CHAPTER 8

Properties of Solids and Liquids

Chapter Highlights

Elastic behaviour. Stress-strain relationship. Hooke's Law. Young's modulus. Bulk modulus. Modulus of rigidity. Pressure due to a fluid column. Pascal's law and its applications. Viscosity. Stokes' law. Terminal velocity. Streamline and turbulent flow. Reynolds number. Bernoulli's principle and its applications. Surface energy and surface tension. Angle of contact. Application of surface tension - drops. Bubbles and capillary rise.

ELASTICITY

Elasticity and Plasticity

The property of a material body by virtue of which it regains its original configuration (i.e. shape and size) when the external deforming force is removed is called elasticity. The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Deforming Force

An external force applied to a body which changes its size or shape or both is called deforming force.

Perfectly Elastic Body

A body is said to be perfectly elastic if it completely regains its original form when the deforming force is removed. Since no material can regain completely its original form, the concept of perfectly elastic body is only an ideal concept. A quartz fibre is the nearest approach to the perfectly elastic body.

Perfectly Plastic Body

A body is said to be perfectly plastic if it does not regain its original form even slightly when the deforming force is removed. Since every material partially regain its original form on the removal of deforming force, the concept of perfectly plastic body is only an ideal concept. Paraffin wax, wet clay are the nearest approach to a perfectly plastic bodies.

Cause of Elasticity

In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e. body) remains in its equilibrium position, and the intermolecular forces between the molecules of the solid are at its maximum.

On applying the deforming force on the body, the molecules either come closer or go far apart from each other. As a result, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

Stress

When deforming force is applied on the body, an equal restoring force in opposite direction is developed inside the body. The restoring forces per unit area of the body are called stress.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area of the body}} = \frac{F}{A}$$

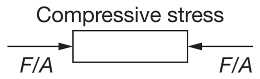
The unit of stress is N/m^2 or N/m^{-2}. There are three types of stress.

Longitudinal or Normal Stress

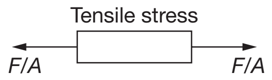
When object is one-dimensional, then force acting per unit area is called longitudinal stress.

This is of two types:

1. Compressive stress



2. Tensile stress



Examples:
Consider a block of solid as shown in Fig. 8.1. Let a force F be applied to the face which has area A. Resolve $\vec{F}$ into two components:

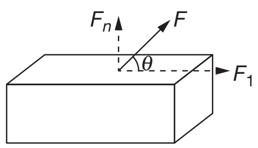


Fig. 8.1

$F_n = F \sin\theta$ called normal force and $F_t = F \cos\theta$ called tangential force.

$$\text{Normal (tensile) stress} = \frac{F_n}{A} = \frac{F \sin\theta}{A}$$

Tangential or Shear Stress

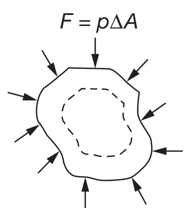
It is defined as the restoring force acting per unit area tangential to the surface of the body. Refer to Fig. 8.1.

$$\textbf{Tangential (shear) stress} = \frac{F_t}{A} = \frac{F \cos\theta}{A}$$

The effect of stress is to produce distortion or a change in size, volume, and shape (i.e., configuration of the body).

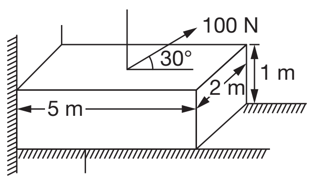
Bulk Stress or all Around Stress or Pressure

When force is acting all along the surface normal to the area, then force acting per unit area is known as pressure. The effect of pressure is to produce volume change. The shape of the body may or may not change depending upon its homogeneity.



SOLVED EXAMPLES

1. Find out longitudinal stress and tangential stress on a fixed block.



Solution:

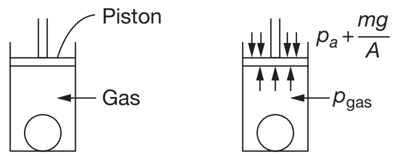
Longitudinal or normal stress

$$\Rightarrow \quad s_1 = \frac{100 \sin 30°}{5 \times 2} = 5 \text{ N/m}^2$$

Tangential stress

$$\Rightarrow \quad s_t = \frac{100 \cos 30°}{5 \times 2} = 5\sqrt{3} \text{ N/m}^2.$$

2. Find out bulk stress on the spherical object of radius $\frac{10}{\pi}$ cm if area and mass of piston is 50 cm^2 and 50 kg, respectively, for a cylinder filled with gas.



Solution:

$$p_{gas} = \frac{mg}{A} + p_a = \frac{50 \times 10}{50 \times 10^{-4}} + 1 \times 10^5 = 2 \times 10^5 \text{ N/m}^2$$

$$\text{Bulk stress} = p_{gas} = 2 \times 10^5 \text{ N/m}^2.$$

Strain

The ratio of the change in configuration (i.e., shape, length, or volume) to the original configuration of the body is called strain.

That is, $$\text{Strain, } \hat{I} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

It has no unit.

Types of Strain

There are three types of strain.

1. **Longitudinal Strain:** This type of strain is produced when the deforming force causes a change in length of the body. It is defined as the ratio of the change in length to the original length of the body.
   Consider a wire of length L: When the wire is stretched by a force F, then the change in length of the wire ΔL shown in Fig. 8.2.

   $$\text{Longitudinal strain,} \quad \hat{I}_1 = \frac{\text{Change in length}}{\text{Original length}}$$

   or $$\text{Longitudinal strain} = \frac{\Delta L}{L}$$

2. **Volume Strain:** This type of strain is produced when the deforming force produces a change in volume of the body shown in Fig. 8.2. It is defined as the ratio of the change in volume to the original volume of the body.

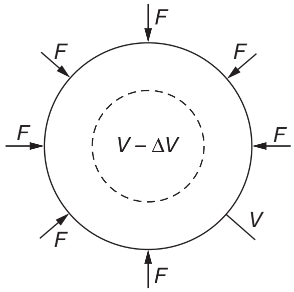


Fig. 8.2

If ΔV = change in volume
V = original volume
$\hat{I}_v$ = volume strain = $\dfrac{\Delta V}{V}$

3. **Shear Strain:** This type of strain is produced when the deforming force causes a change in the shape of the body. It is defined as the angle (ϕ) through which a face originally perpendicular to the fixed face is turned as shown in Fig. 8.3.

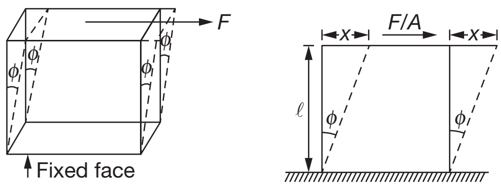


Fig. 8.3

$$\tan\phi \text{ or } \phi = \frac{x}{\ell}$$

HOOKE'S LAW AND MODULUS OF ELASTICITY

According to this law, within the elastic limit, stress is proportional to the strain.

i.e., Stress ∝ Strain

or Stress = constant × strain

or $\dfrac{\text{Stress}}{\text{Strain}}$ = modulus of elasticity.

This constant is called modulus of elasticity.

Thus, modulus of elasticity is defined as the ratio of the stress to the strain.

Modulus of elasticity depends on the nature of the material of the body and is independent on its dimensions (i.e. length, volume, etc.).

Unit: The Sl unit of modulus of elasticity is N/m^{-2} or Pascal (Pa).

Types of Modulus of Elasticity

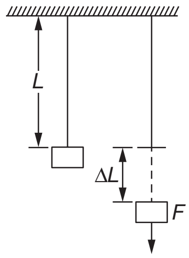
Corresponding to the three types of strain, there are three types of modulus of elasticity.

1. Young's modulus of elasticity (Y)
2. Bulk modulus of elasticity (K)
3. Modulus of rigidity (h)

Young's Modulus of Elasticity

It is defined as the ratio of the normal stress to the longitudinal strain.

i.e., Young's modulus (Y) = $\dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$



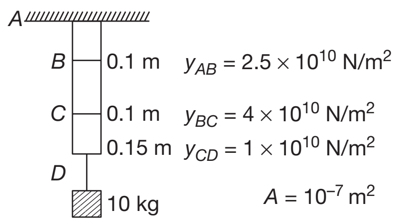
Normal stress = F/A

Longitudinal strain = $\Delta L/L$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

SOLVED EXAMPLE

3. Find out the shift in point B, C, and D



Solution:

$$\Delta L_B = \Delta L_{AB} = \frac{FL}{AY} = \frac{MgL}{AY}$$

$$= \frac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}} = 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$

$$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \frac{100 \times 0.2}{10^{-7} \times 4 \times 10^{10}}$$

$$= 4 \times 10^{-3} + 5 \times 10^{-3} = 9 \text{ mm}$$

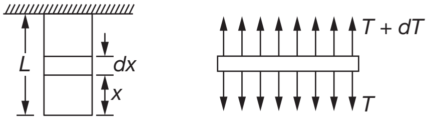
$$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \frac{100 \times 0.15}{10^{-7} \times 1 \times 10^{10}}$$

$$= 9 \times 10^{-3} + 15 \times 10^{-3} = 24 \text{ mm.}$$

Elongation of Rod Under its Self-weight

Let us take a rod having self-weight W, area of cross-section A, and length L. Considering one element at a distance x from bottom,

then $$T = \frac{W}{L}x$$



$$\text{Elongation in } dx \text{ element} = \frac{T.dx}{Ay}$$

$$\text{Total elongation } s = \int_0^L \frac{Td\,x}{Ay} = \int_0^L \frac{W\,x\,d\,y}{LAy} = \frac{WL}{2Ay}$$

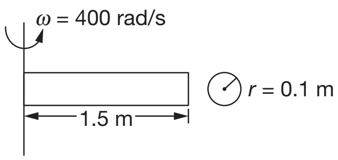
NOTE

One can do directly by considering total weight at C.M. and using effective length $l/2$.

$$\text{Total elongation} = \frac{WL}{2Ay}$$

SOLVED EXAMPLES

4. A rod of length 1.5 m and cross-sectional radius 0.1 m is rotated about one end with angular speed 100 rad/s. Find out the elongation of rod.
[Given: $y = 2 \times 10^{11}$ N/m^2; $r = 10^4$ kg/m^3]



Solution:

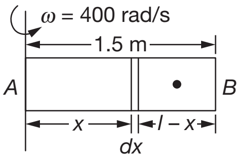
Mass of shaded portion

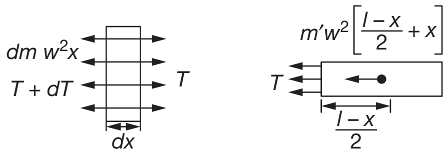
$$m' = \frac{m}{\ell}(\ell - x) \qquad \text{[where } m = \text{total mass} = r\,A\,l]$$

$$T = m'\,w^2\left[\frac{\ell - x}{2} + x\right]$$

$$\Rightarrow \qquad T = \frac{m}{\ell}\,(\ell - x)\,w^2\left(\frac{\ell + x}{2}\right)$$

$$T = \frac{mw^2}{2\ell}\,(\ell^2 - x^2)$$





The tension will be maximum at $\left(\frac{mw^2\ell}{2}\right)$ A and minimum at B (zero), elongation in element of width $dx = \frac{Tdx}{Ay}$.

Total elongation,

$$\delta = \int \frac{Tdx}{Ay} = \int_0^\ell \frac{mw^2(\ell^2 - x^2)}{2\ell\,Ay}\,dx$$

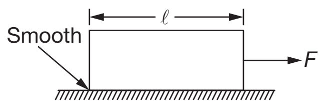
$$\delta = \frac{mw^2}{2\ell Ay}\left[\ell^2 x - \frac{x^3}{3}\right]_0^\ell$$

$$= \frac{mw^2 \times 2\ell^3}{2\ell Ay \times 3} = \frac{mw^2\ell^2}{3Ay} = \frac{\rho A\ell w^2\ell^2}{3Ay}$$

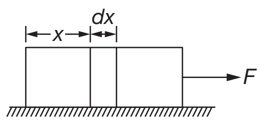
$$\delta = \frac{\rho w^2\ell^3}{3y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}}$$

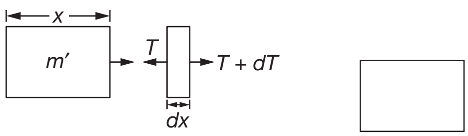
$$= 9 \times 10^{-3} \text{ m} = 9 \text{ mm}.$$

5. Find out the elongation in block. If mass, area of cross-section and Young modulus of block are m, A, and y, respectively.



Solution:





$$\text{Acceleration,} \qquad a = \frac{F}{m}$$

then $\qquad T = m'a$

where $\Rightarrow \qquad m' = \dfrac{m}{\ell}x$

$$T = \frac{m}{\ell} \times \frac{F}{m} = \frac{Fx}{\ell}$$

Elongation in element $dx = \dfrac{Tdx}{Ay}$

Total elongation,

$$d = \int_o^\ell \frac{Tdx}{Ay} \quad d = \int_o^\ell \frac{Fxdx}{A\ell y} = \frac{F\ell}{2Ay}.$$

NOTE

Try this problem, if friction is given between block and surface (μ = friction coefficient) and cases (I) and (II)

Case: (I) $F < \mu mg$

(II) $F > \mu mg$

Ans. In both cases, answer will be $\dfrac{F\ell}{2Ay}$

Bulk Modulus

It is defined as the ratio of the normal stress to the volume strain

i.e., $\qquad K = \dfrac{\text{Pressure}}{\text{Volume strain}}$

The stress is the normal force applied per unit area and is equal to the pressure applied (p).

$$K = \frac{p}{\frac{-\Delta V}{V}} = -\frac{pV}{\Delta V}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

Compressibility The reciprocal bulk modulus of elasticity is called compressibility. Unit of compressibility in Sl is $N^{-1}\,m^2$ or Pascal^{-1} (Pa^{-1}).

Bulk modulus of solids is about fifty times that of liquids and for gases it is 10^{-8} times of solids.

$$K_{\text{solids}} > K_{\text{liquids}} > K_{\text{gases}}$$

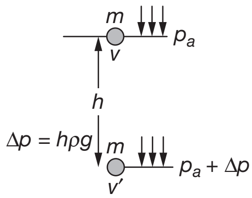
Isothermal modulus of elasticity of gas $K = P$ (pressure of gas)

Adiabatic modulus of elasticity of gas $K = g \times P$, where $g = \dfrac{C_p}{C_v}$.

SOLVED EXAMPLE

6. Find the depth of lake at which density of water is 1% greater than the surface. Given compressibility $k = 50 \times 10^{-6}$ /atm.

Solution:



$$B = \frac{\Delta p}{-\frac{\Delta V}{V}} \qquad \frac{\Delta V}{V} = -\frac{\Delta p}{B}.$$

We know that

$$p = p_{\text{atm}} + hrg$$

or $\qquad m = rV = \text{Constant}$

$$dr \cdot v + dv \cdot r = 0$$

$$dr\, V + dV \cdot r = 0$$

$\Rightarrow \qquad \dfrac{d\rho}{\rho} = -\dfrac{dV}{V}$

i.e., $\qquad \dfrac{\Delta\rho}{\rho} = \dfrac{\Delta p}{B}$

$\Rightarrow \qquad \dfrac{\Delta\rho}{\rho} = \dfrac{1}{100}$

$$\frac{1}{100} = \frac{h\rho g}{B} \qquad \text{[assuming } r = \text{Constant]}$$

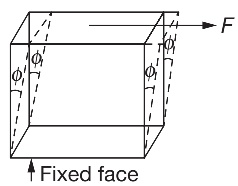
$$hrg = \frac{B}{100} = \frac{1}{100\,k}$$

$\Rightarrow \qquad hrg = \dfrac{1 \times 1 \times 10^5}{100 \times 50 \times 10^{-6}}$

$$h = \frac{10^5}{5000 \times 10^{-6} \times 1000 \times 10} = \frac{100 \times 10^3}{50} = 2 \text{ km}.$$

Modulus of Rigidity

It is defined as the ratio of the tangential stress to the shear strain. Let us consider a cube whose lower face is fixed and a tangential force F acts on the upper face whose area is A.



$\therefore \qquad$ Tangential stress $= F/A$.

Let the vertical sides of the cube shifts through an angle ϕ, called shear strain.

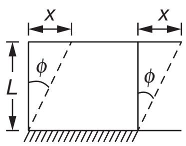
$\therefore$ Modulus of rigidity is given by

$$h = \frac{\text{Tangential stress}}{\text{Shear strain}}$$

or
$$h = \frac{F/A}{\phi} = \frac{F}{A\phi}.$$

SOLVED EXAMPLE

7. A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m^2.



Solution:

$$L = 5 \times 10^{-2}\text{ m} \quad \Rightarrow \quad \frac{F}{A} = \eta \frac{x}{L}$$

$$\text{Strain } \theta = \frac{F}{A\eta} = \frac{1800}{25 \times 10^{-4} \times 2.4 \times 10^6}$$

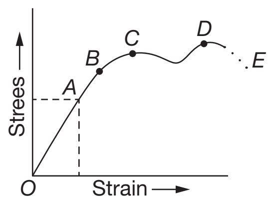
$$= \frac{180}{25 \times 24} = \frac{3}{10} = 0.3 \text{ radian}$$

$$\frac{x}{L} = 0.3$$

$$\Rightarrow \quad x = 0.3 \times 5 \times 10^{-2} = 1.5 \times 10^{-2}\text{ m} = 1.5\text{ mm}.$$

Variation of Strain with Stress

When a wire is stretched by a load, it is seen that for small value of load, the extension produced in the wire is proportional to the load. On removing the load, the wire returns to its original length. The wire regains its original dimensions only when load applied is less or equal to a certain limit. This limit is called elastic limit. Thus, elastic limit is the maximum stress on whose removal the bodies regain their original dimensions. In Fig. 8.4, this type of behaviour is represented by OB portion of the graph. Till A, the stress is proportional to strain and from A to B, if deforming forces are removed, then the wire comes to its original length, but here stress is not proportional to strain.



$OA \rightarrow$ Limit of proportionality
$OB \rightarrow$ Elastic limit
$C \rightarrow$ Yield point
$CD \rightarrow$ Plastic behaviour
$D \rightarrow$ Ultimate point
$DE \rightarrow$ Fracture

Fig. 8.4

As we go beyond the point B, then even for a very small increase in stress, the strain produced is very large. This type of behaviour is observed around point C and at this stage the wire begins to flow like a viscous fluid. The point C is called yield point. If the stress is further increased, then the wire breaks off at a point D called the breaking point. The stress corresponding to this point is called breaking stress or tensile strength of the material of the wire. A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as elastic limit is crossed.

IMPORTANT POINTS

- Breaking stress = Breaking force/area of cross-section.
- Breaking stress is constant for a material.
- Breaking force depends upon the area of the section of the wire of a given material.
- The working stress is always kept lower than that of a breaking stress so that safety factor = breaking stress/working stress may have a large value.
- Breaking strain = elongation or compression/original dimension.
- Breaking strain is constant for material.

Elastic After-effect

We know that some material bodies take some time to regain their original configuration when the deforming force is removed. The delay in regaining the original configuration by the bodies on the removal of deforming force is called elastic after-effect. The elastic after-effect is negligibly small for quartz fibre and phosphor bronze. For this reason, the suspensions made from quartz and phosphor-bronze are used in galvanometers and electrometers.

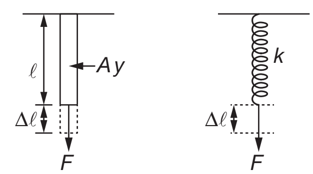
For glass fibre elastic after-effect is very large. It takes hours for glass fibre to return to its original state on removal of deforming force.

Elastic Fatigue

Thus, the loss of strength of the material due to repeated strains on the material is called elastic fatigue. That is why bridges are declared unsafe after a years of use.

Analogy of Rod as a Spring

$$y = \frac{\text{Stress}}{\text{Strain}} \quad \Rightarrow \quad y = \frac{F\ell}{A\Delta\ell}$$



or $$F = \frac{Ay}{\ell}\ \Delta l$$

$\frac{Ay}{\ell}$ = constant depends on type of material and geometry of rod.

$$F = k\Delta\ell$$

where $k = \frac{Ay}{\ell}$ = equivalent spring constant.

For the system of rods shown in Fig. 8.5(a). The replaced spring system is shown in Fig. 8.5(b) two spring in series. Figure. 8.5(c) represents equivalent spring system.

Figure 8.5(d) represents another combination of rods and their replaced spring system.

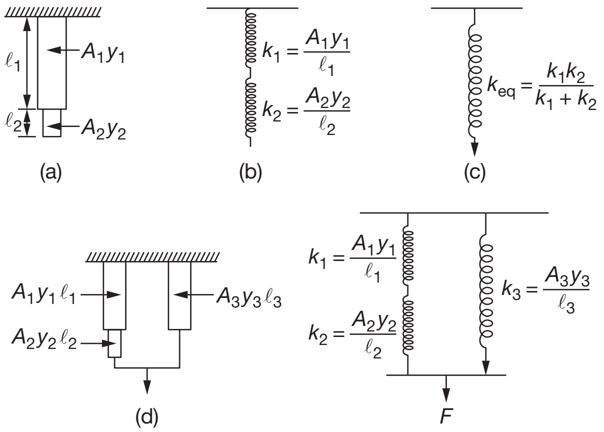


Fig. 8.5

SOLVED EXAMPLE

8. A mass m is attached with rods as shown in Fig. 8.6. This mass is slightly stretched and released whether the motion of mass is simple harmonic motion (SHM). If yes, then find out the time period.

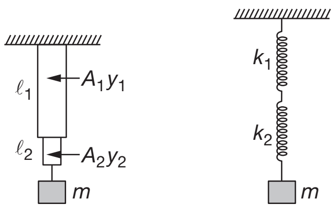


Fig. 8.6

Solution:

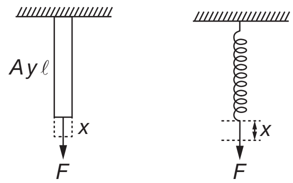
$$k_{eq} = \frac{k_1k_2}{k_1 + k_2}$$

$$T = 2p\sqrt{\frac{m}{k_{eq}}} = 2p\sqrt{\frac{m(k_1 + k_2)}{k_1k_2}}$$

where $k_1 = \frac{A_1y_1}{\ell_1}$ and $k_2 = \frac{A_2y_2}{\ell_2}$.

Elastic potential energy stored in a stretched wire or in a rod

Strain energy stored in equivalent spring



$$U = \frac{1}{2}kx^2$$

where $$x = \frac{F\ell}{Ay},\quad k = \frac{Ay}{\ell}$$

$$U = \frac{1}{2}\frac{Ay}{\ell}\frac{F^2\ell^2}{A^2y^2} = \frac{1}{2}\frac{F^2\ell}{Ay}.$$

Equation can be re-arranged

$$U = \frac{1}{2}\frac{F^2}{A^2} \times \frac{\ell A}{y}$$

[ℓA = volume of rod, F/A = stress]

$$U = \frac{1}{2}\frac{(\text{stress})^2}{y} \times \text{volume}$$

again, $$U = \frac{1}{2}\frac{F}{A} \times \frac{F}{Ay} \times A\ell \qquad \left[\text{Strain} = \frac{F}{Ay}\right]$$

$$U = \frac{1}{2}\ \text{stress} \times \text{strain} \times \text{volume}$$

again, $$U = \frac{1}{2}\frac{F^2}{A^2y^2}A\ell y$$

$$U = \frac{1}{2}y\,(\text{strain})^2 \times \text{volume}$$

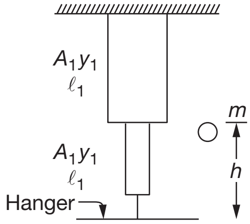
$$\text{Strain energy density} = \frac{\text{Strain energy}}{\text{Volume}}$$

$$= \frac{1}{2}\frac{(\text{stress})^2}{y} = \frac{1}{2}y(\text{strain})^2$$

$$= \frac{1}{2}\ \text{stress} \times \text{strain}$$

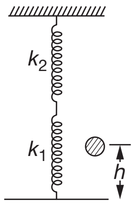
SOLVED EXAMPLE

9. Hanger is mass-less A ball of mass m drops from a height h, which sticks to hanger after striking. Neglect over turning, find out the maximum extension in rod. Assuming rod is massless.



Solution:

Applying energy conservation,



$$mg\,(h+x) = \frac{1}{2}\,\frac{k_1 k_2}{k_1+k_2}\,x^2$$

where $$k_1 = \frac{A_1 y_1}{\ell_1}$$

$$k_2 = \frac{A_2 y_2}{\ell_2}$$

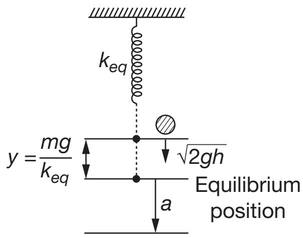
and $$k_{eq} = \frac{A_1 A_2 y_1 y_2}{A_1 y_1 \ell_2 + A_2 y_1 \ell_1}$$

$$k_{eq}x^2 - 2mgx - 2mgh = 0$$

$$x = \frac{2mg \pm \sqrt{4m^2 g^2 + 8mghk_{eq}}}{2k_{eq}}$$

$$x_{\max} = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2 g^2}{k_{eq}^2} + \frac{2mgh}{k_{eq}}}.$$

ALTERNATIVES OF SHM



$$w = \sqrt{\frac{k_{eq}}{m}}$$

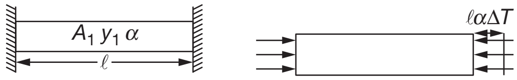
$$v = w\,\sqrt{a^2 - y^2}$$

$$\sqrt{2gh} = \sqrt{\frac{k_{eq}}{m}}\sqrt{a^2 - y^2}$$

$$\Rightarrow \quad \sqrt{\frac{2mgh}{k_{eq}} + \frac{m^2 g^2}{k_{eq}^2}} = a$$

$$\text{Maximum extension} = a + y = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2 g^2}{k_{eq}} + \frac{2mgh}{k_{eq}}}.$$

Thermal Stress



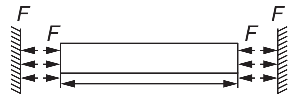
If temp of rod is increased by ΔT, then change in length

$$\Delta\ell = \ell\alpha\Delta T$$

$$\text{Strain} = \frac{\Delta\ell}{\ell} = \alpha\,\Delta T$$

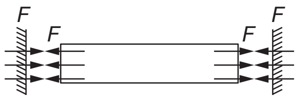
But due to rigid support, there is no strain. Supports provide force on stresses to keep the length of rod same

$$y = \frac{\text{Stress}}{\text{Strain}}$$



If ΔT = (+)ve

Thermal stress = y strain = $y\alpha\Delta T$



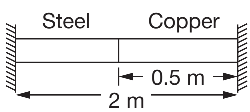
If ΔT = (–)ve

$$\frac{F}{A} = y\,\alpha\,\Delta T$$

$$F = Ay\alpha\Delta T.$$

SOLVED EXAMPLE

10. When composite rod is free, then composite length increases to 2.002 m for temp 20ºC to 120ºC. When composite rod is fixed between the support, there is no change in component length. Find y and a of steel, if $y_{cu} = 1.5 \times 10^{13}$ N/m^2 $a_{cu} = 1.6 \times 10^{-5}$/ºC.



Solution:

$$\Delta\ell = \ell_s\, a_s\, \Delta T + \ell_c\, a_c \Delta T$$

$$0.002 = [1.5\, a_s + 0.5 \times 1.6 \times 10^{-5}] \times 100$$

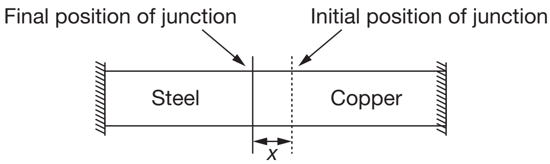
$$a_s = \frac{1.2\times10^{-5}}{1.5} = 8 \times 10^{-6}/\text{ºC}.$$

There is no change in component length

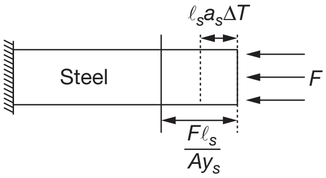
For steel,

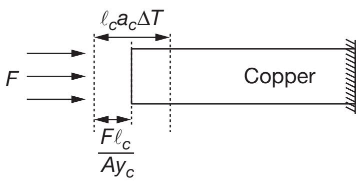
$$x = \ell_s a_s \Delta T - \frac{F\ell_s}{AY_s} = 0$$

$$\frac{F}{AY_s} = \alpha_s \, \Delta T \qquad (1)$$



For copper,





$$x = \frac{F\ell_c}{Ay_c} - \ell_c a_c \Delta T = 0$$

$$\frac{F}{Ay_c} = a\Delta T \qquad (2)$$

$$B/A \quad \Rightarrow \quad \frac{y_s}{y_c} = \frac{\alpha_c}{\alpha_s}$$

$$y_s = y_c \frac{\alpha_c}{\alpha_s} = \frac{1.5\times10^{13}\times16\times10^{-5}}{8\times10^{-6}}$$

$$y_s = 3 \times 10^{13} \text{ N/m}^2.$$

APPLICATIONS OF ELASTICITY

Some of the important applications of the elasticity of the materials are discussed as follows:

1. The material used in bridges loses its elastic strength with time, thus, bridges are declared unsafe after long use.
2. **To estimate the maximum height of a mountain**
   The pressure at the base of the mountain = hrg = stress.
   The elastic limit of a typical rock is 3×10^8 N/m^{-2}
   The stress must be less than the elastic limits, otherwise the rock begins to flow.

$$h < \frac{3\times10^8}{\rho g} < \frac{3\times10^8}{3\times10^3\times10} < 10^4 \text{ m}$$

$$(\because r = 3 \times 10^3 \text{ kg m}^{-3}; g = 10 \text{ ms}^{-2})$$

or $\qquad h = 10$ km

It may be noted that the height of Mount Everest is nearly 9 km.

Torsion Constant of a Wire

$C = \dfrac{\pi\eta r^4}{2\ell}$, where h is modulus of rigidity r and ℓ is radius and length of wire, respectively.

1. Toque required for twisting by angle θ, $\tau = C\theta$.
2. Work done in twisting by angle θ, $W = \dfrac{1}{2} C\theta^2$.

VISCOSITY

When a solid body slides over another solid body, a frictional force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig. 8.7). The layer of the liquid which is in contact with the surface is at rest, while the velocity of other layers increases with distance from the fixed surface. In the figure, the lengths of the arrows represent the increasing velocity of the layers. Thus, there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers: a, b, and c. Their velocities are in the increasing order. Layer a tends to retard Layer b, while b tends to retard c. Thus, each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external

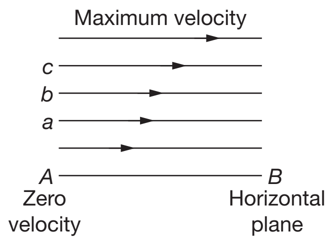


Fig. 8.7

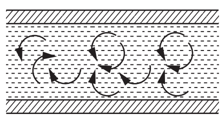
force must be applied to overcome the dragging viscous forces. In the absence of an external force, the viscous forces would soon bring the liquid to rest. The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.

The property of viscosity is seen in the following examples:

1. A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerine, etc., have a larger viscosity than thinner ones like water. If we pour coaltar and water on a table, the coaltar will stop soon while the water will flow up to quite a large distance.
2. If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.
3. We can walk fast in air but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.
4. The cloud particles fall down very slowly because of the viscosity of air and hence appear floating in the sky. Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

Flow of Liquid In a Tube: Critical Velocity

When a liquid flows in a tube, the viscous forces oppose the flow of the liquid. Hence, a pressure difference is applied between the ends of the tube which maintains the flow of the liquid. If all particles of the liquid passing through a particular point in the tube move along the same path, the flow of the liquid is called 'stream-lined flow'. This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'. When the velocity of flow exceeds the critical velocity, the flow is no longer streamlined but becomes turbulent. In this type of flow, the motion of the liquid becomes zig-zag, and eddy-currents are developed in it.



Reynold proved that the critical velocity for a liquid flowing in a tube is $v_c = kh/ra$, where r is density and h is viscosity of the liquid, a is radius of the tube, and k is 'Reynold's number' (whose value for a narrow tube and for water is about 1000). When the velocity of flow of the liquid is less than the critical velocity, then the flow of the liquid is controlled by the viscosity – the density having no effect on it. But when the velocity of flow is larger than the critical velocity, then the flow is mainly governed by the density, the effect of viscosity becoming less important. It is because of this reason that when a volcano erupts, the lava coming out of it flows at lightening speed in spite of it being very thick (of large viscosity).

Velocity Gradient and Coefficient of Viscosity

The property of a liquid by virtue of which an opposing force (internal friction) comes into play whenever there is a relative motion between the different layers of the liquid is called viscosity. Consider a flow of a liquid over the horizontal solid surface as shown in Fig. 8.8. Let us consider two layers AB and CD moving with velocities $\vec{v}$ and $\vec{v} + \vec{dv}$ at a distance x and $(x + dx)$, respectively, from the fixed solid surface.

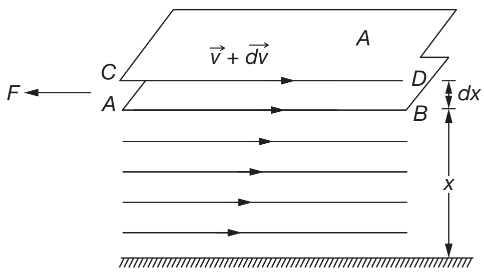


Fig. 8.8

According to Newton, the viscous drag or backward force (F) between these layers

1. Directly proportional to the area (A) of the layer and
2. Directly proportional to the velocity gradient $\left(\dfrac{dv}{dx}\right)$ between the layers.

   That is, $$F \propto A\frac{dv}{dx} \quad \text{or} \quad F = -hA\frac{dv}{dx} \qquad (1)$$

   h is called coefficient of viscosity. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.

Similarities and Differences Between Viscosity and Solid Friction

Similarities

Viscosity and solid friction are similar as:

1. Both oppose relative motion. While viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.

2. Both come into play whenever there is relative motion between layers of liquid or solid surfaces, as the case may be.
3. Both are due to molecular attractions.

Differences between them are

Viscosity	Solid Friction
(i) Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	(i) Friction between two solids is independent of the area of solid surfaces in contact.
(ii) Viscous drag is proportional to the relative velocity between two layers of liquid.	(ii) Friction is independent of the relative velocity between two surfaces.
(iii) Viscous drag is independent of normal reaction between two layers of liquid.	(iii) Friction is directly proportional to the normal reaction between two surfaces in contact.

Some Applications of Viscosity

Viscosity of various liquids and gases can be observed in our daily life. Some of its applications are discussed as follows:

1. As the viscosity of liquids vary with temperature, proper choice of lubricant can be made depending upon the season.
2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.
3. The viscosity of air and liquid dampens the strings vibration of some instruments.
4. The coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.
5. It finds an important use in the circulation of blood through arteries and veins of human body.

Units of Coefficient of Viscosity

From the above formula,

we have $$\eta = \frac{F}{A(\Delta v_x / \Delta z)}$$

$\therefore$ Dimensions of $$\eta = \frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \frac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$$

Its unit is kg/(meter-second)*

In CGS system, the unit of coefficient of viscosity is dyne s cm^{-2} and is called poise. In SI, the unit of coefficient of viscosity is N/sm^{-2} and is called decapoise.

$$1 \text{ decapoise} = 1 \text{ N/sm}^{-2} = (10^5 \text{ dyne}) \times s \times (10^2 \text{ cm})^{-2} = 10 \text{ dyne s cm}^{-2} = 10 \text{ poise.}$$

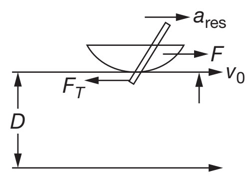
SOLVED EXAMPLES

11. A man is rowing a boat with a constant velocity v_0 in a river. The contact area of boat is A and coefficient of viscosity is h. The depth of river is D. Find the force required to row the boat.

Solution:

$$F - F_T = ma_{res}$$

As boat moves with constant velocity $a_{res} = 0$



$$F = F_T$$

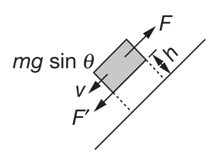
but $$F_T = hA\frac{dv}{dz},$$

but $$\frac{dv}{dz} = \frac{v_0 - 0}{D} = \frac{v_0}{D}$$

then $$F_T = hv/AD.$$

12. A cubical block (of side 2 m) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $h = 10^{-1}$ poise with constant velocity of 10 m/s. ($g = 10$ m/s^2)
Find out the thickness of layer of liquid.

Solution:



$$F = F' = hA\frac{dv}{dz} = mg\sin\theta$$

$$\frac{dv}{dz} = \frac{v}{h}$$

$$20 \times 10 \times \sin 30° = h \times 4 \times \frac{10}{h}$$

$$h = \frac{40 \times 10^{-2}}{100} \quad - [h = 10^{-1} \text{ poise} = 10^{-2} \text{ N/sm}^{-2}]$$
$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}.$$

13. As per Fig. 8.9, the central solid cylinder starts with initial angular velocity w_0. Find out the time after which the angular velocity becomes half.

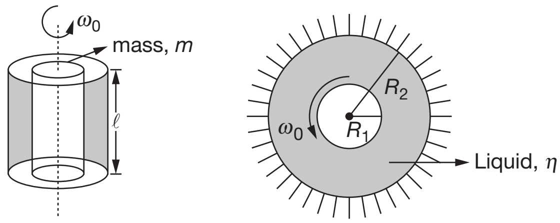


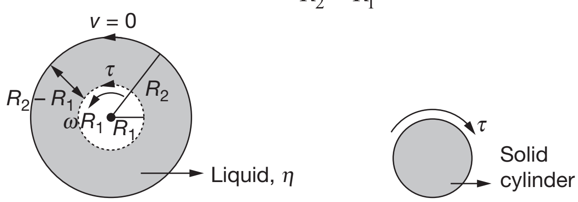
Fig. 8.9

Solution:

$$F = hA\,\frac{dv}{dz},$$

where $$\frac{dv}{dz} = \frac{\omega R_1 - 0}{R_2 - R_1}$$

$$F = h\,\frac{2\pi\, R_1\, \ell\, \omega\, R_1}{R_2 - R_1}$$



and $$t = FR_1 = \frac{2\pi\,\eta\, R_1^3\,\omega\,\ell}{R_2 - R_1}$$

$$\ell a = \frac{2\pi\,\eta\, R_1^3\,\omega\,\ell}{R_2 - R_1}$$

$$\Rightarrow \quad \frac{MR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\,\eta R_1^3\,\omega\ell}{R_2 - R_1}$$

or $$-\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \frac{4\pi\eta R_1\,\ell}{m\,(R_2 - R_1)} \int_0^t dt$$

$$\Rightarrow \quad t = \frac{m\,(R_2 - R_1)\ell n2}{4\,\pi\,\eta\,\ell\, R_1}.$$

Effect of Temperature on the Viscosity

The viscosity of liquids decreases with increase in temperature and increases with the decrease in temperature. That is, $h \propto \frac{1}{\sqrt{T}}$. On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $h \propto \frac{1}{\sqrt{T}}$.

STOKES' LAW

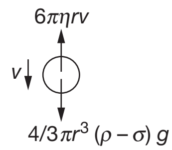
Stokes proved that the viscous drag (F) on a spherical body of radius r moving with velocity v in a fluid of viscosity h is given by $F = 6\,\pi\eta r v$. This is called Stokes' law.

Terminal Velocity

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

Calculation of Terminal Velocity

Let us consider a small ball, whose radius is r and density is r, falling freely in a liquid (or gas), whose density is s and coefficient of viscosity η. When it attains a terminal velocity v, it is subjected to two forces:



1. Effective force acting downward
$= v\,(\rho - \sigma)\,g = \frac{4}{3}\,\pi r^3\,(\rho - \sigma)g,$
2. Viscous force acting upward
$= 6\,\pi\eta rv.$
Since the ball is moving with a constant velocity v, that is there is no acceleration in it, the net force acting on it must be zero. That is,

$$6\,\pi\,\eta\, rv = \frac{4}{3}\,\pi r^3\,(\rho - \sigma)\,g$$

or $$v = \frac{2}{9}\,\frac{r^2(\rho - \sigma)g}{\eta}.$$

Thus, terminal velocity of the ball is directly proportional to the square of its radius.

Important Point

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

SOLVED EXAMPLES

14. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Solution:

Rate of heat loss = power = $F \times v = 6\,\pi\eta r v \times v$

$$= 6\,\pi\eta r v^2 = 6\,\pi\eta r \left[\frac{2}{9}\,\frac{gr^2(\rho_0 - \rho_\ell)}{\eta}\right]^2$$

Rate of heat loss ar^5.

15. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg/(m-s), what will be the terminal velocity of the drop? (density of water = 1.0×10^3 kg/m^2 and $g = 9.8$ N/kg.) Density of air can be neglected.

Solution:

By Stokes' law, the terminal velocity of a water drop of radius r is given by

$$n = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

where r is the density of water, σ is the density of air, and η the coefficient of viscosity of air. Here σ is negligible and r = 0.0015 mm = 1.5×10^{-3} mm = 1.5×10^{-6} m. Substituting the values

$$n = \frac{2}{9} \times \frac{(1.5\times10^{-6})^2 \times (1.0\times10^3)\times 9.8}{1.8\times10^{-5}}$$

$$= 2.72 \times 10^{-4} \text{ m/s.}$$

16. A metallic sphere of radius 1.0×10^{-3} m and density 1.0×10^4 kg/m^3 enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given: coefficient of viscosity of water = 1.0×10^{-3} Ns/m^2, $g = 10$ m/s^2 and density of water = 1.0×10^3 kg/m^3.

Solution:

The velocity attained by the sphere in falling freely from a height h is

$$n = \sqrt{2gh} \qquad (1)$$

This is the terminal velocity of the sphere in water. Hence from Stokes' law, we have

$$n = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere

σ (= 1.0×10^3 kg/m^3) is the density of water and η is coefficient of viscosity of water.

$$\therefore \quad n = \frac{2\times(1.0\times10^{-3})^2(1.0\times10^4 - 1.0\times10^3)\times 10}{9\times1.0\times10^{-3}}$$

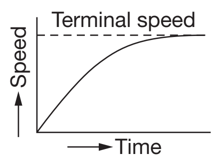
$$= 20 \text{ m/s.}$$

From Equation (1), we have

$$h = \frac{v^2}{2g} = \frac{20\times20}{2\times10} = 20 \text{ m.}$$

Applications of Stokes' Formula

1. **In Determining the Electronic Charge by Millikan's Experiment:** Stokes' formula is used in Millikan's method for determining the electronic charge. In this method, the formula is applied for finding out the radii of small oil drops by measuring their terminal velocity in air.
2. **Velocity of Rain Drops:** Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning, the raindrops are very small in size and thus they fall with such a small velocity that they appear as if they are floating in the sky as cloud. As they grow in size by further condensation, they reach the earth with appreciable velocity.
3. **Parachute:** When a soldier with a parachute jumps from a flying aeroplane, he descends very slowly in air.



In the beginning, the soldier falls with gravity acceleration g, but soon the acceleration goes on decreasing rapidly until in parachute is fully opened. Therefore, in the beginning, the speed of the falling soldier increases somewhat rapidly but then very slowly. Due to the viscosity of air, the acceleration of the soldier becomes ultimately zero and the soldier then falls with a constant terminal speed. The graph shows the speed of the falling soldier and time.

SURFACE TENSION

Explanation of Some Observed Phenomena

1. Lead balls are spherical in shape.
2. Raindrops and a globule of mercury placed on glass plate are spherical.
3. Hair of a shaving brush/painting brush when dipped in water spread out, but as soon as it is taken out, its hair sticks together.
4. A greased needle placed gently on the free surface of water in a beaker does not sink.
5. Similarly, insects can walk on the free surface of water without drowning.
6. Bits of camphor gum move irregularly when placed on water surface.

Surface tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in Fig. 8.10.

i.e., Surface tension

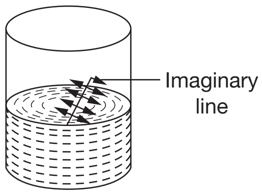


Fig. 8.10

$$(T) = \frac{\text{Total force on either of the imginary line } (F)}{\text{Length of the line } (\ell)}$$

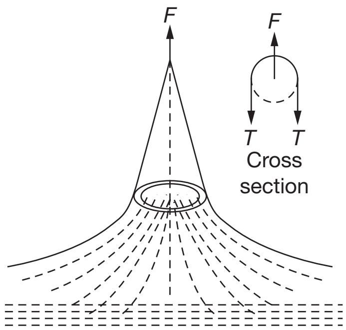
Units of Surface Tension

In CGS system, the unit of surface tension is dyne/cm (dyne cm^{-1}) and in Sl system, its units is N/m^{-1}.

SOLVED EXAMPLE

17. A ring is cut form a platinum tube of 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water? ($g = 980$ cm/s^2).

Solution:



The ring is in contact with water along its inner and outer circumference; so when pulled out, the total force on it due to surface tension will be

$$F = T(2\pi r_1 + 2\pi r_2)$$

So, $$T = \frac{mg}{2\pi(r_1 + r_2)} \quad [\because F = mg]$$

i.e., $$T = \frac{3.97 \times 980}{3.14 \times (8.5 + 8.7)} = 72.13 \text{ dyne/cm.}$$

Excess Pressure Inside a Liquid Drop and a Bubble

Inside a Bubble

Consider a soap bubble of radius r.

Let p be the pressure inside the bubble and p_a outside. The excess pressure = $p - p_a$. Imagine the bubble broken into two halves consider one half of it as shown in Fig. 8.11. Since there are two surfaces, inner and outer, the force due to surface tension will be

$$\begin{aligned} F &= \text{surface tension} \times \text{length} \\ &= T \times 2 \text{ (circumference of the bubble)} \\ &= T \times 2 \ (2\ Tpr) \qquad (1) \end{aligned}$$

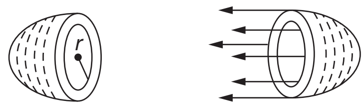


Fig. 8.11

The excess pressure ($p - p_a$) acts on a cross-sectional area pr^2, so the force due to excess pressure is

$$\Rightarrow \qquad F = (p - p_a)\, pr^2 \qquad (2)$$

The surface tension force given by equation (1) must balance the force due to excess pressure given by equation (2) to maintain the equilibrium. That is,

$$(p - p_a)\, pr^2 = T \times 2\ (2pr)$$

or $$(p - p_a) = \frac{4T}{r} = p_{\text{excess}}.$$

The above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.

Inside the Drop

In a drop, there is only one surface and hence excess pressure can be written as

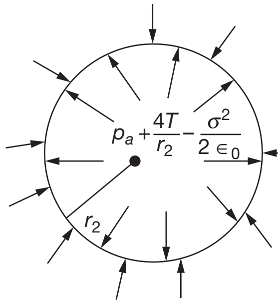
$$(p - p_a) = \frac{2T}{r} = p_{\text{excess}}.$$

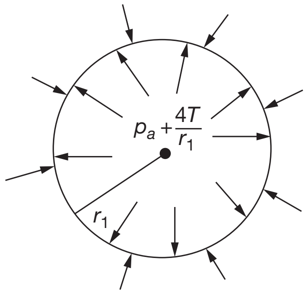
Inside Air Bubble in a Liquid

$$(p - p_a) = \frac{2T}{r} = p_{\text{excess}}.$$

A Charged Bubble

If bubble is charged, its radius increases.





Bubble has pressure excess due to charge too. Initially, pressure inside the bubble

$$= p_a + \frac{4T}{r_1}.$$

For charge bubble, pressure inside $= p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}$, where s is surface charge density. If temperature remains constant, then from Boyle's law,

$$\left(p_a + \frac{4T}{r_1}\right)\frac{4}{3}pr_1^3 = \left[p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}\right]\frac{4}{3}pr_2^3.$$

From above expression, the radius of charged drop may be calculated. It can be concluded that radius of charged bubble increases, i.e., $r_2 > r_1$.

SOLVED EXAMPLE

18. A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

Solution:

The total pressure inside the bubble at depth h_1 is (P is atmospheric pressure)

$$= (P + h_1 \rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is =

$$(P + h_2 \rho g) + \frac{2T}{r_2} = P_2$$

Now, according to Boyle's Law,

$$P_1V_1 = P_2V_2,$$

where $V_1 = \frac{4}{3}pr_1^3$ and $V_2 = \frac{4}{3}pr_2^3$

Hence we get,

$$\left[(P + h_1 \rho g) + \frac{2T}{r_1}\right]\frac{4}{3}pr_1^3$$

$$= \left[(P + h_2 \rho g) + \frac{2T}{r_2}\right]\frac{4}{3}pr_2^3$$

or $$\left[(P + h_1 \rho g) + \frac{2T}{r_1}\right]r_1^3 = \left[(P + h_2 \rho g) + \frac{2T}{r_2}\right]r_2^3$$

Given that: h_1 = 100 cm, r_1 = 0.1 mm = 0.01 cm, r_2 = 0.126 mm = 0.0126 cm, T = 567 dyne/cm, P = 76 cm of mercury. Substituting all the values, we get

$$h_2 = 9.48 \text{ cm}.$$

Force of Cohesion

The force of attraction between the molecules of the same substance is called cohesion.

In case of solids, the force of cohesion is very large and due to this, solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence, liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. As a result, gases have neither fixed shape nor volume.

Examples:

1. Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.
2. It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.
3. It is very difficult to break a drop of mercury into small droplets because of the large cohesive force between mercury molecules.

Force of Adhesion

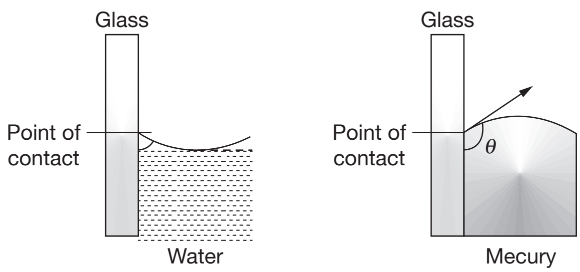
The force of attraction between molecules of different substances is called adhesion.

Examples:

1. Adhesive force enables us to write on the black board with a chalk.
2. Adhesive force helps us to write on the paper with ink.
3. Large force of adhesion between cement and bricks helps us in construction work.
4. Due to force of adhesive, water wets a glass plate.
5. Fevicol and gum are used in gluing two surfaces together because of the adhesive force.

Angle of Contact

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact. Those liquids which wet the walls of the container (as in the case of water and glass) have meniscus concave upwards, and their value of angle of contact is less than 90° (also called acute angle). However, those liquids which do not wet the walls of the container (as in the case of mercury and glass) have meniscus convex upwards, and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass about 140°, whereas the angle of contact of water with glass is about 8°. But for pure water, the angle of contact θ with glass is taken as 0°.



(a) (b)

Shape of Liquid Meniscus

When a capillary tube or a tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. This curved surface is due to the two forces:

1. The force of cohesion.
2. The force of adhesion. The curved surface of the liquid is called **meniscus of the liquid**.
   Various forces acting on molecule *A* are as follows:
3. Force F_1 due to force of adhesion which acts outwards at right angle to the wall of the tube. This force is represented by *AB*.
4. Force F_2 due to force of cohesion which acts at an angle of 45° to the vertical. This force is represented by *AD*.
5. The weight of the molecule *A* which acts vertically downward along the wall of the tube.

Since the weight of the molecule is negligible as compared to F_1 and F_2 it can be neglected. Thus, there are only two forces (F_1 and F_2) acting on the liquid molecules. These forces are inclined at an angle of 135°.

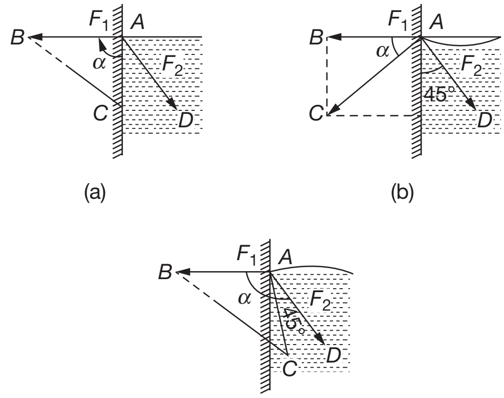
The resultant force represented by *AC* will depend upon the values of F_1 and F_2. Let the resultant force make an angle a with F_1.

According to parallelogram law of vectors,

$$\tan\alpha = \frac{F_2 \sin 135°}{F_1 + F_2 \cos 135°} = \frac{F_2/\sqrt{2}}{F_1 - F_2/\sqrt{2}} = \frac{F_2}{\sqrt{2}\,F_1 - F_2}$$

Special Cases

1. If $F_2 = \sqrt{2}\,F_1$, then tan $a = \infty$ and $a = 90°$
   The resultant force will act vertically downward and hence the meniscus will be plane or horizontal shown in Fig. 8.12(a). Example includes pure water contained in silver capillary tube.



(c)

Fig. 8.12

2. If $F_2 < \sqrt{2}\,F_1$, then tan a is positive and *a* is an acute angle.
   Thus, the resultant will be directed outside the liquid and hence the meniscus will be concave upward shown in Fig. 8.12(b). This is possible in case of liquids which wet the walls of the capillary tube. Example includes water in glass capillary tube.
3. If $F_2 > \sqrt{2}\,F_1$, then tan a is negative and *a* is an obtuse angle.
   Thus, the resultant will be directed inside the liquid and hence the meniscus will be convex upward shown in Fig. 8.12 (c). This is possible in case of liquids which do not wet the walls of the capillary tube. Example includes mercury in glass capillary tube.

Relation between Surface Tension, Radii of Curvature, and Excess Pressure on a Curved Surface

Let us consider a small element ABCD (Fig. 8.13) of a curved liquid surface, which is convex on the upper side. R_1 and R_2 are the maximum and minimum radii of curvature, respectively. They are called the 'principal radii of curvature' of the surface. Let p be the excess pressure on the concave side.

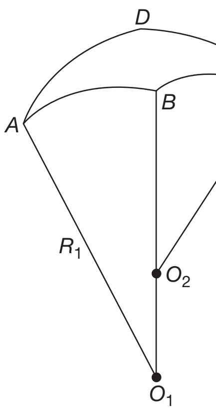


Fig. 8.13

Then $p = T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$. If instead of a liquid surface, we have a liquid film, the above expression will be

$p = 2T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$, because a film has two surfaces.

Excess of Pressure Inside a Curved Surface

Plane Surface

If the surface of the liquid is plane [as shown in Fig. 8.14(a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore, on the liquid surface is normal.

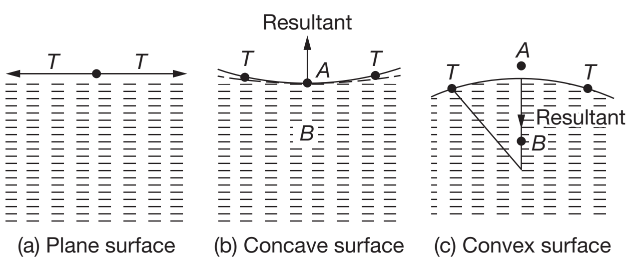


Fig. 8.14

Concave Surface

If the surface is concave upwards [as shown in Fig. 8.14(b)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side in the downward direction to balance the resultant force of surface tension $p_A - p_B = \frac{2T}{r}$.

Convex Surface

If the surface is convex [as shown in Fig. 8.14(c)], the resultant force due to surface tension acts in the downward direction. Since the molecule on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension. Hence there is always an excess of pressure on the concave side of a curved surface over that of the convex side.

$$p_B - p_A = \frac{2T}{r}.$$

SOLVED EXAMPLE

19. A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in the narrow tube is 0.2 m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3}, N/m and $g = 9.8$ m/s^2.

Solution:

Let the pressure in the wide and narrow capillaries of radii r_1 and r_2, respectively, be P_1 and P_2.

Then pressure just below the meniscus in the wide and narrow tubes, respectively, are

$$\left(P_1 - \frac{2T}{r_1}\right) \text{ and } \left(P_2 - \frac{2T}{r_2}\right)$$

[excess pressure = $\frac{2T}{r}$].

Difference in these pressures

$$= \left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = \text{hrg}$$

∴ True pressure difference

$$= P_1 - P_2$$

$$= hrg + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}$$

$$\left[\frac{1}{1.44\times10^{-3}} - \frac{1}{7.2\times10^{-4}}\right]$$

$$= 1.86 \times 10^3 = 1860 \text{ N/m}^2.$$

Capillarity

A glass tube of very fine bore throughout the length of the tube is called capillary tube. If the capillary tube is dipped in water, the water wets the inner side of the tube and rises in it [shown in Fig. 8.15(a)]. If the same capillary tube is dipped in the mercury, then the mercury is depressed [shown in Fig. 8.15(b)]. The phenomenon of rise or fall of liquids in a capillary tube is called capillarity.

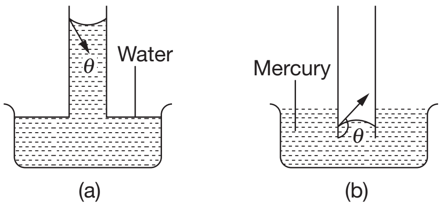


Fig. 8.15

Practical Applications of Capillarity

1. The oil in a lamp rises in the wick by capillary action.
2. The tip or nib of a pen is split up, to make a narrow capillary so that the ink rises up to the tip or nib continuously.
3. Sap and water rise up to the top of the leaves of the tree by capillary action.
4. If one end of the towel dips into a bucket of water and the other end hangs over the bucket, the towel soon becomes wet throughout due to capillary action.
5. Ink is absorbed by the blotter due to capillary action.
6. Sandy soil is drier than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture *m* in the soil, capillaries must be broken up. This is done by ploughing and leveling the fields.
8. Bricks are porous and behave like capillaries.

Capillary Rise (Height of a Liquid in a Capillary Tube) Ascent Formula

Consider the liquid, which wets the walls of the tube, that forms a concave meniscus as shown in Fig. 8.16. Consider a capillary tube of radius r dipped in a liquid of surface tension T and density p. Let h be the height through which the liquid rises in the tube. Let p be the pressure on the concave side of the meniscus and p_a be the pressure on the convex side of the meniscus. The excess pressure

$$(p - p_a) \text{ is given by } (p - p_a) = \frac{2T}{R},$$

where R is the radius of the meniscus. Due to this excess pressure, the liquid will rise in the capillary tube till it becomes equal to the hydrostatic pressure hpg. Thus in equilibrium state.

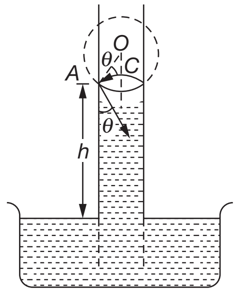


Fig. 8.16

Excess pressure = Hydrostatic pressure

or
$$\frac{2T}{R} = hpg.$$

Let θ be the angle of contact and r be the radius of the capillary tube shown in Fig. 8.16.

From DOAC,

$$\frac{OC}{OA} = \cos\theta \quad \text{or} \quad R \frac{r}{\cos\theta}$$

$$\Rightarrow \qquad h = \frac{2T\cos\theta}{r\rho g}.$$

This expression is called ascent formula.

Discussion

For liquids which wet the glass tube or capillary tube, angle of contact $\theta < 90°$. Hence, $\cos\theta$ = positive. $\Rightarrow h$ = positive. It means that these liquids rise in the capillary tube.

Hence, the liquids which wet capillary tube rise in the capillary tube. For example, water, milk, kerosene oil, patrol, etc.

SOLVED EXAMPLES

20. A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm, and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0º.

Solution:

The surface tension of the liquid is

$$T = \frac{rh\rho g}{2}$$

$$= \frac{(0.025 \text{ cm})(3.0 \text{ cm})(1.5 \text{ gm/cm}^3)(980 \text{ cm/s}^2)}{2}$$

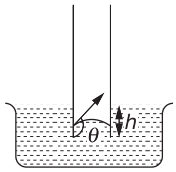
$$= 55 \text{ dyne/cm}.$$

Hence excess pressure inside a spherical bubble,

$$p = \frac{4T}{R} = \frac{4 \times 55 \text{ dyne/cm}}{(0.5 \text{ cm})} = 440 \text{ dyne/cm}^2.$$

For liquids which do not wet the glass tube or capillary tube, angle of contact $\theta > 90°$.

Hence, $\cos\theta$ = negative $\Rightarrow h$ = negative. Hence, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.

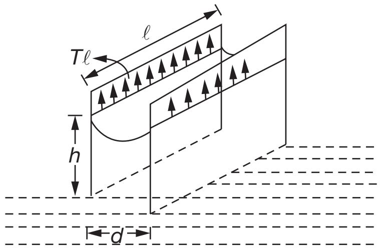


T, θ, r, and g are constant and hence $h \infty \frac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. This is called Jurin's law.

If two parallel plates with the spacing d are placed in water reservoir, then height of rise

$$\Rightarrow \qquad 2T\ell = r\ell h d g$$

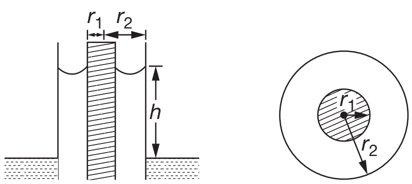
$$h = \frac{2T}{\rho d g}.$$



If two concentric tubes of radius r_1 and r_2 (inner one is solid) are placed in water reservoir, then height of rise

$$\Rightarrow \qquad T[2pr_1 + 2pr_2] = [pr^2{}_2 h - pr_1{}^2 h]\rho g$$

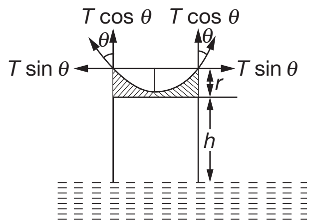
$$h = \frac{2T}{(r_2 - r_1)\rho g}.$$



If weight of the liquid in the meniscus is to be considered:

$$T\cos\theta \times 2pr = [pr^2 h + \frac{1}{3}pr^2 \times r]\rho g$$

$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}.$$



When capillary tube (radius, r) is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = 2T/R_1$, where R_1 = radius of curvature of upper meniscus.

The hydrostatic pressure $p_2 = hrg$ is always directed downwards.

If $p_1 > p_2$, then resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (Fig. 8.17(a)). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

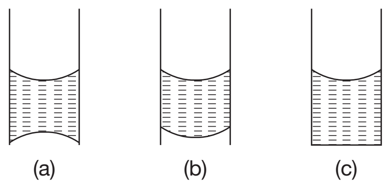


Fig. 8.17

If $p_1 < p_2$, then resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (Fig. 8.17(b)).

The radius of lower meniscus can be given by $\frac{2T}{R_2} = p_2 - p_1$.

If $p_1 = p_2$, there is no resulting pressure. Then, $p_1 - p_2 = \frac{2T}{R_2} = 0$ or $R_2 = \infty$. That is, lower surface will be flat (Fig. 8.17(c)).

Liquid between two plates: When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let R and r be the radii of curvature of the enclosed film in two perpendicular directions.

Hence, the pressure inside the film is less than the atmospheric pressure outside it by an amount p given by $p = T\left(\frac{1}{r} + \frac{1}{R = \infty}\right)$ and we have $p = \frac{T}{r}$.

If d is the distance between the two plates and θ the angle of contact for water and glass, then, from Fig. 8.18, $\cos\theta = \frac{\frac{1}{2}d}{r}$ or $\frac{1}{r} = \frac{2\cos\theta}{d}$.

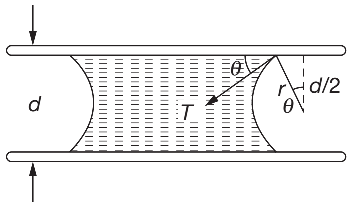


Fig. 8.18

Substituting for $\frac{1}{r}$ in, we get $p = \frac{2T}{d}\cos\theta$.

θ can be taken zero for water and glass, i.e., $\cos\theta = 1$. Thus, the upper plate is pressed downward by the atmospheric pressure minus $\frac{2T}{d}$. Hence the resultant downward pressure acting on the upper plate is $\frac{2T}{d}$. If A is the area of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by F = resultant pressure × area = $\frac{2TA}{d}$.

For a very nearly plane surface, d will be very small and hence the pressing force F will be very large. Therefore, it will be difficult to separate the two plates normally.

21. A drop of water with volume 0.05 cm^3 is pressed between two glass plates. As a result, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to separate the two glass plates in Newton.

Solution:

Pressure inside the film is less than outside by an amount, $P = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$, where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = t/2$ and $r_2 = \infty$, then the force required to separate the two glass plates, between which a liquid film is enclosed (Fig. 8.19) is,

$F = P \times A = \frac{2AT}{t}$, where t is the thickness of the film, A = area of film.

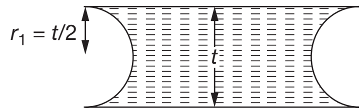


Fig. 8.19

$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}}$$

$= 45$ N.

22. A glass plate of length 10 cm, breadth 1.54 cm, and thickness 0.20 cm weighs 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water = 73 dyne per cm, $g = 980$ cm/s^2.

Solution:

Volume of the portion of the plate immersed in water is

$$10 \times \frac{1}{2}(1.54) \times 0.2 = 1.54 \text{ cm}^3.$$

Therefore, if the density of water is taken as 1, then up thrust

= wt. of the water displaced

= 1.54 × 1 × 980 = 1509.2 dynes.

Now, the total length of the plate in contact with the water surface is $2(10 + 0.2) = 20.4$ cm, downward pull upon the plate due to surface tension

= 20.4 × 73 = 1489.2 dynes

Resultant upthrust

= 1509.2 – 1489.2

$= 20.0 \text{ dynes} = \frac{20}{980}$

= 0.0204 gm of wt.

Apparent weight of the plate in water

= weight of the plate in air – resultant upthrust

= 8.2 – 0.0204 = 8.1796 gm.

23. A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Given: outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension of water 73 dyne/cm and $g = 980$ cm/sec^2.

Solution:

Let l be the length of the tube inside water. The forces acting on the tube are:

(A) Upthrust of water acting upward

$= \pi r^2 l \times 1 \times 980 = \frac{22}{7} \times (0.14)^2\, 1 \times 980$

$= 60.368$ dyne.

(B) Weight of the system acting downward

$= mg = 0.2 \times 980 = 196$ dyne.

(C) Force of surface tension acting downward

$= 2\pi r T$

$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24$ dyne.

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,
60.368 l = 196 + 64.24 = 260.24.

$$l = \frac{260.24}{60.368} = 4.31 \text{ cm.}$$

24. A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07 N/m^{-1}. Assume that the angle of contact between water and glass is 0º.

Solution:

Let pressures at the points A, B, C, and D be P_A, P_B, P_C and P_D, respectively.

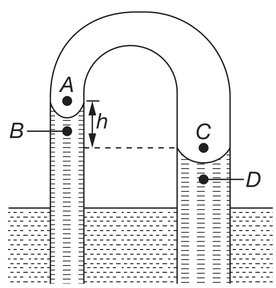
The pressure on the concave side of the liquid surface is greater than that of the other side by $2T/R$.

An angle of contact θ is given to be 0º, hence,

$R \cos 0º = r$ or $R = r$

$\therefore \quad P_A = P_B + 2T/r_1$ and $P_C = P_D + 2T/r_2$

where r_1 and r_2 are the radii of the two limbs.



But $\quad P_A = P_C$

$$\therefore \quad P_B + \frac{2T}{r_1} = P_D + \frac{2T}{r_2}$$

or $$P_D - P_B = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

where h is the difference in water levels in the two limbs.

Now, $$h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Given that $\quad T = 0.07$ N/m^{-1}, $r = 1000$ kgm^{-3}

$$r_1 = \frac{3}{2} \text{ mm} = \frac{3}{20} \text{ cm} = \frac{3}{20 \times 100} \text{ m} = 1.5 \times 10^{-3} \text{ m},$$

$r_2 = 3 \times 10^{-3}$ m

$$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right) \text{m}$$

$= 4.76 \times 10^{-3}$ m $= 4.76$ mm.

25. Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} N/m^{-1}. Take the angle of contact to be zero and density of water to be 1.0×10^3 kg m^{-3} ($g = 9.8$ ms^{-2}).

Solution:

Given that

$$r_1 = \frac{3.0}{2} = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m},$$

$$r_2 = \frac{6.0}{2} = 3.0 \text{ mm} = 3.0 \times 10^{-3} \text{ m},$$

$T = 7.3 \times 10^{-2}$ N/m^{-1},

$\theta = 0º$ $r = 1.0 \times 10^3$ kg m^{-1}, $g = 9.8$ ms^{-2}.

When angle of contact is zero degree, the radius of the meniscus equals radius of bore.

Excess pressure in the first bore,

$$P_1 = \frac{2T}{r_2} = \frac{2 \times 7.3 \times 10^{-2}}{1.5 \times 10^{-3}} = 97.3 \text{ Pascal.}$$

Excess pressure in the second bore,

$$P_2 = \frac{2T}{r_2} = \frac{2 \times 7.3 \times 10^{-2}}{3 \times 10^{-3}} = 48.7 \text{ Pascal.}$$

Hence, pressure difference in the two limbs of the tube

$$\Delta P = P_1 - P_2 = hrg$$

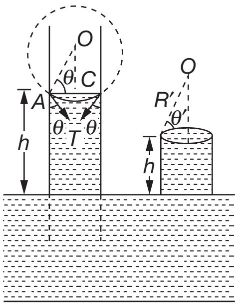
or $$h = \frac{P_1 - P_2}{\rho g} = \frac{97.3 - 48.7}{1.0 \times 10^3 \times 9.8} = 5.0 \text{ mm.}$$

Capillary Rise in a Tube of Insufficient Length

We know that the height through which a liquid rises in the capillary tube of radius r is given by

$$h = \frac{2T}{R\rho g} \quad \text{or} \quad hR = \frac{2T}{\rho g} = \text{constant.}$$

When the capillary tube is cut and its length is less then h (i.e., h'), then the liquid rises up to the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus increases and it becomes more flat so that $hR = h'R'$ = constant. Hence, the liquid does not overflow.



If $\quad h' < h$ then $R' > R$

or $\quad \dfrac{r}{\cos\theta'} > \dfrac{r}{\cos\theta}$

$\Rightarrow \quad \cos\theta' < \cos\theta \quad \Rightarrow \theta' > \theta.$

SOLVED EXAMPLE

26. If a 5 cm long capillary tube with 0.1 mm internal diameter open at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether

(A) water will rise half way in the capillary,
(B) water will rise up to the upper end of capillary,
(C) water will overflow out of the upper end of capillary. Give reasons for your answer.

Solution:

Given that surface tension of water, $T = 75$ dyne/cm

Radius $\quad r = \dfrac{0.1}{2}$ mm = 0.05 mm = 0.005 cm,

Density $\quad \rho = 1$ gm/cm^3, angle of contact, $\theta = 0°$.

Let h be the height to which water rise in the capillary tube. Then

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times 75\times\cos 0°}{0.005\times 1\times 981}\text{ cm} = 30.58\text{ cm}.$$

But length of capillary tube, $h' = 5$ cm

(A) Because $h > \dfrac{h'}{2}$ the first possibility does not exist.
(B) Because the tube is of insufficient length, the water will rise up to the upper end of the tube.
(C) The water will not overflow out of the upper end of the capillary. It will rise only up to the upper end of the capillary.

The liquid meniscus will adjust its radius of curvature R' in such a way that

$$R'h' = Rh \qquad \left[\because\ hR = \frac{2T}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature. The liquid meniscus would possess if the capillary tube were of sufficient length

$$R' = \frac{Rh}{h'} = \frac{rh}{h'} \qquad \left[\because\ R = \frac{r}{\cos\theta} = \frac{r}{\cos 0°} = r\right]$$

$$= \frac{0.005\times 30.58}{5} = 0.0306\text{ cm}.$$

Applications of Surface Tension

1. The wetting property is made use of detergents and waterproofing. When detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.
2. Antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension, the antiseptics spread evenly over the wound. Lubricating oils and paints also have low surface tension and they also spread evenly.
3. Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.
4. Oil spreads over the surface of water, because the surface tension of oil is less than the surface tension of cold water.
5. A rough sea can be calmed by pouring oil on its surface.

Effect of Temperature and Impurities on Surface Tension

The surface tension of a liquid decreases with the rise in temperature and vice-versa. According to Ferguson, $T = T_0\left(1-\dfrac{\theta}{\theta_c}\right)^n$, where T_0 is surface tension at 0ºC, θ is absolute temperature of the liquid, θ_c is the critical temperature, and n is a constant varies slightly from liquid and has mean value 1.21. This formula shows that the surface tension becomes zero at the critical temperature, where the interface between the liquid and its vapour disappears. It is for this reason that hot soup tastes better, while machinery parts get jammed in winter.

The surface tension of a liquid changes appreciably, with addition of impurities. For example, surface tension of water increases with addition of highly soluble substances like NaCI, $ZnSO_4$, etc. On the other hand, surface tension of water gets reduced with addition of sparingly soluble substances like phenol, soap, etc.

SURFACE ENERGY

We know that the molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy. Unit of surface energy is erg cm^{-2} in CGS system and Jm^{-2} in Sl system. Dimensional formula of surface energy is $[ML^{\circ}T^{-2}]$. Surface energy depends on a number of surfaces. For example, a liquid drop has only one liquid air surface while a bubble has two liquid air surfaces.

Relation Between Surface Tension and Surface Energy

Consider a rectangular frame PQRS of wire, whose arm RS can slide on the arms PR and QS. If this frame is dipped in a soap solution, then a soap film is produced in the frame PQRS as in Fig. 8.20. Due to surface tension (T), the film exerts a force on the frame (towards the interior of the film). Let ℓ be the length of the arm RS, then the force acting on the arm RS towards the film be $F = T \times 2\ell$ [Since soap film has two surfaces, the length is taken twice].

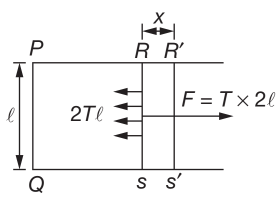


Fig. 8.20

Let the arm RS be displaced to a new position $R'S'$ through a distance x

$\therefore$ Work done, $W = Fx = 2T\ell x$

Increase in potential energy of the soap film

$= EA = 2E\ell x =$ work done in increasing the area (ΔW),

where $E =$ surface energy of the soap film per unit area.

According the law of conservation of energy, the work done must be equal to the increase in the potential energy.

$$\therefore \quad 2T\ell x = 2E\ell x \quad \text{or} \quad T = E = \frac{\Delta W}{A}$$

Thus, surface tension is numerically equal to surface energy or work done per unit to increase surface area.

SOLVED EXAMPLES

27. A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded if surface tension of mercury is 35×10^{-3} N/m.

Solution:

If drop of radius R is sprayed into n droplets of equal radius r, as a drop has only one surface, the initial surface area will be $4\pi R^2$, while final area is $n(4\pi r^2)$. So the increase in area,

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

so energy expended in the process,

$$W = T\Delta S = 4\pi T\,[nr^2 - R^2] \qquad (1)$$

now since the total volume of n droplets is the same as that of initial drop, i.e.,

$$\frac{4}{3}\pi R^3 = n[(4/3)\pi r^3]$$

or $$r = R/n^{1/3} \qquad (2)$$

putting the value of r from Equation (2) in (1)

$$W = 4\pi R^2 T\,(n)^{1/3} - 1].$$

28. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, shows that the rise in temperature will be given by

$$\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

where T is the surface tension of water and J is the mechanical equivalent of heat.

Solution:

Let n be the number of little droplets.

Since the volume will remain constant, volume of n little droplets = volume of single drop

$$\therefore \quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

or $$nr^3 = R^3$$

Decrease in surface area $= n \times 4\pi r^2 - 4\pi R^2$

or $$\Delta A = 4\pi\,[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right]$$

$$= 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Energy evolved $W = T \times$ decrease in surface area

$$= T \times 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Heat produced, $$Q = \frac{W}{J} = \frac{4\pi T R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

But Q = $msdq$

where m is the mass of big drop, s is the specific heat of water, and dq is the rise in temperature.

$$\therefore \quad \frac{4\pi T R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right] = \text{volume of big drop} \times \text{density}$$

of water × sp. heat of water × dq

or $$\frac{4}{3}\pi R^3 \times 1 \times 1 \times dq = \frac{4\pi T R^3}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

or $$dq = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right].$$

29. A film of water is formed between two straight parallel wires each 10 cm long separated by 0.5 cm. Calculate the work required to increase 1 mm distance between them.

Surface tension of water = 72×10^{-3} N/m.

Solution:

Here the increase in area is shown by the shaded portion in Fig. 8.21.

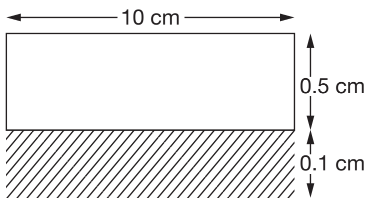


Fig. 8.21

Since this is a water film, it has two surfaces, therefore increase in area,

$$\Delta S = 2 \times 10 \times 0.1 = 2 \text{ cm}^2.$$

Work required to be done,

$$W = \Delta S \times T$$
$$= 2 \times 10^{-4} \times 72 \times 10^{-3}$$
$$= 144 \times 10^{-7} \text{ J}$$
$$= 1.44 \times 10^{-5} \text{ J}.$$

FLUID MECHANICS

Definition of Fluid

The term ‘fluid’ refers to a substance that can flow and does not have a shape of its own. For example, liquid and gases.

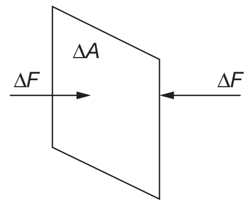
The properties of fluid are:

1. Density
2. Viscosity
3. Bulk modulus of elasticity
4. Pressure
5. Specific gravity

Pressure in a Fluid

The pressure P is defined as the magnitude of the normal force acting on a unit surface area.

$$P = \frac{\Delta F}{\Delta A}$$ ΔF = normal force on a surface area ΔA.



The pressure is a scalar quantity. This is because hydrostatic pressure is transmitted equally in all directions when force is applied, which shows that there is no definite direction associated with pressure.

Thrust

The total force exerted by a liquid on any surface in contact with it is called thrust of the liquid.

Consequences of Pressure

1. Railway tracks are laid on large-sized wooden or iron sleepers. This is because the weight (force) of the train is spread over a large area of the sleeper. This reduces the pressure acting on the ground and hence prevents the yielding of ground under the weight of the train.
2. A sharp knife is more effective in cutting the objects than a blunt knife.
   The pressure exerted = force / area. The sharp knife transmits force over a small area as compared to a blunt knife. Hence, the pressure exerted in case of sharp knife is more than the pressure exerted by a blunt knife.
3. A camel walks easily on sand but a man cannot in spite of the fact that a camel is much heavier than man.

This is because the area of camel's feet is large as compared to man's feet. So the pressure exerted by camel on the sand is very small as compared to the pressure exerted by man. Due to large pressure, sand under the feet of man yields and hence he cannot walk easily on sand.

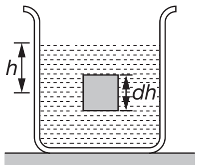
Variation of Pressure with Height

Assumptions:

1. Unaccelerated liquid
2. Uniform density of liquid
3. Uniform gravity

Weight of the small element dh is balanced by the excess pressure. It means $\frac{dp}{dh} = \rho g$.

$$\int_{P_a}^{P} dp = \rho g \int_0^h dh$$



$\Rightarrow$ $$P = P_a + \rho gh$$

PASCAL'S LAW

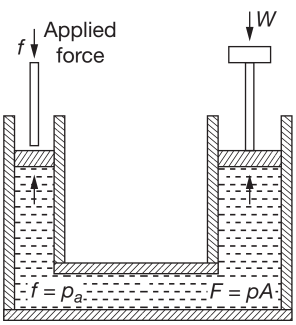
If the pressure in a liquid is changed at a particular point, the change is transmitted to the entire liquid without being diminished in magnitude. In the above case, if P_a is increased by some amount than P must increase to maintained the difference $(P - P_a) = h\rho g$. This is Pascal's Law, which states that hydraulic lift is a common application of Pascal's Law.

Hydraulic Press

$$p = \frac{f}{a} = \frac{W}{A} \text{ or } f = \frac{W}{A} \times a$$

as $A >> a$

then $f << W$.



This can be used to lift a heavy load placed on the platform of larger piston or to press the things placed between the piston and the heavy platform. The work done by applied force is equal to change in potential energy of the weight in hydraulic press.

SOLVED EXAMPLE

30. The areas of cross-section of the two arms of a hydraulic press are 1 cm^2 and 10 cm^2, respectively, (Fig. 8.22). A force of 5 N is applied on the water in the thinner arm. What force should be applied on the water in the thicker arms so that the water may remain in equilibrium?

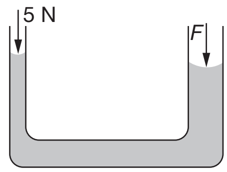


Fig. 8.22

Solution:

In equilibrium, the pressures at the two surfaces should be equal as they lie in the same horizontal level. If the atmospheric pressure is P and a force F is applied to maintain the equilibrium, the pressures are

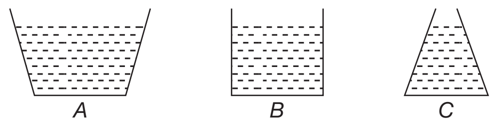
$$P_0 + \frac{5\text{ N}}{1\text{ cm}^2} \quad \text{and} \quad P_0 + \frac{F}{10\text{ cm}^2} \quad \text{respectively.}$$

This gives $F = 50$ N.

Hydraulic Brake

Hydraulic brake system is used in automobiles to retard the motion.

Hydrostatic Paradox



Pressure is directly proportional to depth and by applying Pascal's law, it can be seen that pressure is independent of the size and shape of the containing vessel. (In all the three cases, the heights are same.)

$$P_A = P_B = P_C$$

ATMOSPHERIC PRESSURE

Definition

The atmospheric pressure at any point is numerically equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere.

At 0°C, density of mercury = 13.595 g cm^{-3}, and at sea level, g = 980.66 cm/s^{-2}

Now $P = h\rho g.$

Atmospheric pressure = $76 \times 13.595 \times 980.66$ dyne cm^{-2} = 1.013×10^{-5} N/m^2 (p_a).

Height of Atmosphere

The standard atmospheric pressure is $1.013\times10^5 P_a$ (N/m^{-2}). If the atmosphere of earth has a uniform density ρ = 1.30 kg m^{-3}, then the height h of the air column which exerts the standard atmospheric pressure is given by

$$\Rightarrow \quad h\rho g = 1.013 \times 10^5$$

$$h = \frac{1.013\times10^5}{\rho g} = \frac{1.013\times10^5}{1.13\times9.8}\text{ m}$$

$$= 7.95 \times 10^3 \text{ m} \sim 8 \text{ km}.$$

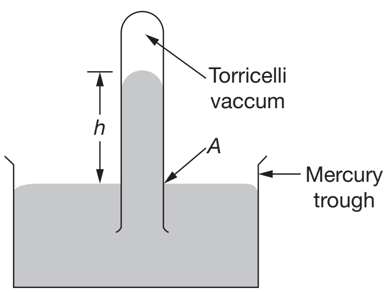
In fact, density of air is not constant but decreases with height. The density becomes half at about 6 km high, $\frac{1}{4}th$ at about 12 km, and so on. Therefore, we cannot draw a clear cut line above where there is no atmosphere. However, the atmosphere extends up to 1200 km. This limit is set for all practical purposes.

Measurement of Atmospheric Pressure

Mercury Barometer

To measure the atmospheric pressure experimentally, Torricelli invented a mercury barometer in 1643.

$$P_a = h\rho g$$



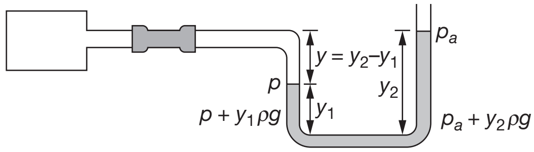
The pressure exerted by a mercury column of 1 mm high is called 1 Torr.

1 Torr = 1 mm of mercury column

Open Tube Manometer

Open tube manometer is used to measure the pressure gauge.

When equilibrium is reached, the pressure at the bottom of left limb is equal to the pressure at the bottom of right limb.



i.e., $$p + y_1 \rho g = p_a + y_2 \rho g$$

$$p - p_a = \rho g (y_2 - y_1) = \rho g y$$

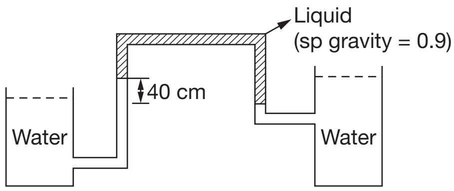
$$p - p_a = \rho g (y_2 - y_1) = \rho g y$$

p = absolute pressure, $p - p_a$ = gauge pressure.

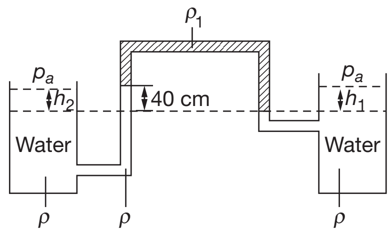
Thus, from y and ρ (density of liquid), we can measure the gauge pressure.

SOLVED EXAMPLE

31. The manometer shown below is used to measure the difference in water level between the two tanks. Calculate this difference for the conditions indicated.



Solution:



$$p_a + h_1 \rho g - 40\rho_1 g + 40\rho g = p_a + h_2 \rho g$$

$$h_2 \rho g - h_1 \rho g = 40 \rho g - 40 \rho_1 g$$

as $$\rho\rho_1 = 0.9\rho$$

$$(h_2 - h_1) \rho\rho g = 40\rho g - 36\rho g$$

$$h_2 - h_1 = 4 \text{ cm}.$$

Water Barometer

Suppose water is used in the barometer instead of mercury.

$$h\rho g = 1.013 \times 10^5$$

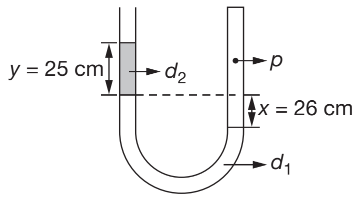
or $$h = \frac{1.013\times10^5}{\rho g}$$

The height of the water column in the tube will be 10.3 m. Such a long tube cannot be managed easily, thus water barometer is not feasible.

SOLVED EXAMPLES

32. In a given U-tube (open at one-end), find out relation between p and p_a.

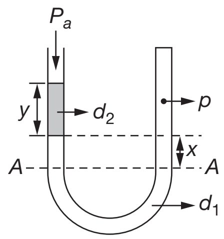
Given $d_2 = 2 \times 13.6$ gm/cm^3 $d_1 = 13.6$ gm/cm^3



Solution:

Pressure in a liquid at same level is same, i.e., at $A - A$–,

$$p_a + d_2 yg + xd_1 g = p$$



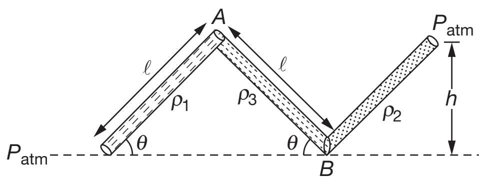
In CGS,

$$p_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = p$$

$$p_a + 13.6 \times g\,[50 + 26] = p$$

$$2p_a = p. \qquad [p_a = 13.6 \times g \times 76]$$

33. Find out pressure at points A and B. Also find angle 'θ'.



Solution:

Pressure at A

$$P_A = P_{\text{atm}} - \rho_1 g\ell \sin\theta$$

Pressure at B

$$P_B = P_{\text{atm}} + \rho_2 gh\ \theta$$

But P_B is also equal to

$$P_B = P_A + \rho_3 g\ell \sin\theta$$

Hence, $P_{\text{atm}} + \rho_2 gh = P_A + \rho_3 g\ell \sin\theta$

$$P_{\text{atm}} + \rho_2 gh = P_{\text{atm}} - \rho_1 g\ell \sin\theta + \rho_3 g\ell \sin\theta$$

$$\sin\theta = \frac{\rho_2 h}{(\rho_3 - \rho_1)\ell}.$$

34. In Fig. 8.23, the container slides down with acceleration a on an incline of angle θ. Liquid is stationary with respect to container. Find out

(A) angle made by surface of liquid with horizontal plane.

(B) angle if $a = g \sin\theta$.

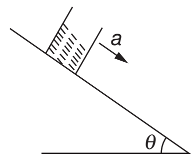
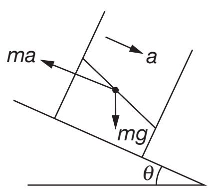
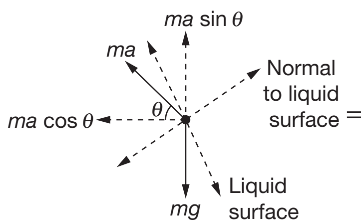


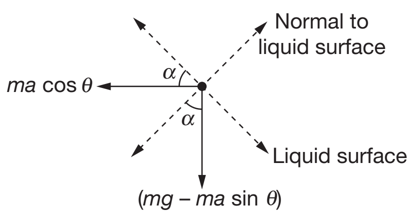
Fig. 8.23

Solution:

Consider a fluid particle on surface. The forces acting on it are shown below.







Resultant force acting on liquid surface will always be normal to it

$$\tan\alpha = \frac{ma\cos\theta}{mg - ma\sin\theta} = \frac{a\cos\theta}{(g - a\sin\theta)}$$

Thus, angle of liquid surface with the horizontal is equal to

$$\alpha = \tan^{-1}\frac{a\cos\theta}{(g - a\sin\theta)}.$$

(B) If $a = g \sin \theta$, then

$$\alpha = \tan^{-1}\left(\frac{a\cos\theta}{g - g\sin^2\theta}\right) = \tan^{-1}\frac{g\sin\theta\cos\theta}{g\cos^2\theta}$$

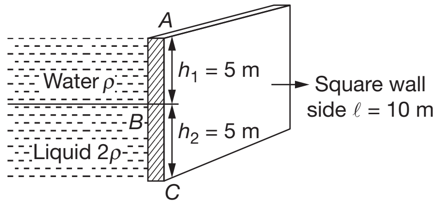
$$= \tan^{-1}(\tan\theta)\ \alpha = \theta.$$

35. Water and liquid are filled up behind a square wall of side ℓ. Find out

(A) pressures at A, B and C

(B) forces in part AB and BC

(C) total force and point of application of force (Neglect atmosphere pressure in every calculation.)



Solution:

(A) As there is no liquid above A,

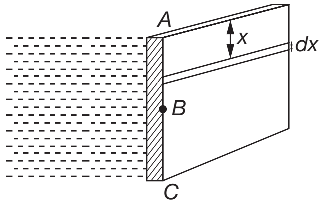
So pressure at A, $p_A = 0$

Pressure at B, $p_B = \rho g h_1$

Pressure at C, $p_C = \rho g h_1 + 2\rho g h_2$.

(B) Force at $A = 0$

Take a strip of width dx at a depth x in part AB.



Pressure is equal to $\rho g x$.

Force on strip = pressure × area

$$dF = \rho g x\ \ell dx$$

Total force up to B

$$F = \int_0^{h_1} \rho g x \ell dx \quad = \frac{\rho g x \ell h_1^2}{2}$$

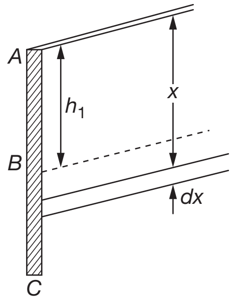
$$= \frac{1000\times10\times10\times5\times5}{2} = 1.25\times10^6\ \text{N}$$

In part BC for force, take an elementary strip of width dx in portion BC. Pressure is equal to

$$= \rho g h_1 + 2\rho g(x - h_1)$$

Force on elementary strip = pressure × area

$$dF = [\rho g h_1 + 2\rho g(x - h_1)]\ \ell\ dx.$$



Total force on part BC

$$F = \int_{h_1}^{\ell}[\rho g h_1 + 2\rho g(x - h_1)]\ell\, dx$$

$$= \left[\rho g h_1 x + 2\rho g\left[\frac{x^2}{2} - h_1 x\right]\right]_{h_1}^{\ell} \ell$$

$$= \rho g h_1 h_2\ \ell + 2\rho g\ell\left[\frac{\ell^2 - h_1^2}{2} - h_1\ell + h_1^2\right]$$

$$= \rho g h_1 h_2 \ell + \frac{2\rho g\ell}{2}\ [\ell^2 + h_1^{\,2} - 2h_1\ell]$$

$$= \rho g h_1 h_2 \ell + \rho g\ell\ (\ell - h_1)^2$$

$$= \rho g h_2 \ell\ [h_1 + h_2] = \rho g h_2 \ell^2$$

$$= 1000\times10\times5\times10\times10 = 5\times10^6\ \text{N}.$$

(C) Total force $= 5\times10^6 + 1.25\times10^6$

$$= 6.25\times10^6\ \text{N}$$

Taking torque about A

Total torque of force in $AB = \int dF\cdot x\ = \int_0^{h_1}\rho g x\ell dx.x$

$$= \left[\frac{\rho g\ell x^3}{3}\right]_0^{h_1} = \frac{\rho g\ell h_1^3}{3} = \frac{1000\times10\times10\times125}{3}$$

$$= \frac{1.25\times10^7}{3}\ \text{N/m}$$

Total torque of force in $BC = \int dF\cdot x$

On solving we get

$$= \rho g h_1 h_2 \ell[h_1 + \frac{h_2}{2}\ ] + \rho g h_2^{\,2}\ell[h_1 + \frac{2h_2}{3}\ ]$$

$$= 1000\times10\times5\times5\times10\ [5 + 2.5]$$

$$+ 1000\times10\times25\times10\ [5 + \frac{10}{3}]$$

$$= 2.5\times7.5\times10^6 + \frac{62.5}{3}\times10^6 = \frac{118.75}{3}\times10^6$$

$$\text{Total torque} = \frac{11.875\times10^7}{3} + \frac{1.25\times10^7}{3}$$

$$= \frac{13.125\times10^7}{3}$$

Total torque = total force × distance of point of application of force from top

$$= F \times x_p$$

$$6.25 \times 10^6 \, x_p = \frac{13.125 \times 10^7}{3}$$

$$x_p = 7 \text{ m.}$$

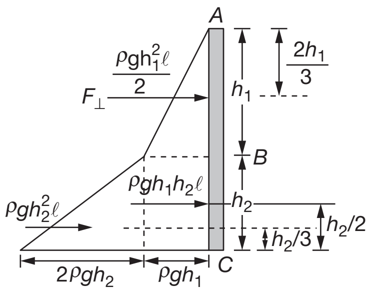
Alternatively,

We can solve this problem by pressure diagram also.

Force on AB part is area of triangle ABC

$$F_{AB} = \rho g h_1 \times \frac{h_1}{2} \times \ell = \frac{\rho g h_1^2 \ell^2}{2}$$

Torque of force of AB part about A :



$$\tau_{AB} = \frac{\rho g h_1^2 \ell}{2} \times \frac{2h_1}{3}$$

$$= \frac{\rho g h_1^3 \ell}{3} = \frac{\rho g \ell^4}{24}$$

Force on BC part is area of trapezium,

$$F_{BC} = \rho g h_1 h_2 \ell + 2\rho g h_2 \times \frac{h_2}{2} \ell$$

$$= \rho g h_1 h_2 \ell + \rho g h_2{}^2 \ell$$

Torque of force of BC part about A,

$$\tau_{BC} = \rho g h_1 h_2 \ell \left(h_1 + \frac{h_2}{2}\right) + \rho g h_2{}^2 \ell \left(h_1 + \frac{2h_2}{3}\right)$$

$$= \frac{\rho g \ell^3}{4}\left[\frac{\ell}{2} + \frac{\ell}{4}\right] + \rho g \frac{\ell^3}{4}\left[\frac{\ell}{2} + \frac{\ell}{3}\right]$$

$$= \frac{\rho g \ell^3}{4}\left[\ell + \frac{\ell}{3} + \frac{\ell}{4}\right] = 19\frac{\rho g \ell^4}{48}$$

$$\text{Total force} = \frac{\rho g h_1^2 \ell}{2} + \rho g h_1 h_2 \ell + \rho g h_2{}^2 \ell$$

$$= \frac{\rho g \ell^3}{8} + \rho g \frac{\ell^3}{4} + \frac{\rho g \ell^3}{4}\left[1 + 1 + \frac{1}{2}\right]$$

$$= \frac{5\rho g \ell^3}{8}$$

$$\text{Total torque} = \frac{19\rho g \ell^4}{48} + \frac{\rho g \ell^4}{24} = \frac{21\rho g \ell^4}{48}$$

But $$F\,x_p = \frac{21\rho g \ell^4}{48}$$

$$\frac{5\rho g \ell^3}{8} \times p = \frac{21\rho g \ell^4}{48}$$

$$x_p = \frac{21\ell}{30} = \frac{21 \times 10}{30} = 7 \text{ m}$$

Thus, total force is acting at 7 m below point A.

ARCHIMEDES' PRINCIPLE

According to this principle, when a body is immersed wholly or partially in a fluid, it loses its weight which is equal to the weight of the fluid displaced by the body.

$$\text{Up thrust} = \text{buoyancy} = V\rho\ell g$$

V = volume submerged

$\rho\ell$ = density of liquid

Relation between Density of Solid and Liquid

Weight of the floating solid = weight of the liquid displaced

$$V_1\,\rho_1\,g = V_2\,\rho_2\,g \;\Rightarrow\; \frac{\rho_1}{\rho_2} = \frac{V_2}{V_1}$$

or $$\frac{\text{Density of solid}}{\text{Density of liquid}}$$

$$= \frac{\text{Volume of the immeresed portion of the solid}}{\text{Total volume of the solid}}$$

This relationship is valid in accelerating fluid also. Thus, the force acting on the body are as follows:

1. Its weight mg which acts downward.
2. Net upward thrust on the body or the buoyant force (mg).

Hence, the apparent weight of the body = $Mg - mg$ = weight of the body – weight of the displaced liquid.

Or actual weight of body – Apparent weight of body = weight of the liquid displaced.

The point through which the upward thrust or the buoyant force acts when the body is immersed in the liquid is called its centre of buoyancy. This will coincide with the centre of gravity if the solid body is homogeneous. On the other hand, if the body is not homogeneous, then the centre of gravity may not lie on the line of the upward thrust and hence there may be a torque that causes rotation in the body.

If the centre of gravity of the body and the centre of buoyancy lie on the same straight line, the body is in equilibrium.

If the centre of gravity of the body does not coincide with the centre of buoyancy (i.e., the line of up thrust), then torque acts on the body. This torque causes the rotational motion of the body.

SOLVED EXAMPLES

36. A copper piece of mass 10 g is suspended by a vertical spring. The spring elongates 1cm over its natural length to keep the piece in equilibrium. A beaker containing water is now placed below the piece so as to immerse the piece completely in water. Find the elongation of the spring. Density of copper = 9000 kg/m^3. Take g = 10 m/s^2.

Solution:
Let the spring constant be k. When the piece is hanging in air, the equilibrium condition gives

$$k\,(1\text{ cm}) = (0.01\text{ kg})\,(10\text{ m/s})$$

or $$k\,(1\text{ cm}) = 0.1\text{ N}. \quad (1)$$

The volume of the copper piece

$$= \frac{0.01\text{ kg}}{9000\text{ kg/m}^3} = \frac{1}{9}\times 10^{-5}\text{ m}^3.$$

This is also the volume of water displaced when the piece is immersed in water. The force of buoyancy

= weight of the liquid displaced

$$= \frac{1}{9}\times 10^{-5}\text{ m}^3\times(1000\text{ kg/m}^3)\times(10\text{ m/s}^2)$$

= 0.011N.

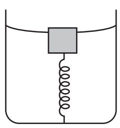
If the elongation of the spring is x when the piece is immersed in water, the equilibrium condition of the piece gives,

$$kx = 0.1\text{ N} - 0.011\text{ N} = 0.089\text{ N}. \quad (2)$$

By (1) and (2),

$$x = \frac{0.089}{0.1}\text{ cm} = 0.89\text{ cm}.$$

37. A cubical block of wood of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of wood = 800 kg/m^3 and spring constant of the spring = 50 N/m. Take g = 10 m/s^2.



Solution:
The specific gravity of the block = 0.8. Hence, the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside after wetting = 3 cm – 2.4 = 0.6 cm. Suppose, the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water

= volume of the block = 27 × 10^{-6} m^3.

Hence, the force of buoyancy

$$= (27\times 10^{-6}\text{ m}^3)\times 1(1000\text{ kg/m3})\times(10\text{ m/s}^2)$$

= 0.27 N.

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring

= 50 N/m × 0.6 cm = 0.3 N.

The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is

$$W' = (27\times 10^{-6}\text{ m})\times(800\text{ kg/m}^3)\times(10\text{ m/s}^2)$$

= 0.22 N.

Thus, $W = 0.27\text{ N} + 0.3\text{ N} - 0.22\text{ N} = 0.35\text{ N}$.

38. A wooden plank of length 1m and uniform cross-section is hinged at one end to the bottom of a tank as shown in Fig. 8.24. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position.
(Exclude the case $\theta = 0$).

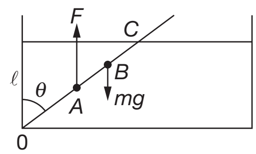


Fig. 8.24

Solution:
The forces acting on the plank are shown in Fig. 8.24. The height of water level is ℓ = 0.5 m. The length of the plank is 1.0 m = 2ℓ. The weight of the plank acts through the centre B of the plank. We have $OB = \ell$. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have $$OA = \frac{OC}{2} = \frac{\ell}{2\cos\theta}$$

Let the mass per unit length of the plank be ρ.
Its weight mg = $2\ell\rho g$.

The mass of the part OC of the plank $= \left(\dfrac{\ell}{\cos\theta}\right)\rho.$

The mass of water displaced $= \dfrac{1}{0.5}\,\dfrac{\ell}{\cos\theta}\,\rho = \dfrac{2\ell\rho}{\cos\theta}$

The buoyant force F is, therefore, $F = \dfrac{2\ell\rho g}{\cos\theta}$

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, $\quad mg\,(OB)\sin\theta = F(OA)\sin\theta$

or $\quad (2\ell\rho)\ell = \left(\dfrac{2\ell\rho}{\cos\theta}\right)\left(\dfrac{\ell}{2\cos\theta}\right)$

or $\quad \cos^2\theta = \dfrac{1}{2}$

or $\quad \cos\theta = \dfrac{1}{\sqrt{2}}$

or $\quad \theta = 45°.$

39. A cylindrical block of wood of mass m is floating in water with its axis vertical. It is depressed a little and then released. Show that the motion of the block is simple harmonic and find its frequency.

Solution:

Suppose a height h of the block is dipped in the water in equilibrium position. If r be the radius of the cylindrical block, the volume of the water displaced $= \pi r^2 h$. For floating in equilibrium,

$$\pi r^2 h\rho g = W \qquad (1)$$

where ρ is the density of water and W the weight of the block.

Suppose during the vertical motion, the block is further dipped through a distance x at some instant. The volume of the displaced water is $\pi r^2 (h + x)$. The forces acting on the block are the weight W vertically downward and the buoyancy $\pi r^2 (h + x)\rho g$ vertically upward.

Net force on the block at displacement x from the equilibrium position is

$$F = W - \pi r^2 (h + x)\rho g = W - \pi r^2 h\rho g - \pi r^2 \rho x g$$

Using (1)

$$F = -\pi r^2 \rho g x = -kx,$$

where $k = \pi r^2 \rho g$.

Thus, the block executes SHM with frequency.

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\pi r^2 \rho g}{m}}.$$

40. A cylindrical bucket with one end open is observed to be floating on a water ($\rho = 1000$ kg/m^3) with open and down. It is of 10 N weight and is supported by air that is trapped inside it as shown in Fig. 8.25. The bucket floats with a height 10 cm above the liquid surface. If the air trapped is assumed to follow isothermal law, then determine the force F necessary just to submerge the bucket. The internal area of cross-section of bucket is 21 cm^2. The thickness of the wall is assumed to negligible and the atmospheric pressure must be neglected. ($g = 10$ m/s^2)

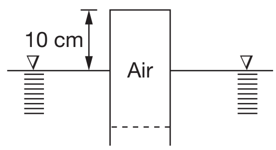
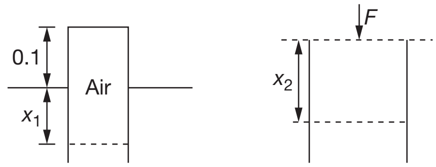


Fig. 8.25

Solution:

Weight of bucket

$$W = Ax_1\rho g \qquad (1)$$



pressure at liquid – air interface = pressure of air = $\rho g x_1$

From (1)

$$p_1 = \rho g x_1 = \rho g \frac{W}{A\rho g} = \frac{W}{A}$$

$$v_1 = A[h + x_1] = A\left[h + \frac{W}{A\rho g}\right]$$

Let force F be applied

Downward force = $F + W$ = Buoyant = $Ax_2\,\rho g \qquad (2)$

$$p_2 = x_2\rho g,\; v_2 = Ax_2$$

$$p_1 v_1 = p_2 v_2$$

$$\frac{W}{A} \times A\left[h + \frac{W}{A\rho g}\right] = x_2\,\rho g A\, x_2$$

$$\Rightarrow \qquad x_2 = \sqrt{\frac{W}{A\rho g}\left[h + \frac{W}{A\rho g}\right]}$$

From (2)

$$F + W = A\rho g\sqrt{\frac{W}{A\rho g}\left[h + \frac{W}{A\rho g}\right]}$$

$$\Rightarrow \qquad F + W = \sqrt{WA\rho gh + W^2}$$

$$F = \sqrt{WA\rho gh + W^2} - W$$

substituting values,

$$W = 10 \text{ N}, \rho = 1000 \text{ kg/m}^3, A = 2.1 \times 10^{-3} \text{ m}^2$$

$$F = \sqrt{10 \times 2.1 \times 10^{-3} \times 1000 \times 10 \times 10^{-1} + 100} - 10$$

$$= 11 - 10 = 1 \text{ N}.$$

41. A large block of ice cuboid of height ℓ and density $\rho_{ice} = 0.9\ \rho_w$, has a large vertical hole along its axis. This block is floating in a lake. Find out the length of the rope required to raise a bucket of water through the hole.

Solution:

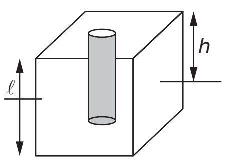
Let area of ice-cuboid excluding hole = A

Weight of ice block = weight of liquid displaced

$$A\,\rho_{ice}\,\ell\,g = A\,\rho_w\,(\ell - h)\,g$$

$$\frac{9\ell}{10} = \ell - h$$

$$\Rightarrow \qquad h = \ell - \frac{9\ell}{10} = \left(\frac{\ell}{10}\right).$$



PRESSURE IN CASE OF ACCELERATING FLUID

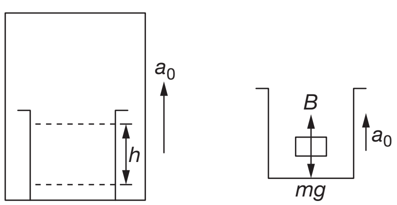
Liquid Placed in Elevator

When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by,

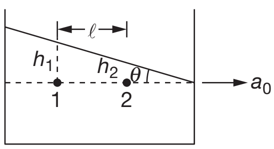
$$p = h\rho\,[g + a_0]$$

and force of buoyancy,

$$B = m\,(g + a_0)$$



Free Surface of Liquid in Horizontal Acceleration



$$\tan\theta = \frac{a_0}{g}$$

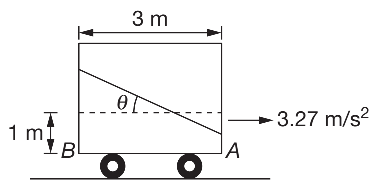
$p_1 - p_2 = \ell\rho\,a_0$, where p_1 and p_2 are pressures at points 1 and 2. Then $h_1 - h_2 = \dfrac{\ell a_0}{g}$.

SOLVED EXAMPLE

42. An open rectangular tank 1.5 m wide 2 m deep and 2 m long is half filled with water. It is accelerated horizontally at 3.27 m/s^2 in the direction of its length. Determine the depth of water at each end of tank. [$g = 9.81$ m/s^2]

Solution:

$$\tan\theta = \frac{a}{g} = \frac{1}{3}$$



Depth at corner A

$$= 1 - 1.5\tan\theta$$

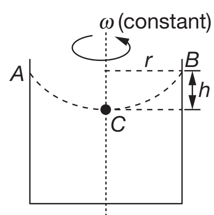
$$= 0.5 \text{ m}$$

Depth at corner B

$$= 1 + 1.5\tan\theta = 1.5 \text{ m}.$$

Free Surface of Liquid in Case of Rotating Cylinder

$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$



STREAMLINE FLOW

The path taken by a particle in flowing fluid is called its line of flow. In the case of steady flow, all the particles passing through a given point follow the same path and hence we have a unique line of flow passing through a given point, which is also called streamline.

Characteristics of Streamline

1. A tangent at any point on the streamline gives the direction of the velocity of the fluid particle at that point.
2. Two streamlines never intersect each other.
   Laminar flow: If the liquid flows over a horizontal surface in the form of layers of different velocities, then the flow of liquid is called laminar flow. The particles of one layer do not go to another layer. In general, laminar flow is a streamline flow.
   Turbulent flow: The flow of fluid in which velocity of all particles crossing a given point is not same, and the motion of the fluid becomes disorderly or irregular is called turbulent flow.

Reynold's Number

According to Reynold, the critical velocity (v_c) of a liquid flowing through a long narrow tube is

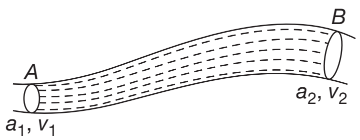
1. directly proportional to the coefficient of viscosity (η) of the liquid.
2. inversely proportional to the density (ρ) of the liquid.
3. inversely proportional to the diameter (D) of the tube.

That is $v_c \propto \dfrac{\eta}{\rho D}$ or $v_c = \dfrac{R\eta}{\rho D}$ or $= \dfrac{v_c \rho D}{\eta}$ (1)

where R is the Reynold number.
If $R < 2000$, the flow of liquid is streamline or laminar. If $R > 3000$, the flow is turbulent. If R lies between 2000 and 3000, the flow is unstable and may change from streamline flow to turbulent flow.

Equation of Continuity

The equation of continuity expresses the law of conservation of mass in fluid dynamics.



$$a_1 v_1 = a_2 v_2$$

In general, av = constant. This is called equation of continuity and states that as the area of cross-section of the tube of flow becomes larger, the liquid's (fluid) speed becomes smaller and vice-versa.

Illustrations

1. Velocity of liquid is greater in the narrow tube as compared to the velocity of the liquid in a broader tube.
2. Deep waters run slow. This can be explained from the equation of continuity, i.e., av = constant. When water is deep, the area of cross-section increases hence velocity decreases.

Energy of a Liquid

A liquid can possess three types of energies.

Kinetic Energy

The energy possessed by a liquid due to its motion is called kinetic energy. The kinetic energy of a liquid of mass m moving with speed v is $\frac{1}{2}mv^2$.

$$\therefore \quad \text{KE per unit mass} = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2.$$

Potential Energy

The potential energy of a liquid of mass m at a height h is mgh.

$$\therefore \quad \text{PE per unit mass} = \frac{mgh}{m} = gh$$

Pressure Energy

The energy possessed by a liquid by virtue of its pressure is called pressure energy.

Consider a vessel fitted with piston at one side (Fig. 8.26). Let this vessel be filled with a liquid. Let A be the area of cross-section of the piston and P be the pressure experienced by the liquid.

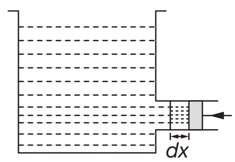


Fig. 8.26

The force acting on the piston = PA.

If dx is the distance moved by the piston, then work done by the force = $PA\,dx = PdV$,
where $dV = Adx$, volume of the liquid swept.

This work done is equal to the pressure energy of the liquid.

$\therefore$ Pressure energy of liquid in volume $dV = PdV$.

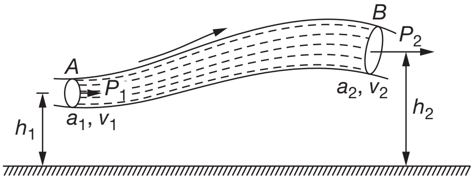
The mass of the liquid having volume $dV = \rho dV$,
ρ is the density of the liquid.

$\therefore$ Pressure energy per unit mass of the liquid $= \dfrac{PdV}{\rho dV} = \dfrac{P}{\rho}$.

BERNOULLI'S THEOREM

It states that the sum of pressure energy, kinetic energy, and potential energy per unit mass or per unit volume or per unit weight is always constant for an ideal (i.e., incompressible and non-viscous) fluid having streamline flow.

i.e., $$\frac{P}{\rho}+\frac{1}{2}v^2+gh=\text{constant.}$$



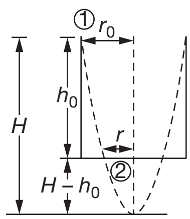
SOLVED EXAMPLES

43. A circular cylinder of height $h_0 = 10$ cm and radius $r_0 = 2$ cm is opened at the top and filled with liquid. It is rotated about its vertical axis. Determine the speed of rotation so that half the area of the bottom gets exposed ($g = 10$ m/s^2).

Solution:

Area of bottom $= \pi r_0^2$

If r is radius of the exposed bottom, then



$$\pi r^2 = \frac{1}{2}\pi r_0^2 \qquad r = \frac{r_0}{\sqrt{2}}$$

Applying Bernoulli's equation between points (1) and (2),

$$P_{atm}+\frac{1}{2}\rho v_1^2-\rho gH = P_{atm}+\frac{1}{2}\rho v_2^2-\rho g(H-h_0)$$

$$-\rho g h_0 = \frac{1}{2}\rho(v_2^2-v_1^2)$$

$$\Rightarrow \quad 2gh_0 = [v_1^2-v_2^2] = [w^2r_0^2-w^2r^2]$$

$$r_0 = 2\times10^{-2}\text{ m} \quad \Rightarrow \quad 2gh_0 = w^2[r_0^2-r^2]$$

$$w=\frac{2}{r_0}\sqrt{gh} = \frac{2}{2\times10^{-2}}\sqrt{10\times0.1} = 100 \text{ rad/s.}$$

44. Water flows in a horizontal tube as shown in Fig. 8.27. The pressure of water changes by 600 N/m^2 between A and B, where the areas of cross-section are 30 cm^2 and 15 cm^2, respectively. Find the rate of flow of water through the tube.

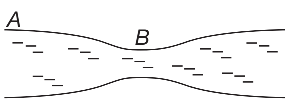


Fig. 8.27

Solution:

Let the velocity at $A = v_A$ and that at $B = v_B$.

By the equation of continuity, $\frac{v_B}{v_A} = \frac{30\text{ cm}^2}{15\text{ cm}^2} = 2.$

By Bernoulli's equation,

$$P_A+\frac{1}{2}\rho v_A^2 = P_B+\frac{1}{2}\rho v_B^2$$

or $$P_A-P_B = \frac{1}{2}\rho(2v_A)^2-\frac{1}{2}\rho v_A^2 = \frac{3}{2}\rho v_A^2$$

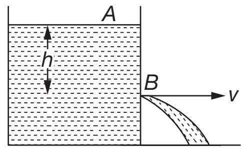
or $$600\,\frac{\text{N}}{\text{m}^2} = \frac{3}{2}\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)v_A^2$$

or $$v_A = \sqrt{0.4\text{ m}^2/\text{s}^2} = 0.63\text{ m/s.}$$

The rate of flow = (30 cm^2) (0.63 m/s) = 1800 cm^3/s.

Applications of Bernoulli's Theorem

1. Bunsen burner
2. Lift of an airfoil
3. Spinning of a ball (Magnus effect)
4. The sprayer
5. A ping pong ball in an air jet
6. Torricelli's theorem (speed of efflux)



At point A, $P_1 = P$, $v_1 = 0$ and $h_1 = h$

At point B, $P_2 = P$, $v_2 = v$ (speed of efflux) and $h = 0$

Using Bernoulli's theorem

$$\frac{P_1}{\rho}+gh_1+\frac{1}{2}v_1^2 = \frac{P_2}{\rho}+gh_2=\frac{1}{2}v_2^2,$$

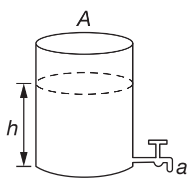
we have

$$\frac{P}{\rho}+gh+0 = \frac{P}{\rho}+0+\frac{1}{2}v^2$$

$$\Rightarrow \quad \frac{1}{2}v^2 = gh \quad \text{or} \quad v=\sqrt{2gh}$$

SOLVED EXAMPLES

45. A cylindrical container of cross-section area, A, is filled up with water up to height h. Water may exit through a tap of cross-section area a in the bottom of container. Find out:



(A) Velocity of water just after opening of tap.

(B) The area of cross-section of water stream coming out of tape at depth h_0 below tap in terms of a just after opening of tap.

(C) Time in which container becomes empty. (Given, $\left(\frac{a}{A}\right)^{1/2} = 0.02$, $h = 20$ cm, $h_0 = 20$ cm)

Solution:

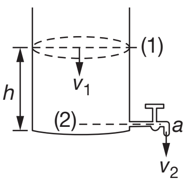
(A) Applying Bernoulli's equation between (1) and (2),

$$P_a + \rho g h + \frac{1}{2}\rho v_1^2 = P_a + \frac{1}{2}\rho v_2^2$$

Through continuity equation,

$$Av_1 = av_2,\ v_1 = \frac{av_2}{a}$$

$$\rho g h + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$$



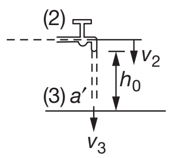
On solving,

$$v_2 = \sqrt{\frac{2gh}{1-\frac{a^2}{A^2}}} = 2 \text{ m/s.} \quad (1)$$

(B) Applying Bernoulli's equation between (2) and (3),

$$\frac{1}{2}\rho v_2^2 + \rho g h_0 = \frac{1}{2}\rho v_3^2$$

Through continuity equation,



$$av_2 = a' v_3$$

$$\Rightarrow \quad v_3 = \frac{av_2}{a'}$$

$$\Rightarrow \quad \frac{1}{2}\rho v_2^2 + \rho g h_0 = \frac{1}{2}\rho\left(\frac{av_2}{a'}\right)^2$$

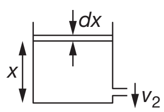
$$\frac{1}{2}\times 2\times 2 + gh_0 = \frac{1}{2}\left(\frac{a}{a'}\right)^2 \times 2\times 2$$

$$\left(\frac{a}{a'}\right)^2 = 1 + \frac{9.8\times .20}{2}$$

$$\Rightarrow \quad \left(\frac{a}{a'}\right)^2 = 1.98$$

$$\Rightarrow \quad a' = \frac{a}{\sqrt{1.98}}.$$

(C) From (1) at any height h of liquid level in container, the velocity through tap,



$$v = \sqrt{\frac{2gh}{0.98}} = \sqrt{20h}$$

we know, volume of liquid coming out of tap = decrease in volume of liquid in container.

For any small time interval dt,

$$av_2 dt = -A \cdot dx$$

$$a\sqrt{20x}\, dt = -A\, dx$$

$$\Rightarrow \quad \int_0^t dt = -\frac{A}{a}\int_h^0 \frac{dx}{\sqrt{20x}}$$

$$t = \frac{A}{a\sqrt{20}}\left[2\sqrt{x}\right]_h^0$$

$$\Rightarrow \quad t = \frac{A}{a\sqrt{20}}\, 2\sqrt{h} = \frac{A}{a}\times 2\times\sqrt{\frac{h}{20}}$$

$$= \frac{2A}{a}\sqrt{\frac{0.20}{20}} = \frac{2A}{a}\times 0.1$$

Given $\left(\frac{a}{A}\right)^{1/2} = 0.02$

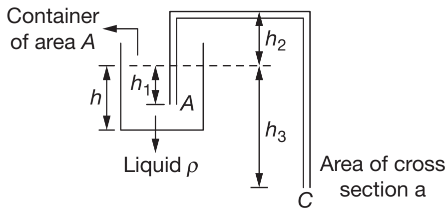
or $\frac{A}{a} = \frac{1}{0.0004} = 2500$

Thus, $t = 2\times 2500\times 0.1 = 500$ s.

46. In a given arrangement,

(A) Find out velocity of water coming out of C.

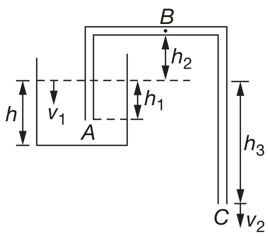
(B) Find out pressure at A, B, and C.



Solution:

(A) Applying Bernoulli's equation between liquid surface and point C.

$$p_a + \frac{1}{2}\rho v_1^2 = p_a - \rho g h_3 + \frac{1}{2}\rho v_2^2$$



Through continuity equation,

$$Av_1 = av_2,\ v_1 = \frac{av_2}{A}$$

$$\Rightarrow \quad \frac{1}{2}\rho\frac{a^2}{A^2}v_2^2 = -\rho g h_3 + \frac{1}{2}\rho v_2^2$$

$$v_2^2 = \frac{2gh_3}{1-\frac{a^2}{A^2}}, \quad v_2 = \sqrt{\frac{2gh_3}{1-\frac{a^2}{A^2}}}.$$

(B) Pressure at A just outside the tube, $p_A = p_{\text{atm}} + \rho g h_1$

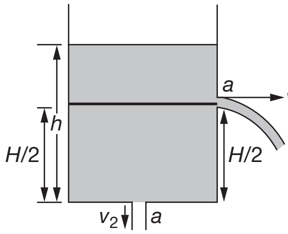
For pressure at B,

$$P_A + 0 + 0 = p_B + \rho g h_2 + \frac{1}{2}\rho v_B^2$$

$$P_B = P_A - \rho g h_2 - \frac{1}{2}\rho\left(\frac{2gh_3}{1-\frac{a^2}{A^2}}\right)$$

Pressure at C, $p_C = p_{atm}$.

47. A fixed container of height H with large cross-sectional area A is completely filled with water. Two small orifice of cross-sectional area a are made, one at the bottom and the other on the vertical side of the container at a distance $H/2$ from the top of the container. Find the time taken by the water level to reach a height of $H/2$ from the bottom of the container.



Solution:

$$v_1 = \sqrt{2g\,(h - H/2)}$$

$$v_2 = \sqrt{2g\,h}$$

$\therefore$ By continuity equation,

$$A\left(-\frac{dh}{dt}\right) = a\,(v_1 + v_2)$$

$$\Rightarrow \quad A\left(-\frac{dh}{dt}\right) = a\left\{\sqrt{2g\,(h-H/2)} + \sqrt{2gh}\right\}$$

$$\text{or} \quad -\frac{A}{a\sqrt{2g}}\int_H^{H/2}\frac{dh}{\sqrt{h}+\sqrt{h-H/2}} = \int_0^t dt$$

$$\Rightarrow \quad t = \frac{2A}{3a}\left(\sqrt{2}-1\right)\sqrt{\frac{H}{g}}.$$

48. An L-shaped glass tube is kept inside a bus that is moving with constant acceleration. During the motion, the level of the liquid in the left arm is at 12 cm, whereas in the right arm, it is at 8 cm when the orientation of the tube is as shown in Fig. 8.28. Assuming that the diameter of the tube is much smaller than levels of the liquid and neglecting effect of surface tension, acceleration of the bus will be ($g = 10$ m/s^2).

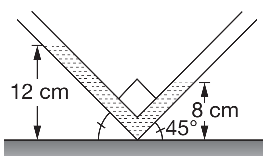


Fig. 8.28

Solution:

$$\tan\theta = \frac{a}{g} = \frac{h_2 - h_1}{h_2\tan 45^\circ + h_1\tan 45^\circ} = \frac{4\text{ cm}}{20\text{ cm}}$$

$$\Rightarrow \quad a = 2\text{ m/s}^2.$$

49. A cube (density 0.5 gm/cc) of side 10 cm is floating in water kept in a cylindrical beaker of base area 1500 cm^2. When a mass m is kept on wooden block, the level of water rises in the beaker by 2 mm. Find the mass m.

Solution:

Let the cube dips further by $y$cm and water level rises by 2 mm.

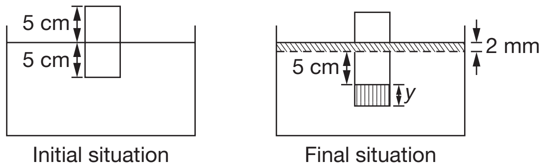


Fig. 8.29

Then equating the volumes (/// volume = \\\ volume in Fig. 8.29)

$\Rightarrow$ volume of water raised = volume of extra depth of wood

$$\Rightarrow \quad 100\,y = (1500 - 100)\,\frac{2}{10} = 1400 \times \frac{2}{10} = 280$$

$$\therefore \quad y = 2.8\text{ cm}$$

$\therefore$ Extra upthrust

$$\rho_{water} \times (2.8 + 0.2) \times 100\text{ g} = mg$$

$$\Rightarrow \quad m = 300\text{ gm}.$$

50. An open water tanker moving on a horizontal straight road has a cubical block of cork floating over its surface. If the tanker has an acceleration of a as shown in Fig. 8.30, the acceleration of the cork with respect to container is (ignore viscosity).

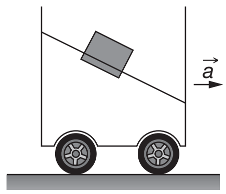
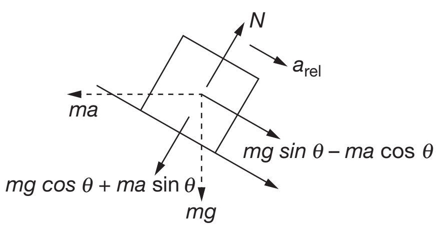


Fig. 8.30

Solution:



$$ma_{rel} = mg\sin\theta - ma\cos\theta$$

But for water surface, $\tan\theta = a/g$

$\Rightarrow$ $$a_{rel} = 0.$$

51. Water (density ρ) is flowing through the uniform tube of cross-sectional area A with a constant speed v as shown in Fig. 8.31. The magnitude of force exerted by the water on the curved corner of the tube is (neglecting viscous forces).

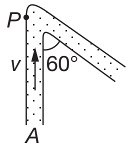
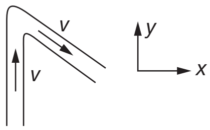


Fig. 8.31

(A) $\sqrt{3}\,\rho A v^2$ (B) $2\rho A v^2$

(C) $\sqrt{2}\rho A v^2$ (D) $\dfrac{\rho A v^2}{\sqrt{2}}$

Solution: (A)



$$|\overrightarrow{\Delta P}_x| = mv\sin 60^\circ = \frac{\sqrt{3}}{2}\,mv$$

$$|\overrightarrow{\Delta P}_y| = \frac{mv}{2} + mv = \frac{3}{2}\,mv$$

$$\Rightarrow \quad |\overrightarrow{\Delta P}_{net}| = \sqrt{\Delta P_x^2 + \Delta P_y^2} = \sqrt{\left(\frac{9}{4}+\frac{3}{4}\right)}\,mv$$

$$\Rightarrow \quad |\overrightarrow{\Delta F}_{net}| = \sqrt{3}\left(\frac{dm}{dt}\right)\cdot v = \sqrt{3}\,\rho A v^2.$$

$$\left(\begin{array}{l}\text{Since, } dm = A(v\,dt)\rho \\ \Rightarrow \dfrac{dm}{dt} = A\rho v\end{array}\right)$$

Venturimeter

It is a gauge put on a flow pipe to measure the flow of speed of a liquid (Fig. 8.32). Let the liquid of density ρ be flowing through a pipe of area of cross-section A_1. Let A_2 be the area of cross-section at the throat and a manometer is attached as shown in Fig. 8.32. Let v_1 and P_1 be the velocity of the flow and pressure at point A, v_2 and P_2 be the corresponding quantities at point B.

Using Bernoulli's Theorem

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2,$$

we get

$$\frac{P_1}{\rho} + gh + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh + \frac{1}{2}v_2^2$$

(Since, $h_1 = h_2 = h$)

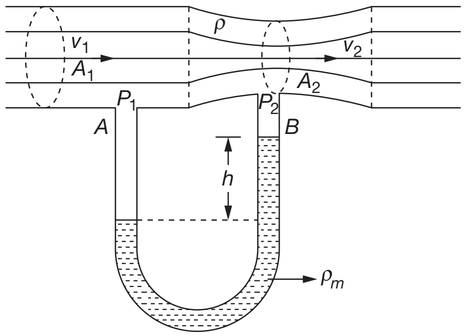


Fig. 8.32

or $$(P_1 - P_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad (1)$$

According to continuity equation, $A_1 v_1 = A_2 v_2$

or $$v_2 = \left(\frac{A_1}{A_2}\right)v_1$$

Substituting the value of v_2 in Equation (1), we have

$$(P_1 - P_2) = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right]\frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

Since $A_1 > A_2$, $P_1 > P_2$

or $$v_1^2 = \frac{2(P_1 - P_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]} = \frac{2A_2^2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}$$

where $(P_1 - P_2) = \rho_m gh$ and h is the difference in heights of the liquid levels in the two tubes.

$$v_1 = \sqrt{\frac{2\rho_m gh}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

The flow rate (R), i.e., the volume of the liquid flowing per second is given by $R = v_1 A_1$.

During Wind Storm

The velocity of air just above the roof is large, so according to Bernoulli's theorem, the pressure just above the roof is less than pressure below the roof. Due to this pressure difference, an upward force acts on the roof which is blown off without damaging other parts of the house.

When a fast moving train crosses a person standing near a railway track, the person has a tendency to fall towards the train. This is because a fast moving train produces large velocity in air between the person and the train and hence pressure decreases according to Bernoulli's theorem. Thus, the excess pressure on the other side pushes the person towards the train.

BRAIN MAP I

FLUID and SOLID

1. **Stress** $= \dfrac{F}{A}$

   Normal stress $= \dfrac{F_n}{A}$

   Tangential stress $= \dfrac{F_t}{A}$

2. **Strain**

   Longitudinal strain $= \dfrac{\Delta L}{L}$

   Volume strain $= \dfrac{\Delta V}{V}$

   Shearing strain $= \dfrac{x}{L}$

3. **Hook's Law:** within elastic limit stress α strain

   (i) Young's modulus (Y)

   $\dfrac{F}{A} = Y\dfrac{l}{L}$

   (ii) Bulk modulus (B)

   $\dfrac{F}{A} = -B\dfrac{\Delta V}{V}$

   (iii) Modulus of rigidity (η)

   $\dfrac{F}{A} = \eta\dfrac{x}{L}$

4. Energy stored in a deformed body per unit volume $= \dfrac{1}{2} \times \text{stress} \times \text{strain}$

Fluid Statics

1. Variation of pressure in a liquid → Pressure varies along the depth of a liquid as ρgh.
2. Force of Buoyancy = Weight of displaced liquid $= V\rho g$
3. Center of Buoyancy is a point from which force of buoyancy acts. It is center of gravity of displaced liquid.

Fluid Dynamics

1. Continuity equation $A_1V_1 = A_2V_2$
2. Bernoullis theorem $P + \rho gh + \dfrac{1}{2}\rho v^2 = \text{constant}$
3. Velocity of efflux

   $$= \sqrt{\frac{2gh}{1-\frac{A_2^2}{A_1^2}}} \approx \sqrt{2gh}$$

BRAIN MAP 2

SURFACE TENSION

and

VISCOSITY

1. $T = \dfrac{F}{L}$

$T = \dfrac{W}{\Delta A}$

Angle of contact:

(i) Acute: Adhesion > cohesion; Meniscus is concave

(ii) Right angle: Adhesion = cohesion; Meniscus is plane

(iii) Obtuse: Adhesion < cohesion; Meniscus is convex

2. Capillarity:

(i) $h = \dfrac{2T\cos\theta}{R\rho g} = \dfrac{2T}{r\rho g}$

(ii) hr = constant

3. Excess pressure:

(i) Inside a drop

$p = \dfrac{2T}{r} = p_i - p_o$

(ii) Inside a bubble

$p = \dfrac{4T}{r} = p_i - p_o$

4. Newton's Law:

$F = -\eta A \dfrac{dv}{dx}$

5. Stokes Law:

(i) $F = 6\pi\eta r v$

(ii) Terminal velocity

$v_T = \dfrac{2}{9} r^2 \dfrac{(\rho - \sigma)}{\eta} g$

EXERCISES

Single Option Correct Type

1. The breaking stress of a wire depends on
   (A) Material of the wire
   (B) Length of the wire
   (C) Radius of the wire
   (D) Length of the wire

2. A wire can be broken by applying a load of 20 kg wt. The force required to break the wire of twice the diameter is
   (A) 20 kg wt (B) 5 kg wt
   (C) 80 kg wt (D) 160 kg wt

3. An elongation of 0.1% in a wire of cross-sectional area 10^{-6} m^2 causes a tension of 100 N. The Young's modulus is
   (A) 10^{12} N/m^2 (B) 10^{11} N/m^2
   (C) 10^{10} N/m^2 (D) 10^2 N/m^2

4. A vessel of 1×10^{-3} m^3 volume contains oil, when a pressure of 1.2×10^5 N/m^2 is applied on it, then volume decreases by 0.3×10^{-6} m^3. The bulk modulus of oil is
   (A) 1×10^6 N/m^2 (B) 2×10^7 N/m^2
   (C) 4×10^8 N/m^2 (D) 6×10^{10} N/m^2

5. A wooden cylinder floats in water with two-third of its volume inside the water. The density of wood is
   (A) $\frac{1000}{3}$ kg/m^3 (B) $\frac{2000}{3}$ kg/m^3
   (C) $\frac{500}{3}$ kg/m^3 (D) 250 kg/m^3

6. Two blocks A and B made of iron and aluminium, respectively, have exactly the same weight. They are completely immersed in water and weighed. If the densities of iron and aluminium are 8000 kg/m^3 and 2700 kg/m^3, then
   (A) A will weigh more than B
   (B) B will weigh more than A
   (C) A and B will weigh the same as before
   (D) Data insufficient

7. A body of mass 0.5 kg is attached to a thread and it just floats in a liquid. The tension in the thread is
   (A) 0.5 kg wt
   (B) More than 0.5 kg wt
   (C) Less than 0.5 kg wt
   (D) Zero

8. Equal volumes of water and alcohol are mixed together. The density of water is 1000 kg/m^3 and the density of alcohol is 800 kg/m^3. The density of the mixture is
   (A) 900 kg/m^3 (B) 1100 kg/m^3
   (C) 875 kg/m^3 (D) 950 kg/m^3

9. A water tank is 20 m deep. If the water barometer reads 10 m, the pressure at the bottom of the tank is
   (A) 2 atmosphere (B) 1 atmosphere
   (C) 3 atmosphere (D) 4 atmosphere

10. The three vessels shown in Fig. 8.33(A–C) have exactly the same base area. Equal volumes of a liquid are poured in the three vessels. The force on the base will be

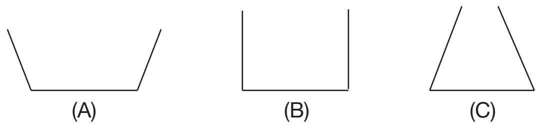
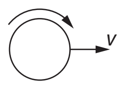
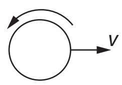
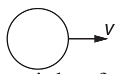


Fig. 8.33

   (A) Maximum in vessel A
   (B) Maximum in vessel B
   (C) Maximum in vessel C
   (D) Equal in all the vessels

11. An incompressible, non-viscous fluid flows steadily through a cylindrical pipe, which has radius $2R$ at point A and radius R at point B farther along the flow direction. If the velocity of flow at point A is v, the velocity of flow at point B will be
   (A) $2v$ (B) v (C) $v/2$ (D) $4v$

12. A barometer kept in an elevator accelerating upward reads 76 cm. The air pressure in the elevator is
   (A) 76 cm (B) < 76 cm
   (C) > 76 cm (D) Zero

13. To get the maximum flight, a ball must be thrown as
   (A)  (B) 
   (C)  (D) Any of (A), (B), and (C)

14. The weight of a body in water is one-third of its weight in air. The density of the body is
   (A) 0.5 gm/cm^3 (B) 1.5 gm/cm^3
   (C) 2.5 gm/cm^3 (D) 3.5 gm/cm^3

15. The volume of a liquid flowing per sec out of an orifice at the bottom of a tank does not depend upon

(A) the height of the liquid above the orifice.
(B) the acceleration due to gravity.
(C) the density of the liquid.
(D) the area of the orifice.

16. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof, and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height $(3L/4)$ from its lower end is

(A) W_1/S (B) $[W_1 + (W/4)]/S$
(C) $[W_1 + (3W/4)]/S$ (D) $[W_1 + W]/S$

17. A wire can sustain the weight of 20 kg before breaking. If the wire is cut into two equal parts, each part can sustain a weight of

(A) 10 kg (B) 20 kg (C) 40 kg (D) 80 kg

18. A solid uniform ball having volume V and density ρ floats at the interface of two immiscible liquids as shown in Fig. 8.34. The densities of the upper and the lower liquids are ρ_1 and ρ_2, respectively, such that $\rho_1 < \rho < \rho_2$. The fraction of the volume of the ball in the lower liquid is

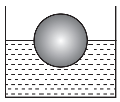


Fig. 8.34

(A) $\dfrac{\rho - \rho_2}{\rho_1 - \rho_2}$ (B) $\dfrac{\rho_1}{\rho_1 - \rho_2}$
(C) $\dfrac{\rho_1 - \rho}{\rho_1 - \rho_2}$ (D) $\dfrac{\rho_1 - \rho_2}{\rho_2}$

19. The deformation of a wire under its own weight compared to the deformation of same wire subjected to a load equal to the weight of the wire is

(A) Same (B) One-third
(C) Half (D) One-fourth

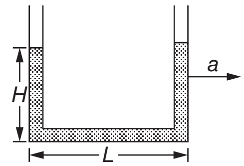
20. The weight of a body in air is 100 N. Its weight in water, if it displaces 400 cc of water is

(A) 90 N (B) 94 N
(C) 98 N (D) None of these

21. The specific gravity of ice is 0.9. The area of the smallest slab of ice of height 0.5 m floating in fresh water that will just support a 100 kg man is

(A) 1.5 m^2 (B) 2 m^2 (C) 3 m^2 (D) 4 m^2

22. A liquid stands at the same level in the U-tube when at rest. If area of cross-section of both the limbs are equal, the difference in heights h of the liquid in the two limbs of U-tube, when the system is given an acceleration a in horizontal direction as shown, is



(A) $\dfrac{gL^2}{aH}$ (B) $\dfrac{La}{g}$ (C) $\dfrac{L^2 a}{H g}$ (D) $\dfrac{Hg}{a}$

23. Figure 8.35 shows a semi-cylindrical massless gate (of width R) pivoted at the point O holding a stationary liquid of density ρ. A horizontal force F is applied at its lowest position to keep it stationary. The magnitude of the force is

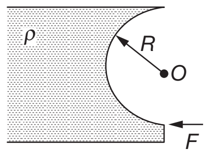


Fig. 8.35

(A) $\dfrac{9}{2}\rho g R^3$ (B) $\dfrac{3}{2}\rho g R^3$
(C) $\rho g R^3$ (D) Zero

24. A block A of mass 10 kg, connected to another hollow block B of same size and negligible mass, by a spring of spring constant 500 N/m, floats in water as shown in Fig. 8.36. The compression in the spring is
($\rho_{water} = 1 \times 10^3$ kg/m^3, $g = 10$ m/s^2)

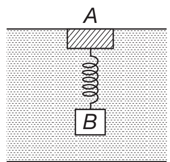
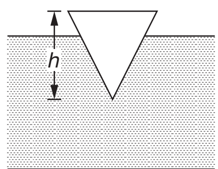


Fig. 8.36

(A) 10 cm (B) 20 cm (C) 50 cm (D) 100 cm

25. A conical block floats in water with 90% height immersed in it. Height h of the block is equal to the diameter of the block, i.e., 20 cm. The mass to be kept on the block, so that the block just floats at the surface of water, is



(A) 568 g (B) 980 g (C) 112 g (D) 196 g

26. A large cylindrical tank has a hole of area A at its bottom. Water is poured in the tank by a tube of equal cross-sectional area A ejecting water at the speed v.

(A) The water level in the tank will keep on rising.
(B) No water can be stored in the tank.
(C) The water level will rise to a height $v^2/2g$ and then stop.
(D) The water level will oscillate.

27. A tank is filled with water and an orifice is made in the wall so that the horizontal range x of water rushing out is maximum. If H is the height of water above the orifice in the tank, then

(A) $x = H$ (B) $x = 2H$
(C) $2x = H$ (D) $4x = H$

28. There is a small hole at the bottom of a large open vessel. If water is filled up to a height h, velocity of water coming out of hole is v. Then velocity of water coming out of hole when water is filled to a height $4h$ is

(A) $4v$ (B) $3v$ (C) $2v$ (D) v

29. A gas having density ρ flows with a velocity v along a pipe of cross-sectional area s and bent at an angle of 90° at a point A. The force exerted by the gas on the pipe at A is

(A) $\dfrac{\sqrt{2}\,sv}{\rho}$ (B) $sv^2\rho$
(C) $\dfrac{\sqrt{3}\,sv^2\rho}{2}$ (D) $sv^2\rho$

30. Water flows through a frictionless duct with a cross-section varying as shown in Fig. 8.37. Pressure p at points along the axis is represented by

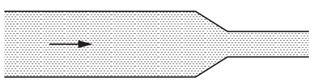
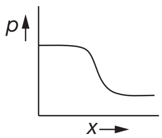
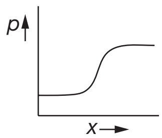
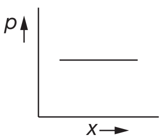
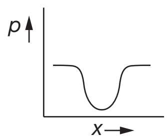


Fig. 8.37

(A)


(B)


(C)


(D)


31. The dimensional formula for coefficient of viscosity

(A) $[MLT^{-1}]$ (B) $[ML^{-1}T^{-1}]$
(C) $[MLT^{-2}]$ (D) $[ML^2T^{-1}]$

32. Which of the following has greatest viscosity?

(A) Hydrogen (B) Air
(C) Water (D) Ammonia

33. With increase in temperature, the viscosity of

(A) both gases and liquids increases.
(B) both gases and liquids decreases.
(C) gases increases and of liquids decreases.
(D) gases deceases and of liquids increases.

34. The profile of advancing liquid in a tube is a

(A) Straight line (B) Circle
(C) Parabola (D) Hyperbola

35. Terminal velocity of water drops depends upon the

(A) Radius of drop (B) Charge of drop
(C) Temperature of drop (D) Velocity of light

36. Water rises in a vertical capillary tube up to a length of 10 cm. If the tube is inclined at 45°, the length of water arisen in the tube will be

(A) $10\sqrt{2}$ cm (B) 10 cm
(C) $10/\sqrt{2}$ cm (D) None of these

37. The surface tension of a liquid is 5 N/m. If a film is held on a ring of area 0.02 m^2, its total surface energy is about

(A) 5×10^{-2} J (B) 2.5×10^{-2} J
(C) 2×10^{-1} J (D) 3×10^{-1} J

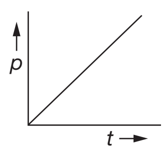
38. If the angle of contact is 0°, the shape of meniscus is

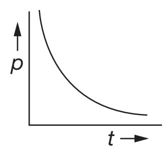
(A) Plane (B) Parabolic
(C) Cylindrical (D) Hemispherical

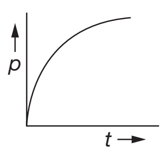
39. The pressure just below the meniscus of water

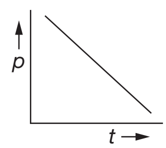
(A) is greater than just above it.
(B) is lesser than just above it.
(C) is same as just above it.
(D) is always equal to atmospheric pressure.

40. A soap bubble is blown slowly at the end of a tube by a pump supplying air at a constant rate. Which one of the following graphs represents the correct variation of the excess of pressure inside the bubble with time?

(A)


(B)


(C)


(D)


41. The viscous force on a small sphere of radius R moving in a fluid varies as

(A) $\propto R^2$ (B) $\propto R$
(C) $\propto (1/R)$ (D) $\propto (1/R)^2$

42. If a small sphere is let to fall vertically in a large quantity of a still liquid of density smaller than that of the material of the sphere

(A) At first its velocity increases, but soon approaches a constant value.
(B) It falls with constant velocity all along from the very beginning.
(C) At first it falls with a constant velocity which after some time goes on decreasing.
(D) Nothing can be said about its motion.

43. A small spherical solid ball is dropped in a viscous liquid. Its journey in the liquid is best described in Fig. 8.38 drawn by

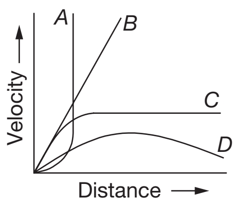


Fig. 8.38

(A) Curve A (B) Curve B
(C) Curve C (D) Curve D

44. Rain drops fall from a height under gravity, we observe that

(A) Their velocities go on increasing until they hit the ground but the velocity with which the drops hit the ground differs with the radius of the rain drop.
(B) Their velocities go on increasing until they hit the ground, velocity being independent of the radius of the drop.
(C) They fall with a terminal velocity which is dependent of the radius of the rain drop.
(D) They fall with a terminal velocity which depends upon the radius of the rain drop.

45. Air is pushed into a soap bubble of radius r to double its radius. If the surface tension of the soap solution is S, the work done in the process is

(A) $8\pi r^2 S$ (B) $12\pi r^2 S$
(C) $16\pi r^2 S$ (D) $24\pi r^2 S$

46. Water flows in a continuous stream down a vertical pipe, whereas it breaks into drops when falling freely because of

(A) Viscosity
(B) Surface tension
(C) Atmospheric pressure
(D) Hydrostatic pressure

47. A soap bubble (surface tension 30 dyne/cm) has a radius of 2 cm. The work done in doubling its radius is

(A) Zero (B) 2261 erg
(C) 1135.5 erg (D) 9043 erg

48. An air bubble of radius r in water is at a depth h below the water surface at some instant. If P is atmospheric pressure and d and T are the density and surface tension of water, respectively, the pressure inside the bubble will be

(A) $P+hdg-\frac{4T}{r}$ (B) $P+hdg+\frac{2T}{r}$
(C) $P+hdg-\frac{2T}{r}$ (D) $P+hdg+\frac{4T}{r}$

49. Water rises in a capillary tube to a certain height such that the upward force due to surface tension is balanced by 75×10^{-4} N, force due to the weight of the liquid. If the surface tension of water is 6×10^{-2} N/m, the inner circumference of the capillary must be

(A) 1.25×10^{-2} m (B) 0.50×10^{-2} m
(C) 6.5×10^{-2} m (D) 12.5×10^{-2} m

50. In a surface tension experiment with a capillary tube, water rises up to 0.1 m. If the same experiment is repeated on an artificial satellite, which is revolving around the earth, water will rise in the capillary tube up to a height of

(A) 0.1 m (B) 0.2 m
(C) 0.98 m (D) Full length of tube

51. A mercury barometer reads 75 cm. If the tube be inclined by 60° from vertical, the length of mercury in the tube will be

(A) 37.5 cm (B) 150 cm
(C) $\frac{75\sqrt{3}}{2}$ cm (D) 100 cm

52. Two circular metal plates of radius 1 m and 2 m are placed horizontally in a liquid at rest at the same depth. The ratio of thrusts on them is

(A) 1 : 2 (B) 1 : 1 (C) 1 : 4 (D) 4 : 1

53. A ball of mass m and radius r is released in viscous liquid. The value of its terminal velocity is proportional to

(A) $(1/r)$ only (B) m/r
(C) $(m/r)^{1/2}$ (D) m only

54. Two capillary tubes of same length l but radii r_1 and r_2 are fitted in parallel to the bottom of a vessel. The pressure head is P. What should be the radius r of the single tube that can replace the two tubes, so that the rate of flow is same as before?

(A) $r = r_1 + r_2$ (B) $r = r_1^2 + r_2^2$

(C) $r^4 = r_1^4 + r_2^4$ (D) $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$

55. If two soap bubbles of radii r_1 and r_2 ($>r_1$) are in contact, the radius of their common interface is

(A) $r_1 + r_2$ (B) $(r_1 + r_2)^2$

(C) $\frac{r_1 r_2}{r_2 - r_1}$ (D) $\sqrt{r_1 r_2}$

56. On putting a capillary tube in a pot filled with water, the level of water rises up to a height of 4 cm in the tube. If a tube of half the diameter is used, the water will rise to the height of nearly

(A) 2 cm (B) 5 cm
(C) 8 cm (D) 11 cm

57. A small spherical solid ball is dropped in a viscous liquid. Its journey in the liquid is best described in Fig. 8.39 drawn by

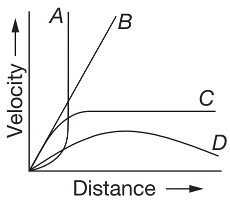


Fig. 8.39

(A) Curve A (B) Curve B
(C) Curve C (D) Curve D

58. A boat with wood is floating in a lake. If the wood is thrown in the lake, the water level will

(A) Go up (B) Go down
(C) Remain unchanged (D) None of the above

59. The amount of work done in increasing the size of a soap film 10 cm × 6 cm to 10 cm × 10 cm is (S.T. = 30×10^{-3} N/m)

(A) 2.4×10^{-2} J (B) 1.2×10^{-2} J
(C) 2.4×10^{-4} J (D) 1.2×10^{-4} J

60. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at height ($3L/4$) from its lower end is

(A) W_1/S (B) $\{W_1 + (W/4)\}\, S$

(C) $\left(W_1 + \frac{3W}{4}\right)/S$ (D) $(W_1 + W)/S$

61. If Young's modulus of iron is 2×10^{11} N/m^2 and the interatomic spacing between two molecules is 3×10^{-10} m, the interatomic force constant is

(A) 60 N/m (B) 120 N/m
(C) 3 N/m (D) 180 N/m

62. Two rods of identical dimensions, with Young's modulus Y_1 and Y_2 are joined end to end. The equivalent Young's modulus for the composite rod is

(A) $\frac{2Y_1Y_2}{Y_1 + Y_2}$ (B) $\frac{Y_1Y_2}{Y_1 + Y_2}$

(C) $\frac{1}{2(Y_1 + Y_2)}$ (D) $Y_1 + Y_2$

63. A uniform rod of length L has a mass per unit length λ and area of cross-section A. The elongation in the rod is l due to its own weight if it is suspended from the ceiling of a room. The Young's modulus of the rod is

(A) $\frac{2\lambda gL^2}{Al}$ (B) $\frac{\lambda gL^2}{2Al}$ (C) $\frac{2\lambda gL}{Al}$ (D) $\frac{\lambda gl^2}{AL}$

64. If A denotes the area of free surface of a liquid and h the depth of an orifice of area of cross-section a, below the liquid surface, then the velocity v of flow through the orifice is given by

(A) $v = \sqrt{(2gh)}$

(B) $v = \sqrt{(2gh)}\sqrt{\left(\frac{A^2}{A^2 - a^2}\right)}$

(C) $v = \sqrt{2gh}\sqrt{\left(\frac{A}{A - a}\right)}$

(D) $v = \sqrt{2gh}\sqrt{\left(\frac{A^2 - a^2}{A^2}\right)}$

65. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done is

(A) $\frac{YAx^2}{2L}$ (B) $\frac{YAx}{2L^2}$

(C) $\frac{YAx}{2L}$ (D) $\frac{YAx^2}{L}$

66. For the arrangement shown in Fig. 8.40, the force at the bottom of the vessel is

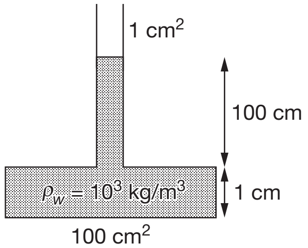
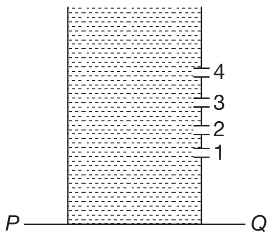


Fig. 8.40

(A) 200 N (B) 100 N (C) 20 N (D) 2 N

67. A cylindrical vessel of 90 cm height is kept filled up to its brim. It has four holes 1, 2, 3, 4 which are, respectively, at heights of 20 cm, 30 cm, 45 cm, and 50 cm from the horizontal floor *PQ*. The water falling at the maximum horizontal distance from the vessel comes from



(A) Hole number 4 (B) Hole number 3
(C) Hole number 2 (D) Hole number 1

68. An open vessel containing water is given a constant acceleration *a* in the horizontal direction. Then the free surface of water gets sloped with the horizontal at an angle θ given by

(A) $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ (B) $\theta = \tan^{-1}\left(\frac{g}{a}\right)$

(C) $\theta = \sin^{-1}\left(\frac{a}{g}\right)$ (D) $\theta = \cos^{-1}\left(\frac{g}{a}\right)$

69. The stress–strain graph for two materials is shown in Fig. 8.41. If the Young's modulus for two materials are Y_A and Y_B then

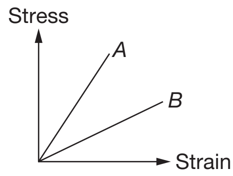


Fig. 8.41

(A) $Y_A = Y_B$
(B) $Y_A > Y_B$
(C) $Y_A < Y_B$
(D) Can't be predicted from the graph

More than One Option Correct Type

70. A metal wire of length *L*, area of cross-section A, and Young's modulus *Y* is stretched by a variable force *F* such that *F* is always slightly greater than the elastic forces of resistance in the wire. When the elongation of the wire is ℓ

(A) The work done by *F* is $\frac{YA\ell^2}{2L}$

(B) The work done by *F* is $\frac{YA\ell^2}{L}$

(C) The elastic potential energy stored in the wire is $\frac{YA\ell^2}{2L}$

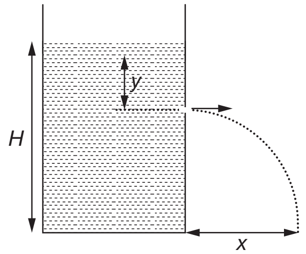
(D) No heat is produced during the elongation

71. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. River is 1 m deep and the water in contact with the bed is stationary. Then choose the correct statements.
(Coefficient of viscosity of water = 10^{-3} Ns/m^2)

(A) Velocity gradient is 2 s^{-1}.
(B) Velocity gradient is 1 s^{-1}.
(C) Force required to keep the plate moving with constant speed is 0.02 N.
(D) Force required to keep the plate moving with constant speed is 0.01 N.

72. A tank, which is open at the top, contains a liquid up to a height *H*. A small hole is made in the side of the tank at a distance *y* below the liquid surface. The liquid emerging from the hole lands at a distance *x* from the tank

(A) If *y* is increased from zero to *H*, *x* will first increase and then decrease.
(B) *x* is maximum for $y = H/2$.
(C) The maximum value of *x* is *H*.
(D) The maximum value of *x* will depend on the density of the liquid.



73. An object is floating in a liquid, kept in a container. The container is placed in a lift. Choose the correct option(s).

(A) Buoyant force increases as lift accelerates up.
(B) Buoyant force decreases as lift accelerates up.
(C) Buoyant force remains constant as lift accelerates.
(D) The fraction of solid submerged into liquid does not change.

74. A small solid ball of density ρ is held inside at point A. A cubical container of side L, filled with an ideal liquid of density 4ρ as shown in Fig. 8.42. Now, if the container starts moving with constant acceleration a horizontally and the ball is released from point A simultaneously, then

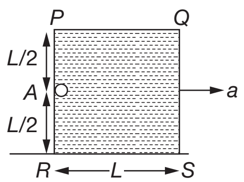


Fig. 8.42

(A) For ball to hit the top of container at end Q, $a = 3g$.
(B) For ball to hit the top of container at end Q, $a = 2g$.
(C) Ball hits the top of container at end Q after a time $t = \sqrt{\frac{L}{3g}}$.
(D) Ball hits the top of container at end Q after a time $t = \sqrt{\frac{2L}{3g}}$.

75. A block of mass m is attached by means of a spring to the bottom of a tank of water as shown in Fig. 8.43. At equilibrium, the spring is under compression. If the tank is now allowed to fall freely, then choose the correct alternative(s).

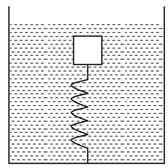


Fig. 8.43

(A) The spring comes to its relaxed position.
(B) The spring compresses more than its equilibrium compression.
(C) The buoyant force becomes zero.
(D) There will be some elongation is the spring.

Passage Based Questions

Passage 1

A block of mass m and height h is hanging in a liquid kept in a container up to height 5 h as shown in Fig. 8.44. Density of block is two times of density of liquid. A very small hole is made at the bottom of container. Tension in the string varies with height x of the liquid surface from bottom as

$$T = a\left(b - \frac{x}{h}\right) \quad \text{for} \quad 2h \le x \le 3h$$

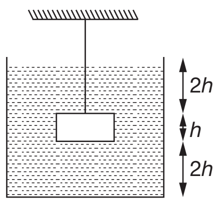


Fig. 8.44

76. What is the value of a?

(A) 2 mg (B) mg (C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

77. What is the value of b?

(A) 1 (B) 2 (C) 3 (D) 4

78. If tension in the string at $x = 4\,h$ is T_1 and T_2 at $x = h$ then T_1/T_2 is

(A) 2 (B) 1 (C) 1/2 (D) 1/4

Passage 2

A heavy rod of length ℓ is released from a height h above the water surface as shown in Fig. 8.45. The rod falls in air and then in water such that it always remains in vertical position. Neglecting the viscous force and drag force on rod while moving in air and water. Also the relative density of the rod is 2 neglecting the loss in energy when rod hits the surface. Assuming the water level remains unchanged due to fall of the rod and it touches the bottom of the tank at same speed. The cross-sectional area of the rod is A and is constant. ρ_w = density of water

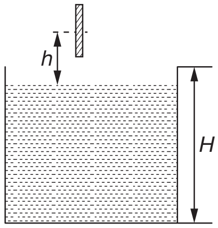


Fig. 8.45

79. The average body and force until rod sinks completely

(A) $2\rho_w 1gA$ (B) $\dfrac{\rho_w 1gA}{2}$

(C) $\rho_w 1gA$ (D) $\dfrac{\rho_w lAg}{3}$

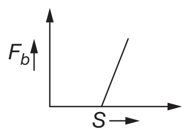
80. The work done by rod against buoyant fore until rod sinks completely

(A) $\rho_w Al^2 g$ (B) $\dfrac{\rho_w Al^2 g}{2}$

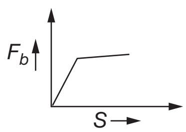
(C) $2\rho_w Al^2 g$ (D) Zero

81. Buoyant force varies as the displacement of the rod is best represented by

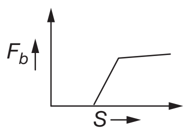
(A)



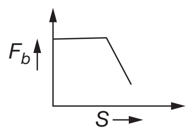
(B)



(C)



(D)



Passage 3

A wire is bend at an angle θ. A rod of mass m can slide along the bend wire without friction as shown in Fig. 8.46. If a soap film is maintained in the frame and frame is kept in a vertical position, the rod is in equilibrium.

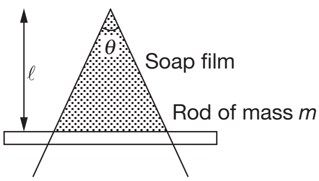


Fig. 8.46

82. Surface tension of the soap solution is

(A) $\dfrac{mg}{4\ell \tan\dfrac{\theta}{2}}$ (B) $\dfrac{mg}{2\ell \tan\dfrac{\theta}{2}}$

(C) $\dfrac{mg}{4\ell \tan\theta}$ (D) None of these

83. If rod is displaced slightly in vertical direction, then

(A) Motion is periodic but not SHM.
(B) Motion is periodic and SHM.
(C) The rod always moves vertically downward.
(D) None of these.

84. If rod is displaced slightly in vertical direction, find the time period of small oscillation.

(A) $\pi\sqrt{\dfrac{l}{g}}$ (B) $2\pi\sqrt{\dfrac{l}{g}}$

(C) $\dfrac{\pi}{2}\sqrt{\dfrac{l}{g}}$ (D) None of these

Passage 4

Figure 8.47 shows a container having ideal liquid of variable density. The density of liquid varies as $\rho = \rho_0\left(4 - \dfrac{3h}{h_0}\right)$, where h_0 is height of liquid in container, ρ_0 is constant and h is height from bottom. A solid block of small dimensions whose density $\dfrac{5}{2}\rho_0$ and mass m is released from bottom of tank.

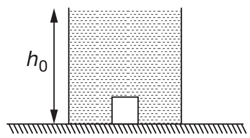


Fig. 8.47

85. The motion of the block is

(A) Periodic but not SHM.
(B) SHM.
(C) Oscillatory but not SHM.
(D) Oscillatory but not periodic.

86. After what time does the block reach its initial position?

(A) $2\pi\sqrt{\dfrac{5h_0}{6g}}$ (B) $2\pi\sqrt{\dfrac{h_0}{g}}$

(C) $\pi\sqrt{\dfrac{h_0}{3g}}$ (D) $\pi\sqrt{\dfrac{5h_0}{6g}}$

87. Maximum speed of block is

(A) $\sqrt{\dfrac{gh_0}{5}}$ (B) $\sqrt{2gh_0}$

(C) $\sqrt{\dfrac{3gh_0}{10}}$ (D) $\sqrt{\dfrac{10}{3}gh_0}$

88. Maximum height reached by the block

(A) $\dfrac{h_0}{2}$ (B) h_0

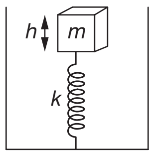
(C) $\dfrac{3h_0}{4}$ (D) $\dfrac{h_0}{4}$

Match the Column Type

89. Assuming that all the liquid drop or air bubble have surface tension T and radius R

Column-I	Column-II
(A) Excess pressure of liquid drop in air is	(1) $\dfrac{4T}{R}+\rho gh$
(B) Excess pressure of bubble in air is	(2) $\dfrac{2T}{R}+\rho gh$
(C) Excess pressure of air bubble in liquid at its free surface is	(3) $\dfrac{4T}{R}$
(D) Excess pressure of air bubble in liquid at depth h from free surface is	(4) $2\dfrac{T}{R}$

90. A rectangular block of density ρ, base area A, and height h is kept on a spring. The lower end of spring is fixed on the bottom of an empty vessel of base area $2A$. The block compresses the spring by $h/4$ at equilibrium. The vessel is then slowly filled by a liquid of density 2ρ till the spring becomes relaxed. The block is then slowly pushed inside the liquid till it is immersed completely. Work done to push the block completely inside is W_1, work done by gravity on the block is W_2, and work done by upthrust is W_3.



Match the following based on the above statements.

Column-I	Column-II
(A) $\lvert W_1\rvert$	(1) $\dfrac{5\rho gh^2 A}{8}$
(B) $\lvert W_2\rvert$	(2) $\dfrac{\rho gh^2 A}{8}$
(C) $\lvert W_3\rvert$	(3) $\dfrac{3\rho gh^2 A}{4}$
	(4) $\dfrac{\rho gh^2 A}{4}$

Assertion-Reason Type

91. Assertion: In case of motion of an ideal fluid in a horizontal tube, where the area of cross-section is minimum, pressure is maximum.

Reason: Hydrostatic pressure in different ideal liquids at points of different depth can be same.

(A) A (B) B (C) C (D) D

92. Assertion: Force of buoyancy due to atmosphere on a body is almost zero (or negligible).

Reason: The body is completely submerged in the atmosphere.

(A) A (B) B (C) C (D) D

93. Assertion: Up to elastic limit of a stress–strain curve the steel wire tends to regain its original shape when stress is removed.

Reason: Within elastic limit the wire follows Hook's law.

(A) A (B) B (C) C (D) D

94. Assertion: When an ideal fluid flows through a horizontal tube of variable cross-section, the pressure becomes different at different points.

Reason: Raindrops falling from a great height reach the ground with a relatively small velocity. This phenomenon involves the viscosity of air.

(A) A (B) B (C) C (D) D

Integer Type

95. A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force F stretches it along its length. The strain in rod is 1.59×10^{-n} then the value of n is. Given that the Young's modulus E of the structural steel is 2.0×10^{11} N/m^{-2}.

96. A hollow conical vessel floats in water with its vertex downwards with a certain depth of its axis immersed. When water is poured into it up to the level originally immersed, it sinks till its mouth is on level with the surface of water. The fraction of its height that was originally under water is $\left(\frac{h}{H}\right)=\left(\frac{1}{2}\right)^{1/n}$. Find the value of n.

97. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in Fig. 8.48. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. The angle θ in radian that the plank makes with the vertical in the equilibrium position is $\frac{\pi}{n}$, then the value of n is.

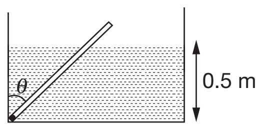


Fig. 8.48

98. A glass rod of diameter d_1 = 1.5 mm is inserted symmetrically into a glass capillary of inside diameter d_2 = 2 mm. Then the whole arrangement is vertically oriented and brought in contact with the surface of water. To what height (in mm) will the water rise in the capillary (in mm)? (Density of water = 1 gm/cc, surface tension = 0.07 N/m, angle of contact = 0^0, g = 10 m/s^2)

99. Two opposite forces F_1 = 120 N and F_2 = 80 N act on an elastic plank of modulus of elasticity $Y = 2\times10^{11}$ N/m^2 and length ℓ = 1 m placed over a smooth horizontal surface. The cross-sectional area of the plank is S = 0.5 m^2. The change in length of the plank is $x\times10^{-11}$ m, then find the value of x.

100. A U-tube having uniform cross-section but unequal arm lengths ℓ_1 and ℓ_2 ($<\ell_1$) has same liquid of density ρ_1 filled in it up to a height h as shown in Fig. 8.49. Another liquid of density $\rho_2 = (\rho_1/2)$ is poured in arm A. Both liquids are immiscible. What length of the second liquid should be poured in A so that first overtone of A is in unison with fundamental tone of B? (Take ℓ_1 = 5 m, ℓ_2 = 1 m and h = 0.5 m)

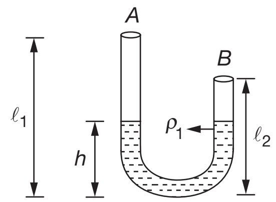


Fig. 8.49

101. A piece of brass (Cu and Zn) weighs 12.9 g in air. When completely immersed in water, it weighs 11.3 g. The relative densities of Cu and Zn are 8.9 and 7.1, respectively. Calculate the mass of copper in the alloy (in decigram).

102. A drop of water of mass m = 0.4 g is placed between two clean glass plates, the distance between which is 0.01 cm. Find the force of attraction between the plates. Surface tension of water = 0.01 N/m.

Previous Years' Questions

103. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is **[2005]**

(A) $2S^2Y$ (B) $\frac{S^2}{2Y}$ (C) $\frac{2Y}{S^2}$ (D) $\frac{S}{2Y}$

104. A wire is fixed at the upper end and stretches by length ℓ by applying a force F. The work done in stretching is **[2004]**

(A) $\frac{F}{2\ell}$ (B) $F\ell$

(C) $2F\ell$ (D) $\frac{F\ell}{2}$

105. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then, the elastic energy stored in the wire is **[2003]**

(A) 0.2 J (B) 10 J (C) 20 J (D) 0.1 J

106. A wire elongates by ℓmm when a load w is hung from it. If the wire goes over a pulley and two weights w each are hung at the two ends, the elongation of the wire will be (in mm) **[2006]**

(A) ℓ (B) 2ℓ (C) Zero (D) $\frac{\ell}{2}$

107. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2(\rho_2 < \rho_1)$ [Assuming that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2 (k > 0)$]. The terminal speed of the ball is **[2008]**

(A) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$ (B) $\frac{Vg\rho_1}{k}$

(C) $\sqrt{\frac{Vg\rho_1}{k}}$ (D) $\frac{Vg(\rho_1 - \rho_2)}{k}$

108. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in Fig. 8.50. Which of the following is true for ρ_1, ρ_2 and ρ_3? **[2007]**

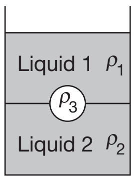


Fig. 8.50

(A) $\rho_3 < \rho_1 < \rho_2$ (B) $\rho_1 > \rho_3 > \rho_2$

(C) $\rho_1 > \rho_2 > \rho_3$ (D) $\rho_1 < \rho_3 < \rho_2$

109. If the terminal speed of a sphere of gold (density = 19.5 kgm^{-3}) is 0.2 ms^{-1} in a viscous liquid (density = 1.5 kgm^{-3}), find the terminal speed of a sphere of silver (density = 10.5 kg/m^{-3}) of the same size in the same liquid. **[2006]**

(A) 0.4 ms^{-1} (B) 0.133 ms^{-1}

(C) 0.1 ms^{-1} (D) 0.2 ms^{-1}

110. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be **[2005]**

(A) 8 cm (B) 10 cm (C) 4 cm (D) 20 cm

111. A spherical balls of a radius R is falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is **[2004]**

(A) directly proportional to R but inversely proportional to v.

(B) directly proportional to both radius R and velocity v.

(C) inversely proportional to both radius R and velocity v.

(D) inversely proportional to R but directly proportional to velocity v.

112. If two soap bubbles of different radii are connected by a tube **[2004]**

(A) Air flows from the bigger bubble to the smaller bubble till the sizes become equal.

(B) Air flows from bigger bubble to the smaller bubble till the sizes are interchanged.

(C) Air flows from the smaller bubble to the bigger bubble.

(D) There is no flow of air.

113. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is **[2004]**

(A) 10 (B) 20 (C) 25.5 (D) 5

114. A uniform cylinder of length ℓ and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density at equilibrium position. The extension of the spring when it is in equilibrium is: **[2013]**

(A) $\frac{Mg}{k}\left(1 - \frac{\ell A\sigma}{M}\right)$ (B) $\frac{Mg}{k}\left(1 - \frac{\ell A\sigma}{2M}\right)$

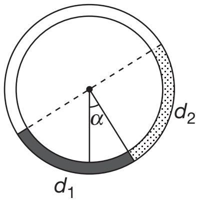
(C) $\frac{Mg}{k}\left(1 + \frac{\ell A\sigma}{M}\right)$ (D) $\frac{Mg}{k}$

115. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ, and L is its latent heat of vaporization. **[2013]**

(A) $\sqrt{T/\rho L}$ (B) $T/\rho L$

(C) $2T/\rho L$ (D) $\rho L/T$

116. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90º angle at the centre. Radius joining their interface makes an angle α with vertical plane. Ratio $\frac{d_1}{d_2}$ is **[2014]**

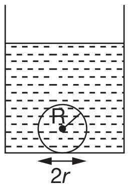


(A) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (B) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(C) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (D) $\frac{1+\sin\alpha}{1-\cos\alpha}$

117. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$, and the surface tension of water is T, value of r just before bubbles detach is

(Density of water is ρ_w) **[2014]**



(A) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (B) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(C) $R^2\sqrt{\frac{\rho_w g}{T}}$ (D) $R^2\sqrt{\frac{3\rho_w g}{T}}$

118. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to TM. If the Young's modulus of the material of the wire is Y, then $\frac{1}{Y}$ is equal to

(g = gravitational acceleration) **[2015]**

(A) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$ (B) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$

(C) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (D) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$

ANSWER KEYS

Single Option Correct Type

1. (A)	**2.** (C)	**3.** (B)	**4.** (C)	**5.** (B)	**6.** (A)	**7.** (D)	**8.** (A)	**9.** (C)	**10.** (C)
11. (D)	**12.** (C)	**13.** (B)	**14.** (B)	**15.** (C)	**16.** (C)	**17.** (B)	**18.** (C)	**19.** (C)	**20.** (D)
21. (B)	**22.** (B)	**23.** (D)	**24.** (A)	**25.** (A)	**26.** (C)	**27.** (B)	**28.** (C)	**29.** (B)	**30.** (A)
31. (B)	**32.** (C)	**33.** (C)	**34.** (C)	**35.** (A)	**36.** (A)	**37.** (C)	**38.** (D)	**39.** (B)	**40.** (B)
41. (B)	**42.** (A)	**43.** (C)	**44.** (D)	**45.** (D)	**46.** (B)	**47.** (D)	**48.** (B)	**49.** (D)	**50.** (D)
51. (B)	**52.** (C)	**53.** (B)	**54.** (C)	**55.** (C)	**56.** (C)	**57.** (C)	**58.** (C)	**59.** (C)	**60.** (C)
61. (A)	**62.** (A)	**63.** (B)	**64.** (B)	**65.** (A)	**66.** (B)	**67.** (B)	**68.** (A)	**69.** (B)	

More than One Option Correct Type

70. (A), (C) and (D) **71.** (A) and (C) **72.** (A), (B) and (C)
73. (A) and (D) **74.** (B) and (C) **75.** (B) and (C)

Passage Based Questions

Passage 1

76. (C) **77.** (D) **78.** (C)

Passage 2

79. (B) **80.** (B) **81.** (C)

Passage 3

82. (A) **83.** (B) **84.** (B)

Passage 4

85. (B) **86.** (A) **87.** (C) **88.** (B)

Match the Column Type

89. (A) → 4; (B) → 3; (C) → 4; (D) → 2
90. (A) → 1; (B) → 4; (C) → 3;

Assertion-Reason Type

91. (D) **92.** (B) **93.** (C) **94.** (B)

Integer Type

95. 1.59 mm **96.** 3 **97.** 4 **98.** 56 mm **99.** 100 **100.** 2 m
101. 76 decigram **102.** 8 N

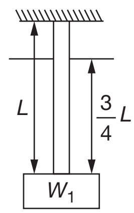
Previous Years' Questions

103. (B) **104.** (D) **105.** (D) **106.** (A) **107.** (A) **108.** (B) **109.** (C) **110.** (D)
111. (B) **112.** (C) **113.** (B) **114.** (B) **115.** (C) **116.** (C) **118.** (D)

HINTS AND SOLUTIONS

Single Option Correct Type

1. The correct option is (A)
2. $B = \frac{20}{A} = \frac{F}{4A} \Rightarrow F = 80$ kg*wt*
   The correct option is (C)
3. $y = \frac{FL}{A\Delta L} = \frac{100\times100}{10^{-6}\times0.1} = 10^{11}\,\text{N/m}^2$
   The correct option is (B)
4. $B = \frac{1.2\times10^5\times1\times10^{-3}}{0.3\times10^{-6}} = 4\times10^8\,\text{N/m}^2$
   The correct option is (C)
5. $\frac{\rho_s}{\rho_l} = \frac{h}{H} = \frac{2}{3} \Rightarrow \rho_s = \frac{2}{3}\times10^3\ \text{kg/m}^3$
   The correct option is (B)
6. $F_u \propto$ volume of material $\propto \frac{1}{\text{density of material}}$
   The correct option is (A)
7. The correct option is (D)
8. $\rho_{\text{mix}} = \frac{\rho_w+\rho_a}{2} = \frac{1000+800}{2} = 900\,\text{kg/m}^3$
   The correct option is (A)
9. 10 m corresponds to approximately 1 atm
   So 20 m corresponds to additional 2 atm
   The correct option is (C)
10. Height in C is maximum, thus force will be maximum in C
    The correct option is (C)
11. $A_1v_1 = A_2v_2$
    So $v_2 = \frac{v\pi(2R)^2}{\pi R^2} = 4v$
    The correct option is (D)
12. Because of upward acceleration, pressure will increase
    The correct option is (C)
13. To get maximum lift, velocity of air at the top of ball with respect to ball should be maximum.
    The correct option is (B)
14. $\frac{F_u}{W} = \frac{\rho_l}{\rho_s} = \frac{2}{3}$
    or $\rho_s = \frac{3}{2}\rho_l$
    The correct option is (B)
15. $V = \sqrt{2gh}$
    Independent of density
    The correct option is (C)
16. Net beared weight $= W_1 + \frac{3}{4}W$
    Stress $= \left(\frac{W_1+\frac{3}{4}W}{S}\right)$
    The correct option is (C)



17. Breaking stress is independent of length
    The correct option is (B)

18. $v\rho g = v_1\rho_1 g + v_2\rho_2 g$, $v_1\rho_1 + v_2\rho_2 = v\rho$ and $v_1 + v_2 = v$

$(v - v_2)\rho_1 + v_2\rho_2 = v\rho$, $v_2(\rho_2 - \rho_1) = v(\rho - \rho_1)$,

$\frac{v_2}{v} = \frac{\rho - \rho_1}{\rho_2 - \rho_1} = \frac{\rho_1 - \rho}{\rho_1 - \rho_2}$

The correct option is (C)

19. $\Delta_1 = \int_0^l \frac{Mgxdx}{LAY} = \frac{Mg\ L}{2AY}$ and $\Delta_2 = \frac{MgL}{AY}$

$\Delta_2 = 2\Delta_1$

The correct option is (C)

20. Upthrust $= 400 \times 10^{-6} \times 10^3 \times g = 3.92$ N.

Apparent weight = 96.08 N

The correct option is (D)

21. $\frac{H}{10}$ part of ice slab will be out of water. Volume of ice slab outside the water $= \frac{A(0.5)}{10}\ \text{m}^3$

(A = area of slab)

$A \times \left(\frac{0.5}{10}\right) \times 10^3\ \text{g} = 100\ \text{g},\ A = \frac{10}{5} = 2\ \text{m}^2$

The correct option is (B)

22. $P_2 + a\rho L = P_1$, $P_1 - P_2 = a\rho L$

$h_1 - h_2 = \frac{aL}{g}$

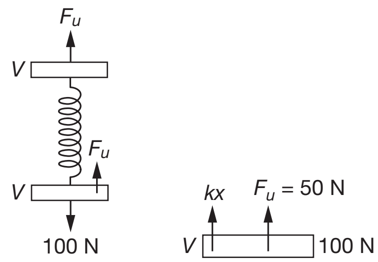
The correct option is (B)

23. Since the force due to fluid acts normal to the surface, at about point O, no net torque is acting on the gate. Hence $F = 0$.

The correct option is (D)

24. 2 F_u = 100 N (F_u is upthrust force)

F_u = 50 N



Upper block

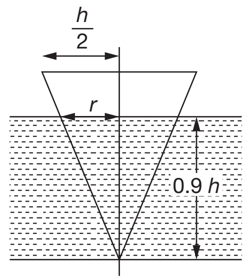
$kx = 50$ N

$x = \frac{50}{500} = 0.1$ m

The correct option is (A)

25. $\frac{h}{h/2} = \frac{0.9h}{r} \Rightarrow r = \frac{0.9h}{2}$ (1)

Upthrust $= \frac{1}{3}\pi r^2.(0.9h)\ g\rho_l = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h.\rho_s g$ (2)



When totally submerged $w + \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 gh\rho_s = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 g\rho_l h$ (3)

On solving (1) and (2) and (3)

W = 568 gm

The correct option is (A)

26. When rate of inflow = rate of outflow, height will be constant

$Av_{\text{in}} = Av_{\text{out}}$, $v_{\text{in}} = v_{\text{out}}$.

$v_{\text{out}} = v \Rightarrow \sqrt{2gh} = v,\ h = \frac{v^2}{2g}$

The correct option is (C)

27. For maximum range $x = h = 2$H

h is height of container

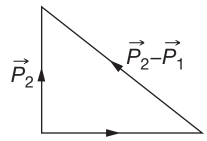
The correct option is (B)

28. $v = \sqrt{2gh}$, $v_1 = \sqrt{2g(4h)} = 2$v

The correct option is (C)

29. Rate of change of momentum

$|\vec{P}_2 - \vec{P}_1| = \sqrt{2}sv^2\rho$



The correct option is (B)

30. Using Bernoulli's equation,

The correct option is (A)

31. $\eta = \frac{F/A}{dv/dx} = [\text{ML}^{-1}\text{T}^{-1}]$

The correct option is (B)

32. The correct option is (C)

33. The correct option is (C)

34. The correct option is (C)

35. Terminal velocity of water drops depends upon radius.

The correct option is (A)

36. Pressure difference in both cases should be same.

$10\ \rho g = L \cos 45° \rho g$

or $L = 10\sqrt{2}$ cm

The correct option is (A)

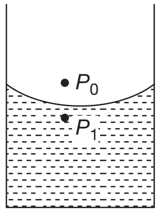
37. Total surface energy $= T \cdot (\text{Area}) = 5 \times (2 \times 0.02) = 0.2$ J

The correct option is (C)

38. Shape will be hemispherical

The correct option is (D)

39. Shape of meniscus is convex, if seen from inside the water, hence pressure inside the water is $P_1 = P_0 - \frac{2T}{r}$



where r = radius of curvature of meniscus

The correct option is (B)

40. Excess pressure $\Delta P = \frac{4T}{R}$

$\Delta P.R = 4\ T$ = constant, R is increasing linearly with time t. Hence ΔPt = constant

The correct option is (B)

41. $F_v = 6\pi\eta Rv$

The correct option is (B)

42. Since viscous force depends upon velocity, viscous force on sphere will increase till it achieves terminal velocity.

The correct option is (A)

43. Explanation is same as Q.No.2

The correct option is (C)

44. Terminal velocity depends upon radius of rain drop.

The correct option is (D)

45. $\Delta A = 4\pi\left(R_2^2 - R_1^2\right) = 12\pi R^2$

$W = \Delta E = 2S\Delta A = 24\pi R^2 S$

The correct option is (D)

46. Because of surface tension, the liquid tries to occupy the minimum surface area. Hence, it breaks into drops.

The correct option is (B)

47. $\Delta A = 4\pi\left(R_2^2 - R_1^2\right) = 4\pi(16-4) = 48\pi\ \text{cm}^2$

work done $= T \cdot (2\Delta A) = 30 \times 96\ \pi = 9043$ erg

The correct option is (D)

48. Pressure in the concave surface is $\frac{2T}{r}$ greater than convex surface; hence,

P inside $= P + hdg + \frac{2T}{r}$

The correct option is (B)

49. $6\times10^{-2}\times(2\pi r) = 75\times10^{-4}$,

$(2\pi r) = \frac{75}{6}\times10^{-2} = 12.5\times10^{-2}$ m

The correct option is (D)

50. In satellite, $g_{\text{eff}} = 0$

Thus due to surface tension, the tube will be completely filled

The correct option is (D)

51. We have

$\rho gh = \rho gh' \cos 60°$ (where h' is the length of column

$\therefore\ h' = 2h = 150$ cm inside the tube)

The correct option is (B)

52. $\frac{F_1}{F_2} = \frac{P\pi(1)^2}{P\pi(2)^2} = \frac{1}{4}$

The correct option is (C)

53. $6\pi\eta rv = mg - F_{\text{thrust}} = mg - mg\frac{\sigma}{\rho}$

where σ = density of liquid

and ρ = density of ball, $6\pi\eta rv = mg\left(1-\frac{\sigma}{\rho}\right)$

$v = \left(\frac{m}{r}\right)\frac{g(1-\sigma/\rho)}{6\pi\eta}$

$\therefore\ v \propto \frac{m}{r}$

The correct option is (B)

54. $Q = Q_1 + Q_2 = \frac{\pi P r_1^4}{8\eta l_0} + \frac{\pi P r_2^4}{8nl} = \frac{\pi P r^4}{8\eta l}$

$\therefore\ r^4 = r_1^4 + r_2^4$

The correct option is (C)

55. For 1st bubble, $P_1 = P_0 + \frac{4T}{r_1}$

For 2nd bubble, $P_2 = P_0 + \frac{4T}{r_2}$

$\therefore$ Pressure inside smaller bubble will be greater than the larger bubble, so for interface

$P = P_1 - P_2 = \frac{4T}{R} = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$\therefore\ R = \frac{r_1 r_2}{r_2 - r_1}$

The correct option is (C)

56. $\therefore\ hr$ = constant

$h_1 r_1 = h_2 r_2$

$\therefore\ h_2 = h_1\left(\frac{r_1}{r_2}\right) = 2h_1 = 8$ cm

The correct option is (C)

57. The correct option is (C)

58. The correct option is (C)

59. $\therefore\ \Delta W = 2T \times \Delta A$

$= 2 \times 30 \times 10^{-3} \times [(100 - 60) \times 10^{-4}] = 2.4 \times 10^{-4}$ J

The correct option is (C)

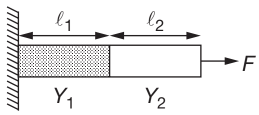
60. Stress $= \frac{\text{Force}}{\text{Area}} = \frac{\left(W_1 + \frac{3W}{4}\right)}{S}$

The correct option is (C)

61. $K = Yr_o$, $K = 60$ N/m

The correct option is (A)

62. Let ℓ_1 and ℓ_2 be the extensions of the rods



$Y_1 = \frac{FL}{Al_1}$, $Y_2 = \frac{FL}{Al_2}$, $Y = \frac{F2L}{A(l_1+l_2)} = \frac{2Y_1Y_2}{Y_1+Y_2}$

The correct option is (A)

63. $l = \int_0^L \frac{(L-x)mg\,dx}{LAY} = \frac{mgL}{2AY}$ (Here $m = \lambda L$)

$Y = \frac{\lambda g L^2}{2Al}$

The correct option is (B)

64. $Av_1 = av$, $v_1 = \frac{av}{A}$,

$P_0 + \frac{1}{2}\rho v_1^2 + \rho gh = P_0 + \frac{1}{2}\rho v^2$,

$v^2 - v_1^2 = 2gh$, $v^2\left[\frac{A^2-a^2}{A^2}\right] = 2gh$

$v = \sqrt{(2gh)}\sqrt{\left(\frac{A^2}{A^2-a^2}\right)}$

The correct option is (B)

65. Work done = $\frac{1}{2}$ stress × strain × volume = $\frac{1}{2}\frac{YAx^2}{L}$

The correct option is (A)

66. Force on bottom,

$F = (\rho gh)A = (10^3)\times(9.8)\times(1.01)\times(100\times10^{-4})$

$F =$ 100 N (approximately)

The correct option is (B)

67. For maximum range, height of the hole from the ground = $\frac{H}{2} = 45$ cm

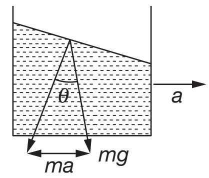
The correct option is (B)

68. $\tan\theta = \frac{ma}{mg} = \frac{a}{g}$

$\theta = \tan^{-1}\left(\frac{a}{g}\right)$

The correct option is (A)

69. Slope of A > slope of B



So, $Y_A > Y_B$

The correct option is (B)

More than One Option Correct Type

70. Work done by force equal to the elastic potential energy stored in the wire.

The correct option is (A), (C) and (D)

71. $\frac{dv}{dx} = \frac{2}{1} = 2\text{s}^{-1}$

$F = \eta A\frac{dv}{dx} = 10^{-3}\times10\times2 = 0.02$ N

The correct option is (A) and (C)

72. The velocity of efflux $= v = \sqrt{2gy}$

The emerging liquid moves as a projectile and reaches the ground in time t, where

$H - y = \frac{1}{2}gt^2$

or $t = \sqrt{\frac{2(H-y)}{g}}$

$\therefore$ $x = vt = 2\sqrt{(H-y)y}$,

For x to be maximum, $\frac{dx}{dy} = 0$ or $y = \frac{H}{2}$

$\therefore$ $x_{max} = H$

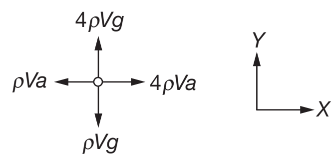
The correct option is (A), (B) and (C)

73. As pressure gradient changes, $\frac{dP}{dy} = \rho(g+a)$

So buoyant force also changes due to pressure difference. But as body also accelerates with lift, the fraction of volume submerged does not change and the extra buoyant force will provide the resultant force ma to the body.

The correct option is (A) and (D)

74. FBD of ball in frame of container



$a_y = \frac{4\rho Vg - \rho Vg}{\rho V} = 3g$

$\frac{L}{2} = \frac{1}{2}3gt^2$

$\Rightarrow$ $t = \sqrt{\frac{L}{3g}}$

$a_x = \frac{4\rho Va - \rho Va}{\rho V} = 3a$

$$L = \frac{1}{2}3at^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2L}{3a}}$$

Ball to collide at point Q, $t = \sqrt{\frac{2L}{3a}} = \sqrt{\frac{L}{3g}}$

$\Rightarrow \quad a = 2g$

The correct option is (B) and (C)

75. As the tank is allowed to fall vertically, buoyant force becomes zero, so the spring compresses more than its equilibrium compression.

The correct option is (B) and (C)

Passage Based Questions

Passage 1

76. At $x = 3h$, $T = mg - F_B = mg - \frac{\rho}{2}Vg = \frac{mg}{2}$

At $x = 2h$, $T = mg$

$$\therefore \quad \frac{mg}{2} = a\left(b - \frac{3h}{h}\right) \text{ and } mg = a\left(b - \frac{2h}{h}\right)$$

$$\Rightarrow \quad a = \frac{mg}{2} \text{ and } b = 4$$

The correct option is (C)

77. The correct option is (D)

78. $T_1 = mg - \frac{mg}{2} = \frac{mg}{2}$,

$T_2 = mg$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{2}$$

The correct option is (C)

Passage 2

79. As buoyant force varies linearly, its average value is

$$F_{ay} = \frac{0 + \rho w \lg A}{2} = \frac{1}{2}\rho w \lg A$$

The correct option is (B)

80. Work done by buoyant force while sinking is

$$W = F_{ay} \times l = \frac{1}{2}\rho l^2 gA$$

The correct option is (B)

81. As for some time, rod falls freely so no buoyant force acts during that interval and then it varies linearly and after sinking completely, it becomes constant.

The correct option is (C)

Passage 3

82. $2T\left(2l\tan\frac{\theta}{2}\right) = mg, T = \dfrac{mg}{4l\tan\frac{\theta}{2}}$

The correct option is (A)

83. The correct option is (B)

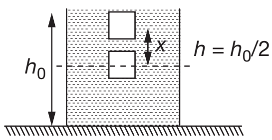
84. If x be the displacement in vertical direction of the rod from equilibrium position

$$F_{res} = -2x\tan\frac{\theta}{2}T \times 2, \quad a = \frac{-4T\tan\frac{\theta}{2}}{m}x,$$

$$T = 2\pi\sqrt{\frac{m}{4T\tan\frac{\theta}{2}}} = 2\pi\sqrt{\frac{m}{4\frac{mg}{4l}}} = 2\pi\sqrt{\frac{l}{g}}$$

The correct option is (B)

Passage 4



$F_{net} = U_p(\text{thrust}) - \text{Weight}$

$$= \left(\frac{m}{(5/2)\rho_0}\right)\rho_0\left(4 - \frac{3h}{h_0}\right)g - mg$$

$F_{net} = 0$ at $h = \frac{h_0}{2}$

So, at $h = \frac{h_0}{2}$ block is equilibrium.

For upward displacement x from mean position.

$$F_{net} = -\left[mg - \left(\frac{m}{(5/2)\rho_0}\right)\rho_0\left(4 - \frac{3(h+x)}{h_0}\right)\right]g$$

$$= -\frac{6mgx}{5h_0}$$

$F \propto -x$

Motion is SHM.

Time period $= 2\pi\sqrt{\frac{5h_0}{6g}}$

Amplitude $= \frac{h_0}{2}$, so maximum height reached by block is h_0.

$$v_{max} = \frac{h_0}{2}\sqrt{\frac{6g}{5h_0}} = \sqrt{\frac{3}{10}gh_0}$$

85. The correct option is (B)

86. The correct option is (A)

87. The correct option is (C)

88. The correct option is (B)

Match the Column Type

89. (A) → (4); (B) → (3); (C) → (4); (D) → (2)

90. $k\dfrac{h}{4} = mg,\ k = \dfrac{4mg}{h} = 4\rho gA\dfrac{h}{h} = 4\rho gA$

When liquid is filled and spring is relaxed,

$2\rho xgA = \rho ghA$

$x = h/2$ (inside the liquid).

As external force pushes the block in the liquid, liquid also rises from remaining area.

Remaining area = $2A - A = A$

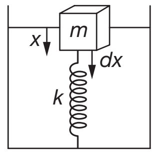
$A_1x_1 = A_2x_2,\ x_1 = x_2,\ x_1 + x_2 = \dfrac{h}{2},\ x_1 = x_2 = \dfrac{h}{4}$

Displacement of block $x_1 = \dfrac{h}{4}$,

$$W_{mg} = W_2 = \rho ghA\frac{h}{4} = \frac{\rho gh^2 A}{4}$$

Spring energy $E = \dfrac{1}{2}kx^2 = \dfrac{1}{2}4\rho gA \times \dfrac{h^2}{16} = \dfrac{\rho gAh^2}{8}$

$$W_{\text{upthrust}} = \int_{h/2}^{h} -2\rho gxA\,dx = W_3$$



$$-2\rho g\frac{A}{2}\left(h^2 - \frac{h^2}{4}\right) = W_3 = -\frac{3}{4}\rho gh^2 A$$

$W_{ext} + W_{mg} + W_{upth} = E$

$W_{ext} = W_1 = \rho gAh^2\left(\dfrac{1}{8} - \dfrac{1}{4} + \dfrac{3}{4}\right),\ W_1 = \dfrac{5}{8}\rho gAh^2$

(A) → (1); (B) → (4); (C) → (3)

Assertion-Reason Type

91. The correct option is (B)

92. Force of buoyancy on a body is due to pressure difference in fluid. Atmospheric pressure is nearly constant for a very small change in altitude.

The correct option is (B)

93. The correct option is (C)

94. The correct option is (B)

Integer Type

95. We assume that the rod is held by a clamp at one end. Then the force F is applied at the other end, parallel to the length of the rod. Now, the stress on the rod is given by

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{100\times 10^3\ \text{N}}{3.14\times(10^{-2}\ \text{m})} = 3.18\times 10^8\ \text{N/m}^{-2}$$

$= 3.18 \times 10^8\ \text{N/m}^{-2}$

Elongation, $\Delta L = \dfrac{(F/A)L}{E} = 1.59\times 10^{-3}\ \text{m} = 1.59\ \text{mm}$

Strain = $\Delta L/L = 1.59 \times 10^{-3}$ m/1 m $= 1.59 \times 10^{-3}$, $n = 3$

96. Let W be the weight of the vessel; V be the total volume of the vessel.

Now, $W = v\rho g$, where v is the volume of the vessel submerged in water.

Also $W + v\rho g = V\rho g$ or $\dfrac{v}{V} = \dfrac{1}{2}$

Volume of the cone = $\dfrac{1}{2}\pi r^2 h = \dfrac{1}{2}\pi \tan^2 \alpha h^3$, where α is the semi-vertical angle

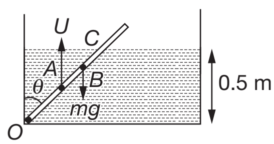
$$\Rightarrow \frac{v}{V} = \left(\frac{h}{H}\right)^3 \text{ or } \frac{1}{2} = \left(\frac{h}{H}\right)^3$$

$$\therefore \left(\frac{h}{H}\right) = \left(\frac{1}{2}\right)^{1/3}$$

$n = 3$

97. We have $OA = \dfrac{OC}{2} = \dfrac{0.5}{2\cos\theta}$

Let the mass per unit length of the plank be ρ.



Its weight $mg = l\rho g$.

The mass of the part OC of the plank $= \left(\dfrac{0.5}{\cos\theta}\right)\rho$.

The mass of water displaced $= \dfrac{1}{0.5}\dfrac{0.5}{\cos\theta}\rho$

The buoyant force F is, therefore, $F = \dfrac{\rho g}{\cos\theta}$

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, $mg(OB)\sin\theta = F(OA)\sin\theta$

or $\cos\theta = \dfrac{1}{\sqrt{2}}$

or $\theta = 45° = \dfrac{\pi}{4}$

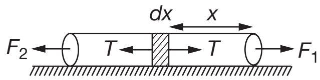
$n = 4$

98. Net force upward = weight of liquid in capillary.

$$\left[\frac{2\pi d_2}{2}T+\frac{2\pi d_1}{2}T\right]=\pi\left(\frac{d_2^2}{4}-\frac{d_1^2}{4}\right)\rho gh$$

$h=$ 56 mm

99. $dl=\frac{T}{S}\frac{dx}{Y}$ and $T=F_1-(F_1-F_2)\frac{x}{\ell}$



$$\int_0^{\Delta\ell} dl=\int_0^{\ell}\frac{F_1-\frac{(F_1-F_2)x}{\ell}dx}{SY}$$

$$\Delta\ell=\frac{(F_1+F_2)}{2SY}\ell=\frac{200\times 1}{2\times 0.5\times 2\times 10^{11}}=100\times 10^{-11}\text{ m}$$

$x=100$

100. Let the length of second liquid passed in A be y.

Let the first liquid come down to a level x in arm A.

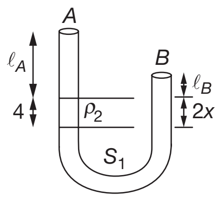
$\rho_2 gy=\rho_1 g2x$

$x=\frac{y}{4}$

$\ell_A=(\ell_1-h)-\left(y-\frac{y}{4}\right)$

$=\ell_1-h-\frac{3y}{4}$

$\ell_A=\ell_2-h+\frac{y}{4}$



According to problem,

$l_A=3l_B$ (I = overtone of A = fundamental tone of B)

$\Rightarrow\ y=\frac{2(l_1-3l_2+2h)}{3}$

$y=2$ m.

101. Loss of weight = (12.9 –11.3) gf = 1.6 gf

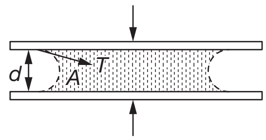
Weight of water displaced = 1.6 gf

If m is the mass of Cu, then $\left[\frac{m}{8.9}+\frac{12.9-m}{7.1}\right]g=1.6$ g

$7.1\text{ m}+12.9\times 8.9-8.9\text{ m}=1.6\times 8.9\times 7.1$

$1.8\text{ m}=114.8-101.1\Rightarrow m=7.6$ grm = 76 decigram

102. Let R be the radius of the circular layer of water.



Then, $\pi R^2 d\times\rho=m$ (1)

Pressure at $A=p_0-\frac{2T}{d}$ (2)

Thus, pressure between the plates is less than the atmospheric pressure and so the plates are pressed together as though attracted towards each other.

F, force of attraction = $\Delta p\times$ area $\Rightarrow F=\frac{2T}{d}\times\pi R^2$

$$\Rightarrow\ F=\frac{2T}{d}\times\frac{m}{d\cdot\rho}=\frac{2Tm}{d^2\cdot\rho}=\frac{2\times 0.4\times 10^{-3}\times 0.01}{0.01^2\times 10^{-4}\times 1000}=8\text{ N}$$

Previous Years' Questions

103. Energy stored per unit volume $=\frac{1}{2}\times\text{Stress}\times\text{Strain}$

$$=\frac{\text{Stress}\times\text{Strain}}{2Y}=\frac{S^2}{2Y}$$

The correct option is (B)

104. Young's modulus $Y=\frac{FL}{A\ell}$ (1)

$\therefore\ F=\frac{YA\ell}{L}$

or $dW=Fd\ell=\frac{YA\ell(dl)}{L}$

or $\int dW=\frac{YA}{L}\int_0^{\ell}\ell d\ell=\frac{YA\ell^2}{2L}$

or workdone $=\frac{YA\ell^2}{2L}$

Workdone $=\frac{F\ell}{2}$

The correct option is (D)

105. Elastic energy per unit volume

$=\frac{1}{2}\times\text{stress}\times\text{strain}$

$\therefore$ Elastic energy

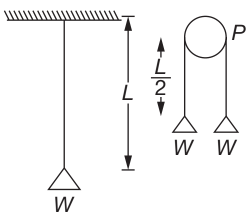
$=\frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume}$

$=\frac{1}{2}\times\frac{F}{A}\times\frac{\Delta L}{L}\times(AL)$

$=\frac{1}{2}F\Delta L\frac{1}{2}\times 200\times 10^{-3}=0.1$ J

The correct option is (D)

106.



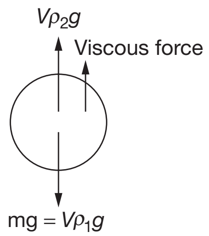
$$Y = \frac{\text{Force} \times L}{A \times \ell} = \frac{WL}{A\ell}$$

$$\therefore \ \ell = \frac{WL}{AY}$$

Due to pulley arrangement, the length of wire is $L/2$ on each side and so the elongation will be $\ell/2$. For both sides, elongation $= \ell$

The correct option is (A)

107. The forces acting on the solid ball when it is falling through a liquid are mg downwards, thrust by Archimedes principle upwards and the force due to the force of friction also acting upwards. The viscous force rapidly increases with velocity, attaining a maximum when the ball reaches the terminal velocity. Then the acceleration is zero. $mg - V\rho_2 g - kv^2 = ma$, where V is volume and v is the terminal velocity. When the ball is moving with terminal velocity, a = 0. Therefore, $V\rho_1 g - V\rho_2 g - kv^2 = 0$.



The correct option is (A)

108. Liquid 1 is over liquid 2. Therefore, $\rho_1 < \rho_2$. If ρ_3 was greater then ρ_2, it will not be partially inside but anywhere inside liquid 2 if $\rho_3 = \rho_2$ or it would have sunk totally, if ρ_3 had been greater than ρ_2.

$\therefore \quad \rho_1 < \rho_3 < \rho_2$.

The correct option is (B)

109. Terminal velocity = v

Viscous force upwards = weight of sphere downwards

or $\quad 6\pi\eta r v = \left(\frac{4}{3}\pi r^3\right)(\rho - \sigma)g$

For gold and silver spheres falling in viscous liquid,

$$\therefore \quad \frac{v_g}{v_s} = \frac{\rho_g - \sigma}{\rho_s - \sigma} = \frac{19.5 - 1.5}{10.5 - 1.5} = \frac{18}{9} = \frac{2}{1}$$

or $\quad \therefore \ v_s = \frac{v_g}{2} = \frac{0.2}{2} = 0.1\,\text{m/s}$

The correct option is (C)

110. In a freely falling elevator, $g = 0$

Water will rise to the fall length, i.e., 20 cm to tube.

The correct option is (D)

111. Retarding viscous force $= 6\pi\eta R v$

Obviously, option (B) holds goods.

The correct option is (B)

112. Pressure inside the bubble $= P_0 + \frac{4T}{r}$

Smaller the radius, greater will be the pressure. Air flows from higher pressure to lower pressure. Hence, air flows from the smaller bubble to the bigger bubble.

The correct option is (C)

113. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s

The correct option is (B)

114. Let k be the spring constant of spring and it gets extended by length L in equilibrium position. In equilibrium,

$$kx_0 + F_B = Mg$$

$$kx_0 + \sigma \frac{L}{2} Ag = Mg$$

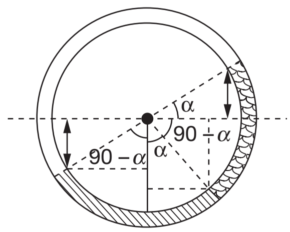
$$x_0 = \frac{Mg - \dfrac{\sigma L A g}{2}}{k}$$

$$= \frac{Mg}{k}\left(1 - \frac{\sigma L A}{2M}\right)$$

The correct option is (B)

115. The correct option is (C)

116.



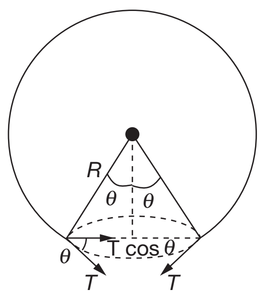
$$d_1 gR(1 - \sin\alpha) = d_2 gR\cos\alpha + d_2 gR\sin\alpha + d_1 gR(1 - \cos\alpha)$$

$$d_1(1 - \sin\alpha - 1 + \cos\alpha) = d_2(\cos\alpha + \sin\alpha)$$

$$\frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

The correct option is (C)

117.



$$\frac{4}{3}\pi R^3 \rho_w g = 2\pi r \sin\theta$$

$$\frac{4}{3}\pi R^3 \rho_w g = 2\pi r \frac{r}{R}$$

$$r = \sqrt{\frac{2}{3} R^4 \rho_w g} = \left(\sqrt{\frac{2}{3}\rho_w g}\right) R^2$$

No option is correct

118. $T = 2\pi\sqrt{\frac{\ell}{g}}$; $T_M = 2\pi\sqrt{\frac{\ell'}{g}}$

$$\Delta\ell = \frac{Mg\ell}{YA}$$

$$\ell' - \ell = \frac{Mg\ell}{YA}$$

$$\frac{1}{Y} = (\ell' - \ell)\frac{A}{Mg\ell}$$

$$= \left[\frac{T_M^2}{T^2} - 1\right]\frac{A}{Mg}$$

The correct option is (D)