 18.45 shows the method of determining the nth maxima and minima on the screen. Let P be a point on the screen located at a distance y_n from the origin O. The optical path difference between the two waves S_2P and S_1P reaching the point P is given by

$$p = S_2P - S_1P = d\sin\theta$$

since $D >> d$ and $D >> y_n$, therefore,

$$\sin\theta = \tan\theta = \frac{y_n}{D}$$

or $$p = \frac{dy_n}{D}$$

If a maxima is to be formed at the point P, then

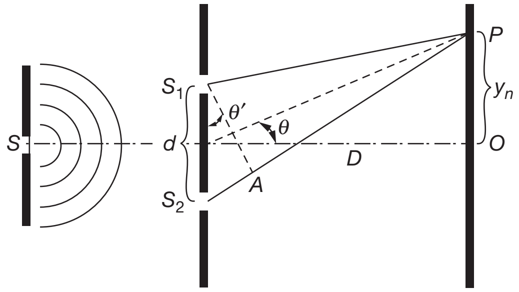


Fig. 18.45

$$p = n\ \lambda$$

∴ $$\frac{dy_n}{D} = n\lambda$$

or $$y_n = \frac{n\lambda D}{d}$$

where $n = 0, 1, 2, 3, \ldots$

If a minima is to be formed at the point P, then

$$p = \left(n - \frac{1}{2}\right)\lambda$$

∴ $$\frac{dy_n}{D} = \left(n - \frac{1}{2}\right)\lambda$$

or $$y_n = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d}$$

where $n = 1, 2, 3, \ldots$

Fringe Width

The distance between the two successive maxima or minima is called the fringe width ()

Thus, $$\beta = y_{n+1} - y_n = (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$$

or $$\beta = \frac{\lambda D}{d}$$

Intensity Distribution on the Screen

If the two slits are identical in size, then they act as two coherent sources of equal intensities,

i.e. $$I_1 = I_2 = I_0$$

By using equation, the intensity at the bright fringe or maxima is

$$I_{max} = 4I_0$$

and that at the dark fringe or minima is

$$I_{min} = 0$$

The variation of light intensity on the screen is

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi$$

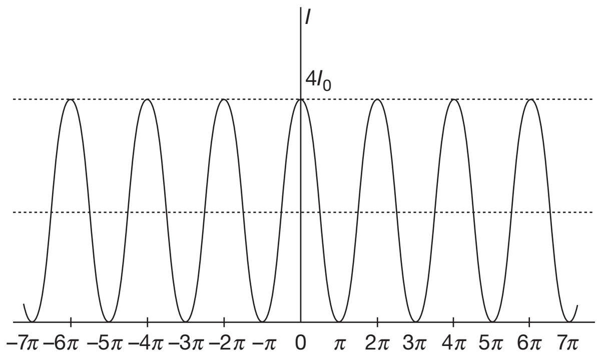
or $$I = 2I_0(1 + \cos\phi)$$

or $$I = 4I_0\cos^2\frac{\phi}{2}$$

The figure shows the variation of light intensity on the screen as a function of phase difference ϕ.

NOTE

According to the principle of energy conservation, energy is not destroyed at the positions of minima as it appears to be destroyed; nor is it created at the positions of minima as it appears to be created. The average intensity on the screen is exactly that which would exist in the absence of interference.



Important Points about Young's Experiment

1. If one of the slits S_1 or S_2, is covered up, the fringes disappear.
2. If the distance between the slits d is diminished, the separation of the fringes (i.e. fringe width) increases; and if d is increased, the separation of the fringes decreases.
3. If the screen is moved away from the slits, the separation of fringes increases, and if the screen is moved towards the slits, the separation of the fringes decreases.
4. If white light is used the central fringe is white and the fringes on either side are coloured. The coloured fringe nearest to the central is blue and the farthest fringe is red.
5. If the source slit S is moved nearer to the double slits S_1 and S_2, the separation of the fringes is unaffected but their intensity increases.

6. If the source slit S is widened the fringes gradually disappear. The slit S is then equivalent to a large number of narrow slits, each producing its own fringe system at different places. The bright and dark fringes of different systems overlap, therefore, giving rise to uniform illumination.

7. If the slit width of S is more than $\frac{\lambda D'}{d}$, where D' is the distance between the source slit and the double slits, then the fringe pattern disappears.

SOLVED EXAMPLE

65. In a YDSE, $D = 1$ m, $d = 1$ mm and $\lambda = 1/2$ mm
(i) Find the distance between the first and central maxima on the screen.
(ii) Find the no of maxima and minima obtained on the screen.

Solution:

(i) D >> d
Hence $P = d \sin \theta$

$$\frac{d}{\lambda} = 2,$$

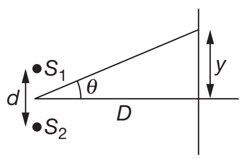
Clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $P = \frac{dy}{D}$ cannot be used for first maxima,

$$P = d \sin \theta = \lambda$$

$$\Rightarrow \quad \sin q = \frac{\lambda}{d} = \frac{1}{2}$$

$$\Rightarrow \quad q = 30^{\circ}$$



Hence, $y = \Delta \tan \theta = \frac{1}{\sqrt{3}}$ meter

(ii) Maximum path difference
$\Delta P_{max} = d = 1$ mm

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$

and highest order minima, $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total number of maxima $= 2n_{max} + 1 = 5$
Total number of minima $= 2n_{min} = 4$

Displacement of Fringe

An introduction of a glass slab in the path of the light coming out of the slits.

On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by $t\ (\mu - 1)$. Now the path difference between waves coming from S_1 and S_2 at any point P is

$$P = S_2P - (S_1P + t\,(\mu - 1)) = (S_2P - S_1P) - t(\mu - 1)$$

$$\Rightarrow \quad P = d \sin \theta - t\,(\mu - 1)$$

if $\quad d << D$

and $\quad \mathrm{P} = \frac{yd}{D} - t(\mu - 1)$

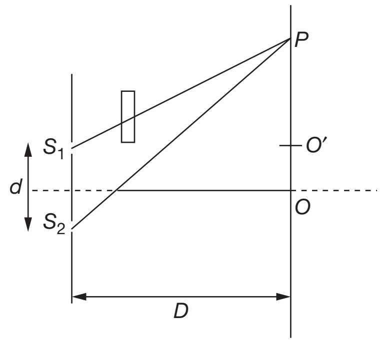
If $\quad y << D$ as well.

For central bright fringe,

$$P = 0$$

$$\Rightarrow \quad \frac{yd}{D} = t(\mu - 1).$$

$$\Rightarrow \quad y = OO' = (\mu - 1)\,t\,\frac{D}{d} = (\mu - 1)\,t\,\frac{\beta}{\lambda}.$$



The whole fringe pattern gets shifted by the same distance

$$= (\mu - 1) \cdot \frac{D}{d} = (\mu - 1)\,t\,\frac{B}{\lambda}.$$

Notice that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

SOLVED EXAMPLE

66. In YDSE with $d = 1$ mm and $D = 1$ m, slabs of ($t = 1\ \mu$m, $\mu = 3$) and ($t = 0.5\ \mu$m, $\mu = 2$) are introduced in front of upper and lower slit respectively. Find the shift in the fringe pattern.

Solution:

Optical path for light coming from upper slit S_1 is

$$S_1P + 1\ \mu\text{m}\ (2 - 1) = S_2P + 0.5\ \mu\text{m}$$

Similarly optical path for light coming from S_2 is

$$S_2P + 0.5\ \mu\text{m}\ (2 - 1) = S_2P + 0.5\ \mu\text{m}$$

Path difference

$$P = (S_2P + 0.5\ \mu\text{m}) - (S_1P + 2\mu\text{m})$$

$$= (S_2P - S_1P) - 1.5\ \mu\text{m}.$$

$$= \frac{yd}{D} - 1.5\ \mu\text{m}$$

for central bright fringe $P = 0$

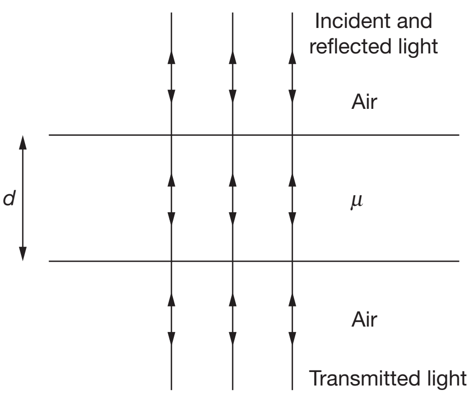
$$\Rightarrow \quad y = \frac{1.5\,\mu m}{1\,mm} \times 1\text{m} = 1.5\ \mu\text{m}.$$

The whole pattern is shifted by 1.5 μm upwards.

THIN-FILM INTERFERENCE

In YDSE we obtained two coherent sources from a single (incoherent) source by division of wavefront. Here we do the same by division of amplitude (into reflected and refracted wave).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d then waves reflected from the upper surface interfere with waves reflected from the lower surface.



Clearly the wave reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film.

Further if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

Consequently condition for constructive and destructive interference in the reflected light is given by,

$$2\mu d = n\lambda \text{ for destructive interference}$$

and

$$2\mu d = (n + \frac{1}{2})\text{l for constructive interference} \qquad (1)$$

where $n = 0, 1, 2$ .... and λ = wavelength in free space.

Interference will also occur in the transmitted light and here condition of constructive and destructive interference will be the reverse of (1)

$$\text{i.e.} \quad 2\mu d = \begin{cases} n\lambda & \text{for constructive interference} \\ (n + \frac{1}{2}\lambda) & \text{for destructive interference} \end{cases} \qquad (2)$$

This can be easily explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light) however, the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

SOLVED EXAMPLE

67. White light, with a uniform intensity across the visible wavelength range 430–690 nm is perpendicularly incident on a water film, of index of refraction m = 1.33 and thickness d = 320 nm, that is suspended in air. At what wavelength λ is the light reflected by the film is brighter to an observer?

Solution:

For the interference maxima,

$$\lambda = \frac{2\mu d}{m + 1/2}$$

$$= \frac{(2)(1.33)(320\,\text{nm})}{m + 1/2} = \frac{851\,\text{nm}}{m + 1/2}$$

for m = 0, this gives us λ = 1700 nm, which is in the infrared region. For m = 1, we find λ = 567 nm, which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 nm, which is in the ultraviolet region. So the wavelength at which the light seen by the observer is brightest is

$$\lambda = 567\text{ nm}.$$

$$\Rightarrow \qquad \psi = -1\text{ cm}.$$

(ii) for point 0, $\theta = 0$

Hence, $\Delta p = d \sin \theta_0$; $d\theta_0 = 1$ mm $\times$ (10^{-2} rad)

$= 10{,}000$ nm $= 20 \times (500$ nm)

$\Rightarrow \quad \Delta p = 20\,\lambda$

Hence point O corresponds to 20th maxima

$\Rightarrow$ intensity at $O = I_0$

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

DIFFRACTION OF LIGHT

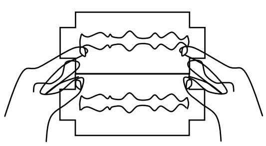
The phenomenon of bending of light around the corners of an obstacle or aperture is called the diffraction of light. On account of diffraction, light deviates from is linear path and enters into the regions of geometrical shadow of the obstacle.

In fact, diffraction is a characteristic of wave motion and all types of wave motions exhibit this effect.

A person sitting in a room can hear the sound of another person sitting outside the room, even though the speaker may not be visible. This is due to the bending of sound waves around the corners of the open door or window i.e. diffraction of sound. Similarly, radio waves too exhibit diffraction easily.

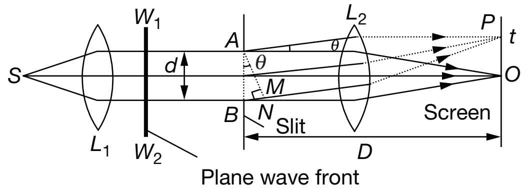
Experimental studies have revealed that diffraction occurs more prominently when the wavelength of the waves is comparable with the size of the obstacle or aperture. Since the wavelength of visible light is very small ($\lambda = 10^{-6}$m) and the size of the common obstacles or aperture is very large, the diffraction of visible light is not very common. However, when the size of the obstacle or aperture (slit) is made very small (comparable with the wavelength) even visible light shows this phenomenon. Diffraction is common in case of sound wave and radio waves because their wavelength is large and comparable to the size of common obstacle.

According to Fresnel, diffraction occurs on account of mutual interference of secondary wavelets starting from portions of the wavefront which are not blocked by the obstacle or from portions of the wavefront which are allowed to pass through the aperture.



Diffraction at a Single Slit

Consider a narrow slit AB of width d kept perpendicular to the plane of the paper. Let a plane wavefront of monochromatic light be incident normally on this slit. As the width d of the slit is comparable to the wavelength λ of the light, the light gets diffracted on passing through the slit. The converging lens L_2 helps in focusing the diffraction pattern on the screen. The diffraction pattern consists of a bright band at its centre (O) with alternate dark and bright bands on both sides. The intensity of the bright bands decreases very sharply as one move away from O on both sides.



Explanation

As the plane wavefront W_1 W_2 is incident normally on the slit, the entire slit AB gets, illuminated. According to Huygen's principle, each point on the portion AB of the wavefront becomes the source of secondary disturbance. The secondary wavelets start spreading from AB in all directions.

Position of Central Maxima

The secondary wavelets travelling parallel to MO are focussed at O by the converging lens I_2. Actually, the secondary wavelets originating from any two points equidistant from the centre of the slit M (one each in MA and MB) cover equal distances before converging at O. Hence, the path difference between them is zero. As a result, they produce the brightest band at O on the screen. This is called the central maximum.

Position of Secondary Maxima and Minima

Consider the secondary wave travelling in a direction making an angle θ with MO. All these waves are focussed by lens L_1 at point P. Such waves start from all the points of slit AB in the same phase but travel different distances in reaching the point P. The intensity at point P will, therefore, depend on the path difference between the secondary waves emitted from the corresponding points (equidistant from M) of the wavefront AB.

Let us calculate the path difference between two secondary waves emitting from points A and B of the wavefront. Draw AN perpendicular to the wave coming from B.

Path difference between the secondary wavelets which originate from A and B and reach point P on the screen, $BN = AB \sin \theta = d \sin \theta$.

Secondary Minima

Let this path difference be equal to 1 λ i.e.

$$\therefore \quad d\sin\theta = \lambda$$

Hence the path difference between the wavelets originating from points A and M and reaching point P will be $\lambda/2$. Similarly, the path difference between the wavelets originating from M and B and reaching P will also be $\lambda/2$. Thus for each point in the upper half of the slit AM, there exists a corresponding point in its lower half MB such that the wavelets originating from such points reach point P out of phase i.e. having a phase difference of π. Therefore, destructive interference takes place at point P and it represents the first secondary minima.

Similarly, if $\quad d\sin\theta = 2\lambda.$

The slit AB can be imagined to be divided into four equal parts. The path difference between the wavelets originating from corresponding points in two adjacent parts will be $2\lambda/4 = \lambda/2$. Hence, the wavelets cause destructive interference and point P again represents the second minima.

In general, for n^{th} minima, we have

$$d\sin\theta = n\lambda$$

Where $n = 1, 2, 3,\ldots\ldots$

Hence, the direction of n^{th} secondary minima

$$\sin\theta = n\frac{\lambda}{d}$$

$\therefore$ For small values θ, $\sin\theta_n = \theta_n = n\dfrac{\lambda}{d}$

Secondary Maxima

Let the path difference between the wavelets originating from A and B and reaching an off –point P_1 (not shown in the figure) be $3\lambda/2$.

$$\therefore \quad d\sin\theta = 3\lambda/2$$

We assume that the slit AB is divided into three equal parts so that the difference between the wavelets originating from the corresponding points in the first two parts is $\lambda/2$. Such wavelets cause destructive interference at P_1. However, the wavelets originating from different point of the third part reinforce each other (not completely as for them, $0 > d\sin\theta < \lambda/2$) and give rise to the first secondary maxima at P_1. The intensity at the first secondary maxima is much smaller than that of the central maxima.

Similarly, $d\sin\theta = 5\lambda/2$ give the second secondary maxima with much lower intensity than the first maxima.

In general, for n^{th} secondary maxima the have

$$d\sin\theta = (2n+1)\lambda/2$$

Where $n = 1, 2, 3\ \ldots.$

The direction of n^{th} secondary maxima is

$$\sin\theta = \theta_n = \frac{(2n+1)\lambda}{2d}$$

Thus, the diffraction pattern due to a single slit consists of central maxima flanked by alternate minima and secondary maxima. The intensity distribution in the pattern on the screen in shown in Fig. 18.46.

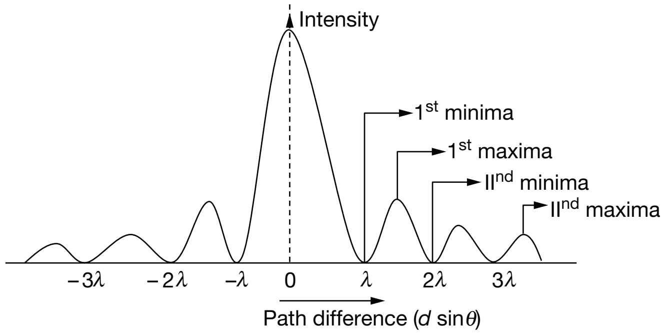


Fig. 18.46

The graph shows that as we go away from the central maxima O on both sides, the intensity of secondary maxima decreases very rapidly.

Width of Central Maxima

It is the distance between the first secondary minimum an either side of the central maxima

Let y be the distance of a first minimum from the centre (O) of the central maxima.

$$\therefore \quad d\sin\theta = \lambda \text{ and } \sin\theta = \theta = \frac{\lambda}{d}D$$

Let f be the focal length of the converging lens L_2 (placed very close to slit AB) and D is the distance of the screen from AB.

$$\therefore \quad D = f$$

Hence, $\quad \theta = \dfrac{y}{f} = \dfrac{y}{D}$

Comparing equation, we get $\dfrac{y}{D} = \dfrac{\lambda}{d},\ y = \dfrac{\lambda}{d}$

$$\therefore \text{ width of central maxima} = 2y = \frac{2\lambda D}{d} = \frac{2\lambda f}{d}$$

Figure shows the widths of the central maxima (CD) and the first secondary maximum (DE or CF)

Let y_1 and y_2 respectively be the distance of the first minima and the second minima from O. Then width of the first secondary maximum $= y_2 - y_1 = \dfrac{2\lambda D}{d} - \dfrac{\lambda D}{d} = \dfrac{\lambda D}{d}.$

The width of the central maxima is twice the width of the first secondary maximum.

Diffraction Pattern at Single Slit due to Monochromatic Light and White Light

When a parallel beam of monochromatic light is incident normally on the slit, the diffraction pattern obtained on the screen consists of alternate bright and dark bands of equal width. The intensity is maximum at the central bright band and it decreases very rapidly for successive secondary bright bands i.e. maximum.

When the beam of white light is incident on the slit, the diffraction pattern is coloured. The central maxima is white but the other bands are coloured. As the band width ∝ λ the red band will be wider than the violet band (as $\lambda_R > \lambda_V$).

Fresnel Distance (Z_F)

It is defined as the distance of the screen from the slit at which the spreading of light due to diffraction at single slit becomes equal to the width of the slit.

From the figure $\theta = \frac{y}{D} = \frac{d}{D}$ and for 1st minima $\theta = \frac{\lambda}{d}$

$$\therefore \quad \frac{\lambda}{d} = \frac{d}{D}$$

$$\therefore \quad D = Z_F = \frac{d^2}{\lambda}$$

Diffraction between Interference and Diffraction of Light

Inspite of the fact that interference and diffraction both are the consequence of the superposition of wave and also that mostly both occur simultaneously, they are different. Following are the main points of the difference between them.

INTERFERENCE	DIFFRACTION
1. It occurs due to the superposition of the secondary wavelets originating from two coherent sources.	1. It occurs due to the superposition of the secondary wavelets originating from different points of the same wavefront.
2. The width of all fringes is equal.	2. The width of all fringes is not equal.
3. All maxima are of the same intensity.	3. Intensity of secondary maxima fall off rapidly as we go away from the central maxima.
4. Interference pattern consists of a large number of bands.	4. In diffraction pattern there are only a few bands.

Resolving Power of Microscope or Telescope

Due to diffraction of light a point source can never give a point image. It gives an image in the from of a circular disc. This fact puts a limit on resolving two neighbouring points imaged by a lens, when two point objects are close to each other, their images i,e, diffraction patterns as seen through optical instruments, will also be close and overlap each other. If the overlapping is small both the point objects are seen separate in the optical instrument. However if the overlapping is large, the two points objects will not be seen as separate i.e. optical instrument is not able to resolve them.

Hence resolving power of an optical instrument is the power or ability of the instrument to produce distinct separate image of two close objects.

The minimum distance between the two objects which can just be seen as separate by the optical instrument is called the limit of resolution of the instrument. Smaller the limit of resolution of the optical instrument, greater is its resolving power and vice-versa.

Resolving Power of a Microscope

The resolving power of microscope is its ability to form separate images of two point objects lying close together. It is determined by the least distance between the two points which can be distinguished. This distance is given by $d = \frac{\lambda}{2\mu \sin\theta}$, where λ is the wavelength of light used to illuminate the object and μ is the refractive index of the medium between the object and the objective. The angle θ is the half angle of the cone of light from the point object.

$$\text{Resolving power } = \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$$

Resolving Power of a Telescope

The resolving power of a telescope is the reciprocal of the smallest angular separation between two distant objects whose images are separated in the telescope. This is given by $\theta = \frac{1.22\lambda}{a}$, where θ is the angle subtended by the point object at the objective, λ is the wavelength of light used and a is the diameter of the telescope objective. A telescope with a larger aperture objective gives a high resolving power.

POLARISATION AND PLANE POLARISED LIGHT

In writing equation for light wave, we assumed that the direction of electric field is fixed and the magnitude varies sinusoidally with space and time. The electric field in a light wave propagating in free space is perpendicular to the direction of propagation. However, there are infinite numbers of directions perpendicular to the direction of propagation and the electric field may be along any of these directions. For example, if the light propagates along the X-axis, the electric field may be along the Y-axis, or along the Z-axis or along any direction in the Y-Z plane. If the electric field at a point always remains parallel to a fixed direction as the time passes, the light is called linearly polarized along the direction. For example, if the electric field at a point is always parallel to the Y-axis, we say that the light is linearly polarized along the Y-axis. The same is also called plane polarized light. The plane containing the electric field and the direction of propagation is called the plane of polarization.

The phenomenon of restricting the vibrations of light in a particular direction perpendicular to direction of wave motion is called polarisation of light.

Polarisation of Light by Reflection

When unpolarised light is reflected from a surface, the reflected light may be completely polarised, partially polarised or non-polarised. This would depend on the angle of incidence.

If angle of incidences is 0° or 90°, the reflected beam remains unpolarised. For angles of incidence between 0° and 90°, the reflected beam is polarized to varying degree.

The angle of incidence at which the reflected light is completely plane polarized is called polarising angle or Brewster's angle. It is represented by i_p. The value of i_p depends on the wavelength of light used. Therefore, complete polarisation is possible only for monochromatic light. If non-polarised light is incident along AO at angle i_p on the interface XY separating air from a medium of refractive index μ, the light reflected along OB is completely plane polarised. The light refracted along OC continues to be unpolarised.

Infact, the non-polarised light has two electric field components, one perpendicular to the plane of incidence (represented by dots) and the other in the plane of incidence (represented by arrows).

The vibrations of electric vector perpendicular to the plane of incidence remain always parallel to the reflecting surface, whatever be the angle of incidence. Therefore, condition of their reflection is not changed with the change in the angle of incidence. However, the other set of oscillations of electric vector in the plane of incidence make different angles with the reflecting surface, as angle of incidence of unpolarised light is changed. At the polarising angle (i_p), most of these vibration of electric vector get transmitted and are not reflected.

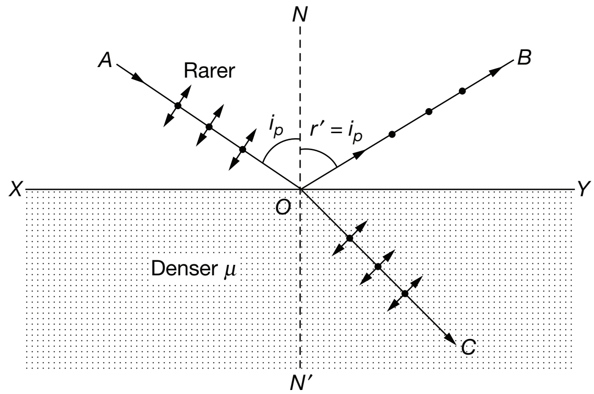
The reflected light therefore, contains vibrations of electric vector perpendicular to the plane of incidence. Hence the reflected light is completely plane polarised in a direction perpendicular to the plane of incidence.

BREWSTER'S LAW

According to this law, when unpolarised light is incident at polarising angle i_p on an interface separating air from a medium of refractive index μ, then the reflected light is fully polarized (perpendicular to the plane of incidence), provided.

$$\mu = \tan i_p$$

This relation represents Brewster's Law



$$\angle BOY + \angle YOC = 90°$$

$$(90° - i_p) + (90° - r) = 90°$$

where r is the angle of refraction

or $$90° - i_p = r \quad (1)$$

According to Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

When $$i = i_p, \quad i = (90° - i_p)$$

$\therefore$ $$\mu = \frac{\sin i_p}{\sin(90° - i_p)} = \frac{\sin i_p}{\cos i_p} = \tan i_p$$

This proves Brewster's law.

SOLVED EXAMPLES

68. A slit of width 3 mm is illuminated by light of $\lambda = 600$ nm at normal incidence. If the distance of the screen from the slit is 60 cm, the distance between the first order minimum on both sides of central maximum is $y \times 10^{-2}$ mm. The value of y is

Solution:

Given $d = 3 \text{ mm} = 3 \times 10^{-3}$ m

$\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m, $D = 60$ cm $= 0.60$ cm

Distance of first order minima from central maximum

$$x_1 = 1 \times \frac{\lambda D}{d}$$

$\therefore$ distance between first order minimum on both sides of the central maximum

$$= 2x_1 = \frac{2\lambda D}{d} = \frac{2 \times 6 \times 10^{-7} \times 0.6}{3 \times 10^{-3}}$$

$$= 0.24 \times 10^{-3} \text{ m} \quad \text{or} \quad y = 24$$

69. A beam of wavelength 6000 Å is incident on a slit of width 0.2 mm. The angular spread of the central maximum is $x \times 10^{-3}$ radius. The value of x is

Solution:

Given $\lambda = 6000$ Å $= 6000$ Å $= 6000 \times 10^{-10}$ m

$d = 0.2 \text{ mm} = 0.2 \times 10^{-3}$ m

Angular spread of the central maximum

$$\theta = \frac{\lambda}{d} = \frac{6000 \times 10^{-10}}{0.2 \times 10^{-3}} = 3 \times 10^{-3} \text{ radian.}$$

$\therefore \quad x = 3$

70. (A) A ray of light is incident on a glass surface at an angle of 45°. If the reflected and refracted rays are perpendicular to each other, find the refractive index of glass.

(B) what is the value of the refractive index of a medium of polarising angle 45°.

Solution:

(A) and (B) we know that if the reflected and refracted rays are perpendicular, the angle of incidence is equal to polarising angle.

$\therefore \quad i_p = 45°$

$\therefore \quad \mu = \tan i_p = \tan 45° = 1$

or $\quad \mu = 1$

SCATTERING OF LIGHT

When light passes though a medium, a part of it appears in directions other than the incident direction. The phenomenon is called scattering of light. The basic process in scattering is absorption of light by the molecules of gas and other particles of the medium followed by re-radiation in different directions. The strength of scattering can be measured by the loss of energy in the light beam as it passes through the medium. In absorption, the light energy is converted into internal energy of the medium whereas in scattering, the light energy is radiated in other directions.

There are two types of scattering. If these particles are smaller than the wavelength of incident light, then scattering is proportional to $\frac{1}{\lambda^4}$. This is known as Rayleigh's law of scattering. Thus, red light is scattered the least and violet is scattered the most law of scattering. This is the reason why danger signals are made of red colour so that they can be seen from far away. The blue appearance of sky is due to scattering of sunlight from the atmosphere. When we look at the sky, it is the scattered light that enters the eyes. Among the shorter wavelengths, the colour blue is present in larger proportion in sunlight. Light of shorter wavelengths are scattered by air molecules which because of there smaller size follow Rayleigh's scattering. Blue light is strongly scattered by the air molecules and reach the observer. This explains the blue colour of sky.

The appearance of red colour of sun at the sunset and at the sunrise is also because of scattering of sunlight due to air molecules. The blue and neighbouring colours are scattered away in the path and light reaching the observer is predominantly red.

The sky would appear black and stars could be seen during day hours, if the earth had no atmosphere and hence no scattering.

Clouds contain a high concentration of water droplets or ice crystals which due to their comparatively larger size scatter light uniformly of all wavelengths. Since light of all wavelength is eventually scattered out of the cloud, so the cloud looks white.

BRAIN MAP

OPTICS

1. Reflection of light:

- In case of reflection at a plane or curved mirror, $\angle i = \angle r$.
- The image formed by a plane mirror is at the same distance behind the mirror as the object is in front of it. It is virtual, erect and of the same size as the object. The image formed by a plane mirror is laterally inverted.
- Derivation produced by a plane mirror $= 180° - 2i = 2g$, where g is the glancing angle.
- Keeping the direction of the incident ray fixed, if the plane mirror is rotated through angle θ, then reflected ray gets rotated through 2θ in the same sense.
- If two plane mirrors are inclined at angle θ, then number of images formed

  $= \frac{360°}{\theta} - 1$, if $\left(\frac{360°}{\theta}\right)$ is an even integer.

  $= \frac{360°}{\theta}$, if $\left(\frac{360°}{\theta}\right)$ is an odd integer, and object is placed unsymmetrical to the mirrors.

  $= \frac{360°}{\theta} - 1$, if $\left(\frac{360°}{\theta}\right)$ is an odd integer, and object is placed symmetrical to the mirrors.
- In case of a spherical mirror, $f = R/2$.
- The images formed by a concave mirror can be real as well as virtual. But the images formed by a convex mirror is always virtual.

  Mirror formula
- $1/v + 1/u = 1/f$
- Magnification produced by a spherical mirror

  $m = \frac{I}{O} = -\frac{v}{u}$

2. Refraction of light at a plane surface:

- In case of refraction of a ray at plane or spherical surface,

  $$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = 1\mu_2$$
- Lateral shift produced by a glass slab is $d = \frac{t}{\cos r}\sin(i - r)$
- When an object in denser medium is viewed almost normally by an observer in rarer medium, we have

  $$R\mu D = \frac{\text{Real depth}}{\text{Apparent depth}}$$
- When an object in rarer medium is viewed almost normally by an observer in denser medium, we have

  $$R\mu D = \frac{\text{Apparent depth}}{\text{Real depth}}$$
- $\sin c = 1/R\mu D$
- In case of refraction through prism, $r_1 + r_2 = A$

  Deviation produced by a prism $\delta = (I + e) - A$
- Condition for minimum deviation for a prism $i = e$ and $r_1 = r_2$
- $\mu = \frac{\sin(A + \delta_m/2)}{\sin(A/2)}$
- Deviation produced by a prism of small angle $= (\mu - 1)A$
- Angular dispersion $= (\mu_V - \mu_R)A$; Dispersive power $= \frac{\mu_V - \mu_R}{\mu - 1}$.
- Condition for dispersion without deviation is $\frac{A'}{A} = -\left(\frac{\mu - 1}{\mu' - 1}\right)$

  Net angular dispersion produced $= (\mu - 1)$ A $[\omega - \omega']$.
- Deviation without dispersion is $\frac{A'}{A} = -\left(\frac{\mu_V - \mu_R}{\mu'_V - \mu'_R}\right)$

  or, $\frac{\omega}{\omega'} = -\frac{\delta'}{\delta}$. Net deviation $= (\mu - 1)A\left[1 - \frac{\omega}{\omega'}\right]$.

3. Refraction of light at spherical surfaces:

- Refraction at a single spherical surface

  $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

  Transverse magnification produced is $m = \frac{\mu_1}{\mu_2}\frac{v}{u}$
- Lens maker's formula $\frac{1}{f} = \left(\frac{\mu}{\mu_0} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$.
- Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
- Linear magnification produced by a lens $m = \frac{I}{O} = \frac{v}{u}$.
- Power of a lens $P = 1/f$.
- Two thin lenses in contact $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots$

  Two thin lenses separated by a distance d

  $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$
- Lenses with a one surface silvered behaves as a spherical mirror. The equivalent focal length of the spherical mirror is given by $\frac{1}{F} = \Sigma\frac{1}{f_i}$, where f_i is to be repeated as many times as reflection or refraction is taking place.
- Displacement method:

  $f = \frac{D^2 - L^2}{4D}, O = \sqrt{I_1, I_2}$

4. Wave optics:

- In case of superposition of two waves,

  $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, or $I_R \alpha\ (A_1^2 + A_2^2 + 2A_1A_2\cos\phi)$.
- Condition for constructive interference

  $\phi = \pm 2\pi n$, $n = 1, 2, 3, \ldots$

  $(\Delta x) = \pm n\lambda$, $n = 1, 2, 3, \ldots$
- Condition for destructive interference

  $\phi = \pm(2n - 1)\pi$, $n = 1, 2, 3, \ldots$

  $(\Delta x) = \pm(2n - 1)\lambda/2$, $n = 1, 2, 3, \ldots$
- Distance of n th bright fringe from C.B.F. is

  $(y_n)_B = \pm n\lambda\frac{D}{d}$, $n = 0, 1, 2, 3, \ldots$

  Distance of nth dark fringe from C.B.F. is

  $(y_n)_D = \pm(2n - 1)\frac{\lambda D}{2d}$, $n = 0, 1, 2, 3, \ldots$
- Fringe width $(\omega) = \lambda D/d$, Angular fringe width $= \omega/D = \lambda/d$
- Equivalent optical path of a medium of R.I. μ and distance d is μd.
- Displacement of fringe pattern due to introduction of a transparent sheet of R.I. μ and thickness t in YDSE $= Y_0 = (\mu - 1)\,t\,(D/d)$, in the same side in which the transparent sheet is introduced.

EXERCISES

Single Option Correct Type

1. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When lens is placed in a medium of refractive index (4/3), focal length will be now
   (A) 30 cm (B) 60 cm
   (C) – 60 cm (D) – 30 cm

2. An object of mass m is moving with velocity $\vec{u}$ towards a plane mirror kept on a stand as shown in the Fig. 18.43. The mass of the mirror and stand system is m. A head-on elastic collision takes place between the object and the mirror stand, the velocity of image before and after the collision is
   (A) $\vec{u}, 2\vec{u}$ (B) $-\vec{u}, -2\vec{u}$
   (C) $-\vec{u}, 2\vec{u}$ (D) $\vec{u}, -2\vec{u}$

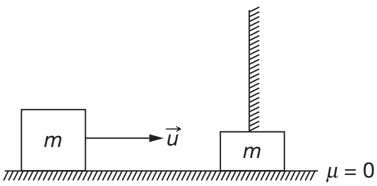


Fig. 18.43

3. A concave mirror of focal length f produces an image n times the size of the object. If the image is real then the distance of the object from the mirror is
   (A) $(n-1)f$ (B) $\frac{(n-1)}{n}f$
   (C) $\frac{(n+1)}{n}f$ (D) $(n+1)f$

4. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O, and $PO = OQ$. The distance PO is equal to
   (A) $5R$ (B) $3R$ (C) $2R$ (D) $1.5R$

5. A ray of light falls normally on a refracting face of a prism of refractive index 1.5. If the ray just fails to emerge from the prism. Then the angle of prism is
   (A) $\sin^{-1}\left(\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{2}{3}\right)$
   (C) $\sin^{-1}\left(\frac{1}{2}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

6. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3, and μ_4 as shown in the Fig. 18.44. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, then
   (A) $\mu_1 = \mu_2$ (B) $\mu_2 = \mu_3$
   (C) $\mu_3 = \mu_4$ (D) $\mu_4 = \mu_1$

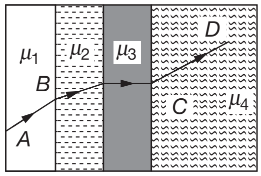


Fig. 18.44

7. Two plane mirrors A and B are aligned parallel to each other as shown in Fig. 18.45. A light ray is incident at an angle of 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the Fig. 18.45. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is
   (A) 28 (B) 30 (C) 32 (D) 34

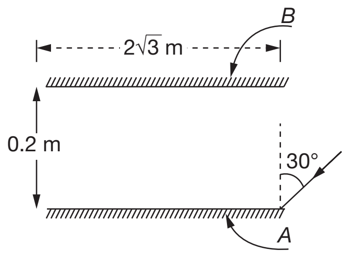


Fig. 18.45

8. An object 1 cm tall is placed in front of a mirror at a distance of 4 cm. In order to produce an upright image of 3 cm height, one needs a
   (A) convex mirror of radius of curvature 12 cm.
   (B) concave mirror of radius of curvature 12 cm.
   (C) concave mirror of radius of curvature 4 cm.
   (D) plane mirror of height 12 cm.

9. Light travels through a glass plate of thickness t and having refractive index μ. If c be the velocity of light in vacuum, the time taken by the light to travel this thickness of glass is

(A) $\frac{t}{\mu c}$ (B) $t \mu c$ (C) $\frac{\mu t}{c}$ (D) $\frac{t c}{\mu}$

10. The Young's double slits experiment is performed with blue and green light of wavelengths 4360 Å and 5460 Å, respectively. If x is the distance of fourth maximum from the central one, then

(A) x (blue) = x (green)
(B) x (blue) > x (green)
(C) x (blue) < x (green)
(D) x (blue) / x (green) = 5460 / 4360

11. Monochromatic light from a narrow slit illuminates two parallel narrow slits producing an interference pattern on a screen. The separation between the two slits is now doubled and the distance between the screen and the slits is reduced to half. The fringe width

(A) Is doubled
(B) Becomes four times
(C) Becomes one-fourth
(D) Remains the same

12. A thin sheet of glass (μ = 1.5) of thickness 6 microns introduced in the path of one of interfering beams of a double slit experiment shifts the central fringes to a position previously occupied by fifth bright fringe. Then the wavelength of the light used is

(A) 6000 Å (B) 3000 Å
(C) 4500 Å (D) 7500 Å

13. A concave lens of focal length F produces an image equal to $1/n$ of size of object, the distance of the image, from the lens is

(A) $(n+1)F$ (B) $(n-1)F$
(C) $\left(\frac{n+1}{n}\right)F$ (D) $\left(\frac{n-1}{n}\right)F$

14. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then the distance d in cm will be

(A) 25 (B) 15 (C) 30 (D) 50

15. If ε_0 and μ_0 represent the permittivity and permeability of vacuum, ε and μ represent the permittivity and permeability of medium, then refractive index of the medium is given by

(A) $\sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}}$ (B) $\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$
(C) $\sqrt{\frac{\varepsilon}{\mu_0\varepsilon_0}}$ (D) $\sqrt{\frac{\mu_0\varepsilon_0}{\mu}}$

16. A ray of light strikes a glass plate at an angle of 60° with the vertical. If the reflected and refracted rays are perpendicular to each other, the refractive index of glass is

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) $\sqrt{3}$

17. An optically active compound

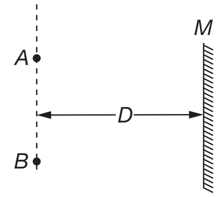
(A) Rotates the plane polarized light
(B) Changes the direction of polarized light
(C) Do not allow plane polarized light to pass through
(D) None of the above

18. The principle of optical fibre is

(A) Diffraction
(B) Polarization
(C) Interference
(D) Total internal reflection

19. A beam of light starting from point A reflects from a mirror M and forms an image at B as shown in the same horizontal plane as A. For small angle of incidence i, AB is equal to

(A) iD (B) $2iD$ (C) $4iD$ (D) $iD/2$



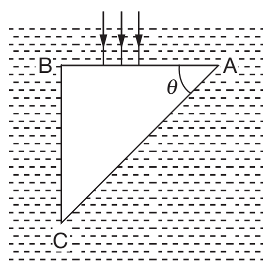
20. In Young's double slit experiment, the width of one slit is double that of the other. The ratio of intensity of a bright band to that of a dark band in the interference pattern will be

(A) 2 : 1 (B) 3 : 1 (C) 6 : 1 (D) 9 : 1

21. The critical angle of a medium is $\sin^{-1}$ (3/5). The polarizing angle of the medium will be

(A) $\sin^{-1}$ (4/5) (B) $\tan^{-1}$ (5/3)
(C) $\tan^{-1}$ (3/4) (D) $\tan^{-1}$ (4/3)

22. A glass prism of refractive index 1.5 is immersed in water ($\mu = 4/3$). A light beam incident normally on the face AB is totally reflected to reach the face BC, if



(A) $\sin\theta \geq \frac{8}{9}$ (B) $\sin\theta \leq \frac{2}{3}$

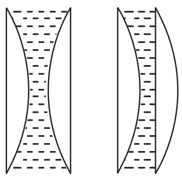
(C) $\sin\theta = \frac{4}{5}$ (D) $\frac{2}{3} < \sin\theta < \frac{8}{9}$

23. An object is placed at a distance of $3f$ from a convex lens of focal length f. A slab of refractive index μ is placed in between lens and object. The image of the object will be formed nearest to the object if thickness of the slab is

(A) f (B) $2f$ (C) $\frac{f}{\mu-1}$ (D) $\frac{\mu f}{\mu-1}$

24. A liquid of refractive index 1.33 is placed between two identical plano-convex lenses, with refractive index 1.50. Two possible arrangements, *P* and *Q,* are shown. The system is

(A) divergent in *P*, convergent in *Q*.
(B) convergent in *P*, divergent in *Q*.
(C) convergent in both.
(D) divergent in both.



25. A concave lens of focal length 10 cm and a convex lens of focal length 20 cm are placed certain distance apart. If parallel rays incident on one lens become converging after passing through other lens, then the separation between the lenses must be greater than

(A) Zero (B) 5 cm
(C) 10 cm (D) 9 cm

26. Light is incident normally on face *AB* of a prism as shown in Fig. 18.46. A liquid of refractive index μ is placed on face *AC* of the prism. The prism is made of glass of refractive index 3/2. The limits of μ for which total internal reflection cannot takes place on face *AC* is

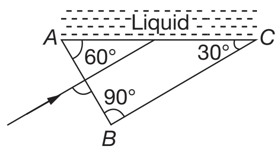


Fig. 18.46

(A) $\frac{3\sqrt{3}}{4} > \mu > \frac{\sqrt{3}}{2}$ (B) $\mu < \frac{3\sqrt{3}}{4}$

(C) $\mu > \sqrt{3}$ (D) $\mu < \frac{\sqrt{3}}{2}$

27. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then in the interference pattern

(A) The intensities of both the maximum and minimum increase
(B) The intensity of the maximum increases and the minimum has zero intensity
(C) The intensity of the maximum decreases and that of minimum increases
(D) The intensity of the maximum decreases and the minimum has zero intensity

28. A bulb is placed at a depth of $2\sqrt{7}$ m in water and a floating opaque disc is placed over the bulb so that the bulb is not visible from the surface. The radius of the disc should be at least ($\mu_{water} = 4/3$)

(A) 42 m (B) 6 m (C) $2\sqrt{7}$ m (D) 12 m

29. A ray of light falls on a prism *ABC* (*AB* = *BC*) and travels as shown in Fig. 18.47. The refractive index of the prism material should be greater than

(A) 4/3 (B) $\sqrt{2}$
(C) 1.5 (D) $\sqrt{3}$

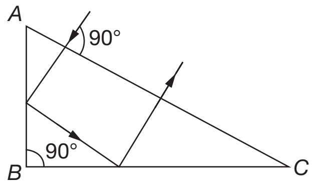


Fig. 18.47

30. On introducing a thin sheet of mica (thickness 12×10^{-5}cm) in path of one of the interfering beams in Young's double slit experiment, the central fringe is shifted through a distance equal to the spacing between successive bright fringes. The refractive index of mica is (wavelength of light used $\lambda = 6 \times 10^{-5}$ cm)

(A) 1.33 (B) 1.5 (C) 2.5 (D) 1.478

31. A plane glass plate is kept on a paper on which letters are printed in various colours; colour of the letters which will be more close to upper surface is

(A) yellow (B) red (C) blue (D) green

32. Focal length of an equiconvex lens is 20 cm. If we cut it once perpendicular to principle axis, and then along principal axis, then focal length of each part will be

(A) 20 cm (B) 10 cm (C) 40 cm (D) 5 cm

33. Rays are converging towards a convex mirror, final image will be

(A) Real
(B) Virtual
(C) May be real or virtual
(D) Image will not form

34. In YDSE distance between the slits plane and screen is 1 m and distance between two slits is 5 mm. If slabs of thickness 2 mm and 1.5 mm having refractive index 1.5 and 1.4 are placed in front of two slits, the shift of central maximum will be

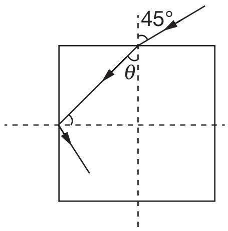
(A) 2 m (B) 8 cm
(C) 20 cm (D) 80 cm

35. We have two equilateral prisms *A* and *B*. They are made of materials having refractive index 1.5 and 1.6. For minimum deviation, incident angle will be

(A) Small for prism *A*
(B) Small for prism *B*
(C) Equal for both the prisms
(D) Can't be predicted

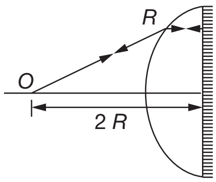
36. A light ray falls on a square slab at an angle 45°. What must be the minimum index of refraction of glass, if total internal reflection takes place at the vertical face?

(A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\frac{3}{2}$ (D) $\frac{3}{\sqrt{2}}$

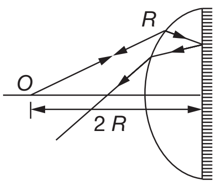


37. A thin plano-convex glass lens ($\mu = 1.5$) has its plane surface silvered, and *R* is the radius of curvature of curved part, then which of the following ray diagram is true for an object placed at *O*?

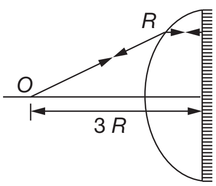
(A)



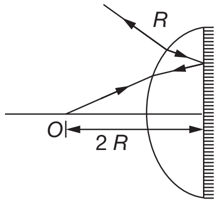
(B)



(C)



(D)



38. Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of thickness 2.5×10^{-3} cm was placed in front of one of the slits, the fringe pattern shifted by a distance equal to 20 fringe-widths. The refractive index of the material of the film is

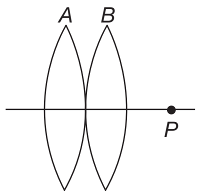
(A) 1.25 (B) 1.35
(C) 1.4 (D) 1.5

39. A thin convergent glass lens ($\mu_g = 1.5$) has a power of +5.0 *D*. When this lens is immersed in a liquid of refractive index μ_l, it acts as a divergent lens of focal length 100 cm. The value of μ_l is

(A) 4/3 (B) 5/3
(C) 5/4 (D) 6/5

40. Two convex lenses placed in contact form the image of a distant object at *P*. If the lens *B* is moved to the right, the image will

(A) move to the left.
(B) move to the right.
(C) remain at *P*.
(D) move either to the left or right, depending upon focal lengths of the lenses.



41. A thin film of thickness *t* and index of refraction 1.33 coats a glass with index of refraction 1.50. Which of the following thickness *t* will not reflect normally incident light with wavelength 640 nm in air?

(A) 120 nm (B) 240 nm
(C) 300 nm (D) 480 nm

42. For a concave mirror of focal length 20 cm, if the object is at a distance of 30 cm from the pole, then the nature of the image and its magnification will be

(A) real and –2 (B) virtual and –2
(C) real and +2 (D) virtual and +2

43. The relation between lateral magnification *m*, object distance *u*, and focal length *f* of a spherical mirror is

(A) $m = \frac{f-u}{f}$ (B) $m = \frac{f}{f+u}$

(C) $m = \frac{f+u}{f}$ (D) $m = \frac{f}{f-u}$

44. In a Young's double slit experiment, the fringe width is found to be 0.4 mm. If the whole apparatus is immersed in water of refractive index (4/3), without disturbing the geometrical arrangement, the new fringe width will be

(A) 0.30 mm (B) 0.40 mm
(C) 0.53 mm (D) 450 microns

45. Light of wavelength 6328 Å is incident normally on a slit having a width of 0.2 mm. The width of the central maximum measured from minimum to minimum of diffraction pattern on a screen 9.0 metres away will be about

(A) 0.36° (B) 0.18°
(C) 0.72° (D) 0.09°

46. Two polaroids are kept crossed to each other. Now one of them is rotated through an angle of 45°. The percentage of unpolarized incident light now transmitted through the system is

(A) 15 % (B) 25 %
(C) 50 % (D) 60 %

47. A ray incident at a point at an angle of incidence of 60° enters a glass sphere of R.I. $n = \sqrt{3}$ and gets reflected and refracted at the farther surface of the sphere. The angle between the reflected and refracted rays at this surface is

(A) 50° (B) 60° (C) 90° (D) 40°

48. A microscope has an objective of focal length 1.5 cm and an eye-piece of focal length 2.5 cm. If the distance between objective and eye-piece is 25 cm, what is the approximate value of magnification produced for relaxed eye?

(A) 75 (B) 110 (C) 140 (D) 25

49. A circular beam of light of diameter $d = 2$ cm falls on a plane surface of glass. The angle of incidence is 60° and refractive index of glass is $\mu = 3/2$. The diameter of the refracted beam is

(A) 4.0 cm (B) 3.0 cm
(C) 3.26 cm (D) 2.52 cm

50. The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10\ d$?

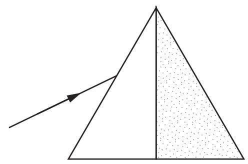
(A) $\frac{I_0}{2}$ (B) $\frac{3I_0}{4}$ (C) I_0 (D) $\frac{I_0}{4}$

51. Light wave enters from medium 1 to medium 2. Its velocity in second medium is double from first. For total internal reflection, the angle of incidence must be greater than

(A) 30° (B) 60° (C) 45° (D) 90°

52. A light ray is incident upon a prism in minimum deviation position and suffers a deviation of 34°. If the shaded half of the prism is knocked off, the ray will

(A) suffer a deviation of 34°.
(B) suffer a deviation of 68°.
(C) suffer a deviation of 17°.
(D) not come out of the prism.



53. The light ray is incident at angle of 60° on a prism of angle 45°. When the light ray falls on the other surface at 90°, the refractive index of the material of prism μ and the angle of deviation δ are given by

(A) $\mu = \sqrt{\frac{3}{2}}, \delta = 30°$ (B) $\mu = 1.5, \delta = 15°$

(C) $\mu = \frac{\sqrt{3}}{2}, \delta = 30°$ (D) $\mu = \sqrt{\frac{3}{2}}, \delta = 15°$

54. In a Young's experiment, two coherent sources are placed 0.90 mm apart and the fringes are observed 1 m away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of monochromatic light used would be

(A) 60×10^{-4} cm (B) 10×10^{-4} cm
(C) 10×10^{-5} cm (D) 6×10^{-5} cm

55. If two lenses of power +5 diopters are mounted at some distance apart, the combination will always behave like a diverging lens if the distance between them is

(A) Greater than 40 cm (B) Equal than 40 cm
(C) Equal to 10 cm (D) Less than 10 cm

56. When the angle of incidence on a material is 60°, the reflected light is completely polarized. The velocity of the refracted ray inside the material is (in m/s)

(A) 3×10^{8} (B) $\left(\frac{3}{\sqrt{2}}\right)\times10^{8}$
(C) $\sqrt{3}\times10^{8}$ (D) 0.5×10^{8}

57. An object is placed at 20 cm from a convex mirror of focal length 10 cm. The image formed by the mirror is
(A) real and at 20 cm from the mirror.
(B) virtual and at 20 cm from the mirror.
(C) virtual and at (20/3) cm from the mirror.
(D) real and at (20/3) cm from the mirror.

58. A convex lens of focal length 12 cm is made of glass of $\mu = \frac{3}{2}$. What will be its focal length when immersed in liquid of $\mu = \frac{5}{4}$
(A) 6 cm (B) 12 cm
(C) 24 cm (D) 30 cm

59. For a material, the refractive indices for red, violet, and yellow colour light are, respectively, 1.52, 1.64, and 1.60. The dispersive power of the material is
(A) 2 (B) 0.45 (C) 0.2 (D) 0.045

60. The refractive index of water is 4/3 and that of glass is 5/3. Then the critical angle for a ray of light entering in water from glass will be
(A) $\sin^{-1}\left(\frac{4}{5}\right)$ (B) $\sin^{-1}\left(\frac{5}{4}\right)$
(C) $\sin^{-1}\left(\frac{20}{9}\right)$ (D) $\sin^{-1}\left(\frac{9}{20}\right)$

61. In Young's double slit experiment, when sodium light of wavelength 5893 Å is used, then 62 fringes are seen in the field of view. Instead of sodium light, if violet light of wavelength 4358 Å is used, then the number of fringes that will be seen in the field of view will be
(A) 54 (B) 64 (C) 74 (D) 84

62. A prism (μ = 1.5) has a refracting angle of 30°. The deviation of a monochromatic ray incident normally on its one surface will be (sin48°36′ = 0.75)
(A) 18°36′ (B) 22°38′
(C) 18° (D) 22°1

63. The length of a vertical pole above the surface of a lake of water (n = 4/3) is 24 cm. To an underwater fish, just below the water surface, the tip of the pole appears to be
(A) 18 cm above the surface.
(B) 24 cm above the surface.
(C) 32 cm above the surface.
(D) 36 cm above the surface.

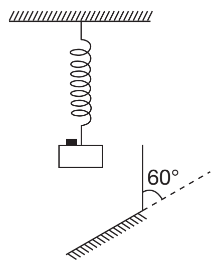
64. An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, there is no parallax between the images formed by the two mirrors. The radius of curvature of the convex mirror (in cm) is
(A) 60 (B) 50
(C) 30 (D) 25

65. A plano-convex lens has a thickness of 4 cm. When placed on a horizontal table with curved surface in contact with it, the apparent depth of the bottom-most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of plane face is found to be 25/8 cm. The focal length of the lens is
(A) 50 cm (B) 75 cm
(C) 100 cm (D) 150 cm

66. A lens of power +2 diopters is placed in contact with a lens of power –1 diopter. The combination will behave like
(A) a convergent lens of focal length 50 cm.
(B) a divergent lens of focal length 100 cm.
(C) a convergent lens of focal length 100 cm.
(D) a convergent lens of focal length 200 cm.

67. In a Young's double slit experiment, the slit separation is 1 mm and the screen is 1 m from the slit. For a monochromatic light of wavelength 500 nm, the distance of third minimum from the central maximum is
(A) 0.50 mm (B) 1.25 mm
(C) 1.50 mm (D) 1.75 mm

68. An insect of negligible mass is sitting on a block of mass M, tied with a spring of force constant k. The block performs simple harmonic motion with amplitude A in front of a plane mirror placed as shown. The maximum speed of insect relative to its image will be

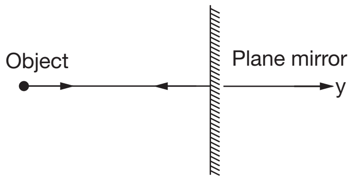


(A) $A\sqrt{\frac{k}{M}}$ (B) $\frac{A\sqrt{3}}{2}\sqrt{\frac{k}{M}}$
(C) $A\sqrt{3}\sqrt{\frac{k}{M}}$ (D) $\frac{A}{2}\frac{k}{\sqrt{M}}$

69. In Young's double slit experiment, double slit of separation 0.1 cm is illuminated by white light. A coloured interference pattern is formed on a screen 100 cm away. If a pinhole is located on this screen at a distance of 2 mm from the central fringe, the wavelength in the visible spectrum which will be absent in the light transmitted through the pin hole are

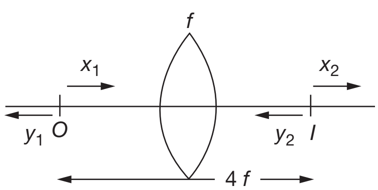
(A) 5714 Å and 4444 Å
(B) 6000 Å and 5000 Å
(C) 5500 Å and 4500 Å
(D) 5200 Å and 4200 Å

70. In the diagram shown, the object is performing SHM according to the equation $y = 2A\sin(\omega t)$ and the plane mirror is performing SHM according to the equation $Y = -A\sin\left(\omega t - \frac{\pi}{3}\right)$. The diagram shows the state of the object and the mirror at time $t = 0$ s. The minimum time from $t = 0$ s after which the velocity of the image becomes equal to zero is



(A) $\frac{\pi}{3\omega}$ (B) $\frac{3\pi}{\omega}$ (C) $\frac{\pi}{6\omega}$ (D) $\frac{2\pi}{3\omega}$

71. In a converging lens of focal length f and the distance between real object and its real image is $4f$. If the object moves x_1 distance towards lens, its image moves x_2 distance away from the lens and when object moves y_1 distance away from the lens its image moves y_2 distance towards the lens, then choose the correct option



(A) $x_1 > x_2$ and $y_1 > y_2$
(B) $x_1 < x_2$ and $y_1 < y_2$
(C) $x_1 < x_2$ and $y_1 > y_2$
(D) $x_1 > x_2$ and $y_2 > y_1$

72. A thin film of thickness t and index of refraction 1.33 coats a glass with index of refraction 1.50. Which of the following thickness t will not reflect normally incident light with wavelength 640 nm in air?

(A) 120 nm (B) 240 nm
(C) 300 nm (D) 480 nm

73. When a thin transparent sheet of refractive index $\mu = \frac{3}{2}$ is placed near one of the slits in Young's double slits experiment, the intensity at the centre of the screen reduces to half of the maximum intensity. The minimum thickness of the sheet should be

(A) $\frac{\lambda}{4}$ (B) $\frac{\lambda}{8}$ (C) $\frac{\lambda}{2}$ (D) $\frac{\lambda}{3}$

74. A thin equiconvex lens of glass has radii of curvature of its surfaces 30 cm each. This lens has different medium on its two sides as shown in the Fig. 18.48. The refractive indices of the mediums on the two sides of the lens are 1.2 and 1.6, and refractive index of the glass is 1.5. The focal length of the lens in the shown Fig. 18.48 is

(A) 30 cm (B) 60 cm
(C) 120 cm (D) 240 cm

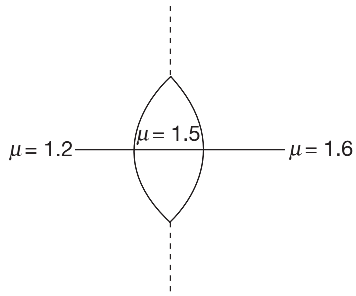
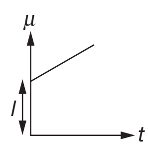


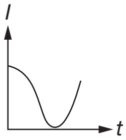
Fig. 18.48

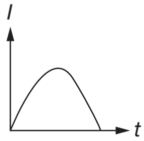
75. In an usual Young's double slit experiment with $\lambda = 5000$ Å, $d = 2$ mm, and $D = 1$ m, the number of maximum obtained on the screen if it is large enough to receive any number of fringes

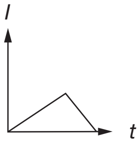
(A) 8001 (B) 6001
(C) 4001 (D) 2001

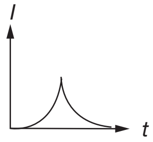
76. In YDSE, one of the slits is covered by a thin sheet of variable refractive index, whose variation with time is represented by graph as shown. The variation of intensity (I) at a point on the screen with time, where first minimum is obtained at $t = 0$ will be best represented by



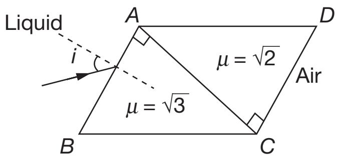
(A) 

(B) 

(C) 

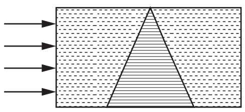
(D) 

77. A ray of light from a liquid $(\mu = \sqrt{3})$ is incident on a system of two right-angled prisms of refractive indices $\sqrt{3}$ and $\sqrt{2}$ as shown. The ray suffers zero deviation when emerges into air from *CD*. The angle of incidence *i* is



(A) 45° (B) 35° (C) 20° (D) 10°

78. A thin isosceles prism with angle 4° and refractive index 1.5 is placed inside a transparent tube with water $\left(\text{refractive index} = \dfrac{5}{4}\right)$ as shown. The deviation of light due to prism will be



(A) 0.8° upward (B) 0.8° downward
(C) 0.67° upward (D) 0.67° downward

79. A ray of light is incident on the plane mirror at rest. The mirror starts turning at a uniform angular acceleration of 2π rad s^{-2}. The reflected ray, at the end of $\dfrac{1}{4}$ s must have turned through

(A) 90° (B) 45° (C) 22.5° (D) 11.25°

80. A thin rod of length $f/3$ is placed along the optic axis of a concave mirror of focal length f such that its image, which is real and elongated, just touches the rod. The magnification is

(A) 2 (B) 4 (C) 2.4 (D) 1.5

81. The plane face of plano-convex lens of focal length 20 cm is silvered. This combination is equivalent to the type of mirror and its focal length is

(A) convex, f = 20 cm
(B) concave, f = 20 cm
(C) convex, f = 10 cm
(D) concave, f = 10 cm

82. The magnifying power of an astronomical telescope in normal adjustment is 8 and the distance between the two lenses is 54 cm. The focal length of eye lens and objective lens will be, respectively.

(A) 6 cm and 48 cm (B) 48 cm and 6 cm
(C) 8 cm and 64 cm (D) 64 cm and 8 cm

83. An astronomical telescope has magnifying power 10. The focal length of the eyepiece is 20 cm. The focal length of the objective is

(A) $\dfrac{1}{200}$ cm (B) $\dfrac{1}{2}$ cm
(C) 2 cm (D) 200 cm

84. In a displacement method using convex lens, two images are obtained for a separation of *d* between the positions of the lens. One image is magnified and the other is diminished. If *m* is the magnification of one image, the focal length of the lens is $(m > 1)$

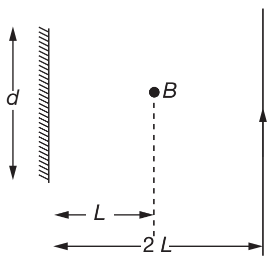
(A) $d/(m - 1)$ (B) $md/(m^2 - 1)$
(C) $d/(m^2 - m)$ (D) $(m - 1)d$

85. If a graph is drawn between the separation of slits and bandwidth in Young's double slit experiment, the graph will be

(A) a straight line having positive slope.
(B) a straight line having negative slope.
(C) a rectangular hyperbola.
(D) a parabola.

86. A point source of light *B* is placed at a distance *L* in front of the centre of a mirror of width *d* hung vertically on a wall. A man walks in front of the mirror at a distance 2*L* from it as shown. The greatest distance over which he can see the image of the light source in the mirror is

(A) $d/2$ (B) d (C) $2d$ (D) $3d$



87. A ray of light falls on the surface of a spherical glass paperweight making an angle α with the normal and is refracted in the medium at an angle β. The angle of deviation of the emergent ray from the direction of the incident ray is

(A) $(\alpha - \beta)$ (B) $2(\alpha - \beta)$
(C) $(\alpha - \beta)/2$ (D) $(\alpha + \beta)$

88. An observer can see through a pinhole the top end of a thin rod of height h, placed as shown in the Fig. 18.49. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is

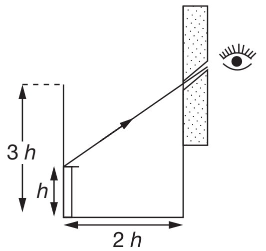
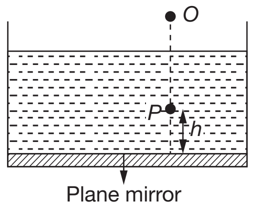


Fig. 18.49

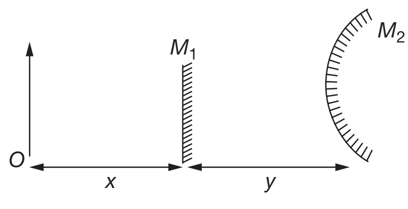
(A) 5/2 (B) $\sqrt{5/2}$
(C) $\sqrt{3/2}$ (D) 3/2

89. A plane mirror is placed at the bottom of a tank containing a liquid of refractive index μ, P is a small object at a height h above the plane mirror. An observer O, vertically above P, outside the liquid, observers P and its image in the mirror. The apparent distance between object and its image will be



(A) $2\mu h$ (B) $\frac{2h}{\mu}$ (C) $\frac{2h}{\mu - 1}$ (D) $h\left(\frac{1}{\mu}\right)$

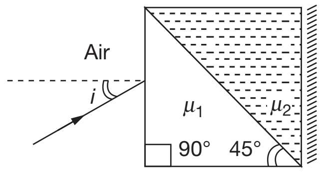
90. An object O is placed in front of a small plane mirror M_1 and a large convex mirror M_2 of focal length f. The distance between O and M_1 is x, and the distance between M_1 and M_2 is y. The images of O formed by M_1 and M_2 coincide. The magnitude of f is



(A) $\frac{x^2 - y^2}{2y}$ (B) $\frac{x^2 + y^2}{2y}$

(C) $x - y$ (D) $\frac{x^2 + y^2}{x - y}$

91. In the given situation, for what value of i, the incidence ray will retrace its initial path



(A) $\cos^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$

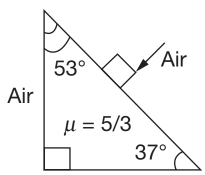
(B) $\sin^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$

(C) $\sin^{-1}\left[\mu_2 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_1}{\sqrt{2}\mu_2}\right)\right]$

(D) $\cos^{-1}\left[\mu_2 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_1}{\sqrt{2}\mu_2}\right)\right]$

92. In the given situation, what is the angle of deviation?

(A) 53° (B) 127° (C) 37° (D) 143°



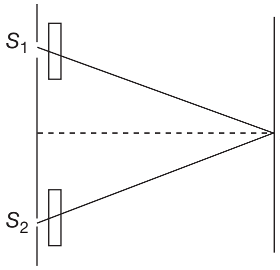
93. One slit in a YDSE set up is covered by a glass plate (Refractive index = μ_1) and other by another glass plate (Refractive index = μ_2) of same thickness. If I_0 is the intensity of light through each slit and λ is the wavelength of light, then intensity at point P is (P is symmetrical with respect to slits S_1 and S_2)

(A) $4I_0 \cos^2 \frac{\pi}{\lambda}(\mu_1 - \mu_2)t$

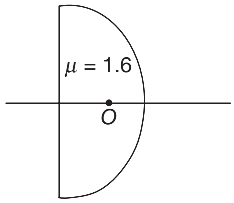
(B) $4I_0 \cos^2 \frac{\pi}{\lambda}(\mu_1 + \mu_2)t$

(C) $4I_0 \sin \frac{\pi}{\lambda}(\mu_1 - \mu_2)t$

(D) $4I_0 \sin \frac{\pi}{\lambda}(\mu_1 + \mu_2)t$



94. A plastic hemisphere has a radius of curvature of 8 cm and an index of refraction of 1.6. On the axis half way between the plane surface and the spherical one (4 cm from each) is a small object *O*. The distance between the two images when viewed along the axis from the two sides of the hemisphere is approximately.



(A) 1.0 cm (B) 1.5 cm
(C) 3.75 cm (D) 2.5 cm

95. *x*-axis is normal to the reflecting surface of a plane mirror. If object is moving with velocity $(3\hat{i}+4\hat{k})$m/s The relative velocity of image with respect to object will be along
(A) –*x* axis (B) +*x* axis
(C) –*z* axis (D) +*z* axis

96. A monochromatic light of wavelength 4500 Å fall on a single slit of width 1.5 mm and resulting diffraction pattern is observed with the help of a lens of focal length 10 m. Angular width of central bright fringe will be
(A) $6 \times 10^{-4\circ}$ (B) 0.034°
(C) 0.034 radian (D) 2.4°

97. A simple telescope, consisting of an objective of focal length 30 cm and a single eye lens of focal length 6 cm. Eye observes final image in relaxed condition. If the angle subtended on objective is 1.5° then image will subtend angle of
(A) 6.5° (B) 7.5° (C) 0.3° (D) 2.5°

98. When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1. The object is moved 15 cm away with respect to the earlier position along principal axis, magnification becomes m_2. If $m_1 \times m_2 = 4$, the focal length of the mirror is
(A) 10 cm (B) 30 cm
(C) 15 cm (D) 20 cm

99. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. The radius of curvature of the curved part is approximately.
(A) 15 cm (B) 20 cm
(C) 30 cm (D) 10 cm

100. η number of identical equilateral prisms are kept in contact as shown in Fig. 18.50 . If deviation through a single prism is δ then consider the following statements (*m* is an integer).

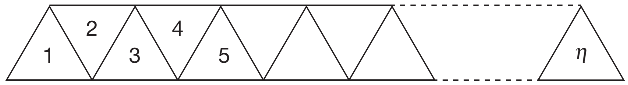


Fig. 18.50

(i) if (η = 2m) deviation through η prisms is zero
(ii) if (η = 2m + 1) deviation through system of η prisms is δ
(iii) if η = 2m deviation through system of η prisms is δ
(iv) if η = 2m + 1 deviation through system of η prisms is zero
(A) statements (i) and (iii) are true
(B) statements (i) and (ii) are true
(C) statements (i) and (iv) are true
(D) statements (iii) and (iv) are true

101. At a point on the screen in YDSE, third maximum is observed at *t* = 0. Now screen is slowly moved with constant speed away from the slits in such a way that the centre of slits and centre of screen lie on same line always and at *t* = 1s, the intensity at that point is observed to be $\left(\frac{3}{4}\right)^{th}$ of maximum intensity in between second and third maximum. The speed of screen will be (*D* = 1m)
(A) $\frac{5}{13}$ m/s (B) $\frac{5}{12}$ m/s
(C) $\frac{12}{5}$ m/s (D) $\frac{13}{5}$ m/s

102. A thin converging lens of refractive index 1.5 has power of +5D. When this lens is immersed in a liquid, it acts as a diverging lens of focal length 100 cm. The refractive index of the liquid is
(A) 2 (B) $\frac{15}{11}$
(C) $\frac{4}{3}$ (D) $\frac{5}{3}$

103. In an astronomical telescope, the focal length of objective lens and eyepiece are 150 cm and 6 cm, respectively. In case when final image is formed at least distance of clear vision (*D* = 25 cm). The magnifying power is

(A) 29 (B) 30
(C) 31 (D) 32

104. Path difference for the first secondary maximum in the Fraunhofer diffraction pattern of a single slit is given by (a is the width of the slit)

(A) $a\sin\theta = \frac{\lambda}{2}$ (B) $a\cos\theta = \frac{3\lambda}{2}$
(C) $a\sin\theta = \lambda$ (D) $a\sin\theta = \frac{3\lambda}{2}$

105. A telescope of diameter 2 m uses light of wavelength 5000 Å for viewing stars. The minimum angular separation between two stars whose image is just resolved by this telescope is

(A) 4×10^{-4} rad
(B) 0.25×10^{-6} rad
(C) 0.31×10^{-6} rad
(D) 5.0×10^{-3} rad

106. A microscope is focused on a needle lying in an empty tank. Now, the tank is filled with benzene to a height 120 mm. The microscope is moved 40 mm to focus the needle again. The refractive index of benzene is

(A) 1.5 (B) 2.5
(C) 3.0 (D) 4.5

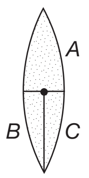
More than One Option Correct Type

107. For which of the pairs of u and f for a mirror image is smaller in size than the object

(A) $u = -10$ cm, $f = 20$ cm
(B) $u = -20$ cm, $f = -30$ cm
(C) $u = -45$ cm, $f = -10$ cm
(D) $u = -60$ cm, $f = 30$ cm

108. A thin, symmetric double-convex lens of power P is cut into three parts A, B, and C as shown. The power of

(A) A is P (B) A is $2P$
(C) B is $\frac{P}{2}$ (D) B is $\frac{P}{4}$



109. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen

(A) half of the image will disappear.
(B) complete image will be formed.
(C) intensity of the image will increase.
(D) intensity of the image will decrease.

110. For a concave mirror

(A) virtual image is always larger in size.
(B) real image is always smaller in size.
(C) real image is always larger in size.
(D) real image may be smaller or larger in size.

111. Consider the situation shown in the Fig. 18.51. Two slits S_1 and S_2 are placed symmetrically about the line OP which is perpendicular to screen and bisector to line joining the slits. The space between screen and slits is filled with a liquid of refractive index μ_3. A plate of thickness t and refractive index μ_2 is placed in front of one of the slit. A source S is placed above OP at a distance d in front of slit.

Given that $D = 1$ m, $d = 2$ mm, $t = 6 \times 10^{-6}$m, $\mu_2 = 1.2$, $\mu_3 = 1.8$, Choose the correct alternatives

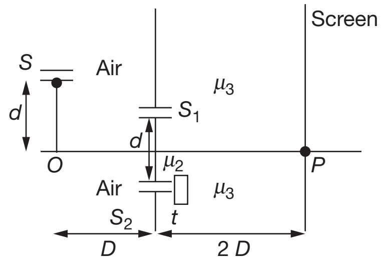


Fig. 18.51

(A) Position of central maximum from point P is 2 mm.
(B) Position of central maximum from point P is 1 mm.
(C) If slab is removed, the central maximum shift by a distance of 2 mm.
(D) If slab is removed, the central maximum shift by a distance of 1 mm.

112. An object of length 1cm is placed on the principal axis of biconvex lens of radius 5 cm. Distance between the lens and object is 20 cm. Space between the lens and object is filled with medium of two different refractive indices 2 and 1 as shown in the Fig. 18.52. Refractive index is 1 on the left of the object and on the right side of the lens. Boundary of both medium is midway between the object and lens as shown in Fig. 18.52.

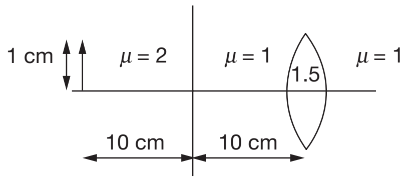
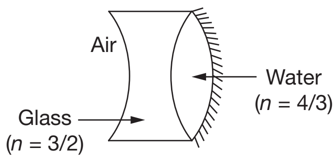


Fig. 18.52

(A) The image will be formed at distance of 7.5 cm from the optical centre.
(B) The image will be formed at distance of 10 cm from the optical centre.
(C) The size of the image is 0.5 cm.
(D) The size of the image is 0.4 cm.

113. The radius of curvature of the left and right surface of the concave lens are 10 cm and 15 cm, respectively. The radius of curvature of the mirror is 15 cm
(A) Equivalent focal length of the combination is –18 cm
(B) Equivalent focal length of the combination is +36 cm
(C) The system behaves like a concave mirror
(D) The system behaves like a convex mirror



114. A point object is placed at 30 cm from a convex glass lens $\left(\mu_g = \frac{3}{2}\right)$ of focal length 20 cm. The final image of object will be formed at infinity if
(A) Another concave lens of focal length 60 cm is placed in contact with the previous lens.
(B) Another convex lens of focal length 60 cm is placed at a distance of 30 cm from the first lens.
(C) The whole system is immersed in a liquid of refractive index $\frac{4}{3}$.
(D) The whole system is immersed in a liquid of refractive index $\frac{9}{8}$.

115. In an interference arrangement, similar to Young's double slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by distance $d = 150.0$ m. The intensity $I_{(\theta)}$ is measured as a function of θ, where θ is defined as shown in the Fig. 18.53. If I_0 is maximum intensity, then $I_{(\theta)}$ for $0 \le \theta \le 90$ is given by

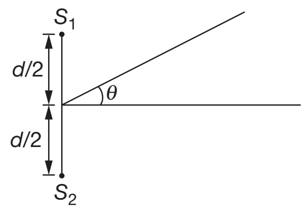


Fig. 18.53

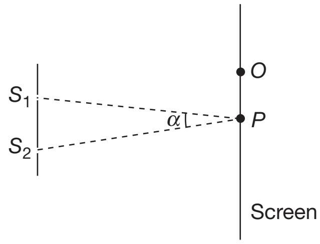
(A) $I_{(\theta)} = I_0$ for $\theta = 0°$
(B) $I_{(\theta)} = (I_0 / 2)$ for $\theta = 30°$
(C) $I_{(\theta)} = (I_0 / 4)$ for $\theta = 90°$
(D) $I_{(\theta)}$ is constant for all values of θ

116. A ray is incident on a spherical body ($\mu = \sqrt{3}$) making an angle 60° with the normal drawn at that point. The ray after passing through sphere gets incident on the farther surface of sphere and gets reflected and refracted. Then choose the correct alternative
(A) The angle of refraction at first surface is 60°.
(B) The angle of refraction at first surface is 30°.
(C) The angle of incidence at second surface is 30°.
(D) The angle between reflected ray and refracted ray at second surface is 90°.

Passage Based Questions

Passage 1

In Young's double slit experimental arrangement, two slits S_1 and S_2 are illuminated by a source of monochromatic light of wavelength $\lambda = 5000$ Å. Both the slits transmit light of intensity I_0. Distance between plane of slits and screen is $D = 1$ m and $S_1 S_2$ subtends an angle $\alpha = 10^{-3}$ radian at the centre of central bright fringe (O). Answer the following questions related to above experiment.



117. Total numbers of fringes can be seen on the screen will be

(A) 2001 (B) 4001
(C) 6001 (D) 8001

118. Distance between centre of second bright fringe on one side to centre of third bright fringe on other side of central bright fringe (O) will be

(A) 0.25 mm (B) 0.75 mm
(C) 1.75 mm (D) 2.50 mm

119. At point P, which is y mm above the centre of central bright fringe (O) intensity is found to be I_0. The value of y can be nearly

(A) 0.17 mm (B) 0.5 mm
(C) 1.2 mm (D) 1.5 mm

Passage 2

In physics laboratory, two friends Peter and Paul are trying to calculate the focal lengths of two thin lenses. One is convex lens and the other is plano-convex lens. But only one optical bench is available. Peter started experiment with convex lens. He measures the distance between a screen and a light source lined up on the optical bench to be 120 cm. When Peter shifts the convex lens along the axis of optical bench, sharp images of the source is obtained at two lens positions. He also measures the ratio of these two magnifications to be 1 : 9. Meanwhile, Paul splits the plano-convex lens into two equal halves along its optical axis. One of the halves is shifted along optical axis. Paul measures, the separation between object and image planes to be 1.8 m. The magnification of the image formed by one of the half lens is 2.

120. The focal length of the convex lens measured by Peter is

(A) 22.5 cm (B) 30 cm
(C) 45 cm (D) None of these

121. Which image, as seen by Peter, is brighter?

(A) Smaller
(B) Bigger
(C) Both are equally bright
(D) Cannot be judged

122. The separation between the two halves as measured by Paul is

(A) 50 cm (B) 40 cm
(C) 60 cm (D) 70 cm

Passage 3

A ray of light travelling in air is incident at incident angle 90° on a long rectangular slab of a transparent medium of thickness $t = 1.0$ m as shown in the Fig. 18.54. The point of incidence is the origin A (0, 0). The medium has a variable index of refraction $n(y)$ given by $n(y) = \left[ky^{3/2} + 1\right]^{1/2}$, where $k = 1.0\ (\text{m})^{-3/2}$. The refractive index of air is 1.0.

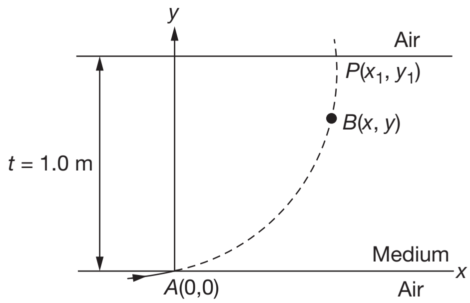


Fig. 18.54

123. The relation between θ (if slope of the trajectory of the path of ray at a point B (x, y) in the medium is tan θ) and refractive index n at point B is

(A) $\dfrac{\sin\theta}{\theta} = \dfrac{\sqrt{n^2-1}}{\sin^{-1}(1/n)}$

(B) $\dfrac{\cos\theta}{\theta} = \dfrac{\sqrt{n^2-1}}{\sin^{-1}(1/n)}$

(C) $\dfrac{\tan\theta}{\theta} = \dfrac{\sqrt{n^2-1}}{\sin^{-1}(1/n)}$

(D) $\dfrac{\cot\theta}{\theta} = \dfrac{\sqrt{n^2-1}}{\sin^{-1}(1/n)}$

124. The equation of the trajectory of the ray in the medium is

(A) $4y^{1/4} = k^{1/2}x$ (B) $4y^{1/2} = k^{1/2}x$
(C) $4y^{1/4} = k^{1/2}x^{1/2}$ (D) $4y^{1/2} = k^{1/2}x^{1/2}$

125. The coordinates (x_1, y_1) of the point P, where the ray intersects upper surface of the slab-air boundary are

(A) $P(x_1, y_1) = P(1, 4)$

(B) $P(x_1, y_1) = P(4, 1)$

(C) $P(x_1, y_1) = P(1, 2)$

(D) $P(x_1, y_1) = P(2, 1)$

126. After coming out of medium, ray of light will move

(A) Parallel to x-axis
(B) Parallel to y-axis
(C) At an angle 45° with x-axis
(D) On a zig-zag path

Passage 4

An object is placed at a distance 3 m from an equiconvex lens. Its image is formed at 2 m from the lens as shown in the Fig. 18.55.

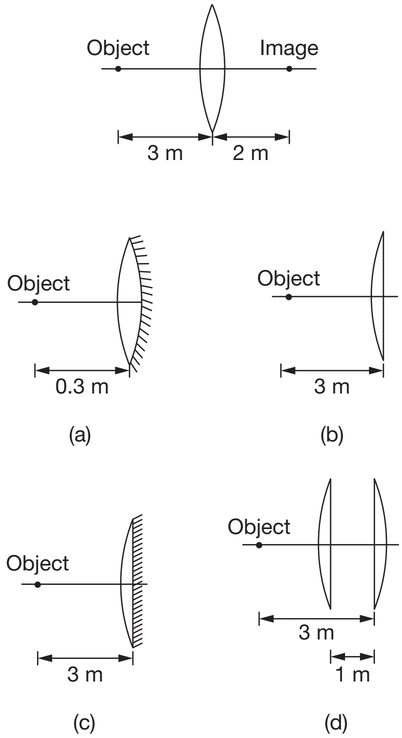


Fig. 18.55

127. Find the position of image if one face of the lens is silvered [Fig. 18.55 (a)].

(A) 12 m from the lens towards right
(B) 12 m from the lens towards left
(C) 6 m from the lens towards right
(D) At infinity

128. Find the position of image if lens is cut into two symmetrical plano-convex lens and one of the plano-convex lens is removed [Fig. 18.55 (b)].

(A) 12 m from the lens towards right
(B) 12 m from the lens towards left
(C) 6 m from the lens towards right
(D) 6 m from the lens towards left

129. Find the position of image if plano-convex surfaces are displaced by 100 cm [Fig. 18.55 (d)].

(A) 2.94 m towards right of second lens
(B) 2.94 m towards right of first lens
(C) 3.31 m towards right of second lens
(D) 4.31 m towards right of first lens

Passage 5

In mathematics, equation $\frac{1}{y} + \frac{1}{x} = \frac{1}{c}$ represents a rectangular hyperbola, which has the centre at (c, c). If we want to shift this hyperbola by c units towards negative x-axis and c units towards negative y-axis, that is to shift its centre at (0, 0), we have to put $y = Y + c$ and $x = X + c$. After substituting these values of x and y, we get a rectangular hyperbola with x- and y-axis as its asymptotes. We can use same substitution in mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ to make simple formula for solving problems of mirror.

130. If we put $v = V + f$ and $u = U + f$, the mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ becomes

(A) $(V + f)(U + f) = f^2$ (B) $VU = f^2$
(C) $(V - f)(U - f) = f^2$ (D) $VU = 2f^2$

131. To use the formula, which we get in above question, we have to shift the origin, from pole at

(A) $+f$ (B) $-f$ (C) $2f$ (D) $-2f$

132. Assuming that we have taken the origin at the same place as in previous question. A point object is placed on the principal axis 18 cm from the origin in front of a convex mirror of focal length 6 cm. The distance of image from the pole of mirror is

(A) 6 cm (B) 10 cm
(C) 25 cm (D) 4 cm

Assertion-Reason Type

133. **Assertion:** A lens has two principal focal lengths which may differ.

Reason: Light can fall on either surface of the lens. The two principal focal lengths differ when medium on the two sides have different refractive indices.

(A) A (B) B
(C) C (D) D

134. **Assertion:** Image formed by concave lens is not always virtual.

Reason: Image formed by a lens is real if the image is formed in the direction of ray of light with respect to the lens.

(A) A (B) B
(C) C (D) D

135. **Assertion:** Different colours of light have same velocity in vacuum, but they have different velocities in any other transparent medium.

Reason: $v = c/\mu$, where symbols have standard meaning. For different colours, refractive index, μ of transparent medium has different values. Therefore, v is different

(A) A (B) B
(C) C (D) D

136. **Assertion:** Light from an object falls on a concave mirror forming a real image of the object. If both the object and mirror are immersed in water, there is no change in the position of the image.

Reason: The formation of image by reflection does not depend on surrounding medium, so there is no change in position of image, provided it is also formed in water.

(A) A (B) B
(C) C (D) D

137. **Assertion:** If a convex lens is placed in water, its converging power decreases.

Reason: The focal length of a lens is independent of the refractive index of the material of the lens.

(A) A (B) B
(C) C (D) D

138. **Assertion:** If lower half of a lens is covered with a black paper; the full image of the object is formed.

Reason: Every portion of lens forms the full image of the object.

(A) A (B) B
(C) C (D) D

139. **Assertion:** The images formed by total internal reflections are much brighter than those formed by mirrors or lenses.

Reason: There is no loss of intensity in total internal reflection.

(A) A (B) B
(C) C (D) D

140. **Assertion:** No interference pattern is detected when two sources are infinitely close to each other.

Reason: The fringe width is inversely proportional to the distance between the two slits.

(A) A (B) B
(C) C (D) D

141. **Assertion:** The coating of optical fibres of a light pipe is of a material having refractive index more than that of the fibre, to optically insulate from each other.

Reason: The optical fibre works on the principal of total internal reflection.

(A) A (B) B (C) C (D) D

142. **Assertion:** A plano-convex lens is silvered on plane surface. It can act as a diverging mirror.

Reason: Focal length of concave mirror is independent of medium.

(A) A (B) B (C) C (D) D

Match the Column Type

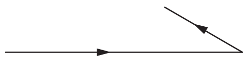
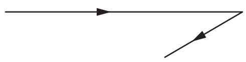
143. Match the following

Column-I	Column-II
(A) Young's double slit experiment uses	1. Incoherent sources
(B) Sources of variable phase difference	2. Coherent sources
(C) A point on a wave front behaves as a light source	3. Superposition principle
(D) Net displacement is the vector sum of individual displacement.	4. Huygens principle

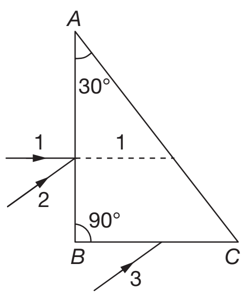
144. Match the statement given in Column-I with those given in Column-II.

Column-I	Column-II
(A) In refraction	1. Speed of wave does not change
(B) In reflection	2. Wavelength is decreased
(C) In refraction of a ray moving from rarer to a denser medium.	3. Frequency doesn't change
(D) In reflection of a ray moving in rarer from a denser medium.	4. Phase change of π radians takes place

145. Four rays of light parallel to optic axis and their path after passing through an optical system are shown in Column-I. Match the corresponding optical instrument from Column-II

Column-I	Column-II
(A) 	1. Convex lens
(B) 	2. Concave lens
(C) 	3. Convex mirror
(D) 	4. Concave mirror

146. *ABC* is a right-angled prism kept in air. A ray (1) is incident on the face *AB* along the normal. Refractive index of the material of prism is the minimum value that will be required so that ray (1) undergoes total internal reflection at the face *AC*. Another ray (2) is incident on the face *AB* such that it emerges from face *AC* along the normal to *AC*. A third ray (3) falls on the face *BC* and emerges from face *AC* such that its angle of emergence is the same as that of incidence. Assuming light (1), (2), and (3) have the same wavelength, then match the following.



Column-I	Column-II
(A) Refractive index of the material of prism is	1. 120
(B) Angle of incidence in degree of ray (2) is	2. 90
(C) Deviation in degree suffered by ray (2) is	3. 2
(D) Deviation in degree suffered by ray (3) is	4. 60
	5. 1.5

147. We have a thin plano-convex lens of focal length 40 cm and reflective index $\mu = 1.25$. It is used in various experiments as given in Column-I and results obtained are given in Column-II.

Column-I	Column-II
(A) If we obtain virtual two times magnified image, then object distance from lens (in cm) is	1. 10
(B) Curved surface of lens is polished, then lens behave like a concave mirror. Focal length of mirror (in cm) is	2. 4
(C) If an object is placed at a distance of 8 cm in front of polished lens of part (B), then distance of image from lens (in cm) is	3. 1
(D) If a slab of thickness 5cm and refractive index = 1.25 is placed between object and pole perpendicular to the principal axis in part (C), then distance (in cm) by which object should be shifted to coincide image with object is	4. 20
	5. 8

Integer Type

148. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same set-up, the two slits are source of equal amplitude A and wavelength λ, but are incoherent. The ratio of the intensity of light at the midpoint of the screen in the first case to that of second case is

149. In the given Fig. 18.56, ABC is a right-angled isosceles prism kept in air. A ray of light is incident on it normally as shown in Fig. 18.56. Refractive index of the prism is varying with time t as $\mu = 1 + 0.4\,t$, here t is in seconds. The angular velocity of the emergent ray at time $t = 1$ s is

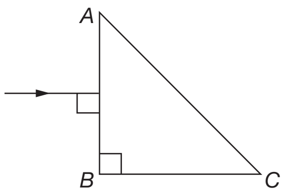
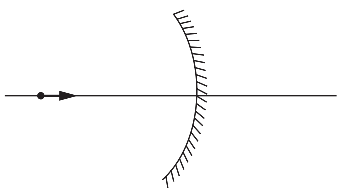


Fig. 18.56

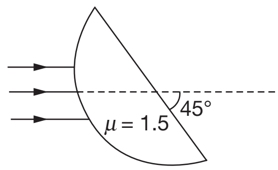
150. Unpolarized light of intensity 32 Wm^{-2} passes through three polarizers such that transmission axes of the first and second polarizer makes an angle 30° with each other and the transmission axis of the last polarizer is crossed with that of the first. The intensity of final emerging light will be

151. Magnification of a compound microscope is 30. Focal length of eyepiece is 5 cm, and the image is formed at a distance of distinct vision of 25 cm. The magnification of objective lens is

152. A point object is moving along the principal axis of a concave mirror at rest of focal length 30 cm with speed 5 m/s towards the mirror. Find the speed of image of object when object is at a distance 60 cm from mirror.



153. A parallel narrow beam of light is incident on the surface of a transparent hemisphere of radius R = 1 m and refractive index $\mu = 1.5$ as shown. The position of the image formed by refraction at the spherical surface only is



154. Two coherent sources S_1 and S_2 emitting light of wavelength 5000 Å are placed at 0.1 mm apart, as shown in the Fig. 18.57. A detector is moved along a line perpendicular to S_1S_2 and passing through S_1. The position of farthest maximum from S_1 is approximately at a distance of (in cm)

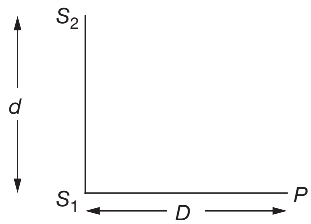


Fig. 18.57

155. A ray of light enters into a transparent liquid from air as shown in the Fig. 18.58. The refractive index of the liquid varies with depth x from the topmost surface as $\mu = \sqrt{2} - \frac{1}{\sqrt{2}}x$, where x in metres. The depth of the liquid medium is sufficiently large. The maximum depth reached by the ray inside the liquid is

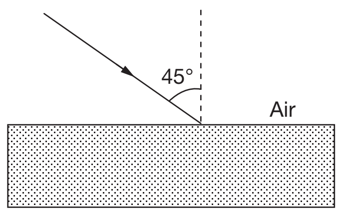


Fig. 18.58

156. What will be the angular width of central maximum in Fraunhofer diffraction when light of wavelength 6000 Å is used and slit width is 12×10^{-5} cm?

157. The distance between an object and the screen is 100 cm. A lens produces an image on the screen when placed at either of the positions 40 cm apart. The power of the lens is nearly

Previous Years' Questions

158. If two mirrors are kept at 60° to each other, then the number of images formed by them is **[2002]**

(A) 5 (B) 6 (C) 7 (D) 8

159. Wavelength of light used in an optical instrument are $\lambda_1 = 400$ Å and $\lambda_2 = 500$ Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is **[2002]**

(A) 16 : 25 (B) 9 : 1
(C) 4 : 5 (D) 5 : 4

160. Which of the following is used in optical fibres? **[2002]**

(A) Total internal reflection
(B) Scattering
(C) Diffraction
(D) Refraction

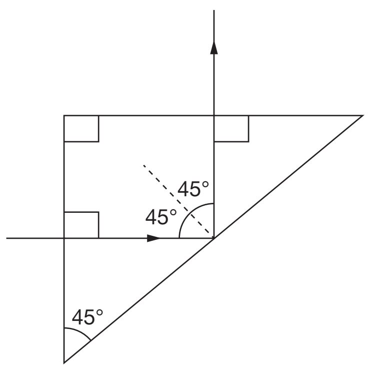
161. To demonstrate the phenomenon of interference, we require two sources which emit radiation **[2003]**

(A) of nearly the same frequency.
(B) of the same frequency.
(C) of different wavelengths.
(D) of the same frequency and having a definite phase relationship.

162. To get three images of single object, one should have two plane mirrors at an angle of **[2003]**

(A) 60° (B) 90° (C) 120° (D) 30°

163. A light ray is incident perpendicularly to one face of a 90° prism and is totally internally reflected at the glass–air interface. If the angle of reflection is 45°, we conclude that the refractive index **[2004]**



(A) $n > \frac{1}{\sqrt{2}}$ (B) $n > \sqrt{2}$

(C) $n < \frac{1}{\sqrt{2}}$ (D) $n > \sqrt{2}$

164. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of size of the object **[2004]**

(A) 60 cm (B) 30 cm
(C) 20 cm (D) 80 cm

165. The maximum number of possible interference maximum for slit separation equal to twice the wavelength in Young's double-slit experiment is **[2004]**

(A) Three (B) Five
(C) Infinite (D) Zero

166. A fish looks up through the water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the surface, the radius of this circle in cm is **[2005]**

(A) $\frac{36}{\sqrt{7}}$ (B) $36\sqrt{7}$

(C) $4\sqrt{5}$ (D) $36\sqrt{5}$

167. Two pointed white dots are 1 mm apart on a block paper. They are viewed by eye of pupil diameter 3 mm approximately. What is the maximum distance at which these dots can be resolved by the eyes? [Take wavelength of light = 500 nm] **[2005]**

(A) 1 m (B) 5 m (C) 3 m (D) 6 m

168. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is **[2005]**

(A) Circle (B) Hyperbola
(C) Parabola (D) Straight line

169. If I_0 is the intensity of the principle maximum in the single-slit diffraction pattern, then what will be its intensity when the slit width is doubled? **[2005]**

(A) $4I_0$ (B) $2I_0$ (C) $\frac{I_0}{2}$ (D) I_0

170. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is **[2005]**

(A) $\frac{1}{4}I_0$ (B) $\frac{1}{2}I_0$

(C) I_0 (D) Zero

171. The refractive index of a glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light, respectively, in a prism of this glass. Then, **[2006]**

(A) $D_1 < D_2$
(B) $D_1 = D_2$
(C) D_1 can be less than or greater than D_2 depending upon the angle of prism
(D) $D_1 > D_2$

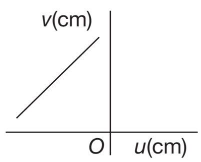
172. In a Young's double slit experiment, the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to **[2007]**

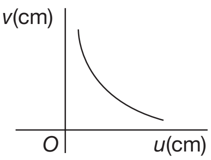
(A) $\frac{3}{4}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

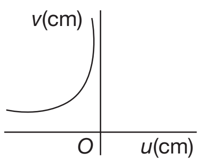
173. Two lenses of power –15 D and + 5 D are in contact with each other. The focal length of the combination is **[2007]**

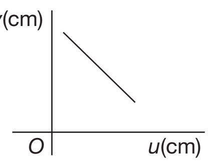
(A) +10 cm (B) –20 cm
(C) –10 cm (D) + 20 cm

174. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like **[2008]**

(A)


(B)


(C)


(D)


175. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is **[2009]**

(A) 885.0 nm (B) 442.5 nm
(C) 776.8 nm (D) 393.4 nm

176. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the Fig. 18.59. **[2009]**

The incident angle θ for which the light ray grazes along the wall of the rod is

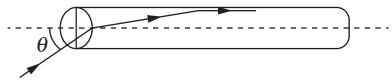


Fig. 18.59

(A) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (B) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(C) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\sin^{-1}\left(\frac{1}{2}\right)$

177. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by **[2012]**

(A) $\frac{I_m}{9}(4+5\cos\phi)$ (B) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$
(C) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (D) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

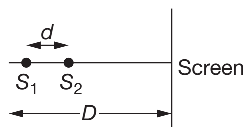
178. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film? **[2012]**

(A) 7.2 m (B) 2.4 m
(C) 3.2 m (D) 5.6 m

179. A beam of unpolarized light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is **[2013]**

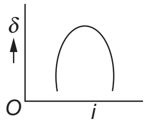
(A) $I_0/2$ (B) $I_0/4$
(C) $I_0/8$ (D) I_0

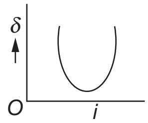
180. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be **[2013]**

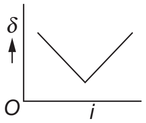


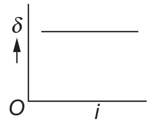
(A) Straight lines (B) Semi-circles
(C) Concentric circles (D) Points

181. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **[2013]**

(A) 

(B) 

(C) 

(D) 

182. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **[2013]**

(A) 20 cm (B) 30 cm
(C) 10 cm (D) 15 cm

183. A thin convex lens made from crown glass $\left(\mu=\frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2, respectively. The correct relation between the focal lengths is **[2014]**

(A) $f_1 = f_2 < f$
(B) $f_1 > f$ and f_2 becomes negative
(C) $f_2 > f$ and f_1 becomes negative
(D) f_1 and f_2 both become negative

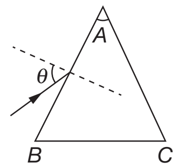
184. A green light is incident from the water to the air–water interface at the critical angle (θ). Select the correct statement. **[2014]**

(A) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.
(B) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.
(C) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.
(D) The entire spectrum of visible light will come out of the water at various angles to the normal.

185. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B, respectively, then $\frac{I_A}{I_B}$ equals **[2014]**

(A) 3 (B) 3/2
(C) 1 (D) 1/3

186. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided



(A) $\theta < \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(B) $\theta > \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(C) $\theta < \cos^{-1}\left[\mu\sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(D) $\theta > \sin^{-1}\left[\mu\sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

187. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is **[2015]**

(A) 30 μm (B) 100 μm
(C) 300 μm (D) 1 μm

188. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam **[2015]**

(A) goes horizontally without any deflection
(B) bends downwards
(C) bends upwards
(D) becomes narrower

189. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears **[2016]**

(A) 10 times nearer (B) 20 times taller
(C) 20 times nearer (D) 10 times taller

190. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when **[2016]**

(A) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\dfrac{2\lambda^2}{L}\right)$

(B) $a = \sqrt{\lambda L}$ and $b_{\min} = \sqrt{4\lambda L}$

(C) $a = \dfrac{\lambda^2}{L}$ and $b_{\min} = \sqrt{4\lambda L}$

(D) $a = \dfrac{\lambda^2}{L}$ and $b_{\min} = \left(\dfrac{2\lambda^2}{L}\right)$

191. In an experiment for determination of refractive index of glass of a prism by $i - \delta$ plot, it was found that a ray incident at an angle 35°, suffers a deviation of 40° and that it emerges at an angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index? **[2016]**

(A) 1.6 (B) 1.7
(C) 1.8 (D) 1.5

ANSWER KEYS

Single Option Correct Type

1. (B)	**2.** (C)	**3.** (C)	**4.** (A)	**5.** (A)	**6.** (D)	**7.** (B)	**8.** (B)	**9.** (C)	**10.** (C)
11. (C)	**12.** (A)	**13.** (D)	**14.** (B)	**15.** (B)	**16.** (D)	**17.** (A)	**18.** (D)	**19.** (B)	**20.** (D)
21. (B)	**22.** (A)	**23.** (D)	**24.** (C)	**25.** (C)	**26.** (B)	**27.** (A)	**28.** (B)	**29.** (B)	**30.** (B)
31. (C)	**32.** (C)	**33.** (A)	**34.** (B)	**35.** (A)	**36.** (B)	**37.** (A)	**38.** (C)	**39.** (B)	**40.** (B)
41. (A)	**42.** (A)	**43.** (D)	**44.** (A)	**45.** (A)	**46.** (B)	**47.** (C)	**48.** (C)	**49.** (C)	**50.** (A)
51. (A)	**52.** (C)	**53.** (D)	**54.** (D)	**55.** (A)	**56.** (C)	**57.** (C)	**58.** (D)	**59.** (C)	**60.** (A)
61. (D)	**62.** (A)	**63.** (C)	**64.** (D)	**65.** (B)	**66.** (C)	**67.** (B)	**68.** (C)	**69.** (A)	**70.** (D)
71. (C)	**72.** (A)	**73.** (C)	**74.** (D)	**75.** (A)	**76.** (B)	**77.** (A)	**78.** (A)	**79.** (C)	**80.** (D)
81. (D)	**82.** (A)	**83.** (D)	**84.** (B)	**85.** (C)	**86.** (D)	**87.** (B)	**88.** (B)	**89.** (B)	**90.** (A)
91. (B)	**92.** (B)	**93.** (A)	**94.** (D)	**95.** (A)	**96.** (B)	**97.** (B)	**98.** (D)	**99.** (A)	**100.** (B)
101. (A)	**102.** (D)	**103.** (C)	**104.** (D)	**105.** (C)	**106.** (A)				

More than One Option Correct Type

107. (A), (C) and (D) **108.** (A) and (C) **109.** (B) and (D)
110. (A) and (D) **111.** (A) and (C) **112.** (A) and (C)
113. (A) and (C) **114.** (A) and (D) **115.** (A) and (B)
116. (B), (C) and (D)

Passage Based Questions

Passage 1

117. (B) **118.** (D) **119.** (A)

Passage 2

120. (A) **121.** (A) **122.** (C)

Passage 3

123. (D) **124.** (A) **125.** (B)
126. (A)

Passage 4

127. (D) **128.** (A) **129.** (A)

Passage 5

130. (B) **131.** (A) **132.** (D)

Assertion-Reason Type

133. (A) **134.** (B) **135.** (A) **136.** (A) **137.** (C) **138.** (A) **139.** (A) **140.** (B) **141.** (D) **142.** (B)

Match the Column Type

143. (A) → 2, 3; (B) → 1; (C) → 4; (D) → 3
144. (A) → 3; B → 1, 3; (C) → 2, 3; (D) → 1,3, 4
145. (A) → 3, 4; (B) → 1, 2; (C) → 1, 2; (D) → 3, 4
146. (A) → 3; (B) → 2; (C) → 4; (D) → 1
147. (A) → 4; (B) → 2; (C) → 5; (D) → 3

Integer Type

148. Ratio = 2
149. = 2 rad/sec
150. 3 W/m^2
151. $m_0 = 5$
152. $\frac{dv}{dt} = -5$ m/s
153. $v = \frac{\mu}{\mu - 1} R = 3$
154. $D = 1$ cm
155. $x = 1$ m
156. = 1 rad
157. ≈ 5D

Previous Years' Questions

158. (A)	**159.** (D)	**160.** (A)	**161.** (D)	**162.** (B)	**163.** (B)	**164.** (C)	**165.** (B)	**166.** (B)	**167.** (B)
168. (D)	**169.** (D)	**170.** (B)	**171.** (A)	**172.** (A)	**173.** (C)	**174.** (C)	**175.** (B)	**176.** (C)	**177.** (D)
178. (D)	**179.** (B)	**180.** (C)	**181.** (B)	**182.** (B)	**183.** (B)	**184.** (B)	**185.** (D)	**186.** (D)	**187.** (A)
188. (C)	**189.** (B)	**190.** (B)	**191.** (D)						

HINTS AND SOLUTIONS

Single Option Correct Type

1. In first case, $\frac{1}{15} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

In second case, $\frac{1}{f} = \left(\frac{1.5}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

dividing, we get $f = 60$ cm

The correct option is (B)

2. Before collision,

$\vec{V}_{\text{image}} = \vec{V}_{\text{object}} = -\vec{u}$

After collision,

$\vec{V}_{\text{image}} = 2\vec{V}_{\text{object}} = 2\vec{u}$

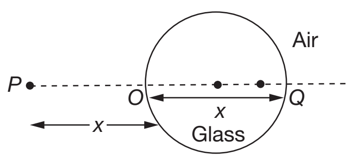
The correct option is (C)

3. $-\frac{1}{nu} - \frac{1}{u} = \frac{-1}{f}, \; u = \left(\frac{n+1}{n}\right) f$

The correct option is (C)

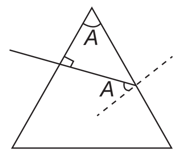
4. $\frac{1.5}{x} + \frac{1}{x} = \frac{1.5 - 1}{R}, \; \frac{2.5}{x} = \frac{0.5}{R}$

$x = 5R$



The correct option is (A)

5. $\sin A = \frac{1}{\mu} = \frac{2}{3}, \; A = \sin^{-1}\left(\frac{2}{3}\right)$



The correct option is (A)

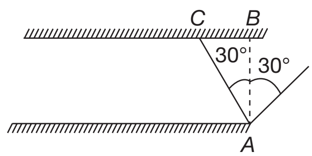
6. Ray inside medium *AB* is parallel to ray inside medium *CD*

The correct option is (D)

7. From the law of refection,

$\tan 30° = \frac{BC}{AB} = \frac{BC}{0.2}; \; BC = 0.2 \times \frac{1}{\sqrt{3}} = 0.115$

Total number of reflection = 30



The correct option is (B).

8. Erect and enlarged image can be produced by concave mirror

$\frac{I}{O} = \frac{f}{f - u} \Rightarrow \frac{+3}{+1} = \frac{f}{f - (-4)}$

$\Rightarrow \; f = -6$ cm $\Rightarrow R = 12$ cm

The correct option is (B)

9. $\mu = \dfrac{\text{Velocity of light in vacuum}}{\text{Velocity of light in glass plate}} = \dfrac{c}{c'}$ or $c' = \dfrac{c}{\mu}$

time taken $= \dfrac{\text{distance}}{\text{velocity}} = \dfrac{t}{(c/\mu)} = \dfrac{\mu t}{c}$

The correct option is (C)

10. Fringe width, $\omega \propto \lambda$

The correct option is (C)

11. $\omega = \dfrac{\lambda D}{d}, \omega' = \dfrac{\lambda D/2}{2d} = \dfrac{\omega}{4}$

The correct option is (C)

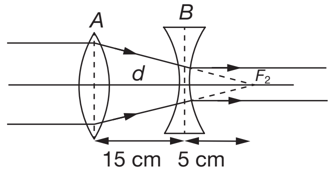
12. $S = \dfrac{D}{d}(\mu - 1)t = \dfrac{5\lambda D}{d} \quad \therefore \ \lambda = \dfrac{(\mu-1)t}{5} = 6000 \text{ Å}.$

The correct option is (A)

13. $m = \dfrac{f - v}{f} \Rightarrow \dfrac{1}{n} = \dfrac{-F + v}{-F} \quad \therefore \ v = \left(\dfrac{n-1}{n}\right)F$

The correct option is (D)

14. In the absence of concave lens, the parallel beam will be focused at F_2, i.e., at a distance 20 cm from lens A. The focal length of concave lens is 5 cm, i.e., if this lens is placed at 15 cm from A, then beam will become parallel. So, $d = 15$ cm



The correct option is (B)

15. Refractive index $= \dfrac{c_v}{c} = \dfrac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}} = \sqrt{\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

The correct option is (B)

16. $\mu = \tan i = \tan 60° = \sqrt{3}$

The correct option is (D)

17. The correct option is (A)

18. The correct option is (D)

19. The correct option is (B)

20. $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \dfrac{9}{1} \quad (\because \ I_1 = 4I_2)$

The correct option is (D)

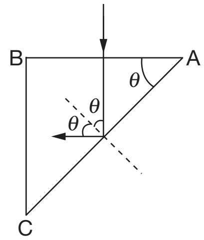
21. $\sin C = \dfrac{3}{5}$, Hence $n = \dfrac{5}{3}$

$\tan i_p = n$ or $i_p = \tan^{-1}\left(\dfrac{5}{3}\right)$.

The correct option is (B)

22. $\sin\theta > \sin\theta_C = \dfrac{1}{\mu}$

$\sin\theta > \dfrac{1}{\dfrac{3/2}{4/3}}, \ \sin\theta > \dfrac{8}{9}$



The correct option is (A)

23. The correct option is (D)

24. For arrangement P,

$\dfrac{1}{f_{eq}} = \dfrac{1}{2R} + \dfrac{1}{2R} - \dfrac{2}{3R} = \dfrac{1}{3R}$ (R = radius of curvature)

For arrangement Q, $\dfrac{1}{f_{eq}} = \dfrac{1}{2R} + \dfrac{1}{2R} - \dfrac{1}{3R} = \dfrac{2}{3R}$

The correct option is (C)

25. If rays are parallel to principal axis, then rays will converge if equivalent local length is positive.

i.e. $\dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2} > 0$, $\dfrac{1}{-10} + \dfrac{1}{20} - \dfrac{d}{(-10)(20)} > 0$

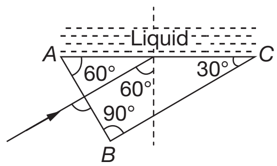
$\Rightarrow \ d > 10$ cm

The correct option is (C)

26. For total internal reflection to take place,

$\sin 60° > \sin C$

$\sin 60° > \dfrac{2\mu}{3} \Rightarrow \mu < \dfrac{3\sqrt{3}}{4}$

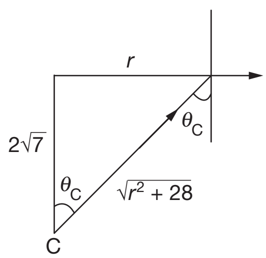


The correct option is (B)

27. The correct option is (A)

28. $\sin\theta_C = \dfrac{1}{\mu} = \dfrac{r}{\sqrt{r^2 + 28}}$

$\mu = \dfrac{4}{3} \Rightarrow r = 6\,\text{m}$



The correct option is (B)

29. $\sin\theta_C \geq \frac{1}{\mu} \Rightarrow \sin 45 \geq \frac{1}{\mu} \Rightarrow \mu \geq \sqrt{2}$

The correct option is (B)

30. $S = \frac{(\mu-1)t\,D}{d}$ and $w = \frac{\lambda D}{d}$ given, $S = w$

$\Rightarrow \mu = \left(\frac{\lambda}{t}+1\right) = 1.5$

The correct option is (B)

31. As $d_{app} = \frac{d_{oct}}{\mu}$

Also,

$\mu = A + \frac{B}{\lambda^2}$

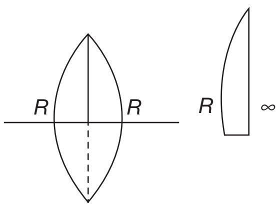
for least d_{app}. μ should be large

The correct option is (C)

32. $\frac{1}{20} = (\mu-1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

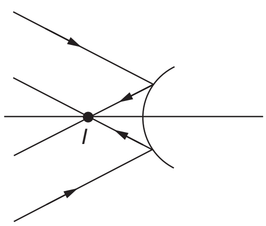
$= \frac{1}{f} = (\mu-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$f = 40$ cm



The correct option is (C)

33. Real image formed,



The correct option is (A)

34. $D = 1$ m, $d = 5\times10^{-3}$ m

shift $= \frac{D}{d}\left[t_1(\mu_1-1) - t_2(\mu_2-1)\right]$

$= \frac{1}{5\times10^{-3}}\left[2(1.5-1) - 1.5(1.4-1)\right]\times10^{-3}$

$= \frac{0.4}{5} = 0.08$ m

The correct option is (B)

35. $\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin A/2} \Rightarrow \sin i \;\alpha\; \mu$

$\frac{i_A}{i_B} \approx \frac{\mu_A}{\mu_B} = \frac{1.5}{1.6} < 1$

$\Rightarrow \; i_A < i_B$

The correct option is (A)

36. Applying Snell's law at horizontal surface,

$$\sin 45° = \mu \sin\theta \qquad (1)$$

Applying Snell's law at vertical surface,

$$\mu\cos\theta = 1 \qquad (2)$$

From (1) and (2),

$$\mu = \sqrt{\frac{3}{2}}$$

The correct option is (B)

37. Focal length of the convex lens

$\frac{1}{f} = \left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{1}{2R} \Rightarrow f = 2R$

Thus, the ray would become parallel to the principal axis after the refraction and fall ⊥ to the mirror and hence would get reflected back along the same path.

The correct option is (A)

38. Shift in the interference pattern due to insertion of slab is

$= \frac{t(\mu-1)D}{d} = 20$

or $\frac{t(\mu-1)D}{d} = 20\frac{\lambda D}{d}$

$\mu - 1 = 20\times\frac{\lambda}{t} = \frac{20\times5000\times10^{-10}}{2.5\times10^{-5}} = 4\times10^{-1} = 0.4$

$\mu = 1.4$

The correct option is (C)

39. Power = 5 $\Rightarrow f = 20$ cm

$\frac{1}{f} = \left(\frac{\mu_g-\mu_a}{\mu_a}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{20} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (1)$$

$$\text{and } \frac{1}{-100} = \left(\frac{1.5-\mu_l}{\mu_l}\right)\left(\frac{1}{R_1} - \frac{r}{R_2}\right) \qquad (2)$$

Dividing 1 by 2, we get $-5 = \frac{\frac{1}{2}}{\left(\frac{1.5}{\mu_l} - 1\right)}$

$$\Rightarrow 1-\frac{1.5}{\mu_l}=\frac{1}{10} \Rightarrow \frac{9}{10}=\frac{3}{2\mu_l}$$

$$\Rightarrow \quad \mu_l=\frac{5}{3}$$

The correct option is (B)

40. If B is moved to right, focus will also move to the right with respect to initial focus

The correct option is (B)

41. $2\mu_1 t=(2n-1)\frac{\lambda}{2}$, $t=\frac{(2n-1)\frac{\lambda}{2}}{2\mu_1} \Rightarrow t=120$ nm, 360 nm, …

The correct option is (A)

42. $f=-20$ cm, $u=-30$ cm, $\frac{1}{v}=\frac{1}{f}-\frac{1}{4}=\frac{1}{-20}-\frac{1}{-30}=-\frac{1}{60}$, $v=-60$ cm

$$m=-\frac{v}{u}=-\left(\frac{-60}{-30}\right)=-2$$

The correct option is (A)

43. $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{u-f}{fu}$, $m=-\frac{v}{u}=\frac{f}{f-u}$

The correct option is (D)

44. $\beta'=\frac{\beta}{\mu}=\frac{0.4}{4/3}=0.3\,\text{mm}$

The correct option is (A)

45. $B=\frac{2D\lambda}{d}$, $\frac{B}{D}=\frac{2\lambda}{d}$ radian

$$=\frac{2\times 6328\times 10^{-10}}{0.2\times 10^{-3}}\times\frac{180}{\pi}=0.36°$$

The correct option is (A)

46. $I_1=\frac{I_0}{2}$, $I_2=I_1\cos^2\phi=\frac{I_0}{2}\times\cos^2 45=\frac{I_0}{4}=0.25I_0$

The correct option is (B)

47. The correct option is (C)

48. The correct option is (C)

49. The correct option is (C)

50. $I=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$, $\phi=\frac{2\pi}{\lambda}\Delta x=\frac{\pi}{2}\Rightarrow I=\frac{I_{\max}}{2}$

The correct option is (A)

51. $\frac{\mu_2}{\mu_1}=\frac{v_1}{v_2}=\frac{1}{2} \Rightarrow \frac{\mu_1}{\mu_2}=2(\mu_1>\mu_2)$

For total internal reflection,

$${}_2\mu_1=\frac{1}{\sin C}=2\Rightarrow C=30°$$

The correct option is (A)

52. In the case of minimum deviation, ray inside the prism is parallel to base.

Therefore, ray is deviated equally from both refracting faces

If, $\delta=34°$, $\delta'=\frac{\delta}{2}=17°$

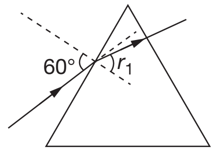
The correct option is (C)

53. $r_1=45°$

$$\therefore\ \mu=\frac{\sin i}{\sin r_1}=\frac{\sin 60°}{\sin 45°}=\sqrt{\frac{3}{2}}$$

$$\delta=i+e-A=60°+0-45°$$

$$\Rightarrow \ \delta=15°$$



The correct option is (D)

54. Distance of second dark fringe from central fringe $(x_2)=\frac{3\lambda D}{2d}$

$$\Rightarrow \ 1\times 10^{-3}=\frac{3\times\lambda\times 1}{2\times 0.9\times 10^{-3}}$$

$$\Rightarrow \ \lambda=6\times 10^{-7}\text{ m}=6\times 10^{-5}\text{ cm}$$

The correct option is (D)

55. Using $P=P_1+P_2-d\times P_1P_2$

For equivalent power to be negative

$$d\times P_1P_2>P_1+P_2 \Rightarrow d\times 25>10$$

$$\Rightarrow \ d>\frac{10}{25}\text{ m}\Rightarrow d>40\text{ cm}$$

The correct option is (A)

56. From Brewster's law, $\mu=\tan i_p$

$$\Rightarrow \ \frac{c}{v}=\tan 60°=\sqrt{3}$$

$$\Rightarrow \ v=\frac{c}{\sqrt{3}}=\frac{3\times 10^8}{\sqrt{3}}=\sqrt{3}\times 10^8\text{ m/s}$$

The correct option is (C)

57. $u=-20$ cm, $f=+10\,\text{cm}$ also $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

$$\Rightarrow \ \frac{1}{+10}=\frac{1}{v}+\frac{1}{(-20)}$$

$$\Rightarrow \ v=\frac{20}{3}\text{ cm; virtual image.}$$

The correct option is (C)

58. $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{3/2}{5/4}-1\right)\left(\frac{1}{12}+\frac{1}{12}\right)=\frac{1}{30}$

$f=30$ cm

The correct option is (D)

59. $\omega=\frac{1.64-1.52}{1.6-1}=\frac{0.12}{0.6}=0.2$

The correct option is (C)

60. $\mu_w = \frac{4}{3},\ \mu_g = \frac{5}{3}$

$\sin c = \frac{1}{{}_w\mu_g} = \frac{\mu_w}{\mu_g} = \frac{4}{5}$

$\Rightarrow \quad C = \sin^{-1}\left(\frac{4}{5}\right)$

The correct option is (A)

61. $n_1 \frac{D\lambda_1}{d} = n_2 \frac{D\lambda_2}{d} \Rightarrow n_1\lambda_1 = n_2\lambda_2 \Rightarrow n_2 = \frac{n_1\lambda_1}{\lambda_2} \approx 84$

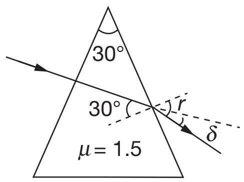
The correct option is (D)

62. 1.5 sin 30° = 1 sin r

$\Rightarrow \quad \sin r = \frac{3}{4} = 0.75 = \sin 48°36'$

$\therefore \quad r = 48°36'$

$\therefore \quad \delta = r - i = 48°36' - 30° = 18°36'$



The correct option is (A)

63. Apparent length = $\mu \times$ original length

The correct option is (C)

64. For convex mirror,

$u = -50$ cm, $v = 10$ cm

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$f = 12.5$ cm

The correct option is (D)

65. $3 = \frac{4}{\mu} \Rightarrow \mu = \frac{4}{3}$

$R = 25$ cm

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$f = 75$ cm

The correct option is (B)

66. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{P_1}{100} + \frac{P_2}{100} \Rightarrow f = 100\,\text{cm}$

$\therefore$ A converging lens of focal length 100 cm.

The correct option is (C)

67. Distance of n^{th} minimum from central bright fringe

$x_n = \frac{(2n-1)\lambda D}{2d}$

For $n = 3$, i.e., 3^{rd} minimum

$x_3 = \frac{(2\times 3 - 1)\times 500\times 10^{-9}\times 1}{2\times 1\times 10^{-3}} = 1.25\times 10^{-3}\text{ m} = 1.25\text{ mm}$

The correct option is (B)

68. The maximum velocity of the insect is $A\sqrt{\frac{k}{M}}$.

Its component perpendicular to the mirror is $A\sqrt{\frac{k}{M}}\sin 60°$.

Thus maximum relative speed $= \sqrt{3}A\sqrt{\frac{k}{M}}$.

The correct option is (C)

69. Absent wavelengths correspond to interference minimum

$\therefore \quad d\sin\theta = (2n-1)\frac{\lambda}{2} \Rightarrow d\frac{y}{D} = (2n-1)\frac{\lambda}{2}$

$\Rightarrow \lambda = \frac{2yd}{D(2n-1)} = \frac{40000}{2(n-1)}$

$= 13333\,\text{Å},\ 80000\,\text{Å},\ 5714\,\text{Å},\ 4444\,\text{Å},\ 3636\,\text{Å}$

The correct option is (A)

70. Velocity of image = $v_0 - 2v_m = 0$

$2A\omega\cos\omega t = -2A\omega\cos\left(\omega t - \frac{\pi}{3}\right)$

$\Rightarrow \quad t = \frac{2\pi}{3\omega}$

The correct option is (D)

71. When distance between real object and its real image is $4f$ for a converging lens, both object and its image are at a distance of $2f$ from the lens.

The correct option is (C)

72. $2\mu_1 t = (2n-1)\frac{\lambda}{2},\ t = \frac{(2n-1)\frac{\lambda}{2}}{2\mu_1} \Rightarrow t = 120$ nm, 360 nm, …

The correct option is (A)

73. $I = 4I_0\cos^2\frac{\phi}{2} \Rightarrow 2I_0 = 4I_0\cos^2\frac{\phi}{2}$

$\Rightarrow \quad \cos\frac{\phi}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \phi = 2\times\frac{\pi}{4} = \frac{\pi}{2}$

Path difference $\Delta x = \frac{\lambda}{2\pi}\times\phi = \frac{\lambda}{2\pi}\frac{\pi}{2} = \frac{\lambda}{4}$

$\Rightarrow \quad (\mu - 1)t = \frac{\lambda}{4}$ or $0.5t = \frac{\lambda}{4} \Rightarrow t_{\min} = \frac{\lambda}{2}$

The correct option is (C)

74. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_0 - \mu_1}{R_1} + \frac{\mu_2 - \mu_0}{R_2}$

When $u = \infty$ $V = f$

$$\frac{1.6}{f}=\frac{1.5-1.2}{+30}+\frac{1.6-1.5}{-30}$$

$f = 240$ cm

The correct option is (D)

75. Total number of fringes $= 2\times\frac{2\times10^{-3}}{5\times10^{-7}}+1 = 8001$

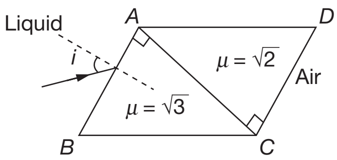
The correct option is (A)

76. The correct option is (B)

77. $\sqrt{3}\sin(90-i)=\sqrt{2}\sin r$

$1\sin i=\sqrt{2}\sin(90°-r)$

on solving we get, $i = 45°$



The correct option is (A)

78. $\delta=\left(\frac{\mu_2}{\mu_1}-1\right)A=\left(\frac{1.5}{5/4}-1\right)\times4°=0.8°$ downward

The correct option is (B)

79. As the mirror rotates by an angle θ, the reflected ray rotates by 2θ.

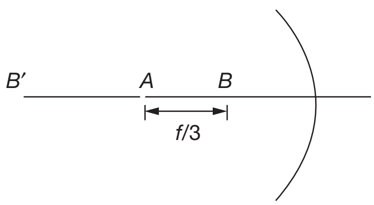
The correct option is (C)

80. For B

$$\frac{1}{V}+\frac{1}{-(2f-\frac{f}{3})}=-\frac{1}{f},\ \frac{1}{V}=\frac{3}{5f}-\frac{1}{f}=\frac{3-5}{5f}$$

$$V=-\frac{5f}{2},\ |AB'|=\frac{5f}{2}-2f=\frac{f}{2}$$

$$\text{Magnification}=\frac{\frac{f}{2}}{\frac{f}{3}}=1.5$$



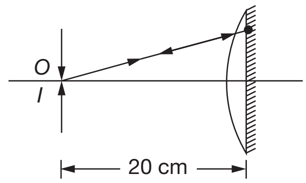
The correct option is (D)

81. When the object is placed at the focus of the lens, the refracted rays will be incident normally on the silvered surface. Thus, they will retrace their path.

Hence, the image will be formed at the location of the object.

Hence, the combination behaves as a concave mirror of radius of curvature (R) = 20 cm

$\therefore\ f=\frac{R}{2}=10$ cm



The correct option is (D)

82. $m = f_0/f_e$; $f_0+f_e = 54$ cm $\therefore$ $f_e = 6$ cm and $f_0 = 48$ cm

The correct option is (A)

83. $\frac{f_0}{fe}=10 \Rightarrow f_0 = 200$

The correct option is (D)

84. $\because\ f=\frac{d}{m_1-m_2}$

If m_1 is taken as m, $m_2=\frac{1}{m}$

Thus, f becomes $\frac{md}{m^2-1}$

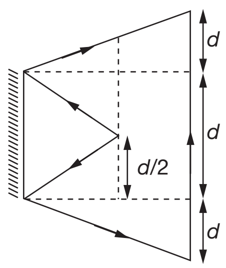
The correct option is (B)

85. Fringe width (B) $=\frac{\lambda D}{d}$

$\Rightarrow\ \beta\propto\frac{1}{d}$

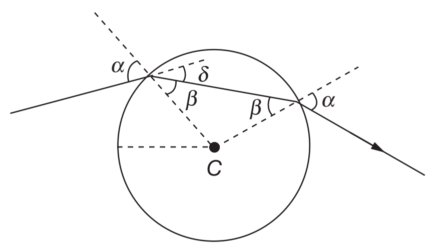
The correct option is (C)

86.



The correct option is (D)

87. Total angle of deviation $= 2(\alpha-\beta)$

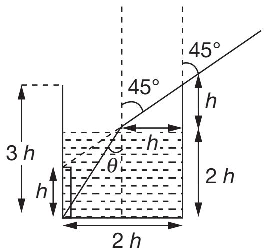


The correct option is (B)

88. $\mu \sin\theta = \sin 45°$

$$\frac{\mu h}{h\sqrt{5}} = \frac{1}{\sqrt{2}}$$

$$\mu = \sqrt{\frac{5}{2}}$$



The correct option is (B)

89. x is distance of object from surface.

Apparent depth of object from surface = $\frac{x}{\mu}$

Apparent depth of image from surface = $\frac{x+2h}{\mu}$

Distance between the apparent depths of object and image $= \frac{2h}{\mu}$

The correct option is (B)

90. Due to M_1, an image is formed at a distance x from M_1, i.e., at a distance $(x - y)$ behind M_2. Thus, for M_2

$u = -(x+y),\ v = x - y$

Use $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

The correct option is (A)

91. Since incident ray retraces its path, it must strike the plane mirror perpendicularly.

From Snell's law, $\sin i = \mu_1 \sin r_1$

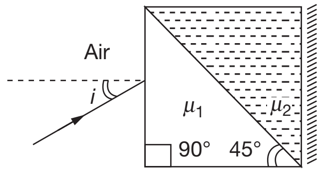
and $\mu_1 \sin r_2 = \mu_2 \sin 45° \Rightarrow \mu_1 \sin r_2 = \frac{\mu_2}{\sqrt{2}}$

$$\Rightarrow \quad r_2 = \sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right)$$

Also, $r_1 + r_2 = \frac{\pi}{4}$

$$\therefore r_1 = \frac{\pi}{4} - \sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right)$$

$$\therefore \quad i = \sin^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$$



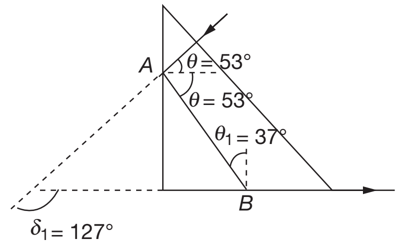
The correct option is (B)

92. At point A

$\mu \sin\theta > 1$ Hence total internal reflection will take place

At point B

$\mu \sin\theta_1 = 1$. Therefore, light ray will graze the surface hence $\delta = 127°$



The correct option is (B)

93. $\Delta x = (\mu_1 - \mu_2)\,t$

$$I_P = 2I_0\left[1 + \cos\left(\frac{2\pi}{\lambda}\right)(\mu_1 - \mu_2)\,t\right]$$

$$= 4I_0 \cos^2 \frac{\pi}{\lambda}(\mu_1 - \mu_2)\,t$$

The correct option is (A)

94. Distance of image from the plane surface is

$$x_1 = \frac{4}{1.6} = 2.5\,\text{cm} \quad \left(d_{\text{app}} = \frac{d_{\text{actual}}}{\mu}\right)$$

For the curved side $\frac{1.6}{4} + \frac{1}{x_2} = \frac{1-1.6}{-8}$

$x_2 \approx -3.0\,\text{cm}$

The minus sign means the image is on the object side

$\therefore \quad I_1 I_2 = (8 - 2.5 - 3.0)\,\text{cm} = 2.5\,\text{cm}$

The correct option is (D)

95. $\vec{v}_{\text{om}} = (3i + 4k)$

$\vec{v}_{\text{Im}} = -3i + 4k$

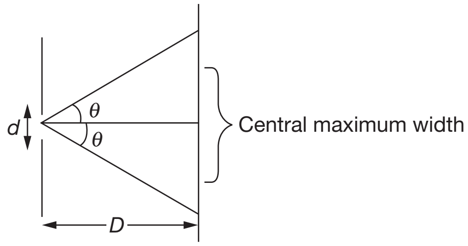
$\Rightarrow \quad v_{\text{Io}} = \vec{v}_{\text{Im}} - \vec{v}_{\text{om}} = -6i$

$\Rightarrow$ negative x-axis

The correct option is (A)

96. $\Delta\theta = \frac{2\lambda}{d} = \frac{2 \times 4500 \times 10^{-10}}{1.5 \times 10^{-3}} = 0.0006$ radian

$= 0.034°$



The correct option is (B)

97. $M = \dfrac{f_0}{f_e} = \dfrac{\beta}{\alpha}$

$\dfrac{30}{6} = \dfrac{\beta}{1.5}$

$\beta = 7.5°$

The correct option is (B)

98. Case - I: $u_1 = -25\text{cm}, f = f \Rightarrow v_1 = \dfrac{25f}{f+25}$

Case - II: $u_2 = -40\text{cm}, f = f \Rightarrow v_2 = \dfrac{40f}{f+40}$

$$m_1 \times m_2 = 4 \Rightarrow \frac{-\left(\dfrac{25f}{f+25}\right)}{-25} \times \frac{-\left(\dfrac{40f}{f+40}\right)}{-40} = 4$$

$\Rightarrow f = -20$ cm

The correct option is (D)

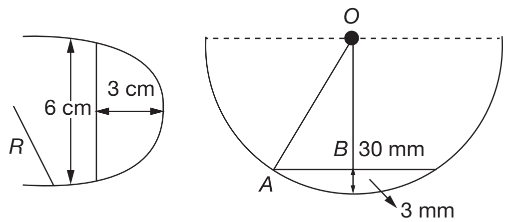
99. In $\Delta\, OAB$, $OA = R$ mm

$\therefore \quad OB = (R - 3)$ mm and $AB = 30$ mm

$OA^2 = OB^2 + AB^2$

$R^2 = (R-3)^2 + (30)^2 \Rightarrow R^2 = R^2 + 9 - 6R + 900$

$\Rightarrow R = \dfrac{909}{6} = 151.5 \text{ mm} \approx 15 \text{ cm.}$



The correct option is (A)

100. Since consecutive prisms are kept in inverted position with respect to each other, they would (a pair of prisms) cancel each others deviations. So if even number of prisms are there, deviation would be zero and if odd number of prisms are there, it would be δ.

The correct option is (B)

101. $\dfrac{2\pi}{\lambda}\left(\dfrac{yd}{D+v}\right) = 4\pi + \dfrac{\pi}{3}$; $\dfrac{2\pi}{\lambda}\dfrac{D(3\lambda)}{D+v} = 4\pi + \dfrac{\pi}{3}$

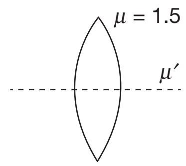
$\Rightarrow v = \dfrac{5}{13}$ m/s

The correct option is (A)

102. $P = \dfrac{1}{f} = (\mu - 1)\dfrac{2}{R}$

$P = (1.5 - 1)\dfrac{2}{R} = 5$

$R = 20$ cm



Focal length of this lens in medium (μ')

$\dfrac{1}{f'} = \left(\dfrac{\mu}{\mu'} - 1\right)\dfrac{2}{R} = \left(\dfrac{1.5}{\mu'} - 1\right)\dfrac{2}{20}$

$\dfrac{-1}{10} = \left(\dfrac{1.5}{\mu'} - 1\right)\dfrac{1}{1} \Rightarrow = \dfrac{1.5}{\mu'} = 1 - \dfrac{1}{10} = \dfrac{9}{10}$

$\mu' = \dfrac{15}{9} = \dfrac{5}{3}$

The correct option is (D)

103. $m = \dfrac{f_0}{f_e}\left(1 + \dfrac{f_e}{D}\right) = \dfrac{150}{6}\left(1 + \dfrac{6}{25}\right) = 31$

The correct option is (C)

104. For n^{th} secondary maximum path difference

$d\sin\theta = (2n+1)\dfrac{\lambda}{2} \Rightarrow a\sin\theta = \dfrac{3\lambda}{2}$

The correct option is (D)

105. Minimum angular separation $\Delta\theta = \dfrac{1}{\text{R.P.}} = \dfrac{1.22\lambda}{d}$

$= \dfrac{1.22 \times 5000 \times 10^{-10}}{2} = 0.31 \times 10^{-6}$ rad

The correct option is (C)

106. $\mu = \dfrac{\text{Real depth}}{\text{Apparent depth}} = \dfrac{120}{120 - 40} = 1.5$

The correct option is (A)

More than One Option Correct Type

107. For convex mirror (positive focal length), image is always smaller in size. For concave mirror (negative focal length), image is smaller when object lies beyond $2f$.

The correct option is (A) (C) and (D)

108. Let f be the focal length of lens.

Then focal length of part A is f and of part B and C is $2f$ each.

$\therefore$ Power of A, B, and C is P, $P/2$, and $P/2$, respectively.

The correct option is (A) and (C)

109. When upper half of the lens is covered, image is formed by the rays coming from lower half of the lens or image will be formed by less number of rays. Therefore, intensity of image will decrease. But complete image will be formed.

The correct option is (B) and (D)

110. Real image is smaller in size if object lies beyond $2f$ and it is larger if object lies between f and $2f$

The correct option is (A) and (D)

111. Path difference = 0

$$\frac{d^2}{D} = \frac{yd}{2D} - \left(\frac{\mu_2}{\mu_3} - 1\right)t$$

$y = 2$ mm

When slab is removed, then path difference $= \frac{d^2}{D} - \frac{y_1 d}{2D} = 0$,

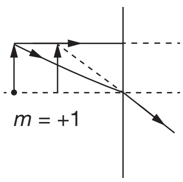
$y_1 = 2d = 4$mm

The correct option is (A) and (C)

112. Apparent position of the object with respect to lens.

$$u = \left(\frac{10}{1} + \frac{10}{2}\right) = 15\,\text{cm}$$

$$\frac{1}{15} + \frac{1}{v} = \frac{2(1.5) - 1}{R}$$



$v = 7.5$ cm

Image size $= \frac{7.5}{15} \times 1 = 0.5$ cm

The correct option is (A) and (C)

113. $$-\frac{1}{F} = P = 2P_{l_1} + 2P_{l_2} + P_m \qquad (1)$$

$$P_{l_1} = \frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$P_{l_1} = [(1.5 - 1)]\left[-\frac{1}{10} - \frac{1}{15}\right] = -\frac{1}{12} \qquad (2)$$

$$P_{l_2} = \frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$P_{l_2} = \left(\frac{4}{3} - 1\right)\left[\frac{2}{15}\right] = \frac{2}{45} \qquad (3)$$

$$P_m = -\frac{1}{f} = +\frac{2}{15} \qquad (4)$$

$$-\frac{1}{F} = P = 2\left[-\frac{1}{12} + \frac{2}{45}\right] + \frac{2}{15} = -\frac{1}{6} + \frac{4}{45} + \frac{2}{15} = \frac{1}{18}$$

$F = -18\,\text{cm}$. If focus is negative, system will behave as concave mirror.

The correct option is (A) and (C)

114. Focal length of combination should be $f_{comb} = 30$ cm,

For image formed at ∞

$$\frac{1}{30} = \frac{1}{20} + \frac{1}{f}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$f = -60$ cm

Let the lens be immersed in a liquid having refractive index μ to have effective focal length 30 cm

$$\frac{1}{f}\alpha(\mu - 1)$$

$$\frac{1}{20}\alpha\left(\frac{3}{2} - 1\right) \qquad (1)$$

$$\frac{1}{30}\alpha\left(\frac{3}{2\mu} - 1\right) \qquad (2)$$

Dividing (2) by (1) and solving $\mu = \frac{9}{8}$

The correct option is (A) and (D)

115. For microwaves,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^6} = 300 \text{ m.}$$

$\Delta x = d \sin\theta$

$$\phi = \frac{2\pi}{\lambda} d\sin\theta = \frac{2\pi}{300}(150\sin\theta) = \pi\sin\theta$$

$$I_\theta = I_0 \cos^2\left(\frac{\pi\sin\theta}{2}\right)$$

The correct option is (A) and (B)

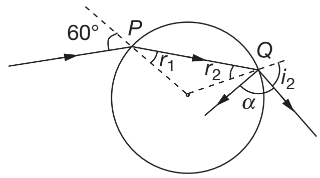
116. At P

$$\frac{\sin 60}{\sin r_1} = \sqrt{3}$$

$$r_1 = 30° = r_2$$

$$\frac{\sin i_2}{\sin r_2} = \sqrt{3}$$

$$i_2 = 60°$$



$\alpha = 180 - (30 + 60) = 90°$

The correct option is (B), (C), and (D)

Passage Based Questions

Passage 1

117. Distance between slits S_1 and S_2, $d = \alpha D = 1 \times 10^{-3}$ m

Total number of fringes $N = \frac{2d}{\lambda} + 1 = 4001$

The correct option is (B)

118. For bright $y_n = \frac{n\lambda D}{d}$, $y_2 = \frac{2\lambda D}{d}$, $y_3 = \frac{3\lambda D}{d}$

The distance asked $= y_2 + y_3$ (being opposite side)

$= \frac{5\lambda D}{d} = 2.50$ mm

The correct option is (D)

119. $I_r = 4I_0 \cos^2\left(\frac{\phi}{2}\right) \Rightarrow \phi = \frac{2\pi}{3}$

$y = \frac{Dx}{d} = \frac{D}{d}\left(\frac{\phi}{2\pi}\right)\lambda = 0.167 = 0.17$ mm (approximatelly)

The correct option is (A)

Passage 2

120. $m_1 \times m_2 = 1$, $\frac{m_1}{m_2} = \frac{1}{9}$

$\Rightarrow m_1 = \frac{1}{3} = \frac{v}{u}$, $u + v = 120$ cm

Solving $u = 90$ cm and $v = 30$ cm $\therefore \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\Rightarrow f = 22.5$ cm

The correct option is (A)

121. Smaller image will be brighter as intensity $\propto \frac{1}{\text{Area}}$

The correct option is (A)

122. $\frac{v}{u} = 2$, $v + u = 180\,\text{cm} \Rightarrow f = 40\,\text{cm}$

Using the formula $f = \frac{D^2 - L^2}{4D} \Rightarrow L = 60$ cm

The correct option is (C)

Passage 3

123. From Snell's law

$n \sin\theta = \text{constant} = C$

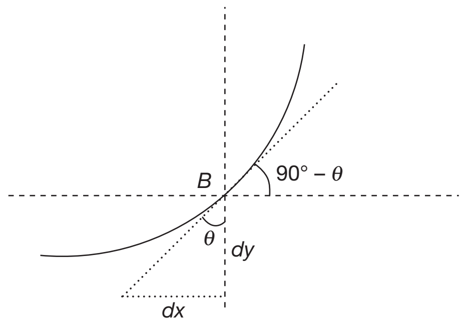
At A (0, 0), $y = 0$

$\therefore \quad n(0) = \left[k(0)^{3/2} + 1\right]^{1/2} = 1$

Now, $n(0)\sin 90° = C \Rightarrow C = 1$

$\therefore \quad \theta = \sin^{-1}\left(\frac{1}{n}\right)$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

or $\quad \frac{\cot\theta}{\theta} = \frac{\sqrt{1-(1/n)^2}}{\theta(1/n)} = \frac{\sqrt{n^2-1}}{\sin^{-1}(1/n)}$



The correct option is (D)

124. We have, $n^2 \sin^2\theta = 1$ or $\frac{n^2}{1+\cot^2\theta} = 1$

$$\Rightarrow \frac{n^2}{1+\left(\frac{dy}{dx}\right)^2} = 1$$

Also $n^2 = \left(ky^{3/2} + 1\right)$

Hence, $\left(ky^{3/2} + 1\right) = 1 + \left(\frac{dy}{dx}\right)^2$ or $\frac{dy}{y^{3/4}} = k^{1/2}dx$

Integrating on both sides, $4y^{1/4} = k^{1/2}x + B$

At $x = 0, y = 0$ ∴ $B = 0$

Thus, $4y^{1/4} = k^{1/2}x$

The correct option is (A)

125. At $y = 1, x = 4$ m, hence $P(x_1, y_1) = P(4, 1)$

The correct option is (B)

126. The correct option is (A)

Passage 4

127. The correct option is (D)

128. For lens, $(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2} + \frac{1}{3}$

$R = \frac{12}{5}(\mu - 1)$

For half lens, $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right]$

$f = \frac{12}{5}$ m, $\quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \quad v = 12$ m.

The correct option is (A)

129. Distance of first image from left lens $\frac{1}{v_1} + \frac{1}{2} = \frac{5}{12}$

$\Rightarrow \quad v_1 = -12$ m

Distance of first image from right lens = 12 + 1 = 13 m

Distance of final image from right lens = $\frac{1}{v} - \frac{1}{-13} = \frac{5}{12}$

$\Rightarrow \quad v = 2.94$ from right lens.

The correct option is (A)

Passage 5

130. $\frac{1}{V + f} + \frac{1}{U + f} = \frac{1}{f}$

$\Rightarrow (U + f + V + f)f = (V + f)(U + f) \Rightarrow VU = f^2$

The correct option is (B)

131. The correct option is (A)

132. $VU = f^2 \Rightarrow 18 \times V = 36 \Rightarrow V = 2$ cm

So distance from pole = (6 –2) = 4 cm

The correct option is (D)

Assertion-Reason Type

133. The correct option is (A)

134. The correct option is (B)

135. The correct option is (A)

136. The correct option is (A)

137. The correct option is (C)

138. The correct option is (A)

139. The correct option is (A)

140. The correct option is (B)

141. The coating of optical fibres of a light pipe is of a material having refractive index less than that of the fibre, to optically insulate from each other. It works on the principle of total internal reflection.

The correct option is (D)

142. The correct option is (B)

Match the Column Type

143. (A) → 2, 3; (B) → 1; (C) → 4; (D) → 3

144. (A) → 3; (B) → 1, 3; (C) → 2, 3; (D) → 1, 3, 4

145. (A) → 3, 4; (B) → 1, 2; (C) → 1, 2; (D) → 3, 4

146. (A) Refractive index of the prism is the minimum value required for ray (1) to undergo total internal reflection at face *AC*. Ray (1) falls on face *AC* at an angle of incidence 30°

∴ $30° > i_C$

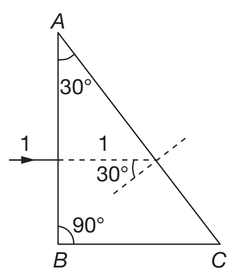
$\sin 30° > \sin i_C$

∴ $\mu > 2$

Minimum value of μ can be taken as 2.

(B) For ray 2, refractive angle of prism is 30°. Apply Snell's law for refraction at face *AB*.

$1 \sin i = \mu \sin r$



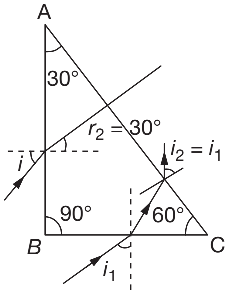
$i = 90°$

(C) Using the relation $i_1 + i_2 = A + \delta$ for ray 2.

$90° + 0° = 30° + \delta$

$\delta = 60°$

(D) $\mu = \dfrac{\sin\left(\dfrac{A+\delta m}{2}\right)}{\sin\dfrac{A}{2}} \Rightarrow \delta m = 120°$



(A) → 3; (B) → 2; (C) → 4; (D) → 1

147. (A) $m = 2 = \dfrac{v}{u} \Rightarrow \dfrac{1}{2u} - \dfrac{1}{u} = \dfrac{1}{40}$

$\Rightarrow \quad u = -20$ cm

(B) $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

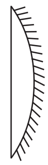
$\dfrac{1}{40} = (1.25 - 1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right)$

$R = 10$ cm

$P = 2P_l + P_m$

$P = \dfrac{2}{40} + \dfrac{2}{10} = \dfrac{1}{4}$

$F_{eq} = -4$ cm



(C) $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} - \dfrac{1}{8} = \dfrac{1}{-4}$

$v = -8$ cm

(D) Shift $= \left(1 - \dfrac{1}{\mu}\right)t = \left(1 - \dfrac{1}{1.25}\right)5 = 1$ cm

(A) → 4; (B) → 2; (C) → 5; (D) → 3

Integer Type

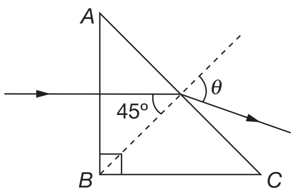
148. For coherent source $I_{max.} = 4I$

∴ for incoherent source $I = I_1 + I_2 = 2I$

∴ Ratio = 2

149. $\mu \sin 45 = 1 \sin\theta \Rightarrow \theta = \sin^{-1}\left(\dfrac{\mu}{\sqrt{2}}\right)$

$$\frac{d\theta}{dt} = \frac{1}{\sqrt{1 - \dfrac{\mu^2}{2}}} \times \frac{1}{\sqrt{2}}\left(\frac{d\mu}{dt}\right) = 2 \text{ rad/s}$$



150. $I_0 = 32\,\text{Wm}^{-2}$

After passing through first polarizer, $I_1 = \dfrac{I_0}{2} = 16\ \text{W/m}^2$

After passing through second polarizer, $I_2 = I_1 \cos^2 30° = 12\ \text{W/m}^2$

After passing through third polarizer, $I_3 = I_2 \cos^2 60° = 3\ \text{W/m}^2$

151. $m = \dfrac{v_0}{u_0}\left(1 + \dfrac{D}{f_e}\right) = m_0\left(1 + \dfrac{D}{f_e}\right)$

$\Rightarrow \quad 30 = m_0\left(1 + \dfrac{25}{5}\right) = m_0 \times 6$

$\Rightarrow \quad m_0 = 5$

152. $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$, $u = -60$ $f = -30$,

$\dfrac{1}{-60} + \dfrac{1}{v} = -\dfrac{1}{30}, \ \dfrac{1}{v} = -\dfrac{1}{30} + \dfrac{1}{60} = -\dfrac{1}{60}$

$v = -60$ cm and $\dfrac{1}{v^2}\dfrac{dv}{dt} + \dfrac{1}{u^2}\dfrac{du}{dt} = 0, \ \dfrac{du}{dt} = -\dfrac{v^2}{u^2}\left(\dfrac{du}{dt}\right)$

$\dfrac{dv}{dt} = -5$ m/s

153. For refraction at spherical surface $\dfrac{\mu}{v} - \dfrac{1}{\infty} = \dfrac{\mu - 1}{R}$

$\Rightarrow v = \dfrac{\mu}{\mu - 1}R = 3$

154. $\sqrt{d^2 + D^2} - D = \lambda$

$d^2 + D^2 = D^2 + \lambda^2 + 2\lambda D$

$D = \dfrac{d^2}{2\lambda} = \dfrac{10^{-8}}{2 \times 5 \times 10^{-7}} = 10^{-2}\,\text{m}$

$D = 1$ cm

155. At maximum depth, the ray graze the surface (i.e. the angle made by the ray with normal will become 90°)

Applying Snell's law, $1\times\sin 45° = \left(\sqrt{2}-\frac{1}{\sqrt{2}}x\right)\sin 90°$

$\Rightarrow \quad \sqrt{2}-\frac{1}{\sqrt{2}}x = \frac{1}{\sqrt{2}}$ or $x = 1$ m

156. Angular width $= \frac{2\lambda}{d} = \frac{2\times 6000\times 10^{-10}}{12\times 10^{-5}\times 10^{-2}} = 1$ rad

157. $f = \frac{D^2 - x^2}{4D}$ (Focal length by displacement method)

$\Rightarrow \quad f = \frac{(100)^2-(40)^2}{4\times 100} = 21\,\text{cm}$

$\therefore \quad P = \frac{100}{f} = \frac{100}{21} \approx 5\text{D}$

Previous Years' Questions

158. $n = \frac{360°}{\theta°} - 1 = \frac{360°}{60°} - 1 = 5$

The correct option is (A)

159. Resolving power is proportional to λ^{-1}

$\therefore \quad \frac{\text{R.P. for }\lambda_1}{\text{R.P. for }\lambda_2} = \frac{\lambda_1}{\lambda_2} = \frac{5000}{4000} = \frac{5}{4}$

The correct option is (D)

160. Total internal reflection is used in optical fibres.

The correct option is (A)

161. For interference phenomenon, two sources should emit radiation of the same frequency and have a definite phase relationship.

The correct option is (D)

162. $n = \frac{360°}{\theta°} - 1$

$\therefore \quad 3 = \frac{360°}{\theta°} - 1 \Rightarrow 4\theta° = 360° \Rightarrow \theta° = 90°$

The correct option is (B)

163. Total internal reflection occurs in a denser medium when light is incident at surface of separation at angle exceeding critical angle of the medium.

Given: $i = 45°$ in the medium and total internal reflection occurs at the glass air interface

$\therefore \quad n > \frac{1}{\sin C} > \frac{1}{\sin 45°} > \sqrt{2}$

The correct option is (B)

164. A plano-convex lens behaves like a concave mirror when its curved surface is silvered.

$\therefore$ F of concave mirror so formed.

$= \frac{R}{2\mu} = \frac{30}{2\times 1.5} = 10\,\text{cm}$

To form an image of object size, the object should be placed at (2F) of the concave mirror

$\therefore$ distance of object from lens $= 2\times F$

$= 2\times 10 = 20$ cm

The correct option is (C)

165. For interference maximum, $d\sin\alpha = n\lambda$

$\therefore \quad 2\lambda\sin\theta = n\lambda$ or $\sin\theta = \frac{n}{2}$

This equation is satisfied if $n = -2, -1, 0, 1, 2$

$\sin\theta$ is never greater than (+1), less than (–1)

$\therefore$ maximum number of maximum can be five.

The correct option is (B)

166. For total internal reflection,

$\mu = \frac{1}{\sin\theta_C} \Rightarrow \sin\theta_C = \frac{1}{\mu} = \frac{3}{4}$

$\therefore \quad \tan\theta_C = \frac{\sin\theta_C}{\sqrt{1-\sin^2\theta_C}}$

$= \frac{3/4}{\sqrt{1-\frac{9}{16}}} = \frac{3}{4}\times\frac{4}{\sqrt{7}} = \frac{3}{\sqrt{7}}$

$\therefore \quad \frac{R}{12} = \frac{3}{\sqrt{7}} \Rightarrow R = \frac{36}{\sqrt{7}}$ cm

The correct option is (B)

167. Resolution limit $= \frac{1.22\lambda}{d}$

Again resolution limit $= \sin\theta = \theta = \frac{y}{D}$

$\therefore \quad \frac{y}{D} = \frac{1.22\lambda}{d}$

or $\quad D = \frac{yd}{1.22\lambda}$

or $\quad D = \frac{(10^{-3})\times(3\times 10^{-3})}{(1.22)\times(5\times 10^{-7})} = \frac{30}{6.1} \approx 5$ m

The correct option is (B)

168. Straight line fringes are formed on screen.

The correct option is (D)

169. For diffraction pattern,

$I = I_0\left(\frac{\sin\phi}{\phi}\right)^2$, where ϕ denotes path difference

For principal maximum, $\phi = 0$. Hence $\left(\frac{\sin\phi}{\phi}\right) = 1$

Hence, intensity remains constant at I_0

The correct option is (D)

170. Intensity of polarized light $= I_0/2$

$\therefore$ Intensity of light not transmitted

$$I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

The correct option is (B)

171. Angle of minimum deviation $D = A(\mu - 1)$

$$\frac{D_1 \text{ for red}}{D_2 \text{ for blue}} = \frac{\mu_R - 1}{\mu_B - 1}$$

Since $\mu_B > \mu_R$

$$\therefore \frac{D_1}{D_2} < 1$$

$$\therefore D_1 < D_2$$

The correct option is (A)

172. In Young's double slits experiment, intensity at a point is given by.

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

where $\phi =$ phase difference, $I_0 =$ maximum intensity

$$\text{or} \quad \frac{I}{I_0} = \cos^2\left(\frac{\phi}{2}\right) \qquad (1)$$

Phase difference $\phi = \frac{2\pi}{\lambda} \times \text{path difference}$

$$\therefore \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} \text{ or } \phi = \frac{\pi}{3} \qquad (2)$$

Substitute Equation (2) in Equation (1), we get

$$\frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right) \text{ or } \frac{I}{I_0} = \frac{3}{4}$$

The correct option is (A)

173. Power of combination $= P_1 + P_2$

$= -15D + 5D = -10D$

Focal length of combination $F = \frac{1}{F} = \frac{1}{-10D}$

$= -0.1 \text{ m} = -10 \text{ cm}$

The correct option is (C)

174. According to the new Cartesian,

System used in schools, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for a convex lens, u has to be negative.

If $v = \infty, u = f$ and $u = \infty, v = f$

A parallel beam $(u = \infty)$ is focused at f and if the object is at f, the rays are parallel. The point which meets the curve at $u = v$ given $2f$. Therefore v is +ve, u is negative, both are symmetrical and this curve satisfies all the conditions for a convex lens.

The correct option is (C)

175. For interference, by Young's double slits, the path difference $\frac{xd}{D} = n\lambda$ for bright fringes and $\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$ for getting dark fringes

The central fringes when $x = 0$, coincide for all wavelengths. The third fringe of $\lambda_1 = 590$ nm coincides with the fourth bright fringe of unknown wavelength λ.

$$\therefore \frac{xd}{D} = 3 \times 590 \text{ nm} = 4 \times 1 \text{ nm}$$

$$\therefore \lambda = \frac{3 \times 590}{4} = 442.5 \text{ nm}$$

The correct option is (B)

176. If θ_c has to be the critical angle, $\theta_c = \sin^{-1}\frac{1}{\mu}$

But $\theta_c = 90° - \phi, \theta_i = \theta$

$$\frac{\sin\theta}{\sin\phi} = \mu = \frac{2}{\sqrt{3}} \Rightarrow \frac{\sin\theta}{\cos\theta_C} = \mu$$

But

$$\cos\theta_C = \frac{\sqrt{\mu^2 - 1}}{\mu} \therefore \sin\theta = \mu\frac{\sqrt{\mu^2 - 1}}{\mu} = \sqrt{\mu^2 - 1}$$

$$\therefore \theta = \sin^{-1}\sqrt{\frac{4}{3} - 1} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

So that θ_C is making total internal reflection.

The correct option is (C)

177. $I \propto A^2, I_1 = I_0, I_2 = 4I_0$

$$I = I_0 + 4I_0 + 2.\sqrt{I_0}\ \sqrt{(4I_0)}\cos\phi$$

$$= (I_m/9)[1 + 4\cos^2(\phi/2)]$$

The correct option is (D)

178. $\Delta t = [(1/15)] \times 1 = (2/3) \text{ cm } (1/v) - (1/u) = (1/f)$

$\Rightarrow (1/12) - (1/-240) = (1/f)$

$\Rightarrow f = (240/21)\text{cm}$

Again

$(1/v) - (1/u) = (1/f)$

$\Rightarrow (3/35) - (1/-u) = (21/240)$

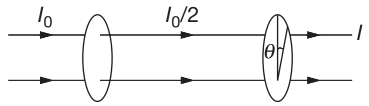
$u = [(35 \times 240)/(3 \times 240 - 21 \times 35)]$

$= [8400/(720 - 735)] = (8400/-15)$

$= -560 \text{ cm} = -5.6 \text{ m}.$

The correct option is (D)

179. $I = \left(\frac{I_0}{2}\right)\cos^2\theta$

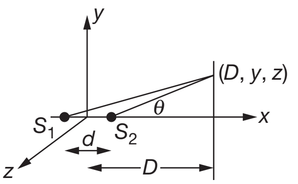


$$I = \frac{I_0}{2}\cos^2\theta = \frac{I_0}{4}$$

The correct option is (B)

180. Path difference = $\Delta x = d\cos\theta$

$$d\cdot\frac{D}{\sqrt{y^2+z^2+D^2}} = n\lambda^2$$



$$y^2+z^2+D^2 = \frac{d^2D^2}{n^2\lambda^2}$$

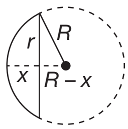
$y^2+z^2 =$ constant

The correct option is (C)

181. $\delta = i+e-A$

The correct option is (B)

182. $R^2 = (R-x)^2 + r^2$



$$R^2 = R^2 + x^2 - 2Rx + r^2$$

$$2Rx = r^2 \;(x << R) \qquad R = \frac{r^2}{2x} = \frac{(3)^2}{2\times3\times10^{-1}} = 15 \text{ cm}$$

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R}\right), \qquad f = \frac{R}{(\mu-1)} = 30 \text{ cm}$$

The correct option is (B)

183. $\frac{1}{f} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f} = \left(\frac{\mu_r}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)(x)$$

$$\frac{1}{f_1} = \left(\frac{9}{8} - 1\right)(x)$$

$$\frac{1}{f_2} = \left(\frac{9}{10} - 1\right)(x) \text{ where } x = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The correct option is (B)

184. $\mu = A + \frac{B}{\lambda^2} + \ldots\ldots$ (1)

Also, $\mu = \frac{1}{\sin C}$ (2)

$\Rightarrow$ On using white light in place of green at θ, green will be along surface.

Y, O, R will be above surface VIB below surface.

The correct option is (B)

185. We know that

$I = I_{max}\cos^2\theta$ (Law of Malus)

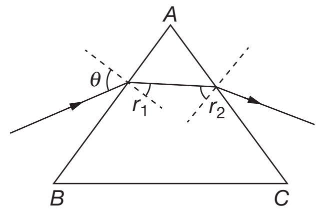
According to question,

$$I_A\left(\frac{3}{4}\right) = I_B\left(\frac{1}{4}\right) \Rightarrow \frac{I_A}{I_B} = \frac{1}{3}$$

The correct option is (D)

186. For transmission through face AC $r_2 < \theta_C$

As $r_1 + r_2 = A$



So $r_1 > A - \theta_C$

From Snell's law,

$$\sin\theta = \mu\sin r_1$$

$$\sin\theta > \mu\sin(A-\theta_C)$$

$$\theta > \sin^{-1}\left[\mu\sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

The correct option is (D)

187. The limit of resolution of a simple microscope is given by

$$d = \frac{1.22\lambda}{2\mu\sin\theta}$$

Here d is the least distance between two objects, which can be just resolved by eye. θ is semi-vertical angle of cone.

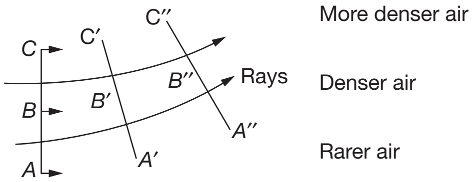
$$\sin\theta \approx \tan\theta = \frac{0.25 \text{ cm}}{25 \text{ cm}} \approx \frac{1}{100}$$

$$d = \frac{1.22\times500\times10^{-9}}{2\times10^{-2}}$$

$$d = 30.5\ \mu\text{m}$$

The correct option is (A)

188. Consider the wave front *ABC* moving towards right side. The speed of secondary wavelets emitted from this wave front will decrease continuously from *A* to *C*. Hence, upper portions of wave front will move slower than lower portions. So the shape of wave front is likely to bend as it moves forward. Hence, the rays of beam will bend upward.



The correct option is (C)

189. $m = \dfrac{\theta}{\theta_0} \quad \alpha \quad \dfrac{h_I}{h_0}$

$h_I = 20\,h_0$

The correct option is (B)

190. Total spread (Radius), $b = a + \dfrac{\lambda L}{a}$

for maximum value of b

$$\frac{db}{da} = 0$$

$$1 - \frac{\lambda L}{a^2} = 0$$

$$a = \sqrt{\lambda L}$$

$$\therefore \quad b = \sqrt{4\lambda L}$$

The correct option is (B)

191. $i = 35°, \quad \delta = 40°, \quad e = 79°$

$$\delta = i + e - A$$

$$\Rightarrow \quad A = 74°$$

As $\quad \delta_{\text{given}} > \delta_{\text{min}}$

$$\left(\frac{\delta_{\text{given}} + A}{2}\right) > \left(\frac{\delta_{\text{min}} + A}{2}\right)$$

$$\sin\left(\frac{\delta_{\text{given}} + A}{2}\right) > \sin\left(\frac{\delta_{\text{min}} + A}{2}\right)$$

$$\frac{\sin\left(\frac{\delta_{\text{given}} + A}{2}\right)}{\sin\frac{A}{2}} > \frac{\sin\left(\frac{\delta_{\text{min}} + A}{2}\right)}{\sin\frac{A}{2}}$$

$$\frac{\sin\left(\frac{40 + 74}{2}\right)}{\sin 37°} > \mu$$

$$\mu < \frac{\sin 57°}{\sin 37°}$$

$$\mu < (\approx 1.4)$$

The correct option is (D)