CHAPTER 11

Heat Transfer

Chapter Highlights

Conduction, Convection, Radiation, Newton's law of cooling, Wien's displacement law.

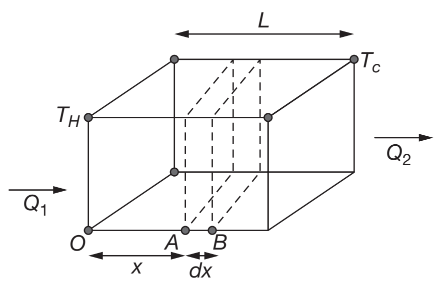
INTRODUCTION

Heat is energy in transit, which flows due to temperature difference, from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three routes.

1. Conduction
2. Convection
3. Radiation

CONDUCTION

The process of transmission of heat energy in which heat is transferred from one particle of the medium to the other, but each particle of the medium stays at its own position is called conduction; for example, if you hold an iron rod with one of its end on a fire for some time, the handle will get hot. The heat is transferred from the fire to the handle by conduction along the length of iron rod. The vibrational amplitude of atoms and electrons of the iron rod at the hot end takes on relatively higher values due to the higher temperature of their environment. These increased vibrational amplitudes are transferred along the rod, from atom to atom during collision between adjacent atoms. In this way, a region of rising temperature extends itself along the rod to your hand.



Consider a slab of face area A, lateral thickness L, whose faces have temperatures T_H and T_C ($T_H > T_C$).

Now consider two cross-sections in the slab at positions A and B separated by a lateral distance of dx. Let temperature of face A be T and that of face B be $T + \Delta T$. Thus, experiments show that Q, the amount of heat crossing the area A of the slab at position x in time t is given by

$$\frac{Q}{t} = -KA\frac{dt}{dx} \quad (11.1)$$

Here, K is a constant depending on the material of the slab and is named thermal conductivity of the material, and the quantity $\left(\frac{dT}{dx}\right)$ is called temperature gradient. The (–) sign in Equation (11.1) shows heat flows from high to low temperature (ΔT is a –ve quantity).

STEADY STATE

If the temperature of a cross-section at any position x in the above slab remains constant with time (remember, it does vary with position x), the slab is said to be in steady state.

Remember steady state is distinct from thermal equilibrium for which temperature at any position (x) in the slab must be same.

For a conductor in steady state, there is no absorption or emission of heat at any cross-section (as temperature at each point remains constant with time). The left and right face are maintained at constant temperatures T_H and T_C, respectively, and all other faces must be covered with adiabatic walls so that no heat escapes through them and same amount of heat flows through each cross-section in a given interval of time. Hence, $Q_1 = Q = Q_2$. Consequently, the temperature gradient is constant throughout the slab.

Hence, $$\frac{dT}{dx} = \frac{\Delta T}{L} = \frac{T_f - T_i}{L} = \frac{T_C - T_H}{L}$$

and $$\frac{Q}{t} = -KA\frac{\Delta T}{L}$$

$$\Rightarrow \quad \frac{Q}{t} = KA\,Q\left(\frac{T_H - T_C}{L}\right)$$

Here Q is the amount of heat flowing through a cross-section of slab at any position in a time interval of t.

SOLVED EXAMPLE

1. One face of an aluminium cube of edge 2 metre is maintained at 100°C and the other end is maintained at 0°C. All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cube in 5 s. (Thermal conductivity of aluminium is 209 W/m–°C.)

Solution:

Heat will flow from the end at 100°C to the end at 0°C.

Area of cross-section perpendicular to direction of heat flow,

$$A = 4\text{ m}^2$$

then $$\frac{Q}{t} = KA\frac{(T_H - T_C)}{L}$$

$$Q = \frac{(209\text{W/m°C})(4\text{ m}^2)(100\text{°C} - 0\text{°C})(5\text{ s})}{2\text{ m}} = 209\text{ KJ}$$

THERMAL RESISTANCE TO CONDUCTION

If you are interested in insulating your house from cold weather or for that matter keeping the meal hot in your tiffin box, you are more interested in poor heat conductors, rather than good conductors. For this reason, the concept of thermal resistance R has been introduced.

For a slab of cross-section A, lateral thickness L, and thermal conductivity K,

$$R = \frac{L}{KA}$$

In terms of R, the amount of heat flowing though a slab in steady state (in time t)

$$\frac{Q}{t} = \frac{(T_H - T_L)}{R}$$

If we name $\frac{Q}{t}$ as thermal current i_T

then $$i_T = \frac{T_H - T_L}{R}$$

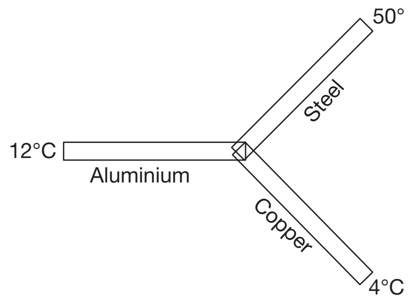
This is mathematically equivalent to OHM's law, with temperature donning the role of electric potential. Hence, results derived from OHM's law are also valid for thermal conduction.

Moreover, for a slab in steady state as we have seen earlier, the thermal current i_L remains same at each cross-section. This is analogous to Kirchhoff's current law in electricity, which can now be very conveniently applied to thermal conduction.

SOLVED EXAMPLE

2. Three identical rods of length 1m each, having cross-section area of 1 cm^2 each and made of aluminium, copper, and steel, respectively, are maintained at temperatures of 12°C, 4°C, and 50°C, respectively, at their separate ends.

Find the temperature of their common junction.



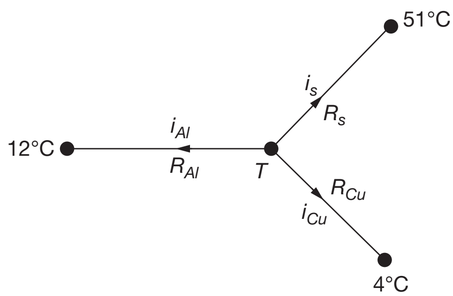
[K_{Cu} = 400 W/m-K, K_{Al} = 200 W/m-K, K_{steel} = 50 W/m-K]

Solution:

$$R_{Al} = \frac{L}{KA} = \frac{1}{10^{-4} \times 200} = \frac{10^4}{200}$$

Similarly, $$R_{\text{steel}} = \frac{10^4}{50} \text{ and } R_{\text{copper}} = \frac{10^4}{400}$$

Let temperature of common junction be T



Then from Kirchhoff's current laws,

$$i_{Al} + i_{\text{steel}} + i_{Cu} = 0$$

$$\Rightarrow \quad \frac{T-12}{R_{Al}} + \frac{T-51}{R_{\text{steel}}} + \frac{T-4}{R_{Cu}} = 0$$

$$\Rightarrow \quad (T-12)\,200 + (T-50)\,50 + (T-4)\,400$$

$$\Rightarrow \quad 4(T-12) + (T-50) + 8\,(T-4) = 0$$

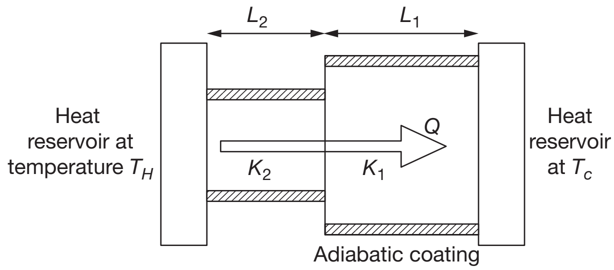
$$\Rightarrow \quad 13T = 48 + 50 + 32 = 130$$

$$\Rightarrow \quad T = 10°\text{C}$$

SLABS IN PARALLEL AND SERIES

Slabs in Series (In Steady State)

Consider a composite slab consisting of two materials having different thickness L_1 and L_2 different cross-sectional areas A_1 and A_2 and different thermal conductivities K_1 and K_2. The temperatures at the outer surface of the states are maintained at T_H and T_C, and all lateral surfaces are covered by an adiabatic coating.



Let temperature at the junction be T, since steady state has been achieved thermal current through each slab will be equal. Then thermal current through the first slab,

$$i = \frac{Q}{t} = \frac{T_H - T}{R_1} \text{ or } T_H - T = iR_1 \qquad (11.2)$$

and that of the second slab,

$$i = \frac{Q}{t} = \frac{T - T_C}{R_2}$$

or $$T - T_C = iR_2 \qquad (11.3)$$

adding Equation (11.2) and (11.3)

$$T_H - T_L = (R_1 + R_2)\, i$$

or $$i = \frac{T_H - T_C}{R_1 + R_2}$$

Thus, these two slabs are equivalent to a single slab of thermal resistance $R_1 + R_2$.

If more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$$R = R_1 + R_2 + R_3 + \ldots$$

SOLVED EXAMPLES

3. Figure 11.1 shows the cross-section of the outer wall of a house built in a hill resort to keep the house insulated from the freezing temperature of outside. The wall consists of teak wood of thickness L_1 and brick of thickness ($L_2 = 5L_1$), sandwiching two layers of an unknown material with identical thermal conductivities and thickness. The thermal conductivity of teak wood is K_1 and that of brick is ($K_2 = 5K$). Heat conduction through the wall has reached a steady state with the temperature of three surfaces being known ($T_1 = 25°\text{C}$, $T_2 = 20°\text{C}$, and $T_5 = -20°\text{C}$). Find the interface temperature T_4 and T_3.

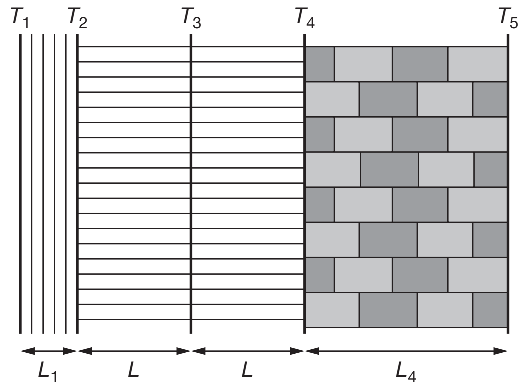


Fig. 11.1

Solution:

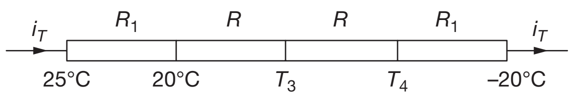
Let interface area be A, then thermal resistance of wood,

$$R_1 = \frac{L_1}{K_1 A}$$

and that of brick wall

$$R_2 = \frac{L_2}{K_2 A} = \frac{5L_1}{5K_1 A} = R_1$$

Let thermal resistance of the each sandwich layer be R. Then the above wall can be visualized as a circuit



Thermal current through each wall is same.

Hence $$\frac{25-20}{R_1} = \frac{20-T_3}{R} = \frac{T_3 - T_4}{R} = \frac{T_4 + 20}{R_1}$$

$$\Rightarrow \quad 25 - 20 = T_4 + 20$$

$$\Rightarrow \quad T_4 = -15°\text{C}$$

also $20 - T_3 = T_3 - T_4$

$$\Rightarrow \quad T_3 = \frac{20 + T_4}{2} = 2.5°\text{C}.$$

4. In Example 3, K_1 = 0.125 W/m–°C, $K_2 = 5K_1$ = 0.625 W/m–°C and thermal conductivity of the unknown material is K = 0.25 W/m°C. L_1 = 4 cm, $L_2 = 5L_1$ = 20 cm, and L = 10 cm. If the house consists of a single room of total wall area of 100 m^2, then find the power of the electric heater being used in the room.

Solution:

$$R_1 = R_2 = \frac{(4\times10^{-2}\text{ m})}{(0.125\text{W/m-°C})(100\text{ m}^2)} = 32 \times 10^{-4}\text{ °C/W}$$

$$R = \frac{(10\times10^{-2}\text{ m})}{(0.25\text{W/m-°C}(100\text{ m}^2)} = 40 \times 10^{-4}\text{ °C/W}$$

The equivalent thermal resistance of the entire wall $= R_1 + R_2 + 2R$

$$= 144 \times 10^{-4}\text{ °C/W}$$

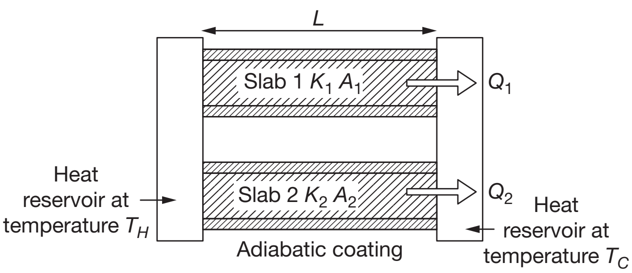
∴ Net heat current, i.e., amount of heat flowing out of the house per second $= \frac{T_H - T_C}{R}$

$$= \frac{25°\text{C} - (-20°\text{C})}{144\times10^{-4}\,°\text{C/W}} = \frac{45\times10^4}{144}\text{ W}$$

$$= 3.12\text{ kW}$$

Hence, the heater must supply 3.12 kW to compensate for the outflow of heat.

Slabs in Parallel



Consider two slabs held between the same heat reservoirs, their thermal conductivities are K_1 and K_2 and cross-sectional areas are A_1 and A_2.

then $$R_1 = \frac{L}{K_1A_1}, \quad R_2 = \frac{L}{K_2A_2}$$

Thermal current through slab 1

$$i_1 = \frac{T_H - T_C}{R_1}$$

and that of slab 2

$$i_2 = \frac{T_H - T_C}{R_2}$$

Net heat current from the hot to cold reservoir

$$i = i_1 + i_2 = (T_H - T_C)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Comparing with

$$i = \frac{T_H - T_C}{R_{eq}},$$

we get,

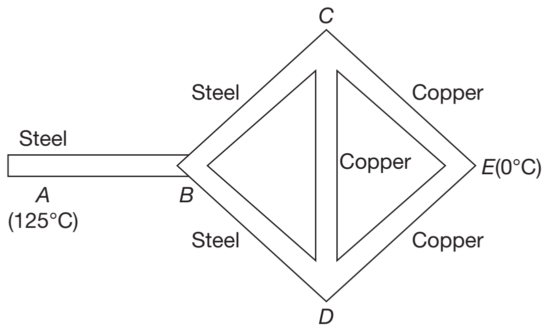
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

If more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

SOLVED EXAMPLES

5. Three copper rods and three steel rods each of length $\ell = 10$ cm and area of cross-section 1 cm^2 are connected as shown



If ends A and E are maintained at temperatures 125°C and 0°C, respectively, calculate the amount of heat flowing per second from the hot to cold function [K_{Cu} = 400 W/m-K, K_{steel} = 50 W/m-K].

Solution:

$$R_{steel} = \frac{L}{KA} = \frac{10^{-1}\text{ m}}{50(\text{W/m-°C})\times10^{-4}\text{ m}^2} = \frac{1000}{50}\text{ °C/W.}$$

Similarly, $$R_{Cu} = \frac{1000}{400}\text{ °C/W}$$

Junction C and D are identical in every respect and both will have same temperature. Consequently, the rod CD is in thermal equilibrium and no heat will flow through it. Hence, it can be neglected in further analysis.

Now rod BC and CE are in series their equivalent resistance is $R_1 = R_S + R_{Cu}$ similarly rods BD and DE are in series with same equivalent resistance $R_1 = R_S + R_{Cu}$

These two are in parallel giving an equivalent resistance of

$$\frac{R_1}{2} = \frac{R_S + R_{Cu}}{2}$$

This resistance is connected in series with rod AB. Hence, the net equivalent of the combination is

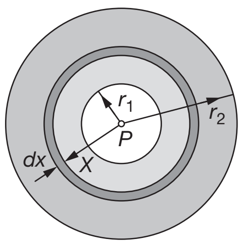
$$R = R_{\text{steel}} + \frac{R_1}{2} = \frac{3R_{\text{steel}} + R_{Cu}}{2}$$

$$= 500\left(\frac{3}{50} + \frac{1}{400}\right) \text{°C/W}$$

Now $$i = \frac{T_H - T_C}{R} = \frac{125\text{°C}}{500\left(\frac{3}{50} + \frac{1}{400}\right)\text{°C/W}}$$

$$= 4 \text{ W}$$

6. Two thin concentric shells made from copper with radius r_1 and r_2 $(r_2 > r_1)$ have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C, respectively, by keeping a heater of power P at the centre of the two spheres. Find the value of P.



Solution:

Heat flowing per second through each cross-section of the sphere $= P = i$.

Thermal resistance of the spherical shell of radius x and thickness dx,

$$dR = \frac{dx}{K \cdot 4\pi x^2}$$

$$\Rightarrow \quad R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2 \cdot K} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

thermal current

$$i = P = \frac{T_H - T_C}{R} = \frac{4\pi K(T_H - T_C) r_1 r_2}{(r_2 - r_1)}.$$

7. A container of negligible heat capacity contains 1 kg of water. It is connected by a steel rod of length 10 m and area of cross-section 10 cm^2 to a large steam chamber which is maintained at 100°C. If initial temperature of water is 0°C, find the time after which it becomes 50°C. (Neglect heat capacity of steel rod and assume no loss of heat to surroundings) (take specific heat of water = 4180 J/kg °C)

Solution:

Let temperature of water at time t be T, then thermal current at time t,

$$i = \left(\frac{100 - T}{R}\right)$$

This increases the temperature of water from T to $T + dT$

$$\Rightarrow \quad i = \frac{dH}{dt} = ms\frac{dT}{dt}$$

$$\Rightarrow \quad \frac{100 - T}{R} = ms\frac{dT}{dt}$$

$$\Rightarrow \quad \int_0^{50} \frac{dT}{100 - T} = \int_0^t \frac{dT}{Rms}$$

$$\Rightarrow \quad -\ell n\left(\frac{1}{2}\right) = \frac{t}{Rms}$$

or $$t = Rms\, \ell n2 \text{ s}$$

$$= \frac{L}{KA} \text{ ms } \ell n2 \text{ s}$$

$$= \frac{(10\text{ m})(1\text{ kg})(4180\text{ J/kg-°C})}{46(\text{W/m°C}) \times (10 \times 10^{-4}\text{ m}^2)} \ell n2$$

$$= \frac{418}{46}(0.69) \times 10^5$$

$$= 6.27 \times 10^5 \text{ s}$$

$$= 174.16 \text{ hours}$$

Can you now see how the following facts can be explained by thermal conduction?

1. In winter, iron chairs appear to be colder than the wooden chairs.
2. Ice is covered in gunny bags to prevent melting.
3. Woolen clothes are warmer.
4. We feel warmer in a fur coat.
5. Two thin blankets are warmer than a single blanket of double the thickness.
6. Birds often swell their feathers in winter.
7. A new quilt is warmer than an old one.
8. Kettles are provided with wooden handles.
9. Eskimos make double-walled ice houses.
10. Thermos flask is double walled.

CONVECTION *(Not in JEE Syllabus)

When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection. Convection occurs through the aid of earth's gravity. Normally, the portion of fluid at greater temperature is less dense, while that at lower temperature is denser. Hence, hot fluids rises up white colder fluid sink down, accounting for convection. In the absence of gravity, convection would not be possible.

Also, the anomalous behaviour of water (its density increases with temperature in the range 0–4°C) give rise to interesting consequences vis-a-vis the process of convection. One of these interesting consequences is the presence of aquatic life in temperate and polar waters. The other is the rain cycle.

Can you now see how the following facts can be explained by thermal convection?

1. Oceans freeze top-down and not bottom-up. (This fact is singularly responsible for presence of aquatic life is temperate and polar waters.)
2. The temperature in the bottom of deep oceans is invariably 4°C, whether it is winter or summer.
3. You cannot illuminate the interior of a lift in free fall or an artificial satellite of earth with a candle.
4. You can illuminate your room with a candle.

RADIATION

The process of the transfer of heat from one place to another without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Properties of Radiation

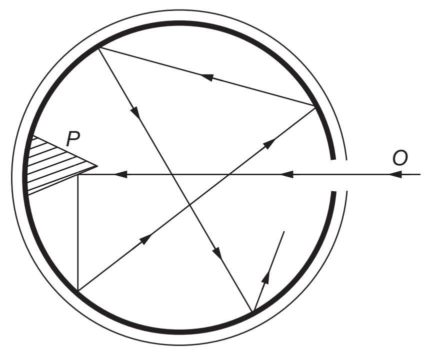
1. All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of the radiation that falls on them from other objects.
2. Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.
3. More radiations are emitted at higher temperature of a body and lesser at lower temperature.
4. The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to which the colour of a body appears to be changing. Radiations from a body at NTP have predominantly infrared waves.
5. Thermal radiations travel with the speed of light and move in a straight line.
6. Radiations are electromagnetic waves and can also travel through vacuum.
7. Similar to light, thermal radiations can be reflected, refracted, diffracted, and polarized.
8. Radiation from a point source obeys inverse square law (intensity $\propto \frac{1}{r^2}$).

PREVOST THEORY OF EXCHANGE

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area, and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more then what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings, it radiates at the same rate as it absorbs.

PERFECTLY BLACK BODY AND BLACK BODY RADIATION (FERRY'S BLACK BODY)

A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black, whatever be the colour of the incident radiation.



In actual practice, no natural object possesses strictly the properties of a perfectly black body. But the lamp black and platinum black are good approximation of black body. They absorb about 99% of the incident radiation. The most simple and commonly used black body was designed by Ferry. It consists of an enclosure with a small opening which is painted black from inside. The opening acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little chance of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.

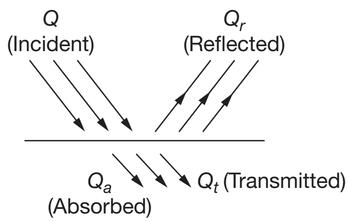
ABSORPTION, REFLECTION, AND EMISSION OF RADIATIONS

$$Q = Q_r + Q_t + Q_a$$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}$$

$$1 = r + t + a$$

where r = reflecting power
and a = absorptive power
and t = transmission power.



1. $r = 0, t = 0, a = 1$, perfect black body
2. $r = 1, t = 0, a = 0$, perfect reflector
3. $r = 0, t = 1, a = 0$, perfect transmitter

Absorptive Power

In particular, absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body.

$$a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$$

As all the radiations incident on a black body are absorbed, $a = 1$ for a black body.

Emissive Power

Energy radiated per unit time per unit area along the normal to the area is known as emissive power.

$$E = \frac{Q}{\Delta A\, \Delta t}$$

(Notice that unlike absorptive power, emissive power is not a dimensionless quantity.)

Spectral Emissive Power (E_λ)

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, E_λ. If E is the total emissive power and E_λ is spectral emissive power, they are related as follows,

$$E = \int_0^\infty E_\lambda \, d\lambda$$

and $$\frac{dE}{d\lambda} = E_\lambda$$

Emissivity

$$e = \frac{\text{Emissive power of a body at temperature } T}{\text{Emissive power of a black body at same temperature } T}$$

$$= \frac{E}{E_0}.$$

KIRCHOFF'S LAW

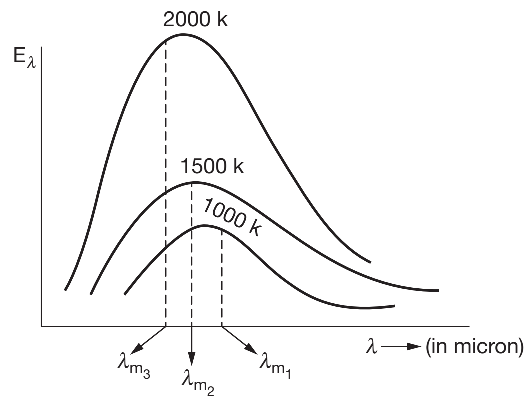
The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of a perfectly black body for the same wavelength and temperature.

$$\frac{E\,(\text{body})}{a\,(\text{body})} = E(\text{black body})$$

Hence, we can conclude that good emitters are also good absorbers.

NATURE OF THERMAL RADIATIONS: (WIEN'S DISPLACEMENT LAW)

From the energy distribution curve of black body radiation, the following conclusions can be drawn:



1. The higher the temperature of a body, the higher is the area under the curve, i.e., more amount of energy is emitted by the body at higher temperature.
2. The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.
3. For a given temperature, there is a particular wavelength (λ_m) for which the energy emitted (E_λ) is maximum.
4. With an increase in the temperature of the black body, the maxima of the curves shift toward shorter wavelengths.

From the study of energy distribution of black body radiation, discussed as above, it was established experimentally that the wavelength (λ_m) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body. That is,

$$\lambda_m \propto \frac{1}{T} \quad \text{or} \quad \lambda_m T = b$$

This is called Wien's displacement law.

Here $b = 0.282$ cm-K is the Wien's constant.

SOLVED EXAMPLE

8. Solar radiation is found to have an intensity maximum near the wavelength range of 470 nm. Assuming the surface of sun to be perfectly absorbing ($a = 1$), calculate the temperature of solar surface.

Solution:

Since $a = 1$, sun can be assumed to be emitting as a black body from Wien's law for a black body

$$\lambda_m \cdot T = b$$

$$\Rightarrow \qquad T = \frac{b}{\lambda_m} = \frac{0.282\,(\text{cm-K})}{(470 \times 10^{-7}\ \text{cm})}$$

$$\simeq 6125\ \text{K}.$$

STEFAN-BOLTZMANN'S LAW

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$$u = \sigma A T^4 \qquad (11.4)$$

where σ is Stefan's constant = 5.67×10^{-8} W/m^2 k^4

A body which in not a black body absorbs and hence emits less radiation then that given is by Equation (11.4).

For such a body,

$$u = e\sigma A T^4$$

where e = emissivity (which is equal to absorptive power) which lies between 0 and 1

With the surroundings of temperature T_0, net energy radiated by an area A per unit time.

$$\Delta u = u - u_0 = e\sigma A\,(T^4 - T_0^4)$$

SOLVED EXAMPLES

9. A body of emissivity (e = 0.75), surface area of 300 cm^2, and temperature 227°C is kept in a room at temperature 27°C. Calculate the initial value of net power emitted by the body.

Solution:

Using Eequation (11.4)

$$P = \rho\sigma A\,(T^4 - T_0^4)$$
$$= (0.75)\,(5.67 \times 10^{-8}\ \text{W/m}^2\text{–k}^4)\,(300 \times 10^{-4}\ \text{m}^2)$$
$$\times \{(500\ \text{K})^4 - (300\ \text{K})^4\}$$
$$= 69.4\ \text{W}$$

10. A hot black body emits the energy at the rate of 16 J m^{-2} s^{-1} and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in J m^{-2} s^{-1}.

Solution:

Wien's displacement law is:

$$\lambda_m \cdot T = b$$

i.e., $$T \propto \frac{1}{\lambda_m}$$

Here, λ_m becomes half.

∴ temperature doubles.

Also $$e = \sigma T^4$$

$$\Rightarrow \qquad \frac{e_1}{e_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\Rightarrow \qquad e_2 = \left(\frac{T_1}{T_2}\right)^4 \cdot e_1 = (2)^4 \cdot 16$$

$$= 16.16 = 256\ \text{J m}^{-2}\ \text{s}^{-1}$$

NEWTON'S LAW OF COOLING

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

$$\frac{dQ}{dt} \propto (\theta - \theta_0),$$

where θ and θ_0 are temperatures corresponding to object and surroundings.

From above expression,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

This expression represents Newton's law of cooling. It can be derived directly from Stefan's law, which gives,

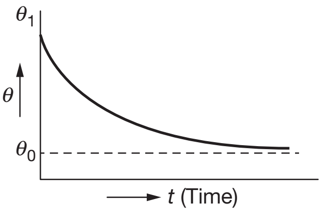
$$k = \frac{4e\sigma\theta_0^3}{mc} A$$

Now $$\frac{d\theta}{dt} = -k\ [\theta - \theta_0]$$

$$\Rightarrow \int_{\theta_i}^{\theta_f} \frac{d\theta}{(\theta - \theta_0)} = \int_0^t -k\ dt$$

where θ_i = initial temperature of object

θ_f = final temperature of object



$$\Rightarrow \ell n \frac{(\theta_f - \theta_0)}{(\theta_i - \theta_0)} = -kt$$

$$\Rightarrow (\theta_f - \theta_0) = (\theta_i - \theta_0)\, e^{-kt}$$

$$\Rightarrow \theta_f = \theta_0 + (\theta_i - \theta_0)\, e^{-kt} \qquad (11.5)$$

Limitations of Newton's Law of Cooling

1. The difference in temperature between the body and surroundings must be small.
2. The loss of heat from the body should be by radiation only.
3. The temperature of surroundings must remain constant during the cooling of the body.

Approximate Method for Applying Newton's Law of Cooling

Sometime when we need only approximate values from Newton's law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

$$\left\langle \frac{d\theta}{dt} \right\rangle = -k(\langle\theta\rangle - \theta_0) \qquad (11.6)$$

If θ_i and θ_f are initial and final temperature of the body then,

$$\langle\theta\rangle = \frac{\theta_i + \theta_f}{2} \qquad (11.7)$$

Remember Equation (11.7) is only an approximation and Equation (11.6) must be used for exact values.

SOLVED EXAMPLE

11. A body at temperature 40°C is kept in a surrounding of constant temperature 20°C. It is observed that its temperature falls to 35°C in 10 min. Find how much more time will it take for the body to attain a temperature of 30°C.

Solution:

From Equation (11.5)

$$\Delta\theta_f = \Delta\theta_i\, e^{-kt}$$

For the interval in which temperature falls from 40 to 35°C,

$$(35 - 20) = (40 - 20)\, e^{-k.\,10}$$

$$\Rightarrow e^{-10k} = \frac{3}{4}$$

$$\Rightarrow K = \frac{\ell n \frac{4}{3}}{10}$$

For the next interval,

$$(30 - 20) = (35 - 20)e^{-kt}$$

$$\Rightarrow e^{-10k} = \frac{2}{3}$$

$$\Rightarrow kt = \ell n \frac{3}{2}$$

$$\Rightarrow \frac{\left(\ell n \frac{4}{3}\right)t}{10} = \ell n \frac{3}{2}$$

$$\Rightarrow \quad t = 10\,\frac{\left(\ell n\frac{3}{2}\right)}{\left(\ell n\frac{4}{3}\right)}\ \text{min}$$

$$= 14.096 \text{ min}$$

Aliter: (by approximate method)
For the interval in which temperature falls from 40 to 35°C,

$$<\theta> = \frac{40+35}{2} = 37.5°\text{C}$$

From Equation (11.6),

$$\left(\frac{d\theta}{dt}\right) = -k(<\theta> - \theta_0)$$

$$\Rightarrow \quad \frac{(35°\text{C} - 40°\text{C})}{10(\text{min})} = -K(37.5°\text{C} - 20°\text{C})$$

$$\Rightarrow \quad K = \frac{1}{35}(\text{min}^{-1})$$

For the interval in which temperature falls from 35°C to 30°C,

$$<\theta> = \frac{35+30}{2} = 32.5°\text{C}$$

From Equation (11.6),

$$\frac{(30°\text{C} - 35°\text{C})}{t} = -(32.5°\text{C} - 20°\text{C})$$

$\Rightarrow$ required time,

$$t = \frac{5}{12.5} \times 35\,\text{min} = 14 \text{ min}$$

BRAIN MAP

1. Rate of Heat flow through conduction in steady state
   $$\frac{\Delta Q}{\Delta t} = \frac{KA\,(T_1 - T_2)}{l}$$
2. Net rate of loss of energy by radiation per unit area per second = $e\sigma(T_1^4 - T_2^4)$
   Newton's law of cooling
   $$\frac{dT}{dt} = -bA\,(T - T_0)$$
3. Weins displacement law:
   $\lambda_m T = b$ (constant)

EXERCISES

Single Option Correct Type

1. The ends of a uniform metre stick of iron are maintained at 80°C and 30°C. One end of another rod is maintained at 50°C, where its other end should be touched on the metre stick so that there is no heat current in the rod in steady state?

(A) 40 cm from hot end
(B) 40 cm from cold end
(C) 50 cm from cold end
(D) 70 cm from cold end

2. A body cools from 60°C to 50°C in 10 min. If the room temperature is 25°C and assuming Newton's law of cooling to hold good, the temperature of the body at the end of the next 10 min will be

(A) 38.5°C (B) 40°C
(C) 42.85°C (D) 45°C

3. Two rods of equal length and area of cross-section are kept parallel and lagged between temperature 20°C and 80°C. The ratio of the effective thermal conductivity to that of the first rod is $(K_1/K_2) = 3 : 4$

(A) 7 : 4 (B) 7 : 6 (C) 4 : 7 (D) 7 : 8

4. If the temperature of the sun is increased from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be

(A) 4 (B) 16 (C) 32 (D) 64

5. A hot body is being cooled in air according to Newton's law of cooling, the rate of fall of temperature being k times the difference of its temperature with respect to that of surroundings. The time, after which the body will lose half the maximum heat it can lose, is

(A) $\frac{1}{k}$ (B) $\frac{\ln 2}{k}$ (C) $\frac{\ln 3}{k}$ (D) $\frac{2}{k}$

6. A block body is at a temperature 2880 K. The energy radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 mm and 1000 nm is U_2, and between 1499 nm and 1500 nm is U_3, then (Wien's constant $b = 2.88 \times 10^6$ nm-K)

(A) $U_1 > U_2$ (B) $U_2 > U_1$
(C) $U_1 = 0$ (D) $U_3 = 0$

7. On the surface of lake when the atmospheric temperature is –15ºC, 1.5 cm thick layer of ice is formed in 20 min, time taken to change its thickness from 1.5 cm to 3 cm will be

(A) 20 min
(B) less than 20 min
(C) greater than 40 min
(D) less than 40 min and greater than 20 min

8. A diatomic molecule having atoms of masses m_1 and m_2 has its potential energy function about the equilibrium position r_0 as given by $U(r) = -A + B(r - r_0)^2$, where A and B are constants. When the atom vibrate at high temperature condition, the square of angular frequency of vibration will be

(A) $\frac{2B}{m_1}$ (B) $\frac{2B}{m_2}$

(C) $\frac{2B(m_1+m_2)}{m_1 m_2}$ (D) $\frac{B(m_1+m_2)}{2m_1 m_2}$

9. It is known that the temperature in the room is +20ºC when the outdoor temperature is –20ºC and +10ºC when the outdoor temperature is –40ºC. Then what is the temperature T of the radiator heating the room? (Assuming that radiated by the heater is proportional to the temperature difference with the room.)

(A) 40ºC (B) 60ºC (C) 30ºC (D) 20ºC

10. One mole of an ideal gas with heat capacity at constant pressure C_P undergoes the process $T = T_0 + \alpha V$, where T_0 and α are constants. If its volume increases from V_1 to V_2, the amount of heat transferred to the gas is

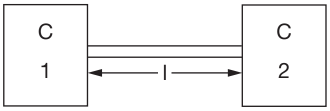
(A) $C_P R T_0 \ln\left(\frac{V_2}{V_1}\right)$

(B) $\alpha C_P \frac{(V_2 - V_1)}{RT_0} \ln\left(\frac{V_2}{V_1}\right)$

(C) $\alpha C_P (V_2 - V_1) + RT_0 \ln\left(\frac{V_2}{V_1}\right)$

(D) $RT_0 \ln\left(\frac{V_2}{V_1}\right) + \alpha C_P (V_1 - V_2)$

11. Two bodies each having a heat capacity of $C = 500$ J/K are joined together by a rod of length L = 40.0 cm, thermal conductivity 20 W/mK, and cross-sectional area of $S = 3.00$ cm^2. The bodies are joined with the help of a thermally insulated rod. The time after which temperature difference diminishes η = 2 times is (Disregard the heat capacity of the rod.)



(A) 193 min (B) 240 min
(C) 77 min (D) 144 min

12. A rod of length l (laterally thermally insulated) of uniform cross-sectional area A consists of a material whose thermal conductivity varies with temperature as $K = \frac{K_o}{a+bT}$, where K_0, a and b are constants. T_1 and T_2 ($<T_1$) are the temperature of two ends of rod. Then rate of flow of heat across the rod is

(A) $\frac{AK_0}{b\,l}\left(\frac{a+bT_1}{a+bT_2}\right)$

(B) $\frac{AK_0}{b\,l}\left(\frac{a+bT_2}{a+bT_1}\right)$

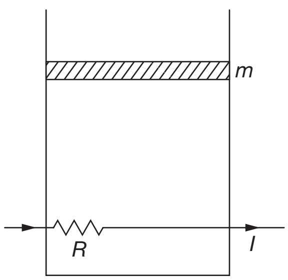
(C) $\frac{AK_0}{b\,l}\ln\left[\frac{a+bT_1}{a+bT_2}\right]$

(D) $\frac{AK_0}{a\,l}\ln\left[\frac{a+bT_2}{a+bT_1}\right]$

13. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at constant temperature of 900 K by heating electrically. A total of 300 W electric power is needed to do this. When half of the surface of the copper sphere is completely blackened, 600 W is needed to maintain the same temperature of sphere. The emissivity of copper is

(A) 1/4 (B) 1/3
(C) 1/2 (D) 1

14. A coil of resistance R connected to an external battery is placed inside an adiabatic cylinder fitted with a frictionless piston and containing an ideal gas. A current $I = a_0 t$ flows through the coil (a_0 is a +ve constant). For time interval $t = 0$ to $t = t_0$, the piston goes up to a height of (Assume $\Delta U = 0$)



(A) $\frac{a_0^2 R^2 t_0^2}{2mg}$ (B) $\frac{a_0^2 R t_0^3}{2mg}$

(C) $\frac{a_0^2 R t_0^3}{3mg}$ (D) $\frac{a_0^2 R t_0^2}{3mg}$

15. In a composite rod, when two rods of different lengths and of the same area of cross-section are joined end to end, then if K is the effective coefficient of thermal conductivity, $\frac{l_1 + l_2}{K}$ is equal to

(A) $\frac{l_1}{K_1} - \frac{l_2}{K_2}$ (B) $\frac{l_1}{K_2} - \frac{l_2}{K_1}$

(C) $\frac{l_1}{K_1} + \frac{l_2}{K_2}$ (D) $\frac{l_1}{K_2} + \frac{l_2}{K_1}$

16. The SI unit of thermal conductivity is

(A) $Js^{-1}\,m\,K^{-1}$ (B) $Jsm^{-1}\,K^{-1}$
(C) $Jsm^{-1}\,K$ (D) $Js^{-1}\,m^{-1}\,K^{-1}$

17. According to Wien's displacement law,

(A) λ_m = constant
(B) $\lambda_m T$ = constant
(C) $\lambda_m T^2$ = constant
(D) $\lambda_m^2 T$ = constant

18. A bucket full of hot water is kept in a room and it cool from 75°C to 70°C in T_1 min, from 70°C to 65°C in T_2 min and from 65°C to 60°C in T_3 min. Then

(A) $T_1 = T_2 = T_3$
(B) $T_1 < T_2 < T_3$
(C) $T_1 > T_2 > T_3$
(D) $T_1 < T_2 > T_3$

19. Three rods made of the same material and having the same cross-section have been joined as shown in Fig. 11.2. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C, respectively. The temperature of the junction of the three rods will be

(A) 45°C (B) 60°C
(C) 30°C (D) 20°C

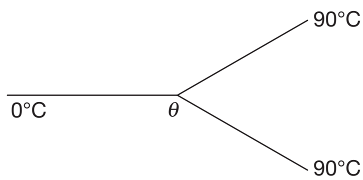


Fig. 11.2

More than One Option Correct Type

20. When a hollow and a solid sphere of same material with same outer radius and identical surface finish are heated to the same temperature

(A) In the beginning, both will emit equal amount of radiation per unit time

(B) In the beginning, both will absorb equal amount of radiation per unit time

(C) Both spheres will have same rate of fall of temperature $\left(\dfrac{dT}{dt}\right)$

(D) Both spheres will have equal temperatures at any moment

21. A 10 g body is kept in an enclosure of 27°C. For body's temperature 127°C, the specific heat 0.1 K cal/kg°C and surface area 10^{-3}m^2. The ($\sigma = 5.67 \times 10^{-8}$ W/m^{2}k^4]

(A) Rate of cooling is 0.227 ks^{-1}

(B) Rate of cooling will be zero at 400 K enclosure

(C) Cooling does not take place

(D) Cooling will be faster at 127°C enclosure

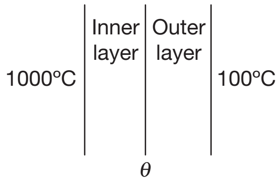
22. The temperature drop through a two-layered furnace wall is 900ºC. Each layer is of equal area of cross-section. Which of the following actions will result in lowering the temperature θ of the interface?

(A) By increasing the thermal conductivity of outer layer

(B) By increasing the thermal conductivity of inner layer

(C) By increasing thickness of outer layer

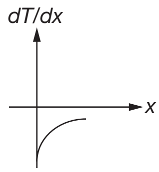
(D) By increasing thickness of inner layer

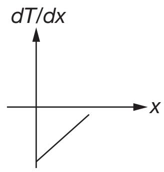


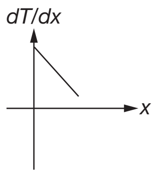
23. If the temperature of the sun is increased from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be

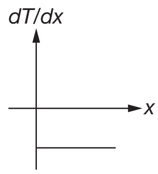
(A) 4 (B) 16 (C) 32 (D) 64

24. The curved surface of uniform rod is thermally isolated from surrounding. Its ends are maintained at temperature T_1 and T_2 $(T_1 > T_2)$. If in steady state, temperature gradient at a distance x from hot end is equal to $\dfrac{dT}{dx}$, then which one of the following graphs is correct?

(A) 

(B) 

(C) 

(D) 

Passage Based Questions

The rate of flow of thermal current depends on the nature of material (thermal conductivity), cross-sectional area A, and temperature gradient. More the temperature difference, higher the thermal current flow. This fact identifies the thermal resistance offered by the material while conducting heat. One can find by equivalent resistance in heat flow using the same principles as for current. A body may transfer energy better by radiation. The nature of radiating surfaces play a role in the power radiated from it. On covering a surface by non-conducting/radiating media, the loss of the heat energy can be controlled.

25. A rod of length l and conductivity k is placed between two reservoirs maintained at 0°C and 100°C. At a distance $\dfrac{l}{3}$ and $\dfrac{l}{5}$ from 100°C reservoir the rate of flow of thermal current are I_1 and I_2. The temperatures at $\dfrac{l}{5}$ and $\dfrac{l}{3}$ are θ_1 and θ_2 then

(A) $I_1 = I_2$, $2\theta_1 = 3\theta_2$

(B) $5\theta_1 - 3\theta_2 = 200$

(C) $I_1 = I_2$, $5\theta_1 - 3\theta_2 = 200$

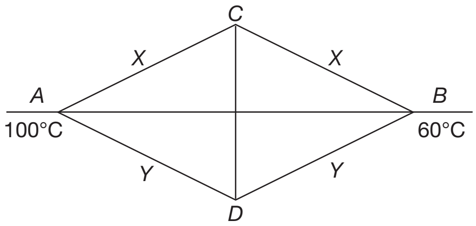
(D) $I_1 \neq I_2$, $5\theta_2 = 3\theta_1$

26. Two spheres with emissive powers 0.6 and 0.8 of radii 2 cm and 4 cm are heated to temperatures of 27°C and 157°C and placed in a room at absolute temperature 0 K. The ratio of heat radiated per second is

(A) 0.187 (B) 1.6×10^{-4}

(C) 0.079 (D) 0.831

27. Two rods of material X and two rods of material Y are placed with a rod of material Z as shown with the junctions A and D at 100°C and 60°C. Then all rods have equal length



(A) Thermal current in AC and CB are equal
(B) Thermal current in AC and AD are equal
(C) Temperature of C and D are unequal
(D) Thermal current in AC and BD is same

Assertion-Reason Type

28. **Assertion:** Water can be boiled inside satellite by convection.

Reason: Convection is the process in which heat is transmitted from a place of higher temperature to a place of lower temperature by means of particles with their migrations from one place to another.

(A) A (B) B (C) C (D) D

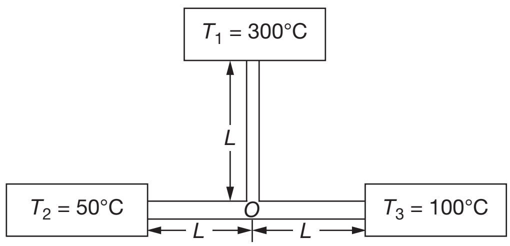
29. **Assertion:** If liquid in a container is heated at top rather than at the bottom. The main process by which the rest of the liquid becomes hot is conduction.

Reason: Two spheres of the same material have radii a and $2a$, temperature $2T$ and T, respectively. The energy radiated per second by the first sphere is greater than that of the second.

(A) A (B) B (C) C (D) D

Integer Type

30. Heat energy is transferred from a heat source maintained at temperature T_1 = 300°C to two lower temperature heat reservoirs maintained at temperatures T_2 = 50°C and T_3 = 100°C. Three identical solid steel rods, each of length L = 1 m and cross-sectional area 0.01 m^2, are used to pipe the heat, as shown. Find the ratio of power in watts delivered by the source to each of the lower temperature reservoirs and also find the temperature of the junction O. Consider only axial heat flow. Thermal conductivity of each of the rod is 46 SI units.



31. A liquid cools from 70°C to 60°C in 5 min. Calculate the time taken by the liquid to cool from 60°C to 50°C, if the temperature of the surrounding is constant at 30°C.

32. Inner surface of a cylindrical shell of length l and of material of thermal conductivity k is kept at constant temperature T_1 and outer surface of the cylinder is kept at constant temperature T_2 such that $T_1 > T_2$ as shown in Fig. 11.3. Heat flows from inner surface to outer surface radially outward. Inner and outer radii of the shell are R and $2R$, respectively. Due to lack of space, this cylinder has to be replaced by a smaller cylinder of length $\frac{l}{2}$, inner and outer radii $\frac{R}{4}$ and R, respectively, and thermal conductivity of material nk. If rate of radial outward heat flow remains same for same temperatures of inner and outer surface, i.e., T_1 and T_2, then find the value of n.

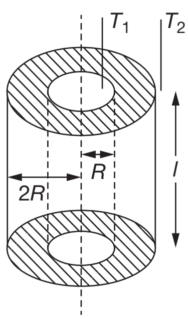


Fig. 11.3

Previous Years' Questions

33. Which of the following is more close to a black body? **[2004]**

(A) Black board paint
(B) Green leaves
(C) Black holes
(D) Red roses

34. Infrared radiations are detected by **[2002]**

(A) spectrometer (B) pyrometer
(C) nanometer (D) photometer

35. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K, respectively. The ratio of the energy radiated per second by the first sphere to that by the second is **[2002]**

(A) 1:1 (B) 16:1 (C) 4:1 (D) 1:9

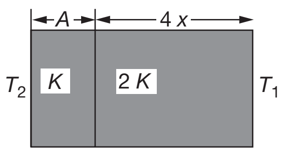
36. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$ where $\Delta\theta$ is the difference between the temperature of the body and the surrounding. Then n is equal to **[2003]**

(A) 2 (B) 3 (C) 4 (D) 1

37. If the temperature of the sun was to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be **[2004]**

(A) 4 (B) 16 (C) 32 (D) 64

38. The temperatures of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab in a steady state is $\left[\dfrac{A(T_2 - T_1)K}{x}\right] f$ with f equal to **[2004]**



(A) 1 (B) 1/2 (C) 2/3 (D) 1/3

39. The Figure 11.4 shows a system of two concentric spheres of radii r_1 and r_2 kept at temperature T_1 and T_2. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to **[2005]**

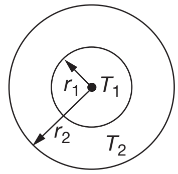


Fig. 11.4

(A) $\dfrac{(r_2 - r_1)}{(r_1 r_2)}$ (B) $In\left(\dfrac{r_2}{r_1}\right)$

(C) $\dfrac{r_1 r_2}{(r_2 - r_1)}$ (D) $(r_2 - r_1)$

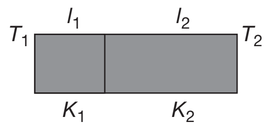
40. Assuming the sun to be a spherical body of radius R at a temperature of T K. Evaluate the total radiant power, incident on earth, at a distance r from the sun **[2006]**

(A) $\dfrac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$ (B) $\dfrac{\pi r_0^2 R^2 \sigma T^4}{r^2}$

(C) $\dfrac{r_0^2 R^2 \sigma T^4}{4\pi r^2}$ (D) $\dfrac{R^2 \sigma T^4}{r^2}$

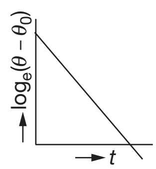
where r_0 is the radius of the earth and s is Stefan's constant.

41. One end of a thermally insulated rod is kept at a temperature T_1 and the other end at T_2. The rod is composed of two sections of length l_1 and l_2 and thermal conductivities K_1 and K_2, respectively. The temperature at the interface of the two sections is **[2007]**



(A) $(K_2 l_2 T_1 + K_1 l_1 T_2)/(K_1 l_1 + K_2 l_2)$
(B) $(K_2 l_1 T_1 + K_1 l_2 T_2)/(K_2 l_1 + K_1 l_2)$
(C) $(K_1 l_2 T_1 + K_2 l_1 T_2)/(K_1 l_2 + K_2 l_1)$
(D) $(K_1 l_1 T_1 + K_2 l_2 T_2)/(K_1 l_1 + K_2 l_2)$

42. A liquid in a beaker has temperature $q(t)$ at time t, and if θ_0 is the temperature of the surrounding, then according to Newton's law of cooling, the correct graph between log$e$ $(\theta - \theta_0)$ and t is **[2012]**

(A)


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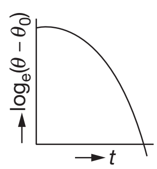
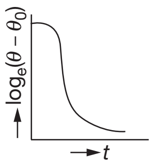
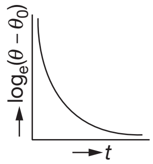
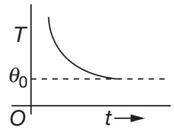
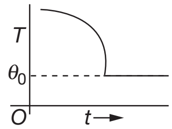


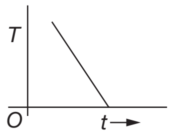
(C) 

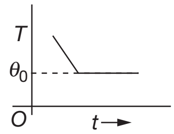
(D) 

43. If a piece of metal is heated to temperature θ and then allowed to cool in a room, which is at temperature θ_0, then the graph between the temperature T of the metal and time t will be closed to

(A) 

(B) 

(C) 

(D) 

ANSWER KEYS

Single Option Correct Type

1. (B) **2.** (C) **3.** (B) **4.** (D) **5.** (B) **6.** (B) **7.** (C) **8.** (C) **9.** (B) **10.** (C)
11. (A) **12.** (C) **13.** (B) **14.** (C) **15.** (C) **16.** (D) **17.** (B) **18.** (B) **19.** (B)

More than One Option Correct Type

20. (A) and (B) **21.** (A) and (B) **22.** (A) and (D) **23.** (D) **24.** (D)

Passage Based Questions

25. (C) **26.** (A) **27.** (A)

Assertion-Reason Type

28. (D) **29.** (B)

Integer Type

30. 46 W **31.** 7 min **32.** $n = 4$

Previous Years' Questions

33. (A) **34.** (B) **35.** (A) **36.** (D) **37.** (D) **38.** (D) **39.** (C) **40.** (B) **41.** (C) **42.** (D)
43. (B)

HINTS AND SOLUTIONS

Single Option Correct Type

1. If there is no heat current, temperature at both end of rod are equal and temperature at a distance 60 cm from hot end of the rod is 50°C.

The correct option is (B)

2. According to Newton's law of cooling, rate of cooling $\propto (T - T_0)$, where T is the average temperature in the given time interval. Hence

$$\frac{(60-50)}{10} \propto \left(\frac{60+50}{2} - 25\right)$$

and $$\frac{(50-T)}{10} \propto \left(\frac{50+T}{2} - 25\right)$$

Solving we get,

$T = 42.85$°C

The correct option is (C)

3. For parallel combination of two rods of equal length and equal area of cross-section,

$$K = \frac{K_1 + K_2}{2} = \frac{K_1 + \frac{4K_1}{3}}{2} = \frac{7K_1}{6}$$

Hence $$\frac{K}{K_1} = \frac{7}{6}$$

The correct option is (B)

4. $P \propto AT^4$ and $A \propto r^2$

$\therefore \quad P \propto r^2T^4$

Now, $\quad T' = 2T,\ r' = 2r$

Hence, $\quad P' = 4 \times 16P = 64P$

The correct option is (D)

5. The correct option is (B)

6. $\lambda_m T = \text{constant}\ \ \lambda_m = \dfrac{2.88 \times 10^6 \text{ nmK}}{2880} = 1000 \text{ nm}$

U_2 is maximum also $U_1 \neq 0, U_3 \neq 0$

The correct option is (B)

7. $\left(\dfrac{dm}{dt}\right)L = \dfrac{kA\theta}{x}$

$$A\rho \frac{dx}{dt}L = \frac{kA[0-(-15)]}{x}$$

$$\int_{x_1}^{x_2} x\,dx = \int_0^t \frac{15kA}{A\rho L}dt$$

$$t = \frac{L}{30k}[x_2^2 - x_1^2]$$

$$\frac{t_2}{t_1} = \frac{[3.0^2 - 1.5^2]}{[1.5^2 - 0^2]}$$

$\Rightarrow \quad t_2 = 20 \times 3 = 60$ min

The correct option is (C)

8. $F = -\dfrac{dU}{dr} = -2B(r - r_0)$

$$\omega^2 = \frac{K}{m_{\text{reduced}}} = \frac{2B}{m_1m_2}(m_1 + m_2)$$

The correct option is (C)

9. We must take into account here that the heat transferred per unit time is proportional to the difference in temperature. Let us introduce the following notation: $T_{\text{out1}}, T_{\text{out2}}$ and T_{r1} and T_{r2} are the temperatures outdoors and in the room in the first and second cases, respectively. The thermal power dissipated by the radiator in the room is $k_1(T - T_r)$, where k_1 is a certain coefficient. The thermal power dissipated from the room is $k_2(T_r - T_{\text{out}})$, where k_2 is another coefficient. In thermal equilibrium, the power dissipated by the radiator is equal to the power dissipated from the room. Therefore, we can write

$$k_1(T - T_{r1}) = k_2(T_{r1} - T_{\text{out1}})$$

Similarly, in the second case,

$$k_1(T - T_{r2}) = k_2(T_{r2} - T_{\text{out2}})$$

Dividing the first equation by the second, we obtain

$$\frac{T - T_{r1}}{T - T_{r2}} = \frac{T_{r1} - T_{\text{out1}}}{T_{r2} - T_{\text{out2}}}$$

Hence, we can determine T:

$$T = \frac{T_{r2}T_{\text{out1}} - T_{r1}T_{\text{out2}}}{T_{r2} + T_{\text{out1}} - T_{\text{out2}} - T_{r1}} = 60°\text{C}$$

The correct option is (B)

10. $\Delta Q = dU + \Delta W$

$$= nC_V dT + \int P dV = \alpha C_P(V_2 - V_1) + RT_0\frac{V_2}{V_1}$$

The correct option is (C)

11. $-mc\dfrac{dT_1'}{dt} = \dfrac{KA}{l}(T_1' - T_2') \qquad (1)$

$mc\dfrac{dT_2'}{dt} = \dfrac{KA}{l}(T_1' - T_2') \qquad (2)$

From (1) and (2)

$$t = \frac{mcl\ln 2}{2kA} = 193 \text{ min}$$

The correct option is (A)

12. $\dfrac{dQ}{dt} = -KA\dfrac{dT}{dx}$

$$\frac{dQ}{dt} = -\frac{K_0A}{a + bT}\frac{dT}{dx}$$

$$\frac{dQ}{dt}\int_0^l dx = -K_0A\int_{T_1}^{T_2}\frac{dT}{a + bT}$$

$$\frac{dQ}{dt} = \frac{AK_0}{b\,l}\ln\left[\frac{a + bT_1}{a + bT_2}\right]$$

The correct option is (C)

13. $300 = e\sigma A\left(900^4 - 300^4\right) \qquad (1)$

$600 = \dfrac{\sigma A}{2}\left(900^4 - 300^4\right) + \dfrac{e\sigma A}{2}\left(900^4 - 300^4\right) \qquad (2)$

$$e = \frac{1}{3}$$

The correct option is (B)

14. $I^2 R\,dt = mg\,dx,\ \int_o^t (a_0t)^2 R dt = \int_o^h mg\,dx$,

$$a_o^2 R\int_o^{t_o} dt\,t^2 = mgh\ ,\ \frac{a_o^2Rt_o^3}{3} = mgh\ ,\ h = \frac{a_0^2Rt_0^3}{3mg}$$

The correct option is (C)

15. $H = H_1 \qquad (1)$

Also $(\theta_1 - \theta_2) = (\theta_1 - \theta) + (\theta - \theta_2) \qquad (2)$

As $\quad H = \dfrac{KA(\theta_1 - \theta_2)}{l_1 + l_2}$

$\therefore \quad (\theta_1 - \theta_2) = \dfrac{(l_1 + l_2)H}{KA} \qquad (3)$

Also $\quad H_1 = \dfrac{K_1A(\theta_1 - \theta)}{l_1}$

$\therefore \quad \theta_1 - \theta = \dfrac{l_1H_1}{K_1A} \qquad (4)$

Similarly, $\theta - \theta_2 = \dfrac{l_2H_2}{K_2A} \qquad (5)$

Putting Equations (3), (4), and (5) in Equation (2),

$$\frac{(l_1+l_2)H}{K}=\frac{l_1}{K_1}+\frac{l_2}{K_2}$$

The correct option is (C)

16. The correct option is (D)

17. The correct option is (B)

18. The rate of cooling decreases with the decrease in temperature difference between the body and surroundings.

The correct option is (B)

19. Let θ be the temperature of the junction

$$\frac{2KA}{l}(90-\theta)=\frac{KA\theta}{l}$$

$\therefore \quad \theta = 60°$

The correct option is (B)

More than One Option Correct Type

20. $E = e\sigma A[T^4]$

Both have same surface area and temperature so both will initially emit same radiation in unit time. Both have same nature of surface so they will absorb equal radiation.

$$\frac{dT}{dt}=\frac{eA\sigma}{mc}[T^4-T_0^4] \qquad \frac{dT}{dt}\propto\frac{1}{m}$$

Spheres have different mass so rate of fall of temperature is different and obviously has different temperature at any moment.

The correct option is (A) and (B)

21. Rate of cooling is given by $\frac{dT}{dt}=\frac{\sigma\left(T^4-T_s^4\right)A}{ms}$ can be calculated to be 0.227 k/s^{-1}

Cooling will not happen, if temperatures are same.

The correct option is (A) and (B)

22. $H =$ rate of heat flow $= \dfrac{900}{\dfrac{l_i}{K_iA}+\dfrac{l_0}{K_0A}}$ (1)

Now, $1000-\theta=\dfrac{Hl_i}{K_iA}$ (2)

or $\theta = 1000-\left[\dfrac{900}{\dfrac{l_i}{K_iA}+\dfrac{l_0}{K_0A}}\right]\dfrac{l_i}{K_iA}=1000-\dfrac{900}{1+\dfrac{l_0}{K_0}\dfrac{K_i}{l_i}}$

Now, we can see that θ can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (l_i).

The correct option is (A) and (D)

23. $P \propto AT^4$ and $A \propto r^2$

$\therefore \quad P \propto r^2T^4$

Now, $T' = 2T$, $r' = 2r$

Hence, $P' = 4\times 16P = 64P$

The correct option is (D)

24. $H = -KA\dfrac{dT}{dx}$

$$\frac{dT}{dx}=-\frac{H}{KA}$$

The correct option is (D)

Passage Based Questions

25. The correct option is (C)

26. The correct option is (A)

27. The correct option is (A)

Assertion-Reason Type

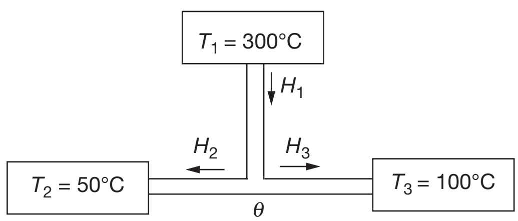
28. The correct option is (D)

29. The correct option is (B)

Integer Type

30. Let θ be the temperature of the junction

we have, $H_1 = H_2 + H_3$



or $(300-\theta)\dfrac{KA}{L}=(\theta-100)\dfrac{KA}{L}+(\theta-50)\dfrac{KA}{L}$

$$\Rightarrow \quad \theta=\frac{300+100+50}{3}=150°\text{ C}$$

$$H_2=\frac{KA(\theta-100)}{L}=\frac{46\times .01\times 50}{1}=23\text{ W}$$

$$H_3=\frac{KA[\theta-50]}{L}=\frac{46\times .01\times 100}{1}=46\text{ W}$$

Ratio = 2

31. In the first case, $-\dfrac{d\theta}{dt} = k\left(\dfrac{\theta_1+\theta_2}{2} - \theta_0\right)$

$-\dfrac{10}{5} = k(65-30),$, $k = -\dfrac{2}{35}$

In the second case

$-\dfrac{10}{t} = -\dfrac{2}{35}(55-30)$, $t = \dfrac{35\times 10}{2\times 25} = 7$ min

32. Since rate of heat flow remains same in both the cases, so

$$\int_R^{2R} \frac{dx}{k2\pi x l} = \int_{\frac{R}{4}}^{R} \frac{dx}{nk(2\pi x)\frac{l}{2}} \quad \Rightarrow nk = 4k$$

$\Rightarrow \quad n = 4$

Previous Years' Questions

33. A black body has maximum ability to absorb and emit radiation. However, black hole only absorbs radiation, hence black hole is not a black body.

The correct option is (A)

34. Spectrometer is an instrument which is used to split white light into component colours. Pyrometer is used to detect infrared radiations. Nanometer is a small unit of distance and is not a device.

Photometer is used to measure photometric quantities such as luminous intensity, luminance, and so on.

The correct option is (B)

35. Energy radiated per second by a body that has surface area A at temperature T is given by Stefan's law, $E = e\sigma AT^4$

Therefore,

$$\frac{E_1}{E_2} = \left(\frac{r_1}{r_2}\right)^2\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{1}{40}\right)^2\left(\frac{4000}{2000}\right)^4$$

(Since bodies are of same material, so $e_1 = e_2$)

$\Rightarrow \dfrac{E_1}{E_2} = \dfrac{16}{16} = \dfrac{1}{1} = 1:1$

The correct option is (A)

36. According to Newton's law of cooling

$\dfrac{dQ}{dt} \propto \Delta\theta$

$\therefore \quad n = 1$

The correct option is (D)

37. From Stefan's law, the energy radiated by sun is given by

$P = \sigma e A T^4$

In 1st case, $P_1 = \sigma e \times 4\pi R^2 \times T^4$

In 2nd case, $P_2 = \sigma e \times 4\pi(2R^2)\times(2T)^4$

$= \sigma e \times 4\pi R^2 \times T^4 \times 64 = 64P_1$

Since the energy radiated by sun increases 64 times, therefore, the radiant energy received on earth also increases 64 times.

The correct option is (D)

38. The two slabs are in series

Thus, $\quad R_{eq} = R_1 + R_2 = \dfrac{x}{KA} + \dfrac{4x}{2KA} = \dfrac{3x}{KA}$

Rate of heat flow, $\dfrac{d\theta}{dt} = \dfrac{T_2+T_1}{R_{eq}} = \dfrac{KA(T_2+T_1)}{3x}$

Thus, $\quad f = \dfrac{1}{3}$

The correct option is (D)

39. For hollow shell, resistance R is given by

$= \dfrac{r_2 - r_1}{4\pi K(r_1 r_2)}$

Thus, the rate of heat flow $= \dfrac{dQ}{dt}$

$= \dfrac{T_1 - T_2}{R}$

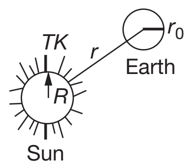
$= \dfrac{T_1 - T_2}{r_2 - r_1} \times 4\pi K(r_1 r_2) \propto \dfrac{r_1 r_2}{r_2 - r_1}$

The correct option is (C)

40. From Stefan's law, the rate of energy radiated by sun is

$P = \sigma 4\pi R^2 \times T^4$

At earth



$I = \dfrac{P}{4\pi r^2}\left(I = \dfrac{\text{Energy}}{\text{Area}}\right)$

$= \dfrac{\sigma \times 4\pi R^2 T^4}{4\pi r^2} = \dfrac{\sigma R^2 T^4}{r^2}$

$\therefore$ Total radiant power as received by earth $= \pi r_0^2 \times I$ (πr_0^2 is the area that receives energy on earth.)

$\dfrac{\pi r_0^2 R^2 \sigma T^4}{r^2}$

The correct option is (B)

41. Let temperature at the interface be T. For part AB,

$\dfrac{Q_1}{t} = \dfrac{(T_1 - T)K_1}{l_1}A$

For part BC,

$\dfrac{Q_2}{t} = \dfrac{(T - T_2)K_2}{l_2}A$

At equilibrium, $\dfrac{Q_1}{t} = \dfrac{Q_2}{t}$

$\Rightarrow \quad \dfrac{(T_1 - T)K_1 A}{l_1} = \dfrac{(T - T_2)K_2 A}{l_2}$

or $T = \frac{T_1K_1l_2 + T_2K_2l_1}{K_1l_2 + K_2l_1}$

The correct option is (C)

42. According to Newton's law of cooling.

$$-\frac{d\theta}{dt} = k(\theta - \theta_0)$$

$$\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -k\,dt$$

In $(\theta - \theta_0) = kt + C$

The correct option is (D)

43. Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\int \frac{d\theta}{\theta - \theta_0} = -k\int dt$$

The correct option is (B)