CHAPTER 10

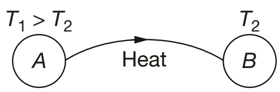
Heat and Thermal Expansion

Chapter Highlights

Heat, Temperature, Thermal expansion, Specific heat capacity, Latent heat, Calorimetry, Kinetic theory of gases, Concept of pressure, Kinetic energy and temperature, RMS speed of gas molecules, Law of equipartition of energy

HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of the receiving body. It should be clearly understood that the word 'heat' is meaningful only as long as the energy is being transferred. Thus, expressions like 'heat in a body' or 'heat of a body' are meaningless.



When we say that a body is heated it means that its molecules begin to move with greater kinetic energy.

SI unit of heat energy is joule (J). Another common unit of heat energy is calorie (cal).

$$1 \text{ calorie} = 4.18 \text{ J}$$

1 calorie: The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5°C at 1 atmospheric pressure is 1 calorie.

Mechanical Equivalent of Heat

In early days, heat was not recognized as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured as to how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write,

$$W = JH$$

where J is called mechanical equivalent of heat. J is expressed in joule/calorie. The value of J gives the joules of mechanical work needed to raise the temperature of 1 g of water by 1°C.

SOLVED EXAMPLE

1. What is the change in potential energy (in calories) of a 10 kg mass after 10 m fall?

Solution:

Change in potential energy

$$\Delta U = mgh = 10 \times 10 \times 10$$

$$= 1000 \text{ J}$$

$$= \frac{1000}{4.186} \text{ cal}$$

SPECIFIC HEAT

Specific heat of substance is equal to heat gain or released by that substance to raise or fall its temperature by 1°C for a unit mass of substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to:

1. The mass of the body $\Delta Q \propto m$
2. Rise or fall of temperature of the body $\Delta Q \propto \Delta T$

$\Delta Q \propto m\Delta T$ or $\Delta Q = ms\Delta T$

or $dQ = msdT$ or $Q = m\int sdT$

where s is a constant and is known as the specific heat of the body $s = \dfrac{Q}{m\,\Delta T}$. SI unit of s is joule/kg-kelvin and CGS unit is cal/gm°C.

Specific heat of water: S = 4200 J/kg°C = 1000 cal/kg°C = 1 kcal/kg°C = 1 cal/gm°C

Specific heat of steam = half of specific heat of water = specific heat of ice

SOLVED EXAMPLE

2. Heat required to increase the temperature of 1 kg water by 20°C

Solution:

Heat required = $\Delta Q = ms\Delta\theta$

$\because \quad S = 1$ cal/gm°C = 1 kcal/kg°C

$= 1 \times 20 = 20$ kcal.

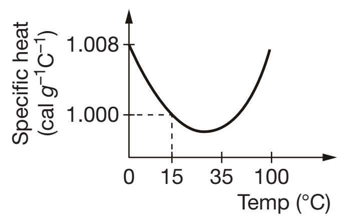
Heat Capacity or Thermal Capacity

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°. If m is the mass and s the specific heat of the body, then Heat capacity = ms.

Units of heat capacity in: CGS system is cal°C^{-1} and SI unit is JK^{-1}

IMPORTANT POINTS

- We know $s = \dfrac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$) , then $s = Q/0 = \infty$. Thus, the specific heat of a substance when it melts or boils at constant temperature is infinite.
- If the temperature of the substance changes without the transfer of heat ($Q = 0$) then $s = \dfrac{Q}{m\Delta T} = 0$. Thus, when liquid in the thermos flask is shaken, its temperature increases without the transfer of heat, and hence the specific heat of liquid in the thermos flask is zero.
- To raise the temperature of saturated water vapours, heat (Q) is withdrawn. Hence, specific heat of saturated water vapours is negative. (This is for your information only and not in the course.)
- The slight variation of specific heat of water with temperature is shown in the graph at 1 atmosphere pressure. Its variation is less than 1% over the interval from 0 to 100°C.



Relation between Specific Heat and Water Equivalent

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$$ms\,\Delta T = m_W\, s_W\, \Delta T \quad \Rightarrow \quad m_W = \frac{ms}{s_W}$$

In calorie $s_W = 1$

$$\therefore \quad m_W = ms$$

m_w is also represent by W

so $\quad W = ms.$

Phase change

Heat required for the change of phase or state,

$$Q = mL,\ L = \text{latent heat.}$$

Latent heat (*L*)

The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

Latent heat of fusion (L_f)

The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm pressure is called latent heat of fusion. Latent heat of fusion of ice is 80 kcal/kg.

Latent heat of vaporization (L_v)

The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization. Latent heat of vaporization of water is 540 kcal kg^{-1}.

If in question latent heat of water is not mention and to solve the problem it is required to assume, then we should consider the following value.

Latent heat of ice

$L = 80$ cal/gm = 80 kcal/kg = 4200×80 J/kg

Latent heat of steam

$L = 540$ cal/gm = 540 kcal/kg = 4200×540 J/kg

The given Fig. 10.1 represents the change of state by different lines

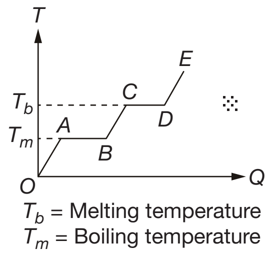


Fig. 10.1

OA – solid state, *AB* – solid + liquid state (Phase change)

BC – liquid state, *CD* – liquid + vapour state (Phase change)

DE – vapour state

$\Delta Q = ms\Delta T$

$$\text{slope } \frac{\Delta T}{\Delta Q} = \frac{1}{ms} \Rightarrow \frac{\Delta T}{\Delta Q} \mu \frac{1}{S}$$

where mass (m) of substance constant slope of $T - Q$ graph is inversely proportional to specific heat, as in given Fig. 10.1.

(slope)	$OA >$ (slope) DE
then	$(s)_{OA} < (s)_{DE}$
when	$\Delta Q = mL$
If	(length of AB) > (length of CD)
then	(latent heat of AB) > (latent heat of CD)

SOLVED EXAMPLE

3. Find the amount of heat released if 1 kg steam at 200°C is converted into –20°C ice.

Solution:

Heat required ΔQ = heat release to convert steam at 200°C into 100°C steam + heat release to convert 100°C steam into 100°C water + heat release to convert 100° water into 0°C water + heat release to convert 0°C water into –20°C ice.

$$\Delta Q = 1 \times \frac{1}{2} \times 100 + 540 \times 1 + 1 \times 1 \times 100 + 1 \times 80 + 1 \times \frac{1}{2} \times 20$$

$$= 780 \text{ kcal.}$$

CALORIMETRY

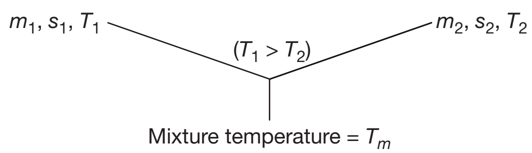
The branch of thermodynamics which deals with the measurement of heat is called calorimetry.

A simple calorimeter is a vessel generally made of copper with a stirrer of the same material. The vessel is kept in a wooden box to isolate it thermally from the surrounding. A thermometer is used to measure the temperature of the contents of the calorimeter. Object at different temperatures are made to come in contact with each other in the calorimeter. As a result, heat is exchanged between the object as well as with the calorimeter neglecting any heat exchange with the surrounding.

Law of Mixture

When two substances at different temperatures are mixed together, then exchange of heat continues to take place till their temperatures become equal. This temperature is then called final temperature of mixture. Here, Heat taken by one substance = Heat given by another substance

$$\Rightarrow \quad m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$$



SOLVED EXAMPLE

4. An iron block of mass 2 kg, falls from a height of 10 m. After colliding with the ground, it loses 25% energy to surroundings. Then find the temperature rise of the block. (Take sp. heat of iron 470 J/kg°C.)

Solution:

$$mS\Delta\theta = \frac{1}{4} mgh \Rightarrow \Delta\theta = \frac{10 \times 10}{4 \times 470}$$

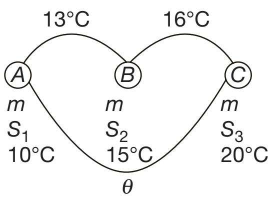
Zeroth Law of Thermodynamics

If objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

SOLVED EXAMPLES

5. The temperature of equal masses of three different liquids A, B, and C are 10°C, 15°C, and 20°C, respectively. The temperature when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?

Solution:



when A and B are mixed

$$mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13)$$

$$3S_1 = 2S_2 \quad (1)$$

when B and C are mixed

$$S_2 \times 1 = S_3 \times 4 \quad (2)$$

when C and A are mixed

$$S_1(\theta - 10) = S_3 \times (20 - \theta) \quad (3)$$

by using equation (1), (2), and (3)

we get $\theta = \dfrac{140}{11}$ °C

6. If three different liquid of different masses specific heats and temperature are mixed with each other, then what is the temperature mixture at thermal equilibrium.

$m_1, s_1, T_1 \rightarrow$ specification for liquid

$m_2, s_2, T_2 \rightarrow$ specification for liquid

$m_3, s_3, T_3 \rightarrow$ specification for liquid

Solution:

Total heat lost or gained by all substance is equal to zero

$$\Delta Q = 0$$

$$m_1s_1(T - T_1) + m_2s_2(T - T_2) + m_3s_3\,(T - T_3) = 0$$

So $$T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$$

7. In following equation, calculate value of H 1 kg ice at –20°C = H + 1 kg water at 100°C, here H means heat required to change the state of substance.

Solution:

Heat required to convert 1 kg ice at – 20°C into 1 kg water at 100°C

= 1 kg ice at – 20°C to 1 kg ice at 0°C ice at 0°C + 1 kg water

at 0°C + 1 kg water at 0°C to 1 kg water at 100°C

$$= 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 100 = 190 \text{ kcal.}$$

So $H = -190$ kcal

Negative sign indicate that 190 kcal heat is withdrawn from 1 kg water at 100°C to convert it into 1 kg ice at – 20°C

8. 1 kg ice at –20°C is mixed with 1 kg steam at 200°C. Then find equilibrium temperature and mixture content.

Solution:

Let equilibrium temperature be 100°C heat required to convert 1 kg ice at –20°C to 1 kg water at 100°C is equal to

$$H_1 = 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 1 \times 100 = 190 \text{ kcal}$$

Heat release by steam to convert 1 kg steam at 200°C to 1 kg water at 100°C is equal to

$$H_2 = 1 \times \frac{1}{2} \times 100 + 1 \times 540 = 590 \text{ kcal}$$

1 kg ice at – 20°C = H_1 + 1 kg water at 100°C (1)

1 kg steam at 200°C = H_2 + 1 kg water at 100°C (2)

By adding equation (1) and (2)

1 kg ice at –20°C + 1 kg steam at 200°C = $H_1 + H_2 +$ 2 kg water at 100°C.

Here heat required to ice is less than heat supplied by steam so mixture of equilibrium temperature is 100°C, then steam is not completely converted into water.
So mixture has water and steam which is possible only at 100°C mass of steam, when converted into water is equal to

$$m = \frac{190 - 1 \times \frac{1}{2} \times 100}{540} = \frac{7}{27} \text{ kg}$$

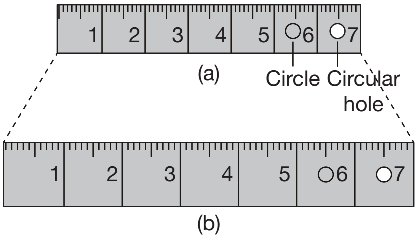
so mixture content

$$\text{mass of steam} = 1 - \frac{7}{27} = \frac{20}{27}\ \text{kg}$$

$$\text{mass of water} = 1 + \frac{7}{27} = \frac{34}{27}\ \text{kg}$$

THERMAL EXPANSION

Most materials expand when their temperature is increased. Rails, roads, tracks, bridges, etc. have some means of compensating for thermal expansion. When a homogeneous object expands, the distance between any two points on the object increases. Fig. 10.2 shows a block of metal with a hole in it. **The expanded object is like a photographic enlargement.** That in the hole expands in the same proportion as the metal, it does not get smaller.



The same steel ruler two different temperatures. When it expands, the scale, the numbers, the thickness, and the diameters of the circle and circular and hole are all increased by the same factor. (The expansion has been exaggerated for clarity.)

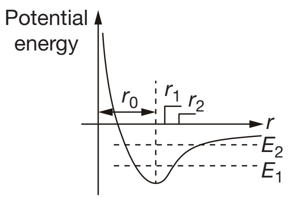


Fig. 10.2

Thermal expansion arises because the well is not symmetrical about the equilibrium position $r0$. As the temperature rise the energy of the atom increases. The average position when the energy is E_2 is not the same as that when the energy is E_1.

At the atomic level, thermal expansion may be understood by considering how the potential energy of the atoms varies with distance. The equilibrium position of an atom will be at the minimum of the potential energy if the well is symmetric. At a given temperature, each atom vibrates about its equilibrium position and its average remains at the minimum point. If the shape of the well is not symmetrical the average position of an atom will not be at the minimum point. When the temperature is raised, the amplitude of the vibrations increases and the average position is located at a greater interatomic separation. This increased separation is manifested as expansion of the material.

Almost all solids and liquids expand as their temperature increases. Gases also expand if allowed. Solids can change in length, area, or volume, while liquids change in their volumes.

SOLVED EXAMPLES

9. A rectangular plate has a circular cavity as shown in Fig. 10.3. If we increase its temperature then which dimension will increase as in Fig. 10.3.

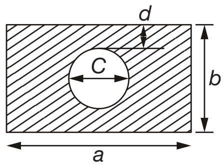


Fig. 10.3

Solution:

Distance between any two point on an object increases with increase in temperature.

So, all dimension a, b, c, and d will increase

10. In Fig. 10.4, when temperature is increased then which of the following increases

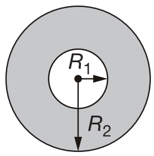


Fig. 10.4

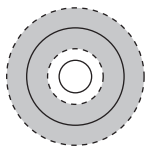
(A) R_1 (B) R_2 (C) $R_2 - R_1$

Solution:

All of the above

- - - - - represents expanded boundary

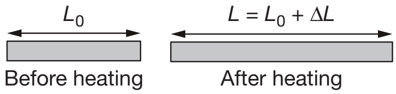
--------- represents original boundary



As the intermolecular distance between atoms increases on heating, the inner and outer perimeter increases. Also if the atomic arrangement in radial direction is observed, then we can say that it also increases. Hence, all A, B, C are true.

LINEAR EXPANSION

When the rod is heated, its increase in length ΔL is proportional to its original length L_0 and change in temperature ΔT, where ΔT is in °C or K.



$$dL = \alpha L_0\, dT \quad \Rightarrow \quad \Delta L = \alpha\, L_0\, \Delta\, T \text{ If } \alpha\, \Delta T << 1$$

$$\alpha = \frac{\Delta\, L}{L_0\, \Delta\, T},$$

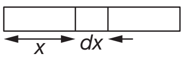
where α is called the coefficient of linear expansion whose unit is $°C^{-1}$ or K^{-1}.

$$L = L_0\,(1 + \alpha\, \Delta T)$$

where L is the length after heating the rod.

Variation of α with Temperature and Distance

1. If α varies with distance, $\alpha = ax + b$.
   Then total expansion $= \int (ax + b)\, \Delta T\, dx$.
2. If α varies with temperature, $\alpha = f(T)$. Then $\Delta L = \int \alpha\, L_0\, dT$.



NOTE

Actually thermal expansion is always 3D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one-dimension and it is known as linear expansion.

SOLVED EXAMPLE

11. What is the percentage change in length of 1m iron rod if its temperature changes by 100°C. α for iron is 2×10^{-5}/°C?

Solution:

Percentage change in length due to temperature change

$$\%\ell = \frac{\Delta\ell}{\ell} \times 100 = \alpha\Delta\,\theta \times 100$$

$$= 2 \times 10^{-5} \times 100 \times 100$$

$$= 0.2\% .$$

Thermal Stress of a Material

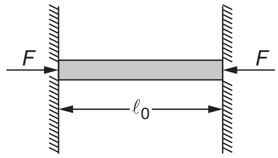
If the rod is free to expand, then there will be no stress and strain. Stress and strain is produced only when an object is restricted to expand or contract according to change in temperature. When the temperature of the rod is decreased or increased under constrained condition, compressive or tensile stresses are developed in the rod. These stresses are known as thermal stresses.

$$\text{Strain} = \frac{\Delta L}{L_0} = \frac{\text{Final length} - \text{Original length}}{\text{Original length}} = \alpha\, \Delta T$$

NOTE

Original and final length should be at same temperature

Consider a rod of length ℓ_0 which is fixed between to rigid end separated at a distance ℓ_0. Now if the temperature of the rod is increased by $\Delta\theta$ then the strain produced in the rod will be:



$$\text{Strain} = \frac{\text{length of the rod at new temperature} - \text{natural length of the rod at new temperature}}{\text{natural length of the rod at new temperature}}$$

SPECIFIC HEAT

The specific heat capacity of a substance is defined as the heat supplied per unit mass of the substance per unit rise in the temperature. If an amount ΔQ of heat is given to a mass m of the substance, and its temperature rises by ΔT, the specific heat capacity s is given by equation

$$s = \frac{\Delta Q}{m\Delta T}$$

The molar heat capacities of a gas are defined as the heat given per mole of the gas per unit rise in the temperature. The molar heat capacity at constant volume, denoted by C_V, is,

$$C_V = \left(\frac{\Delta Q}{n\ \Delta T}\right)_{\text{constant volume}} = \frac{f}{2}R$$

and the molar heat capacity at constant pressure, denoted by C_P is,

$$C_P = \left(\frac{\Delta Q}{n\ \Delta T}\right)_{\text{constant volume}} = \left(\frac{f}{2} + 1\right)R$$

where n is the amount of the gas in number of moles and f is degree of freedom. Quite often, the term specific heat capacity or specific heat is used for molar heat capacity. It is advised that the unit be carefully noted to determine the actual meaning. The unit of specific heat capacity is J/kg–K, whereas that of molar heat capacity is J/mol–K.

Molar Heat Capacity of Ideal Gas in Terms of *R*

1. For a monoatomic gas, $f = 3$

$$C_V = \frac{3}{2}R\,,\; C_P = \frac{5}{2}R \Rightarrow \frac{C_P}{C_V} = \gamma = \frac{5}{3} = 1.67$$

2. For a diatomic gas, $f = 5$

$$C_V = \frac{5}{2}R,\;\; C_P = \frac{7}{2}R \;\; \gamma = \frac{C_P}{C_V} = 1.4$$

3. For a triatomic gas, $f = 6$

$$C_V = 3R,\;\; C_P = 4R$$

$$\gamma = \frac{C_P}{C_V} = \frac{4}{3} = 1.33$$ [Note for CO_2; $f = 5$, it is linear]

In general, if f is the degree of freedom of a molecule, then,

$$C_V = \frac{f}{2}R,\quad C_P = \left(\frac{f}{2}+1\right)R,\;\; \gamma = \frac{C_P}{C_V} = \left[1+\frac{2}{f}\right]$$

SOLVED EXAMPLES

12. Two moles of a diatomic gas at 300 K are enclosed in a cylinder as shown in Fig. 10.5. Piston is light. Find out the heat given if the gas is slowly heated to 400 K in the following three cases.

(A) Piston is free to move

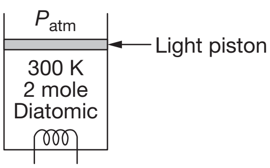


Fig. 10.5

(B) If piston does not move
(C) If piston is heavy and movable

Solution:

(A) Since pressure is constant

$$\therefore \Delta Q = nC_P\Delta T = 2 \times \frac{7}{2} \times R \times (400 - 300)$$
$$= 700\,R$$

(B) Since volume is constant

$\therefore \Delta W = 0$ and $\Delta Q = \Delta u$ (from first law)

$$\Delta Q = \Delta u = nC_V\Delta T = 2 \times \frac{5}{2} \times R \times (400 - 300)$$
$$= 500\,R$$

(C) Since pressure is constant

$$\therefore \Delta Q = nC_P\Delta T = 2 \times \frac{7}{2} \times R \times (400 - 300)$$
$$= 700\,R$$

13. P-V curve of a diatomic gas is shown in Fig. 10.6. Find the total heat given to the gas in the process AB and BC

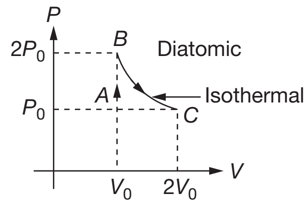


Fig. 10.6

Solution:

From first law of thermodynamics,

$$\Delta Q_{ABC} = \Delta u_{ABC} + \Delta W_{ABC}$$

$$\Delta W_{ABC} = \Delta W_{AB} + \Delta W_{BC} = 0 + nR\,T_B \ln\frac{V_C}{V_B}$$

$$= nR\,T_B \ln\frac{2V_0}{V_0}$$

$$= nRT_B \ln 2 = 2P_0\,V_0 \ln 2$$

$$\Delta u = nC_V\,\Delta T = \frac{5}{2}\,(2P_0V_0 - P_0V_0)$$

$$\Rightarrow \quad \Delta Q_{ABC} = \frac{5}{2}P_0V_0 + 2P_0V_0 \ln 2.$$

14. Calculate the value of mechanical equivalent of heat from the following data. Specific heat capacity of air at constant volume = 170 cal/kg-K, $\gamma = C_P/C_V = 1.4$ and the density of air at STP is 1.29 kg/m^3. Gas constant $R = 8.3$ J/mol-K.

Solution:

Using $pV = nRT$, the volume of 1 mole of air at STP is

$$V = \frac{nRT}{p} = \frac{(1\text{ mol})\times(8.3\text{ J/mol}-\text{K})\times(273\text{ K})}{1.0\times10^5\text{ N/m}^2}$$
$$= 0.0224\text{ m}^3.$$

The mass of 1 mole is, therefore,

$$(1.29\text{ kg/m}^3)\times(0.0224\text{ m}^3) = 0.029\text{ kg}.$$

The number of moles in 1 kg is $\frac{1}{0.029}$. The molar heat capacity at constant volume is

$$C_v = \frac{170 \text{ cal}}{(1/0.029) \text{ mol-K}} = 4.93 \text{ cal/mol-K}.$$

Hence, $C_P = \gamma C_V = 1.4 \times 4.93$ cal/mol-K

or $C_P - C_V = 0.4 \times 4.93$ cal/mol-K

$= 1.97$ cal/mol-K

Also, $C_P - C_V = R = 8.3$ J/mol-K

Thus, 8.3 J = 1.97 cal.

The mechanical equivalent of heat is

$$\frac{8.3 \text{ J}}{1.97 \text{ cal}} = 4.2 \text{ J/cal}.$$

Average Molar Specific Heat of Metals [Dulong and Petit Law]

At room temperature, average molar specific heat of all metals are same and is nearly equal to 3R

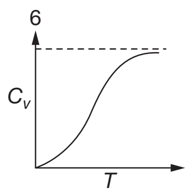
$$(6 \text{ cal. mol}^{-1} \text{ K}^{-1}).$$

Temperature above which the metals have constant C_V is called Debye temperature.

$$= \frac{\ell_0 - \ell_0(1+\alpha\Delta\theta)}{\ell_0(1+\alpha\Delta\theta)} = \frac{-\ell_0\alpha\Delta\theta}{\ell_0(1+\alpha\Delta\theta)}$$

$\because$ α is very small so

Strain $= -\alpha\Delta\theta$ (negative sign in the answer represents that the length of the rod is less than the



Mayer's Equation

$$C_P - C_V = R \qquad \text{(for ideal gases only)}$$

natural length that is compressed by the ends.

SOLVED EXAMPLES

15. In the given Fig. 10.7, a rod is free at one end and other end is fixed. When we change the temperature of rod by $\Delta\theta$, then strain produced in the rod will be

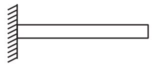


Fig. 10.7

(A) $\alpha\Delta\theta$ (B) $\frac{1}{2}\alpha\Delta\theta$

(C) Zero (D) Information insufficient

Solution:

Here rod is free to expand from one side; so by changing temperature, no strain will be produced in the rod. Hence, answer is (C)

16. An iron ring measuring 15.00 cm in diameter is to be shrunk on a pulley which is 15.05 cm in diameter. All measurements refer to the room temperature 20°C. To what minimum temperature should the ring be heated to make the job possible? Calculate the strain developed in the ring when it comes to the room temperature. Coefficient of linear expansion of iron $= 12 \times 10^{-6}$/°C.

Solution:

The ring should be heated to increase its diameter from 15.00 cm to 15.05 cm.

Using $\ell_2 = \ell_1(1 + \alpha\Delta\theta)$,

$$= \frac{0.05 \text{ cm}}{15.00 \text{ cm} \times 12 \times 10^{-6}/°\text{C}} = 278°\text{C}$$

The temperature = 20°C + 278°C = 298°C.

$$\text{The strain developed} = \frac{\ell_2 - \ell_1}{\ell_1} = 3.33 \times 10^{-3}.$$

17. A steel rod of length 1 m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

Solution:

In absence of external force, no strain or stress will be created. Here rod is free to move.

18. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is in natural length at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel $= 1.2 \times 10^{-5}$/°C.

Solution:

$$\text{As we known that strain} = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{\Delta\ell}{\ell_0}$$

$\therefore$ Strain $= \alpha\Delta\theta$

$$= 1.2 \times 10^{-5} \times (50 - 20)$$
$$= 3.6 \times 10^{-4}$$

Here strain is a compressive strain because final length is smaller than initial length.

19. A steel wire of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is 1.2×10^{-5}/°C and its Young's modulus is 2.0×10^{11} N/m^2.

Solution:

Here final length is more than original length so that strain is tensile and tensile force is given by

$$F = AY\,\alpha\,\Delta t = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20$$
$$= 24 \text{ N}$$

Variation of Time Period of Pendulum Clocks

The time represented by the clock hands of a pendulum depends on the number of oscillation performed by pendulum. Every time it reaches to its extreme position, the second hand of the clock advances by one second that means second hand moves by two seconds when one oscillation in complete.

Let $T = 2\pi\sqrt{\dfrac{L_0}{g}}$ at temperature θ_0

and $T' = 2\pi\sqrt{\dfrac{L}{g}}$ at temperature θ.

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L[1 + \alpha\,\Delta\,\theta]}{L}} = 1 + \frac{1}{2}\,\alpha\,\Delta\,\theta$$

Therefore, change (loss or gain) in time per unit time lapsed is

$$\frac{T'-T}{T} = \frac{1}{2}\,\alpha\,\Delta\theta$$

gain or loss in time in duration of t in

$$\Delta t = \frac{1}{2}\,\alpha\,\Delta\theta t, \text{ if } T \text{ is the correct time then}$$

(a) $\theta < \theta_0$, $T' < T$ clock becomes fast and gain time
(b) $\theta > \theta_0$, $T' > T$ clock becomes slow and lose time

SOLVED EXAMPLE

20. A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at 20°C, how fast or slow will it go in 24 hours at 40°C? Coefficient of linear expansion of iron = 1.2×10^{-6} /°C.

Solution:

The time difference occurred in 24 hours (86400 seconds) is given by

$$\Delta t = \frac{1}{2}\,\alpha\,\Delta\theta\,t$$
$$= \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 86400$$
$$= 1.04 \text{ s}$$

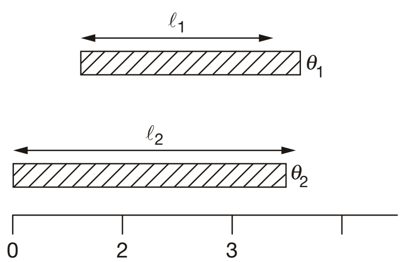
This is loss of time as θ is greater than θ_0. As the temperature increases, the time period also increases. Thus, the clock goes slow.

Measurement of Length by Metallic Scale

Case I: When object is expanded only

$$\ell_2 = \ell_1\,\{1 + \alpha_0(\theta_2 - \theta_1)\}$$

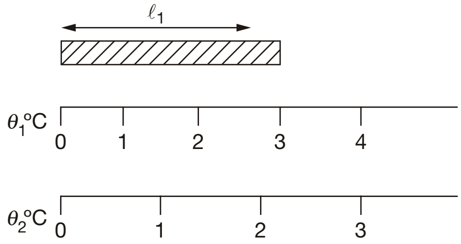
ℓ_1 = actual length of object at θ_1°C = measure length of object at θ_1°C.
ℓ_2 = actual length of object at θ_2°C = measure length of object at θ_2°C.
α_0 = linear expansion coefficient of object.



Case II: When only measurable instrument is expanded, actual length of object will not change but measurable value (MV) decreases.

$$MV = \ell_1\,\{1 - \alpha_S\,(\theta_2 - \theta_1)\}$$

α_S = linear expansion coefficient of measuring instrument.



at θ_1 C $MV = 3$

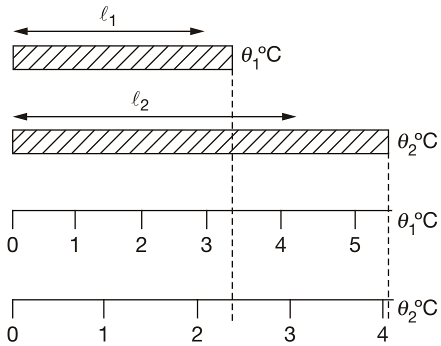
at θ_1 C $MV = 2.2$

Case III: If both expanded simultaneously

$$MV = \{1 + (\alpha_0 - \alpha_s)(\theta_2 - \theta_1)$$

(i) If $\alpha_0 > \alpha_s$, then measured value is more then actual value at θ_1°C

(ii) If $\alpha_0 < \alpha_s$, then measured value is less then actual value at θ_1°C



at θ_1°C $MV = 3.4$

θ_2°C $MV = 4.1$

Measured value = calibrated value × $\{1 + \alpha \Delta \theta\}$

where $\alpha = \alpha_0 - \alpha_s$

α_0 = coefficient of linear expansion of object material,

α_s = coefficient of linear expansion of scale material

$$\Delta\theta = \theta - \theta_C$$

θ = temperature at the time of measurement

θ_C = temperature at the time of calibration.

For scale, true measurement = scale reading $[1 + \alpha(\theta - \theta_0)]$

If $\theta > \theta_0$ true measurement > scale reading

$\theta < \theta_0$ true measurement < scale reading

SOLVED EXAMPLE

21. A bar measured with a vernier caliper is found to be 180 mm long. The temperature during the measurement is 10°C. The measurement error will be if the scale of the vernier caliper has been graduated at a temperature of 20°C ($\alpha = 1.1 \times 10^{-5}$°C^{-1}. Assume that the length of the bar does not change.)

(A) 1.98×10^{-1} mm (B) 1.98×10^{-2} mm
(C) 1.98×10^{-3} mm (D) 1.98×10^{-4} mm

Solution: (B)

True measurement = scale reading $[1 + \alpha(\theta - \theta_0)]$

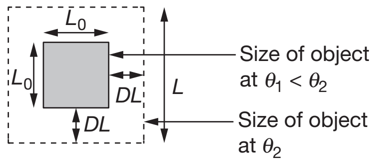
$$= 180 \times \{1 + (10 - 20) \times (-1.1 \times 10^{-5})\}$$

measurement error = true measurement – scale reading

$$= 180 \times \{1 + (10 - 20) \times (-1.1 \times 10^{-5})\} - 180$$
$$= 1.98 \times 10^{-2} \text{ mm}$$

SUPERFICIAL OR AREAL EXPANSION

When a solid is heated and its area increases, then the thermal expansion is called superficial or areal expansion. Consider a solid plate of area A_0. When it is heated, the change in area of the plate is directly proportional to the original area A_0 and the change in temperature ΔT.



$$dA = \beta A_0\, dT \text{ or } \Delta A = \beta A_0 \Delta T$$

$$\beta = \frac{\Delta A}{A_0 \Delta T} \quad \text{Unit of } \beta \text{ is °C}^{-1} \text{ or K}^{-1}.$$

$$A = A_0(1 + \beta \Delta T)$$

where A is area of the plate after heating,

SOLVED EXAMPLE

22. A plane lamina has area 2 m^2 at 10°C then what is its areal at 110°C. It's superficial expansion is 2×10^{-5}/C

Solution:

$$A = A_0(1 + \beta \Delta \theta)$$
$$= 2\{1 + 2 \times 10^{5} \times (110 - 10)\}$$
$$= 2 \times \{1 + 2 \times 10^{-3}\}$$

VOLUME OR CUBICAL EXPANSION

When a solid is heated and its volume increases, then the expansion is called volume expansion or cubical expansion.

Let us consider a solid or liquid whose original volume is V_0. When it is heated to a new volume, then the change ΔV

$$dV = \gamma V_0\, dT \text{ or } \Delta V = \gamma V_0\, \Delta T$$

$$\gamma = \frac{\Delta V}{V_0\, \Delta T} \quad \text{Unit of } \gamma \text{ is} °C^{-1} \text{ or } K^{-1}.$$

$$V = V_0\,(1 + \gamma \Delta T),$$

where V is the volume of the body after heating

SOLVED EXAMPLE

23. The volume of glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficient of cubical expansion of mercury and glass are 1.8×10^{-4}/°C and 9.0×10^{-6}/°C, respectively.

Solution:

Let volume of glass vessel at 20°C is V_g and volume of mercury at 20°C be V_m

So volume of remaining space is $= V_g - V_m$

It is given constant so that

$$V_g - V_m = V_g' - V'_m$$

where V_o' and V_m' are final volumes.

$$V_g - V_m = V_g\,\{1 + \gamma_g\, \Delta\theta\} - V_m\{1 + \gamma_{Hg}\,\Delta\theta\}$$

$$\Rightarrow \; V_g\, \gamma_g = V_m\, \gamma_{Hg}$$

$$\Rightarrow \quad V_m = \frac{100 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}}$$

$$V_m = 50 \text{ cc.}$$

RELATION BETWEEN α, β, AND γ

1. For isotropic solids, $\alpha : \beta : \gamma = 1 : 2 : 3$ or $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$
2. For non-isotropic solid, $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$. Here α_1, α_2, and α_3 are coefficient of linear expansion in X, Y, and Z direction.

SOLVED EXAMPLE

24. If percentage change in length is 1% with change in temperature of a cuboid object ($\ell \times 2\ell \times 3\ell$) then what is the percentage change in its area and volume.

Solution:

Percentage change in length with change in temperature = % ℓ

$$\frac{\Delta \ell}{\ell} \times 100 = \alpha \Delta\theta \times 100 = 1$$

Change in area

$$\Rightarrow \% A = \frac{\Delta A}{A} \times 100 = \beta \Delta\theta \times 100 \Rightarrow 2\,(\,\alpha\,\Delta\,\theta \times 100)$$

$$\% A = 2\,\%$$

Change in volume

$$\% V = \frac{\Delta V}{V} \times 100 = V\,\Delta\,\theta \times 100 = 3\,(\alpha\,\Delta\,\theta \times 100)$$

$$\% V = 3\,\%$$

VARIATION OF DENSITY WITH TEMPERATURE

As we known that mass = volume × density.

Mass of substance does not change with change in temperature, so with increase of temperature, volume increases so density decreases and vice-versa.

$$d = \frac{d_0}{(1 + \gamma\,\Delta T)}.$$

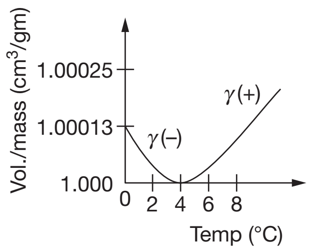
For solids, values of γ are generally small so we can write $d = d_0\,(1 - \gamma \Delta T)$ (using binomial expansion).

NOTE

(i) γ for liquids are in order of 10^{-3}.

(ii) Anomalous expansion of water:

As water density increases from 0°C to 4°C, so γ is negative and for 4°C to higher temperature γ is positive. At 4°C, density is maximum. This anomalous behaviour of water is due to presence of three types of molecules, i.e. H_2O, $(H_2O)_2$, and $(H_2O)_3$ having different volume/mass at different temperatures.



- This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature decreases initially at the surface. The water there sinks because of its increased density. Consequently, the surface reaches 0°C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4°C.

SOLVED EXAMPLE

25. The densities of wood and benzene at 0°C are 880 kg/m^3 and 900 kg/m^3, respectively. The coefficients of volume expansion are 1.2×10^{-3}/°C for wood and 1.5×10^{-3}/°C for benzene. At what temperature will a piece of wood just sink in benzene?

Solution:

At just sink, gravitation force = upthrust force

$$\Rightarrow \quad mg = F_B$$

$$\Rightarrow \quad V\rho_1 g = V\rho_2 g$$

$$\Rightarrow \quad \rho_1 = \rho_2$$

$$\Rightarrow \quad \frac{880}{1+1.2\times10^{-3}\theta} = \frac{900}{1+1.5\times10^{-3}\theta}$$

$$\Rightarrow \quad \theta = 83°C$$

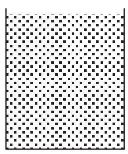
Apparent Expansion of a Liquid in a Container

Initially, container was full. When temperature changes by ΔT,

volume of liquid $\quad V_L = V_0 (1 + \gamma_L \Delta T)$

volume of container $\quad V_C = V_0 (1 + \gamma_C \Delta T)$

So overflow volume of liquid relative to container



$$\Delta V = V_L - V_C \quad \Delta V = V_0 (\gamma_L - \gamma_C) \Delta T$$

So, coefficient of apparent expansion of liquid with respect to container

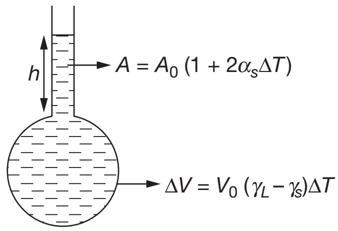
$$\gamma_{\text{apparent}} = \gamma_L - \gamma_C.$$

In case of expansion of liquid + container system,

if $\quad \gamma_L > \gamma_C \rightarrow$ level of liquid rise

if $\quad \gamma_L < \gamma_C \rightarrow$ level of liquid fall

Increase in height of liquid level in tube when bulb was initially completely filled



$$h = \frac{\text{Volume of liquid}}{\text{Area of tube}} = \frac{V_0(1+\gamma_L \Delta T)}{A_0(1+2\alpha_S \Delta T)}$$

$$= h_0 \{1 + (\gamma_L - 2\alpha_S) \Delta T\}$$

$$h = h_0 \{1 + (\gamma_L - 2\alpha_S) \Delta T\}$$

where $\quad h_0$ = original height of liquid in container
α_S = linear coefficient of expansion of container.

SOLVED EXAMPLE

26. A glass vessel of volume 100 cm^3 is filled with mercury and is heated from 25°C to 75°C. What volume of mercury will overflow? Coefficient of linear expansion of glass = 1.8×10^{-6}/°C and coefficient of volume expansion of mercury is 1.8×10^{-4}/°C.

Solution:

$$\Delta V = V_0(\gamma_L - \gamma_C)\Delta T = 100 \times \{1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}\} \times 50$$

$$\Delta V = 0.87 \text{ cm}^3$$

Variation of Force of Buoyancy with Temperature

If body is submerged completely inside the liquid:

For solid, buoyancy force

$$F_B = V_0 d_L g$$

V_0 = Volume of the solid inside liquid

d_L = density of liquid

Volume of body after increase its temperature

$$V = V_0\,[1 + \gamma_S\,\Delta\theta]$$

Density of body after increase its temperature

$$d'_L = \frac{d_L}{[1 + \gamma_L\ \Delta\theta]}.$$

Buoyancy force of body after increase its temperature,

$$F'_B = V\,d'_L\,g,$$

$$\frac{F'_B}{F_B} = \frac{[1 + \gamma_S\,\Delta\theta]}{[1 + \gamma_L\ \Delta\theta]},$$

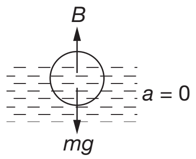
if $\gamma_S < \gamma_L$ then $F'_B < F_B$

Buoyant force decreases or apparent weight of body in liquid gets increased

$$[W - F'_B > W - F_B].$$

SOLVED EXAMPLES

27. A body will float inside liquid if we increase temperature then what changes occur in buoyancy force. (Assume body is always in floating condition.)



Solution:

Body is in equilibrium

so $\quad mg = B$

and gravitational force does not change with change in temperature. So Buoyancy force remains constant. By increasing temperature, density of liquid decreases so volume of body inside the liquid increases to kept the buoyance force constant for equal to gravitational force.

28. In previous question, discuss the case when body move downwards, upwards, and remains at same position when we increase temperature.

Solution:

Let f = fraction of volume of body submerged in liquid.

$$f = \frac{\text{Volume of body submerged in liquid}}{\text{Total volume of body}}$$

$$f_1 = \frac{v_1}{v_0} \text{ at } \theta_1°\text{C}$$

$$f_2 = \frac{v_2}{v_0(1 + 3\alpha_S\Delta\theta)} \text{ at } \theta_2°\text{C}$$

for equilibrium mg $= B = v_1 d_1 g = v_2 d_2 g$.

so $\quad v_2 = \dfrac{v_1 d_1}{d_2}$

$\because \quad d_2 = \dfrac{d_1}{1 + \gamma_L\Delta\theta} = v_1(1 + \gamma_L\,\Delta\theta)$

$\therefore \quad f_2 = \dfrac{v_1(1 + \gamma_L\Delta\theta)}{v_0(1 + 3\alpha_s\Delta\theta)}$

where $\quad \Delta\theta = \theta_2 - \theta_1$

Case I: Body move downward if $f_2 > f_1$

means $\quad \gamma_L > 3\alpha_S$

Case II: Body move upwards if $f_2 < f_1$

means $\quad \gamma_L < 3\alpha_S$

Case III: Body remains at same position

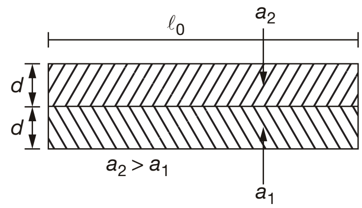
if $\quad f_2 = f_1$

means $\quad \gamma_L = 3\alpha_S$

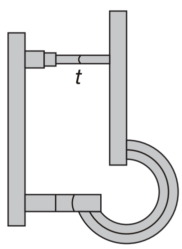
BIMETALLIC STRIP

It two strips of different metals are welded together to form a bimetallic strip, when heated uniformly it bends in form of an arc, the metal with greater coefficient of linear expansion lies on convex side. The radius of arc thus formed by bimetal is:

$$\ell_0\,(1 + \alpha_1\Delta\theta) = \left(R - \frac{d}{2}\right)\theta$$



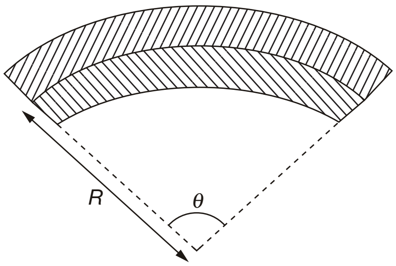
Lower temperature (at θ_1°C)



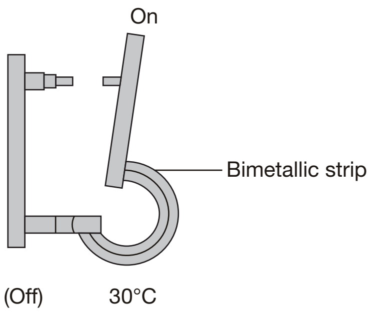
25°C

$$\ell_0 (1 + \alpha_2 \Delta\theta) = \left(R + \frac{d}{2}\right)\theta$$

$$\Rightarrow \qquad \frac{1+\alpha_2\Delta\theta}{1+\alpha_1\Delta\theta} = \frac{R+\frac{d}{2}}{R-\frac{d}{2}}$$



Higher temperature (at θ_2°C)



$$\Rightarrow \qquad R = \frac{d}{(\alpha_2 - \alpha_1)\Delta\theta}$$

$$\Delta\theta = \text{change in temperature} = \theta_2 - \theta_1$$

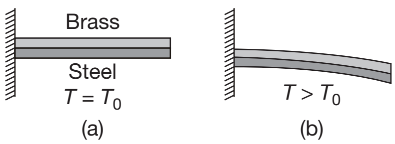


Fig. 10.8

A bimetallic strip, consisting of a strip of brass and a strip of steel welded together, at temperature T_0 in shown in Fig. 10.8 (a) (b). The strip bends as shown at temperatures above the reference temperature. Below the reference temperature, the strip bends the other way. Many thermostats operate on this principle, making and breaking an electrical constant as the temperature rises and falls.

APPLICATIONS OF THERMAL EXPANSION

1. A small gap is left between two iron rails of the railway.
2. Iron rings are slipped on the wooden wheels by heating the iron rings.
3. Stopper of a glass bottle jammed in its neck can be taken out by heating the neck.
4. The pendulum of a clock is made of invar (an alloy of zinc and copper).

TEMPERATURE

Temperature may be defined as the **degree of hotness or coldness** of a body. Heat energy flows from a body at higher temperature to that at lower temperature until their temperatures become equal. At this stage, the bodies are said to be in thermal equilibrium.

Measurement of Temperature

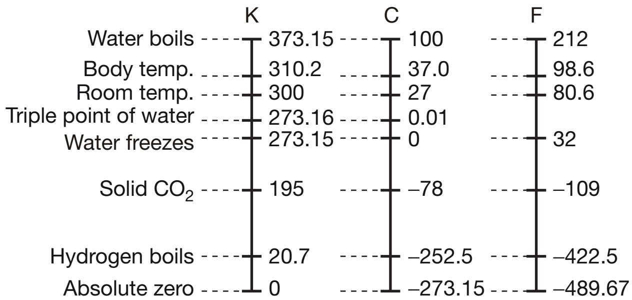
The branch of thermodynamics which deals with the measurement of temperature is called thermometry. A thermometer is a device used to measure the temperature of a body. The substances like liquids and gases which are used in the thermometer are called thermometric substances.

Different Scales of Temperature

A thermometer can be graduated into following scales.

1. The Centigrade or Celsius scale (°C)
2. The Fahrenheit scale (°F)
3. The Reaumur scale (°R)
4. Kelvin scale of temperature (K)

Comparison between Different Temperature Scales



The formula for the conversion between different temperature scales is:

$$\frac{K - 273}{100} = \frac{C}{100} = \frac{F - 32}{180} = \frac{R}{80}$$

General formula for the conversion of temperature from one scale to another:

$$\frac{\text{Temp on one scale}(S_1) - \text{Lower fixed point }(S_1)}{\text{Upper fixed point }(S_2) - \text{Lower fixed point }(S_1)}$$

$$= \frac{\text{Temp. on other scale}(S_2) - \text{Lower fixed point }(S_2)}{\text{Upper fixed point }(S_2) - \text{Lower fixed point }(S_2)}$$

Thermometers

Thermometer is a device that is used to measure temperatures. All thermometers are based on the principle that some physical property of a system changes as the system temperature changes.

Required properties of good thermometric substance.

1. Non-sticky (absence of adhesive force)
2. Low melting point (in comparison with room temperature)
3. High boiling temperature
4. Coefficient of volumetric expansion should be high (to increase accuracy in measurement)
5. Heat capacity should be low
6. Conductivity should be high

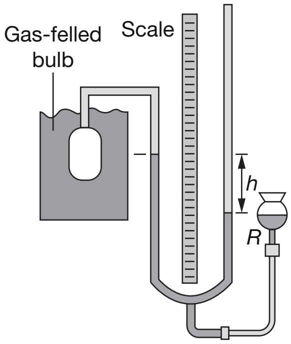
Mercury (Hg) suitably exhibits above properties.

Types of Thermometers (not for JEE)

Type of thermometer and its range	Thermometric property	Advantages	Disadvantages	Particular Uses
Mercury-in-glass – 39°C to 450°C	Length of column of mercury in capillary tube	(i) Quick and easy to (direct reading) (ii) Easily portable	(i) Fragile (ii) Small size limits (iii) Limited range	(i) Every laboratory use where high ccuracy is not required. (ii) Can be calibrated against constant-volume gas thermometer for more accurate work
Constant-volume gas thermometer – 270° to 1500°C	Pressure of a fixed mass of gas at constant volume	(i) Very accurate (ii) Very sensitive (iii) Wide range (iv) Easily reproducible	(i) Very large volume of bulb (ii) Slow to use and inconvenient	(i) Standard against which others calibrated (ii) He, H_2 or N_2 used depending on range (iii) can be corrected to the ideal gas scale (iv) Used as standard below-183°C
Platinum resistance –180° to 1150°C	Electrical resistance of a platinum coil	(i) Accurate (ii) Wide range	Not suitable for varying temperature (i.e., is slow to respond to changes)	(i) Best thermometer for small steady temperature differences (ii) Used as standard between 183°C and 630°C.
Thermocouple –250°C to 1150°C	Emf produced between junctions of dissimilar metals at different temperatures for measurement of emfs	(i) Fast response because of low heat capacity. (ii) wide range (iii) can be employed for remote readings using long leads.	Accuracy is lost if emf is measured using a moving-coil voltmeter (as may be necessary for rapid changes when potentiometer is unsuitable)	(i) Best thermometer for small steady temperature differences (ii) Can be made direct reading by calibrating galvanometer (iii) Used as standard between 630°C and 1063°C
Radiation pyrometer above 1000°C	Colour of radiation emitted by a hot body	Does not come into contact when temperature is measured	(i) Cumbersome (ii) Not direct reading (needs a trained observer)	(i) Only thermometer possible for very high temperatures (ii) Used as standard above 1063°C.

The Constant-Volume Gas Thermometer

The standard thermometer, against which all other thermometers are calibrated, is based on the pressure of a gas in a fixed volume. Fig. 10.9 shows such a constant volume gas thermometer; it consists of a gas-filled bulb connected by a tube to a mercury monometer.



A constant volume gas thermometer, its bulb immersed in a liquid whose temperature T is to be measured.

Fig. 10.9

$$T = (273.16\text{ K})\left(\lim_{\text{gas}\to 0} \frac{p}{p_3}\right)$$

P = Pressure at the temperature being measured, P_3 = pressure when bulb in a triple point cell.

SOLVED EXAMPLES

29. The readings of a thermometer at 0°C and 100°C are 50 cm and 75 cm of mercury column, respectively. Find the temperature at which its reading is 80 cm of mercury column?

Solution:

By using formula,

$$\frac{80-50}{75-50} = \frac{T-0}{100-0}$$

$$\Rightarrow \quad T = 120°\text{C}$$

30. A bullet of mass 10 gm in moving with speed 400 m/s. Find its kinetic energy in calories?

Solution:

$$\Delta k = \frac{1}{2} \times \frac{10}{1000} \times 400 \times 400 = 800$$

$$\frac{800}{4.2} = 191.11\text{ Cal.}$$

31. Calculate amount of heat required to convert 1 kg steam from 100°C to 200°C steam

Solution:

$$\text{Heat required} = 1 \times \frac{1}{2} \times 100 = 50\text{ kcal}$$

32. Calculate heat required to raise the temperature of 1 g of water through 1°C?

Solution:

Heat required $= 1 \times 10^{-3} \times 1 \times 1 = 1 \times 10^{-3}$ kcal

33. 420 J of energy supplied to 10 g of water will raise its temperature by

Solution:

$$\frac{420 \times 10^{-3}}{4.20} = 10 \times 10^{-3} \times 1 \times \Delta t = 10°\text{C}$$

34. The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3. Find the ratio of their thermal capacities for unit volume?

Solution:

$$\frac{\rho_1}{\rho_2} = \frac{3}{4}, \quad \frac{s_1}{s_2} = \frac{4}{3}$$

$$\theta = \frac{m \times s}{m/\rho}$$

$$\Rightarrow \quad \frac{\theta_1}{\theta_2} = \frac{s_1}{s_2} \times \frac{\rho_1}{\rho_2} = 1 : 1.$$

35. Heat releases by 1 kg steam at 150°C if it is converted into 1 kg water at 50°C.

Solution:

$$\text{H} = 1 \times \frac{1}{2} \times 50 + 1 \times 540 + 1 \times 1 \times 50$$

$$= 540 + 75 = 615\text{ kcal}$$

Heat release = 615 kcal.

36. 200 gm water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature increases by 20°C. Find the heat supplied to the water.

Solution:

$$\text{H} = 200 \times 10^{-3} \times 1 \times 20 = 4\text{ kcal.}$$

Heat supplied = 4000 cal

37. A bullet of mass 5 gm is moving with speed 400 m/s. strike a target and energy. Then calculate rise of temperature of bullet. Assuming all the loss in kinetic energy is converted into heat energy of bullet if its specific heat is 500J/kg°C.

Solution:

$$\text{Kinetic energy} = \frac{1}{2} \times 5 \times 10^{-3} \times 400 \times 400$$

$$= 5 \times 10^{-3} \times 500 \times \Delta T$$

$$\Delta T = 160°C$$

Rise in temperature is 160°C

38. 1 kg ice at –10°C is mixed with 1 kg water at 100°C. Then find equilibrium temperature and mixture content.

Solution:

$$\text{Heat given by 1 kg ice} = 1 \times \frac{1}{2} \times 10 = 5 \text{ kcal}$$

$$5K + 1 \times 80 + 1 \times T = 1 \times (100 - T)$$

$$85 = 100 - 2T$$

$$\Rightarrow \quad 2T = 15$$

$$\theta = \frac{15}{2} = 7.5°C, \text{ water}$$

39. 1 kg ice at –10° is mixed with 1kg water at 50°C. Then find equilibrium temperature and mixture content.

Solution:

Heat given by ice = $1 \times \frac{1}{2} \times 10$ = 5 kcal + 80 kcal = 85 kcal

Heat taken by water = 1 × 1 × 50 = 50 kcal

Heat given > Heat taken so, ice will not complete melt. Let *m* g ice melt then

$$1 \times \frac{1}{2} \times 10 + 80 \text{ m} = 50 \text{ m}$$

$$80 \text{ m} = 45$$

$$\Rightarrow \quad m = \frac{45}{80}$$

$$\text{Content of mixture} \begin{cases} \text{water} & \left(1+\frac{45}{80}\right)\text{kg} \\ \text{ice} & \left(1-\frac{45}{80}\right)\text{kg} \end{cases} \text{ and}$$

temperature is 0°C

40. A small ring having small gap is shown in Fig. 10.10. On heating what will happen to size of gap.

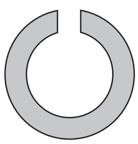


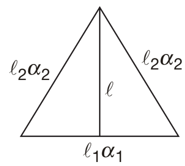
Fig. 10.10

Solution:

Gap will also increase. The reason is same as in above.

41. An isosceles triangle is formed with a thin rod of length ℓ_1 and coefficient of linear expansion α_1, as the base and two thin rods each of length ℓ_2 and coefficient of linear expansion α_2 as the two sides. If the distance between the apex and the midpoint of the base remain unchanged as the temperature is varied show that

Solution:



$$\ell = \sqrt{\left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2}$$

$$\ell^2 = \left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2$$

$$O = \frac{\ell_1 2}{2}\frac{d\ell_1}{dT} + 2\ell_2 \times \frac{d\ell_2}{dt}$$

$$\ell_1 \times \frac{\ell_1\alpha_1}{2} \times \Delta T = 2\ell_2 \times \ell_2\, \alpha_2 \Delta T$$

$$\frac{\ell_1}{\ell_2} = 4\frac{\alpha_2}{\alpha_1} = \frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}.$$

42. A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is 1.0×10^{-5} /°C.

Solution:

$$\ell_t = 10(1 + 1 \times 10^{-5} \times 35)\ 10.0035 \text{ m}$$

43. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = 1.2×10^{-5}/°C.

Solution: $= \frac{\Delta\ell}{\ell} = \frac{\ell_0\alpha\Delta t}{\ell_0} - 3.6 \times 10^{-4}$

44. If rod is initially compressed by $\Delta\ell$ length then what is the strain on the rod when the temperature

(a) is increased by $\Delta\theta$

(b) is decreased by $\Delta\theta$

Solution:

(a) Strain $= \frac{\Delta\ell}{\ell} + \alpha\Delta\theta$

(b) Strain $= \left|\frac{\Delta\ell}{\ell} - \alpha\Delta\theta\right|$

45. A pendulum clock having copper rod keeps correct time at 20°C. It gains 15 seconds per day if cooled to 0°C. Calculate the coefficient of linear expansion of copper.

Solution:

$$\frac{15}{24\times 60\times 60} = \frac{1}{2}\alpha\times 20$$

$$\Rightarrow \quad \alpha = \frac{1}{16\times 3600} = 1.7\times 10^{-5}/°C$$

46. A meter scale made of steel is calibrated at 20°C to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is 1.1 × 10^{-5}/°C.

Solution:

$$\ell_t = 1\,(1 - 1.1\times 10^{-5}\times 10) = 0.99989 \text{ cm}$$

47. A uniform solid brass sphere is rotating with angular speed w_0 about a diameter. If its temperature is now increased by 100°C, what will be its new angular speed? (Given $\alpha_B = 2.0\times 10^{-5}$/°C)

(A) $\frac{\omega_0}{1-0.002}$ (B) $\frac{\omega_0}{1+0.002}$

(C*) $\frac{\omega_0}{1+0.004}$ (D) $\frac{\omega_0}{1-0.004}$

Solution:

$$I_0\,\omega_0 = I_t\,\omega t\; Mr_0^2\,\omega_0 = Mr_0^2\,(1 + 2\alpha\Delta T)\omega_t$$

$$\omega_t = \frac{\omega_0}{1+0.004}.$$

48. The volume occupied by a thin-wall brass vessel and the volume of a solid brass sphere are the same and equal to 1,000 cm^3 at 0°C. How much will the volume of the vessel and that of the sphere change upon heating to 20°C? The coefficient of linear expansion of brass is $\alpha = 1.9\times 10^{-5}$.

Solution:

$$V = V_0\,(1 + 3\alpha\,\Delta T) = 1.14 \text{ cm}^3$$

1.14 cm^3 for both

49. A thin copper wire of length L increases in length by 1%, when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions $2L\times L$ is heated from T_1 to T_2?

(A) 1% (B) 3% (C) 4% (D*) 2%

Solution:

$$\ell_f = L\,(1 + \alpha\Delta t) = \frac{L_f}{L}\times 100$$

$$= (1 + \alpha\Delta t)\times 100 = 1\%$$

$$A = 2L\times L\,(1 + 2\,\alpha\Delta t) = \frac{A_f}{2L\times L}\times 100$$

$$= (1 + 2\alpha\,\Delta t)\times 100 = 2\%$$

50. The density of water at 0°C is 0.998 g/cm^3 and at 4°C it is 1.000 g/cm^3. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

Solution:

$$d_t = \frac{d_0}{1+\gamma\,\Delta t}$$

$$\Rightarrow \quad 1 = \frac{0.998}{1+\gamma\times 4} = -5\times 10^{-4}/°C$$

51. A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. It is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1.6 cm^3 of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass = 6.5×10^{-6}/°C

Solution:

$$\Delta V = V_{Hg} - V_V$$

$$1.6 = 10^3\,(\gamma_\ell\times 10 - 10^3\times 3\times 6.5\times 10^{-6}\times 10$$

$$\gamma_L = (1.6 + 0.195)\times 10^{-4}$$

$$= 1.795\times 10^{-4}\,/°C$$

52. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 50°C. The coefficient of cubical expansion of the metal is less than alcohol. Assuming that density of the metal is large compared to that of the alcohol, find which of W_1 and W_2 is greater?

Solution:

$$\gamma_M < \gamma\ell$$

so, $$\frac{F_B'}{F_B} = \frac{[1+\gamma_S\Delta\theta]}{[1+\gamma_\ell\Delta\theta]}\; F_B' < F_B$$

so, apparent weight increased

so, $W_2 > W_1$

53. In Fig. 10.11, which strip brass or steel have higher coefficient of linear expansion.

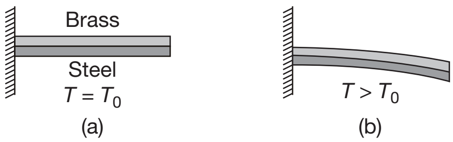


Fig. 10.11

Solution:

Brass strip

54. The upper and lower fixed points of a faulty thermometer are 5°C and 105°C. If the thermometer reads 25°C, what is the actual temperature ?

Solution:

$$\frac{25-5}{100} = \frac{C-0}{100} \quad C = 20°\text{C}$$

55. At what temperature is the Fahrenheit scale reading equal to twice of Celsius ?

Solution:

$$\frac{F-32}{180} = \frac{C-0}{100}$$

$$\frac{2x-32}{180} = \frac{x-0}{100}$$

$$1x - 160 = 9x$$

$$x = 160°\text{C}$$

56. Temperature of a patient is 40°C. Find the temperature on Fahrenheit scale?

Solution:

$$\frac{F-32}{180} = \frac{40-0}{100}$$

$$F = 104°\text{ F}$$

KINETIC THEORY OF GASES

Kinetic theory of gases is based on the following basic assumptions.

1. A gas consists of very large number of molecules. These molecules are identical, perfectly elastic, and hard spheres. They are so small that the volume of molecules is negligible as compared with the volume of the gas.
2. Molecules do not have any preferred direction of motion; motion is completely random.
3. These molecules travel in straight lines and in free motion most of the time. The time of the collision between any two molecules is very small.
4. The collision between molecules and the wall of the container is perfectly elastic. It means kinetic energy is conserved in each collision.
5. The path travelled by a molecule between two collisions is called free path and the mean of this distance travelled by a molecule is called mean free path.
6. The motion of molecules is governed by Newton's law of motion.
7. The effect of gravity on the motion of molecules is negligible.

Expression for the Pressure of a Gas

Let us suppose that a gas is enclosed in a cubical box having length ℓ. Let there be N identical molecules, each having mass m. Since the molecules are of same mass and perfectly elastic, their mutual collisions result in the interchange of velocities only. Only collisions with the walls of the container contribute to the pressure by the gas molecules. Let us focus on a molecule having velocity v_1 and components of velocity $v_{x_1}, v_{y_1}, v_{z_1}$ along x, y, and z-axis as shown in Fig. 10.12.

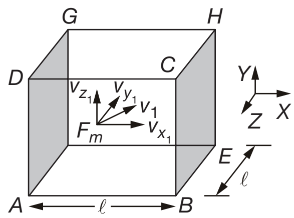


Fig. 10.12

$$v_1^2 = v_{x_1}^2 + v_{y_1}^2 + v_{z_1}^2$$

The change in momentum of the molecule after one collision with wall BCHE

$$= mv_{x_1} - (-mv_{x_1}) = 2\,mv_{x_1}.$$

The time taken between the successive impacts on the face

$$\text{BCHE} = \frac{\text{distance}}{\text{velocity}} = \frac{2\ell}{v_{x_1}}$$

Time rate of change of momentum due to collision

$$= \frac{\text{change in momentum}}{\text{time taken}} = \frac{2mv_{x_1}}{2\ell/v_{x_1}} = \frac{mv_{x_1}^2}{\ell}$$

Hence, the net force on the wall BCHE due to the impact of n molecules of the gas is:

$$F_x = \frac{mv_{x_1}^2}{\ell} + \frac{mv_{x_2}^2}{\ell} + \frac{mv_{x_3}^2}{\ell} + \ldots\ldots\ldots + \frac{mv_{x_n}^2}{\ell} = \frac{m}{\ell}$$

$$\left(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + \ldots\ldots\ldots\ldots + v_{x_n}^2\right) = \frac{mN}{\ell} < v_x^2 >$$

where $< v_x^2 >$ = mean square velocity in x-direction. Since molecules do not favour any particular direction, therefore $< v_x^2 > = < v_y^2 > = < v_z^2 >$. But $< v^2 > = < v_x^2 > + < v_y^2 > + < v_z^2 >$

$$\Rightarrow \qquad < v_x^2 > = \frac{< v^2 >}{3}.$$

Pressure is equal to force divided by area.

$$P = \frac{F_x}{\ell^2} = \frac{M}{3\ell^3} < v^2 > = \frac{M}{3V} < v^2 >.$$

Pressure is independent of x, y, z directions.

Where ℓ^3 = volume of the container = V

M = total mass of the gas, $<c^2>$ = mean square velocity of molecules

$$\Rightarrow \qquad P = \frac{1}{3} \rho < v^2 >$$

As $PV = n\,RT$, then total translational KE of gas $= \frac{1}{2} M < v^2 > = \frac{3}{2} PV = \frac{3}{2} n\,RT$

Translational kinetic energy of 1 molecule $= \frac{3}{2}$kT (It is independent of the nature of gas.)

$$< v^2 > = \frac{3P}{\rho} \text{ or } v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_{\text{mole}}}} = \sqrt{\frac{3kT}{m}}$$

Where v_{rms} is root mean square velocity of the gas.
Pressure exerted by the gas is

$$P = \frac{1}{3} \rho <v^2> = \frac{2}{3} \times \frac{1}{2} \rho <v^2> \text{ or } P = \frac{2}{3} E, E = \frac{3}{2} P$$

Thus, total translational kinetic energy per unit volume (it is called energy density) of the gas is numerically equal to $\frac{3}{2}$ times the pressure exerted by the gas.

IMPORTANT POINTS

- $v_{\text{rms}} \propto \sqrt{T}$ and $v_{\text{rms}} \propto \frac{1}{\sqrt{M_{\text{mole}}}}$
- At absolute zero, the motion of all molecules of the gas stops.
- At higher temperature and low pressure or at higher temperature and low density, a real gas behaves as an ideal gas.

MAXWELL'S DISTRIBUTION LAW

Distribution Curve

A plot of $\frac{dN(v)}{dv}$ (number of molecules per unit speed interval) against c is known as Maxwell's distribution curve. The total area under the curve is given by the integral

$$\int_0^\infty \frac{dN(v)}{dv} dv = \int_0^\infty dN(v) = N\ .$$

The actual formula of $\frac{dN(v)}{dv}$ is not in JEE syllabus.

Figure 10.13 shows the distribution curves for two different temperatures. At any temperature, the number of molecules in a given speed interval dv is given by the area under the curve in that interval (shaded in Fig. 10.13). This number increases, as the speed increases, up to a maximum and then decreases asymptotically toward zero. Thus, maximum numbers of the molecules have speed lying within a small range centered about the speed corresponding the peak (A) of the curve. This speed is called the 'most probable speed' v_p or v_{mp}.

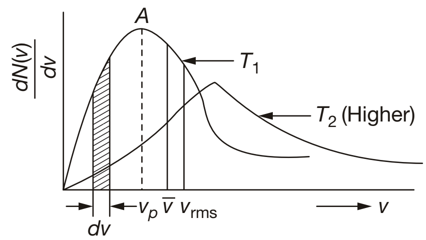


Fig. 10.13

The distribution curve is asymmetrical about its peak (the most probable speed v_p) because the lowest possible speed is zero, whereas there is no limit to the upper speed a molecule can attain. Therefore, the average speed $\bar{v}$ is slightly larger than the most probable speed v_p. The root-mean-square speed, v_{rms}, is still larger ($v_{\text{rms}} > \bar{v} > v_p$).

Average (or Mean) Speed

$$\bar{v} = \sqrt{\frac{8}{\pi}\frac{kT}{m}} = 1.59\sqrt{kT/m}$$

(Derivation is not in the course).

RMS Speed

$$v_{\text{rms}} = \sqrt{< v^2 >} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}\ .$$

Most Probable Speed

The most probable speed v_p or v_{mp} is the speed possessed by the maximum number of molecules and corresponds to the maximum (peak) of the distribution curve. Mathematically, it is obtained by the condition.

$\frac{dN(v)}{dv} = 0$ [by substitution of formula of $dN(v)$ (which is not in the course)]

Hence, the most probable speed is

$$v_p = \sqrt{\frac{2kT}{m}} = 1.41 \,.\, \sqrt{kT/m}$$

From the above expression, we can see that

$$v_{\text{rms}} > \bar{v} > v_p.$$

The laws which can be deduced with the help of kinetic theory of gases are as follows:

1. Boyle's law
2. Charles's law
3. Avogadro's hypothesis
4. Graham's law of diffusion of gases
5. Regnault's or Gay Lussac's law
6. Dalton's law of partial pressure
7. Ideal gas equation or equation of state

DEGREE OF FREEDOM

Total number of independent co-ordinates which must be known to completely specify the position and configuration of dynamical system is known as 'degree of freedom f'. Maximum possible translational degrees of freedom are three, i.e. $\left(\frac{1}{2}mV_x^2 + \frac{1}{2}mV_y^2 + \frac{1}{2}mV_z^2\right)$.

Maximum possible rotational degrees of freedom are three, i.e. $\left(\frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2\right)$.

Vibrational degrees of freedom are two, i.e., kinetic energy of vibration and potential energy of vibration.

Mono atomic: (all inert gases, He, Ar, etc.) $f = 3$ (translational)

Diatomic: (gases like H_2, N_2, O_2, etc.) $f = 5$ (3 translational + 2 rotational)

If temp < 70 K for diatomic molecules, then $f = 3$

If temp in between 250 K to 5000 K, then $f = 5$

If temp > 5000 K $f = 7$ (3 translational + 2 rotational + 2 vibrational)

MAXWELL'S LAW OF EQUPARTITION OF ENERGY

Energy associated with each degree of freedom = $\frac{1}{2}$kT.

If degree of freedom of a molecule is f, then total kinetic energy of that molecule $U = \frac{1}{2} f$ kT.

Internal Energy

The internal energy of a system is the sum of kinetic and potential energies of the molecule of the system. It is denoted by U. Internal energy (U) of the system is the function of its absolute temperature (T) and its volume (V), i.e., $U = f(T, V)$.

In case of an ideal gas, intermolecular force is zero. Hence, its potential energy is also zero. In this case, the internal energy is only due to kinetic energy, which depends on the absolute temperature of the gas, i.e., $U = f(T)$. For an ideal gas, internal energy $U = \frac{f}{2} nRT$.

SOLVED EXAMPLES

57. A light container having a diatomic gas enclosed within is moving with velocity v. Mass of the gas is M and number of moles is n.

(i) What is the kinetic energy of gas with respect to centre of mass of the system?

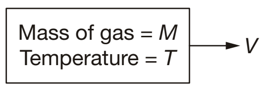
(ii) What is kinetic energy of gas with respect to ground?

Solution:

(i) KE = $\frac{5}{2} nRT$

(ii) Kinetic energy of gas with respect to ground = Kinetic energy of centre of mass with respect to ground + Kinetic energy of gas with respect to centre of mass.

$$\text{KE} = \frac{1}{2}MV^2 + \frac{5}{2}\,nRT$$



58. Two non-conducting containers having volume V_1 and V_2 contain monoatomic and diatomic gases, respectively. They are connected as shown in Fig. 10.14. Pressure and temperature in the two containers are P_1, T_1, and P_2, T_2, respectively. Initially, stop cock is closed, and if the stop cock is opened find the final pressure and temperature.

Solution:

$$n_1 = \frac{P_1V_1}{RT_1} \quad n_2 = \frac{P_2V_2}{RT_2}$$

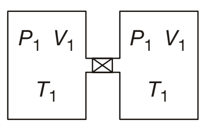


Fig. 10.14

$$n = n_1 + n_2$$
(number of moles are conserved)

Finally, pressure in both parts and temperature of the both the gases will be become equal.

$$\frac{P(V_1 + V_2)}{RT} = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}$$

From energy conservation,

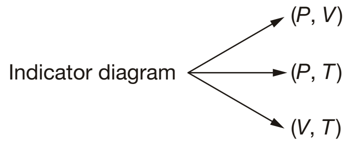
$$\frac{3}{2}\, n_1RT_1 + \frac{5}{2}\, n_2RT_2 = \frac{3}{2}\, n_1RT + \frac{5}{2}\, n_2RT$$

$$\Rightarrow T = \frac{(3P_1V_1 + 5P_2V_2)T_1T_2}{3P_1V_1T_2 + 5P_2V_2T_1}$$

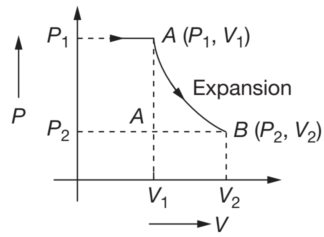
$$\Rightarrow P = \left(\frac{3P_1V_1 + 5P_2V_2}{3P_1V_1T_2 + 5P_2V_2T_1}\right)\left(\frac{P_1V_1T_2 + P_2V_2T_2}{V_1 + V_2}\right)$$

INDICATOR DIAGRAM

A graph representing the variation of pressure or variation of temperature or variation of volume with each other is called or indicator diagram.



1. Every point of indicator diagram represents a unique state (P, V, T) of gases.
2. Every curve on indicator diagram represents a unique process.



BRAIN MAP

1. Thermal Expansion

$l = l_0(1 + \alpha\Delta T)$
$S = S_0(1 + \beta\Delta T)$
$V = V_0(1 + \gamma\Delta T)$
$\alpha : \beta : \gamma :: 1 : 2 : 3$

2. $C = C' + 3\alpha$

where C is coefficient of real expansion and C' is coefficient of apparent expansion of liquid and α is coefficient of linear expansion of solid

3. Thermometry

$$\frac{C}{100} = \frac{K - 273}{100} = \frac{F - 32}{180} = \frac{R}{4}$$

4. Calorimetry

Heat given = Heat taken
$Q = ms\Delta\theta$ is used for heat given to raise temperature.
$Q = mL$ is used for phase change.

EXERCISES

Single Option Correct Type

1. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape. The real length of the given piece of wood must be

(A) 25 cm (B) Less than 25 cm
(C) More than 25 cm (D) None of these

2. Heat required to melt 1 gm of ice is 80 cal. A man melts 60 gm of ice by chewing it in 1 minute. Power supplied by the man to melt ice is

(A) 4800 W (B) 336 W
(C) 80 W (D) 0.75 W

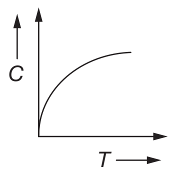
3. The temperature of cold junction of a thermocouple is –20°C and the temperature of inversion is 560°C. The neutral temperature is

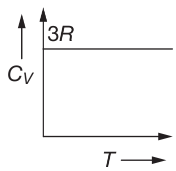
(A) 270°C (B) 560°C
(C) 1120°C (D) 290°C

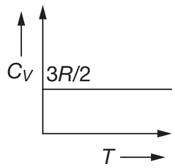
4. Two litres of water at initial temperature of 27°C is heated by a heater of power 1 kW in a kettle. If the lid of the kettle is open, then heat energy is lost at a constant rate of 160 J/s. The time in which the temperature will rise from 27°C to 77°C is (specific heat of water = 4.2 kJ/kg)

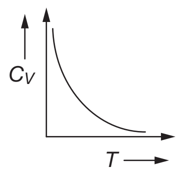
(A) 5 min 20 s (B) 8 min 20 s
(C) 10 min 40 s (D) 12 min 50 s

5. Graph of specific heat at constant volume for a monoatomic gas is

(A) 

(B) 

(C) 

(D) 

6. The average translational kinetic energy of 1 mole of O_2 molecules (molar mass = 32) at a particular temperature is 0.048 eV. The internal energy of 1 mole of N_2 molecules (molar mass = 28) in eV at same temperature is

(A) 0.048 (B) 0.003
(C) 0.0288 (D) 0.080

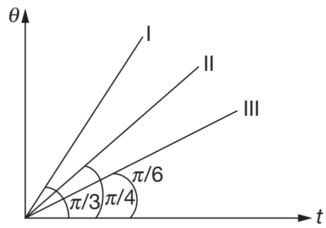
7. The ratio of coefficients of cubical expansion and linear expansion is

(A) 1 : 1 (B) 3 : 1
(C) 2 : 1 (D) None of these

8. Minimum amount of steam of 100°C required to melt 12 gm ice completely will be

(A) 1.5 gm (B) 1 gm (C) 2 gm (D) 5 gm

9. Three bodies A, B, and C of masses m, m, and $\sqrt{3}\,m$, respectively, are supplied heat at a constant rate. The change in temperature θ versus time t graph for A, B, and C are shown by I, II, and III, respectively. If their specific heat capacities are S_A, S_B, and S_C, respectively, then which of the following relation is correct? (Initial temperature of each body is 0°C)



(A) $S_A > S_B > S_C$ (B) $S_B = S_C < S_A$
(C) $S_A = S_B = S_C$ (D) $S_B = S_C > S_A$

10. A container X contains 1 mole of O_2 gas (molar mass 32) at a temperature T and pressure P. Another identical container Y contains 1 mole of He gas (molar mass 4) at temperature $2T$, then

(A) pressure in the container Y is $P/8$.
(B) pressure in container Y is P.
(C) pressure in the container Y is $2P$.
(D) pressure in container Y is $P/2$.

11. A 2gm bullet moving with a velocity of 200 m/s is brought to a sudden stoppage by an obstacle. The total heat produced goes to the bullet. If the specific heat of the bullet is 0.03 cal/gm-°C, the rise in its temperature will be

(A) 158.0°C (B) 15.80°C
(C) 1.58°C (D) 0.1580°C

12. We have a jar A filled with gas characterized by parameters P, V, and T and another jar B filled with gas with parameters $2P$, $V/4$, and $2T$, where the symbols have their usual meanings. The ratio of the number of molecules of jar A to those of jar B is

(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 4 : 1

13. If γ be the ratio of specific heats of a perfect gas, the number of degrees of freedom of a molecule of the gas is

(A) $(\gamma - 1)$ (B) $\frac{3\gamma - 1}{2\gamma - 1}$

(C) $\frac{2}{\gamma - 1}$ (D) $\frac{9}{2}(\gamma - 1)$

14. The root mean square velocity of the gas molecules is 300 m/s. What will be the root mean square speed of the molecules if the atomic weight is double and absolute temperature is halved?

(A) 300 m/s (B) 150 m/s
(C) 600 m/s (D) 75 m/s

15. 100 g of ice at 0°C is mixed with 100 g of water at 100°C. What will be the final temperature of the mixture?

(Latent of fusion for ice = 80 cal/gm and specific heat of water is 1 cal/gm °C)

(A) 10°C (B) 20°C
(C) 30°C (D) 0°C

16. The temperature of a substance increases by 27°C. On the Kelvin scale, this increase is equal to

(A) 300 K (B) 2.46 K
(C) 27 K (D) 7 K

17. The amount of heat required will be minimum when a body is heated through

(A) 1 K
(B) 1°C
(C) 1°F
(D) It will be the same in all the three cases

18. A constant volume gas thermometer shows pressure reading of 50 cm and 90 cm of mercury at 0°C and 100°C, respectively. When the pressure reading is 60 cm of mercury, the temperature is

(A) 25°C (B) 40°C
(C) 15°C (D) 12.5°C

19. At what temperature will the resistance of a copper wire become three times its value at 0°C (Temperature coefficient of resistance for copper = 4×10^{-3}/°C)

(A) 400°C (B) 450°C
(C) 500°C (D) 550°C

20. 1000 drops of a liquid of surface tension σ and radius r join together to form a big single drop. The energy released raises the temperature of the drop. If ρ be the density of the liquid and S be the specific heat, the rise in temperature of the drop would be (J = Joule's equivalent of heat)

(A) $\frac{\sigma}{JrS\rho}$ (B) $\frac{10\sigma}{JrS\rho}$

(C) $\frac{100\sigma}{JrS\rho}$ (D) $\frac{27\sigma}{10JrS\rho}$

21. A soap bubble in vacuum has a radius of 3 cm and another soap bubble in vacuum has a radius of 4 cm. If two bubbles coalesce under isothermal conditions, then the radius of the new bubble is

(A) 2.3 cm (B) 4.5 cm
(C) 5 cm (D) 7 cm

22. 1 kg water of specific heat 1 cal/gm °C is kept in a container at 10°C. If 50 gm of ice at 0°C is required to cool down the water from 10°C to 0°C, the water equivalent of container is

(Latent of fusion for ice = 80 cal/gm and specific heat of water is 1 cal/gm °C)

(A) 1 kg (B) 2 kg (C) 3 kg (D) $\frac{1}{2}$ kg

23. The temperature of a monoatomic gas in an uniform container of length L varies linearly from T_0 to T_L as shown in Fig. 10.15. If the molecular weight of the gas is M, then the time taken by a wave pulse in traveling from end A to end B is

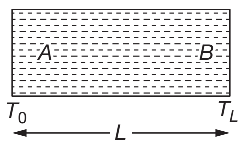


Fig. 10.15

(A) $\frac{2L}{\sqrt{T_L} + \sqrt{T_0}}\sqrt{\frac{3M}{5R}}$ (B) $\sqrt{\frac{3(T_L - T_0)}{5RML}}$

(C) $\frac{2L}{\sqrt{T_L} - \sqrt{T_0}}\sqrt{\frac{3M}{5R}}$ (D) $L\sqrt{\frac{M}{2R(T_L - T_0)}}$

24. An iron tyre is to be fitted onto a wooden wheel 1.0 m in diameter. The diameter of the tyre is 6 mm smaller than that of wheel. The tyre should be heated so that its temperature increases by a minimum of (coefficient of volumetric expansion of iron is 3.6×10^{-5}/°C)

(A) 167°C (B) 334°C
(C) 500°C (D) 1000°C

25. At what temperature, the Fahrenheit and the Celsius scales will give numerically equal (but opposite in sign) values?

(A) –40 °F and 40 °C
(B) 11.43 °F and –11.43 °C
(C) –11.43 °F and +11.43 °C
(D) + 40 °F and –40 °C

26. Two rods of length L_1 and L_2 are made of materials whose coefficients of linear expansion are α_1 and α_2. If the difference between the two lengths is independent of temperature

(A) $(L_1/L_2) = (\alpha_1/\alpha_2)$ (B) $(L_1/L_2) = (\alpha_2/\alpha_1)$
(C) $L_1^2\alpha_1 = L_2^2\alpha_2$ (D) $\alpha_1^2 L_1 = \alpha_2^2 L_2$

27. The ratio of coefficients of cubical expansion and linear expansion is

(A) 1 : 1 (B) 3 : 1
(C) 2 : 1 (D) None of these

28. On the Celsius scale, the absolute zero of temperature is at

(A) 0°C (B) –32°C
(C) 100°C (D) –273.15°C

29. At what temperature, the Fahrenheit and the Celsius scales will give numerically equal (but opposite in sign) values?

(A) –40°F and 40°C
(B) 11.43°F and –11.43°C
(C) –11.43°F and +11.43°C
(D) +40°F and –40°C

30. Two liquids A and B are at 32°C and 24°C. When mixed in equal masses, the temperature of the mixture is found to be 28°C. Their specific heats are in the ratio of

(A) 3 : 2 (B) 2 : 3
(C) 1 : 1 (D) 4 : 3

31. The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_v, respectively. Further, $\frac{C_p}{C_v} = \gamma$ and R is the gas constant for 1 gm mole of a gas. Then C_v is equal to

(A) R (B) γR (C) $\frac{R}{\gamma - 1}$ (D) $\frac{\gamma R}{\gamma - 1}$

32. If the degree of freedom of a gas molecule is f, then the ratio of two specific heats C_p/C_v is given by

(A) $\frac{2}{f} + 1$ (B) $1 - \frac{2}{f}$
(C) $1 + \frac{1}{f}$ (D) $1 - \frac{1}{f}$

33. On the Celsius scale, the absolute zero of temperature is at

(A) 0°C (B) –32°C
(C) 100°C (D) –273.15°C

34. At what temperature, the Fahrenheit and the Celsius scales will give numerically equal (but opposite in sign) values?

(A) –40°F and 40°C
(B) 11.43°F and –11.43°C
(C) –11.43°F and +11.43°C
(D) +40°F and –40°C

35. A faulty thermometer has its fixed points body as shown on Celsius scale is 55°, then its temperature shown on this faulty thermometer is

(A) 50 (B) 55 (C) 60 (D) 65

36. Two rods of length L_1 and L_2 are made of materials whose coefficients of linear expansion are α_1 and α_2 If the difference between the two lengths is independent of temperature

(A) $(L_1/L_2) = (\alpha_1/\alpha_2)$ (B) $(L_1/L_2) = (\alpha_2/\alpha_1)$
(C) $L_1^2\alpha_1 = L_2^2\alpha_2$ (D) $\alpha_1^2 L_1 = \alpha_2^2 L_2$

37. A substance of mass m kg requires is power input of P watts to remain in the molten state at its melting point. When the power is turned off, the sample completely solidifies in time t second. What is the latent heat of fusion of the substance?

(A) $\frac{Pm}{t}$ (B) $\frac{Pt}{m}$ (C) $\frac{m}{Pt}$ (D) $\frac{t}{Pm}$

38. Two liquids A and B are at 32°C and 24°C. When mixed in equal masses, the temperature of the mixture is found to be 28°C. Their specific heats are in the ratio of

(A) 3 : 2 (B) 2 : 3
(C) 1 : 1 (D) 4 : 3

39. Two rods of length L_1 and L_2 are made of materials whose coefficients of linear expansion are α_1 and α_2. If the difference between the two lengths is independent of temperature.

(A) $(L_1/L_2) = (\alpha_1/\alpha_2)$ (B) $(L_1/L_2) = (\alpha_2/\alpha_1)$
(C) $L_1^2\alpha_1 = L_2^2\alpha_2$ (D) $\alpha_1^2 L_1 = \alpha_2^2 L_2$

40. The number of degrees of freedom for each atom of a monoatomic gas is

(A) 3 (B) 5 (C) 6 (D) 1

41. If the degrees of freedom of a gas molecule be f, then the ratio of two specific heats C_p/C_v is given by

(A) $\frac{2}{f} + 1$ (B) $1 - \frac{2}{f}$
(C) $1 + \frac{1}{f}$ (D) $1 - \frac{1}{f}$

42. The internal energy U is a unique function of any state because change in U
(A) does not depend upon path.
(B) depends upon path.
(C) corresponds to an adiabatic process.
(D) corresponds to an isothermal process.

43. The rms speed of a gas molecule is
(A) $\sqrt{(M/3RT)}$ (B) $(M/3RT)$
(C) $\sqrt{(3RT/M)}$ (D) $(3RT/M)^2$

44. A bimetallic strip is made of aluminium and steel $(\alpha_{Al} > \alpha_{\text{steel}})$. On heating, the strip will
(A) remain straight.
(B) get twisted.
(C) bend with aluminium on concave side.
(D) bend with steel on concave side.

45. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly,
(A) its speed of rotation increases.
(B) its speed of rotation decreases.
(C) its speed of rotation remains same.
(D) its speed in increases because its moment of inertia increases.

46. The graph between two temperature scales A and B is shown in Fig. 10.16 between upper fixed point and lower fixed point there are 150 equal division on scale A and 100 on scale B. The relationship for conversion between the two scales is given by

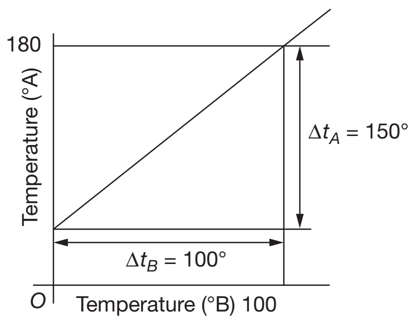


Fig. 10.16

(A) $\frac{t_A - 180}{100} = \frac{t_B}{150}$ (B) $\frac{t_A - 30}{150} = \frac{t_B}{100}$
(C) $\frac{t_B - 180}{150} = \frac{t_A}{100}$ (D) $\frac{t_B - 40}{100} = \frac{t_A}{180}$

47. An aluminium sphere is dipped into water. Which of the following is true?
(A) Buoyancy will be less in water at 0°C than that in water at 4°C
(B) Buoyancy will be more in water at 0°C than that in water at 4°C
(C) Buoyancy in water at 0°C will be same as that in water at 4°C
(D) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere

48. As the temperature is increased, the period of pendulum,
(A) Increases as its effective length increases even though its centre of mass still remains at the centre of the bob.
(B) Decreases as its effective length increases even through its centre of mass still remains at the centre of the bob.
(C) Increases as its effective length increases due to shifting to centre of mass below the centre of the bob.
(D) Decreases as its effective length remains same but the centre of mass shifts above the centre of the bob.

49. Heat is associated with,
(A) Kinetic energy of random motion of molecules.
(B) Kinetic energy of orderly motion of molecules.
(C) Total kinetic energy of random and orderly motion of molecules.
(D) Kinetic energy of random motion in some cases and kinetic energy of orderly motion in other.

50. The radius of a metal sphere at room temperature T is R and the coefficient of linear expansion of the metal is α. The sphere heated a little by a temperature ΔT so that its new temperature is $T + \Delta T$. The increase in the volume of the sphere is approximately.
(A) $2\pi R\alpha\,\Delta T$ (B) $\pi R^2\alpha\,\Delta T$
(C) $4\pi R^3\alpha\,\Delta T/3$ (D) $4\pi R^3\alpha\,\Delta T$

51. A sphere, a cube, and a thin circular plate, all of same material and same mass are initially heated to same high temperature.
(A) Plate will cool fastest and cube the slowest.
(B) Sphere will cool fastest and cube the slowest.
(C) Plate will cool fastest and sphere the slowest.
(D) Cube will cool fastest and plate the slowest.

52. A cubic vessel (with face horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 ms^{-1} in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground

(A) Remains the same because 500 ms^{-1} is very much smaller than v_{rms} of the gas.
(B) Remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.
(C) Will increase by a factor equal to $(v_{rms}^2 + (500)^2)/v_{rms}^2$, where v_{rms} was the original mean square velocity of the gas.
(D) Will be different on the top wall and bottom wall of the vessel.

53. 1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300 K (Fig. 10.18). One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,

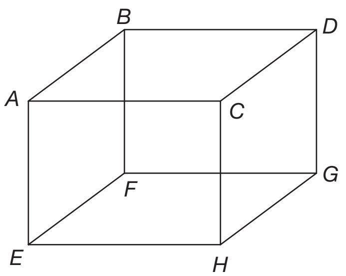


Fig. 10.18

(A) The pressure on *EFGH* would be zero.
(B) The pressure on all the faces will be equal.
(C) The pressure of *EFGH* would be double the pressure on *ABCD.*
(D) The pressure of *EFGH* would be half that on *ABCD.*

54. Boyle's law is applicable for an
(A) adiabatic process. (B) isothermal process.
(C) isobaric process. (D) isochoric process.

55. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction (Fig. 10.19). If the temperature is increased.

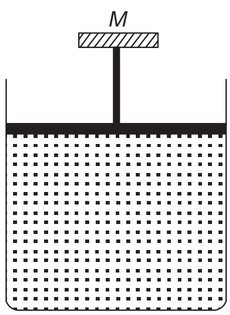


Fig. 10.19

(A) Both p and V of the gas will change
(B) Only p will increase according to Charles' law
(C) V will change but not p
(D) p will change but not V

56. Volume versus temperature graphs for a given mass of an ideal gas are shown in Fig. 10.20. At two different values of constant pressure, what can be inferred about relation between p_1 and p_2 ?

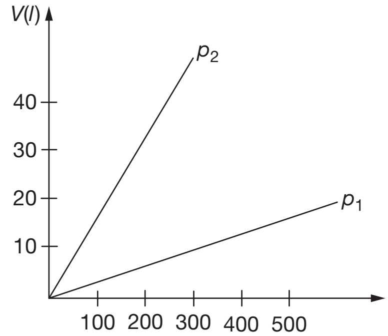


Fig. 10.20

(A) $p_1 > p_2$ (B) $p_1 = p_2$
(C) $p_1 < p_2$ (D) Data is insufficient

57. 1 mole of H_2 gas is contained in a box of volume $V = 1.00\ m^3$ at $T = 300$ K. The gas is heated to a temperature of $T = 3000$ K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)
(A) same as the pressure initially.
(B) two times the pressure initially.
(C) ten times the pressure initially.
(D) twenty times the pressure initially.

58. A vessel of volume V contains a mixture of 1 mole of hydrogen and 1 mole oxygen (both considered as ideal). Let $f_1(v)dv$ denote the fraction of molecules with speed between v and $(v + dv)$ with $f_2(v)dv$, similarly for oxygen. Then,
(A) $f_1(v) + f_2(v) = f(v)$ obeys the Maxwell's distribution law.
(B) $f_1(v), f_2(v)$ will obey the Maxwell's distribution law separately.
(C) neither $f_1(v)$ nor $f_2(v)$ will obey Maxwell's distribution law.
(D) $f_2(v)$ and $f_1(v)$ will be the same.

59. An inflated rubber balloon contains 1 mole of an ideal gas, has a pressure p, volume V, and temperature T. If the temperature rises to 1.1 T, and the volume is increased to 1.05 V, the final pressure will be
(A) 1.1 p (B) p
(C) less than p (D) between p and 1.1

More than One Option Correct Type

60. 10 gm of ice at 0°C is mixed with 5 gm of steam at 100°C. If latent heat of fusion of ice is 80 cal/gm and latent heat of vaporization is 540 cal/gm. Then at thermal equilibrium

(A) temperature of mixture is 0°C.
(B) temperature of mixture is 100°C.
(C) mixture contains 13.33 gm of water and 1.67 gm of steam.
(D) mixture contains 5.3 gm of ice and 9.7 gm of water.

61. A bimetallic strip is formed by two identical strips, one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is

(A) proportional to ΔT.
(B) inversely proportional to ΔT.
(C) proportional to $|\alpha_B - \alpha_C|$.
(D) inversely proportional to $|\alpha_B - \alpha_C|$.

62. Refer to the plot of temperature versus time (Fig. 10.21) showing the changes in the state if ice on heating (not to scale). Which of the following is correct?

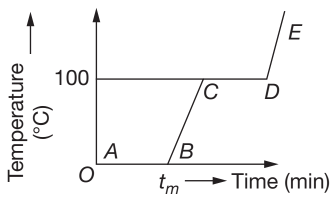


Fig. 10.21

(A) The region AB represents ice and water in thermal equilibrium.
(B) At B, water starts boiling.
(C) At C, all the water gets converted into steam.
(D) C to D represents water and steam in equilibrium at boiling point.

63. Mark the correct options

(A) A system X is in thermal equilibrium with Y but not with Z. The system Y and Z may be in thermal equilibrium with each other.
(B) A system X is in thermal equilibrium with Y but not with Z. The system Y and Z are not in thermal equilibrium with each other.
(C) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z must be in thermal equilibrium with each other.
(D) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z may be in thermal equilibrium with each other.

64. Gulab jamuns (assumed to be spherical) are to be heated in an oven. They are available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice bigger (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following.

(A) Both size gulab jamuns will get heated in the same time.
(B) Smaller gulab jamuns are heated before bigger ones.
(C) Smaller pizzas are heated before bigger ones.
(D) Bigger pizzas are heated before smaller.

Passage Based Questions

Passage 1

A ring of radius r is to be mounted on a wheel of radius R. The coefficient of linear expansion of the material of the ring is α, Young's modulus is Y, area of cross-section is A, and mass is m. Initially, ring and wheel are at same temperature. $(r < R)$

65. The temperature through which ring should be heated so that it can be mounted on the wheel is

(A) $\dfrac{R-r}{r\alpha}$ (B) $\dfrac{R-r}{R\alpha}$

(C) $\dfrac{R}{r\alpha}$ (D) $\dfrac{r}{R\alpha}$

66. After mounting, ring cools down to its initial temperature. The strain in the ring is

(A) $\dfrac{R}{r}$ (B) $\dfrac{R-r}{R}$ (C) $\dfrac{R-r}{r}$ (D) $\dfrac{r}{R}$

67. The wheel with mounted ring starts rotating with angular velocity ω. The value of ω for which tension in the ring becomes zero is

(A) $\sqrt{\dfrac{2\pi}{m}\dfrac{(R-r)}{Rr}YA}$ (B) $\sqrt{\dfrac{YA(R-r)}{R^2 m}}$

(C) $\sqrt{\dfrac{YA(R-r)}{Rrm}}$ (D) $\sqrt{\dfrac{YA(R-r)2\pi}{mr^2}}$

Passage 2

A non-conducting piston of mass m and area S_0 divides a non-conducting, closed cylinder as shown in Fig. 10.22. Piston having mass m is connected with top wall of cylinder by a spring of force constant k. Top part is evacuated and bottom part contains an ideal gas at pressure P_0 in equilibrium position. Adiabatic constant γ and in equilibrium length of each part is l. (neglect friction)

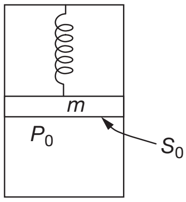


Fig. 10.22

68. Find compression in the spring at equilibrium position (Assuming $S_0P_0 > mg$).

(A) Zero (B) $\frac{2P_0S_0 - mg}{k}$

(C) $\frac{P_0S_0 - mg}{k}$ (D) $\frac{P_0S_0 - mg}{2k}$

69. Find angular frequency for small oscillation.

(A) $\sqrt{\frac{kl + \gamma P_0 S_0}{2ml}}$ (B) $\sqrt{\frac{2kl + \gamma P_0 S_0}{ml}}$

(C) $\sqrt{\frac{k}{m}}$ (D) $\sqrt{\frac{kl + \gamma P_0 S_0}{ml}}$

70. If spring is disconnected and top part of cylinder is removed, then find the angular frequency for small oscillation. (Assuming pressure of gas at equilibrium position is P_1 and length of gas column is l_1.)

(A) $\sqrt{\frac{\gamma P_1 S_0}{ml_1}}$ (B) $\sqrt{\frac{2\gamma P_1 S_0}{ml_1}}$

(C) $\sqrt{\frac{\gamma P_1 S_0}{4ml_1}}$ (D) $\sqrt{\frac{\gamma P_1 S_0}{2ml_1}}$

71. Find the angular frequency of oscillation. If process is isothermal. Length of gas column at equilibrium position is l_1 and gas pressure is P_1 at equilibrium position.

(A) $\sqrt{\frac{P_1 S_0}{4ml_1}}$ (B) $\sqrt{\frac{2P_1 S_0}{ml_1}}$

(C) $\sqrt{\frac{P_1 S_0}{ml_1}}$ (D) $\sqrt{\frac{P_1 S_0}{2ml_1}}$

Match the Column Type

72. A cubical block is in a floating equilibrium in a liquid with half of its volume submerged as shown in Fig. 10.23 at temperature T.

$\alpha_S \rightarrow$ coefficient of linear expansion of block
$\gamma_L \rightarrow$ coefficient of volume expansion of liquid
$\rho_S \rightarrow$ density of block at temperature T
$\rho_L \rightarrow$ density of liquid at temperature T

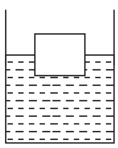


Fig. 10.23

Column-I	Column-II
I. The ratio of densities of solid and liquid at temperature T is	A. $\frac{1}{2}$
II. If the depth of the block submerged in the liquid does not change on increasing temperature, then ratio of α_s to γ_L is	B. 3
III. If fraction submerged does not change on increasing temperature, then the ratio of γ_L to α_s is	C. 2
IV. The ratio of buoyant force to weight of body at temperature $2T$ is	D. 1

Assertion–Reason Type

73. **Assertion:** A block of ice is placed inside a closed room where only walls of the room radiate heat energy.

Reason: If ideal gas is compressed isobarically, then energy is always rejected by the gas.

(A) A (B) B (C) C (D) D

74. **Assertion:** We cannot change the temperature of a body without giving (or taking) heat to (or from) it

Reason: According to principal of conservation of energy, total energy is conserved.

(A) A (B) B (C) C (D) D

75. **Assertion:** It is convenient to define two specific heats C_P and C_V in case of a gas. However, it is not generally necessary to define two specific heats in case of a solid or liquid.

Reason: For a given temperature rise, the expansion of a solid or liquid is negligible as compared to that of a gas.

(A) A (B) B (C) C (D) D

76. **Assertion:** Water cannot be liquefied at a temperature greater than 374.1ºC, no matter how large pressure is.

Reason: The critical temperature of water is 374.1ºC. Water in its gas form at a temperature lower than 374.1ºC is called water vapour and above 374.1ºC is called water gas.

(A) A (B) B (C) C (D) D

77. **Assertion:** At very high temperatures, molar specific heat at constant volume for an monoatomic ideal gas is greater than $\frac{3}{2}R$ (R is gas constant)

Reason: At ordinary temperature, the molecules of an ideal gas may have only translational and rotational kinetic energy, and at high temperature, they may also have vibrational kinetic energy.

(A) A (B) B (C) C (D) D

Integer Type

78. A copper plate of length 1 m is riveted to two steel plates of same length and same cross-section area at 0°C. Calculate tension (in kilo newton) generated in copper plate when heated to 20°C.

$Y_{copper} = \frac{1}{2} \times Y_{steel} = 2 \times 10^{11}$ N/m^2 Y = Young's modules

$\alpha_{copper} = 18 \times 10^{-6}$ K^{-1}

$\alpha_{steel} = 11 \times 10^{-6}$ K^{-1} α = coefficient of linear expansion

Area of each plate = 50 cm^2

79. Water flows at the rate of 0.1500 kg/min through a tube and is heated by a heater dissipating 25.2 W. The inflow and outflow water temperatures are 15.2°C and 17.4°C, respectively. When the rate of flow is increased to 0.2318 kg/min and the rate of heating to 37.8 W, the inflow and outflow temperatures are unaltered. Find the rate of loss of heat from the tube

Previous Years' Questions

80. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. The elastic energy stored in the wire is **[2003]**

(A) 0.2 J (B) 10 J (C) 20 J (D) 0.1 J

81. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is **[2004]**

(A) $F/2l$ (B) Fl (C) $2Fl$ (D) $Fl/2$

82. If S is stress and Y is Young's modulus of material of a wire, then the energy stored in the wire per unit volume is **[2005]**

(A) $2S^2Y$ (B) $S^2/2Y$ (C) $2Y/S^2$ (D) $S/2Y$

83. A wire elongates by l mm when a load w is hanged from it. If the wire goes over a pulley and two weights w each are hung at the two ends, then the elongation of the wire will be (in mm) **[2006]**

(A) l (B) $2l$ (C) Zero (D) 1/2

84. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire-2 has cross-sectional area 3 A. If the length of wire 1 increases by ΔX on applying force F, then how much force is needed to stretch wire 2 by the same amount? **[2009]**

(A) F (B) $4F$ (C) $6F$ (D) $9F$

85. A metal rod of Young's modulus Y and coefficient of thermal expansion a is held at its two ends such that its length remains invariant. If its temperature is raised by t°C, then the linear stress developed in it is **[2011]**

(A) $\dfrac{\alpha t}{Y}$ (B) $Y\alpha t$ (C) $\dfrac{Y}{\alpha t}$ (D) $\dfrac{1}{Y\alpha t}$

86. An aluminium sphere of 20 cm diameter is heated from 0°C to 100°C. Its volume changes by (given that the coefficient of linear expansion for aluminium $\alpha_{Al} = 23\times10^{-6}/°C$) **[2011]**

(A) 28.9 cc (B) 2.89 cc
(C) 9.28 cc (D) 49.8 cc

87. A wooden wheel of radius R is made of two semi-circular parts (see Fig. 10.24). The two parts are held together by a ring made of a metal strip of cross-sectional area S and length L. L is slightly less than $2p\ R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semi-circular parts together. If the coefficient of linear expansion of the metal is a and its Young's modulus is Y, then the force that one part of the wheel applies on the other part is **[2012]**

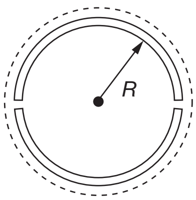


Fig. 10.24

(A) $2\pi SY\alpha\Delta T$ (B) $SY\alpha\Delta T$
(C) $\pi SY\alpha\Delta T$ (D) $2SY\alpha\Delta T$

88. Heat given to a body which raises its temperature by 1°C is **[2002]**

(A) water equivalent. (B) thermal capacity.
(C) specific heat. (D) temperature gradient.

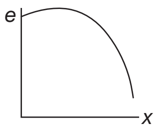
89. Cooking gas containers are loaded on to a truck moving with uniform speed. The temperature of the gas molecules inside the containers will **[2002]**

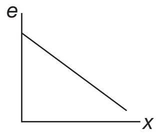
(A) increase.
(B) decrease.
(C) remain same.
(D) decrease for some, whereas increase for others.

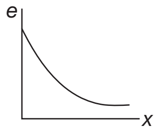
90. If mass–energy equivalence is taken into account, when water is cooled to form ice, then the mass of water should **[2002]**

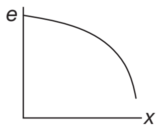
(A) increase.
(B) remain unchanged.
(C) decrease.
(D) first increase then decrease.

91. A long metallic bar is carrying heat from one end to the other under steady state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following. **[2009]**

(A) 

(B) 

(C) 

(D) 

92. A solid body of constant heat capacity 1 J/ °C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the case body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is **[2015]**

(A) $ln2$, $ln2$ (B) $ln2$, $2ln2$
(C) $2ln2$, $8ln2$ (D) $ln2$, $4ln2$

93. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively. **[2016]**

(A) 60°C; $\alpha = 1.85\times10^{-4}$ / °C

(B) 30°C; $\alpha = 1.85\times10^{-3}$ / °C

(C) 55°C; $\alpha = 1.85\times10^{-2}$ / °C

(D) 25°C; $\alpha = 1.85\times10^{-5}$ / °C

ANSWER KEYS

Single Option Correct Type

1. (B)	**2.** (B)	**3.** (A)	**4.** (B)	**5.** (C)	**6.** (D)	**7.** (B)	**8.** (C)	**9.** (D)	**10.** (C)
11. (A)	**12.** (D)	**13.** (C)	**14.** (B)	**15.** (A)	**16.** (C)	**17.** (C)	**18.** (A)	**19.** (C)	**20.** (D)
21. (C)	**22.** (C)	**23.** (A)	**24.** (C)	**25.** (B)	**26.** (B)	**27.** (B)	**28.** (D)	**29.** (B)	**30.** (C)
31. (C)	**32.** (A)	**33.** (D)	**34.** (B)	**35.** (A)	**36.** (B)	**37.** (B)	**38.** (C)	**39.** (B)	**40.** (A)
41. (A)	**42.** (A)	**43.** (C)	**44.** (D)	**45.** (B)	**46.** (B)	**47.** (A)	**48.** (A)	**49.** (A)	**50.** (D)
51. (C)	**52.** (B)	**53.** (D)	**54.** (B)	**55.** (C)	**56.** (A)	**57.** (D)	**58.** (B)	**59.** (D)	

More than One Option Correct Type

60. (B) and (C) **61.** (B) and (D) **62.** (A) and (D) **63.** (B) and (D) **64.** (B) and (C)

Passage Based Questions

Passage 1

65. (A) **66.** (C) **67.** (A)

Passage 2

68. (C) **69.** (D) **70.** (A) **71.** (C)

Match the Column Type

72. I $\to$ A, II $\to$ A, III $\to$ B, IV $\to$ D

Assertion-Reason Type

73. (D) **74.** (D) **75.** (A) **76.** (A) **77.** (D)

Integer Type

78. 112 KN **79.** $P \simeq 2$ W

Previous Years' Questions

80. (D)	**81.** (D)	**82.** (B)	**83.** (A)	**84.** (D)	**85.** (C)	**86.** (A)	**87.** (D)	**88.** (B)	**89.** (C)
90. (A)	**91.** (B)	**92.** No Option is Correct			**93.** (D)				

HINTS AND SOLUTIONS

Single Option Correct Type

1. On heating, the distance between the divisions increases and hence it measures less than actual value. But when temperature decreases, distance between divisions decreases and it measures more than actual value.
   The correct option is (B)

2. $P = \frac{Q}{t} = \frac{mL}{t} = \frac{60 \times 80 \times 4.2}{60} = 336$ W
   The correct option is (B)

3. $\theta_n = \frac{\theta_i + \theta_c}{2} = \frac{560 - 20}{2} = 270°C$
   The correct option is (A)

4. Heat gained by water = Heat supplied – Heat loss
   $ms\Delta\theta = 1000\,t - 160\,t$
   $\Rightarrow \quad t = \frac{2 \times 4200 \times 50}{840} = 8\,\text{min}.20\,\text{s}$
   The correct option is (B)

5. The correct option is (C)

6. For O_2, $K_T = \frac{3}{2}RT = 0.048$
   For N_2, $U = \frac{5}{2}RT = \frac{5}{3} \times 0.048 = 0.08$ eV
   The correct option is (D)

7. The correct option is (B)

8. Heat required = Heat supplied
   $12 \times 80 = m[540 + 1 \times 100]$
   $m = \frac{12 \times 80}{640} = 1.5$ gm
   The correct option is (C)

9. If R is rate of heating, $\Delta Q = ms(\theta)$
   $Rt = ms\theta \Rightarrow \theta = \left(\frac{R}{ms}\right)t$
   Slope of $\theta - t$ curve $= \frac{R}{ms} = \tan\phi$
   $\therefore \quad S_B = S_C > S_A$
   The correct option is (D)

10. $P_{O_2} = \frac{nRT}{V} = \frac{RT}{V}$ (1) $[n = 1]$
    $P_{He} = \frac{nRT}{V} = \frac{R(2T)}{V}$ (2) $[n = 1]$
    By (1) and (2), $P_{He} = 2P_{O_2}$
    The correct option is (C)

11. The correct option is (A)

12. $PV = \frac{N}{N_A}RT$
    $\frac{N_1}{N_2} = \frac{P_1}{P_2} \times \frac{V_1}{V_2} \times \frac{T_2}{T_1} = \frac{1}{2} \times 4 \times 2$
    $\therefore \quad \frac{N_1}{N_2} = \frac{4}{1}$
    The correct option is (D)

13. $\gamma = 1 + \frac{2}{f}$ or $f = \frac{2}{\gamma - 1}$
    The correct option is (C)

14. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ or $300 = \sqrt{\frac{3RT}{M}}$
    and $v'_{\text{rms}} = \sqrt{\frac{3R(T/2)}{2M}} = \frac{1}{2} \times 300 = 150$ m/s
    The correct option is (B)

15. Let the final temperature of mixture be θ
    Then $100 \times 80 + 100(\theta - 0) = 100 \times 1 \times (100 - \theta)$
    Solving, we get $\theta = 10°C$
    The correct option is (A)

16. In case of change of temperature, $\frac{\Delta T_C}{100} = \frac{\Delta T_K}{100}$
    $\therefore \quad \Delta T_K = \Delta T_C = 27$ K
    The correct option is (C)

17. The amount of heat required to raise the temperature through $1°F\left(= \frac{5}{9}°C\right)$ is minimum.
    The correct option is (C)

18. $t = \frac{P_t - P_0}{P_{100} - P_0} \times 100 = \frac{60 - 50}{90 - 50} \times 100 = 25°C$
    The correct option is (A)

19. By using $R_t = R_0(1 + \alpha t)$
    $3 \times R_0 = R_0(1 + 4 \times 10^{-3}t) \quad \Rightarrow \quad t = 500°C$
    The correct option is (C)

20. $1000 \times \left(\frac{4\pi}{3}\right)r^3 = \left(\frac{4\pi}{3}\right)R^3 \Rightarrow R = 10r$
    Heat released $= \sigma(\Delta A) = (4\pi r^2 \times 1000 - 4\pi R^2) \times \sigma$
    $= 3600\,\pi r^2 \sigma$
    $J(mS\Delta T)$ = Heat released
    $\therefore \quad \Delta T = \frac{3600\pi r^2 \sigma}{mSJ} = \frac{3600\pi r^2 \sigma}{(\frac{4}{3}\pi r^3 \times 1000)\,\rho SJ} = \frac{27\sigma}{10 J r S \rho}$
    The correct option is (D)

21. Pressure inside the bubble in vacuum, $P = \frac{4S}{r}$; Volume of bubble, $V = \frac{4}{3}\pi r^3$

Under isothermal conditions, $PV = P_1V_1 + P_2V_2$

$$\therefore \frac{4S}{R} \times \frac{4}{3}\pi R^3 = \frac{4S}{r_1} \times \frac{4}{3}\pi r_1^3 + \frac{4S}{r_2} \times \frac{4}{3}\pi r_2^2$$

or $R^2 = r_1^2 + r_2^2 = 3^2 + 4^2 = 25$ or $R = 5$ cm.

The correct option is (C)

22. $(m + 1000) \times 1 \times 10 = 50 \times 80$

$(m + 1000) = 4000$

$m = 3000$ gm $= 3$ kg

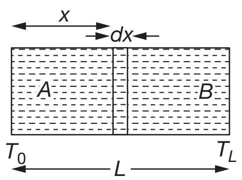
The correct option is (C)

23.

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{5RT}{3M}}$$

$$dx = C \cdot dt = \sqrt{\frac{5R}{3M}\left[T_0 + \left(\frac{T_L - T_0}{L}\right)x\right]}\,dt$$

$$t = \frac{2L}{\sqrt{T_L} + \sqrt{T_0}}\sqrt{\frac{3M}{5R}}$$



The correct option is (A)

24. $\Delta d = d\left(\frac{\gamma}{3}\right)\Delta\theta$

$$0.006 = 0.994 \times \frac{3.6 \times 10^{-5}}{3} \times \Delta\theta$$

$\therefore \quad \Delta\theta \approx 500°C$

The correct option is (C)

25. If $F = +\theta$ then $C = -\theta$

Now $\frac{F - 32}{180} = \frac{C}{100}$ or $\frac{F - 32}{9} = \frac{C}{5}$ or $\frac{\theta - 32}{9} = -\frac{\theta}{5}$

$\therefore \quad \theta = 1.43$

Hence, + 11.43°F = – 11.43°C

The correct option is (B)

26. $\Delta L_1 = \Delta L_2$

$L_1\, \alpha_1\, \Delta\theta = L_2 \alpha_2 \Delta\theta$ or $\left(\frac{L_1}{L_2}\right) = \left(\frac{\alpha_2}{\alpha_1}\right)$

The correct option is (B)

27. The correct option is (B)

28. The correct option is (D)

29. If $F = +\theta$ then $C = -\theta$

Now, $\frac{F - 32}{180} = \frac{C}{100}$ or $\frac{F - 32}{9} = \frac{C}{5}$ or $\frac{\theta - 32}{9} = -\frac{\theta}{5}$

$\therefore \quad \theta = 1.43$

Hence, + 11.43°F = – 11.43°C

The correct option is (B)

30. $ms_1 \times 4 = ms_2 \times 4 \Rightarrow s_1 : s_2 = 1:1$

The correct option is (C)

31. The correct option is (C)

32. $C_P - C_V = R$

$\therefore \quad \gamma C_V - C_V = R$

$\therefore \quad C_V = \frac{R}{\gamma - 1}$

The correct option is (C)

33. The correct option is (D)

34. If $F = +\theta$ then $C = -\theta$

Now, $\frac{F - 32}{180} = \frac{C}{100}$ or $\frac{F - 32}{9} = \frac{C}{5}$ or $\frac{\theta - 32}{9} = -\frac{\theta}{5}$

$\therefore \quad \theta = 1.43$

Hence, + 11.43°F = – 11.43°C

The correct option is (B)

35. The correct option is (A)

36. $\Delta L_1 = \Delta L_2$

$L_1\, \alpha_1\, \Delta\theta = L_2 \alpha_2 \Delta\theta$ or $\left(\frac{L_1}{L_2}\right) = \left(\frac{\alpha_2}{\alpha_1}\right)$

The correct option is (B)

37. Heat lost in t second $= mL$

or heat lost per second $= \frac{mL}{t}$

This must be the heat supplied for keeping the substance in molten state per second.

$\therefore \quad \frac{mL}{t} = P$ or $L = \frac{Pt}{m}$

The correct option is (B)

38. $ms_1 \times 4 = ms_2 \times 4 \Rightarrow s_1 : s_2 = 1:1$

The correct option is (C)

39. $\Delta L_1 = \Delta L_2$

$L_1 \alpha_1\, \Delta\theta = L_2\, \alpha_2 \Delta\theta$ or $\left(\frac{L_1}{L_2}\right) = \left(\frac{\alpha_2}{\alpha_1}\right)$

The correct option is (B)

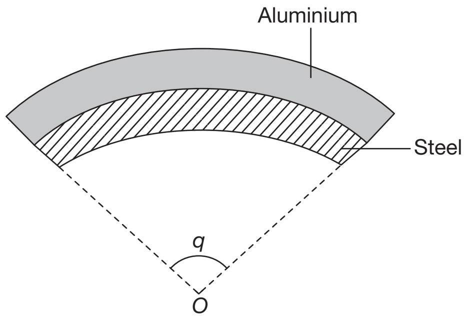
40. The correct option is (A)

41. The correct option is (A)

42. The correct option is (A)

43. The correct option is (C)

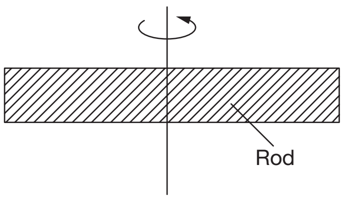
44.



As $\alpha_{Al} > \alpha_{steel}$, aluminium will expand more. So, it should have larger radius of curvature. Hence, aluminium will be on convex side.

The correct option is (D)

45.



As the rod is heated, it expands. No external torque is acting on the system so angular momentum should be conserved.

L = Angular momentum = $I\omega$ = constant

$$\Rightarrow \quad I_1\omega_1 = I_2\omega_2$$

Due to expansion of the rod, $I_2 > I_1$

$$\Rightarrow \quad \frac{\omega_2}{\omega_1} = \frac{I_1}{I_2} < 1$$

$$\Rightarrow \quad \omega_2 < \omega_1$$

So, angular velocity (speed of rotation) decreases.

The correct option is (B)

46. It is clear from the graph that lowest point for scale A is 30° and lowest point for scale B is 0°. Highest point for the scale A is 180° and for scale B is 100°. Hence, correct relation is

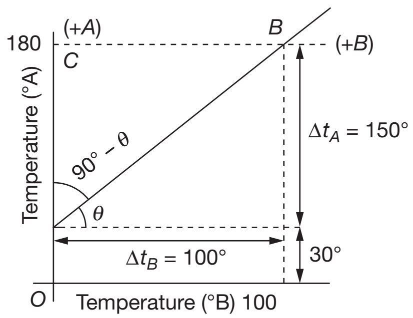


Fig. 10.17

$$\frac{t_A - (LFP)_A}{(UFP)_4 - (LFP)_A} = \frac{t_B - (LFP)_B}{(UFP)_B - (LFP)_B}$$

$$\Rightarrow \quad \frac{t_A - 30}{180 - 30} = \frac{t_B - 0}{100 - 0}$$

$$\Rightarrow \quad \frac{t_A - 30}{150} = \frac{t_B}{100}$$

where, $LFP \to$ Lower fixed point

$UFP \to$ Upper fixed point

The correct option is (B)

47. Let volume of the sphere be V and ρ is its density, then we can write buoyant force

$F = V\rho G$ (g = acceleration due to gravity)

$\Rightarrow \; F \propto \rho$ ($\because$ V and g are almost constant)

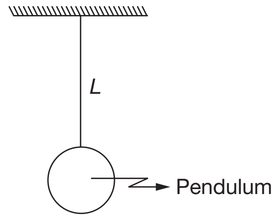
$$\Rightarrow \quad \frac{F_{4°C}}{F_{0°C}} = \frac{\rho_{4°C}}{\rho_{0°C}} > 1 \qquad (\because \rho_{4°C} > \rho_{0°C})$$

$$\Rightarrow \quad F_{4°C} > F_{0°C}$$

Hence, buoyancy will be less in water at 0°C than that in water at 4°C

The correct option is (A)

48.



As the temperature is increased, length of the pendulum increases. We know that time period of pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$\Rightarrow \quad T \propto \sqrt{L}$ as L, increases.

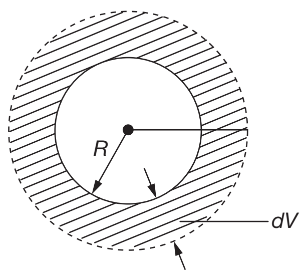
So, time period (T) also increases

The correct option is (A)

49. We know that as temperature increases, vibration of molecules about their mean position increases; hence, kinetic energy associated with random motion of molecules increases.

The correct option is (A)

50.



Let the radius of the sphere be R. As the temperature increases, radius of the sphere increases as shown.

Original volume $v_0 = \frac{4}{3}\pi R^3$

Coefficient of linear expansion = α

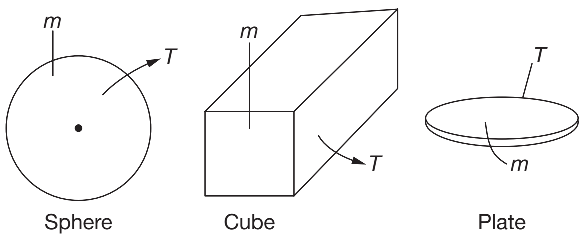
Coefficient of volume expansion = 3α

$$\frac{1}{V}\frac{dV}{dT} = 3\alpha \Rightarrow dV = 3V\alpha dt = 4\pi R^3 \alpha \Delta T$$

= Increases in the volume

The correct option is (D)

51. Consider the diagram where all the three objects are heated to same temperature T. We know that density, $\rho = \frac{\text{mass}}{\text{volume}}$ as ρ is same for all the three objects hence, volume will also be same.



As thickness of the plate is least, surface area of the plate is maximum.

We know that, according to Stefan's law of heat loss $H \alpha AT^4$, where A is surface area of object and T is temperature.

Hence, $H_{\text{sphere}} : H_{\text{cube}} : H_{\text{plate}}$

$= A_{\text{sphere}} : A_{\text{cube}} : A_{\text{plate}}$

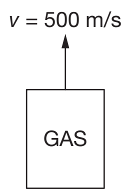
As A_{plate} is maximum

Hence, the plate will cool fastest.

As the sphere is having minimum surface area, the sphere cools slowest.

The correct option is (C)

52. As the motion of the vessel as a whole does not affect the relative motion of the gas molecules with respect to the walls of the vessel, pressure of the gas inside the vessel, as observed by us, on the ground remains the same.



The correct option is (B)

53. In an ideal gas, when a molecule collides elastically with a wall, the momentum transferred to each molecule will be twice the magnitude of its normal momentum. For the face $EFGH$, it transfers only half of that.

The correct option is (D)

54. Boyle's law is applicable when temperature is constant

i.e., $pV = nRT$ = constant

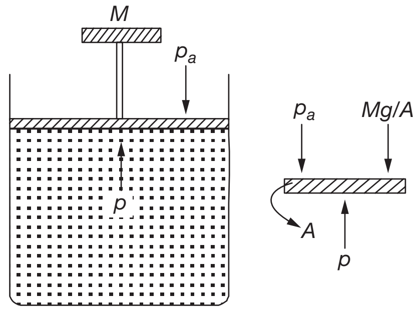
$\Rightarrow$ pV = constant (at constant temperature)

i.e., $p \propto \frac{1}{v}$ [where p = pressure, v = volume]

So, this process can be called as isothermal process.

The correct option is (B)

55. Consider the diagram where an ideal gas is contained in a cylinder, having a piston of mass m. friction is absent.



the pressure inside the gas will be

$p = p_a + Mg/A$

where p_a = atmospheric pressure

A = area of cross-section of the piston

Mg = weight of piston

Hence, p = constant.

When temperature increases

as $pV = nRT \Rightarrow$ volume (V) increases at constant pressure.

The correct option is (C)

56. We know for an ideal gas,

$$pV = nRT \Rightarrow V = \left(\frac{nR}{p}\right)T$$

Slope of the $V - T$ graph, $m = \frac{dV}{dT} = \frac{nR}{p}$

[m = slope of $V - t$ graph]

$\Rightarrow \quad m \propto \frac{1}{p}$ [$\therefore nR$ = constant]

$\Rightarrow \quad p \propto \frac{1}{m}$

hence, $\frac{p_1}{p_2} = \frac{m_2}{m_1} < 1$

[P = pressure
V = volume
n = number of moles of gases
R = gas constant
T = temperature]

where m_1 is slope of the graph corresponding to p_1 and similarly m_2 is slope corresponding to p_2.

$\Rightarrow \quad p_2 < p_1$ or $p_1 < p_2$

The correct option is (A)

57. Consider the diagram, when the molecules breaks into atoms, the number of moles would become twice.

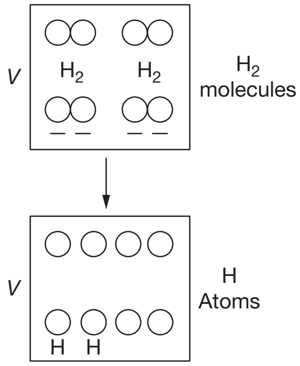
Now, by ideal gas equation,

P = Pressure of gas, n = Number of moles

R = Gas constant, T = Temperature

$pV = nRT$

As volume (V) of the container is constant.



As gases breaks, number of moles becomes twice of initial, so $n_2 = 2n_1$

So, $p \propto nT$

$$\Rightarrow \quad \frac{p_2}{p_1} = \frac{n_2T_2}{n_1T_1} = \frac{(2n_1)(3000)}{n_1(300)} = 20$$

$\Rightarrow \quad p_2 = 20p_1$

Hence, final pressure of the gas would be twenty times the pressure initially.

The correct option is (D)

58. For a function $f(v)$, the number of molecules $n = f(v)$, which are having speeds between v and $v + dv$.

For each function $f_1(v)$ and $f_2(v)$, n will be different, hence each function $f_1(v)$ and $f_2(v)$ will obey Maxwell's distribution law separately.

The correct option is (B)

59. We know for an ideal gas, $pV = nRT$ (Ideal gas equation)

$\Rightarrow$ n = Number of moles, p = Pressure, V = Volume

R = Gas constant, T = Temperature

$$= \frac{pV}{RT}$$

As number of moles of the gas remains fixed, we can write

$$\frac{p_1V_1}{RT_1} = \frac{p_2V_2}{RT_2}$$

$$\Rightarrow \quad p_2 = (p_1V_1)\left(\frac{T_2}{V_2T_1}\right)$$

$$= \frac{(p)(V)(1.1T)}{(1.05)\,V(T)}$$

$$\begin{bmatrix} p_1 = p \\ V_2 = 1.05V \text{ and } T_2 = 1.1T \end{bmatrix}$$

$$= p \times \left(\frac{1.1}{1.05}\right)$$

$$= p(1.0476) = 1.05\,p$$

Hence, final pressure p_2 lies between p and $1.1\,p$

The correct option is (D)

More than One Option Correct Type

60. Heat required to raise the temperature of ice from 0°C to 100°C

$= 10 \times 80 + 10 \times 1 \times 100 = 1800$ cal

Heat given by steam when it converts into water at 100°C $= 5 \times 540 = 2700$ cal

$\therefore$ temperature of mixture is 100°C at thermal equilibrium

Amount of steam converted into water at 100°C by 1800 cal

$$= \frac{1800}{540} = 3.33 \text{ gm}$$

The correct option is (B) and (C)

61. $R = \dfrac{l}{|\alpha_B - \alpha_C|\Delta t}$

The correct option is (B) and (D)

62. During the process, AB temperature of the system is 0°C. Hence, it represents phase change, that is, transformation of ice into water while temperature remains 0°C.

BC represents rise in temperature of water from 0°C to 100°C (at C).

Now, water starts converting into steam which is represented by CD.

The correct option is (A) and (D)

63. According to question,

$T_x = T_y$ ($\because x$ and y are in thermal equilibrium)

$T_x \neq T_z$ ($\because x$ is not in thermal equilibrium with z)

Clearly, $T_y \neq T_z$

Hence, y and z are not in thermal equilibrium.

(D) Given, $T_x \neq T_y$

and $T_x \neq T_z$

We cannot say about equilibrium of Y and Z, they may or may not be in equilibrium.

The correct option is (B) and (D)

64. Smaller gulab jamuns are having least surface area hence they will be heated first.

As in case of smaller gulab jamun, heat radiation will be less

Similarly, smaller pizzas are heated before bigger ones because they are of small surface areas.

The correct option is (B) and (C)

Passage Based Questions

Passage 1

65. $R = r(1+\alpha\Delta\theta)$

$$\Delta\theta = \frac{R-r}{r\alpha}$$

The correct option is (A)

66. Strain $= \dfrac{2\pi R - 2\pi r}{2\pi r} = \dfrac{R-r}{r}$

The correct option is (C)

67. Tension $= YA$ strain $= \dfrac{YA(R-r)}{r}$

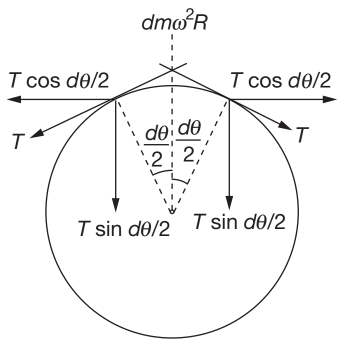
If wheel starts rotating, then tension becomes zero

$$2T\sin\frac{d\theta}{2} = dm\omega^2 R$$

$$T = \frac{m}{2\pi}\omega^2 R$$

$$YA\frac{(R-r)}{r} = \frac{m}{2\pi}\omega^2 R$$

$$\omega = \sqrt{\frac{2\pi YA}{m}\left(\frac{R-r}{Rr}\right)}$$



The correct option is (A)

Passage 2

68. $kx_0 + mg = P_0S_0$

$$x_0 = \frac{P_0S_0 - mg}{k}$$

The correct option is (C)

69. $PV^\gamma = c$

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0 \quad \Rightarrow \quad dP = -\frac{\gamma P_0}{V}dV$$

Restoring force $F = -kdx + S_0dP$

$$\Rightarrow \quad a = -\left(\frac{kl + \gamma S_0P_0}{ml}\right)dx$$

$$\omega = \sqrt{\frac{kl + \gamma P_0S_0}{ml}}$$

The correct option is (D)

70. $PV^\gamma = C$

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

$$dP = -\frac{\gamma P}{V}dV = -\frac{\gamma P}{l_0}dx$$

Restoring force is $= -\dfrac{\gamma P_1}{l_1}S_0dx$

$$\omega = \sqrt{\frac{\gamma P_1S_0}{ml_1}}$$

The correct option is (A)

71. $PV = c$

$$PdV + VdP = 0$$

$$dP = -P\frac{dV}{V} = -P_1\frac{dx}{l_1}$$

Restoring force $= -\dfrac{P_1S_0}{l_1}dx$

$$a = -\frac{P_1S_0}{ml_0}dx$$

$$\omega = \sqrt{\frac{P_1S_0}{ml_1}}$$

The correct option is (C)

Match the Column Type

72. $mg = \dfrac{v}{2}\rho_L g,\ v\rho_s g = \dfrac{v}{2}\rho_L g,\ \rho_s = \dfrac{\rho_L}{2}$

$mg = v'\rho_L g,\ mg = Ax\rho_L g$ (1)

when temperature is raised by ΔT, $mg = A'x\rho'_L g$ (2)

From (1) and (2), $A\rho_L = A'\rho'_L$

$$A\rho_L = A(1+2\alpha_s\Delta T)\frac{\rho_L}{1+\gamma_L\Delta T}$$

$\gamma_L = 2\alpha_s$ for fraction inside the liquid to be same $\dfrac{\rho_s}{\rho_L} = \dfrac{\rho'_s}{\rho'_L}$

$$\frac{\rho_s}{\rho_L} = \frac{\rho_s(1+\gamma_L\Delta T)}{\rho_L(1+3\alpha_s\Delta T)} \quad \therefore \quad \rho_L = 3\alpha_s$$

$\therefore$ I → A, II → A, III → B, IV → D

Assertion-Reason Type

73. The correct option is (D)

74. The correct option is (D)

75. The correct option is (A)

76. The correct option is (A)

77. The correct option is (D)

Integer Type

78. Copper expands more than steel, so increase in temperature will lead to elastic compression of the copper. Let F be the force with which the copper plate is controlled.

Net expansion of copper, $x = l\alpha_c\Delta\theta - \frac{Fl}{Y_C A}$

Net expansion of steel, $x = l\alpha_s\Delta\theta + \frac{Fl}{Y_S 2A}$

On solving, $F = \frac{2(\alpha_c - \alpha_s)A\Delta\theta Y_s Y_c}{Y_c + 2Y_s} = 112 \times 10^3$ N

$= 112$ kN

79. $\Delta m = \frac{0.15}{60}$ kg/s $= 2.5 \times 10^{-3}$ kg/s

Let P be the rate of loss of heat from the tube, and C be the specific heat capacity of water, then

$P + \Delta m \times C \times (17.4 - 15.2) = 25.2$ (1)

Also, $P + \Delta m' \times C \times (17.4 - 15.2) = 37.8$ (2)

where $\Delta m' = \frac{0.2318}{60} = 3.8633 \times 10^{-3}$ kg/s

Equation (2) – Equation (1)

$2.2 \times C\,[3.86 - 2.5] \times 10^{-3} = 12.6$

$C = 4.2 \times 10^3$ J/kg –°C

Putting the value of C in Equation (1)

$P + 23.1 = 25.2$

$P \simeq 2$ W

Previous Years' Questions

80. Elastic energy stored in the wire is

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2}\frac{F}{A} \times \frac{\Delta l}{L} \times AL$$

$$= \frac{1}{2}F\Delta l$$

$$= \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1 \text{ J}$$

The correct option is (D)

81. Work done in stretching the wire = potential energy stored

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL = \frac{1}{2}Fl$$

The correct option is (D)

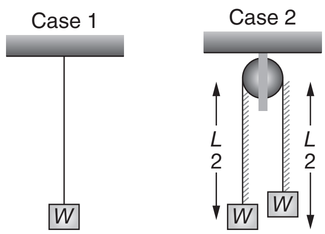
82. Energy per unit volume $= \frac{1}{2}\text{stress} \times \text{string}$

$$\left(\text{Putting strain} = \frac{\text{Stress}}{Y}\right)$$

$$\frac{U}{V} = \frac{1}{2}S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

The correct option is (B)

83. In the both cases, tension in wire is W. Also, the length L and area A are equal in both.



Tush, ΔL is also same.

The correct option is (A)

84. $A_1 l_1 = A_2 l_2$

$$\Rightarrow l_2 = \frac{A_1 l_1}{A_2} = \frac{A \times l_1}{3A} = \frac{l_1}{3}$$

$$\Rightarrow \frac{l_1}{l_2} = 3$$

It is given that Δx is same, thus using

$$\frac{F}{A} = Y\frac{\Delta x}{l}$$

$$\Delta x = \frac{F_2}{3Ay}l_2 = \frac{F_1}{Ay}l_1$$

Thus, $F_2 = 3F \times \frac{l_1}{l_2} = 3F_1 \times 3 = 9F$

The correct option is (D)

85. $\Delta L = \alpha L \Delta T = \frac{FL}{AY}$

$\Rightarrow$ Stress $= \frac{F}{A} = Y\alpha\Delta T$

The correct option is (C)

86. Given $d = 20$ cm

$V = V_0(1+\gamma t) = V_0(1+3at)$ (since $= \gamma = 3\alpha$)

Change in volume $= V - V_0$

$$= 3V_0\alpha t$$

$$3\times\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times 23\times10^{-6}\times100$$

$$3\times\frac{4}{3}\pi\left(\frac{0.2}{2}\right)^3 \times 23\times10^{-6}\times100$$

$$= 28.9\ cc\,(1\ cc = 10^{-6}\,\text{m}^3)$$

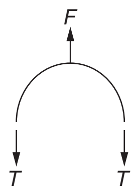
The correct option is (A)

87. Increase in length

$$\Delta L = \alpha L \Delta T$$

$$\frac{\Delta L}{L} = \alpha\Delta T$$

The thermal stress developed is



$\frac{T}{S} = Y\frac{\Delta L}{L} = Y\alpha\Delta T$

$T = SY\alpha\Delta T$

From FBD of one part of the whell, $F = 2T$

Where F is the force applied by one part of whell on the other part

$$F = 2SY\alpha\Delta T$$

The correct option is (D)

88. The thermal capacity of a substance is defined as the amount of heat required to raise its temperature by 1°C.

The correct option is (B)

89. Temperature of a gas is determined by the total translational kinetic energy measured with respect to the centre of mass of the gas. Therefore, the motion of centre of mass of the gas does not affect its temperature. Hence, the temperature of gas will remain same.

The correct option is (C)

90. When water is cooled to form ice, it loses energy. Since energy and mass are equivalent, it loses mass, and thus, mass decreases.

The correct option is (A)

91. Rate of heat flow is given by

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

$$\Rightarrow \frac{dT}{dx} = \frac{-1}{KA}\frac{dQ}{dt}$$

Since K and A are constants and $\frac{dQ}{dt}$ is same at all points in steady state.

Thus, $\frac{dT}{dx}$ or slope is constant (negative)

The correct option is (B)

92. Change in Entropy

$$(\Delta s) = \int\frac{dq}{T} = \int_{T_1}^{T_2}\frac{CdT}{T} = C\ell n\frac{T_2}{T_1} = 1\times\ell n\frac{473}{373}\,\text{J/K}$$

No Option is Correct

93. $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$

$\frac{12}{1day} = \frac{1}{2}\alpha(40-T)$ (1)

$\frac{4}{1day} = \frac{1}{2}\alpha(T-20)$ (2)

(1) / (2)

$\frac{12}{4} = \frac{40-T}{T-20}$

$3T - 60 = 40 - T$

$4T = 100$

$T = 25\,°\text{C}$

From (1)

$\frac{12}{24\times3600} = \frac{1}{2}\alpha(40-25)$

$\alpha = \frac{1}{3600\times15} = 1.85\times10^{-5}\,/\,°\text{C}$