CHAPTER 15

Magnetism and Magnetic Effect of Current

Chapter Highlights

Biot – Savart law and its application to current carrying circular loop. Ampere's law and its applications to infinitely long current carrying straight wire and solenoid. Force on a moving charge in uniform magnetic and electric fields. Cyclotron. Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field; Moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter. Current loop as a magnetic dipole and its magnetic dipole moment. Bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements. Para-, dia- and ferro- magnetic substances. Magnetic susceptibility and permeability, Hysteresis, Electromagnets and permanent magnets.

MAGNET

Two bodies, in spite of being neutral (showing no electric interaction), may attract/repel strongly if they have a special property. This property is known as magnetism. This force is called magnetic force. Those bodies are called magnets. Later on, we will see that it is due to circulating currents inside the atoms. Magnets are found in different shapes but for many experimental purposes, a bar magnet is frequently used. When a bar magnet is suspended at its middle, as shown, and it is free to rotate in the horizontal plane, it always comes to equilibrium in a fixed direction.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason, the end A, which points towards north direction is called 'North Pole' and the other end which points towards south direction is called 'South Pole'. They can be marked as 'N' and 'S' on the magnet. This property can be used to determine the north or south direction anywhere on the earth and indirectly east and west also if they are not known by other method (like rising and setting of the sun). This method is used by navigators of ships and aeroplanes. The directions are as shown in the Fig. 15.1. All directions, E, W, N, S, are in the horizontal plane.

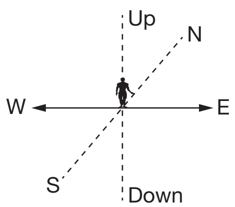
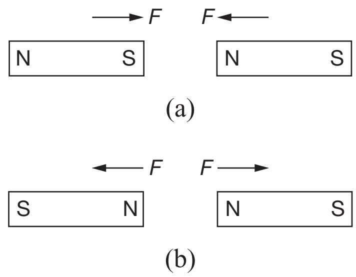


Fig. 15.1

The magnet rotates due to the earths magnetic field about which we will discuss later in this chapter.

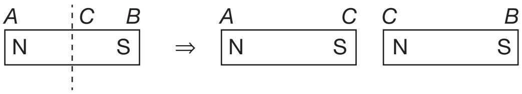
Pole Strength Magnetic Dipole and Magnetic Dipole Moment

A magnet always has two poles 'N' and 'S'; the like poles of two magnets repel each other and the unlike poles of two magnets attract each other, and they form action reaction pair.



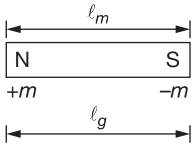
The poles of the same magnet do not come to meet each other due to attraction. They are maintained; we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.

∴ they are



Known as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their 'Pole Strength' $+m$ and $-m$, respectively (similar to charges $+q$ and $-q$ in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole and also of the magnet).

A magnet can be treated as a dipole since it always has two opposite poles (as in electric dipole where we have two opposite charges $-q$ and $+q$). It is called 'Magnetic Dipole' and it has a 'Magnetic Dipole Moment'. It is represented by $\vec{M}$. It is a vector quantity. Its direction is from $-m$ to $+m$ that means from 'S' to 'N')



$M = m \cdot \ell_m$ here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end-to-end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes, we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].

The units of m and M will be mentioned afterwards where you can remember and understand.

Magnetic Field and Strength of Magnetic Field

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called 'Magnetic Field' and that force is called 'Magnetic Force'. This field is qualitatively represented by 'Strength of Magnetic Field' or 'Magnetic Induction' or 'Magnetic Flux Density'. It is represented by $\vec{B}$. It is a vector quantity.

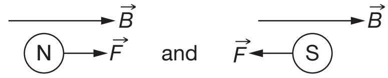
Definition of $\vec{B}$: The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \dfrac{\vec{F}}{m}$

Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

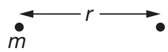
SI unit of $\vec{B}$ is Tesla or Wb/m^2 (abbreviated as T and Wb/m^2).

We can also write $\vec{F} = m\vec{B}$. Accordingly, direction of +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$.



The field generated by sources does not depend on the test pole (for it has any value and any sign).

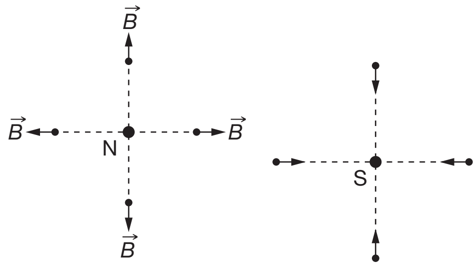
1. $\vec{B}$ due to various sources
   (i) Due to a single pole
   (Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}.$$

This is magnitude

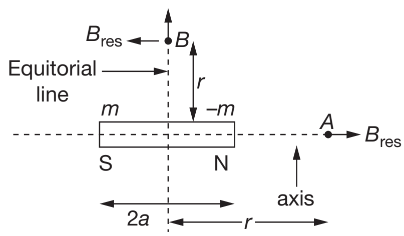
Direction of B due to north pole and due to south poles are as shown



in vector form $\vec{B} = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{r^3}\vec{r}$

Here m is with sign and $\vec{r}$ = position vector of the test point with respect to the pole.

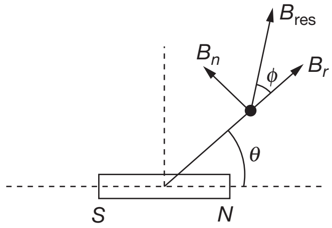
2. **Due to a bar magnet**
   (Same as the case of electric dipole in electrostatics). Independent case never found. Always 'N' and 'S' exist together as magnet.



at A (on the axis) = $\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{\vec{M}}{r^3}$ for $a << r$

at B (on the equatorial) = $-\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{\vec{M}}{r^3}$ for $a << r$

At general point,



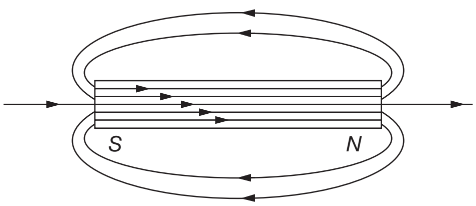
$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$$

$$B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

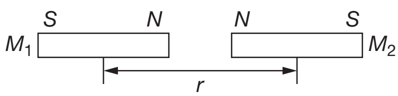
$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$

Magnetic Lines of Force of a Bar Magnet



SOLVED EXAMPLES

1. Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.

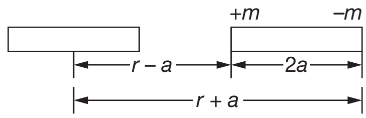


Solution:

To find the magnetic force, we will use the formula of B due to a magnet. We will also assume m and $-m$ as pole strengths of 'N' and 'S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at $+m$ and $-m$, respectively. They experience magnetic forces F_1 and F_2 as shown.

$$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m$$

and $$F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$$



$$\therefore\ F_{res} = F_1 - F_2$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)M_1 m\left[\left(\frac{1}{(r-a)^3}\right)-\left(\frac{1}{(r+a)^3}\right)\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3}-\left(1+\frac{a}{r}\right)^{-3}\right]$$

By using acceleration, binomial expansion, and neglecting terms of high power we get,

$$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r}$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4}$$

$$= 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$$

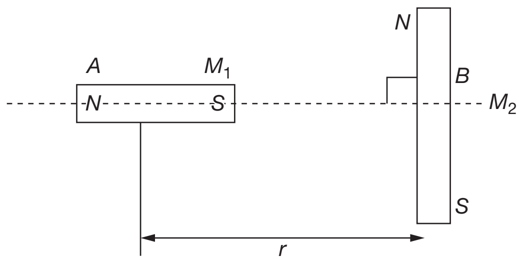
Direction of F_{res} is towards right.

Alternative method:

$$B = \frac{\mu_0}{4\pi}.\frac{2M_1}{r^3} \Rightarrow \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$$

$$F = -M_2\times\frac{dB}{dr} \Rightarrow F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1M_2}{r^4}$$

2. Two short magnets A and B of magnetic dipole moments M_1 and M_2, respectively are placed as shown. The axis of A and the equatorial line of B are the same. Find the magnetic force on one magnet due to the other.



Solution:

$$F = 3\left(\frac{\mu_0}{4\pi}\right)\frac{M_2M_1}{r^4}$$ upwards on M_1 downwards on M_2

3. A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Solution:

The pole strength is m = 120 CGS units = 12A-m.

Magnetic length is 2ℓ = 10 cm or ℓ = 0.05 m.

Distance from the magnet is d = 20 cm = 0.2 m.

The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2\,Md}{(d^2-\ell^2)^2}$$

$$= \frac{\mu_0}{4\pi}\frac{4\,m\ell d}{(d^2-\ell^2)^2}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12\text{A}-\text{m})\times(0.05\text{ m})\times(0.2\text{ m})}{[(0.2\text{ m})^2-(0.05\text{ m})^2]^2}$$

$$= 3.4\times10^{-5}\text{ T}.$$

4. Find the magnetic field due to a dipole of magnetic moment 1.2 A-m^2 at a point 1 m away from it in a direction making an angle of 60º with the dipole-axis.

Solution:

The magnitude of the field is

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{1.2\,\text{A}-\text{m}^2}{1\text{ m}^3}\sqrt{1+3\cos^2 60^\circ}$$

$$= 1.6\times10^{-7}\text{ T}.$$

The direction of the field makes an angle α with the radial line, where

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}.$$

5. A bar magnet has a pole strength of 3.6 A-m and magnetic length 8 cm. Find the magnetic field at

(A) a point on the axis at a distance of 6 cm from the centre towards the north pole and

(B) a point on the perpendicular bisector at the same distance.

Solution:

(A) 8.6×10^{-4} T (B) 7.7×10^{-5} T

6. Figure15.2 shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.

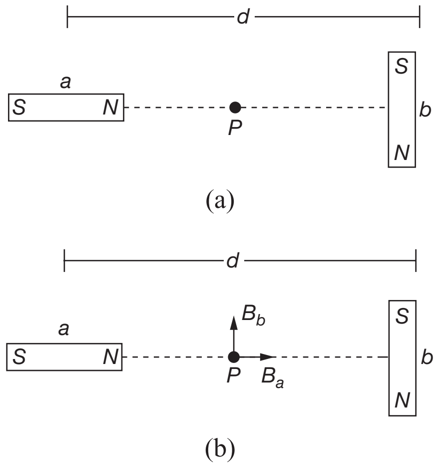


Fig. 15.2

Solution:

The point P is in end-on position for the dipole a and in broadside-on position for the dipole b. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that of b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in the figure. The resultant field at P is, therefore.

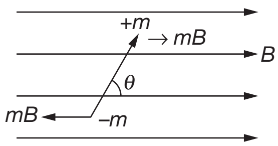
$$B = \sqrt{B_a^2 + B_b^2}$$

$$= \frac{\mu_0\,M}{4\pi(d/2)^3}\sqrt{1^2+2^2}$$

$$= \frac{2\sqrt{5}\mu_0 M}{\pi d^2}.$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

Magnet in an External Uniform Magnetic Field



(Same as case of electric dipole)

$$F_{res} = 0 \qquad \text{(for any angle)}$$

$$\tau = \text{MB}\sin\theta$$

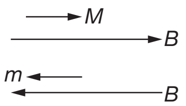
here θ is angle between $\vec{B}$ and $\vec{M}$

NOTE

- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.
- $\vec{\tau}$ is same about every point of the dipole and its potential energy is

$$U = -MB\cos\theta$$
$$= -\vec{M}\cdot\vec{B}$$

$\theta = 0^\circ$ is stable equilibrium



$\theta = \theta$ is unstable equilibrium

for small θ, the dipole performs SHM about $\theta = 0^\circ$ position

$$\tau = -\text{MB}\sin\theta;$$
$$I\alpha = -\text{MB}\sin\theta$$

for small θ, $\sin\theta\ \theta \simeq$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

Angular frequency of SHM,

$$\omega = \sqrt{\frac{MB}{I}}$$
$$= \frac{2\pi}{T}$$

$\Rightarrow$ $$T = 2\pi\sqrt{\frac{I}{MB}}$$

here $I = I_{cm}$ if the dipole is free to rotate

$= I_{hinge}$ if the dipole is hinged

SOLVED EXAMPLES

7. A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60º from the field.

Solution:

The work done by the external agent = change in potential energy

$$= (-MB\cos\theta_2) - (-MB\cos\theta_1)$$
$$= -MB(\cos 60^\circ - \cos 0^\circ)$$
$$= \frac{1}{2}MB$$
$$= \frac{1}{2} \times (1.0 \times 10^4 \text{ J/T})(4 \times 10^{-5}\text{ T}) = 0.2 \text{ J}$$

8. A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in Fig. 15.3. Find

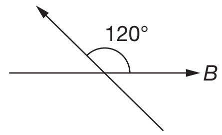


Fig. 15.3

(i) Its kinetic energy at $\theta = 90^\circ$

(ii) Its maximum kinetic energy during the motion.

(iii) Will it perform SHM, Oscillation, and periodic motion? What is its amplitude?

Solution:

(i) Apply energy conservation at $\theta = 120^\circ$ and $\theta = 90^\circ$

$$-MB\cos 120^\circ + 0 = -MB\cos 90^\circ + (\text{KE})$$
$$\text{KE} = \frac{MB}{2}$$

(ii) KE will be maximum, where PE is minimum. PE is minimum at $\theta = 0^\circ$. Now apply energy conservation between $\theta = 120^\circ$ and $\theta = 0^\circ$.

$$-MB\cos 120^\circ + 0 = -mB\cos 0^\circ + (\text{KE})_{max}$$
$$(\text{KE})_{max} = \frac{3}{2}\text{ MB.}$$

The KE is max at $\theta = 0^\circ$ and can also be proved by torque method. From $\theta = 120^\circ$ to $\theta = 0^\circ$, the torque always acts on the dipole in the same direction (here it is clockwise) so its KE keeps on increasing till $\theta = 0^\circ$. Beyond that, τ reverses its direction and then KE starts decreasing.

$\therefore$ $\theta = 0^\circ$ is the orientation of M to here the maximum K.E.

(iii) Since θ is not small.

$\therefore$ the motion is not SHM but it is oscillatory and periodic amplitude is 120º.

9. A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm, and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25 μT.

(A) Find the magnetic moment of the magnet.

(B) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Solution:

(A) The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2)$$

$$= \frac{100\times10^{-3}}{12}\,[(7\times10^{-2})^2 + (1\times10^{-2})^2]\ \text{kg/m}^2.$$

$$= \frac{25}{6}\times10^{-5}\ \text{kg/m}^2.$$

We have, $T = 2\pi\sqrt{\frac{I}{MB}}$

or $$M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2\times25\times10^{-5}\ \text{kg/m}^2}{6\times(25\times10^{-6}\ \text{T})\times\frac{\pi^2}{4}s^2}$$

$$= 27\ \text{A/m}^2.$$

(B) In this case, the moment of inertia becomes

$$I' = \frac{m'}{12}(L^2 + b^2),$$

where $b' = 0.5$ cm.
The time period would be

$$T' = \sqrt{\frac{I'}{MB}} \qquad (2)$$

Dividing by Equation (1),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2+b'^2)}}{\sqrt{\frac{m'}{12}(L^2+b^2)}}$$

$$= \frac{\sqrt{(7\,\text{cm})^2 + (0.5\,\text{cm})^2}}{\sqrt{(7\,\text{cm})^2 + (1.0\,\text{cm})^2}}$$

$$= 0.992$$

or, $$T' = \frac{0.992\times\pi}{2}\ \text{s}$$

$$= 0.496\pi\ \text{s}.$$

Magnet in an External Non-uniform Magnetic Field

No special formula are applied is such problems. Instead see the force on individual poles and calculate the resistant force torque on the dipole.

SOLVED EXAMPLE

10. Find the torque on M_1 due to M_2 in Q. 1

Solution:

Due to M_2, magnetic fields at 'S' and 'N' of M_1 are B_1 and B_2, respectively. The forces on $-m$ and $+m$ are F_1 and F_2 as shown in Fig. 15.4. The torque (about the centre of the dipole m_1) will be

$$= F_1 a + F_2 a = (F_1 + F_2)\,a$$

$$= \left[\left(\frac{\mu_0}{4\pi}\right)\frac{M_2}{(r-a)}m + \frac{\mu_0}{4\pi}\frac{M_2}{(r+a)}m\right]a$$

$$\cong \frac{\mu_0}{4\pi}M_2 m\left(\frac{1}{r^3}+\frac{1}{r^3}\right)a \qquad \because a << r$$

$$= \frac{\mu_0 M_2 m}{4\pi}\,\frac{2}{r^3}a = \frac{\mu_0 M_1 M_2}{4\pi r^3}$$

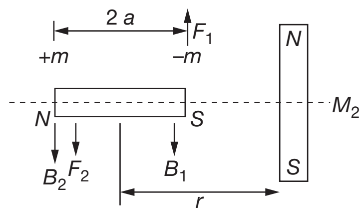


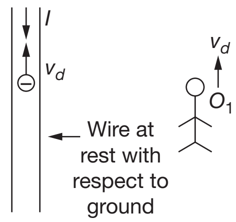
Fig. 15.4

MAGNETIC EFFECTS OF CURRENT (AND MOVING CHARGE)

It was observed by O ersted that a current carrying wire produces magnetic field nearly it. It can be tested by placing a magnet in the near by space, it will show some movement (deflection or rotation of displacement). This observation shows that current or moving charge produces magnetic field.

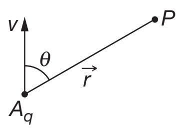
Frame Dependence of $\vec{B}$

1. The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at same velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observers O_3 then $\vec{B}$ due to that charge with respect to O_1 will be zero and with respect to O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different).



2. In a current carrying wire, electron moves in the opposite direction to that of the current and +ve ions (of the metal) are static with respect to the wire. Now if some observer (O_1) moves with velocity v_d in the direction of motion of the electrons, then electrons will have zero velocity and +ve ions will have velocity v_d in the downward direction with respect to O_1. The density (n) of +ve ions is same as the density of free electrons, and their charges are of the same magnitudes.
   So, with respect to O_1, electrons will produce zero magnetic field but +ve ions will produce +ve same $\vec{B}$ due to the current carrying wire does not depend on the reference frame (this is true for any velocity of the observer).
3. **$\vec{B}$ due to magnet**
   $\vec{B}$ produced by the magnet does not contain the term of velocity.
   So, we can say that the $\vec{B}$ due magnet does not depend on frame.

$\vec{B}$ Due to a Point Charge



A charge particle q has velocity v as shown in Fig. 15.4. It is at position A at some time. $\vec{r}$ is the position vector of point P with respect to position of the charge. Then $\vec{B}$ at P due to q is

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2};$$

here θ = angle between $\vec{v}$ and $\vec{r}$

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{v}\times\vec{r}}{r^3}; \text{ with sign}$$

$$\vec{B}\perp\vec{v} \text{ and also } \vec{B}\perp\vec{r}.$$

Direction of $\vec{B}$ will be found by using the rules of vector product.

Biot–Savart's law ($\vec{B}$ due to a wire)

It is an experimental law. A current i flows in a wire (may be straight or curved). Due to $d\ell$ length of the wire, the magnetic field at P is

$$dB \propto i d\ell$$

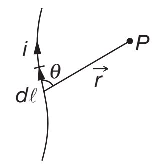
$$\propto \frac{1}{r^2}$$

$$\propto \sin\theta$$

$$\Rightarrow \quad dB \propto \frac{i d\ell \sin\theta}{r^2}$$

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i d\ell\sin\theta}{r^2}$$

$$\Rightarrow \quad \vec{dB} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\vec{d\ell}\times\vec{r}}{r^3}$$



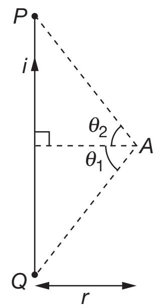
Here $\vec{r}$ = position vector of the test point with respect to $\vec{d\ell}$
θ = angle between $\vec{d\ell}$ and $\vec{r}$. The resultant $\vec{B} = \int\vec{dB}$

Using this fundamental formula, we can derive the expression of $\vec{B}$ due a long wire.

$\vec{B}$ Due to a Straight Wire

Due to a straight wire, PQ carrying a current i the $\vec{B}$ at A is given by the formula,

$$B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$



(Derivation can be seen in a standard text book like your school book or concept of physics of HCV part-II)

Direction: Due to every element of PQ $\vec{B}$ at A is directed inwards. So its resultant is also directed inwards. It is represented by (x)

The direction of $\vec{B}$ at various points is shown in Fig. 15.5 (a).

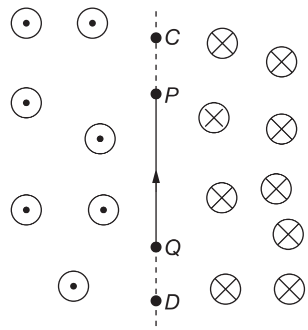


Fig. 15.5 (a)

At points C and D $\vec{B} = 0$ (think how).
For the case shown in Fig. 15.5 (b)

$$B \text{ at } A = \frac{\mu_0 i}{4\pi r}(\sin\theta_2 - \sin\theta_1) \otimes$$

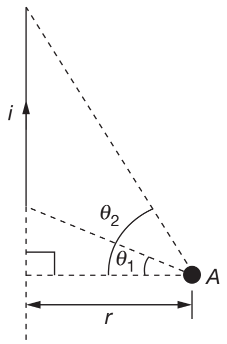
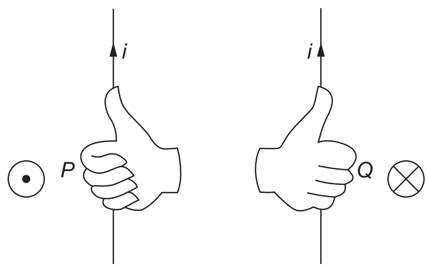


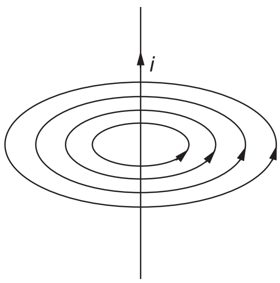
Fig. 15.5 (b)

Shortcut for Direction

The direction of the magnetic field at a point P due to a straight wire can be found by a slight variation in the right-hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through the point P, the direction of the fingers at P gives the direction of the magnetic field there.



We can draw magnetic field lines on the pattern of electric field lines. A tangent to a magnetic field line gives the direction of the magnetic field existing at that point. For a straight wire, the field lines are concentric circles with their centres on the wire and in plane perpendicular to the wire. There will be infinite number of such lines in the planes parallel to the above-mentioned plane.



SOLVED EXAMPLES

11. Find resultant magnetic field at C in Fig. 15.6.

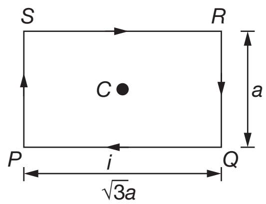


Fig. 15.6

Solution:

It is clear that B at C due all the wires is directed ⊗.
Also B at C due PQ and SR is same.

$\vec{B}$ also due to QR and PS is same

$$\therefore \quad B_{res} = 2(B_{PQ} + B_{SP})$$

$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}}(\sin 60^\circ + \sin 60^\circ)$$

$$B_{sp} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}}(\sin 30^\circ + \sin 30^\circ)$$

$$\Rightarrow \quad B_{res} = 2\left(\frac{\sqrt{3}\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

12. A loop in the shape of an equilateral triangle of side a carries a current I as shown in Fig. 15.7. Find out the magnetic field at the centre C of the triangle.

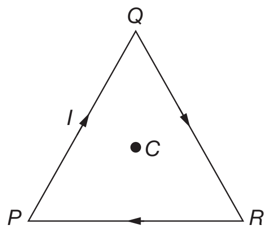


Fig. 15.7

Solution:

$$\frac{9\mu_0 i}{2\pi a}$$

13. Figure 15.8 shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points A and C.

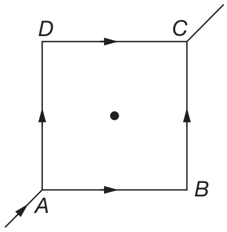


Fig. 15.8

Solution:

The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.

Special case

(i) If the wire is infinitely long, then the magnetic field at P (as shown in Fig. 15.9) is given by using $\theta_1 = \theta_2 = 90^\circ$ and the formula of B due to straight wire

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{I}{r}$$

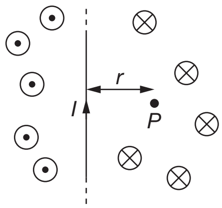


Fig. 15.9

The direction of magnetic field $\vec{B}$ at various point is as shown in Fig. 15.10. The magnetic lines of force will be concentric circles around the wire (as shown earlier).

(ii) If the wire is infinitely long but P is as shown in Fig. 15.10. The direction of $\vec{B}$ at various points is as shown in the figure. At P

$$B = \frac{\mu_0 I}{4\pi r}$$

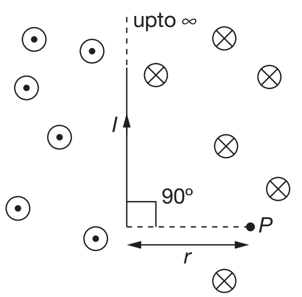


Fig. 15.10

14. As shown in Fig. 15.11, there are two parallel long wires (placed in the plane of paper) are carrying currents $2I$ and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find

(i) $\vec{B}$ at A, C, D

(ii) position of point on line A, C, D, where $\vec{B}$ is O.

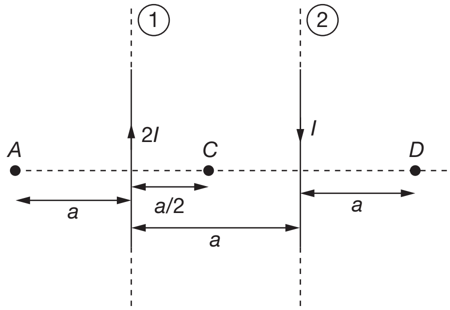


Fig. 15.11

Solution:

(i) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$, respectively. Then

at A $\vec{B}_1$ ⊙ is and $\vec{B}_2$ is ⊗

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$$\therefore \quad B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$$

at C $\quad \vec{B}_1$ is ⊗ and $\vec{B}_2$ also ⊗

$$\therefore \quad B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi \frac{a}{2}} + \frac{\mu_0 I}{2\pi \frac{a}{2}}$$

$$= \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes$$

at D $\vec{B}_1$ is ⊗ and $\vec{B}_2$ is ⊙ and both are equal in magnitude.

$$\therefore \quad B_{res} = 0$$

(ii) It is clear from the above solution that $B = 0$ at point D.

15. In Fig. 15.12, two long wires W_1 and W_2 each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find $\vec{B}$ at P and Q.

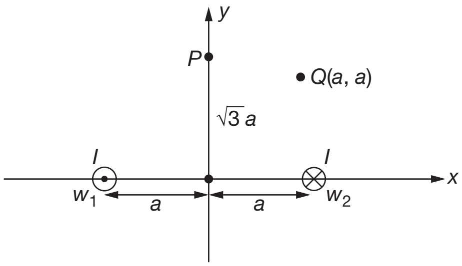
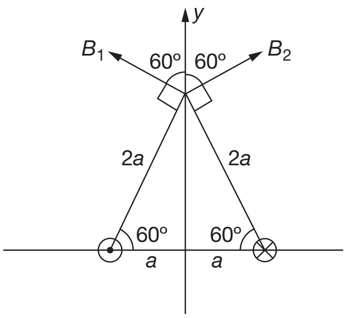


Fig. 15.12

Solution:

Let $\vec{B}$ due to W_1 be $\vec{B_1}$ and due to W_2 be $\vec{B_2}$.

By symmetry, $|\vec{B_1}| = |\vec{B_2}| = B$



$$B_p = 2\,B\cos 60^\circ = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

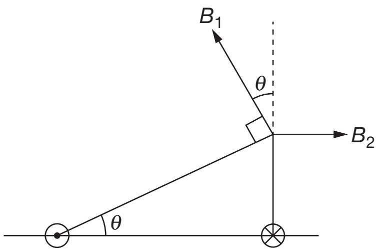
$$\therefore \quad \vec{B_p} = \frac{\mu_0 I}{4\pi a}\hat{j}$$

For θ $\quad B_1 = \dfrac{\mu_0 I}{2\pi\sqrt{5}a}$,

$$\Rightarrow \quad B_2 = \frac{\mu_0 I}{2\pi a}$$

$$\tan\theta = \frac{a}{2a} = \frac{1}{2}$$

$$\Rightarrow \quad \vec{B} = (B_1\cos\theta\,\hat{j}) + (B_2 - B_1\sin\theta)\,\hat{i}$$



$$\sin\theta = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{5\pi a}\,\hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a}\right)\hat{i}$$

$$\cos\theta = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \quad \vec{B} = \frac{2\mu_0 I}{5\pi a}\hat{i} + \frac{\mu_0 I}{5\pi a}\hat{j}$$

16. In Fig. 15.13, a large metal sheet of width w carries a current I (uniformly distributed in its width w). Find the magnetic field at point P which lies in the plane of the sheet.

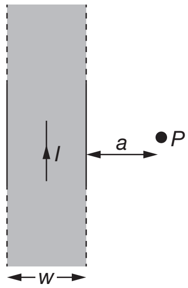
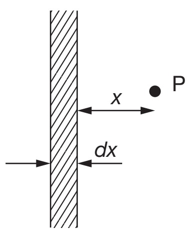


Fig. 15.13

Solution:

To find B at P the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance x from P and of width dx. Due to this, the magnetic field at P is 'dB'

$$dB = \frac{\mu_0\left(\dfrac{I}{w}dx\right)}{2\pi x} \otimes$$



due to each such wire, $\vec{B}$ will be directed inwards

$$\therefore \quad B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}$$

$$\int_{x=a}^{a+w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}\,.\ \ln\frac{a+w}{a}$$

17. Two long wires are kept along x and y axes and they carry currents I and J, respectively in +ve x and +ve y directions, respectively. Find $\vec{B}$ at a point (0, 0, d).

Solution:

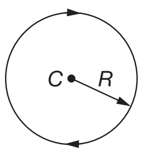
$$\frac{\mu_0 I}{2\pi d}(\hat{i} - \hat{j})$$

$\vec{B}$ *Due to Circular Loop*

1. **At centre:** Due to each $\vec{d\ell}$, element of the loop $\vec{B}$ at C is inwards (in this case).

   $\therefore \quad \vec{B_{res}}$ at C is $\otimes$.

$$B = \frac{\mu_0 NI}{2R}$$



N = Number of turns in the loop.

$= \frac{\ell}{2\pi R} \quad \ell$ = length of the loop.

N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} \text{ etc.}\right)$ or integer.

Direction of $\vec{B}$: The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field. (Fig. 15.14).

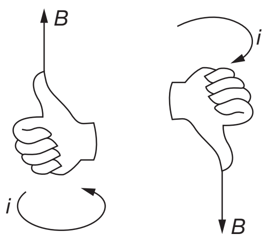
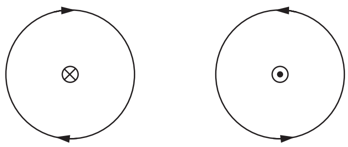
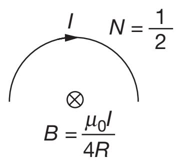


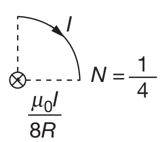
Fig. 15.14

Another way to find the direction is to look into the loop along its axis. If the current is in anti-clockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.



Semicircular and quarter of a circle:

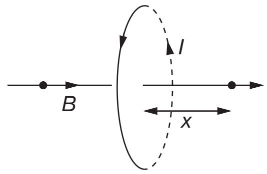




2. **On the axis of the loop:**

$$B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}$$

N = Number of turns (integer)



Direction can be obtained by right-hand thumb rule. Curl your fingers in the direction of the current then the direction of the thumb points to the direction of $\vec{B}$ at the points on the axis.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in Fig. 15.15 the magnetic field lines due to a circular current which will give some idea of the field.

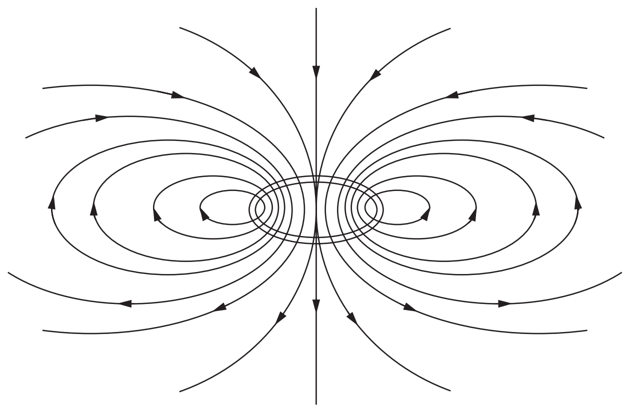
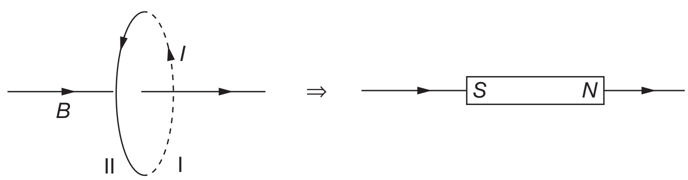


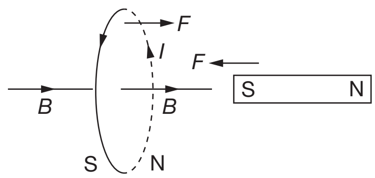
Fig. 15.15

A Loop as a Magnet

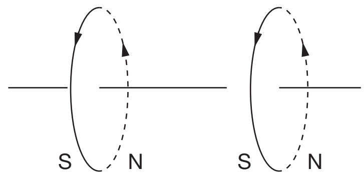
The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



The side I (the side from which the $\vec{B}$ emerges out) of the loop acts as 'North Pole' and side II (the side in which the $\vec{B}$ enters) acts as the 'South Pole'. It can be verified by studying force on one loop due to a magnet or a loop.



The loop and the magnet attract each other



The two loops attract each other.

Mathematically,

$$B_{\text{axis}} = \frac{\mu_0 NIR^2}{2(R^2+x^2)^{3/2}} \cong \frac{\mu_0 NIR^2}{2x^3} \quad \text{for } x >> R$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IN\pi R^2}{x^3}\right)$$

it is similar to B_{axis} due to magnet $= 2\left(\frac{\mu_0}{4\pi}\right)\frac{m}{x^3}$

Magnetic dipole moment of the loop

$$M = IN\pi R^2$$

$M = INA$ for any other shaped loop.

Unit of M is Amp. m^2.

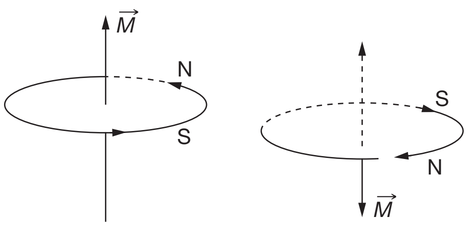
Unit of m (pole strength) = Amp. m

$\{\because$ in magnet $M = m\ell\}$

$$\vec{M} = IN\vec{A},$$

$\vec{A}$ = unit normal vector for the loop.

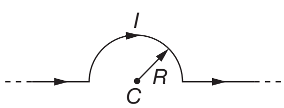
To be determined by right-hand rule, which is also used to determine direction of $\vec{B}$ on the axis. It is also from 'S' side to 'N' side of the loop.



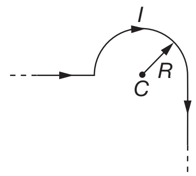
SOLVED EXAMPLE

18. Find B at centre C in the following cases:

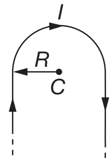
(i)



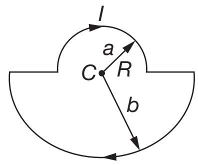
(ii)



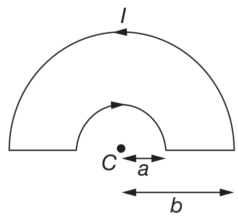
(iii)



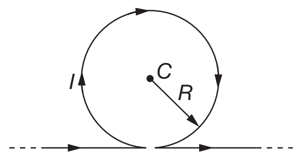
(iv)



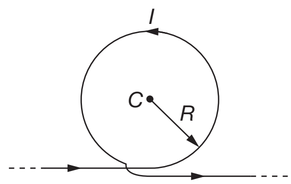
(v)



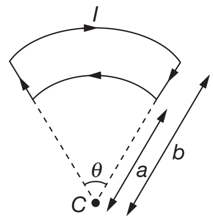
(vi)



(vii)



(viii)



Solution:

(i) $\frac{\mu_0 I}{4R} \otimes$

(ii) $\frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right) \otimes$

(iii) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right) \otimes$

(iv) $\frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right) \otimes$

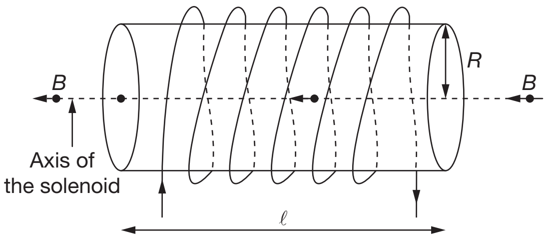
(v) $\frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right) \otimes$

(vi) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right) \otimes$

(vii) $\dfrac{\mu_0 I}{2R}\left(1+\dfrac{1}{\pi}\right)\odot$ (viii) $\dfrac{\mu_0 I\theta}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)\odot$

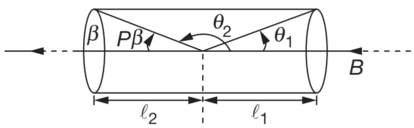
Solenoid

1. Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (It may be a hollow cylinder or it may be a solid cylinder.)



2. The winding of the wire in uniform direction of the magnetic field is same at all points of the axis.
3. $\vec{B}$ on axis (turns should be very close to each others).

$$B=\frac{\mu_0 ni}{2}\ (\cos\theta_1-\cos\theta_2)$$



where n : number of turns per unit length.

$$\cos\theta_1=\frac{\ell_1}{\sqrt{\ell_1^2+R^2}}$$

$$\cos\beta=\frac{\ell_2}{\sqrt{\ell_2^2+R^2}}=-\cos\theta_2$$

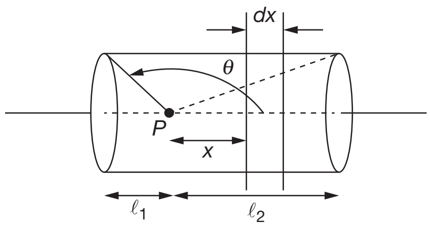
$$B=\frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R^2}}+\frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right]$$

$$=\frac{\mu_0 ni}{2}\ (\cos\theta_1+\cos\beta)$$

Derivation

Take an element of width dx at a distance x from point P. Point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element $dn = ndx$, where n: number of turns per unit length.

$$dB=\frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}\ (ndx)$$



$$B=\int dB=\int_{-\ell_1}^{\ell_2}\frac{\mu_0 i R^2\, ndx}{2(R^2+x^2)^{3/2}}$$

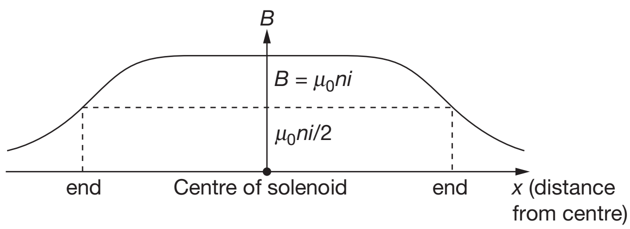
$$=\frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R^2}}+\frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right]$$

$$=\frac{\mu_0 ni}{2}[\cos\theta_1-\cos\theta_2]$$

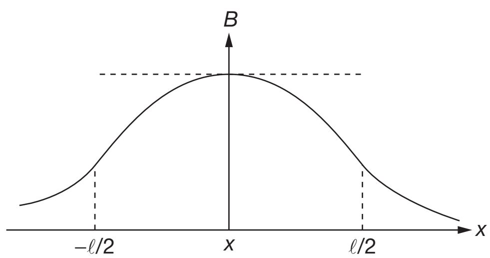
4. **For 'Ideal Solenoid'**
   ***Inside** (at the mid point)
   $\ell >> R$ or length is infinite
   $\theta_1 \to 0$
   $\theta_2 \to \pi$
   $B=\dfrac{\mu_0 ni}{2}\ [1-(-1)]$
   $B=\mu_0 ni$
   If material of the solid cylinder has relative permeability 'μ_r', then $B=\mu_0\mu_r ni$

   At the ends, $B=\dfrac{\mu_0 ni}{2}$

5. Comparison between ideal and real solenoid:
   (i) Ideal solenoid



   (ii) Real solenoid



SOLVED EXAMPLES

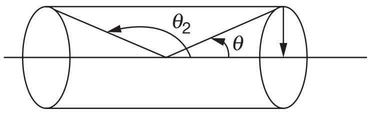
19. A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0×10^{-3} A. Find the magnetic field on the axis at the middle and at the ends of the solenoid (Given $\mu_0 = 4\pi \times 10^{-7}\ \frac{V-s}{A-m}$).

Solution:

$$B = \frac{1}{2}\mu_0 ni\,[\cos\theta_1 - \cos\theta_2]$$

$$\Rightarrow \quad n = \frac{1000}{0.4} = 2500 \text{ per meter}$$

$$i = 5 \times 10^{-3}\text{ A.}$$



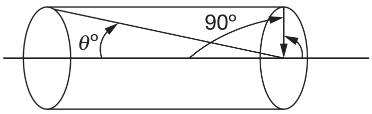
(i) $\cos\theta_1 = \dfrac{0.2}{\sqrt{(0.3)^2 + (0.2)^2}} = \dfrac{0.2}{\sqrt{0.13}}$

$$\cos\theta_2 = \frac{-0.2}{\sqrt{0.13}}$$

$$\Rightarrow B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3}\ \frac{2 \times 0.2}{\sqrt{0.13}}$$

$$= \frac{\pi \times 10^{-5}}{\sqrt{13}}\text{T}$$

(ii) At the end



$$\cos\theta_1 = \frac{0.4}{\sqrt{(0.3)^2 + (0.4)^2}} = 0.8$$

$$\cos\theta_2 = \cos 90° = 0$$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$$

$$\Rightarrow \quad B = 2\pi \times 10^{-6}\text{ Wb/m}^2$$

20. A thin solenoid of length 0.4 m and having 500 turns of wire carries a current 1A; then find the magnetic field on the axis inside the solenoid.

Solution:

$$5\pi \times 10^{-4}\text{ T}$$

Ampere's Circuital Law

The line integral $\oint \vec{B}\cdot\vec{d\ell}$ on a closed curve of any shape is equal to μ_0 (permeability of free space) times the net current I through the area bounded by the curve.

$$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I$$

NOTE

- Line integral is independent of the shape of path and position of wire with in it.
- The statement $\oint \vec{B}\cdot\vec{d\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is passing through the path.
- Sign of current: The current due to which $\vec{B}$ is produced in the same sense as $\vec{d\ell}$ (i.e., $\vec{B}\cdot\vec{d\ell}$ positive will be taken positive and the current which produces $\vec{B}$ in the sense opposite to $\vec{d\ell}$ will be negative).

SOLVED EXAMPLE

21. Find the values of $\oint \vec{B}\cdot\vec{d\ell}$ for the loops L_1, L_2, L_3 in Fig. 15.16 as shown.

The sense of $\vec{d\ell}$ is mentioned in the Fig. 15.16.

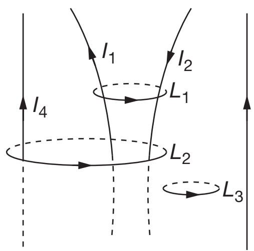


Fig. 15.16

Solution:

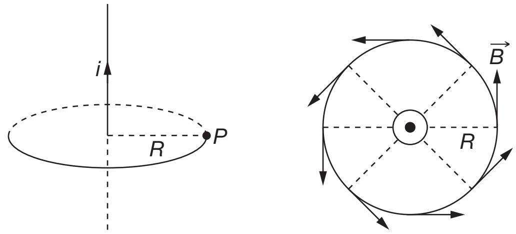
for $\quad L_1 \oint \vec{B}\cdot\vec{d\ell} = \mu_0(I_1 - I_2)$

here I_1 is taken positive because magnetic lines of force produced by I_1 is anti-clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{d\ell}$ is anti-clockwise as seen from top.

for $L_2 : \oint \vec{B}\cdot\vec{d\ell} = \mu_0\,(I_1 - I_2 + I_4)$

for $L_3 : \oint \vec{B}\cdot\vec{d\ell} = 0$

Uses: 2.4.1 To find out magnetic field due to infinite current carrying wire



By B.S.L., $\vec{B}$ will have circular lines. $\vec{d\ell}$ is also taken tangent to the circle.

$$\oint \vec{B}\cdot\vec{d\ell} = \oint B.d\ell$$

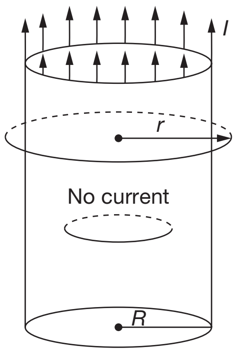
$\because\ \theta = 0^\circ$ so B $\oint d\ell = B\,2\pi R$ ($\because\ B$ = constant)

Now by amperes law:

$$B\,2\pi R = \mu_0 I$$

$$\therefore \qquad B = \frac{\mu_0 i}{2\pi R}.$$

Hollow Current Carrying Infinitely Long Cylinder: (I is Uniformly Distributed on the whole Circumference)



1. For $r > R$

By symmetry, the amperian loop is a circle.

$$\oint \vec{B}\cdot\vec{d\ell} = \oint B\,d\ell \quad \because\ \theta = 0$$

$$= B\int_0^{2\pi r} d\ell \quad \because\ B = \text{const.}$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r}$$

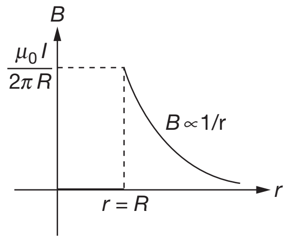
2. $r < R$

$$= \oint \vec{B}\cdot\vec{d\ell} = \oint B\,d\ell$$

$$= B(2\pi r) = 0$$

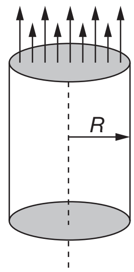
$$\Rightarrow \qquad B_{in} = 0$$

Graph:



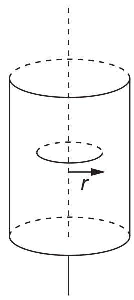
Solid Infinite Current Carrying Cylinder

Assume current is uniformly distributed on the whole cross-sectional area



Current density $\qquad J = \dfrac{I}{\pi R^2}$

Case (I): $\qquad r \le R$



Take an amperian loop inside the cylinder. By symmetry, it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.

$$\oint \vec{B}\cdot\vec{d\ell} = \oint B\cdot d\ell = B\oint d\ell$$

$$= B\cdot 2\pi r = \mu_0 \frac{I}{\pi R^2}\pi r^2$$

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2}$$

$$\Rightarrow \qquad \vec{B} = \frac{\mu_0 \vec{J}\times\vec{r}}{2}$$

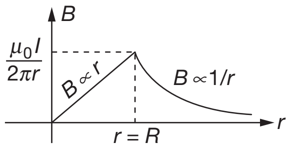
Case (II): $r \geq R \oint \vec{B}\cdot d\vec{\ell} = \oint B\, d\ell$

$$= B\oint d\ell = B\cdot(2\pi r) = \mu_0 \cdot I$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r}$$

also $$\vec{B}\,\frac{\mu_0 I}{2\pi r}(\hat{J}\times\hat{r}) = \frac{\mu_0 J\pi R^2}{2\pi r}$$

$$\vec{B} = \frac{\mu_0 R^2}{2r^2}(\vec{J}\times\vec{r})$$



SOLVED EXAMPLES

22. Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c, respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where (a) $x < a$, (b) $a < x < b$, (c) $b < x < c$, and (d) $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Solution:

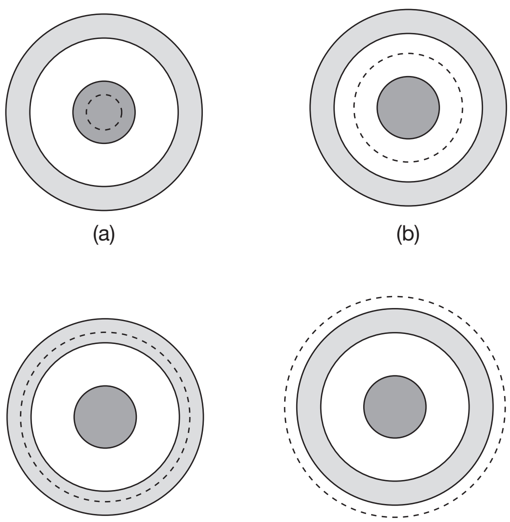


Fig. 15.17

A cross-section of the cable is shown in Fig 15.17. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c, and d of Fig 15.17 correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B}.d\vec{\ell} = B2\pi x$$

in each of the four parts of Fig. 15.17.

(A) The current enclosed within the circle in part b is i_0 so that

$$\frac{i_0}{\pi a^2}\,.\,\pi x^2 = \frac{i_0}{a^2}x^2.$$

Ampere's law

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i \text{ gives}$$

$$B\cdot 2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or,} \quad B = \frac{\mu_0 i_0 x}{2\pi a^2}.$$

The direction will be along the tangent to the circle.

(B) The current enclosed within the circle in part b is i_0 so that

$$B\cdot 2\pi x = \mu_0\, i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{2\pi x}.$$

(C) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part (c) of the Fig 15.17 is $\pi x^2 - \pi b^2$.

Thus, the current through this part is $\frac{i_0(x^2-b^2)}{(c^2-b^2)}$.

This is in the opposite direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

$$i_{net} = i_0 - \frac{i_0(x^2-b^2)}{c^2-b^2} = \frac{i_0(c^2-x^2)}{c^2-b^2}.$$

From ampere's law,

$$B\cdot 2\pi x = \frac{\mu_0 i_0(c^2-x^2)}{c^2-b^2}$$

or, $$B = \frac{\mu_0 i_0(c^2-x^2)}{2\pi x(c^2-b^2)}$$

(D) The net current enclosed by the circle in part (d) of the Fig. 15.17 is zero and hence

$$B\,2\pi x = 0 \quad \text{or,} \quad B = 0.$$

23. Figure 15.18 shows a cross-section of a large metal sheet carrying an electric current along its surface.

The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.

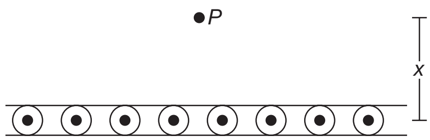


Fig. 15.18

Solution:

Consider two strips A and C of the sheet situated symmetrically on the two sides of P (Fig. 15.19). The magnetic field at P due to the strip A is B_0 perpendicular to AP and that due to the strip C is B_C perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.

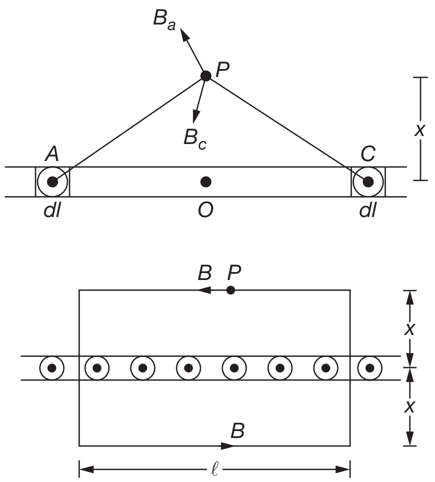


Fig. 15.19

The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying ampere's law to the rectangle shown in Fig. 15.19.

$$2B\ell = \mu_0 K\ell \quad \text{or,} \quad B = \frac{1}{2}\mu_0 K.$$

Note that it is independent of x.

24. Three identical long solenoids P, Q, and R are connected to each other as shown in Fig. 15.20. If the magnetic field at the centre of P is 2.0 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.

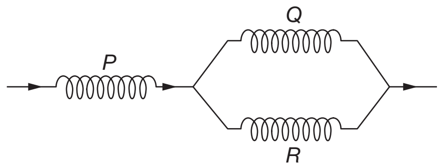


Fig. 15.20

Solution:

As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence, the field in Q will be equal to the field in R and will be half the field in P, i.e., 1.0 T.

MAGNETIC FORCE ON MOVING CHARGE

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula :

$$\vec{F} = q(\vec{v} \times \vec{B}) \text{ Put } q \text{ with sign.}$$

$\vec{v}$: Instantaneous velocity
$\vec{B}$: Magnetic field at that point

SOLVED EXAMPLES

25. A charged particle of mass 5 mg and charge $q = +2\mu C$ has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field. $\vec{B} = 3\hat{j} - 2\hat{k} \cdot \vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2, respectively.

Solution:

$$\vec{F} = q\vec{v} \times \vec{B} = 2 \times 10^{-6} (2\hat{i} - 3\hat{j} \times 4\hat{k}) \times (3\hat{j} - 2\hat{k})$$

$$= 2 \times 10^{-6} [-6\ \hat{i} + 4\ \hat{j} + 6\hat{k}] \text{ N}$$

$$\text{By Newton's law, } \vec{a} = \frac{\vec{F}}{m} = \frac{2\times10^{-6}}{5\times10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k})$$

$$= 0.8\,(-3\hat{i} + 2\hat{j} + 3\hat{k}) \text{ m/s}^2$$

26. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

Solution:

$\because \quad \vec{F} \perp \vec{B}$

$\therefore \quad \vec{a} \perp \vec{B}$

$\therefore \qquad \vec{a}\cdot\vec{B} = 0$

$\therefore \qquad (2\hat{i} + x\hat{j})\cdot(-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0$

$\Rightarrow \qquad -6 + 2x = 0 \quad \Rightarrow \quad x = 3.$

27. A charged particle of charge 2C is thrown vertically upwards with velocity 10 m/s. Find the magnetic force on this charge due to earth's magnetic field. Given vertical component of the earth = 3 μT and angle of dip = 37º.

Solution:

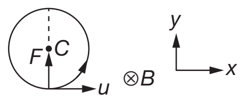
$$2 \times 10 \times 4 \times 10^{-6} = 8 \times 10^{-5} \text{ N towards west.}$$

28. A charged particle of charge 1C and mass 1 kg has initial velocity $\vec{v} = 2\hat{i} + 3\hat{j} - 3\hat{k}$ in a uniform magnetic field $\vec{B} = -4\hat{i} - 6\hat{j} + 6\hat{k}$. Find at $t = 2$s (A) velocity, (B) acceleration, (C) position vector of the particle.

Solution:
(A) $2\hat{i} + 3\hat{j} - 2\hat{k}$ (B) 0 (C) $4\hat{i} + 6\hat{j} - 6\hat{k}$

Motion of Charged Particles under the Effect of Magnetic Force

- Particle released if $v = 0$ then $f_m = 0$
  $\therefore$ particle will remain at rest
- $\vec{v} \parallel \vec{B}$ here $\theta = 0$ or $\theta = 180º$
  $\therefore F_m = 0 \therefore \vec{a} = 0 \therefore \vec{v}$ = constant
  $\therefore$ particle will move in a straight line with constant velocity
- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform



In this case, $\because$ B is in z direction so the magnetic force in z-direction will be zero $\left(\because \vec{F_m} \perp \vec{B}\right)$.

Now there is no initial velocity in z-direction.

$\therefore$ particle will always move in xy plane.

$\therefore$ velocity vector is always $\perp \vec{B}$ $\therefore$ $F_m = quB$ = constant

now $quB = \dfrac{mu^2}{R} \Rightarrow R = \dfrac{mu}{qB}$ = constant.

The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle.

$\therefore$ path of the particle is circular.

$$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$$

here p = linear momentum; k = kinetic energy

now $v = \omega R \Rightarrow \omega = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f$

Time period $T = 2\pi m/qB$

Frequency $f = qB/2\pi m$

SOLVED EXAMPLES

29. A proton (p), α-particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

Solution:

$$R = \frac{\sqrt{2mK}}{qB}$$

$$\therefore \quad R_p : R\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4\,mK}}{2qB} : \frac{\sqrt{2.2\,mK}}{qB}$$

$$= 1 : 1 : \sqrt{2}$$

$$T = \frac{2\pi m}{qB}$$

$$\therefore \quad T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB}$$

$$= 1 : 2 : 2$$

30. A positive charge particle of charge q and mass m enters into a uniform magnetic field with velocity v as shown in Fig. 15.21. There is no magnetic field to the left of PQ. Find

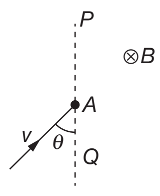


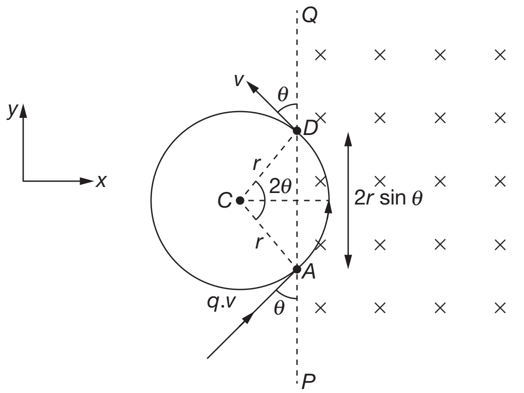
Fig. 15.21

(A) time spent.
(B) distance travelled in the magnetic field.
(C) impulse of magnetic force.

Solution:

The particle will move in the field as shown.

Angle subtended by the arc at the centre = 2θ



(A) Time spent by the charge in magnetic field

$$\omega t = \theta \Rightarrow \frac{qB}{m} \; t = \theta \Rightarrow t = \frac{m\theta}{qB}$$

(B) Distance travelled by the charge in magnetic field

$$= r(2\theta) = \frac{mv}{qB} \; . \, 2\theta$$

(C) Impulse = change in momentum of the charge

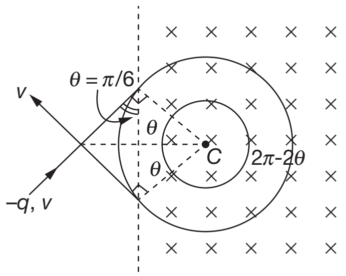
$= (-mv \sin \theta \, \hat{i} \; + mv \cos \theta \; \hat{j}\,) - (mv \sin \theta \; \hat{i} \; + mv \cos \theta \; \hat{j}\,) = -2mv \sin \theta \; \hat{i}$

31. Repeat above question if the charge is –ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

Solution:

(i) $2\pi - 2\theta = 2\pi - 2.\frac{\pi}{6} \; = 2\pi - \frac{\pi}{3} \; = \frac{5\pi}{3}$

$$= \omega t = \frac{qBt}{m} \; \Rightarrow t = \frac{5\pi m}{3qB}$$



(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

(iii) Impulse = charge in linear momentum

$= m(-v \sin \theta \; \hat{i} + v \cos \theta \, \hat{j}\,) - m(v \sin \theta \; \hat{i} + v \cos \theta \, \hat{j}\,)$

$= -2mv \sin \theta \; \hat{i} = -2mv \sin \frac{\pi}{6} \; \hat{i} \; = -mv\,\hat{i}$

32. P, α, and D are accelerated by the potential difference from rest and then sent to a magnetic field where they move in circular orbits. Neglecting interaction between them find the ratio of their time periods and ratio of their radii.

Solution:

(i) 1 : 2 : 2 (ii) $1 : \sqrt{2} : \sqrt{2}$

33. In Fig. 15.22 shown, the magnetic field on the left on PQ is zero and on the right of PQ is uniform. Find the time spent in the magnetic field.

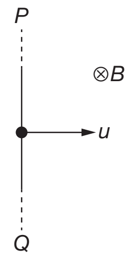
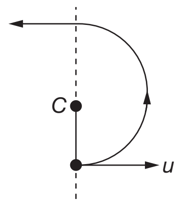


Fig. 15.22

Solution:

The path will be semicircular time spent $= \frac{T}{2} = \frac{\pi m}{qB}$



34. A uniform magnetic field of strength B exists in a region of width d. A particle of charge q and mass m is shot perpendicularly (as shown in Fig. 15.23) into the magnetic field. Find the time spend by the particle in the magnetic field if

(i) $d > \frac{mu}{qB}$ (ii) $d < \frac{mu}{qB}$

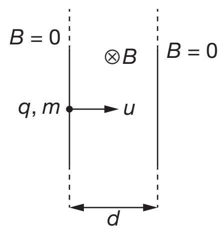
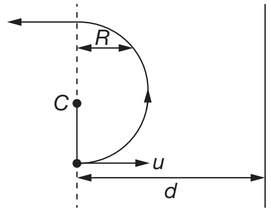


Fig. 15.23

Solution:

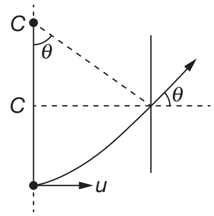
(i) $d > \frac{mu}{qB}$ means $d > R$

$\therefore \quad t = \frac{T}{2} = \frac{\pi m}{qB}$



(ii) $\sin\theta = \dfrac{d}{R}$

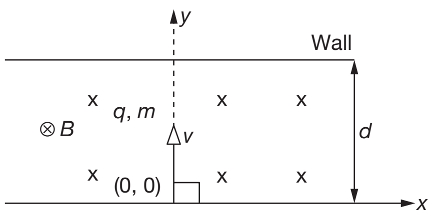
$$\theta = \sin^{-1}\left(\frac{d}{R}\right)$$



$$\omega t = \theta$$

$$\Rightarrow \qquad t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$$

35. What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.

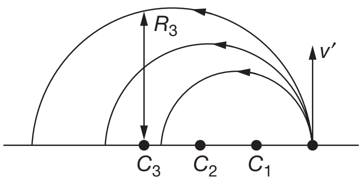


Solution:

(i) The path of the particle will be circular; larger the velocity, larger will be the radius.

For particle not to s strike $R < d$

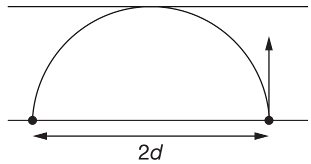
$$\therefore \qquad \frac{mv}{qB} < d \Rightarrow v < \frac{qBd}{m}.$$



(ii) for limiting case $v = \dfrac{qBd}{m}$

$$R = d$$

$\therefore$ coordinate $= (-2d, 0, 0)$

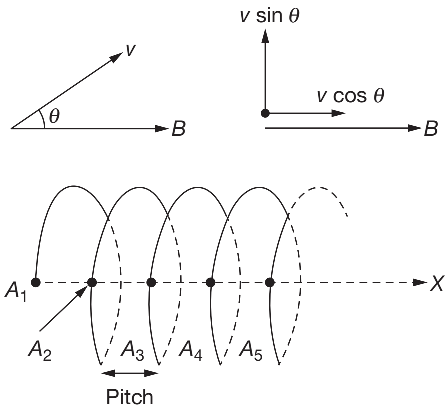


Helical Path

If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity into two components: $v_{\parallel}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The components $v_{\parallel}$ remains unchanged as the force $q\vec{v}\times\vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \dfrac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Complete Analysis

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.



The particle starts from point A_1.

It completes its first revolution at A_2 and second revolution at A_3 and so on. x-axis is the tangent to the helix points

$A_1, A_2, A_3,$ ....all are on the x-axis.

distance $A_1A_2 = A_3A_4 =$ .... $= v\cos\theta.\ T =$ pitch

where $T =$ Time period

Let the initial position of the particle be (0, 0, 0) and v sin$q$ in $+y$ direction. Then in x : $F_x = 0$, $a_x = 0$, $v_x =$ constant $= v\cos\theta$, $x = (v\cos\theta)\,t$

In y-z plane

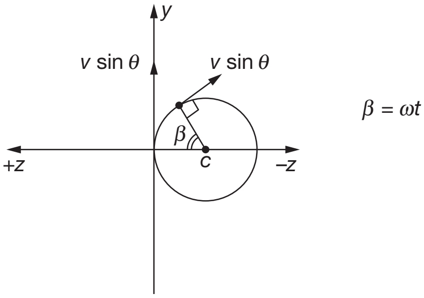


Fig. 15.24

From Fig. 15.24, it is clear that

$$y = R\sin\beta,\ v_y = v\sin\theta\cos\beta$$

$$z = -(R - R\cos\beta)$$

$$v_z = v\sin\theta\sin\beta$$

acceleration towards centre = $(v\sin\theta)^2/R = \omega^2 R$

$$\therefore \qquad a_y = -\,\omega^2 R\sin\beta,\ a_z = -\,\omega^2 R\cos\beta$$

At any time, the position vector of the particle
(or its displacement with respect to initial position)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad , x, y, z \text{ already found}$$

velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, v_x, v_y, v_z already found

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k},\ a_x, a_y, a_z \text{ already found}$$

Radius $\quad q(v\sin\theta)B = \dfrac{m(v\sin\theta)^2}{R}$

$$\Rightarrow \qquad R = \frac{mv\sin\theta}{qB}$$

$$\omega = \frac{v\sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f.$$

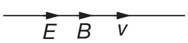
Charged Particle in $\vec{E}$ and $\vec{B}$

When a charged particle moves with velocity $\vec{V}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$, then net force experienced by it is given by following equation.

$$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$$

Combined force is known as Lorentz force.

$$\vec{E} \parallel \vec{B} \parallel \vec{v}$$



In above situation, particle passes undeviated but its velocity will change due to electric field. Magnetic force on it equal to 0.

SOLVED EXAMPLES

35. Which of the following combination of E and B is possible if a charged particle passes undeviated from a region?

(A) $E = 0\ \ B = 0$ (B) $E \neq 0\ \ B = 0$
(C) $E = 0\ \ B \neq 0$ (D) $B \neq 0\ \ E \neq 0$

Solution:

A, B, C, D

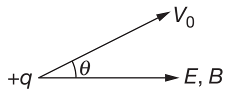
36. In above question, the charged particle passes undeviated without changing its velocity.

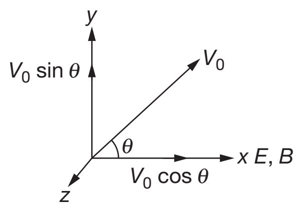
Solution:

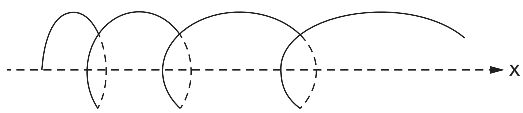
A, B, C, D, D when $\vec{E} = -(\vec{V} \times \vec{B})$

Case(i):

- $\vec{E} \parallel \vec{B}$ are uniform $\theta \neq 0, 180°$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)

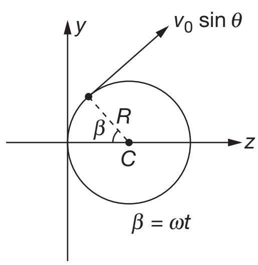






in x : $F_x = qE$, $a_x = \dfrac{qE}{m}$, $v_x = v_0\cos\theta + a_x t$, $x = v_0 t + \dfrac{1}{2}a_x t^2$

in yz plane



$$qv_0 \sin\theta B = \frac{m(v_0 \sin\theta)^2}{R}$$

$$\Rightarrow \quad R = \frac{mv_0 \sin\theta}{qB}$$

$$\omega = \frac{v_0 \sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$$

$$\vec{r} = \{(v_0 \cos\theta)t + \frac{1}{2}\frac{qE}{m}t^2\}\,\hat{i} + R\sin\omega t\,\hat{j}$$

$$+ (R - R\cos\omega t)(-\hat{k})$$

$$\vec{v} = \left(v_0\cos\theta + \frac{qE}{m}t\right)\hat{i} + (v_0\sin\theta)\cos\omega t\,\hat{j}$$

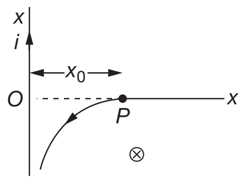
$$+ v_0 \sin\theta \sin\omega t\,(-\hat{k})$$

$$\vec{a} = \frac{qE}{m}\,\hat{i} + \omega^2 R[-\sin\beta\,\hat{j} - \cos\beta\,\hat{k}]$$

MISCELLANEOUS EXAMPLES

37. A long, straight wire carries a current i. A particle having a positive charge q and mass m kept at a distance x_0 from the wire is projected towards it with a speed v. Find the minimum separation between the wire and the particle

Solution:



Let the particle be initially at P. Take the wire as the y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the x-axis. We have, $OP = x_0$. The magnetic field B at any point to the right of the wire is along the negative z-axis. The magnetic force on the particle is, therefore, in the x–y plane. As there is no initial velocity along the z-axis, the motion will be in the x–y plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle.

The force at time t is $\vec{F} = q\vec{v} \times \vec{B}$

$$= q(\vec{i}v_x + \vec{j}v_y) \times \left(-\frac{\mu_0 i}{2\pi x}\vec{k}\right)$$

$$= \vec{j}qv_x\frac{\mu_0 i}{2\pi x} - \vec{i}\,qv_y\frac{\mu_0 i}{2\pi x}.$$

Thus, $$a_x = \frac{F_x}{m} = -\frac{\mu_0 qi}{2\pi m}\frac{v_y}{x} = -\lambda\frac{v_y}{x} \qquad (1)$$

where $$\lambda = \frac{\mu_0 qi}{2\pi m}.$$

Also, $$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{v_x dv_x}{dx} \qquad (2)$$

As, $$v_x^2 + v_y^2 = v^2,$$

giving $$v_x\,dv_x = -v_y\,dv_y. \qquad (3)$$

From (1), (2), and (3),

$$\frac{v_y dv_y}{dx} = \frac{\lambda v_y}{x}$$

or, $$\frac{dx}{x} = \frac{dv_y}{\lambda}$$

Initially, $x = x_0$ and $v_y = 0$. At minimum separation from the wire, $v_x = 0$ so that $v_y = -v$.

Thus $$\int_{x_0}^{x}\frac{dx}{x} = \int_0^{-v}\frac{dv_y}{\lambda}$$

or $$\ln\frac{x}{x_0} = -\frac{v}{\lambda}$$

or $$x = x_0\,e^{-v/\lambda} = x_0\,e^{-\frac{2\pi mv}{\mu_0 qi}}.$$

38. An electron is released from the origin at a place where a uniform electric field E and a uniform magnetic field B exist along the negative y-axis and the negative z-axis, respectively. Find the displacement of the electron along the y-axis when its velocity becomes perpendicular to the electric field for the first time.

Solution:

Let us take axes as shown in Fig. 15.25. According to the right-handed system, the z-axis is upward in the Fig. 15.25 and hence the magnetic field is shown downwards. At any time, the velocity of the electron may be written as

$$\vec{u} = u_x\vec{i} + u_y\vec{j}$$

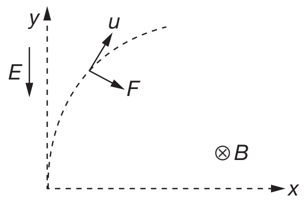


Fig. 15.25

The electric and magnetic fields may be written as

$$\vec{E} = -E\vec{j}$$

and $$\vec{B} = -B\vec{k}$$

respectively. The force on the electron is

$$\vec{F} = -e(\vec{E} + \vec{u} \times \vec{B})$$
$$= eE\vec{j} + eB(u_y\vec{i} - u_x\vec{j})$$

Thus, $F_x = eu_y B$

and $F_y = e(E - u_x B)$.

The components of the acceleration are

$$a_x = \frac{du_x}{dt} = \frac{eB}{m}u_y \quad (1)$$

and $$a_y = \frac{du_y}{dt} = \frac{e}{m}(E - u_x B) \; . \quad (2)$$

We have, $$\frac{d^2u_y}{dt^2} = -\frac{eB}{m}\frac{du_x}{dt}$$

$$= -\frac{eB}{m} \cdot \frac{eB}{m} u_y = -\omega^2 u_y$$

where $$\omega = \frac{eB}{m} \quad (3)$$

This equation is similar to that of a simple harmonic motion. Thus,

$$u_y = A \sin(\omega t + \delta) \quad (4)$$

and hence, $$\frac{du_y}{dt} = A\,\omega \cos(\omega t + \delta) \quad (5)$$

At $t = 0$, $u_y = 0$ and $\dfrac{du_y}{dt} = \dfrac{F_y}{dt} = \dfrac{eE}{m}$.

Putting in (4) and (5),

$$\delta = 0 \text{ and } A = \frac{eE}{m\omega}\frac{E}{B} \; .$$

Thus, $$u_y = \frac{E}{B} \sin \omega t.$$

The path of the electron will be perpendicular to the y-axis when $u_y = 0$. This will be the case for the first time at t, where

$$\sin \omega t = 0$$

or, $$\omega t = \pi$$

or, $$t = \frac{\pi}{\omega} = \frac{\pi m}{eB}$$

Also, $$u_y = \frac{dy}{dt} = \frac{E}{B} \sin \omega t$$

or, $$\int_0^y dy = \frac{E}{B} \sin \omega t \; dt$$

or, $$y = \frac{E}{B\omega}(1 - \cos \omega t).$$

At $$t = \frac{\pi}{\omega},$$

$$y = \frac{E}{B\omega}(1 - \cos \pi) = \frac{2E}{B\omega}$$

Thus, the displacement along the y-axis is

$$\frac{2E}{B\omega} = \frac{2Em}{BeB} = \frac{2Em}{eB^2} \; .$$

Magnetic Force on a Current Carrying Wire

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element $d\ell$ of the wire (Fig. 15.26). The free electrons drift with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is

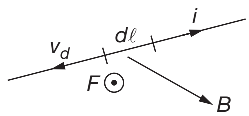


Fig. 15.26

$$i = jA = nev_d A \quad (1)$$

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$$\vec{f} = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered as $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$$

If we denote the length $d\ell$ along the direction of the current by $d\vec{\ell}$, the above equation becomes

$$d\vec{F} = nAev_d \, d\vec{\ell} \times \vec{B}.$$

Using (1),

$$d\vec{F} = i\,d\vec{\ell} \times \vec{B}.$$

The quantity is called a *current element*.

$$\overrightarrow{F_{\text{res}}} = \int \overrightarrow{dF} = \int \overrightarrow{id\ell} \times \vec{B} \ = i \int \overrightarrow{d\ell} \times \vec{B}$$

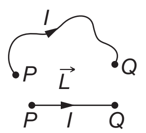
($\because$ i is same at all points of the wire.)

If $\vec{B}$ is uniform then $\overrightarrow{F_{\text{res}}} = i(\int \overrightarrow{d\ell}) \times \vec{B}$

$$\overrightarrow{F_{\text{res}}} = i\,\vec{L} \times \vec{B}$$

Here $\vec{L} = \int \overrightarrow{d\ell}$ = vector length of the wire = vector connecting the end points of the wire.

$\Rightarrow$

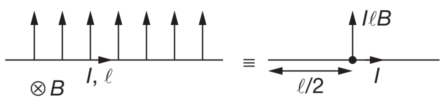


NOTE

If a current loop of any shape is placed in a uniform $\vec{B}$ then $\overrightarrow{F_{\text{res}}}\Big)_{\text{magnetic}}$ on it = 0 ($\because$ $\vec{L} = 0$).

Point of Application of Magnetic Force

On a straight current carrying wire, the magnetic force in a uniform magnetic field can be assumed to be acting at its midpoint.



This can be used for calculation of torque.

SOLVED EXAMPLES

39. A wire is bent in the form of an equilateral triangle *PQR* of side 10 cm and carries a current of 5.0 A. It is placed in a magnetic field *B* of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

Solution:

Suppose the field and the current have directions as shown in Fig. 15.27. The force on *PQ* is

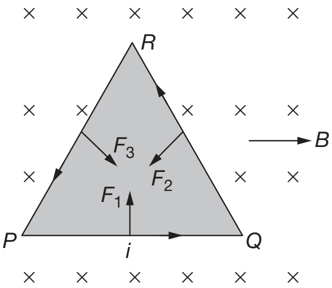


Fig. 15.27

$$\vec{F_1} = i\vec{\ell} \times \vec{B}$$

or $\qquad F_1 = 5.0 \text{ A} \times 10 \text{ cm} \times 2.0 \text{ T} = 1.0 \text{ N}$

The rule of vector product shows that the force F_1 is perpendicular to *PQ* and is directed towards the inside of the triangle.

The forces $\vec{F_2}$ and $\vec{F_3}$ on *QR* and *RP* can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalized. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

40. Two long wires, carrying currents i_1 and i_2, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length $d\ell$ of the second wire situated at a distance ℓ from the first wire.

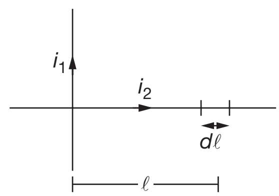


Fig. 15.28

Solution:

The situation is shown in Fig. 15.28. The magnetic field at the site of $d\ell$, due to the first wire is

$$B = \frac{\mu_0 i_1}{2\pi\ell}$$

This field is perpendicular to the plane of the figure going into it. The magnetic force on the length $d\ell$ is,

$$dF = i_2\, d\ell\, B \sin 90^{\circ} = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

This force is parallel to the current i_1.

41. Figure 15.29 shows two long metal rails placed horizontally and parallel to each other at a separation ℓ. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.

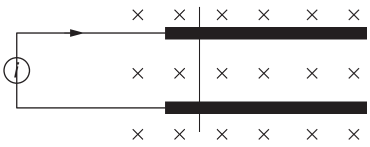


Fig. 15.29

(A) What soluble the minimum value of μ which can prevent the wire from sliding on the rails?
(B) Describe the motion of the wire if the value of μ is half the value found in the previous part.

Solution:

(A) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell}\times\vec{B}$$

or $$F = i\ell B$$

It acts towards right in the given Fig. 15.29. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0\, mg$. Thus,

$$\mu_0\, mg = i\ell B$$

or, $$\mu_0 = \frac{i\ell B}{mg}$$

(B) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{i\ell B}{2\,mg}$, the wire will slide towards right. The frictional force by the rails is

$$f = \mu mg = \frac{i\ell B}{2} \text{ towards left.}$$

The resultant force is $i\ell B - \frac{i\ell B}{2} = \frac{i\ell B}{2}$ towards right. The acceleration will be $a = \frac{i\ell B}{2m}$. The wire will slide towards right with this acceleration.

42. In Fig. 15.30 shown a semicircular wire is placed in a uniform $\vec{B}$ directed towards right. Find the resultant magnetic force and torque on it.

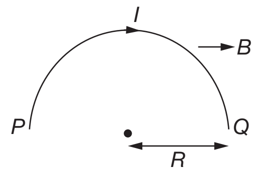


Fig. 15.30

Solution:

The wire is equivalent to

$\because$ $$\theta = 0$$

$\therefore$ $$F_{res} = 0$$

forces on individual parts are marked in Fig. 15.31 by ⊗ and ⊙.

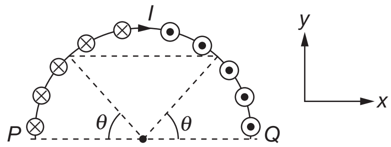


Fig. 15.31

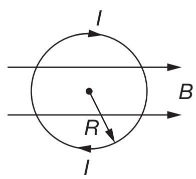
By symmetry, there will be pair of forces forming couples.

$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta)\cdot 2R\cos\theta$$

$$\tau = \frac{i\pi R^2}{2}B$$

$\Rightarrow$ $$\vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$$

43. Find the resultant magnetic force and torque on the loop.



Solution:

$$\overrightarrow{F_{res}} = 0, (\because \text{ loop })$$

and $\vec{\tau} = i\pi R^2 B(-\hat{j})$ using the above method

44. In Fig. 15.32, find the resultant magnetic force and torque about C and P.

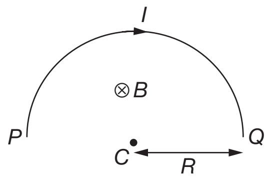
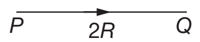


Fig. 15.32

Solution:

$$\vec{F}_{\text{nett}} = I \cdot 2R \cdot B$$

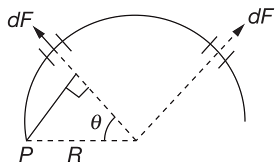
$\because$ wire is equivalent to



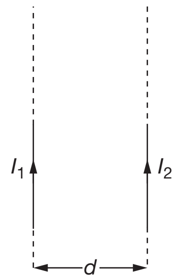
Force on each element is radially outward: $\tau_c = 0$ point about

$$P = \int_0^{\pi} [i(Rd\theta)B\sin 90^0]\ R\sin\theta$$

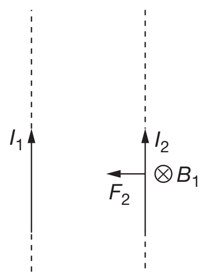
$$= 2IBR^2$$



45. Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2/2\pi d$. Here it is attractive also.



Solution:



On (2), B due to (1) is $= \dfrac{\mu_0 I_1}{2\pi d} \otimes$

$\therefore$ F on (2) on 1m length

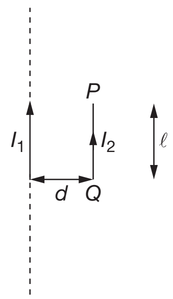
$$= I_2 \cdot \frac{\mu_0 I_1}{2\pi d} \cdot 1 \text{ towards left it is attractive}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d} \text{ (hence proved)}$$

Similarly, on the other wire also.

NOTE

- Definition of ampere (fundamental unit of current) using the above formula.
  If $I_1 = I_2 = 1\text{A}$, $d = 1\text{m}$ then $F = 2 \times 10^{-7}$ N
  $\therefore$ 'When two very long wires carrying equal currents and separated by 1m distance exert on each other a magnetic force of 2×10^{-7} N on 1m length then the current is 1 A'.
- The above formula can also be applied if one wire is infinitely long and the other is of finite length. In this case, the force per unit length on each wire will not be same.
  Force per unit length on $PQ = \dfrac{\mu_0 I_1 I_2}{2\pi d}$ (attractive)



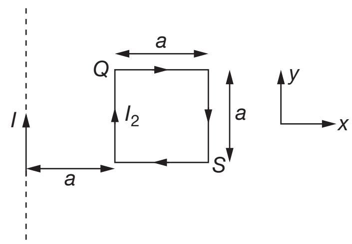
(3) If the currents are in the opposite direction, then the magnetic force on the wires will be repulsive.

46. Find the magnetic force on the loop $PQRS$ due to the loop wire.

Solution:

$$F_{\text{res}} = \frac{\mu_0 I_1 I_2}{2\pi a}\ a(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i})$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$$



47. In Fig. 15.33, the wires *AB* and *PQ* carry constant currents I_1 and I_2, respectively. *PQ* is of uniformly distributed mass *m* and length ℓ. *AB* and *PQ* are both horizontal and kept in the same vertical plane. The wire *PQ* is in equilibrium at height *h*. Find

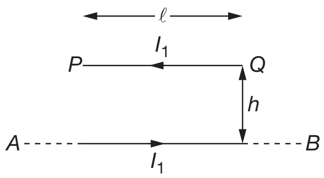


Fig. 15.33

(i) *h* in terms of I_1, I_2, ℓ, *m*, *g*, and other standard constants.

(ii) If the wire *PQ* is displaced vertically by small distance prove that it performs SHM. Find its time period in terms of *h* and *g*.

Solution:

(i) Magnetic repulsive force balances the weight.

$$\frac{\mu_0 I_1 I_2}{2\pi h}\ell\,\text{mg} \quad\Rightarrow\quad h = \frac{\mu_0 I_1 I_2 \ell}{2\pi\, mg}$$

(ii) Let the wire be displaced downward by distance $x (<< h)$.

Magnetic force on it will increase, so it goes back towards its equilibrium position. Hence, it performs oscillations.

$$F_{\text{res}} = \frac{\mu_0 I_1 I_2}{2\pi(h-x)}\ell - mg = \frac{mgh}{h-x} - mg$$

$$= \frac{mg(h-h+x)}{h-x}$$

$$= \frac{mg}{h-x}x \cong \frac{mg}{h}x \text{ for } x << h$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}}.$$

TORQUE ON A CURRENT LOOP

When a current-carrying coil is placed in a uniform magnetic field, the net force on it is always zero. However, as its different parts experience forces in different directions, the loop may experience a torque (or couple) depending on the orientation of the loop and the axis of rotation. For this, consider a rectangular coil in a uniform field *B* which is free to rotate about a vertical axis *PQ* and normal to the plane of the coil making an angle θ with the field direction as shown in Fig. 15.34 (A).

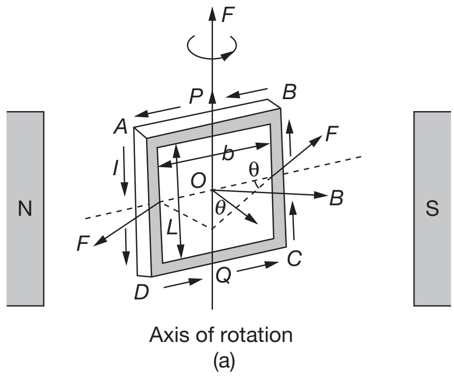


Fig. 15.34

The arms *AB* and *CD* will experience forces *B*(NI)*b* vertically up and down, respectively. These two forces together will give zero net force and zero torque (as are collinear with axis of rotation), so will have no effect on the motion of the coil.

Now the forces on the arms *AC* and *BD* will be *BINL* in the direction out of the page and into the page, respectively, resulting in zero net force, but an anti-clockwise couple of value

$$\tau = F \times Arm = BINL \times (b\sin\theta)$$

i.e., $\tau = BIA\sin\theta$

with $A = NLb$ (1)

Now treating the current-carrying coil as a dipole of moment $\vec{M} = I\vec{A}$. Equation (1) can be written in vector form as

$$\vec{\tau} = \vec{M}\times\vec{B} \quad [\text{with } \vec{M} = I\vec{A} = NIA\vec{n} \qquad (2)]$$

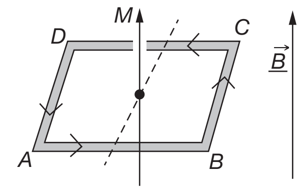
This is the required result and from this it is clear that:

1. Torque will be minimum (= 0) when $\sin\theta = \min = 0$, i.e., $\theta = 0^\circ$, i.e., 180º i.e., the plane of the coil is perpendicular to magnetic field, i.e., normal to the coil is collinear with the field [Fig. 15.35 (a) and (c)]
2. Torque will be maximum (= BINA) when $\sin\theta = \max = 1$, i.e., $\theta = 90^\circ$, i.e., the plane of the coil is parallel to the field, i.e., normal to the coil is perpendicular to the field [Fig. 15.35 (b)].
3. By analogy, with dielectric or magnetic dipole in a field, in case of current-carrying in a field.

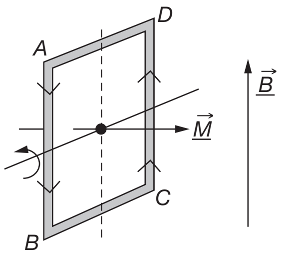
$$U = \vec{M}\cdot\vec{B} \text{ with } F = \frac{dU}{dr}$$

and $W = MB(1-\cos\theta)$

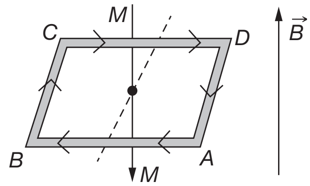
The values of *U* and *W* for different orientations of the coil in the field are shown in Fig. 15.35.



$\theta = 0°$ ($\vec{M}$ is parallel to $\vec{B}$)
$\tau = 0 = \text{min}$
$W = 0 = \text{min}$
$U = -MB = \text{min}$
Stable equilibrium
(a)



$\theta = 90°$ ($\vec{M}$ is $\perp$ to $\vec{B}$)
$\tau = MB = \text{max}$
$W = MB$
$U = 0$
No equilibrium
(b)



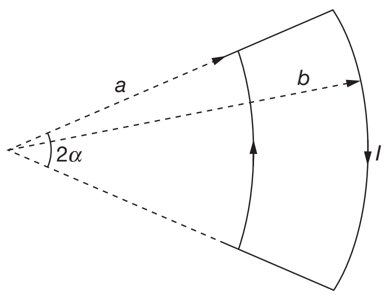
$\theta = 180°$ ($\vec{M}$ is antiparallel to $\vec{B}$)
$\tau = 0$
$W = 2MB = \text{max}$
$U = MB = \text{max}$
Unstable equilibrium
(c)

Fig. 15.35

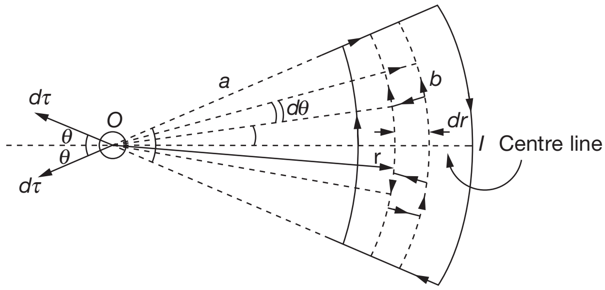
4. Instruments such as electric motor, moving coil galvanometer, tangent galvanometers, etc., are based on the fact that a current-carrying coil in a uniform magnetic field experiences a torque (or couple).

SOLVED EXAMPLE

48. A loop with current I is in the field of a long straight wire with current I_0. The plane of the loop is perpendicular to the straight wire. Find torque acting on the loop.



Solution:



$$d\vec{s} = (r d\theta\, dr) \text{ (inwards)}$$

$$d\vec{M} = (rI\, d\theta\, dr) \text{ (inwards)}$$

$$\vec{B} = \frac{\mu_0 I_0}{2\pi r} \quad \text{(tangential clockwise)}$$

$$d\tau = |d\vec{M} \times \vec{B}| = \frac{\mu_0 I I_0\, d\theta\, dr}{2\pi} \quad \text{(towards centre)}$$

$$\therefore \quad \tau = \int_{-\alpha}^{\alpha} \int_a^b d\tau \cos\theta$$

$$= \frac{\mu_0 I I_0}{2\pi} \int_{-\alpha}^{\alpha} \int_a^b \cos\theta\, d\theta\, dr$$

$$= \frac{\mu_0 I I_0 (b-a)\sin\alpha}{\pi} \quad \text{(to the left).}$$

TERRESTRIAL MAGNETISM (EARTH'S MAGNETISM)

Introduction

The idea that earth is magnetized was first suggested towards the end of the sixteenth century by Dr. William Gilbert. The origin of earth's magnetism is still a mater of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.5°) to the earth's axis of rotation with its south pole pointing north. The lines of force of earth's magnetic field are shown in Fig. 15.36 which are parallel to the earth's surface near the equator and perpendicular to it near the poles. While discussing magnetism of the

Earth, one should keep in mind that:

1. The magnetic meridian at a place is not a line but a vertical plane passing through the axis of a freely suspended magnet, i.e., it is a plane which contains the place and the magnetic axis.

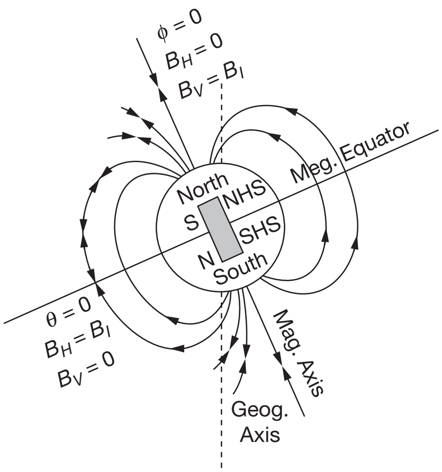


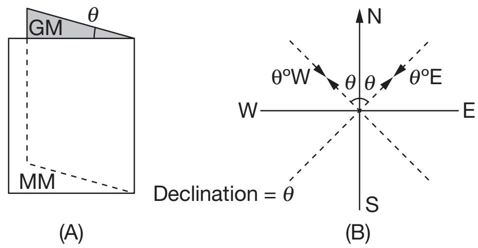
Fig. 15.36

2. The geographical meridian at a place is a vertical plane which passes through the line joining the geographical north and south, i.e., it is a plane which contains the place and earth's axis of rotation, i.e., geographical axis.
3. The magnetic equator is a great circle (a circle with the centre at earth's centre) on earth's surface which is perpendicular to the magnetic axis. The magnetic equator passing through Trivandrum in South India divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called the northern hemisphere (NHS), while the other, the southern hemisphere (SHS).
4. The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

Elements of the Earth's Magnetism

The magnetism of earth is completely specified by the following three parameters called elements of earth's magnetism:

1. **Variation or Declination θ:** At a given place, the angle between the geographical meridian and the magnetic meridian is called declination, i.e., at a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle. Declination at a place is expressed at $\theta°$ E or $\theta°$ W depending upon whether the north pole of the compass needle lies to the east (right) or to the west (left) of the geographical north-south direction. The declination at London is 10° W means that at London, the north pole of a compass needle points 10° W, i.e., left of the geographical north.



2. **Inclination or Angle of Dip ϕ:** It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place. Actually, it is the angle which the axis of a freely suspended magnet (up or down) subtends with the horizontal in magnetic meridian at a given place.
Here, it is worthy to note that as the northern hemisphere contains south polarity of earth's magnetism, in it the north pole of a freely suspended magnet (or pivoted compass needle) will dip downwards, i.e., towards the earth while the opposite will take place in the southern hemisphere.

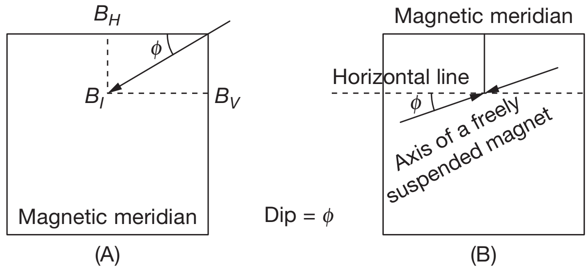


Fig. 15.37

Angle of dip at a place is measured by the instrument called 'Dip-Circle' in which a magnetic needle is free to rotate in a vertical plane which can be set in any vertical direction. Angle of dip at Delhi is 42°.

3. **Horizontal Component of Earth's Magnetic Field B_H:** At a given place, it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by B_H and is measured with the help of a vibration or deflection magnetometer. At Delhi, the horizontal component of the earth's magnetic field is 35 μT, i.e., 0.35 G.
If at a place magnetic field of earth is B_I and angle of dip ϕ, then in accordance with Fig. 15.37 (a).

$$B_H = B_I \cos \phi$$

and $$B_v = B_I \sin \phi \quad (1)$$

so that, $$\tan \phi = \frac{B_v}{B_H}$$

and $$I = \sqrt{B_H^2 + B_v^2} \quad (2)$$

BRAIN MAP

MAGNETICS

1. Biot-Savart law

It gives the magnetic induction due to an infinitesimal current element

$$dB = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$$

2. Field due to straight current carrying wire

(a) When the wire is of finite length

$$B = \frac{\mu_0}{4\pi}\frac{I}{d}[\sin\alpha + \sin\beta]$$

(b) When the wires of infinite length

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$

3. Field due to circular current carrying wire

(a) At an axial point

$$B = \frac{\mu_0}{4\pi}\frac{2\pi IR^2}{(R^2 + x^2)^{3/2}}$$

(b) At the centre

$$B = \frac{\mu_0}{2}\frac{I}{R}$$

4. Field due to current carrying arc

(a) At the centre making an angle ϕ

$$B = \frac{\mu_0}{4\pi}\frac{I\phi}{R} = \frac{\mu_0}{4\pi}\frac{Il}{R^2}$$

(b) At the centre of semi circular wire

$$B = \frac{\mu_0}{4}\frac{I}{R}$$

5. Field due to solenoid

(a) At a point on the axis of solenoid of finite length

$$B = \frac{\mu_0}{2}nI(\sin\alpha + \sin\beta)$$

(b) At a point inside the solenoid of infinite length $B = \mu_0 nI$

6. (a) Force on a moving charge

(i) When it is moving in a magnetic field

$$\vec{F} = q(\vec{v}\times\vec{B})$$

(ii) When it is moving in a combined electric and magnetic field

$$\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$$

(b) Force on a current carrying straight wire when it is in a uniform magnetic field

$$F = I(\vec{l}\times\vec{B})$$

7. Motion of charged particle in a uniform magnetic field

(a) When it enters at right angle to the field, Path will be circular and

$$r = \frac{mv}{qB},\ T = \frac{2\pi m}{qB}$$

(b) When it enters at some angle θ with the field, Path will be helical

$$r = \frac{mv\sin\theta}{qB},\ T = \frac{2\pi m}{qB}$$

$$\text{Pitch} = \frac{2\pi m}{qB}v\cos\theta$$

8. When a current carrying loop is in a uniform field

(a) Force = 0

(b) Magnetic moment $\vec{M} = \vec{I}.\vec{A}$

(c) Torque, $(\vec{\tau}) = \vec{M}\times\vec{B}$

(d) Wark done $= MB(1 - \cos\theta)$

9. Force between two long straight parallel wire per unit length carrying current I_1 and I_2

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

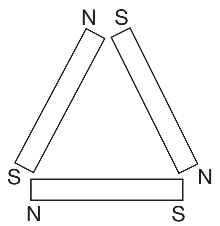
EXERCISES

Single Option Correct Type

1. The line on the earth's surface joining the points where the earth's magnetic field is horizontal is called
(A) Magnetic meridian
(B) Magnetic axis
(C) Magnetic line
(D) Magnetic equator

2. A homogeneous electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ are pointing in the same direction. A proton is projected with its velocity parallel to $\vec{E}$. It will
(A) go on moving in the same direction with increasing velocity.
(B) go on moving in the same direction with constant velocity.
(C) turn to its right.
(D) turn to its left.

3. Three identical bar magnets, each of magnetic moment *M*, are placed in the form of an equilateral triangle with north pole of one touching the south pole of the other as shown. The net magnetic moment of the system is



(A) Zero
(B) $3M$
(C) $\frac{3M}{2}$
(D) $M\sqrt{3}$

4. Which of the following is true?
(A) Diamagnetism is temperature dependent
(B) Paramagnetism is temperature dependent
(C) Paramagnetism is temperature independent
(D) None of the above

5. The magnetic induction field at the centre *C* of the arrangement shown in Fig. 15.38 is

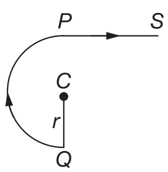


Fig. 15.38

(A) $\frac{\mu_0 i}{4\pi r}(1+\pi)$
(B) $\frac{\mu_0 i}{2\pi r}(1+\pi)$
(C) $\frac{\mu_0 i}{\pi r}(1+\pi)$
(D) $\frac{\mu_0 i}{r}(1+\pi)$

6. A cube made of wires of equal length is connected to a battery as shown in Fig. 15.39. The side of cube is *L*. The magnetic field at the centre of cube will be

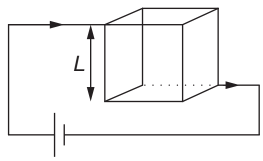


Fig. 15.39

(A) $\frac{12}{\sqrt{2}}\frac{\mu_0 I}{\pi L}$
(B) $\frac{6}{\sqrt{2}}\frac{\mu_0 I}{\pi L}$
(C) $6\frac{\mu_0 I}{\pi L}$
(D) Zero

7. A circular coil *A* has a radius *R* and the current flowing through it is *I*. Another circular coil *B* has radius 2*R* and if 2*I* is the current flowing through it, then the magnetic field at the centre of the circular coil are in the ratio of
(A) 4 : 1 (B) 2 : 1 (C) 3 : 1 (D) 1 : 1

8. Earth's magnetic induction at a certain point is 7×10^{-5} Wb/m^2. This field is to be annulled by the magnetic induction at the centre of a circular conducing loop 5.0 cm in radius. The required current is
(A) 0.056 A (B) 6.5A (C) 5.6 A (D) 12.8 A

9. A current *i* is flowing in an equilateral triangle of side *a* as shown in Fig. 15.40. The magnetic field at the centroid will be

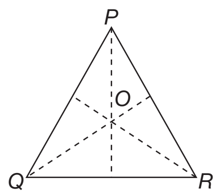


Fig. 15.40

(A) $\frac{9\mu_0 i}{2\pi a}$
(B) $\frac{5\sqrt{2}\mu_0 i}{3\pi a}$
(C) $\frac{3\mu_0 i}{2\pi a}$
(D) $\frac{\mu_0 i}{3\sqrt{3}\pi a}$

10. A uniform horizontal magnetic field $\vec{B}$ exists in the region. A rectangular loop of mass *m*, horizontal side *l* (perpendicular to the magnetic field), and resistance *R* is placed in the region. The velocity with which it should be pushed down so that it continues to fall without acceleration will be

(A) $\frac{mgR}{B^2l^2}$ (B) $\frac{B^2l^2}{mgR}$ (C) $\frac{mg}{BlR}$ (D) $\frac{RBl}{mg}$

11. A magnet makes 30 oscillations per minute at a plane where intensity is 32 T. At another place it takes 1 s to complete one oscillation. The value of horizontal intensity at the second place is,

(A) 12.8 T (B) 25.6 T (C) 128 T (D) 256 T

12. A bar magnet has a magnetic moment of 2.5 JT^{-1} and is placed in a magnetic field of 0.2 T. Work done in turning the magnet from parallel to anti-parallel position relative to field direction is

(A) 0.5 J (B) 1 J (C) 2 J (D) 0 J

13. Curie temperature is the temperature above which

(A) a paramagnetic material becomes diamagnetic.
(B) a ferromagnetic material becomes diamagnetic.
(C) a paramagnetic material becomes ferromagnetic.
(D) a ferromagnetic material becomes paramagnetic.

14. A proton is rotating along a circular path with kinetic energy *K* in a uniform magnetic field *B*. If the magnetic field is made four times, the kinetic energy of rotation of the proton will become

(A) 16 *K* (B) 8 *K* (C) 4 *K* (D) *K*

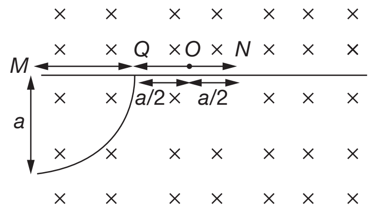
15. A charged particle moves in a uniform magnetic field of induction $\vec{B}$ with a velocity $\vec{v}$. The change in kinetic energy in the magnetic field is zero when the velocity $\vec{v}$ is

(A) parallel to $\vec{B}$ (B) perpendicular to $\vec{B}$
(C) at any angle to $\vec{B}$ (D) None of these

16. An equilateral triangular current loop *PQR* carries a current *I* ampere. Length of each side is *l* metre. A uniform magnetic field of induction $\vec{B}$ exists in a direction parallel to *PQ*. Then the force on the side *PQ* is

(A) IlB (B) $\frac{IlB}{2}$
(C) $\left(\frac{IlB}{2}\right)\sqrt{3}$ (D) Zero

17. There is a uniform magnetic field perpendicular to the plane of paper. A positive charged particle enters in the region, perpendicularly and collides inelastically at point *Q* to the rigid wall *MN*. If co-efficient of restitution $e = \frac{1}{2}$ then particle



(A) Retraces its path
(B) Strikes at point *O*
(C) Strikes at point *M*
(D) Strikes at point *N*

18. Two short bar magnets of magnetic moments *M* each are arranged at the opposite corners of a square of side *d*, such that their centers coincide with the corners and their axes are parallel. If the like poles are in the same direction, the magnetic induction at any of the other corners of the square is

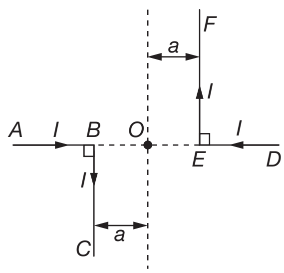
(A) $\frac{\mu_0}{4\pi}\cdot\frac{M}{d^3}$ (B) $\frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3}$

(C) $\frac{\mu_0}{4\pi}\cdot\frac{M\sqrt{5}}{d^3}$ (D) $\frac{\mu_0}{4\pi}\cdot\frac{3M}{d^3}$

19. Two long parallel wires *P* and *Q* are held perpendicular to the plane of the paper at a separation of 5 m. If *P* and *Q* carry currents of 2.5 A and 5 A, respectively, in the same direction, then the magnetic field at a point midway between *P* and *Q* is

(A) $\frac{\mu_0}{\pi}$ (B) $\frac{\sqrt{3}\mu_0}{\pi}$ (C) $\frac{\mu_0}{2\pi}$ (d) $\frac{3\mu_0}{2\pi}$

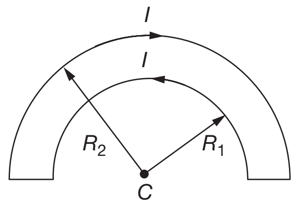
20. Two long thin wires *ABC* and *DEF* are arranged as shown. They carry current *I* as shown. The magnitude of the magnetic field at *O* is



(A) Zero (B) $\frac{\mu_0 I}{4\pi a}$

(C) $\frac{\mu_0 I}{2\pi a}$ (D) $\frac{\mu_0 I}{2\sqrt{2}\pi a}$

21. The wire loop formed by joining two semicircular sections of radii R_1 and R_2, carries a current *I*, as shown. The magnitude of magnetic of field at the centre *C* is



(A) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ (B) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

(C) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (D) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

22. A length of wire carries a steady current. It is bent first to form a circular coil of one turn. The same length is now bent more sharply to give a double loop of smaller radius. The magnetic field at the centre caused by the same current is

(A) a quarter of its first value.
(B) unaltered.
(C) four times of its first value.
(D) half of its first value.

23. A magnetic needle is kept in a non-uniform magnetic field. It experiences

(A) a torque but not a force.
(B) neither a force nor a torque.
(C) a force and a torque.
(D) a force but not a torque.

24. A charged particle moves with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The magnetic force experienced by the particle is

(A) always zero.
(B) never zero.
(C) zero if $\vec{B}$ and $\vec{v}$ are perpendicular.
(D) zero if $\vec{B}$ and $\vec{v}$ are parallel.

25. A particle of charge -16×10^{-18} C moving with velocity 10 m/s along the x-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

(A) 10^3 Wb/m^2 (B) 10^5 Wb/m^2
(C) 10^{16} Wb/m^2 (D) 10^{-3} Wb/m^2

26. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other, 3 A and 4 A are the currents flowing in coils, respectively. The magnetic induction in Wb/m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/Am)

(A) 12×10^{-5} (B) 10^{-5}
(C) 5×10^{-5} (D) 7×10^{-5}

27. An equilateral triangular loop ACD of side length l carries a current i in the direction shown in Fig. 15.41. The loop is kept in a uniform horizontal magnetic field $\vec{B}$ as shown in Fig. 15.41, net force on the loop is

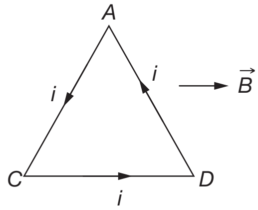


Fig. 15.41

(A) Zero
(B) $\frac{\sqrt{3}}{2} ilB$
(C) Perpendicular to paper inwards.
(D) Perpendicular to paper outwards.

28. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of masses of X and Y is

(A) $\left(\frac{R_1}{R_2}\right)^{1/2}$ (B) $\frac{R_2}{R_1}$

(C) $\left(\frac{R_1}{R_2}\right)^2$ (D) $\left(\frac{R_1}{R_2}\right)$

29. A current-carrying conductor is looped into a circle of radius 10 cm. The magnetic moment of the current loop becomes 0.314 A/m^2. What is the current in the loop?

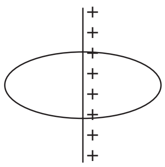
(A) 5 A (B) 8 A (C) 10 A (D) 12 A

30. Two straight long conductors AOB and COD are perpendicular to each other and carry currents i_1 and i_2. The magnitude of the magnetic induction at a point P at a distance a from the point O in a direction perpendicular to the plane $ACBD$ is

(A) $\frac{\mu_0}{2\pi a}(i_1+i_2)$ (B) $\frac{\mu_0}{2\pi a}(i_1-i_2)$

(C) $\frac{\mu_0}{2\pi a}(i_1^2+i_2^2)^{1/2}$ (D) $\frac{\mu_0}{2\pi a}\frac{i_1 i_2}{(i_1+i_2)}$

30. A very long uniformly charged rod falls with a constant velocity V through the centre of a circular loop. Then the magnitude of induced EMF in loop is



(charge per unit length of rod = λ)

(A) $\frac{\mu_0}{2\pi}\lambda v^2$ (B) $\frac{\mu_0}{2}\lambda v^2$

(C) $\frac{\mu_0}{2\lambda}v$ (D) Zero

32. A current i is flowing in a conductor as shown in Fig. 15.42. The magnetic induction at point O will be

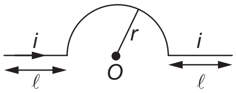


Fig. 15.42

(A) Zero (B) $l\mu_0 i / r$

(C) $\mu_0 i / 4r$ (D) $\mu_0 i / r$

33. The distance between two thin long straight parallel conducting wires is b. On passing the same current i in them, the force per unit length between them will be

(A) Zero (B) $\frac{\mu_0 i^2}{2\pi b}$ (C) $\frac{\mu_0 i}{2\pi b}$ (D) $\frac{\mu_0 i}{2\pi}$

34. The magnetic field strength at O due to current I in Fig. 15.43 is

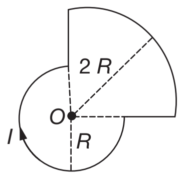


Fig. 15.43

(A) $\frac{7\mu_0 I}{16R}$ (B) $\frac{15\mu_0 I}{16R}$ (C) $\frac{11\mu_0 I}{32R}$ (D) $\frac{13\mu_0 I}{32R}$

35. Ratio of magnetic field at the centre of a current carrying coil of radius R and at a distance of $3R$ on its axis is

(A) $10\sqrt{10}$ (B) $20\sqrt{10}$ (C) $2\sqrt{10}$ (D) $\sqrt{10}$

36. A proton and an alpha-particle enter a uniform magnetic field with the same velocity. The period of rotation of the alpha particle will be

(A) four times that of the proton.
(B) two times that of the proton.
(C) three times that of the proton.
(D) same as that of the proton.

37. Two very long, straight, parallel wires carry steady current I and $-I$, respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires and in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$

(C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) Zero

38. Two very long, straight, parallel wires carry currents I and $-I$, respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to magnetic field acting on the charge at this instant is

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Idq}{\pi d}$

(C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) zero

39. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a

(A) Straight line (B) Circle
(C) Helix (D) Cycloid

40. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is

(A) qbB/m
(B) $q(b - a)B/m$
(C) qaB/m
(D) $q(b + a)B/2m$

41. Two charged particles having charges Q and $-Q$ and masses m and $4m$, respectively, enter in uniform magnetic field B at an angle θ with magnetic field from same point with speed v. The displacement from starting point, where they will meet again, is

(A) $\frac{2\pi m}{QB}v\sin\theta$ (B) $\frac{2\pi m}{QB}v\cos\theta$

(C) $\frac{8\pi m}{QB}v\cos\theta$ (D) $\frac{12\pi m}{QB}v\cos\theta$

42. Two short bar magnets of magnetic moments M each are arranged at the opposite corners of a square of side d such that their centres coincide with the corners and their axes are parallel. If the like poles are in the same direction, the magnetic induction at any of the other corners of the square is

(A) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (B) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$

(C) $\frac{\mu_0}{4\pi}\frac{M}{2d^3}$ (D) $\frac{\mu_0}{4\pi}\frac{M^2}{2d^3}$

43. A current of 5 A is passed through a straight wire of length 6 cm; then the magnetic induction at a point 5 cm from the both end of the wire is

(A) 0.25 gauss (B) 0.125 gauss
(C) 0.15 gauss (D) 0.30 gauss

44. An electron of mass m is accelerated through a potential difference of V and then it enters a magnetic field of induction B normal to the lines. Then the radius of the circular path is

(A) $\sqrt{\frac{2eV}{m}}$ (B) $\sqrt{\frac{2Vm}{eB^2}}$

(C) $\sqrt{\frac{2Vm}{eB}}$ (D) $\sqrt{\frac{2Vm}{e^2B}}$

45. The magnetic field at O due to current in the infinite wire forming a loop as shown in Fig. 15.44 is

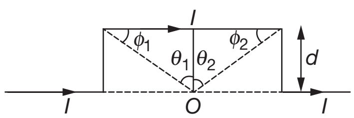


Fig. 15.44

(A) $\frac{\mu_0 I}{4\pi d}(\cos\phi_1+\cos\phi_2)$ (B) $\frac{\mu_0}{4\pi}\times\frac{2I}{d}$

(C) $\frac{\mu_0 I}{4\pi d}(\sin\phi_1+\sin\phi_2)$ (D) $\frac{\mu_0}{4\pi}\times\frac{I}{d}$

46. Shown in Fig. 15.45 is a conductor carrying a current I. The magnetic field intensity at the point O (common centre of all the three arcs) is (θ in radian)

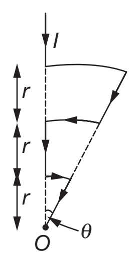


Fig. 15.45

(A) $\frac{5\mu_0 I\theta}{24\pi r}$ (B) $\frac{\mu_0 I\theta}{24\pi r}$

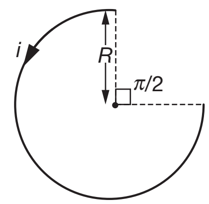
(C) $\frac{11\mu_0 I\theta}{24\pi r}$ (D) Zero

47. A straight wire of mass 200 gm and length 1.5 m carries a current of 2A. It is suspended in mid-air by a uniform horizontal magnetic field B. The magnitude of B (in tesla) is

($g = 9.8$ m/s^2)

(A) 2 (B) 1.5 (C) 0.55 (D) 0.66

48. A current i ampere flows in a circular arc of wire whose radius is R, which subtend an angle $3\pi/2$ radian at its centre. The magnetic induction B at the centre is



(A) $\frac{\mu_0 i}{R}$ (B) $\frac{\mu_0 i}{2R}$ (C) $\frac{2\mu_0 i}{R}$ (D) $\frac{3\mu_0 i}{8R}$

49. The magnetic moment of a small current carrying loop is 2.1×10^{-25} A × m^2. The magnetic field at a point on its axis at a distance of 1 Å is

(A) 4.2×10^{-2} Wb/m^2
(B) 4.2×10^{-3} Wb/m^2
(C) 4.2×10^{-4} Wb/m^2
(D) 4.2×10^{-5} Wb/m^2

50. A conductor in the form of a right angle ABC with AB = 3 cm and BC = 4 cm carries a current of 10 A. There is a uniform magnetic field of 5 T perpendicular to the plane of the conductor. The force on the conductor will be

(A) 1.5 N (B) 2.0 N
(C) 2.5 N (D) 3.5 N

51. Figure 15.46 shows a straight wire of length l carrying a current i. The magnitude of magnetic field produced by the current at point P is

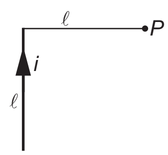


Fig. 15.46

(A) $\frac{\sqrt{2}\mu_0 i}{\pi l}$ (B) $\frac{\mu_0 i}{4\pi l}$

(C) $\frac{\sqrt{2}\mu_0 i}{8\pi l}$ (D) $\frac{\mu_0 i}{2\sqrt{2}\pi l}$

52. Current i is carried in a wire of length L. If the wire is turned into a circular coil, the maximum magnitude of torque in a given magnetic field B will be

(A) $\frac{L^2 iB}{2}$ (B) $\frac{L^2 iB}{\pi}$

(C) $\frac{L^2 iB}{4\pi}$ (D) $\frac{Li^2 B}{4\pi}$

53. In which type of materials the magnetic susceptibility does not depend on temperature?

(A) Diamagnetic (B) Paramagnetic

(C) Ferromagnetic (D) Ferrite

54. A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19}C is projected with a speed of 2×10^6 m/s at an angle of 60° to the x-axis. If a uniform magnetic field of 0.104 T is applied along y-axis, the path of proton is

(A) a circle of radius = 0.2 m and time period $\pi \times 10^{-7}$s.
(B) a circle of radius = 0.1 m and time period $2\pi \times 10^{-7}$s.
(C) a helix of radius = 0.1 m and time period $2\pi \times 10^{-7}$s.
(D) a helix of radius = 0.2 m and time period $4\pi \times 10^{-7}$s.

55. A triangular loop of side l carries a current I. It is placed in a magnetic field B such that the plane of the loop is in the direction of B. The torque on the loop is

(A) Zero (B) IBl^2

(C) $\frac{\sqrt{3}}{2} IBl^2$ (D) $\frac{\sqrt{3}}{4} IBl^2$

56. A wire carrying current I and other carrying $2I$ in the same direction produces a magnetic field B at the mid point. What will be the field when $2I$ wire is switched off?

(A) $B/2$ (B) $2B$ (C) B (D) $4B$

57. Current i is carried in a wire of length L. If the wire is turned into a circular coil, the maximum magnitude of torque in a given magnetic field B will be

(A) $\frac{L^2 iB}{2}$ (B) $\frac{L^2 iB}{\pi}$ (C) $\frac{L^2 iB}{4\pi}$ (D) $\frac{Li^2 B}{4\pi}$

58. A solenoid of 1.5 metre length and 4.0 cm diameter posses 10 turn per cm. A current of 5 ampere is flowing through it. The magnetic induction at axis inside the solenoid is

(A) $2\pi \times 10^{-3}$ T (B) $2\pi \times 10^{-5}$ T

(C) $4\pi \times 10^{-2}$ gauss (D) $2\pi \times 10^{-5}$ gauss

59. Through two parallel wires A and B, 10 and 2 A of currents are passed, respectively, in opposite direction. If the wire A is infinitely long and the length of the wire B is 2 m, the force on the wire B, which is situated at 10 cm distance from A will be

(A) 8×10^{-5} N (B) 4×10^{-7} N

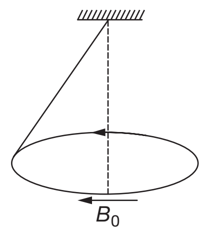
(C) 4×10^{-5} N (D) $4\pi \times 10^{-7}$ N

60. Three infinitely long thin conductors are joined at the origin of coordinates and lies along the x, y, and z axes. A current i flowing along the conductor lying along the negative x-axis divides equally into the other two at the origin. The magnitude of magnetic filed at the point $(0, -a, 0)$ is

(A) $\frac{\mu_0 i}{4\pi a}$ (B) $\frac{3\mu_0 i}{4\sqrt{2}\pi a}$

(C) $\frac{\sqrt{5}\mu_0 i}{8\pi a}$ (D) $\frac{\sqrt{3}\mu_0 i}{2\pi a}$

61. A uniform current carrying ring of mass m and radius R is connected by a massless string as shown. A uniform magnetic field B_0 exist in the region to keep the ring in horizontal position, then the current in the ring is



(l = length of string)

(A) $\frac{mg}{\pi RB_0}$ (B) $\frac{mg}{RB_0}$

(C) $\frac{mg}{3\pi RB_0}$ (D) $\frac{mg\,l}{\pi R^2 B_0}$

62. A particle of mass m and charge q is projected from origin with initial velocity $[u\hat{i} - v\hat{j}]$. Uniform electric and magnetic field exist in the region along $+y$ direction of magnitudes E and B, respectively. Then particle will definitively return to the origin if

(A) $\frac{vB}{2\pi E}$ is an integer

(B) $\frac{(u^2 + v^2)^{1/2}}{[B/\pi E]}$ is an integer

(C) $\frac{vB}{\pi E}$ is an integer

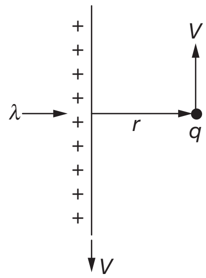
(D) $\frac{vB}{3\pi E}$ is an integer

63. A charge particle enters into a region containing uniform electric field (E) and uniform magnetic field (B) along x-axis and y-axis, respectively. If it passes the region undeviated, the velocity of charge particle is given by

(A) $2\hat{i} + \frac{E}{B}\hat{k}$ (B) $2\hat{j}+\frac{E}{B}\hat{k}$

(C) $-\frac{E}{B}\hat{k}$ (D) None of these

64. A long wire having linear charge density λ moving with constant velocity v along its length. A point charge moving with same speed in opposite direction and at that instant, it is r distance from the wire. The net force acting on the charge is given by



(A) $\frac{\lambda q}{2\pi r}\left[\frac{1}{\varepsilon_0}+v^2\mu_0\right]$ (B) $\frac{\lambda q}{2\pi r}\left[\frac{1}{\varepsilon_0}-\mu_0 v^2\right]$

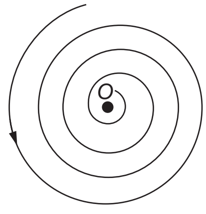
(C) $\frac{\lambda q}{2\pi r}\sqrt{\left(\frac{1}{\varepsilon_0}\right)^2+v^4\mu_0^2}$ (D) Zero

65. Three particles each of mass m and charge q are attached to the vertices of a triangular frame, made up of three light rigid rods of equal length l. The frame is rotated at constant angular speed ω about an axis perpendicular to the plane of the triangle and passing through its centre. The ratio of the magnetic moment of the system and its angular momentum about the axis of rotation is

(A) $\frac{q}{2m}$ (B) $\frac{q}{m}$

(C) $\frac{2q}{m}$ (D) $\frac{4q}{m}$

66. A positive charged particle is moving in a plane spiral path with constant angular velocity and constant centripetal velocity $\left(-\frac{dr}{dt}\right)$. If a current carrying wire is placed at O perpendicular to the plane of spiral path, then path followed by the charged particle is



(A) helical path with constant radius and constant pitch.
(B) helical path with decreasing radius and increasing pitch.
(C) helical path with decreasing radius and decreasing pitch.
(D) path will remain same.

67. A long straight wire carrying a current of 30 A is placed in an external uniform magnetic field of magnitude 4×10^{-4} T. The magnetic field is acting parallel to the direction of current. The magnitude of the resultant magnetic field in tesla at a point 2.0 cm away from the wire is

(A) 10^{-4} (B) 3×10^{-4}
(C) 5×10^{-4} (D) 6×10^{-4}

68. A circular current carrying loop of radius R, carries a current i. The magnetic field at a point on the axis of coil is $\frac{1}{\sqrt{8}}$ times the value of magnetic field at the centre. Distance of point from centre is

(A) $\frac{R}{\sqrt{2}}$ (B) $\frac{R}{\sqrt{3}}$ (C) $R\sqrt{2}$ (D) R

69. A particle of charge per unit mass α is released from origin with a velocity $\vec{v} = v_0\hat{i}$ in a uniform magnetic field $\vec{B} = -B_0\hat{k}$. If the particle passes through $(0, y, 0)$, then y is equal to

(A) $-\frac{2v_0}{B_0\alpha}$ (B) $\frac{v_0}{B_0\alpha}$

(C) $\frac{2v_0}{B_0\alpha}$ (D) $-\frac{v_0}{B_0\alpha}$

70. A point charge of 0.1 C is placed on the circumference of a non-conducting ring of radius 1 m which is rotating with a constant angular acceleration of 1 rad/s^2. If ring starts its motion at $t = 0$, the magnetic field at the centre of the ring, at $t = 10$ s, is

(A) 10^{-6} T (B) 10^{-7} T
(C) 10^{-8} T (D) 10^{7} T

71. A conducting rod of mass m and length l is connected by two identical springs as shown in Fig. 15.47. Initially, the system is in equilibrium. A uniform magnetic field of magnitude B directed perpendicular to the plane of the paper outwards also exists in the region. If a current I is switched on that passes from P to Q through the rod. Further maximum elongation in the spring is [Given: $|mg| = |BIl|$]

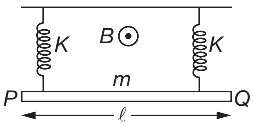


Fig. 15.47

(A) $\frac{BIl}{K}$ (B) $\frac{BIl}{4K}$ (C) $\frac{BIl}{8K}$ (D) $\frac{BIl}{16K}$

72. If a magnet is suspended at an angle of 30° to the magnetic meridian, the dip needle makes an angle of 45° with the horizontal. The real dip is

(A) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (B) $\tan^{-1}\left(\sqrt{3}\right)$

(C) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$ (D) $\tan^{-1}\frac{2}{\sqrt{3}}$

73. The magnetic susceptibility of a material of a rod is 499. Permeability of vacuum is $4\pi \times 10^{-7}$ H/m. Absolute permeability of the material of the rod in henry per metre is

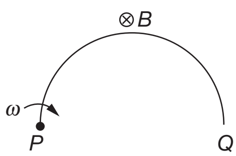
(A) $\pi \times 10^{-4}$ (B) $2\pi \times 10^{-4}$

(C) $3\pi \times 10^{-4}$ (D) $4\pi \times 10^{-4}$

74. Three long straight wires are connected parallel to each other across a battery of negligible internal resistance. The ratio of their resistances are 3 : 4 : 5. What is the ratio of distances of middle wire from the others if the net force experienced by it is zero?

(A) 4 : 3 (B) 3 : 1 (C) 5 : 3 (D) 2 : 3

75. A wire is bent to form a semi-circle of radius a. The wire rotates about its one end with angular velocity ω. Axis of rotation being perpendicular to plane of the semicircle. In the space, a uniform magnetic field of induction B exists along the axis of rotation as shown. The correct statement is



(A) potential difference between P and Q is equal to $2B\omega a^2$.

(B) potential difference between P and Q is equal to $2\pi^2 B\omega a^2$.

(C) P is at higher potential than Q.

(D) potential difference between P and Q is zero.

76. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} Wb/m^2. Another long solenoid has 100 turns per cm and it carries a current $\frac{i}{3}$. The value of the magnetic field at its centre is

(A) 1.05×10^{-4} Wb/m^2

(B) 1.05×10^{-2} Wb/m^2

(C) 1.05×10^{-5} Wb/m^2

(D) 1.05×10^{-3} Wb/m^2

77. A proton with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field $B = -B_0\hat{k}$ which exists to the right of y-axis. The proton exits from the region after some time with the speed v at ordinate y, then

(A) $v > u,\ y < 0$ (B) $v = u,\ y > 0$

(C) $v > u,\ y > 0$ (D) $v = u,\ y < 0$

78. A conducting wire bent in the form of a parabola $y^2 = 2x$ carries a current $i = 2$A as shown in Fig. 15.48. This wire is placed in a uniform magnetic field $\vec{B} = -4\hat{k}$ T. The magnetic force on the wire is

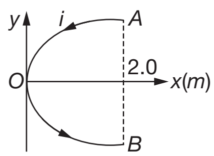


Fig. 15.48

(A) $-16\hat{i}$ (B) $32\hat{i}$ (C) $-32\hat{i}$ (D) $16\hat{i}$

79. In Fig. 15.49, a coil of single turn is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the inclined plane and lies in the equatorial plane of the sphere. Current in the coil is i. The value of B if the sphere is in equilibrium is

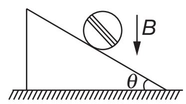


Fig. 15.49

(A) $\frac{mg\cos\theta}{\pi iR}$ (B) $\frac{mg}{\pi iR}$

(C) $\frac{mg\tan\theta}{\pi iR}$ (D) $\frac{mg\sin\theta}{\pi iR}$

80. Three long, straight and parallel wires C, D, and G carrying currents are arranged as shown in Fig. 15.50. The force experienced by a 25 cm length of wire C is

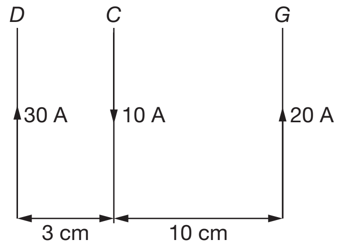


Fig. 15.50

(A) 0.4 N (B) 0.04 N
(C) 4×10^{-3} N (D) 4×10^{-4} N

81. A positive charge q is projected in magnetic field of width $\frac{mv}{\sqrt{2}\,qB}$ with velocity v as shown in Fig. 15.51. Then time taken by charged particle to emerge from the magnetic field is

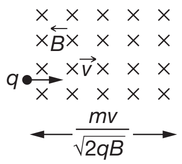


Fig. 15.51

(A) $\frac{m}{\sqrt{2}\,qB}$ (B) $\frac{\pi m}{4qB}$
(C) $\frac{\pi m}{2qB}$ (D) $\frac{\pi m}{\sqrt{2}\,qB}$

82. Magnetic inductance B exists as shown in Fig. 15.52. On the other side of line. A charged particle of charge q and mass m enters the magnetic field at 45°. The displacement of the particle when it emerges out of the magnetic field will be (velocity of the particle is v)

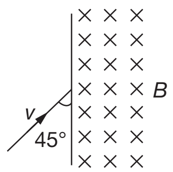


Fig. 15.52

(A) $\frac{\sqrt{2}mv}{qB}$ (B) $\frac{mv}{qB}$ (C) $\frac{mv}{\sqrt{2}qB}$ (D) $\frac{\sqrt{3}mv}{qB}$

83. One conducting U tube can slide inside another as shown in Fig. 15.53, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of Fig. 15.53. If each tube moves towards the other at a constant speed v, then the EMF induced in the circuit in terms of B, l, and v, where l is the width of each tube, will be

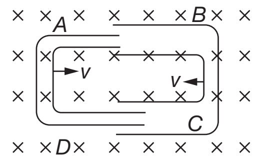


Fig. 15.53

(A) Blv (B) $-Blv$
(C) Zero (D) 2 Blv

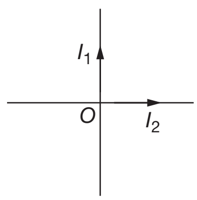
84. When a long wire carrying a steady current is bent into a circular coil of one turn, the magnetic induction at its centre is B. When the same wire carrying the same current is bent to form a circular coil of n turns of a smaller radius, the magnetic induction at the centre will be

(A) B/n (B) nB
(C) B/n^2 (D) n^2B

85. Two infinitely long, thin, insulated, straight wires lies in the x-y plane along the x- and y-axes, respectively. Each wire carries a current I, respectively, in the positive x-direction and positive y-direction. The magnetic field will be zero at all points on the straight line.

(A) $y = x$ (B) $y = -x$
(C) $y = x - 1$ (D) $y = -x + 1$

86. Two long wires carrying current are kept crossed (not joined at O). The locus where magnetic field is zero is



(A) $I_1 = \frac{x}{y} I_2$ (B) $I_1 = \frac{y}{x} I_2$
(C) $x = y$ (D) $x = -y$

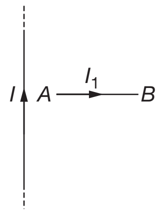
87. A particle of charge q and mass m starts moving from origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{k}$. Its velocity at $(x, 0, 0)$ is $6\hat{i} + 8\hat{j}$. The value of x is

(A) $\frac{25m}{qE_0}$ (B) $\frac{100m}{qB_0}$
(C) $\frac{50m}{qE_0}$ (D) $\frac{14m}{qE_0}$

88. Two long wires are hanging freely. They are joined first in parallel and then in series and they are connected with a battery. In both cases, which type of force acts between the two wires?

(A) attraction force when in parallel and repulsion force when in series.
(B) repulsion force when in parallel and attraction force when in series.
(C) repulsion force in both cases.
(D) attraction force in both cases.

89. A short wire AB carrying I_1 current lies in the plane of long wire which carry I current upward. If wire AB is released from horizontal position and a_A and a_B are magnitude of acceleration of points A and B, respectively, then select the correct alternative. (The space is gravity-free.)

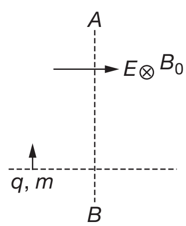


(A) $a_A > a_B$ (B) $a_A < a_B$
(C) $a_A = a_B \neq 0$ (D) $a_A = a_B = 0$

90. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then

(A) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards.
(B) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$.
(C) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$.
(D) $\vec{F_1} = \vec{F_2} = 0$.

91. A uniform electric field E is present horizontally along the paper throughout the region but uniform magnetic field B_0 is present horizontally (perpendicular to plane of paper in inward direction) right to the line AB as shown. A charge particle having charge q and mass m is projected vertically upward and crosses the line AB after time t_0. Find the speed of projection, if particle moves after t_0 with constant velocity (given $qE = mg$).



(A) gt_0 (B) $2gt_0$
(C) $\frac{gt_0}{2}$
(D) Particle can't move with constant velocity after crossing AB

92. There is a small metallic ring of radius l_0 having negligible resistance placed perpendicular to a constant magnetic field B_0. One end of a rod is hinged at the centre of ring O and other end is placed on the ring. Now rod is rotated with constant angular velocity ω_0 by some external agent and circuit is connected as shown in Fig. 15.55, initially switch is open and capacitor is uncharged. If switch S is closed at $t = 0$, then calculate heat loss from the resistor R_2 from $t = 0$ to the instant when voltage across the capacitor becomes v_0 (Assume plane of ring to be horizontal and friction to be an absent at all the contacts). (Assume, $R_2 = 2R_1$, $B_0 l_0^2 \omega_0 = 4v_0$)

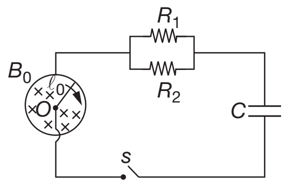


Fig. 15.55

(A) $\frac{1}{2}Cv_0^2$ (B) $\frac{1}{6}Cv_0^2$
(C) $\frac{2}{3}Cv_0^2$ (D) $\frac{1}{3}Cv_0^2$

93. A particle charge q and mass m is projected with a velocity v_0 towards a circular region having uniform magnetic field B perpendicular and into the plane of paper from point P as shown in Fig. 15.56. R is the radius and O is the centre of the circular region. If the line OP makes an angle θ with the direction of v_0 then the value of v_0 so that particle passes through O is

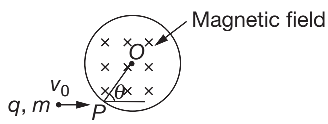


Fig. 15.56

(A) $\frac{qBR}{m\sin\theta}$ (B) $\frac{qBR}{2m\sin\theta}$
(C) $\frac{2qBR}{m\sin\theta}$ (D) $\frac{3qBR}{2m\sin\theta}$

94. A very long current carrying wire is placed along z-axis having current of magnitude i_1 towards negative z-axis. A semicircular wire of radius R and having current i_2 is placed in x-y plane, such that line joining two end points of the semicircular wire passes through long wire as shown in Fig. 15.57. Nearest distance of semicircular wire from long wire is R. Net magnetic force on semicircular wire will be

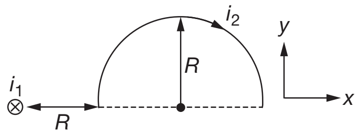


Fig. 15.57

(A) $\frac{\mu_0 i_1 i_2}{2\pi}\ln 3$ (B) $\frac{\mu_0 i_1 i_2}{2\pi}\ln\frac{3}{2}$

(C) zero (D) $\frac{\mu_0 i_1 i_2}{2\pi}$

95. Calculate the magnetic moment associated with the loop carrying current I_0 as shown in Fig. 15.58 is

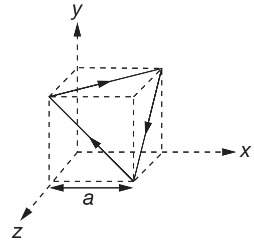


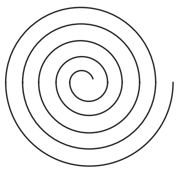
Fig. 15.58

(A) $\frac{3\sqrt{3}}{2}I_0a^2$ (B) $\frac{2\sqrt{2}}{3}I_0a^2$

(C) $\frac{2}{5}I_0a^2$ (D) $\frac{\sqrt{3}}{2}I_0a^2$

More than One Option Correct Type

96. A charged particle enters a region which offers some resistance against its motion, and a uniform magnetic field exists in the region. The particle traces a spiral path as shown. Then



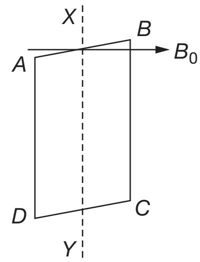
(A) angular velocity of particle remains constant.
(B) speed of particle decreases continuously.
(C) total mechanical energy of the particle remains conserved.
(D) net force on the particle is always perpendicular to its direction of motion.

97. A flat coil $ABCD$, of n turns, area A, and resistance R is placed in a uniform magnetic field of magnitude B_0. The plane of the coil is initially perpendicular to magnitude field B_0. If the coil is rotated by an angle θ about the axis XY (passing through centre and parallel to AD), charge of amount Q flows through it.

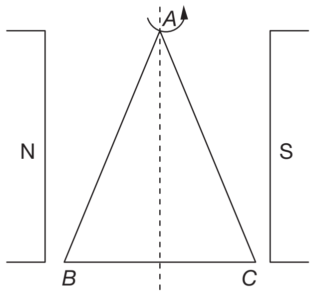
(A) If $\theta = 90^0$, $Q = \frac{BAn}{R}$

(B) If $\theta = 180^0$, $Q = \frac{BAn}{R}$

(C) If $\theta = 180°$, $Q = 0$
(D) If $\theta = 360°$, $Q = 0$



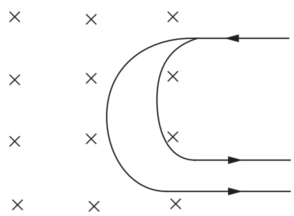
98. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from vertex such that it is hanging in a vertical plane between the pole-pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T. Current in loop is 0.1 A. Then



(A) magnetic moment of the loop is $\sqrt{3}\times10^{-5}$ A/m^2.
(B) magnetic moment of the loop is 1×10^{-6} A/m^2.
(C) couple acting on the coil is $5\sqrt{3}\times10^{-7}$ N/m.
(D) couple acting on the coil is $\sqrt{3}\times10^{-7}$ N/m.

99. Two circular coils of radii 5 cm and 10 cm carry equal currents of 2 A. The coils have 50 and 100 turns, respectively, and are placed in such a way that their planes and their centres coincide. Magnitude of magnetic field at the common centre of coils is,
(A) $8\pi\times10^{-4}$ T if currents in the coil are in same direction.
(B) $4\pi\times10^{-4}$ T if currents in the coil are in opposite direction.
(C) zero, if currents in the coils are in opposite direction.
(D) $8\pi\times10^{-4}$ T if currents in the coil are in opposite direction.

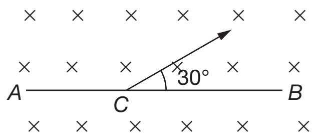
100. A narrow beam of singly charged carbon ions, moving at a constant velocity of 6×10^4 m/s is sent perpendicularly in a rectangular region having uniform magnetic field $B = 0.5$ T. It is found that two beams emerge from the field in the backward direction, the separations from the incident beam being 3 cm and 3.5 cm. If mass of an ion = A (1.66×10^{-27})kg, where A is the mass number then isotopes present in beam are



(A) ^{11}C (B) ^{12}C
(C) ^{13}C (D) ^{14}C

101. Ratio of radii of paths when an electron and a proton enters at right angles to a uniform field with
(A) same velocity is $\frac{1}{1840}$.
(B) same momentum is 1.
(C) same kinetic energy is $\frac{1}{43}$.
(D) same kinetic energy is 43.

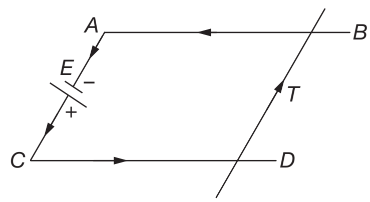
102. A conducting rod AB of length l = 1m is moving at a velocity v = 4 m/s making an angle 30° with its length. C is the middle point of the rod. A uniform magnetic field B = 2 T exists in a direction perpendicular to the plane of motion, then



(A) $v_A - v_B = 8$ v (B) $v_A - v_B = 4$ v
(C) $v_B - v_C = 2$ v (D) $v_B - v_C = -2$ v

103. Velocity and acceleration vector of a charged particle moving in a magnetic field at some instant are $\vec{v} = 3\hat{i} + 4\hat{j}$ and $\vec{a} = 2\hat{i} + x\hat{j}$. Select the correct alternative (s)
(A) $x = -1.5$ (B) $x = 3$
(C) Magnetic field is along z-direction
(D) Kinetic energy of the particle is constant

104. AB and CD are smooth, parallel, horizontal rails on which a conductor T can slide. A cell, E, drives current i through the rails and T



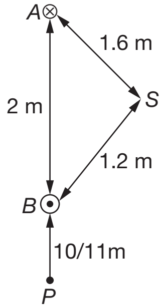
(A) The current in the rails will set up a magnetic field over T.
(B) T will experience a force to the right.
(C) T will experience a force to the left.
(D) T will not experience any force.

105. A conductor AB carries a current i in a magnetic field $\vec{B}$. If $\vec{AB} = \vec{r}$ and the force on the conductor is $\vec{F}$,
(A) $\vec{F}$ does not depend on the shape of AB.
(B) $\vec{F} = i(\vec{r}\times\vec{B})$.
(C) $\vec{F} = i(\vec{B}\times\vec{r})$.
(D) $|\vec{F}| = i(\vec{r}\cdot\vec{B})$.

Passage Based Questions

Passage I

Two long straight parallel wires are 2 m apart, perpendicular to the plane of the paper. The wire *A* carries a current of 9.6 ampere, directed into the plane. The wire *B* carries a current such that magnetic field of induction at the point *P*, at a distance of $\frac{10}{11}$ m from the wire *B*, is zero.



106. The magnitude of the current (amp) in *B* is
(A) 1 (B) 2 (C) 3 (D) 4

107. The magnitude of the magnetic field (tesla) of induction at the point *S* is
(A) 14×10^{-7} (B) 13×10^{-7}
(C) 12×10^{-7} (D) 11×10^{-7}

108. The force per unit length on the wire *B* is
(A) 2.88×10^{-6} N/m (B) 9.6×10^{-6} N/m
(C) 3×10^{-6} N/m (D) 2.52×10^{-6} N/m

109. Magnitude of the magnetic field (tesla) at the midpoint of *AB* is
(A) 2.88×10^{-6} (B) 9×10^{-6}
(C) 3×10^{-6} (D) 2.52×10^{-6}

Passage 2

Torque acting on a current carrying loop in uniform magnetic field is given by $\vec{\tau} = \vec{M} \times \vec{B}$. Net force on it is zero. If it is free to rotate, then it will rotate about an axis passing through its centre of mass and parallel to $\vec{\tau}$. Potential energy of the loop is given by $U = -\vec{M}.\vec{B}$. Assume a current carrying ring with its centre at the origin and moment of inertia $2\times10^{-2}\ \text{kg/m}^2$ about an axis passing through its centre and perpendicular to its plane has magnetic moment $\vec{M} = (3\hat{i} - 4\hat{j})\,\text{A/m}^2$. At time $t = 0$, a magnetic field $\vec{B} = (4\hat{i} + 3\hat{j})T$ is switched on. Then

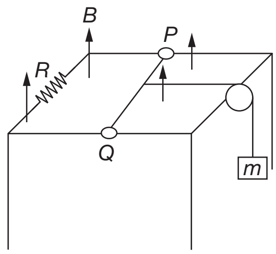
110. Torque acting on the loop is
(A) Zero (B) $25\hat{k}$ N/m
(C) $16\hat{k}$ N/m (D) $10\hat{k}$ N/m

111. Angular acceleration of the ring at time $t = 0$ (in rad/s^2) is
(A) 5000 (B) 1250 (C) 2500 (D) Zero

112. Maximum angular velocity of the ring (in rad/s) will be
(A) $50\sqrt{2}$ (B) $25\sqrt{2}$ (C) $100\sqrt{2}$ (D) $150\sqrt{2}$

Passage 3

A U-shape wire is formed by connecting a resistance *R* with two parallel conducting rods at its both ends and kept on a horizontal table. A sliding wire *PQ* of length *l* and mass *m* can slide over the U-Shape wire without friction. This rod *PQ* is connected to a block of mass *m* through an insulating string going over a smooth pulley as shown. A vertical magnetic field *B* is switched on and rod *PQ* is allowed to move from rest. After some time, it attains a constant velocity v_0 (also called terminal velocity). Now mass of the block is tripled, it attain another terminal velocity v_T.



(Given $B = \sqrt{10}T$, $l = 1$ m, $v_0 = 10$ m/s, $R = 10\,\Omega$, $g = 10$ m/s^2)

113. The value of mass *m* is
(A) 2 kg (B) 0.5 kg (C) 1 kg (D) 1.5 kg

114. The value of terminal velocity v_T is
(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

115. Find the acceleration of block at the instant when its velocity is $\left(\frac{v_0 + v_T}{2}\right)$
(A) 2.5 m/s^2 (B) 5 m/s^2
(C) 10 m/s^2 (D) 1.25 m/s^2

Passage 4

A charge particle of charge 1 C and mass 10 gm is moving with velocity 10 $\hat{i}$ m/s in horizontal plane consisting of magnetic field –0.1 $\hat{k}$ T of width $\frac{\sqrt{3}}{2}$ m and electric field –0.1 $\hat{j}$ N/C of width *d*. Particle enters the magnetic field at *t* = 0 perpendicularly and follows the path as shown in Fig. 15.59 and strikes the wall *AB* perpendicularly at *A′* (neglect gravity).

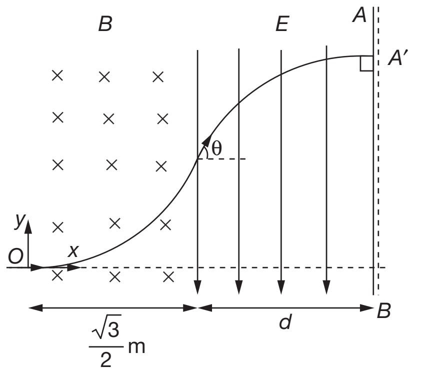


Fig. 15.59

116. The time t after which it strikes the wall

(A) 1.2 s (B) 0.97 s (C) 1.07 s (D) 1.98 s

117. The width d of electric field is

(A) $5\sqrt{3}$ m (B) $\frac{5}{2}\sqrt{3}$ m

(C) $10\sqrt{3}$ m (D) $\frac{5}{4}\sqrt{3}$ m

118. Taking O as origin, y-coordinate of A' will be

(A) 3.75 m (B) 0.50 m

(C) 4.25 m (D) 3.25 m

119. Angle θ at which it enters the electric field will be

(A) 30° (B) 45° (C) 53° (D) 60°

Passage 5

An ammeter and a voltmeter are connected in series to a battery with EMF $E = 6$ V and negligible resistance. When a resistance $R = 3\ \Omega$ is connected in parallel to voltmeter, reading of ammeter increases three times while that of voltmeter reduces to one-third.

120. The resistance of ammeter is

(A) 24 Ω (B) 8 Ω (C) 4 Ω (D) 3Ω

121. The resistance of voltmeter is

(A) 24 Ω (B) 8Ω (C) 4Ω (D) 3Ω

122. Reading of voltmeter after the connection of resistance is

(A) 1 V (B) 3 V

(C) $\frac{9}{2}$ V (D) $\frac{3}{2}$ V

123. Reading of ammeter before the connection of the resistance is

(A) $\frac{3}{4}$ A (B) $\frac{6}{7}$ A (C) $\frac{3}{16}$ A (D) 1 A

Match the Column Type

124.

Column-I	Column-II
(A) A charge particle is moving in uniform electric and magnetic field in gravity-free space.	1. Velocity of the particle may be constant.
(B) A charge particle is moving in uniform electric magnetic and gravitational field.	2. Path of particle may be straight line.
(C) A charge particle is moving in uniform magnetic and gravitational field (where electric field is zero).	3. Path of particle may be circular.
(D) A charge particle is moving in only uniform electric field.	4. Path of particle may be helical.

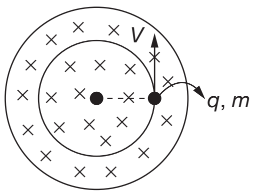
125. Two long parallel wires carrying equal currents in opposite directions are placed at $x = \pm a$ parallel to y-axis with $z = 0$. Then

Column-I	Column-II
(A) Magnetic field B_1 at origin O	1. $\frac{\mu_0 i}{3\pi a}$
(B) Magnetic field B_2 at P $(2a, 0, 0)$	2. $\frac{\mu_0 i}{4\pi a}$
(C) Magnetic field at M $(a, 0, 0)$	3. $\frac{\mu_0 i}{\pi a}$
(D) If wire carries current in the same direction, then magnetic field at origin	4. Zero
	5. None

126.

Column-I	Column-II
(A) Moving charge	1. Produce conservative electric field
(B) Time varying magnetic field must induce	2. Produce non-conservative electric field
(C) Stationary charge	3. Magnetic field
(D) Between the plates of a charged capacitor	4. Current

127. The central cross-section of a long cylindrical region containing uniform but time varying magnetic field B is shown. A particle of constant mass and variable positive charge moves in a circle in the plane, so that the radius of the circle remains constant.



Column-I	Column-II
(A) If the magnetic field is increased by 2%, the speed of the particle will	1. Decrease
(B) If the magnetic field is decreased by 4%, the speed of the particle will	2. Increase
(C) If the magnetic field is increased by 2%, the charge of the particle will	3. Change by 1%
(D) If the magnetic field is decreased by 4%, the charge of the particle will	4. Change by 2%

128.

Column-I	Column-II
(A) Magnetic flux density due to a current carrying circular coil is	1. Zero
(B) Magnetic flux density at a point on a current carrying thin wire is	2. Maximum at the centre
(C) Electric field strength due to an uniformly charged ring is	3. Continuously decreases as we move away from the centre along the axis.
(D) Electric potential due to an uniformly charged ring is	4. Continuously increases as we move away from the centre up to a definite distance along the axis.

Assertion-Reason Type

129. Assertion: A charged particle at rest experiences no electromagnetic force.

Reason: The electric and magnetic field must be zero.

(A) A (B) B (C) C (D) D

130. Assertion: A charged particle moves perpendicular to magnetic field. Its kinetic energy remains constant, but momentum changes.

Reason: Force acts perpendicular to velocity of the particle.

(A) A (B) B (C) C (D) D

131. Assertion: A beam of electron can pass undeflected through a region of $\vec{E}$ and $\vec{B}$.

Reason: Force on moving charge particle due to magnetic field may be zero.

(A) A (B) B (C) C (D) D

132. Assertion: Electric lines of forces are perpendicular to equipotential surface.

Reason: Work done by electric field on moving a positive charge on equipotential surface is always zero.

(A) A (B) B (C) C (D) D

133. Assertion: Magnetic field lines are different from magnetic lines of force.

Reason: $\vec{F} = q\vec{v} \times \vec{B}$

(A) A (B) B (C) C (D) D

134. Assertion: A conducting circular disc of radius R rotates about its own axis with angular velocity ω in uniform magnetic field B_0 along axis of the disc then no EMF is induced in the disc.

Reason: Whenever a conductor cuts across magnetic lines of flux, an EMF is induced in the conductor.

(A) A (B) B (C) C (D) D

135. Assertion: A metal ring is placed in a magnetic field, with its plane perpendicular to the field. If the magnitude of magnetic field begins to change, the ring may experience a tension along its length.

Reason: In above assertion, when the field changes, a current flows along the ring. Each section of the ring now experiences a force acting outward or inward.

(A) A (B) B (C) C (D) D

136. Assertion: The mathematical statement of ampere's law $\oint \vec{B}\cdot\vec{dl} = \mu_0 i$ is true when variable electric field is not present in the medium.

Reason: A variable electric field produces magnetic field.

(A) A (B) B (C) C (D) D

137. Assertion: A charged particle moves perpendicular to a uniform magnetic field then its momentum remains constant.

Reason: Magnetic force acts perpendicular to the velocity of the particle.

(A) A (B) B (C) C (D) D

138. Assertion: If current enclosed by a closed loop is zero, then magnetic field at all the points of loop is zero.

Reason: $\int_C \vec{B}\cdot\vec{dl} = \mu_0 i_{enc}$

(A) A (B) B (C) C (D) D

Integer Type

139. A semicircular loop of mass per unit length $\frac{\pi}{10}$ kg/m is placed on a surface with its plane parallel to the surface in a uniform magnetic field of magnitude 1 T as shown in Fig. 15.60. Find the minimum amount of current that should be passed in the loop so that it just starts to rotate.

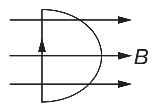
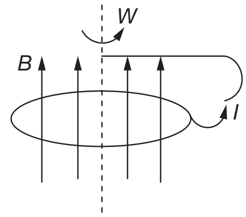


Fig. 15.60

140. An electron accelerated by a potential difference $V = 3.6$ V first enters into a uniform electric field of a parallel-plate capacitor whose plates extend over a length $l = 6$ cm in the direction of initial velocity. The electric field is normal to the direction of initial velocity and its strength varies with time as $E = a \times t$, where $a = 3200\ Vm^{-1}s^{-1}$. Then the electron enters into a uniform magnetic field of induction $B = \pi \times 10^{-9}$ T. Direction of magnetic field is same as that of the electric field. Calculate pitch (in mm) of helical path traced by the electron in the magnetic field (Mass of electron, $m = 9 \times 10^{-31}$ kg). [Neglect the effect of induced magnetic field.]

141. A metal disc of radius $R = 6$ cm is mounted on a frictionless axle. The current can flow through the axle out along the disc to a sliding contact of rim of the disc. A uniform magnetic field $B = 2$ T is parallel to the axis of the disc. When the current is 3 A, the disc rotates with constant angular velocity. The frictional force at the rim between the stationary electrical contact and the rotating rim is $9x \times 10^{-2}$ N. Find the value of x.



142. A circular loop has a radius π cm and carries a current I_2 in a clockwise direction. The centre of the loop is at distance $d = 8$ cm above along straight wire. If the magnetic field at the centre is zero. Find ratio of I_1 and I_2.

143. A thin conducting ring of mass m = 1 gm having total charge $q = 2 \times 10^{-3}$ C can rotate freely about its own axis. Initially, when the ring is stationary, a magnetic field B = 1 T directed perpendicular to the plane is switched on. Find the angular velocity (rad/s) acquired by the ring.

144. A non-conducting non-magnetic rod having circular cross-section of radius R is suspended from a rigid support as shown in Fig. 15.61. A light and small coil of 300 turns is wrapped tightly at the left end of the rod where uniform magnetic field B exists in vertically downward direction. Air of density ρ hits one half of the right part of the rod with velocity v as shown in Fig. 15.61. What should be current in clockwise direction (as seen from O) in the coil so that rod remains horizontal? Give answer in mA, given $\frac{2}{Lv}\sqrt{\frac{\pi RB}{\rho}} = 1\text{A}^{-1/2}$.

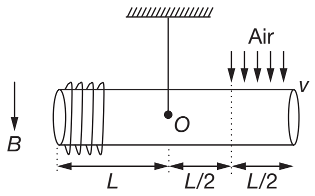


Fig. 15.61

145. The region between $X = 0$ and $X = Lm$ is filled with uniform steady magnetic field $2\,\text{T}\,\hat{k}$. A particle of mass 2 kg, positive charge 1C and velocity $2(\text{m/s})\,\hat{i}$ travels along x-axis and enters the region of the magnetic field (neglect gravity). Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.

146. A proton is fired with a speed of 5×10^7 m/s at an angle of 30° to a magnetic field $\vec{B} = 0.40\,\hat{i}$ T. The pitch of the proton will be (in cm)

147. A particle having mass m and charge q is projected with velocity v_0 along y-axis in a region of uniform magnetic field B_0 which is outward and perpendicular to the plane of the paper as shown in Fig. 15.62. The particle is continuously subjected to a frictional force which varies with velocity as $\vec{F}_r = -\alpha\vec{v}$, where α is a constant. Consequently, the particle moves on a spiral path till it comes to rest at point P. Find the x-co-ordinate of point P.

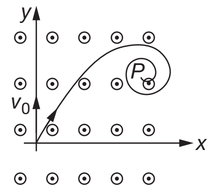


Fig. 15.62

(Take $\alpha = 10^{-3}$ kg/s, $q = 10^{-3}$ C, $B_0 = 1\,\text{T}$, $v_0 = 1\,\text{m/s}$, $m = 20$ gm)

148. A bar magnet has a magnetic moment of 2.5 JT^{-1} and is placed in a magnetic field of 0.2 T. Work done in turning the magnet from parallel to antiparallel position relative to field direction is

Previous Years' Questions

149. The time period of a charged particle undergoing a circular motion in a uniform magnetic filed is independent its **[2002]**

(A) Speed (B) Mass
(C) Charge (D) Magnetic induction

150. If an electron and a proton having same momenta enter perpendicular to a magnetic field, then

(A) curved path of electron and proton will be same (ignoring the sense of revolution). **[2002]**
(B) they will move undeflected.
(C) curved path of electron is more curved than that of the proton.
(D) path of proton is more curved.

151. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields B_A and B_B produced by them will be **[2002]**

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 4

152. A particle of mass M and charge Q moving with velocity $\vec{v}$ describe a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is **[2003]**

(A) $\left(\frac{Mv^2}{R}\right)2\pi R$ (B) Zero
(C) $BQ2\pi R$ (D) $BQv2\pi R$

153. Curie temperature is the temperature above which **[2003]**

(A) a ferromagnetic material becomes paramagnetic.
(B) a paramagnetic material becomes diamagnetic.
(C) a ferromagnetic material becomes diamagnetic.
(D) a paramagnetic material becomes ferromagnetic.

154. The magnetic lines of force inside a bar magnet **[2003]**

(A) are from north pole to south pole of the magnet.
(B) does not exist.
(C) depend upon the area of cross-pole of the magnet.
(D) are from south pole to north pole of the magnet.

155. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this position will be **[2003]**

(A) $\sqrt{3}\,W$ (B) W
(C) $\frac{\sqrt{3}}{2}W$ (D) $2W$

156. The materials suitable for making electromagnets should have **[2004]**

(A) high retentivity and low coercivity.
(B) low retentivity and low coercivity.
(C) high retentivity and high coercivity.
(D) low retentivity and high coercivity.

157. Two long conductors, separated by a distance *d*, carry current I_1 and I_2 in the same direction. They exert a force *F* on each other. Now the current in one of them is doubles and its direction is reversed. The distance is also increased to 3*d*. The new value of the force between them is **[2004]**

(A) $-\frac{2F}{3}$ (B) $\frac{F}{3}$ (C) $-2F$ (D) $-\frac{F}{3}$

158. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is $54\,\mu$T. What will be its value at the centre of loop? **[2004]**

(A) 125 μT (B) 150 μT
(C) 250 μT (D) 75 μT

159. A current *i* ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is **[2004]**

(A) $\frac{\mu_0}{4\pi}, \frac{2i}{r}$T (B) Zero
(C) Infinite (D) $\frac{2i}{r}$T

160. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then **[2005]**

(A) its velocity will increase.
(B) its velocity will decrease.
(C) it will turn towards left of direction of motion.
(D) it will turn towards right of direction of motion.

161. A magnetic needle is kept in a non-uniform magnetic field. It experiences **[2005]**

(A) neither a force nor a torque.
(B) a torque but not a force.
(C) a force but not a torque.
(D) a force and a torque.

162. A charged particle of mass *m* and charge *q* travels on a circular path of radius *r* that is perpendicular to complete one revolution is **[2005]**

(A) $\frac{2\pi q^2 B}{m}$ (B) $\frac{2\pi mq}{B}$
(C) $\frac{2\pi m}{qB}$ (D) $\frac{2\pi qB}{m}$

163. A long solenoid has 200 turns per cm and carries a current *i*. The magnetic field at its centre is 6.28×10^{-2} Wb/m^2 Another long solenoid has 100 turns per cm and it carries a current $\frac{i}{3}$. The value of the magnetic field at its centre is **[2006]**

(A) 1.05×10^{-2} Wb/m^2
(B) 1.05×10^{-5} Wb/m^2
(C) 1.05×10^{-3} Wb/m^2
(D) 1.05×10^{-4} Wb/m^2

164. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a **[2006]**

(A) Helix (B) Straight line
(C) Ellipse (D) Circle

165. Needles N_1, N_2, and N_3 are made of a ferromagnetic, paramagnetic, and a diamagnetic substance, respectively. A magnet when brought close to them will **[2006]**

(A) attract N_1 and N_2 strongly but repel N_3
(B) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(C) attract N_1 strongly, but repel N_2 and N_3 weakly
(D) attract all three of them

166. Two identical conducting wires *AOB* and *COD* are placed at right angles to each other. The wire *AOB* carries an electric current I_1 and *COD* carries a current I_2. The magnetic field on a point lying at a distance *d* from *O*, in a direction perpendicular to the plane of the wires *AOB* and *COD*, will be given by **[2007]**

(A) $\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)$ (B) $\frac{\mu_0}{2\mu}\left(\frac{I_1+I_2}{d}\right)^{\frac{1}{2}}$
(C) $\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)^{\frac{1}{2}}$ (D) $\frac{\mu_0}{2\pi d}\left(I_1+I_2\right)$

167. A current *I* flows along the length of an infinitely long, straight, thin-walled pipe. Then **[2007]**

(A) the magnetic field at all points inside the pipe is the same, but not zero.
(B) the magnetic field is zero only on the axis of the pipe.
(C) the magnetic field is different at different points inside the pipe.
(D) the magnetic field at any point inside the pipe is zero.

168. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $a/2$ and $2a$ is **[2007]**

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 4 (D) 1

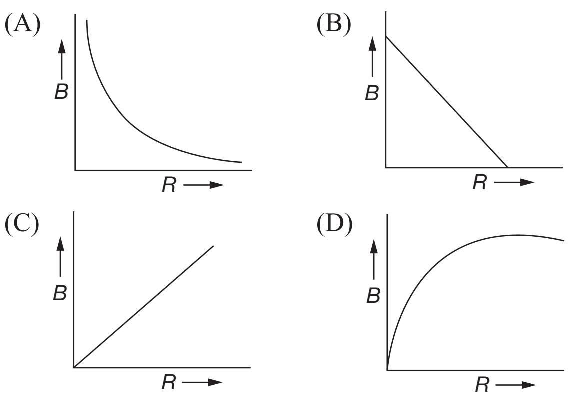
168. Relative permittivity and permeability of a material ε_r and μ_r respectively. Which of the following value of these quantities are allowed for a diamagnetic material? **[2008]**

(A) $\varepsilon_r = 0.5, \mu_r = 1.5$ (B) $\varepsilon_r = 1.5, \mu_r = 0.5$
(C) $\varepsilon_r = 0.5, \mu_r = 0.5$ (D) $\varepsilon_r = 1.5, \mu_r = 1.5$

170. A horizontal overhead powerline is at height of 4m from the ground and carries a current of 100A from east to west. The magnetic field directly below it on the ground is $\left(\mu_0 = 4\pi\times10^{-7}\,TmA^{-1}\right)$ **[2008]**

(A) 2.5×10^{-7} T southward
(B) 5×10^{-6} T northward
(C) 5×10^{-6} T southward
(D) 2.5×10^{-7} T northward

171. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through the centre with an angular velocity ω. As a result of this rotation, a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic induction at the centre of the disc will be represented by the figure **[2012]**



172. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to **[2013]**

(Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

(A) 2.56×10^{-4} Wb/m^2
(B) 3.50×10^{-4} Wb/m^2
(C) 5.80×10^{-4} Wb/m^2
(D) 3.6×10^{-5} Wb/m^2

173. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see Fig. 15.63). For a field $\vec{B} = 3.0\times10^{-4}\,e^{-0.2x}\ \hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis. **[2014]**

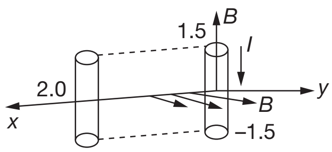


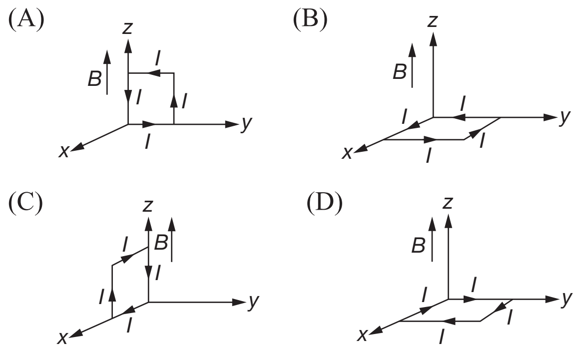
Fig. 15.63

(A) 1.57 W (B) 2.97 W
(C) 14.85 W (D) 29.7 W

174. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 A m^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid is **[2014]**

(A) 30 mA (B) 60 mA
(C) 3 A (D) 6 A

175. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below. **[2015]**



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?
(A) (A) and (C), respectively
(B) (B) and (D), respectively
(C) (B) and (C), respectively
(D) (A) and (B), respectively

176. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in Fig. 15.64, with threads making an angle θ with the vertical. If wires have mass λ per unit length, then the value of I is **[2015]**

(g = gravitational acceleration)

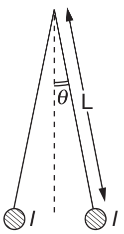


Fig. 15.64

(A) $2\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$ (B) $2\sqrt{\dfrac{\pi gL}{\mu_0}\tan\theta}$

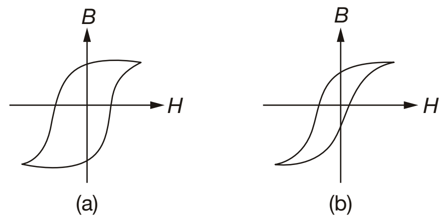
(C) $\sqrt{\dfrac{\pi\lambda gL}{\mu_0}\tan\theta}$ (D) $\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$

177. Two identical wires A and B, each of length 'ℓ', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\dfrac{B_A}{B_B}$ is **[2016]**

(A) $\dfrac{\pi^2}{16\sqrt{2}}$ (B) $\dfrac{\pi^2}{16}$

(C) $\dfrac{\pi^2}{8\sqrt{2}}$ (D) $\dfrac{\pi^2}{8}$

178. Hysteresis loops for two magnetic materials A and B are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use: **[2016]**
(A) A for electromagnets and B for electric generators.
(B) A for transformers and B for electric generators.
(C) B for electromagnets and transformers.
(D) A for electric generators and transformers.

ANSWER KEYS

Single Option Correct Type

1. (D)	**2.** (A)	**3.** (A)	**4.** (B)	**5.** (A)	**6.** (D)	**7.** (D)	**8.** (C)	**9.** (A)	**10.** (A)
11. (C)	**12.** (B)	**13.** (D)	**14.** (D)	**15.** (C)	**16.** (D)	**17.** (D)	**18.** (A)	**19.** (C)	**20.** (C)
21. (D)	**22.** (C)	**23.** (C)	**24.** (D)	**25.** (A)	**26.** (C)	**27.** (A)	**28.** (C)	**29.** (C)	**30.** (C)
31. (D)	**32.** (C)	**33.** (B)	**34.** (A)	**35.** (A)	**36.** (B)	**37.** (D)	**38.** (D)	**39.** (A)	**40.** (B)
41. (C)	**42.** (A)	**43.** (C)	**44.** (B)	**45.** (A)	**46.** (A)	**47.** (D)	**48.** (D)	**49.** (A)	**50.** (C)
51. (C)	**52.** (C)	**53.** (A)	**54.** (C)	**55.** (D)	**56.** (C)	**57.** (C)	**58.** (A)	**59.** (A)	**60.** (C)
61. (A)	**62.** (C)	**63.** (B)	**64.** (A)	**65.** (A)	**66.** (B)	**67.** (C)	**68.** (D)	**69.** (C)	**70.** (B)
71. (A)	**72.** (A)	**73.** (B)	**74.** (C)	**75.** (A)	**76.** (B)	**77.** (B)	**78.** (B)	**79.** (B)	**80.** (D)
81. (B)	**82.** (A)	**83.** (D)	**84.** (D)	**85.** (A)	**86.** (A)	**87.** (C)	**88.** (A)	**89.** (A)	**90.** (D)
91. (B)	**92.** (A)	**93.** (B)	**94.** (A)	**95.** (D)					

More than One Option Correct Type

96. (A) and (B)	**97.** (A) and (D)	**98.** (A) and (C)	**99.** (A) and (C)	**100.** (B) and (D)
101. (A), (B) and (C)	**102.** (B) and (D)	**103.** (A), (C) and (D)	**104.** (A) and (B)	**105.** (A) and (B)

Passage Based Questions

Passage 1

106. (C) **107.** (B) **108.** (A) **109.** (D)

Passage 2

110. (B) **111.** (C) **112.** (A)

Passage 3

113. (C) **114.** (C) **115.** (A)

Passage 4

116. (B) **117.** (B) **118.** (C) **119.** (D)

Passage 5

120. (B) **121.** (A) **122.** (D) **123.** (C)

Match the Column Type

124. (A) $\rightarrow$ 1, 2, 4; (B) $\rightarrow$ 1, 2, 3, 4; (C) $\rightarrow$ 1, 2, 4; (D) $\rightarrow$ 2
125. (A) – 3; (B) $\rightarrow$ 1; (C) – 2; (D) $\rightarrow$ 2
126. (A) $\rightarrow$ 1, 2, 3, 4; (B) $\rightarrow$ (2); (C) $\rightarrow$ 1, 2; (D) $\rightarrow$ 1, 2, 3, 4
127. (A) $\rightarrow$ 2, 3; (B) $\rightarrow$ 1, 4; (C) $\rightarrow$ 1, 3; (D) $\rightarrow$ 2, 4
128. (A) $\rightarrow$ 2, 3; (B) $\rightarrow$ 1; (C) $\rightarrow$ 4; (D) $\rightarrow$ 2, 3

Assertion-Reason Type

129. (C) **130.** (A) **131.** (B) **132.** (A) **133.** (A) **134.** (D) **135.** (A) **136.** (A) **137.** (D) **138.** (D)

Integer Type

139. $I = \dfrac{4\lambda g}{\pi B} = 4$ A
140. $P = 9$ mm
141. 2
142. 8 = 1 rad/s
143. 1 rad/s
144. 10 ma
145. $L = 1$
146. 690 cm
147. 10
148. 1 J

Previous Years' Questions

149. (A) **150.** (A) **151.** (A) **152.** (B) **153.** (A) **154.** (D) **155.** (A) **156.** (B) **157.** (A) **158.** (C)
159. (B) **160.** (B) **161.** (D) **162.** (C) **163.** (A) **164.** (B) **165.** (B) **166.** (C) **167.** (D) **168.** (D)
169. (B) **170.** (C) **171.** (A) **172.** (A) **173.** (B) **174.** (C) **175.** (B) **176.** (A) **177.** (C) **178.** (C)

HINTS AND SOLUTIONS

Single Option Correct Type

1. The correct option is (D)
2. Here magnetic force is zero, but the velocity increases due to electric force.

   The correct option is (A)
3. Magnetic moment vectors of three bar magnets represent three side of a triangle taken in order.

   The correct option is (A)
4. The correct option is (B)
5. $B = \dfrac{\mu_0 I}{4r} + \dfrac{\mu_0 I}{4\pi r} = \dfrac{\mu_0 I}{4\pi r}(1+\pi)$

   The correct option is (A)
6. Due to symmetry of the circuit, field will be zero at centre.

   The correct option is (D)
7. $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi nI}{R}$; so $B \propto \dfrac{I}{R}$;

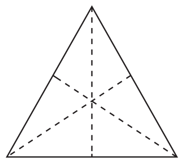
   Hence $\dfrac{B_A}{B_B} = \dfrac{I/R}{2I/2R} = 1.$

   The correct option is (D)
8. $B_0 = \dfrac{\mu_0 I}{2R} \Rightarrow I = \dfrac{7\times10^{-5}\times 2\times 5.0\times10^{-2}}{4\pi\times10^{-7}} = 5.6$ A

   The correct option is (C)

9. $B = 3\left[\frac{\mu_0 i}{4\pi r}(\sin 60° + \sin 60°)\right]$

$= \frac{9\mu_0 i}{2\pi a}$



The correct option is (A)

10. $\Sigma F = 0$; $mg - F_M = 0$; $mg = BIL$

$\Rightarrow \quad v = \frac{mgR}{B^2 l^2}$

The correct option is (A)

11. $f \propto \sqrt{B_H}$; $\quad \frac{f_2}{f_1} = \sqrt{\frac{B_{H_2}}{B_{H_1}}}$

$\Rightarrow \quad B_{H_2} = \left(\frac{f_2}{f_1}\right)^2 B_{H_1} = 32\left(\frac{1}{0.5}\right)^2 = 128 \text{ T}$

The correct option is (C)

12. $W = -MB(\cos\theta_2 - \cos\theta_1) = 1 \text{ J}$

The correct option is (B)

13. The correct option is (D)

14. The correct option is (D)

15. The correct option is (C)

16. The correct option is (D)

17. After collision velocity becomes $v/2$ so radius becomes $a/2$ and hence diameter is a and O will be the centre. So particle strikes at point N.

The correct option is (D)

18. $B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$, $B_2 = \frac{\mu_0}{4\pi}.\frac{M}{d^3}$

Since their like poles are in the same direction, the net magnetic induction at corner is

$B = B_1 - B_2 = \frac{\mu_0}{4\pi}.\frac{2M}{d^3} - \frac{\mu_0}{4\pi}.\frac{M}{d^3} = \frac{\mu_0}{4\pi}.\frac{M}{d^3}$

The correct option is (A)

19. $B = \frac{\mu_0}{4\pi}.\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}.\frac{2I_2}{r_2}$, $I_1 = 2.5\ A$, $I_2 = 5$ A,

and $r_1 = r_2 = 2.5$ m, $B = -\frac{\mu_0}{2\pi}$

The correct option is (C)

20. Sections AB and DE produce no field at O. Sections BC and EF produce equal fields at O, $B = \frac{\mu_0 I}{2\pi a}$.

The correct option is (C)

21. The magnitude of magnetic field due to circular loop at the centre C is

$B = \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

The correct option is (D)

22. $B_0 = \frac{\mu_0 nI}{2r}$, $\quad \frac{B_1}{B_2} = \frac{n_1}{n_2}\left(\frac{r_2}{r_1}\right)$, $n_1 = 1$, $n_2 = 2$, $r_2 = \frac{r_1}{2}$

So, $\frac{B_2}{B_1} = \frac{4}{1}$

The correct option is (C)

23. It experiences force and torque both due to unequal force acting on poles.

The correct option is (C)

24. $F = q(\vec{v} \times \vec{B})$

The correct option is (D)

25. $\vec{F} = \vec{F_e} + \vec{F_m}$, $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = 0$

$\Rightarrow \quad B = \frac{E}{v} = 10^3 \text{ Wb/m}^2$

The correct option is (A)

26. $B_P = \frac{\mu_0 I_2}{2R} = 4 \times 10^{-5} \text{ Wb/m}^2$

$B_Q = \frac{\mu_0 I_1}{2R} = 3 \times 10^{-5} \text{ Wb/m}^2$

$\therefore \quad B = \sqrt{B_P^2 + B_Q^2} = 5 \times 10^{-5} \text{ Wb/m}^2$

The correct option is (C)

27. $\vec{F} = 1(\vec{dl} \times \vec{B})$, $\vec{F} = \vec{F}_{AC} + \vec{F}_{CD} + \vec{F}_{DA} = 0$

The correct option is (A)

28. $R = \frac{mV}{qB} = \frac{\sqrt{2mq\Delta V}}{qB}$ So, $\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

The correct option is (C)

29. $M = i.A$, $3.14 \times 10^{-1} = i \times 3.14 \times 0.01$, $i = 10$ A

The correct option is (C)

30. At P, $B_{\text{net}} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi}\frac{2i_1}{a}\right) + \left(\frac{\mu_0}{4\pi}\frac{2i_2}{a}\right)^2}$

$= \frac{\mu_0}{2\pi a}\left(i_1^2 + i_2^2\right)^{1/2}$

The correct option is (C)

31. Magnetic flux linked with the loop is always zero.

So, induced EMF is also zero

The correct option is (D)

32. $B_0(\text{Half circle}) = \frac{1}{2}\frac{\mu_0 i}{2r} = \frac{\mu_0 i}{4r}$

The correct option is (C)

33. $\dfrac{F}{l}=\dfrac{\mu_0}{4\pi}\dfrac{2i.i}{b}=\dfrac{\mu_0 i^2}{2\pi b}$

The correct option is (B)

34. Small circles ¾ th. Big circles ¼ th part produces effective magnetic fields

$$|\vec{B}_{net}|=\frac{3}{4}\times\frac{\mu_0 I}{2R}+\frac{1}{4}\frac{\mu_0 I}{2(2R)}=\frac{7\mu_0 I}{16R}$$

The correct option is (A)

35. $B\propto\dfrac{1}{(R^2+x^2)^{3/2}}$, $\dfrac{B_0}{B_{3R}}=10\sqrt{10}$

The correct option is (A)

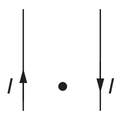
36. $T=\dfrac{2\pi m}{qB}$, $\dfrac{T_\alpha}{T_p}=\dfrac{m_\alpha}{m_p}.\dfrac{q_p}{q_\alpha}=2$

The correct option is (B)

37. $B=\dfrac{\mu_0 I}{2\pi\frac{d}{2}}\times 2=\dfrac{2\mu_0 I}{\pi d}$

Since velocity and magnetic field are parallel or antiparallel

$\therefore\quad F=0$



The correct option is (D)

38. The correct option is (D)

39. Due to electric field E, the force on a particle of charge q is $F=qE$ in the direction of the electric field. Since E is parallel to B, the velocity v of the particle is parallel to B. Hence, B will not affect the motion of the particle since $v\times B=0$.

The correct option is (A)

40. $\dfrac{mv^2}{r}=qvB$ or $v=\dfrac{qB}{m}(r)$

$\therefore\quad v_{min}=\dfrac{qB}{m}(r_{min})=\dfrac{qB}{m}(b-a)$

The correct option is (B)

41. Time period for first $=\dfrac{2\pi m}{QB}$

Time period for second $=\dfrac{8\pi m}{QB}$

They will meet again after $\dfrac{8\pi m}{QB}$

So, displacement in this time interval $=\dfrac{8\pi m}{QB}v\cos\theta$

The correct option is (C)

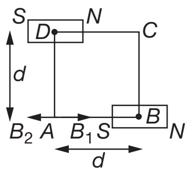
42. Magnetic induction at point A due to magnet at B (axial point)

$B_1=\dfrac{\mu_0}{4\pi}\dfrac{2M}{d^3}$ (along AB)

Magnetic induction at point A due to magnet at D (equatorial point)

$B_2=\dfrac{\mu_0}{4\pi}\dfrac{M}{d^3}$ (along BA)

Resultant $B=B_1-B_2=\dfrac{\mu_0}{4\pi}\left(\dfrac{M}{d^3}\right)$



The correct option is (A)

43. Now $B=\dfrac{\mu_0 I}{4\pi r}(\sin\phi_1+\sin\phi_2)$

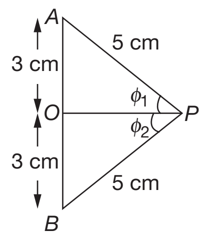
Here $\quad r=OP$

Now, $\quad AO=OB=\dfrac{6}{2}\text{cm}=3\text{ cm}$ and $PB=PA=5\text{cm}$

$\therefore\quad OP=\sqrt{(PB)^2-(OB)^2}=4\text{ cm}$

$\therefore\quad r=4\text{cm}=4\times10^{-2}\text{ m}$

$\sin\phi_1=\dfrac{3}{5}$ and $\sin\phi_2=\dfrac{3}{5}$



The correct option is (C)

44. $Bev=\dfrac{mv^2}{r}$ or $r=\dfrac{mv}{Be}=\dfrac{\sqrt{2Km}}{Be}$

As the electron has been accelerated from rest through a potential difference of V volt, then $K=eV$

$\therefore\quad r=\sqrt{\dfrac{2meV}{B^2e^2}}=\sqrt{\dfrac{2mV}{B^2e}}$

The correct option is (B)

45. Here required angles θ_1 and θ_2 are $(90-\phi_1)$ and $(90-\phi_2)$. Hence

$$B=\frac{\mu_0 I}{4\pi d}[\sin(90-\phi_1)+\sin(90-\phi_2)]=\frac{\mu_0 I}{4\pi d}(\cos\phi_1+\cos\phi_2)$$

The correct option is (A)

46. Since magnetic field at the centre of an arc is equal to

$B=\dfrac{\mu_0 I}{4\pi r}\theta$

Hence, net $B = \frac{\mu_0 I}{4\pi}\left(\frac{1}{r} - \frac{1}{2r} + \frac{1}{3r}\right)\theta = \frac{5\mu_0 I\theta}{24\pi r}$

The correct option is (A)

47. $B = \frac{mg}{Il} = 0.66$ T

The correct option is (D)

48. $B = \frac{\mu_0}{4\pi}\frac{(2\pi - \theta)i}{R} = \frac{\mu_0}{4\pi}\frac{\left(2\pi - \frac{\pi}{2}\right)\times i}{R} = \frac{3\mu_0 i}{8R}$

The correct option is (D)

49. Field at point x from the centre of a current carrying loop on the axis is

$B = \frac{\mu_0}{4\pi}\cdot\frac{2M}{x^3} = \frac{10^{-7}\times 2\times 2.1\times 10^{-25}}{(10^{-10})^3}$

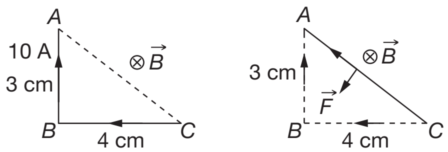
$= 4.2 \times 10^{-32} \times 10^{30} = 4.2 \times 10^{-2}$ W/m^2

The correct option is (A)

50. Force on the conductor ABC

= Force on the conductor AC

$= 5 \times 10 \times (5 \times 10^{-2}) = 2.5$ N

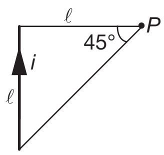


The correct option is (C)

51. $B = \frac{\mu_0}{4\pi}\cdot\frac{i}{l}(\sin\phi_1 + \sin\phi_2)$

Here, $\phi_1 = 0°$, $\phi_2 = 45°$

$\Rightarrow \quad B = \frac{\sqrt{2}\mu_0 i}{8\pi l}$



The correct option is (C)

52. $\tau_{max} = NiAB = 1\times i\times(\pi r^2)\times B \quad \left(2\pi r = L \Rightarrow r = \frac{L}{2\pi}\right);$

$\tau_{max} = \pi i\left(\frac{L}{2\pi}\right)^2 B = \frac{L^2 iB}{4\pi}$

The correct option is (C)

53. The correct option is (A)

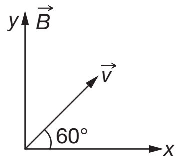
54. Path of the proton will be a helix of radius $r = \frac{mv\sin\theta}{qB}$

(where θ = angle between $\vec{B}$ and $\vec{v}$)

$\Rightarrow \quad r = \frac{1.67\times 10^{-27}\times 2\times 10^6\times\sin 30°}{1.6\times 10^{-19}\times 0.104} = 0.1$ m

Time period $T = \frac{2\pi m}{qB} = \frac{2\pi\times 1.67\times 10^{-27}}{1.6\times 10^{-19}\times 0.104}$

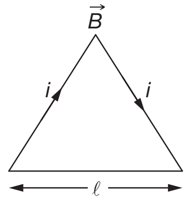
$= 2\pi\times 10^{-7}$ s.



The correct option is (C)

55. Since $\theta = 90°$

Hence $\tau = NIAB = 1\times I\times\left(\frac{\sqrt{3}}{4}l^2\right)B = \frac{\sqrt{3}}{4}l^2 B$

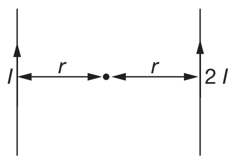


The correct option is (D)

56. When two parallel conductors carrying current I and $2I$ in same direction, then magnetic field at the midpoint is

$$B = \frac{\mu_0 2I}{2\pi r} - \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi r}$$

When current $2I$ is switched off, then magnetic field due to conductor carrying current I is $B = \frac{\mu_0 I}{2\pi r}$.



The correct option is (C)

57. $\tau_{max} = NiAB = 1\times i\times(\pi r^2)\times B \quad \left(2\pi r = L \Rightarrow r = \frac{L}{2\pi}\right)$

$\tau_{max} = \pi i\left(\frac{L}{2\pi}\right)^2 B = \frac{L^2 iB}{4\pi}$

The correct option is (C)

58. $B = \mu_0 ni = 4\pi\times 10^{-7}\times 5\times 1000 = 2\pi\times 10^{-3}$ T

The correct option is (A)

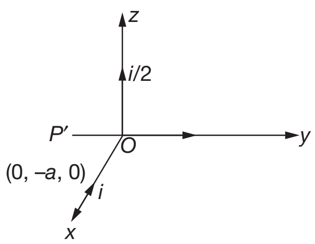
59. $F = \frac{\mu_0}{4\pi}\cdot\frac{2i_1 i_2}{a}\times l$

$$\Rightarrow \quad F = 10^{-7} \times \frac{2\times10\times2}{(10\times10^{-2})}\times 2 = 8\times10^{-5}\text{ N}$$

The correct option is (A)

60. The field at P due to current i along x-axis is $\frac{\mu_0 i}{4\pi a}\hat{k}$, and due to the current $\frac{i}{2}$ along z-axis is $\frac{\mu_0 i}{8\pi a}\hat{i}$.

The resultant field at P is $\frac{\mu_0 i}{4\pi a}\sqrt{1^2+\left(\frac{1}{2}\right)^2} = \frac{\sqrt{5}}{8}\frac{\mu_0 i}{\pi a}$



The correct option is (C)

61. $\tau_{net} = 0$

$mgR = MB_0$ (M = magnetic dipole moment)

$mgR = I(\pi R^2)B_0$

$I = \frac{mg}{\pi R B_0}$

The correct option is (A)

62. $t_1 = \frac{mv}{qE}$

$T = 2t_1 = \frac{2mv}{qE} = \frac{2\pi m}{qB}$

$\frac{vB}{\pi E} = 1$

The correct option is (C)

63. If $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$, $\vec{E} = E\hat{i}$, $\vec{B} = B\hat{j}$

$$\therefore \quad \vec{F} = q\left[E\hat{i} + (V_x\hat{i}+V_y\hat{j}+V_z\hat{k})\times B\hat{j}\right]$$

$$= qE\hat{i} + qV_xB\hat{k} + 0 - qV_zB\hat{i}$$

$$= (E - V_zB)\,\hat{i} + V_xB\,\hat{k}$$

For no deviation, net force should either be zero or in the direction of velocity of particle.

$\therefore$ For $F = 0$, $V_Z = E/B$, $V_x = 0$, $V_y \rightarrow$ has any value

The correct option is (B)

64. Electrostatics force on $q = \frac{\lambda q}{2\pi\varepsilon_0 r}$ away from line charge

Magnetic force $= \frac{\mu_0\lambda v}{2\pi r}\times q\times v$ away from line charge

$$\therefore \quad \text{total force} = \frac{\lambda q}{2\pi r}\left[\frac{1}{\varepsilon_0} + \mu_0 v^2\right]$$

The correct option is (A)

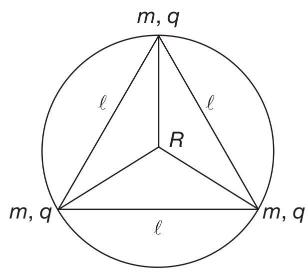
65. $R = \frac{l}{\sqrt{3}}$

$i = \frac{3q}{\frac{2\pi}{\omega}} = \frac{3q\omega}{2\pi}$

$\mu_{magnetic} = \frac{3q\omega}{2\pi}\cdot\pi R^2 = \frac{3q\omega R^2}{2}$

Angular momentum $= (3m)R^2\omega$

$\frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{q}{2m}$



The correct option is (A)

66. Magnetic force acting on charged particle is opposite to the direction of current. So the path will be helical with decreasing radius, and because of magnetic force, pitch will continuously increased.

The correct option is (B)

67. $B_{\text{due to straight wire}} = \frac{\mu_0 i}{2\pi r} = \frac{2\times10^{-7}\times30}{2\times10^{-2}} = 3\times10^{-4}$ T

$B_{external} = 4\times10^{-4}$ T

$B_{net} = \sqrt{3^2+4^2}\times10^{-4} = 5\times10^{-4}$ T

The correct option is (C)

68. $\frac{\mu_0 i R^2}{2\left(R^2+x^2\right)^{3/2}} = \frac{1}{\sqrt{8}}\frac{\mu_0 i}{2R}$, $\left(R^2+x^2\right)^{3/2} = R^3\sqrt{8}$

$\left(R^2+x^2\right)^3 = 8R^6$, $R^2+x^2 = 2R^2$, $x = \pm R$

The correct option is (D)

69. $y = 2r = \frac{2mv_0}{qB_0} = \frac{2v_0}{B_0\alpha}$

The correct option is (C)

70. $\omega = 0 + 1\times10 = 10$ rad/s^2

$\therefore \quad v = r\omega = 1\times10 = 10$ m/s

$$\vec{B} = \frac{\mu_0(q\vec{v}\times\vec{r})}{4\pi r^3}$$

$\Rightarrow \quad |\vec{B}| = \dfrac{\mu_0 qv}{4\pi r^2}; \; B = \dfrac{10^{-7}\times 0.1\times 10}{(1)^2} = 10^{-7}\,\text{T}$

The correct option is (B)

71. By work-energy theorem,

$mgz + BIlz - \int 2k(x+z)dz = 0$

$2mgz = 2k\left[\int_0^z x\,dz + \int_0^z z\,dz\right]$ (where x is elongation in the equilibrium position)

$2mg = mg + kz$

$z = \dfrac{mg}{k} = \dfrac{BIl}{k}$

The correct option is (A)

72. $\tan\delta = \tan\delta'\cos\alpha = \tan 45^\circ \cos 30^\circ$

$\delta = \tan^{-1}\dfrac{\sqrt{3}}{2}$

The correct option is (A)

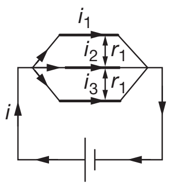
73. $\mu = \mu_0(1+\chi)$; $\mu = 500\times 4\pi\times 10^{-7} = 2\pi\times 10^{-4}$ H/m

The correct option is (B)

74. $i_1 = \dfrac{k}{3}$; $i_2 = \dfrac{k}{4}$; $i_3 = \dfrac{k}{5}$

$\dfrac{\mu_0 i_1 i_2}{2\pi r_1} = \dfrac{\mu_0 i_2 i_3}{2\pi r_2}$

$\therefore \quad \dfrac{r_1}{r_2} = \dfrac{i_1}{i_3} = \dfrac{5}{3}$



The correct option is (C)

75. $E = \dfrac{1}{2}Bl^2\omega \; = \dfrac{1}{2}B(2a)^2\omega \; = 2B\omega a^2$

The correct option is (A)

76. $B_2 = \dfrac{B_1 n_2 i_2}{n_1 i_1} = \dfrac{(6.28\times 10^{-2})(100\times i/3)}{200(i)} = 1.05\times 10^{-2}$ W/m^2

The correct option is (B)

77. The correct option is (B)

78. $\vec{F}_m = 2\left[4\left(-\hat{j}\right)\times 4\left(-\hat{k}\right)\right] = \vec{F}_m = 32\hat{i}$

The correct option is (B)

79. For equilibrium $(mg\sin\theta)R = MB\sin\theta = i\pi R^2 B\sin\theta$

$B = \dfrac{mg}{\pi i R}$

The correct option is (B)

80. $B_D = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{2I}{r} = \dfrac{10^{-7}\times 2\times 30}{0.03} = 2\times 10^{-4}$ T

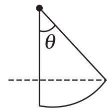
$B_G = \dfrac{10^{-7}\times 2\times 20}{0.1} = 0.4\times 10^{-4}$ T

$B = B_D - B_G = 2\times 10^{-4} - 0.4\times 10^{-4} = 1.6\times 10^{-4}$ T

$F = B\,I\,l\sin 90^\circ = 1.6\times 10^{-4}\times 10\times 0.25 = 4\times 10^{-4}$ N

The correct option is (D)

81. $\sin\theta = \dfrac{\dfrac{mv}{\sqrt{2}\,qB}}{\dfrac{mv}{qB}} = \dfrac{1}{\sqrt{2}}$



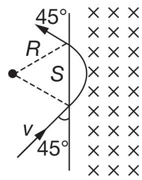
$\Rightarrow \quad \theta = 45^\circ$

$t = \dfrac{T}{8} = \dfrac{\pi m}{4qB}$

The correct option is (B)

82. $S = \sqrt{2}R$

$= \sqrt{2}\dfrac{mv}{qB}$



The correct option is (A)

83. $E_{\text{eff}} = 2BlV$

The correct option is (D)

84. $B_{\text{centre}} = \dfrac{N.\,\mu_0 I}{2R}$, $B_{\text{centre}} \;\alpha\; \dfrac{NI}{R}$

$B = \dfrac{\mu_0 i}{2r}, \quad B' = n\dfrac{\mu_0 i}{2\left(\dfrac{r}{n}\right)} = n^2 B$

The correct option is (D)

85. As $\dfrac{\mu_0 I}{2\pi x} = \dfrac{\mu_0 I}{2\pi y} \Rightarrow x = y$

The correct option is (A)

86. Magnetic field could be zero in the first or third quadrant.

$\dfrac{\mu_0 I_1}{2\pi x} = \dfrac{\mu_0 I_2}{2\pi y}$ or $I_1 = \dfrac{x}{y}I_2$

The correct option is (A)

87. Applying work-energy theorem, $qE_0x = \frac{1}{2}m(10)^2$

$$x = \frac{50m}{qE_0}$$

The correct option is (C)

88. When connected in parallel, the current will be in the same direction and when connected in series, the current will be in the opposite direction.

The correct option is (A)

89. As force on short wire AB acts upward and the torque of magnetic force about cm is clockwise, so acceleration of point A, $a_A = a_{cm} + \alpha l/2$ and for B, $a_B = a_{cm} - \alpha l/2$.

Thus, $a_a > a_B$

The correct option is (A)

90. Magnetic field due to inner enoid in the outside region is zero.

$\therefore$ magnetic force on outer enoid due to inner enoid $(F_2) = 0$

$\therefore$ by Newton's third law, $F_1 = 0$.

The correct option is (D)

91. When crosses AB

$$qvB_0 \cos\theta = mg$$

$$qvB_0 \sin\theta = qE$$

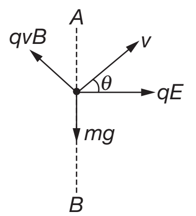
$$\tan\theta = \frac{qE}{mg} = 1$$

$$\theta = \frac{\pi}{4}$$

along horizontal $v\cos\theta = \frac{qE}{m}t_0$

$$u - gt_0 = v\sin\theta$$

$$u = \left(g + \frac{qE}{m}\right)t_0 = 2gt_0$$



The correct option is (B)

92. Voltage across rod $= \frac{1}{2}B_0 l_0^2 \omega_0$

Charge on capacitor $= CV_0$

$$v \times q = \frac{1}{2}CV_0^2 + H_{R_1} + H_{R_2}$$

$$CV_0 \times \frac{1}{2}B_0 l_0^2 \omega_0 = \frac{1}{2}CV_0^2 + \frac{R_2}{R_1}H_{R_2} + H_{R_2}$$

$$\frac{1}{2}CV_0 B_0 l_0^2 \omega_0 - \frac{1}{2}CV_0^2 = \frac{H_{R_2}[R_2 + R_1]}{R_1}$$

$$H_{R_2} = \frac{R_1}{R_1 + R_2}\left[\frac{1}{2}CV_0 B_0 l_0^2 \omega_0 - \frac{1}{2}CV_0^2\right]$$

$$= \frac{R_1}{R_1 + R_2} \times \frac{1}{2}CV_0[B_0 l_0^2 \omega_0 - V_0]$$

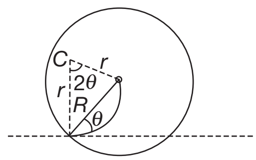
$$= \frac{1}{2}CV_0^2$$

The correct option is (A)

93. $r = \frac{R}{2\sin\theta}$

$$\frac{mv_0}{qB} = \frac{R}{2\sin\theta}$$

$$v_0 = \frac{qBR}{2m\sin\theta}$$

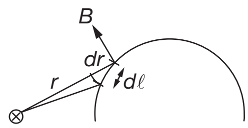


The correct option is (B)

94. Magnetic field at a distance r from the wire will be

$$B = \frac{\mu_0}{2\pi}\frac{i_1}{r}$$

force on the small element of length dl on semicircular wire is



$$dF = i_2\, \vec{dl} \times \vec{B} = i_2(dl_\perp)B$$

$$= i_2 B\, dr \qquad (\because dl_\perp = dr)$$

$$F = \int_R^{3R} i_2\, B\, dr = \frac{\mu_0}{2\pi}i_1 i_2 \ln 3$$

The correct option is (A)

95. Magnetic moment $M = NIA$

$$M = I_0\frac{\sqrt{3}}{4}(\sqrt{2}a)^2 = \frac{\sqrt{3}}{2}I_0 a^2$$

The correct option is (D)

More than One Option Correct Type

96. Time period of revolution of particle in magnetic field is given by $T = \dfrac{2\pi m}{qB}$

Since for a given particle, m and q are constant. Thus in constant magnetic field, T is constant

Angular velocity $\omega = \dfrac{2\pi}{T}$

Hence, angular velocity of particle is constant.

Hence, choice (A) is correct.

Due to resistive force, particle experiences a retardation and an energy is lost during motion against it. Hence, speed and total mechanical energy, both decrease continuously.

Hence, choice (B) is correct and (C) is wrong.

Since speed of electron is decreasing continuously, it is experiencing a tangential retardation. It is possible only when the component of resultant force opposite to the direction of motion has non-zero values. This implies that net force on electron cannot be perpendicular to its direction of motion.

Hence, choice (D) is wrong.

The correct option is (A) and (B)

97. As $Q = \dfrac{|\Delta\phi|}{R}$

$\therefore \quad \theta = 90° \cdot |\Delta\phi| = BAn$ so $Q = \dfrac{BAn}{R}$

$\theta = 180° \cdot |\Delta\phi| = 2\,BAn$ so $Q = \dfrac{2BAn}{R}$

$\theta = 360° \cdot |\Delta\phi| = 0$ so $Q = 0$

The correct option is (A) and (D)

98. Area of triangular loop $= \dfrac{1}{2} \times l \times l \sin 60°$

$= \sqrt{3} \times 10^{-4}\ \text{m}^2$

So, magnetic moment $M = iA = 0.1 \times \sqrt{3} \times 10^{-4}$
$= \sqrt{3} \times 10^{-5} \text{A/m}^2$

Hence, choice (A) is correct and choice (B) is wrong.

Couple acting on loop $\tau = MB \sin\theta$

$= (\sqrt{3} \times 10^{-5})\,(5 \times 10^{-2}) \times \sin 90°$

$= 5\sqrt{3} \times 10^{-7}$ N/m

Hence, choice (C) is correct and choice (D) is wrong.

The correct option is (A) and (C)

99. When currents in the coils are in same direction

$$B = \frac{\mu_0}{2}\left[\frac{50 \times 2}{5 \times 10^{-2}} + \frac{100 \times 2}{10 \times 10^{-2}}\right] = 2\mu_0 \times 10^3 = 8\pi \times 10^{-4}\ \text{T}$$

when currents in the coils are in opposite direction

$$B = \frac{\mu_0}{2}\left[\frac{50 \times 2}{5 \times 10^{-2}} - \frac{100 \times 2}{10 \times 10^{-2}}\right] = 0$$

The correct option is (A) and (C)

100. $R = \dfrac{mv}{qB}$, $1.5 \times 10^{-2} = \dfrac{A\left(1.66 \times 10^{-27}\right) \times 6 \times 10^4}{1.6 \times 10^{-19} \times 0.5} \Rightarrow A = 12$

Similarly for $R = 1.75$ cm, $A = 14$

The correct option is (B) and (D)

101. In uniform magnetic field radius

$r = \dfrac{mv}{qB}$

or $\quad \dfrac{r_e}{r_p} = \dfrac{m_e}{m_p} = \dfrac{1}{1840}$

Hence, choice (A) is correct.

Since, $r = \dfrac{mv}{qB} = \dfrac{p}{qB}$

So, $\dfrac{r_e}{r_p} = 1$

Hence, choice (B) is correct.

Since $\qquad r = \dfrac{p}{qB} = \dfrac{\sqrt{2mK}}{qB}$

$\dfrac{r_e}{r_m} = \sqrt{\dfrac{1}{1840}} \cong \dfrac{1}{43}$

Hence, choice (C) is correct and choice (D) is wrong.

The correct option is (A), (B) and (C)

102. $v_A - v_B = B\left(v \sin 30^0\right) l = 4\text{v}$

$v_B - v_C = -B\left(v \sin 30^0\right)\dfrac{l}{2} = -2\text{v}$

The correct option is (B) and (D)

103. $\vec{v} \perp \vec{a} \quad \therefore \vec{v}.\vec{a} = 0 \quad \therefore\ 6 + 4x = 0 \quad \Rightarrow x = -1.5$

Further, magnetic field is perpendicular to the plane of velocity.

So, magnetic field is along z-direction

Also work done by a magnetic force is zero, i.e., kinetic energy of a particle remains constant if only magnetic force is acting on it.

The correct option is (A), (C), and (D)

104. Current in AB and CD cause magnetic fields in the same direction on T, upward in this case. Hence, there is a net magnetic field over T

The correct option is (A) and (B)

105. $d\vec{F} = I d\vec{l} \times \vec{B}$

For constant $\vec{B}$, $\int d\vec{F} = \left(I \int d\vec{l}\right) \times \vec{B}$

Hence, $\vec{F} = I\left(\vec{l} \times \vec{B}\right)$

Which is independent of shape

The correct option is (A) and (B)

Passage Based Questions

Passage 1

106. $r_{AP} = \frac{32}{11}, r_{BP} = \frac{10}{11}\text{m}, \frac{\mu_0}{2\pi}\frac{9.6}{r_A} = \frac{\mu_0}{2\pi}\frac{I_B}{r_B},$

$$I_B = \frac{10/11}{32/11} \times 9.6 = 3\text{A}$$

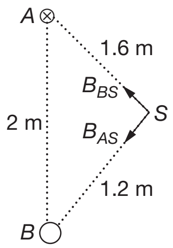
The correct option is (C)

107. Since, $AB^2 = AS^2 + BS^2$ $\therefore$ $\angle ASB$ is 90°

Now $B_{AS} = \frac{\mu_0}{2\pi}\frac{I_A}{r_{AS}} = 12 \times 10^{-7}$ T

$$B_{BS} = \frac{\mu_0}{2\pi}\frac{I_B}{r_{BS}} = 5 \times 10^{-7} \text{ T}$$

$$B_S = \sqrt{B_{AS}^2 + B_{BS}^2} = 13 \times 10^{-7} \text{ T}$$



The correct option is (B)

108. $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{AB} = 2.88 \times 10^{-6}$ N/m

The correct option is (A)

109. B at midpoint $= \frac{\mu_0}{2\pi}[I_A + I_B] = 2.52 \times 10^{-6}$ T

The correct option is (D)

Passage 2

110. Torque acting $\vec{\tau} = \vec{M} \times \vec{B}$

$$\tau = (3\hat{i} - 4\hat{j}) \times (4\hat{i} + 3\hat{j})$$

$$\tau = 25\,\hat{k} \text{ N/m}$$

The correct option is (B)

111. Moment of inertia of the ring about an axis parallel to τ and passing through its centre of mass will be

$$I = \frac{2 \times 10^{-2}}{2} \text{kg m}^2 = 10^{-2} \text{ kg m}^2$$

Hence, angular acceleration $\alpha = \frac{\tau}{I} = \frac{25}{10^{-2}} = 2500 \text{ rad/s}^2$

The correct option is (C)

112. At $t = 0$

$\vec{M} \cdot \vec{B} = 0$

So $M \perp R$

Maximum angular velocity will be when $\vec{M} \parallel \vec{B}$. Magnetic potential energy will decrease and rotational kinetic energy will increase.

Hence, $\frac{1}{2}I\omega^2 = U_i - U_f$

$$= -MB\cos 90° - (-MB\cos 0°)$$

or $\frac{1}{2}I\omega^2 = MB$

$$\omega = \sqrt{\frac{2MB}{I}} = \sqrt{\frac{2 \times 5 \times 5}{10^{-2}}}$$

$$= 50\sqrt{2} \text{ rad/s}$$

The correct option is (A)

Passage 3

Since velocity is constant, $F_{\text{net}} = 0 \Rightarrow mg = ilB = \frac{B^2 l^2 v_0}{R}$

$$\left[\because i = \frac{\varepsilon}{R} = \frac{Blv_0}{R}\right]$$

On solving we get, $m = 1$ kg

Now, when mass is tripled $3mg - T = 3ma$ and

$$T - \frac{B^2 l^2 v}{R} = ma$$

On solving, $3mg - \frac{B^2 l^2 v}{R} = 4ma$ (1)

Now, $v = v_T$ when $a = 0$

$$\Rightarrow v_T = \frac{3mgR}{B^2 l^2} = 30 \text{ m/s}$$

for $v = \frac{v_0 + v_T}{2} = 20$ m/s from Equation (1) $a = 2.5$ m/s^2

113. The correct option is (C)

114. The correct option is (C)

115. The correct option is (A)

Passage 4

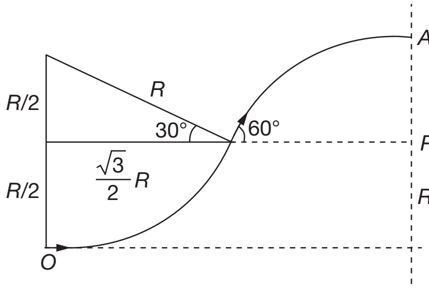
In magnetic field,

$$R = \frac{mv}{qB} = 1 \text{ m}$$

$$T = \frac{2\pi m}{qB} = 0.2\,\pi$$

In electric field,

Acceleration $a = \frac{qE}{m} = 10 \text{ m/s}^2$



Now, PA' = maximum height $= \dfrac{v^2 \sin^2\theta}{2a} = 3.75$ m

$\therefore$ y-co-ordinate of A' is 4.25 m

T' = time elapsed in $\vec{E}$ is half the time of flight

$= \dfrac{v\sin\theta}{a} = \dfrac{\sqrt{3}}{2}$

$\therefore$ time $t = \dfrac{T}{6} + T' = 0.97$ s

116. The correct option is (B)

117. The correct option is (B)

118. The correct option is (C)

119. The correct option is (D)

Passage 5

$V_1 + V_2 = 6$

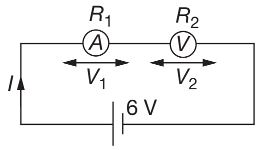
$3V_1 + \dfrac{V_2}{3} = 6$

$V_1 = \dfrac{3}{2}$ V ; $\quad V_2 = \dfrac{9}{2}$ V

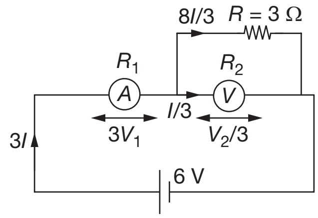
From Figure 15.65(a)

$IR_1 = \dfrac{3}{2}$ and $IR_2 = \dfrac{9}{2}$

$$\therefore \quad R_1 = \frac{R_2}{3} \qquad (1)$$



(a)



(b)

Fig. 15.65

From Figure 15.65(b)

$\dfrac{8I}{3} \times 3 = \dfrac{I}{3} \times R_2$

$R_2 = 24\ \Omega$ and $R_1 = 8\ \Omega$

$I = \dfrac{3}{16}$ A

120. The correct option is (B)

121. The correct option is (A)

122. The correct option is (D)

123. The correct option is (C)

Match the Column Type

124. (A) → 1, 2, 4; (B) → 1, 2, 3, 4; (C) → 1, 2, 4; (D) → 2

125. (A) → 3; (B) →1; (C) → 2; (D) → 2

126. (A) → (1, 2, 3, 4); (B) → (2); (C) → (1, 2); (D) → (1, 2, 3, 4)

127. Let the radius of circle in which particle moves is R. In this magnitude of region, electric field is $E = \dfrac{R}{2}\left(\dfrac{dB}{dt}\right)$ as

$qE = m\dfrac{dV}{dt}$

$\Rightarrow \quad \dfrac{qR}{2}\left(\dfrac{dB}{dt}\right) = m\dfrac{dV}{dt}$

also $R = \dfrac{mV}{Bq}$

$\Rightarrow \quad \dfrac{dV}{V} = \dfrac{1}{2}\dfrac{dB}{B}$

as $\quad R = \dfrac{mV}{Bq}, \dfrac{dV}{V} = \dfrac{dq}{q} + \dfrac{dB}{B}, \dfrac{dq}{q} = -\dfrac{1}{2}\dfrac{dB}{B}$

(A) → 2, 3; (B) → 1, 4; (C) → 1, 3; (D) → 2,4

128. (A) → 2, 3; (B) → 1; (C) → 4; (D) → 2, 3

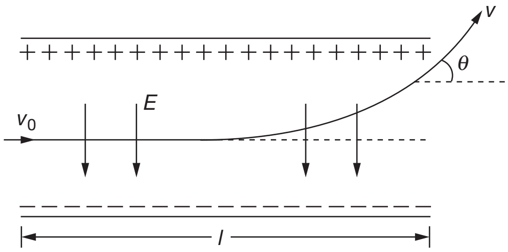
Integer Type

139. $I\frac{\pi R^2}{2}B = (\lambda\pi R)\frac{2R}{\pi}g$

$I = \frac{4\lambda g}{\pi B} = 4\ \text{A}$

140. Since electron is accelerated through a potential difference V, its initial velocity v_0 is given by $\frac{1}{2}mv_0^2 = eV$

or $$v_0 = \sqrt{\frac{2eV}{m}} \qquad (1)$$



Since initial velocity is parallel to plates or normal to the direction of electric field, component of velocity parallel to plates remains constant as v_0.

Hence, time taken by the electron to cross electric field is $t_0 = \frac{l}{v_0}$

Now consider motion of electron, normal to plates.

At some instant t, its acceleration $= \frac{eE}{m} = \frac{eat}{m}$

Let velocity component normal to plates be v_y

$\therefore \quad \frac{d}{dt}v_y = \frac{eat}{m}$

$\therefore \quad \int_0^{v_y} dv_y = \frac{ea}{m}\int_0^{t_0} t\,dt$

or $$v_y = \frac{ea}{2m}t_0^2 = \frac{al^2}{4\text{ V}} \qquad (2)$$

If θ is angular deviation of electron from its initial direction of motion.

Then pitch of its helical path.

$p = \frac{2\pi m}{eB}v\cos(90-\theta)$

$\therefore \quad p = \frac{2\pi m}{eB}v\sin\theta = \frac{2\pi m}{eB}v_y$

or $\quad p = \frac{\pi m a l^2}{2eBV}$

$p = 9$ mm.

141. $\vec{\tau}_f + \vec{\tau}_B = 0$

$0 = \vec{R}\times\vec{f}_f + \int\vec{r}\times(Id\vec{\ell}\times\vec{B})$

$= Rf_r\sin\theta(+\hat{k}) + \int_0 IBr\,dr(-\hat{k})$

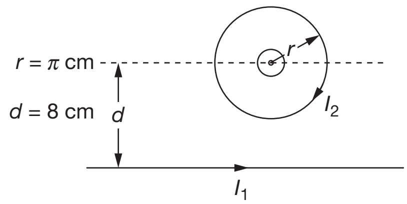
$= Rf_r(+\hat{k}) + \frac{IR^2B}{2}(-\hat{k})$

$f_r = \frac{IRB}{2} = 0.18\text{ N} = 18\times10^{-2}\text{N} = 18$

$= 2$

142. $\frac{\mu_0 I_2}{2r} = \frac{\mu_0 I_1}{2\pi d}$

$\Rightarrow \quad \frac{I_1}{I_2} = \frac{\pi d}{r} = 8$



$\therefore \quad 8$

143. $\varepsilon = -\frac{d\phi}{dt} = \frac{-d}{dt}\int\vec{B}.\vec{ds}$ and $\varepsilon = \oint\vec{E}.\vec{dl}$

or $\quad E2\pi r = -\pi r^2\frac{dB}{dt}$ or $E = \frac{-r}{2}\frac{dB}{dt}$

force $F = qE$ and torque $\tau = Fr = qEr = \frac{-qr^2}{2}\frac{dB}{dt}$

Angular impulse = change in angular momentum

$\frac{-qr^2}{2}\int\frac{dB}{dt}dt = mr^2\omega \Rightarrow |\omega| = qB/2m = 1$ rad/s

144. Force exerted by air on the rod $= \left(\rho\frac{L}{2}2R\right)v^2 = \rho LRv^2$

Balancing torque about point O $NI\left(\pi R^2\right)B = \rho LRv^2\times\frac{3L}{4}$

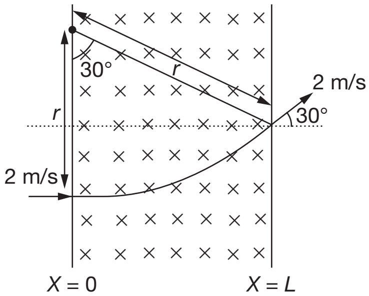
$\Rightarrow \quad 300\,\pi IBR = \frac{3\rho v^2L^2}{4}$

$\Rightarrow \quad I = \frac{\rho L^2v^2}{400\pi BR} = 0.01\text{A} = 10$ mA

145. $\sin 30^0 = \frac{L}{r}$

$$\frac{1}{2} = \frac{L}{\left(\frac{mv_0}{qB}\right)}$$

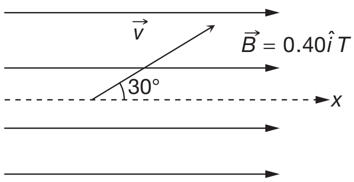
$$L = \frac{mv_0}{2qB} = \frac{2\times 2}{2\times 1\times 2} = 1\text{ m} \therefore L = 1$$



146. The proton will move on a circular path of radius

$$r = \frac{mV\sin\theta}{qB}$$

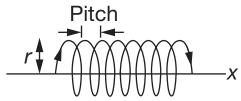
$$= \frac{(1.67\times 10^{-27}\text{ kg})(5\times 10^7\text{ m/s})\times 0.5}{(1.6\times 10^{-19}\text{ C})(0.40\text{ T})}$$



$$= \frac{1.67\times 5\times 0.5}{1.6\times 0.4}\times 10^{-1} = 0.65\text{ m}$$

The proton will spiral along the x-axis

$$\text{Period} = \frac{2\pi r}{V_\perp} = \frac{2\pi r}{V\sin\theta} = \frac{2\pi(0.65)}{5\times 10^7\times 0.5} = 1.6\times 10^{-7}\text{s}$$



Path of a proton

Pitch = ($V_{||}$) (period) = ($V\cos\theta$) (T)

$$= \left(5\times 10^7\times\frac{\sqrt{3}}{2}\right)(1.6\times 10^{-7}) = 690\text{ cm}$$

147. $q[v_x\hat{i} + v_y\hat{j}]\times B_0\hat{k} - \alpha[v_x\hat{i} + v_y\hat{j}] = m\frac{d\vec{v}}{dt}$ (1)

$\therefore \quad a_y = \frac{1}{m}\left[qv_xB_0 + \alpha v_y\right]$

and $a_x = \frac{1}{m}[qB_0v_y - \alpha v_x]$ (3)

$\therefore$ At $t = 0$, $v_y = v_0$ and $v_x = 0$ and $t = t$, $v_x = 0$ $v_y = 0$

$\therefore \quad v_0 = \frac{1}{m}\left[qB_0x + \frac{\alpha^2 x}{qB_0}\right]$

$\Rightarrow \quad x = \frac{qB_0mv_0}{\alpha^2 + q^2B^2} = 10$

148. $W = -MB(\cos\theta_2 - \cos\theta_1) = 1\text{ J}$

Previous Years' Questions

149. $mR\omega^2 = BqR\omega \Rightarrow BqR\omega \Rightarrow \omega = \frac{Bq}{m} \Rightarrow T = \frac{2\pi m}{Bq}$

T is independent of speed.

The correct option is (A)

150. $Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq} = \frac{P}{Bq}$

r will be same for electron and proton as p, B, and q are of same magnitude.

The correct option is (A)

151. $B = \frac{\mu_0}{4\pi}\frac{2\pi\text{I}}{R} = \frac{\mu_0}{2}\frac{\text{I}}{R}$

$\therefore \quad \frac{B_A}{B_B} = \frac{\text{I}_A}{\text{I}_B}\times\frac{R_B}{R_A} = \left(\frac{1}{2}\right)\left(\frac{2}{1}\right) = 1$

The correct option is (A)

152. Work done by the field = zero.

The correct option is (B)

153. A ferromagnetic material becomes paramagnetic above Curie temperature.

The correct option is (A)

154. The magnetic lines of force inside a bar magnet are from south pole to north pole of magnet.

The correct option is (D)

155. $W = -MB(\cos\theta_2 - \cos\theta_1)$

$$= -MB(\cos 60° - \cos 0) = \frac{MB}{2}$$

$\therefore \quad MB - 2W$

Torque $= MB\sin 60° = (2W) = \sin 60°$

$$= \frac{2W \times \sqrt{3}}{2} = \sqrt{3}W$$

The correct option is (A)

156. Materials of low retentivity and low coercivity are suitable for making electromagnets.

The correct option is (B)

157. Initially, $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l$

Finally, $F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{3d} l$

$$\therefore \quad \frac{F'}{F} = \frac{-\mu_0}{2\pi}\frac{2I_1 I_2 l}{3d} \times \frac{2\pi d}{\mu_0 I_1 I_2 l} = -\frac{2}{3}$$

$\therefore \quad F' = -2F/3$

The correct option is (A)

158. Field along axis of coil $B = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$

At the centre of coil, $B' = \frac{\mu_0 i}{2R}$

$$\therefore \quad \frac{B'}{B} = \frac{\mu_0 i}{2R} \times \frac{2(R^2 + x^2)^{3/2}}{\mu_0 i R^2} = \frac{(R^2 + x^2)^{3/2}}{R^3}$$

$$\therefore \quad B' = \frac{B \times (R^2 + x^2)^{3/2}}{R^3} = \frac{54 \times [(3)^2 + (4)^2]^{3/2}}{(3)^3} = \frac{54 \times 125}{27}$$

or $\quad B' = 250\ \mu T$

The correct option is (C)

159. Magnetic field will be zero inside the straight thin-walled tube according to ampere's theorem.

The correct option is (B)

160. Magnetic field applied parallel to motion of electron exerts no force on it as $\theta = 0$ and force $= Bev \sin\theta$ = zero. Electric field opposes motion of electron which carries a negative charge.

$\therefore$ velocity of electron decreases.

The correct option is (B)

161. A force and a torque act on a magnetic needle kept in a non-uniform magnetic field.

The correct option is (D)

162. $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$ (1)

$\because$ centripetal force = magnetic force

$$\therefore \quad \frac{mv^2}{r} = qvB \Rightarrow v = \frac{qBr}{m} \qquad (2)$$

From (1) and (2),

$$\therefore \quad T = \frac{2\pi r \times m}{qBr} = \frac{2\pi m}{qB}$$

The correct option is (C)

163. In first case, $B_1 = \mu_0 n_1 I_1$

In second case, $B_2 = \mu_0 n_2 I_2$

$$\therefore \quad \frac{B_2}{B_1} = \frac{n_2}{n_1} \times \frac{I_2}{I_1} = \frac{100}{200} \times \frac{i/3}{i} = \frac{1}{6}$$

$$\therefore \quad B_2 = \frac{B_1}{6} = \frac{6.28 \times 10^{-2}}{6} = 1.05 \times 10^{-2}\ \text{Wb/m}^2$$

The correct option is (A)

164. Magnetic field exerts a force

$= Bev \sin\theta = Bev \sin\theta = 0$

Electric field exerts force along a straight line.

The path of charged particle will be a straight line.

The correct option is (B)

165. Magnet will attract N_1 strongly, N_2 weakly, and repel N_3 weakly.

The correct option is (B)

166. The field at the same point at the same distance from the mutually perpendicular wires carrying current will be having the same magnitude but in perpendicular direction.

$$\therefore \quad B = \sqrt{B_1^2 + B_2^2} \qquad \therefore \quad B = \frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$$

The correct option is (C)

167. Magnetic field is shielded and no current is inside the pipe to apply ampere's law. (Compare to electric field inside a hollow sphere).

The correct option is (D)

168. Uniform current is flowing. Current enclosed in the first Amperian path is $\frac{I.\pi r_1^2}{\pi R^2} = \frac{I r_1^2}{R^2}$

$$\therefore \quad B = \frac{\mu_0 \times \text{current}}{\text{path}} = \frac{\mu_0 \cdot I r_1^2}{2\pi r_1 R^2} = \frac{\mu_0 \cdot I r_1}{2\pi R^2}$$

Magnetic induction at distance $r_2 = \frac{\mu_0 \cdot I}{2\pi r_2}$

$$\therefore \quad \frac{B_1}{B_2} = \frac{r_1 r_2}{R^2} = \frac{\frac{a}{2} \cdot 2a}{a^2} = 1$$

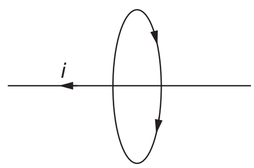
The correct option is (D)

169. The values of relative permeability of diamagnetic materials are slightly less than 1 and ε_r is quite high. According to the table, one takes.

$\varepsilon_r = 1.5$ and $\mu_r = 0.5$. Then the choice (c) is correct.

The correct option is (B)

170.



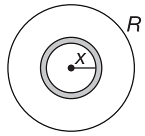
By ampere's theorem, $\vec{B} \cdot 2\pi d = \mu_0 i$

$$\vec{B} = \frac{\mu_0 i}{2\pi d} = \frac{4\pi \times 10^{-7} \times 100\ \text{A}}{2\pi \times 4\ \text{m}} = 50 \times 10^{-7}\ \text{T}$$

$\Rightarrow \quad B = 5 \times 10^{-6}$T southwards.

The correct option is (C)

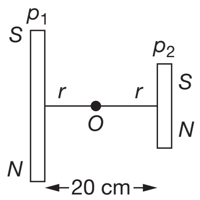
171.



$$B = \int_0^R \frac{\mu_0}{2x} \cdot \frac{\frac{Q}{\pi R^2} \cdot 2\pi x dx}{T} = \frac{\mu_0 Q}{R^2} \cdot \frac{R}{T} = \frac{\mu_0 Q}{RT} = \frac{\mu_0 Q \omega}{2\pi R}$$

The correct option is (A)

172. Magnetic field at 0 is due to two magnets and due to earth magnetism.



$$B = \frac{\mu_0}{4\pi} \cdot \frac{p_1}{r^3} + \frac{\mu_0}{4\pi} \cdot \frac{p_2}{r^3} + B_H$$

$$= \frac{10^{-7}}{(0.10)^3}(1.20 + 1.00) + 3.6 \times 10^{-5} \text{ Wb/m}^2$$

$$= 2.56 \times 10^{-4} \text{ Wb/m}^2$$

The correct option is (A)

173. Force acting on the wire due to the magnetic field at any x,

$$F = \mathrm{I}(\vec{\ell} \times \vec{B})$$

$$= (10)\left[3(-\hat{a}_z) \times 3.0 \times 10^{-4} e^{-0.2x} \, \hat{a}_y\right]$$

$$= \left(9.0 \times 10^{-3}\right) e^{-0.2x} \, \hat{a}_x$$

$\Rightarrow$ required external force will be $\left(9.0 \times 10^{-3}\right) e^{-0.2x} \left(-\hat{a}_x\right)$

$\Rightarrow$ work done in small displacement dx

$$dW = F.dx = -9 \times 10^{-3} \times e^{-0.2x} dx$$

$$\Rightarrow W = \left[-\frac{9 \times 10^{-3} \times e^{-0.2x}}{(-0.2)}\right]_0^2$$

$$= \left[45 \times 10^{-3} e^{-0.2x}\right]_0^2$$

$$= 45 \times 10^{-3} e^{-0.4} - 45 \times 10^{-3}$$

$$= 45 \times 10^{-3}\left[e^{-0.4} - 1\right]$$

$$\Rightarrow \text{Power} = \frac{45 \times 10^{-3}\left[e^{-0.4} - 1\right]}{5 \times 10^{-3}}$$

$$= 9 \times \left[e^{-0.4} - 1\right] = 2.97 \text{ W}$$

The correct option is (B)

174. $H = \dfrac{B}{\mu_0} \Rightarrow B = \mu_0 H$

For the enoid,

$$B = \mu_0 n I$$

$$\Rightarrow \mu_0 H = \mu_0 n I \Rightarrow 3 \times 10^3 = \frac{100}{0.1} \times I$$

$$\Rightarrow 3 \times 10^3 = 10^3 \times I \Rightarrow I = 3A$$

The correct option is (C)

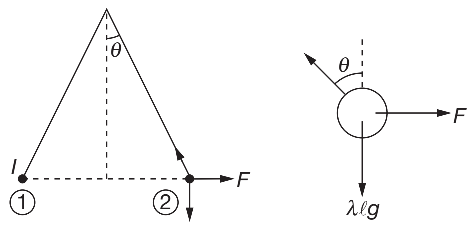
175. $U = -\vec{M}.\vec{B}$

For stable equilibrium, $\theta = 0°$

For unstable equilibrium, $\theta = 180°$

The correct option is (B)

176. Consider length due to wire



For equilibrium,

$$T \sin\theta = F$$

$$T \cos\theta = \lambda \ell g$$

Here, $F = \dfrac{\mu_0}{2\pi} \dfrac{I_1 \cdot I_2}{2(L\sin\theta)} \ell$ (Magnetic force)

$$\tan\theta = \frac{\mu_0 I^2}{4\pi L \sin\theta} \times \frac{1}{\lambda g}$$

$$I^2 = \frac{4\pi\lambda g L}{\mu_0} \frac{\sin^2\theta}{\cos\theta}$$

$$I = 2\sin\theta \sqrt{\frac{\pi \lambda L g}{\mu_0 \cos\theta}}$$

The correct option is (A)

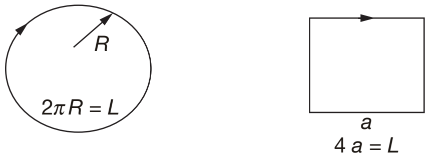
177. $B_A = \dfrac{\mu_0 \mathrm{I}}{2R} = \dfrac{\mu_0 \mathrm{I}\, 2\pi}{2L}$ ....(A)

$$B_B = 4\left[\frac{\mu_0}{4\pi} \frac{2\mathrm{I}}{a} 2\sin 45°\right]$$

$$= 4\left[\frac{\mu_0}{\pi}\frac{4I}{\sqrt{2}L}\right] \quad (2)$$

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

The correct option is (C)



(a) (b)

178. B for electromagnet and transformers since it has low retentivity and low coercivity and area of loop is low, hence low loss of energy in hysteresis per cycle.

The correct option is (C)