CHAPTER 16

Electromagnetic Induction

Chapter Highlights

Electromagnetic induction; Faraday's law, induced emf and current; Lenz's Law, Eddy currents. Self and mutual inductance. Alternating currents, peak and rms value of alternating current/ voltage; reactance and impedance; LCR series circuit, resonance; Quality factor, power in AC circuits, wattless current. AC generator and transformer.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

1. When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire, then an EMF is produced in the loop or in that wire. This EMF is called induced EMF.
   If the circuit is closed, then the current will be called induced current.

$$\text{Magnetic flux} = \int \vec{B}.d\vec{s}$$

2. The magnitude of induced EMF is equal to the rate of change of flux with respect to time in case of loop. In case of a wire, it is equal to the rate at which magnetic lines of force are cut by a wire

$$E = -\frac{d\phi}{dt}$$

   (–) sign indicates that the EMF will be induced in such a way that it will oppose the change of flux.
   SI unit of magnetic flux = Weber.

SOLVED EXAMPLES

1. A coil is placed in a constant magnetic field. The magnetic field is parallel to the plane of the coil as shown in Fig. 16.1. Find the EMF induced in the coil.

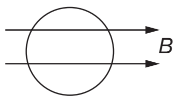


Fig. 16.1

Solution:

$\phi = 0$ (always) since area is perpendicular to magnetic field.

$\therefore$ EMF = 0.

2. Find the EMF induced in the coil shown in Fig. 16.2. The magnetic field is perpendicular to the plane of the coil and is constant.

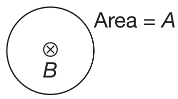


Fig. 16.2

Solution:

$$\phi = \text{BA (always)}$$
$$= \text{constant}$$

$\therefore$ EMF = 0.

3. Find the direction of induced current in the coil shown in Fig. 16.3. Magnetic field is perpendicular to the plane of coil and it is increasing with time.

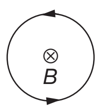


Fig. 16.3

Solution:

Inward flux is increasing with time. To opposite it, outward magnetic field should be induced. Hence, current will flow anti-clockwise.

4. Figure 16.4 shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil. The magnetic field is decreasing at a rate of 10 T/s. Find out current in magnitude and direction.

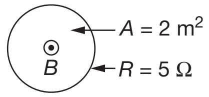


Fig. 16.4

Solution:

$$\phi = B.A$$

$$\text{EMF} = A \cdot \frac{dB}{dt} = 2 \times 10 = 20 \text{ V}$$

$\therefore$ $i = 20/5 = 4$ A

From Lenz's law, direction of current will be anti-clockwise.

5. Figure 16.5 shows a coil placed in a magnetic field decreasing at a rate of 10 T/s. There is also a source of EMF 30 V in the coil. Find the magnitude and direction of the current in the coil.

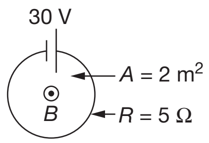
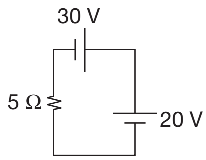


Fig. 16.5

Solution:



Induce EMF = 20 V
equivalent $i = 2$ A clockwise.

6. Figure 16.6 shows a long current carrying wire and two rectangular loops moving with velocity *v*. Find the direction of current in each loop.

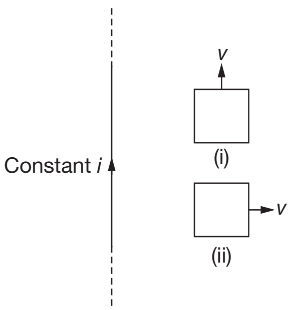


Fig. 16.6

Solution:

In loop (i), no EMF will be induced because there is no flux change.

In loop (ii), EMF will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore, to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. Hence, direction of current should be clockwise.

LENZ'S LAW (CONSERVATION OF ENERGY PRINCIPLE)

According to this law, EMF will be induced in such a way that it will oppose the cause which has produced it. Fig. 16.7 shows a magnet approaching a ring with its north pole towards the ring.

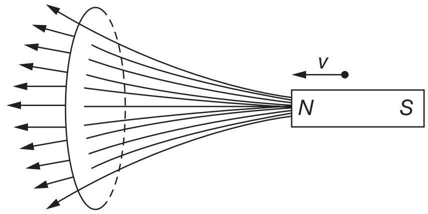


Fig. 16.7

We know that magnetic field lines come out of the north pole and magnetic field intensity decreases as we move away from magnet. So the magnetic flux (here towards left) will increase with the approach of magnet. This is the cause of flux change. To oppose it, induced magnetic field will be towards right. For this, the current must be anti-clockwise as seen by the magnet.

If we consider the approach of north pole to be the cause of flux change, Lenz's law suggests that the side of the coil towards the magnet will behave as north pole and will repel the magnet. We know that a current carrying coil will behave like north pole if it flows anti-clockwise. Thus, as seen by the magnet, the current will be anti-clockwise.

If we consider the approach of magnet as the cause of the flux change, Lenz's law suggest that a force opposite to the motion of magnet will act on the magnet, whatever be the mechanism.

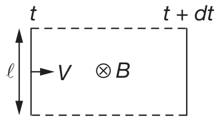
Lenz's law tells that if the coil is set free, it will move away from magnet, because in doing so, it will oppose the 'approach' of magnet.

If the magnet is given some initial velocity towards the coil and is released, it will slow down. It can be explained as follows.

The current induced in the coil will produce heat. From the energy conservation, if heat is produced, there must be an equal decrease of energy in some other form, here it is the kinetic energy of the moving magnet. Thus, the magnet must slow down. So we can justify that Lenz's law is conservation of energy principle.

MOTIONAL EMF

We can find EMF induced in a moving rod by considering the number of lines cut by it per second assuming there are B lines per unit area. Thus, when a rod of length ℓ moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to ℓv and hence it will cut $B \ell v$ lines per unit time.



hence EMF induced between the ends of the rod = $Bv\ell$.

Also EMF $= \dfrac{d\phi}{dt}$. Here ϕ denotes flux passing through the area, swept by the rod. The rod sweeps an area equal to ℓvdt in time interval dt. Flux through this area = $B\ell vdt$.

Thus, $\dfrac{d\phi}{dt} = \dfrac{B\ell vdt}{dt} = Bv\ell$.

If the rod is moving as shown in Fig. 16.8, it will sweep area per unit time = $v\ell\sin\theta$

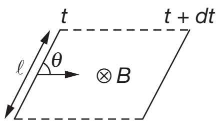
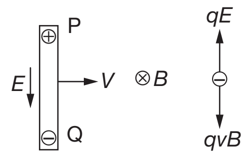


Fig. 16.8

and hence it will cut $Bv\ell \sin\theta$ lines per unit time. Thus, EMF = $Bv\ell\sin\theta$.

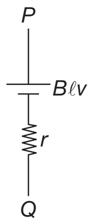
Explanation of EMF Induced in Rod on the Basis of Magnetic Force

If a rod is moving with velocity v in a magnetic field B, as shown, the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be a deficiency of free electrons and hence a surplus of positive charge at the upper end. These charges at the end will produce an electric field in downward direction which will exert an upward force on electron. If the rod has been moving for quite some time, enough charges will accumulate at the ends so that the two forces qE and qvB will balance each other. Thus, $E = v B$.



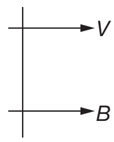
$$V_P - V_Q = VB\ell$$

The moving rod is equivalent to the following diagram, electrically.

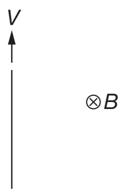


SOLVED EXAMPLES

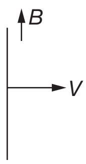
7. Find the EMF induced in the rod in the following cases. The figures are self-explanatory.

(A) 

EMF = 0

(B) 

EMF = 0

(C) 

EMF = 0

Solution:

Figure 16.9 shows a closed coil $ABCA$ moving in a uniform magnetic field B with a velocity v. The flux passing through the coil is a constant and therefore the induced EMF is zero.

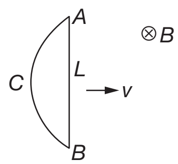
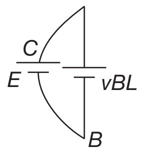


Fig. 16.9

Now consider rod AB, which is a part of the coil. EMF induced in the rod = BLv

Suppose the EMF induced in part ACB is E, as shown.



Since the EMF in the coil is zero, EMF (in ACB) + EMF (in BA) = 0

or $$-E + vBL = 0$$

or $$E = vBL$$

Thus, EMF induced in any path joining A and B is same, provided the magnetic field is uniform. Also the equivalent EMF between A and B is BLv (here the two EMFs are in parallel).

8. Figure 16.10 shows an irregular shaped wire AB moving with velocity v, as shown.

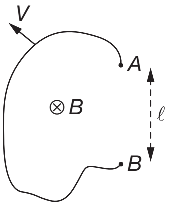
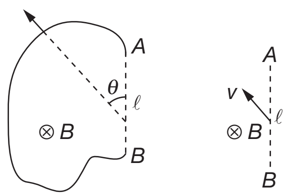


Fig. 16.10

Find the EMF induced in the wire.

Solution:

The same EMF will be induced in the straight imaginary wire joining A and B, which is $Bv\ell\sin\theta$.



9. A circular coil of radius R is moving in a magnetic field B with a velocity v as shown in Fig. 16.11.

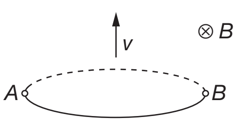


Fig. 16.11

Find the EMF across the diametrically opposite points A and B.

Solution:

$$2\,R\,v\,B.$$

10. Find the EMF across the points P and Q, which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in Fig. 16.12. Also draw the electrical equivalent circuit of each branch.

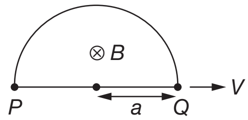


Fig. 16.12

Solution:

Induced EMF = 0.

11. Find the EMF across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown in Fig. 16.13. Also draw the electrical equivalence of each branch.

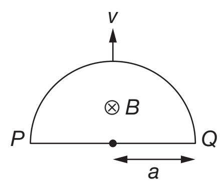
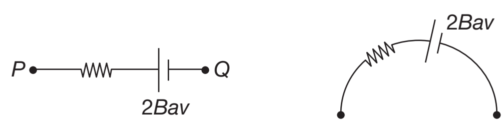


Fig. 16.13

Solution:

Induced EMF = $2Bav$.



12. Figure 16.14 shows a rectangular loop moving in a uniform magnetic field. Show the electrical equivalence of each branch.

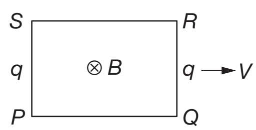
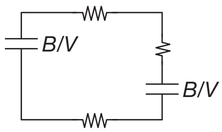


Fig. 16.14

Solution:



13. Figure 16.15 shows a rod of length ℓ and resistance r moving on two rails shorted by a resistance R. A uniform magnetic field B is present normal to the plane of rod and rails. Show the electrical equivalence of each branch.

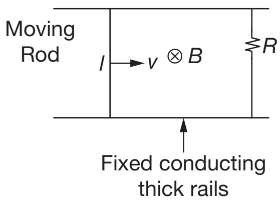
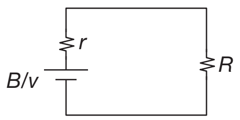


Fig. 16.15

Solution:



14. A rod of length ℓ is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the EMF induced in the wire when its distance from the long wire is x.

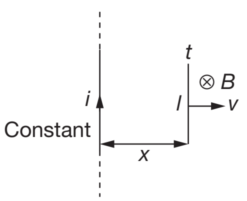
Solution:

$$E = B\ell v = \frac{\mu_0 i l v}{2\pi x}$$

or

EMF is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is $\ell v dt$. The magnetic field lines cut in dt time $= B\ell v dt = \dfrac{\mu_0 i l v dt}{2\pi x}$.

$\therefore$ the rate with which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$.



15. A rectangular loop, as shown in Fig. 16.16, moves away from an infinitely long wire carrying a current i. Find the EMF induced in the rectangular loop.

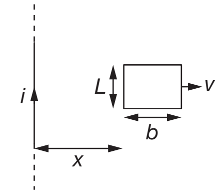
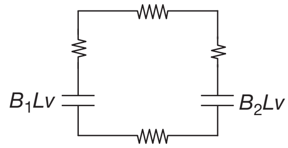


Fig. 16.16

$$E = B_1 Lv - B_2 Lv = \frac{\mu_0 i}{2\pi x} Lv - \frac{\mu_0 i}{2\pi (x+b)} Lv$$

$$= \frac{\mu_0 i L b y}{2\pi x (x+b)}$$

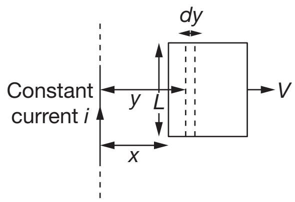


Solution:

Aliter:

Consider a small segment of width dy at a distance y from the wire.

Let flux through the segment be $d\phi$.



$$\therefore \quad d\phi = \frac{\mu_0 i}{2\pi y} L dy$$

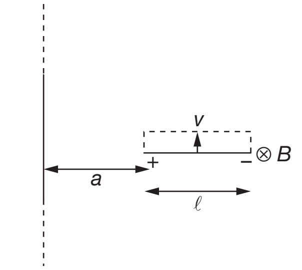
$$\therefore \quad \phi = \frac{\mu_0 i L}{2\pi} \int_x^{x+b} \frac{dy}{y} = \frac{\mu_0 i L}{2\pi} \left(\ell n(x+b) - \ell n x\right)$$

Now $$\frac{d\phi}{dt} = \frac{\mu_0 i L}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]$$

$$= \frac{\mu_0 i L}{2\pi}\left[\frac{(-b)}{x(x+b)}\right] v = \frac{-\mu_0 i b L v}{2\pi x (x+b)}$$

$\therefore$ induced EMF $= \dfrac{\mu_0 i b L v}{2\pi x (x+b)}$.

16. A rod of length ℓ is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the EMF induced in the rod, if its nearest end is at a distance a from the wire.



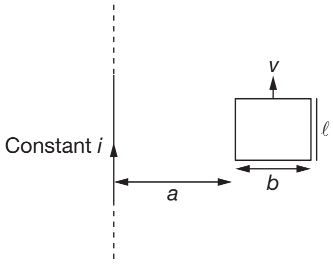
Solution:

Consider a segment of rod of length dx at a distance x from the wire. EMF induced in the segment

$$d\in = \frac{\mu_0 i}{2\pi x}\ dx.v$$

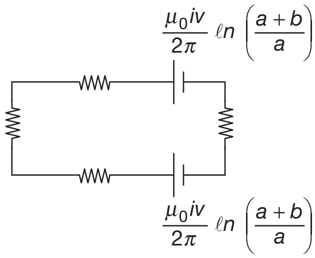
$$\therefore \qquad \in = \int_a^{a+\ell} \frac{\mu_0 i v dx}{2\pi x} = \frac{\mu_0 i v}{2\pi} \ell n\left(\frac{\ell + a}{a}\right).$$

17. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the EMF induced in the loop, if its nearest end is at a distance a from the wire. Draw an equivalent electrical diagram



Solution:

EMF = 0



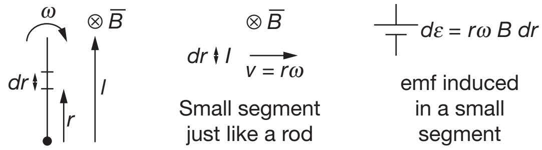
$$i = \frac{\varepsilon}{R+r}$$

$$= \frac{\mu_0 i v}{2\pi(R+r)} \ln\left(\frac{x+\ell}{\ell}\right).$$

INDUCED EMF DUE TO ROTATION

Rotation of the Rod

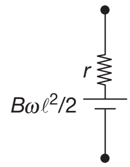
Consider a conducting rod of length ℓ rotating in a uniform magnetic field.



EMF induced in a small segment of length dr, of the rod $= v\ B\ dr = r\omega\ B\ dr$

$$\therefore \text{ EMF induced in the rod} = \omega B \int_0^l r dr = \frac{1}{2} B\omega l^2$$

Equivalent of this rod is as following



or $\quad \varepsilon = \frac{d\Phi}{dt} = \varepsilon = \frac{d\Phi}{dt}$

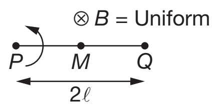
$$= \frac{\text{flux through the area swept by the rod in time } dt}{dt}$$

$$= \frac{B\frac{1}{2}\ell^2\omega dt}{dt}$$

$$= \frac{1}{2}B\omega\ell^2.$$

SOLVED EXAMPLES

18. A rod PQ of length 2 ℓ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced EMF between M and Q if of P and Q = 100 V.



Solution:

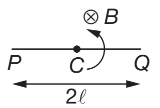
$$E_{MQ} + E_{PM} = E_{PQ} \qquad \text{corner} \rightarrow \frac{B\omega\ell^2}{2} = 100$$

$$E_{MQ} + \frac{B\omega\left(\frac{\ell}{2}\right)^2}{2} = \frac{B\omega\ell^2}{2}$$

$$E_{MQ} = \frac{3}{8}\ B\omega\ell^2$$

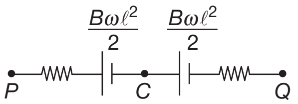
$$= \frac{3}{4}\ \times 100\text{ V} = 75\text{ V}.$$

19. A rod PQ of length 2ℓ is rotating about its mid point C, in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Find the induced EMF between PQ and PC. Draw the circuit diagram of parts PC and CQ.



Solution:

$$\text{EMF}_{PQ} = 0; \quad \text{EMF}_{PC} = \frac{B\omega\ell^2}{2}.$$



20. A rod of length ℓ is rotating with an angular speed ω about its one end which is at a distance a from an infinitely long wire carrying current i. Find the EMF induced in the rod at the instant shown in Fig. 16.17.

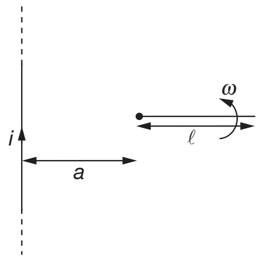
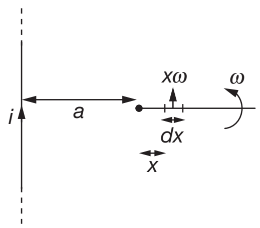


Fig. 16.17

Solution:

Consider a small segment of rod of length dx, at a distance x from one end of the rod. EMF induced in the segment



$$dE = \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx$$

$$\therefore \quad E = \int_0^\ell \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx$$

$$= \frac{\mu_0 i\omega}{2\pi}\left[\ell - a\cdot \ell n\left(\frac{\ell+a}{a}\right)\right].$$

21. A rod of length ℓ is rotating with an angular speed ω about its one end which is at a distance a from an infinitely long wire carrying current i. Find the EMF induced in the rod at the instant shown in Fig. 16.18.

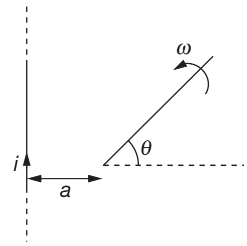
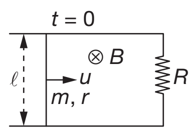


Fig. 16.18

Solution:

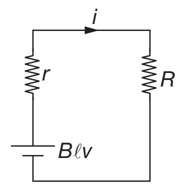
$$E = \frac{\mu_0 i\omega}{2\pi\cos\theta}\left[\ell - \frac{a}{\cos\theta}\ell n\left(\frac{a+\ell\cos\theta}{a}\right)\right].$$

22. A rod of mass m and resistance r is placed on fixed, resistance-less, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u. Find its velocity as a function of time.



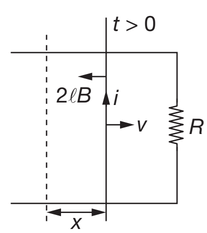
Solution:

Let at an instant the velocity of the rod be v. The EMF induced in the rod will be $vB\ell$. The electrically equivalent circuit is shown in the following diagram.



$\therefore$ Current in the circuit $i = \dfrac{B\ell v}{R+r}$

At time t,



Magnetic force acting on the rod is $F = i\ell B$, opposite to the motion of the rod.

$$i\ell B = -m\,\frac{dv}{dt}$$

$$i = \frac{B\ell v}{R+r}$$

Now solving these two equation,

$$\frac{B^2\ell^2 v}{R+r} = -m \cdot \frac{dv}{dt}$$

$$-\frac{B^2\ell^2}{(R+r)m} \cdot dt = \frac{dv}{v}$$

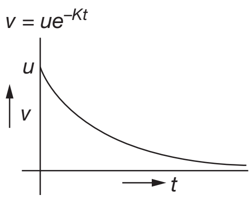
Let $$\frac{B^2\ell^2}{(R+r)m} = k$$

$$-K \cdot dt = \frac{dv}{v}$$

$$\int_u^v \frac{dv}{v} = \int_0^t -K.dt$$

$$\ln\left(\frac{v}{u}\right) = -Kt$$

$$v = ue^{-Kt}.$$



23. In the above question, find the force required to move the rod with constant velocity v and also find the power delivered by the external agent.

Solution:

The force needed to keep the velocity constant $F_{ext} = i\ell B$

$$= \frac{B^2\ell^2 v}{R+r}$$

Power due to external force $= \dfrac{B^2\ell^2 v^2}{R+r} = \dfrac{\varepsilon^2}{R+r} = i^2(R+r)$

Note that the power delivered by the external agent is converted into joule heating in the circuit. That means magnetic field helps in converting the mechanical energy into joule heating.

24. In the above question, if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

Solution:

$$m\frac{dv}{dt} = F - i\ell B$$

$$i = \frac{B\ell v}{R+r}$$

$$m\frac{dv}{dt} = F - \frac{B^2\ell^2 v}{R+r}$$

Let $$K = \frac{B^2\ell^2}{R+r}$$

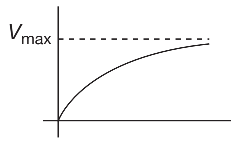
$$\int_0^v \frac{dv}{F-Kv} = \int_0^t \frac{dt}{m}$$

$$-\frac{1}{K}\ln(F-Kv)\int_0^v \frac{t}{m}$$

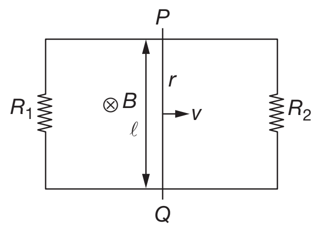
$$\ln\left(\frac{F-kv}{F}\right) = -\frac{Kt}{m}$$

$$F - Kv = Fe^{-kt/m}$$

$$v = \frac{F}{K}\,(1 - e^{-kt/m}).$$



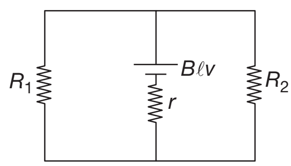
25. A rod PQ of mass m and resistance r is moving on two fixed, resistance-less, smooth conducting rails (closed on both sides by resistances R_1 and R_2). Find the current in the rod at the instant when its velocity is v.



Solution:

$$i = \frac{B\ell v}{r + \dfrac{R_1R_2}{R_1+R_2}}$$

This circuit is equivalent to the following diagram.



26. In the above question, if one resistance is replaced by a capacitor of capacitance C as shown in Fig. 16.19,

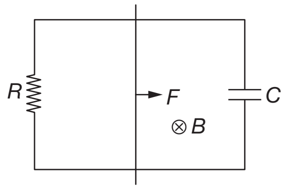
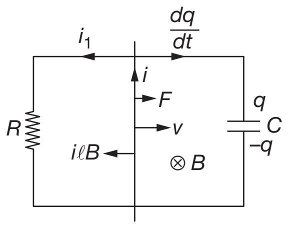


Fig. 16.19

Find the velocity of the moving rod at time t if the initial velocity of the rod is v and a constant force F is applied on the rod. Neglect the resistance of the rod.

Solution:

At any time t, let the velocity of the rod be v.



Applying Newton's law

$$F - i\ell B = ma \qquad (16.1)$$

Also $$B\ell v = i_1 R = \frac{q}{c}$$

Applying $Kc\ell$,

$$i = i_1 + \frac{dq}{dt} = \frac{B\ell v}{R} + \frac{d}{dt}(BlvC)$$

or $$i = \frac{B\ell v}{R} + B\ell C\, a$$

Putting the value of i in Equation (16.1),

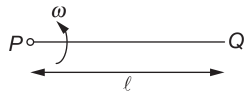
$$F - \frac{B^2\ell^2 v}{R} = (m + B^2\ell^2 C)\, a = (m + B^2\ell^2 C)\frac{dv}{dt}$$

$$(m + B^2\ell^2 C)\ \frac{dv}{F - \dfrac{B^2 l^2 v}{R}} = dt$$

Integrating both sides, and solving we get

$$v = \frac{FR}{B^2\ell^2}\left(1 - e^{-\frac{tB^2\ell^2}{R(m+CB^2\ell^2)}}\right).$$

27. A rod PQ of length ℓ is rotating about end P, with an angular velocity ω. Due to centrifugal forces, the free electrons in the rod move towards the end Q and an EMF is created. Find the induced EMF.



Solution:

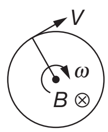
The accumulation of free electrons will create an electric field which will finally balance the centrifugal forces, and a steady state will be reached. In the steady state, $m_e\omega^2 x = e\,E$.

$$V_P - V_Q = \int_{x=0}^{x=\ell} \vec{E}.d\vec{x} = \int_0^\ell \frac{m_e\omega^2 x}{e}\,dx = \frac{m_e\omega^2\ell^2}{2e}.$$

Rotation of a Coil

SOLVED EXAMPLES

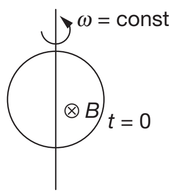
28. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the centre of the ring. A constant magnetic field B exists parallel to the axis. Find the EMF induced in the ring



Solution:

Flux passing through the ring $\phi = \mathrm{B}\cdot\mathrm{A}$ is a constant here; therefore, EMF induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

29. A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the centre of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the EMF induced in the ring as a function of time.



Solution:

At any time t,

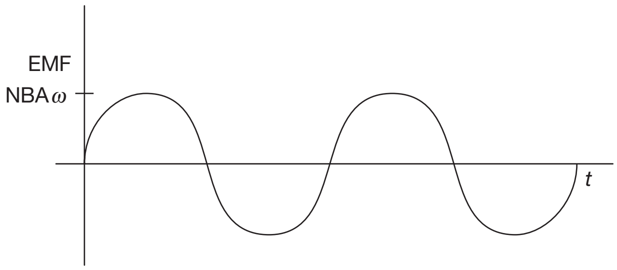
$$\phi = BA\cos\theta = BA\cos\omega t$$

Now induced EMF in the loop

$$e = \frac{-d\phi}{dt} = BA\ \omega \sin \omega t$$

If there are N turns,

$$\text{EMF} = BA\,\omega N \sin \omega t$$



$BA\ \omega$N is the amplitude of the EMF

$$e = e_m \sin \omega t$$

$$i = \frac{e}{R} = \frac{e_m}{R} \sin \omega t = i_m \sin \omega t$$

$$i_m = \frac{e_m}{R}$$

The rotating coil thus produces a sinusoidally varying current, or alternating current. This is also the principle used in the generator.

30. Figure 16. 20 shows a wire frame PQSTXYZ placed in a time varying magnetic field given as $B = \beta t$, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.

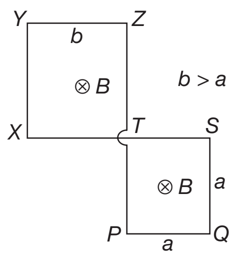


Fig. 16.20

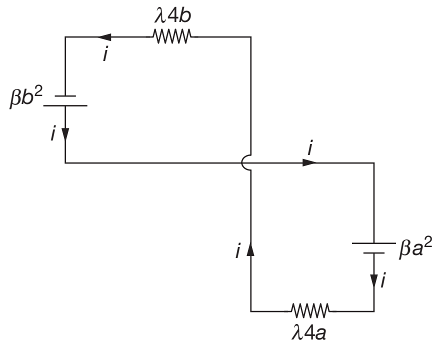
Solution:

Induced EMF in part $PQST = \beta a^2$ (in anti-clockwise direction, from Lenz's Law)

Similarly, induced EMF in part TXYZ = βb^2 (in anti-clockwise direction, from Lenz's Law)

Total resistance of the part $PQST = \lambda 4a$.

Total resistance of the part $PQST = \ell 4b$. The equivalent circuit is as shown in the following diagram.



writing KVL along the current flow

$$\beta b^2 - \beta a^2 - \lambda\, 4ai - \lambda\, 4bi = 0$$

$$i = \frac{\beta}{4\lambda}\ (b - a).$$

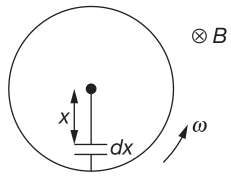
EMF Induced in a Rotating Disc

Consider a disc of radius r rotating in a magnetic field B.

Consider an element dx at a distance x from the centre. This element is moving with speed $v = \omega x$.

∴ Induced EMF across dx

$$= B(dx)\ v + Bdx\omega x = B\omega x dx$$



∴ EMF between the centre and the edge of disc.

$$= \int_0^r B\omega x dx = \frac{B\omega r^2}{2}$$

FIXED LOOP IN A VARYING MAGNETIC FIELD

Now consider a circular loop, at rest in a varying magnetic field. Suppose the magnetic field is directed inside the page and it is increasing in magnitude, the EMF induced in the loop will be

$\varepsilon = -\dfrac{d\phi}{dt}$. Flux through the coil will be $\phi = -\pi r^2 B$;

$$\frac{d\phi}{dt} = -\pi r^2 \frac{dB}{dt}; \quad \varepsilon = -\frac{d\phi}{dt}$$

$$\therefore \quad \varepsilon = \pi r^2 \frac{dB}{dt}$$

$$\therefore \quad E 2\pi r = \pi r^2 \frac{dB}{dt}$$

or $$E = \frac{r}{2}\frac{dB}{dt}$$

Thus, changing magnetic field produces electric field which is non-conservative in nature. The lines of force associated with this electric field are closed curves.

SELF-INDUCTION

Self-induction is induction of EMF in a coil due to its own current change. Total flux $N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = Li$, where L is called coefficient of self-induction or inductance**.** The inductance L is purely a geometrical property, i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt, the average EMF induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\varphi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}.$$

The instantaneous EMF is given as

$$\varepsilon = -\frac{d(N\varphi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$$

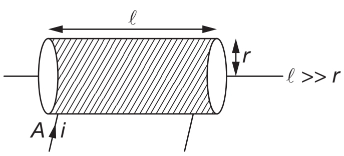
SI unit of inductance is wb/amp or Henry(H)

L – self-inductance is a +ve quality
L depends on:

1. Geometry of loop
2. Medium in which it is kept. L does not depend upon current.

L is a scalar quantity.

Self-inductance of Solenoid



Let the volume of the solenoid be V and the number of turns per unit length be n.

Let a current I be flowing in the solenoid. Magnetic field in the solenoid is given as $B = \mu_0 n I$. The magnetic flux through one turn of solenoid $\phi = \mu_0 n I A$.

The total magnetic flux through the solenoid $= N\phi = N\mu_0 n I A = \mu_0 n^2 \ell A I$

$$\therefore \quad L = \mu_0 n^2 \ell A = \mu_0 n^2 V$$

$$\phi = \mu_0 ni \pi r^2 (n\ell)$$

$$L = \frac{\phi}{i}$$

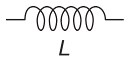
$$= \mu_0 n^2 \pi r^2 \ell.$$

Inductance per unit volume $= \mu_0 n^2$.

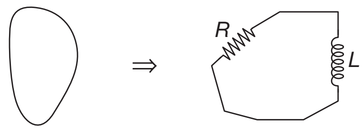
Self-inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant. Current cannot change suddenly in the inductor.

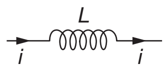
INDUCTOR

It is represented by

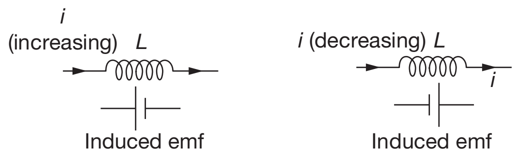


electrical equivalence of loop

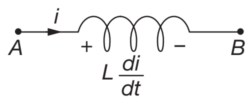




If current i through the inductor is increasing, the induced EMF will oppose the increase in current and hence will be opposite to the current. If current i through the inductor is increasing, the induced EMF will oppose the decrease in current and hence will be in the direction of the current.



Overall result,



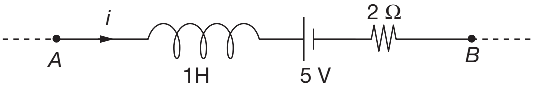
$$V_A - L\frac{di}{dt} = V_B.$$

NOTE

If there is a resistance in the inductor (resistance of the coil of inductor), then:

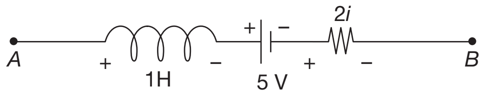
SOLVED EXAMPLE

31. AB is a part of circuit. Find the potential difference $V_A - V_B$ if



(A) current $i = 2$ A and is constant
(B) current $i = 2$ A and is increasing at the rate of 1 amp/s
(C) current $i = 2$ A and is decreasing at the rate 1 amp/s

Solution:



$$L\frac{di}{dt} = 1\frac{di}{dt}$$

writing KVL from A to B

$$V_A - 1\frac{di}{dt} - 5 - 2i = V_B.$$

(A) Put $i = 2$, $\frac{di}{dt} = 0$

$V_A - 5 - 4 = V_B$

$\therefore \quad V_A - V_B = 9$ V

(B) Put $i = 2$, $\frac{di}{dt} = 1$;

$V_A - 1 - 5 - 4 = V_B$

or $\quad V_A - V_B = 10\ V_0$

(C) Put $i = 2$, $\frac{di}{dt} = -1$

$V_A + 1 - 5 - 2 \times 2 = V_B$

$V_A = 8$ V.

Energy Stored in an Inductor

If current in an inductor at an instant is i and is increasing at the rate di/dt, the induced EMF will oppose the current. Its behaviour is as shown in Fig. 16.21.

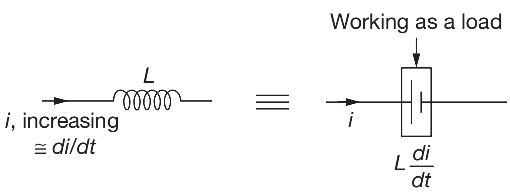


Fig. 16.21

Power consumed by the inductor $= iL\frac{di}{dt}$

Energy consumed in dt time $= iL\frac{di}{dt}\ dt$

$\therefore$ total energy consumed as the current increases from 0 to $I = \int_0^I iLdi = \frac{1}{2}LI^2$

$$= \frac{1}{2}Li^2$$

$$\Rightarrow \qquad U = \frac{1}{2}LI^2.$$

NOTE

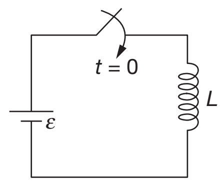
This energy is stored in the magnetic field with energy density.

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy $U = \int \frac{B^2}{2\mu_0\mu_r}dV$

SOLVED EXAMPLES

32. A circuit contains an ideal cell and an inductor with a switch. Initially, the switch is open. It is closed at $t = 0$. Find the current as a function of time.



Solution:

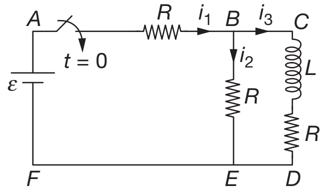
$$\varepsilon = L\frac{di}{dt}$$

$$\Rightarrow \qquad \int_0^i \varepsilon dt = \int_0^i Ldi$$

$$\varepsilon t = Li$$

$$i = \frac{\varepsilon t}{L}.$$

33. In the following circuit, the switch is closed at $t = 0$. Find the currents i_1, i_2, i_3, and $\frac{di_3}{dt}$ at $t = 0$ and at $t = \infty$. Initially, all currents are zero.



Solution:

At $t = 0$

i_3 is zero, since current cannot suddenly change due to the inductor.

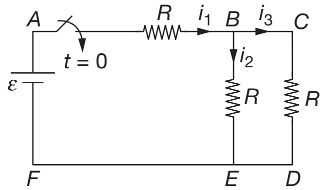
$\therefore \qquad i_1 = i_2$ (from KCL)

applying KVL in part ABEF, we get

$$i_1 = i_2 = \frac{\varepsilon}{2R}.$$

At $t = \infty$

i_3 will become constant and hence potential difference across the inductor will be zero. It is just like a simple wire and the circuit can be solved assuming it to be like shown as in the following diagram.



$$i_2 = i_3 = \frac{\varepsilon}{3R},\ i_1 = \frac{2\varepsilon}{3R}.$$

34. In the circuit shown in Fig. 16.22, S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt just after that moment.

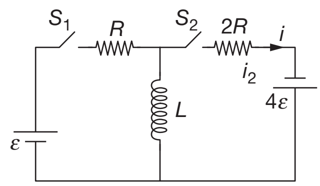
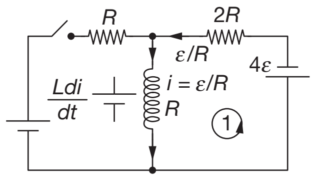


Fig. 16.22

Solution:

Before S_2 is closed and S_1 is opened, current in the left part of the circuit $= \frac{\varepsilon}{R}$.

Now when S_2 closed and S_1 opened, current through the inductor cannot change suddenly, current $\frac{\varepsilon}{R}$ will continue to move in the inductor.



Applying KVL in loop 1,

$$L\frac{di}{dt} + \frac{\varepsilon}{R}(2R) + 4\varepsilon = 0$$

$$\frac{di}{dt} = -\frac{6\varepsilon}{L}.$$

Growth of Current in Series R–L Circuit

Figure 16. 23 shows a circuit consisting of a cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at $t = 0$. Suppose at an instant, current in the circuit be i which is

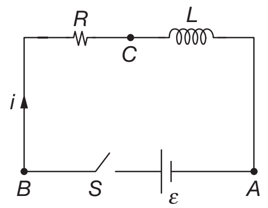


Fig. 16.23

increasing at the rate di/dt.

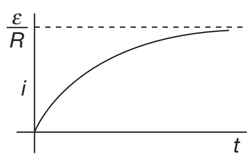
Writing KVL along the circuit, we have

$$\varepsilon - L\frac{di}{dt} - iR = 0$$

On solving we get,

$$i = \frac{\varepsilon}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$$

The quantity L/R is called time constant of the circuit and is denoted by τ. The variation of current with time is as shown.



NOTE

- Final current in the circuit $= \frac{\varepsilon}{R}$, which is independent of L.
- After one time constant, current in the circuit = 63% of the final current (verify yourself).

- More time constant in the circuit implies slower rate of change of current.
- If there is any change in the circuit containing inductor, then there is no instantaneous effect on the flux of inductor.

$$L_1 i_1 = L_2 i_2$$

SOLVED EXAMPLES

35. At $t = 0$ switch is closed (shown in Fig. 16.24) after a long time suddenly the inductance of the inductor is made η times lesser $\left(\frac{L}{\eta}\right)$ then its initial value, find out instant current just after the operation.

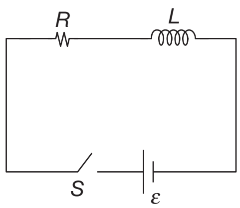


Fig. 16.24

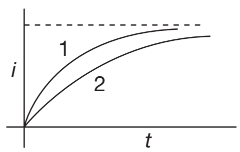
Solution:

Using above result (Note 4)

$$L_1 i_1 = L_2 i_2$$

$$\Rightarrow \quad i_2 = \frac{\eta\varepsilon}{R}.$$

36. Which of the two curves shown has less time constant?



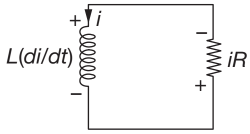
Solution:

Curve 1.

DECAY OF CURRENT IN THE CIRCUIT CONTAINING RESISTOR AND INDUCTOR

Let the initial current in the circuit be I_0. At any time t, let the current be i and let its rate of change at this instant be $\frac{di}{dt}$.

$$L \cdot \frac{di}{dt} + iR = 0$$



$$\frac{di}{dt} = -\frac{iR}{L}$$

$$\int_{I_0}^{i} \frac{di}{i} = -\int_0^t \frac{R}{L} \cdot dt$$

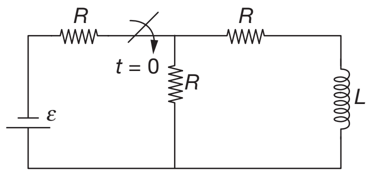
$$\ln\left(\frac{i}{I_0}\right) = -\frac{Rt}{L}$$

or $$i = I_0 e^{\frac{-Rt}{L}}$$

Current after one time constant: $i = I_0 e^{-1} = 0.37\%$ of initial current.

SOLVED EXAMPLES

37. In the following circuit, the switch is closed at $t = 0$. Initially, there is no current in inductor. Find out the current in the inductor coil as a function of time.

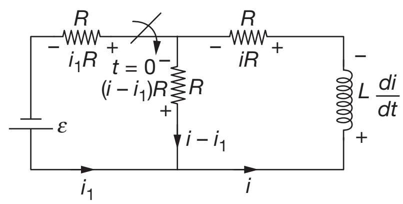


Solution:

At any time t,

$$-\varepsilon + i_1 R - (i - i_1) R = 0$$

$$-\varepsilon + 2i_1 R - iR = 0$$



$$i_1 = \frac{iR + \varepsilon}{2R}$$

Now,

$$-\varepsilon + i_1 R + iR + L\frac{di}{dt} = 0$$

$$-\varepsilon + \left(\frac{iR + \varepsilon}{2}\right) + iR + i\frac{di}{dt} = 0$$

$$-\frac{\varepsilon}{2}+\frac{3IR}{2}=-L\frac{di}{dt}$$

$$\left(\frac{-\varepsilon+3iR}{2}\right)dt=-Ldi$$

$$-\frac{-dt}{2L}=\frac{di}{-\varepsilon+3iR}$$

$$-\int_0^t\frac{dt}{2L}=\int_0^i\frac{di}{-\varepsilon+3iR}$$

$$-\frac{t}{2L}=\frac{1}{3R}\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right)$$

$$-\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right)=\frac{3Rt}{2L}$$

$$i=+\frac{\varepsilon}{3R}\left(1-e^{-\frac{3Rt}{2L}}\right).$$

38. Figure 16. 25 shows a circuit consisting of an ideal cell, an inductor L, and a resistor R, connected in series. Let the switch S be closed at $t = 0$. Suppose at $t = 0$, current in the inductor is i_0 then find out equation of current as a function of time.

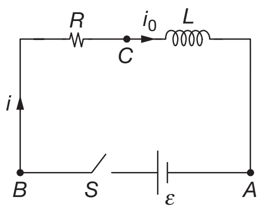
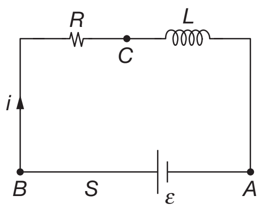


Fig. 16.25

Solution:

Let an instant t current in the circuit be i which is increasing at the rate di/dt.



Writing KVL along the circuit, we have

$$\varepsilon-L\frac{di}{dt}-iR=0$$

$$\Rightarrow\qquad L\frac{di}{dt}=\varepsilon-iR$$

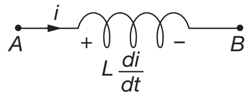
$$\Rightarrow\qquad \int_{i_0}^{i}\frac{di}{\varepsilon-iR}=\int_0^t\frac{dt}{L}$$

$$\Rightarrow\qquad \ln\left(\frac{\varepsilon-iR}{\varepsilon-i_0R}\right)=-\frac{Rt}{L}$$

$$\Rightarrow\qquad \varepsilon-iR=(\varepsilon-i_0R)\,e^{-Rt/L}$$

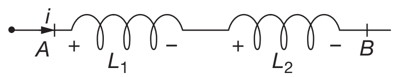
$$\Rightarrow\qquad i=\frac{\varepsilon-(\varepsilon-i_0R)e^{-Rt/L}}{R}.$$

Equivalent Self-inductance:



$$L=\frac{V_A-V_B}{di/dt}\qquad(16.2)$$

Series Combination:



$$V_A-L_1\frac{di}{dt}-L_2\frac{di}{dt}=V_B\qquad(16.3)$$

From (16.2) and (16.3),

$L = L_1 + L_2$ (neglecting mutual inductance)

Parallel Combination:

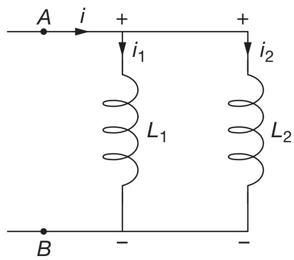


Fig. 16.26

From Fig. 16. 26

$$V_A-V_B=L_1\frac{di_1}{dt}=L_2\frac{di_2}{dt}$$

also $\qquad i=i_1+i_2$

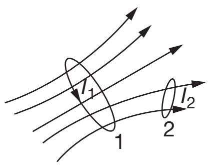
or $\qquad \frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}$

or $\qquad \frac{V_A-V_B}{L}=\frac{V_A-V_B}{L_1}+\frac{V_A-V_B}{L_2}$

$$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}.$$

(neglecting mutual inductance)

MUTUAL INDUCTANCE



Consider two arbitrary conducting loops 1 and 2. Suppose that I_1 is the instantaneous current flowing around loop 1. This current generates a magnetic field B_1 which links the second circuit, giving rise to a magnetic flux ϕ_2 through that circuit. If the current I_1 doubles, then the magnetic field B_1 doubles in strength at all points in space, so the magnetic flux ϕ_2 through the second circuit also doubles. Furthermore, it is obvious that the flux through the second circuit is zero whenever the current flowing around the first circuit is zero. It follows that the flux ϕ_2 through the second circuit is *directly proportional* to the current I_1 flowing around the first circuit. Hence, we can write $\phi_2 = M_{21}I_1$, where the constant of proportionality M_{21} is called the mutual inductance of circuit 2 with respect to circuit 1. Similarly, the flux ϕ_2 through the first circuit due to the instantaneous current I_2 flowing around the second circuit is directly proportional to that current, so we can write $\phi_1 = M_{12}I_2$, where M_{12} is the mutual inductance of circuit 1 with respect to circuit 2. It can be shown that $M_{21} = M_{12}$ (Reciprocity Theorem). Note that M is a purely geometric quantity, depending only on the size, number of turns, relative position, and relative orientation of the two circuits. The SI unit of mutual inductance is called Henry (H). One henry is equivalent to a volt-second per ampere:

Suppose the current flowing around circuit 1 changes by an amount ΔI_1 in a small time interval Δt. The flux linking circuit 2 changes by an amount $\Delta\phi_2 = M\Delta I_1$ in the same time interval. According to Faraday's law, an EMF $\varepsilon_2 = -\dfrac{\Delta\phi_2}{\Delta t}$ is generated around the second circuit due to the changing magnetic flux linking that circuit. Since $\Delta\phi_2 = M\Delta I_1$, this EMF can also be written as $\varepsilon_2 = -M\dfrac{\Delta I_1}{\Delta t}$

Thus, the EMF generated around the second circuit due to the current flowing around the first circuit is directly proportional to the rate at which that current changes. Likewise, if the current I_2 flowing around the second circuit changes by an amount ΔI_1 in a time interval Δt, then the EMF generated around the first circuit is $\varepsilon_1 = -M\dfrac{\Delta I_2}{\Delta t}$.

Note that there is no direct physical connection(coupling) between the two circuits: the coupling is due entirely to the magnetic field generated by the currents flowing around the circuits.

NOTE

- $M \le \sqrt{L_1 L_2}$
- For two coils in series if mutual inductance is considered then

$$L_{eq} = L_1 + L_2 \pm 2M$$

SOLVED EXAMPLES

39. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let I be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of turns the second wire is wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.

Solution:

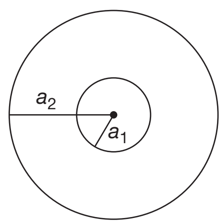
If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \dfrac{\mu_0 N_1 I_1}{\ell}$ is generated in the core. The magnetic field in the region outside the core is of negligible magnitude. The flux linking a single turn of the second wire is B_1A. Thus, the flux linking all N_2 turns of the second wire is

$$\phi_2 = N_2 B_1 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell} = MI_1.$$

$$\therefore \quad M = \frac{\mu_0 N_1 N_2 A}{\ell}.$$

As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

40. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Solution:

Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil $= \dfrac{\mu_0 i}{2a_2} \pi a_1^{\,2}$

or $$Mi = \frac{\mu_0 i}{2a_2} \pi a_1^{\,2}$$

or $$M = \frac{\mu_0 \pi a_1^2}{2a_2}.$$

41. Solve the above question, if the planes of coil are perpendicular.

Solution:

Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence $M = 0$.

42. Solve the above problem if the planes of coils make θ angle with each other.

Solution:

If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

Thus, $$\text{flux} = \vec{B}.\vec{A} = \frac{\mu_0 i}{2a_2} \cdot \pi a_1^2 \cdot \cos\theta$$

or $$M = \frac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}.$$

43. Find the mutual inductance between two rectangular loops, shown in Fig. 16.27

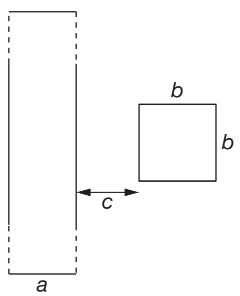
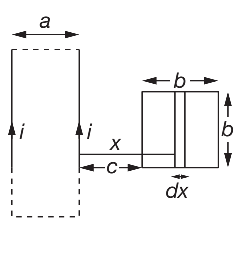


Fig. 16.27

Solution:



Let current i flow in the loop having ∞-by long sides. Consider a segment of width dx at a distance x as shown in flux through the regent

$$d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx\,.$$

$$\Rightarrow \qquad \phi = \int_c^{c+b} \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx\,.$$

$$= \frac{\mu_0 i b}{2\pi}\left[\ln\frac{c+b}{c} - \ln\frac{a+b+c}{a+c}\right].$$

44. Find the mutual inductance of a straight long wire and a rectangular loop, as shown in Fig. 16.28

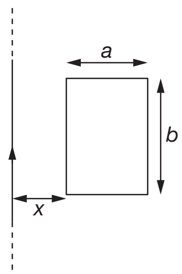


Fig. 16.28

Solution:

$$M = \frac{\mu_0 b}{2\pi} \ln\left(1 + \frac{a}{x}\right).$$

45. Figure 16.29 shows two concentric coplanar coils with radii a and b ($a << b$). A current $i = 2t$ flows in the smaller loop. Neglecting self-inductance of larger loop

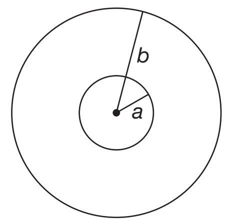


Fig. 16.29

(A) Find the mutual inductance of the two coils.
(B) Find the EMF induced in the larger coil.
(C) If the resistance of the larger loop is R find the current in it as a function of time.

Solution:

(A) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem). Let current i be flowing in the larger coil. Magnetic field at the centre $= \dfrac{\mu_0 i}{2b}$.

Flux through the smaller coil $= \dfrac{\mu_0 i}{2b}\pi a^2$

$\therefore \quad M = \dfrac{\mu_0}{2b}\pi a^2$

(B) $|\text{EMF induced in larger coil}|$

$= M\left[\left(\dfrac{di}{dt}\right) \text{in smaller coil}\right]$

$= \dfrac{\mu_0}{2b}\pi a^2$

$= \dfrac{\mu_0 \pi a^2}{b}$

(C) Current in the larger coil

$= \dfrac{\mu_0 \pi a^2}{bR}.$

46. In above question, if a capacitor of capacitance C is also connected in the larger loop as shown in Fig. 16.30, find the charge on the capacitor as a function of time.

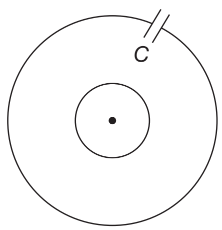


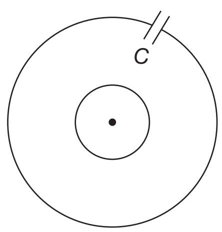
Fig. 16.30

Solution:

$$q = C\varepsilon\left(1 - e^{-t/RC}\right)$$

where $\quad \varepsilon = \dfrac{\mu_0 \pi a^2}{b}.$

47. If the current in the inner loop changes according to $i = 2t^2$, then find the current in the capacitor as a function of time.

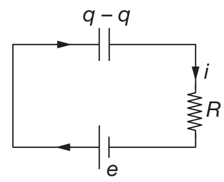


Solution:

$$M = \frac{\mu_0}{2b}\pi a^2$$

$$|\text{EMF induced in larger coil}| = M\left[\left(\frac{di}{dt}\right) \text{in smaller coil}\right]$$

$$e = \frac{\mu_0}{2b}\pi a^2\,(4t) = \frac{2\mu_0 \pi a^2 t}{b}$$



Applying KVL,

$$e - \frac{q}{C} - iR = 0$$

$$\frac{2\mu_0 \pi a^2 t}{b} - \frac{q}{C} - iR = 0$$

differentiate with respect to time

$$\frac{2\mu_0 \pi a^2}{b} - \frac{i}{C} - \frac{di}{dt}R = 0$$

on solving it

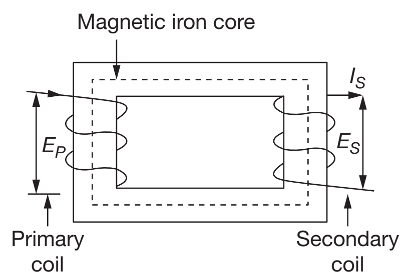
$$i = \frac{2\mu_0 \pi a^2 C}{b}\left[1 - e^{-t/RC}\right].$$

TRANSFORMER

A transformer changes an alternating potential difference from one value to another of greater or smaller value using the principle of mutual induction. Two coils called the primary and secondary windings, which are not connected to one another in any way are wound on a complete soft iron core. When an alternating voltage E_P is applied to the primary winding, the resulting current produces a large alternating magnetic flux which links the secondary and induces an EMF E_S in it. It can be shown that for an ideal transformer,

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s};$$

$\dfrac{N_s}{N_p}$ = turns ratio of the transformer.



E_S, N and I are the EMF, number of turns, and current in the coils, respectively.

$N_S > N_P \Rightarrow \quad E_S > E_P \rightarrow \quad$ step-up transformer.

$N_S < N_P \Rightarrow \quad E_S < E_P \rightarrow \quad$ step-down transformer.

> **NOTE**
>
> Phase difference between the primary and secondary voltage is π.

Energy Losses in Transformers

Although transformers are very efficient devices, small energy losses do occur in them due to the following four main causes.

Resistance of the Windings

The copper wire used for the windings has resistance and so I^2R heat losses occur.

Eddy Current

Eddy current is induced in a conductor when it is placed in a changing magnetic field or when a conductor is moved in a magnetic field and/or both. Any imagined circuit within the conductor will change its magnetic flux linkage and the subsequent induced EMF will drive current around the circuit. Thus, the alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by **laminating** the core, i.e., the core is made of sheets of iron with insulating sheets between them so that the circuits for the eddy currents are broken.

Hysteresis

The magnetization of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

Flux Leakage

The flux due to the primary winding may not link the secondary winding if the core is badly designed or has air gaps in it. Very large transformers have to be oil cooled to prevent overheating.

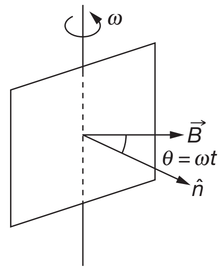
ALTERNATING CURRENT

Voltages and currents that vary symmetrically in magnitude and direction with time are very common. The electric mains supply in our homes and offices is a voltage that varies like a sine function with time. Such a voltage is called alternating voltage (ac voltage) and the current driven through the appliances is called the alternating current (ac current).

BASIC PRINCIPLE OF AC GENERATION

Alternating voltage is generated by rotating a coil of conducting wire in a strong magnetic field. The magnetic flux linked with the coil changes with time, and an alternating EMF is thus induced. Instantaneous flux linked with coil is

$$\phi = (\vec{A}.\vec{B})n$$
$$= ABn\cos(\omega t + \theta_0)$$



where A = area of the coil (in m^2)

B = magnetic field (in tesla)

n = number of turns

ω = angular frequency $= \frac{2\pi}{T} = 2\pi f$ (in rad s^{-1})

f = frequency (in hertz)

θ_0 = initial phase angle.

and the alternating voltage is given by

$$V = -\frac{d\phi}{dt} = V_0 \sin \omega t$$

Where $V_0 = ABn\omega$

The instantaneous value of an AC is given by

$$I = I_0 \sin \omega t.$$

Here, ω is the angular frequency of AC and $(\omega/2\pi)$ is the frequency of AC. $(2\pi/\omega)$ represents the time period of AC.

In one cycle of AC, current increases from zero to a maximum, then decreases to zero and reverses in direction and then decreases to zero. Thus, current is zero twice in one cycle and is numerically maximum also twice in one cycle, once in the forward direction and once in the backward direction in one cycle. Time taken to complete one cycle is called time period. The frequency of AC represents the number of cycles of AC completed in 1 s. AC supplied in India has a frequency of 50 Hz.

AVERAGE VALUES OF AC VOLTAGE AND AC CURRENT

AC voltage or currents are commonly sinusoidal (sine or cosine function) and their mean values for complete cycle is zero. The average values for half cycles are equally positive and negative.

1. Average value for one half cycle (or rectified average value):

$V = V_0 \sin \omega t$

$$\therefore \quad (V)_{av} = \frac{\int_0^{T/2} V\,dt}{\int_0^{T/2} dt} = \frac{2}{T}\int_0^{T/2} V_0 \sin \omega t \; dt$$

$$= \frac{2}{\pi} V_0 = 0.637 V_0.$$

This is also known as the rectified average value of a sinusoidal voltage and is represented as V_{av}.

2. Root Mean Square (RMS) Value (V_{rms} or I_{rms}): Since V or I are equally negative and positive, their squares will always be positive and the square root of the average of their square will give the RMS values.

$V = V_0 \sin \omega t$

$$\left(V^2\right)_{av} = \frac{1}{T}\int_0^T V_0^2 \sin^2 \omega t dt$$

$$= \frac{V_0^2}{2T}\int_0^T (1 - \cos 2\omega t) dt = \frac{V_0^2}{2}$$

Thus $V_{rms} = \sqrt{(V^2)_{av}} = \dfrac{V_0}{\sqrt{2}}$

and $I_{rms} = \sqrt{(I^2)_{av}} = \dfrac{I_0}{\sqrt{2}}$

or RMS value $= \dfrac{\text{Peak value}}{\sqrt{2}}$.

SOLVED EXAMPLE

48. If a domestic appliance draws 2.5 A from a 220-V, 60-Hz power supply, Find

(A) the average current.
(B) the average of the square of the current.
(C) the current amplitude.
(D) the supply voltage amplitude.

Solution:

(A) The average of sinusoidal *AC* values over any whole number of cycles is zero.

(B) RMS value of current $= I_{rms} = 2.5$ A

$\therefore \quad (I^2)_{av} = (I_{rms})^2 = 6.25\,\text{A}^2$

(C) $I_{rms} = \dfrac{I_m}{\sqrt{2}}$

Current amplitude $= \sqrt{2} I_{rms} = \sqrt{2}(2.5\,\text{A}) = 3.5$ A

(D) $V_{rms} = 220\text{ V} = \dfrac{V_m}{\sqrt{2}}$

$\therefore$ Supply voltage amplitude

$V_m = \sqrt{2}(V_{rms}) = \sqrt{2}(220\text{ V}) = 311$ V.

SERIES AC CIRCUIT

When only Resistance is in AC Circuit

Consider a simple AC circuit consisting of a resistor of resistance R and an AC generator, as shown in Fig. 16.31.

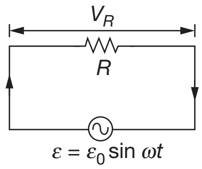


Fig. 16.31

According to Kirchhoff's loop law, at any instant, the algebraic sum of the potential difference around a closed loop in a circuit must be zero.

$$\varepsilon - V_R = 0$$

$$\varepsilon - i_R R = 0$$

$$\varepsilon_0 \sin \omega t - i_R R = 0$$

$$i_R = \frac{\varepsilon_0}{R} \sin \omega t = i_0 \sin \omega t \qquad (16.4)$$

where i_0 is the maximum current.

$$i_0 = \frac{\varepsilon_0}{R}$$

From above equations, we see that the instantaneous voltage drop across the resistor is

$$V_R = i_0 R \sin \omega t \qquad (16.5)$$

We see in equations (16.4) and (16.5), i_R and V_R both vary as $\sin \omega t$ and reach their maximum values at the same time as shown in Fig. 16.32(a), they are said to be in phase. A phasor diagram is used to represent phase relationships. The lengths of the arrows correspond to V_0 and i_0. The projections of the arrows onto the vertical axis give V_R and i_R. In case of the single-loop resistive circuit, the current and voltage phasors lie along the same line, as shown in Fig. 16.32(b), because i_R and V_R are in phase.

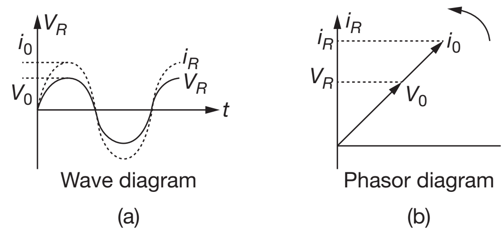


Fig. 16.32

When only Inductor is in an AC Circuit

Now consider an AC circuit consisting only of an inductor of inductance L connected to the terminals of an AC generator, as shown in Fig. 16.33. The induced EMF across the inductor is given by Ldi/dt. On applying Kirchhoff's loop rule to the circuit,

$$\varepsilon - V_L = 0 \quad \Rightarrow \quad \varepsilon - L\frac{di}{dt} = 0$$

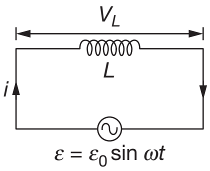


Fig. 16.33

When we rearrange this equation and substitute $\varepsilon = \varepsilon_0 \sin \omega t$, we get

$$L\frac{di}{dt} = \varepsilon_0 \sin \omega t \qquad (16.6)$$

Integration of this expression gives the current as a function of time

$$i_L = \frac{\varepsilon_0}{L}\int \sin \omega t dt = -\frac{\varepsilon_0}{\omega L}\cos \omega t + C$$

For average value of current over one time period to be zero, $C = 0$

$$\therefore \qquad i_L = -\frac{\varepsilon_0}{\omega L}\cos \omega t$$

When we use the trigonometric identity $\cos\omega t = -\sin(\omega t - \pi/2)$, we can express equation as

$$i_L = \frac{\varepsilon_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad (16.7)$$

From Equation (16.7), we see that the current reaches its maximum values when $\cos\omega t = 1$

$$i_0 = \frac{\varepsilon_0}{\omega L} = \frac{\varepsilon_0}{X_L} \qquad (16.8)$$

where the quantity X_L, called the inductive reactance, is

$$X_L = \omega L$$

The expression for the RMS current is similar to Equation (16.8), with ε_0 replaced by ε_{rms}.

Inductive reactance, like resistance, has unit of ohm.

$$V_L = L\frac{di}{dt} = \varepsilon_0 \sin \omega t = I_0 X_L \sin \omega t$$

We can consider Equation (16.8) as Ohm's law for an inductive circuit.

On comparing result of Equation (16.7) with Equation (16.6), we can see that the current and voltage are out of phase with each other by $\pi/2$ rad, or 90°. A plot of voltage and current versus time is given in Fig. 16.34(a). The voltage reaches its maximum value one-quarter of an oscillation period before the current reaches its maximum value. The corresponding phasor diagram for this circuit is shown in Fig. 16.34(b). Thus, we see that for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90°.

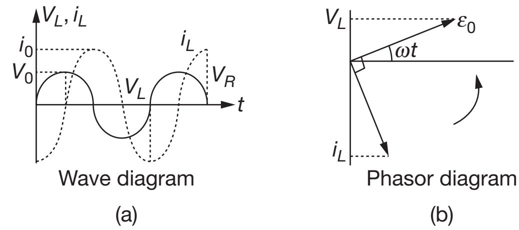


Fig. 16.34

When only Capacitor is in an AC Circuit

Figure 16.35 shows an AC circuit consisting of a capacitor of capacitance C connected across the terminals of an AC generator. On applying Kirchhoff's loop rule to this circuit, gives

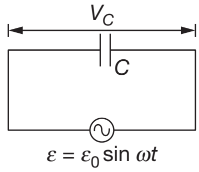


Fig. 16.35

$$\varepsilon - V_C = 0$$

$$V_C = \varepsilon = \varepsilon_0 \sin \omega t$$

where V_C is the instantaneous voltage drop across the capacitor. From the definition of capacitance, $V_C = Q/C$, and this value for V_C substituted into equation gives

$$Q = C\varepsilon_0 \sin \omega t$$

Since $i = dQ/dt$, on differentiating above equation gives the instantaneous current in the circuit.

$$i_C = \frac{dQ}{dt} = C\varepsilon_0 \, \omega \cos \omega t \qquad (16.9)$$

Here again we see that the current is not in phase with the voltage drop across the capacitor, given by Equation (4). Using the trigonometric identity $\cos\omega t = \sin(\omega t + \pi/2)$, we can express this equation in the alternative form

$$i_C = \omega C \, \varepsilon_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad (16.10)$$

From Equation (16.10), we see that the current in the circuit reaches its maximum value when $\cos\omega t = 1$.

$$i_0 = \omega C \varepsilon_0 = \frac{\varepsilon_0}{X_C}$$

where X_C is called the capacitive reactance.

$$X_C = \frac{1}{\omega C}$$

The SI unit of X_C is also ohm. The RMS current is given by an expression similar to equation with V_0 replaced by V_{rms}.

Combining Equation (16.9) and (16.10), we can express the instantaneous voltage drop across the capacitor as

$$V_C = V_0 \sin\omega t = I_0 X_C \sin\omega t$$

Comparing the result of Equation (16.8) with Equation (16.9), we see that the current is $\pi/2$ rad = 90° out of phase with the voltage across the capacitor. A plot of current and voltage versus time shows that the current reaches its maximum value one-quarter of a cycle sooner than the voltage reaches its maximum value. The corresponding phasor diagram is shown in Fig. 16.36(b). Thus, we see that for a sinusoidally applied EMF, the current always leads the voltage across a capacitor by 90°.

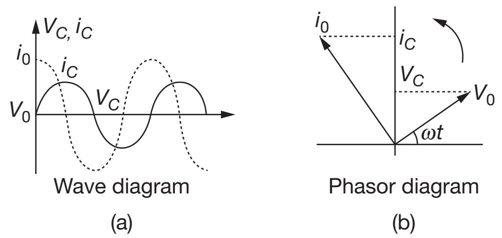


Fig. 16.36

Vector Analysis (Phasor Algebra)

The complex quantities normally employed in AC circuit analysis can be added and subtracted like coplanar vectors. Such coplanar vectors, which represent sinusoidally time varying quantities, are knows as phasors.

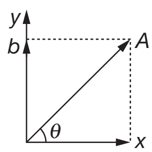
In Cartesian form, a phasor A can be written as,

$$A = a + jb$$

where a is the x-component and b is the y component of phasor A.

The magnitude of A is $|A| = \sqrt{a^2 + b^2}$,

And the angle between the direction of phasor A and the positive x-axis is,



$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

In a given phasor A, the direction of which is along the x-axis is multiplied by the operator j, a new phasor jA is obtained which will be 90° anticlockwise from A, i.e., along y-axis. If the operator j is multiplied now to the phasor jA, a new phasor j^2A is obtained which is along x-axis having same magnitude as of A. Thus,

$$j^2 A = -A$$

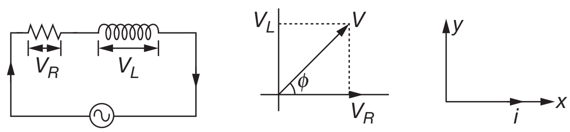
$$j^2 = -1 \quad \text{or} \quad j = \sqrt{-1}$$

Now using the j operator, let us discuss different circuits of an AC.

Series L-R Circuit

Now consider an AC circuit consisting of a resistor of resistance R and an inductor of inductance L in series with an AC source generator.

Suppose in phasor diagram, current is taken along positive x-direction. Then V_R is also along positive x-direction and V_L along positive y-direction. As we know potential difference across a resistance in AC is in phase with current and it leads in phase by 90° with current across the inductor, so we can write



$$V = V_R + jV_L = iR + j(iX_L) = iR + j(i\omega L) = iZ$$

Here, $Z = R + jX_L = R + j(\omega L)$ is called as impedance of the circuit. Impedance plays the same role in AC circuits as the ohmic resistance does in DC circuits. The modulus of impedance is

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

The potential difference leads the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_L}{V_R}\right| = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

SOLVED EXAMPLE

49. An alternating voltage of 220 V RMS at a frequency of 40 cycles/second is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 Ω in series. Calculate

(A) the current,
(B) potential difference across the resistance,
(C) potential difference across the inductance, and
(D) the time lag.

Solution:

The impedance of L-R series circuit is given by

$$Z = [R^2 + (\omega L)^2]^{1/2} = [(R)^2 + (2\pi f L)^2]^{1/2}$$

$$= [62 + (2 \times 3.14 \times 40 \times 0.01)^2]^{1/2} = 6.504 \text{ ohm}$$

(A) RMS value of current

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{220 \text{ V}}{6.504\ \Omega} = 33.83 \text{ A}$$

(B) The potential difference across the resistance is given by

$$V_R = I_{\text{rms}} \times R = 33.83 \times 6 = 202.83 \text{ V}$$

(C) Potential difference across inductance is given by

$$V_L = I_{\text{rms}} \times (\omega L) = 33.83 \times (2 \times 3.14 \times 0.01) = 96.83 \text{ V}$$

(D) Phase angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\therefore \quad \phi = \tan^{-1}(0.4189) = 22°46'$$

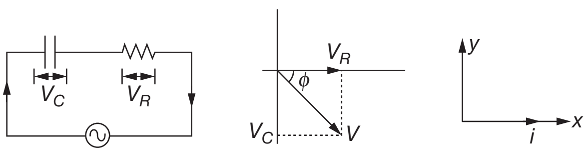
$$\text{Now time lag} = \frac{\phi}{360} \times T = \frac{\phi}{360} \times \frac{1}{f} = \frac{22°46'}{360 \times 40}$$

$$= 0.01579 \text{ s.}$$

Series R-C Circuit

Now consider an AC circuit consisting of a resistor of resistance R and a capacitor of capacitance C in series with an AC source generator.

Suppose in phasor diagram, current is taken along positive x-direction. Then V_R is along positive x-direction but V_C is along negative y-direction, as potential difference across a capacitor in AC lags in phase by 90° with the current in the circuit. So we can write.



$$V = V_R - jV_C = iR - j(iX_C)$$

$$= iR - j\left(\frac{i}{\omega C}\right) = iZ$$

Here, impedance is $Z = R - j\left(\frac{1}{\omega C}\right)$

The modulus of impedance is,

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

and the potential difference lags the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \tan^{-1}\left(\frac{1/\omega C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

SOLVED EXAMPLE

50. An AC source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is i. If now the frequency of the source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance of resistance at the original frequency.

Solution:

At angular frequency ω, the current in R-C circuit is given by

$$i_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{\sqrt{(R^2) + (1/\omega^2 C^2)^2}} \quad (16.11)$$

when frequency is changed to $\omega/3$, the current is halved. Thus

$$\frac{i_{\text{rms}}}{2} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + 1/(\omega/3)^2 C^2}}$$

$$= \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + (9/\omega^2 C^2)}} \quad (16.12)$$

From Equations (16.11) and (16.12), we have

$$\frac{1}{\sqrt{R^2 + (1/\omega^2 C^2)}} == \frac{2}{\sqrt{R^2 + (9/\omega^2 C^2)}}$$

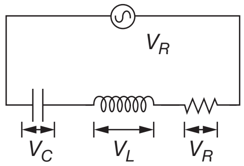
Solving this equation, we get $3R^2 = \frac{5}{\omega^2 C^2}$

Hence, the ratio of reactance to resistance is $\frac{(1/\omega C)}{R} = \sqrt{\frac{3}{5}}$.

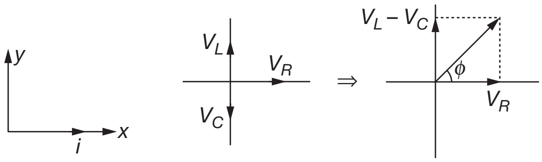
Series L-C-R Circuit and Resonance

Now consider an AC circuit consisting of a resistor of resistance R, a capacitor of capacitance C, and an inductor of inductance L are in series with an AC source generator.

Suppose in a phasor diagram, current is taken along positive x-direction. Then V_R is along positive x-direction, V_L along positive y-direction, and V_C along negative y-direction, as potential difference across an inductor leads the current by 90° in phase while across a capacitor, it lags in phase by 90°.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$



So, we can write,

$$V = V_R + jV_L - iV_C = iR + j(iX_L) - j(iX_C)$$

$$iR + j[i(X_L - X_C)] = iZ$$

Here, impedance is

$$Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The modulus of impedance is $|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

and the potential difference leads the current by an angle.

$$\phi = \tan^{-1}\left|\frac{V_L - V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$= \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \qquad (16.13)$$

The steady current in the circuit is given by

$$i = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi)$$

where ϕ is given from Equation (16.13)

The peak current is $i_0 = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

It depends on angular frequency ω of AC source and it will be maximum when

$$\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega = \sqrt{\frac{1}{LC}}$$

and corresponding frequency is $v = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$.

This frequency is known as resonant frequency of the given circuit. At this frequency, peak current will be $i_0 = \dfrac{V_0}{R}$.

If the resistance R in the LCR circuit is zero, the peak current at resonance is $i_0 = \dfrac{V_0}{0}$.

It means, there can be a finite current in pure LC circuit even without any applied EMF.

This current in the circuit is at frequency, $v = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$.

SOLVED EXAMPLES

51. A resistance R, and inductance L, and a capacitor C all are connected in series with an AC supply. The resistance of R is 16 Ω and for a given frequency, the inductive reactance of L is 24 Ω and capacitive reactance of C is 12 Ω. If the current in the circuit is 5 A, Find

(A) the potential difference across R, L, and C.
(B) the impedance of the circuit.
(C) the voltage of AC supply.
(D) phase angle.

Solution:

(A) Potential difference across resistance

$V_R = iR = 5 \times 16 = 80$ V

Potential difference across inductance

$V_L = i \times (\omega L) = 5 \times 24 = 120$ V

Potential difference across condenser

$V_C = i \times (1/\omega C) = 5 \times 12 = 60$ V

(B) $Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$

$$= \sqrt{(16)^2 + (24 - 12)^2} = 20\ \Omega$$

(C) The voltage of AC supply is given by

$r = IZ = 5 \times 20 = 100$ V

(D) $\phi = \tan^{-1}\left[\dfrac{\omega L - (1/\omega C)}{R}\right]$

$$= \tan^{-1}\left[\frac{24 - 12}{16}\right] = \tan^{-1}(0.75) = 36°46°.$$

52. For the circuit shown in Fig. 16.37. Current in inductance is 0.8 A while in capacitance is 0.6 A. What is the current drawn from the source.

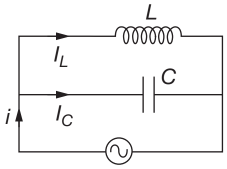


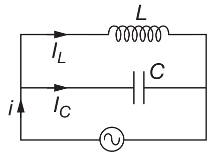
Fig. 16.37

Solution:

In this ac circuit, $\varepsilon = \varepsilon_0 \sin\omega t$ is applied across an inductance and capacitance in parallel, current in inductance will lag the applied voltage while across the capacitor will lead,

and so,
$$I_L = \frac{V}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) = -0.8\cos\omega t$$
$$I_C = \frac{V}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) = 0.6\cos\omega t$$

So the current drawn from the source,



$$I = I_L + I_C = -0.2\cos\omega t,$$

i.e., $|I_0| = 0.2$ A.

BRAIN MAP I

1. **Magnetic flux:**
   $\phi = B \cdot A \cos\theta$
   It can be changed by changing B, A or θ with time
2. **Faraday's law:**
   The emf induced in a closed loop is given by rate of change of magnetic flux with time
   $\varepsilon = -\frac{d\phi}{dt}$
3. **Lenz's law:** Effect of the induced emf is such as to oppose the change in flux that produces it.
4. **Motional Emf:**
   Emf induced across a conductor of length l moving in a uniform magnetic field is
   $\varepsilon = \int (\vec{v} \times \vec{B}) \cdot \vec{dl}$

ELECTROMAGNETIC INDUCTION

5. **(a) Self induction**
   $\phi = Li$
   $\varepsilon = -L\frac{di}{dt}$
   Where L is called self inductance of the coil

   (b) Mutual induction
   $\phi = Mi$
   $\varepsilon = -M\frac{di}{dt}$
   Mutual inductance of pair of solenoids
   $M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$

   (c) Energy stored in an inductor
   $E = \frac{1}{2}Li^2$

   (d) Grouping of inductors
   (i) In series combination
   $L = L_1 + L_2 + L_3 \ldots\ldots$
   (ii) In parallel combination
   $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots\ldots$

   (e) *L-R* circuit
   (i) Growth of current
   $i_R = \frac{E}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$
   (ii) Decay of current
   $i_R = \frac{I_0}{R} e^{\frac{-Rt}{L}}$

   (f) *L-C* oscillation
   When a capcitor is charged up to Q_0 and then connected to an inductor
   (i) $\frac{Ldi}{dt} + \frac{Q}{C} = 0$
   (ii) $Q = Q_0 \cos\omega t$
   (iii) $i = -Q_0\omega\sin\omega t$
   (iv) $\omega = \frac{1}{\sqrt{LC}}$

BRAIN MAP 2

ALTERNATING CURRENT CIRCUITS

1. (a) Instantaneous current

$i = i_0 \sin(\omega t + \phi)$

Instantaneous voltage

$v = v_0 \sin(\omega t + \phi)$

(b) Average current for half cycle

$i_{avg} = \frac{2i_0}{\pi} \cong 0.637\, i_0$

Average voltage for half cycle

$v_{avg} = \frac{2v_0}{\pi} \approx 0.637\, v_0$

(c) RMS current

$i_{rms} = \frac{i_0}{\sqrt{2}} = 0.706\, i_0$

RMS voltage

$v_{rms} = \frac{v_0}{\sqrt{2}} = 0.706\, v_0$

2. Series AC circuit:

(a) When only resistance is in ac circuit

(i) $V_0 = i_0 R$

(ii) Voltage is in phase with current

(b) When only capacitor is in ac circuit

(i) $V_0 = i_0 X_C$, where, $X_c = \frac{1}{\omega C}$

(ii) Voltage is lagging in phase with current by $\frac{\pi}{2}$

(c) When only inductor is in ac circuit

(i) $V_0 = i_0 X_L$ where $X_L = \omega L$

(ii) Voltage is leading in phase with current by $\pi/2$

(d) Series LR circuit

Impedance $(Z) = \sqrt{R^2 + \omega L^2}$

Phase angle, $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

(e) Series CR circuit

Impedance $(Z) = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Phase angle, $\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$

(f) Series LCR – circuit

Impedance $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

Phase angle $\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$

(g) Resonance frequency when *L*, *C*, *R* are in series is, $\omega = \frac{1}{\sqrt{LC}}$

3. Parallel AC circuit:

(a) When *L* and *R* are in series with *C* in parallel

Admittance $(Y) = \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$

(b) Resonance frequency (ω)

$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

(c) Dynamic resistance = $\frac{1}{Y}$

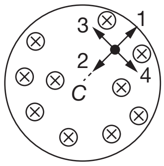
4. Power $= I_{rms} V_{rms} \cos\phi$

where $\cos\phi$ is power factor

EXERCISES

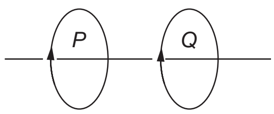
Single Option Correct Type

1. A uniform but time varying magnetic field exists in cylindrical region and directed into the paper. If field decreases with time and a positive charge placed at any point inside the region, then it moves



(A) along 1 (B) along 2
(C) along 3 (D) along 4

2. Two circular coils P and Q are arranged coaxially as shown, and the sign convention adopted that currents are taken as positive when flow in the direction of the arrows



(A) If P carries a steady positive current and P is moved towards Q, a positive current is induced in Q.
(B) If P carries a steady positive current and Q is moved towards P, a negative current is induced in Q.
(C) If a positive current flowing in P is switched off, a negative current is induced momentarily in Q.
(D) If both coils carry positive currents, the coils repel one another.

3. Two coils of self-inductance 4 H and 16 H are wound on the same iron core. The coefficient of mutual inductance for them will be

(A) 8 H (B) 10 H (C) 20 H (D) 64 H

4. Two pure inductors, each of self-inductance L are connected in parallel but are well separated from each other, then the total inductance is

(A) L (B) $2L$ (C) $L/2$ (D) $L/4$

5. SI unit of inductance can be written as

(A) Weber/Ampere (B) Joule/Ampere^2
(C) Ohm/Second (D) All of the above

6. There is a current of 1.344 A in a copper wire whose area of cross-section normal to the length of the wire is 1 mm^2. If the number of free electrons per cm^3 is 8.4×10^{22}, then the drift velocity of electrons will be

(A) 1.0 mm s (B) 1.0 meter s
(C) 0.1 mm s (D) 0.01 mm s

7. Two straight long conductors AOB and COD are perpendicular to each other and carry currents I_1 and I_2, respectively. The magnitude of the magnetic induction at a point P at a distance a from the point O in a direction perpendicular to the plane $ABCD$ is

(A) $\frac{\mu_0}{2\pi a}(I_1 + I_2)$ (B) $\frac{\mu_0}{2\pi a}(I_1 - I_2)$

(C) $\frac{\mu_0}{2\pi a}\left(I_1^2 + I_2^2\right)^{1/2}$ (D) $\frac{\mu_0}{2\pi a}\left(\frac{I_1 I_2}{I_1 + I_2}\right)$

8. A bar magnet, of magnetic moment M, is placed in a magnetic field of induction B. The torque exerted on it is

(A) $\vec{M} \cdot \vec{B}$ (B) $\vec{B} \times \vec{M}$
(C) $\vec{M} \times \vec{B}$ (D) $-\vec{B} \cdot \vec{M}$

9. In a coil when current changes from 10 A to 2 A in time 0.1 s, induced EMF is 3.20 V. The self-inductance of coil is

(A) 4 H (B) 0.4 H (C) 0.04 H (D) 5 H

10. A choke coil has

(A) high inductance and high resistance.
(B) low inductance and low resistance.
(C) high inductance and low resistance.
(D) low inductance and high resistance.

11. The EMF induced in a 1 millihenry inductor in which the current changes from 5 A to 3 A in 10^{-3} second is

(A) 2×10^{-6} V (B) 8×10^{-6} V
(C) 2 V (D) 8 V

12. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic field B, constant in space and time, pointing perpendicular and into the plane of the loop exists everywhere as shown in Fig. 16.38. The current induced in the loop is

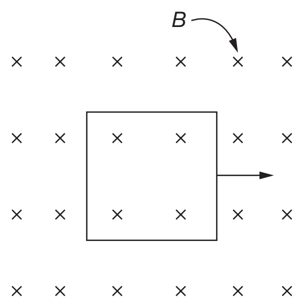


Fig. 16.38

(A) BLv/R clockwise
(B) BLv/R anti-clockwise
(C) $2BLv/R$ anti-clockwise
(D) Zero

13. A magnetic needle is kept in a non-uniform magnetic field. It experiences
(A) A force and torque
(B) A force but not a torque
(C) A torque but not a force
(D) Neither a force nor a torque

14. A current i ampere flows along an infinitely long straight thin-walled tube, then the magnetic induction at any point inside the tube at a distance r from centre is
(A) Infinite (B) Zero
(C) $\frac{\mu_0}{4\pi}\cdot\frac{2i}{r}$ (D) $\frac{2i}{r}$

15. A proton and an alpha particle enter a uniform magnetic field with the same velocity. The period of rotation of the alpha particle will be
(A) four times that of the proton.
(B) two times that of the proton.
(C) three times that of the proton.
(D) same as that of the proton.

16. A coil having an area A_0 is placed in a magnetic field which changes from B_0 to $4B_0$ in time interval t. The average EMF induced in the coil will be
(A) $\frac{3A_0B_0}{t}$ (B) $\frac{4A_0B_0}{t}$
(C) $\frac{3B_0}{A_0t}$ (D) $\frac{4B_0}{A_0t}$

17. When the current changes from +2 A to –2 A in 0.05 s, an EMF of 8 V is induced in a coil. The coefficient of self-induction of the coil is
(A) 0.1 H (B) 0.2 H (C) 0.4 H (D) 0.8 H

18. A circular loop of radius $R = 20$ cm is placed in a uniform magnetic field $\vec{B} = 2\text{T}$ in x-y plane as shown in Fig. 16.39. The loop carries a current $i = 1.0$ A in the direction shown in the Fig. 16.39. Find the magnitude of torque acting on the loop.

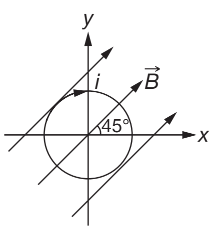
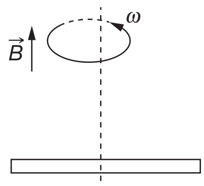


Fig. 16.39

(A) $0.16\,\pi$ N/m. (B) $0.08\,\pi$ N/m.
(C) $\frac{0.08}{\sqrt{2}}\,\pi$ N/m. (D) $\frac{0.16}{\sqrt{2}}\,\pi$ N/m.

19. An average EMF of 20 V is induced in an inductor when the current in it changed from 2.5 A in one direction to the same value in opposite direction in 0.1s, the self-inductance of inductor is
(A) 0.4 H (B) 1 H (C) 2H (D) 0.6 H

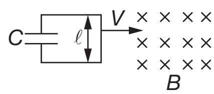
20. A conducting rod of length 2ℓ is rotating with constant angular speed ω about its perpendicular bisector. A uniform magnetic field $\vec{B}$ exists parallel to the axis of rotation. The EMF induced between two ends of the rod is



(A) $B\omega\ell^2$ (B) $\frac{1}{2}B\omega\ell^2$
(C) $\frac{1}{8}B\omega\ell^2$ (D) Zero

21. A 50 mH coil carries a current of 2 A, the energy stored in it in J is
(A) 0.05 (B) 0.1 (C) 0.5 (D) 1

22. A capacitance C is connected to a conducting rod of length ℓ moving with a velocity v in a transverse magnetic field B then the charge developed in the capacitor is
(A) Zero (B) $B\ell vC$
(C) $\frac{BlvC}{2}$ (D) $\frac{BlvC}{3}$



23. A particle of mass M and charge Q moving with a velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes a full circle is
(A) Zero (B) $BQ2\pi R$
(C) $BQv(2\pi R)$ (D) $\left(\frac{Mv^2}{R}\right)(2\pi R)$

24. A rectangular coil of 100 turns and size $0.1\text{ m} \times 0.05\text{ m}$ is placed perpendicular to a magnetic field of 0.1 T.

The induced EMF when the field drops to 0.05 T is 0.05 s is

(A) 0.5 V (B) 1.0 V
(C) 1.5 V (D) 2.0 V

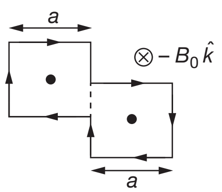
25. If a coil of metal wire is kept stationary in a non-uniform magnetic field,

(A) An EMF and current are both induced in the coil
(B) A current but no EMF is induced in the coil
(C) An EMF but no current is induced in the coil
(D) Neither EMF nor current is induced in the coil

26. A flat coil carrying a current has a magnetic moment $\vec{\mu}$. It is placed in a magnetic field $\vec{B}$. The torque on the coil is $\vec{\tau}$, then

(A) $\vec{\tau} = \vec{\mu} \cdot \vec{B}$
(B) $\vec{\tau} = \vec{B} \times \vec{\mu}$
(C) $|\vec{\tau}| = \vec{\mu} \cdot \vec{B}$
(D) $\vec{\tau}$ is perpendicular to both $\vec{\mu}$ and $\vec{B}$.

27. The torque acting on this loop will be



(A) Zero (B) $\frac{Ia^2 B_0}{2}$
(C) $\frac{2Ia^2 B_0}{3}$ (D) None

28. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane as shown in Fig. 16.40. The loop will have tendency to

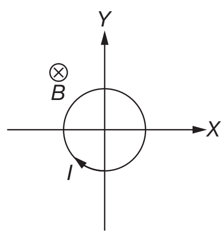


Fig. 16.40

(A) Contract
(B) Expand
(C) Move towards positive x-axis
(D) Move towards negative x-axis

29. A rod of length ℓ is moved with a velocity v in a magnetic field B as shown in Fig. 16.41, the equivalent electrical circuit is

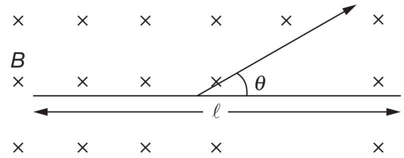
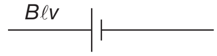
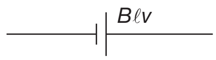
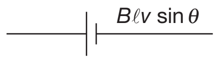
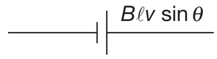


Fig. 16.41

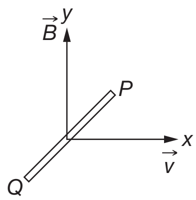
(A) 

(B) 

(C) 

(D) 

30. A conducting rod PQ is moving parallel to x-z-plane in a uniform magnetic field directed in the positive y-direction. The end P of the rod will become



(A) Sometime positive and sometime negative
(B) Positive
(C) Neutral
(D) Negative

31. What is direction of induced current in the coil as shown in Fig. 16.42? (If rate of increase of current is equal to the rate of decrease.)

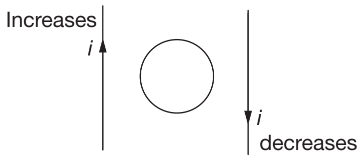


Fig. 16.42

(A) Clockwise
(B) Anti-clockwise
(C) Zero
(D) Not defined with same rate

32. A metallic wire bent into a right Δabc moves with a uniform velocity v as shown in Fig. 16.43, B is the strength of uniform magnetic field perpendicular outwards the plane of triangle. The net EMF is …….. and EMF along ab is ………

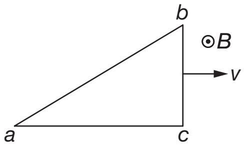


Fig. 16.43

(A) zero, $Bv(bc)$ with b positive
(B) zero, $Bv(bc)$ with a positive
(C) $Bv(bc)$ with c positive, zero
(D) $Bv(bc)$ with b positive, zero

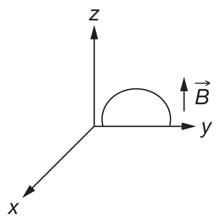
33. The magnetic flux linked with a coil is $\phi = 8t^2 + 3t + 5$Wb. The induced EMF in the fourth second will be

(A) 145 V (B) 139 V (C) 67 V (D) 16 V

34. Flux ϕ (in weber) in a closed circuit of resistance 10 Ω varies with time t (in seconds) according to the equation $\phi = 6t^2 - 5t + 1$. The magnitude of the induced current in the circuit at $t = 0.25$ s is

(A) 0.2 A (B) 0.6 A (C) 0.8 A (D) 1.2 A

35. A semicircular conducting ring is placed in yz-plane in a uniform magnetic field directed along positive z-direction. An induced EMF will be developed in the ring if it is moved along.



(A) Positive x-direction
(B) Positive y-direction
(C) Positive z-direction
(D) None of the above

36. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2.0$ m/s in a uniform magnetic field $B = 4.0$ T direction into the paper. A capacitor of capacity $C = 10$ μF is connected as shown in Fig. 16.44. Then

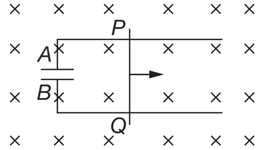


Fig. 16.44

(A) $q_A = +80\ \mu C$ and $q_B = -80\ \mu C$.
(B) $q_A = -80\ \mu C$ and $q_B = +80\ \mu C$.
(C) $q_A = 0 = q_B$.
(D) charge stored in the capacitor increases exponentially with time.

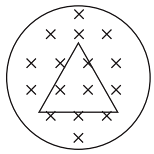
37. The magnetic flux linked with a circuit of resistance 100 ohm increases from 10 to 60 webers. The amount of induced charge that flows in the circuit is (in coulomb)

(A) 0.5 (B) 5 (C) 50 (D) 100

38. In an oscillating L-C circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is

(A) $\frac{Q}{2}$ (B) $\frac{Q}{\sqrt{2}}$ (C) $\frac{Q}{\sqrt{3}}$ (D) $\frac{Q}{3}$

39. An equilateral triangular loop having a resistance R and length of each side ℓ is placed in a magnetic field which is varying at $\frac{dB}{dt} = 1\,\text{T/S}$. The induced current in the loop will be



(A) $\frac{\sqrt{3}}{4}\frac{l^2}{R}$ (B) $\frac{4}{\sqrt{3}}\frac{l^2}{R}$

(C) $\frac{\sqrt{3}}{4}\frac{R}{l^2}$ (D) $\frac{4}{\sqrt{3}}\frac{R}{l^2}$

40. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in Fig. 16.45. An electric field is induced

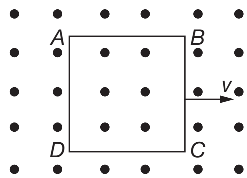


Fig. 16.45

(A) In AD, but not in BC
(B) In BC, but not in AD
(C) Neither in AD nor in BC
(D) In both AD and BC

41. The mutual inductance between two planar concentric rings of radii r_1 and r_2 (with $r_1 >> r_2$) placed in air is given by

(A) $\frac{\mu_0 \pi r_2^2}{2r_1}$ (B) $\frac{\mu_0 \pi r_1^2}{2r_2}$

(C) $\frac{\mu_0 \pi (r_1 + r_2)^2}{2r_1}$ (D) $\frac{\mu_0 \pi (r_1 + r_2)^2}{2r_2}$

42. A coil in the shape of an equilateral triangle of side ℓ is suspended between the pole pieces of a permanent magnet such that $\vec{B}$ is in the plane of the coil. If due to a current i in the triangle, a torque τ acts on it, the side ℓ of the triangle is

(A) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)$ (B) $2\left(\frac{\tau}{\sqrt{3}\,Bi}\right)^{1/2}$

(C) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)^{1/2}$ (D) $\frac{1}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)$

43. When the number of turns in a coil is doubled without any change in the length of the coil, its self-inductance becomes

(A) Four times (B) Doubled

(C) Halved (D) Squared

44. A varying magnetic flux linking a coil is given by $\phi = xt^2$. If at a time $t = 3$ s, the EMF induced is 9 V, then the value of x is

(A) 0.66 Wbs^{-2} (B) 1.5 Wbs^{-2}

(C) –0.66 Wbs^{-2} (D) –1.5 Wbs^{-2}

45. A 100 turns coil shown in Fig. 16.46 carries a current of 2 A in a magnetic field $B = 0.2$ Wb/m^2. The torque acting on the coil is

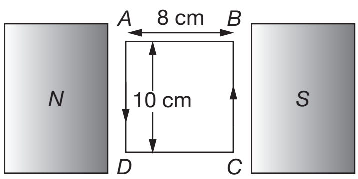


Fig. 16.46

(A) 0.32 Nm tending to rotate the side *AD* out of the page.

(B) 0.32 Nm tending to rotate the side *AD* into the page.

(C) 0.0032 Nm tending to rotate the side *AD* out of the page.

(D) 0.0032 Nm tending to rotate the side *AD* into the page.

46. The magnetic susceptibility of a material of a rod is 499. Permeability of vacuum is $4\pi \times 10^{-7}$ H/m. Absolute permeability of the material of the rod in henry per meter is

(A) $\pi \times 10^{-4}$ (B) $2\pi \times 10^{-4}$

(C) $3\pi \times 10^{-4}$ (D) $4\pi \times 10^{-4}$

47. A conducting circular loop is placed in a uniform magnetic field of induction B tesla with its plane normal to the field. Now the radius of the loop starts shrinking at the rate (dr/dt). Then the induced EMF at the instant when the radius is r will be

(A) $\pi r B\left(\frac{dr}{dt}\right)$ (B) $2\pi r B\left(\frac{dr}{dt}\right)$

(C) $\pi r^2\left(\frac{dB}{dt}\right)$ (D) $B\frac{\pi r^2}{2}\frac{dr}{dt}$

48. If the flux of magnetic induction through a coil of resistance R and having n turns changes from ϕ_1 to ϕ_2, then the magnitude of the charge that passes through the coil is

(A) $\frac{(\phi_2 - \phi_1)}{R}$ (B) $\frac{n(\phi_2 - \phi_1)}{R}$

(C) $\frac{(\phi_2 - \phi_1)}{nR}$ (D) $\frac{nR}{(\phi_2 - \phi_1)}$

49. A varying magnetic flux linking a coil is given by $\phi = 3t^2$. The magnitude of induced EMF in the loop at $t = 3$s is

(A) 3 V (B) 9 V (C) 18 V (D) 27 V

50. A horizontal telegraph wire 0.5 km long running east and west is a part of a circuit whose resistance is 2.5 Ω. The wire falls to the ground from a height of 5 m. If $g = 10.0$ m/s^2 and horizontal component of earth's magnetic field is 2×10^{-5} weber/m^2, then the current induced in the circuit just before the wire hits the ground will be

(A) 0.7 A (B) 0.04 A

(C) 0.02 A (D) 0.01 A

51. A coil of 20 × 20 cm having 30 turns is making 30 rps in a magnetic field of 1 tesla. The peak value of the induced EMF is approximately

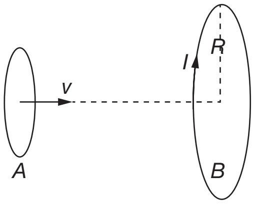
(A) 452 V (B) 226 V

(C) 113 V (D) 339 V

52. Two particles each of mass m and charge q are attached to the two ends of a light rigid rod of length 2ℓ. The rod is rotated at a constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is

(A) $\frac{q}{2m}$ (B) $\frac{q}{m}$ (C) $\frac{2q}{m}$ (D) $\frac{q}{\pi m}$

53. Loop A of radius $r(r << R)$ moves towards a constant current carrying loop B with a constant velocity v in such a way that their planes are parallel and coaxial. The distance between the loops when the induced EMF in loop A is maximum is



(A) R (B) $\frac{R}{\sqrt{2}}$

(C) $\frac{R}{2}$ (D) $R\left(1-\frac{1}{\sqrt{2}}\right)$

54. A uniform current carrying ring of mass m and radius R is connected by a massless string as shown in Fig. 16.47. A uniform magnetic field B_0 exists in the region to keep the ring in horizontal position, then the current in the ring is

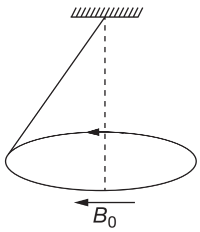


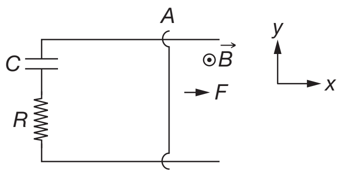
Fig. 16.47

(ℓ = length of string)

(A) $\frac{mg}{\pi RB_0}$ (B) $\frac{mg}{RB_0}$

(C) $\frac{mg}{3\pi RB_0}$ (D) $\frac{mg\,l}{\pi R^2 B_0}$

55. A conducting rod AB moves parallel to x-axis in the x-y plane. A uniform magnetic field B pointing normally out of the plane exists throughout the region. A force F acts perpendicular to the rod, so that the rod moves with uniform velocity v. The force F is given by (neglect resistance of all the wires).



(A) $\frac{vB^2l^2}{R}e^{-t/RC}$

(B) $\frac{vB^2l^2}{R}$

(C) $\frac{vB^2l^2}{R}\left(1-e^{-t/RC}\right)$

(D) $\frac{vB^2l^2}{R}\left(1-e^{-2t/RC}\right)$

56. A conducting rod of mass m and length ℓ is connected by two identical springs as shown in Fig. 16.48. Initially, the system is in equilibrium. A uniform magnetic field of magnitude B directed perpendicular to the plane of the paper outwards also exists in the region. If a current I is switched on, it passes from P to Q through the rod. Further maximum elongation in the spring is [Given: $|mg| = |BI\ell|$]

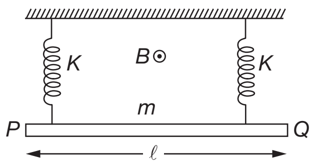


Fig. 16.48

(A) $\frac{BIl}{K}$ (B) $\frac{BIl}{4K}$

(C) $\frac{BIl}{8K}$ (D) $\frac{BIl}{16K}$

57. A uniform but time varying magnetic field is present in a circular region of radius R. The magnetic field is perpendicular and into the plane of the paper and the magnitude of the field is increasing at a constant rate α. There is a straight conducing rod of length $2R$ placed as shown in Fig. 16.49. The magnitude of induced EMF across the rod is

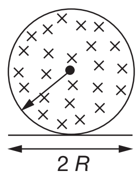
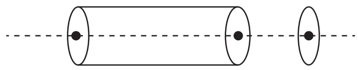


Fig. 16.49

(A) $\pi R^2\alpha$ (B) $\frac{\pi R^2\alpha}{2}$

(C) $\frac{R^2\alpha}{\sqrt{2}}$ (D) $\frac{\pi R^2\alpha}{4}$

58. The diagram shows a solenoid carrying time varying current $I = I_0 t$. On the axis of this solenoid, a ring has been placed. The mutual inductance of the ring and the solenoid is M and the self-inductance of the ring is L. If the resistance of the ring is R then maximum current which can flow through the ring is



(A) $\frac{(2M+L)I_0}{R}$ (B) $\frac{MI_0}{R}$

(C) $\frac{(2M-L)I_0}{R}$ (D) $\frac{(M+L)I_0}{R}$

59. A metallic ring of radius R moves in a vertical plane in the presence of a uniform magnetic field B perpendicular to the plane of the ring. At any given instant of time, its centre of mass moves with a velocity v while ring rotates in its COM frame with angular velocity ω as shown in Fig. 16.50. The magnitude of induced EMF between points O and P is

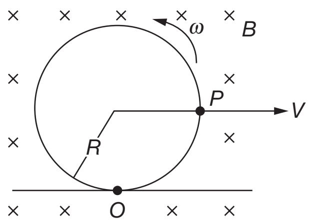
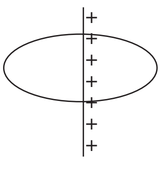


Fig. 16.50

(A) Zero (B) $vBR\sqrt{2}$
(C) vBR (D) $2\,vBR$

60. A very long uniformly charged rod falls with a constant velocity V through the centre of a circular loop. Then the magnitude of induced EMF in loop is



(charge per unit length of rod = λ)

(A) $\frac{\mu_0}{2\pi}\lambda V^2$ (B) $\frac{\mu_0}{2}\lambda V^2$ (C) $\frac{\mu_0}{2\lambda}V$ (D) Zero

61. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L ($L >> \ell$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to

(A) $\frac{l}{L}$ (B) $\frac{l^2}{L}$ (C) $\frac{L}{l}$ (D) $\frac{L^2}{l}$

62. In a uniform magnetic field of induction B, a wire in the form of semicircle of radius r rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per period of rotation is

(A) $\frac{B\pi r^2\omega}{2R}$ (B) $\frac{(B\pi r^2\omega)^2}{2R}$

(C) $\frac{(B\pi r\omega)^2}{2R}$ (D) $\frac{(B\pi r^2\omega)^2}{8R}$

63. A rod of length ℓ, negligible resistance, and mass m slides on two horizontal frictionless rails of negligible resistance by hanging a block of mass m_1 with the help of insulating a massless string passing through fixed massless pulley (as shown in Fig. 16.51). If a constant magnetic field B acts upwards perpendicular to the plane of the figure, the terminal velocity of hanging mass is

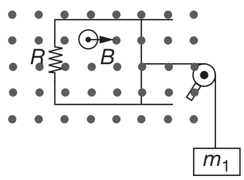


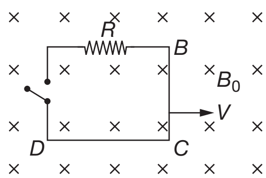
Fig. 16.51

(A) $\frac{m_1 gR}{B^2 l^2}$ upward (B) $\frac{m_1 gR}{B^2 l^2}$ downward

(C) $\frac{m_1 gR}{2B^2 l^2}$ downward (D) $\frac{m_1 gR}{B^2 l}$ downward

64. Magnetic field B_0 exists perpendicular inwards. The resistance of the loop is R. When the switch is closed, the current induced in the circuit is

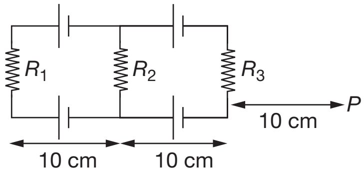
(A) $\frac{Blv}{R}$ (B) $\frac{3Blv}{R}$ (C) $\frac{2Blv}{R}$ (D) Zero



65. In the circuit shown, each battery has EMF = 5 V. Then the magnetic field at P is

(A) Zero (B) $\frac{10\mu_0}{R_1(4\pi)(0.2)}$

(C) $\frac{20\mu_0}{(R_1+R_2)(0.8\pi)}$ (D) None of these



66. The material which shows the effect shown in Fig. 16.52, when placed in a uniform magnetic field is called

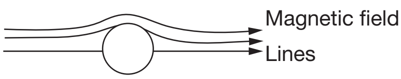


Fig. 16.52

(A) Paramagnetic
(B) Diamagnetic
(C) Ferromagnetic
(D) Anti-ferromagnetic

67. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current $i = 4$A. A horizontal magnetic field $B = 10$ T is switched on at time $t = 0$ as shown in Fig. 16.53. The initial angular acceleration of the ring will be

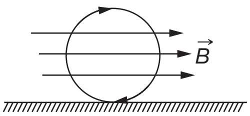


Fig. 16.53

(A) $40\,\pi$ rad/s^2 (B) $20\,\pi$ rad/s^2
(C) $5\,\pi$ rad/s^2 (D) $15\,\pi$ rad/s^2

68. The EMF induced in a 1 millihenry inductor in which the current changes from 5 A to 3 A in 10^{-3} second is

(A) 2×10^{-6} V (B) 8×10^{-6} V
(C) 2 V (D) 8 V

69. Conducting circular loop of radius r is placed in x-y plane in gravity free space as shown in Fig. 16.54, mass of the loop is m and centre of the loop is at the origin. At $t = 0$, a current I starts flowing through the loop and a magnetic field $\vec{B} = B_0\left(\hat{i} + \hat{j}\right)$ is switched on in the region (where B_0 is a constant). The angular acceleration of the loop due to the torque of magnetic field is

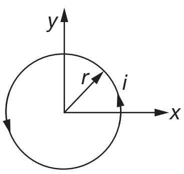


Fig. 16.54

(A) $\dfrac{\sqrt{2}\pi B_0 i}{m}$ (B) $\dfrac{2\sqrt{2}\pi B_0 i}{m}$

(C) $\dfrac{\pi B_0 i}{m}$ (D) $\dfrac{\pi B_0 i}{2m}$

70. A direct current flowing through the winding of a long cylindrical solenoid of radius R produces in it a uniform magnetic field of induction B. An electron flies into the solenoid along the radius between its turns (at right angles to the solenoid axis) at a velocity v as shown in Fig. 16.55. After a certain time, the electron deflected by the magnetic field leaves the solenoid. Then the time t during which the electron moves in the solenoid is

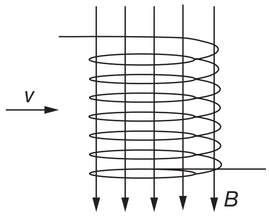


Fig. 16.55

(A) $\dfrac{m}{eB}\tan^{-1}\left(\dfrac{eBR}{mv}\right)$ (B) $\dfrac{2m}{eB}\tan^{-1}\left(\dfrac{eBR}{mv}\right)$

(C) $\dfrac{m}{eB}\tan^{-1}\left(\dfrac{mv}{eBR}\right)$ (D) $\dfrac{2m}{eB}\tan^{-1}\left(\dfrac{mv}{eBR}\right)$

71. A magnetized wire of moment M is bent into an arc of a circle subtending an angle of 60° at the centre, the new magnetic moment is

(A) $\dfrac{2M}{\pi}$ (B) $\dfrac{M}{\pi}$ (C) $\dfrac{3\sqrt{3}M}{\pi}$ (D) $\dfrac{3M}{\pi}$

72. Total energy of electromagnetic waves in vacuum is given by the relation

(A) $\dfrac{1}{2}\cdot\dfrac{E^2}{\varepsilon_0} + \dfrac{B^2}{2\mu_0}$ (B) $\dfrac{1}{2}\varepsilon_0 E^2 + \dfrac{1}{2}\mu_0 B^2$

(C) $\dfrac{E^2 + B^2}{c}$ (D) $\dfrac{1}{2}\varepsilon_0 E^2 + \dfrac{B^2}{2\mu_0}$

73. An iron rod of cross-sectional area 4 sq cm is placed with its length parallel to a magnetic field of intensity 1600 amp/m. The flux through the rod is 4×10^{-4} weber. The permeability of the material of the rod is (In weber/amp-m).

(A) 0.625
(B) 6.25
(C) 0.625×10^{-3}
(D) None of these

74. An electron moves along the line AB which lies in the same plane as a circular loop of conducting wire as shown in Fig. 16.56. The direction of the induced current in loop will be

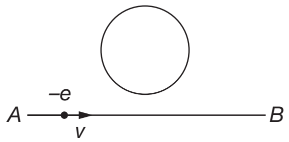


Fig. 16.56

(A) no current induced.
(B) clockwise.
(C) anti-clockwise.
(D) the current will change direction as the electron passes by.

75. A charged particle of mass m and charge q is projected into a uniform magnetic field of induction B with speed v which is perpendicular to B. The width of the magnetic field is d. the impulse imparted to the particle by the field is ($d << mv/qB$)

(A) qBv (B) mv/qB
(C) qBd (D) $2mv^2/qB$

76. A circular ring is fixed in a gravitational field. A bar magnet is projected towards its centre as shown in Fig. 16.57.

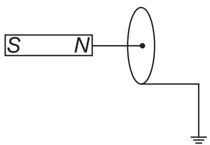


Fig. 16.57

(A) Initially, magnet experiences an acceleration and then it retards.
(B) Magnet starts to oscillate about centre of the ring.
(C) Magnet continues to move along the axis with constant velocity.
(D) The magnet retards and comes to rest finally.

77. Two charge particles are moving parallel to each other with velocities 2×10^8 m/s and 1×10^8 m/s, respectively. The ratio of magnetic force between them and electrostatic force between them is

(A) $\frac{2}{9}$ (B) $\frac{1}{9}$ (C) $\frac{9}{2}$ (D) 9

78. There is a small metallic ring of radius ℓ_0 having negligible resistance placed perpendicular to a constant magnetic field B_0. One end of a rod is hinged at the centre of ring O and other end is placed on the ring. Now rod is rotated with constant angular velocity ω_0 by some external agent and circuit is connected as shown in Fig. 16.58; initially, switch is open and capacitor is uncharged. If switch S is closed at $t = 0$, then calculate heat loss from the resistor R_2 from $t = 0$ to the instant when voltage across the capacitor becomes V_0. (Assume plane of ring to be horizontal and friction to be absent at all the contacts.) (Assume, $R_2 = 2R_1,\ B_0 l_0^{\,2}\omega_0 = 4V_0$)

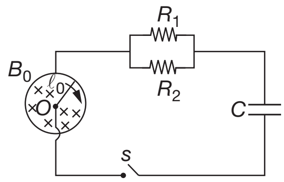


Fig. 16.58

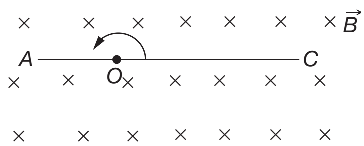
(A) $\frac{1}{2}CV_0^2$ (B) $\frac{1}{6}CV_0^2$
(C) $\frac{2}{3}CV_0^2$ (D) $\frac{1}{3}CV_0^2$

More than One Option Correct Type

79. A positive charge is passing through an electromagnetic field in which $\vec{E}$ and $\vec{B}$ are directed towards y-axis and z-axis, respectively. If a charged particle passes through the region undeviated, then its velocity is/are represented by (here a, b, and c are constant)

(A) $\vec{v} = \frac{E}{B}\hat{i} + a\hat{j}$ (B) $\vec{v} = \frac{E}{B}\hat{i} + b\hat{k}$
(C) $\vec{v} = \frac{E}{B}\hat{i} + c\hat{i}$ (D) $\vec{v} = \frac{E}{B}\hat{i}$

80. A conducting rod AC of length 4ℓ is rotated about a point O in a uniform magnetic field directed into the paper. $AO = \ell$ and $OC = 3\ell$. Then



(A) $V_O - V_A = \frac{B\omega l^2}{2}$ (B) $V_O - V_A = \frac{9}{2} B\omega l^2$

(C) $V_A - V_C = 4B\omega l^2$ (D) $V_O - V_C = \frac{9}{2} B\omega l^2$

81. Two straight conducting rails form a right angle where their ends are joined. A conducting bar in contact with the rails starts at the vertex at $t = 0$ and moves with a constant velocity v along them as shown in Fig. 16.59. A magnetic field B is directed into the page. The induced EMF in the circuit at any time t is proportional to

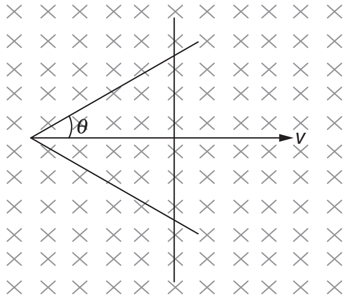


Fig. 16.59

(A) t^0 (B) t (C) v (D) v^2

82. In a cylindrical region of radius R, there exists a time varying magnetic field B such that $\frac{dB}{dt} = k(>0)$. A charged particle having charge q is placed at the point P at a distance d (> R) from its centre O. Now, the particle is moved in the direction perpendicular to OP (see Fig. 16.60) by an external agent up to infinity so that there is no gain in kinetic energy of the charged particle. Choose the correct statement/s.

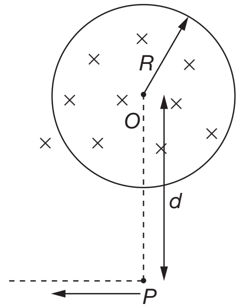


Fig. 16.60

(A) Work done by external agent is $\frac{q\pi R^2}{4} k$ if $d = 2R$

(B) Work done by external agent is $\frac{q\pi R^2}{8} k$ if $d = 4R$

(C) Work done by external agent is $\frac{q\pi R^2}{4} k$ if $d = 4R$

(D) Work done by external agent is $\frac{q\pi R^2}{4} k$ if $d = 6R$

83. The magnetic field perpendicular to the plane of conducting ring of radius r changes at the rate $\frac{dB}{dt} = \alpha$. Then

(A) EMF induced in the ring is $\pi r^2 \alpha$.

(B) EMF induced in the ring is $2\pi r \alpha$.

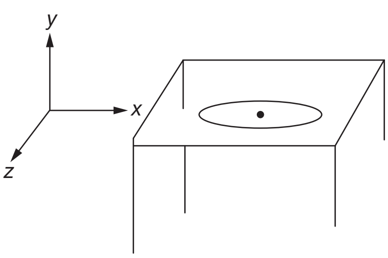
(C) The potential difference between diametrically opposite points on the ring is half of induced EMF.

(D) All points on the ring are at same potential.

Passage Based Questions

Passage I

A uniform conducting ring of mass π kg and radius 1 m is kept on smooth horizontal table. A uniform but time varying magnetic field $B = (\hat{i} + t^2 \hat{j})$ Tesla is present in the region. (Where t is time in seconds). Resistance of ring is 2 Ω. Then, ($g = 10\ m/s^2$)



(*x* - *z* plane is horizontal plane)

84. Net magnetic force (in Newton) on conducting ring as function of time is

(A) $2\pi^2 t$ (B) $2\pi^2 t^2$

(C) $2\pi^2 t^3$ (D) Zero

85. Time (in second) at which ring start toppling is

(A) $\frac{10}{\pi}$ (B) $\frac{20}{\pi}$

(C) $\frac{5}{\pi}$ (D) $\frac{25}{\pi}$

86. Heat generated (in kJ) through the ring till the instant when ring starts toppling is

(A) $\frac{1}{3\pi}$ (B) $\frac{2}{\pi}$ (C) $\frac{2}{3\pi}$ (D) $\frac{1}{\pi}$

87. Induced electric field (in volt/meter) at the circumference of ring at the instant ring starts toppling is

(A) $\frac{10}{\pi}$ (B) $\frac{20}{\pi}$ (C) $\frac{5}{\pi}$ (D) $\frac{25}{\pi}$

Passage 2

A plane loop is shaped as shown in Fig. 16.61 with radii $a = 20$ cm and $b = 10$ cm and is placed in a uniform time varying magnetic field $B = (20+10t)\text{T}$, where t is the time in second. Answer the following questions based on the above statement

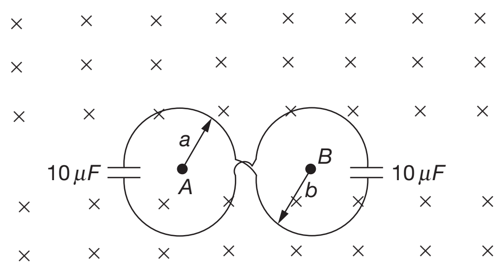


Fig. 16.61

88. The induced EMF in the loop is

(A) 0.942 V (B) 1.57 V
(C) 0.157 V (D) 0.0942 V

89. The maximum charge on each capacitor is

(A) 4.71×10^{-6} C (B) 0.471×10^{-6} C
(C) 52×10^{-6} C (D) 5.2×10^{-6} C

90. The energy stored in each capacitor is

(A) 1.11×10^{-7} J (B) 11.1×10^{-7} J
(C) 111×10^{-7} J (D) 0.111×10^{-7} J

Passage 3

Net force on a current carrying loop kept in uniform magnetic field is zero and the torque on the loop $\vec{\tau} = \vec{M} \times \vec{B}$, where M and B are magnetic dipole moment and magnetic field intensity, respectively. If it is free to rotate, then it will rotates about an axis passing through its centre of mass and parallel to $\vec{\tau}$. Potential energy of the loop is given by $U = -\vec{M}.\vec{B}$. Assume a current carrying ring with its centre at the origin and having moment of inertia 2×10^{-2} kg– m^2 about an axis passing through one of its diameter and magnetic moment $\vec{M} = (3\hat{i} - 4\hat{j})\,\text{Am}^2$. At time $t = 0$, a magnetic field $\vec{B} = (4\hat{i} - 3\hat{j})T$ is switched on. Then

91. Torque acting on the loop is

(A) Zero (B) $25\hat{k}$ Nm
(C) $16\hat{k}$ Nm (D) $10\hat{k}$ Nm

92. Angular acceleration of the ring at time $t = 0$ (in rad/s^2) is

(A) 5000 (B) 1250
(C) 2500 (D) Zero

93. Maximum angular velocity of the ring (in rad/s) will be

(A) $50\sqrt{2}$ (B) $25\sqrt{2}$
(C) $100\sqrt{2}$ (D) $150\sqrt{2}$

Passage 4

Two parallel vertical rails AB and CD, separated by 1m, are connected at two ends by resistances R_1 and R_2 as shown in Fig. 16.62. A horizontal metallic bar of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6T perpendicular to the plane of the rails. It is observed that when the terminal velocity is achieved, the powers dissipated in R_1 and R_2 are 0.76 watt and 1.2 watt, respectively.

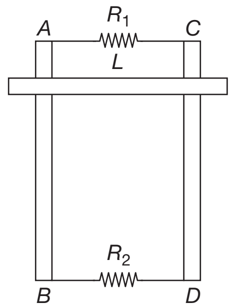


Fig. 16.62

94. Terminal velocity of bar is

(A) 4 m/s (B) 3 m/s
(C) 2 m/s (D) 1 m/s

95. Value of resistance R_1 is

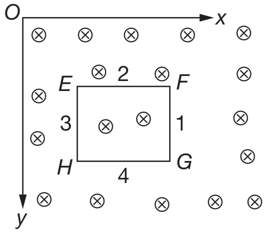
(A) 0.12 Ω (B) 0.24 Ω
(C) 0.47 Ω (D) 0.96 Ω

96. Value of resistance R_2 is

(A) 0.1 Ω (B) 0.2 Ω
(C) 0.3 Ω (D) 0.4 Ω

Passage 5

A magnetic field $\vec{B} = \left(\frac{B_0 y}{a}\right)\hat{k}$ is into the paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m, and resistance R, in x-y plane, starts falling under the influence of gravity. Assume x-axis is horizontal and y is vertically downward.



97. The magnitude and direction of the induced current in the loop when its speed is v is

(A) $\dfrac{B_0 a v}{R}$, anti-clockwise (B) $\dfrac{B_0 a v}{R}$, clockwise

(C) $\dfrac{B_0 v}{aR}$, anti-clockwise (D) None of these

98. The magnitude and direction of the net Lorentz force, acting on the loop when its speed is v, is

(A) $\dfrac{B_0 a^2 v}{R}$, upward

(B) $\dfrac{B_0 a^2 v}{R}$, downward

(C) $\dfrac{B_0^2 a^2 v}{R}$, downward

(D) $\dfrac{B_0^2 a^2 v}{R}$, upward

99. The expression for the speed of the loop $v(t)$ is

(A) $\dfrac{mg}{B_0^2 a^2}\left(1-e^{-\frac{B_0^2 a^2 t}{mR}}\right)$

(B) $\dfrac{Rmg}{B_0^2 a^2}\left(1-e^{-\frac{B_0^2 a^2 t}{mR}}\right)$

(C) $\dfrac{Rmg}{B_0^2 a^2}\left(e^{-\frac{B_0^2 a^2 t}{mR}}\right)$

(D) None of these

100. Acceleration of the loop when its speed is half of its terminal speed is

(A) $\dfrac{g}{2}$ (B) ge^{-2} (C) $ge^{-1/2}$ (D) ge^{-4}

Match the Column Type

101. A uniform but time varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper, as shown in Fig. 16.63. Magnetic field decreases at constant rate inside the region. If r is the distance from axis of cylindrical region, then

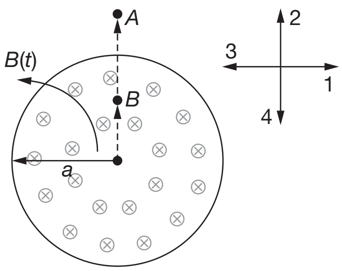
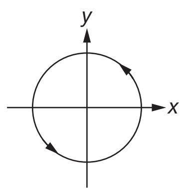


Fig. 16.63

Column-I	Column-II
(A) Induced electric field at point A	1. Directed along 3
(B) Induced electric field at point B	2. Directed along 1
(C) Force on an electron placed at point A	3. Increases as r
(D) Force on an electron placed at point B	4. Decreases as $1/r$

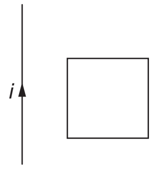
102. A circular current carrying loop of 100 turns and radius 10 cm is placed in x-y plane as shown. A uniform magnetic field $\vec{B} = (-\hat{i} + \hat{k})$ tesla is present in the region. If current in the loop is 5 A,



Column-I	Column-II
(A) Magnitude and direction of magnetic moment (in A-m) of the loop	1. Zero
(B) Magnitude and direction of torque (in N-m) on the loop	2. 5π

(C) Magnitude and direction of net force (in N) on the current loop	3. along positive z-axis
(D) Direction of magnetic field of loop at the centre	4. along negative y-axis
	5. 7π

103. A square loop is placed near a long straight current carrying wire as shown.



Column-I	Column-II
(A) If current is increased	1. Induced current in loop is clockwise
(B) If current is decreased	2. Induced current in loop is anti-clockwise
(C) If loop is moved away from the wire	3. Wire will attract the loop
(D) If loop is moved towards the wire	4. Wire will repel the loop

104. Magnetic flux in a circular coil of resistance 10 Ω and radius 7/44m changes with time as shown in Fig. 16.64. ⊗ direction indicates a direction perpendicular to a paper inwards.

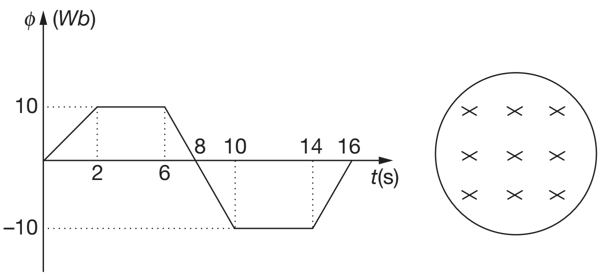


Fig. 16.64

Column-II	Column-II
(A) At 1s induced current is (in A)	1. Clockwise
(B) At 7s induced current is (in A)	2. Anti-clockwise
(C) At 1s induced electric filed at circumference is (in N/C)	3. 5
(D) Maximum induced EMF in coil within $t = 16$s (in volt)	4. 0.5

105. Which of the effect (s) given in column-II will be produced by a loop mentioned in column-I

Column-I	Column-II
(A) Stationary dielectric ring having uniform charge	1. Electric field
(B) Dielectric ring having uniform charge is rotating with constant angular velocity	2. Magnetostatic field
(C) A constant current I_0 in the loop	3. Time-dependent induced electric field outside the loop
(D) Time varying sinusoidal current in the loop $I = I_0 \cos \omega t$	4. Magnetic moment in the loop

Assertion-Reason Type

106. Assertion: In Fig. 16.65, just after closing the switch the potential drop across inductor is maximum.

Reason: The rate of change of current just after closing the switch is maximum.

(A) A (B) B
(C) C (D) D

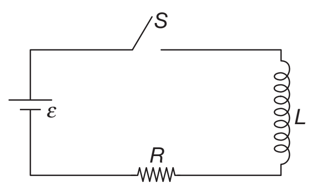


Fig. 16.65

107. **Assertion:** A charged particle is moving in a circle with constant speed in uniform magnetic field. If we increase the speed of particle to twice, its acceleration will become four times.

Reason: In circular path with constant speed, acceleration is given by $\frac{v^2}{R}$.

(A) A (B) B (C) C (D) D

108. **Assertion:** A charged particle is accelerated by a potential difference of V volts. It then enters perpendicularly to a uniform magnetic field. It rotates in a circle. Its angular momentum about centre is say L. Now if V is doubled, L also becomes double.

Reason: If V is doubled, kinetic energy will become two times and therefore, but L remains same.

(A) A (B) B (C) C (D) D

109. **Assertion:** When a test charge moves through a magnetic field, its momentum changes but kinetic energy remains constant.

Reason: The magnetic force acts as a centripetal force, which is perpendicular to the instantaneous velocity and so does no work.

(A) A (B) B (C) C (D) D

110. **Assertion:** When a magnet is made to fall freely through a closed conducting coil, its acceleration is always less than acceleration due to gravity.

Reason: Current induced in the coil opposes the motion of the magnet, as per Lenz's law.

(A) A (B) B (C) C (D) D

111. **Assertion:** An inductor acts as perfect conductor for DC at steady state.

Reason: DC remains constant in magnitude.

(A) A (B) B (C) C (D) D

112. **Assertion:** In the phenomenon of mutual induction, self-induction of each of the coil persists.

Reason: Self-induction arises when strength of current in one coil changes. In mutual induction, current is changing in both the individual coils.

(A) A (B) B (C) C (D) D

113. **Assertion:** A charged particle moves perpendicular to a uniform magnetic field then its momentum remains constant.

Reason: Magnetic force acts perpendicular to the velocity of the particle.

(A) A (B) B (C) C (D) D

114. **Assertion:** A solenoid is connected to an ideal cell. If there is no resistance in the circuit, then rate of change of current $\left(\frac{dI}{dt}\right)$ will decrease.

Reason: If a solenoid is connected to a cell through a resistance, potential difference across the solenoid decreases.

(A) A (B) B (C) C (D) D

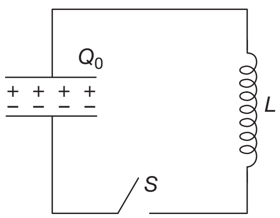
115. **Assertion:** A charged particle at rest experiences no electromagnetic force.

Reason: The electric and magnetic field must be zero.

(A) A (B) B (C) C (D) D

Integer Type

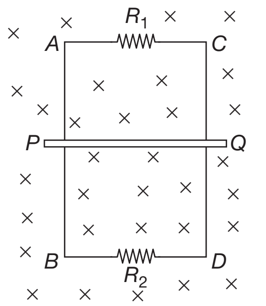
116. A capacitor of capacitance $C = \frac{18}{\pi}$ mH having initial charge Q_0 connected to an inductor of inductance $L = \frac{18}{\pi}$ mH at $t = 0$. Find the time (in milli second) after energy stored in electric field is three times energy stored in magnetic field.



117. A coil of inductance 1H and resistance 10 Ω is connected to a resistance-less battery of EMF 50 V at time $t = 0$. The ratio of rate at which magnetic energy is stored in the coil to the rate at which energy is supplied by the battery at $t = 0.1$ s. is $x \times 10^{-2}$. Find the value of x.

(Given $\frac{1}{e} = 0.37$)

118. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown. A horizontal metallic bar PQ of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is uniform horizontal magnetic field of 0.6 T perpendicular to plane of the rails. It is observed that when the terminal velocity attained, the power dissipated in R_1 and R_2 are 0.76 W and 1.2 W, respectively. Find the terminal velocity of bar in m/s ($g = 9.8$ m/s^2).



119. Figure 16.66 shows four rods having $\lambda = 0.5$ Ω/m resistance per unit length. The arrangement is kept in a magnetic field of constant magnitude $B = 2$T and directed perpendicular to the plane of the figure and directed inwards. Initially, the rods form a square of side length $\ell = 15$ m as shown. Now each wire starts moving with constant velocity $v = 5$ m/s towards opposite wire. Find the force required in newton on each wire to keep its velocity constant at $t = 1$ s.

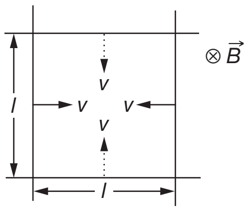
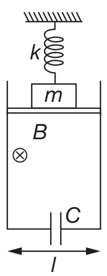
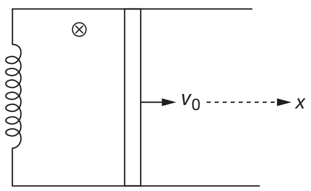


Fig. 16.66

120. A block is attached to the ceiling by a spring that has a force constant $k = 200$ N/m. A conducting rod is rigidly attached to the block. The combined mass of the block and the rod is $m = 0.3$ kg. The rod can slide without friction along two vertical parallel rails, which are a distance $\ell = 1$ m apart. A capacitor of known capacitance $C = 500$ μF is attached to the rails by the wires. The entire system is placed in a uniform magnetic field $B = 20$ T directed as shown. Find the angular frequency (in rad/s) of the vertical oscillations of the block. Neglect the self-inductance and electrical resistance of the rod and all wires.



121. A loop is formed by two parallel conductors connected by a solenoid with inductance $L = 2H$ and a conducting rod of mass $m = 8$ kg which can freely (without friction) slide over the conductors. The conductors are located in a horizontal plane and in a uniform vertical magnetic field $B = \pi T$. The distance between the conductors is $\ell = 2$ m. At the moment, $t = 0$, the rod is imparted on initial velocity $v_0 = 2$ m/s directed to the right. Find the time period of oscillation of rod in second if the resistance of loop is negligible.



122. A non-conducting non-magnetic rod having circular cross-section of radius R is suspended from a rigid support as shown in Fig. 16.67. A light and small coil of 300 turns is wrapped tightly at the left end of the rod where uniform magnetic field B exists in vertically downward direction. Air of density ρ hits the half of the right part of the rod with velocity v as shown in Fig. 16.67. What should be current in clockwise direction (as seen from O) in the coil so that rod remains horizontal? Give answer in mA, given $\frac{2}{Lv}\sqrt{\frac{\pi RB}{\rho}} = 1\ \text{A}^{-1/2}$.

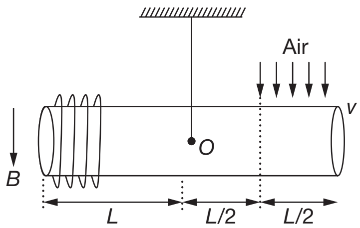


Fig. 16.67

123. A uniform disc of radius R having charge Q distributed uniformly all over its surface is placed on a smooth horizontal surface. A magnetic field, $B = kxt^2$, where k is a constant, x is the distance (in metre) from the centre of the disc, and t is the time (in second) is switched on perpendicular to the plane of the disc. Find the torque (in N-m) acting on the disc after 15 s (Take $4\ kQ = 1$ S.I. unit and $R = 1$ m).

Previous Years' Questions

124. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is **[2002]**

(A) $R/\omega L$ (B) $R/\left(R^2+\omega^2L^2\right)^{1/2}$

(C) $\omega L/R$ (D) $R/\left(R^2-\omega^2L^2\right)^{1/2}$

125. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in Fig. 16.68. The induced EMF is **[2002]**

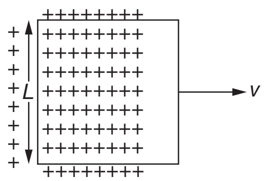
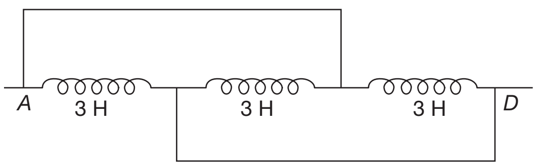


Fig. 16.68

(A) Zero (B) RvB
(C) VBL/R (D) VBL

126. The inductance between A and D is **[2002]**



(A) 3.66 H (B) 9 H
(C) 0.66 H (D) 1 H

127. In a transformer, number of turns in primary coil are 140 and that of the secondary coil are 280. If current in primary coil is 4 A, then that of the secondary coil is **[2002]**

(A) 4 A (B) 2 A (C) 6 A (D) 10 A

128. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon **[2002]**

(A) the rates at which currents are changing in the two coils.
(B) relative position and orientation of the two coils.
(C) the materials of the wires of the coils.
(D) the currents in the two coils.

129. When the current changes from +2 A to –2 A in 0.05 second, an EMF of 8 V is induced in a coil. The coefficient of self-induction of the coil is **[2003]**

(A) 0.2 H (B) 0.4 H
(C) 0.8 H (D) 0.1 H

130. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is **[2003]**

(A) $\frac{Q}{2}$ (B) $\frac{Q}{\sqrt{3}}$ (C) $\frac{Q}{\sqrt{2}}$ (D) Q

131. The core of any transformer is laminated so as to **[2003]**

(A) reduce the energy loss due to eddy currents.
(B) make it light weight.
(C) make it robust and strong.
(D) increase the secondary voltage.

132. Alternating current cannot be measured by DC ammeter because **[2004]**

(A) Average value of current for complete cycle is zero.
(B) AC changes direction.
(C) AC cannot pass through DC ammeter.
(D) DC ammeter will get damaged.

133. In an LCR series AC circuit, the voltage across each of the components, L, C, and R is 50 V. The voltage across the LC combination will be **[2004]**

(A) 100 V (B) $50\sqrt{2}$ V
(C) 50 V (D) 0 V(zero)

134. A coil having n turns and resistance $R\Omega$ is connected with a galvanometer of resistance $4R\Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is **[2004]**

(A) $-\frac{(W_2-W_1)}{Rnt}$ (B) $-\frac{n(W_2-W_1)}{5Rt}$

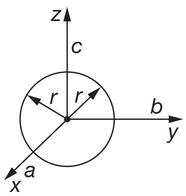
(C) $-\frac{(W_2-W_1)}{5Rnt}$ (D) $-\frac{n(W_2-W_1)}{Rt}$

135. In a uniform magnetic field of induction B, a wire in the form of a semicircle of radius r rotates about the diameter of the circle with an angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is **[2004]**

(A) $\frac{(B\pi r\omega)^2}{2R}$ (B) $\frac{(B\pi r^2\omega)^2}{8R}$

(C) $\frac{B\pi r^2\omega}{2R}$ (D) $\frac{(B\pi r\omega^2)^2}{8R}$

136. In a LCR circuit, capacitance is changed from C to 2 C, For the resonant frequency to remain unchanged, the inductance should be changed from L to **[2004]**



(A) L/2 (B) 2L (C) 4L (D) L/4

137. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the EMF developed between the two ends of the conductor is **[2004]**

(A) 5 mV (B) 50μV

(C) 5μV (D) 50mV

138. One conducting U tube can slide inside another as shown in Fig. 16.69, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of Fig. 16.69. If each tube moves towards the other at a constant speed v, then the EMF induced in the circuit in terms of B, ℓ, and v, where ℓ, the width of each tube, will be **[2005]**

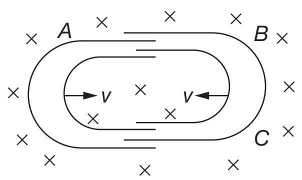


Fig. 16.69

(A) $-Blv$ (B) Blv (C) 2 Blv (D) Zero

139. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[2005]**

(A) 8μF (B) 4μF (C) 2μF (D) 1μF

140. The phase difference between the alternating current and EMF is $\frac{\pi}{2}$. Which of the following cannot be the constituent of the circuit? **[2005]**

(A) R, L (B) C alone

(C) L alone (D) L, C

141. A circuit has a resistance of 12 ohm and an impedance of 15 Ω. The power factor of the circuit will be **[2005]**

(A) 0.4 (B) 0.8 (C) 0.125 (D) 1.25

142. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[2005]**

(A) 0.1 s (B) 0.05 s

(C) 0.3 s (D) 0.15 s

143. Which of the following units denotes the dimension $\frac{ML^2}{Q^2}$, where Q denotes the electric charge? **[2006]**

(A) Wb/m^2 (B) Henry (H)

(C) H/m^2 (D) Weber (Wb)

144. In a series resonant LCR circuit, the voltage across R is 100 V and $R = 1k\Omega$ with $C = 2\mu\text{F}$. The resonant frequency ω is 200 rad/s. At resonance, the voltage across L is **[2006]**

(A) 2.5×10^{-2} V (B) 40V

(C) 250V (D) 4×10^{-3} V

145. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω is a magnetic field B. The maximum value of EMF generated in the coil is **[2006]**

(A) N.A.B.R. ω (B) N.A.B

(C) N.A.B.R. (D) N.A.B ω

146. The flux linked with a coil at my any instant t is given by $\phi = 10t^2 - 50t + 250$

The induced EMF at t = 3 s is **[2006]**

(A) –190 V (B) –10 V

(C) 10 V (D) 190 V

147. An inductor $(L = 100\,\text{mH})$, a resistor $(R = 100\,\Omega)$, and a battery (E = 100 V) are initially connected in series as shown in Fig. 16.70. After a long time, the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is **[2006]**

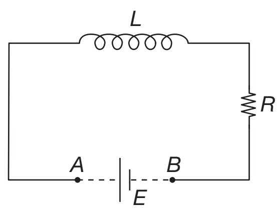


Fig. 16.70

(A) 1/eAq (B) eA
(C) 0.1 A (D) 1A

148. In an AC circuit, the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin(\omega t - \frac{\pi}{2})$. The power consumption in the circuit is given by **[2007]**

(A) $P = \sqrt{2} E_0 I_0$ (B) $P = \frac{E_0 I_0}{\sqrt{2}}$
(C) $P = 0$ (D) $P = \frac{E_0 I_0}{2}$

149. An ideal coil of 10 H is connected in series with a resistance of 5Ω and a battery of 5 V. 2 second after the connection is made. The current flowing in ampere in the circuit is **[2007]**

(A) $(1 - e^{-1})$ (B) $(1 - e)$
(C) e (D) e^{-1}

150. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\,\text{cm}^2$ and length = 20 cm. If one of the solenoid has 300 turns and the other 400 turns, their mutual inductance is $(\mu_0 = 4\pi \times 10^{-7}\,\text{TmA}^{-1})$ **[2008]**

(A) $2.4\pi \times 10^{-5}$ H (B) $4.8\pi \times 10^{-4}$ H
(C) $4.8\pi \times 10^{-5}$ H (D) $2.4\pi \times 10^{-4}$ H

151. An inductor of inductance $L = 400\,\text{mH}$ and resistors of resistance $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of EMF 12 V as shown in Fig. 16.71. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[2009]**

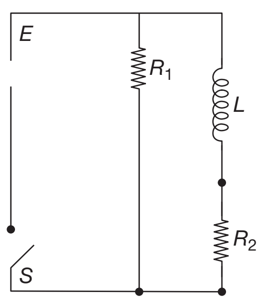
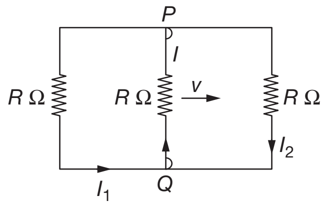


Fig. 16.71

(A) $\frac{12}{t} e^{-3t}$ V (B) $6(1 - e^{-t/0.2})$ V
(C) $12e^{-5t}$ V (D) $6e^{-5t}$ V

152. A rectangular loop has a sliding connector PQ of length ℓ and resistance $R\ \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2, and I are **[2010]**



(A) $I_1 = -I_2 = \frac{Blv}{6R}, I = \frac{2Blv}{6R}$

(B) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(C) $I_1 = I_2 = I = \frac{Blv}{R}$

(D) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

153. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil, it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to **[2012]**

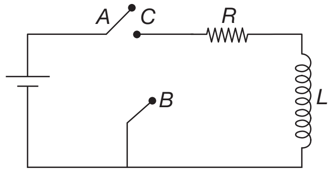
(A) development of air current when the plate is placed.
(B) induction of electrical charge on the plate.
(C) shielding of magnetic lines of force as aluminium is a paramagnetic material.
(D) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

154. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is **[2013]**

(A) 6×10^{-11} weber
(B) 3.3×10^{-11} weber
(C) 6.6×10^{-9} weber
(D) 9.1×10^{-11} weber

155. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly, point C is disconnected from point a and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to

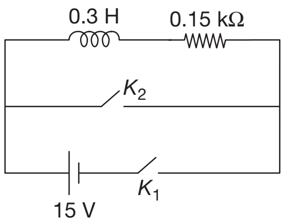
[2014]



(A) $\frac{e}{1-e}$ (B) 1

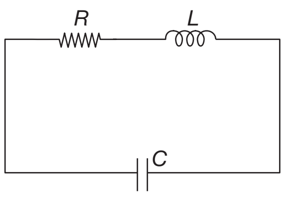
(C) –1 (D) $\frac{1-e}{e}$

156. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be: ($e^5 \cong 150$) **[2015]**

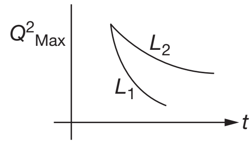


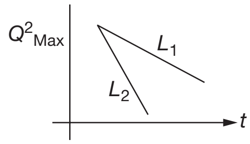
(A) 67 mA (B) 6.7 mA
(C) 0.67 mA (D) 100 mA

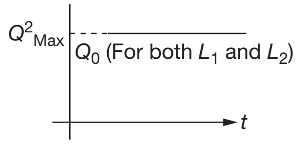
157. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown below **[2015]**

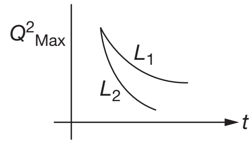


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time(t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly?
(Plots as schematic and not drawn to scale)

(A)


(B)


(C)


(D)


158. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), and a 50 Hz AC supply, the series inductor needed for it to work is close to

(A) 0.08 H
(B) 0.044 H
(C) 0.065 H
(D) 80 H

ANSWER KEYS

Single Option Correct Type

1. (D)	**2.** (B)	**3.** (A)	**4.** (C)	**5.** (D)	**6.** (C)	**7.** (C)	**8.** (C)	**9.** (C)	**10.** (C)
11. (C)	**12.** (D)	**13.** (A)	**14.** (B)	**15.** (B)	**16.** (A)	**17.** (A)	**18.** (B)	**19.** (A)	**20.** (D)
21. (B)	**22.** (B)	**23.** (A)	**24.** (A)	**25.** (D)	**26.** (D)	**27.** (A)	**28.** (B)	**29.** (C)	**30.** (D)
31. (C)	**32.** (B)	**33.** (C)	**34.** (A)	**35.** (A)	**36.** (A)	**37.** (A)	**38.** (B)	**39.** (A)	**40.** (D)
41. (A)	**42.** (B)	**43.** (B)	**44.** (D)	**45.** (A)	**46.** (B)	**47.** (B)	**48.** (B)	**49.** (C)	**50.** (B)
51. (B)	**52.** (A)	**53.** (C)	**54.** (A)	**55.** (A)	**56.** (A)	**57.** (D)	**58.** (B)	**59.** (C)	**60.** (D)
61. (B)	**62.** (D)	**63.** (B)	**64.** (A)	**65.** (A)	**66.** (B)	**67.** (A)	**68.** (C)	**69.** (B)	**70.** (B)
71. (D)	**72.** (D)	**73.** (C)	**74.** (D)	**75.** (C)	**76.** (D)	**77.** (A)	**78.** (A)		

More than One Option Correct Type

79. (B) and (D) **80.** (A), (C) and (D) **81.** (B) and (D)
82. (A), (C) and (D) **83.** (A) and (B)

Passage Based Questions

Passage 1

84. (D) **85.** (A) **86.** (C) **87.** (A)

Passage 2

88. (A) **89.** (A) **90.** (B)

Passage 3

91. (B) **92.** (B) **93.** (A)

Passage 4

94. (D) **95.** (C) **96.** (C)

Passage 5

97. (A) **98.** (D) **99.** (B) **100.** (A)

Match the Column Type

101. (A) → 2, 4; (B) → 2, 3; (C) → 1, 4; (D) → 1, 3
102. (A) → 2, 3; (B) → 2, 4; (C) → 1; (D) → 3
103. (A) → 2, 4; (B) → 1, 3; (C) → 1, 3; (D) → 2, 4
104. (A) → 2, 4; (B) → 1, 4; (C) → 2, 3; (D) → 3
105. (A) → 1; (B) → 1, 2, 4; (C) → 2, 4; (D) → 2, 3, 4

Assertion-Reason Type

106. (A) **107.** (D) **108.** (C) **109.** (A) **110.** (A) **111.** (A) **112.** (A) **113.** (D) **114.** (D) **115.** (C)

Integer Type

116. 3 ms **117.** 0.37 **118.** 1 m/s
119. 200 N **120.** 0.469 A **121.** 4 se
122. 10 ma **123.** AT $T = 15$ sec, $\tau = 1$ N-M

Previous Years' Questions

124. (B) **125.** (D) **126.** (D) **127.** (B) **128.** (B) **129.** (D) **130.** (C) **131.** (A) **132.** (A) **133.** (D)
134. (B) **135.** (B) **136.** (A) **137.** (B) **138.** (C) **139.** (D) **140.** (A) **141.** (B) **142.** (A) **143.** (B)
144. (C) **145.** (D) **146.** (B) **147.** (A) **148.** (C) **149.** (A) **150.** (D) **151.** (C) **152.** (B) **153.** (D)
154. (D) **155.** (C) **156.** (C) **157.** (D) **158.** (C)

HINTS AND SOLUTIONS

Single Option Correct Type

1. Due to the time varying magnetic field, induced electric field will be set-up and its lines are in clockwise, so force on stationary charge q is along (4)

   The correct option is (D)
2. The correct option is (B)
3. $M = K\sqrt{L_1 L_2}$

   Here, $K = 1$

   $\therefore \quad M = \sqrt{4 \times 16} = 8\text{H}$

   The correct option is (A)
4. The correct option is (C)
5. The correct option is (D)

6. $I = neAv_d$

$\Rightarrow \quad 1.344 = 8.4\times10^{22}\times10^{6}\times1.6\times10^{-19}\times1\times10^{-6}\times V_d$

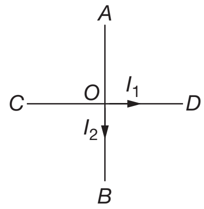
$\therefore \quad v_d = 1\times10^{-4}$ m/s $= 0.1$ mm/s

The correct option is (C)

7. $\vec{B}$ due to AOB and COD will be perpendicular to each other at point P

$B^2 = B_1^{\,2} + B_2^{\,2}$

$B = \dfrac{\mu_0}{2\pi a}\left(I_1^2 + I_2^2\right)^{1/2}$



The correct option is (C)

8. The correct option is (C)

9. $e = L\dfrac{dI}{dt}$

$\Rightarrow \quad L = 0.04$ H

The correct option is (C)

10. The correct option is (C)

11. $E = -L\dfrac{dI}{dt} = -1\times10^{-3}\times\dfrac{3-5}{10^{-3}} = 2$ V.

The correct option is (C)

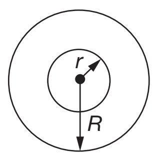
12. Since the magnetic field is constant with time and space and exists everywhere, there is no change in magnetic flux when the loop is moved in it. Hence, no current is induced.

The correct option is (D)

13. The correct option is (A)

14. No current is enclosed in the circle, so from Ampere's circuital law, the magnetic induction at any point inside the infinitely long straight thin-walled tube (cylindrical) is zero.

The correct option is (B)



15. $T = \dfrac{2\pi m}{qB}$, $\dfrac{T_\alpha}{T_p} = \dfrac{m_\alpha}{m_p}.\dfrac{q_p}{q_\alpha} = 2$

The correct option is (B)

16. $e = \left|\dfrac{d\phi}{dt}\right| = \dfrac{4B_0A - B_0A}{t} = \dfrac{3A_0B_0}{t}$

The correct option is (A)

17. $e = L\dfrac{dI}{dt}$, $\quad 8 = L\dfrac{4}{0.05}$, $\quad L = 0.1$H

The correct option is (A)

18. Magnitude of torque is given by $|\vec{\tau}| = MB\sin\theta$

Here, $M = NiA = (1)(1.0)(\pi)(0.2)^2 = (0.04\pi)$A– m^2

and θ = angle between $\vec{M}$ and $\vec{B} = 90°$

$\therefore \quad |\vec{\tau}| = (0.04\pi)(2)\sin 90° = 0.08\,\pi$ N-m.

The correct option is (B)

19. $e = L\dfrac{dI}{dt}$; $\quad 20\,\text{V} = L\dfrac{5}{0.1}$

$L = \dfrac{2}{5} = 0.4$ H

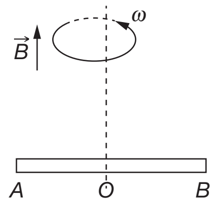
The correct option is (A)

20. Potential difference between

O and A is $V_A - V_O = \dfrac{1}{2}Bl^2\omega$

O and B is $V_B - V_O = \dfrac{1}{2}Bl^2\omega$

So $V_A - V_B = 0$



The correct option is (D)

21. $E = \dfrac{1}{2}LI^2 = \dfrac{1}{2}\times\left(50\times10^{-3}\right)(2)^2 = 0.1$ J

The correct option is (B)

22. $V = B\ell v$, $\quad Q = CV = BlvC$

The correct option is (B)

23. Work done by centripetal force = 0

$\vec{F}_C = \vec{F}_m = q\left(\vec{V}\times\vec{B}\right)$

The correct option is (A)

24. $\varepsilon = \left|-\dfrac{d\phi}{dt}\right| = \left|-\dfrac{d}{dt}(NBA)\right| = 0.5$ V

The correct option is (A)

25. No change in flux linked with the coil

The correct option is (D)

26. We know that $\vec{\tau} = \vec{\mu}\times\vec{B}$; hence, it is perpendicular to both $\vec{\mu}$ and $\vec{B}$

The correct option is (D)

27. Direction of magnetic moment and direction of magnetic field is parallel, then torque acting on the loop is zero.

The correct option is (A)

28. The correct option is (B)

29. The positive charge of the rod shift towards left due to $F = q\left(\vec{v}\times\vec{B}\right)$

The correct option is (C)

30. The correct option is (D)

31. Increasing and decreasing rate of current is same then no net flux change in the loop, then the induced current in the loop is zero.

The correct option is (C)

32. The correct option is (B)

33. $\varepsilon_{\text{net}} = \left|\dfrac{d\phi}{dt}\right| = (16t + 3) = 67$ V at $t = 4$ s

The correct option is (C)

34. $E = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(6t^2 - 5t + 1) = -12t + 5$

At $t = 0.25$ s, $E = -12 \times 0.25 + 5 = -3 + 5 = 2$ V

$I = \dfrac{E}{R} = \dfrac{2}{10} = 0.2$ A.

The correct option is (A)

35. For induced EMF (motional EMF) $\vec{v} \perp \vec{B} \perp \vec{L}$

The correct option is (A)

36. $q = CV = C(Bvl) = 80\mu\text{C} =$ constant

Magnetic force on electrons of the conducting rod PQ is towards Q.

Therefore, A is positively charged and B is negatively charged.

The correct option is (A)

37. The correct option is (A)

38. $\dfrac{q^2}{2C} = \dfrac{Q^2}{4C}$, $\quad q = \dfrac{Q}{\sqrt{2}}$

The correct option is (B)

39. $\phi = \dfrac{\sqrt{3}}{4} l^2 B$;

$\varepsilon = \left|\dfrac{d\varphi}{dt}\right| = \dfrac{\sqrt{3}}{4} l^2 \dfrac{dB}{dt}$;

$i = \dfrac{\varepsilon}{R} = \dfrac{\sqrt{3}\, l^2}{4R}$

The correct option is (A)

40. The correct option is (D)

41. Magnetic field due to the larger coil at its centre is $B = \dfrac{\mu_0 I}{2r_1}$

Flux through the inner coil is $\phi = B \times \pi r_2^2 = \dfrac{\mu_0 I}{2r_1} \times \pi r_2^2$

But $\phi = MI$.

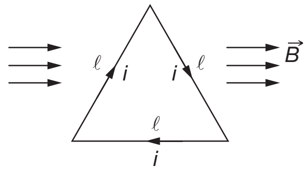
Therefore, $M = \dfrac{\mu_0 \pi r_2^2}{2r_1}$

The correct option is (A)

42. $\tau = BiA\sin\theta = BiA\sin 90°$

$= Bi \times \dfrac{\sqrt{3}}{4} l^2$

$l = 2\left(\dfrac{\pi}{Bi\sqrt{3}}\right)^{1/2}$



The correct option is (B)

43. $L = \dfrac{\mu_0 N^2 \pi r}{2}$

When the number of turns in a coil is doubled without changing the length of the coil, the radius of the coil is half.

Hence, $L' = \dfrac{\mu_0 (2N)^2 \times \pi (r/2)}{2} = 2\left[\dfrac{\mu_0 N^2 \pi r}{2}\right] = 2L$

The correct option is (B)

44. Induced EMF, $e = -\dfrac{d\phi}{dt} = -2xt$

$\therefore \quad x = -\dfrac{e}{2t} = -1.5$ Wb/s^2

The correct option is (D)

45. $\tau = NBiA = 100 \times 0.2 \times 2 \times (0.08 \times 0.1) = 0.32$ N-m

The correct option is (A)

46. $\mu = \mu_0(1 + \chi)$

$\mu = 500 \times 4\pi \times 10^{-7} = 2\pi \times 10^{-4}$ H/m

The correct option is (B)

47. $e = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(\pi r^2 B) = -2\pi Br\left(-\dfrac{dr}{dt}\right)$

The correct option is (B)

48. Induced is EMF is $|e| = n\dfrac{\Delta\phi}{\Delta t}$.

Now $\Delta q = I\Delta t = \dfrac{e}{R}\Delta t = \dfrac{n\Delta\phi}{R\Delta t} \times \Delta t = \dfrac{n\Delta\phi}{R} = \dfrac{n(\phi_2 - \phi_1)}{R}$

The correct option is (B)

49. $|\varepsilon| = \dfrac{d}{dt}(3t^2) = 6t = 18$ V

at $t = 3$ s

The correct option is (C)

50. $\varepsilon = Blv = 2 \times 10^{-5} \times 500 \times \sqrt{2 \times 10 \times 5} = 0.1$ V

$\therefore \quad I = \dfrac{\varepsilon}{R} = \dfrac{0.1}{2.5} = 0.04$ A

The correct option is (B)

51. $e_0 = NAB\omega = 30 \times 400 \times 10^{-4} \times 1 \times 30 \times 2\pi$

$= 226$ V

The correct option is (B)

52. $\dfrac{M}{L} = \dfrac{\dfrac{2q\omega}{2\pi}\pi l^2}{2ml^2\omega}$

The correct option is (A)

53. Magnetic flux through the loop A;

$\phi = B\pi r^2 = \dfrac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} \pi r^2$

Induced EMF in the loop A;

$$\varepsilon = -\frac{d\phi}{dt} = \frac{\mu_0 I R^2 \pi r^2}{2}\left(-\frac{3}{2\left(R^2+x^2\right)^{5/2}}2x\frac{dx}{dt}\right)$$

$$= -\frac{3\mu_0 I R^2 \pi r^2 v}{2}\left[\frac{x}{\left(R^2+x^2\right)^{5/2}}\right]$$

Induced EMF is maximum when $\frac{d\varepsilon}{dx} = 0$

$\Rightarrow \quad \left(R^2+x^2\right) - 5x^2 = 0$ or $x = \frac{R}{2}$

The correct option is (C)

54. $\tau_{net} = 0$

$mgR = MB_0$ (M = magnetic dipole moment)

$mgR = I(\pi R^2)B_0$

$I = \frac{mg}{\pi R B_0}$

The correct option is (A)

55. Induced EMF in the rod $\varepsilon = Blv$

Current in the circuit $I = \frac{\varepsilon}{R}e^{-t/RC} = \frac{Blv}{R}e^{-t/RC}$

Since the net force on the rod should be zero, the external force will be equal in magnitude but opposite to the magnetic force.

$\Rightarrow \quad F = I\,l\,B = \frac{B^2 l^2 v}{R}e^{-t/RC}$

The correct option is (A)

56. By work-energy theorem,

$mgz + BIlz - \int 2k(x+z)dz = 0$

$2mgz = 2k\left[\int_0^z x\,dz + \int_0^z z\,dz\right]$ (where x is elongation in the equilibrium position)

$2mg = mg + kz$

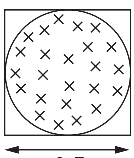
$z = \frac{mg}{k} = \frac{BIl}{k}$

The correct option is (A)

57. Considering a square loop as shown, EMF across the loop

$e = \oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt}$

$\Rightarrow \quad e = -\pi R^2 \frac{dB}{dt} = -\pi R^2 \alpha$



$\therefore$ Magnitude of EMF across one side $= \frac{\pi R^2 \alpha}{4}$

The correct option is (D)

58. The flux linked with the ring due to solenoid is $MI_0 t$. Also the induced EMF (back EMF) due to the inductance in the ring is zero, because when current will be maximum, $\frac{dI}{dt} = 0$.

$\therefore$ Current in the ring $= \frac{MI_0 t}{tR} = \frac{MI_0}{R}$

The correct option is (B)

59. The induced EMF between O and P will be only due to translation motion of the ring. So EMF between O and P is VBR.

The correct option is (C)

60. Magnetic flux linked with the loop is always zero.

So induced EMF is also zero

The correct option is (D)

61. The magnetic flux that links the larger loop with the smaller loop of side ℓ ($\ell << L$) is

$\phi_{12} = B\ell^2 = \frac{2\sqrt{2}\mu_0 I l^2}{\pi L}$

$\therefore$ Mutual inductance $M_{12} = \frac{\phi_{12}}{I} = \frac{2\sqrt{2}\mu_0}{\pi}\left(\frac{l^2}{L}\right)$

i.e., $M_{12} \propto \frac{l^2}{L}$

The correct option is (B)

62. $\phi = BA\cos\theta = \frac{1}{2}B\pi r^2 \cos\omega t$

$\therefore \quad e_{induced} = -\frac{d\phi}{dt} = \frac{1}{2}B\pi r^2 \omega \sin\omega t$

$P = \frac{e^2}{R}$

$\therefore \quad P_{mean} = \frac{\int_0^T P\,dt}{T} = \frac{\left(B\pi r^2\omega\right)^2}{8R}$

The correct option is (D)

63. For terminal velocity,

Net force $= 0$

So, $m_1 g = F_m$, $m_1 g = BIl = B\left(\frac{Blv}{R}\right)l$,

$v = \frac{m_1 g R}{B^2 l^2}$

The correct option is (B)

64. $\varepsilon = Blv$, $\quad I_{inst} = \frac{\varepsilon}{R} = \frac{Blv}{R}$

The correct option is (A)

65. Since there is no current in the circuit, magnetic field at P will be zero.

The correct option is (A)

66. Diamagnetic materials show a feeble repulsion.

The correct option is (B)

67. $\vec{M} = iA\left(-\hat{k}\right) = -\pi\hat{k}$

$\vec{\tau} = \vec{M}\times\vec{B} = -10\pi\hat{j}$

Now, axis of rotation is on y-axis, which is diameter to the ring

$\therefore \quad I = \frac{1}{2}mR^2 = \frac{1}{4}\text{kg} - \text{m}^2$

$\therefore \quad \alpha = \frac{|\vec{\tau}|}{I} = 40\,\pi\,\text{rad/s}^2$

The correct option is (A)

68. $E = -L\frac{dI}{dt} = -1 \times 10^{-3} \times \frac{3-5}{10^{-3}} = 2$ V.

The correct option is (C)

69. $\alpha = \frac{|\vec{\tau}|}{I} = \frac{i\pi r^2 B_0\sqrt{2}}{\frac{1}{2}mr^2}$

$\alpha = \frac{2\sqrt{2}\pi B_0 i}{m}$

Axis of rotation of the loop will be along unit vector $\frac{(\hat{j}-\hat{i})}{\sqrt{2}}$

The moment of inertia of ring about that axis $= \frac{1}{2}mR^2$

The correct option is (B)

70. The magnetic induction of the solenoid is directed along its axis. Therefore, the Lorentz force acting on the electron at any instant of time will lie in the plane perpendicular to the solenoid axis. Since the electron velocity at the initial moment is directed at right angles to the solenoid axis, the electron trajectory will lie in the plane perpendicular to the solenoid axis. The Lorentz force can be found from the formula $F = evB$.

The trajectory of the electron in the solenoid is an arc of the circle whose radius can be deter mined from the relation $evB = mv^2/r$, hence

$r = \frac{mv}{eB}$

The trajectory of the electron is shown in Fig. 16.72, where O_1 is the centre of the arc AC described by the electron, v' is the velocity at which the electron leaves the solenoid. The segments OA and OC are tangents to the electron trajectory at points A and C. The angle between v and v' is obviously $\phi = \angle AO_1C$ since $\angle OAO_1 = \angle OCO_1$.

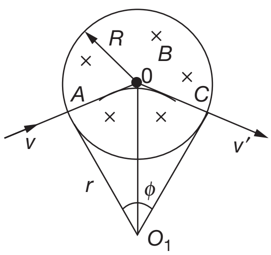


Fig. 16.72

In order to find ϕ, let us consider the right triangle OAO_1; side $OA = R$ and side $AO_1 = r$.

Therefore, $\tan(\phi/2) = R/r = eBR/(mv)$.

Therefore, $\quad \phi = 2\tan^{-1}\left(\frac{eBR}{mv}\right)$

Obviously, the magnitude of the velocity remains unchanged over the entire trajectory since the Lorentz force is perpendicular to the velocity at any instant. Therefore, the transit time of electron in the solenoid can be determined from the relation

$t = \frac{r\phi}{v} = \frac{m\phi}{eB} = \frac{2m}{eB}\tan^{-1}\left(\frac{eBR}{mv}\right)$.

The correct option is (B)

71. New magnetic moment

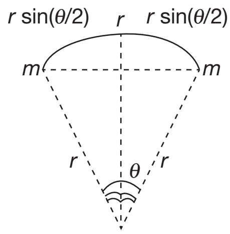
$M' = m \times$ Distance between poles

$= m \times 2r\sin(\theta/2)$

$= m \times 2 \times \frac{\text{arc}}{\text{angle}} \times \sin\frac{\theta}{2}$

$= \frac{2 \times (m \times \text{arc}) \times \sin(\theta/2)}{\theta}$

$M' = \frac{2M\sin(\theta/2)}{\theta} = \frac{2M\sin(\pi/6)}{\pi/3} = \frac{3M}{\pi}$



The correct option is (D)

72. The correct option is (D)

73. $B = \frac{\phi}{A} = \frac{4\times10^{-4}\text{ weber}}{4\times10^{-4}\text{ m}^2} = 1$ tesla

$\mu = \frac{B}{H} = \frac{1}{1600} = 0.625\times10^{-3}\frac{\text{weber}}{\text{amp}\times\text{m}}$

The correct option is (C)

74. As the electron moves from left to right, the flux linked with the loop (which is into the page) will first increase and then decrease. So, the induced current in the loop will be first anti-clockwise and will change direction (i.e., will become clockwise) as the electron passes by.

The correct option is (D)

75. $qvB = \frac{mv^2}{r}$ or $\frac{v}{r} = \frac{qB}{m}$

According to Fig. 16.73,

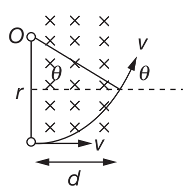


Fig. 16.73

$d = r\sin\theta$

When $d << r$, $\sin\theta \equiv \theta = \dfrac{d}{r}$

Since $\Delta v = 2v\sin\dfrac{\theta}{2} = v\theta$ $\left(\because \sin\dfrac{\theta}{2} \approx \dfrac{\theta}{2}\right)$

The impulse imparted = change in momentum

$$\Delta P = mv\theta = mv\left(\frac{d}{r}\right) = md\frac{qB}{m} = qBd$$

The correct option is (C)

76. The correct option is (D)

77. The correct option is (A)

78. Voltage across rod = $\dfrac{1}{2}B_0 l_0^2 \omega_0$

Charge on capacitor = CV_0

$$v \times q = \frac{1}{2}CV_0^2 + H_{R_1} + H_{R_2}$$

$$CV_0 \times \frac{1}{2}B_0 l_0^2 \omega_0 = \frac{1}{2}CV_0^2 + \frac{R_2}{R_1}H_{R_2} + H_{R_2}$$

$$\frac{1}{2}CV_0 B_0 l_0^2 \omega_0 - \frac{1}{2}CV_0^2 = \frac{H_{R_2}[R_2 + R_1]}{R_1}$$

$$H_{R_2} = \frac{R_1}{R_1 + R_2}\left[\frac{1}{2}CV_0 B_0 l_0^2 \omega_0 - \frac{1}{2}CV_0^2\right]$$

$$= \frac{R_1}{R_1 + R_2} \times \frac{1}{2}CV_0[B_0 l_0^2 \omega_0 - V_0] = \frac{1}{2}CV_0^2$$

The correct option is (A)

More than One Option Correct Type

79. In both cases (B) and (D),

$\vec{F}_{net} = 0$ so it passes the region undeviated.

The correct options are (B) and (D)

80. $V_0 - V_A = B\dfrac{(\omega l)l}{2} = \dfrac{B\omega l^2}{2}$

$V_0 - V_C = \dfrac{B\omega(3l)3l}{2} = \dfrac{9}{2}B\omega l^2$

$V_A - V_C = 4B\omega l^2$ or $V_A > V_C$

The correct option is (A), (C) and (D)

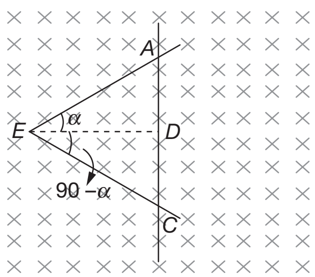
81. At time t, the distance traveled by the rod is

$ED = vt$

$\tan\alpha = \dfrac{AD}{ED}$ or $AD = ED\tan\alpha$

$AD = vt\tan\alpha$

$\tan(90 - \alpha) = \dfrac{DC}{ED}$ or $DC = at\cot\alpha$



So, $AC = AD + DC = vt(\tan\alpha + \cot\alpha)$

Induced EMF = $Bvl = Bv(AC)$

$= Bv \cdot vt(\tan\alpha + \cot\alpha)$

Induced EMF $= Bv^2 t(\tan\alpha + \cot\alpha)$

Hence, induced EMF $\propto t$ and v^2

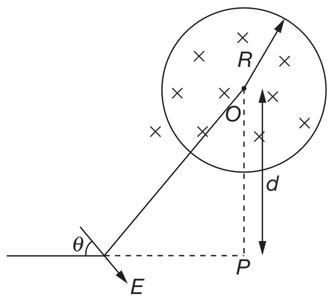
The correct options are (B) and (D)

82. $\int \vec{E} \cdot d\vec{l} = A\dfrac{dB}{dt}$

$$E 2\pi\sqrt{x^2 + d^2} = \pi R^2 k$$

$$E = \frac{\pi R^2 k}{2\sqrt{x^2 + d^2}}$$

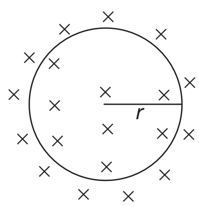
$$W_{ext} = \int_0^{\infty} q\vec{E} \cdot dx = \frac{q\pi R^2}{4}k$$



The correct options are (A), (C), and (D)

83. $E = \dfrac{d\phi}{dt} = \pi r^2 \dfrac{dB}{dt}$ (E = induced EMF)

or $E = \pi r^2 \alpha$



Let i = current in the ring

R = resistance of loop

Consider a small element $d\ell$ of the ring.

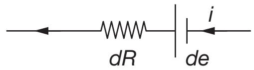
EMF induced in element $de = \left(\frac{E}{2\pi r}\right)dl$

resistance of element $dR = \left(\frac{R}{2\pi r}\right)dl$

Potential difference across the element

$dV = de - i\,dR$

$$= \left(\frac{E}{2\pi r}\right)dl - \left(\frac{e}{R}\right)\left(\frac{R}{2\pi r}\right)dl$$



dV = zero

or V is constant.

So, all points on the ring are at same potential.

The correct options are(A) and (B)

Passage Based Questions

Passage I

84. The correct option is (D)

85. The correct option is (A)

86. The correct option is (C)

87. **Solution:** Induced EMF in the ring is $e = \left|-\frac{d\phi}{dt}\right| = 2\pi t$

Induced current in the ring $= \pi t$

Net force on the ring is always zero.

Ring starts topping when

$\tau_{\text{due to magnetic field}} > \tau_{mg}$ (above one end)

$\pi t \times \pi 1^2 \times 1 \geq \pi \times 10 \times 1$

When ring start toppling. $t = \frac{10}{\pi}$

Heat generated $= \int_0^t I^2 R dt = \frac{2}{3\pi}$ kJ

$\int E.dl$ = induced EMF $E = \frac{2\pi t}{2\pi a} = t = \frac{10}{\pi}$

The correct option is (A)

Passage 2

88. ϕ = instantaneous flux through loop = $BA_1 + BA_2$

Sense of induced EMF is opposite in each loop

Hence, the net EMF induced $= A_1\frac{dB}{dt} - A_2\frac{dB}{dt} = 10\pi\left(a^2 - b^2\right)$

$= 0.942$ V

The correct option is (A)

89. $Q = CV = 5\times10^{-6}\times0.942 = 4.71\times10^{-6}$ C

The correct option is (A)

90. $E = \frac{Q^2}{2C} = \frac{(4.71)^2\times\left(10^{-6}\right)^2}{2\times10\times10^{-6}} = 11.1\times10^{-7}$ J

The correct option is (B)

Passage 3

91. $\vec{\tau} = \vec{M}\times\vec{B} = \left(3\hat{i} - 4\hat{j}\right)\times\left(4\hat{i} + 3\hat{j}\right) = 25\hat{k}$ N-m

The correct option is (B)

92. $\tau = I\alpha \Rightarrow \alpha = \left(\frac{\tau}{I}\right) = \frac{25}{0.02} = 1250$ rad/s^2

The correct option is (B)

93. $(\Delta U)_{\max} = \frac{1}{2}I\omega_{\max}^2$

$\Rightarrow \omega_{\max} = \sqrt{\frac{4MB}{I}} = \left(50\sqrt{2}\right)$ rad/ s

The correct option is (A)

Passage 4

94. At terminal velocity $mgv = P_1 + P_2$

$\Rightarrow v = 1$ m/s

The correct option is (D)

95. $\varepsilon = Bv\ell = 0.6$ V

$\frac{\varepsilon^2}{R_1} = 0.76$ w

$\Rightarrow R_1 = 0.47\ \Omega$

The correct option is (C)

96. $\varepsilon = Bv\ell = 0.6$ V

$\frac{\varepsilon^2}{R_2} = 1.2$ w $\Rightarrow R_2 = 0.3\ \Omega$

The correct option is (C)

Passage 5

97. $e = \left.\frac{d\phi}{dt}\right|_{y=a} = B_0 a.v.$

So, $I = \frac{e}{R} = \frac{B_0 av}{R}$, anti-clockwise

The correct option is (A)

98. Net force, $F = F_2 - F_1 = IB_2 a - IB_1 a = \frac{B_0^2 a^2 v}{R}$, upward.

The correct option is (D)

99. $F = mg - \frac{B_0^2 a^2 v}{R}$, $m.\frac{dv}{dt} = mg - \frac{B_0^2 a^2 v}{R}$

or $\int_0^t dt = \int_0^v \frac{mdv}{mg - \frac{B_0^2 a^2 v}{R}}$

$$v = \frac{R\,mg}{B_0^2 a^2}\left[1 - e^{-B_0^2 a^2 t/mR}\right]$$

The correct option is (B)

100. $v = \frac{v_t}{2} = \frac{Rmg}{2B_0^2 a^2} = \frac{R\,mg}{B_0^2 a^2}\left[1 - e^{-B_0^2 a^2 t/mR}\right]$

$\Rightarrow \left[1 - e^{-B_0^2 a^2 t/mR}\right] = \frac{1}{2}$

Acceleration $a = \left(\frac{dv}{dt}\right)_{v=\frac{v_t}{2}} = \frac{g}{2}$

The correct option is (A)

Match the Column Type

101. (A) → 2, 4; (B) → 2, 3; (C) → 1, 4; (D) → 1, 3

102. (A) → 2, 3; (B) → 2, 4; (C) → 1; (D) → 3

103. (A) → 2, 4; (B) → 1, 3; (C) → 1, 3; (D) → 2, 4

104. At $t = 1$s, $\frac{d\phi}{dt} = 5$

$\therefore \quad i = -0.5$ A

Similarly at $t = 7$ s, $\frac{d\phi}{dt} = -5$, $i = 0.5$ A

Also, $E2\pi r = 5$, $E \times \frac{27}{7} \times \frac{7}{44} = 5$, $E = 5$ N/ C

(A) → 2, 4; (B) → 1, 4; (C) → 2, 3; (D) → 3.

105. (A) → 1; (B) → 1, 2, 4; (C) → 2, 4; (D) → 2, 3, 4

Integer Type

116. $U_C + U_L = \frac{Q_0^2}{2C}$

$\frac{4}{3}U_C = \frac{Q_0}{2C}$

$\Rightarrow \frac{4}{3}\frac{Q^2}{2C} = \frac{Q_0^2}{2C}$, $Q = \frac{\sqrt{3}}{2}Q_0$ and $Q = Q_0 \cos\omega t$

$t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{LC} = \frac{\pi}{6}\sqrt{\left(\frac{18}{\pi}\right)\left(\frac{18}{\pi}\right)\times 10^{-6}} = 3$ ms

117. Let i = Current at any instant then energy stored in magnetic field

$U_B = \frac{1}{2}Li^2$

Rate of energy stored in magnetic field

$\frac{dU_B}{dt} = Li\frac{di}{dt}$

The rate at which energy is supplied by battery = Vi

So, required ratio $= \frac{Li\frac{di}{dt}}{Vi}$

$= \frac{L}{V}\frac{di}{dt}$

$= \frac{L}{V}\frac{i_0}{\tau}e^{-t/\tau}$

$= \frac{1}{50} \times \frac{5}{0.1}e^{-1}$

$= \frac{1}{e} = 0.37$

$\therefore$ Required ratio = 0.37.

118. When bar attains terminals velocity $BIL = mg$

$I = \frac{0.2 \times 9.8}{0.6 \times 1} = \frac{9.8}{3}$,

e is EMF induced in rod and $eI = P_1 + P_2$

$e = \frac{0.76 + 1.2}{9.8/3} = \frac{1.96 \times 3}{9.8} = 0.6$ V and $e = Bv_T l$

$v_T = \frac{e}{B} = 1$ m/s.

119. At any time length of each wire $= l - 2vt$

Induced EMF $= 4\,B\,v(l - 2vt)$

Induced current $= \frac{4B\,v(l - 2vt)}{4\lambda(l - 2vt)} = \frac{B\,v}{\lambda}$,

$F = B\left(\frac{Bv}{\lambda}\right)(l - 2vt) = \frac{B^2 v}{\lambda}(l - 2vt)$

$= \frac{(2)^2 \times 5}{0.5}(15 - 2 \times 5 \times 1) = 200$ N

120. Total energy $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + \frac{1}{2}\frac{q^2}{C}$, $e = Bvl = \frac{q}{C}$

$E = \frac{1}{2}(m + B^2 l^2 C)v^2 + \frac{1}{2}kx^2$ as E = constant $\frac{dE}{dt} = 0$,

$\frac{dv}{dt} = -\left(\frac{k}{m + B^2 l^2 C}\right)x$,

$\omega = \sqrt{\frac{k}{m + B^2 l^2 C}} = 20$ rad/s.

(B) In the closed circuit, there is a resistance of 4 Ω and hence the current in the circuit $I = \frac{E}{R} = \frac{1.875}{4} = 0.469$ A

121. Let any instant of time, velocity of the rod is v towards right.

The current in the circuit is i. In Fig. 16.74,

$V_a - V_b = V_d - V_c$

i.e., $Ldi = Bldx$

Integrating, we get $Li = Blx$

Magnetic force on the rod at this instant is

$$F_m = ilB = \frac{B^2l^2}{L}x \qquad (1)$$

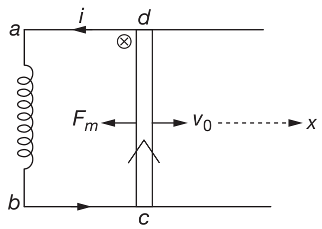


Fig. 16.74

Since, this force is in opposite direction of $\vec{v}$, from Newton's second law we can write,

$$m\left(\frac{d^2x}{dt^2}\right) = -\frac{B^2l^2}{L}x$$

Comparing this with equation of SHM, i.e.

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad (2)$$

We have, $\omega = \dfrac{Bl}{\sqrt{mL}}$

So $T = \dfrac{2\pi}{\omega} = 2\pi\dfrac{\sqrt{mL}}{Bl} = 2\pi\dfrac{\sqrt{8\times 2}}{\pi\times 2} = 4$ s

122. Force exerted by air on the rod $= \left(\rho\frac{L}{2}2R\right)v^2 = \rho L R v^2$

Balancing torque about point O $NI\left(\pi R^2\right)B = \rho L R v^2 \times \dfrac{3L}{4}$

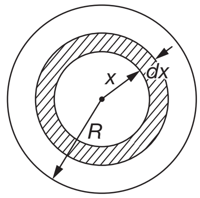
$$\Rightarrow \quad 300\pi IBR = \frac{3\rho v^2 L^2}{4}$$

$$\Rightarrow \quad I = \frac{\rho L^2 v^2}{400\pi BR} = 0.01\text{A} = 10 \text{ mA}$$

123. $d\phi = (2\pi x dx)kxt^2$

$$\phi = \frac{2}{3}kt^2\pi x^3$$

$$\frac{d\phi}{dt} = \frac{4k\pi t x^3}{3}$$



$$E2\pi x = \frac{4\pi k t x^3}{3}; \qquad E = \frac{2}{3}ktx^2;$$

$$d\tau = \left(\frac{2}{3}ktx^2\right)\frac{Q}{\pi R^2}(2\pi x dx)x$$

$$\int d\tau = \frac{4}{3}\frac{ktQ}{R^2}\int_0^R x^4 dx$$

$$\Rightarrow \quad \tau = \frac{4}{3}\frac{ktQ}{R^2}\frac{R^5}{5}$$

At $t = 15$s, $\tau = 1$ N-m

Previous Years' Questions

124. The impedance triangle for resistance (R) and inductor (L) connected in series is shown in Fig. 16.75.

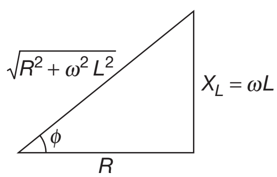


Fig. 16.75

Power factor $\cos\phi = \dfrac{R}{\sqrt{R^2 + \omega^2 L^2}}$

The correct option is (B)

125. The induced EMF is

$$e = \frac{-d\phi}{dt} = -\frac{d(\vec{B}\cdot\vec{A})}{dt} = \frac{-d(BA\cos 0^\circ)}{dt}$$

$$\therefore\ e = -B\frac{dA}{dt} = -B\frac{d(\ell \times \text{x})}{dt}$$

$$\therefore\ e = -B\frac{dx}{dt} = -B\ell v$$

The correct option is (D)

126. These three inductors are connected in parallel. The equivalent inductance L_P is given by

$$\frac{1}{L_p} = \frac{1}{L_l} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

$\therefore\ L_p = 1$

The correct option is (D)

127. $N_p = 140, N_s = 280, I_p = 4\,\text{A}, I_s = ?$

For a transformer $\dfrac{I_s}{I_p} = \dfrac{N_p}{N_s}$

$\Rightarrow \frac{I_s}{4} = \frac{140}{280} \Rightarrow I_s = 2\,A$

The correct option is (B)

128. Mutual conductance depends on the relative position and orientation of both the coils.

The correct option is (B)

129. $e = -\frac{\Delta\phi}{\Delta t} = \frac{-\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$

$\therefore |e| = L\frac{\Delta I}{\Delta t} \Rightarrow 8 = L \times \frac{4}{0.05}$

$\Rightarrow L = \frac{8 \times 0.05}{4} = 0.1\,H$

The correct option is (D)

130. When the capacitor is completely charged, the total energy in the LC circuit is with the capacitor and the energy is

$E = \frac{1}{2}\frac{Q^2}{C}$.

When half energy is with the capacitor in the form of electric field between the plates of the capacitor we get,

$\frac{E}{2} = \frac{1}{2}\frac{Q^2}{C}$, where Q is the charge on one plate of the capacitor

$\therefore \frac{1}{2} \times \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q'^2}{C} \Rightarrow Q' = \frac{Q}{\sqrt{2}}$

The correct option is (C)

131. Laminated core provide less area of cross-section for the current to flow. Because of this, resistance of the core increases and current decreases there by decreasing the eddy current losses.

The correct option is (A)

132. DC ammeter measure average current in AC current average current is zero for complete cycle. Hence, reading will be zero.

The correct option is (A)

133. Since the phase difference between L and C is π,

net voltage difference across LC = 50 – 50 = 0

The correct option is (D)

134. $\frac{\Delta\phi}{\Delta t} = \frac{(W_2 - W_1)}{t}$

$R_{tol} = (R + 4R)\Omega = 5R\Omega$

$i = \frac{nd\phi}{R_{tol}dt} = \frac{-n(W_2 - W_1)}{5Rt}$

($\because W_2$ and W_1 are magnetic flux)

The correct option is (B)

135. $\phi = \vec{B}.\vec{A} : \phi = BA\cos\omega t$

$\varepsilon = -\frac{d\phi}{dt} = \omega BA\sin\omega t;\ i = \frac{\omega BA}{R}\sin\omega t$

$P_{inst} = i^2 R = \left(\frac{\omega BA}{R}\right)^2 \times R\sin^2\omega t$

$$P_{avg} = \frac{\int_0^T P_{inst} \times dt}{\int_0^T dt} = \frac{(\omega BA)^2 \int_0^T \sin^2\omega t dt}{\int_0^T dt} = \frac{1}{2}\frac{(\omega BA)^2}{R}$$

$\therefore P_{avg} = \frac{(\omega B\pi r^2)^2}{8R}\ \left[A = \frac{\pi r^2}{2}\right]$

The correct option is (B)

136. For resonant frequency to remain same, LC should be constant. LC = constant

$\Rightarrow LC = L' \times 2C \Rightarrow L' = \frac{L}{2}$

The correct option is (A)

137. $r = 1\text{m}\ \omega = 5\,\text{rad/s},\ B = 0.2 \times 10^{-4}\,\text{T}$

$\varepsilon = \frac{B\omega'}{2} = \frac{0.2 \times 10^{-4} \times 5 \times 1}{2} = 50\mu\text{V}$

The correct option is (B)

138. Relative velocity = $v + v = 2v$

$\therefore$ EMF = $B \cdot l(2v)$

The correct option is (C)

139. For maximum power, $X_L = X_C$, which yields

$C = \frac{1}{(2\pi n)^2 L} = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$

$\therefore C = 0.1 \times 10^{-5}\,F = 1\mu\text{F}$

The correct option is (D)

140. Phase difference for R – L circuit lies between $\left(0, \frac{\pi}{2}\right)$

The correct option is (A)

141. Power factor $= \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

The correct option is (B)

142. The charging of inductance given by,

$t = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

$\frac{i_0}{2} = i_0\left(1 - e^{-\frac{Rt}{L}}\right) \Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2}$

Taking log on both the sides

$-\frac{Rt}{L} = \log 1 - \log 2$

$\Rightarrow t = \frac{L}{R}\log 2 = \frac{300 \times 10^{-3}}{2} \times 0.69$

$\Rightarrow t = 0.1\,\text{s}$

The correct option is (A)

143. Mutual inductance $= \frac{\phi}{I} = \frac{BA}{I}$

$[\text{Henry}] = \frac{[MT^{-1}Q^{-1}L^2]}{[QT^{-1}]} = ML^2Q^{-2}$

The correct option is (B)

144. Across resistor,

$$I=\frac{V}{R}=\frac{100}{1000}=0.1\,\text{A}$$

At resonance,

$$X_L=X_C=\frac{1}{\omega C}=\frac{1}{200\times 2\times 10^{-6}}=2500$$

Voltage across L is

$$I\,X_L=0.1\times 2500=250\,\text{V}$$

The correct option is (C)

145. $e=-\frac{d\phi}{dt}=-\frac{d(N\vec{B}.\vec{A})}{dt}$

$$=-N\frac{d}{dt}(BA\cos\omega t)=NBA\omega\sin\omega t$$

$$\Rightarrow e_{\max}=NBA\omega$$

The correct option is (D)

146. $\phi=10t^2-50t+250$

$$e=-\frac{d\phi}{dt}=-(20t-50)$$

$$e_{t=3}=-10\,\text{V}$$

The correct option is (B)

147. Initially, when steady state is achieved,

$$i=\frac{E}{R}$$

Let E be short circuited at $t=0$. Then

At $t=0$, $i_0=\frac{E}{R}$

Let during decay of current at any time the current flowing be $-L\frac{di}{dt}-iR=0$

$$\Rightarrow\frac{di}{i}=-\frac{R}{L}dt\Rightarrow\int_{i_0}^{t}\frac{di}{i}=\int_0^t-\frac{R}{L}dt$$

$$\Rightarrow\log_e\frac{i}{i_0}=-\frac{R}{L}t\Rightarrow i=i_0e^{-\frac{R}{L}t}$$

$$\Rightarrow i=\frac{E}{R}e^{-\frac{R}{L}t}=\frac{100}{100}e^{\frac{-100\times10^{-3}}{100\times10^{-3}}}=\frac{1}{e}$$

The correct option is (A)

148. We know that power consumed in AC circuit is given by $P=E_{\text{rms}}\cdot I_{\text{rms}}\cos\phi$

Here, $E=E_0\sin\omega t$

$$I=I_0\sin\left(\omega t-\frac{\pi}{2}\right)$$

which implies that the phase difference,

$$\phi=\frac{\pi}{2}$$

$$\therefore\; P=E_{\text{rms}}\cdot I_{\text{rms}}\cdot\cos\frac{\pi}{2}=0$$

The correct option is (C)

149. $I=I_0\left(1-e^{-\frac{R}{L}t}\right)$

(When current is in growth in LR circuit)

$$=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)=\frac{5}{5}\left(1-e^{-\frac{5}{10}\times 2}\right)$$

$$=\left(1-e^{-1}\right)$$

The correct option is (A)

150. $M=\frac{\mu_0N_lN_2A}{\ell}=\frac{4\pi\times10^{-7}\times300\times400\times100\times10^{-4}}{0.2}$

$$=2.4\pi\times10^{-4}\,\text{H}$$

The correct option is (D)

151. Growth in current in LR2 branch when switch is closed is given by

$$i=\frac{E}{R_2}[1-e^{-R_2t/L}]\Rightarrow\frac{di}{dt}=\frac{E}{R_2}\cdot\frac{R_2}{L}.e^{-R_2t/L}=\frac{E}{L}e^{-\frac{R_2t}{L}}$$

Hence, potential drop across

$$L=\left(\frac{E}{L}e^{-R_2t/L}\right)L=Ee^{-R_2t/L}=12e^{-\frac{2t}{400\times10^{-3}}}$$

The correct option is (C)

152. Due to the movement of resistor R, an EMF equal to Blv will be induced in it as shown in figure clearly,

$$I=I_1+I_2$$

Also, $I_1=I_2$

solving the circuit,

we get

$$I_1=I_2=\frac{Blv}{3R}$$

and $I=2I_1=\frac{2Blv}{3R}$

The correct option is (B)

153. The correct option is (D)

154. $B=\frac{\mu_0niR^2}{2(R^2+x^2)^{3/2}}$

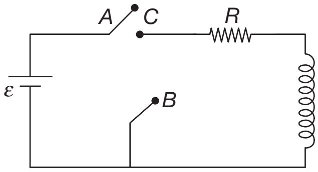
$$\phi=\vec{B}\cdot\vec{A}=\frac{\mu_0niR^2}{2(R^2+x^2)^{3/2}}\cdot\pi r^2$$

$$=\frac{4\pi\times10^{-7}}{2}\times\frac{2\times(0.20)^2}{\left(\frac{20^2+15^2}{100^2}\right)^{3/2}}\times\pi\times(0.3\times10^{-2})^2$$

$$=\frac{4\times\pi^2\times10^{-2}\times0.15^2\times9\times10^{-6}\times10^3}{125}$$

$$=9.1\times10^{-11}\text{ Wb}$$

The correct option is (D)



From KVL

$V_R + V_L = 0$

$\therefore \quad \frac{V_R}{V_L} = -1$

(Note: Ratio of $\frac{V_R}{V_L} = 1$ is also possible, if we consider only magnitude.)

The correct option is (C)

156. $i = i_0 e^{-Rt/L}$

$= \frac{15}{0.15\times10^3} e^{-\frac{0.15\times10^3\times10^{-3}}{0.03}}$

$= \frac{1}{10} e^{-5}$

$= \frac{1}{10\times150} = 0.67\,\text{mA}$

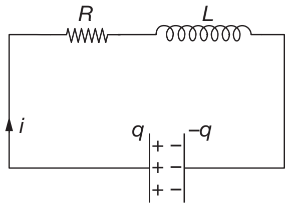
The correct option is (C)

157. $\frac{q}{c} - iR - L\frac{di}{dt} = 0$

$\frac{q}{c} + R\cdot\left(\frac{dq}{dt}\right) + L\left(\frac{d^2q}{dt^2}\right) = 0$

$\left(\frac{d^2q}{dt^2}\right) + \left(\frac{dq}{dt}\right)\frac{R}{L} + \frac{q}{LC} = 0$

$\Rightarrow Q_{\max} \propto e^{-\left(\frac{R}{2L}\right)t}$



The correct option is (D)

158. $V^2 = V_L^2 + V_R^2$

$220^2 = V_L^2 + 80^2$

$\Rightarrow \quad V_L = 10\sqrt{420}$

$= \text{I}X_L = 10\,\omega L = 10\times 2\pi \times L$

$\Rightarrow \quad L = 0.065\,H$

The correct option is (C)