CHAPTER 9

Oscillations and Waves

Chapter Highlights

Periodic motion - period, frequency, Displacement as a function of time, Periodic functions, Simple harmonic motion (SHM) and its equation, Phase, Oscillations of a spring-restoring force and force constant, Energy in SHM - kinetic and potential energies, Simple pendulum - derivation of expression for its time period, Free, forced and damped oscillations, Resonance, Wave motion, Longitudinal and transverse waves, Speed of a wave, Displacement relation for a progressive wave, Principle of superposition of waves, Reflection of waves, Standing waves in strings and organ pipes, Fundamental mode and harmonics, Beats, Doppler effect in sound

SIMPLE HARMONIC MOTION

Periodic Motion

When a body or a moving particle repeats its motion along a definite path after regular intervals of time, its motion is said to be **periodic motion** and interval of time is called **time period** or harmonic motion period (T). The path of periodic motion may be linear, circular, elliptical, or any other curve. For example, rotation of earth around the sun.

Oscillatory Motion

'To and Fro' type of motion is called an **oscillatory motion**. It need not be periodic and need not have fixed extreme positions. For example, motion of pendulum of a wall clock.

The oscillatory motions in which energy is conserved are also periodic.

The force/torque (directed towards equilibrium point) acting in oscillatory motion is called restoring force/torque.

Damped oscillations are those in which energy is consumed due to some resistive forces and hence total mechanical energy decreases.

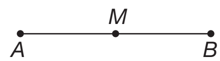
Simple Harmonic Motion

If the restoring force/torque acting on the body in oscillatory motion is directly proportional to the displacement of body/particle and is always directed towards equilibrium position then the motion is called simple harmonic motion (SHM). It is the simplest (easy to analyse) form of oscillatory motion.

Types of SHM

Linear SHM

When a particle moves to and fro about an equilibrium point, along a straight line, A and B are extreme positions. M is mean position. $AM = MB =$ amplitude



Angular SHM

When body/particle is free to rotate about a given axis executing angular oscillations.

Equation of Simple Harmonic Motion

The necessary and sufficient condition for SHM is

$$F = -kx$$

where k = positive constant for a SHM = Force constant
x = displacement from mean position.

or $$m\frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad \text{(differential equation of SHM)}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \omega^2 x = 0,$$

where $$\omega = \sqrt{\frac{k}{m}}$$

Its solution is $x = A\sin(\omega t + \phi)$.

Characteristics of SHM

In Fig. 9.1, path of the particle is on a straight line.

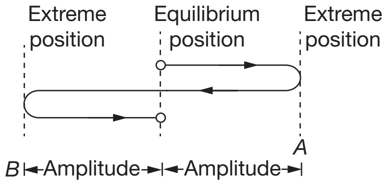


Fig. 9.1

Displacement

It is defined as the distance of the particle from the mean position at that instant.

Displacement in SHM at time t is given by $x = A \sin(\omega t + \phi)$

Amplitude

It is the maximum value of displacement of the particle from its equilibrium position.

Amplitude $= \frac{1}{2}$ (distance between extreme points/position)

It depends on the energy of the system.

Angular Frequency (ω)

$\omega = \frac{2\pi}{T} = 2\pi f$ and its units is rad/s.

Frequency (f)

Number of oscillations completed in unit time interval is called frequency of oscillations, $f = \frac{1}{T} = \frac{\omega}{2\pi}$, its units is $\sec^{-1}$ or Hz.

Time Period (T)

Smallest time interval after which the oscillatory motion gets repeated is called time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

SOLVED EXAMPLE

1. For a particle performing SHM, equation of motion is given as $\frac{d^2x}{dt^2} + 4x = 0$. Find the time period.

Solution:

$$\frac{d^2x}{dt^2} = -4x \quad \omega^2 = 4 \quad \omega = 2$$

Time period,

$$T = \frac{2\pi}{\omega} = \pi.$$

Phase

The physical quantity which represents the state of motion of particle (e.g., its position and direction of motion at any instant).

The argument $(\omega t + \phi)$ of sinusoidal function is called instantaneous phase of the motion.

Phase Constant (ϕ)

Constant ϕ in equation of SHM is called phase constant or initial phase.

It depends on the initial position and direction of velocity.

Velocity (v)

It is the rate of change of particle's displacement with regard to time at that instant.

If the displacement from mean position is given by

$$x = A \sin(\omega t + \phi)$$

Velocity, $$v = \frac{dx}{dt} = \frac{d}{dt}[A\sin(\omega t + \phi)]$$

$$v = A\omega \cos(\omega t + \phi)$$

or $$v = \omega\sqrt{A^2 - x^2}$$

At mean position ($x = 0$), velocity is maximum.

$$v_{max} = \omega A$$

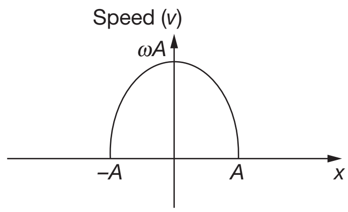
At extreme position ($x = A$), velocity is minimum.

$$v_{min} = \text{zero}$$

Graph of Speed (v) vs Displacement (x)

$$v = \omega\sqrt{A^2 - x^2} \qquad v^2 = \omega^2\left(A^2 - x^2\right)$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2 \qquad \frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1.$$



Graph could be an Ellipse

Acceleration

It is the rate of change of particle's velocity with respect to time at that instant.

Acceleration, $$a = \frac{dv}{dt} = \frac{d}{dt}[A\omega\cos(\omega t + \phi)]$$

$$a = -\omega^2 A \sin(\omega t + \phi)$$

$$a = -\omega^2 x$$

NOTE

Negative sign shows that acceleration is always directed towards the mean position.

At mean position $(x = 0)$, acceleration is minimum.

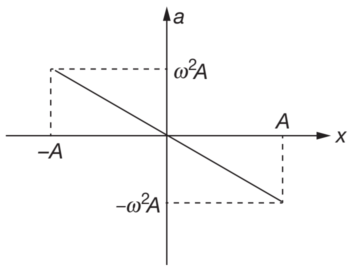
$$a_{min} = \text{zero}$$

At extreme position $(x = A)$, acceleration is maximum.

$$a_{max} = \omega^2 A$$

Graph of acceleration (a) vs displacement (x)

$$a = -\omega^2 x$$



SOLVED EXAMPLES

2. The equation of particle executing SHM is $x = (5\text{ m}) \sin\left[(\pi s^{-1})t + \frac{\pi}{3}\right]$. Write down the amplitude, time period, and maximum speed. Also find the velocity at $t = 1$ s.

Solution:

Comparing with equation $x = A \sin(\omega t + \delta)$, we see that the amplitude = 5 m,

and time period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi s^{-1}} = 2\text{s}.$

The maximum speed $= A\omega = 5\text{ m} \times \pi\text{ s}^{-1} = 5\pi$ m/s.

The velocity at time $t = \frac{dx}{dt} = A\,\omega \cos(\omega t + \delta)$

At $t = 1$ s,

$$v = (5\text{ m})\,(\pi s^{-1}) \cos\left(\pi + \frac{\pi}{5}\right) = -\frac{5\pi}{2}\text{ m/s.}$$

3. A particle executing SHM has angular frequency 6.28 s^{-1} and amplitude 10 cm. Find

(A) the time period.

(B) the maximum speed.

(C) the maximum acceleration.

(D) the speed when the displacement is 6 cm from the mean position.

(E) the speed at $t = 1/6$ s assuming that the motion starts from rest at $t = 0$.

Solution:

(A) Time period $= \frac{2\pi}{\omega} = \frac{2\pi}{6.28}\text{ s} = 1\text{ s}.$

(B) Maximum speed $= A\omega = (0.1\text{ m})\,(6.28\text{ s}^{-1})$

$= 0.628$ m/s.

(C) Maximum acceleration $= A\omega^2$

$= (0.1\text{ m})\,(6.28\text{ s}^{-1})^2$

$= 4\text{ m/s}^2.$

(D) $v = \omega\sqrt{A^2 - x^2} = (6.28\text{ s}^{-1})\sqrt{(10\text{ cm})^2 - (6\text{ cm})^2}$

$= 50.2$ cm/s.

(E) At $t = 0$, the velocity is zero, i.e., the particle is at an extreme. The equation for displacement may be written as

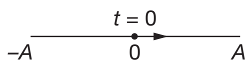
$$x = A\cos\omega t.$$

The velocity is $v = -A\omega \sin \omega t$.

At $t = \frac{1}{6}$ s, $v = -(0.1\text{ m})\,(6.28\text{ s}^{-1}) \sin\left(\frac{6.28}{6}\right)$

$= (-0.628\text{ m/s}) \sin\frac{\pi}{3} = 54.4$ cm/s.

4. A particle starts from mean position and moves towards positive extreme as shown. Find the equation of the SHM. Amplitude of SHM is A.



Solution:

General equation of SHM can be written as $x = A \sin(\omega t + \phi)$

At $t = 0, x = 0$

$\therefore$ $0 = A \sin\phi$

$\therefore$ $\phi = 0, \pi \qquad \phi \in [0, 2\pi)$

Also, at $t = 0$, $v = +\text{ve}$

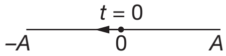
$\therefore$ $A\omega \cos\phi = +\text{ve}$

or $\phi = 0.$

Hence, if the particle is at mean position at $t = 0$ and is moving towards +ve extreme, then the equation of SHM is given by $x = A \sin \omega t$

Similarly,

for



$$\phi = \pi$$

$\therefore$ Equation of SHM is $x = A \sin (\omega t + \pi)$

or $$x = -A \sin \omega t.$$

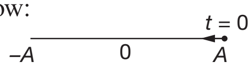
NOTE

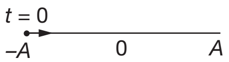
If mean position is not at the origin, then we can replace x by $x - x_0$ and the equation becomes

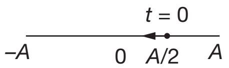
$$x - x_0 = -A \sin \omega t,$$

where x_0 is the position co-ordinate of the mean position.

5. Write the equation of SHM for the situations shown below:

(A) 

(B) 

(C) 

Solution:

(A) $x = A \cos\omega t$;

(B) $x = -A \cos\omega t$;

(C) $x = A \sin(\omega t + 150°)$.

6. A particle is performing SHM of amplitude A and time period T. Find the time taken by the particle to go from 0 to A/2.

Solution:

Let equation of SHM be $x = A \sin \omega t$

when $x = 0, t = 0$

when $x = A/2$; $A/2 = A \sin \omega t$

or $$\sin \omega t = 1/2$$

$$\omega t = \pi/6$$

$$\frac{2\pi}{T} t = \pi/6$$

$$t = T/12$$

Hence, time taken is $T/12$, where T is time period of SHM.

7. A particle of mass 2 kg is moving on a straight line under the action force $F = (8 - 2x)$ N. It is released at rest from $x = 6$ m.

(A) Is the particle moving simple harmonically.

(B) Find the equilibrium position of the particle.

(C) Write the equation of motion of the particle.

(D) Find the time period of SHM.

Solution:

$$F = 8 - 2x \quad \text{or} \quad F = -2(x - 4)$$

For equilibrium position

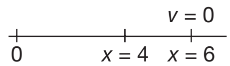
$$F = 0 \quad \Rightarrow \quad x = 4 \text{ is equilibrium position}$$

Hence, the motion of particle is SHM with force constant 2 and equilibrium position $x = 4$.

(A) Yes, motion is SHM.

(B) Equilibrium position is $x = 4$

(C) At $x = 6$ m, particle is at rest, i.e., it is in one of the extreme positions.



Hence, amplitude is $A = 2$ m, and initially particle is at the extreme position.

$\therefore$ Equation of SHM can be written as

$$x - 4 = 2 \cos \omega t,$$

where $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{2}} = 1,$$

i.e., $$x = 4 + 2 \cos t$$

(D) Time period, $T = \dfrac{2\pi}{\omega} = 2\pi$ s.

SHM as a Projection of Uniform Circular Motion

Consider a particle moving on a circle of radius A with a constant angular speed ω as shown in Fig. 9.2.

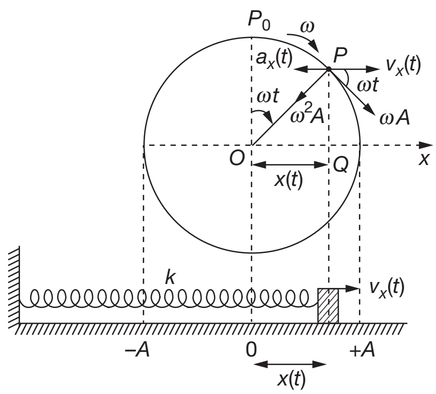


Fig. 9.2

Suppose the particle is on the top of the circle (Y-axis) at $t = 0$. The radius OP make an angle $\theta = \omega t$ with the Y-axis at time t. Drop a perpendicular PQ on X-axis. The components of position vector, velocity vector, and acceleration vector at time t on the X-axis are

$$x(t) = A \sin \omega t$$

$$v_x(t) = A\omega \cos \omega t$$

$$a_x(t) = -\,\omega^2 A \sin \omega t$$

Above equations show that the foot of perpendicular Q executes a SHM on the X-axis. The amplitude is A, and angular frequency is ω. Similarly, the foot of perpendicular on Y-axis will also execute SHM of amplitude A and angular frequency ω [$y(t) = A \cos \omega t$]. The phases of the two SHMs differ by $\pi/2$.

Graphical Representation of Displacement, Velocity, and Acceleration in SHM

Displacement $x = A \sin \omega t$

Velocity $v = A\omega \cos \omega t = A\omega \sin\left(\omega t + \dfrac{\pi}{2}\right)$

or $v = \omega\sqrt{A^2 - x^2}$

Acceleration $a = -\omega^2 A \sin\omega t = \omega^2 A \sin(\omega t + \pi)$

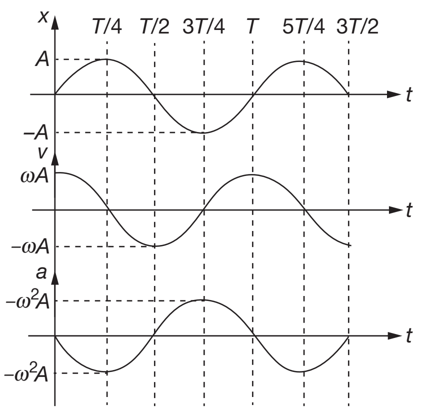
or $a = -\omega^2 x$

Note: $v = \omega\sqrt{A^2 - x^2}$

$a = -\omega^2 x$

These relations are true for any equation of x.

time t	0	$T/4$	$T/2$	$3T/4$	T
displacement x	0	A	0	$-A$	0
velocity v	$A\omega$	0	$-A\omega$	0	$A\omega$
acceleration a	0	$-\omega^2 A$	0	$\omega^2 A$	0



1. All the three quantities – displacement, velocity, and acceleration – vary harmonically with time, having same period.
2. The velocity amplitude is ω times the displacement amplitude ($v_{max} = \omega A$).
3. The acceleration amplitude is ω^2 times the displacement amplitude ($a_{max} = \omega^2 A$).
4. In SHM, the velocity is ahead of displacement by a phase angle of $\dfrac{\pi}{2}$.
5. In SHM, the acceleration is ahead of velocity by a phase angle of $\dfrac{\pi}{2}$.

Energy of SHM

Kinetic Energy (KE)

$$\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$= \frac{1}{2}k(A^2 - x^2) \quad \text{(as a function of } x\text{)}$$

$$= \frac{1}{2}m A^2\omega^2 \cos^2(\omega t + \theta)$$

$$= \frac{1}{2}KA^2 \cos^2(\omega t + \theta) \quad \text{(as a function of } t\text{)}$$

$$\text{KE}_{max} = \frac{1}{2}kA^2; \quad \langle \text{KE} \rangle_{0-T} = \frac{1}{4}kA^2; \quad \langle \text{KE} \rangle_{0-A} = \frac{1}{3}kA^2$$

Frequency of KE = 2 (frequency of SHM)

Potential Energy (PE)

$$\frac{1}{2}Kx^2 \quad \text{(as a function of } x\text{)}$$

$$= \frac{1}{2}kA^2 \sin^2(\omega t + \theta) \quad \text{(as a function of time)}$$

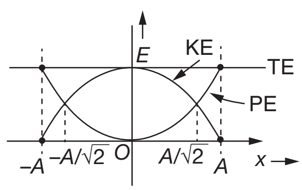
Total Mechanical Energy (TME)

Total mechanical energy = Kinetic energy + Potential energy

$$= \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}Kx^2 = \frac{1}{2}KA^2$$

Hence, total mechanical energy is constant in SHM.

Graphical Variation of Energy of SHM



SOLVED EXAMPLE

8. A particle of mass 0.50 kg executes a SHM under a force $F = -(50\ \text{N/m})x$. If it crosses the centre of oscillation with a speed of 10 m/s, find the amplitude of the motion.

Solution:

The kinetic energy of the particle when it is at the centre of oscillation is

$$E = \frac{1}{2}\, mv^2 = \frac{1}{2}(0.50\ \text{kg})\ (10\ \text{m/s})^2$$
$$= 25\ \text{J}.$$

The potential energy is zero here. At the maximum displacement $x = A$, the speed is zero and hence the kinetic energy is zero. The potential energy here is $\frac{1}{2}kA^2$. As there is no loss of energy,

$$\frac{1}{2}kA^2 = 25\ \text{J} \qquad (1)$$

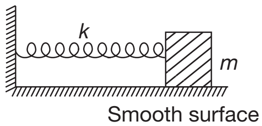
The force on the particle is given by

$$F = -(50\ \text{N/m})x.$$

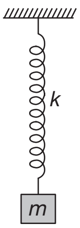
Thus, the spring constant is k = 50 N/m.
Equation (1) gives

$$\frac{1}{2}(50\ \text{N/m})\ A^2 = 25\ \text{J} \quad \text{or} \quad A = 1\ \text{m}.$$

Spring–Mass System

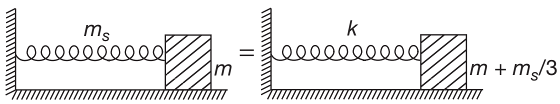
1. 

$$T = 2\pi\sqrt{\frac{m}{k}}$$

2. 

$$T = 2\pi\sqrt{\frac{m}{k}}$$

3. If spring has mass m_s, then



$$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$ **[Not in JEE, for other exams]**

SOLVED EXAMPLES

9. A particle of mass 200 g executes a SHM. The restoring force is provided by a spring of spring constant 80 N/m. Find the time period.

Solution:

The time period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{200\times10^{-3}\ \text{kg}}{80\ \text{N/m}}}$$
$$= 2\pi\times 0.05\ \text{s} = 0.31\ \text{s}.$$

10. The friction coefficient between the two blocks shown in Fig. 9.3 is μ and the horizontal plane is smooth.

(A) If the system is slightly displaced and released, find the time period.

(B) Find the magnitude of the frictional force between the blocks when the displacement from the mean position is x.

(C) What can be the maximum amplitude if the upper block does not slip relative to the lower block?

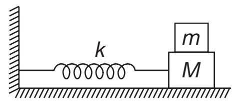


Fig. 9.3

Solution:

(A) For small amplitude, the two blocks oscillate together. The angular frequency is

$$\omega = \sqrt{\frac{k}{M+m}}$$

and so the time period

$$T = 2\pi\sqrt{\frac{M+m}{k}}\ .$$

(B) The acceleration of the blocks at displacement x from the mean position is

$$a = -\omega^2 x = \left(\frac{-kx}{M+m}\right)$$

The resultant force on the upper block is, therefore,

$$ma = \left(\frac{-mkx}{M+m}\right)$$

This force is provided by the friction of the lower block. Hence, the magnitude of the frictional force is $\left(\frac{mk\,|x|}{M+m}\right)$.

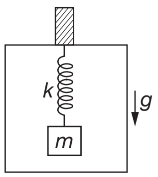
(C) Maximum force of friction required for SHM of the upper block is $\frac{mk\,A}{M+m}$ at the extreme positions. But the maximum frictional force can only be μ mg. Hence,

$$\frac{mk\,A}{M+m} = \mu\text{mg} \quad \text{or} \quad A = \frac{\mu(M+m)g}{k}.$$

11. A block of mass m is suspended from the ceiling of a stationary elevator through a spring of spring constant k when it is in equilibrium. Suddenly, the cable breaks and the elevator starts falling freely. Shows that block now executes a SHM of amplitude mg/k in the elevator.

Solution:

When the elevator is stationary, the spring is stretched to support the block. If the extension is x, the tension is kx, which should balance the weight of the block.



Thus, x = mg/k. As the cable breaks, the elevator starts falling with acceleration g. We shall work in the frame of reference of the elevator. Then we have to use a pseudo force mg upward on the block. This force will 'balance' the weight. Thus, the block is subjected to a net force kx by the spring when it is at a distance x from the position of unstretched spring. Hence, its motion in the elevator is simple harmonic with its mean position corresponding to the unstretched spring. Initially, the spring is stretched by x = mg/k, where the velocity of the block (with respect to the elevator) is zero. Thus, the amplitude of the resulting SHM is mg/k.

12. The left block in Fig. 9.4 collides inelastically with the right block and sticks to it. Find the amplitude of the resulting SHM.

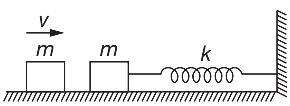


Fig. 9.4

Solution:

Assuming the collision to last just for a small interval, we can apply the principle of conservation of momentum. The common velocity after the collision is $\frac{v}{2}$. The kinetic energy $= \frac{1}{2}\,(2\text{m})\left(\frac{v}{2}\right)^2 = \frac{1}{4}\,mv^2$. This is also the total energy of vibration as the spring is unstretched at this moment. If the amplitude is A, the total energy can also be written as $\frac{1}{2}\,kA^2$. Thus,

$$\frac{1}{2}kA^2 = \frac{1}{4}\,mv^2,$$

giving $$A = \sqrt{\frac{m}{2k}}\;v.$$

13. Two blocks of mass m_1 and m_2 are connected with a spring of natural length l and spring constant k. The system is lying on a smooth horizontal surface. Initially, spring is compressed by x_0 as shown in Fig. 9.5.

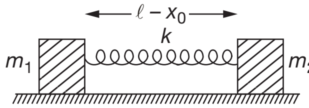


Fig. 9.5

Show that the two blocks will perform SHM about their equilibrium position. Also

(A) find the time period,
(B) find amplitude of each block, and
(C) length of spring as a function of time.

Solution: (A)

(A) Here both the blocks will be in equilibrium at the same time when spring is in its natural length. Let EP_1 and EP_2 be equilibrium positions of block A and B as shown in Fig. 9.6.

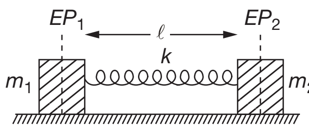
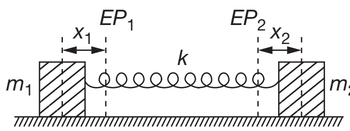


Fig. 9.6

If at any time during oscillations, blocks are at a distance of x_1 and x_2 from their equilibrium positions.



As no external force is acting on the spring block system

$$\therefore \quad (m_1 + m_2)\,\Delta x_{cm} = m_1 x_1 - m_2 x_2 = 0$$

or $$m_1 x_1 = m_2 x_2$$

For 1st particle, force equation can be written as

$$k\,(x_1 + x_2) = -m_1 \frac{d^2 x_1}{dt^2}$$

or, $$k\,(x_1 + \frac{m_1}{m_2} x_1) = -m_1 a_1$$

or, $$a_1 = -\frac{k(m_1 + m_2)}{m_1 m_2} x_1$$

$\therefore$ $$\omega^2 = \frac{k(m_1 + m_2)}{m_1 m_2}$$

Hence, $$T = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} = 2\pi\sqrt{\frac{\mu}{K}},$$

where $\mu = \dfrac{m_1 m_2}{(m_1 + m_2)}$ which is known as reduced mass

Similarly time period of 2nd particle can be found. Both will have the same time period.

(B) Let the amplitude of blocks be A_1 and A_2.

$$m_1 A_1 = m_2 A_2$$

By energy conservation;

$$\frac{1}{2} k(A_1 + A_2)^2 = \frac{1}{2} k x_0^2$$

or $$A_1 + A_2 = x_0$$

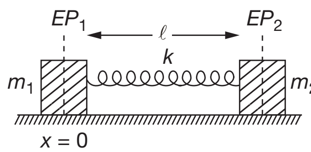
or $$A_1 + A_2 = x_0$$

or $$A_1 + \frac{m_1}{m_2} A_1 = x_0$$

or $$A_1 = \frac{m_2 x_0}{m_1 + m_2}.$$

Similarly, $$A_2 = \frac{m_1 x_0}{m_1 + m_2}$$

(C) If equilibrium position of 1st particle has origin, i.e., $x = 0$.



x co-ordinate of particles can be written as

$$x_1 = A_1 \cos \omega t$$

and $$x_2 = \ell - A_2 \cos \omega t$$

Hence, length of spring can be written as,

$$\text{length} = x_2 - x_1$$
$$= \ell - (A_1 + A_2)\cos \omega t.$$

14. Block A of mass m is performing SHM of amplitude a. Another block B of mass m is gently placed on A when it passes through mean position and B sticks to A. Find the time period and amplitude of new SHM.

Solution:

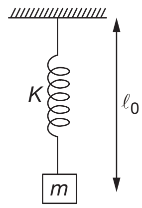
$$T = 2\pi\sqrt{\frac{2m}{K}}, \qquad \text{Amplitude} = \frac{a}{\sqrt{2}}.$$

15. Repeat the above problem assuming B is placed on A at a distance $\frac{a}{2}$ from mean position.

Solution:

$$T = 2\pi\sqrt{\frac{2m}{K}}, \qquad \text{Amplitude} = a\sqrt{\frac{5}{8}}.$$

16. The block is allowed to fall, slowly from the position where spring is in its natural length. Find maximum extension of the string.



Solution:

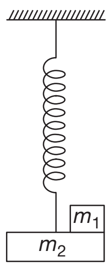
$$\frac{mg}{K}.$$

17. In the above problem, if block is released from there, what would be the maximum extension.

Solution:

$$\frac{2\,mg}{K}.$$

18. The system is in equilibrium and at rest. Now mass m_1 is removed from m_2. Find the time period and amplitude of resultant motion. Spring constant is K.

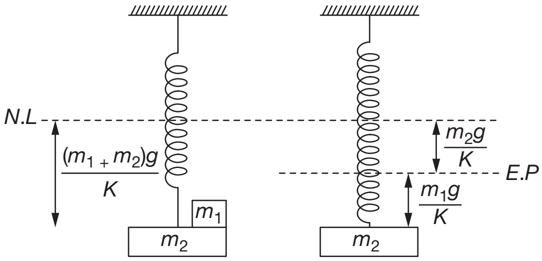


Solution:

Initial extension in the spring

$$x = \frac{(m_1 + m_2)\,g}{K}$$

Now, if we remove m_1, equilibrium position (EP) of m_2 will be $\frac{m_2 g}{K}$ below natural length of spring.



At the initial position, since velocity is zero, i.e., it is in the extreme position.

Hence amplitude = $\frac{m_1 g}{K}$

Time period = $2\pi\sqrt{\frac{m_2}{K}}$.

19. Block of mass m_2 is in equilibrium as shown in Fig. 9.7. Another block of mass m_1 is kept gently on m_2. Find the time period of oscillation and amplitude.

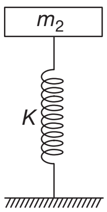
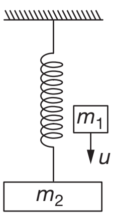


Fig. 9.7

Solution:

$$T = 2\pi\sqrt{\frac{m_1+m_2}{K}} \qquad \text{Amplitude} = \frac{m_1 g}{K}.$$

20. Block of mass m_2 is in equilibrium and at rest. The mass m_1 moving with velocity u vertically downwards collides with m_2 and sticks to it. Find the energy of oscillation.



Solution:

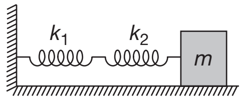
$$\frac{1}{2}\left[\frac{m_1^2 u^2}{m_1+m_2} + \frac{m_1^2 g^2}{K^2}\right].$$

Combination of Springs

Series Combination

Total displacement $x = x_1 + x_2$

Tension in both springs = $k_1 x_1 = k_2 x_2$



$\therefore$ Equivalent constant in series combination K_{eq} is given by:

$$1/k_{eq} = 1/k_1 + 1/k_2 \Rightarrow T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

NOTE

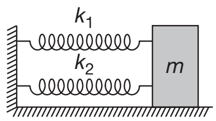
- In series combination, tension is same in all the springs and extension will be different. (If k is same then deformation is also same.)
- In series combination, extension of springs will be reciprocal of its spring constant.
- Spring constant of spring is reciprocal of its natural length.

  $\because k \propto 1/\ell$

  $\therefore k_1 \ell_1 = k_2 \ell_2 = k_3 \ell_3$
- If a spring is cut in 'n' pieces, then spring constant of one piece will be nk.

Parallel Combination

Extension is same for both springs, but force acting will be different.



Force acting on the system = F

$\therefore \quad F = -(k_1 x + k_2 x)$

$\Rightarrow \quad F = -(k_1 + k_2)x$

$\Rightarrow \quad F = -k_{eq} x$

$\therefore \quad k_{eq} = k_1 + k_2$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

Methods to Determine Time Period and Angular Frequency in SHM

1. Force/torque method
2. Energy method

SOLVED EXAMPLES

21. The string, the spring, and the pulley shown in Fig. 9.8 are light. Find the time period of the mass *m*.

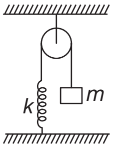
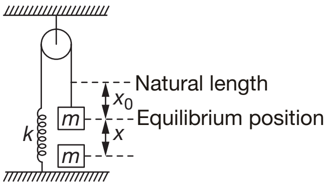


Fig. 9.8

Solution:

(A) **Force Method:** In equilibrium position of the block, extension in spring is x_0.

$\therefore \quad kx_0 = mg \qquad (1)$



Now if we displace the block by *x* in the downward direction, net force on the block towards mean position is

$$F = k(x + x_0) - mg = kx \qquad \text{using (1)}$$

Hence, the net force is acting towards mean position and is also proportional to *x*. Thus, the particle will perform SHM and its time period would be

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(B) **Energy Method:** Let gravitational potential energy be zero at the level of the block when spring is in its natural length.

Now at a distance *x* below that level, let speed of the block be *v*.

Since total mechanical energy is conserved in SHM

$$\therefore \quad -mgx + \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{constant}$$

Differentiating with respect to time, we get

$$-mgv + kxv + mva = 0,$$

where a is acceleration.

$$\therefore \quad F = ma = -kx + mg$$

$$\text{or} \quad F = -k\left(x - \frac{mg}{k}\right)$$

This shows that for the motion, force constant is *k* and equilibrium position is $x = \frac{mg}{k}$.

So, the particle will perform SHM and its time period would be

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

22. Solve the above problem if the pulley has a moment of inertia *I* about its axis and the string does not slip over it.

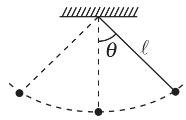
Solution:

$$2\pi\sqrt{\frac{(m + I/r^2)}{k}}.$$

Simple Pendulum

If a heavy point-mass is suspended by a weightless, inextensible, and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum.

Time period of a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$



(Sometimes we can take $g = \pi^2$ for making calculation simple.)

NOTE

- If angular amplitude of simple pendulum is more, then time period

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(1 + \frac{\theta_0^2}{16}\right) \quad \text{(For other exams)}$$

where θ_0 is in radians.

- General formula for time period of simple pendulum,

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R} + \frac{1}{\ell}\right)}}$$

- On increasing length of simple pendulum, time period increases, but time period of simple pendulum of infinite length is 84.6 min, which is maximum and is equal to $T = 2\pi\sqrt{\frac{R}{g}}$

(Where *R* is radius of earth)

- Time period of seconds pendulum is 2s and $\ell = 0.993$ m.
- Simple pendulum performs angular SHM but due to small angular displacement, it is considered as linear SHM.

- If time period of clock based on simple pendulum increases, then clock will be slow but if time period decreases then clock will be fast.
- If g remains constant and $\Delta\ell$ changes in length, then $\frac{\Delta T}{T}\times 100 = \frac{1}{2}\frac{\Delta\ell}{\ell}\times 100$
- If ℓ remain constant and Δg changes in acceleration, then $\frac{\Delta T}{T}\times 100 = -\frac{1}{2}\frac{\Delta g}{g}\times 100$
- If $\Delta\ell$ is change in length and Δg is change in acceleration due to gravity, then $\frac{\Delta T}{T}\times 100 = \left[\frac{1}{2}\frac{\Delta\ell}{\ell} - \frac{1}{2}\frac{\Delta g}{g}\right]\times 100$.

SOLVED EXAMPLE

23. A simple pendulum of length 40 cm oscillates with an angular amplitude of 0.04 rad. Find
(A) the time period,
(B) the linear amplitude of the bob,
(C) the speed of the bob when the string makes 0.02 rad with the vertical, and
(D) the angular acceleration when the bob is in momentary rest. Take $g = 10\ \text{m/s}^2$.

Solution:

(A) The angular frequency is

$$\omega = \sqrt{g/\ell} = \sqrt{\frac{10\ \text{m/s}^2}{0.4\ \text{m}}} = 5\ \text{s}^{-1}$$

The time period is

$$\frac{2\pi}{\omega} = \frac{2\pi}{5\,\text{s}^{-1}} = 1.26\ \text{s}.$$

(B) Linear amplitude = 40 cm × 0.04 = 1.6 cm
(C) Angular speed at displacement 0.02 rad is

$$\Omega = (5\ \text{s}^{-1})\sqrt{(0.04)^2 - (0.02)^2}\ \text{rad} = 0.17\ \text{rad/s},$$

where speed of the bob at this instant

$$= (40\ \text{cm}) \times 0.175^{-1} = 6.8\ \text{cm/s}.$$

(D) At momentary rest, the bob is in extreme position. Thus, the angular acceleration

$$\alpha = (0.04\ \text{rad})\ (25\ \text{s}^{-2}) = 1\ \text{rad/s}^2.$$

Time Period of Simple Pendulum in Accelerating Reference Frame

$$T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}},$$

where g_{eff} = effective acceleration due to gravity in reference system $= |\vec{g} - \vec{a}|$

$\vec{a}$ = acceleration of the point of suspension with respect to ground.

Condition for applying this formula: $|\vec{g} - \vec{a}|$ = constant

SOLVED EXAMPLES

24. A simple pendulum is suspended from the ceiling of a car accelerating uniformly on a horizontal road. If the acceleration is a_0 and the length of the pendulum is ℓ, find the time period of small oscillations about the mean position.

Solution:

We shall work in the car frame. As it is accelerated with respect to the road, we shall have to apply a pseudo force ma_0 on the bob of mass m.

For mean position, the acceleration of the bob with respect to the car should be zero. If θ be the angle made by the string with the vertical, the tension, weight, and the pseudo force will add to zero in this position.

Hence, resultant of mg and ma_0 (say $F = m\sqrt{g^2 + a_0^2}$) has to be along the string.

$$\therefore \quad \tan\theta_0 = \frac{ma_0}{mg} = \frac{a_0}{g}$$

Now, suppose the string is further deflected by an angle θ as shown in Fig. 9.9.

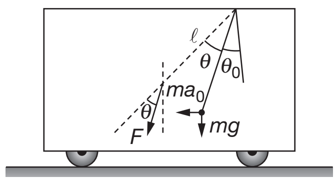


Fig. 9.9

Now, restoring torque can be given by

$$(F\sin\theta)\ \ell = -m\ell^2\alpha$$

Substituting F and using $\sin\theta = \theta$, for small θ.

$$(m\sqrt{g^2 + a_0^2})\,\ell\,\theta = -m\ell^2\alpha$$

or

$$\alpha = -\frac{\sqrt{g^2 + a_0^2}}{\ell}\theta$$

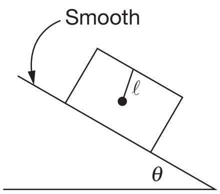
so

$$\omega^2 = \frac{\sqrt{g^2 + a_0^2}}{\ell}$$

This is an equation of SHM with time period

$$T = \frac{2\pi}{\omega} = 2\pi\frac{\sqrt{\ell}}{(g^2 + a_0^2)^{1/4}}.$$

25. A block is placed on a smooth inclined plane, and it is free to move. A simple pendulum is attached in the block. Find its time period.



Solution:

$$T = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}$$

If forces other than $m\vec{g}$ acts, then

$$T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}},$$

where $\quad g_{\text{eff}} = \left|\vec{g} + \frac{\vec{F}}{m}\right|$

$\vec{F}$ = constant force acting on m.

26. A simple pendulum of length ℓ having bob of mass m is in angular SHM inside water. A constant buoyant force equal to half the weight of the bob is acting on the ball. Find the time period of oscillations?

Solution:

Here

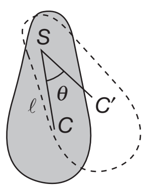
$$g_{\text{eff}} = g - \frac{mg/2}{m} = g/2.$$

Hence

$$T = 2\pi\sqrt{\frac{2\ell}{g}}.$$

Compound Pendulum/ Physical Pendulum

When a rigid body is suspended from an axis and made to oscillate about, then it is called compound pendulum.



C = Initial position of centre of mass

C' = Position of centre of mass after time t

S = Point of suspension

ℓ = Distance between point of suspension and centre of mass (it remains constant during motion)

For small angular displacement θ from mean position

The restoring torque is given by

$$\tau = -mgl\theta$$

or $\quad I\alpha = -mgl\theta,$

where I = Moment of inertia about point of suspension.

or $\quad \alpha = -\frac{mgl}{I}\theta$

or $\quad \omega^2 = \frac{mgl}{I}$

Time period, $\quad T = 2\pi\sqrt{\frac{I}{mg\ell}}$

$$I = I_{\text{CM}} + m\ell^2$$

where I_{CM} = moment of inertia relative to the axis which passes from the center of mass and parallel to the axis of oscillation.

$$T = 2\pi\sqrt{\frac{I_{\text{CM}} + m\ell^2}{mg\ell}}$$

where $I_{\text{CM}} = mk^2$

k = gyration radius (about axis passing from centre of mass)

$$T = 2\pi\sqrt{\frac{mk^2 + m\ell^2}{mg\ell}}$$

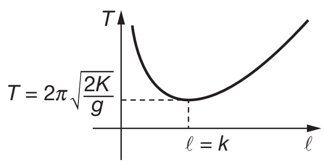
$$T = 2\pi\sqrt{\frac{k^2 + \ell^2}{\ell g}} = 2\pi\sqrt{\frac{L_{\text{eq}}}{g}}$$

$L_{\text{eq}} = \frac{k^2}{\ell} + \ell$ = equivalent length of simple pendulum.

T is minimum when $\ell = k$.

$$T_{\min} = 2\pi\sqrt{\frac{2K}{g}}$$

Graph of T vs l



SOLVED EXAMPLE

27. A uniform rod of length 1.00 m is suspended through an end and is set into oscillation with small amplitude under gravity. Find the time period of oscillation ($g = 10$ m/s^2).

Solution:

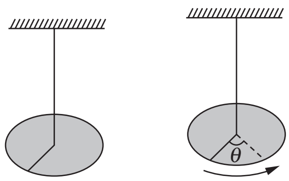
For small amplitude, the angular motion is nearly simple harmonic and the time period is given by

$$T = 2\pi\sqrt{\frac{I}{mg(\ell/2)}} = 2\pi\sqrt{\frac{(m\ell^2/3)}{mg(\ell/2)}}$$

$$= 2\pi\sqrt{\frac{2\ell}{3g}} = 2\pi\sqrt{\frac{2\times 1.00\text{ m}}{3\times 10\text{ m/s}^2}} = \frac{2\pi}{\sqrt{15}}\text{ s.}$$

Torsional Pendulum

In torsional pendulum, an extended object is suspended at the centre by a light torsion wire. A torsion wire is essentially inextensible, but is free to twist about its axis. When the lower end of the wire is rotated by a slight amount, the wire applies a restoring torque causing the body to oscillate rotationally when released.



The restoring torque produced is given by

$$\tau = -C\theta,$$

where C = Torsional constant

or, $$I\alpha = -C\theta,$$

where I = Moment of inertia about the vertical axis

or, $$\alpha = -\frac{C}{I}\theta$$

∴ Time period, $$T = 2\pi\sqrt{\frac{I}{C}}$$

SOLVED EXAMPLE

28. A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.

Solution:

The situation is shown in Fig. 9.10. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200\text{ kg})(5.0\times 10^{-2}\text{ m})^2}{2}$$

$$= 2.5\times 10^{-4}\text{ kg-m}^2.$$

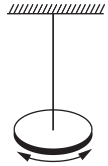


Fig. 9.10

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}}$$

or, $$C = \frac{4\pi^2 I}{T^2}$$

$$= \frac{4\pi^2(2.5\times 10^{-4}\text{ kg-m}^2)}{(0.20\text{ s})^2} = 0.25\ \frac{\text{kg-m}^2}{\text{s}^2}.$$

Superposition of Two SHMs

In same Direction and of same Frequency

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \theta),$$

then resultant displacement

$$x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \theta) = A \sin(\omega t + \phi),$$

where $$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

and $$\phi = \tan^{-1}\left[\frac{A_2\sin\theta}{A_1 + A_2\cos\theta}\right]$$

If $\theta = 0$, both SHMs are in phase and $A = A_1 + A_2$

If $\theta = \pi$, both SHMs are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHMs of this case can also be found by phasor diagram.

In same Direction but of different Frequencies

$$x_1 = A_1 \sin \omega_1 t$$

$$x_2 = A_2 \sin \omega_2 t$$

Then resultant displacement $x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$

This resultant motion is not SHM.

In Two Perpendicular Directions

$$x = A \sin \omega t$$

$$y = B \sin(\omega t + \theta)$$

Case (I): If $\theta = 0$ or π then $y = \pm(B/A)x$. So path will be straight line, and resultant displacement will be $r = (x^2 + y^2)^{1/2} = (A^2 + B^2)^{1/2} \sin\omega t$, which is equation of SHM having amplitude $\sqrt{A^2 + B^2}$

Case (II): If $\theta = \frac{\pi}{2}$, then $x = A \sin \omega t$

$$y = B \sin (\omega t + \pi/2) = B \cos \omega t$$

Thus, resultant will be $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$. That is, equation of an ellipse and if $A = B$, then superposition will be an equation of circle.

Superposition of SHMs along the same Direction (Using Phasor Diagram)

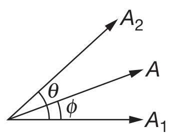
If two or more SHMs are along the same line, their resultant can be obtained by vector addition by making phasor diagram.

1. Amplitude of SHM is taken as length (magnitude) of vector.
2. Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant of vectors give resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.

For example,

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin (\omega t + \theta)$$



If equation of resultant SHM is taken as $x = A \sin (\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\theta}$$

$$\tan\phi = \frac{A_2 \sin\theta}{A_1 + A_2 \cos\theta}.$$

SOLVED EXAMPLES

29. Find the amplitude of the SHM obtained by combining the motions

$$x_1 = (2.0 \text{ cm}) \sin\omega t$$

and $$x_2 = (2.0 \text{ cm}) \sin (\omega t + \pi/3).$$

Solution:

The two equations given represent SHMs along X-axis with amplitudes $A_1 = 2.0$ cm and $A_2 = 2.0$ cm. The phase difference between the two SHMs is $\pi/3$. The resultant SHM will have an amplitude A given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\delta}$$

$$= \sqrt{(2.0\text{ cm})^2 + (2.0\text{ cm})^2 + 2(2.0\text{ cm})^2 \cos\frac{\pi}{3}}$$

$$= 3.5 \text{ cm}.$$

30. $x_1 = 3 \sin \omega t$
$x_2 = 4 \cos \omega t$

Find
(A) amplitude of resultant SHM and
(B) equation of the resultant SHM.

Solution:

First write all SHMs in terms of sine functions with positive amplitude. Keep ωt with positive sign.

$\therefore$ $$x_1 = 3 \sin \omega t$$

$$x_2 = 4 \sin (\omega t + \pi/2)$$

$$A = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\frac{\pi}{2}}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

$$\tan\phi = \frac{4\sin\frac{\pi}{2}}{3+4\cos\frac{\pi}{2}} = \frac{4}{3}$$

$$\phi = 53°$$

Equation $x = 5 \sin (\omega t + 53°)$.

31. $x_1 = 5 \sin (\omega t + 30°)$
$x_2 = 10 \cos (\omega t)$

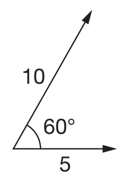
Find amplitude of resultant SHM.

Solution:

$$x_1 = 5 \sin (\omega t + 30°)$$

$$x_2 = 10 \sin (\omega t + \frac{\pi}{2})$$

Phasor diagram



$$A = \sqrt{5^2 + 10^2 + 2\times5\times10\cos 60°}$$

$$= \sqrt{25+100+50} = \sqrt{175} = 5\sqrt{7}.$$

32. $x_1 = 5 \sin \omega t$
$x_2 = 5 \sin (\omega t + 53°)$
$x_3 = -10 \cos \omega t$

Find amplitude of resultant SHM

Solution: 10.

33. A particle is subjected to two SHMs

$$x_1 = A_1 \sin \omega t$$

and $$x_2 = A_2 \sin (\omega t + \pi/3).$$

Find
(A) the displacement at $t = 0$,
(B) the maximum speed of the particle and
(C) the maximum acceleration of the particle.

Solution:

(A) At $t = 0, x_1 = A_1 \sin \omega t = 0$

and $x_2 = A_2 \sin (\omega t + \pi/3)$

$$= A_2 \sin (\pi/3) = \frac{A_2\sqrt{3}}{2}.$$

Thus, the resultant displacement at $t = 0$ is

$$x = x_1 + x_2 = A_2 \frac{\sqrt{3}}{2}$$

(B) The resultant of the two motions is a SHM of the same angular frequency ω. The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\pi/3)}$$

$$= \sqrt{A_1^2 + A_2^2 + A_1A_2}.$$

The maximum speed is

$$u_{max} = A\omega = \omega \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

(C) The maximum acceleration is

$$a_{max} = A\omega^2 = \omega^2 \sqrt{A_1^2 + A_2^2 + A_1A_2}.$$

34. A particle is subjected to two SHMs in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.

Solution:

Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. If the phase difference between the two motions is δ,

$$A = \sqrt{A^2 + A^2 + 2A.\ A.\cos\delta}$$

or $$= A\sqrt{2(1+\cos\delta)} = 2A\cos\frac{\delta}{2}$$

or $$\cos\frac{\delta}{2} = \frac{1}{2}$$

or $$\delta = 2\pi/3.$$

WAVE ON A STRING

Waves

Wave motion is the phenomenon that can be observed almost everywhere around us, and it also appears in almost every branch of physics. Surface waves on bodies of matter are commonly observed. Sound waves and light waves are essential to our perception of the environment. All waves have a similar mathematical description, which makes the study of one kind of wave useful for the study of other kinds of waves. In this chapter, we will concentrate on string waves, which are type of mechanical waves. Mechanical waves require a medium to travel through. Sound waves, water waves are other examples of mechanical waves. Light waves are not mechanical waves; these are electromagnetic waves which do not require a medium to propagate.

Mechanical waves originate from a disturbance in the medium (such as a stone dropping in a pond) and the disturbance that propagates through the medium. The forces between the atoms in the medium are responsible for the propagation of mechanical waves. Each atom exerts a force on the atoms near it, and through this force, the motion of the atom is transmitted to the others. The atoms in the medium do not, however, experience any net displacement. As the wave passes, the atoms simply move back and forth. Again for simplicity, we concentrate on the study of harmonic waves (i.e., those that can be represented by sine and cosine functions).

Types of Mechanical Waves

Mechanical waves can be classified according to the physical properties of the medium, as well as in other ways.

Direction of Particle Motion

Waves can be classified by considering the direction of motion of the particles in the medium as wave passes. If the disturbance travels in the x-direction but the particles move in a direction perpendicular to the x-axis as the wave passes, it is called a transverse wave. If the motion of the particles were parallel to the x-axis, then it is called a longitudinal wave. A wave pulse in a plucked guitar string is a transverse wave. A sound wave is a longitudinal wave.

Number of Dimensions

Waves can propagate in one, two, or three dimensions. A wave moving along a taut string is a one-dimensional wave. A water wave created by a stone thrown in a pond is a two-dimensional wave. A sound wave created by a gunshot is a three-dimensional wave.

Periodicity

A stone dropped into a pond creates a wave pulse, which travels outward in two dimensions. There may be more than one ripple created, but there is still only one wave pulse. If similar stones are dropped in the same place at even time intervals, then a periodic wave is created.

Shape of Wave Fronts

The ripples created by a stone dropped into a pond are circular in shape. A sound wave propagating outward from a point source has spherical wavefronts. A plane wave is a three-dimensional wave with flat wave fronts.

(Far away from a point source emitting spherical waves, the waves appear to be plane waves.)

A solid can sustain transverse as well as longitudinal wave. A fluid has no well-defined form or structure to maintain and offer far more resistance to compression than to a shearing force. Consequently, only longitudinal wave can propagate through a gas or within the body of an ideal (non-viscous) liquid.

However, transverse waves can exist on the surface of a liquid. In the case of ripples on a pond, the force restoring the system to equilibrium is the surface tension of the water, whereas for ocean waves, it is the force of gravity.

Also, if disturbance is restricted to propagate only in one direction and there is no loss of energy during propagation, then shape of disturbance remains unchanged.

Describing Waves

Two kinds of graph may be drawn: displacement-distance and displacement-time.

A displacement-distance graph for a transverse mechanical wave shows the displacement y of the vibrating particles of the transmitting medium at different distance x from the source at a certain instant, i.e., it is like a photograph showing shape of the wave at that particular instant.

The maximum displacement of each particle from its undisturbed position is the amplitude of the wave. In Fig 9.11, it is OA or OB.

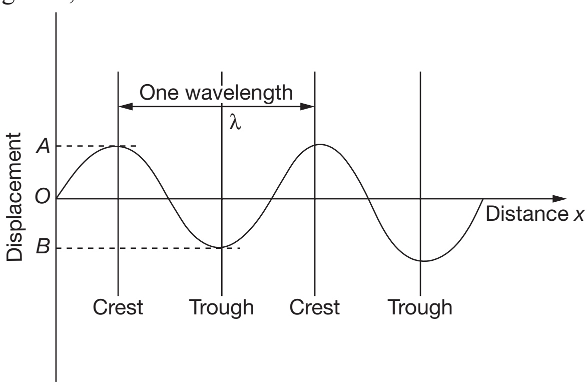


Fig. 9.11

The wavelength λ of a wave is generally taken as the distance between two successive crests or two successive troughs. To be more specific, it is the distance between two consecutive points on the wave which have same phase.

A displacement-time graph may also be drawn for a wave motion, showing how the displacement of one particle at a particular distance from the source varies with time. If this is simple harmonic variation, then the graph is a sine curve.

Wave Length, Frequency, Speed

If the source of a wave makes f vibrations per second, so too will the particles of the transmitting medium. That is, the frequency of the waves equals frequency of the source.

When the source makes one complete vibration, one wave is generated and the disturbance spreads out a distance λ from the source. If the source continues to vibrate with constant frequency f, then f waves will be produced per second and the wave advances a distance $f\lambda$ in one second. If v is the wave speed, then

$$v = f\lambda$$

This relationship holds for all wave motions.

Travelling Waves

Imagine a horizontal string stretched in the x-direction. Its equilibrium shape is flat and straight. Let y measure the displacement of any particle of the string from its equilibrium position, perpendicular to the string. If the string is plucked on the left end, a pulse will travel to the right. The vertical displacement y of the left end of the string ($x = 0$) is a function of time.

i.e., $$y\,(x = 0, t) = f(t).$$

If there are no frictional losses, the pulse will travel undiminished, retaining its original shape. If the pulse travels with a speed v, the 'position' of the wave pulse is $x = vt$. Therefore, the displacement of the particle at point x at time t was originated at the left end at time $t - \dfrac{x}{v}$. [y, (x, t) is function of both x and t.] But the displacement of the left end at time t is $f(t)$ thus at time $t - \dfrac{x}{v}$, it is $f(t - \dfrac{x}{v})$.

Therefore,

$$y(x, t) = y\left(x = 0, t - \frac{x}{v}\right) = f\left(t - \frac{x}{v}\right)$$

This can also be expressed as,

$$\Rightarrow \qquad \frac{f}{v}(vt - x)$$

$$\Rightarrow \qquad -\frac{f}{v}(x - vt)$$

$$y(x, t) = \frac{x}{v}\, g(x - vt)$$

using any fixed value of t (i.e., at any instant), this shows shape of the string.

If the wave is travelling in $-x$ direction, then wave equation is written as

$$y(x, t) = f(t + \frac{x}{v})$$

The quantity $x - vt$ is called phase of the wave function. As phase of the pulse has fixed value

$$x - vt = \text{const.}$$

Taking the derivative with respect to time $\frac{dx}{dt} = v$

where v is the phase velocity although often called wave velocity. It is the velocity at which a particular phase of the disturbance travels through space.

In order for the function to represent a wave travelling at speed v, the three quantities x, v, and t must appear in the combination $(x + vt)$ or $(x - vt)$. Thus, $(x - vt)^2$ is acceptable but $x^2 - v^2t^2$ is not.

SOLVED EXAMPLES

35. A wave pulse is travelling on a string at 2 m/s. Displacement y of the particle at $x = 0$ at any time t is given by

$$y = \frac{2}{t^2 + 1}$$

Find

(A) Expression of the function $y = (x, t)$, i.e., displacement of a particle at position x and time t.

(B) Shape of the pulse at $t = 0$ and $t = 1$ s.

Solution:

(A) By replacing t by $\left(t - \frac{x}{v}\right)$, we can get the desired wave function, i.e.,

$$y = \frac{2}{\left(t - \frac{x}{2}\right)^2 + 1}$$

(B) We can use wave function at a particular instant, say $t = 0$, to find shape of the wave pulse using different values of x.

at $t = 0$ $\quad y = \dfrac{2}{\frac{x^2}{4} + 1}$

at $x = 0$ $\quad y = 2$

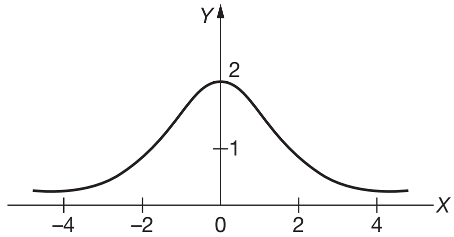
$x = 2$ $\quad y = 1$

$x = -2$ $\quad y = 1$

$x = 4$ $\quad y = 0.4$

$x = -4$ $\quad y = 0.4$

Using these value, shape is drawn.



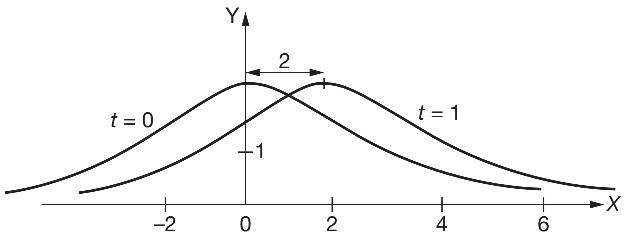
Similarly for $t = 1$ s, shape can be drawn. What do you conclude about direction of motion of the wave from the graphs? Also check how much the pulse has moved in 1 s time interval. This is equal to wave speed. Here is the procedure:

$$y = \frac{2}{\left(1 - \frac{x}{2}\right)^2 + 1} \quad \text{at} \quad t = 1 \text{ s}$$

at $x = 2$ $\quad y = 2$ (maximum value)

at $x = 0$ $\quad y = 1$

at $x = 4$ $\quad y = 1$



The pulse has moved to the right by 2 units in 1 s interval.

Also as $\quad t - \frac{x}{2} = \text{constant.}$

Differentiating with respect to time

$$1 - \frac{1}{2} \cdot \frac{dx}{dt} = 0 \quad \Rightarrow \quad \frac{dx}{dt} = 2.$$

36. A wave pulse moving along the x-axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3t)^2 + 1}$$

where x and y are measured in cm and t is in seconds.

(A) In which direction is the wave moving?

(B) Find speed of the wave.

(C) Plot the waveform at $t = 0$, $t = 2$ s.

Solution:

(A) Positive x-axis $\qquad$ (B) 3 cm/s.

37. At $t = 0$, a transvalue's wave pulse in a wire is described by the function $y = \dfrac{6}{x^3 + 3}$, where x and y are in metres. Write the function $y(x, t)$ that describes this wave if it is travelling in the positive x-direction with a speed of 4.5 m/s.

Solution:

$$\frac{6}{(x - 4.5t)^2 + 3}.$$

Travelling Sine Wave in One-dimension (Wave on String)

The wave equation $y = f\left(t - \dfrac{x}{v}\right)$ is quite general. If holds for arbitrary wave shapes, and for transverse as well as for longitudinal waves.

A complete description of the wave requires specification $f(x)$. The most important case, by far, in physics and engineering is when $f(x)$ is sinusoidal, that is, when the wave has the shape of a sine or cosine function. This is possible when the source, that is moving the left end of the string, vibrates the left end $x = 0$ in a simple harmonic motion. For this, the source has to continuously do work on the string, and energy is continuously supplied to the string.

The equation of motion of the left end may be written as

$$f(t) = A \sin \omega t$$

where A is amplitude of the wave, that is, maximum displacement of a particle in the medium from its equilibrium position ω is angular frequency, that is, $2\pi f$, where f is frequency of SHM of the source.

The displacement of the particle at x at time t will be

$$y = f\left(t - \frac{x}{v}\right)$$

or

$$y = A \sin \omega\left(t - \frac{x}{v}\right)$$

$$y = A \sin (\omega t - kx)$$

where $k = \dfrac{\omega}{v} = \dfrac{2\pi}{\lambda}$ is called wave number. $T = \dfrac{2\pi}{\omega} = \dfrac{1}{f}$ is period of the wave, that is, the time it takes to travel the distance between two adjacent crests or through (it is wavelength λ).

The wave equation $y = A \sin (\omega t - kx)$ says that at $x = 0$ and $t = 0$, $y = 0$. This is not necessarily the case of source. For the same condition, y may not be equal to zero. Therefore, the most general expression would involve a phase constant ϕ, which allows for other possibilities,

$$y = A \sin (\omega t - kx + \phi)$$

A suitable choice of ϕ allows any initial condition to be met. The term $kx - \omega t + \phi$ is called the phase of the wave. Two waves with the same phase (on phase differing by a multiple of 2π) are said to be 'in phase'. They execute the same motion at the same time.

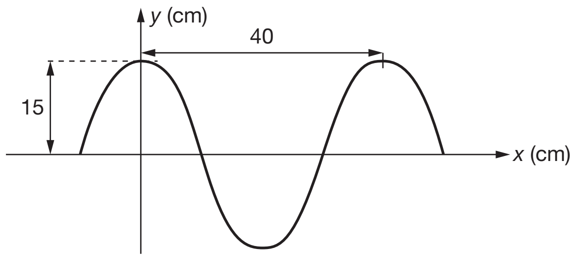
The velocity of the particle at position x and at time t is given by

$$\frac{\partial y}{\partial t} = A\omega \cos (\omega t - kx + \phi)$$

The wave equation has been partially differentiated keeping x as constant, to specify the particle. Note that wave velocity $\dfrac{dx}{dt}$ is different from particle velocity, while waves velocity is constant for a medium and it is along the direction of string, whereas particle velocity is perpendicular to wave velocity and is dependent upon x and t.

SOLVED EXAMPLES

38. A sinusoidal wave travelling in the positive x-direction has an amplitude of 15 cm, wavelength 40 cm, and frequency 8 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15 cm, as shown.



(A) Find the angular wave number, period, angular frequency, and speed of the wave.
(B) Determine the phase constant ϕ and write a general expression for the wave function.

Solution:

(A) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi \text{ rad}}{40 \text{ cm}} = \dfrac{\pi}{20}$ rad/cm

$T = \dfrac{1}{f} = \dfrac{1}{8}$ s

$\omega = 2\pi f = 16 \text{ s}^{-1}$

$v = f\lambda = 320$ cm/s

(B) It is given that $A = 15$ cm
and also $y = 15$ cm at $x = 0$ and $t = 0$
then using $y = A \sin (\omega t - kx + \phi)$
$15 = 15 \sin \phi$
$\Rightarrow \quad \sin \phi = 1$

or $\phi = \dfrac{\pi}{2}$ rad.

Therefore, the wave function is

$$y = A \sin\left(\omega t - kx + \frac{\pi}{2}\right)$$

$$= (15 \text{ cm}) \sin\left[(16\pi \text{s}^{-1})t - \left(\frac{\pi \text{ rad}}{20 \text{ cm}}\right)\cdot x + \frac{\pi}{2}\right].$$

39. A sinusoidal wave is travelling along a rope. The oscillator that generates the wave completes 60 vibrations in 30 s. Also, a given maximum travels 425 cm along the rope in 10.0 s. What is the wavelength?

Solution:

$$v = \frac{425}{10} = 42.5 \text{ cm/s}.$$

$$f = \frac{60}{30} = 2 \text{ Hz}$$

$$\lambda = \frac{v}{f} = 21.25 \text{ cm}.$$

40. The wave function for a travelling wave on a string is given as

$$y(x, t) = (0.350 \text{ m}) \sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$$

(A) What are the speed and direction of travel of the wave?

(B) What is the vertical displacement of the string at $t = 0$, $x = 0.1$ m?

(C) What are wavelength and frequency of the wave?

Solution:

(A) $3.33\hat{i}$ m/s

(B) –5.48 cm

(C) 0.67 m, 5 Hz.

The Linear Wave Equation

By using wave function $y = A \sin(\omega t - kx + \phi)$, we can describe the motion of any point on the string. Any point on the string moves only vertically, and so its x coordinate remains constant. The transverse velocity v_y of the point and its transverse acceleration a_y are therefore

$$v_y = \left[\frac{dy}{dt}\right]_{x=\text{constant}}$$

$$\Rightarrow \quad \frac{\partial y}{\partial t} = \omega A \cos(\omega t - kx + \phi) \quad (9.1)$$

$$a_y = \left[\frac{dv_y}{dt}\right]_{x=\text{constant}}$$

$$\Rightarrow \quad \frac{\partial v_y}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A \sin(\omega t - kx + \phi) \quad (9.2)$$

and hence

$$v_{y,\max} = \omega A$$

$$a_{y,\max} = \omega^2 A$$

The transverse velocity and transverse acceleration of any point on the string do not reach their maximum value simultaneously. In fact, the transverse velocity reaches its maximum value (ωA) when the displacement $y = 0$, whereas the transverse acceleration reaches its maximum magnitude ($\omega^2 A$) when $y = \pm A$ further

$$\left[\frac{dy}{dx}\right]_{t=\text{constant}} \Rightarrow \frac{\partial y}{\partial x} = -kA \cos(\omega t - kx + \phi) \quad (9.3)$$

$$= \frac{\partial^2 y}{\partial x^2} = -k^2 A \sin(\omega t - kx + \phi) \quad (9.4)$$

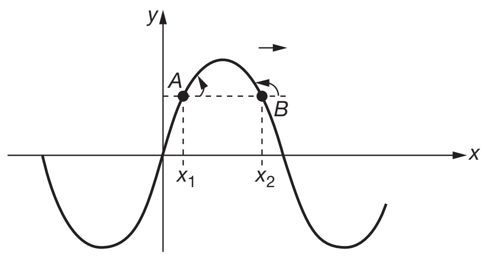
From (9.1) and (9.3),

$$\frac{\partial y}{\partial t} = -\frac{\omega}{k}\frac{\partial y}{\partial x}$$

$$\Rightarrow \quad v_P = -v_w \times \text{slope}$$

i.e., if the slope at any point is negative, particle velocity is positive and vice-versa, for a wave moving along positive x-axis, i.e., v_w is positive.

For example, consider two points A and B on the y-x curve for a wave, as shown. The wave is moving along positive x-axis.



Slope at A is positive therefore at the given moment, its velocity is negative. That means it is coming downward. Reverse is the situation for particle at point B.

Now using Equation (9.2) and (9.4),

$$\frac{\partial^2 y}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

This is known as the linear wave equation or differential equation representation of the travelling wave model. We have developed the linear wave equation from a sinusoidal mechanical wave travelling through a medium, but it is much more general. The linear wave equation successfully describes waves on strings, sound waves, and also electromagnetic waves.

SOLVED EXAMPLES

41. Verify that wave function

$$y = \frac{2}{(x-3t)^2 + 1}$$

is a solution to the linear wave equation. x and y are in cm.

Solution:

By taking partial derivatives of this function with respect to x and to t

$$\frac{\partial^2 y}{\partial x^2} = \frac{12(x-3t)^2 - 4}{[(x-3t)^2+1]^3},$$

and
$$\frac{\partial^2 y}{\partial t^2} = \frac{108(x-3t)^2 - 36}{[(x-3t)^2+1]^3}$$

or
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{9}\frac{\partial^2 x}{\partial t^2}$$

Comparing with linear wave equation, we see that the wave function is a solution to the linear wave equation if the speed at which the pulse moves is 3 cm/s. It is apparent from wave function, therefore, it is a solution to the linear wave equation.

The Speed of Transverse Waves on Strings

The speed of a wave on a string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

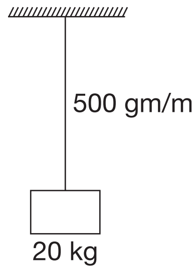
where T is tension in the string (in Newtons) and μ is mass per unit length of the string (kg/m).

It should be noted that v is speed of the wave with respect to the medium (string).

In case the tension is not uniform in the string or string has non-uniform linear mass density, then v is speed at a given point and T and μ are corresponding values at that point.

SOLVED EXAMPLES

42. Find speed of the wave generated in the string as in the situation shown. Assume that the tension is not affected by the mass of the cord.

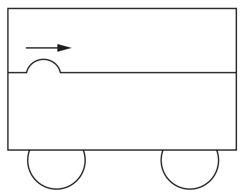


Solution:

$$T = 20 \times 10 = 200 \text{ N}$$

$$v = \sqrt{\frac{200}{0.5}} = 20 \text{ m/s}.$$

43. A taut string having tension 100 N and linear mass density 0.25 kg/m is used inside a cart to generate a wave pulse starting at the left end, as shown. What should be the velocity of the cart for the pulse to remain stationary with respect to the ground?



Solution:

$$\text{Velocity of pulse} = \sqrt{\frac{T}{\mu}} = 20 \text{ m/s}$$

Now
$$\vec{v}_{PG} = \vec{v}_{PC} + \vec{v}_{CG}$$

$$0 = 20i + \vec{v}_{CG}$$

$$\vec{v}_{CG} = -20i \text{ m/s}.$$

44. A uniform rope of mass m and length L hangs from a ceiling.

(A) Show that the speed of a transverse wave on the rope is a function of y, the distance from the lower end, and is given by $v = \sqrt{gy}$.

(B) Show that the time a transverse wave takes to travel the length of the rope is given by $t = 2\cdot\sqrt{L/g}$.

Power Transmitted along the String by a Sine Wave

When a travelling wave is established on a string, energy is transmitted along the direction of propagation of the wave, in form of potential energy and kinetic energy

Average power $\quad \langle P \rangle = 2\pi^2 f^2 A^2 \mu v$

$$\text{Energy transferred} = \int_0^t P_{av}\,dt$$

Energy transferred in one time period = $P_{av}\,T$

This is also equal to the energy stored in one wavelength.

Intensity

Energy transferred per second per unit cross-sectional area is called intensity of the wave.

$$I = \frac{\text{Power}}{\text{Cross sectional area}} = \frac{P}{s}$$

$$I = \frac{1}{2}\,\rho\omega^2 A^2 v$$

This is average intensity of the wave.

Energy density of a wave is energy per unit volume.

$$= \frac{Pdt}{svdt} = \frac{I}{v}$$

SOLVED EXAMPLES

45. A string with linear mass density $m = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution:

The wave speed on the string is

$$v = \sqrt{\frac{T}{\mu}} = \left(\frac{80.0\text{ N}}{5.00\times10^{-2}\text{ kg/m}}\right)^{1/2} = 40.0\text{ m/s}$$

Because ϕ = 60 Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0\text{ Hz}) = 377\text{ s}^{-1}$$

Using these values in following equation for the power, with $A = 6.00 \times 10^{-2}$ m, gives

$$p = \frac{1}{2}\,\mu\omega^2 A^2 v$$

$$= (5.00 \times 10^{-2}\text{ kg/m})\,(377\text{s}^{-1})^2$$

$$\times (6.00 \times 10^{-2}\text{ m})^2\,(40.0\text{ m/s})$$

$$= 512\text{ W.}$$

46. Two waves in the same medium are represented by *y*-*t* curves in Fig. 9.12. Find ratio of their average intensities?

Solution:

$$\frac{I_1}{I_2} = \frac{\omega_1^2 A_1^2}{\omega_2^2 A_2^2} = \frac{f_1^2 \cdot A_1^2}{f_2^2 \cdot A_2^2} \,. = \frac{1\times25}{4\times4} = \frac{25}{16}\,.$$

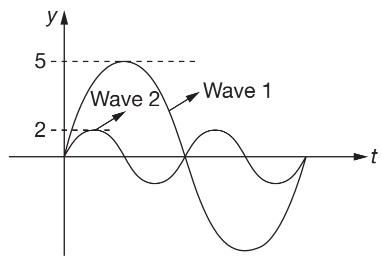


Fig. 9.12

47. A transverse wave of amplitude 0.50 mm and frequency 100 Hz is produced on a wire stretched to a tension of 100 N. If the wave speed is 100 m/s, what average power is the source transmitting to the wire?

Solution:

49 mW.

The Principle of Superposition

When two or more waves simultaneously pass through a point, the disturbance at the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s).

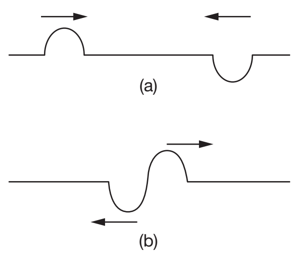
In general, the principle of superposition is valid for small disturbances only. If the string is stretched too far, the individual displacements do not add to give the resultant displacement. Such waves are called non-linear waves. In this course, we will be discussing linear waves which obey the superposition principle.

To put this rule in a mathematical form, let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that any element of the string would experience if each wave travelled alone. The displacement $y(x, t)$ of an element of the string when the waves overlap is then given by

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

The principal of superposition can also be expressed by stating that overlapping waves algebraically add to produce a resultant wave. The principle implies that the overlapping waves do not in any way alter the travel of each other.

If we have two or more waves moving in the medium, the resultant waveform is the sum of wave functions of individual waves.



(c)

A sequence of pictures showing two pulses travelling in opposite directions along a stretched string. When the two disturbances overlap, they give a complicated pattern as shown in (b). In (c), they have passed each other and proceed unchanged.

An Illustrative example of this principle is phenomena of interference and reflection of waves.

SOLVED EXAMPLE

48. Two waves passing through a region are represented by

$$y = (1.0 \text{ m}) \sin [(3.14 \text{ cm}^{-1})\, x - (157 \text{ s}^{-1})\, t]$$

and $y = (1.5 \text{ cm}) \sin [(1.57 \text{ cm}^{-1})\, x - (314 \text{ s}^{-1})\, t]$.

Find the displacement of the particle at $x = 4.5$ cm at time $t = 5.0$ ms.

Solution:

According to the principle of superposition, each wave produces its disturbance independent of the other, and the resultant disturbance is equal to the vector sum of the individual disturbance. The displacements of the particle at $x = 4.5$ cm at time $t = 5.0$ ms due to the two waves are

$$y_1 = (1.0 \text{ cm}) \sin [(3.14 \text{ cm}^{-1})(4.5 \text{ cm}) - (157 \text{ s}^{-1})(5.0 \times 10^{-3} \text{ s})]$$

$$= (1.0 \text{ cm}) \sin \left[4.5\pi - \frac{\pi}{4}\right]$$

$$= (1.0 \text{ cm}) \sin \left[4\pi + \frac{\pi}{4}\right] = \frac{1.0 \text{ cm}}{\sqrt{2}}$$

and

$$y_2 = (1.5 \text{ cm}) \sin [(1.57 \text{ cm}^{-1})(4.5 \text{ cm}) - (314 \text{ s}^{-1})(5.0 \times 10^{-3} \text{ s})]$$

$$= (1.5 \text{ cm}) \sin \left[2.25\pi - \frac{\pi}{2}\right]$$

$$= (1.5 \text{ cm}) \sin \left[2\pi - \frac{\pi}{4}\right]$$

$$= (1.5 \text{ cm}) \sin \frac{\pi}{4} = -\frac{1.5 \text{ cm}}{\sqrt{2}}$$

The net displacement is

$$y = y_1 + y_2 = \frac{-0.5 \text{ cm}}{\sqrt{2}} = -0.35 \text{ cm}.$$

Interference of Waves going in same Direction

Suppose two identical sources send sinusoidal waves of same angular frequency ω in positive x-direction. Also, the wave velocity and the wave number k is same for the two waves. One source may be situated at different points. The two waves arriving at a point then differ in phase. Let the amplitudes of the two waves be A_1 and A_2 and the two waves differ in phase by an angle ϕ. Their equations may be written as

$$y_1 = A_1 \sin (kx - \omega t)$$

$$y_2 = A_2 \sin (kx - \omega t + \phi).$$

According to the principle of superposition, the resultant wave is represented by

$$y = y_1 + y_2 = A_1 \sin (kx - \omega t) + A_2 \sin (kx - \omega t + \phi).$$

we get, $\quad y = A \sin (kx - \omega t + \alpha)$

where $\quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

(A is amplitude of the resultant wave)

Also, $\quad \tan\alpha = \dfrac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$

(α is phase difference of the resultant wave with the first wave)

Constructive and Destructive Interferences

Constructive Interference

When resultant amplitude A is maximum

$$A = A_1 + A_2$$

when $\quad \cos\phi = +1 \quad$ or $\quad \phi = 2n\pi$

where n is an integer.

Destructive Interference

When resultant amplitude A is minimum

or $\quad A = |A_1 - A_2|$

When $\quad \cos\phi = -1 \quad$ or $\quad \phi = (2n + 1)\pi$

where n is an integer.

SOLVED EXAMPLES

49. Two sinusoidal waves of the same frequency travel in the same direction along a string. If $A_1 = 3.0$ cm, $A_2 = 4.0$ cm, $\phi_1 = 0$, and $\phi_2 = \pi/2$ rad, what is the amplitude of the resultant wave?

Solution:

$$\text{Resultant amplitude} = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos 90^\circ}$$

$$= 5 \text{ cm.}$$

50. Two sinusoidal waves of the same frequency are to be sent in the same direction along a taut string. One wave has an amplitude of 5.0 mm, the other 8.0 mm.

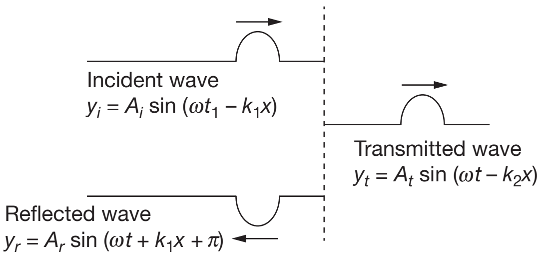
(A) What phase difference ϕ_1 between the two waves results in the smallest amplitude of the resultant wave?
(B) What is that smallest amplitude?
(C) What phase difference ϕ_2 results in the largest amplitude of the resultant wave?
(D) What is that largest amplitude?
(E) What is the resultant amplitude if the phase angle is $(\phi_1 - \phi_2)/2$?

Solution:

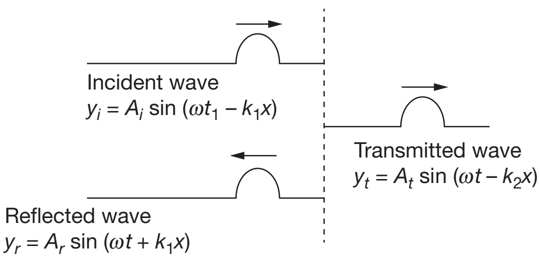
(A) π rad (B) 3.0 mm (C) 0 rad
(D) 13 mm (E) 9.4 mm.

Reflection and Transmission of Waves

A travelling wave, at a rigid or denser boundary, is reflected with a phase reversal but the reflection at an open boundary (rarer medium) takes place without any phase change. The transmitted wave is never inverted, but propagation constant k is changed.



Reflection at denser boundary



Reflection at rarer boundary

Amplitude of Reflected and Transmitted Waves

v_1 and v_2 are speeds of the wave in incidenting and reflecting mediums, respectively, then

$$A_r = \frac{v_2 - v_1}{v_1 + v_2} A_i$$

$$A_t = \frac{2v_2}{v_1 + v_2} . A_i$$

A_r is positive if $v_2 > v_1$, i.e., wave is reflected from a rarer medium.

SOLVED EXAMPLE

51. A harmonic wave is travelling on string 1. At a junction with string 2, it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string, and that the boundary between the two strings is at $x = 0$. If the expression for the incident wave is

$$y_i = A_i \cos(k_1 x - \omega_1 t)$$

What are the expressions for the transmitted and the reflected waves in terms of A_i, k_1, and ω_1?

Solution:

Since $v = \sqrt{T/\mu}$, $T_2 = T_1$, and $\mu_2 = 4\mu_1$

we have, $$v_2 = \frac{v_1}{2} \quad (1)$$

The frequency does not change, that is,

$$\omega_1 = \omega_2 \quad (2)$$

Also, because $k = \omega/v$, the wave numbers of the harmonic waves in the two strings are related by

$$k_2 = \frac{\omega_2}{v_2} = \frac{\omega_1}{v_1/2} = 2\frac{\omega_1}{v_1} = 2k_1 \quad (3)$$

The amplitudes are

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_u = \left[\frac{2(v_1/2)}{v_1 + (v_1/2)}\right] A_i = \frac{2}{3} A_i \quad (4)$$

and $$A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_u = \left[\frac{(v_1/2) - v_1}{v_1 + (v_1/2)}\right] A_i = \frac{A_i}{3} \quad (5)$$

Now with Equation (2), (3), and (4), the transmitted wave can be written as,

$$y_t = \frac{2}{3} A_i \cos(2k_1 x - \omega_1 t)$$

Similarly the reflected wave can be expressed as,

$$= \frac{A_i}{3} \cos(k_1 x + \omega_1 t + \pi).$$

Standing Waves

In case two sine waves of equal amplitude and frequency propagate on a long string in opposite directions. The equations of the two waves are given by

$$y_1 = A \sin (\omega t - kx)$$

and $$y_2 = A \sin (\omega t + kx + \phi).$$

These waves interfere to produce what we call standing waves. To understand these waves, let us discuss the special case when $\phi = 0$.

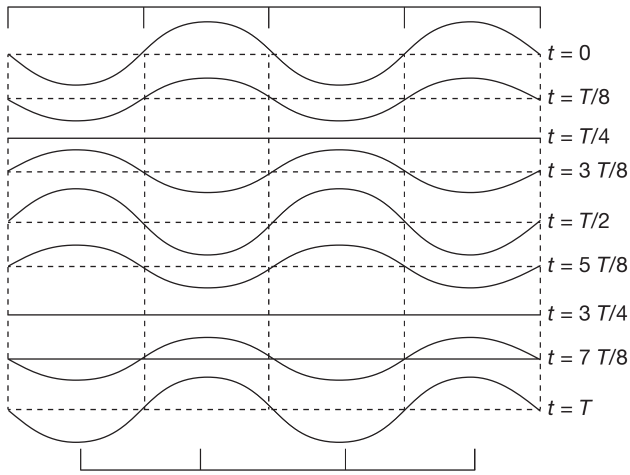
The resultant displacements of the particles of the string are given by the principle of superposition as

$$y = y_1 + y_2$$
$$= A [\sin (\omega t - kx) + \sin(\omega t + kx)]$$
$$= 2A \sin \omega t \cos kx$$

or, $$y = (2A \cos kx) \sin \omega t.$$

This is the required result and from this it is clear that:

1. As this equation satisfies the wave equation,
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$
it represents a wave. However, as it is not of the form $f(ax \pm bt)$, the wave is not travelling and so is called standing or stationary wave.
2. The amplitude of the wave
$$A_s = 2A \cos kx$$
is not constant but varies periodically with position (and not with time as in beats).
3. The points for which amplitude is minimum are called nodes and for these
$$\cos kx = 0, \quad \text{i.e.,} \quad kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$
i.e., $$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \qquad \left[\text{as} \quad k = \frac{2\pi}{\lambda}\right]$$
in a stationary wave, nodes are equally spaced.
4. The points for which amplitude is maximum are called antinodes and for these,
$$\cos kx = \pm 1, \quad \text{i.e.,} \quad kx = 0, \pi, 2\pi, 3\pi, \ldots$$
i.e., $$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots \qquad \left[\text{as} \quad k = \frac{2\pi}{\lambda}\right]$$
like nodes, antinodes are also equally spaced with spacing ($\lambda/2$) and $A_{max} = \pm 2A$. Furthermore, nodes and antinodes are alternate with spacing ($\lambda/4$).
5. The nodes divide the medium into segments (or loops). All the particles in a segment vibrate in same phase, but in opposite phase with the particles in the adjacent segment. Twice in one period all the particles pass through their mean position simultaneously with maximum velocity ($A_s\omega$), the direction of motion being reversed after each half cycle.



(a)

6. Standing waves can be transverse or longitudinal, e.g., in strings (under tension) if reflected wave exists, the waves are transverse-stationary, while in organ pipes waves are longitudinal-stationary.
7. As in stationary waves, nodes are permanently at rest, so no energy can be transmitted across them, i.e., energy of one region (segment) is confined in that region. However, this energy oscillates between elastic potential energy and kinetic energy of the particles of the medium. When all the particles are at their extreme positions, KE is minimum while elastic PE is maximum (as shown in Fig. 9.13 (a)), and when all the particles (simultaneously) pass through their mean position, KE will be maximum while elastic PE minimum (Fig. 9.13 (b)). The total energy, confined in a segment (elastic PE + KE), always remains the same.

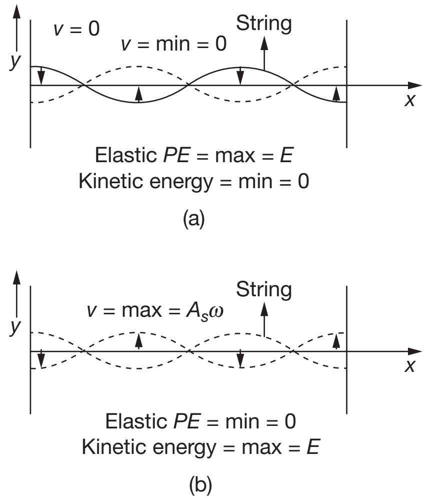


Fig. 9.13

SOLVED EXAMPLES

52. Two waves travelling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = (4.0 \text{ cm}) \sin (3.0x - 2.0t)$$
$$y_2 = (4.0 \text{ cm}) \sin (3.0x + 2.0t)$$

where x and y are in centimeter.

(A) Find the maximum displacement of a particle of the medium at $x = 2.3$ cm.

(B) Find the position of the nodes and antinodes.

Solution:

(A) When the two waves are summed, the result is a standing wave whose mathematical representation is given by equation, with $A = 4.0$ cm and $k = 3.0$ rad/cm;

$$y = (2A \sin kx) \cos \omega t = [(8.0 \text{ cm}) \sin 3.0\,x] \cos 2.0t$$

Thus, the maximum displacement of a particle at the position $x = 2.3$ cm is

$$y_{max} = [(8.0 \text{ cm}) \sin 3.0x]_{x = 2.3 \text{ cm}}$$
$$= (8.0 \text{ m}) \sin (6.9 \text{ rad}) = 4.6 \text{ cm}$$

(B) Because $k = 2\pi/\lambda = 3.0$ rad/cm, we see that $\lambda = 2\pi/3$cm. Therefore, the antinodes are located at

$$x = n\left(\frac{\pi}{6.0}\right) \text{ cm} \qquad (n = 1, 3, 5, \ldots)$$

and the nodes are located at

$$x = n\,\frac{\lambda}{2}\left(\frac{\pi}{3.0}\right) \text{ cm} \qquad (n = 1, 2, 3, \ldots).$$

53. Two travelling waves of equal amplitudes and equal frequencies move in opposite direction along a string. They interfere to produce a standing wave having the equation:

$$y = A \cos kx \sin \omega t$$

in which $A = 1.0$ mm, $k = 1.57$ cm^{-1}, and $\omega = 78.5$ s^{-1}.

(A) Find the velocity and amplitude of the component travelling waves.

(B) Find the node closest to the origin in the region $x > 0$.

(C) Find the antinode closest to the origin in the region $x > 0$.

(D) Find the amplitude of the particle at $x = 2.33$ cm.

Solution:

(A) The standing wave is formed by the superposition of the waves

$$y_1 = \frac{A}{2} \sin (\omega t - kx)$$

and $$y_2 = \frac{A}{2} \sin (\omega t + kx).$$

The wave velocity (magnitude) of either of the waves is

$$v = \frac{\omega}{k} = \frac{78.5 \text{ s}^{-1}}{1.57 \text{ cm}^{-1}} = 50 \text{ cm/s};$$

Amplitude = 0.5 mm.

(B) For a node, $\cos kx = 0$.

The smallest positive x satisfying this relation is given by

$$kx = \pi/2$$

or $$x = \frac{\pi}{2k} = \frac{3.14}{2 \times 1.57 \text{ cm}^{-1}} = 1 \text{ cm}$$

(C) For an antinode, $|\cos kx| = 1$.

The smallest positive x satisfying this relation is given by

$$kx = \pi$$

or, $$x = \frac{\pi}{k} = 2 \text{ cm}$$

(D) The amplitude of vibration of the particle at x is given by $|A \cos kx|$. For the given point,

$$kx = (1.57 \text{ cm}^{-1})(2.33 \text{ cm}) = \frac{7}{6}\pi = \pi + \frac{\pi}{6}$$

Thus, the amplitude will be

$$|(1.0 \text{ mm})| \cos (\pi + \pi/6)| = \frac{\sqrt{3}}{3} \text{ mm} = 0.86 \text{ mm}.$$

54. A string fixed at both ends is 8.40 m long and has a mass of 0.120 kg. It is subjected to a tension of 96.0 N and set oscillating.

(A) What is the speed of the waves on the string?

(B) What is the longest possible wavelength for a standing wave?

(C) Give the frequency of the wave.

Solution:

(A) 82.0 m/s (B) 16.8 m (C) 4.88 Hz

Vibration of String

Fixed at both Ends

Suppose a string of length L is kept fixed at the ends $x = 0$ and $x = L$. In such a system, suppose we send a continuous sinusoidal wave of a certain frequency, say, towards the right. When the wave reaches the right end, it gets reflected and begins to travel back. The left-going wave then overlaps the wave, which is still travelling to the right. When the left-going wave reaches the left end, it gets reflected again, and the newly reflected wave begins to travel to the right, overlapping the left-going wave. This process will continue and, therefore, very soon we have many overlapping waves, which interfere with one another. In such a system, at any point x and at any time t, there are always two waves, one moving to the left and another to the right. We, therefore, have

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

(wave travelling in the positive direction of x-axis)

and $$y_2(x, t) = y_m \sin(kx + \omega t)$$

(wave travelling in the negative direction of x-axis).

The principle of superposition given for the combined wave

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$
$$= y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$
$$= (2y_m \sin kx) \cos \omega t$$

It is seen that the points of maximum or minimum amplitude stay at one position.

Nodes

The amplitude is zero for values of kx that give $\sin kx = 0$, i.e., for,

$$kx = n\pi, \quad \text{for} \quad n = 0, 1, 2, 3, \ldots$$

Substituting $k = 2\pi/\lambda$ in this equation, we get

$$x = n\frac{\lambda}{2}, \quad \text{for} \quad n = 0, 1, 2, 3, \ldots$$

The positions of zero amplitude are called the nodes. Note that a distance of $\frac{\lambda}{2}$ or half a wavelength separates two consecutive nodes.

Antinodes

The amplitude has a maximum value of $2y_m$, which occurs for the values of kx that give $|\sin kx| = 1$. Those values are

$$kx = (n + 1/2)\pi \quad \text{for} \quad n = 0, 1, 2, 3, \ldots$$

Substituting $k = 2\pi/\lambda$ in this equation, we get.

$$x = (n + 1/2)\frac{\lambda}{2} \quad \text{for} \quad n = 0, 1, 2, 3, \ldots$$

as the positions of maximum amplitude. These are called antinodes. The antinodes are separated by $\lambda/2$ and are located half way between pairs of nodes.

For a stretched string of length L, fixed at both ends, the two ends of the ends are chosen as position $x = 0$, then the other end is $x = L$. In order that this end is a node, the length L must satisfy the condition

$$L = n\frac{\lambda}{2}, \quad \text{for} \quad n = 1, 2, 3, \ldots$$

This condition shows that standing waves on a string of length L have restricted wavelength given by

$$\lambda = \frac{2L}{n}, \quad \text{for} \quad n = 1, 2, 3, \ldots$$

The frequencies corresponding to these wavelengths follow from equation as

$$f = n\frac{v}{2L}, \quad \text{for} \quad n = 1, 2, 3, \ldots$$

where v is the speed of travelling waves on the string. The set of frequencies given by equation are called the natural frequencies or **modes** of oscillation of the system. This equation tells us that the natural frequencies of a string are integral multiples of the lowest frequency $f = \frac{v}{2L}$, which corresponds to $n = 1$. The oscillation mode with that lowest frequency is called the fundamental mode or the first harmonic. The second harmonic or first overtone is the oscillation mode with $n = 2$. The third harmonic and second overtone corresponds to $n = 3$, and so on. The frequencies associated with these modes are often labeled as v_1, v_2, v_3, and so on. The collection of all possible modes is called the harmonic series, and n is called the harmonic number.

Some of the harmonic of a stretched string fixed at both the ends are shown in Fig. 9.14.

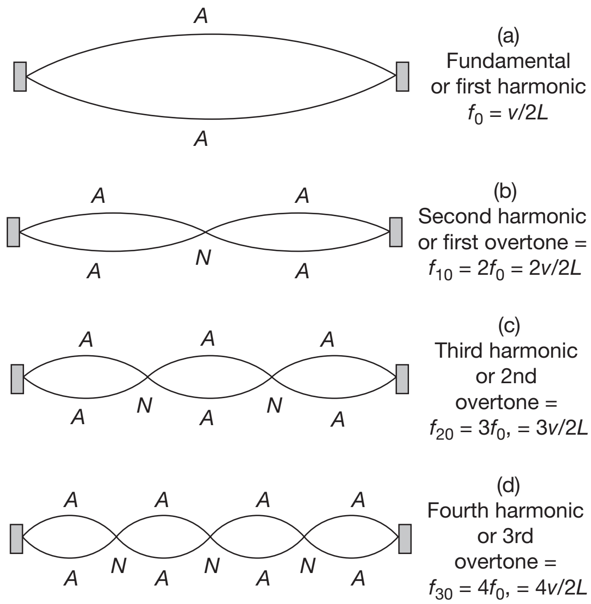


Fig. 9.14

SOLVED EXAMPLES

55. A middle C string on a piano has a fundamental frequency of 262 Hz, and the A note has fundamental frequency of 440 Hz.

(A) Calculate the frequencies of the next two harmonics of the C string.

(B) If the strings for the A and C notes are assumed to have the same mass per unit length and the same length, determine the ratio of tensions in the two strings.

Solution:

(A) Because $f_1 = 262$ Hz for the C string, we can use Equation to find the frequencies f_2 and f_3;

$$f_2 = 2f_1 = 524 \text{ Hz}$$

$$f_3 = 3f_1 = 786 \text{ Hz}$$

Using Equation for the two strings vibrating at their fundamental frequencies gives

$$f_{1A} = \frac{1}{2L}\sqrt{\frac{T_A}{\mu}}$$

$$f_{1C} = \frac{1}{2L}\sqrt{\frac{T_C}{\mu}}$$

$$\therefore \qquad \frac{f_{1A}}{f_{1C}} = \sqrt{\frac{T_A}{T_C}}$$

$$\frac{T_A}{T_C} = \left(\frac{f_{1A}}{f_{1C}}\right)^2 = \left(\frac{440\text{ Hz}}{262\text{ Hz}}\right)^2$$

$$= 2.82.$$

56 A wire having a linear mass density 5.0×10^{-3} kg/m is stretched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.

Solution:

Suppose the wire vibrates at 420 Hz in its nth harmonic and at 490 Hz in its $(n + 1)$th harmonic.

$$420 \text{ s}^{-1} = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \qquad (1)$$

and

$$490 \text{ s}^{-1} = \frac{(n+1)}{2L}\sqrt{\frac{F}{\mu}} \qquad (2)$$

This gives $$\frac{490}{420} = \frac{n+1}{n}$$

or $$n = 6.$$

Putting the value in (1),

$$420 \text{ s}^{-1} = \frac{6}{2L}\sqrt{\frac{450\text{ N}}{5.0\times10^{-3}\text{ kg/m}}} = \frac{900}{L} \text{ m/s}$$

or, $$L = \frac{900}{420}$$

$$m = 2.1 \text{ m.}$$

Fixed at One End

Standing waves can be produced on a string which is fixed at one end and whose other end is free to move in a transverse direction. Such a free end can be nearly achieved by connecting the string to a very light thread.

If the vibrations are produced by a source of 'correct' frequency, standing waves are produced. If the end $x = 0$ is fixed and $x = L$ is free, the equation is again given by

$$y = 2A \sin kx \cos \omega t$$

with the boundary condition that $x = L$ is an antinode. The boundary condition that $x = 0$ is a node is automatically satisfied by the above equation. For $x = L$ to be an antinode,

$$\sin kL = \pm 1$$

or, $$kL = \left(n + \frac{1}{2}\right)\pi$$

or $$\frac{2\pi L}{\lambda} = \left(n + \frac{1}{2}\right)\pi$$

or, $$\frac{2Lf}{v} = n + \frac{1}{2}$$

or $$f = \left(n + \frac{1}{2}\right)\frac{v}{2L} = \frac{n + \frac{1}{2}}{2L}\sqrt{T/\mu} \quad \ldots$$

These are the normal frequencies of vibration. The fundamental frequency is obtained when $n = 0$, i.e.,

$$f_0 = v/4L$$

The overtone frequencies are

$$f_1 = \frac{3v}{4L} = 3f_0$$

$$f_2 = \frac{5v}{4L} = 5f_0$$

We see that all the harmonic of the fundamental are not the allowed frequencies for the standing waves. Only the odd harmonics are the overtones. Fig. 9.15 shows shapes of the string for some of the normal modes.

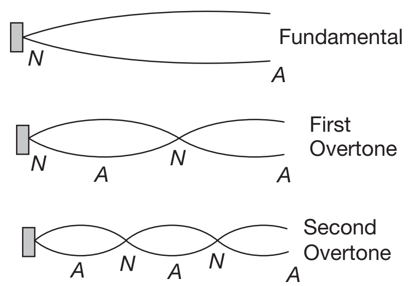


Fig. 9.15

Laws of Transverse Vibrations of a String Sonometer Wire

1. Law of length

$$f \propto \frac{1}{L} \qquad \text{so } \frac{f_1}{f_2} = \frac{L_2}{L_1}; \quad \text{if } T \text{ and } \mu \text{ are constant}$$

2. Law of tension

$$f \propto \sqrt{T} \qquad \text{so } \frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}; \quad \text{if } L \text{ and } \mu \text{ are constant}$$

3. Law of mass

$$f \propto \frac{1}{\sqrt{\mu}} \qquad \text{so } \frac{f_1}{f_2} = \sqrt{\frac{\mu_2}{\mu_1}}; \quad \text{if } T \text{ and } L \text{ are constant}$$

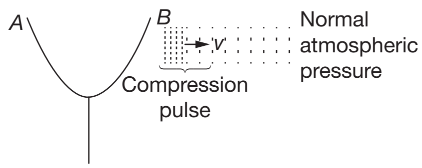
SOUND WAVES

Propagation of Sound Waves

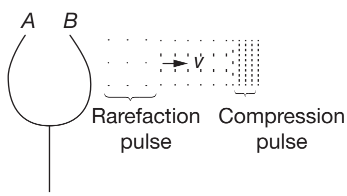
Sound is a mechanical three-dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork, or the diaphragm of a loudspeaker. Being a mechanical wave, sound needs a medium having properties of inertia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

Consider a tuning fork producing sound waves.

When prong *B* moves outward towards right, it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a compression pulse, and it travels away from the prong with the speed of sound.



After producing the compression pulse, the prong *B* reverses its motion and moves inward. This drags away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called a rarefaction pulse. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.



If the prongs vibrate in SHM, the pressure variations in the layer close to the prong also varies simple harmonically, and hence the increase in pressure above normal value can be written as

$$\delta P = \delta P_0 \sin \omega t$$

where δP_0 is the maximum increase in pressure above normal value.

As this disturbance travel towards right with wave velocity v, the excess pressure at any position x at time t will be given by

$$\delta P = \delta P_0 \sin \omega\,(t - x/v)$$

Using $p = \delta P$, $p_0 = \delta P_0$, the above equation of sound wave can be written as

$$p = p_0 \sin \omega\,(t - x/v)$$

SOLVED EXAMPLE

57. The equation of a sound wave in air is given by $p = (0.02\ ) \sin [(3000\ )\, t - (9.0\ )\, x]$, where all variables are in SI units.

(A) Find the frequency, wavelength, and the speed of sound wave in air.

(B) If the equilibrium pressure of air is 1.0×10^5 N/m^2, what are the maximum and minimum pressures at a point as the wave passes through that point?

Solution:

(A) Comparing with the standard form of a travelling wave

$$p = p_0 \sin [\omega\,(t - x/v)]$$

we see that $\omega = 3000$ s^{-1}. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{3000}{2\pi} \text{ Hz}$$

Also from the same comparison, $\omega/\text{v} = 9.0$ m^{-1}.

or $$v = \frac{\omega}{9.0 \text{ m}^{-1}} = \frac{3000 \text{ s}^{-1}}{9.0 \text{ m}^{-1}} = \frac{1000}{3} \text{ m/s}^{-1}$$

The wavelength is $$\lambda = \frac{v}{f} = \frac{1000/3 \text{ m/s}}{3000/2\pi \text{ Hz}} = \frac{2\pi}{9} \text{ m.}$$

(B) The pressure amplitude is $p_0 = 0.02$ N/m^2. Hence, the maximum and minimum pressures at a point in the wave motion will be $(1.01 \times 10^5 \pm 0.02)$ N/m^2.

Frequency and Pitch of Sound Waves

Frequency

Each cycle of a sound wave includes one compression and one rarefaction, and frequency is the number of cycles per second that passes by a given location. This is normally equal to the frequency of vibration of the (tuning fork) source producing sound. If the source vibrates in SHM of a single frequency, sound produced has a single frequency and it is called a pure tone.

However, a sound source may not always vibrate in SHM (this is the case with most of the common sound sources, e.g., guitar string, human vocal card, surface of drum, etc.) and hence the pulse generated by it may not have the shape of a sine wave. But even such a pulse may be considered to be obtained by superposition of a large number of sine waves of different frequency and amplitudes. We say that the pulse contains all these frequencies.

Audible Frequency Range for Human

A normal person hears all frequencies between 20 and 20 KHz. This is a subjective range (obtained experimentally) which may vary slightly from person to person. The ability to hear the high frequencies decreases with age and a middle-age person can hear only up to 12–14 KHz.

Infrasonic Sound

Sound can be generated with frequency below 20 Hz called **infrasonic sound.**

Ultrasonic Sound

Sound can be generated with frequency above 20 Hz called **infrasonic sound.**

Even through humans cannot hear these frequencies, other animals may. For instance, Rhinos communicate through infrasonic frequencies as low as 5Hz, and bats use ultrasonic frequencies as high as 100 KHz for navigating.

Pitch

Frequency as we have discussed till now is an objective property measured in units of Hz and which can be assigned a unique value. However, a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called p**itch.** A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound.

SOLVED EXAMPLE

58. A wave of wavelength 4 mm is produced in air and it travels at a speed of 300 m/s. Will it be audible?

Solution:

From the relation $v = \nu\lambda$, the frequency of the wave is

$$\nu = \frac{v}{\lambda} = \frac{300 \text{ m/s}}{4 \times 10^{-3} \text{ m}} = 75000 \text{ Hz.}$$

This is much above the audible range. It is an ultrasonic wave and will not be audible to humans, but it will be audible to bats.

Pressure Wave and Displacement Wave

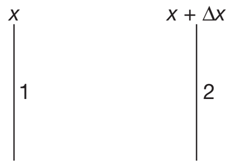
We can describe sound waves either in terms of excess pressure (Equation (9.5)) or in terms of the longitudinal displacement suffered by the particles of the medium with respect to mean position.

If $s = s_0 \sin \omega(t - x/v)$ represents a sound wave,

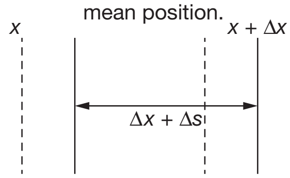
where s = displacement of medium particle from its mean position at x,

$$s = s_0 \sin(\omega t - kx) \qquad (9.5)$$

When sound is not propagating particle are at mean position 1 and 2



When particles are displaced from mean position.



Change in volume = $\Delta V = (\Delta x + \Delta s)A - \Delta xA = \Delta sA$

$$\frac{\Delta V}{V} = \frac{\Delta sA}{\Delta xA} = \frac{\Delta s}{\Delta x}$$

$$\Delta P = -\frac{B\Delta V}{V}$$

$$\Delta P = -\frac{B\Delta s}{\Delta x}$$

$$dp = -\frac{Bds}{dx}$$

$$dp = -B(-k s_0) \cos(\omega t - kx)$$

$$dp = Bks_0 \cos(\omega t - kx)$$

$$dp = (dp)_{max} \cos(\omega t - kx)$$

$$p = p_0 \sin(\omega t - kx + \pi/2) \qquad (9.6)$$

where $p = dp$ = variation in pressure at position x

and $p_0 = Bks_0$ = maximum pressure variation

Equation (9.6) represents that same sound wave, where P is excess pressure at position x, over and above the average atmospheric pressure and pressure amplitude p_0 is given by

$$P_0 = BKs_0 \quad (9.7)$$

(B = Bulk modulus of the medium, K = angular wave number)

Note from Equation (9.5) and (9.6) that the displacement of a medium particle and excess pressure at any position are out of phase by $\frac{\pi}{2}$ Hence a displacement maxima corresponds to a pressure minima and vice-versa.

SOLVED EXAMPLES

59. A sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is 2.0×10^{-3} N/m^2, find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m^2.

Solution:

The pressure amplitude is

$$p_0 = \frac{2.0\times10^{-3}\ \text{N/m}^2}{2} = 10^{-3}\ \text{N/m}^2.$$

The displacement amplitude s_0 is given by

$$p_0 = B\,k\,s_0$$

or, $$s_0 = \frac{p_0}{Bk} = \frac{p_0\lambda}{2\,\pi\,B}$$

$$= \frac{10^{-3}\ \text{N/m}^2\times(40\times10^{-2}\ \text{m})}{2\times\pi\times14\times10^4\ \text{N/m}^2} = \frac{100}{7\pi}\ \text{Å}$$

$$= 6.6\ \text{Å}.$$

60. What will be phase difference between general equation of pressure wave and displacement wave.

Solution:

If $\quad s = s_0 \sin(\omega t - kx)$

then $\quad p = p_0 \sin\left(\omega t - kx + \frac{\pi}{2}\right)$

So phase difference is $\frac{\pi}{2}$.

Speed of Sound Waves

1. Velocity of sound waves in a linear solid medium is given by

$$v = \sqrt{\frac{Y}{\rho}} \quad (9.8)$$

where Y = young's modulus of elasticity and ρ = density.

2. Velocity of sound waves in a fluid medium (liquid or gas) is given by

$$v = \sqrt{\frac{B}{\rho}} \quad (9.9)$$

where ρ = density of the medium and B = Bulk modulus of the medium given by,

$$B = -V\frac{dP}{dV} \quad (9.10)$$

Newton's Formula

Newton assumed propagation of sound through a gaseous medium to be an isothermal process.

$$PV = \text{constant}$$

$\Rightarrow$ $$\frac{dP}{dV} = \frac{-P}{V}$$

and

Hence $B = P$ using Equation (9.10)

and thus he obtained for velocity of sound in a gas,

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}},$$

where M = molar mass

The density of air at 0º and pressure 76 cm of Hg column is $\rho = 1.293$ kg/m^3. This temperature and pressure is called standard temperature and pressure at STP. Speed of sound in air is 280 m/s. This value is somewhat less than measured speed of sound in air 332 m/s then Laplace suggested a correction.

Laplace's Correction

Later Laplace established that propagation of sound in a gas is not an isothermal but is an adiabatic process and hence PV^{γ} = constant

$\Rightarrow$ $$\frac{dP}{dV} = -\gamma\frac{P}{V}$$

where $$B = -V\frac{dP}{dV} = gP$$

and hence speed of sound in a gas,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \quad (9.11)$$

Factors Affecting Speed of Sound in Atmosphere

1. **Effect of Temperature:** as temperature (T) increases, velocity (v) increases.

$$v \propto \sqrt{T}$$

For small change in temperature above room temperature v increases linearly by 0.6 m/s for every 1°C rise in temp.

$$v = \sqrt{\frac{\gamma R}{M}} \times T^{1/2}$$

$$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\Delta v = \left(\frac{1}{2}\frac{v}{T}\right)\Delta T$$

$$\Delta v = (0.6)\,\Delta T$$

2. **Effect of Pressure:** The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

3. **Effect of Humidity:** With increase in humidity, density decreases. This is because the molar mass of water vapour is less than the molar mass of air.

So at constant temperature, if P changes, then ρ also changes in such a way that P/ρ remains constant. Hence, pressure does not have any effect on velocity of sound as long as temperature is constant.

SOLVED EXAMPLE

61. The constant γ for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 450 m/s, what will be the speed of hydrogen at the same temperature and pressure?

Solution:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Since temperature T is constant,

$$\frac{v_{(H_2)}}{v_{(O_2)}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$$

$\Rightarrow$ $v\,(H_2) = 4 \times 450 = 1800$ m/s

Aliter: The speed of sound in a gas is given by $u = \sqrt{\frac{\gamma P}{\rho}}$. At STP, 22.4 litres of oxygen has a mass of 32 g, whereas the same volume of hydrogen has a mass of 2 g. Thus, the density of oxygen is 16 times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases,

$$\frac{f_{(\text{hydrogen})}}{f_{(\text{oxygen})}} = \sqrt{\frac{\rho_{(\text{oxygen})}}{\rho_{(\text{hydrogen})}}}$$

or $f_{(\text{hydrogen})} = 4f_{(\text{oxygen})}$

$= 4 \times 450$ m/s $= 1800$ m/s.

Intensity of Sound Waves

Like any other progressive wave, sound waves also carry energy from one point of space to the other. This energy can be used to do work, for example, forcing the eardrums to vibrate or in the extreme case of a sonic boom, created by a supersonic jet, can even cause glass panes of windows to crack.

The amount of energy carried per unit time by a wave is called its power, and power per unit area held perpendicular to the direction of energy flow is called intensity.

For a sound wave travelling along positive x-axis described by the equation,

$$s = s_0 \sin(\omega t - kx + \phi)$$

$$P = p_0 \cos(\omega t - kx + \phi)$$

$$\frac{\delta s}{\delta t} = \omega s_0 \cos(\omega t - kx + \phi)$$

Instantaneous power

$$P = F \cdot v = pA\,\frac{\delta s}{\delta t}$$

$$P = p_0 \cos(\omega t - kx + \phi)\, A\omega s_0 \cos(\omega t - kx + \phi)$$

$$P_{\text{average}} = \langle P\rangle$$

$$= p_0 A\omega s_0 \langle\cos^2(\omega t - kx + \phi)\rangle$$

$$= \frac{p_0 \omega s_0 A}{2}$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$B = \rho v^2$$

$$p_0 = Bks_0 == \rho v^2 ks_0$$

$$P_{\text{average}} = \frac{1}{2}\omega p_0 A\left(\frac{p_0}{\rho v^2 k}\right)$$

$$= \frac{p_0^2 A}{2\rho V} = \frac{pAv\omega^2 s_0^2}{2}$$

Maximum power $= P_{\max} = \dfrac{p_0^2 A}{\rho V} = (pA)\, v\, v_{p,\max}^2 = pAv\omega^2 s_0{}^2$

Total energy transfer $= P_{av} \times t = \dfrac{\rho A v\omega^2 s_0^2}{2} \times t$

Average intensity = average power/area

The average intensity at position x is given by

$$\langle I\rangle = \frac{1}{2}\frac{\omega^2 s_0^2 B}{v} = \frac{P_0^2 v}{2B}$$

Substituting $B = \rho v^2$, intensity can also be expressed as

$$I = \frac{P_0^2}{2\rho v}$$

NOTE

- If the source is a point source, then $I \propto \frac{1}{r^2}$ and $s_0 \propto \frac{1}{r}$ and $s = \frac{a}{r}\sin(\omega t - kr + \theta)$
- If a sound source is a line source, then $I \propto \frac{1}{r}$ and $s_0 \propto \frac{1}{\sqrt{r}}$ and
$$s = \frac{a}{\sqrt{r}}\sin(\omega t - kr + \theta)$$

SOLVED EXAMPLES

62. The pressure amplitude in a sound wave from a radio receiver is 2.0×10^{-3} N/m^2 and the intensity at a point is 10^{-6} W/m^2. If by turning the volume knob, the pressure amplitude is increased to 3×10^{-3} N/m^2, evaluate the intensity.

Solution:

The intensity is proportional to the square of the pressure amplitude.

Thus, $$\frac{I'}{I} = \left(\frac{p_0'}{p_0}\right)^2$$

or $$I' = \left(\frac{p_0'}{p_0}\right)^2 I = \left(\frac{3}{2.0}\right)^2 \times 10^{-6}\ \text{W/m}^2$$

$$= 2.25 \times 10^{-16}\ \text{W/m}^2.$$

63. A microphone of cross-sectional area 0.40 cm^2 is placed in front of a small speaker emitting p W of sound output. If the distance between the microphone and the speaker is 2.0 m, how much energy falls on the microphone in 5.0 s?

Solution:

The energy emitted by the speaker in 1 second is p J. Let us consider a sphere of radius 2.0 m centered at the speaker. The energy p J falls normally on the total surface of this sphere in 1 second. The energy falling on the area 0.4 cm^2 of the microphone in 1 second

$$= \frac{0.4\ \text{cm}^2}{4\pi(2.0\ \text{m})^2} \times p\ \text{J} = 25 \times 10^{-6}\ \text{J}.$$

The energy falling on the microphone in 5.0 is

$$25 \times 10^{-6}\ \text{J} \times 5 = 125\ \mu\text{J}.$$

64. Find the amplitude of vibration of the particles of air through which a sound wave of intensity 8.0×10^{-6} W/m^2 and frequency 5.0 kHz is passing. Density of air = 1.2 kg/m^3 and speed of sound in air = 330 m/s.

Solution:

The relation between the intensity of sound and the displacement amplitude is

$$I = \frac{\omega^2 s_0^2 B}{2v},$$

where

$$B = \rho v^2$$

and $$\omega = 2\pi v$$

$$\Rightarrow \quad I = 2\pi^2 s_0{}^2 n^2 \rho_0 v$$

or $$s_0{}^2 = \frac{I}{2\pi^2 v^2 \rho_0 v}$$

$$= \frac{8.0 \times 10^{-6}\ \text{W/m}^2}{2\pi^2 \times (25.0 \times 10^6\ \text{s}^{-2}) \times (1.2\ \text{kg/m}^3) \times (330\ \text{m/s})}$$

or $$s_0 = 6.4\ \text{nm}.$$

Loudness

Audible Intensity Range for Humans

The ability of human to perceive intensity at difference frequency is different. The perception of intensity is maximum at 1000 Hz and perception of intensity decreases as the frequency decreases or increases from 1000 Hz.

For a 1000 Hz tone, the smallest sound intensity that a human ear can detect is 10^{-12} W/m^2. On the other hand, continuous exposure to intensities above 1 W/m^2 can result in permanent hearing loss.

The overall perception of intensity of sound to human ear is called loudness.

Human ear do not perceive loudness on a linear intensity scale rather it perceives loudness on logarithmic intensity scale.

For example, if intensity is increased ten times, human ear does not perceive ten times increase in loudness. It roughly perceived that loudness is doubled where intensity is increased by ten times. Hence, it is prudent to define a logarithmic scale for intensity.

Decibel Scale

The logarithmic scale which is used for comparing to sound intensity is called **a decible scale**.

The intensity level β described in terms of decibels is defined as

$$\beta = 10 \log \left(\frac{I}{I_0}\right) \text{ (dB)}$$

Here I_0 is the threshold intensity of hearing for human ear

i.e., $I = 10^{-12}$ W/m^2.

In terms of decibel, threshold of human hearing is 1 dB

Note that intensity level β is a dimensionless quantity and is not same as intensity expressed in W/m^2.

SOLVED EXAMPLES

65. If the intensity is increased by a factor of 20, by how many decibels is the intensity level increased.

Solution:

Let the initial intensity be I and the intensity level be β_1 and when the intensity is increased by 20 times, the intensity level increases to β_2.

Then $\beta_1 = 10 \log (I/I_0)$

and $\beta_2 = 10 \log (20I/I_0)$

Thus, $\beta_2 - \beta_1 = 10 \log (20I/I) = 10 \log 20 = 13$ dB.

66. A bird is singing on a tree. A person approaches the tree and perceives that the intensity has increased by 10 dB. Find the ratio of initial and final separation between the man and the bird.

Solution:

$$b_1 = 10 \log \frac{I_1}{I_0}$$

$$b_2 = 10 \log \frac{I_2}{I_0}$$

$$\Rightarrow \quad b_2 - b_1 = 10 \log \frac{I_2}{I_1}$$

or $$10 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$$

$$\Rightarrow \quad \frac{I_2}{I_1} = 10^1 = 10$$

For point source

$$I \propto \frac{1}{r^2}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10}.$$

67. The sound level at a point is increased by 40 dB. By what factor is the pressure amplitude increased?

Solution:

The sound level in dB is

$$\beta = 10 \log_1\left(\frac{I}{I_0}\right).$$

If β_1 and β_2 are the sound levels and I_1 and I_2 are the intensities in the two cases,

$$\beta_2 - \beta_1 = 10 \left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]$$

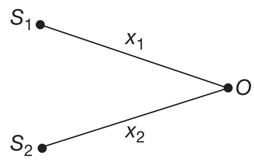
or, $$40 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$$

or, $$\frac{I_2}{I_1} = 10^4.$$

As the intensity is proportional to the square of the pressure amplitude,

we have $$\frac{p_{02}}{p_{01}} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10000} \approx 100.$$

Interference of Sound Waves



If $$p_1 = p_{m_1} \sin (\omega t - kx_1 + \theta_1)$$

and $$p_2 = p_{m_2} \sin (\omega t - kx_2 + \theta_2)$$

resultant excess pressure at point O is

$$p = p_1 + p_2$$

$$\Rightarrow \quad p = p_0 \sin (\omega t - kx + \theta)$$

where $$p_0 = \sqrt{p_{m_1}^2 + p_{m_2}^2 + 2p_{m_1}p_{m_2}\cos\phi},$$

$$\phi = |k(x_1 - x_2) + (\theta_1 - \theta_2)| \qquad (9.12)$$

1. For constructive interference,

$$\phi = 2n\pi \Rightarrow p_0 = p_{m_1} + p_{m_2}$$

2. For destructive interference,

$$\phi = (2n+1)\pi \Rightarrow p_0 = |p_{m_1} - p_{m_2}|$$

If ϕ is only due to path difference, then $\phi = \frac{2\pi}{\lambda}\Delta x$, and

Condition for constructive interference: $\Delta x = n\lambda$, $n = 0, \pm1, \pm2$

Condition for destructive interference: $\Delta x = (2n+1)\frac{\lambda}{2}$, $n = 0, \pm1, \pm2$

From Equation (9.12),

$$P_0^2 = P_{m_1}^2 + P_{m_2}^2 + 2P_{m_1}P_{m_2}\cos\phi$$

Since intensity, $I \propto$ (Pressure amplitude)^2,
we have, for resultant intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

If $$I_1 = I_2 = I_0$$

$$I = 2I_0(1 + \cos\phi)$$

$\Rightarrow$ $$I = 4I_0\cos^2\frac{\phi}{2}$$

Hence in this case,

For constructive interference: $\phi = 0, 2\pi, 4\pi\ldots$ and $I_{max} = 4I_0$

For destructive interference: $\phi = \pi, 3\pi\ldots$ and $I_{min} = 0$

Coherence

Two sources are said to be coherent if the phase difference between them does not change with time. In this case, their resultant intensity at any point in space remains constant with time. Two independent sources of sound are generally incoherent in nature, i.e. phase difference between them changes with time and hence the resultant intensity due to them at any point in space changes with time.

SOLVED EXAMPLES

68. Figure 9.16 shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 10.0 cm. The frequency of the sound source can be varied from 1 to 10 kHz. Find the frequencies at which the ear perceives maximum intensity. The speed of sound in air = 342 m/s.

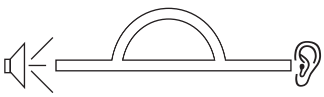


Fig. 9.16

Solution:

The sound wave bifurcates at the junction of the straight and the semicircular parts. The wave through the straight part travels a distance $p_1 = 2 \times 10$ cm and the wave through the curved part travels a distance $p_2 = \pi\ 10$ cm = 31.4 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore,

$$\Delta p = p_2 - p_1 = 31.4\text{ cm} - 20\text{ cm} = 11.4\text{ cm}$$

The wavelength of either wave is $\frac{v}{\upsilon} = \frac{330\text{ m/s}}{\upsilon}$. For constructive interference, $\Delta p = n\lambda$, where n is an integer.

or $$\Delta p = n\cdot\frac{v}{\nu}$$

$\Rightarrow$ $$n = \frac{n.v}{\Delta p}$$

$\Rightarrow$ $$\frac{n\cdot 342}{(0.114)} = 3000\text{ n}$$

Thus, the frequencies within the specified range which cause maximum of intensity are

3000×1, 3000×2, and 3000×3 Hz.

69. A source emitting sound of frequency 165 Hz is placed in front of a wall at a distance of 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Speed of sound in air = 330 m/s.

Solution:

The situation is shown in Fig. 9.17. Suppose the detector is placed at a distance of x meter from the source. The direct wave received from the source travels a distance of x meter. The wave reaching the detector after reflection from the wall has travelled a distance of $2[(2)^2 + x^2/4]^{1/2}$ metre. The path difference between the two waves is

$$\Delta = \left\{2\left[(2)^2 + \frac{x^4}{4}\right]^{1/2} - x\right\}\text{ metre.}$$

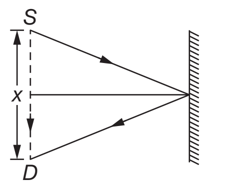


Fig. 9.17

Constructive interference will take place when $\Delta = \lambda, 2\lambda, \ldots$. The minimum distance x for a maximum corresponds to

$$\Delta = \lambda \qquad (1)$$

The wavelength is $\lambda = \frac{\upsilon}{v} = \frac{330\text{ m/s}}{165\text{ s}^{-1}} = 2$ m.

Thus, by Equation (1),

$$2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x = 2$$

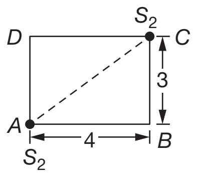
or $$\left[4 + \frac{x^2}{4}\right]^{1/2} = 1 + \frac{x}{2}$$

or $$4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$$

or $$3 = x.$$

Thus, the detector should be placed at a distance of 3 m from the source. Note that there is no abrupt phase change.

70. Two coherent sources are placed at the corners of a rectangular track of sides 3 m and 4 m. The source S_1 lags S_2 by phase angle π. A detector is moved along path ABC. Then find:



(A) Position of detector where the phase difference between the sound waves of sources S_1 and S_2 is zero.

(B) Find the ratio of total number of minima detected on line AB to the total number of minima on line BC.

[Velocity of sound = 330 m/s; Frequency of sources S_1; and S_2 = 165 Hz]

Solution:

(A) at D on ADC (B) ratio 2 : 3.

Reflection of Sound Waves

Reflection of sound waves from a rigid boundary (e.g. closed end of an organ pipe) is analogous to reflection of a string wave from rigid boundary; reflection accompanied by an inversion, i.e., an abrupt phase change of p. This is consistent with the requirement of displacement amplitude to remain zero at the rigid end, since a medium particle at the rigid end cannot vibrate. As the excess pressure and displacement corresponding to the same sound wave vary by $\pi/2$ in term of phase, displacement minima at the rigid end will be a point of pressure maxima. This implies that the reflected pressure wave from the rigid boundary will have same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse.

On the other hand, reflection of sound wave from a low pressure region (like open end of an organ pipe) is analogies to reflection of string wave from a free end. This point corresponds to displacement maxima, so that the incident and reflected displacement wave at this point must be in phase. This would imply that this point would be a minima for pressure wave (i.e. pressure at this point remains at its average value), and hence the reflected pressure wave would be out of phase by π with respect to the incident wave. That is, a compression pulse is reflected as a rarefaction pulse and vice-versa.

Longitudinal Standing Waves

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering waves are given by

$$p_1 = p_0 \sin(\omega t - kx)$$

and $$p_2 = p_0 \sin(\omega t + kx + \phi)$$

then the equation of the resultant standing wave would be given by

$$p = p_1 + p_2 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

$$\Rightarrow \quad p = p'_0 \sin\left(\omega t + \frac{\phi}{2}\right) \qquad (9.13)$$

This is equation of SHM* in which the amplitude p'_0 depends on position as

$$p'_0 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \qquad (9.14)$$

Points were pressure remains permanently at its average value; i.e., pressure amplitude is zero is called a pressure node, and the condition for a pressure node would be given by

$$p'_0 = 0$$

i.e., $$\cos\left(kx + \frac{\phi}{2}\right) = 0$$

i.e., $$kx + \frac{\phi}{2} = 2n\pi \pm \frac{\pi}{2}, \quad n = 0, 1, 2, \ldots \qquad (9.15)$$

Similarly, points where pressure amplitude is maximum is called a pressure antinode and condition for a pressure antinode would be given by

$$p'_0 = \pm 2p_0$$

i.e., $$\cos\left(kx + \frac{\phi}{2}\right) = \pm 1$$

or, $$\left(kx + \frac{\phi}{2}\right) = n\pi, \quad n = 0, 1, 2, \ldots \qquad (9.16)$$

Note that a pressure node in a standing wave would correspond to a displacement antinode, and a pressure anti-node would correspond to a displacement node.

when we label Equation (9.13) as SHM, we mean that excess pressure at any point varies simple-harmonically. If the sound waves were represented in terms of displacement waves, then the equation of standing wave corresponding to Equation (9.12) would be

$$s = s'_0 \cos(\omega t + \frac{\phi}{2})$$

where $$s'_0 = 2s_0 \sin(kx + \frac{\phi}{2})$$

This can be easily observed to be an equation of SHM. It represents the medium particles moving simple harmonically about their mean position at x.

SOLVED EXAMPLES

71. A certain organ pipe resonates in its fundamental mode at a frequency of 1 kHz in air. What will be the fundamental frequency if the air is replaced by hydrogen at the same temperature? (take molar mass of air = 29 g)

Solution:

Suppose the speed of sound in hydrogen is v_h and that in air is v_a. The fundamental frequency of an organ pipe is proportional to the speed of sound in the gas contained in it. If the fundamental frequency with hydrogen in the tube is n, we have

$$\frac{v}{1000\text{ Hz}} = \frac{v_h}{v_a} = \sqrt{\frac{M_{\text{Air}}}{M_{\text{H}_2}}}$$

(Since both air and H_2 are diatomic, y is same for both)

or, $$\frac{v}{1\text{ kHz}} = \sqrt{\frac{29}{2}}$$

$\Rightarrow$ $$n = \sqrt{\frac{29}{2}}\text{ kHz.}$$

72. A tube open at only one end is cut into two tubes of non-equal lengths. The piece open at both ends has fundamental frequency of 450 Hz and of 675 Hz. What is the first overtone frequency of the original tube.

Solution:

$$450 = \frac{v}{2\ell_1} \qquad 675 = \frac{v}{4\ell_2}$$

Length of original tube = $(\ell_1 + \ell_2)$

Its first obtained frequency,

$$n_1 = \frac{3v}{(\ell_1 + \ell_2)} = \frac{3v}{\frac{v}{900} + \frac{v}{675 \times 4}}$$

$$= \frac{3(2700 \times 900)}{2700 + 900} = 2700 \times \frac{3}{4} = 2025\text{ Hz.}$$

73. The range audible frequency for humans is 20 Hz to 20,000 Hz. If speed of sound in air is 336 m/s, what can be the maximum and minimum length of a musical instrument, based on a resonance pipe?

Solution:

For an open pipe, $$f = \frac{v}{2\ell}\, n$$

$\Rightarrow$ $$\ell = \frac{v}{2f} \cdot n$$

Similarly for a closed pipe,

$$\ell = \frac{v}{4f}(2n+1)$$

$$\ell_{\min} = \frac{v}{4f_{\max}}(2n+1)_{\min} = \frac{336}{4 \times 20000} = 4.2\text{ mm}$$

$$\ell_{\max} = \frac{v}{2f_{\min}}\, n_{\max} = \frac{v}{2 \times 20}\, n_{\max} = 8.4\text{ (m)} \times n_{\max}$$

Clearly, there is no upper limit on the length of such an musical instrument.

Vibration of Air Columns

Standing waves can be set up in air columns trapped inside cylindrical tubes if frequency of the tuning fork sounding the air column matches one of the natural frequencies of air columns. In such a case, the sound of the tuning fork becomes markedly louder, and we say there is resonance between the tuning fork and air column. To determine the natural frequency of the air column, notice that there is a displacement node (pressure antinode) at each closed end of the tube as air molecules there are not free to move, and a displacement antinode (pressure-node) at each open end of the air column.

In reality, antinodes do not occurs exactly at the open end but a little distance outside. However, if diameter of tube is small compared to its length, this end correction can be neglected.

Closed Organ Pipe

(In Fig. 9.18, A_p = Pressure antinode, A_s = displacement antinode, N_p = pressure node, N_s = displacement node.)

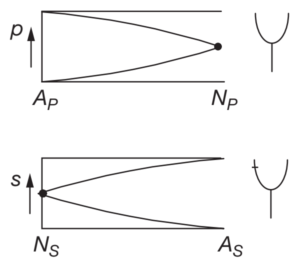
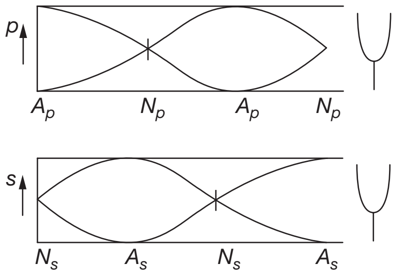


Fig. 9.18

Fundamental Mode

The smallest frequency (largest wavelength) that satisfies the boundary condition for resonance (i.e. displacement node at left end and antinode at right end is $\lambda_0 = 4\ell$, where ℓ = length of closed pipe the corresponding frequency.)

$n_0 = \frac{v}{\lambda} = \frac{v}{4L}$ is called the fundamental frequency.



First Overtone

Here there is one node and one antinode apart from the nodes and antinodes at the ends.

$$l_1 = \frac{4\ell}{3} = \frac{\lambda_0}{3}$$

and corresponding frequency,

$$n_1 = \frac{v}{\lambda_1} = 3n_0$$

This frequency is three times the fundamental frequency and hence is called the 3rd harmonic.

nth Overtone

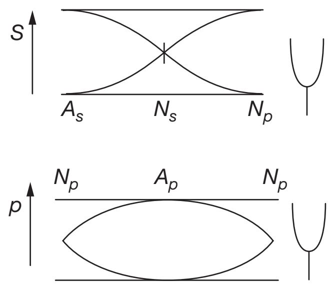
In general, the *n*th overtone will have *n* nodes and *n* antinodes between the two nodes. The corresponding wavelength is

$$l_n = \frac{4\ell}{2n+1} = \frac{\lambda_0}{2n+1}$$

and $$v_n(2n+1)n_0$$

This corresponds to the $(2n + 1)^{th}$ harmonic. Clearly, only odd harmonic are allowed in a closed pipe.

Open Organ Pipe



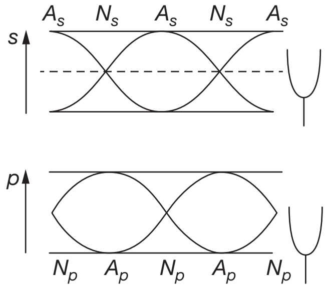
Fundamental Mode

The smallest frequency (largest wave length) that satisfies the boundary condition for resonance (i.e. displacement antinodes at both ends) is

$$l_0 = 2\ell$$

corresponding frequency is called the fundamental frequency

$$n_0 = \frac{v}{2\ell}$$



1st Overtone

Here there is one displacement antinode between the two antinodes at the ends.

$$l_1 = \frac{2\ell}{2} = 1 = \frac{\lambda_0}{2}$$

and corresponding frequency

$$n_1 = \frac{v}{\lambda_1} = 2n_0$$

This frequency is two times the fundamental frequency and is called the 2nd harmonic.

nth Overtone

The *n*th overtone has *n* displacement antinodes between the two antinodes at the ends.

$$l_n = \frac{2\ell}{n+1} = \frac{\lambda_0}{n+1}$$

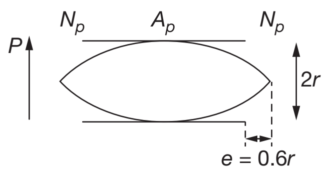
and $$n_n = (n + 1)\, n_0$$

This corresponds to $(n + 1)^{th}$ harmonic: clearly both even and odd harmonics are allowed in an open pipe.

End Correction

As mentioned earlier, the displacement antinode at an open end of an organ pipe lie slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by

$$e = 0.6\, r$$



where r = radius of the organ pipe.

With end correction, the fundamental frequency of a closed pipe (f_c) and an open argon pipe (f_0) will be given by

$$f_c = \frac{v}{4(\ell + 0.6r)}$$

and $$f_0 = \frac{v}{2(\ell + 1.2r)}$$

SOLVED EXAMPLES

74. A tuning fork is vibrating at frequency 100 Hz. When another tuning fork is sounded simultaneously, 4 beats per second are heard. When some mass is added to the tuning fork of 100 Hz, beat frequency decreases. Find the frequency of the other tuning fork.

Solution:

$$|f - 100| = 4$$

$\Rightarrow$ $$f = 95 \quad \text{or} \quad 105$$

when 1st tuning fork is loaded, its frequency decreases and so does the beat frequency

$\Rightarrow$ $$100 > f$$

$\Rightarrow$ $$f = 95 \text{ Hz.}$$

75. A closed organ pipe has length ℓ. The air in it is vibrating in third overtone with maximum amplitude a. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

Solution:

Figure 9.19 shows variation of displacement of particles in a closed organ pipe for a third overtone.

For third overtone,

$$\ell = \frac{7\lambda}{4} \quad \text{or} \quad \lambda = \frac{4\ell}{7} \quad \text{or} \quad \frac{\lambda}{4} = \frac{\ell}{7}$$

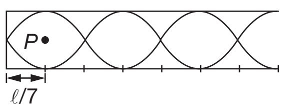


Fig. 9.19

Hence the amplitude of P at a distance $\frac{\ell}{7}$ from closed end is a because there is an antinode at that point.

76. A steel rod having a length of 1 m is fastened at its middle. Assuming Young's modulus to be 2×10^{11} Pa and density to be 8 gm/cm^3 the fundamental frequency of the longitudinal vibration is_______ and frequency of first overtone is_____________.

Solution:

2.5 kHz, 7.5 kHz.

Interference in Time: Beats

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are

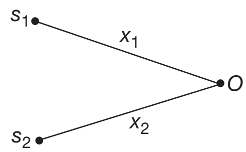
$$p_1 = p_0 \sin(2\pi f_1 t - kx_1 + \theta_1)$$

$$p_2 = p_0 \sin(2\pi f_2 t - kx_2 + \theta_2)$$

By principle of superposition

$$p = p_1 + p_2$$

$$= 2p_0 \cos\left\{p(f_1 - f_2)t + \frac{\theta_1 + \theta_2}{2}\right\} \sin\left\{p(f_1 + f_2)t + \frac{\theta_1 + \theta_2}{2}\right\}$$



i.e., the resultant sound at point O has frequency $\left(\frac{f_1 + f_2}{2}\right)$ while pressure amplitude $p'_0(t)$ varies with time as

$$p'_0(\text{t}) = 2p_0 \cos\left\{\pi(f_1 - f_2)t + \frac{\phi_1 - \phi_2}{2}\right\}$$

Hence, pressure amplitude at point O varies with time with a frequency of $\left(\frac{f_1 - f_2}{2}\right)$.

Hence, sound intensity will vary with a frequency $f_1 - f_2$.

This frequency is called beat frequency (f_B) and the time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$$f_B = f_1 - f_2$$

$$T_B = \frac{1}{f_1 - f_2}$$

IMPORTANT POINTS

- The frequency $|f_1 - f_2|$ should be less than 16 Hz, for it to be audible.
- Beat phenomenon can be used for determining an unknown frequency by sounding it together with a source of known frequency.
- If the arm of a tuning fork is waxed or loaded, then its frequency decreases.
- If the arm of tuning fork is filed, then its frequency increases.

SOLVED EXAMPLES

77. Two strings *X* and *Y* of a sitar produce a beat of frequency 4 Hz. When the tension of string *Y* is slightly increased, the beat frequency is found to be 2 Hz. If the frequency of *X* is 300 Hz, then the original frequency of *Y* was

(A) 296 Hz (B) 298 Hz
(C) 302 Hz (D) 304 Hz

Solution: (A)

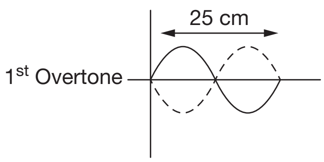
$f_y = 304$ or 296, on increasing tension f_y will increase in place of in

$\therefore \quad f_y = 296$ Hz.

78. A string 25 cm long fixed at both ends and having a mass of 2.5 g is under tension. A pipe closed from one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s. Find tension in the string.

Solution:

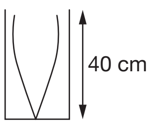
$$\mu = \frac{2.5}{25} = 0.1 \text{ g/cm} = 10^{-2} \text{ kg/m}$$



$$\lambda_s = 25 \text{ cm} = 0.25 \text{ m}$$

$$f_s = \frac{1}{\lambda_s}\sqrt{\frac{T}{\mu}}$$

Pipe in fundamental frequency



$$\lambda_p = 160 \text{ cm} = 1.6 \text{ m}$$

$$f_p = \frac{V}{\lambda_p}$$

$\because$ By decreasing the tension, beat frequency is decreased

$\therefore \qquad f_s > f_p$

$\Rightarrow \qquad f_s - f_p = 8$

$$\Rightarrow \qquad \frac{1}{0.25}\sqrt{\frac{T}{10^{-2}}} - \frac{320}{1.6} = 8$$

$\Rightarrow \qquad T = 27.04$ N.

79. The wavelengths of two sound waves are 49 cm and 50 cm, respectively. If the room temperature is 30°C, then the number of beats produced by them is approximately (velocity of sound in air at 0°C = 332 m/s).

(A) 6 (B) 10 (C) 14 (D) 18

Solution:

$$v = 332\sqrt{\frac{303}{273}}$$

$$\Rightarrow \quad \text{Beat frequency} = f_1 - f_2 = v\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= 332\sqrt{\frac{303}{273}}\left(\frac{1}{49} - \frac{1}{50}\right) \times 100$$

$$\cong 14.$$

Doppler's Effect

When there is relative motion between the source of a sound/light wave and an observer along the line joining them, the actual frequency observed is different from the frequency of the source. This phenomenon is called Doppler's Effect. If the observer and source are moving towards each other, the observed frequency is greater than the frequency of the source. If the observer and source move away from each other, the observed frequency is less than the frequency of source.

(v = velocity of sound with respect to ground, c = velocity of sound with respect to medium, v_m = velocity of medium, v_O = velocity of observer, v_s = velocity of source.)

Sound Source is Moving and Observer Is Stationary

If the source emitting a sound of frequency *f* is travelling with velocity v_s along the line joining the source and observer,

observed frequency, $$f' = \left(\frac{v}{v - v_s}\right) \cdot f \qquad (9.17)$$

and apparent wavelength $$\lambda' = \lambda\left(\frac{v - v_s}{v}\right) \qquad (9.18)$$

In the above expression, the positive direction is taken along the velocity of sound, that is, from source to observer. Hence, v_s is positive if source is moving towards the observer, and negative if source is moving away from the observer.

Sound Source is Stationary and Observer is moving with Velocity v_0 along the Line Joining them

The source (at rest) is emitting a sound of frequency *f* travelling with velocity v so that wavelength is $\lambda = v/f$, i.e., there is no change in wavelength. However since the observer is moving with a velocity v_0 along the line joining the source and observer, the observed frequency is

$$f' = f\left(\frac{v - v_0}{v}\right) \tag{9.19}$$

In the above expression, the positive direction is taken along the velocity of sound, i.e., from source to observer. Hence, v_O is positive if observer is moving away from the source, and negative if observer is moving towards the source.

The Source and Observer both are moving with Velocities v_s and v_0 along the Line Joining them

The observed frequency, $$f' = f\left(\frac{v - v_0}{v - v_s}\right) \tag{9.20}$$

and apparent wavelength $$\lambda' = \lambda\left(\frac{v - v_s}{v}\right) \tag{9.21}$$

In the above expression also, the positive direction is taken along the velocity of sound, i.e., from source to observer.

In all of the above expressions from Equation (9.17) to (9.21), v stands for velocity of sound with respect to ground.

If velocity of sound with respect to medium is c and the medium is moving in the direction of sound from source to observer with speed v_m, $v = c + v_m$, and if the medium is moving opposite to the direction of sound from observer to source with speed v_m, $v = c - v_m$

SOLVED EXAMPLES

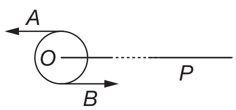
80. A whistle of frequency 540 Hz is moving in a circle of radius 2 ft at a constant angular speed of 15 rad/s. What are the lowest and height frequencies heard by a listener standing at rest, far away from the centre of the circle? (Velocity of sound in air is 1100 ft/sec.)

Solution:

The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by

$$v_s = \omega R$$

where R is radius of the circle.



At points A and B, the velocity v_s of whistle is parallel to line OP; i.e., with respect to observer at P, whistle has maximum velocity v_s away from P at point A, and towards P at point B. (Since distance OP is large compared to radius R, whistle may be assumed to be moving along line OP).

Observer, therefore, listens to maximum frequency when source is at B moving towards observer

$$f_{max} = f\,\frac{v}{v - v_s}$$

where v is speed of sound in air. Similarly, observer listens minimum frequency when source is at A, moving away from observer

$$f_{min} = \frac{f}{v + v_s}$$

For $f = 540$ Hz, $v_s = 2$ ft × 15 rad/s = 30 ft/s,

and $v = 1100$ ft/s,

we get (approx.)

$$f_{max} = 555 \text{ Hz}$$

and $$f_{min} = 525 \text{ Hz.}$$

81. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 600 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find

(A) the frequency of the whistle as heard by an observer on the hill.

(B) the distance from the hill at which the echo from the hill is heard by the driver and its frequency. (Velocity of sound in air = 1200 km/hr.)

Solution:

A train is moving towards a hill with speed v_s with respect to the ground. The speed of sound in air, i.e., the speed of sound with respect to medium (air) is c, while air itself is blowing towards the hill with velocity v_m (as observed from ground). For an observer standing on the ground, which is the inertial frame, the speed of sound towards hill is given by

$$v = c + v_m$$

(A) The observer on the hill is stationary while source is approaching him. Hence, frequency of whistle heard by him is

$$f' = f\frac{v}{v - v_s}$$

for $f = 600$ Hz, $v_s = 40$ km/hr, and $v = (1200 + 40)$ km/hr, we get

$$f' = 600\,\frac{1240}{1240 - 40} = 620 \text{ Hz.}$$

(B) The train sounds the whistle when it is at a distance x from the hill. Sound moving with velocity v with respect to ground takes time t to reach the hill, such that

$$t = \frac{x}{v} = \frac{x}{c + v_m} \tag{1}$$

After reflection from hill, sound waves move backwards, towards the train. The sound is now moving opposite to the wind direction. Hence, its velocity with respect to the ground is

$$v' = c - v_m$$

Suppose when this reflected sound (or echo) reaches the train, it is at distance x' from hill. The time taken by echo to travel distance x' is given by

$$t' = \frac{x'}{v} = \frac{x'}{c - v_m} \qquad (2)$$

Thus, total time $(t + t')$ elapses between sounding the whistle and echo reaching back. In the same time, the train moves a distance $(x - x')$ with constant speed v_s, as observed from ground. That is,

$$x - x' = (t + t')\, v_s.$$

Substituting from (1) and (2), for t and t', we get

$$x - x' = \frac{v_s}{c + v_m} x + \frac{v_s}{c + v_m} x'$$

or, $$\frac{c + v_m - v_s}{c + v_m} = \frac{v_s + c - v_m}{c - v_m} x'$$

For $x = 1$ km, $c = 1200$ km/hr, $v_s = 40$ km/hr, and $v_m = 40$ km/hr, we get

$$\frac{1200 + 40 - 40}{1200 - 40} \times 1 = \frac{40 + 1200 - 40}{1200 - 40} x'$$

or, $$x' = \frac{1160}{1240} = 0.935 \text{ km.}$$

Thus, the echo is heard when train is 935 m from the hill.

(C) Now, for the observer moving along with train, echo is a sound produced by a stationary source, i.e., the hill. Hence as observed from ground, source is stationary and observer is moving towards source with speed 40 km/hr. Hence, $v_O = -40$ km/hr. On the other hand, reflected sound travels opposite to wind velocity. That is, velocity of echo with respect to ground is v'. Further, the source (hill) is emitting sound of frequency f', which is the frequency observed by the hill.

Thus, frequency of echo as heard by observer on train is given by

$$f'' = f' \frac{v' + v_O}{v'}$$

$$\Rightarrow f'' = \frac{(1160 - (-40))}{1160} \times 620 = \frac{18600}{29} \text{ Hz.}$$

82. A train approaching a railway crossing at a speed of 72 km/h sounds a short whistle at frequency 640 Hz when it is 1 km away from the crossing. The speed of sound in air is 330 m/s. A road intersects the crossing perpendicularly. What is the frequency heard by a person standing on the road at the distance of 1732 m from the crossing.

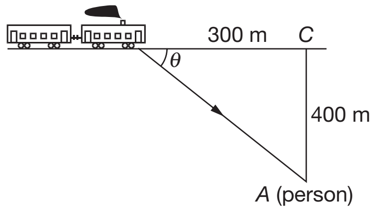


Fig. 9.20

Solution:

The observer A is at rest with respect to the air and the source is travelling at a velocity of 72 km/h, i.e., 20 m/s. As is clear from Fig. 9.20, the person receives the sound of the whistle in a direction BA making an angle θ with the track, where $\tan\theta = \frac{1732}{1000} = \sqrt{3}$, i.e., $\theta = 60°$. The component of the velocity of the source (i.e., of the train) along this direction is 20 cos q = 10 m/s. As the source is approaching the person with this component, the frequency heard by the observer is

$$\upsilon' = \frac{v}{v - u\cos\theta}$$

$$\upsilon = \frac{330}{330 - 10} \times 640 \text{ Hz} = 660 \text{ Hz.}$$

83. In Fig. 9.21 a source of sound of frequency 510 Hz moves with constant velocity $v_s = 20$ m/s in the direction shown. The wind is blowing at a constant velocity $v_w = 20$ m/s towards an observer who is at rest at point B. The frequency detected by the observer corresponding to the sound emitted by the source at initial position A, will be (speed of sound relative to air = 330 m/s)

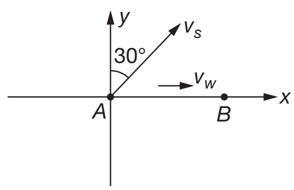


Fig. 9.21

(A) 485 Hz (B) 500 Hz
(C) 512 Hz (D) 525 Hz

Solution:

Apparent frequency

$$n' = n\,\frac{(u+v_w)}{(u+v_w-v_s\cos 60°)}$$

$$= \frac{510\,(330+20)}{330+20-20\cos 60°} = 510 \times \frac{350}{340} = 525 \text{ Hz.}$$

84. A source S, receiver R, and air are moving relative to the ground as shown in Fig. 9.22. $v_s = v_r = v_w = 20$ m/s and velocity of sound in still air is 320 m/s. If source generates sound of frequency 3600 Hz then the frequency of sound received by the receiver will be_____ (v_w = velocity of wind)

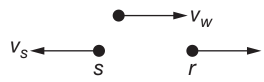


Fig. 9.22

Solution:

3200 Hz.

BRAIN MAP I

SHM

1. Equation of SHM:
- (i) Linear: $a = -\omega^2 x$
- (ii) Angular: $\alpha = -\omega^2\theta$

2. Linear SHM:
- (i) Displacement of particle: $x = A\sin(\omega t+\phi)$
- (ii) Velocity: $\frac{dx}{dt} = A\omega\cos(\omega t+\phi) = \omega\sqrt{A^2-x^2}$
- (iii) Acceleration: $\frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t+\phi) = -\omega^2 x$
- (iv) Phase: $\omega t+\phi$
- (v) Phase constant: ϕ

3. Angular SHM:
- (i) Displacement: $\theta = \theta_0\sin(\omega t+\phi)$
- (ii) Angular velocity: $\frac{d\theta}{dt} = \theta_0\omega\cos(\omega t+\phi)$
- (iii) Angular acceleration: $\frac{d^2\theta}{dt^2} = -\theta_0\omega^2\sin(\omega t+\phi)$
- (iv) Phase: $\omega t+\phi$
- (v) Phase constant: ϕ

4. Energy in SHM:
- (i) $K = \frac{1}{2}m\omega^2(A^2-x^2)$
- (ii) $U = \frac{1}{2}m\omega^2x^2$
- (iii) $E = K+U = \frac{1}{2}m\omega^2A^2$ = constant

5. Time Period: Pendulums:
- (i) Simple pendulum: $T = 2\pi\sqrt{l/g}$
- (ii) Physical pendulum: $T = 2\pi\sqrt{I/mgl}$
- (iii) Torsional pendulum: $T = 2\pi\sqrt{I/k}$

6. Mass-spring system:
- (i) $T = 2\pi\sqrt{m/k}$
- (ii) Two bodies system: $T = 2\pi\sqrt{\mu/k}$; Where $\mu = \frac{m_1m_2}{m_1+m_2}$

7. Composition of 2 SHMs:

$x_1 = A_1\sin\omega t$
$x_2 = A_2\sin(\omega t+\phi)$
$x = x_1 + x_2$
$x = A\sin(\omega t+\delta)$

where, $A = \sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$

and $\tan\delta = \frac{A_2\sin\phi}{A_1+A_2\cos\phi}$

8. Combination of springs:
- (i) Series – $\frac{1}{K_{eff}} = \frac{1}{K_1}+\frac{1}{K_2}$
- (ii) Parallel – $K_{eff} = K_1+K_2$
- (iii) Spring cut into two parts $m:n$: $K_1 = \frac{(m+n)K}{m}$, $\frac{(m+n)K}{n}$

BRAIN MAP 2

WAVE MOTION and SOUND

1. Wave function:

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

Sinusoidal wave:

$y = A \sin(\omega t - kx)$

$\omega = 2\pi v = \frac{2\pi}{T}$

$k = \frac{2\pi}{\lambda}$

2. Energy of a plane progressive wave:

$P = 2\pi^2 f^2 A^2 \rho S v$ (J/s)

Intensity of the wave

$I = 2\pi^2 f^2 A^2 \rho v$ (W/m^2)

3. Stationary waves in strings:

(i) String fixed at both the ends

$y = -2a \sin kx \cos\omega t$

(ii) $f = \frac{n}{2L}\left(\sqrt{\frac{T}{\mu}}\right)$;

μ = mass/length

(iii) Velocity of transverse wave

$v = \sqrt{\frac{T}{\mu}}$

4. Amplitude of reflected and transmitted wave:

$A_r = \frac{v_2 - v_1}{(v_2 + v_1)} A_i$

$A_t = \frac{2v_2}{v_2 + v_1} A_i$

Phase change on reflection from:

(i) Rigid end is π

(ii) Free end is zero.

5. Sound Wave:

$y_P = A \sin(\omega t - kx)$

$P = P_0 \cos(\omega t - kx)$

Volume strain: $= \frac{\delta y}{\delta x}$

Pressure: $- \frac{P\delta y}{\delta x}$

Velocity of longitudinal wave:

(i) Solid: $v = \sqrt{\frac{Y}{\rho}}$

(ii) Liquid: $v = \sqrt{\frac{B}{\rho}}$

(iii) Gas: $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

(iv) Intensity: $I = \frac{P_0^2 v}{2B}$

(v) Loudness:

$\beta = 10 \log10 (I/I0)$

6. Organ Pipe:

(i) Closed pipe:

$f = \frac{(2n-1)}{4l} v$

(ii) Open pipe:

$f = \frac{nv}{2l}$

7. Beat frequency

$f = f_1 - f_2$

8. Doppler Effect:

$f' = \frac{[(v \pm v_W) \pm v_0)]}{[(v \pm v_W) \pm v_S)]} f$

EXERCISES

Single Option Correct Type

1. A simple harmonic motion (SHM) has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = A/2$ is

(A) $T/6$ (B) $T/4$
(C) $T/3$ (D) $T/2$

2. A mass m is suspended from two springs of spring constant k_1 and k_2 as shown in Fig. 9.23. The time period of vertical oscillations of the mass will be

(A) $2\pi\sqrt{\left(\dfrac{k_1+k_2}{m}\right)}$ (B) $2\pi\sqrt{\dfrac{m}{(k_1+k_2)}}$
(C) $2\pi\sqrt{\dfrac{m(k_1k_2)}{(k_1+k_2)}}$ (D) $2\pi\sqrt{\dfrac{m(k_1+k_2)}{(k_1k_2)}}$

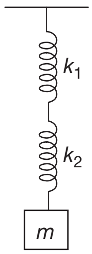


Fig. 9.23

3. Two SHMs are represented by the equations $Y_1 = 10 \sin\left(3\pi t+\dfrac{\pi}{4}\right)$ and $Y_2 = 5\ (\sin 3\pi t + \sqrt{3}\ \cos 3\pi t)$. Their amplitudes are in the ratio of

(A) 2 : 1 (B) 3 : 1
(C) 1 : 3 (D) 1 : 4

4. A mass M attached to a spring oscillates with a period of 2 seconds. If the mass is increased by 2 kg the period increases by 1 second. The initial mass M will be

(A) 1.6 kg (B) 1 kg (C) 1.5 kg (D) 2 kg

5. The ratio of kinetic energy at mean position to the potential energy when the displacement is half of the amplitude is

(A) $\dfrac{4}{1}$ (B) $\dfrac{2}{3}$
(C) $\dfrac{4}{3}$ (D) $\dfrac{1}{2}$

6. If the displacement (x) and velocity (v) of a particle executing SHM are related through the expression $4v^2 = 25 - x^2$, then its time period is

(A) π (B) 2π (C) 4π (D) 6π

7. In a simple pendulum at mean position,

(A) KE is maximum and PE is minimum.
(B) KE is minimum and PE is maximum.
(C) Both PE and KE are maximum.
(D) Both PE and KE are minimum.

8. Maximum velocity in SHM is v_m. The average velocity during the motion from one extreme point to the other extreme point will be

(A) $\dfrac{\pi}{2}v_m$ (B) $\dfrac{2}{\pi}v_m$
(C) $\dfrac{4}{\pi}v_m$ (D) $\dfrac{\pi}{4}v_m$

9. When a particle oscillates simple harmonically, its kinetic energy varies periodically. If frequency of the particle is n, the frequency of the kinetic energy is

(A) $n/2$ (B) n (C) $2n$ (D) $4n$

10. A mass M suspended from a spring of negligible mass. The spring is pulled a little and then released, so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$. The ratio of m/M is

(A) $\dfrac{5}{3}$ (B) $\dfrac{3}{5}$ (C) $\dfrac{16}{9}$ (D) $\dfrac{25}{9}$

11. What will be the displacement of a particle in SHM when its velocity is half the maximum velocity (A = amplitude)

(A) $\dfrac{3}{\sqrt{2}}A$ (B) $\sqrt{2}A$
(C) $\dfrac{3}{4}A$ (D) $\dfrac{\sqrt{3}}{2}A$

12. Two blocks of mass m_1 and m_2 are kept on a smooth horizontal table as shown in Fig. 9.24. Block of mass m_1 but not m_2 is fastened to the spring. If now both the blocks are pushed to the left so that the spring is compressed a distance d. The amplitude of oscillation of block of mass m_1, after the system is released is

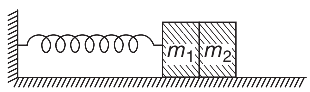


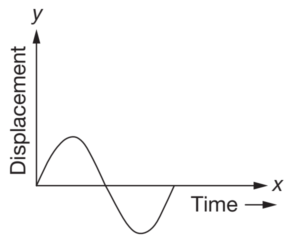
Fig. 9.24

(A) $d\sqrt{\dfrac{m_1}{m_1+m_2}}$ (B) $d\sqrt{\dfrac{m_2}{m_1+m_2}}$

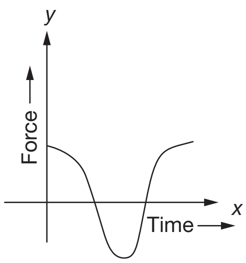
(C) $d\sqrt{\dfrac{2m_2}{m_1+m_2}}$ (D) $d\sqrt{\dfrac{2m_1}{m_1+m_2}}$

13. Displacement–time graph of a particle executing SHM is shown.

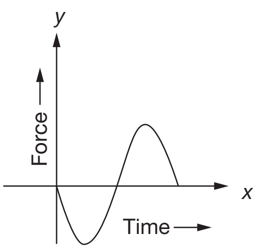
The corresponding force–time graph of the particle is



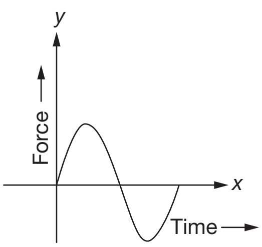
(A)



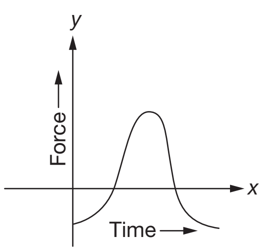
(B)



(C)



(D)



14. A particle starts executing SHM of amplitude a and total energy E. At the instant, its kinetic energy is $\dfrac{3E}{4}$ and its displacement y is given by

(A) $y=\dfrac{a}{\sqrt{2}}$ (B) $y=\dfrac{a}{2}$

(C) $y=\dfrac{a\sqrt{3}}{2}$ (D) $y=a$

15. The periodic time of a mass suspended by a spring (force constant k) is T. If the spring is cut in three equal pieces, the force constant of each part and the periodic time if the same mass is suspended from one piece are

(A) $k, T/\sqrt{3}$ (B) $3k, T$

(C) $3k, \sqrt{3}\,T$ (D) $3k, T/\sqrt{3}$

16. $x = A\sin\omega t$, represents the equation of a SHM. If displacements of the particle are x_1 and x_2 and velocities are v_1 and v_2, respectively, then amplitude of SHM is

(A) $\left[\dfrac{(v_2-v_1)(x_2-x_1)}{v_2^2-v_1^2}\right]^{\frac{1}{2}}$

(B) $\left[\dfrac{(v_2x_1)^2-(v_1x_2)^2}{v_2^2-v_1^2}\right]^{\frac{1}{2}}$

(C) $\dfrac{v_1x_2}{(v_2-v_1)^2}$

(D) $\left[\dfrac{v_1x_2-v_2x_1}{v_1^2-v_2^2}\right]^{\frac{1}{2}}$

17. On smooth inclined plane, a body of mass m is attached between two massless springs. The other ends of the springs are fixed to firm supports. If each spring has force constant k, the period of oscillation of the body is

(A) $2\pi\sqrt{\dfrac{m}{2k}}$ (B) $2\pi\sqrt{\dfrac{2m}{k}}$

(C) $2\pi\sqrt{\dfrac{mg\sin\theta}{2k}}$ (D) $2\pi\sqrt{\dfrac{2mg\sin\theta}{k}}$

18. A person measures the time period of a simple pendulum inside a stationary lift and finds it to be T. If the lift starts accelerating upwards with an acceleration of g/3, the time period of the pendulum will be

(A) $\sqrt{3}\ T$ (B) $\dfrac{\sqrt{3}}{2}T$

(C) $T/\sqrt{3}$ (D) $T/3$

19. The displacement x (in centimeters) of an oscillating particle varies with time t (in seconds) as $x = 2\cos\left(0.5\pi t + \frac{\pi}{3}\right)$. The magnitude of the maximum acceleration of the particle in cms^{-2} is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi^2}{2}$ (D) $\frac{\pi^2}{4}$

20. A particle is executing SHM with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is

(A) $\sqrt{3}$ cm (B) $\sqrt{5}$ cm
(C) $2\sqrt{3}$ cm (D) $2\sqrt{5}$ cm

21. A source x of unknown frequency produces 8 beats with a source of 250 Hz and 12 beats with a source of 270 Hz. The frequency of source x is

(A) 258 Hz (B) 242 Hz
(C) 262 Hz (D) 282 Hz

22. A sonometer wire of density d and radius r is held between two bridges at a distance L apart. The wire has a tension T. The fundamental frequency of the wire will be

(A) $f = \frac{1}{2Lr}\sqrt{\frac{T}{\pi d}}$ (B) $f = \frac{r}{2L}\sqrt{\frac{\pi d}{T}}$

(C) $f = \frac{1}{2Lr}\sqrt{\frac{d}{\pi T}}$ (D) $f = \frac{1}{2L}\sqrt{\frac{d}{T}}$

23. The amplitude of a wave disturbance propagating in the positive y-direction is given by $y = \frac{1}{1+x^2}$ at t = 0 and $y = \frac{1}{[1+(x-1)^2]}$ at t = 2 second, where x and y are in m. If the shape of the wave disturbance does not change during the propagation, what is the velocity of the wave?

(A) 1 m/s (B) 1.5 m/s
(C) 0.5 m/s (D) 2 m/s

24. Two sinusoidal plane waves of the same frequency having intensities I_0 and $4I_0$ are traveling in the same direction. The resultant intensity at a point at which waves meet with a phase difference of zero radian is

(A) I_0 (B) $5I_0$

(C) $9I_0$ (D) $3I_0$

25. An open pipe is suddenly closed which results in the second overtone of the closed pipe to be higher in frequency by 100 Hz than the first overtone of the original pipe. The fundamental frequency of open pipe will be

(A) 100 Hz (B) 300 Hz
(C) 150 Hz (D) 200 Hz

26. A fast train moving at 40 m/s passes by a stationary observer, emitting a whistle of frequency 300 Hz. If the velocity of sound waves is 340 m/s, then the change in the apparent frequency of the sound, just before and just after the train passes by the observer, will be nearly

(A) 32 Hz (B) 40 Hz
(C) 72 Hz (D) 8 Hz

27. Which of the following represents a standing wave?

(A) $y = A\sin(\omega t - kx)$
(B) $y = A\sin kx \sin(\omega t - \theta)$
(C) $y = Ae^{-bx}\sin(\omega t - kx + \alpha)$
(D) $y = (ax + b)\sin(\omega t - kx)$

28. A man on the platform is watching two trains, one leaving and the other entering the station with equal speed of 4 m/s. If they sound their whistles each of natural frequency 240 Hz, the number of beats heard by the man (velocity of sound in air = 320 m/s) will be

(A) 6 (B) 3 (C) 0 (D) 12

29. The two waves having intensities in the ratio 1 : 9 produce interference. The ratio of the maximum to the minimum intensities is equal to

(A) 10 : 8 (B) 9 : 1 (C) 4 : 1 (D) 2 : 1

30. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s then the ratio $\frac{f_1}{f_2}$ is

(A) $\frac{18}{19}$ (B) $\frac{1}{2}$

(C) 2 (D) $\frac{19}{18}$

31. A uniform cord has a mass of 0.3 kg and length of 6 m. (see Fig. 9.25). The speed of a pulse on this cord is ($g = 10$ m/s^2)

(A) 20 m/s (B) 10 m/s
(C) 40 m/s (D) 5 m/s

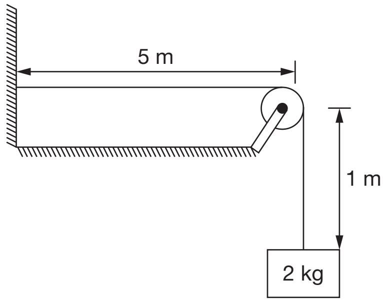


Fig. 9.25

32. A closed-organ pipe of length L is placed in a container having gas of density ρ_1 and an open organ pipe is placed in another container having gas of density ρ_2, both the gases are of same compressibility. If the frequency of first overtone for both the pipes is same, the length of open organ pipe is

(A) L (B) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$

(C) $L\sqrt{\frac{\rho_1}{\rho_2}}$ (D) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

33. The frequency of sound emitted from a source in water is 600 Hz. If speed of sound in water is 1500 m/s and in air is 300 m/s, then the frequency of sound heard above the surface of water is

(A) 300 Hz (B) 750 Hz
(C) 600 Hz (D) 1200 Hz

34. An organ pipe opens at both ends and another organ pipe closed at one end will resonate with each other if their lengths are in the ratio of

(A) 1 : 1 (B) 1 : 4
(C) 2 : 1 (D) None of these

35. If n_1, n_2 and n_3 are the fundamental frequencies of three segments into which a string is divided, then the original fundamental frequency n of the string is given by

(A) $n = n_1 + n_2 + n_3$

(B) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

(C) $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} + \frac{1}{\sqrt{n_3}}$

(D) $\sqrt{n} = \sqrt{n_1} + \sqrt{n_2} + \sqrt{n_3}$

36. A longitudinal wave sent by a ship to the bottom of the sea returns after a lapse of 2.64 s. Elasticity of water is 220 kg/mm^2 and density of sea water is 1.1 gm/cc. The depth of the sea is (in metres)

(A) 1400 (B) 1848 (C) 924 (D) 700

37. When the speed of sound in air is 330 m/s, the shortest air column, closed at one end that will respond to a tuning for k with a frequency of 440 vibs/s has a length of (approximately).

(A) 19 cm (B) 33 cm (C) 38 cm (D) 67 cm

38. The speed of a longitudinal wave in a gas is given by

(A) $v = \sqrt{p/d}$ (B) $v = (1/\gamma)\sqrt{p/d}$

(C) $v = \sqrt{\gamma p/d}$ (D) $v = \sqrt{p/\gamma d}$

39. For a wave displacement amplitude is 10^{-8} m, density of air 1.3 kg m^{-3}, velocity in air 340 ms^{-1}, and frequency is 2000 Hz. The intensity of wave is

(A) 5.3×10^{-4} W/m^{-2} (B) 5.3×10^{-6} W/m^{-2}
(C) 3.5×10^{-8} W/m^{-2} (D) 3.5×10^{-6} W/m^{-2}

40. The displacement of a particle is represented by the equation $y = 3\cos\left(\frac{\pi}{2} - 2\omega t\right)$. The motion of the particle is

(A) simple harmonic with period $2\pi/\omega$.
(B) simple harmonic with period π/ω.
(C) periodic but not simple harmonic.
(D) non-periodic.

41. The displacement of a particle is represented by the equation $y = \sin^3 \omega t$

The motion is

(A) non-periodic.
(B) periodic but not simple harmonic.
(C) simple harmonic with periodic $2\pi/\omega$.
(D) simple harmonic with periodic π/ω.

42. The relation between acceleration and displacement of four particles are given below.

(A) $a_x = +2x$ (B) $a_x = +2x^2$

(C) $a_x = -2x^2$ (D) $a_x = -2x$

43. Motion of an oscillating liquid column in an U-tube is

(A) periodic but not simple harmonic.
(B) non-periodic.
(C) simple harmonic and time period is independent of the density of the liquid.
(D) simple harmonic and time period is directly proportional to the density of the liquid.

44. A particle is acted simultaneously by mutually perpendicular SHM $x = a\cos\omega t$ and $y = a\sin\omega t$ The trajectory of motion of the particle will be.

(A) An ellipse (B) A parabola
(C) A circle (D) A straight line

45. The displacement of a particle varies with time according to the relation. $y = a \sin \omega t + b \cos \omega t$
(A) The motion is oscillatory but not SHM
(B) The motion is SHM with amplitude $a + b$
(C) The motion is SHM with amplitude $a^2 + b^2$
(D) The motion is SHM with amplitude $\sqrt{a^2 + b^2}$

46. For pendulums A, B, C, and D are suspended from the same

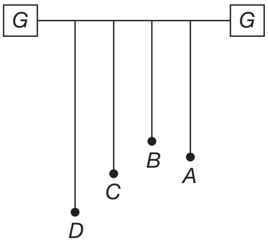


Fig. 9.26

elastic support as shown in Fig. 9.26. A and C are of the same length, while B is smaller than A, and D is larger than A. If A is given a transverse displacement,
(A) D will vibrate with maximum amplitude.
(B) C will vibrate with maximum amplitude.
(C) B will vibrate with maximum amplitude.
(D) All four will oscillate with equal amplitude.

47. Figure 9.27 shows the circular motion of a particle. The radius of the circle, the period, sense of revolution, and the initial position are indicated in the Fig. 9.27. The SHM of the x-projection of the radius vector of the rotating particle P is

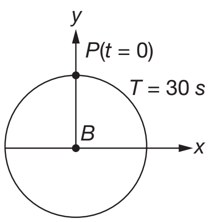


Fig. 9.27

(A) $x(t) = B \sin\left(\frac{2\pi t}{30}\right)$

(B) $x(t) = B \cos\left(\frac{\pi t}{15}\right)$

(C) $x(t) = B \sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

(D) $x(t) = B \cos\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$

48. The equation of motion of a particle is $x = a\cos(\alpha t)^2$. The motion is
(A) periodic but not oscillatory.
(B) periodic and oscillatory.
(C) oscillatory but not periodic.
(D) neither periodic nor oscillatory.

49. A particle executing SHM has a maximum speed of 30 cm/s and maximum acceleration of $60\,\text{cm/s}^2$. The period of oscillation is
(A) π s (B) $\frac{\pi}{2}$ s
(C) 2π s (D) $\frac{\pi}{t}$ s

50. In a common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(A) 48 (B) 49 (C) 50 (D) 51

51. Doppler effect can be observed for the following case (s)
(A) Supersonic speed (B) Ultrasonic waves
(C) Both of these (D) None of these

52. Transverse waves are generated in two uniform wires A and B of the same material by attaching their free ends to a vibrating source of frequency 200 Hz. The cross-section of A is half that of B while the tension on A is twice that on B. The ratio of wavelengths of the transverse waves in A and B is
(A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$
(C) 1 : 2 (D) 2 : 1

53. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was
(A) 261 Hz (B) 258 Hz
(C) 254 Hz (D) 251 Hz

54. Two waves having the intensities in the ratio of 9 : 1 produce interference. The ratio of maximum to minimum intensity is equal to
(A) 10 : 8 (B) 9 : 1
(C) 4 : 1 (D) 2 : 1

55. Velocity of sound at 0°C is 330 m/s. When pressure increases by 1 atmosphere and temperature increases by 1°C, the velocity of sound

(A) Increases by 0.6 m s^{-1}
(B) Decreases by 0.6 m s^{-1}
(C) Increases by 60 m s^{-1}
(D) Decreases by 60 m s^{-1}

56. A tuning fork of frequency 256 Hz is excited and held at the mouth of resonance column of frequency 254 Hz. Choose the correct statement;
(A) 2 beats per second will be heard
(B) 4 beats per second will be heard
(C) 1 beat per second will be heard
(D) No beat will be heard

57. The frequency of sound emitted from a source in water is 600 Hz. If speed of sound in water is 1500 m/s and in air is 300 m/s, then the frequency of sound heard above the surface of water is
(A) 300 Hz (B) 750 Hz
(C) 600 Hz (D) 1200 Hz

58. The equation $y = a \sin \frac{2\pi}{\lambda}(vt - x)$ is expression for
(A) Stationary wave of single frequency along x-axis.
(B) A simple harmonic motion.
(C) A progressive wave of single frequency along x-axis.
(D) The resultant of two SHMs of slightly different frequencies.

59. A SHW is represented by the equation $y(x,t) = a_0 \sin 2\pi\left(vt - \frac{x}{\lambda}\right)$. If the maximum particle velocity is three times the wave velocity, the wavelength λ of the wave is
(A) $\frac{\pi a_0}{3}$ (B) $\frac{2\pi a_0}{3}$
(C) πa_0 (D) $\frac{\pi a_0}{2}$

60. Two waves are represented by the equations $y_1 = A\sin\left(10\pi x - 15\pi t + \frac{\pi}{2}\right)$ and $y_2 = 2A\sin(30\pi x + 45\pi t)$. Which of the following statements is correct?
(A) The maximum particle velocity of the second wave is twice that of first.
(B) Their superposition will produce a standing wave.
(C) Maximum particle acceleration for the second wave is eighteen times that of the first wave.
(D) Their wave velocities are different.

61. The equation of stationary wave in a stretched string is given by $y = 5 \sin(\pi x/3) \cos(40\pi t)$, where x and y are in cm and t is in sec. The separation between two adjacent nodes is
(A) 1.5 cm (B) 3 cm
(C) 6 cm (D) 4 cm

62. A string of mass m and length L is hung vertically from a ceiling, and a mass M is attached at its lower end. A wave pulse is generated at the lower end. The velocity of the generated pulse as it moves up towards the ceiling will
(A) remain constant.
(B) increase.
(C) decrease linearly.
(D) decrease non-linearly.

63. When two tuning forks A and B sounded together produce 4 beats per second. After filing of A and waxing of B, the number of beats remains unaltered. If initial frequency of A is 250 Hz, then the initial frequency of B is
(A) 246 Hz (B) 250 Hz
(C) 254 Hz (D) 242 Hz

64. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is (speed of sound = 330 m/s)
(A) 409 (B) 429 (C) 517 (D) 500

65. Two waves represented by $y_1 = 10\sin 2000\pi t$, $y_2 = 20\sin\left(2000\pi t + \frac{\pi}{2}\right)$ are superimposed at any point at a particular instant. The amplitude of the resultant wave is
(A) 200 (B) 30 (C) $10\sqrt{5}$ (D) $10\sqrt{3}$

66. Which of the following is wrong?
(A) Velocity of sound is more in denser medium.
(B) Sound propagation is an adiabatic process.
(C) Frequencies of standing wave and its constituent wave are same.
(D) Frequency of resonance tube will change if we change the liquid maintaining same level.

67. The ratio of maximum to minimum intensity at a place due to superposition of two waves represented by $y_1 = 3\sin(200t)$ cm and $y_2 = 4\cos(208t)$ cm will be
(A) 7 : 1 (B) 49 : 1
(C) 4 : 3 (D) 16 : 9

68. 2nd overtone of an open organ pipe resonates with 3rd harmonics of a closed organ pipe. The ratio of their length will be

(A) $\frac{2}{1}$ (B) $\frac{1}{2}$

(C) $\frac{6}{5}$ (D) $\frac{5}{6}$

69. If λ_1, λ_2, and λ_3 are the wavelengths of the waves giving resonance with the fundamental, first and second overtones, respectively, of a closed organ pipe, then the ratio of wavelengths $\lambda_1 : \lambda_2 : \lambda_3$ is

(A) $1 : 2 : 3$ (B) $1 : \frac{1}{3} : \frac{1}{5}$

(C) $1 : 3 : 5$ (D) $5 : 3 : 1$

70. A transverse wave is described by the equation $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity if

(A) $\lambda = \frac{\pi y_0}{4}$ (B) $\lambda = \frac{\pi y_0}{2}$

(C) $\lambda = \pi y_0$ (D) $\lambda = 2\pi y_0$

71. Two waves are represented by the following equations $y_1 = 5\sin 2\pi(10t - 0.1x)$; $y_2 = 10\sin 2\pi(20t - 0.2x)$ Ratio of intensities I_2/I_1 will be

(A) 1 (B) 2

(C) 4 (D) 16

72. For the stationary wave $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$, ($x$ and y are in cm and t in second) the distance between a node and the next anti-nodes is

(A) 7.5 cm (B) 15 cm

(C) 22.5 cm (D) 30 cm

73. The equation of a plane progressive wave is $y = 0.09\sin 8\pi\left(t - \frac{x}{20}\right)$. When it is reflected at rigid support, its amplitude becomes two-third of its previous value. The equation of the reflected wave is

(A) $y = -0.09\sin 8\pi\left(t - \frac{x}{20}\right)$

(B) $y = -0.06\sin 8\pi\left(t - \frac{x}{20}\right)$

(C) $y = 0.06\sin 8\pi\left(t + \frac{x}{20}\right)$

(D) $y = -0.06\sin 8\pi\left(t + \frac{x}{20}\right)$

74. If the temperature is raised by 1 K from 300 K the percentage change in the speed of sound in a gaseous mixture is ($R = 8.31$ J/mole-K)

(A) 0.167% (B) 2%

(C) 1% (D) 0.334%

75. A wave is represented by the equation: $y = 0.1\sin(100\pi t - kx)$. If wave velocity is 100 m/s, its wave number is equal to

(A) 1 m^{-1} (B) 2 m^{-1}

(C) $\pi\, m^{-1}$ (D) $2\pi\, m^{-1}$

76. A racing car moving towards a cliff sounds its horn. The driver observes that the sound reflected from the cliff has a frequency one octave higher than the actual frequency of the horn. If v is the velocity of sound, then the velocity of the car is

(A) $\frac{v}{2}$ (B) $\frac{v}{\sqrt{2}}$ (C) $\frac{v}{4}$ (D) $\frac{v}{3}$

77. The power of sound from the speaker of a radio is 20 mW. By turning the knob of volume control, the power of sound is increased to 400 mW. The power increase in dB as compared to the original power is ($\log_{10} 2 = 0.3$)

(A) 1.3 dB (B) 3.1 dB

(C) 13 dB (D) 30.1 dB

78. For a carrier frequency of 100 kHz and a modulating frequency of 5 kHz, what is the band width of AM transmission?

(A) 5 kHz (B) 10 kHz

(C) 20 kHz (D) 200 kHz

79. The phase difference between two points separated by 0.8 m in a wave of frequency 120 Hz is 0.5π. The wave velocity is

(A) 144 m/s

(B) 256 m/s

(C) 384 m/s

(D) 720 m/s

80. The end correction of a resonance column is 1.0 cm. If the shortest length resonating with a tuning fork is 14.0 cm, the next resonating length is

(A) 44 cm (B) 45 cm

(C) 46 cm (D) 47 cm

81. A ball is dropped into a well in which the water level is at a depth h below the top ($t = 0$). If the speed of sound be c, then the time after which the splash is heard will be given by

(A) $h\left[\sqrt{\frac{2}{gh}}+\frac{1}{c}\right]$ (B) $h\left[\sqrt{\frac{2}{gh}}-\frac{1}{c}\right]$

(C) $h\left[\frac{2}{g}+\frac{1}{c}\right]$ (D) $h\left[\frac{2}{g}-\frac{1}{c}\right]$

82. A sonometer wire is in unison with a tuning fork in fundamental mode. Keeping the same tension, the length of wire between the bridges is doubled. The tuning fork can still be in resonance with the wire, provided the wire now vibrates in

(A) 4 segments (B) 6 segments
(C) 3 segments (D) 2 segments

83. The driver of a car approaching a vertical wall notices that the frequency of the horn of his car changes from 400 Hz to 450 Hz after being reflected from the wall. Assuming speed of sound to be 340 ms^{-1}, the speed of approach of car towards the wall is

(A) 10 ms^{-1} (B) 20 ms^{-1}
(C) 30 ms^{-1} (D) 40 ms^{-1}

84. Following are equations of four waves:

(i) $y_1 = a\sin\omega\left(t-\frac{x}{v}\right)$

(ii) $y_2 = a\sin\omega\left(t+\frac{x}{v}\right)$

(iii) $z_1 = a\sin\omega\left(t-\frac{x}{v}\right)$

(iv) $z_2 = a\cos\omega\left(t+\frac{x}{v}\right)$

Which of the following statement is correct?

(A) On superposition of waves (i) and (iii), a traveling wave having amplitude a will be formed.
(B) Superposition of waves (ii) and (iii) is not possible.
(C) On superposition of (i) and (ii), a stationary wave having amplitude $a\sqrt{2}$ will be formed.
(D) On superposition of (iii) and (iv), a transverse stationary wave will be formed.

85. A wave is represented by the equation: $y = 0.1\sin(100\pi t - kx)$. If wave velocity is 100 m/s, its wave number is equal to

(A) 1 m^{-1} (B) 2 m^{-1}
(C) $\pi\,m^{-1}$ (D) $2\pi\,m^{-1}$

86. A stretched wire of some length under a tension is vibrating with its fundamental frequency. Its length is decreased by 45% and tension is increased by 21%. Now its fundamental frequency (assuming linear mass density remains the same)

(A) increases by 50%
(B) increases by 100%
(C) decreases by 50%
(D) decreases by 25%

87. Two trains, one coming towards and another going away from an observer both at 4 m/s produce a whistle simultaneously of frequency 300 Hz. The number of beats heard by observer will be (velocity of sound = 340 m/s)

(A) 5 (B) 6
(C) 7 (D) 12

88. Speed of sound wave is v. If a reflector moves towards a stationary source emitting waves of frequency f with speed u, the frequency of reflected wave will be

(A) $\frac{v-u}{v+u}f$ (B) $\frac{v+u}{v}f$

(C) $\frac{v+u}{v-u}f$ (D) $\frac{v-u}{v}f$

89. The intensity of sound after passing through a slab decreases by 20%. On passing through two such slabs, the intensity will decrease by

(A) 50% (B) 40%
(C) 36% (D) 30%

90. The length of a sonometer wire AB is 110 cm. Where should the two bridges be placed from A to divide the wire in three segments whose fundamental frequencies are in the ratio of 1 : 2 : 3 ?

(A) 30 cm, 90 cm (B) 60 cm, 90 cm
(C) 40 cm, 70 cm (D) None of these

91. A sound wave of wavelength λ travels towards the right horizontally with a velocity v. It strikes and reflects from a vertical plane surface, traveling at a speed v towards the left. The number of positive crests striking in a time interval of three seconds on the wall is

(A) $3(V+v)/\lambda$ (B) $3(V-v)/\lambda$
(C) $(V+v)/3\lambda$ (D) $(V-v)/3\lambda$

92. A stationary source of sound is emitting waves of frequency 30 Hz towards a stationary wall. There is an observer standing between the source and the wall. If the wind blows from the source to the wall with a speed 30 m/s, then the number of beats heard by the observer is (velocity of sound with respect to wind is 330 m/s)

(A) 10 (B) 3
(C) 6 (D) Zero

93. The driver of a car traveling with speed 30 m/s towards a hill sound a horn of frequency 600 Hz. If the velocity of sound in air is 330 m/s, the frequency of reflected sound as heard by the driver is

(A) 720 Hz (B) 555.5 Hz
(C) 550 Hz (D) 500 Hz

94. A wave disturbance in a medium is described by

$$y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$$

where x and y are in meter and t is in second. Then

(A) First node occurs at $x = 0.15$ m.
(B) First anti-node occurs at $x = 0.3$ m.
(C) The speed of interfering waves is 5.0 m/s.
(D) The wavelength is 0.5 m.

95. A train is moving with a constant speed along a circular track. The engine of the train emits a sound of frequency f. The frequency heard by the guard at rear end of the train

(A) is less than f.
(B) is greater than f.
(C) is equal to f.
(D) may be greater than, less than or equal to f depending on the factors like speed of train, length of train and radius of circular track.

96. A massless rod AB of length L is hung from two identical wires of equal length. A block of mass m is attached at point O on the rod as shown in Fig. 9.28; the value of AO so that a tuning fork excites the wire on the left in its fundamental tone and the wire on the right in its second harmonic is

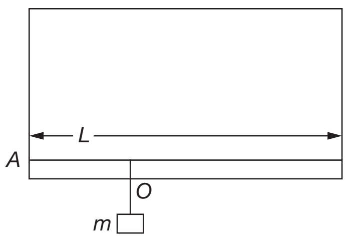


Fig. 9.28

(A) $\frac{4L}{5}$ (B) $\frac{L}{4}$
(C) $\frac{3L}{4}$ (D) $\frac{L}{5}$

97. The frequency of a sonometer wire is 100 Hz. When the weights producing the tensions are completely immersed in water, the frequency becomes 80 Hz and on immersing the weights in a certain liquid, the frequency becomes 60 Hz. The specific gravity of the liquid is

(A) 1.42 (B) 1.77 (C) 1.82 (D) 1.21

98. A siren creates a sound level of 60 dB at a location of 500 m from the speaker. The siren is powered by a battery that delivers a total energy of 1 kJ. The efficiency of siren is 30%. The total time for which the siren sound is

(A) 95 s (B) 95.5 s
(C) 96 s (D) 96.5 s

99. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz, while the train approaches the siren. During his return journey in a different train B, he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is

(A) 242/252 (B) 2
(C) 5/6 (D) 11/6

100. A stationary wave is set up on a string fixed at both ends. The distance between two consecutive nodes is 18 cm at a particular mode of vibration and for the next higher mode of vibration in the same string the distance between two consecutive nodes is 16 cm. The length of string is

(A) 144 cm (B) 140 cm
(C) 36 cm (D) 32 cm

101. Two waves are represented as $y_1 = 2a\sin(\omega t + \pi/6)$ and $y_2 = -2a\cos\left(\omega t - \frac{\pi}{6}\right)$. The phase difference between the two waves is

(A) $\frac{\pi}{3}$ (B) $\frac{4\pi}{3}$
(C) $\frac{3\pi}{3}$ (D) $\frac{5\pi}{6}$

102. Two cars are moving towards each other with same speed, if frequency of horn blown by driver of one car and frequency appeared to another driver differ by 4% from the frequency of horn, then find out speed of cars (speed of sound = 300 m/s)

(A) 12 m/s (B) 6.6 m/s
(C) 4.2 m/s (D) 5.9 m/s

103. If the velocity of sound in air is 320 m/s, then (maximum and minimum audible frequency are 20 Hz and 20000 Hz, respectively), the maximum and minimum lengths of a closed pipe that would produce a just audible sound are

(A) 2.6 m and 3.6×10^{-3} m
(B) 4 m and 4.2×10^{-3} m
(C) 3 m and 3×10^{-3} m
(D) 4 m and 4×10^{-3} m

104. Four sources of sound each of sound level 10 dB are sounded together; the resultant intensity level will be (log 2 = 0.3)

(A) 40 dB (B) 26 dB
(C) 16 dB (D) 13 dB

105. The air in an open pipe of length 36 cm long is vibrating with 2 nodes and 2 antinodes. The temperature of the air inside the pipe is 51°C. What is the wavelength of waves produced in air outside the tube where the temperature of air is 16°C?

(A) 32.1 cm (B) 68 cm
(C) 34 cm (D) 10.2 cm

106. A dog while barking delivers about 1 mW of power. If this power is uniformly over a hemispherical area, what is the sound level at a distance $\frac{5}{\sqrt{\pi}}$ m?

(A) 73 dB (B) 96 dB
(C) 32 dB (D) 40 dB

107. Two identical wires are stretched by the same tension of 100N and each emits a note of frequency 200Hz. If tension in one wire is increased by 1N, the number of beats heard per second when the wires are plucked is

(A) 2 (B) 1 (C) 3 (D) 4

108. An open organ pipe is vibrating in its fifth overtone. The distance between two consecutive points where pressure amplitude is $\frac{1}{\sqrt{2}}$ times pressure amplitude at pressure antinodes is 40 cm. Then the length of organ pipe is (Neglect end correction)

(A) 3 m (B) 3.6 m (C) 4.2 m (D) 4.8 m

109. A particle is subjected to two SHM along x and y axis, according to $x = 6 \sin 100\pi t$ and $y = 8 \cos\left(100\pi t - \frac{\pi}{2}\right)$, then motion of particle is

(A) Ellipse
(B) Circle
(C) Straight line
(D) None of these

110. Two wave pulses are generated in a string. One of the pulses is given by equation $y_1 = A\sin(\omega t - kx)$. If average power transmitted by both the pulses along the string are same and is given by $P = \frac{TA^2\omega^2}{2v}$, where T is the tension in the string, A is amplitude of a pulse, ω is angular frequency of the source, and v is wave velocity, then which one of the following equations may represent the other wave pulse?

(A) $y_2 = \frac{A}{\sqrt{2}}\sin(2\omega t - kx)$

(B) $y_2 = \frac{A}{\sqrt{2}}\sin(\omega t - 2kx)$

(C) $y_2 = 2A\sin\left(\frac{\omega t}{2} - kx\right)$

(D) $y_2 = 2A\sin\left(\frac{\omega t}{2} - \frac{kx}{2}\right)$

111. A knife edge divides a sonometer wire into two parts, which differ in length by 2 mm. The whole length of the wire is 1 metre. The two parts of the string when sounded together produce one beat per second, then the frequencies of the smaller and longer parts are

(A) 250.5 and 249.5
(B) 249.5 and 250.5
(C) 124.5 and 125.5
(D) 125.5 and 124.5

112. Oxygen is 16 times heavier than hydrogen. Equal volumes of hydrogen and oxygen are mixed. The ratio of the velocity of sound in the mixture to that of oxygen is

(A) $\sqrt{\frac{1}{8}}$ (B) $\sqrt{\frac{32}{17}}$

(C) $\sqrt{\frac{17}{32}}$ (D) $\sqrt{8}$

113. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad/sec in the horizontal plane. Then the range of frequencies heard by an observer stationed at a large distance from the whistle will be ($v = 330$ m/s)

(A) 400.0 Hz to 487.0 Hz
(B) 403.3 Hz to 480.0 Hz
(C) 400.0 Hz to 480.0 Hz
(D) 403.3 Hz to 484.0 Hz

114. The frequency and wavelength of the wave shown in Fig. 9.29 are (wave speed = 320 m/s)

(A) 8 cm, 400 Hz
(B) 80 cm, 40 Hz
(C) 8 cm, 4000 Hz
(D) 40 cm, 8000 Hz

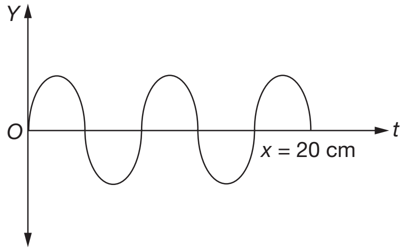


Fig. 9.29

115. Which frequency can be reflected from ionosphere?
(A) 5 MHz (B) 6 GHz
(C) 5 KHz (D) 500 MHz

116. The range of frequencies allotted for FM radio is
(A) 88 to 108 kHz
(B) 88 to 108 MHz
(C) 47 to 230 kHz
(D) 47 to 230 MHz

117. A stretched sonometer wire is in unison with a tuning fork. When the length of the wire is increased by 2%, the number of beats heard per second is 5. Then the frequency of the fork is
(A) 245 Hz (B) 250 Hz
(C) 255 Hz (D) 260 Hz

118. A transverse wave is travelling along a string from left to right. Fig. 9.30 represents the shape of the string at a given instant. At this instant, among the following, choose the wrong statement
(A) Points *D*, *E*, *F* have upwards positive velocity
(B) Points *A*, *B,* and *H* have downwards negative velocity
(C) Point *C* and *G* have zero velocity
(D) Points *A* and *E* have minimum velocity

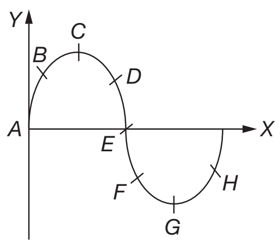


Fig. 9.30

119. Let a disturbance *y* be propagated as a plane wave along the *x*-axis. The wave profiles at the instants $t = t_1$ and $t = t_2$ are represented, respectively, as $y_1 = f(x_1 - vt_1)$ and $y_2 = f(x_2 - vt_2)$. The wave is propagating without change of shape.
(A) The velocity of the wave is $2v$.
(B) The velocity of the wave is $v = \frac{x_2 - x_1}{t_2}$.
(C) The particle velocity is $v_p = v$.
(D) None of these.

120. A tuning fork of frequency 340 Hz is vibrated just above a cylindrical tube of length 120 cm. Water is slowly poured in the tube. If the speed of sound is 340 m/s, then the minimum height of water required for resonance is
(A) 25 cm (B) 45 cm
(C) 75 cm (D) 95 cm

121. Two waves traveling in opposite directions produce a standing wave. The individual wave functions are given by $y_1 = 4\sin(3x - 2t)$ cm and $y_2 = 4\sin(3x + 2t)$ cm, *x* and *y* are in cm. Now, select the correct statement:
(A) Nodes are formed at $x = 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}$.....
(B) Anti-nodes are formed at $x = 0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}$.....
(C) Nodes are formed at $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$.....
(D) Anti-nodes are formed at $x = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$.....

122. The area of region covered by the TV broadcast by a TV tower of 100 m height will be (radius of the earth = 6.4×10^6 m)
(A) $12.8\pi \times 10^8$ km^2
(B) $1.28\pi \times 10^3$ km^2
(C) $0.64\pi \times 10^3$ km^2
(D) 1.28×10^3 km^2

123. For a particular mode of vibration of string, the distance between two consecutive nodes is 18 cm. For the next higher mode, the distance becomes 16 cm. The length of the string is
(A) 18 cm (B) 16 cm
(C) 144 cm (D) 72 cm

124. An organ pipe of length 33 cm closed at one end vibrates in its 5th overtone. If amplitude of a particle at anti-nodes is 6 mm, then amplitude of a particle which is at a distance 18 cm from closed end is
(A) 3 cm (B) $3\sqrt{2}$ mm
(C) 2 mm (D) Zero

125. A stationary source of sound is emitting waves of frequency 30Hz towards a stationary wall. There is an observer standing between the source and the wall. If the wind blows from the source to the wall with a speed 30 m/s then the number of beats heard by the observer is (velocity of sound with respect to wind is 330 m/s)

(A) 10 (B) 3 (C) 6 (D) Zero

126. Two sources of sound are moving in opposite directions with velocities v_1 and v_2 ($v_1 > v_2$). Both are moving away from a stationary observer. The frequency of both the source is 1700 Hz. What is the value of ($v_1 - v_2$) so that the beat frequency observed by the observer is 10 Hz. $v_{sound} = 340$ m/s and assume that v_1 and v_2 both are very much less than v_{sound}.

(A) 1 m/s (B) 2 m/s
(C) 3 m/s (D) 4 m/s

127. A 3.6 m long vertical pipe is filled completely with a liquid. A small hole is drilled at the base of the pipe due to which liquids starts leaking out. This pipe resonates with a tuning fork. The first two resonances occur when height of water column is 3.22 m and 2.34 m, respectively. The area of cross-section of pipe is

(A) 25 π cm^2 (B) 100 π cm^2
(C) 200 π cm^2 (D) 400 π cm^2

128. An organ pipe of length 3.9 π m open at both ends is driven to third harmonic standing wave pattern. If the maximum amplitude of pressure oscillations is 1% of mean atmospheric pressure (P_0 = 105 N/m^2), the maximum displacement of the particle from mean position will be (Velocity of sound = 200 m/s and density of air = 1.3 kg/m^3)

(A) 2.5 cm (B) 5 cm
(C) 1 cm (D) 2 cm

129. Two trains (*A* and *B*) are moving towards each other on two parallel tracks at the same speed with respect to the ground. The whistle of train *A* blows. In which of the following cases, the frequency of the sound heard by a passenger on the other train *B* will be greatest?

(A) If the air is still.
(B) If a wind blows in the same direction and at the same speed as the other train *B*.
(C) If a wind blows in the opposite direction and at the same speed as the other train *B*.
(D) Frequency will be same in the above three cases.

130. A tuning fork of known frequency is held at the open end of a long tube, which is dipped into water as shown in Fig. 9.31. The tuning fork of frequency 165 Hz resonates with air column, when air column is vibrating in 1st and 3rd harmonic with air column lengths l_1 = (50 ± 0.5) cm and l_2 = (150 ± 0.1) cm, respectively. The speed of sound in air column is

(A) (320 ± 1.98) m/s
(B) (330 ± 1.98) m/s
(C) (320 ± 0.99) m/s
(D) (330 ± 0.99) m/s

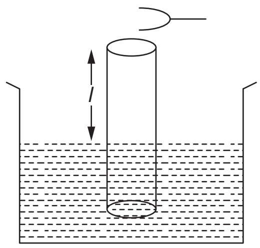


Fig. 9.31

131. A string of length *l* is fixed at both ends. It is vibrating in its third overtone. Maximum amplitude of the particles on the string is *A*. The amplitude of the particle at a distance *l*/3 from one end is

(A) *A* (B) 0
(C) $\frac{\sqrt{3}A}{2}$ (D) $\frac{A}{2}$

132. A closed organ pipe of length 99.4 cm is vibrating in its first overtone and in always resonance with a tuning fork having frequency $f = (300 - 2t)$ Hz, where *t* is time in second. The rate by which radius of organ pipe changes when its radius is 1 cm is (speed of sound in organ pipe = 320 m/s)

(A) $\frac{1}{72}$ m/s (B) $\frac{1}{36}$ m/s
(C) $\frac{1}{18}$ m/s (D) $\frac{1}{9}$ m/s

133. A closed organ pipe of length *L* is vibrating in its first overtone. There is a point *Q* inside the pipe at a distance 7*L*/9 from the open end. The ratio of pressure amplitude at *Q* to the maximum pressure amplitude in the pipe is

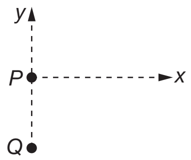
(A) 1 : 2 (B) 2 : 1
(C) 1 : 1 (D) 2 : 3

134. The general wave equation can be written as

$$y = m(x - vt), x \in \left[vt, vt + \frac{a}{2}\right];$$

$$y = -m[(x - vt) - a], \ x \in \left[vt + \frac{a}{2}, vt + a\right]$$

135. Two identical sources P and Q emit waves in same phase and of same wavelength. Spacing between P and Q is 3λ. The maximum distance from P along the x-axis at which a minimum intensity occurs is given by

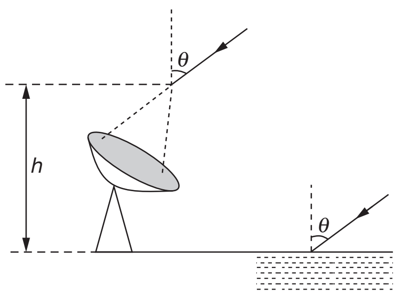


(A) 6.58λ (B) 2.25λ
(C) 8.75λ (D) 0.55λ

136. Two longitudinal waves propagating in the X and Y directions superimpose. The wave equations are as below $\psi_1 = A\cos(\omega t - kx)$ and $\psi_2 = A\cos(\omega t - ky)$. Trajectory of the motion of a particle lying on the line $y = x + \frac{(2n+1)\lambda}{2}$ will be

(A) Straight line
(B) Circle
(C) Ellipse
(D) None of these

137. Radio waves of wavelength λ at an angle θ to vertical are received by a radar after reflecting from a nearby water surface and directly. If the radar records a maximum intensity, the height of antenna h from water surface can be



(A) $\frac{\lambda}{2\cos\theta}$ (B) $\frac{\lambda}{2\sin\theta}$

(C) $\frac{\lambda}{4\sin\theta}$ (D) $\frac{\lambda}{4\cos\theta}$

138. Water waves produced by a motorboat sailing in water are

(A) neither longitudinal no transverse.
(B) both longitudinal and transverse.
(C) only longitudinal.
(D) only transverse.

139. Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where its speed in $2v$ m/s. Wavelength of sound waves in the second medium is

(A) λ (B) $\frac{\lambda}{2}$

(C) 2λ (D) 4λ

140. Speed of sound wave in air

(A) is independent of temperature.
(B) increases with pressure.
(C) increases with increase in humidity.
(D) decreases with increase in humidity.

141. Change in temperature of the medium changes

(A) frequency of sound waves.
(B) amplitude of sound waves.
(C) wavelength of sound waves.
(D) loudness of sound waves.

142. With propagation of longitudinal waves through a medium, the quantity transmitted is

(A) Matter
(B) Energy
(C) Energy and matter
(D) Energy, matter, and momentum

143. Which of the following statements are true for wave motion?

(A) Mechanical transverse waves can propagate through all mediums.
(B) Longitudinal waves can propagate through solids only.
(C) Mechanical transverse waves can propagate through solids only.
(D) Longitudinal waves can propagate through vacuum.

144. A sound wave is passing through air column in the form of compression and rarefaction. In consecutive compressions and rarefactions,

(A) density remains constant.
(B) Boyle's law is obeyed.
(C) bulk modulus of air oscillates.
(D) there is no transfer of heat.

145. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t - \frac{x}{2}\right)$. On reflection from a denser medium, its amplitude becomes $\frac{2}{3}$ of the amplitude of the incident wave. The equation of the reflected wave is

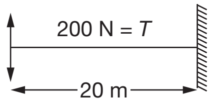
(A) $y = 0.6 \sin 2\pi\left(t + \frac{x}{2}\right)$

(B) $y = -0.4 \sin 2\pi\left(t + \frac{x}{2}\right)$

(C) $y = 0.4 \sin 2\pi\left(t + \frac{x}{2}\right)$

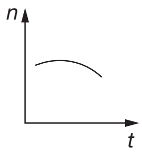
(D) $y = -0.4 \sin 2\pi\left(t - \frac{x}{2}\right)$

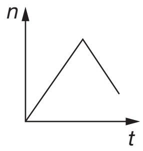
146. A string of mass 2.5 kg is under tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in

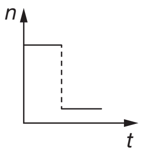


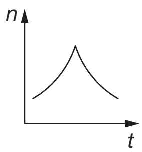
(A) 1 s
(B) 0.5 s
(C) 2 s
(D) Data given is insufficient

147. A train whistling at constant frequency is moving towards a station at a constant speed v. The train goes past a stationary observer on the station. The frequency n of the sound as heard by the observer is plotted as a function of time t (Fig. 9.32). Identify the expected curve.

(A) 

(B) 

(C) 

(D) 

More than One Option Correct Type

148. A particle moves on the x-axis as per the equation $x = x_0 \sin^2 \omega t$. The motion is simple harmonic

(A) With amplitude x_0
(B) With amplitude $2x_0$
(C) With time period $\frac{2\pi}{\omega}$
(D) With time period $\frac{\pi}{\omega}$

149. A particle starts SHM at time $t = 0$. Its amplitude is A and angular frequency is ω. At time $t = 0$, its kinetic energy is $\frac{E}{4}$, where E is total energy. Assuming potential energy to be zero at mean position, the displacement-time equation of the particle can be written as

(A) $x = A\cos\left(\omega t + \frac{\pi}{6}\right)$

(B) $x = A\sin\left(\omega t + \frac{\pi}{3}\right)$

(C) $x = A\sin\left(\omega t - \frac{2\pi}{3}\right)$

(D) $x = A\cos\left(\omega t - \frac{\pi}{6}\right)$

150. A particle moves along the x-axis as per the equation $x = 4 + 3\sin(2\pi t)$. Here x is in cm and t in seconds. Select the correct alternative(s)

(A) The motion of the particle is simple harmonic with mean position at $x = 0$.
(B) The motion of the particle is simple harmonic with mean position at $x = 4$ cm.
(C) The motion of the particle is simple harmonic with mean position at $x = -4$ cm.
(D) Amplitude of oscillation is 3 cm.

151. If y, u, and a represent displacement, velocity, and acceleration at any instant for a particle executing SHM, which of the following statements are true?

(A) v and y may have same direction.
(B) v and a have same direction twice in each cycle.

(C) a and y may have same direction.
(D) a and v never have same direction.

152. A book with many printing errors contains four different expressions for the displacement y of a particle executing SHM. Which of the following expressions are wrong?

(A) $y = A\sin\left(\frac{2\pi t}{T}\right)$

(B) $y = A\sin vt$

(C) $y = \frac{A}{T}\sin\left(\frac{t}{A}\right)$

(D) $y = \frac{A}{\sqrt{2}}(\sin\omega t + \cos\omega t)$

153. If force (F) versus displacement (x) and displacement (x) versus time graph of a particle performing SHM is shown in Fig. 9.33. Then choose correct statement.

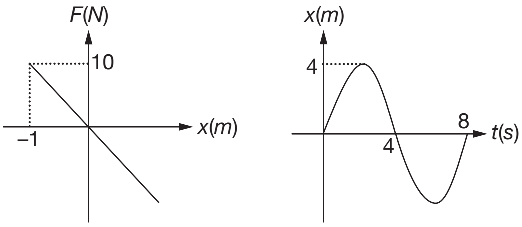


Fig. 9.33

(A) Mass of the particle $\frac{160}{\pi^2}$ kg.
(B) Mass of the particle 160 π^2 kg.
(C) Maximum kinetic energy of particle is 80 J.
(D) Maximum kinetic energy of particle is 40 J.

154. Two SHMs are represented by the equations:
$Y_1 = 10\sin[3\pi t + \pi/4]$
$Y_2 = 5[\sin 3\pi t + \sqrt{3}\cos 3\pi t]$
(A) The amplitude ratio of the two SHM is 1 : 1.
(B) The amplitude ratio of the two SHM is 2 : 1.
(C) Time periods of both the SHMs are equal.
(D) Time periods of two SHMs are different.

155. In Fig. 9.34, the block of mass m is in equilibrium initially. Now the block is pushed down by a slight distance and released (springs are identical and massless having spring constant k). Then

(A) Initial elongation of the spring is $\frac{mg}{2k\cos\theta}$.

(B) Initial elongation of the spring is $\frac{mg}{2k\cos^2\theta}$.

(C) Time period of oscillation of the block is $2\pi\sqrt{\frac{m}{2k\cos^2\theta}}$.

(D) Time period of oscillation of the block is $2\pi\sqrt{\frac{m}{2k}}$.

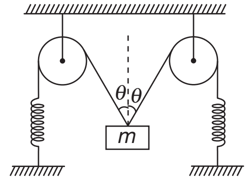


Fig. 9.34

156. Two pendulums of same amplitude but time period 3 s and 7 s start oscillating simultaneously from two opposite extreme positions. After how much time they will be in phase

(A) $\frac{21}{8}$ s (B) $\frac{21}{4}$ s
(C) $\frac{21}{2}$ s (D) $\frac{21}{10}$ s

157. A particle of mass m is moving in a field where the potential energy is given by $U(x) = U_0(1-\cos ax)$, where U_0 and a are constants and x is the displacement from mean position. Then (for small oscillations)

(A) The time period is $T = 2\pi\sqrt{\frac{m}{aU_0}}$.

(B) The speed of particle is maximum at $x = 0$.

(C) The amplitude of oscillations is $\frac{\pi}{a}$.

(D) The time period is $T = 2\pi\sqrt{\frac{m}{a^2U_0}}$.

158. Two SHMs are represented by the equations:
$Y_1 = 10\sin[3\pi t + \pi/4]$
$Y_2 = 5\cos\pi t$
(A) The amplitude ratio of the two SHM is 1 : 1.
(B) The amplitude ratio of the two SHM is 2 : 1.
(C) Time periods of both the SHMs are equal.
(D) Time periods of two SHMs are different.

159. A particle of mass m moves in a straight line. If v is the velocity at a distance x from a fixed point on the line and $v^2 = a - bx^2$, where a and b are constant, then

(A) The motion continues along the positive x-direction only.
(B) The motion is simple harmonic.
(C) The particle oscillates with a frequency equal to $\frac{\sqrt{b}}{2\pi}$.
(D) The total energy of the particle is ma.

160. A particle of mass m is attached to three identical springs A, B, and C each of force constant k as shown in Fig. 9.35. If the particle of mass m is pushed slightly against the spring A and released, then the time period of oscillation

(A) Extension in springs are same

(B) $2\pi\sqrt{\dfrac{m}{2k}}$

(C) Extension in A is different from B and C

(D) $2\pi\sqrt{\dfrac{m}{3k}}$

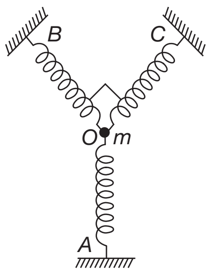
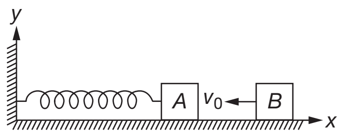


Fig. 9.35

161. A particle vibrates in SHM along a straight line. Its greatest acceleration is $5\pi^2$ cm s^{-2} and its distance from the equilibrium position is 4 cm, the velocity of the particle is 3π cms^{-1}, then

(A) The amplitude is 10 cm
(B) The period of oscillation 2 s
(C) The amplitude is 5 cm
(D) The period of oscillation 4 s

162. A block A of mass m connected with a spring of force constant k is executing SHM. The position (x) and time (t) equation of the block is $x = x_0 + a\sin\omega t$. An identical block B moving towards negative x-axis with velocity v_0 collides elastically with block A at time $t = 0$. Then



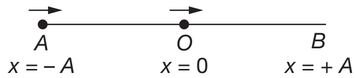
(A) Displacement time equation of A after collision will be $x = x_0 - v_0\sqrt{\dfrac{m}{k}}\sin\omega t$.

(B) Displacement time equation of A after collision will be $x = x_0 + v_0\sqrt{\dfrac{m}{k}}\sin\omega t$.

(C) Velocity of B just after collision will be $a\,\omega$ towards positive x-direction.

(D) Velocity of B just after collision will be v_0 towards positive x-direction.

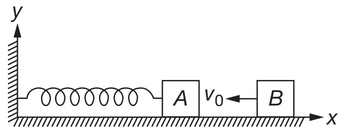
163. Two particles undergo SHM along the same line with the same time period (T) and equal amplitudes (A). At a particular instant, one particle is at $x = -A$ and the other is at $x = 0$. They move in the same direction. They will cross each other at time t and at position x then



(A) $t = \dfrac{4T}{3}$ (B) $t = \dfrac{3T}{8}$

(C) $x = \dfrac{A}{2}$ (D) $x = \dfrac{A}{\sqrt{2}}$

164. A block A of mass m connected with a spring of force constant k is executing SHM. The position (x) and time (t) equation of the block is $x = x_0 + a\sin\omega t$. An identical block B moving towards negative x-axis with velocity v_0 collides elastically with block A at time $t = 0$. Then



(A) Displacement time equation of A after collision will be $x = x_0 - v_0\sqrt{\dfrac{m}{k}}\sin\omega t$.

(B) Displacement time equation of A after collision will be $x = x_0 + v_0\sqrt{\dfrac{m}{k}}\sin\omega t$.

(C) Velocity of B just after collision will be $a\omega$ towards positive x-direction.

(D) Velocity of B just after collision will be v_0 towards positive x-direction.

165. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound at frequency

(A) 80 Hz (B) 240 Hz
(C) 320 Hz (D) 400 Hz

166. A sound wave of frequency ν travels horizontally to the right. It is reflected from a large vertical plane surface moving to the left with a speed V. The speed of sound in the medium is c.

(A) The number of wave pulse striking the surface per second is $\frac{v(c+V)}{c}$

(B) The wavelength of the reflected wave is $\frac{c(c-V)}{v(c+V)}$

(C) The frequency of the reflected wave is $\frac{v(c+V)}{(c-V)}$

(D) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vV}{c-V}$

167. A traveling wave pulse is given by $y=\frac{6}{2+(x+3t)^2}$, where symbols have their usual meanings, x, y are in metre and t is in second. Then

(A) The pulse is traveling along +ve x-axis with velocity 3 m/s.

(B) The pulse is traveling along –ve x-axis with velocity 3 m/s.

(C) The amplitude of the wave pulse is 3 m.

(D) The pulse is a symmetric pulse.

168. $Y(x,t)=\frac{0.8}{\left[(4x+5t)^2+5\right]}$ represents a moving pulse, where x and y are in metres and t in second. Then

(A) Pulse is moving in positive x-direction

(B) In 2s it will travel a distance of 2.5 m

(C) Its maximum displacement is 0.16 m

(D) It is a symmetric pulse at $t = 0$

169. Two monochromatic coherent point sources S_1 and S_2 are separated by a distance L. Each source emits light of wavelength λ, where $L >> \lambda$. The line S_1S_2 when extended meets a screen perpendicular to it at a point A.

(A) The interference fringes on the screen are circular in shape.

(B) The interference fringes on the screen are straight lines perpendicular to the line S_1S_2A.

(C) The point A is an intensity maxima if $L = n\lambda$.

(D) The point A is always an intensity maxima for any separation L.

170. For a certain stretched string, three consecutive resonance frequencies are observed as 105, 175, 245 Hz, respectively. Then select the correct alternatives

(A) The string is fixed at both ends.

(B) The string is fixed at one end only.

(C) The fundamental frequency is 35 Hz.

(D) The fundamental frequency is 52.5 Hz.

171. A source of sound moves along a circle of radius 2 m with constant angular velocity 40 rad/s. Frequency of the source is 300 Hz. A detector is kept at some distance from the circle in the same plane of the circle (as shown in Fig. 9.36). Which of the following is not the possible value of frequency registered by the detector? (Speed of sound = 320 m/s)

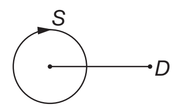


Fig. 9.36

(A) 250 Hz (B) 360 Hz
(C) 410 Hz (D) 220 Hz

172. A wave disturbance in a medium is described by $y(x,t)=0.02\cos\left(50\pi t+\frac{\pi}{2}\right)\cos(10\pi x)$, where x and y are in meter and t is in second. Then

(A) First node occurs at $x = 0.15$ m

(B) First anti-node occurs at $x = 0.3$ m

(C) The speed of interfering waves is 5.0 m/s

(D) The wavelength is 0.2 m

173. A 10 m long horizontal stainless steel wire AB of mass 1 kg, whose end A is fixed, is connected to a massless string BC passing over a smooth pulley. String BC is connected to a container of mass 2 kg at end C. Water (density = 1 × 103 kg/m^3) is poured in the container at a constant rate of 2.25 litre/sec at t = 0. Also at t = 0, a pulse is generated at end A.

(A) Time taken by the pulse to reach point B is 0.612 s

(B) Time taken by the pulse to reach point B is 0.212 s

(C) Tension in the string at this moment is 33.77 N

(D) Tension in the string at this moment is 24.77 N

174. The (x, y) co-ordinates of the corners of a square plate are (0, 0), (L, 0), (L, L), and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If u (x, y) denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is (are) (a = positive constant)

(A) $a\cos(\pi x/2L)\cos(\pi y/2L)$

(B) $a\sin(\pi x/L)\sin(\pi y/L)$

(C) $a\sin(\pi x/L)\sin(2\pi y/L)$

(D) $a\cos(2\pi x/L)\sin(\pi y/L)$

Passage Based Questions

Passage 1

When more than one force (say two forces) acts on a system, it might produce more than one SHM. The combination may form another SHM depending on the direction of such SHM, the amplitude will vary. The phase difference between the two also has a role to play in deciding the resultant amplitude. However, when the superimposed SHMs are in perpendicular direction, the pattern may change not only with phase but also with frequencies. When different oscillating systems are connected, there can be an influence of one on another.

175. When two SHMs in the same direction with amplitude A_1 and A_2 are superimposed, the resultant amplitude will be

(A) $|A_1 + A_2|$ always
(B) $|A_1 + A_2|$ for $\delta = \pi$
(C) $|A_1 - A_2|$ for $\delta = 0$
(D) between $|A_1 - A_2|$ and $(A_1 + A_2)$ if $0 \le \delta \le \pi$

176. The track followed for two perpendicular SHMs is a perfect ellipse when (δ-phase difference, A_1, A_2 amplitudes)

(A) $\delta = \frac{\pi}{4}, A_1 \neq A_2$ (B) $\delta = \frac{3\pi}{4}, A_1 \neq A_2$
(C) $\delta = \frac{\pi}{2}, A_1 \neq A_2$ (D) $\delta = \pi, A_1 = A_2$

177. If $Y_1 = 5\sin(\omega t)$ and $Y_2 = 5\left[\sqrt{3}\sin\omega t + \cos\omega t\right]$ are two SHMs, the ratio of their amplitude is

(A) $1:\sqrt{3}$ (B) 1 : 3
(C) 1 : 2. (D) $1:\cos\left(\frac{\pi}{6}\right)$

Passage 2

A particle of mass m is attached to one end of the light inextensible string and other end of the string is fixed in vertical plane as shown in Fig. 9.37. Particle is given the horizontal velocity $u = \sqrt{\frac{5}{2}gl}$.

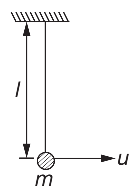


Fig. 9.37

178. The maximum angle made by the particle with downward vertical is

(A) $\cos^{-1}\left(\frac{1}{4}\right)$ (B) $\sin^{-1}\left(\frac{1}{4}\right)$
(C) $\frac{\pi}{2} + \cos^{-1}\left(\frac{1}{4}\right)$ (D) $\pi - \cos^{-1}\left(\frac{1}{4}\right)$

179. The tension in string at an instant when acceleration of the particle is horizontal is

(A) mg (B) 2 mg
(C) 4 mg (D) 6 mg

180. The string makes an angle θ with downward vertical when acceleration of the particle is horizontal, where θ is

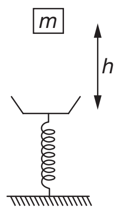
(A) 30° (B) 60°
(C) 120° (D) $\pi - \cos^{-1}\left(\frac{1}{4}\right)$

181. The particle will

(A) Oscillate about mean position.
(B) Leave the vertical circle at some point.
(C) Complete the vertical circle.
(D) None of these.

Passage 3

A body of mass m fell from a height h at $t = 0$ onto the pan of a spring balance. The masses of the pan and the spring are negligible. The spring constant of the spring is $k = \frac{3mg}{2h}$. Having stuck to the pan, the body starts performing harmonic oscillations in vertical direction.



182. Find the time period of oscillations.

(A) $2\pi\sqrt{\frac{2h}{3g}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{3g}{2h}}$
(C) $\pi\sqrt{\frac{2h}{3g}}$ (D) $\frac{1}{\pi}\sqrt{\frac{3g}{2h}}$

183. Speed of the block when acceleration of the block is zero is

(A) $\sqrt{2gh}$ (B) $\sqrt{\frac{8}{3}gh}$
(C) $2\sqrt{gh}$ (D) $\frac{8}{3}$

184. Amplitude of SHM is

(A) h (B) $\frac{4}{3}h$ (C) $\frac{3h}{4}$ (D) $2h$

185. Time after block reaches its extreme position for first time is (when block performing SHM)

(A) $\sqrt{2gh}$

(B) $\sqrt{2gh}+\frac{7}{6}\pi\sqrt{\frac{2h}{3g}}$

(C) $\sqrt{2gh}+\frac{6}{7}\sqrt{\frac{3g}{2h}}$

(D) $\sqrt{2gh}+\pi\sqrt{\frac{3g}{2h}}$

Passage 4

A pendulum inside a stationary elevator has a time period T where acceleration due to gravity is g. When elevator moves up by acceleration a, its time period is measured as T_1, and when it moves down by same acceleration its time period is T_2.

Now instead of time period, experiment is done to find out acceleration due to gravity in stationary elevator. Suppose while doing the experiment, a student made a tiny error dT in measuring time period. Then corresponding error in g is found to be dg.

Again the pendulum is in stationary elevator but due to temperature change, its length changes from l to $l + a$ ($a << l$) and due to change in place, acceleration due to gravity changes from g to $g - b$ ($b << g$). Due to this, its percentage change in time period is found to be η. Now to restore its original time period, its length is decreased by l_1.

186. Relation between T_1, T_2, and T is

(A) $T=\left(\frac{T_2\sqrt{T_1T_2}}{T_1^2+T_2^2}\right)$

(B) $T=\frac{\sqrt{2}(T_1T_2)}{\sqrt{T_1^2+T_2^2}}$

(C) $T=\frac{\sqrt{5}(T_1T_2)}{2\sqrt{T_1^2+T_2^2}}$

(D) $T=\frac{T_1^2T_2^2}{(T_1^2+T_2^2)^{3/2}}$

187. Error dg is

(A) $\frac{-gdT}{\pi\sqrt{gl}}$

(B) $\frac{-g\sqrt{g}\,dT}{\pi\sqrt{l}}$

(C) $\frac{-g^{3/2}dT}{\pi l}$

(D) $\frac{-4g\sqrt{g}}{\pi\sqrt{l}}dT$

188. Value of l_1 is

(A) $a^2+\frac{lb^2}{g}$

(B) $a+\frac{lb}{g}$

(C) $a-\frac{l^2b}{g}$

(D) $\frac{al}{g}-b$

Passage 5

An incident wave $y = A\sin\left(ax+bt+\frac{\pi}{2}\right)$ is reflected by a rigid obstacle at $x = 0$, which reduces intensity of reflected wave by 36%. Due to superposition, the resulting wave consists of a standing wave and a traveling wave, which is given by $Y=-dA\sin ax\cdot\sin bt+cA\cos(bt+ax)$, where A, a, b, c are positive constants.

189. Amplitude of reflected wave is

(A) 0.6 A (B) 0.8 A
(C) 0.4 A (D) 0.2 A

190. Value of c is

(A) 0.2. (B) 0.4
(C) 0.6. (D) 0.3

191. Maximum displacement of a medium particle is

(A) A (B) 0.2 A
(C) 0.8 A (D) 1.8 A

Passage 6

An oscillator of frequency 680 Hz drives two speakers. The speakers are fixed on a vertical pole at a distance 3 m from each other as shown in Fig. 9.38. A person whose height is almost the same as that of the lower speaker walks towards the lower speaker in a direction perpendicular to the pole. Assuming that there is no reflection of sound from the ground and speed of sound is $v = 340$ m/s, answer the following questions.

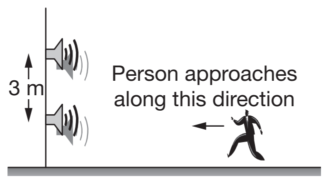


Fig. 9.38

192. As the person walks towards the pole, his minimum distance from the pole when he first hears a minimum in sound intensity is nearly

(A) 14.6 m (B) 17.9 m
(C) 10.1 m (D) 22.4 m

193. As the person walks toward the pole, the total number of times that the person hears a minimum in sound intensity will be

(A) 2. (B) 8 (C) 4 (D) 6

194. At some instant, when the person is at a distance 4 m from the pole, the wave function (at the person's location) that describes the waves coming from the lower speaker is $y = A\cos(kx-\omega t)$ (where A is the amplitude, $\omega = 2\pi v$ with $v = 680$ Hz (given) and $k = \frac{2\pi}{\lambda}$)

Then wave function (at the person's location) that describes waves coming from the upper speaker can be expressed as

(A) $y = A\cos(kx - \omega t + 2\pi)$

(B) $y = A\cos(kx - \omega t + \pi)$

(C) $y = A\cos(kx - \omega t + 4\pi)$

(D) $y = A\cos\left(kx - \omega t + \frac{3\pi}{2}\right)$

195. Suppose at one instant of time, the person is at certain position and the frequency of upper speaker is changed (by connecting it with some other oscillator) without change in its intensity. Then the net intensity of sound heard by the person at P will

(A) increase.
(B) decrease.
(C) remains same.
(D) may increase or decrease depending upon the position of the person.

Passage 7

The equations of two plane progressive sound waves are given as $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$. Answer the following questions based on above equations

196. How many times the value of $y_1 + y_2$ becomes zero at $x = 0$ in 1 second?

(A) 46 (B) 42 (C) 100 (D) 184

197. Wave speed of the louder wave is

(A) 192 m/s (B) 200 m/s
(C) 100 m/s (D) 184 m/s

198. When the given waves superimpose the number of times the intensity of sound becomes maximum in 1 second is

(A) 4 (B) 6 (C) 8 (D) 12

Passage 8

A sound source S of frequency 600 Hz is performing SHM with amplitude 300 cm between AA' along x-axis, about mean position as origin O. There is a detector with another stationary sound source S' of sound of same frequency lying near to point A', i.e., at point B as shown in Fig. 9.39. If the maximum number of beats detected by the detector is 60 at time $T/2$, where T is the time period of SHM of source S. (velocity of sound is 330 m/s)

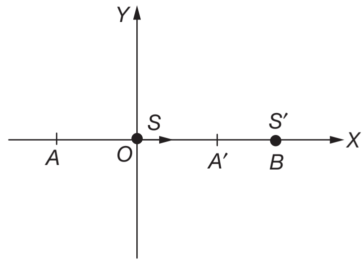


Fig. 9.39

199. The maximum velocity of the source is

(A) 20 m/s (B) 30 m/s
(C) 40 m/s (D) 60 m/s

200. The equation of SHM of the source S is

(A) 300 sin 20 t (B) 300 sin (10 $t + \pi$)
(C) 300 cos (10 $t + \pi$) (D) 300 cos 20 t

201. The minimum number of beats detected by the detector is

(A) Zero (B) 45 (C) 50 (D) 55

Passage 9

The vibration of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4 \sin\left[\frac{\pi x}{15}\right] \cos(96\pi t)$$

where x and y are in cm and t in second.
Answer the following questions based on the above statement.

202. The maximum displacement at $x = 5$ cm is

(A) $2\sqrt{3}$ cm (B) $3\sqrt{2}$ cm
(C) Zero (D) $2\sqrt{3}$ m

203. Where are the nodes located along the string?

(A) 0, 15 cm, 30 cm, 45 cm, 60 cm
(B) 7.5 cm, 22.5 cm, 37.5 cm, 52.5 cm
(C) Both (A) and (B)
(D) None

204. The equation of components wave whose superposition gives the above wave are

(A) $2\sin\left(96\pi t + \frac{\pi x}{15}\right)$ cm ; $2\cos\left(96\pi t + \frac{\pi x}{15}\right)$ cm

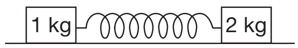
(B) $2\cos\left(96\pi t + \frac{\pi x}{15}\right)$ cm ; $2\cos\left(96\pi t - \frac{\pi x}{15}\right)$ cm

(C) $2\sin\left(96\pi t + \frac{\pi x}{15}\right)$ cm ; $2\sin\left(96\pi t - \frac{\pi x}{15}\right)$ cm

(D) None of these

Match the column Type

205. In the two block springs, force constant of spring is $K = 6$ N/m. Spring is stretched by 12 cm and then left.



Column-I	Column-II
(A) Angular frequency of oscillation	1. 28.8×10^{-3} SI unit
(B) Maximum kinetic energy of 1 kg block	2. 3 SI unit
(C) Maximum energy of 2 kg	3. 14.4×10^{-3} SI unit
(D) Maximum acceleration of 2 kg block	4. 36.0×10^{-2} SI unit
	5. 30.0×10^{-3} SI unit

206. In Fig. 9.40, a block of mass $M = 1$ kg is attached to one end of massless spring of spring constant $k = 100$ N/m and other end of spring is fixed. Initially, spring in its natural length. A horizontal force $F = 10$ N at $t = 0$ is applied on the block.

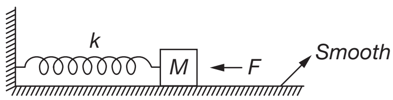


Fig. 9.40

Column-I	Column-II
(A) Amplitude of SHM in cm is	1. $\pi/30$
(B) Maximum speed of block in cm/s is	2. 100
(C) The time in second after which compression in the spring is half of amplitude if block starts from extreme position	3. 10
(D) Maximum compression in the spring in cm is	4. 20

207. The speed (v) of a particle of mass 1 kg moving along a straight line, when it is at a distance (x) from a fixed point on the line is given by $v^2 = 144 - 9x^2$.

Column-I	Column-II
(A) Motion is simple harmonic with time period	1. $2\pi/3$ units
(B) Max. displacement from the fixed point is	2. 27 units
(C) Magnitude of acceleration at a distance 3 units from the fixed point is	3. 4 units
(D) Potential energy at a distance $x = 2$ unit from fixed point is.	4. 2 units
	5. 18 units

208. A particle of mass 4 kg is tied with a spring of spring constant '64' N/m. The system is kept on a horizontal frictionless surface and performing SHM of amplitude $A = 25$ cm.

Column-I	Column-II
(A) Time for the particle to go from mean position to 25 cm (in seconds)	1. $\frac{\pi}{8}$
(B) Value of kinetic energy when kinetic energy and potential energy are equal (in Joule)	2. $\frac{\pi}{12}$
(C) Time period of oscillation (in seconds)	3. $\frac{\pi}{2}$
(D) Time for the particle to go from mean position to half of the amplitude position is (in seconds)	4. 1

209. A uniform ring of mass $M = 1$ kg has massless spokes. A spring of stiffness constant $K = 1$ N/m is attached to the centre of the ring at one end and the other end is fixed to the wall as shown in Fig. 9.41. The ring is given an angular velocity ω and released from point A. As it reaches point B, its velocity of centre of mass becomes $V = 1$ m/s, where $V = R\omega$. The surface to the left of point B is perfectly rough, so that no slipping takes place. There is a point O on the rough part which corresponds to zero deformation of spring.

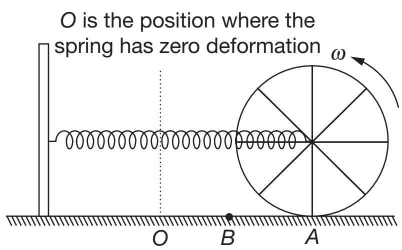


Fig. 9.41

Column-I	Column-II
(A) The time taken by the ring to go from A to B is (in sec)	1. $\sqrt{2}\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(B) The time taken by the ring to go from B to O is (in sec)	2. $\frac{\pi}{4}$
(C) Velocity of centre of mass at O is (m/s)	3. $\sqrt{3}$
(D) The maximum compression of the spring (in m)	4. $\sqrt{\frac{3}{2}}$
	5. $\sqrt{\frac{1}{2}}$

210. From a single source, two wave trains sent in two different strings of same length. String-2 is four times heavy than string-1. The two wave equations are (area of cross-section and tension of both strings is same).

$y_1 = A\sin(\omega_1 t - k_1 x)$ and $y_2 = 2A\sin(\omega_2 t - k_2 x)$

Suppose u = energy density, P = power transmitted, and I = intensity of wave, v = velocity of wave, then match the following:

Column-I	Column-II
(A) $\frac{u_1}{u_2} =$	1. $\frac{1}{8}$
(B) $\frac{P_1}{P_2} =$	2. $\frac{1}{16}$
(C) $\frac{v_1}{v_2} =$	3. 2
(D) $\frac{k_1}{k_2} =$	4. $\frac{1}{2}$
	5. 5

211. A closed organ pipe of length L vibrating in second overtone, then match the following

Column-I	Column-II
(A) Displacement node	1. Closed end
(B) Displacement anti-node	2. Open end
(C) Pressure node	3. $4L/5$ from closed end
(D) Pressure anti-node	4. $L/5$ from closed end

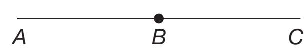
212. Match the information given in Column-I with that given in Column-II

Column-I	Column-II
(A) Mechanical waves	1. Transverse only
(B) Electromagnetic waves	2. Can be transverse or longitudinal
(C) Longitudinal waves	3. Required a medium to propagate
(D) Pressure waves	4. Must be elastic parameters dependent

213.

Column-I	Column-II
(A) Beats	1. Redistribution of energy
(B) Standing waves	2. Multiple reflection
(C) Interference	3. Varying amplitude
(D) Echo	4. Reflection from a rigid support

214. A long wire ABC is made by joining two wires AB and BC of equal area of cross-section. AB has length 4.8 m and mass 0.12 kg while BC has length 2.56 m and mass 0.4 kg. The wire is under a tension of 160 N. A wave Y (in cm) = 3.5 sin ($kx - wt$) is sent along ABC from end A. No power is dissipated during propagation of wave.



Column-I	Column-II
(A) Amplitude of reflected wave	1. 2.0
(B) Amplitude of transmitted wave	2. 1.5
(C) Maximum displacement of antinodes in the wire AB	3. 5
(D) Percentage fraction of power transmitted in the wire BC	4. 82
	5. 92

Assertion–Reason Type

215. Assertion: Average speed of a particle performing SHM (of amplitude A with time period T) in one time period is $\frac{4A}{T}$.

Reason: In case of SHM, displacement of particle in one time period is zero.

(A) A (B) B (C) C (D) D

216. Assertion: Speed of a particle and magnitude of its acceleration in its SHM (time period 2π second) are equal when its displacement from mean position is $\frac{A}{\sqrt{2}}$, where A is amplitude of SHM.

Reason: Speed of particle is maximum when acceleration of particle is zero.

(A) A (B) B (C) C (D) D

217. Assertion: In SHM, total mechanical energy is always equal to sum of kinetic energy and potential energy.

Reason: At the mean position, the particle has only kinetic energy.

(A) A (B) B (C) C (D) D

218. Assertion: The scalar product of the displacement and the acceleration in SHM is never greater than zero.

Reason: Acceleration is linearly proportional to and opposite to displacement.

(A) A (B) B (C) C (D) D

219. Assertion: Tension in the string remains same irrespective of the position of bob in oscillation.

Reason: Tension is maximum at mean position.

(A) A (B) B (C) C (D) D

220. Assertion: A heavy mass is hanging from a string in equilibrium without breaking it. When this same mass is set into oscillation, then string can break.

Reason: In above assertion, tension in string can never be greater than weight of hanging mass.

(A) A (B) B (C) C (D) D

221. Assertion: Maximum speed of a particle performing SHM is 2 m/s, then average speed of particle during it move from one extreme to other extreme position is $\frac{4}{\pi}$ m/s.

Reason: $< v > = \dfrac{\int v dt}{\int dt}$

(A) A (B) B (C) C (D) D

222. Assertion: Two SHMs are given by

$y_1 = 10\sin\left(3\pi t + \frac{\pi}{4}\right)$ and $y_2 = 5\sin(3\pi t) + \sqrt{3}\cos(3\pi t)$.

Reason: y_2 represents two SHMs each of amplitude 5 and so total amplitude is 10, same as that of y_1.

(A) A (B) B (C) C (D) D

223. Assertion: The function $Y = \cos^2\omega t + \sin\omega t$ does not represent a SHM.

Reason: Sum of two harmonic functions may not be a harmonic motion.

(A) A (B) B (C) C (D) D

224. Assertion: Time period of a simple pendulum having a hollow sphere, filled with water, as bob then its time period increases continuously as water drains out.

Reason: Effective length of above mentioned pendulum changes until all the water drains out.

(A) A (B) B (C) C (D) D

225. Assertion: For two identical sources, the maximum intensity in interference pattern is four times the intensity due to each wave.

Reason: The intensity is directly proportional to the square of amplitude.

(A) A (B) B (C) C (D) D

226. Assertion: A closed organ pipe is vibrating in its first overtone with frequency 340 Hz. Speed of sound in organ pipe is 340 m/s. Length of organ pipe is less than 75 cm.

Reason: In case of standing wave in closed organ pipe, pressure amplitude is maximum at closed end.

(A) A (B) B (C) C (D) D

227. Assertion: In Young's double slit experiment, the two slits are at distance d apart. Interference pattern is observed on a screen at distance D from the slits. At a point on the screen when it is directly opposite to one of the slits, a dark fringe is observed. Then the wavelength of wave proportional to square of distance between the two slits.

Reason: For a dark fringe, intensity is zero.

(A) A (B) B (C) C (D) D

228. Assertion: When two vibrating tuning forks having frequencies 300 Hz and 246 Hz are placed near each other, beats cannot be heard by normal human ear.

Reason: The principle of superposition is valid only if the frequencies of the oscillators are nearly equal.

(A) A (B) B (C) C (D) D

229. **Assertion:** $Y = 2A\sin kx\cos\omega t$ refers to a standing wave.

Reason: When a continuous traveling wave interacts with its reflected wave from a rigid support, it may form a standing wave.

(A) A (B) B (C) C (D) D

230. **Assertion:** If two transverse pulses are generated in the same string given by $y = A\sin(kx - \omega t + \varphi)$ and $y = 2A\sin(2kx - \omega t + \varphi)$, then the ratio of average power for the pulses will be 1/8.

Reason: Average power for transverse wave is $\frac{T\omega KA^2}{2}$.

(A) A (B) B (C) C (D) D

231. **Assertion:** A sound wave can be studied as any of the three waves, namely, pressure wave, displacement wave, and density wave.

Reason: In a sound wave pressure, displacement and density change simultaneously to a maximum or minimum.

(A) A (B) B (C) C (D) D

232. **Assertion:** There are two sound waves propagating in same medium having amplitudes and frequencies $2A$, f and A, $2f$, respectively. The intensity of first wave is four times that of the other.

Reason: Intensity of a wave $I = \frac{1}{2}\rho v\omega^2 A^2$

(A) A (B) B (C) C (D) D

233. **Assertion:** No interference pattern is detected when two sources are infinitely close to each other

Reason: The fringe width is inversely proportional to the distance between the two slits.

(A) A (B) B (C) C (D) D

234. **Assertion:** When two vibrating tuning forks having frequencies 256 Hz and 512 Hz are held near each other, beats cannot be heard.

Reason: The principle of superposition is valid only if the frequencies of the oscillations are nearly equal.

(A) A (B) B (C) C (D) D

Integer Type

235. Find the time period of the motion of the particle shown in Fig. 9.42. (Neglect the small effect of the bend near the bottom)

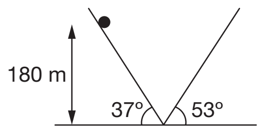


Fig. 9.42

236. A simple pendulum is suspended from the ceiling of an empty box falling in air near earth surface. The total mass of system is M. The box experiences air resistance $\vec{R} = -k\vec{v}$, where v is the velocity of box and k is a positive constant. After some time, it is found that period of oscillation of pendulum becomes double the value when it would have suspended from a point on earth. The velocity of box at that moment $v = \frac{Mg}{nk}$, then the value of n is. (Take g in air same as on earth's surface.)

237. In Fig. 9.43, string, spring, and pulleys are massless. Block A, performing SHM of amplitude 1 m and time period $\pi/2$ s. If block B remains at rest, then minimum value of co-efficient of friction between block B and surface will be $\frac{13}{5n}$, then the value of n is. ($g = 10$ m/s^2)

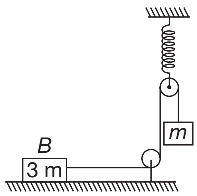


Fig. 9.43

238. A particle of mass M is attached to four springs as shown in Fig.9.44. Initial tension in each spring is F_0 and length of each spring is l, if the period of small oscillations of the particle along a line perpendicular to the plane containing the springs is $T = 2\pi\sqrt{\frac{Ml}{nF_0}}$, then the value of n is (Neglect effect of gravity and assume that the force developed in springs due to displacement is much smaller than the original tension F_0), then

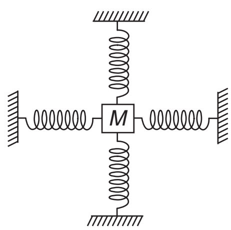


Fig. 9.44

239. Two blocks A and B, each of mass m are connected by means of a pulley-spring system on a smooth inclined plane of inclination θ as shown in Fig 9.45. All the pulleys and spring are ideal. Now, B is slightly displaced from its equilibrium position. It starts to oscillate. Time period of oscillation of B will be $T = 2\pi\sqrt{\frac{5m}{nk}}$, then the value of n is.

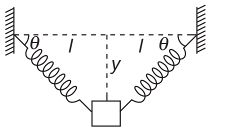


Fig. 9.45

240. The equation of a particle executing SHM is given by $x = 3\cos\left(\frac{\pi}{2}\right)t$ cm, where t is in second. The distance travelled by the particle in the first 8.5 s is $\left(24 + \frac{n}{\sqrt{2}}\right)$ then the value of n is.

241. An open organ pipe has a fundamental frequency of 240 vib/s. The first overtone of a closed organ pipe has the same frequency as the first overtone of the open pipe. How long is each pipe? Velocity of sound at the room temperature is 350 ms.

242. A wire of length 1.5 m under tension emits a fundamental note of frequency 120 Hz.
(A) What would be its fundamental frequency if the length is increased by half under the same tension?
(B) By how much should the length be shortened so that the frequency is increased three-fold?

243. A bus is moving towards a huge wall with a velocity of 5 m/s. The driver sounds a horn of frequency 200 Hz. What is the frequency of beats heard by a passenger of the bus, if the speed of sound in air is 330 m/s.

244. A U-tube having uniform cross-section but unequal arm lengths l_1 and l_2 ($< l_1$) has same liquid of density ρ_1 filled in it upto a height h as shown in Fig. 9.46. Another liquid of density $\rho_2 = (\rho_1/2)$ is poured in arm A. Both liquids are immiscible. What length of the second liquid should be poured in A so that first overtone of A is in unison with fundamental tone of B. (Take $l_l = 5$ m, $l_2 = 1$ m and $h = 0.5$ m)

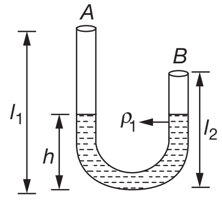


Fig. 9.46

245. A pipe of length L closed at one end is located along x-axis with closed end at origin and open end at (l, 0). The pipe resonates in its n^{th} overtone with maximum amplitude of air molecules to be equal to a_0. Calculate the x-co-ordinates of those points, where maximum pressure change (ΔP_m) occurs and calculate (ΔP_m). Density of air is equal to ρ and velocity of sound in air is v.

246. A rope, under tension of 200 N and fixed at both ends, oscillates in a second harmonic standing wave pattern. The displacement of the rope is given by $y = (0.10\text{ m})\sin\left(\frac{\pi x}{2}\right)\sin(12\pi t)$, where $x = 0$ at one end of the rope, x is in metres, and t is in seconds. Find
(A) the length of the rope
(B) the speed of waves on the rope
(C) the mass of the rope
(D) if the rope oscillates in a third harmonic standing wave pattern, what will be the period of oscillation?

247. A bus B is moving with a velocity v_B in the positive x-direction along a road as shown in Fig. 9.47. A shooter S is at a distance l from the road. He has a detector which can detect signals only of frequency 1500Hz. The bus blows horn of frequency 1000 Hz. When the detector detects a signal, the shooter immediately shoots towards the road along SC and the bullet hits the bus. Find the velocity of the bullet if velocity of sound in air is $v = 340$ m/s and $\frac{v_B}{v} = \frac{2}{3\sqrt{3}}$.

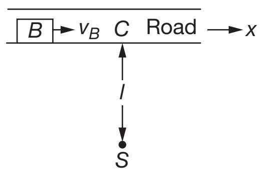


Fig. 9.47

248. Sound waves of frequency 16 kHz are emitted by two coherent point sources of sound placed 2 m apart at the centre of a circular train track of large radius. A person riding the train observes 2 maxima per second when the train is running at a speed of 36 km/h. Calculate the radius of the track. [Velocity of sound in air is 320 m/s.]

248. A sound source S emitting a sound of frequency 500 Hz and receiver R of mass m are at the same point. R is performing SHM with the help of a spring of force constant k. At a time $t = 0$, R is at mean position and moving towards right extreme position as shown in Fig. 9.48. At the same time, source starts moving away from the R with an acceleration 18.75 m/s^2.

Find the frequency (in Hz) registered by receiver at a time t = 10 s. Given that $\frac{m}{k}=\frac{100}{\pi^2}$, amplitude of oscillation of $R=\frac{150}{\pi}$ m, v_{sound} = 300 m/s.

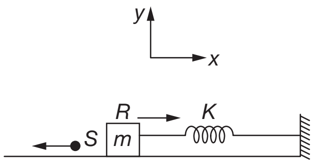


Fig. 9.48

250. A column of air at 16°C and a tuning fork produces 1 beat per second when sounded together. When temperature is raised to 51°C the two produces 4 beats per second. Find the frequency of tuning fork?

Previous years' Questions

251. In a simple harmonic oscillator, at the mean position **[2002]**

(A) kinetic energy is minimum, potential energy is maximum.
(B) both kinetic and potential energies are maximum.
(C) kinetic energy is maximum, potential energy is minimum.
(D) both kinetic and potential energies are minimum.

252. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be **[2002]**

(A) $T\sqrt{n}$ (B) $T/\sqrt{n}$ (C) nT (D) T

253. A child swinging on a swing in sitting position, stands up, then the time period of the swing will **[2002]**

(A) increase.
(B) decrease.
(C) remains same.
(D) increase of the child is long and decreases if the child is short.

254. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the radio of $\frac{m}{M}$ is **[2003]**

(A) $\frac{3}{5}$ (B) $\frac{25}{9}$ (C) $\frac{16}{9}$ (D) $\frac{5}{3}$

255. Two particles A and B of equal masses are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of A and B is **[2003]**

(A) $\sqrt{\frac{k_1}{k_2}}$ (B) $\frac{k_2}{k_1}$ (C) $\sqrt{\frac{k_2}{k_1}}$ (D) $\frac{k_1}{k_2}$

256. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is **[2003]**

(A) 11% (B) 21%
(C) 42% (D) 10%

257. The displacement of a particle varies according to the relation. $x=4(\cos\pi t+\sin\pi t)$ The amplitude of the particle is **[2003]**

(A) –4 (B) 4 (C) $4\sqrt{2}$ (D) 8

258. A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as a function of displacement x. Which of the following statements is true ?

(A) KE is maximum when x = 0.
(B) TE is zero when $x = 0$
(C) KE is maximum when x is maximum
(D) PE is maximum when $x = 0$

259. The total energy of a particle, executing simple harmonic motion is **[2004]**

(A) independent of x (B) $\propto x^2$
(C) $\propto x$ (D) $\propto x^{1/2}$

where x is the displacement from the mean position.

260. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t(\omega\neq\omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to **[2004]**

(A) $\frac{1}{m(\omega_0^2+\omega^2)}$ (B) $\frac{1}{m(\omega_0^2-\omega^2)}$

(C) $\frac{m}{\omega_0^2-\omega^2}$ (D) $\frac{m}{(\omega_0^2+\omega^2)}$

261. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force while the energy is maximum for a frequency ω_2 of the force; then **[2004]**

(A) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(B) $\omega_1 > \omega_2$
(C) $\omega_1 = \omega_2$
(D) $\omega_1 < \omega_2$

262. Two simple harmonic motions are represented by the equations $y_1 = 0.1\sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1\cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is **[2005]**

(A) $\frac{\pi}{3}$ (B) $\frac{-\pi}{6}$ (C) $\frac{\pi}{6}$ (D) $\frac{-\pi}{3}$

263. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would **[2005]**

(A) first decrease and then increase to the original value.
(B) first increase and then decrease to the original value.
(C) increase towards a saturation value.
(D) remain unchanged.

264. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is **[2005]**

(A) $\frac{2\pi}{\sqrt{\alpha}}$ (B) $\frac{2\pi}{\alpha}$ (C) $2\pi\sqrt{\alpha}$ (D) $2\pi\alpha$

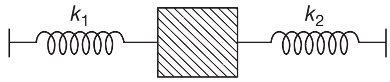
265. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is **[2006]**

(A) 0.01 s (B) 10 s (C) 0.1 s (D) 100 s

266. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?

(A) $\frac{1}{6}$ s (B) $\frac{1}{4}$ s (C) $\frac{1}{3}$ s (D) $\frac{1}{12}$ s

267. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[2007]**



(A) $2f$ (B) $\frac{f}{2}$ (C) $\frac{f}{4}$ (D) $4f$

268. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2\times10^{-2}$ meter. The time at which the maximum speed first occurs is **[2007]**

(A) 0.25 s (B) 0.5 s (C) 0.75 s (D) 0.125 s

269. While measuring the speed of sound by performing a resonance column experiment, a student gets the first reconance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then **[2008]**

(A) $18 > x$ (B) $x > 54$
(C) $54 > x > 36$ (D) $36 > x > 18$

270. A wave travelling along the x-axis is described by the equation $y(x,t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are **[2008]**

(A) $\alpha = 25.00\pi,\ \beta = \pi$
(B) $\alpha = \frac{0.08}{\pi},\ \beta = \frac{2.0}{\pi}$
(C) $\alpha = \frac{0.04}{\pi},\ \beta = \frac{1.0}{\pi}$
(D) $\alpha = 12.50\pi,\ \beta = \frac{\pi}{2.0}$

271. Three sound waves of equal amplitudes have frequencies $(\nu - 1), \nu, (\nu + 1)$. They superpose to give beats. The number of beats produced per second will be **[2009]**

(A) 3 (B) 2 (C) 1 (D) 4

272. A motor cycle starts from rest and accelerates along a straight path at $2\,\text{m/s}^2$. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the direiver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? **[2009]**

(Speed of sound $= 330\ \text{ms}^{-1}$)

(A) 98 m (B) 147 m
(C) 196 m (D) 49 m

273. The equation of a wave on a string of linear mass density $0.04\ \text{kg m}^{-1}$ is given by

$$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$$

The tension in the string is **[2010]**

(A) 4.0 N (B) 12.5 N
(C) 0.5 N (D) 6.25 N

274. A cylindrical tube, open at both ends has a fundamental frequency, *f* in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now **[2012]**

(A) *f* (B) *f*/2
(B) 3*f*/4 (D) 2*f*

275. If a simple pendulum has significant amplitude (up to a factor 1/e of original) only in the period between $t = 0s$ to $t = \tau s$, then τ may be called the average lift of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with *b* as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds: **[2012]**

(A) $\frac{0.693}{b}$ (B) b
(C) $\frac{1}{b}$ (D) $\frac{2}{b}$

276. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5 s. In another 10 s it will decrease to α times its original magnitude, where α equals **[2013]**

(A) 0.81 (B) 0.729 (C) 0.6 (D) 0.7

276. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance *a*, and in next τ s it travels 2*a*, in same direction, then **[2014]**

(A) amplitude of motion is 3*a*.
(B) time period of oscillation is 8τ.
(C) amplitude of motion is 4*a*.
(D) time period of oscillation is 6τ.

278. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? **[2014]**

(Atmospheric pressure = 76 cm of Hg)

(A) 16 cm (B) 22 cm
(C) 38 cm (D) 6 cm

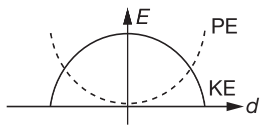
279. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s **[2014]**

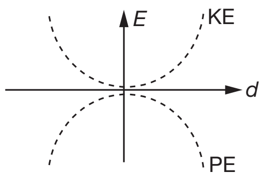
(A) 12. (B) 8 (C) 6 (D) 4

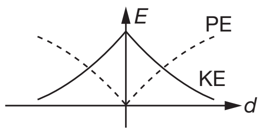
280. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resulting signal is/are **[2015]**

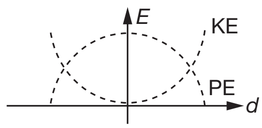
(A) 2005 kHz, and 1995 kHz
(B) 2005 kHz, 2000 kHz and 1995 kHz
(C) 2000 kHz and 1995 kHz
(D) 2 MHz only

281. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against displacement *d*. Which one of the following represents these correctly?
(graphs are schematic and not drawn to scale) **[2015]**

(A)


(B)


(C)


(D)


282. A train is moving on a straight track with speed $20\ \text{ms}^{-1}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = $320\ \text{ms}^{-1}$) close to **[2015]**

(A) 12% (B) 18%
(C) 24% (D) 6%

283. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is **[2002]**

(A) 20 (B) 80 (C) 40 (D) 120

284. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is **[2002]**

(A) 1 : 2 (B) 1 : 4 (C) 2 : 1 (D) 4 : 1

285. A tuning fork arrangement (pair) produces 4 beats/s with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is **[2002]**

(A) 286 cps (B) 292 cps
(C) 294 cps (D) 288 cps

286. When temperature increase, the frequency of a tuning fork **[2002]**

(A) increase.
(B) decrease.
(C) remains same.
(D) increase or decreases depending on the material.

287. The displacement y of a wave travelling in the x-direction is given by $y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$ metres where x is expressed in meters and t in seconds, The speed of the wave-motion, ms^{-1} in, is **[2003]**

(A) 300 (B) 600 (C) 1200 (D) 200

288. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 meter apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is **[2003]**

(A) 50 Hz (B) 100 Hz
(C) 200 Hz (D) 25 Hz

289. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[2003]**

(A) 256 + 2Hz (B) 256 – 2 Hz
(C) 256 – 5 Hz (D) 256 + 5 Hz

290. The displacement y of a particle in a medium can be expressed as, $y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right) m$ where t is in second and x in meter. The speed of the wave is **[2004]**

(A) 20 m/s (B) 5 m/s
(C) 2000 m/s (D) 5π m/s

291. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2? **[2005]**

(A) 202 Hz (B) 200 Hz
(C) 204 Hz (D) 196 Hz

292. An observer moves towards a stationary source of sound, with velocity one-fifth of the velocity of sound. What is the percentage increases in the in the apparent frequency? **[2006]**

(A) 0.5 % (B) Zero (C) 20% (D) 5 %

293. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of v upto which he can hear whistle is **[2006]**

(A) $15\sqrt{2}$ ms^{-1} (B) $\frac{15}{\sqrt{2}}$ ms^{-1}
(C) 15 ms^{-1} (D) 30 ms^{-1}

294. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of **[2007]**

(A) 100 (B) 1000
(C) 10000 (D) 10

295. A wave travelling along the x-axis is described by the equation $y(x,t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are **[2008]**

(A) $\alpha = 25.00\pi, \beta = \pi$

(B) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$

(C) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$

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$$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$ The tension in the string is **[2010]**

(A) 4.0 N (B) 12.5 N
(C) 0.5 N (D) 6.25 N

298. A cylindrical tube, open at both ends has a fundamental frequency, f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now **[2012]**

(A) f (B) $\frac{f}{2}$ (C) $\frac{3f}{4}$ (D) $2f$

299. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? **[2014]**

(Atmospheric pressure = 76 cm of Hg)
(A) 16 cm (B) 22 cm (C) 38 cm (D) 6 cm

300. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s **[2014]**

(A) 12 (B) 8 (C) 6 (D) 4

301. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resulting signal is/are **[2015]**

(A) 2005 kHz, and 1995 kHz
(B) 2005 kHz, 2000 kHz and 1995 kHz
(C) 2000 kHz and 1995 kHz
(D) 2 MHz only

302. A train is moving on a straight track with speed $20\ \text{ms}^{-1}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = $320\ \text{ms}^{-1}$) close to **[2015]**

(A) 12% (B) 18% (C) 24% (D) 6%

303. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is **[2016]**

(A) 2s (B) $2\sqrt{2}$ s
(C) $\sqrt{2}$ s (D) $2\pi\sqrt{2}$ s

304. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now **[2016]**

(A) $\frac{3f}{4}$ (B) $2f$
(C) f (D) $\frac{f}{2}$

305. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is **[2016]**

(A) $3A$ (B) $A\sqrt{3}$
(C) $\frac{7A}{3}$ (D) $\frac{A}{3}\sqrt{41}$

ANSWER KEYS

Single Option Correct Type

1. (A)	**2.** (D)	**3.** (D)	**4.** (A)	**5.** (A)	**6.** (C)	**7.** (A)	**8.** (B)	**9.** (C)	**10.** (C)
11. (D)	**12.** (A)	**13.** (B)	**14.** (B)	**15.** (D)	**16.** (B)	**17.** (A)	**18.** (B)	**19.** (C)	**20.** (C)
21. (A)	**22.** (A)	**23.** (C)	**24.** (C)	**25.** (D)	**26.** (C)	**27.** (B)	**28.** (A)	**29.** (C)	**30.** (D)
31. (A)	**32.** (B)	**33.** (C)	**34.** (C)	**35.** (B)	**36.** (B)	**37.** (A)	**38.** (C)	**39.** (D)	**40.** (B)
41. (B)	**42.** (D)	**43.** (C)	**44.** (C)	**45.** (D)	**46.** (B)	**47.** (A)	**48.** (C)	**49.** (A)	**50.** (B)
51. (B)	**52.** (D)	**53.** (D)	**54.** (C)	**55.** (A)	**56.** (D)	**57.** (C)	**58.** (C)	**59.** (B)	**60.** (C)
61. (B)	**62.** (B)	**63.** (C)	**64.** (D)	**65.** (C)	**66.** (D)	**67.** (B)	**68.** (A)	**69.** (B)	**70.** (B)
71. (D)	**72.** (A)	**73.** (D)	**74.** (A)	**75.** (C)	**76.** (D)	**77.** (C)	**78.** (B)	**79.** (C)	**80.** (A)

81. (A)	**82.** (D)	**83.** (B)	**84.** (D)	**85.** (C)	**86.** (B)	**87.** (C)	**88.** (C)	**89.** (C)	**90.** (B)
91. (A)	**92.** (D)	**93.** (A)	**94.** (C)	**95.** (C)	**96.** (D)	**97.** (B)	**98.** (B)	**99.** (B)	**100.** (A)
101. (D)	**102.** (D)	**103.** (D)	**104.** (C)	**105.** (C)	**106.** (A)	**107.** (B)	**108.** (D)	**109.** (C)	**110.** (D)
111. (A)	**112.** (B)	**113.** (D)	**114.** (C)	**115.** (A)	**116.** (B)	**117.** (B)	**118.** (D)	**119.** (D)	**120.** (B)
121. (C)	**122.** (B)	**123.** (C)	**124.** (D)	**125.** (D)	**126.** (B)	**127.** (B)	**128.** (A)	**129.** (B)	**130.** (B)
131. (C)	**132.** (A)	**133.** (A)	**134.** (B)	**135.** (C)	**136.** (A)	**137.** (D)	**138.** (B)	**139.** (C)	**140.** (C)
141. (C)	**142.** (B)	**143.** (C)	**144.** (D)	**145.** (B)	**146.** (B)	**147.** (C)			

More than One Option Correct Type

148. (D)	**149.** (A), (B), (C) and (D)	**150.** (B) and (D)	**151.** (A) and (B)
152. (B) and (C)	**153.** (A) and (C)	**154.** (A) and (C)	**155.** (A) and (C)
156. (A)	**157.** (B), (C) and (D)	**158.** (B) and (D)	**159.** (B) and (C)
160. (B) and (C)	**161.** (B) and (C)	**162.** (A) and (C)	**163.** (B) and (D)
164. (A) and (C)	**165.** (A), (B) and (D)	**166.** (A) (B) and (C)	**167.** (B), (C) and (D)
168. (B), (C) and (D)	**169.** (A) and (C)	**170.** (B) and (C)	**171.** (C) and (D)
172. (C) and (D)	**173.** (A) and (C)	**174.** (B) and (C)	

Passage Based Questions

Passage 1

175. (D) **176.** (C) **177.** (C)

Passage 2

178. (D) **179.** (B) **180.** (B) **181.** (B)

Passage 3

182. (A) **183.** (B) **184.** (B) **185.** (B)

Passage 4

186. (B) **187.** (B) **188.** (B)

Passage 5

189. (B) **190.** (A) **191.** (D)

Passage 6

192. (B) **193.** (D) **194.** (C) **195.** (D)

Passage 7

196. (C) **197.** (B) **198.** (A)

Passage 8

199. (B) **200.** (B) **201.** (A)

Passage 9

202. (B) **203.** (A) **204.** (C)

Match the Column Type

205. (A) $\rightarrow$ 2; (B) $\rightarrow$ 1; (C) $\rightarrow$ 3; (D) $\rightarrow$ 4

206. (A) $\rightarrow$ 3, (B) $\rightarrow$ 2, (C) $\rightarrow$ 1, (D) $\rightarrow$ 4

207. (A) $\rightarrow$ 1, (B) $\rightarrow$ 3, (C) $\rightarrow$ 2, (D) $\rightarrow$5

208. (A) $\rightarrow$ (1); (B) $\rightarrow$ (4); (C) $\rightarrow$ (3); (D) $\rightarrow$ (2)

209. (A) $\rightarrow$ 2, (B) $\rightarrow$ 1, (C) $\rightarrow$ 4, (D) $\rightarrow$ 3

210. (A) $\rightarrow$ 2; (B) $\rightarrow$ 1; (C) $\rightarrow$ 3; (D) $\rightarrow$ 4

211. (A) $\rightarrow$ 1, 3; (B) $\rightarrow$ 2, 4; (C) $\rightarrow$ 2, 4; (D) $\rightarrow$ 1, 3

212. (A) $\rightarrow$ 2,3; (B) $\rightarrow$ 1; (C) $\rightarrow$ 3,4; (D) $\rightarrow$ 3,4

213. (A) $\rightarrow$ (3); (B) $\rightarrow$ (4); (C) $\rightarrow$ (1); (D) $\rightarrow$ (2)

214. (A) $\rightarrow$ 2, (B) $\rightarrow$ 1, (C) $\rightarrow$ 3, (D) $\rightarrow$ 4

Assertion–Reason Type

215. (B)	**216.** (B)	**217.** (C)	**218.** (A)	**219.** (D)	**220.** (C)	**221.** (A)	**222.** (C)	**223.** (B)	**224.** (D)
225. (B)	**226.** (B)	**227.** (C)	**228.** (C)	**229.** (B)	**230.** (D)	**231.** (C)	**232.** (D)	**233.** (B)	**234.** (C)

Integer Type

235. = 35 s	**236.** $n = 4$	**237.** $n = 3$	**238.** $n = 4$	**239.** $n = 4$
240. $N = 3$	**241.** 73.9 cm	**242.** $m = 0.5$ m	**243.** = 206 – 200 = 6 HZ	

244. $Y = 2$ m

245. $X = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda$

246. Time Period $= \frac{1}{\nu} = \frac{1}{9}$ s

247. $= 136$ m/s

248. $\frac{1000}{\pi}$ m

249. $f' = 500\left(\frac{300+15}{300+150}\right) = 350\,\text{Hz}$

250. $f = 50$ HZ or 86 HZ

Previous Years' Questions

251. (C)	**252.** (B)	**253.** (B)	**254.** (C)	**255.** (C)	**256.** (D)	**257.** (C)	**258.** (A)	**259.** (A)	**260.** (B)
261. (C)	**262.** (B)	**263.** (B)	**264.** (A)	**265.** (A)	**266.** (A)	**267.** (A)	**268.** (B)	**269.** (B)	**270.** (A)
271. (B)	**272.** (A)	**273.** (D)	**274.** (A)	**275.** (C)	**276.** (B)	**277.** (D)	**278.** (A)	**279.** (C)	**280.** (B)
281. (A)	**282.** (A)	**283.** (B)	**284.** (C)	**285.** (B)	**286.** (B)	**287.** (A)	**288.** (A)	**289.** (C)	**290.** (B)
291. (D)	**292.** (B)	**293.** (C)	**294.** (A)	**295.** (A)	**296.** (A)	**297.** (D)	**298.** (A)	**299.** (A)	**300.** (C)
301. (B)	**302.** (A)	**303.** (B)	**304.** (C)	**305.** (C)					

HINTS AND SOLUTIONS

Single Option Correct Type

1. $y = \cos\omega t$, $\frac{A}{2} = A\cos\omega t \Rightarrow t = \frac{T}{6}$

The correct option is (A)

2. $K_{eq} = \frac{k_1 k_2}{k_1 + k_2}$

The correct option is (D)

3. The correct option is (D)

4. Period of the mass attached to the spring is given by

$$T = 2\pi\sqrt{\frac{M}{k}} \quad \therefore\; 2 = 2\pi\sqrt{\frac{M}{k}} \qquad (1)$$

In the second case, $3 = 2\pi\sqrt{\frac{M+2}{k}}$ (2)

Dividing (1) by (2), $\frac{2}{3} = \sqrt{\frac{M}{M+2}} \quad \therefore \quad \frac{4}{9} = \frac{M}{M+2}$

$9M = 4M + 8$; $5M = 8$; $M = 1.6$ kg

The correct option is (A)

5. KE at mean position $= \frac{1}{2}m\omega^2 A^2$

PE at $\frac{A}{2} = \frac{1}{2}m\omega^2 \frac{A^2}{4} \quad \therefore$ Ratio = 4 : 1

The correct option is (A)

6. $4v^2 = 25 - x^2$

Differentiating $4\left(2v\frac{dv}{dt}\right) = -2x\frac{dx}{dt}$

$\therefore \quad \omega^2 = \frac{1}{4}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{1/2} = 4\,\pi$

The correct option is (C)

7. The correct option is (A)

8. $T = \frac{2\pi}{\omega}$ and $A\omega = v_m$, $v_{av} = \frac{2A}{(T/2)} = \frac{4A}{T} = \frac{4Av_m}{2\pi A} = \frac{2}{\pi}v_m$

The correct option is (B)

9. $K = \frac{1}{2}m\omega^2\left(A^2 - y^2\right) = \frac{1}{2}m\omega^2 A^2\cos^2\omega t$

$= \frac{1}{4}m\omega^2 A^2\left(1 + \cos 2\omega t\right) = \frac{1}{2}E\left(1 + \cos\omega' t\right)$

i. e., $\omega' = 2\omega$

The correct option is (C)

10. $m_1 = M$, $T_1 = T$, $m_2 = M + m$, $T_2 = \frac{5T}{3}$,

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{m_1/k}}{2\pi\sqrt{m_2/k}} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{M}{M+m}}$$

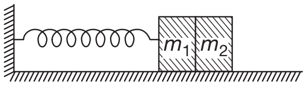
$\therefore \ \dfrac{m}{M} = \dfrac{16}{9}$

The correct option is (C)

11. The correct option is (D)

12. Velocity at equilibrium position, $v = \sqrt{\dfrac{k}{m_1 + m_2}}\,d$

$$\frac{1}{2}kA^2 = \frac{1}{2}m_1 \ \frac{k}{(m_1 + m_2)}d^2 , \ A = d\sqrt{\left(\frac{m_1}{(m_1 + m_2)}\right)}$$



The correct option is (A)

13. $F \ \alpha \left(-x\right)$

The correct option is (B)

14. Total energy $E = \dfrac{1}{2}\,m\,(\omega a)^2$

$$\text{KE} = \frac{1}{2}\,m\omega^2\,\sqrt{a^2 - y^2}\ ;$$

$$\text{PE} = \frac{1}{2}\,m\omega^2 y^2$$

Since its kinetic energy = $\dfrac{3}{4}\,E$, the potential energy = $\dfrac{E}{4}$

$$\therefore \ \frac{\frac{1}{2}\,m\,\omega^2 y^2}{\frac{1}{2}\,m\,\omega^2 a^2} = \frac{1}{4} \Rightarrow y^2 = \frac{a^2}{4}\,; y = \frac{a}{2}$$

The correct option is (B)

15. The correct option is (D)

16. $x = A\,\sin\,\omega t$

$v = A\,\omega\cos\,\omega t = \,\omega\sqrt{A^2 - x^2}$

$v_1^2 = \omega^2\left(A^2 - x_1^2\right)$

$v_2^2 = \omega^2\left(A^2 - x_2^2\right)$

Solving $A = \sqrt{\dfrac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}}$

The correct option is (B)

17. Time period of SHM will be $T = 2\pi\sqrt{\dfrac{m}{K_{\text{eq}}}}$

The correct option is (A)

18. $T = 2\pi\sqrt{\dfrac{l}{g}}$

When the lift accelerates upwards with an acceleration of $g/3$,

The effective acceleration is $g' = g + \dfrac{g}{3} = 4\dfrac{g}{3}$

Therefore, the new time period is $T' = 2\pi\sqrt{\dfrac{l}{g'}}$

$\therefore \quad T' = \dfrac{\sqrt{3}}{2}T$

The correct option is (B)

19. Given $x = 2\cos\left(0.5\pi t + \dfrac{\pi}{3}\right)$

$\therefore$ Velocity $v = \dfrac{dx}{dt} = -2 \times 0.5\pi\,\sin\left(0.5\pi t + \dfrac{\pi}{3}\right)$

$\therefore$ Acceleration $a = \dfrac{dV}{dt} = -2 \times 0.5\pi \times 0.5\pi\,\cos\left(0.5\pi t + \dfrac{\pi}{3}\right)$

$\therefore$ Maximum acceleration

$$a_{\max} = -2 \times 0.5\pi \times 0.5\pi = -\frac{\pi^2}{2}\text{cms}^{-2}$$

The correct option is (C)

20. $y = A\sin\left(\omega t + \theta\right), \ v = v_{\max}\cos\left(\omega t + \theta\right)$

$\Rightarrow \quad 5 = 10\cos\left(\omega t + \theta\right), \ \omega t + \theta = \dfrac{\pi}{3} = 60°$

$y = 4 \times \sin\left(\omega t + \theta\right) \ = 4 \times \dfrac{\sqrt{3}}{2} = 2\sqrt{3}$ cm

The correct option is (C)

21. The correct option is (A)

22. $f = \dfrac{v}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{T}{\pi r^2 d}}$

The correct option is (A)

23. At $t = 0$, $y = \dfrac{1}{1 + x^2}$ or $x = \sqrt{\dfrac{1 - y}{y}}$

At $t = 2$ second, $y = \dfrac{1}{[1 + (x - 1)^2}$ or $x = 1 + \sqrt{\dfrac{1 - y}{y}}$

$$\therefore \ v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{1 + \sqrt{\frac{1 - y}{y}}\ \sqrt{\frac{1 - y}{y}}}{2 - 0} = \frac{1}{2} = 0.5 \ \text{m/s}$$

The correct option is (C)

24. $I = I_1 + I_2 + 2\ (\sqrt{I_1 I_2})\cos\phi$

$= I_0 + 4I_0 + 2\ \sqrt{(I_0)(4I_0)}\cos\phi \ = 5I_0 + 4I_0\,\cos\phi$

As $\ \phi = 0$, so $\cos\phi = 1$

$\therefore \quad I = 5I_0 + 4I_0 = 9I_0$

The correct option is (C)

25. The correct option is (D)

26. The correct option is (C)

27. The correct option is (B)

28. Apparent frequency of the approaching train is

$$n_a = (240)\left(\frac{320}{320-4}\right) = \left(\frac{240 \times 320}{316}\right)$$

Apparent frequency of the leaving train is

$$n_l = (240)\left(\frac{320}{320+4}\right) = \left(\frac{240\times 320}{324}\right)$$

Hence, the number of beats heard by the man per second

$$= (n_a - n_l) = 240 \times 320\left[\frac{1}{316} - \frac{1}{324}\right] = \frac{240 \times 320 \times 8}{316 \times 324}$$

$= 6$

The correct option is (A)

29. $\frac{I_1}{I_2} = \frac{1}{9}$ the $\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{1}{3}$, $\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{1+3}{1-3} = \frac{4}{-2} = -2$

or $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \frac{4}{1}$

The correct option is (C)

30. Let f_0 be natural frequency of source, then $f_1 = f_0\left[\frac{v}{v-34}\right]$,

$f_2 = f_0\left[\frac{v}{v-17}\right]$ $\frac{f_1}{f_2} = \frac{19}{18}$

The correct option is (D)

31. $F = mg = 20$ N, $\mu = \frac{0.3}{6} = 0.05$ kg/m, $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20}{0.05}}$

$= 20$ m/s

The correct option is (A)

32. $2\frac{V_0}{2l_0} = \frac{3V_C}{4l_C} \Rightarrow l_0 = \frac{4}{3}\frac{l_C V_0}{V_C} = \frac{4}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$

The correct option is (B)

33. Frequency is independent of medium

The correct option is (C)

34. $\frac{v}{2L_{open}} = \frac{v}{4L_{closed}}$, $L_{open} = 2L_{closed}$, $\frac{L_{open}}{L_{closed}} = \frac{2}{1}$

The correct option is (C)

35. $L = L_1 + L_2 + L_3$ and $n \alpha \frac{1}{L}$, $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

The correct option is (B)

36. The velocity of sound in seawater is $v = \sqrt{\frac{E}{d}}$

$$v = \sqrt{\frac{E}{d}} = \sqrt{\frac{220 \times 10^6 \times 9.8}{1.1 \times 10^3}} = \sqrt{196 \times 10^4} \text{ m/s}$$

$= 14 \times 10^2$ m/s.

The depth of the well is therefore $= \left(\frac{2.64}{2}\right) \times 14 \times 10^2$ m

$= 1848$ m

The correct option is (B)

37. Frequency $n = \frac{v}{4l}$

$\Rightarrow \quad 440 = \frac{330}{4l}$, where l is in metres

$\Rightarrow \quad l = \frac{3}{16}$ m

$\Rightarrow \quad l = \frac{3}{16} \times 100 \approx 19$ cm.

The correct option is (A)

38. The correct option is (C)

39. Using $I = \frac{1}{2}\rho A^2 \omega^2 c$, we get

$$I = \frac{1}{2} \times 1.3 \times \left(10^{-8}\right)^2 (2\pi \times 2000)^2 (340)$$

$$= 3.5 \times 10^{-6} \text{ Wm}^{-2}$$

The correct option is (D)

40. Given $y = 3\cos\left(\frac{\pi}{2} - 2\omega t\right)$

Velocity of the particle

$$v = \frac{dy}{dt} = \frac{d}{dt}\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 3(-2\omega)\left[-\sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$$= 6\omega \sin\left(\frac{\pi}{4} - 2\omega t\right)$$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left[6\omega\sin\left(\frac{\pi}{4} - 2\omega t\right)\right]$

$$= 6\omega \times (-2\omega)\left(\frac{\pi}{4} - 2\omega t\right)$$

$$= -12\omega\cos\left(\frac{\pi}{4} - 2\omega t\right)$$

$$= -4\omega^2\left[3\cos\left(\frac{\pi}{4} - 2\omega t\right)\right]$$

$\Rightarrow \quad a = -4\omega^2 y$

$\Rightarrow$ As acceleration, $a \propto -y$

Hence, due to negative sign, motion is SHM.

Clearly, from the equation,

$\omega' = 2\omega \quad$ [$\because$ standard equation $y = a\cos(\omega t + \phi)$]

$\Rightarrow \quad \frac{2\pi}{T'} = \omega \Rightarrow T' \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

$\left[\text{and given equation } y = 3\cos\left(-2\omega t + \frac{\pi}{4}\right)\right]$

So, motion in SHM with periodic $\frac{\pi}{\omega}$

The correct option is (B)

41. Given equation of motion is

$y = \sin^3 \omega t$

$= (3\sin\omega t - 4\sin 3\omega t)4$

$\Rightarrow \frac{dy}{dt} = \left[\frac{d}{dt}(3\sin\omega t) - \frac{d}{dt}(4\sin 3\omega t)\right]/4$

$\Rightarrow 4\frac{dy}{dt} = 3\omega\cos\omega t - 4\times[3\omega\cos 3\omega t]$

$\Rightarrow 4\times\frac{d^2y}{dt^2} = -3\omega^2\sin\omega t + 12\omega\sin 3\omega t$

$\Rightarrow \frac{d^2y}{dt^2} = -\frac{3\omega^2\sin\omega t + 12\omega^2\sin 3\omega t}{4}$

$\Rightarrow \frac{d^2y}{dt^2}$ is not proportional to y.

Hence, motion is not SHM.

As the expression is involving sine function, it will be periodic.

The correct option is (B)

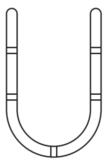
42. For motion to be SHM, acceleration of the particles must be proportional to negative of displacement.

That is, $a \propto -(y \text{ or } x)$

We should be clear that y has to be linear.

The correct option is (D)

43. Consider the diagram in which a liquid column oscillates. In this case, restoring force acts on the liquid due to gravity. Acceleration of the liquid column can be calculated in terms of restoring force.



Restoring force f = Weight of liquid column of height $2y$

$\Rightarrow f = -(A\times 2y\times\rho)\times g = -2A\rho gy$

$\Rightarrow f \propto -y \Rightarrow$ Motion is SHM with force constant $k = 2A\rho g$

$\Rightarrow$ Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{A\times 2h\times\rho}{2A\rho g}} = 2\pi\sqrt{\frac{h}{g}}$

$T = 2\pi\sqrt{\frac{l}{g}}$, where $l = h$

Which is independent of the density of the liquid.

The correct option is (C)

44. Given, $x = a\cos\omega t$ (1)

$y = a\sin\omega t$ (2)

Squaring and adding Equation (1) and (2)

$x^2 + y^2 = a^2 \quad (\cos^2\omega t + \sin^2\omega t)$

$= a^2 \Rightarrow x^2 + y^2 = a^2 \quad [\because \cos^2\omega t + \sin^2\omega t = 1]$

This is the equation of a circle.

Clearly, the locus is a circle of constant radius a.

The correct option is (C)

45. According to the question, the displacement

$y = a\sin\omega t + b\cos\omega t$

Let $a = A\sin\phi$ and $b = A\cos\phi$

Now, $a^2 + b^2 = A^2\sin^2\phi + A^2\cos^2\phi$

$= A^2 \Rightarrow A = \sqrt{a^2 + b^2}$

$y = A\sin\phi\cdot\sin\omega t + A\cos\phi\cdot\cos\omega t$

$= A\sin(\omega t + \phi)$

$\frac{dy}{dt} = A\omega\cos(\omega t + \phi)$

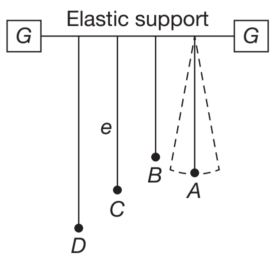
$\frac{d^2y}{dt^2} = -A\omega\sin(\omega t + \phi) = -Ay\omega^2 = (-A\omega^2)y$

$\Rightarrow \frac{d^2y}{dt^2} \propto (-y)$

Hence, it is an equation of SHM with amplitude $A = \sqrt{a^2 + b^2}$

The correct option is (D)

46. According to the question, A is given a transverse displacement



Through the elastic support, the disturbance is transferred to all the pendulums. A and C are having same length, hence they will be in resonance, because their time period is off oscillation.

$T = 2\pi\sqrt{\frac{l}{g}}$. Hence, frequency is same.

So, amplitude of A and C will be maximum.

The correct option is (B)

47. Let angular velocity of the particle executing circular motion be ω and when it is at Q, it makes an angle θ as shown in Fig. 9.49.

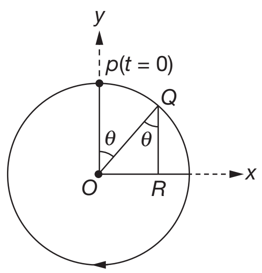


Fig. 9.49

Clearly, $\theta = \omega t$

Now, we can write $OR = OQ \cos(90 - \theta)$

$= OQ \sin\theta = OQ \sin \omega t$

$= r \sin \omega t$ $[\because OQ = r]$

$\Rightarrow x = r \sin \omega t = B \sin \omega t$ $[\because r = B]$

$= B \sin \frac{2\pi}{T} t = B \sin\left(\frac{2\pi}{30} t\right)$

Clearly, this equation represents SHM.

The correct option is (A)

48. As the given equation is

$x = a\cos(\alpha t)^2$

is a cosine function. Hence, it is an oscillatory motion.

Now, putting $t + T$ in place of t

$x(t+T) = a\cos[\alpha(t+T)]^2$ $[\because x(t) = a\cos(\alpha t)^2]$

$= a\cos[\alpha t^2 + \alpha T^2 + 2\alpha t T] \neq x(t)$

where T is supposed to be period of the function $\omega(t)$

Hence, it is not periodic.

The correct option is (C)

49. Let equation of an SHM be represented by $y = a \sin \omega t$

$v = \frac{dy}{dt} = a\omega \cos \omega t$

$\Rightarrow (V)_{max} = a\omega = 30$ (1)

Acceleration $(A) = \frac{dx^2}{dt^2} = -a\omega^2 \sin \omega t$

$A_{max} = \omega^2 a = 60$ (2)

From Equations (1) and (2), we get $\omega(\omega a) = 60 \Rightarrow \omega(30) = 60$

$\Rightarrow \omega = 2$ rad/s

$\Rightarrow \frac{2\pi}{T} = 2$ rad/s $\Rightarrow T = \pi$ s.

The correct option is (A)

50. $I_b = I_e - I_c$

$\beta = \frac{I_c}{I_b} = 49$

The correct option is (B)

51. The correct option is (B)

52. $\frac{\lambda_A}{\lambda_B} = \frac{v_A}{v_B} = \sqrt{\frac{T_A \times A_B}{T_B \times A_A}}$ $[\because f$ and ρ is constant]

$\frac{\lambda_A}{\lambda_B} = \sqrt{2 \times 2} = 2$

The correct option is (D)

53. The correct option is (D)

54. $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{4}{1}$

The correct option is (C)

55. $v = v_0\left(1 + \frac{t}{546}\right)$

$\Rightarrow \Delta v = \frac{v_0 t}{546} = 0.6$ m/s

The correct option is (A)

56. The correct option is (D)

57. Frequency is independent of medium

The correct option is (C)

58. The correct option is (C)

59. $2\pi v a_0 = 3v\lambda$; $\lambda = \frac{2\pi a_0}{3}$

The correct option is (B)

60. Maximum particle acceleration $(a_{max}) = \omega^2 A$

$\frac{(a_{max})_2}{(a_{max})_1} = \left(\frac{45\pi}{15\pi}\right)^2 \frac{2A}{A} = 18$

The correct option is (C)

61. Separation between adjacent nodes is $\lambda/2$ and $K = \frac{\pi}{3} = \frac{2\pi}{\lambda}$, $\lambda = 6$ cm

The correct option is (B)

62. As the pulse goes up, the tension in the string increases and $v \alpha \sqrt{T}$

$\therefore$ velocity of the pulse will keep increasing as the pulse moves upwards

The correct option is (B)

63. $|250 - v_B| = 4$ $\therefore$ v_B can be 254 Hz or 246 Hz

Now on filing of A, v_A will increase and on waxing of B, v_B will decrease

$\therefore$ For same number of beats initially, $v_B > v_A$

$\therefore$ $v_B = 254$ Hz

The correct option is (C)

64. $n_1 = n\left(\frac{v}{v - v_s}\right) = 450\left(\frac{330}{330 - 33}\right) = 500$ Hz

The correct option is (D)

65. $A=\sqrt{(10)^2+(20)^2+2(10)(20)\cos\frac{\pi}{2}}=10\sqrt{5}$

The correct option is (C)

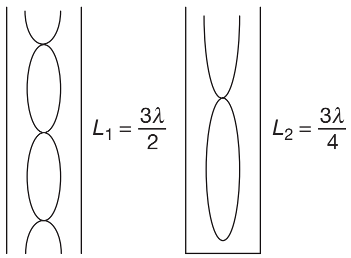
66. Frequency depends on the unfilled length of tube.

The correct option is (D)

67. $\frac{I_{\max}}{I_{\min}}=\left(\frac{A_1+A_2}{A_1-A_2}\right)^2=\left(\frac{4+3}{4-3}\right)^2=\frac{49}{1}$

The correct option is (B)

68.



$\frac{L_1}{L_2}=\frac{3\lambda/2}{3\lambda/4}=\frac{2}{1}$

The correct option is (A)

69. $l=\frac{\lambda_1}{4}\ \therefore\ \lambda_1=4l$

$l=\frac{3\lambda_2}{4}\ \therefore\ \lambda_2=\frac{4l}{3}$

$l=\frac{5\lambda_2}{4}\ \therefore\ \lambda_3=\frac{4l}{5}$

$\therefore\ \lambda_1:\lambda_2:\lambda_3=1:\frac{1}{3}:\frac{1}{5}$

The correct option is (B)

70. $2\pi f y_0=4\,f\lambda,\ \lambda=\frac{\pi y_0}{2}$

The correct option is (B)

71. $\frac{I_2}{I_1}=\frac{f_2^2A_2^2v_2}{f_1^2A_1^2v_1}=\frac{400\times100\times100}{100\times25\times100}=16$

The correct option is (D)

72. $k=\frac{2\pi}{\lambda}=\frac{\pi}{15}\ \Rightarrow\lambda=30$ cm

Distance between a node and next anti-node $=\frac{\lambda}{4}=\frac{30}{4}$ $=7.5$ cm

The correct option is (A)

73. At rigid support, after reflection phase changes by π.

The correct option is (D)

74. As $v=\sqrt{\frac{\gamma RT}{M}}$, $\therefore\ \frac{\Delta v}{v}=\frac{1}{2}\frac{\Delta T}{T}$

$100\times\frac{\Delta v}{v}=\frac{1}{2}\times\frac{1}{300}\times100=0.167\%$

The correct option is (A)

75. $y=0.1\sin(100\pi t-kx)$

$\omega=100\pi\ \therefore\ k=\frac{\omega}{v}=\frac{100\pi}{100}=\pi\,\text{m}^{-1}$

The correct option is (C)

76. In case of reflection from a stationary cliff,

$f'=\left(\frac{v+v_0}{v-v_0}\right)f=2f\ \therefore\ v_0=\frac{v}{3}$

The correct option is (D)

77. As $P\propto I$,

$\therefore\ SL_2-SL_1=10\log\left(\frac{I_2}{I_1}\right)=10\log\left(\frac{P_2}{P_1}\right)$

$=10\log\left(\frac{400}{20}\right)=10\log20=13$ dB

The correct option is (C)

78. Band width $=2f_m=10$ kHz

The correct option is (B)

79. $\Delta x=0.8\,\text{m},\ \ n=120$ Hz, $\ \Delta\phi=0.5\pi$

$\Delta\phi=\frac{2\pi}{\lambda}\cdot\Delta x$

or $\lambda=2\pi.\frac{\Delta x}{\Delta\phi}=\frac{2\pi}{0.5\pi}\times0.8=3.2$ m

Wave velocity, $v=n\lambda=120\times3.2=384$ m/s

The correct option is (C)

80. $e=\frac{1}{2}(l_2-3l_1)$

$1=\frac{1}{2}(l_2-3\times14)\ \Rightarrow\ l_2=44$ cm

The correct option is (A)

81. Time of fall $=\sqrt{\frac{2h}{g}}$

Time taken by the sound to come out $=\frac{h}{c}$

$\therefore$ Total time $=\sqrt{\frac{2h}{g}}+\frac{h}{c}=h\left[\sqrt{\frac{2}{gh}}+\frac{1}{c}\right]$

The correct option is (A)

82. $f=\frac{nv}{2L}$, if L is doubled, n should also be two for same f.

The correct option is (D)

83. $f'=\left(\frac{v+v_0}{v-v_0}\right)\times f$; $450=\frac{340+v_0}{340-v_0}\times400$;

$v_0=20$ m/s

The correct option is (B)

84. The correct option is (D)

85. $y = 0.1\sin(100\pi t - kx)$

$$\omega = 100\pi \quad \therefore\ k = \frac{\omega}{v} = \frac{100\pi}{100} = \pi\,\text{m}^{-1}$$

The correct option is (C)

86. $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$$f' = \frac{1}{2\times 0.55l}\sqrt{\frac{1.21T}{\mu}} = \frac{1}{l}\sqrt{\frac{T}{\mu}} = 2f$$

The correct option is (B)

87. Number of beats $= f\left(\frac{v}{v-v_s}\right) - f\left(\frac{v}{v+v_s}\right) \approx \frac{2f\,v_s}{v^2}$

$(\because\ v_s << v)$

$$= \frac{2\times 300\times 4}{340} \approx 7$$

The correct option is (C)

88. Apparent frequency for reflector would be

$$f_1 = \left(\frac{v+u}{v}\right)f$$

After being reflected, the apparent frequency will further change and the reflector will now behave as a source. The apparent frequency will now become

$$f_2 = \left(\frac{v}{v-u}\right)f_1$$

Finally, we get $f_2 = \left(\frac{v+u}{v-u}\right)f$

The correct option is (C)

89. Intensity after passing through first slab = 80% of $I_0 = 0.8I_0$

Intensity after passing through second slab = 80% of 0.8 I_0 = 0.64 I_0

Decrease in intensity $= I_0 - 0.64I_0 = 0.36I_0$

The correct option is (C)

90. As $n_1 : n_2 : n_3 = 1:2:3$

$$\therefore\quad l_1 : l_2 : l_3 = \frac{1}{1}:\frac{1}{2}:\frac{1}{3} = 6\ :3:2$$

$$\therefore\quad l_1 = \frac{110}{11}\times 6 = 60\ \text{cm}\ ;\ l_2 = \frac{110}{11}\times 3 = 30\ \text{cm}$$

∴ Bridges should be placed from A at 60 cm and 90 cm.

The correct option is (B)

91. The relative velocity of sound waves with respect to the walls is $V + v$.

Hence, the apparent frequency of the waves striking the surface of the wall is $\frac{(V+v)}{\lambda}$.

The number of positive crests striking per second is the same as frequency.

In three seconds, the number is $[3(V+v)]/\lambda$.

The correct option is (A)

92. Doppler's effect depends upon velocity of approach and separation of source and observer. Hence no change in frequency received by the observer.

∴ No beat is heard.

The correct option is (D)

93. For reflected sound,

$$f' = \left(\frac{v+v_0}{v-v_0}\right)f = \frac{360}{300}\times 600 = 720\ \text{Hz}$$

The correct option is (A)

94. $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$

$$\equiv A\cos\left(\omega t + \frac{\pi}{2}\right)\cos kx$$

Node occurs when $kx = \frac{\pi}{2} \Rightarrow 10\pi x = \frac{\pi}{2} \Rightarrow x = 0.05$ m

Antinode occurs when $kx = \pi \Rightarrow 10\pi x = \pi \Rightarrow x = 0.1$ m

Speed of wave $(v) = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s

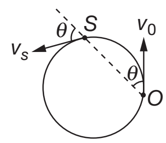
Wavelength $(\lambda) = \frac{2\pi}{k} = 0.2$ m

The correct option is (C)

95. Let v = speed of sound and u = speed of train

Then $v_s = v_0 = u$

and $f' = f\left(\frac{v + v_0\cos\theta}{v + v_s\cos\theta}\right) = f$



The correct option is (C)

96. The correct option is (D)

97. $f \propto \sqrt{g}$

In water, $f_w = 0.8 f_{air}$

$$\therefore\quad \frac{g'}{g} = (0.8)^2 = 0.64$$

or $\frac{\rho_w}{\rho_m} = 0.36$ (1)

In liquid, $\frac{g'}{g} = (0.6)^2 = 0.36$ or $\frac{\rho_L}{\rho_m} = 0.64$

$$\frac{\rho_L}{\rho_w} = \frac{0.64}{0.36}$$

$\therefore \quad S_L = \rho_L/\rho_w = 1.77$

The correct option is (B)

98. $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, $I = 10^6 I_0 = 10^{-6}$ w/m^2

$P = I4\pi(r)^2 = 10^{-6} \times 4\pi \times (500)^2$, $P = \pi W = 3.14$ W.

$$t = \frac{10^3 \times \frac{30}{100}}{3.14} = 95.5$$

The correct option is (B)

99. Using the formula, $f' = f\left(\frac{v + v_0}{v}\right)$

we get, $5.5 = 5\left(\frac{v + v_A}{v}\right)$ (1)

and $6.0 = 5\left(\frac{v + v_B}{v}\right)$ (2)

From (1) and (2), $\frac{v_B}{v_A} = 2$

The correct option is (B)

100. $n\frac{\lambda_1}{2} = (n+1)\frac{\lambda_2}{2}$ (1)

(Given $\lambda_1 = 36, \lambda_2 = 32$)

$\therefore n = 8$

$\frac{n\lambda_1}{2} = l \ \therefore \ l = 144$ cm

The correct option is (A)

101. $y_1 = 2a\sin\left(\omega t + \frac{\pi}{6}\right) = -2a\cos\left(\frac{\pi}{2} + \omega t + \frac{\pi}{6}\right)$

$\Rightarrow \phi_1 = \frac{\pi}{2} + \omega t + \frac{\pi}{6}$

$y_2 = -2a\cos\left(\omega t - \frac{\pi}{6}\right) \Rightarrow \phi_2 = \omega t - \frac{\pi}{6}$

phase diff. $= \phi_1 - \phi_2 \ = \frac{\pi}{2} + \omega t + \frac{\pi}{6} - \left(\omega t - \frac{\pi}{6}\right)$

$= \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

The correct option is (D)

102. $v' = 1.04\,V$

$= v\left(\frac{300 + v_0}{300 - v_0}\right)$

$300 + v_0 = 300 \times 1.04 - 1.04v_0$

$2.04v_0 = 0.04 \times 300$

$v_0 = 5.88m/\sec$

The correct option is (D)

103. $f = \frac{n}{4l}v$

$l = \frac{nv}{4f} = \frac{n \times 320}{4 \times f} = \frac{80n}{f}$

$l_{max} = \frac{80}{20} = 4$ m

$l_{min} = \frac{80}{20 \times 10^3} = 4 \times 10^{-3}$ m

The correct option is (D)

104. Sound level : $= 10\log\frac{I}{I_0}$

$10 = 10\log\frac{I}{I_0} \ \Rightarrow \ I = I_0 10^1 - 10I_0$

When four sources are sounds together, intensity $= 4I = 40I_0$

$\text{SL} = 10\ \log\frac{40I_0}{I_0} = 10\ \log 40 \approx 16\ dB$

The correct option is (C)

105. In this mode of vibrations,

Wavelength λ = length of the pipe = 36 cm

Frequency of wave will be same inside and outside the pipe

$\Rightarrow \quad \frac{v}{\lambda} = \frac{v'}{\lambda'} \Rightarrow \lambda' = \lambda\sqrt{\frac{v'}{v}} = \lambda\sqrt{\frac{T'}{T}}$

or $\quad \lambda' = 36\sqrt{\frac{289}{324}} = 34$ cm

The correct option is (C)

106. $I = \frac{P}{S} = \frac{P}{2\pi r^2} = \frac{10^{-3}}{2\pi \times \frac{25}{\pi}} = \frac{10^{-3}}{50} = 2 \times 10^{-5}$

Sound level $= 10\log\frac{I}{I_0} = 10\log\frac{2 \times 10^{-5}}{10^{-12}}$

$= 10\log(2 \times 10^7) = 10[7 + 0.3010] = 73$ dB

The correct option is (A)

107. $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$\frac{df}{f} = \frac{1}{2}\frac{dT}{T}$

$df = \frac{1}{2}\left(\frac{1}{100}\right)(200) = 1$

The correct option is (B)

108. Distance between two consecutive nodes is 80 cm.

So, length = 6 × 80 = 480 cm = 4.8 m

The correct option is (D)

109. $\frac{y}{x}=\frac{8}{6}$

$y=\frac{4}{3}x$

The correct option is (C)

110. Velocity of wave remains same for both pulses for the same string.

The correct option is (D)

111. The correct option is (A)

112. $v=\sqrt{\frac{\gamma P}{\rho}},\ \frac{v_{O_2}}{v_m}=\sqrt{\frac{\rho_m}{\rho_{O_2}}}$

The correct option is (B)

113. $f_{\max}=\frac{v}{v-v_s}f,\ f_{\min}=\frac{v}{v+v_s}f$

The correct option is (D)

114. According to Fig. 9.30, $\frac{5\lambda}{2}=20$

or $\lambda=\frac{20\times 2}{5}=8\ \text{cm}$

$\therefore\quad n=\frac{v}{\lambda}=\frac{320\times 100}{8}=4000\ \text{Hz}$

The correct option is (C)

115. The correct option is (A)

116. The correct option is (B)

117. $f=\frac{nv}{2l}$, If l increases by 2%, f decreases by 2%.

$\therefore\quad$ 2% of $f=5$, $f=250\,\text{Hz}$

The correct option is (B)

118. Particle velocity $=\frac{dy}{dt}=-v\left(\frac{dy}{dx}\right)$

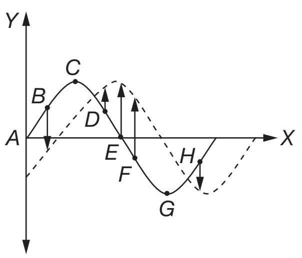
or $\frac{dy}{dt}=-$ wave velocity $\times$ slope of the wave

(A) For upward velocity, $v_{Pa}=+ve$, so slope must be negative which is at the points D, E, and F.

(B) For downward velocity, $v_{Pa}=-ve$, so slope must be negative which is at the points A, B and H.

(C) For zero velocity, slope must be zero which is at C and G.

(D) For maximum magnitude of velocity, | slope | = maximum, which is at A and E. Hence, alternative is wrong.



The correct option is (D)

119. The correct option is (D)

120. $f=\frac{(2n-1)v}{4l}$

$l=25$ cm, 75 cm, 125 cm

Minimum height of water is 45 cm

The correct option is (B)

121. $y=y_1+y_2=8\sin 3x\cos 2t$

Nodes are formed $\sin 3x=0\ \Rightarrow x=0,\ \frac{\pi}{3},\ \frac{2\pi}{3}$ ....

The correct option is (C)

122. Area covered = $2\pi Rh$ = $2\pi\times 6.4\times 10^6\times 100\ \text{m}^2$ = 1.28 $\pi\times 10^3\ \text{km}^2$

The correct option is (B)

123. $18n=l$; $16(n+1)=l\ \Rightarrow l=144$ cm

The correct option is (C)

124. For fifth overtone, $\frac{5\lambda}{2}+\frac{\lambda}{4}=33$ cm, $\lambda=12$ cm

At a distance 18 cm (= $3\lambda/2$) from closed end node is formed

The correct option is (D)

125. Doppler's effect depends upon velocity of approach and separation of source and observer. Hence, no change in frequency received by the observer.

$\therefore$ no beat is heard.

The correct option is (D)

126. $1700\left\{\frac{340}{340+v_2}-\frac{340}{340+v_1}\right\}=10$

$\Rightarrow\quad \frac{1700\times 340}{340}\left\{\left(1-\frac{v_2}{340}\right)-\left(1-\frac{v_1}{340}\right)\right\}=10$

or $\quad \frac{1700(v_1-v_2)}{340}=10$ or $v_1-v_2=2$ m/s

The correct option is (B)

127. $f=\frac{(2n+1)v}{4(l+e)}$; $(l_1+e)=\frac{v}{4f}$; $(l_2+e)=\frac{3v}{4f}$

$\Rightarrow\quad \frac{l_2+e}{l_1+e}=3$

$l_2=(3.6-2.34)$ m and $l_1=(3.6-3.22)$

$\Rightarrow\quad e=0.06\ \text{m}=0.6\,r\quad\Rightarrow\quad r=0.1$ m

$A=100\ \pi\,\text{cm}^2$

The correct option is (B)

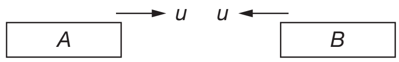
128. $y=2A\cos kx\sin\omega t$ (assuming $t=0, y=0$), $\lambda=\frac{2l}{3}$

as $\Delta P=B\frac{dy}{dx}=B\,2Ak\sin kx\sin\omega t$,

$\Delta P_{\max}=B(2A)k$ also $v=\sqrt{\frac{B}{\rho}}\ \Rightarrow 2A=2.5$ cm.

The correct option is (A)

129. Let velocity of sound in air is v and u is velocity of each train



If air in still, then $n' = n_0\left(\frac{v+u}{v-u}\right) = n_0\left(1+\frac{2u}{v-u}\right)$

If wind blows in the same direction and at the same speed as that of B

$$n' = n_0\left(\frac{v-u+u}{v-u-u}\right) = n_0\left(1+\frac{2u}{v-2u}\right)$$

If wind blows in the opposite direction and at the same speed as that of B

$$n' = n_0\left(\frac{v+u+u}{v+u-u}\right) = n_0\left(1+\frac{2u}{v}\right)$$

The correct option is (B)

130. $l_1 = \frac{\lambda}{4}, l_2 = \frac{3}{4}\lambda \Rightarrow \lambda = 2(l_2 - l_1)$, $v = n\lambda = 330$ m/s,

also $\frac{dv}{v} = \frac{dl_2 + dl_1}{l_2 - l_1}$, $dv = 1.98$ m/s.

The correct option is (B)

131. In third overtone, $\frac{4\lambda}{2} = l \Rightarrow \lambda = \frac{l}{2}$

Amplitude at a distance $\frac{l}{3}$ from one end

$$= \left|A\sin\frac{2\pi}{\lambda}\frac{l}{3}\right| = \left|A\sin\frac{\pi}{3}\right| = \frac{\sqrt{3}A}{2}$$

The correct option is (C)

132. $f = \frac{3v}{4(L+0.6r)}$

$$\frac{df}{dt} = \frac{3v}{4}\left(-\frac{1}{(L+0.6r)^2}\cdot(0.6)\frac{dr}{dt}\right)$$

$$-2 = -\frac{3v}{4}\left(0.6\frac{dr}{dt}\right)$$

$$\frac{8}{3v\times 0.6} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{72} \text{ m/s}$$

The correct option is (A)

133. $\Delta Pm = 2\Delta P_0 \cos kx$ (assuming closed end as origin)

At point Q, $x = L - \frac{7L}{9} = \frac{2L}{9}$

$$\Delta Pm = 2\Delta P_0 \cos\left(\frac{2\pi}{\lambda}\times\frac{2L}{9}\right) = \Delta P_0$$

$\therefore$ Required ratio = 1 : 2

The correct option is (A)

134. The energy stored in the first part of the wave is equal to energy stored in the second part of the wave

Instantaneous power $= -T\frac{\partial y}{\partial x}.\frac{\partial y}{dt} = Tm^2v$ (constant)

Where $\frac{\partial y}{\partial t} = -mv$ and $\frac{\partial y}{\partial x} = +m$

$\therefore$ Energy in wave pulse = instantaneous power $\times \frac{a/2}{v}\times 2$
$= Tm^2a$

The correct option is (B)

135. On the horizontal, the path difference is decreasing so at the maximum distance from P where minima occurs, path difference is $\frac{\lambda}{2}$.

Hence $\sqrt{9\lambda^2 + x^2} - x = \frac{\lambda}{2} \Rightarrow x = 8.75\,\lambda$

The correct option is (C)

136. $\psi_1 = A\cos(\omega t - kx)$ (1)

$\psi_2 = A\cos(\omega t - ky)$ (2)

Put $y = x + \frac{(2n+1)}{2}\lambda$ in (2)

$$\psi_2 = A\cos\left[\omega t - k\left(x + (2n+1)\frac{\lambda}{2}\right)\right]$$

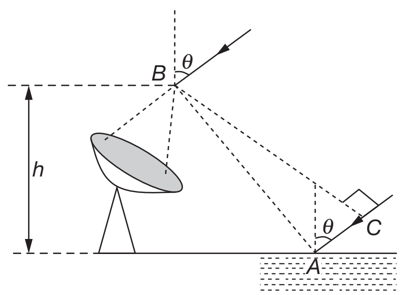
$\therefore$ $\psi_2 = A\cos(\omega t - kx - (2n+1)\pi)$

$\therefore$ $\psi_2 = -A\cos(\omega t - kx) = -\psi_1$

$\therefore$ Shape of path is a straight line.

The correct option is (A)

137. Path difference of the two rays (one coming directly to antenna and other after reflection from water surface)



$$\Delta x = AB + AC + \frac{\lambda}{2}$$

$$\Rightarrow \Delta x = \frac{h}{\cos\theta} + \frac{h}{\cos\theta}\cos 2\theta + \frac{\lambda}{2}$$

$$= \frac{h}{\cos\theta}(1+\cos 2\theta) + \frac{\lambda}{2}$$

or $\Delta x = 2h\cos\theta + \frac{\lambda}{2}$

For constructive interference $\Delta x = n\lambda$

$$\Rightarrow \quad 2h\cos\theta + \frac{\lambda}{2} = n\lambda$$

$$\text{or } h = \frac{\left(n-\frac{1}{2}\right)\lambda}{2\cos\theta} = \frac{\lambda}{4\cos\theta}, \frac{3\lambda}{4\cos\theta}, \ldots\ldots \text{ etc}$$

The correct option is (D)

138. Water waves produced by a motorboat sailing in water are both longitudinal and transverse, because the waves produce transverse as well as lateral vibrations in the particles of the medium.

The correct option is (B)

139. Let the frequency in the first medium be ν and in the second medium be ν'

Frequency remains same in both the mediums

$$\text{So,} \quad \nu = \nu' \Rightarrow \frac{v}{\lambda} = \frac{v'}{\lambda'}$$

$$\Rightarrow \quad \lambda' = \left(\frac{v'}{v}\right)\lambda$$

λ and λ', v and v' are wavelengths and speeds in first and second medium, respectively.

$$\text{So,} \quad \lambda' = \left(\frac{2v}{v}\right)\lambda = 2\lambda$$

The correct option is (C)

140. Due to presence of moisture, density of air decreases.

We know that speed of sound in air is given by $v = \sqrt{\frac{\gamma p}{\rho}}$

For air γ and p are constants.

$v \propto \frac{1}{\sqrt{\rho}}$, where ρ is density of air.

$$\frac{v_2}{v_1} = \sqrt{\frac{\rho_2}{\rho_1}}$$

where ρ_1 is density of dry air and ρ_2 is density of moist air.

$$\text{As,} \quad \rho_2 < \rho_1 = \frac{v_2}{v_1} > 1 \Rightarrow v_2 > v_1$$

Hence, speed of sound wave in air increases with increase in humidity.

The correct option is (C)

141. Speed of sound wave in a medium $v \propto \sqrt{T}$ (where T is temperature of the medium) Clearly, when temperature changes, speed also changes.

As, $v = \nu\lambda$

where ν is frequency and is wavelength.

Frequency (ν) remains fixed.

$\Rightarrow \quad v \propto \lambda$ or $\lambda \propto v$

As frequency does not change, wavelength (λ) changes.

The correct option is (C)

142. Propagation of longitudinal waves through a medium leads to transmission of energy through the medium without matter being transmitted. There is no movement of matter (mass) and hence momentum

The correct option is (B)

143. When mechanical transverse wave propagates through a medium, the constituent of the medium oscillate perpendicular to wave motion causing change in shape. That is, each element of the medium is subjected to shearing stress. Solids and strings have shear modulus, that is, way sustain shearing stress.

Fluids have no shape of their own, they yield to shearing stress. This is why transverse waves are possible in solids and strings but not in fluids.

The correct option is (C)

144. (A) Due to compression and rarefactions, density of the medium (air) changes. At compressed regions, density is maximum and at rarefactions density is minimum

(B) As density is changing, so Boyle's law is not obeyed

(C) Bulk modulus remains same

(D) The time of compression and rarefaction is too small, i.e., we can assume adiabatic process and hence no transfer of heat

The correct option is (D)

145. Amplitude of reflected wave

$$A_r = \frac{2}{3} \times A_r = \frac{2}{3} \times 0.6 = 0.4 \text{ units}$$

Given equation of incident wave

$$y_i = 0.6\sin 2\pi\left(t - \frac{x}{2}\right)$$

Equation of reflected wave is

$$y_r = A_r \sin 2\pi\left(t + \frac{x}{2} + \pi\right)$$

[$\because$ at denser medium, phase changes by π]

The positive sign is due to reversal of direction of propagation

$$\text{Thus,} \quad y_r = -0.4\sin 2\pi\left(t + \frac{x}{2}\right) \qquad [\because \sin(\pi+\theta) = -\sin\theta]$$

The correct option is (B)

146. Mass $m = 2.5$ kg

μ = mass per unit length

$$= \frac{m}{l} = \frac{2.5 \text{ kg}}{20} = \frac{125}{10} = 0.125 \text{ kg/m}$$

$$\text{Speed } v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}}$$

[speed of transverse waves in any string]

$$l = v \times t \Rightarrow 20 = \sqrt{\frac{200}{0.125}} \times t$$

$$\Rightarrow \quad t = 20 \times \sqrt{\frac{125}{2\times10^5}} = 20\times\sqrt{\frac{25\times5}{2\times10^5}}$$

$$= 20\times\sqrt{25\times\frac{1}{0.4\times10^5}}$$

$$= 20\times5\sqrt{\frac{1}{4\times10^4}} = \frac{20\times5}{2\times10^2}$$

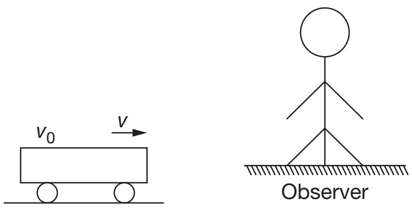
$$= \frac{1}{2} = 0.5$$

The correct option is (B)

147. Let the original frequency of the source be n_0.

Let the speed of sound wave in the medium be v.

As observer is stationary,



Apparent frequency $n_a = \left(\frac{v}{v - v_s}\right) n_0$

[when train is approaching]

$$= \left(\frac{v}{v - v_s}\right) n_o = n_a > n_o$$

When the train is going away from the observer

Apparent frequency $n_a = \left(\frac{v}{v - v_s}\right) n_o = n_a < n_o$

The correct option is (C).

More than One Option Correct Type

148. The correct option is (D)

149. At $t = 0$, kinetic energy is $\frac{1}{4}th$ the maximum kinetic energy, i.e., speed of the particle is half the maximum speed or $v = \frac{\pm Aw}{2}$ at $t = 0$.

$$v = \frac{-A\omega}{2} \text{ for option (A)}$$

$$v = \frac{A\omega}{2} \text{ for option (B)}$$

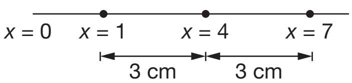
$$v = \frac{-A\omega}{2} \text{ for option (C)}$$

$$v = \frac{A\omega}{2} \text{ for option (D)}$$

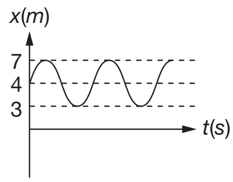
Therefore, all the four options are correct.

The correct option is (A), (B), (C) and (D)

150. The motion of the particle is somewhat like



minimum value of x = 1 cm and maximum value 7 cm, i.e., the particle oscillates simple harmonically about point $x = 4$ cm with amplitude 3 cm.



The correct option is (B) and (D)

151. The correct option is (A) and (B)

152. Argument of sine function must be dimensionless options (b) and (c) are wrong expressions.

The correct option is (B) and (C)

153. Time period T = 8s and force constant = 10 N/m (slope of F-x graph)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$m = \frac{(8)^2}{4\pi^2}\times10 = \frac{160}{\pi^2}\text{ kg}$$

Maximum kinetic energy $= \frac{1}{2}m\omega^2A^2 = 80$ J

The correct option is (A) and (C)

154. The correct option is (A) and (C)

155. $2kx_0\cos\theta = mg$ (1)

$$F_R = -2k\cos^2\theta\, x$$

$$\omega = \sqrt{\frac{2k\cos^2\theta}{m}}$$

The correct option is (A) and (C)

156. $y_1 = a\cos\omega_1 t$ and $y_2 = a\cos(\pi + \omega_2 t)$

For same phase $\omega_1 t = \pi + \omega_2 t$

$$t = \frac{\pi}{\omega_1 - \omega_2} = \frac{\pi}{\frac{2\pi}{3} - \frac{2\pi}{7}} = \frac{21}{8}s$$

The correct option is (A)

157. $U = U_0(1 - \cos ax)$,

$$F = \frac{-dU}{dx} = -aU_0 \sin ax$$

$$m\frac{d^2x}{dt^2} = -a^2U_0 x \text{ (For small oscillations)}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{a^2U_0}}$$

Potential energy is maximum at $x = \frac{\pi}{a}$

Amplitude is $\frac{\pi}{a}$

Speed of particle is maximum at $x = 0$.

($\because$ PE is minimum)

The correct option is (B), (C) and D)

158. The correct option is (B) and (D)

159. $v^2 = a - bx^2$

differentiating both side with respect to time

$$2v\frac{dv}{dt} = -2bx\left(\frac{dx}{dt}\right) \quad \frac{dv}{dt} = A \text{ (acceleration)}$$

$$A = -bx$$

Hence motion will be SHM $\omega^2 = b$, $4\pi^2 f^2 = b$, $f = \frac{\sqrt{b}}{2\pi}$

$$\text{T.E.} = \frac{1}{2}m\omega^2 \text{ (Amplitude)}^2 = \frac{1}{2}m\, b\, \frac{a}{b} = \frac{1}{2}ma$$

The correct option is (B) and (C)

160. When the particle of mass m at O is pushed by y in the direction of A, spring A will be compressed by y, while B and C will stretched by $y' = y\cos 45°$. So the total restoring force on the mass m along AO

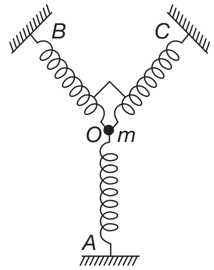
$$RF = F_A + F_B\cos 45 + F_C\cos 45$$

i.e. $RF = ky + 2(ky')\cos 45$

or $RF = ky + 2k(y\cos 45)\cos 45$

i.e. $F = -k'y$ with $k' = 2k$

so $T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$



The correct option is (B) and (C)

161. $(\text{Acceleration})_{\max} = \omega^2 a = 5\pi^2$

Velocity at $y = 4$ cm is

$$v = \omega\sqrt{a^2 - y^2} = \omega\sqrt{a^2 - 4^2} = 3\pi$$

or $\omega\sqrt{a^2 - 16} = 3\pi$

Squaring, $\omega^2\left(a^2 - 16\right) = 9\pi^2$

So, $\frac{\omega^2\left(a^2 - 16\right)}{\omega^2 a} = \frac{9\pi^2}{5\pi^2} = \frac{9}{5}$

or $a = \frac{9 \pm 41}{10} = 5$ or -3.2 cm

$\therefore \quad a = 5$ cm

from (1), $\omega^2 \times 5 = 5\pi^2$ or $\omega = \pi$

$\therefore \quad T = 2$ s

The correct option is (B) and (C)

162. Velocity of A just after collision is v_0 in –ve x-axis and velocity of B is $a\omega$.

and $\quad v_0 = A\omega = A\sqrt{\frac{k}{m}}$, $A = v_0\sqrt{\frac{m}{k}}$

So equation is $x = x_0 - v_0\sqrt{\frac{m}{k}}\sin\omega t$

The correct option is (A) and (C)

163. Equation of SHM for particles are $X = -A\cos\omega t$, $X = A\sin\omega t$

They will meet when $A\sin\omega t = -A\cos\omega t$, $A\left[\sin\omega t + \cos\omega t\right] = 0$

$$t = \frac{3T}{8} \text{ and } X = A\sin\frac{2\pi}{T}.\frac{3T}{8} = A\sin\frac{3\pi}{4} = \frac{A}{\sqrt{2}}$$

The correct option is (B) and (D)

164. Velocity of A just after collision is v_0 in –ve x-axis and velocity of B is $a\omega$.

and $v_0 = A\omega = A\sqrt{\frac{k}{m}}$, $A = v_0\sqrt{\frac{m}{k}}$

So equation is $x = x_0 - v_0\sqrt{\frac{m}{k}}\sin\omega t$

The correct option is (A) and (C)

165. Possible frequencies of pipe is $\frac{(2n+1)v}{4L}$ = 80, 240, 400, 560 ....

The correct option is (A), (B) and (D)

166. Number of wave pulse encountered by the moving plane per unit time is given by

$$v' = \frac{\text{distance travelled}}{\text{wavelength}} = \frac{c+V}{\lambda} = \frac{c}{\lambda}\left(1+\frac{V}{c}\right) = v\left(1+\frac{V}{c}\right)$$

The stationary observer meets the frequency v of the incident wave and receives the reflects wave of frequency v'' emitted by the moving plane as

$$v'' = \frac{v'}{1-\frac{V}{c}} = \frac{v\left(1+\frac{V}{c}\right)}{\left(1-\frac{V}{c}\right)} = \frac{v(c+V)}{(c-V)}$$

wavelength $\lambda'' = \frac{c}{v''} = \frac{c}{v}\left(\frac{c-V}{c+V}\right)$

Beat frequency $v'' - v = \frac{v\left(1+\frac{V}{c}\right)}{\left(1-\frac{V}{c}\right)} - v$

$$= v\left[\frac{1+\frac{V}{c}}{1-\frac{V}{c}} - 1\right] = \frac{2vV}{c-V}$$

The correct option is (A), (B) and (C)

167. Sign between x and t is $\oplus$ hence wave is propagating in negative x direction.

Wave velocity $= \frac{\text{coefficient of } t}{\text{coefficient of } x} = \frac{3}{1}$

Max value of y is amplitude, y has maximum value when denominator is least, that is

when $(x+3t)^2$ is zero,

$y = 3m$

Shape of pulse is given by $y = \frac{6}{2+x^2}$ $(t = 0)$

Shape of pulse is symmetrical out y-axis

The correct option is (B), (C) and (D)

168. If equation of a wave pulse is $y = f(ax \pm bt)$

The speed of wave is $\frac{b}{a}$ in negative x direction for $y = f(ax+bt)$ and positive x direction for $y = f(ax-bt)$. Comparing this from given equation, we can find the speed of wave is 5/4 = 1.25 m/s and it is travelling in negative x-direction.

Since $y(x) = y(-x)$ at $t = 0$

$\therefore$ pulse is symmetric, also maximum value of Y is $\frac{0.8}{5}$ = 0.16 m

The correct option is (B), (C) and (D)

169. If screen is perpendicular to the line joining the sources, the fringes will be circular and central fringe will be bright if $S_1S_2 = n\lambda$.

The correct option is (A) and (C)

170. Ratio of frequencies is 3 : 5 : 7

So string is fixed at one end and fundamental frequency is 35 Hz.

The correct option is (B) and (C)

171. $f_{\max} = \frac{320}{320-80} \times 300 = 400$ Hz,

$f_{\min} = \frac{320}{320+80} \times 300 = 240$ Hz

The correct option is (C) and (D)

172. $y(x,t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$

$$\equiv A\cos\left(\omega t + \frac{\pi}{2}\right)\cos kx$$

Node occurs when $kx = \frac{\pi}{2} \Rightarrow 10\pi x = \frac{\pi}{2}$

$\Rightarrow$ $x = 0.05$ m

Antinode occurs when $kx = \pi \Rightarrow 10\pi x = \pi \Rightarrow x = 0.1$ m

Speed of wave $(v) = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s

Wavelength $(\lambda) = \frac{2\pi}{k} = 0.2$ m

The correct option is (C) and (D)

173. Mass per unit length of the wire = 0.1 kg/m.

At any instant, tension in the string is (2 + 2.25 t) g N

$$v = \sqrt{\frac{(2+2.25t)g}{0.1}}$$

$$\int_0^{10} dx = 10\int_0^t \sqrt{2+2.25t}\,dt$$

$t = 0.612\ s$

$T = (2 + 2.25 \times 0.612) \times 10 = 33.77\ N$

The correct option is (A) and (C)

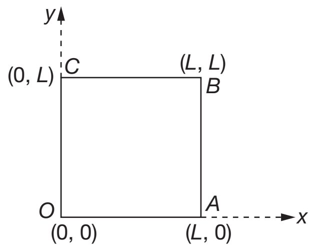
174. The edges are clamped, displacement of the edges $u(x, y) = 0$ for

OA, i.e., $y = 0 \;\; 0 \le x \le L$

AB, i.e., $x = L \;\; 0 \le y \le L$

BC, i.e., $y = L \;\; 0 \le x \le L$

OC, i.e., $x = 0 \;\; 0 \le y \le L$



The correct option is (B) and (C)

Passage Based Questions

Passage 1

175. The correct option is (D)

176. The correct option is (C)

177. The correct option is (C)

Passage 2

178. $\frac{1}{2}mu^2 = mgh$

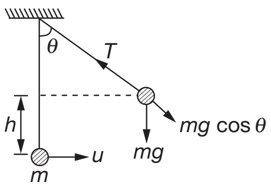
$$h = \frac{u^2}{2g} = \frac{5/2gl}{2g} = \frac{5}{4}l$$

$$h = l(1 - \cos\theta)$$

$$\frac{5}{4}l = l(1 - \cos\theta)$$

$$\cos\theta = -\frac{1}{4}$$

$$\theta = \pi - \cos^{-1}\left(\frac{1}{4}\right)$$



The correct option is (D)

179. If acceleration is horizontal

$$T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta} \quad (1)$$

$$T - mg\cos\theta = \frac{mv^2}{l} \quad (2)$$

$$v^2 = u^2 - 2gl\,(1 - \cos\theta) \quad (3)$$

(1), (2), and (3) $\Rightarrow$

$$\frac{mg}{\cos\theta} - mg\cos\theta = \frac{m}{l}\left[\frac{5gl}{2} - 2gl(1 - \cos\theta)\right]$$

$$\frac{1}{\cos\theta} - \cos\theta = \frac{5}{2} - 2 + 2\cos\theta$$

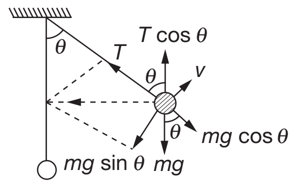
$$\frac{1}{\cos\theta} = 3\cos\theta + \frac{1}{2}$$

$$2 = 6\cos^2\theta + \cos\theta$$

$$6\cos^2\theta + \cos\theta - 2 = 0$$

$$\Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60°$$

(1) $\Rightarrow$ $\quad T = 2mg$



The correct option is (B)

180. (As shown in previous solution)

$\theta = 60°$

The correct option is (B)

181. $\sqrt{2gl} < 4 < \sqrt{5gl}$

$\therefore$ Particle will leave the vertical circle at some point

The correct option is (B)

Passage 3

182. The correct option is (A)

183. The correct option is (B)

184. The correct option is (B)

185. The correct option is (B)

Passage 4

186. Elevator at rest, $T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$

Elevator moves up $T_1 = 2\pi\sqrt{\frac{l}{g + a}} \quad (2)$

When elevator moves down $T_2 = 2\pi\sqrt{\frac{l}{g-a}}$ (3)

Solving given equations (2) and (3) and adding

$$(g+a)+(g-a) = 4\pi^2 l\left(\frac{1}{T_1^2}+\frac{1}{T_2^2}\right)$$

Substitute value of g from equation (1),

we get $T = \sqrt{2}\frac{T_1T_2}{\sqrt{T_1^2+T_2^2}}$

The correct option is (B)

187. Since $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore \quad dT = 2\pi\sqrt{l}\left(-\frac{1}{2}\right)g^{-3/2}dg$

$\Rightarrow \quad dg = \frac{-g\sqrt{g}}{\pi\sqrt{l}}dT$

The correct option is (B)

188. Let length $l + a$ be decreased by l_1 to restore period

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l+a-l_1}{g-b}}$$

Which on solving $l_1 = a + \frac{lb}{g}$

The correct option is (B)

Passage 5

189. Intensity of reflected wave is 64% of incident wave

$$\frac{A_R}{A_I} = \sqrt{\frac{I_R}{I_I}} = 0.8$$

Amplitude of reflected wave $A_R = 0.8\,\text{A}$

The correct option is (B)

190. $d = 2\times 0.8A = 1.6\,\text{A}$

Given resultant wave is constituted by the following waves.

$$y = 0.8A\cos(bt+ax) - 0.8A\cos(bt-ax) + 0.2A\cos(bt+ax)$$

First two waves constitute stationary wave

Hence, $c = 0.2$

The correct option is (A)

191. Maximum displacement = *A + 0.8 A = 1.8 A*

The correct option is (D)

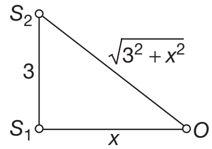
Passage 6

192. $\lambda = \frac{V}{f} = \frac{340}{680} = \frac{1}{2}$

$\Delta x = \frac{\lambda}{2}$

$\sqrt{(3^2+x^2)} - x = \frac{1}{4}$, $3^2 + x^2 = \left(\frac{1}{4}+x\right)^2$

$x = \frac{143}{8} = 17.9$ m



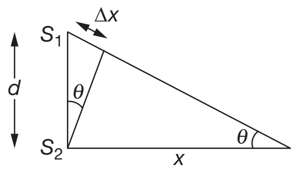
The correct option is (B)

193. $\Delta x = d\sin\theta = (2n-1)\frac{\lambda}{2}$

$\sin\theta = \frac{(2n-1)}{12} \le 1$

$n \le \frac{13}{2} = 6.5$

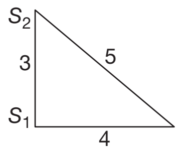
$n = 6$



The correct option is (D)

194. $\Delta x = 5 - 4 = 1$ m

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = 4\pi$



$\therefore$ The correct option is (C)

195. If initially minimum is produced at *P* then by changing the frequency of upper speaker, the net intensity at *P* will *d* increase. But if initially a maxima is produced at *P* then by changing the frequency of upper speaker, the net intensity at *P* will decrease.

The correct option is (D)

Passage 7

196. $y = y_1 + y_2$

$y = A\cos(100\pi t) + A\cos(92\pi t) = 2A\cos 96\pi t.\cos 4\pi t$

$y = 0 \Rightarrow \cos 96\pi t = 0$ or $\cos 4\pi t = 0$

In 1 second, the value of $y_1 + y_2 = 0$, 100 times

The correct option is (C)

197. $v = \frac{\omega}{k} = 200$ m/s

The correct option is (B)

198. $|f_1 - f_2| = 50 - 46 = 4$ Hz

The correct option is (A)

Passage 8

199. Number of beats = $\left| v - \left(\frac{330}{330 \pm v_s}\right) v \right| = \left|\frac{\pm 30 v}{330 \pm v_s}\right|$

$\therefore$ maximum number of beats = $\left(\frac{30v}{330 - v_s}\right) - v = 60$

$\Rightarrow v_s = 30$ m/s

The correct option is (B)

200. $A\omega = 3\ \omega = 30.\ \therefore\ \omega = 10$

Since at $t = 0$, $X = 0$ and at $t = T/2\ v_s > 0$

Hence, equation of SHM will be $X = -300 \sin 10t = 300 \sin (10t + \pi)$

The correct option is (B)

201. Minimum number of beats will be 0.

The correct option is (A)

Passage 9

202. For $x = 5$, $y = 4 \sin (5\pi/15) \cos (96\pi t)$

or $y = 2\sqrt{3} \cos (96\pi t)$

So y will be max when $\cos (96\pi t) = \max = 1$, i.e., $(y_{max})_x = 5 = 2\sqrt{3}$ cm

$\therefore$ The correct option is (B)

203. At nodes amplitude of wave is zero,

i.e., $4\sin\left[\frac{\pi x}{15}\right] = 0$ or $\frac{\pi x}{15} = 0, \pi, 2\pi, 3\pi, \ldots$

So, $x = 0, 15, 30, 45\ 60$ cm [as length of string = 60 cm].

The correct option is (A)

204. As $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

So, $y = 4 \sin\left[\frac{\pi x}{15}\right]\cos(96\pi t) = 2\left[\sin\left(\frac{\pi x}{15} + 96\pi t\right) + \sin\left(\frac{\pi x}{15} - 96\pi t\right)\right]$

or, $y = 2 \sin\left[96\pi t + \frac{\pi x}{15}\right] - 2\sin\left[96\pi t - \frac{\pi x}{15}\right]$

[as $\sin(-\theta) = -\sin\theta$]

i.e., $y = y_1 + y_2$ with $y_1 = 2\sin\left[96\pi t + \frac{\pi x}{15}\right]$

and $y_2 = 2\sin\left[96\pi t - \frac{\pi x}{15}\right]$

The correct option is (C)

Match the Column Type

205. $\omega = \sqrt{\frac{k}{\mu}}$ (μ = reduced mass)

$\omega = \sqrt{\frac{6}{2/3}} = 3$ rad/s

Amplitude 12 cm distributes in inverse ratio of mass

$\therefore\ A_1 = 8$ cm, $A_2 = 4$ cm.

$k_1 = \frac{1}{2} \times 1 \times 3 \times 3 \times \frac{8 \times 8}{100 \times 100} = 28.8 \times 10^{-3}$ J

$k_2 = \frac{1}{2} \times 2 \times 3 \times 3 \times \frac{4 \times 4}{100 + 100} = 14.4 \times 10^{-3}$ J

$\therefore$ (A) $\to$ 2; (B) $\to$ 1; (C) $\to$ 3; (D) $\to$ 4

206. $\frac{1}{2}Kx_{max}^2 = Fx_{max}$

$x_{max} = \frac{2F}{K} = 20$ cm

At equilibrium $Kx_0 = F$

$X_0 = \frac{10}{100} = 0.1$ m = 10 cm

Amplitude = 20 – 10 = 10 cm

Maximum speed = $A\omega = 0.1 \times \sqrt{\frac{100}{1}} = 1$ m/s

Particle starts from extreme

$x = A\cos\omega t$, $\frac{A}{2} = A\cos\omega t$, $t = \frac{\pi}{30} s$

$\therefore$ (A) $\to$ 3, (B) $\to$ 2, (C) $\to$ 1, (D) $\to$ 4

207. $v^2 = 144 - 9x^2$

$\omega^2 = 9 \Rightarrow \frac{2\pi}{T} = 3 \Rightarrow T = \frac{2\pi}{3}$

when $v = 0$, $x = \sqrt{\frac{144}{9}} = \frac{12}{3} = 4$ unit

Acceleration = $v\frac{dv}{dx} = -9x$

Potential energy = $\frac{1}{2}m\omega^2 x^2 = 18$ units

$\therefore$ (A) $\to$ 1, (B) $\to$ 3, (C) $\to$ 2, (D) $\to$ 5

208. (A) $\to$ (1); (B) $\to$ (4); (C) $\to$ (3); (D) $\to$ (2)

209. From A to B:

Angular frequency $\omega = \sqrt{K/M}$

$$V=\omega\sqrt{A_1^2-X^2} \Rightarrow V^2=\frac{K}{M}\left(A_1^2-L^2\right)$$

$$\Rightarrow A_1=\left[\frac{MV^2}{K}+L^2\right]^{1/2}=\sqrt{2} \quad \text{(i)}$$

Also, $L=A_1\cos\omega t$, $t_1=\sqrt{\frac{M}{K}}\cos^{-1}\left[\frac{L}{\left(L^2+\frac{MV^2}{K}\right)^{1/2}}\right]=\pi/4$

From B to O

Now angular frequency $\omega_2=\sqrt{\frac{K}{2M}}$,

$$V=\omega_2\left(A_2^2-L^2\right)^{1/2},$$

$$A_2=\left[L^2+\frac{2MV^2}{K}\right]=\sqrt{3}$$

Also $L=A_2\sin\omega t_2$, $t_2=\sqrt{\frac{2M}{K}}\sin^{-1}\frac{L}{\left[L^2+\frac{2MV^2}{K}\right]^{1/2}}$

$$=\sqrt{2}\sin^{-1}\frac{1}{\sqrt{3}}$$

Now V_{cm} at O is $A_2\omega_2=\sqrt{\frac{3}{2}}$

The maximum compression of the spring is $\sqrt{3}$

$\therefore$ (A) → 2, (B) → 1, (C) → 4, (D) → 3

210. $u=\frac{1}{2}\rho\omega^2A^2$, $\frac{u_1}{u_2}=\frac{\rho_1}{\rho_2}\left(\frac{A_1}{A_2}\right)^2=\frac{1}{16}$,

$$P=\frac{1}{2}\rho\omega^2A^2sv=usv,\ v\propto\frac{1}{\sqrt{\mu}}$$

$\therefore \frac{v_1}{v_2}=2$

$\frac{P_1}{P_2}=\frac{u_1}{u_2}\times\frac{v_1}{v_2}=\frac{1}{8}$, $\quad I=\frac{P}{s}$

$\therefore$ (A) → 2; (B) → 1; (C) → 3; (D) → 4

211. (A) → 1, 3; (B) → 2, 4; (C) → 2, 4; (D) → 1, 3

212. (A) → 2,3; (B) → 1; (C) → 3,4; (D) → 3,4

213. (A) → (3); (B) → (4); (C) → (1); (D) → (2)

214. Velocity of the wave in the wire AB

$$v_1=\sqrt{\frac{T}{\mu_1}}=\sqrt{\frac{160}{0.12/4.8}}=80 \text{ m/s}$$

Velocity of the wave in the wire BC

$$v_2=\sqrt{\frac{T}{\mu_2}}=\sqrt{\frac{160}{0.4/2.56}}=32 \text{ m/s}$$

Amplitude of the reflected wave

$$a_r=\left(\frac{v_2-v_1}{v_1+v_2}\right)a_i=\frac{32-80}{32+80}\times3.5=-1.5 \text{ cm}$$

Negative sign signifies that reflected wave is inverted.

Amplitude of the transmitted wave

$$a_t=\frac{2v_2}{v_1+v_2}a_t=\frac{2\times32}{32+80}\times3.5=2.0 \text{ cm}$$

Equation of the reflected wave $y_r=-1.5\sin(\omega t+kx)$

$\therefore$ Resultant wave equation in the wire AB

$$y=3.5\sin(\omega t-kx)-1.5\sin(\omega t+kx)$$

$$y=1.5\sin(\omega t-kx)-1.5\sin(\omega t+kx)+2.0\sin(\omega t-kx)$$

$$y=-3.0\sin kx\cos\omega t+2.0\sin(\omega t-kx)$$

Maximum displacement of the nodes in the wire AB = 2.0 cm

Fraction of power transmitted in the wire

$$BC=1-\frac{a_r^2}{a_i^2}=1-\frac{1.5\times1.5}{3.5\times3.5}=0.82$$

$\therefore$ (A) → 2, (B) → 1, (C) → 3, (D) → 4

Assertion–Reason Type

215. The correct option is (B)
216. The correct option is (B)
217. The correct option is (C)
218. The correct option is (A)
219. The correct option is (D)
220. The correct option is (C)
221. The correct option is (A)
222. The correct option is (C)
223. The correct option is (B)
224. The correct option is (D)
225. The correct option is (B)
226. The correct option is (B)

227. Here $S_1P=D$ and $S_2P=\sqrt{D^2+d^2}=D\left[1+\frac{d^2}{2D^2}\right]$

$$\therefore \Delta x=S_2P-S_1P=\frac{d^2}{2D}=\frac{\lambda}{2}(2n-1)$$

$$\therefore \lambda=(2n-1)\frac{d^2}{D}$$

$\therefore$ The correct option is (C)

228. The principle of superposition does not state that the frequencies of the oscillation should be nearly equal. For beats to be heard, the condition is that difference in frequencies of the two oscillations should not be more than 10 times per second for a normal human ear to recognize it.

The correct option is (C)

229. The correct option is (B)

230. The correct option is (D)

231. A sound wave can be studied as any of the three waves. All the three quantities are never at their maximum or minimum simultaneously.

The correct option is (C)

232. The correct option is (D)

233. The correct option is (B)

234. The correct option is (C)

Integer Type

235. $T = 2t_1 + 2t_2$

$$= 2\left[\sqrt{\frac{2h}{g\sin^2\theta_1}} + \sqrt{\frac{2h}{g\sin^2\theta_2}}\right]$$

$$= 2\sqrt{\frac{2h}{g}}\left[\frac{1}{\sin\theta_1} + \frac{1}{\sin\theta_2}\right] = 35 \text{ s}$$

236. $T = 2\pi\sqrt{\dfrac{l}{g-a}}$, a is the downward acceleration of box

$$T_0 = 2\pi\sqrt{\frac{l}{g}} \Rightarrow a = \frac{3g}{4}$$

$$Mg - R = Ma \Rightarrow R = \frac{Mg}{4},\quad v = \frac{Mg}{4k}$$

$n = 4$

237. Maximum tension in string $= T = mg + m\omega^2 A$

$$\mu(3mg) = mg + m\left(\frac{2\pi}{\pi/2}\right)^2 \Rightarrow \mu = \frac{13}{15}$$

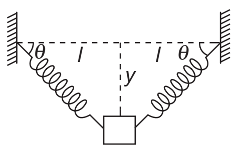
$n = 3$

238. $F_{net} = -4(F_0 + f)\sin\theta$

$$= -4F_0\sin\theta \approx -4F_0\tan\theta = -4F_0\frac{y}{l}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{Ml}{4F_0}}$$

$n = 4$



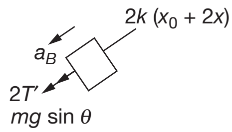
239. Let elongation of spring be x_0 in equilibrium. Then,

$$2T + mg\sin\theta = 2kx_0 \qquad (1)$$

and $\qquad T = mg \qquad (2)$

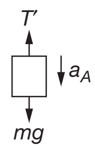
Let Block B is displaced by x down the inclination

FBD of B



$$-ma_B = 2k(x_0 + 2x) - 2T' - mg\sin\theta \qquad (3)$$

FBD of A



$mg - T' = ma_A$

Also, $\qquad a_A = 2a_B$

$$T' = mg - 2ma_B$$

$$-ma_B = 2kx_0 + 4kx - 2mg + 4ma_B - mg\sin\theta$$

$$-ma_B = 4kx + 4ma_B$$

$$a_B = -\frac{4k}{5m}x$$

$$\therefore \qquad T = 2\pi\sqrt{\frac{5m}{4k}}$$

$n = 4$

240. In one oscillation, distance covered $= 4 \times 3 = 12$ cm

Here, $\omega = \dfrac{2\pi}{T} \Rightarrow T = 4\text{s}.$

Now, $8.5 = 4 \times 2 + 0.5$

So, the distance covered

$$= 2 \times 12 + 3\cos\left\{\left(\frac{\pi}{2}\right)\times 0.5\right\} = \left(24 + \frac{3}{\sqrt{2}}\right)\text{ cm}$$

$n = 3$

241. Let n be the fundamental frequency of a closed organ pipe, the frequency of its first overtone which is also the third harmonic is $3n$.

$\therefore \quad 3n = 480$ or $n = 160$ vib/s

The length of a closed organ pipe is given by $n = \dfrac{V}{4V}$

$$\therefore \quad L = \frac{V}{4n} = \frac{350}{4\times 160} = 54.7 \text{ cm}$$

The length of the open organ pipe is given by $n = \dfrac{V}{2L}$

$\therefore \quad L = \dfrac{V}{2n} = \dfrac{350}{2 \times 240} = 73.9$ cm

242. (A) $v = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

Both T and m are constants $\therefore$ vl = constant

$v_2 l_2 = v_1 l_1$ or $v_2\left(l_1 + \dfrac{l_1}{2}\right) = v_1 l_1$

$v_2 = \dfrac{2}{3} \times 120 \text{ Hz} = 80 \text{ Hz}$

(B) Now, $v_3 l_3 = v_1 l_1$, $l_3 = \dfrac{l_1}{3} = \dfrac{1.5}{3} \text{ m} = 0.5 \text{ m}$

The wire should be shortened by (1.5 – 0.5) m, i.e., 1.0 m

243. As source (horn of bus) is approaching stationary wall (say, listener), apparent frequency striking the wall is

$$\nu' = \frac{\upsilon \nu}{\upsilon - \upsilon_s} \qquad (1)$$

Sound of this frequency will be reflected by the wall (now, source). The passenger is the listener moving towards source. Therefore, frequency heard by the listener

$\nu'' = \dfrac{(\upsilon + \upsilon_L)\nu'}{\upsilon}$

using (1)

$$\nu'' = \frac{\upsilon + \upsilon_L}{\upsilon} \times \frac{\upsilon \nu}{\upsilon - \upsilon_s} = \frac{(\upsilon + \upsilon_L)\nu}{\upsilon - \upsilon_s}$$

$$= \frac{(330+5) \times 200}{330 - 5} = \frac{335}{325} \times 200$$

$\nu'' = 206$ Hz

$\therefore$ Beat frequency $= (\nu'' - \nu)$

$= 206 - 200 = 6$ Hz

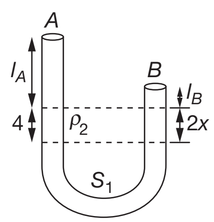
244. Let the length of second liquid passed in A be y.

If the first liquid comes down to a level x in arm A.

$\rho_2 g y = \rho_1 g 2x$

$x = \dfrac{y}{4}$

$l_A = (l_1 - h) - \left(y - \dfrac{y}{4}\right) = l_1 - h - \dfrac{3y}{4}$



$l_A = l_2 - h + \dfrac{y}{4}$

According to problem,

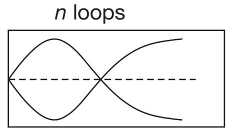
$l_A = 3l_B$ (I overtone of A = fundamental tone of B)

$\Rightarrow \quad y = \dfrac{2(l_1 - 3l_2 + 2h)}{3}$

$y = 2$ m.

245. $(2n+1)\dfrac{\lambda}{4} = l$, $\lambda = \dfrac{4l}{(2n+1)}$, $f = (2n+1)\dfrac{v}{4l}$

Equation of standing wave $y = a_0 \sin \dfrac{2\pi}{\lambda} x \cdot \cos 2\pi f t$



Equation for pressure variation

$$\Delta P = B\frac{\partial y}{\partial x} = \left(v^2 \rho\right)\left(\frac{2\pi}{\lambda}\right) a_0 \cos \frac{2\pi}{\lambda} x \cos 2\pi f t$$

$$(\Delta P)_{\max} = v^2 \rho \frac{2\pi\, a_0}{4l/(2n+1)} = \frac{(2n+1)v^2 \rho \pi a_0}{2l}$$

For location of maximum pressure variation $\cos \dfrac{2\pi}{\lambda} x = \pm 1$

So $x = 0, \dfrac{\lambda}{2}, \lambda, \dfrac{3\lambda}{2}, 2\lambda$........................

246. The equation of the standing wave formed in the rope is given as

$y = (0.10) \sin \left(\dfrac{\pi x}{2}\right) \sin (12\pi t)$

So, we have, $k = \dfrac{\pi}{2}$

or $\quad \dfrac{2\pi}{\lambda} = \dfrac{\pi}{2}$

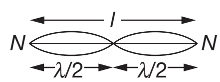
$\Rightarrow \quad \lambda = 4$ m

(A) Let l be the length of the rope,

We have,

$$l = \frac{\lambda}{2} + \frac{\lambda}{2} = \lambda$$

or, $\quad l = 4$ m



(B) We have, $\omega = 12\pi$

or, $\quad 2\pi v = 12\pi$

$v = 6$ Hz

$\therefore \quad V = v\lambda = 4 \times 6 = 24$ m/s

(C) Now $V = \sqrt{\dfrac{T}{\mu}}$, where T is the tension in the rope, μ is the mass per unit length of the rope.

or $\quad V^2 = \frac{T}{\mu} \Rightarrow \mu = \frac{T}{V^2} = \frac{200}{24\times 24} = \frac{25}{72}$ kg/m

$\therefore$ Mass of the rope $= 4\mu = 4\times\frac{25}{72} = \frac{25}{18}$ kg

(D) For the standing wave in the rope, $\nu = \frac{nV}{2l}$

For the third harmonic standing wave pattern, $n = 3$

$$\therefore \quad \nu = \frac{3\times 24}{2\times 4} = 9 \text{ Hz}$$

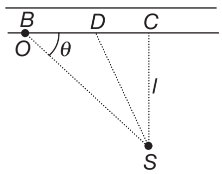
$\therefore$ Time period $= \frac{1}{\nu} = \frac{1}{9}s$

247. Let the bus be at O when it sends a signal that is detected by the detector as of frequency = 1500 Hz

$$\therefore \quad f' = \left(\frac{v}{v - v_B\cos\theta}\right)\times 1000 = 1500$$

$$\therefore \quad \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$$

By the time signal reaches at S, the bus reaches at D. Let this time be t_0



$$\therefore \quad t_0 = \frac{OS}{v} = \frac{l\,\text{cosec}\,\theta}{v} \quad (1)$$

Now man fires and the bullet reaches C in time t_1 (say). In the same time, bus moves from D to C

$\therefore \quad t_1 = \frac{l}{u}$, where u = speed of bullet

Also, $OD + DC = l \cot\theta$

$v_B\, t_0 + v_B t_1 = l\cot\theta$

$$v_B\left(\frac{l\,\text{cosec}\,\theta}{v}\right) + v_B\,(l/u) = l\cot\theta$$

$$\frac{2}{3\sqrt{3}}\times 2 + \frac{2}{3\sqrt{3}}\frac{v}{u} = \sqrt{3}$$

$$\therefore \quad \frac{v}{u} = \frac{5}{2}$$

$$\therefore \quad u = \frac{2}{5}v = \frac{2}{5}\times 340 = 136 \text{ m/s}$$

248. The wavelength of the sound wave in air is $\lambda = \frac{320}{16\times 10^3} = 2\times 10^{-2}$ m. The positions of maxima on the circumference of the circular track will be given by

$$d\sin\theta = n\lambda$$

When d is the separation between the sources and θ is the angular position of n[th] maximum as shown in Fig. 9.50.

$\Rightarrow \quad 2\sin\theta = n\,(2\times 10^{-2})$

$\sin\theta = \frac{n}{100}$

Since $\sin\theta$ lies between 0 and 1, there are 400 maxima on the entire circle.

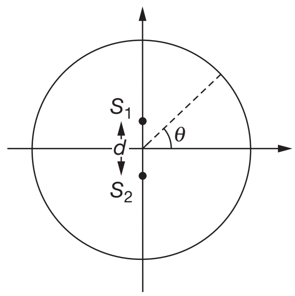


Fig. 9.50

These 400 maximas will be heard by the person in the time

$$t = \frac{400}{2\,\text{beats}/s} = 200s$$

Speed of the train = 36 km/h

$$= 36\times\frac{5}{18} = 10 \text{ m/s}$$

From the obtained values so far, we get length of track

l = (10 m/s) (200 s) = 2000 m

So radius of the track $= \frac{2000}{2\pi} = \frac{1000}{\pi}$ m

249. Time period of oscillation of R

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\times\frac{10}{\pi} = 20s$$

At a time t = 10 s, R will be at mean position and moving along negative x-axis.

$$v_R = A\omega = 15 \text{ m/s}$$

The sound which is received at t = 10 s is emitted at $t = t_0$ s.

$$\frac{1}{2}at_0^2 = v(10 - t_0)$$

$$\frac{1}{2}\times 18.75t_0^2 = 300(10 - t_0) \Rightarrow t_0 = 8 \text{ s}$$

$$v_s = at = 18.75\times 8 = 150 \text{ m/s}$$

$$f' = 500\left(\frac{300+15}{300+150}\right) = 350\,\text{Hz}$$

250. $\frac{V_{16}}{V_{51}} = \frac{f_{16}}{f_{51}} = \sqrt{\frac{273+16}{273+51}} = \frac{17}{18}$

$f_{15} = f \pm 1, \quad f_{51} = f + 4$

$\frac{f\pm 1}{f+4} = \frac{17}{18}$, $18f \pm 18 = 17f + 68$, f = 50 Hz or 86 Hz

Previous Years' Questions

251. The kinetic (KE) and potential energy (U) of a simple harmonic oscillator is given by,

$$\text{KE} = \frac{1}{2}k(A^2 - x^2);\ U = \frac{1}{2}kx^2$$

Where A = amplitude an K = mw

x = displacement from the mean position

At the mean position x = 0

$$\therefore\ \text{KE} = \frac{1}{2}kA^2 = \text{Maximum and } U = 0$$

The correct option is (C)

252. Let the spring constant of the original spring be k. Then its time period $T = 2\pi\sqrt{\frac{m}{k}}$ where m is the mass of oscillating body.

When the spring is cut into n equal parts, the spring constan of one part becomes nk. Therefore the new time period.

$$T' = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}$$

The correct option is (B)

253. The time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ where ℓ = distance between the point of surpension and the centre of mas of the child.

As shown in the figure, $\ell < \ell$

$\therefore\ T' < T$ *i.e.*, the period decreases.

The correct option is (B)

254.

$$T = 2\pi\sqrt{\frac{M}{k}}$$

$$T' = 2\pi\sqrt{\frac{M+m}{k}} = \frac{5T}{3}$$

$$\therefore\ 2\pi\sqrt{\frac{M+m}{k}} = \frac{5}{3}\times 2\pi\sqrt{\frac{M}{k}}$$

$$M + m = \frac{25}{9}\times M$$

$$1 + \frac{m}{M} = \frac{25}{9} \Rightarrow \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$$

The correct option is (C)

255. Maximum velocity during SHM = Aω

But $k = m\omega^2$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

$$\therefore\ \text{Maximum velocity} = A\sqrt{\frac{k}{m}}$$

Here the maximum velocity is same and m is also same

$$\therefore A_1\sqrt{k_1} = A_2\sqrt{k_2} \qquad \therefore \frac{A_1}{A_2}\sqrt{\frac{k_2}{k_1}}$$

The correct option is (C)

256. $T = 2\pi\sqrt{\frac{\ell}{g}}$ and $T' = 2\pi\sqrt{\frac{1.21\ell}{g}}$

$(\because \ell' = \ell + 21\%\ of\ \ell)$

$$\%\ \text{increase} = \frac{T'-T}{T}\times 100$$

$$= \frac{\sqrt{1.21\ell} - \sqrt{\ell}}{\sqrt{\ell}}\times 100 = (\sqrt{1.21} - \sqrt{1})\times 100$$

$$= (1.1 - 1)\times 100 = 10\%$$

The correct option is (D)

257. $x = 4(\cos\pi t + \sin\pi t)$

$$= \sqrt{2}\times 4\left(\frac{\sin\pi t}{\sqrt{2}} + \frac{\cos\pi t}{\sqrt{2}}\right)$$

$$= 4\sqrt{2}(\sin\pi t\cos 45^\circ + \cos\pi t\sin 45^\circ)$$

$$x = 4\sqrt{2}\sin(\pi t + 45^\circ)$$

on comparing it with $x = A\sin(\omega t + \phi)$

we get $A = 4\sqrt{2}$

The correct option is (C)

258. $\text{KE} = \frac{1}{2}m\omega^2(a^2 - x^2)$

When x = 0, KE is maximum and is equal to $\frac{1}{2}m\omega^2a^2$

The correct option is (A)

259. At any instant the total energy is $\frac{1}{2}kA_0^2$ = constant , where A_0 = amplitude

Hence total energy is independent of x,

The correct option is (A)

260. Equation of displacement is given by

$x = A\sin(\omega t + \phi)$

$$\text{where } A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2}} = \frac{F_0}{m(\omega_0^2 - \omega^2)}$$

Here damping effect is considered to be zero

$$\therefore\ x \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$

The correct option is (B)

261. Since energy $\propto$ (Amplitude)^2, the maximum for both of them occure at the same frequency

$\therefore\ \omega_1 = \omega_2$

The correct option is (C)

262. $v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$

$v_2 = \frac{dy_2}{dt} = -0.1\pi \sin \pi t = 0.1\pi \cos\left(\pi t + \frac{\pi}{2}\right)$

$\therefore$ Phase diff. $= \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$

The correct option is (B)

263. Centre of mass of combination of liquid and hollow portion (at position ℓ), first goes down (to $\ell + \Delta\ell$) and when total water is drained out, centre of mass regain its original posiion (to ℓ),

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\therefore$ T first increases and then decreases to original value.

The correct option is (B)

264. $\frac{d^2x}{dth2} = -\alpha x = -\omega^2 x$

$\Rightarrow \omega = \sqrt{x}$ or $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$

The correct option is (A)

265. Maximum velocity,

$v_{max} = a\omega$

$v_{max} = a \times \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01\text{s}$

The correct option is (A)

266. KE of a body undergoing SHM is given by,

$\text{KE} = \frac{1}{2} ma^2\omega^2 \cos^2 \omega t$

$\text{TE} = \frac{1}{2} ma^2\omega^2$

Given KE = 0.75 TE

$\Rightarrow 0.75 = \cos^2 \omega t \Rightarrow \omega t = \frac{\pi}{6}$

$\Rightarrow t = \frac{\pi}{6 \times \omega} \Rightarrow t = \frac{\pi \times 2}{6 \times 2\pi} \Rightarrow t = \frac{1}{6}\text{s}$

The correct option is (A)

267. The two springs are in parallel

$\therefore$ Effective spring constant,

$K = K_1 + K_2$

Now, frequency of oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

or, $$f = \frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}} \quad (1)$$

When both K_1 and K_2 are made four times their original values,

the new frequency is given by

$f' = \frac{1}{2\pi}\sqrt{\frac{4K_1 + 4K_2}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{4(K_1 + K_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{K_1 + K_2}{m}}\right)$

$= 2f$ from Equation (1)

The correct option is (A)

268. Here, $x = 2 \times 10^{-2} \cos \pi t$

Speed is given by

$v = \frac{dx}{dt} = 2 \times 10^{-2}\, \pi \sin \pi t$

For the first time, the speed to be maximum,

$\sin \pi t = 1$ or, $\sin \pi t = \sin \frac{\pi}{2}$

$\Rightarrow \pi t = \frac{\pi}{2}$ or, $t = \frac{1}{2} = 0.5$ s

The correct option is (B)

269. For first resonant length $v = \frac{v}{4\ell_1} = \frac{v}{4 \times 18}$ (in winter)

For second resonant length

$v' = \frac{3v'}{4\ell_2} = \frac{3v'}{4x}$ (in summer) $\therefore \frac{v}{4 \times 18} = \frac{3v'}{4 \times x}$

$\therefore x = 3 \times 18 \times \frac{v'}{v}$ $\quad \therefore x = 54 \times \frac{v'}{v}$ cm

$v' > v$ because velocity of light is greater in summer as compared to winter $(v \propto \sqrt{T})$

$\therefore\ x > 54$ cm

The correct option is (B)

270. $y(x,t) = 0.005 \cos(\alpha x - \beta t)$ (Given)

Comparing it with the standard equation of wave

$y(x,t) = a \cos(kx - \omega t)$ we get

$k = \alpha$ and $\omega = \beta$

But $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$

$\Rightarrow \frac{2\pi}{\lambda} = \alpha$ and $\frac{2\pi}{T} = \beta$

Given that $\lambda = 0.08$ m and $T = 2.0$ s

$\therefore\ \alpha = \dfrac{2\pi}{0.08} = 25\pi$ and $\beta = \dfrac{2\pi}{2} = \pi$

The correct option is (A)

271. Maximum number of beats $= (v+1) - (v-1) = 2$

The correct option is (B)

272. $v_m^2 - u^2 = 2as$

$\therefore\ v_m^2 = 2 \times 2 \times s$

$\therefore\ v_m = 2\sqrt{s}$

According to Doppler's effect

$$v' = v\left[\frac{v - v_m}{v}\right]$$

$$0.94v = v\left[\frac{330 - 2\sqrt{s}}{330}\right]$$

$\Rightarrow s = 98.01$ m

The correct option is (A)

273. $y = 0.02(m)\sin\left[2\pi\left(\dfrac{t}{0.04(s)} - \dfrac{x}{0.50(m)}\right)\right]$

Comparing this equation with the standard wave equation

$y = a\sin(\omega t - kx)$

we get

$$\omega = \frac{2\pi}{0.04}$$

$$\Rightarrow v = \frac{1}{0.04} = 25 \text{ Hz}$$

$$k = \frac{2\pi}{0.05} \Rightarrow \lambda = 0.5 \text{ m}$$

$\therefore$ velocity, $v = v\lambda = 25 \times 0.5\,\text{m/s} = 12.5\,\text{m/s}$

Velocity on a string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

$\therefore T = v^2 \times \mu = (12.5)^2 \times 0.04 = 6.25$ N

The correct option is (D)

274. Tube open at both ends:

$(\lambda/2) = L, \qquad \lambda = 2L = (C/f)$

Tube portion dipped in water:

$(\lambda'/4) = L/2, \qquad \lambda' = 2L = (C/f')$

$\therefore \qquad f' = f$

The correct option is (A)

275. $F_v = bv - \dfrac{dv}{dt} = \dfrac{b}{m}v$

$\displaystyle\int_{A_0}^{A_0/e} \frac{dv}{v} = \frac{b}{m}\int_0^{\tau} dt$ on integration $\tau = \dfrac{1}{b}$

The correct option is (C)

276. The amplitude of domped oscillation is given be $A = A_0 e^{-\frac{bt}{2}}$

$0.9 A_0 = A_0 e^{-b/5/2}$ (1)

$A = A_0 e^{-b\times\frac{(5+10)}{2}}$ (2)

$= A_0.(0.9)^3 = 0.729 A_0$

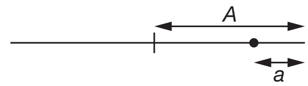
The correct option is (B)

277. Without loss in generality we can consider the particle to be at positive extreme initially

So the equation of SHM performed by the particle can be written as

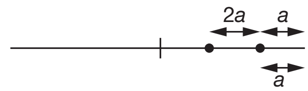
$x = A\cos\omega t$

At $t = \tau$



$A - a = A\cos\omega\tau$ (1)

At $t = 2\tau$



$A - 3a = A\cos\omega(2\tau)$

$A - 3a = A\left[2\cos^2\omega\tau - 1\right]$ (2)

From (1) and (2) we have

$$A - 3a = A\left[2\left(\frac{A-a}{A}\right)^2 - 1\right]$$

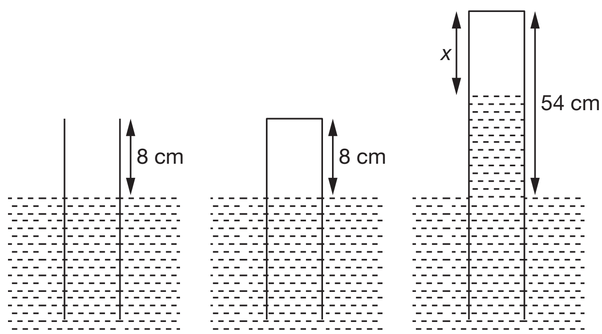
Putting in (1)

$$\omega\tau = \frac{\pi}{3}$$

$$\therefore\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{3\tau}} = 6\tau$$

The correct option is (D)

278.



$P(Ax) = 76(A \times 8)$ (1)

Also $P + (54 - x) = 76$ (2)

From (1) and (2) we have

$x = 16$ cm

The correct option is (A)

279. $(2n+1)\frac{v}{4L} = (2n+1)\frac{340}{4 \times \frac{85}{100}} = (2n+1) \times 100$

$= 100, 300, 500, 700, 900, 1100$

The correct option is (C)

280. LBF $= f_c - f_m = 1995$ kHz

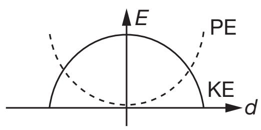
UBF $= f_c + f_m = 2005$ kHz

Hence the frequencies are

2005 kHz, 2000 kHz and 1995 kHz

The correct option is (B)

281.



$\text{KE} = \frac{1}{2}m\omega^2(A^2 - d^2)$

and $\text{PE} = \frac{1}{2}m\omega^2 d^2$

The correct option is (A)

282. When train was coming towards person

$$v' = \frac{v}{v-u}(v_0) = \frac{320}{320-20} \times 1000 = \frac{320}{300} \times 1000$$

When train recedes

$$v'' = \frac{v}{v+u}(v_0) = \frac{320}{320+20} \times 1000 = \frac{320}{340} \times 1000$$

$$\text{change } = |v'' - v'| = \left|1000\left(\frac{320}{340} - \frac{320}{300}\right)\right| = \frac{320 \times 1000 \times 40}{340 \times 300}$$

$$\% \text{ change} = \frac{|v'' - v'|}{v'} \times 100$$

The correct option is (A)

283. $\frac{\lambda_{\max}}{2} = 40 \Rightarrow \lambda_{\max} = 80$ cm

The correct option is (B)

284. In tube A, $\lambda_A = 2l$

In tube B, $\lambda_B = 4l$

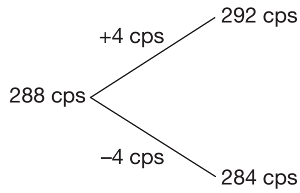
$\therefore \upsilon_A = \frac{v}{\lambda_A} = \frac{v}{2l}$

$\upsilon_B = \frac{v}{\lambda_B} = \frac{v}{4l}$

$\therefore \frac{\upsilon_A}{\upsilon_B} = \frac{2}{1}$

The correct option is (C)

285. The wax decreases the frequency of unknown fork. The possible unknown frequencies are (288 + 4) cps and (288 – 4) cps.



Wax reduces 284 cps and so beats should increases. It is not given in the question. This frequency is ruled out. Wax reduced 292 cps and so beats should decrease. It is given that the beats decrease to 2 from 4.

Hence unknown fork has frequency 292 cps.

The correct option is (B)

286. When temperature increases, l increases

Hence frequency decreases

The correct option is (B)

287. Given wave equation

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right) \text{ m}$$

Standard wave equaiton: $y = a\sin(\omega t - kx + \phi)$

Compare them

Angular speed $= \omega = 600 \text{ s}^{-1}$

Propagation constant $= k = 2 \text{ m}^{-1}$

$$\frac{\omega}{k} = \frac{2\pi\upsilon}{2\pi/\lambda} = \upsilon\lambda = \text{velocity}$$

$$\therefore \text{ Velocity} = \frac{\omega}{k} = \frac{600}{2} = 300 \text{ m/s}$$

The correct option is (A)

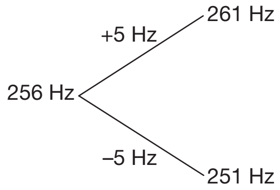
288. At resonace, frequency of vibration of wire become equal to frequency of a.c.

For vibration of wire, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$$\therefore \quad \upsilon = \frac{1}{2 \times 1}\sqrt{\frac{10 \times 9.8}{9.8 \times 10^{-3}}} = \frac{100}{2} = 50 \text{ Hz}$$

The correct option is (A)

289. The possible frequencies of piano are $(256+5)\text{Hz}$ and $(256-5)\text{Hz}$



For piano string, $\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

When tension T increases, υ increases

(A) If 261 Hz increases, beats/sec increase. This is not given in the question.

(B) If 251 Hz increases due to tension, beats per second decrease. This is given in the question.

Hence frequency of piano $=(256-5)\,\text{Hz}$.

The correct option is (C)

290. $y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)\text{m}$

Standard equation: $y = a\sin(\omega t + kx + \phi)$

Compare the two

$\therefore\ \omega = 100$ and $k = 20$

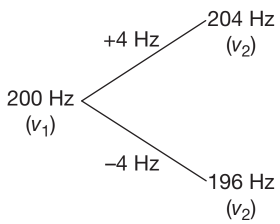
$\therefore\ \frac{\omega}{k} = \frac{100}{20} \Rightarrow \frac{2\pi n}{2\pi/\lambda} = n\lambda = v = 5$

$\therefore\ v = 5\,\text{m/s}$

The correct option is (B)

291. Let the two frequencies be υ_1 and υ_2

υ_2 may be either 204 Hz or 196 Hz.



As mass of second fork increases, υ_2 decreases.

Iff υ_2 = 204 Hz, a decrease in υ_2 decreases beats/sec.

But this is not given in question

If $\upsilon_2 = 196$ Hz, a decreases in υ_2 increased beats/sec.

This is given in the question when beats increase to 6

$\therefore$ Original frequency of second fork = 196 Hz.

The correct option is (D)

292. By Doppler's effect

$\frac{\upsilon'}{\upsilon} = \frac{v_s + v_0}{v_s}$ (where v_s is the velocity of sound)

$= \frac{v + (v/5)}{v} = \frac{6}{5}$

$\therefore$ Fractional increase $= \frac{\upsilon' - \upsilon}{\upsilon} = \left(\frac{\upsilon'}{\upsilon} - 1\right) = \left(\frac{6}{5} - 1\right) = \frac{1}{5}$

$\therefore$ Percentage increase $= \frac{100}{5} = 20\%$

The correct option is (B)

293. $\frac{\upsilon'}{\upsilon} = \frac{v_s}{v_s - v}$

where v_s is the velocity of sound in air.

$\frac{10000}{9500} = \frac{300}{300 - v}$

$\Rightarrow (300 - v) = 285 \Rightarrow v = 15\,\text{m/s}$

The correct option is (C)

294. $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$; $I_2 = 10\log\left(\frac{I_2}{I_0}\right)$

$\therefore\ L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$

or $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$ or $20\,dB = 10\log\left(\frac{I_1}{I_2}\right)$

or $10^2 = \frac{I_1}{I_2}$

or $I_2 = \frac{I_1}{100}$

The correct option is (A)

295. The wave travelling along the x-axis is given by

$y(x,t) = .0005\cos(\alpha x - \beta t)$

Therefore $\alpha = k = \frac{2\pi}{\lambda}$. As $\lambda = 0.08$ m

$\therefore\ \alpha = \frac{2\pi}{0.08} = \frac{\pi}{0.04} \Rightarrow \alpha = \frac{\pi}{4} \times 100.00 = 25.00\,\pi$

$\omega = \beta \Rightarrow \frac{2\pi}{2.0} = \beta \Rightarrow \pi$

$\therefore\ \alpha = 25.00\pi,\ \beta = \pi$

The correct option is (A)

296. The source is at rest, the observer is moving away from the source.

$\therefore\ f' = f\frac{(v_{\text{sound}} - v_{\text{obs}})}{v_{\text{sound}}}$

$\Rightarrow \frac{f'}{f} = v_{\text{sound}} = v_{\text{sound}} - v_{\text{obs}} \Rightarrow \frac{f'}{f} \times v_{\text{sound}} - v_{\text{sound}} = -v_{\text{obs}}$

$$v_{\text{sound}}\left(\frac{f'}{f}-1\right)=-v_{\text{obs}}$$

$$330(0.94-1)=-v_{\text{obs}}$$

$$\Rightarrow\ v_{\text{obs}}=330\times0.06=19.80\,\text{ms}^{-1}$$

$$\therefore\ s=\frac{v^2-u^2}{2a}=\frac{(19.80)^2}{2\times2}=98\text{ m}$$

The correct option is (A)

297. Here, linear mass density $\mu = 0.04\,\text{kg m}^{-1}$

The given equation of a wave is

$$y=0.02\sin\left[2\pi\left(\frac{t}{0.04}-\frac{x}{0.50}\right)\right]$$

Compare it with the standard wave equation

$y = A\sin(\omega t - kx)$

we get,

$$\omega=\frac{2\pi}{0.04}\text{rads}^{-1};$$

$$k=\frac{2\pi}{0.5}\text{rad m}^{-1}$$

Wave velocity, $v=\dfrac{\omega}{k}=\dfrac{(2\pi/0.04)}{(2\pi/0.5)}\text{ms}^{-1}$ (1)

Also $v=\sqrt{\dfrac{T}{\mu}}$ (2)

where T is the tension in the string and μ s the linear mass density

Equating Equations (1) and (2), we get

$$\frac{\omega}{k}=\sqrt{\frac{T}{\mu}}\ \text{ or }\ T=\frac{\mu\omega^2}{k^2}$$

$$T=\frac{0.04\times\left(\dfrac{2\pi}{0.04}\right)^2}{\left(\dfrac{2\pi}{0.05}\right)^2}=6.25\ \text{N}$$

The correct option is (D)

298. Tube open at both ends

$(\lambda/2)=L,\qquad \lambda=2L=(C/f)$

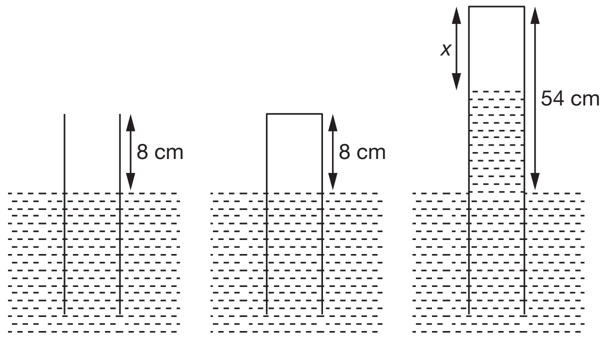
Tube portion dipped in water:

$(\lambda'/4)=L/2,\qquad \lambda'=2L=(C/f')$

$\therefore\qquad f'=f$

The correct option is (A)

299.



$P(Ax)=76(A\times8)$ (1)

Also $P+(54-x)=76$ (2)

From (1) and (2) we have

$x = 16$ cm

The correct option is (A)

300. $$(2n+1)\frac{v}{4L}=(2n+1)\frac{340}{4\times\dfrac{85}{100}}=(2n+1)\times100$$

$$=100,\ 300,\ 500,\ 700,\ 900,\ 1100$$

The correct option is (C)

301. LBF $= f_c - f_m = 1995$ kHz

UBF $= f_c + f_m = 2005$ kHz

Hence the frequencies are

2005 kHz, 2000 kHz and 1995 kHz.

The correct option is (B)

302. When train was coming towards person

$$v'=\frac{v}{v-u}(v_0)=\frac{320}{320-20}\times1000=\frac{320}{300}\times1000$$

When train recedes

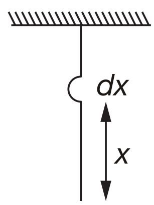
$$v''=\frac{v}{v+u}(v_0)=\frac{320}{320+20}\times1000=\frac{320}{340}\times1000$$

$$\text{change}=|v''-v'|=\left|1000\left(\frac{320}{340}-\frac{320}{300}\right)\right|=\frac{320\times1000\times40}{340\times300}$$

$$\%\text{ change}=\frac{|v''-v'|}{v'}\times100$$

The correct option is (A)

303. Let λ be the mass per unit length.



$$v=\sqrt{\frac{T}{\lambda}}=\sqrt{\frac{\lambda gx}{\lambda}}$$

$$\int_0^{20}\frac{dx}{\sqrt{x}}=\sqrt{10}\int_0^{t}dt$$

$$\left[\frac{\sqrt{x}}{\frac{1}{2}}\right]_0^{20} = \sqrt{10}\,t$$

$$t = \frac{2\sqrt{20}}{\sqrt{10}} = 2\sqrt{2} \text{ s.}$$

The correct option is (B)

304. $f = \dfrac{v}{2\ell}$

$$f' = \frac{v}{4(\ell/2)} = \frac{v}{2\ell} = f$$

The correct option is (C)

305. In SHM $v = \omega\sqrt{A^2 - x^2}$

At $x = \dfrac{2A}{3}$, $v = \omega\sqrt{A^2 - \left(\dfrac{2A}{3}\right)^2} = \dfrac{\sqrt{5}A\omega}{3}$

Now, when speed is trebled

then $3v = \omega\sqrt{(A')^2 - \left(\dfrac{2A}{3}\right)^2}$

$$A' = \frac{7A}{3}$$

The correct option is (C)