CHAPTER 5

Impulse and Momentum

Chapter Highlights

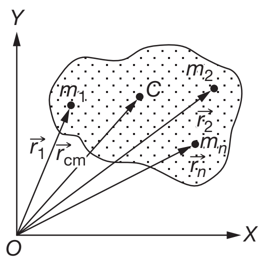
Centre of mass of a two-particle system, Centre of mass of a rigid body, Motion of centre of mass, Momentum, Law of conservation of linear momentum and its applications, Impulse, Elastic and inelastic collisions in one and two dimensions.

CENTRE OF MASS

Every physical system has associated with it a certain point whose motion characterizes the motion of the whole system. When the system moves under some external forces, then this point moves as if the entire mass of the system is concentrated at this point and also the external force is applied at this point for translational motion. This point is called the centre of mass of the system.

Centre of Mass of a System of 'N' Discrete Particles

Consider a system of N point masses m_1, m_2, m_3, … m_n whose position vectors from origin O are given by $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, … $\vec{r}_n$ respectively. Then the position vector of the centre of mass C of the system is given by.



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n}; \quad \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$

$$\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

where, $m_i \vec{r}_i$ is called the moment of mass of the particle with respect to O.

$$M = \left(\sum_{i=1}^{n} m_i\right) \text{ is the total mass of the system.}$$

NOTE

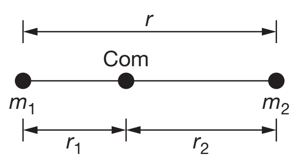
If the origin is taken at the centre of mass then $\sum_{i=1}^{n} m_i \vec{r}_i = 0$.

Hence, the COM is the point about which the sum of "mass moments" of the system is zero.

Position of COM of Two Particles

Centre of mass of two particles of masses m_1 and m_2 separated by a distance r lies in between the two particles. The distance of centre of mass from any of the particle (r) is inversely proportional to the mass of the particle (m)

i.e. $$r \propto \frac{1}{m}$$



or $$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

or $$m_1 r_1 = m_2 r_2$$

or $$r_1 = \left(\frac{m_2}{m_2 + m_1}\right) r \text{ and } r_2 = \left(\frac{m_1}{m_1 + m_2}\right) r$$

Here, r_1 = distance of COM from m_1
and r_2 = distance of COM from m_2

From the above discussion, we see that

$r_1 = r_2 = \frac{1}{2}$ if $m_1 = m_2$, i.e., COM lies midway between the two particles of equal masses.

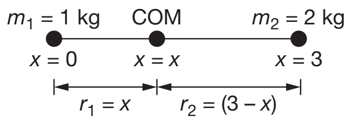
Similarly, $r_1 > r_2$ if $m_1 < m_2$ and $r_1 < r_2$ if $m_2 < m_1$, i.e., COM is nearer to the particle having larger mass.

SOLVED EXAMPLES

1. Two particles of mass 1 kg and 2 kg are located at $x = 0$ and $x = 3$ m. Find the position of their centre of mass.

Solution:

Since, both the particles lies on x-axis, the COM will also lie on x-axis. Let the COM is located at $x = x$, then r_1 = distance of COM from the particle of mass 1 kg = x and r_2 = distance of COM from the particle of mass 2 kg



$= (3 - x)$

Using $$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

or $$\frac{x}{3-x} = \frac{2}{1}$$

or $$x = 2 \text{ m}$$

Thus, the COM of the two particles is located at $x = 2$ m.

2. The position vector of three particles of masses m_1 = 1 kg, m_2 = 2 kg and m_3 = 3 kg are $\vec{r_1} = (\hat{i} + 4\hat{j} + \hat{k})$ m, $\vec{r_2} = (\hat{i} + \hat{j} + \hat{k})$ m and $\vec{r_3} = (2\hat{i} - \hat{j} - 2\hat{k})$ m respectively. Find the position vector of their centre of mass.

Solution:

The position vector of COM of the three particles will be given by

$$\vec{r}_{COM} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3}}{m_1 + m_2 + m_3}$$

Substituting the values, we get

$$\vec{r}_{COM} = \frac{(1)(\hat{i} + 4\hat{j} + \hat{k}) + (2)(\hat{i} + \hat{j} + \hat{k}) + (3)(2\hat{i} - \hat{j} - 2\hat{k})}{1+2+3}$$

$$= \frac{1}{2}(3\hat{i} + \hat{j} - \hat{k})\text{ m.}$$

3. Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices *A, B, C* and *D* of a square of side 1 m. Find the position of centre of mass of the particles.

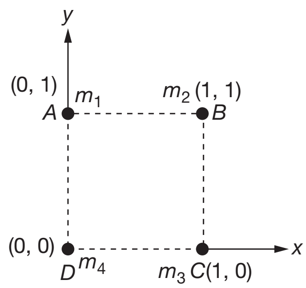
Solution:

Assuming D as the origin, DC as x-axis and DA as y-axis, we have

$$m_1 = 1 \text{ kg}, (x_1, y_1) = (0, 1 \text{ m})$$
$$m_2 = 2 \text{ kg}, (x_2, y_2) = (1 \text{ m}, 1 \text{ m})$$
$$m_3 = 3 \text{ kg}, (x_3, y_3) = (1 \text{ m}, 0)$$

and $$m_4 = 4 \text{ kg}, (x_4, y_4) = (0, 0)$$

Co-ordinates of their COM are



$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3m_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(1)(0) + 2(1) + 3(1) + 4(0)}{1+2+3+4}$$

$$= \frac{5}{10} = \frac{1}{2} \text{ m} = 0.5 \text{ m}$$

Similarly, $$y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(1)(1) + 2(1) + 3(0) + 4(0)}{1+2+3+4}$$

$$= \frac{3}{10} \text{ m}$$

$$= 0.3 \text{ m}$$

$\therefore$ $$(x_{COM}, y_{COM}) = (0.5 \text{ m}, 0.3 \text{ m}).$$

Thus, position of COM of the four particles is as shown in Fig. 5.1.

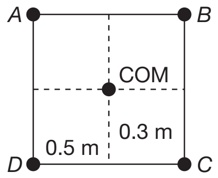


Fig. 5.1

4. Three particles of masses 0.5 kg, 1.0 kg and 1.5 kg are placed at the three corners of a right angled triangle of sides 3.0 cm, 4.0 cm and 5.0 cm as shown in Fig. 5.2. Locate the centre of mass of the system.

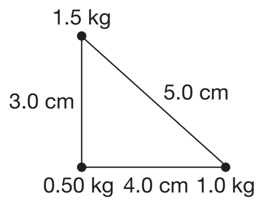


Fig. 5.2

Solution:

The centre of mass is 1.3 cm to the right and 1.5 cm above the 0.5 kg particle.

5. Consider a two-particle system with the particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved so as to keep the centre of mass at the same position?

Solution:

Consider Fig. 5.3. Suppose the distance of m_1 from the centre of mass C is x_1 and that of m_2 from C is x_2. Suppose the mass m_2 is moved through a distance d' towards C so as to keep the centre of mass at C.

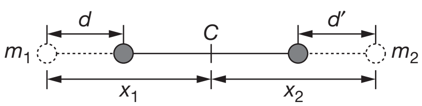


Fig. 5.3

Then,

$$m_1x_1 = m_2x_2 \quad (1)$$

and $$m_1(x_1 - d) = m_2(x_2 - d'). \quad (2)$$

Subtracting (2) from (1)

$$m_1d = m_2 d'$$

or, $$d' = \frac{m_1}{m_2} d,$$

Centre of Mass of a Continuous Mass Distribution

For continuous mass distribution the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

$$x_{cm} = \frac{\int x\,dm}{\int dm},\; y_{cm} = \frac{\int y\,dm}{\int dm},\; z_{cm} = \frac{\int z\,dm}{\int dm}$$

$$\int dm = M \text{ (mass of the body)}$$

$$\vec{r}_{cm} = \frac{1}{M} \cdot \int \vec{r}dm$$

NOTE

If an object has symmetric mass distribution about x axis then y coordinate of COM is zero and vice-versa.

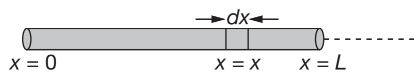
Centre of Mass of a Uniform Rod

Suppose a rod of mass M and length L is lying along the x-axis with its one end at $x = 0$ and the other at $x = L$.

Mass per unit length of the rod $= \frac{M}{L}$

Hence, dm, (the mass of the element dx situated at $x = x$ is) $= \frac{M}{L}\, dx$

The coordinates of the element dx are $(x, 0, 0)$. Therefore, x-coordinate of COM of the rod will be



$$x_{COM} = \frac{\int_0^L x\,dm}{\int dm}$$

$$= \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M}$$

$$= \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

The y-coordinate of COM is

$$y_{COM} = \frac{\int y\,dm}{\int dm} = 0$$

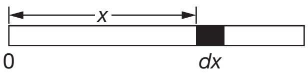
Similarly,

$$z_{COM} = 0$$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$, i.e. it lies at the centre of the rod.

SOLVED EXAMPLE

6. The linear mass density of a straight rod of length L varies as $\rho = A + Bx$ where x is the distance from the left end. Locate the centre of mass.



Solution:

$$\frac{3AL + 2BL^2}{3(2A + BL)}$$

Centre of Mass of a Semicircular Ring

Figure 5.4 shows the object (semicircular ring). By observation we can say that the x-coordinate of the centre of mass of the ring is zero as the half ring is symmetrical about y-axis on both sides of the origin. Only we are required to find the y-coordinate of the centre of mass.

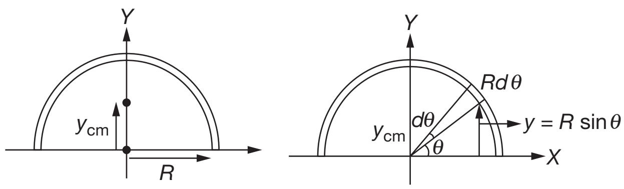


Fig. 5.4

To find y_{cm} we use

$$y_{cm} = \frac{1}{M}\int dm\, y \qquad (1)$$

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width $d\theta$. If radius of the ring is R then its y-coordinate will be $R\sin\theta$, here dm is given as

$$dm = \frac{M}{\pi R} \times R\, d\theta$$

So from Equation (1), we have

$$y_{cm} = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi R} R d\theta\,(R\sin\theta)$$

$$= \frac{R}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

$$y_{cm} = \frac{2R}{\pi} \qquad (2)$$

Centre of Mass of Semicircular Disc

Figure 5.5 shows the half disc of mass M and radius R. Here, we are only required to find the y-coordinate of the centre of mass of this disc as centre of mass will be located on its half vertical diameter. Here to find y_{cm}, we consider a small elemental ring of mass dm of radius x on the disc (disc can be considered to be made up such thin rings of increasing radii) which will be integrated from 0 to R. Here dm is given as

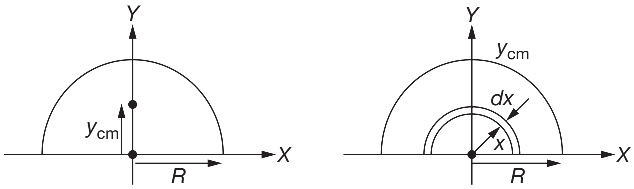


Fig. 5.5

$$dm = \frac{2M}{\pi R^2}(\pi x)\, dx$$

Now the y-coordinate of the element is taken as $\frac{2x}{\pi}$, as in previous section, we have derived that the centre of mass of a semicircular ring is concentrated at $\frac{2R}{\pi}$

Here y_{cm} is given as

$$y_{cm} = \frac{1}{M}\int_0^R dm\, \frac{2x}{\pi}$$

$$= \frac{1}{M}\int_0^R \frac{4M}{\pi R^2} x^2\, dx$$

$$y_{cm} = \frac{4R}{3\pi}.$$

SOLVED EXAMPLE

7. Find the centre of mass of an annular half disc shown in Fig. 5.6.

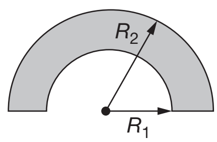


Fig. 5.6

Solution:

Let ρ be the mass per unit area of the object. To find its centre of mass we consider an element as a half ring of mass dm as shown in Fig 5.7 of radius r and width dr and there we have

Now, $\qquad dm = \rho\pi r\, dr$

Centre of mass of this half ring will be at height $\frac{2r}{\pi}$

$$y_{cm} = \frac{1}{M}\int_{R_1}^{R_2} (\rho\cdot\pi r\, dr)\cdot\frac{2r}{\pi}$$

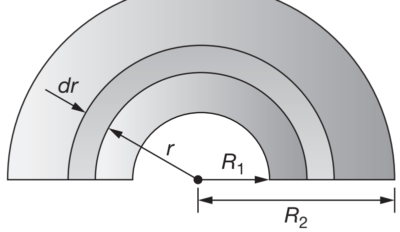


Fig. 5.7

$$y_{cm} = \frac{2\rho}{\rho \frac{\pi}{2}(R_2^2 - R_1^2)} \int_{R_1}^{R_2} r^2\, dr = \frac{4(R_2^3 - R_1^3)}{3\pi(R_2^2 - R_1^2)}$$

Alternative Solution:

We can also find the centre of mass of this object by considering it to be complete half disc of radius R_2 and a smaller half disc of radius R_1 cut from it. If y_{cm} be the centre of mass of this disc we have from the mass moments.

$$\left(\rho \cdot \frac{\pi R_1^2}{2}\right) \times \left(\frac{4R_1}{3\pi}\right) + \left(\rho \cdot \frac{\pi}{2}(R_2^2 - R_1^2)\right)(y_{cm})$$

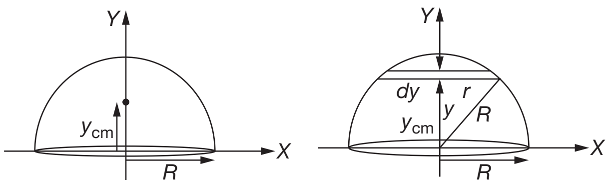
$$= \left(\rho \cdot \frac{\pi R_2^2}{2}\right) \times \left(\frac{4R_2}{3\pi}\right)$$

$$y_{cm} = \frac{4(R_2^3 - R_1^3)}{3\pi(R_2^2 - R_1^2)}$$

Centre of Mass of a Solid Hemisphere

The hemisphere is of mass M and radius R. To find its centre of mass (only y-coordinate), we consider an element disc of width dy, mass dm at a distance y from the centre of the hemisphere. The radius of this elemental disc will be given as

$$R = \sqrt{R^2 - y^2}$$



The mass dm of this disc can be given as

$$dm = \frac{3M}{2\pi R^3} \times \pi r^2\, dy$$

$$= \frac{3M}{2R^3}\,(R^2 - y^2)\, dy$$

y_{cm} of the hemisphere is given as

$$y_{cm} = \frac{1}{M} \int_0^R dm\, y$$

$$= \frac{1}{M} \int_0^R \frac{3M}{2R^3}\,(R^2 - y^2)\, dy\, y$$

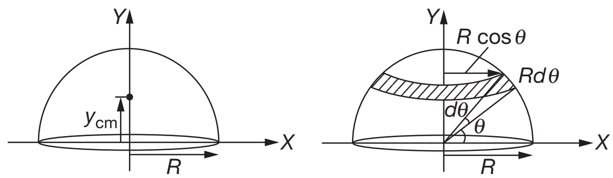
$$= \frac{3}{2R^3} \int_0^R (R^2 - y^2)\, y\, dy$$

$$y_{cm} = \frac{3R}{8}.$$

Centre of Mass of a Hollow Hemisphere

Take a hollow hemisphere of mass M and radius R. Now we consider an elemental circular strip of angular width $d\theta$ at an angular distance θ from the base of the hemisphere. This strip will have an area.

$$dS = 2\pi R \cos\theta\, R d\theta$$



Its mass dm is given as

$$dm = \frac{M}{2\pi R^2}\, 2\pi R \cos\theta\, R d\theta$$

Here y-coordinate of this strip of mass dm can be taken as $R \sin\theta$. Now we can obtain the centre of mass of the system as.

$$y_{cm} = \frac{1}{M} \int_0^{\frac{\pi}{2}} dm\, R\sin\theta$$

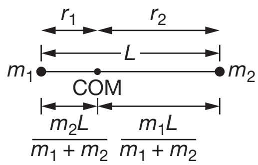
$$= \frac{1}{M} \int_0^{\frac{\pi}{2}} \left(\frac{M}{2\pi R^2} 2\pi R^2 \cos\theta\, d\theta\right) R \sin\theta$$

$$= R \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta\, d\theta$$

$$y_{cm} = \frac{R}{2}$$

Proceeding in the similar manner, we can find the COM of certain rigid bodies. Centre of mass of some well-known rigid bodies are given below:

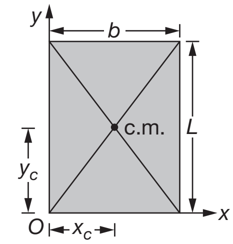
Centre of Mass of Some Common Systems



A system of two point masses $m_1 r_1 = m_2 r_2$

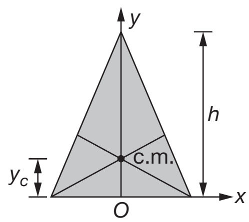
The centre of mass lies closer to the heavier mass.

Rectangular plate (By symmetry)



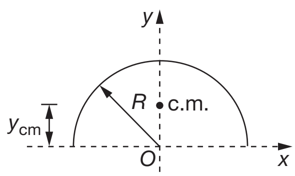
$$x_c = \frac{b}{2} \qquad y_c = \frac{L}{2}$$

A triangular plate (By qualitative argument)



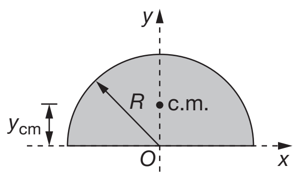
at the centroid: $y_c = \frac{h}{3}$

A semi-circular ring



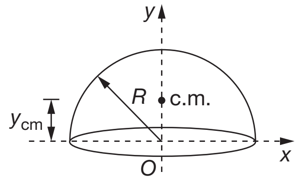
$$y_c = \frac{2R}{\pi} \qquad x_c = 0$$

A semi-circular disc



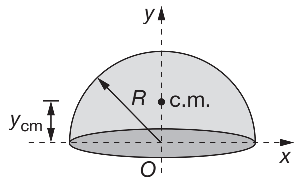
$$y_c = \frac{4R}{3\pi} \qquad x_c = 0$$

A hemispherical shell



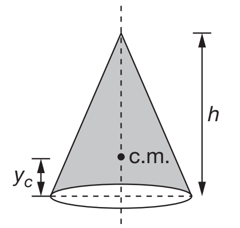
$$y_c = \frac{R}{2} \qquad x_c = 0$$

A solid hemisphere



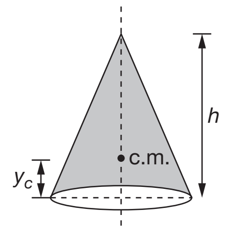
$$y_c = \frac{3R}{8} \qquad x_c = 0$$

A circular cone (solid)



$$y_c = \frac{h}{4}$$

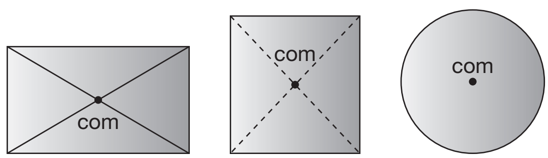
A circular cone (hollow)



$$y_c = \frac{h}{3}$$

NOTE

1. Centre of mass of a uniform rectangular, square or circular plate lies at its centre. Axis of symmetry plane of symmetry.



2. If some mass of area is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formulae:

(i) $\vec{r}_{COM} = \dfrac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2}$ or $\vec{r}_{COM} = \dfrac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$

(ii) $x_{COM} = \dfrac{m_1x_1 - m_2x_2}{m_1 - m_2}$ or $x_{COM} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

$y_{COM} = \dfrac{m_1y_1 - m_2y_2}{m_1 - m_2}$ or $y_{COM} = \dfrac{A_1y_1 - A_2y_2}{A_1 - A_2}$

and $z_{COM} = \dfrac{m_1z_1 - m_2z_2}{m_1 - m_2}$ or $z_{COM} = \dfrac{A_1z_1 - A_2z_2}{A_1 - A_2}$

Here, m_1, A_1, $\vec{r}_1$, x_1, y_1 and z_1 are the values for the whole mass while m_2, A_2, $\vec{r}_2$, $\vec{x}_2$, y_2 and z_2 are the values for the mass which has been removed. Let us see two examples in support of the above theory.

SOLVED EXAMPLES

8. Find the position of centre of mass of the uniform lamina shown in Fig. 5.8.

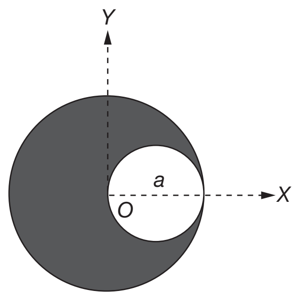


Fig. 5.8

Solution:

Here,

A_1 = area of complete circle = πa^2

A_2 = area of small circle = $\pi\left(\dfrac{a}{2}\right)^2 = \dfrac{\pi a^2}{4}$

(x_1, y_1) = coordinates of centre of mass of large circle = (0, 0)

and (x_2, y_2) = coordinates of centre of mass of small circle = $\left(\dfrac{a}{2}, 0\right)$

Using

$$x_{COM} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

we get

$$x_{COM} = \frac{-\dfrac{\pi a^2}{4}\left(\dfrac{a}{2}\right)}{\pi a^2 - \dfrac{\pi a^2}{4}} = \frac{-\left(\dfrac{1}{8}\right)}{\left(\dfrac{3}{4}\right)}a = -\frac{a}{6}$$

and $y_{COM} = 0$ as y_1 and y_2 both are zero.

Therefore, coordinates of COM of the lamina shown in Fig. 5.8 are $\left(-\dfrac{a}{6}, 0\right)$.

9. The centre of mass of rigid body always lies inside the body. Is this statement true or false?

Solution:

False

10. The centre of mass always lies on the axis of symmetry if it exists. Is this statement true of false?

Solution:

True

11. If all the particles of a system lie in y-z plane, the x-coordinate of the centre of mass will be zero. Is this statement true or not?

Solution:

True

MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM

Velocity of Centre of Mass of System

$$\vec{v}_{cm} = \frac{m_1\dfrac{d\vec{r}_1}{dt} + m_2\dfrac{d\vec{r}_2}{dt} + m_3\dfrac{d\vec{r}_3}{dt}\,.............+ m_n\dfrac{d\vec{r}_n}{dt}}{M}$$

$$= \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3\,.........+ m_n\vec{v}_n}{M}$$

Here numerator of the right hand side term is the total momentum of the system i.e., summation of momentum of the individual component (particle) of the system.

Hence velocity of centre of mass of the system is the ratio of momentum of the system to the mass of the system.

$$\therefore \quad \vec{P}_{System} = M\vec{v}_{cm}$$

Acceleration of Centre of Mass of System

$$\vec{a}_{cm} = \frac{m_1\dfrac{d\vec{v}_1}{dt} + m_2\dfrac{d\vec{v}_2}{dt} + m_3\dfrac{d\vec{v}_3}{dt}\,.............+ m_n\dfrac{d\vec{v}_n}{dt}}{M}$$

$$= \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 ....+ m_n\vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M}$$

$$= \frac{\text{Net External Force} + \text{Net internal Force}}{M}$$

$$= \frac{\text{Net External Force}}{M}$$

($\because$ action and reaction both of an internal force must be within the system. Vector summation will cancel all internal forces and hence net internal force on system is zero)

$$\therefore \qquad \vec{F}_{ext} = M\vec{a}_{cm}$$

where $\vec{F}_{ext}$ is the sum of the 'external' forces acting on the system. The internal forces in which the particles exert force on one another play absolutely no role in the motion of the centre of mass.

If no external force is acting on a system of particles, the acceleration of centre of mass of the system will be zero. If $a_c = 0$, it implies that v_c must be a constant and if v_{cm} is a constant, it implies that the total momentum of the system must remain constant. It leads to the principal of conservation of momentum in absence of external forces.

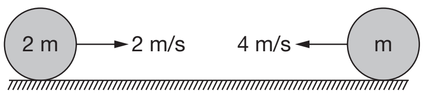
If $\vec{F}_{ext} = 0$ then $\vec{v}_{cm}$ = constant

'If resultant external force is zero on the system, then the net momentum of the system must remain constant'. Motion of COM in a moving system of particles:

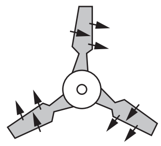
COM at Rest

If $F_{ext} = 0$ and $v_{cm} = 0$, then COM remains at rest. Individual components of the system may move and have non-zero momentum due to mutual forces (internal), but the net momentum of the system remains zero.

1. All the particles of the system are at rest.
2. Particles are moving such that their net momentum is zero.
   Example:



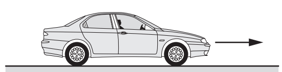
3. A bomb at rest suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal and there is no external force on the system for explosion therefore, the COM of the bomb will remain at the original position and the fragment fly such that their net momentum remains zero.
4. Two men standing on a frictionless platform push each other and then also their net momentum remains zero because the push forces are internal for the two men system.
5. A boat floating in a lake, also has net momentum zero if the people on it changes their position, because the friction force required to move the people is internal of the boat system.
6. Objects initially at rest, if moving under mutual forces (electrostatic or gravitation) also have net momentum zero.
7. A light spring of spring constant k kept compressed between two blocks of masses m_1 and m_2 on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions, such that the net momentum is zero.
8. In a fan, all particles are moving but COM is at rest



COM Moving with Uniform Velocity

If $F_{ext} = 0$, then v_{cm} remains constant therefore, net momentum of the system also remains conserved. Individual components of the system may have variable velocity and momentum due to mutual forces (internal), but the net momentum of the system remains constant and COM continues to move with the initial velocity.

1. All the particles of the system are moving with same velocity.
   Example: A car moving with uniform speed on a straight road has its COM moving with a constant velocity.



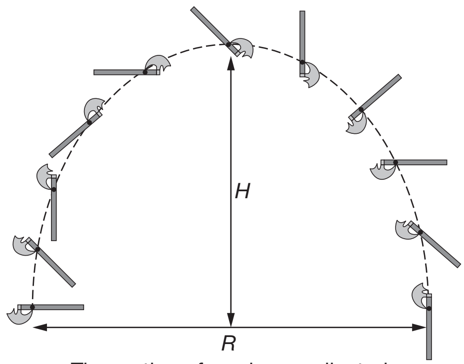
2. Internal explosions/breaking do not change the motion of COM and net momentum remains conserved. A bomb moving in a straight line suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal and there is no external force on the system for explosion therefore, the COM of the bomb will continue the original motion and the fragment fly such that their net momentum remains conserved.
3. Man jumping from cart or buggy also exerts internal forces therefore net momentum of the system and hence, Motion of COM remains conserved.

4. Two moving blocks connected by a light spring on a smooth horizontal surface. If the acting force is only due to spring then COM will remain in its motion and momentum will remain conserved.
5. Particles colliding in absence of external impulsive forces also have their momentum conserved.

COM Moving with Acceleration

If an external force is present then COM continues its original motion as if the external force is acting on it, irrespective of internal forces.

Example:
Projectile Motion: An axe thrown in air at an angle θ with the horizontal will perform a complicated motion of rotation as well as parabolic motion under the effect of gravitation

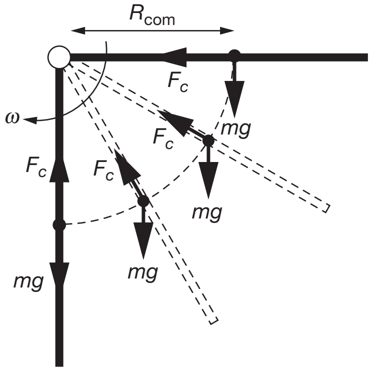


The motion of axe is complicated but the COM is moving in a parabolic motion.

$$H_{\text{com}} = \frac{u^2 \sin^2 \theta}{2g} \qquad R_{\text{com}} = \frac{u^2 \sin 2\theta}{g} \qquad T = \frac{2u \sin\theta}{g}$$

Example:
Circular Motion: A rod hinged at an end, rotates, than its COM performs circular motion. The centripetal force (F_c) required in the circular motion is assumed to be acting on the COM.

$$F_c = m\omega^2 R_{\text{COM}}$$

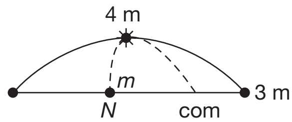


SOLVED EXAMPLES

12. A projectile is fired at a speed of 100 m/s at an angle of 37° above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1: 3, the lighter piece coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Solution:

Internal force do not affect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is,



$$x_{\text{COM}} = \frac{2u^2 \sin\theta \cos\theta}{g} = \frac{2\times10^4 \times \frac{3}{5} \times \frac{4}{5}}{10} \text{ m}$$

$= 960$ m.

The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at $x = 480$ m. If the heavier block hits the ground at x_2, then

$$x_{\text{COM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$960 = \frac{(m)(480) + (3m)(x_2)}{(m + 3m)}$$

$x_2 = 1120$ m.

13. In a boat of mass 4 M and length ℓ on a frictionless water surface. Two men A (mass = M) and B (mass $2M$) are standing on the two opposite ends. Now A travels a distance $\ell/4$ relative to boat towards its centre and B moves a distance $3\ell/4$ relative to boat and meet A. Find the distance travelled by the boat on water till A and B meet.

Solution:

$5\ell/28$.

14. A block A (mass = 4 M) is placed on the top of a wedge B of base length ℓ (mass = 20 M) as shown in Fig. 5.9. The system is released from rest. Find the distance moved by the wedge B till the block A reaches ground. Assume all surfaces are frictionless.

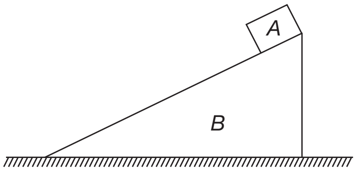


Fig. 5.9

Solution:

$$\ell/6$$

15. An isolated particle of mass m is moving in a horizontal xy plane, along x-axis. At a certain height above ground, it suddenly explodes into two fragments of masses m/4 and 3m/4. An instant later, the smaller fragment is at $y = +15$ cm. Find the position of heavier fragment at this instant.

Solution:

$$y = -5 \text{ cm}$$

Momentum Conservation

The total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.

$$\vec{P} = M\vec{v}_{cm}$$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

If $\quad \vec{F}_{ext} = 0 \Rightarrow \frac{d\vec{P}}{dt} = 0; \quad \vec{P} = \text{constant}$

When the vector sum of the external forces acting on a system is zero, the total linear momentum of the system remains constant.

$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \ldots + \vec{P}_n = \text{constant.}$$

SOLVED EXAMPLES

16. A shell is fired from cannon with a speed of 100 m/s at an angle 60º with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal fragments. One of the fragments moves along the negative x-direction with a speed of 50 m/s. What is the speed of the other fragment at the time of explosion?

Solution:

As we know in absence of external force the motion of centre of mass of a body remains unaffected. Thus, here the centre of mass of the two fragments will continue to follow the original projectile path. The velocity of the shell at the highest point of trajectory is

$$v_M = u\cos\theta = 100 \times \cos 60° = 50 \text{ m/s.}$$

Let v_1 be the speed of the fragment which moves along the negative x-direction and the other fragment has speed v_2, which must be along positive x-direction. Now from momentum conservation, we have

$$mv = \frac{-m}{2} v_1 + \frac{m}{2} v_2$$

or $\quad 2v = v_2 - v_1$

or $\quad v_2 = 2v + v_1 = (2 \times 50) + 50 = 150$ m/s

17. A boy of mass 25 kg stands on a board of mass 10 kg which in turn is kept on a frictionless horizontal ice surface. The boy makes a jump with a velocity component 5 m/s in horizontal direction with respect to the ice. With what velocity does the board recoil? With what rate are the boy and the board separating from each other?

Solution:

$$v = 12.5 \text{ m/s; } 17.5 \text{ m/s.}$$

18. A man of mass m is standing on a platform of mass m kept on smooth ice. If the man starts moving on the platform with a speed v relative to the platform, with what velocity relative to the ice does the platform recoil?

Solution:

Consider the situation shown in Fig. 5.10. Suppose the man moves at a speed w towards right and the platform recoils at a speed V towards left, both relative to the ice. Hence, the speed of the man relative to the platform is $V + w$. By the question,

$$V + w = v, \quad \text{or} \quad w = v - V \qquad (1)$$

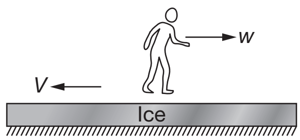


Fig. 5.10

Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant. Initially both the man and the platform were at rest. Thus,

$$0 = MV - mw$$

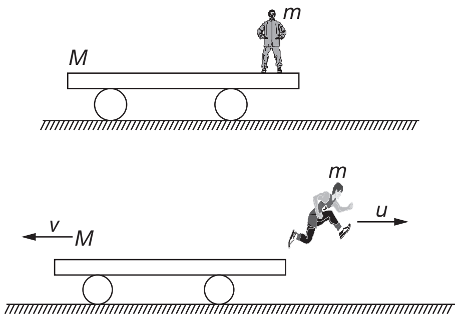
or, $\quad MV = m(v - V) \qquad$ [Using (1)]

or, $\quad V = \frac{mv}{M + m}.$

19. A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, now find the velocity of the car after its jump.

Solution:

Let car attains a velocity v, and the net velocity of the child with respect to earth will be $u - v$, as u is its velocity with respect to car.



Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero, as

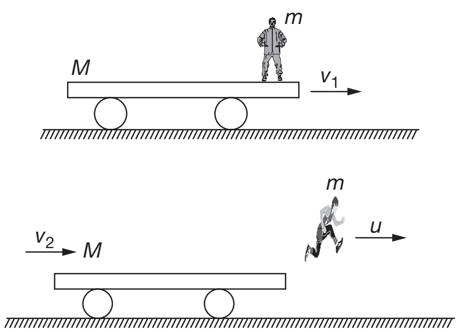
$$m\,(u - v) = M\,v$$

$$v = \frac{mu}{m + M}$$

20. A flat car of mass M with a child of mass m is moving with a velocity v_1 on a friction less surface. The child jumps in the direction of motion of car with a velocity u with respect to car. Find the final velocities of the child and that of the car after jump.

Solution:

This case is similar to the previous example, except now the car is moving before jump. Here also no external force is acting on the system in horizontal direction, hence momentum remains conserved in this direction. After jump car attains a velocity v_2 in the same direction, which is less than v_1, due to backward push of the child for jumping. After jump child attains a velocity $u + v_2$ in the direction of motion of car, with respect to ground.



According to momentum conservation

$$(M + m)v_1 = Mv_2 + m\,(u + v_2)$$

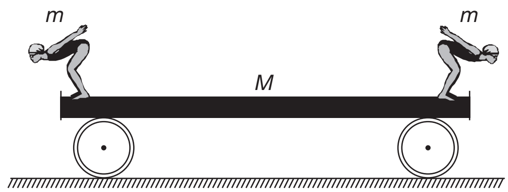
Velocity of car after jump is

$$v_2 = \frac{(M+m)v_1 - mu}{M+m}$$

Velocity of child after jump is

$$u + v_2 = \frac{(M+m)v_1 + (M)u}{M+m}$$

21. Two persons A and B, each of mass m are standing at the two ends of rail-road car of mass M. The person A jumps to the left with a horizontal speed u with respect to the car. Thereafter, the person B jumps to the right, again with the same horizontal speed u with respect to the car. Find the velocity of the car after both the persons have jumped off.



Solution:

$$\frac{m^2 u}{(M+2m)(M+m)}$$

22. Two identical buggies move one after the other due to inertia (without friction) with the same velocity v_0. A man of mass m jumps into the front buggy from the rear buggy with a velocity u relative to his buggy. Knowing that the mass of each buggy is equal to M, find the velocities with which the buggies will move after that.

Solution:

$$v_F = v_0 + \frac{Mmu}{(M+m)^2}\ ;\ v_A = v_0 - \frac{mu}{(M+m)}$$

23. Each of the blocks shown in Fig. 5.11 has mass 1 kg. The rear block moves with a speed of 2 m/s towards the front block kept at rest. The spring attached to the front block is light and has a spring constant 50 N/m. Find the maximum compression of the spring. Assume, on a friction less surface

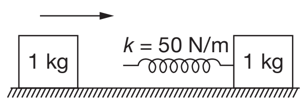


Fig. 5.11

Solution:

Maximum compression will take place when the blocks move with equal velocity. As no net external horizontal force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have,

$$(1 \text{ kg})\,(2 \text{ m/s}) = (1 \text{ kg})V + (1 \text{ kg})V$$

or, $$V = 1 \text{ m/s}.$$

Initial kinetic energy $= \frac{1}{2}\,(1 \text{ kg})\,(2 \text{ m/s})^2 = 2$ J.

Final kinetic energy

$$= \frac{1}{2}\,(1 \text{ kg})\,(1\text{m/s})^2 + \frac{1}{2}\,(1 \text{ kg})\,(1 \text{ m/s})^2$$

$$= 1 \text{ J}$$

The kinetic energy lost is stored as the elastic energy in the spring.

Hence,

$$\frac{1}{2}(50 \text{ N/m})\,x^2 = 2\text{J} - 1\text{J} = 1 \text{ J}$$

or, $$x = 0.2 \text{ m}.$$

24. Figure 5.12 shows two blocks of masses 5 kg and 2 kg placed on a frictionless surface and connected with a spring. An external kick gives a velocity 14 m/s to the heavier block towards the lighter one. Deduce

(A) velocity gained by the centre of mass and

(B) the separate velocities of the two blocks with respect to centre of mass just after the kick.

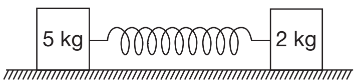


Fig. 5.12

Solution:

(A) Velocity of centre of mass is

$$v_{cm} = \frac{5\times 14 + 2\times 0}{5+2} = 10 \text{ m/s}$$

(B) Due to kick on 5 kg block, it starts moving with a velocity 14 m/s immediately, but due to inertia 2 kg block remains at rest, at that moment. Thus, velocity of 5 kg block with respect to the centre of mass is $v_1 = 14 - 10 = 4$ m/s and the velocity of 2 kg block with respect to centre of mass is $v_2 = 0 - 10 = -10$ m/s

25. A light spring of spring constant k is kept compressed between two blocks of masses m and M on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions. The spring loses contact with the blocks when it acquires natural length. If the spring was initially compressed through a distance x, find the final speeds of the two blocks.

Solution:

Consider the two blocks plus the spring to be the system. No external force acts on this system in horizontal direction. Hence, the linear momentum will remain constant. Suppose, the block of mass M moves with a speed v_1 and the other block with a speed v after losing contact with the spring. From conservation of linear momentum in horizontal direction we have

$$Mv_1 - mv_2 = 0 \quad \text{or} \quad v_1 = \frac{m}{M}\,v_2 \qquad (1)$$

Initially, the energy of the system $= \frac{1}{2}\,kx^2$

Finally, the energy of the system $= \frac{1}{2}mv_2^{\,2} + \frac{1}{2}Mv_1^{\,2}$

As there is no friction, mechanical energy will remain conserved.

Therefore,

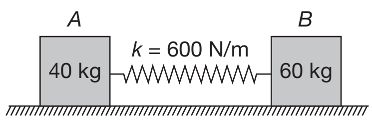
$$\frac{1}{2}mv_2^{\,2} + \frac{1}{2}Mv_1^{\,2} = \frac{1}{2}\,kx^2 \qquad (2)$$

Solving Equations (1) and (2), we get

or, $$v_2 = \left[\frac{kM}{m(M+m)}\right]^{\frac{1}{2}} x$$

and $$v_1 = \left[\frac{km}{M\,(M+m)}\right]^{\frac{1}{2}} x.$$

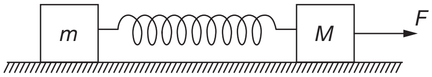
26. Blocks A and B have masses 40 kg and 60 kg respectively. They are placed on a smooth surface and the spring connected between them is stretched by 1.5 m. If they are released from rest, determine the speeds of both blocks at the instant when the spring is not stretched.



Solution:

3 m/s, 4.5 m/s

27. A block of mass m is connected to another block of mass M is a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane and are at rest. The spring is not stretched when a constant force F starts acting on the block of mass M to pull it. Find the maximum extension of the spring.



Solution:

We solve the situation in the reference frame of centre of mass. As only F is the external force acting on the system, due to this force, the acceleration of the centre of mass is $F/(M + m)$. Thus with respect to centre of mass there is a Pseudo force on the two masses in opposite direction, the free body diagram of m and M with respect to centre of mass (taking centre of mass at rest) is shown in Fig. 5.13.

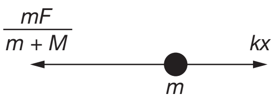


Fig. 5.13

Taking centre of mass at rest, if m moves maximum by a distance x_1 and M moves maximum by a distance x_2, then the work done by external forces (including Pseudo force) will be

$$W = \frac{mF}{m+M} \cdot x_1 + \left(F - \frac{MF}{m+M}\right) \cdot x_2$$

$$= \frac{mF}{m+M} \cdot (x_1 + x_2)$$

This work is stored in the form of potential energy of the spring as

$$U = \frac{1}{2} k(x_1 + x_2)^2$$

Thus on equating we get the maximum extension in the spring, as after this instant the spring starts contracting.

$$\frac{1}{2} k(x_1 + x_2)^2 = \frac{mF}{m+M} \cdot (x_1 + x_2)$$

$$x_{max} = x_1 + x_2 = \frac{2mF}{k(m+M)}$$

28. Two blocks of equal mass m are connected by a spring (not stretched) and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on one of the blocks pulling it away from the other as shown in Fig. 5.14.

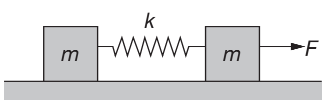


Fig. 5.14

(A) Find the displacement of the centre of mass at time t

(B) if the extension of the spring is x_0 at time t, find the displacement of the two blocks at this instant.

Solution:

(A) The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2} a_{COM}\, t^2 = \frac{Ft^2}{4m}.$$

(B) Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m}$$

or, $$\frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$$

or, $$x_1 + x_2 = \frac{Ft^2}{2m} \quad (1)$$

Further, the extension of the spring is $x_1 - x_2$. Therefore,

$$x_1 - x_2 = x_0 \quad (2)$$

From Equations (1) and (2),

$$x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$$

and $$x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$$

IMPULSE

Impulse of a force $\vec{F}$ acting on a body for the time interval $t = t_1$ to $t = t_2$ is defined as:

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt$$

$$\vec{I} = \int \vec{F} dt = \int m \frac{d\vec{v}}{dt} dt = \int m\, d\vec{v}$$

$$\vec{I} = m(\vec{v}_2 - \vec{v}_1) = \Delta\vec{P}$$

= change in momentum due to force $\vec{F}$

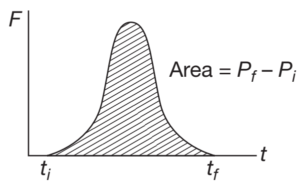
Also,

$$\vec{I}_{Res} = \int_{t_1}^{t_2} \vec{F}_{Res}\, dt = \Delta\vec{P}$$

(impulse – momentum theorem)

NOTE

Impulse applied to an object in a given time interval can also be calculated from the area under force time (F–t) graph in the same time interval.



Instantaneous Impulse

There are many cases when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknown but the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write.

$$\vec{I} = \int \vec{F}\,dt = \Delta\vec{P} = \vec{P}_{fi} - \vec{P}$$

IMPORTANT POINTS

- It is a vector quantity.
- Dimensions = [MLT^{-1}]
- SI unit = kg m/s
- Direction is along change in momentum.
- Magnitude is equal to area under the F-t. graph.
- $\vec{I} = \int \vec{F}dt = \vec{F}_{av}\int dt = \vec{F}_{av}\Delta t$
- It is not a property of a particle, but it is a measure of the degree to which an external force changes the momentum of the particle.

SOLVED EXAMPLES

29. The hero of a stunt film fires 50 g bullets from a machine gun, each at a speed of 1.0 km/s. If he fires 20 bullets in 4 seconds, what average force does he exert against the machine gun during this period?

Solution:
The momentum of each bullet

$$= (0.050 \text{ kg})\ (1000 \text{ m/s}) = 50 \text{ kg-m/s}.$$

The gun has been imparted this much amount of momentum by each bullet fired. Thus, the rate of change of momentum of the gun

$$= \frac{(50 \text{ kg-m/s}) \times 20}{4 \text{ s}} = 250 \text{ N}.$$

In order to hold the gun, the hero must exert a force of 250 N against the gun.

30. A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Find the impulse it receives.

Solution:
Using

Impulse = change in momentum

$$\vec{J} = m(\hat{i} + 3\hat{j}) - m(4\hat{i} - \hat{j})$$
$$= m(-3\hat{i} + 4\hat{j}).$$

IMPULSIVE FORCE

A force, of relatively higher magnitude and acting for relatively shorter time, is called impulsive force.

An impulsive force can change the momentum of a body in a finite magnitude in a very short time interval. Impulsive force is a relative term. There is no clear boundary between an impulsive and non-impulsive force.

NOTE

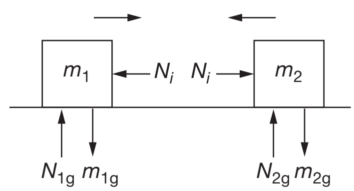
Usually colliding forces are impulsive in nature.

Since, the application time is very small, hence, very little motion of the particle takes place.

IMPORTANT POINTS

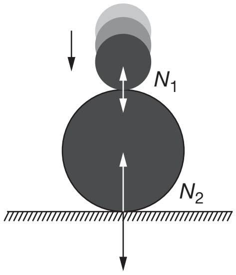
- Gravitational force and spring force are always non-Impulsive.
- Normal, tension and friction are case dependent.
- An impulsive force can only be balanced by another impulsive force.

1. **Impulsive Normal:** In case of collision, normal forces at the surface of collision are always impulsive

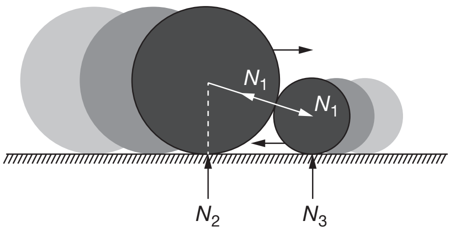


Example: N_i = Impulsive
N_g = Non-impulsive

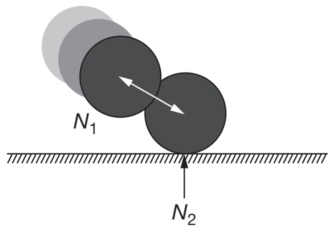
Both normals are Impulsive



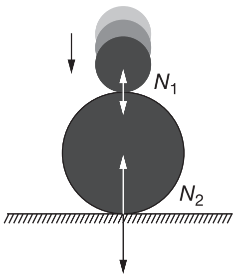
N_1, N_3 = Impulsive; N_2 = non-impulsive



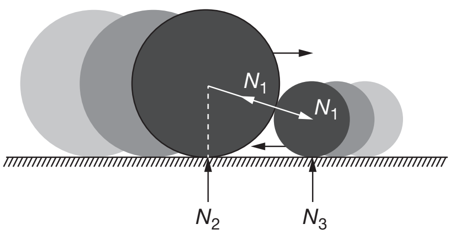
Both normals are Impulsive



2. **Impulsive Friction:** If the normal between the two objects is impulsive, then the friction between the two will also be impulsive.



Friction at both surfaces is impulsive and N are



Friction due to N_2 is non-impulsive and due to N_3

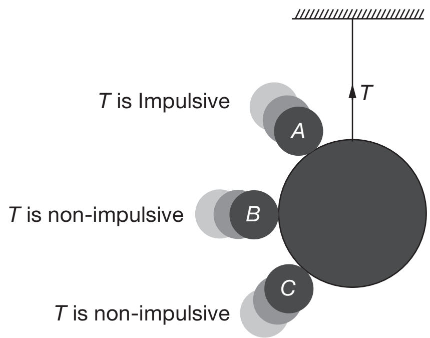
3. **Impulsive Tensions:** When a string jerks, equal and opposite tension act suddenly at each end. Consequently equal and opposite impulses act on the bodies attached with the string in the direction of the string. There are two cases to be considered.

One End of the String is Fixed

The impulse which acts at the fixed end of the string cannot change the momentum of the fixed object there. The object attached to the free end however will undergo a change in momentum in the direction of the string. The momentum remains unchanged in a direction perpendicular to the string where no impulsive forces act.

Both Ends of the String attached to Movable Objects

In this case equal and opposite impulses act on two objects and it produces equal and opposite changes in momentum. The total momentum of the system therefore remains constant, although the momentum of each individual object is changed in the direction of the string. Perpendicular to the string however, no impulse acts and the momentum of each particle in this direction is unchanged.

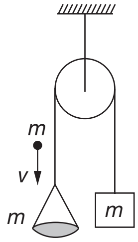


All normal are impulsive but tension T is impulsive only for the ball A

Example:
In case of rod, Tension is always impulsive and in case of spring, Tension is always non-impulsive.

SOLVED EXAMPLES

31. A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v, find the speed with which the system moves just after the collision.



Solution:

Let the required speed is V.

Further, let J_1 = impulse between particle and pan and J_2 = impulse imparted to the block and the pan by the string

Using,

$$\text{impulse} = \text{change in momentum}$$

For particle

$$J_1 = mv - mV \quad (1)$$

For pan

$$J_1 - J_2 = mV \quad (2)$$

For block

$$J_2 = mV \quad (3)$$

Solving, these three equation, we get

$$V = \frac{v}{3}.$$

Alternative Solution:

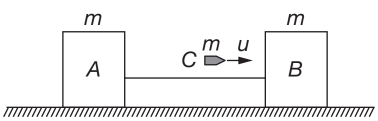
Applying conservation of linear momentum along the string;

$$mv = 3mV$$

we get,

$$V = \frac{v}{3}.$$

32. Two identical blocks A and B, connected by a massless string are placed on a frictionless horizontal plane. A bullet having same mass, moving with speed u strikes block B from behind as shown. If the bullet gets embedded into the block B then find:



(A) The velocity of A, B and C after collision.
(B) Impulse on A due to tension in the string
(C) Impulse on C due to normal force of collision.
(D) Impulse on B due to normal force of collision.

Solution:

(A) By Conservation of linear momentum $v = \frac{u}{3}$

(B) $\int T\,dt = \frac{mu}{3}$

(C) $\int N\,dt = m\left(\frac{u}{3} - u\right) = \frac{-2mu}{3}$

(D) $\int (N - T)\,dt = \int N dt - \int T\,dt = \frac{mu}{3}$

$$\Rightarrow \quad \int N\,dt = \frac{2mu}{3}.$$

COLLISION OR IMPACT

Collision is an event in which an impulsive force acts between two or more bodies for a short time, which results in change of their velocities.

NOTE

- In a collision, particles may or may not come in physical contact.
- The duration of collision, Δt is negligible as compared to the usual time intervals of observation of motion.
- In a collision the effect of external non impulsive forces such as gravity are not taken into account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external forces acting on the system.

The collision is in fact a redistribution of total momentum of the particles. Thus, law of conservation of linear momentum is indispensable in dealing with the phenomenon of collision between particles.

Line of Impact

The line passing through the common normal to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both the bodies.

Direction of line of impact can be determined by:

1. Geometry of colliding objects like spheres, discs, wedge etc.
2. Direction of change of momentum.

If one particle is stationary before the collision then the line of impact will be along its motion after collision.

Classification of Collisions

On the Basis of Line of Impact

1. Head-on collision: If the velocities of the colliding particles are along the same line before and after the collision.

2. Oblique collision: If the velocities of the colliding particles are along different lines before and after the collision.

On the Basis of Energy

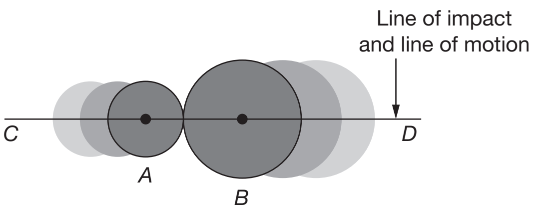
1. **Elastic collision:** In an elastic collision, the colliding particles regain their shape and size completely after collision. i.e., no fraction of mechanical energy remains stored as deformation potential energy in the bodies. Thus, kinetic energy of system after collision is equal to kinetic energy of system before collision. Thus in addition to the linear momentum, kinetic energy also remains conserved before and after collision.
2. **Inelastic collision:** In an inelastic collision, the colliding particles do not regain their shape and size completely after collision. Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles after collision is not equal to that of before collision. However, in the absence of external forces, law of conservation of linear momentum still holds good.
3. **Perfectly inelastic:** If velocity of separation along the line of impact just after collision becomes zero then the collision is perfectly inelastic. Collision is said to be perfectly inelastic if both the particles stick together after collision and move with same velocity,

NOTE

Actually collision between all real objects are neither perfectly elastic nor perfectly inelastic, its inelastic in nature.

Examples of Line of Impact and Collisions Based on Line of Impact

1. Two balls *A* and *B* are approaching each other such that their centres are moving along line *CD*.



Head on Collision

2. Two balls *A* and *B* are approaching each other such that their centre is moving along dotted lines as shown in Fig. 5.15.

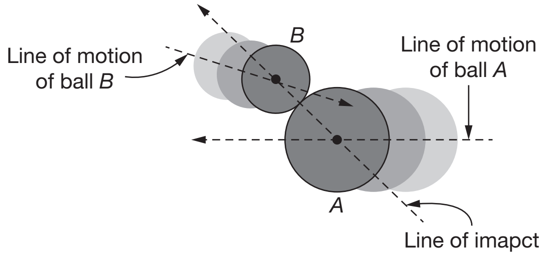
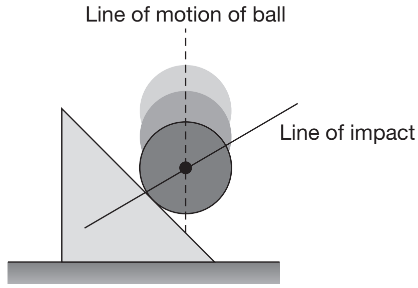


Fig. 5.15 Oblique Collision

3. Ball is falling on a stationary wedge.



Oblique Collision

Coefficient of Restitution (e)

The coefficient of restitution is defined as the ratio of the impulses of reformation and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r\,dt}{\int F_d\,dt}$$

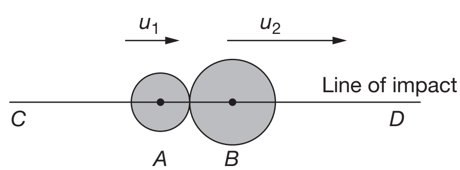
$$= \frac{\text{Velocity of seperation along line of impact}}{\text{Velocity of approach along line of impact}}$$

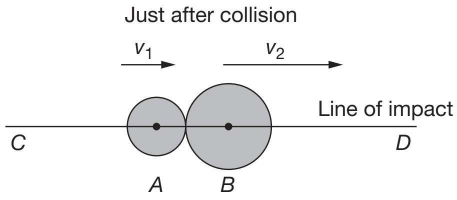
The most general expression for coefficient of restitution is

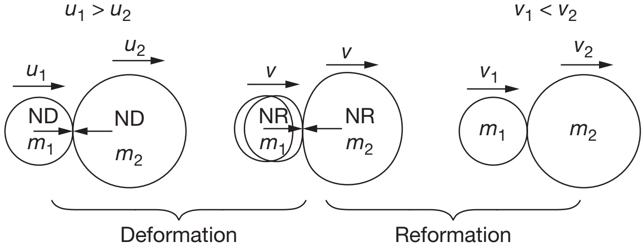
$$e = \frac{\text{Velocity of separation of points of contact along line of impact}}{\text{Velocity of approach of point of contact along line of impact}}$$

Example for Calculation of e

Two smooth balls *A* and *B* approaching each other such that their centres are moving along line *CD* in absence of external impulsive force. The velocities of *A* and *B* just before collision be u_1 and u_2 respectively. The velocities of *A* and *B* just after collision be v_1 and v_2 respectively.







$\because$ $F_{ext} = 0$ momentum is conserved for the system.

$\Rightarrow$ $m_1u_1 + m_2u_2 = (m_1 + m_2)v = m_1v_1 + m_2v_2$

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \tag{1}$$

Impulse of Deformation

J_D = change in momentum of any one body during Deformation.

$$= m_2(v - u_2) \quad \text{for } m_2$$

$$= m_1(-v + u_1) \quad \text{for } m_1$$

Impulse of Reformation

J_R = change in momentum of any one body during Reformation.

$$= m_2(v_2 - v) \quad \text{for } m_2$$

$$= m_1(v - v_1) \quad \text{for } m_1$$

$$e = \frac{\text{Impulse of Reformation } (\vec{J}_R)}{\text{Impulse of Deformation } (\vec{J}_D)} = \frac{v_2 - v_1}{u_1 - u_2}$$

$$= \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

> **NOTE**
>
> *e* is independent of shape and mass of object but depends on the material.
>
> The coefficient of restitution is constant for a pair of materials.

1. $e = 1$
- Impulse of Reformation = Impulse of Deformation
- Velocity of separation = Velocity of approach
- Kinetic energy of particles after collision may be equal to that of before collision.
- Collision is elastic.

2. $e = 0$
- Impulse of Reformation = 0
- Velocity of separation = 0
- Kinetic energy of particles after collision is not equal to that of before collision.
- Collision is perfectly inelastic.

3. $0 < e < 1$
- Impulse of Reformation < Impulse of Deformation
- Velocity of separation < Velocity of approach
- Kinetic energy of particles after collision is not equal to that of before collision.
- Collision is Inelastic.

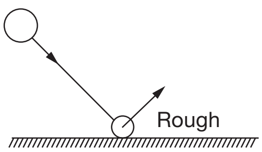
> **NOTE**
>
> In case of contact collisions *e* is always less than unity.
>
> $\therefore \quad 0 \le e \le 1$

Important Points

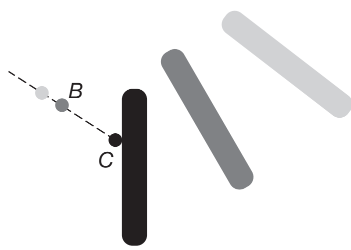
In case of elastic collision, if rough surface is present then

$$k_f < k_i \quad \text{(because friction is impulsive)}$$

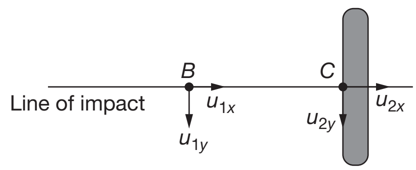
Where, *k* is Kinetic Energy.



A particle *B* moving along the dotted line collides with a rod also in state of motion as shown in the Fig. 5.16. The particle *B* comes in contact with point *C* on the rod.



Just before collision



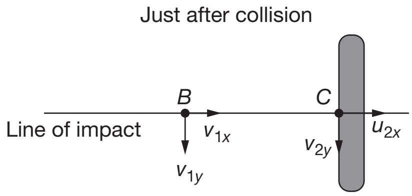
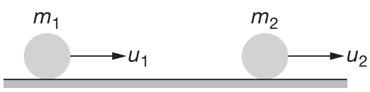


Fig. 5.16

To write down the expression for coefficient of restitution e, we first draw the line of impact. Then we resolve the components of velocities of points of contact of both the bodies along line of impact just before and just after collision.

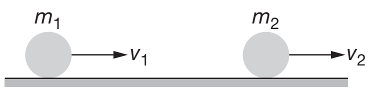
Then $$e = \frac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}}$$

Collision in One Dimension (Head On)



(a)
Before collision

$$u_1 > u_2$$



(b)
After collision

$$v_2 > v_1$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\Rightarrow \quad (u_1 - u_2)e = (v_2 - v_1)$$

By momentum conservation,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$v_2 = v_1 + e(u_1 - u_2)$$

and $$v_1 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$$

$$v_2 = \frac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2}$$

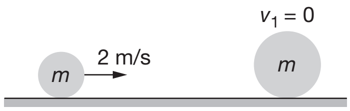
Special Case:

1. $e = 0 \quad \Rightarrow \quad v_1 = v_2$
   $\Rightarrow$ for perfectly inelastic collision, both the bodies, move with same vel. after collision.
2. $e = 1$
   and $$m_1 = m_2 = m,$$
   we get
   $$v_1 = u_2 \text{ and } v_2 = u_1$$

i.e., when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities., e.g.

Before collision

After collision



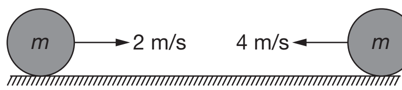
Before collision

After collision

3. $m_1 >>>> m_2$
   $$m_1 + m_2 \approx m_1 \quad \text{and} \quad \frac{m_2}{m_1} \approx 0$$
   $\Rightarrow$ $$v_1 = u_1 \quad \text{No change}$$
   and $$v_2 = u_1 + e(u_1 - u_2)$$

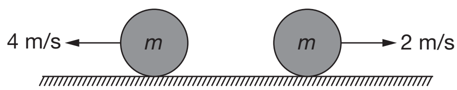
SOLVED EXAMPLES

33. Two identical balls are approaching towards each other on a straight line with velocity 2 m/s and 4 m/s respectively. Find the final velocities, after elastic collision between them.

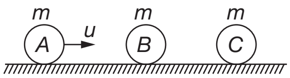


Solution:

The two velocities will be exchanged and the final motion is reverse of initial motion for both.

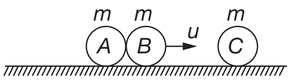


34. Three balls A, B and C of same mass m are placed on a frictionless horizontal plane in a straight line as shown. Ball A is moved with velocity u towards the middle ball B. If all the collisions are elastic then, find the final velocities of all the balls.

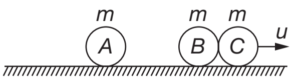


Solution:

A collides elastically with B and comes to rest but B starts moving with velocity u

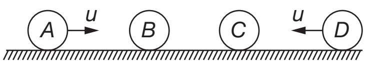


After a while B collides elastically with C and comes to rest but C starts moving with velocity u



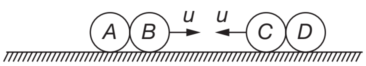
$\therefore$ Final velocities $v_A = 0$; $v_B = 0$ and $v_C = u$.

35. Four identical balls A, B, C and D are placed in a line on a frictionless horizontal surface. A and D are moved with same speed u towards the middle as shown. Assuming elastic collisions, find the final velocities.

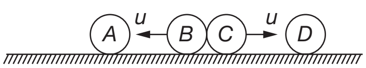


Solution:

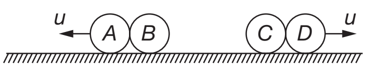
A and D collides elastically with B and C respectively and come to rest but B and C starts moving with velocity u towards each other as shown



B and C collide elastically and exchange their velocities to move in opposite directions

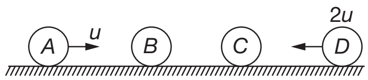


Now, B and C collides elastically with A and D respectively and come to rest but A and D starts moving with velocity u away from each other as shown

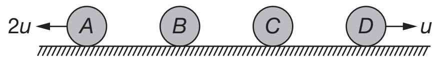


$\therefore$ Final velocities $v_A = u$ (←); $v_B = 0$; $v_C = 0$ and $v_D = u$ (→).

36. Four identical balls are placed on a frictionless surface as shown. A is moved with velocity u towards right and D is moved with $2u$ towards left. Assuming all collisions to be perfectly elastic and A and C do not collide with B simultaneously. What will be the final velocities now be?



Solution:



37. Two particles of mass m and $2m$ moving in opposite directions on a frictionless surface collide elastically with velocity v and $2v$ respectively. Find their velocities after collision, and also find the fraction of kinetic energy lost by the colliding particles.

Solution:

Let the final velocities of m and $2m$ be v_1 and v_2 respectively as shown in the Fig. 5.17

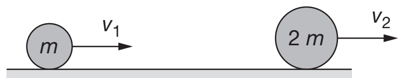


Fig. 5.17

By conservation of momentum:

$$m(2v) + 2m(-v) = m(v_1) + 2m(v_2)$$

or $$0 = mv_1 + 2mv_2$$

or $$v_1 + 2v_2 = 0 \qquad (1)$$

and since the collision is elastic:

$$v_2 - v_1 = 2v - (-v)$$

or $$v_2 - v_1 = 3v \qquad (2)$$

Solving the above two equations, we get,

$$v_2 = v \quad \text{and} \quad v_1 = -2v.$$

i.e., the mass $2m$ returns with velocity v while the mass m returns with velocity $2v$ in the direction shown in Fig. 5.18:

Fig. 5.18

The collision was elastic therefore, no kinetic energy is lost, KE loss $= KE_i - KE_f$

or, $$\left(\frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(-v)^2\right) - \left(\frac{1}{2}m(-2v)^2 + \frac{1}{2}(2m)v^2\right) = 0$$

38. On a frictionless surface, a ball of mass m moving at a speed v makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4th of the original. Find the coefficient of restitution.

Solution:

As we have seen in the above discussion, that under the given conditions:

Before collision

After collision

By using conservation of linear momentum and equation of e, we get,

$$v_1' = \left(\frac{1+e}{2}\right)v \quad \text{and} \quad v_2' = \left(\frac{1-e}{2}\right)v$$

Given that

$$K_f = \frac{3}{4}K_i$$

or $$\frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 = \frac{3}{4}\left(\frac{1}{2}mv^2\right)$$

Substituting the value, we get

$$\left(\frac{1+e}{2}\right)^2 + \left(\frac{1-e}{2}\right)^2 = \frac{3}{4}$$

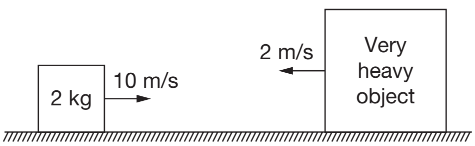
or $$e = \frac{1}{\sqrt{2}}.$$

39. A block of mass m moving at speed v collides with another block of mass 2 m at rest. The lighter block comes to rest after the collision. Find the coefficient of restitution.

Solution:

$$\frac{1}{2}$$

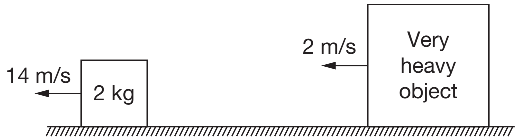
40. A block of mass 2 kg is pushed towards a very heavy object moving with 2 m/s closer to the block (as shown). Assuming elastic collision and frictionless surfaces, find the final velocities of the blocks.



Solution:

Let v_1 and v_2 be the final velocities of 2 kg block and heavy object respectively then,

$$v_1 = u_1 + 1\,(u_1 - u_2) = 2u_1 - u_2$$
$$= -14 \text{ m/s}$$
$$v_2 = -2 \text{ m/s}$$



41. A ball is moving with velocity 2 m/s towards a heavy wall moving towards the ball with speed 1 m/s as shown in Fig. 5.19. Assuming collision to be elastic, find the velocity of the ball immediately after the collision.

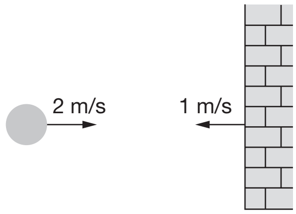
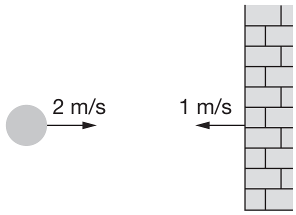


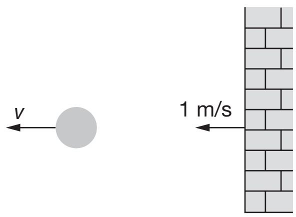
Fig. 5.19

Solution:

The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision in the direction shown in Fig. 5.20. Since collision is elastic ($e = 1$),



Before collision



After collision

Fig. 5.20

separation speed = approach speed

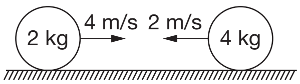
or $$v - 1 = 2 + 1$$

or $$v = 4 \text{ m/s}.$$

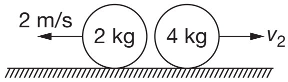
42. Two balls of masses 2 kg and 4 kg are moved towards each other with velocities 4 m/s and 2 m/s respectively on a frictionless surface. After collision, the 2 kg ball returns back with velocity 2 m/s. Then find:

(A) Velocity of 4 kg ball after collision.
(B) Coefficient of restitution e.
(C) Impulse of deformation J_D.
(D) Maximum potential energy of deformation.
(E) Impulse of reformation J_R.

Just before collision



Just after collision



Solution:

(A) By momentum conservation,

$$2(4) - 4(2) = 2(-2) + 4(v_2)$$

$$\Rightarrow \quad v_2 = 1 \text{ m/s}$$

(B) $$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{1-(-2)}{4-(-2)} = \frac{3}{6}$$

$$= 0.5$$

(C) At maximum deformed state, by conservation of momentum, common velocity is $v = 0$.

$$J_D = m_1(v - u_1) = m_2(v - u_2)$$
$$= 2(0 - 4) = -8 \text{ N/s}$$
$$= 4(0 - 2) = -8 \text{ N/s}$$
or $$= 4(0 - 2) = -8 \text{ N/s}$$

(D) Potential energy at maximum deformed state U = loss in kinetic energy during deformation.

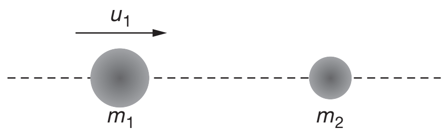
or $$U = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \left(\frac{1}{2}2(4)^2 + \frac{1}{2}4(2)^2\right) - \frac{1}{2}(2 + 4)(0)^2$$

or $$U = 24 \text{ J}$$

(E) $$J_R = m_1(v_1 - v) = m_2(v - v_2)$$
$$= 2(-2 - 0) = -4 \text{ N/s}$$
or $$= 4(0 - 1) = -4 \text{ N/s}$$
or $$e = \frac{J_R}{J_D}$$
$$\Rightarrow \quad J_R = eJ_D$$
$$= (0.5)(-8)$$
$$= -4 \text{ N/s}$$

43. The sphere of mass m_1 travels with an initial velocity u_1 directed as shown and strikes the stationary sphere of mass m_2 head on. For a given coefficient of restitution e, what condition on the mass ratio $\frac{m_1}{m_2}$ ensures that the final velocity of m_2 is greater than u_1?



Solution:

$$\frac{m_1}{m_2} > \frac{1}{e}$$

Collision in Two Dimension (Oblique)

1. A pair of equal and opposite impulses acts along common normal direction. Hence, linear momentum of individual particles does change along common normal direction. If mass of the colliding particles remain constant during collision, then we can say that linear velocity of the individual particles change during collision in this direction.
2. No component of impulse act along common tangent direction. Hence, linear momentum or linear velocity of individual particles (if mass is constant) remain unchanged along this direction.
3. Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remains conserved before and after collision in any direction.
4. Definition of coefficient of restitution can be applied along common normal direction,
   i.e., along common normal direction we can apply
   Relative speed of separation = e (relative speed of approach)

SOLVED EXAMPLES

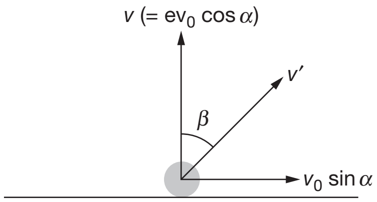
44. A ball of mass m hits a floor with a speed v_0 making an angle of incidence α with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection of the ball.

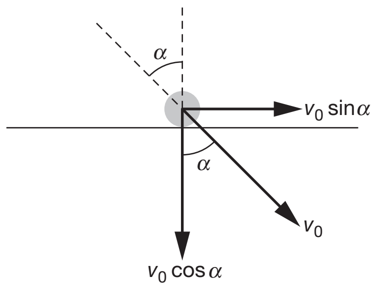
Solution:

The component of velocity v_0 along common tangential direction $v_0 \sin \alpha$ will remain unchanged. Let v be the component along common normal direction after collision. Applying,

Relative speed of separation = e (Relative speed of approach) along common normal direction, we get

$$v = ev_0 \cos \alpha$$

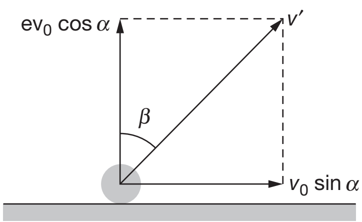




Thus, after collision components of velocity v' are $v_0 \sin \alpha$ and $ev_0 \cos \alpha$

$\therefore$ $$v' = \sqrt{(v_0 \sin \alpha)^2 + (ev_0 \cos \alpha)^2}$$

and $$\tan \beta = \frac{v_0 \sin \alpha}{ev_0 \cos \alpha}$$



or $$\tan \beta = \frac{\tan \alpha}{e}$$

NOTE

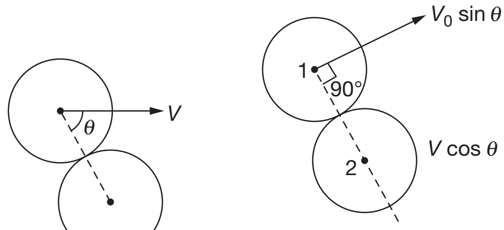
For elastic collision, $e = 1$

$\therefore$ $v' = v_0$ and $\beta = \alpha$

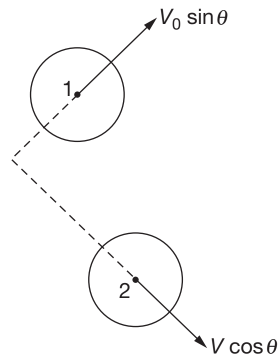
45. A ball of mass m makes an elastic collision with another identical ball at rest. Show that if the collision is oblique, the bodies go at right angles to each other after collision.

Solution:

In head on elastic collision between two particles, they exchange their velocities. In this case, the component of ball 1 along common normal direction, $v \cos \theta$ becomes zero after collision, while that of 2 becomes $v \cos \theta$. While the components along common tangent direction of both the particles remains unchanged. Thus, the components along common tangent and common normal direction of both the balls in tabular form are given below.



Before collision



After collision

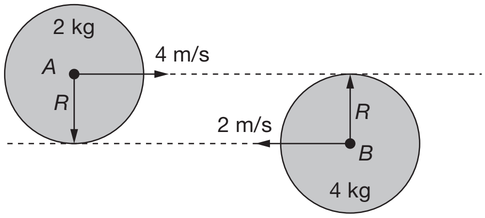
Ball	Component along common tangent direction		Component along common normal direction	
	Before collision	After collision	Before collision	After collision
1	$v \sin \theta$	$v \sin \theta$	$v \cos \theta$	0
2	0	0	0	$v \cos \theta$

From the above table and figure, we see that both the balls move at right angle after collision with velocities $v \sin \theta$ and $v \cos \theta$.

NOTE

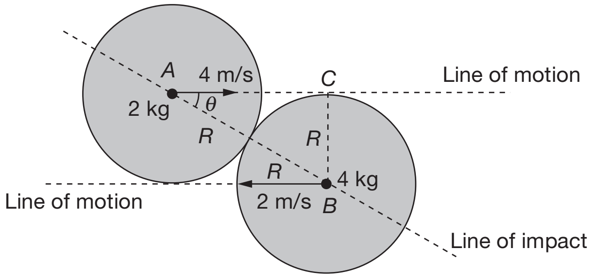
When two identical bodies have an oblique elastic collision, with one body at rest before collision, then the two bodies will go in $\perp$ directions.

46. Two spheres are moving towards each other. Both have same radius but their masses are 2 kg and 4 kg. If the velocities are 4 m/s and 2 m/s respectively and coefficient of restitution is $e = \frac{1}{3}$, find



(A) the common velocity along the line of impact.
(B) the final velocities along line of impact.
(C) the impulse of deformation.
(D) the impulse of reformation.
(E) the maximum potential energy of deformation.
(F) the loss in kinetic energy due to collision.

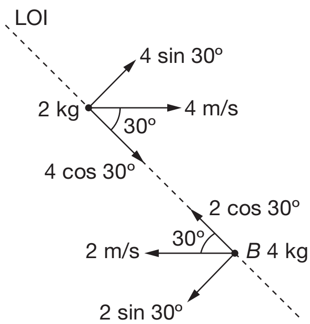
Solution:



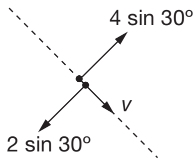
In ΔABC $\quad \sin\theta = \dfrac{BC}{AB} = \dfrac{R}{2R} = \dfrac{1}{2}$

or $\quad \theta = 30°$

(A) By conservation of momentum along line of impact.



Just before collision along LOI

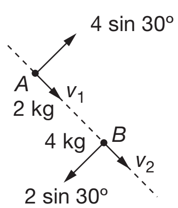


Maximum deformed state

$2(4\cos 30°) - 4(2\cos 30°) = (2+4)v$

or $\quad v = 0 \quad$ (common velocity along LOI)

(B)



Just after collision along LOI

Let v_1 and v_2 be the final velocity of A and B respectively then, by conservation of momentum along line of impact,

$2(4\cos 30°) - 4(2\cos 30°) = 2(v_1) + 4(v_2)$

or $\quad 0 = v_1 + 2v_2 \qquad (1)$

By coefficient of restitution,

$$e = \frac{\text{Velocity of separation along LOI}}{\text{Velocity of approach along LO I}}$$

or $\quad \dfrac{1}{3} = \dfrac{v_2 - v_1}{4\cos 30° + 2\cos 30°}$

or $\quad v_2 - v_1 = \sqrt{3} \qquad (2)$

from the above two equations,

$v_1 = \dfrac{-2}{\sqrt{3}}$ m/s and $v_2 = \dfrac{1}{\sqrt{3}}$ m/s.

(C) $J_D = m_1(v - u_1)$

$\quad = 2(0 - 4\cos 30°) = -\,4\sqrt{3}$ N/s

(D) $J_R = eJ_D = \dfrac{1}{3}(-4\sqrt{3}) = -\dfrac{4}{\sqrt{3}}$ N/s

(E) Maximum potential energy of deformation is equal to loss in kinetic energy during deformation up to maximum deformed state,

$$U = \frac{1}{2}m_1(u_1\cos\theta)^2 + \frac{1}{2}m_2(u_2\cos\theta)^2 - \frac{1}{2}(m_1+m_2)v^2$$

$$= \frac{1}{2}2(4\cos 30°)^2 + \frac{1}{2}4(-2\cos 30°)^2 - \frac{1}{2}(2+4)(0)^2$$

or $\quad U = 18$ J

(F) Loss in kinetic energy,

$$\Delta KE = \frac{1}{2}m_1(u_1\cos\theta)^2 + \frac{1}{2}m_2(u_2\cos\theta)^2 - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$= \frac{1}{2}2(4\cos 30°)^2 + \frac{1}{2}4(-2\cos 30°)^2 - \left(\frac{1}{2}2\left(\frac{2}{\sqrt{3}}\right)^2 + \frac{1}{2}4\left(\frac{1}{\sqrt{3}}\right)^2\right)$$

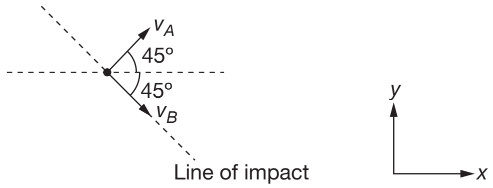
$\Delta KE = 16$ J

47. Two point particles A and B are placed in line on a frictionless horizontal plane. If particle A (mass 1 kg) is moved with velocity 10 m/s towards stationary particle B (mass 2 kg) and after collision the two move at an angle of 45° with the initial direction of motion, then find

(A) the velocities of A and B just after collision.
(B) the coefficient of restitution.

Solution:

The very first step to solve such problems is to find the line of impact which is along the direction of force applied by A on B, resulting the stationary B to move. Thus, by watching the direction of motion of B, line of impact can be determined. In this case line of impact is along the direction of motion of B. i.e. 45º with the initial direction of motion of A.



(A) By conservation of momentum, along x direction:

$$m_A u_A = m_A v_A \cos 45° + m_B v_B \cos 45°$$

or $\quad 1(10) = 1(v_A \cos 45°) + 2(v_B (\cos 45°)$

or $\quad v_A + 2v_B = 10\sqrt{2} \qquad (1)$

along y direction

$$0 = m_A v_A \sin 45° + m_B v_B \sin 45°$$

or $\quad 0 = 1(v_A \sin 45°) – 2(v_B \sin 45°)$

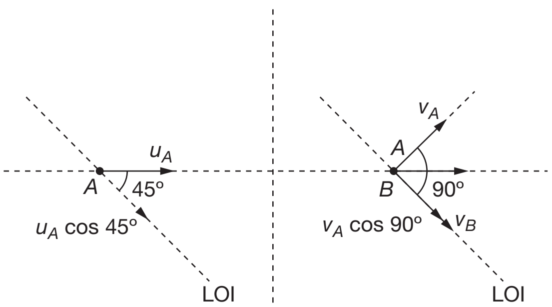
or $\quad v_A = 2v_B \qquad (2)$

solving the two equations,

$$v_A = \frac{10}{\sqrt{2}} \text{ m/s}$$

and $\quad v_B = \dfrac{5}{\sqrt{2}}$ m/s.

(B) $e = \dfrac{\text{Velocity of separation along LOI}}{\text{Veloctiy of approach along LOI}}$



Before collision   After collision

or $\quad e = \dfrac{v_B - v_A \cos 90°}{u_A \cos 45°}$

$$= \frac{\frac{5}{\sqrt{2}} - 0}{\frac{10}{\sqrt{2}}} = \frac{1}{2}.$$

48. A smooth sphere of mass m is moving on a horizontal plane with a velocity $3\hat{i} + \hat{j}$ when it collides with a vertical wall which is parallel to the vector $\hat{j}$. If the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, find

(A) the velocity of the sphere after impact,
(B) the loss in kinetic energy caused by the impact.
(C) the impulse $\vec{J}$ that acts on the sphere.

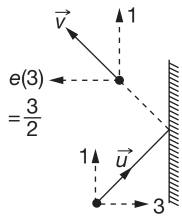
Solution:

Let $\vec{v}$ be the velocity of the sphere after impact.

To find $\vec{v}$ we must separate the velocity components parallel and perpendicular to the wall.

Using the law of restitution the component of velocity parallel to the wall remains unchanged while component perpendicular to the wall becomes e times in opposite direction.

Thus, $\qquad \vec{v} = -\dfrac{3}{2}\hat{i} + \hat{j}$



(A) Therefore, the velocity of the sphere after impact is $= -\dfrac{3}{2}\hat{i} + \hat{j}$.

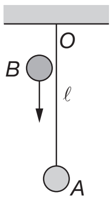
(B) The loss in KE $= \dfrac{1}{2}mv^2 - \dfrac{1}{2}mu^2 = \dfrac{1}{2}m(3^2 + 1^2)$

$$-\frac{1}{2}\, m\left(\left\{\frac{3}{2}\right\}^2 + 1^2\right) = \frac{27}{8}\text{m}.$$

(C) $\vec{J} = \Delta\vec{P} = \vec{P}_{fi} - \vec{P} = m(\vec{v}) - m(\vec{u}) = m\left(-\dfrac{3}{2}\hat{i} + \hat{j}\right)$

$$- \text{m}\left(3\hat{i} + \hat{j}\right) = -\frac{9}{2}m\hat{i}.$$

49. A small steel ball A is suspended by an inextensible thread of length $\ell = 1.5$ from O. Another identical ball is thrown vertically downwards such that its surface remains just in contact with thread during downward motion and collides elastically with the suspended ball. If the suspended ball just completes vertical circle after collision, calculate the velocity of the falling ball just before collision. ($g = 10\ \text{ms}^{-2}$)

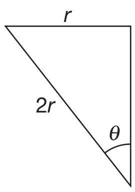


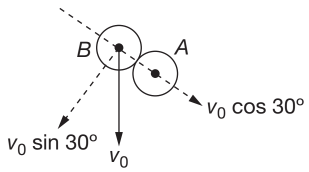
Solution:

Velocity of ball A just after collision is $\sqrt{5gl}$

Let radius of each ball be r and the joining centres of the two balls makes an angle θ with the vertical at the instant of collision, then

$$\sin\theta = \frac{r}{2r} = \frac{1}{2} \quad \text{or} \quad \theta = 30°$$





Let velocity of ball B (just before collision) be v_0. This velocity can be resolved into two components, (i) $v_0 \cos 30°$, along the line joining the centre of the two balls and (ii) $v_0 \sin 30°$ normal to this line. Head-on collision takes place due to $v_0 \cos 30°$ and the component $v_0 \sin 30°$ of velocity of ball B remains unchanged.

Since, ball A is suspended by an inextensible string, therefore, just after collision, it can move along horizontal direction only. Hence, a vertically upward impulse is exerted by thread on the ball A. This means that during collision two impulses act on ball A simultaneously. One is impulsive interaction J between the balls and the other is impulsive reaction J' of the thread.

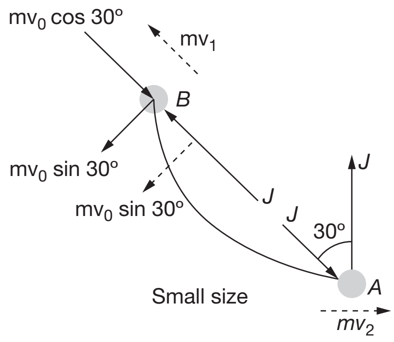
Velocity v_1 of ball B along line of collision is given by

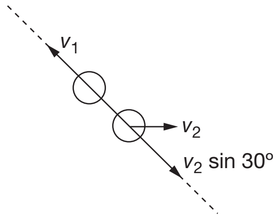
$$J - mv_0 \cos 30° = mv_1$$

or $$v_1 = \frac{J}{m} - v_0 \cos 30° \qquad (1)$$

Horizontal velocity v_2 of ball A is given by $J \sin 30° = mv_2$

or $$v_2 = \frac{J}{2m} \qquad (2)$$





Since, the balls collide elastically, therefore, coefficient of restitution is $e = 1$.

Hence, $$e = \frac{v_2 \sin 30° - (-v_1)}{v_0 \cos 30° - 0} = 1 \qquad (3)$$

Solving Equations (1), (2), and (3),

$$J = 1.6\, mv_0 \cos 30°$$

$\therefore$ $$v_1 = 0.6\, v_0 \cos 30°$$

and $$v_2 = 0.8\, v_0 \cos 30°$$

Since, ball A just completes vertical circle, therefore $v_2 = \sqrt{5g\ell}$

$\therefore$ $$0.8 v_0 \cos 30° = \sqrt{5g\ell}$$

or $$v_0 = 12.5\ \text{ms}^{-1}.$$

Variable Mass System

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity $\vec{v}_{\text{rel}}$ (with return to the system), then the force exerted by this mass on the system has magnitude $\mu|\vec{v}_{\text{rel}}|$.

Thrust Force ($\vec{F}_t$)

$$\vec{F}_t = \vec{v}_{\text{rel}}\left(\frac{dm}{dt}\right)$$

Suppose at some moment $t = t$ mass of a body is m and its velocity is $\vec{v}$. After some time at $t = t + dt$ its mass becomes $(m - dm)$ and velocity becomes $\vec{v} + d\vec{v}$. The mass dm is ejected with relative velocity $\vec{v}_r$. Absolute velocity of mass dm is therefore $(\vec{v} + \vec{v}_r)$. If no external forces are acting on the system, the linear momentum of the system will remain conserved, or

$$\vec{P}_i = \vec{P}_f$$

or $$m\vec{v} = (m - dm)(\vec{v} + d\vec{v}) + dm(\vec{v} + \vec{v}_r)$$

or $$m\vec{v} = m\vec{v} + md\vec{v} - (dm)\vec{v} - (dm)(d\vec{v}) + (dm)\vec{v} + \vec{v}_r\, dm$$

The term $(dm)(d\vec{v})$ is too small and can be neglected.

$\therefore$ $$md\vec{v} = -\vec{v}_r dm$$

or $$m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$$

Here, $$m\left(-\frac{d\vec{v}}{dt}\right) = \text{thrust force } \left(\vec{F}_t\right)$$

and $$-\frac{dm}{dt} = \text{rate at which mass is ejecting}$$

or $$\vec{F}_t = \vec{v}_r\left(\frac{dm}{dt}\right)$$

Problems related to variable mass can be solved in following four steps

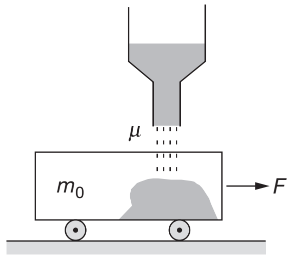
1. Make a list of all the forces acting on the main mass and apply them on it.
2. Apply an additional thrust force $\vec{F}_t$ on the mass, the magnitude of which is $\left|\vec{v}_r\left(\pm\frac{dm}{dt}\right)\right|$ and direction is given by the direction of $\vec{v}_r$ in case the mass is increasing and otherwise the direction of $-\vec{v}_r$ if it is decreasing.
3. Find net force on the mass and apply
   $$\vec{F}_{\text{net}} = m\frac{d\vec{v}}{dt} \quad (m = \text{mass at the particular instant})$$
4. Integrate it with proper limits to find velocity at any time t.

NOTE

Problems of one-dimensional motion (which are mostly asked in JEE) can be solved in easier manner just by assigning positive and negative signs to all vector quantities. Here are few example in support of the above theory.

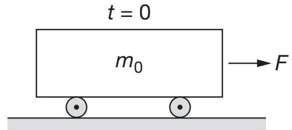
SOLVED EXAMPLES

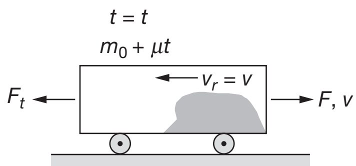
50. A flat car of mass m_0 starts moving to the right due to a constant horizontal force F. Sand spills on the flat car from a stationary hopper. The rate of loading is constant and equal to μ kg/s. Find the time dependence of the velocity and the acceleration of the flat car in the process of loading. The friction is negligibly small.



Solution:

Initial velocity of the flat car is zero. Let v be its velocity at time t and m its mass at that instant. Then





At $\quad t = 0,\ v = 0$

and $\quad m = m_0$ at $t = t,\ v = v$

and $\quad m = m_0 + \mu t$

Here, $\quad v_r = v \quad$ (backwards)

$$\frac{dm}{dt} = \mu$$

$\therefore$ $$F_t = v_r\frac{dm}{dt} = \mu v \quad \text{(backwards)}$$

Net force on the flat car at time t is $F_{\text{net}} = F - F_t$

or $$m\frac{dv}{dt} = F - \mu v \tag{1}$$

or $$(m_0 + \mu t)\frac{dv}{dt} = F - \mu v$$

or $$\int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$$

$\therefore$ $$-\frac{1}{\mu}[\ell n(F - \mu v)]_0^v = \frac{1}{\mu}[\ell n(m_0 + \mu t)]_0^t$$

$\Rightarrow$ $$\ell n\left(\frac{F}{F - \mu v}\right) = \ell n\left(\frac{m_0 + \mu t}{m_0}\right)$$

$\therefore$ $$\frac{F}{F - \mu v} = \frac{m_0 + \mu t}{m_0}$$

or $$v = \frac{Ft}{m_0 + \mu t}.$$

From Equation (1),

$\frac{dv}{dt}$ = acceleration of flat car at time t

or $$= \frac{F - \mu v}{m}$$

$$a = \left(\frac{F - \frac{F\mu t}{m_0 + \mu t}}{m_0 + \mu t} \right)$$

or $$a = \frac{F m_0}{(m_0 + \mu t)^2}.$$

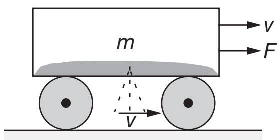
51. A cart loaded with sand moves along a horizontal floor due to a constant force F coinciding in direction with the cart's velocity vector. In the process sand spills through a hole in the bottom with a constant rate μ kg/s. Find the acceleration and velocity of the cart at the moment t, if at the initial moment $t = 0$ the cart with loaded sand had the mass m_0 and its velocity was equal to zero. Friction is to be neglected.

Solution:

In this problem the sand spills through a hole in the bottom of the cart. Hence, the relative velocity of the sand v_r will be zero because it will acquire the same velocity as that of the cart at the moment.

$$v_r = 0$$

Thus, $$F_t = 0 \qquad \left(\text{as } F_t = v_r \frac{dm}{dt} \right)$$



and the net force will be F only.

$\therefore$ $$F_{\text{net}} = F$$

or $$m \left(\frac{dv}{dt} \right) = F \qquad (1)$$

But here

$$m = m_0 - \mu t$$

$\therefore$ $$(m_0 - \mu t) \frac{dv}{dt} = F$$

or $$\int_0^v dv = \int_0^t \frac{F\, dt}{m_0 - \mu t}$$

$\therefore$ $$v = \frac{F}{-\mu} \left[\ell n(m_0 - \mu t) \right]_0^t$$

or $$v = \frac{F}{\mu} \ln \left(\frac{m_0}{m_0 - \mu t} \right).$$

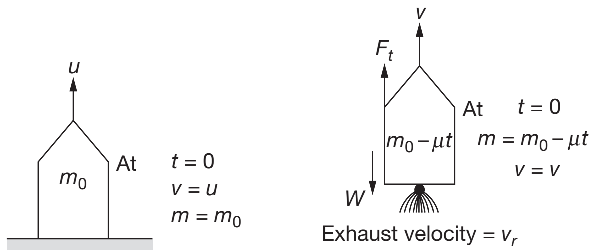
From Equation (1), acceleration of the cart

$$a = \frac{dv}{dt} = \frac{F}{m}$$

or $$a = \frac{F}{m_0 - \mu t}.$$

Rocket Propulsion

Let m_0 be the mass of the rocket at time $t = 0$. m its mass at any time t and v its velocity at that moment. Initially, let us suppose that the velocity of the rocket is u.



Further, let $\left(\frac{-dm}{dt} \right)$ be the mass of the gas ejected per unit time and v_r the exhaust velocity of the gases with respect to rocket. Usually $\left(\frac{-dm}{dt} \right)$ and v_r are kept constant throughout the journey of the rocket. Now, let us write few equations which can be used in the problems of rocket propulsion. At time $t = t$,

1. Thrust force on the rocket

   $$F_t = v_r \left(\frac{-dm}{dt} \right) \qquad \text{(upwards)}$$

2. Weight of the rocket

   $W = \text{mg}$ (downwards)

3. Net force on the rocket

   $$F_{\text{net}} = F_t - W \qquad \text{(upwards)}$$

   or $$F_{\text{net}} = v_r \left(\frac{-dm}{dt} \right) - \text{mg}$$

4. Net acceleration of the rocket

   $$a = \frac{F}{m}$$

   or $$\frac{dv}{dt} = \frac{v_r}{m} \left(\frac{-dm}{dt} \right) - g$$

or $$dv = \frac{v_r}{m}(-dm) - gdt$$

or $$\int_u^v dv = v_r \int_{m_0}^{m} \frac{-dm}{m} - g\int_0^t dt$$

Thus, $$v = u - gt + v_r \ell n\left(\frac{m_0}{m}\right) \quad (1)$$

NOTE

- $F_t = v_r\left(-\frac{dm}{dt}\right)$ is upwards, as v_r is downwards and $\frac{dm}{dt}$ is negative.
- If gravity is ignored and initial velocity of the rocket $u = 0$, Equation (1) reduces to $v = v_r \ln\left(\frac{m_0}{m}\right)$.

52. A rocket, with an initial mass of 1000 kg, is launched vertically upwards from rest under gravity. The rocket burns fuel at the rate of 10 kg per second. The burnt matter is ejected vertically downwards with a speed of 2000 ms^{-1} relative to the rocket. If the burning stops after one minute, find the maximum velocity of the rocket. (Take g as at 10 ms^{-2})

Solution:

Using the velocity equation

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

Here

$u = 0$, $t = 60$ s, $g = 10$ m/s^2, $v_r = 2000$ m/s, $m_0 = 1000$ kg

and $$m = 1000 - 10 \times 60 = 400 \text{ kg}$$

We get $$v = 0 - 600 + 2000 \ln\left(\frac{1000}{400}\right)$$

or $$v = 2000 \ln 2.5 - 600$$

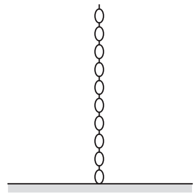
The maximum velocity of the rocket is 200(10 ln 2.5 – 3) = 1232.6 ms^{-1}

53. Find the mass of the rocket as a function of time, if it moves with a constant acceleration a, in absence of external forces. The gas escapes with a constant velocity u relative to the rocket and its mass initially was m_0.

Solution:

$$m = m_0 e^{-at/u}$$

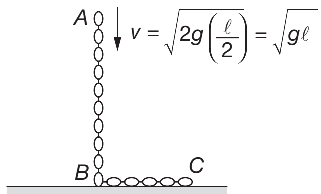
54. A uniform chain of mass m and length ℓ hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the table on the chain when half of its length has fallen on the table. The fallen part does not form heap.



Solution:

1. Weight of the portion BC of the chain lying on the table, $W = \frac{mg}{2}$ (downwards)

Using $v = \sqrt{2gh}$



2. Thrust force $F_t = v_r\left(\frac{dm}{dt}\right)$

$$v_r = v$$

$$\frac{dm}{dt} = \lambda v$$

$$F_t = \lambda v^2$$

(where, $\lambda = \frac{m}{\ell}$, is mass per unit length of chain)

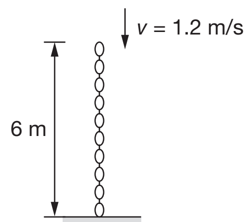
$$v^2 = (\left(\sqrt{g\ell}\right)^2 = g\ell$$

$\therefore$ $$F_t = \left(\frac{m}{\ell}\right)(g\ell) = mg \quad \text{(downwards)}$$

$\therefore$ Net force exerted by the chain on the table is

$$F = W + F_t = \frac{mg}{2} + mg = \frac{3}{2}mg$$

So, from Newton's third law the force exerted by the table on the chain will be $\frac{3}{2}$ mg (vertically upwards).



55. If the chain is lowered at a constant speed $v = 1.2$ m/s, determine the normal reaction exerted on the floor as a function of time. The chain has a mass of 80 kg and a total length of 6 m.

Solution:

$$(19.2 + 16\,t)\text{ N}$$

Linear Momentum Conservation in Presence of External Force

$$\vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt}$$

$\Rightarrow$ $$\vec{F}_{\text{ext}}\,dt = d\vec{P}$$

$\Rightarrow$ $$d\vec{P} = (\vec{F}_{\text{ext}})_{\text{mpulsive}}\,dt$$

$\therefore$ If $(\vec{F}_{\text{ext}})_{\text{mpulsive}} = 0$

$\Rightarrow$ $$d\vec{P} = 0$$

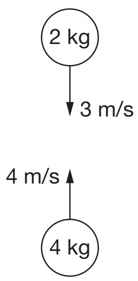
or $\vec{P}$ is constant

NOTE

Momentum is conserved if the external force present is non-impulsive. For example, gravitation or spring force

SOLVED EXAMPLES

56. Two balls are moving towards each other on a vertical line collides with each other as shown. Find their velocities just after collision.



Solution:

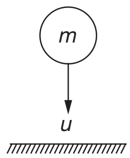
Let the final velocity of 4 kg ball just after collision be v. Since, external force is gravitational which is non-impulsive, hence, linear momentum will be conserved.

Applying linear momentum conservation:

$$2(-3) + 4(4) = 2(4) + 4(v)$$

or $$v = \frac{1}{2}\text{ m/s}$$

57. A ball is approaching ground with speed u. If the coefficient of restitution is e then find out:



(A) the velocity just after collision.
(B) the impulse exerted by the normal due to ground on the ball.

Solution:

(A) $v = eu$;
(B) $J = mu(1 + e)$

58. A bullet of mass 50 g is fired from below into the bob of mass 450 g of a long simple pendulum as shown in Fig. 5.21. The bullet remains inside the bob and the bob rises through a height of 1.8 m. Find the speed of the bullet. Take $g = 10$ m/s^2.

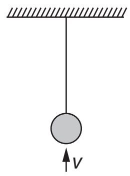


Fig. 5.21

Solution:

Let the speed of the bullet be v. Let the common velocity of the bullet and the bob, after the bullet is embedded into the bob, is v. By the principle of conservation of the linear momentum,

$$v = \frac{(0.05\text{ kg})\,v}{0.45\text{ kg} + 0.05\text{ kg}} = \frac{v}{10}$$

The string becomes loose and the bob will go up with a deceleration of $g = 10$ m/s^2. As it comes to rest at a height of 1.8 m, using the equation $v^2 = u^2 + 2ax$,

$$1.8 \text{ m} = \frac{(v/10)^2}{2\times 10 \text{ m/s}^2}$$

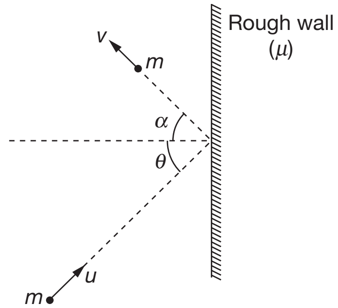
or, $\quad v = 60$ m/s.

59. A small ball of mass m collides with a rough wall having coefficient of friction μ at an angle θ with the normal to the wall. If after collision the ball moves with angle α with the normal to the wall and the coefficient of restitution is e then find the reflected velocity v of the ball just after collision.

Solution:

$$mv \cos \alpha - (m(-u \cos \theta)) = \int N\, dt$$

$$mv \sin \alpha - mu \sin \theta = -\mu \int N\, dt$$



and $\quad e = \dfrac{v\cos\alpha}{u\cos\theta}$

$\Rightarrow \quad v \cos \alpha = eu \cos \theta$

or $\quad mv \sin \alpha - mu \sin \theta = -\mu(mv \cos \alpha + mu \cos \theta)$

or $\quad v = \dfrac{u}{\sin\alpha}[\sin \theta - \mu \cos \theta (e + 1)]$.

BRAIN MAP

IMPULSE AND MOMENTUM

1. Important formulae
- $\vec{P} = M\vec{v}$
- $\vec{F} = \dfrac{\Delta \vec{P}}{\Delta t}, F = \dfrac{d\vec{p}}{dt}$
- $\vec{J} = \int \vec{F} \cdot dt = \Delta \vec{p}$

2. Conservation of linear momentum
- If net force acting on a body or system of bodies is zero, the momentum of body or system of body remains conserved.

3. Classification of impact on the basis of direction of force
- Central
  - Direct or head-on
  - Indirect or oblique
- Eccentric

4. Classification of impact on the basis of nature of colliding bodies
- Elastic
- Inelastic
- Perfectly inelastic

5. Analysis of collision
- Apply conservation of momentum along the line of collision.
- Apply law of restitution along the line of collision i.e., $v_2 - v_1 = e(u_1 - u_2)$
- $e = 1$ for perfectly elastic collision.
- $e = 0$ for perfectly inelastic collision
- $0 < e < 1$ for other collisions.

6. Equation of motion for variable mass system
- $\vec{F}_{ext} + \vec{F}_{th} = M\vec{a}$
- where, $\vec{F}_{th} = \dfrac{-dM}{dt}\vec{v}_{rel}$

EXERCISES

Single Option Correct Type

1. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, −1) is

(A) $-F(\hat{i}-\hat{j})$ (B) $F(\hat{i}-\hat{j})$

(C) $-F(\hat{i}+\hat{j})$ (D) $F(\hat{i}+\hat{j})$

2. Three identical spheres each of radius 10 cm and mass 1 kg are placed touching one another on a horizontal surface. Where is their centre of mass located?

(A) On the horizontal surface
(B) At the point of contact of any two spheres
(C) At the centre of one ball
(D) None of these

3. A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northward with the same speed v. The two particles coalesce on collision. The new particle of mass 2 m will move in the north-east direction with a velocity

(A) $\sqrt{2}\,v$ (B) $\frac{v}{2}$

(C) $\frac{v}{\sqrt{2}}$ (D) v

4. A moving body collides elastically with another body of equal mass at rest. Then

(A) a part of the energy is dissipated as heat.
(B) momentum is conserved but KE is not conserved.
(C) both masses start moving with the same velocity.
(D) the moving mass transfers whole of its energy to the mass at rest.

5. A particle of mass m describes a circle of radius r. The centripetal acceleration of the particle is $4/r^2$. The momentum of the particle is

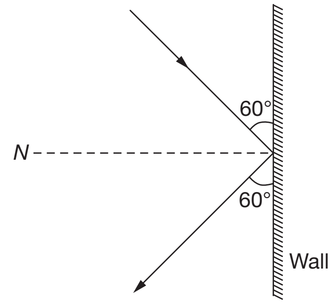
(A) $\frac{4m}{r}$ (B) $\frac{2m}{r}$

(C) $\frac{4m}{\sqrt{r}}$ (D) v

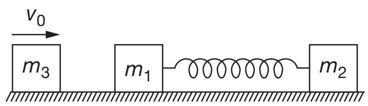
6. A 3 kg ball strikes a heavy rigid wall with a speed of 10 m/s at an angle of 60° with the wall. It gets reflected with the same speed at 60° with the wall. If the ball is in contact with the wall for 0.2 s, the average force exerted on the ball by the wall is

(A) 300 N (B) Zero

(C) $150\sqrt{3}$ N (D) 150 N



7. Two blocks of masses m_1 and m_2 are connected with an ideal spring and kept on a frictionless plane at rest. Another block of mass m_3 making elastic head on collision with the block of mass m_1. After the collision the centre of mass of $(m_1 + m_2)$ as a system will



(A) move with non-uniform acceleration.
(B) move with a uniform velocity.
(C) remain at rest.
(D) move with uniform acceleration.

8. In the head on elastic collision of a heavy vehicle moving with a velocity of 10 ms^{-1} and a small stone at rest, the stone will fly away with a velocity equal to

(A) 5 ms^{-1} (B) 10 ms^{-1}
(C) 20 ms^{-1} (D) 40 ms^{-1}

9. A ball P of mass 2 kg undergoes an elastic collision with another ball Q at rest. After collision, ball P continues to move in its original direction with a speed one-fourth of its original speed. What is the mass of ball Q?

(A) 0.9 kg (B) 1.2 kg
(C) 1.5 kg (D) 1.8 kg

10. A ball hits a floor and rebounds after an inelastic collision. In this case

(A) the momentum of the ball just after the collision is the same as that just before the collision.
(B) the mechanical energy of the ball remains the same is the collision.
(C) the total momentum of the ball and the earth is conserved.
(D) the total energy of the ball and the earth is conserved.

11. A particle of mass m moving eastward with a speed v collides with another particle of same mass moving northward with same speed v. The two particles coalesce on collision. The new particle of mass 2 m will move in the north-east direction with a velocity of

(A) $v\sqrt{2}$ (B) $\frac{v}{\sqrt{2}}$
(C) $\frac{v}{2}$ (D) v

12. A metal ball of mass 2 kg moving with speed of 36 km/h has a head-on collision with a stationary ball of mass 3 kg. If after collision, both the balls move together, then the loss in kinetic energy due to collision is

(A) 40 J (B) 60 J
(C) 100 J (D) 140 J

13. An inelastic ball is dropped from a height of 100 m. If 20% of its energy is lost, to what height will the ball rise?

(A) 80 m (B) 40 m
(C) 60 m (D) 20 m

14. A ball weighing 10 g hits a hard surface vertically with a speed of 5 m/s and rebounds with the same speed. The ball remains in contact with the surface for 0.01 s. The average force exerted by the surface on the ball is

(A) 100 N (B) 10 N
(C) 1 N (D) 0.1 N

15. For same braking force the stopping distance of a vehicle increases from 15 m to 60 m. By what factor the velocity of vehicle has been changed

(A) 2 (B) 3
(C) 4 (D) $3\sqrt{5}$

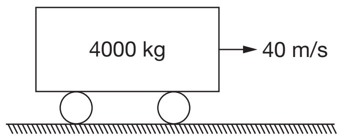
16. A particle of mass m moving with a speed v hits elastically another identical and stationary particle inside a smooth horizontal circular tube of radius r. The time in which the next collision will take place is equal to

(A) $\frac{2\pi r}{v}$ (B) $\frac{4\pi r}{v}$
(C) $\frac{3\pi r}{2v}$ (D) $\frac{\pi r}{v}$

17. Two balls of masses $m_1 = 3$ kg and $m_2 = 2$ kg are moving towards each other with speeds u_1 and u_2. The ball m_1 stops after collision and m_2 starts moving with speed u_1. The co-efficient of restitution between the balls is

(A) Zero (B) 1 (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

18. A trolley containing water has total mass 4000 kg and is moving at a speed of 40 m/s. Now water start coming out of a hole at the bottom of the trolley at the rate of 8 kg/s. Speed of trolley after 50 s is



(A) 44.44 m/s (B) 40 m/s
(C) 44 m/s (D) 54.44 m/s

19. An isolated particle of mass m is moving in horizontal plane (x–y), along the x-axis, at a certain height above ground. It suddenly explodes into two fragments of masses m / 4 and 3 m / 4. An instant later, the smaller fragment is at y = +15 cm. The larger fragment at this instant is at

(A) y = –5 cm (B) y = +5 cm
(C) y = +5 cm (D) y = –20 cm

20. Particle A makes a perfectly elastic head-on collision with another stationary particle B. They fly apart in opposite directions with equal velocities. Ratio of their masses $\frac{M_A}{M_B}$ will be

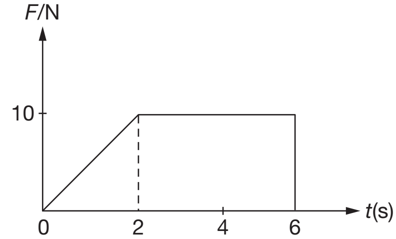
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{1}{\sqrt{3}}$

21. A bullet weighing 50 gm leaves the gun with a velocity of 30 ms^{-1}. If the recoil speed imparted to the gun is 1 ms^{-1}, the mass of the gun

(A) 1.5 kg (B) 15 kg
(C) 20 kg (D) 30 kg

22. A body of mass 3 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is given by (given initial momentum = 0)

(A) Zero (B) 5 N/s
(C) 30 N/s (D) 50 N/s



23. A projectile of mass m is thrown with velocity v making an angle of 30° with vertical. Neglecting air resistance the magnitude of change in momentum between the starting point and at the maximum height is

(A) $\frac{mv}{2}$ (B) $\frac{\sqrt{3}mv}{2}$
(C) mv (D) $\frac{\sqrt{7}mv}{2}$

24. A graph between kinetic energy and momentum of a particle is plotted as shown in the Fig. 5.22. The mass of the moving particle is

(A) 1 kg (B) 2 kg
(C) 3 kg (D) 4 kg

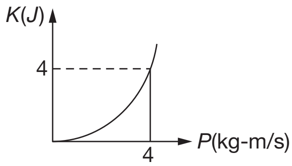


Fig. 5.22

25. The acceleration of centre of mass of the two block system shown in Fig. 5.23 will be

(A) 10 m/s^2 (B) $-\frac{10}{3}$ m/s^2
(C) $\frac{5}{3}$ m/s^2 (D) –5 m/s^2

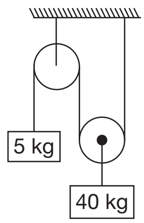


Fig. 5.23

26. A particle moves in the x-y plane under the action of force $\vec{F}$ such that the value of its linear momentum $\vec{P}$ at time t is $P_x = 2\cos t$ and $P_y = 2\sin t$. The angle θ between $\vec{F}$ and $\vec{P}$ at time t will be

(A) 90° (B) 0°
(C) 180° (D) 30°

27. A body of mass 10 kg moving with velocity of 10 m/s hits another body of mass 30 kg moving with velocity 3 m/s in same direction. The co-efficient of restitution is 1/4. The velocity of centre of mass after collision will be

(A) 20 m/s (B) 40 m/s
(C) $\frac{19}{4}$ m/s (D) $\frac{23}{4}$ m/s

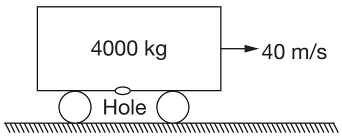
28. A steel ball of volume 0.02 m^3 is sinking at a speed of 10 m/s in a closed jar filled with a liquid of density 2000 kg/m^3. The momentum of the liquid is

(A) 400 N/s (B) 200 N/s
(C) 100 N/s (D) 300 N/s

29. A body of mass m_1 collides elastically with a stationary body of mass m_2 and return with one third speed, then $\frac{m_1}{m_2}$ =

(A) 1 (B) 2
(C) 0.5 (D) 0.33

30. A trolley containing water has total mass 4000 kg and is moving at a speed of 40 m/s. Now water starts coming out of a hole at the bottom of the trolley at the rate of 8 kg/s. Speed of trolley after 50 s is



(A) 44.44 m/s (B) 40 m/s
(C) 44 m/s (D) 54.44 m/s

31. A particle of mass 2 kg starts moving in a straight line with an initial velocity of 2 m/s at a constant acceleration of 2 m/s^2. The rate of change of kinetic energy is

(A) four times the velocity at any moment.
(B) two times the displacement at any moment.
(C) four times the rate of change of velocity at any moment.
(D) constant throughout.

32. A wooden block of mass 0.9 kg is suspended from the ceiling of a room by a long thin wire. A bullet of mass 0.1 kg moving horizontally with a speed of 100 m/s strikes the block and gets embedded in it. The height to which the block rises will be (g = 10 m/s^2)

(A) 2.5 m (B) 5.0 m
(C) 7.5 m (D) 10.0 m

33. A bullet of mass 20 g traveling horizontally with a speed of 500 m/s passes through a wooden block of mass 8.0 kg initially at rest on a level surface. The bullet emerges with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest. The coefficient of friction between the block and the surface is (g = 10 ms^{-2})

(A) 0.4 (B) 0.25
(C) 0.2 (D) 0.16

34. A 9 kg block is originally at rest on a horizontal smooth surface. If a horizontal force F is applied such that it varies with time as shown in Fig. 5.24, the speed of block in 4 s is

(A) 5 m/s (B) 15 m/s
(C) 25 m/s (D) 30 m/s

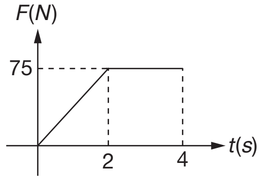


Fig. 5.24

35. Two bodies of masses 2 kg and 4 kg are moving with velocities 10 m/s and 2 m/s towards each other. The velocity of their centre of mass is

(A) Zero (B) 1 m/s
(C) 2 m/s (D) 4 m/s

36. Two bodies of mass 1 kg and 2 kg move towards each other in mutually perpendicular direction with the velocities 3 m/s and 2 m/s respectively. If the bodies stick together after collision the energy loss will be

(A) 13 J (B) $\frac{13}{3}$ J
(C) 8 J (D) 7 J

37. Two particles of masses 4 kg and 8 kg are separated by a distance of 6 m. If they are moving towards each other under the influence of a mutual force of attraction, then the two particles will meet each other at a distance of

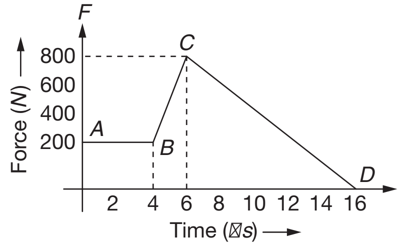
(A) 6 m from 8 kg mass
(B) 2 m from 8 kg mass
(C) 4 m from 8 kg mass
(D) 8 m from 8 kg mass

38. A body of mass 20 kg is moving with a velocity of $2v$ and another body of mass 10 kg is moving with velocity v along same direction. The velocity of their centre of mass is

(A) $\frac{5v}{3}$ (B) $\frac{2v}{3}$
(C) v (D) Zero

39. The magnitude of the force (in N) acting on a body varies with time t (in μs) as shown. AB, BC and CD are straight line segments. The magnitude of the total impulse of the force on the body from $t = 4$ μs to $t = 16$ μs is

(A) 5×10^{-3} N/s (B) 5.8×10^{-3} N/s
(C) 5.8×10^{3} N/s (D) 5×10^{3} N/s



40. Two skaters of masses 40 kg and 60 kg respectively stand facing each other at S_1 and S_2 where S_1S_2 is 5 m. They pull on a massless rope stretched them, then they meet at

(A) 2.5 m from S_1 and S_2
(B) 3 m from S_1 and 2 m from S_2
(C) 2 m from S_1 and 3 m from S_2
(D) 3 m from S_1 and 8 m from S_2

41. Figure 5.25 the force-time graph for a body of mass 10 kg initially at rest. The velocity gained by the body in 6 seconds will be

(A) Zero (B) 6 m/s
(C) 3 m/s (D) 4 m/s

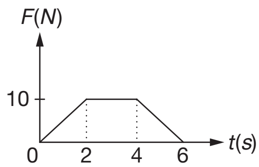


Fig. 5.25

42. Three particles A, B and C of equal mass, move with equal speed v along the medians of an equilateral triangle as shown in the Fig. 5.26. They collide at the centroid G of the triangle. After collision, A comes to rest and B retraces its path with speed v. What is the speed of C after collision?

(A) 0 (B) $\frac{v}{2}$
(C) v (D) $2v$

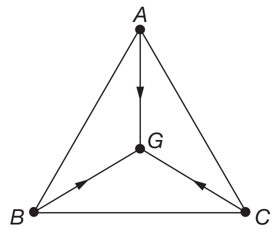


Fig. 5.26

43. A proton moving with velocity v collides elastically with a stationary α-particle. The velocity of the proton after the collision is

(A) $-\frac{3v}{5}$ (B) $\frac{3v}{5}$
(C) $\frac{2v}{5}$ (D) $-\frac{2v}{5}$

44. A bomb of mass $3m$ kg explodes into two pieces of mass m kg and $2m$ kg. If the velocity of m kg mass is 16 m/s, the total energy released in the explosion is

(A) 192 m J (B) 96 m J
(C) 384 m J (D) 768 m J

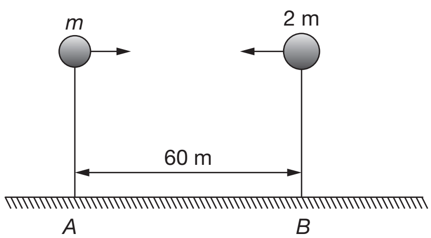
45. A uniform metre scale balances at the 40 cm mark when weight of 10 g and 20 g are suspended from the 10 cm and 20 cm marks. The weight of the metre scale is

(A) 50 g (B) 60 g
(C) 70 g (D) 80 g

46. A body of mass m_1 moving with uniform velocity of 40 m/s collides with another mass m_2 at rest and then the two together begin to move with uniform velocity of 30 m/s. The ratio of their masses $\frac{m_1}{m_2}$ is

(A) 0.75 (B) 1.33
(C) 3.0 (D) 4.0

47. Two particles one of mass m and the other of mass $2m$ are projected horizontally towards each other from the same level above the ground with velocities 10 m/s and 5 m/s respectively. They collide in air and stick to each other. The distance from A, where the combined mass finally land is



(A) 40 m (B) 20 m
(C) 30 m (D) 45 m

48. Position of two particles are given by $x_1 = 2t$ and $x_2 = 2 + 3t$. The velocity of centre of mass at $t = 2$ s is 2 m/s. Velocity of centre of mass at $t = 4$ s will be

(A) 2 m/s (B) 4 m/s
(C) 1 m/s (D) Zero

49. A block of metal weighting 2 kg is resting on a frictionless plane. It is stuck by a jet releasing water at a rate of 1 kg/sec and at a speed of 5 m/s. The initial acceleration of the block will be

(A) 2.5 m/s^2 (B) 5.0 m/s^2
(C) 7.5 m/s^2 (D) 10 m/s^2

50. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of force constant k. They are placed on a smooth horizontal plane. They are stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is

(A) $x\sqrt{\frac{3k}{2m}}$ (B) $x\sqrt{\frac{2k}{3m}}$
(C) $x\sqrt{\frac{2k}{m}}$ (D) $x\sqrt{\frac{3k}{m}}$

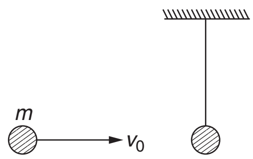
51. Position of two particles are given by $x_1 = 2t$ and $x_2 = 2 + 3t$. The velocity of centre of mass at $t = 2$ s is 2 m/s. Velocity of centre of mass at $t = 4$ s will be

(A) 2 m/s (B) 4 m/s
(C) 1 m/s (D) Zero

52. A ball of mass m moving with a speed u undergoes a head-on elastic collision with a ball of mass nm initially at rest. The fraction of initial energy transferred to the heavier ball is

(A) $\frac{n}{1+n}$ (B) $\frac{n}{(1+n)^2}$
(C) $\frac{2n}{(1+n)^2}$ (D) $\frac{4n}{(1+n)^2}$

53. A sphere of mass m moving horizontally with velocity v_0 collides against a pendulum bob of mass M. If the two masses stick together after the collision, then the maximum height attained is



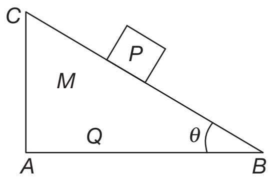
(A) $\frac{v_0^2}{2g}$ (B) $\frac{v_0^2}{4g}$
(C) $\frac{v_0^2}{6g}$ (D) $\frac{v_0^2}{8g}$

54. A bullet of mass m moving with velocity v strikes a suspended wooden block of mass M. If the block rises to a height h, the initial velocity of the block will be

(A) $\sqrt{2gh}$ (B) $\frac{M+m}{m}\sqrt{2gh}$

(C) $\frac{m}{M+m}2gh$ (D) $\frac{M+m}{M}\sqrt{2gh}$

55. A block Q of mass M is placed on a horizontal frictionless surface AB and a body P of mass m is released on its frictionless slope. As P slides by a length L on this slope of inclination θ, the block Q would slide by a distance

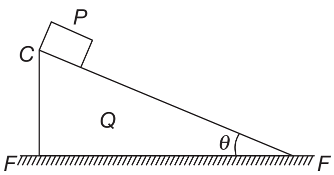


(A) $\frac{m}{M}L\cos\theta$ (B) $\frac{m}{M+m}L$

(C) $\frac{M+m}{mL\cos\theta}$ (D) $\frac{mL\cos\theta}{m+M}$

56. A block Q of mass M is placed on a horizontal frictionless surface FF, and a body P of mass m is released on its frictionless slope. As P slides by length L on this slope of inclination θ, the block Q would slide by distance

(A) $(m/M)L\cos\theta$
(B) $mL/(m+M)$
(C) $(mL\cos\theta)/(m+M)$
(D) None of these



57. A man of mass m stands on a ladder which is tied to a free balloon of mass M. The balloon is at rest initially. If the man starts to climb the ladder at a constant velocity v relative to the ladder, then initial speed of balloon will be (neglect mass of ladder)

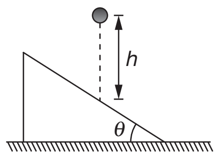
(A) $\frac{mv}{M+m}$ (B) $\frac{mv}{M+2m}$

(C) $\frac{Mv}{M+m}$ (D) $\frac{mv}{M}$

58. A ball after falling through a distance h collides with an inclined plane of inclination θ as shown. It moves horizontally after the impact. The co-efficient of restitution between inclined plane and ball is

(A) 1 (B) $\tan^2\theta$

(C) $\cot^2\theta$ (D) $\sin^2\theta$



59. A ball of mass m approaches a wall of mass M ($>>m$) with speed 4 m/s along the normal to the wall. The speed of wall is 1 m/s towards the ball. The speed of the ball after an elastic collision with the wall is

(A) 5 m/s away from the wall.
(B) 9 m/s away from the wall.
(C) 3 m/s away from the wall.
(D) 6 m/s away from the wall.

60. A gun fires a shell and recoils horizontally. If the shell travels along the barrel with speed v, the ratio of speeds with which the gun recoils, if the barrel is (i) horizontal (ii) inclined at an angle of 30° with horizontal, is

(A) 1 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

61. A glass marble dropped from a certain height above the horizontal surface reaches the surface in time t and then continues to bounce up and down. The time in which the marble finally comes to rest is? (Note: The coefficient of restitution is e)

(A) $e^n t$ (B) $e^2 t$

(C) $t\left[\frac{1+e}{1-e}\right]$ (D) $t\left[\frac{1-e}{1+e}\right]$

62. A block of mass m slides without friction down a fixed inclined board of inclination α with the horizontal. After leaving the inclined, the block falls on a cart of mass M. Initial height of the block above the level of the cart is h as shown in the Fig. 5.27. The velocity of cart when block drops on it will be

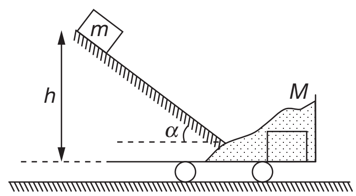
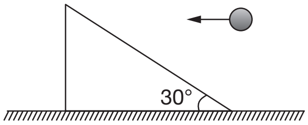


Fig. 5.27

(A) $\dfrac{m\sqrt{2gh}}{M+m}$ (B) $\dfrac{m\sqrt{2gh}\sin\alpha}{M+m}$

(C) $\dfrac{m\sqrt{2gh}\cos\alpha}{M+m}$ (D) $\dfrac{m\sqrt{2gh}\cos\alpha}{M}$

63. A ball of mass 1 kg strikes a wedge of mass 4 kg horizontally with a velocity of 10 m/s. Just after collision velocity of wedge becomes 4 m/s. Friction is absent everywhere and collision is elastic. Then



(A) the speed of ball after collision is 6 m/s.
(B) the speed of ball after collision is 8 m/s.
(C) the speed of ball after collision is 4 m/s.
(D) the speed of ball after collision is 10 m/s.

64. Two blocks of equal mass are tied with a light string, which passes over a massless pulley as shown in Fig. 5.28. The magnitude of acceleration of centre of mass of both blocks is: (wedge is fixed and smooth)

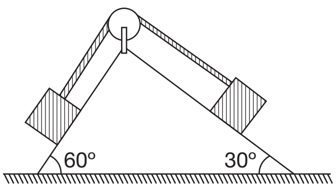


Fig. 5.28

(A) $\left(\dfrac{\sqrt{3}-1}{4\sqrt{2}}g\right)$ (B) $(\sqrt{3}-1)g$

(C) $\dfrac{g}{2}$ (D) $\left(\dfrac{\sqrt{3}-1}{\sqrt{2}}\right)g$

65. A particle of mass m and velocity $\vec{v}$ collides elastically with a stationary particle of same mass m. If the collision is oblique, then the angle between the velocity vectors of the two particles after the collision is

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{2}$ (D) π

66. A plate remains in equilibrium in air when n bullets are fired per second on it. The mass of each bullet is m and it strikes the plate with speed v. The coefficient of restitution is $e = 0.5$. The mass of the plate is

(A) $M = \dfrac{3mnv}{2g}$ (B) $M = \dfrac{mnv}{g}$

(C) $M = \dfrac{2mnv}{3g}$ (D) $M = \dfrac{2mnv}{g}$

67. A ball is dropped from a height h. It rebounds from the ground a number of times. Given that the coefficient of restitution is e, to what height does it go after nth rebounding?

(A) h/e^n (B) h/e^{2n}

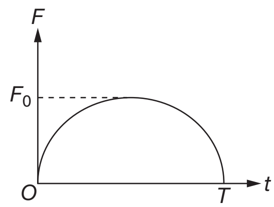
(C) he^n (D) he^{2n}

68. A ball A of mass m moving with velocity u, collides head on with another ball B of the same mass at rest. If the co-efficient of restitution between balls is e, the ratio of the final and initial velocities of ball A will be

(A) $\dfrac{1+e}{1-e}$ (B) $\dfrac{1+e}{2}$

(C) $1-e$ (D) $\dfrac{1-e}{2}$

69. A particle of mass m, initially at rest, is acted upon by a variable force F for a brief interval of time T. The force (F) – time (t) curve is a semicircle as shown. Its velocity u after the force stops acting is given by



(A) $u = \dfrac{\pi F_0 T}{2m}$ (B) $u = \dfrac{\pi F_0 T}{8m}$

(C) $u = \dfrac{\pi F_0 T}{4m}$ (D) $u = \dfrac{F_0 T}{2m}$

70. Two identical billiard balls are in contact on a table. A third identical ball strikes them symmetrically with velocity v and remains at rest after impact. The speed of balls after collision will be

(A) $\dfrac{v}{\sqrt{3}}$ (B) $\dfrac{v}{3}$

(C) $\dfrac{v}{2}$ (D) v

71. A T shaped object, having uniform linear mass density, with dimensions shown in the Fig. 5.29 is lying on a smooth floor. A force F is applied at the point P parallel to AB, such that the object has only translational motion without rotation. The distance of P with respect to C is

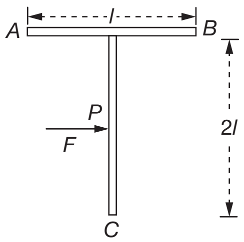


Fig. 5.29

(A) $\frac{4l}{3}$ (B) l

(C) $\frac{2l}{3}$ (D) $\frac{3l}{2}$

72. You are given a metre scale and a rubber ball. Using this which of the following can be experimentally found?

(A) Acceleration due to gravity.
(B) Modulus of elasticity of rubber.
(C) Time taken by the ball to strike the ground.
(D) Coefficient of restitution.

73. If a metal wire is stretched a little beyond its elastic limit (or yield point) and released, it will

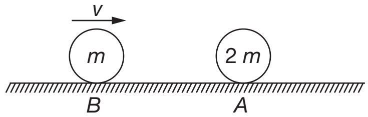
(A) lose its elastic property completely.
(B) not contract.
(C) contract, but its final length will be greater than its initial length.
(D) contract only up to its initial length at the elastic limit.

74. A pendulum consists of a wooden bob of mass m and length l. A bullet of mass m_1 is fired towards the pendulum with a speed v_1. The bullet emerges out of the bob with a speed $v_1/3$ and the bob just completes motion along a vertical circle. Then v_1 is

(A) $\left(\frac{m}{m_1}\right)\sqrt{5gl}$ (B) $\frac{3}{2}\left(\frac{m}{m_1}\right)\sqrt{5gl}$

(C) $\frac{2}{3}\left(\frac{m_1}{m}\right)\sqrt{5gl}$ (D) $\left(\frac{m_1}{m}\right)\sqrt{gl}$

75. A ball A of mass $2m$ is kept at rest on a smooth horizontal surface. Another ball B of same size and having mass m moving with velocity v, collides with ball A. If during collision an impulse $J = mv$ is imparted to the ball A by the ball B then coefficient of restitution between the balls is



(A) 1 (B) $\frac{3}{4}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

76. A body of mass m makes an elastic collision with another identical body at rest. Just after collision the angle between the velocity vectors of one body with the initial line of motion is 15° then the angle between velocity vectors of the other body with the initial line of motion is

(A) 75° (B) 60°
(C) 45° (D) 30°

77. Let $\vec{P}$ be the linear momentum of a particle whose position vector is $\vec{r}$ with respect to the origin and $\vec{L}$ be the angular momentum of this particle about the origin, then

(A) $\vec{r}\cdot\vec{L}=0$ and $\vec{P}\cdot\vec{L}=0$

(B) $\vec{r}\cdot\vec{L}=0$ and $\vec{P}.\vec{L}\neq 0$

(C) $\vec{r}\cdot\vec{L}\neq 0$ and $\vec{P}\cdot\vec{L}=0$

(D) $\vec{r}\cdot\vec{L}\neq 0$ and $\vec{P}\cdot\vec{L}\neq 0$

78. A man A, mass 60 kg and other man B, mass 70 kg are sitting at two extremes of 2m long boat of mass 70 kg, standing still in water as shown in Fig. 5.30. They come to the middle of boat. (Neglect friction). How far does the boat move on the water during the process?

(A) 5 cm Left ward
(B) 5 cm Right ward
(C) 7 cm Left ward
(D) 7 cm Right ward

Fig. 5.30

79. Two point masses connected by an ideal string are placed on a smooth horizontal surface as shown in Fig. 5.31. A sharp impulse of 10 kg-m/s is given to the 5 kg mass. The velocity of the 10 kg mass will be

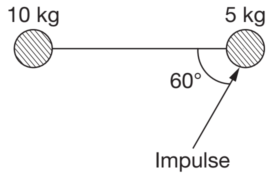


Fig. 5.31

(A) $\frac{2}{3}$ m/s (B) $\frac{1}{3}$ m/s
(C) 2 m/s (D) Zero

80. A small ball thrown at an initial velocity u directed at an angle $\theta = 37°$ above the horizontal collides inelastically (e = 1/4) with a vertical massive wall moving with a uniform horizontal velocity $u/5$ towards ball. After collision with the wall, the ball returns to the point from where it was thrown. Neglect friction between ball and wall. The time t from beginning of motion of the ball till the moment of its impact with the wall is (tan37° = 3/4)

(A) $\frac{3u}{5g}$ (B) $\frac{18u}{25g}$
(C) $\frac{54u}{125g}$ (D) $\frac{54u}{25g}$

81. A ball of mass m falls vertically from a height h and collides with a block of equal mass moving horizontally with velocity v on a smooth surface. The co-efficient of kinetic friction between the block and ball is 0.2 and co-efficient of restitution is 0.5. The difference in velocity of block before and after collision, is

(A) $0.1\sqrt{2gh}$ (B) $0.2\sqrt{2gh}$
(C) $0.3\sqrt{2gh}$ (D) $0.4\sqrt{2gh}$

82. A particle is moving with constant velocity has initial momentum P is given an impulse of magnitude I, If there is no change is its kinetic energy of the particle then

(A) angle between its initial momentum and impulse must be < 90º.
(B) angle between its initial momentum and impulse must be > 90º.
(C) angle between its initial momentum and impulse is 90º.
(D) Not possible.

83. If the thrust acting on a rocket moving with a velocity of 300 m/s is 210 N, then the rate of combustion of fuel is

(A) 0.7 kg/s (B) 1.4 kg/s
(C) 0.07 kg/s (D) 10.7 kg/s

84. Figure 5.32 shows a block A of mass $6m$ having a smooth semi-circular groove of radius a placed on a smooth horizontal surface. A block B of mass m is released from a position in groove where its radius is horizontal. The speed of block A when block B reaches its bottom is

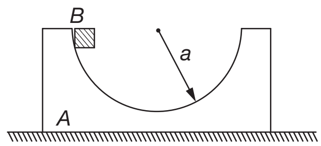
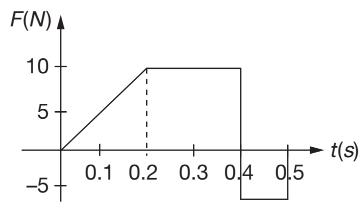


Fig. 5.32

(A) $\sqrt{ga}$ (B) $\sqrt{\frac{2ga}{7}}$
(C) $\sqrt{\frac{ga}{21}}$ (D) Zero

85. A body of mass 5 kg is moving along a straight line with a velocity of 2 ms^{-1}. A variable force F started acting on it for a time period of 0.5s as shown in force-time curve. The final velocity of the block is

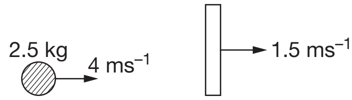
(A) 2 ms^{-1} (B) 2.2 ms^{-1}
(C) 2.5 ms^{-1} (D) 2.7 ms^{-1}



86. A 20 gm bullet moving with speed v gets embedded in a 10 kg block suspended from the ceiling by a massless rope. The block and the bullet swing to a height of 45 cm above the equilibrium position. The initial speed of the bullet is (g = 10 ms^{-2})

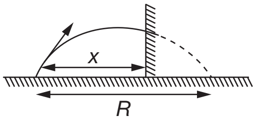
(A) 1000 ms^{-1} (B) 1100 ms^{-1}
(C) 1500 ms^{-1} (D) 1503 ms^{-1}

87. A ball of mass 2.5 kg moving with a velocity of 4 ms^{-1} heading towards a wall. The wall is also moving in the same direction with a speed of 1.5 ms^{-1} as shown. If coefficient of restitution between ball and wall is 0.4, the speed of ball after collision is



(A) 0.5 ms^{-1} (B) 1 ms^{-1}
(C) 2.5 ms^{-1} (D) 4 ms^{-1}

88. A particle is projected with such a velocity that its range is R. The particle undergoes elastic collision with a wall placed at a distance of x (> R/2). The horizontal distance of point of projection and point where particle hits the ground is



(A) $R + x$ (B) $x - R$
(C) $2x - R$ (D) $R + 2x$

89. A geometry consists of a square and a disc as shown in Fig. 5.33. It is made up of an iron sheet of uniform density. The y-co-ordinate of centre of mass of the geometry with respect to point O is

(A) $\frac{4a+3\pi a}{4+\pi}$ (B) $\frac{4a+3\pi a}{4+3\pi}$
(C) $(4+3\pi)a$ (D) $3a$

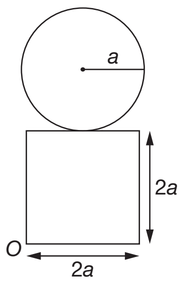


Fig. 5.33

90. A particle of mass 15 kg has an initial velocity $\vec{v}_i = x_{rel} = U_{x_{rel}} t$ m/s. It collides with another body, resulting in a velocity $\vec{v}_f = 6\hat{i} + 4\hat{j} + 5\hat{k}$ m/s after impact. The average force of impact on the particle if impact time is 0.1 s will be

(A) $15(5\hat{i}+6\hat{j}+5\hat{k})$ N (B) $x_{rel} = U_{x_{rel}} t$ N
(C) $150(5\hat{i}+6\hat{j}+5\hat{k})$ N (D) $150(5\hat{i}+6\hat{j}-5\hat{k})$ N

91. A ball of mass m moving at a speed u makes a head on collision with an identical ball kept at rest. The kinetic energy of the balls after the collision is 3/4th of the original. The co-efficient of restitution will be

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$
(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{3}$

92. A solid sphere of radius R is rolled by a force F acting at the top of the sphere as shown in the Fig. 5.34. There is no slipping and initially sphere is at rest. Then

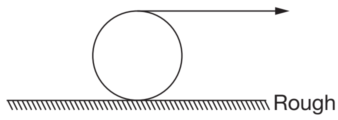


Fig. 5.34

(A) Work done by force F when the centre of mass (c.m) moves a distance S is $2FS$
(B) Speed of the c.m. when c.m. moves a distance S is $\sqrt{\frac{20}{9}\frac{RS}{M}}$
(C) Work done by the force F when c.m. moves a distance S is FS
(D) Speed of the c.m. when c.m. moves a distance S is $\sqrt{\frac{4RS}{M}}$

93. In the Fig. 5.35 shown, a spring mass system is placed on a horizontal smooth surface in between two vertical rigid walls W_1 and W_2. One end of spring is fixed with wall W_1 and other end is attached with mass m which is free to move. Initially, spring is tension free and having natural length l_o. Mass m is compressed through distance a and released. Taking the collision between wall W_2 and mass m as elastic and K as spring constant, the average force exerted by mass m on wall W_2 is

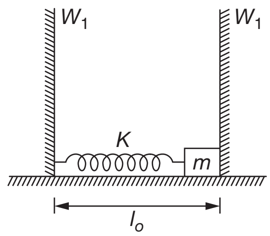
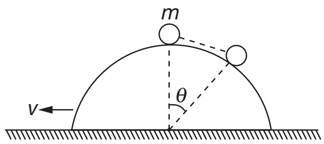


Fig. 5.35

(A) $\frac{2aK}{\pi}$ (B) $\frac{aK}{\pi}$
(C) $\frac{aK}{2\pi}$ (D) $\frac{2aK}{3\pi}$

94. A particle of mass m is placed on top of a smooth hemispherical wedge of mass $4m$ at rest. The hemispherical wedge is kept on a frictionless horizontal surface. The particle is given a gentle push. The angular velocity of the particle relative to the wedge if wedge has velocity v when particle has fallen an angle θ with respect to the wedge is



(A) $\dfrac{5v}{R\cos\theta}$ (B) $\dfrac{v}{R\cos\theta}$

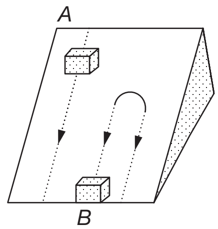
(C) $\dfrac{4v}{R\cos\theta}$ (D) $\dfrac{5v}{R\sin\theta}$

95. A small ball thrown at an initial velocity u directed at an angle $\theta = 37°$ above the horizontal collides inelastically (e = 1/4) with a vertical massive wall moving with a uniform horizontal velocity $u/5$ towards ball. After collision with the wall, the ball returns to the point from where it was thrown. Neglect friction between ball and wall. The time t from beginning of motion of the ball till the moment of its impact with the wall is (tan 37° = 3/4)

(A) $\dfrac{3u}{5g}$ (B) $\dfrac{18u}{25g}$

(C) $\dfrac{54u}{125g}$ (D) $\dfrac{54u}{25g}$

96. A mass A is released from the top of a frictionless inclined plane 18 m long and reaches the bottom 3 second later. At the instant when A is released, a second mass B is projected upwards along the plane from the bottom with a certain initial velocity. The mass B travels a distance up the plane, stops and returns to the bottom simultaneously with A. The two masses do not collide. Initial velocity of B is



(A) 4 ms^{-1} (B) 5 ms^{-1}
(C) 6 ms^{-1} (D) 7 ms^{-1}

97. A bomb of mass 12 kg is dropped by a fighter plane moving horizontally with a speed of 100 ms^{-1} from a height of 1 km from the ground. The bomb exploded after 10s into two pieces of masses in the ratio 1:5. If the small part started moving horizontally with a speed of 600 ms^{-1} the speed of bigger part will be (g = 10 ms^{-2})

(A) 100 ms^{-1} (B) $10\sqrt{65}$ ms^{-1}

(C) 120 ms^{-1} (D) $100\sqrt{2}ms^{-1}$

98. After a totally inelastic collision, two objects of the same mass and same initial speed are found to move together at half of their initial speed. The angle between the initial velocities of the objects is

(A) 120° (B) 90°
(C) 60° (D) 30°

More than One Option Correct Type

99. Two uniform solid spheres having unequal masses and unequal radii are released from rest from the same height on a rough incline. Then, if the spheres roll without slipping.

(A) the heavier sphere reaches the bottom first.
(B) the bigger sphere reaches the bottom first.
(C) the two spheres reach the bottom together.
(D) velocity of the both sphere will be same at the bottom.

100. A body moving towards a finite body at rest collides with it. It is possible that

(A) both the bodies come to rest.
(B) both the bodies move after collision.
(C) the moving body comes to rest and the stationary body starts moving.
(D) the stationary body remains stationary; the moving body changes its velocity.

101. In the adjoining Fig. 5.36 block A is of mass m and block B is of mass $2m$. The spring has a force constant k. All the surfaces are smooth and the system is released from rest with the spring not being stretched

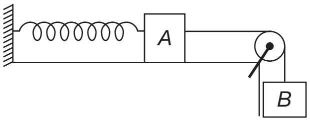


Fig. 5.36

(A) The maximum extension of the spring is $\frac{4mg}{k}$

(B) The speed of block A when extension in spring is $\frac{2mg}{k}$, is $2g\sqrt{\frac{2m}{3k}}$

(C) Net acceleration of block B when the extension in the spring is maximum, is $\frac{2}{3}g$.

(D) Tension in the thread for extension of $\frac{2mg}{k}$ in spring is mg.

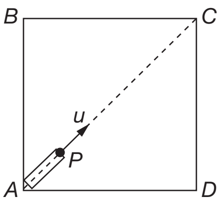
102. A large rectangular box $ABCD$ falls vertically with an acceleration a. A toy gun fixed at A and aimed towards C, fires a particle P

(A) P will hit C if $a = g$

(B) P will hit the roof BC if $a > g$

(C) P will hit the wall CD or the floor AD if $a < g$.

(D) may be either (A), (B) or (C), depending on the speed of projection of P.



103. A block of mass m is placed on a rough horizontal surface. The coefficient of friction between them is μ. An external horizontal force is applied to the block and its magnitude is gradually increased. The force exerted by the block on the surface is R, then which of the following statement/s is/are correct.

(A) The magnitude of R will gradually increase.

(B) $R \le mg\sqrt{\mu^2 + 1}$.

(C) The angle made by R with the vertical will gradually increase.

(D) The angle made by R with the vertical $\le \tan^{-1}\mu$.

104. Two particles, P of mass $2m$ and Q of mass m, are subjected to mutual force of attraction and no other force acts on them. At $t = 0$, P is at rest at point O and Q is moving away from O with a speed $5u$. At a later instant $t = T$ (before any collision has taken place), Q is moving towards O with speed u. Then

(A) momentum of particle P at $t = T$ is zero.

(B) momentum of particle P at $t = T$ is $6\,mu$.

(C) work done by the force of attraction during $0 \le t \le T$ is $12\,mu^2$.

(D) work done by the force of attraction during $0 \le t \le T$ is $-3\,mu^2$.

105. Which of the following statements is/are correct?

(A) In head on elastic collision of two bodies of equal masses, velocities of colliding bodies interchange.

(B) In case of elastic collision kinetic energy remains conserved before collision and during collision.

(C) In case of elastic collision momentum is conserved before and during the collision (assuming that no other force acts on the bodies).

(D) In case of oblique elastic collision of two bodies of equal masses the velocities of colliding bodies interchange.

106. A uniform rod of mass m and length l is placed in gravity free space and linear impulse J is given to the rod at a distance $x = \frac{l}{4}$ from centre and perpendicular to the rod. Point A is at a distance $\frac{l}{3}$ from centre as shown in the Fig. 5.37. Then

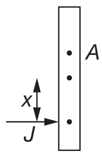


Fig. 5.37

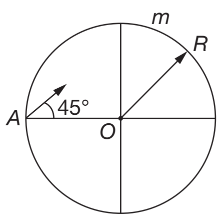
(A) speed of centre of rod is $\frac{J}{m}$.

(B) speed of point A is zero.

(C) speed of upper end of rod is $\frac{J}{2m}$.

(D) speed of lower end of rod is $\frac{5}{2}\frac{J}{m}$.

107. A ring of mass m and radius R is placed on a frictionless horizontal surface. A particle of mass m is projected from point A with velocity v at an angle of 45° with AO as shown. The correct statements are



(A) The particle reaches the same point A on the ring after time $\frac{4R\sqrt{2}}{v}$.

(B) Magnitude of impulse transformed during first collision is $\frac{mv}{\sqrt{2}}$.

(C) Magnitude of impulse transformed during second collision is $\frac{mv}{\sqrt{2}}$.

(D) Particle reaches diametrically opposite point on the ring in time $\frac{2R}{v}$.

Passage Based Questions

Passage 1

A horizontal frictionless string is threaded through a bead of mass m. The string is kept in tension between two vertical opposite sides of a cart of mass M (see Fig. 5.38). Length of thread is L and radius of bead is r ($r << L$). Initially, the bead is at right edge of the cart. At $t = 0$, the cart is imparted a velocity v_0. All collisions are elastic.

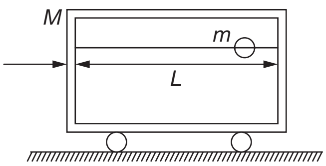


Fig. 5.38

108. The velocity of C.O.M of the cart and the bead after 3 successive collisions is

(A) v_0

(B) $v_0 - \frac{Mv_0}{M+m}$

(C) $v_0 + \frac{Mv_0}{M+m}$

(D) $\frac{Mv_0}{M+m}$

109. The velocity of bead after first collision will be

(A) $\frac{2Mv_0}{m+M}$

(B) $\frac{Mv_0}{m+M}$

(C) $\left(\frac{M-m}{M+m}\right)v_0$

(D) $\left(\frac{M+M}{M-m}\right)v_0$

110. Time internal between second and third collision will be

(A) $\frac{L}{v_0}$

(B) $\frac{2L}{v_0}$

(C) $\frac{3L}{v_0}\frac{M}{(M+m)}$

(D) $\frac{3L}{v_0}\frac{m}{(M+m)}$

111. Distance travelled by cart before second collision is

(A) $2L$

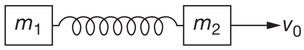
(B) $2L\left(\frac{m}{M+m}\right)$

(C) $2L\left(\frac{M}{M+m}\right)$

(D) $L + L\frac{m}{M+m}$

Passage 2

Two blocks of masses m_1 and m_2 are connected by a spring of spring constant k. The block of mass m_2 is given a sharp impulse so that it acquires a velocity v_0 toward right.



112. Velocity of centre of mass will be

(A) Zero

(B) $\frac{m_1 v_0}{m_1 + m_2}$

(C) $\frac{m_2 v_0}{m_1 + m_2}$

(D) $\left(\frac{m_1 v_0}{m_1 + m_2}\right)v_0$

113. Find maximum elongation of spring

(A) $v_0\left[\frac{m_1 m_2}{(m_1+m_2)k}\right]^{\frac{1}{2}}$

(B) $v_0\left(\frac{m_1^2}{(m_1+m_2)k}\right)^{\frac{1}{2}}$

(C) $v_0\left[\frac{m_2^2}{(m_1+m_2)k}\right]$

(D) Data insufficient

114. If spring break when velocity of m_2 is $\frac{3v_0}{4}$ the velocity of other block will be

(A) $\frac{1}{4}\left(\frac{m_2}{m_1}\right)v_0$

(B) $\frac{3}{4}\left(\frac{m_2}{m_1}\right)v_0$

(C) $\frac{1}{4}\left(\frac{m_1}{m_2}\right)v_0$

(D) $\frac{3}{4}\left(\frac{m_1}{m_2}\right)v_0$

Passage 3

When two objects collide, the mutual impulsive forces acting over the collision time Δt cause a change in their respective momenta,

$$\Delta\vec{P_1} = \vec{F}_{12}\Delta t \qquad (1)$$

$$\Delta\vec{P_2} = \vec{F}_{21}\Delta t \qquad (2)$$

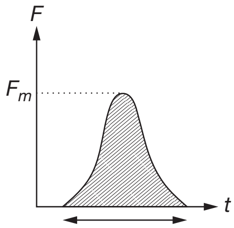
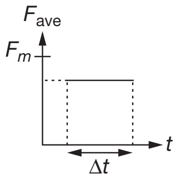
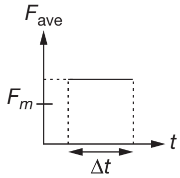


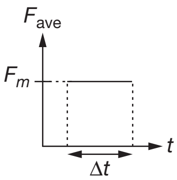
Fig. 5.39

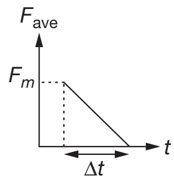
where $\vec{F}_{12}$ is the force exerted on the first particle by the second. $\vec{F}_{21}$ is likewise the force exerted on the second particle by the first particle. The variation of this force during collision is shown in Fig. 5.39. The average force (F_{ave}) during collision time is defined as a constant force which brings same change in momentum as the force indicated in Fig. 5.39. The total kinetic energy of the system is not necessarily conserved. The impact and deformation during collision may generate heat and sound. A useful way to picture the deformation during collision is in terms of a "compressed spring". If the "spring" connecting the two bodies regains its original shape it is said to be elastic collision. If the deformation is partly relieved the potential energy changes permanently and some of the initial kinetic energy is lost, then the collision is called inelastic collision. (here the potential energy is due to the force acting during collision, not the gravitational potential energy)

115. The average force during collision is correctly defined by the curve

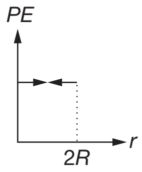
(A) 

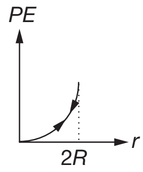
(B) 

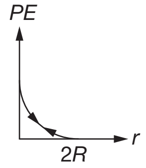
(C) 

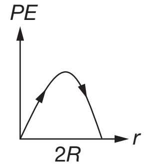
(D) 

116. Which of the following potential energy curves possibly describe elastic collision between two spherical balls? (Here r is the distance between the centres of the balls and R is the radius of each sphere)

(A) 

(B) 

(C) 

(D) 

117. As per the compressed spring analogy, the initial length of spring before collision is l_1 and it is l_2 after collision. Then for inelastic collision

(A) $l_1 = l_2$ (B) $l_1 < l_2$

(C) $l_1 > l_2$ (D) None

Passage 4

A smooth horizontal surface acquires a circular shape of radius R at one of the extremes. A number of small elastic balls are placed at rest on the linear part of the horizontal surface. Let the number of such balls be n. The balls are not of same mass n^{th} ball of mass $\frac{M}{2^{n-1}}$ lies at the planar surface near the circular extreme whereas M is the mass of first ball at the starting end of the horizontal position of the surface. The masses of the balls follow a sequence given by. Displacement of ball i.e. ball of the mass may result into the completion of vertical motion i.e. loop in the loop of the n^{th} ball.

Ball	I	II	III	IV	……	n^{th}
Mass	M	$M/2$	$M/2^2$	$M/2^3$	……	$\frac{M}{2^{n-1}}$

118. Velocity of the second ball when u is the minimum velocity given to the first ball so that the n^{th} ball may result into completion of vertical motion is

(A) $\frac{3}{4}u$ (B) $\frac{4}{3}u$ (C) $\frac{1}{3}u$ (D) $\frac{2}{3}u$

119. Velocity of the third ball when v is the minimum initial velocity given to the first ball so that the n^{th} ball may result into completion of vertical motion is

(A) $\left(\frac{4}{3}\right)^2 u$ (B) $\left(\frac{3}{4}\right)^2 u$

(C) $\left(\frac{1}{3}\right)^2 u$ (D) $\left(\frac{2}{3}\right)^2 u$

120. Velocity of the first ball of mass M so that the n^{th} ball may loop is

(A) $\left(\frac{2}{4}\right)^{n-1}\sqrt{5gR}$ (B) $\left(\frac{1}{3}\right)^{n+1}\sqrt{2gR}$

(C) $\left(\frac{2}{3}\right)^{n-1}\sqrt{gR}$ (D) $\left(\frac{3}{4}\right)^{n-1}\sqrt{gR}$

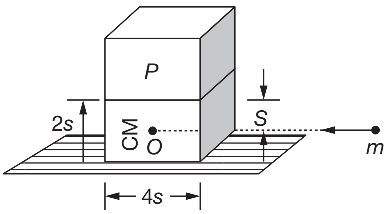
121. Vertical circular motion is an example of non-uniform motion and for completion of vertical circular motion the maximum velocity at the start of vertical motion from lowest point is

(A) $\sqrt{gR}$ (B) $\geq\sqrt{5gR}$

(C) $=\sqrt{5gR}$ (D) $\leq\sqrt{5gR}$

Passage 5

Consider a block P of mass 2 M placed on another block Q of mass 4 M lying on a fixed rough horizontal surface of coefficient of friction μ between this surface and the block Q. Surfaces of P and Q interacting with each other are smooth.



A mass M moving in the horizontal direction along a line passing through the centre of mass of block Q is normal to the face. Speed of mass M is v_0 when it collides elastically with the block Q at a height s from the rough surface. Both the blocks have same length 4 s. Velocity v_0 is enough to make the block P topple.

122. When mass M collides with block Q with velocity v_0 the block P.

(A) Does not move (B) Moves forward
(C) Moves backward (D) Either (B) or (C)

123. Work done in moving the block Q through a distance 2s is

(A) 6 μMgs (B) 12 μMgs
(C) 18 μMgs (D) 24 μMgs

124. When mass M is colliding with velocity v_0 KE acquired by block Q is

(A) $\frac{8}{25}Mv_0^2$ (B) $\frac{25}{8}Mv_0^2$

(C) $\frac{1}{2}Mv_0^2$ (D) $\frac{3}{20}Mv_0^2$

Assertion-Reason Type

125. **Assertion:** In projectile motion, the rate of change in magnitude of potential energy of a particle first decreases and then increases during motion.

Reason: In projectile motion, the rate of change in momentum of a particle remains constant during motion.

(A) A (B) B
(C) C (D) D

126. **Assertion:** In head on elastic collision of two bodies of equal masses the velocities are interchanged.

Reason: In elastic collisions both momentum and kinetic energy are conserved.

(A) A (B) B (C) C (D) D

127. **Assertion:** Linear momentum of a body changes even when it is moving uniformly in a circular path.

Reason: Uniform circular motion has a constant speed but variable velocity.

(A) A (B) B (C) C (D) D

128. **Assertion:** Two different masses released from same height and air resistance is same for two masses, then both masses reaches at the same time on the ground.

Reason: Lighter and heavier bodies moving with same momenta and experiencing same retarding force have equal stopping time.

(A) A (B) B (C) C (D) D

129. **Assertion:** A person left on smooth floor can get away by throwing some object in a direction opposite to the direction in which he wants to move.

Reason: Principle of conservation of momentum says that total momentum of an isolated system always remains conserved.

(A) A (B) B (C) C (D) D

130. **Assertion:** Impulse and momentum have different dimensions.

Reason: From Newton's second law of motion, impulse is equal to change in momentum.

(A) A (B) B (C) C (D) D

131. Assertion: A body can have energy without having momentum but it cannot have momentum without having kinetic energy

Reason: Momentum and energy have same dimensions.

(A) A (B) B (C) C (D) D

132. Assertion: As per law of conservation of momentum, the momentum can never change.

Reason: Momentum is quantity of motion possessed by a body so there is no question of its change.

(A) A (B) B (C) C (D) D

133. Assertion: In an elastic collision of two bodies, the linear momentum and energy of each body is conserved.

Reason: In elastic collision the total linear momentum and energy of the system is conserved before and after the collision.

(A) A (B) B (C) C (D) D

134. Assertion: Two photons having the same kinetic energy must have the same momentum.

Reason: Kinetic energy and momentum are related as $KE = \dfrac{p^2}{2m}$

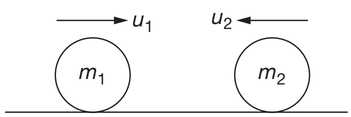
(A) A (B) B (C) C (D) D

Match the Column Type

135. An open truck is loaded with 2×10^3 kg of corn from a hopper. If the truck travels at 3 m/s under the hopes during the 4 s it takes to load the corn

Column-I	Column-II
(A) Average force required to keep the truck moving forward	1. 1500 N
(B) Work done by average force	2. 18000
(C) Change in kinetic energy	3. 9000
(D) Heat produce in this process	4. Does not depend on reference frame

136. Two balls of masses m_1 and m_2 are moving towards each other with speeds u_1 and u_2. They collide head-on and their speeds are v_1 and v_2 after collision. ($m_1 = 8$ kg, $m_2 = 2$kg, $u_2 = 3$ m/s)



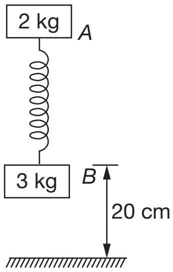
Column-I	Column-II
(A) The speed u_1(in m/s) so that both balls move in same direction if co-efficient of restitution is $e = 0.5$	1. $\dfrac{1}{14}$
(B) The speed u_1(in m/s) so that maximum energy is transformed to m_2 (assume elastic collision)	2. $\dfrac{1}{8}$
(C) Co-efficient of restitution if m_2 stops after collision and $u_1 = 0.5$ m/s	3. 2
(D) If collision is inelastic and $u_1 = 3$ m/s, the loss of kinetic energy (in J) after collision may be	4. 4

137. A particle of mass m_1 moving with velocity u_1 strikes another particle of mass m_2 moving with velocity u_2. After collision the velocities of the particles are v_1 and v_2 respectively. Both the particles are moving on a frictionless horizontal surface. Column-I represents the nature of collision between the particles and Column-II represents the equations for physical quantities. (Symbols have their usual meanings)

Column-I	Column-II
(A) Head on elastic collision	1. $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$
(B) Head on inelastic collision	2. $e = \dfrac{\lvert\vec{v}_2\rvert + \lvert\vec{v}_1\rvert}{\lvert\vec{u}_1\rvert + \lvert\vec{u}_2\rvert}$

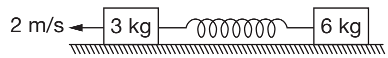
(C) Oblique elastic collision	3. $\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2$ $=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$
(D) Oblique inelastic collision	4. $\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2$ $>\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$

138. A massless spring constant $k = 100$ N/m is connected to two blocks A and B of masses 2 kg and 3 kg respectively held at rest with the spring relaxed at height 20 cm above the ground. The system is released and after hitting the ground block B comes to rest.



Column-I	Column -II
(A) Loose of energy during collision	1. Zero
(B) Spring energy when block A is in equilibrium	2. 6 J
(C) Work done by gravitational force on A from initial to till block A comes in equilibrium	3. 8 J
(D) Spring energy at the time of collision of B with surface	4. 2 J
	5. –8 J

139. Two blocks of masses 3 kg and 6 kg are connected by an ideal spring and are placed on a frictionless horizontal surface. The 3 kg block is imparted a speed of 2 m/s towards left. (consider left as positive direction)

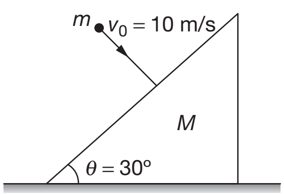


Column-I	Column-II
(A) When the velocity of 3 kg block is $\frac{2}{3}$ m/s.	1. Velocity of centre of mass is $\frac{2}{3}$ m/s.
(B) When the speed of 3 kg block is $\frac{2}{3}$ m/s.	2. Deformation of the spring is zero.
(C) When the speed of 3 kg block is minimum.	3. Deformation of the spring is maximum.
(D) When the velocity of 6 kg block is maximum.	4. Both the blocks are at rest with respect to each other.

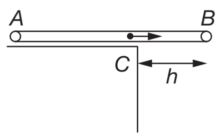
Integer type

140. A body of mass m_1 makes a head on collision perfectly elastic with a body of mass m_2 initially at rest. (i) What fraction of initial energy of mass m_1 is lost in collision? (ii) For what ratio of $\frac{m_2}{m_1}=\eta$, the fraction of energy loss is maximum?

141. A ball of mass 1 kg moving with velocity 10 m/s collides perpendicularly on a smooth stationary wedge of mass 2 kg. If the coefficient of restitution is $e = 7/20$ then find the velocity of ball after the collision. [in ms^{-1}]



142. A straw is placed on the corner of a table of length 3 cm and some part of it is off the table. An insect of half the mass of the straw move from C towards end B. The maximum value of length BC projected from table so that the insect reach the end B safely is equal to h, then find the value of h.



143. A monkey of mass m is sitting on a platform of mass M. Monkey can jump with a velocity of 5 m/s making an angle 37° with the horizontal with respect to platform. If value of m/M is $x \times 10^{-1}$, to jump the monkey 1 meter with respect to the ground. Find out the value of x.

144. In the Fig. 5.40 shown a pendulum having bob of mass $m = 1$ kg is hanging from the roof of a box of mass M, kept over a smooth horizontal plane. Box and a block of mass m' is connected by a light string going over a massless light pulley. When system is in motion, string makes angle 37° from the vertical at equilibrium with respect to box. Find out the value of m' in kg

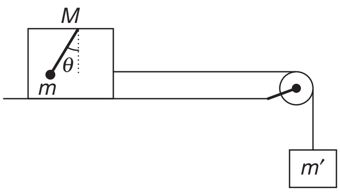
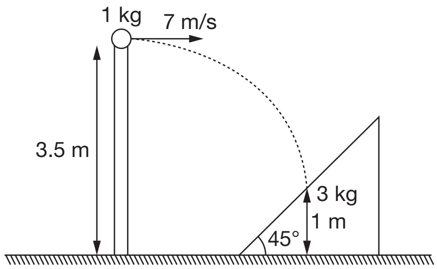


Fig. 5.40

145. A ball of mass 1 kg is projected with velocity 7 m/s horizontally from a tower of height 3.5 m. It collides elastically with a wedge kept on ground of mass 3 kg and inclination 45°. The Ball does collide with the wedge at a height of 1 m above the ground. Find the velocity of the wedge and the ball after collision. (Neglect friction at any contact)

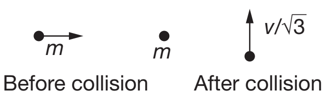


Previous Years' Questions

146. A machine gun fires a bullet of mass 40 g with a velocity $1200\,\text{ms}^{-1}$. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? **[2004]**

(A) Two (B) Four
(C) One (D) Three

147. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the first mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision. **[2005]**



(A) $\sqrt{3}v$ (B) v
(C) $\frac{v}{\sqrt{3}}$ (D) $\frac{2}{\sqrt{3}}v$

148. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is $4\,\text{ms}^{-1}$. The kinetic energy of the other mass is **[2006]**

(A) 144 J (B) 288 J
(C) 192 J (D) 96 J

149. Statement-1: Two particles moving in the same direction do not lose all their energy in a completely inelastic collision. **[2010]**

Statement-2: Principle of conservation of momentum holds true for all kinds of collisions.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1
(B) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(C) Statement-1 is false, Statement-2 is true.
(D) Statement-1 is true, Statement-2 is false.

150. The Fig. 5.41 shows the position – time $(x - t)$ graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[2010]**

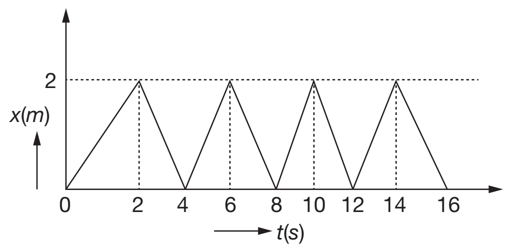


Fig. 5.41

(A) 0.4 Ns (B) 0. 8 Ns
(C) 1.6 Ns (D) 0.2 Ns

151. This question has Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements. **[2013]**

Statement-I: A point particle of mass M moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement-II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(A) Statement-I is true, Statement-II is true not a correct explanation of Statement-I.
(B) Statement-I is true, Statement-II is false.
(C) Statement-I is false, Statement-II is true.
(D) Statement-I is true, Statement-II is true, Statement-II is a correct explanation of Statement-I.

152. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to **[2015]**

(A) $\frac{3h}{4}$ (B) $\frac{5h}{8}$
(C) $\frac{3h^2}{8R}$ (C) $\frac{h^2}{4R}$

153. A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **[2015]**

(A) 50% (B) 56%
(C) 62% (D) 42%

ANSWER KEYS

Single Option Correct Type

1. (C)	**2.** (D)	**3.** (C)	**4.** (D)	**5.** (D)	**6.** (C)	**7.** (B)	**8.** (C)	**9.** (B)	**10.** (C)
11. (B)	**12.** (B)	**13.** (A)	**14.** (B)	**15.** (A)	**16.** (A)	**17.** (C)	**18.** (A)	**19.** (A)	**20.** (A)
21. (A)	**22.** (D)	**23.** (B)	**24.** (B)	**25.** (B)	**26.** (A)	**27.** (C)	**28.** (A)	**29.** (C)	**30.** (A)
31. (A)	**32.** (B)	**33.** (B)	**34.** (C)	**35.** (C)	**36.** (B)	**37.** (B)	**38.** (A)	**39.** (A)	**40.** (B)
41. (D)	**42.** (C)	**43.** (A)	**44.** (A)	**45.** (C)	**46.** (C)	**47.** (A)	**48.** (A)	**49.** (A)	**50.** (A)
51. (A)	**52.** (D)	**53.** (D)	**54.** (B)	**55.** (D)	**56.** (C)	**57.** (A)	**58.** (B)	**59.** (D)	**60.** (B)
61. (C)	**62.** (C)	**63.** (A)	**64.** (A)	**65.** (C)	**66.** (A)	**67.** (D)	**68.** (D)	**69.** (C)	**70.** (A)
71. (A)	**72.** (D)	**73.** (C)	**74.** (B)	**75.** (C)	**76.** (A)	**77.** (A)	**78.** (B)	**79.** (B)	**80.** (C)
81. (C)	**82.** (B)	**83.** (A)	**84.** (C)	**85.** (C)	**86.** (D)	**87.** (A)	**88.** (C)	**89.** (A)	**90.** (C)
91. (A)	**92.** (A)	**93.** (A)	**94.** (A)	**95.** (C)	**96.** (C)	**97.** (C)	**98.** (A)		

More than One Option Correct Type

99. (C) and (D)	**100.** (B) and (C)	**101.** (A), (B) and (C)
102. (A), (B) and (C)	**103.** (A), (B), (C) and (D)	**104.** (B) and (D)
105. (A) and (C)	**106.** (A), (B), (C) and (D)	**107.** (A), (B) and (C)

Passage Based Questions

Passage 1

108. (D) **109.** (A) **110.** (A) **111.** (C)

Passage 2

112. (C) **113.** (A) **114.** (A)

Passage 3

115. (A) **116.** (C) **117.** (C)

Passage 4

118. (D) **119.** (B) **120.** (C) **121.** (C)

Passage 5

122. (A) **123.** (B) **124.** (A)

Assertion–Reason Type

125. (B) **126.** (A) **127.** (A) **128.** (D) **129.** (A) **130.** (D) **131.** (B) **132.** (D) **133.** (D) **134.** (D)

Match the Column type

135. (A) → 1, 4; (B) → 2; (C) → 3; (D) → 3, 4
136. (A) → 3, 4; (B) → 3; (C) → 1; (D) → 1, 2, 3, 4
137. (A) → 1, 2, 3; (B) → 1, 2, 4; (C) → 1, 3; (D) → 1, 4
138. (A) → 2; (B) → 4; (C) → 3; (D) → 1
139. (A) → 1, 3, 4; (B) → 1, 2, 3, 4; (C) → 1; (D) → 1, 2

Integer type

140. = 1 **141.** $v_1 = 2$ m/s **142.** = 1 cm **143.** $x = 14$ **144.** 7 m/s
145. 4 m/s

Previous Years' Questions

146. (D) **147.** (D) **148.** (B) **149.** (A) **150.** (B) **151.** (C) **152.** (A) **153.** (B)

HINTS AND SOLUTIONS

Single Option Correct Type

1. $\vec{\tau} = (-\hat{i} + \hat{j}) \times (-F\hat{k}) = -F(\hat{i} + \hat{j})$
The correct option is (C)

2. The correct option is (D)

3. $P_i = P_f$; $\sqrt{2}\, mv = 2mv'$; $v' = \dfrac{v}{\sqrt{2}}$
The correct option is (C)

4. The correct option is (D)

5. $\dfrac{v^2}{r} = \dfrac{4}{r^2}$ i.e. $v = \dfrac{2}{\sqrt{r}}$

Hence $p = \dfrac{2m}{\sqrt{r}}$

The correct option is (D)

6. $F = \dfrac{2mv\sin\theta}{t} = \dfrac{2\times3\times10\sin60°}{0.2} = 150\sqrt{3}$ N

The correct option is (C)

7. The correct option is (B)

8. If $m_1 >> m_2$
Then $v_1 \approx u_1$ and $v_2 = 2u_1 - u_2$
$v_2 = 2u_1 - 0 = 2 \times 10 = 20$ m/s
The correct option is (C)

9. The correct option is (B)

10. In an inelastic collision only momentum of the system may remain conserved. Some energy can be lost in the form of heat, sound etc.

The correct option is (C)

11. By momentum conservation, $mv = 2mv' \cos(45°)$

$\therefore \quad v' = \dfrac{v}{\sqrt{2}}$

The correct option is (B)

12. Loss in kinetic energy $= \dfrac{1}{2}\mu v_{rel}^2 = \dfrac{1}{2}\left(\dfrac{6}{5}\right)(10)^2 = 60$ J

The correct option is (B)

13. $mgh = \dfrac{80}{100}(mg)(100)$

$h = 80$ m
The correct option is (A)

14. $F = \dfrac{\Delta P}{\Delta t} = \dfrac{2mv}{\Delta t}$

$= \dfrac{2\times10\times10^{-3}\times5}{0.01} = 10$ N

The correct option is (B)

15. Stopping distance $\propto v^2$
$\therefore$ v has increased by factor of 2
The correct option is (A)

16. Time of collision $= \dfrac{\text{Separation distance}}{\text{Relative velocity of approach}}$

The correct option is (A)

17. By conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3u_1 - 2u_2 = 0 + 2u_1\,,\; u_2 = \frac{u_1}{2}\,,\; e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{u_1 - 0}{u_1 + \frac{u_1}{2}} = \frac{2}{3}$

The correct option is (C)

18. The momentum of the system in horizontal direction will not change.

$(4000)40 = (4000 - 400)v$

$v = 44.44$ m/s

The correct option is (A)

19. Before explosion, particle was moving along x-axis, i.e., it has no y-component of velocity. Therefore, the centre of mass will not move in y-direction or we can say $y_{com} = 0$.

Now, $y_{com} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$

Therefore, $0 = \frac{(m/4)(+15) + (3m/4)(y)}{(m/4 + 3m/4)}$ or $y = -5$ cm

The correct option is (A)

20. By conservation of linear momentum, $M_A u = -M_A v + M_B v$

$e = 1 \Rightarrow 2v = u$

$\therefore\; M_A \times 2v = -M_A v + M_B v$

$\therefore\; \frac{M_A}{M_B} = \frac{1}{3}$

The correct option is (A)

21. $mv = MV \;\Rightarrow\; M = 1.5$ kg

The correct option is (A)

22. The correct option is (D)

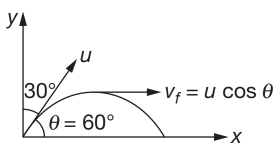
23. $\vec{v}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v}_f = u\cos\theta\hat{i}$

$\Delta\vec{p} = m(\vec{v}_{fi} - \vec{v})$

$\Delta\vec{p} = -mu\sin\theta\,\hat{j}$

$|\Delta\vec{p}| = mu\sin 60° = \frac{\sqrt{3}\,mu}{2}$



The correct option is (B)

24. $K = \frac{P^2}{2m}$. From graph; $4 = \frac{4^2}{2m}$, $m = 2$ kg

The correct option is (B)

25. By constraint relation,

If $a_{40} = a$, then $a_5 = 2a$

$40g - 2T = 40a$ (1)

$T - 5g = 5(2a)$ (2)

From (1) and (2) $a = 5$ m/s^2

$a_{cm} = \frac{40(-5) + 5(10)}{40 + 5} = -\frac{10}{3}$ m/s^2

The correct option is (B)

26. $\vec{P} = (2\cos t)\hat{i} + (2\sin t)\hat{j}$

$\vec{F} = \frac{d\vec{P}}{dt} = -(2\sin t)\hat{i} + (2\cos t)\hat{j}$

$\Rightarrow\; \vec{F} \perp \vec{P}$

The correct option is (A)

27. Before collision, $v_{cm} = \frac{(10)(10) + (30)(3)}{10 + 30} = \frac{19}{4}$ m/s

Since no external force acts on system, velocity of centre of mass remains constant.

The correct option is (C)

28. $p = mv = 0.02 \times 2000 \times 10 = 400$ N/s

The correct option is (A)

29. $m_1u = m_1\left(-\frac{u}{3}\right) + m_2v_2\,;\; u = v_2 + \frac{u}{3}$

Solving $\frac{m_1}{m_2} = 0.5$

The correct option is (C)

30. The momentum of the system in horizontal direction will not change.

$(4000)40 = (4000 - 400)v$

$v = 44.44$ m/s

The correct option is (A)

31. Rate of change of kinetic energy = Fv (For constant acceleration)

$= mav = 4v$

The correct option is (A)

32. $M = 0.9$ kg, $m = 0.1$ kg, $u = 100$ m/s,

$(M + m)v = mu \;\Rightarrow\; v = 10$ m/s, $h = \frac{v^2}{2g} = 5$ m

The correct option is (B)

33. $m = 20$ g, $M = 8$ kg, $u = 500$ ms^{-1}, $v = 100$ ms^{-1}

By the law of conservation of momentum,

$mu = mv + MV \Rightarrow V = 1$ ms^{-1}

$a = \mu g = \frac{V^2}{2s} \;\therefore\; \mu = 0.25$

The correct option is (B)

34. Area of F-t curve = change in momentum

$\frac{1}{2}(4 + 2) \times 75 = m(v - u)$

$225 = 9v$ $(\because u = 0)$

$v = 25$ m/s

The correct option is (C)

35. $v_{cm} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2} = \dfrac{2\times10 + 4\times(-2)}{2+4} = 2$ m/s

The correct option is (C)

36. $m_1 = 1$ kg, $m_2 = 2$ kg, $u_1 = 3\,\text{ms}^{-1}$, $u_2 = 2\,\text{ms}^{-1}$

Initial momentum $= \sqrt{(m_1u_1)^2 + (m_2u_2)^2}$

$= \sqrt{9+16} = 5$ kg ms^{-1}

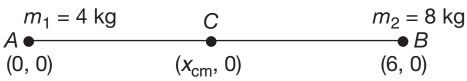
If combined velocity is v, $(m_1 + m_2)v = 5$, $v = \dfrac{5}{3}$ ms^{-1}

Loss in energy $= \dfrac{1}{2}m_1u_1{}^2 + \dfrac{1}{2}m_2u_2{}^2 - \dfrac{1}{2}(m_1 + m_2)v^2 = \dfrac{13}{3}$ J

The correct option is (B)

37. $x_{cm} = \dfrac{8\times6}{4+8} = 4\,\text{m}$

$\therefore$ particle will meet at a distance of 2 m from 8 kg mass.



The correct option is (B)

38. Velocity of centre of mass

$\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \dfrac{20\times2v + 10\times v}{30} = \dfrac{5}{3}v$

The correct option is (A)

39. Impulse $= \begin{pmatrix}\text{Area of force-time graph} \\ \text{under the specified interval}\end{pmatrix}$

$= \dfrac{1}{2}(800 + 200)\times2\times10^{-6} + \dfrac{1}{2}\times800\times10\times10^{-6}$

$= 5\times10^{-3}$ N/s

The correct option is (A)

40. They will meet at the position of CM.

The correct option is (B)

41. Change in momentum = area of F-t curve

$mv = \dfrac{1}{2}(6+2)\times10 = 40$ N/s $\therefore\ v = 4$ m/s

The correct option is (D)

42. In order to conserve momentum, C should move with speed v in a direction opposite to that of B.

The correct option is (C)

43. $v_1 = \dfrac{m - 4m}{m + 4m}v = -\dfrac{3}{5}v$.

The correct option is (A)

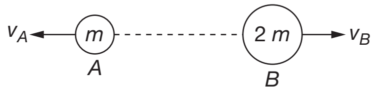
44. By the conservation of momentum,

$m_Av_A = m_Bv_B$

$\Rightarrow\ m\times16 = 2m\times v_B$

$\Rightarrow\ v_B = 8$ m/s

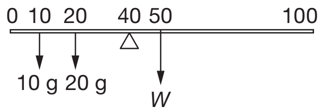
Kinetic energy of system $= \dfrac{1}{2}m_Av_A^2 + \dfrac{1}{2}m_Bv_B^2 = 192$ m J



The correct option is (A)

45. $30\times10 + 20\times20 = W\times10$

$\Rightarrow\ W = 70$ g



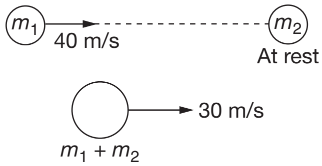
The correct option is (C)

46. By the law of conservation of momentum

$m_1\times40 + m_2\times0 = (m_1 + m_2)\times30$

$\Rightarrow\ 40m_1 = 30m_1 + 30m_2$

$\Rightarrow\ 10m_1 = 30m_2 = \dfrac{m_1}{m_2} = 3$



The correct option is (C)

47. Time of collision $= \dfrac{60}{15} = 4$ s

Therefore, distance from $A = 10\times4 = 40$ m

The correct option is (A)

48. Since $a_1 = \dfrac{d^2x_1}{dt^2} = 0$ and $a_2 = \dfrac{d^2x_2}{dt^2} = 0$

$\therefore$ Velocity of centre of mass remains constant

The correct option is (A)

49. Force on the block $F = u\left(\dfrac{dm}{dt}\right) = 5\times1 = 5$ N

$\therefore$ Acceleration of block $a = \dfrac{F}{m} = \dfrac{5}{2} = 2.5$ m/s^2

The correct option is (A)

50. $mv_1 = 2mv_2$

$\dfrac{1}{2}kx^2 = \dfrac{1}{2}mv_1^2 + \dfrac{1}{2}(2m)v_2^2$

$v_1 = 2x\sqrt{\dfrac{k}{6m}}$, $v_2 = x\sqrt{\dfrac{k}{6m}}$

$v_1 + v_2 = x\sqrt{\dfrac{3k}{2m}}$

The correct option is (A)

51. Since $a_1 = \frac{d^2x_1}{dt^2} = 0$ and $a_2 = \frac{d^2x_2}{dt^2} = 0$

$\therefore$ Velocity of centre of mass remains constant

The correct option is (A)

52. $K_i = \frac{1}{2}mu^2$

By conservation of momentum $mu = mv_1 + nm\,v_2$

$$u = v_1 + nv_2 \quad (1)$$

$$v_2 - v_1 = u_1 - u_2$$

$$\therefore \quad v_2 - v_1 = u \quad (2)$$

$$(1) + (2) \Rightarrow 2u = (n+1)\,v_2$$

$$v_2 = \frac{2u}{n+1}$$

Kinetic energy transferred $K_t = \frac{1}{2}(nm)v_2^{\,2}$, $K_t = \frac{2nmu^2}{(n+1)^2}$,

$$\frac{K_t}{K_i} = \frac{4n}{(n+1)^2}$$

The correct option is (D)

53. Applying conservation of momentum, $m \times v_0 + m \times 0 = (2m)$ v or $v = \frac{v_0}{2}$

Now, $(2m)gh = \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2$ or $h = \frac{v_0^2}{8g}$.

The correct option is (D)

54. $\frac{1}{2}(m+M)\left\{\frac{mv}{m+M}\right\}^2 = (m+M)gh$

$$\therefore \quad v = \frac{\mu + M}{m}\sqrt{2gh}$$

The correct option is (B)

55. Here, the x co-ordinate of centre of mass of the system remains unchanged when the mass m moved a distance $L\cos\theta$, let the mass $(m+M)$ moves a distance x in the backward direction.

$$\therefore \quad (M+m)x - mL\cos\theta = 0$$

$$\therefore \quad x = \frac{mL\cos\theta}{m+M}$$

The correct option is (D)

56. Consider $(M + m)$ moving to the left by x and m moving to the right by $L\cos\theta$.

The correct option is (C)

57. Let the balloon descend with a velocity u.

The velocity of man relative of earth $(v') = v - u$

By conservation of linear momentum,

$$m(v-u) - Mu = 0$$

$$u = \frac{mv}{M+m}$$

The correct option is (A)

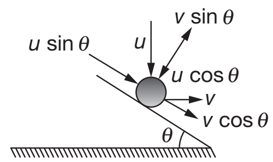
58. Impact takes place along the normal to the inclined plane

$$\therefore \quad u\sin\theta = v\cos\theta$$

$$v = u\tan\theta \quad (1)$$

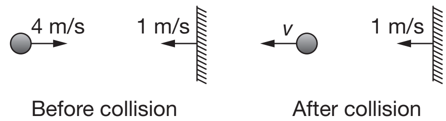
$$e = \frac{v\sin\theta}{u\cos\theta} = \frac{u\tan\theta \cdot \sin\theta}{u\cos\theta}$$

$$e = \tan^2\theta$$



The correct option is (B)

59. Let v be the velocity of ball after collision, collision is elastic



$\therefore \quad e = 1$

by relative velocity of separation = relative velocity of approach

$\therefore \quad v - 1 = 4 + 1$ or $v = 6$ m/s (away from the wall)

The correct option is (D)

60. $mv = Mv_1$

$$\therefore \quad v_1 = \frac{mv}{M} \quad (1)$$

$$mv\cos 30° = Mv_2$$

$$\therefore \quad v_2 = \frac{\sqrt{3}mv}{2M} \quad (2)$$

From equation (1) and (2) we get, $\frac{v_1}{v_2} = \frac{2}{\sqrt{3}}$

The correct option is (B)

61. Let height be h, so $t = \sqrt{\frac{2h}{g}}$

and $\quad v = \sqrt{2gh} = gt$

Now $\quad T = t + \frac{2ev}{g} + \frac{2e^2v}{g} + ...$

$$= t + \frac{2ev}{g}(1 + e + e^2 + ...)$$

$$= t + \frac{2gt}{g}\left(\frac{e}{1-e}\right) = t\left(\frac{1+e}{1-e}\right)$$

The correct option is (C)

62. Speed of block at the bottom of board = $\sqrt{2gh}$

Applying conservation of linear momentum in horizontal direction,

$$m\sqrt{2gh}\cos\alpha = (M+m)v$$

$$v = \frac{m\sqrt{2gh}\cos\alpha}{M+m}$$

The correct option is (C)

63. Collision is elastic. Therefore, kinetic energy will b conserved. Let v be the speed of ball after collision.

Then, $\frac{1}{2}(1)(10)^2 = \frac{1}{2}(4)(4)^2 + \frac{1}{2}(1)v^2$, $v = 6$ m/s

The correct option is (A)

64. Acceleration of system $a = \dfrac{mg\sin 60° - mg\sin 30°}{2m}$

Here, m = mass of each block

or $a = \left(\dfrac{\sqrt{3}-1}{4}\right)g$

Now $\vec{a}_{com} = \dfrac{m\,\vec{a}_1 + m\,\vec{a}_2}{2m}$

Here, $\vec{a}_1$ and $\vec{a}_2$ are $\left(\dfrac{\sqrt{3}-1}{4}\right)g$ at right angles.

Hence, $|\vec{a}_{com}| = \dfrac{\sqrt{2}}{2}a = \left(\dfrac{\sqrt{3}-1}{4\sqrt{2}}\right)g$

The correct option is (A)

65. Let $\vec{v}_1$ and $\vec{v}_2$ are the velocity vector of particles after collision.

From conservation of linear momentum;

$m\vec{v} = m\vec{v}_1 + m\vec{v}_2$ i.e., $\vec{v} = \vec{v}_1 + \vec{v}_2$

from energy conservation $\frac{1}{2}\,mv^2 = \frac{1}{2}\,mv_1{}^2 + \frac{1}{2}\,mv_2{}^2$ i.e., $v^2 = v_1{}^2 + v_2{}^2$

$\Rightarrow (\vec{v}_1 + \vec{v}_2)^2 = v_1{}^2 + v_2{}^2$

$\Rightarrow v_1{}^2 + v_2{}^2 + 2\vec{v}_1.\vec{v}_2 = v_1{}^2 + v_2{}^2$

$\therefore \vec{v}_1 \cdot \vec{v}_2 = 0$

The correct option is (C)

66. The change in momentum of one bullet = final momentum – initial momentum

$= m\left(\dfrac{v}{2}\right) - (-mv) = \dfrac{3}{2}mv$

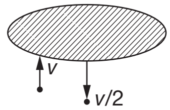
Also, given that n bullet strikes per second.

$\therefore$ Force exerted by the bullets $= \dfrac{3}{2}mnv$

Since, the plate remains in equilibrium

$\therefore$ Force exerted by the bullets = weight of the plate

$\dfrac{3}{2}mnv = Mg$ or $M = \dfrac{3mnv}{2g}$ (where M = mass of the plate)



The correct option is (A)

67. After nth rebound velocity $= e^n u$, height

$$h_1 = \frac{v^2}{2g} = \frac{e^{2n}u^2}{2g} = \frac{e^{2n}u^2}{2g} = \frac{e^{2n}\times 2gh}{2g} = he^{2n}$$

The correct option is (D)

68. $mu = mv_1 + mv_2$

$u = v_1 + v_2$ (1)

$e = \dfrac{v_2 - v_1}{u}$

$v_2 = eu + v_1$ (2)

from Equation (1) and (2)

$u = v_1 + eu + v_1$

$\dfrac{v_1}{u} = \dfrac{1-e}{2}$

The correct option is (D)

69. Change in linear momentum = area of F–t curve

$mu = \dfrac{1}{2}\left(\pi \times F_0 \times \dfrac{T}{2}\right)$

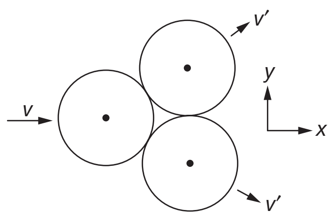
$u = \dfrac{\pi F_0 T}{4m}$

The correct option is (C)

70. $(P_i)x = (P_f)x$

$mv = mv'\cos 30° + mv'\cos 30°$

$v' = \dfrac{v}{\sqrt{3}}$



The correct option is (A)

71. For pure translational motion, the force F should act at centre of mass

$Y_{cm} = \dfrac{m(2l) + 2(m)l}{3m} = \dfrac{4l}{3}$

The correct option is (A)

72. The correct option is (D)

73. The correct option is (C)

74. Using conservation of linear momentum, we get

$$m_1 v_1 + 0 = m_1 \frac{v_1}{3} + m v_2 \Rightarrow v_1 = \frac{3mv_2}{2m_1} = \frac{3m\sqrt{5gl}}{2m_1}$$

($\because$ $v_2 = \sqrt{5gl}$ to complete motion along vertical circle)
The correct option is (B)

75. $v_2' = \frac{J}{2m} = \frac{v}{2}$ (1)

$v_1' = v - \frac{J}{m} = 0$ (2)

where v_2' and v_1' are velocities of A and B just after collision.

$\therefore \frac{v}{2} = ev \therefore e = \frac{1}{2}$

The correct option is (C)

76. Since collision is elastic $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$\Rightarrow v_1^2 + v_2^2 = v^2$ (1)

and momentum conservation given $m\vec{v} = m\vec{v_1} + m\vec{v_2}$

$\Rightarrow \vec{v_1} + \vec{v_2} = \vec{v}$ (2)

Equation (1) and (2) shows that angle between the velocity of both bodes after collision is 90° thus angle made by another body = 90 – 15 = 75°
The correct option is (A)

77. Since $\vec{L} = \vec{r} \times \vec{P}$ thus $\vec{L}$ is perpendicular to both $\vec{r}$ and $\vec{P}$
The correct option is (A)

78. $\Sigma mx = 0$; $70x + 60(1 + x) + 70(x - 1) = 0 \Rightarrow x = 5$ cm.
The correct option is (B)

79. 10 kg mass will gain velocity along the string only. Using momentum conservation along the string

$$v = \frac{10\cos 60°}{10+5} = \frac{1}{3} \text{ m/s}$$

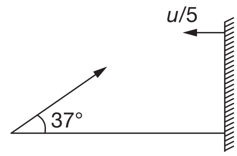
The correct option is (B)

80. Let the ball collides with the wall after time t. Let velocity of ball after collision is v.

$$\frac{-v - \left(-\frac{u}{5}\right)}{-\frac{u}{5} - u\cos 37} = \frac{1}{4}, \quad -v + \frac{u}{5} = -\frac{u}{4}, \quad v = \frac{u}{5} + \frac{u}{4} = \frac{9u}{20}$$

Also, $(u\cos 37)t = \frac{9u}{20}(T - t)$

$$\frac{4ut}{5} = \frac{9u}{20}\left(\frac{2u}{g}\frac{3}{5} - t\right) \Rightarrow t = \frac{54u}{125g}$$



The correct option is (C)

81. $N\,dt = \frac{mu}{2} + mu$, $-\mu N\,dt = mv' - mv$,

$-\mu \frac{3m}{2}\sqrt{2gh} = m(v' - v)$, $v - v' = 0.3\sqrt{2gh}$

The correct option is (C)

82. As KE remains same $P_i = P_f$ also $\vec{P_{fi}} = \vec{P} + \vec{I}$

$$P_f = \sqrt{P_i^2 + I^2 + 2P_i I\cos\theta}$$

$\Rightarrow I = -2P_i\cos\theta$

$\therefore \cos\theta \to$ –ve i.e. $\theta > 90°$
The correct option is (B)

83. From $F = u\frac{dm}{dt}$, $\frac{dm}{dt} = \frac{F}{u} = \frac{210}{300} = 0.7$ kg/s.
The correct option is (A)

84. v_2 = speed of block A; v_1 = speed of block B with respect to A.
By conservation of momentum,
$6mv_2 = m(v_1 - v_2)$

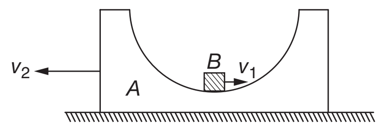
$7v_2 = v_1$ (1)
By conservation of energy

$$mga = \frac{1}{2}\times 6m \times v_2^2 + \frac{1}{2}\times m \times (v_1 - v_2)^2$$

$$ga = 3v_2^2 + \frac{1}{2}(7v_2 - v_2)^2$$

$$2ga = 6v_2^2 + 36v_2^2$$

$$v_2 = \sqrt{\frac{ga}{21}}$$



The correct option is (C)

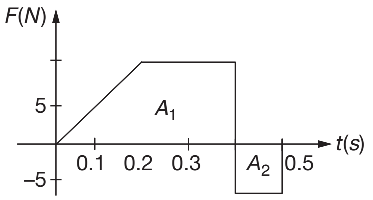
85. Impulse = area of F–t curve

$= A_1 - A_2 = \frac{1}{2}(0.44 + 0.2)\times 10 - 5 \times 0.1$

$= 3 - 0.5 = 2.5$ N/s

$m(v - u) = 2.5$

$5(v - 2) = 2.5$, $v = 2.5\text{ ms}^{-1}$



The correct option is (C)

86. Mass of bullet (m) = 0.02 kg

Mass of block (M) = 10 kg

Let the speed of bullet-block system after collision is v

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.45} = 3 \text{ ms}^{-1}$$

If initial speed of bullet is u, $mu = (M + m)\, v$

$0.02\, u = 10.02 \times 3$

$$u = \frac{1002 \times 3}{2}$$

$u = 501 \times 3$, $u = 1503 \text{ ms}^{-1}$

The correct option is (D)

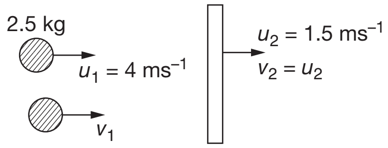
87. $$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$0.4 = \frac{1.5 - v_1}{4 - 1.5}$$

$0.4 \times 2.5 = 1.5 - v_1$

$1 = 1.5 - v_1$

$v_1 = 0.5 \text{ ms}^{-1}$



The correct option is (A)

88. Since collision is elastic, the horizontal velocity reverses its direction.

The correct option is (C)

89. For two dimensional bodies,

$$y_{cm} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(4a^2)(a) + \pi a^2 (3a)}{4a^2 + \pi a^2}$$

$$\therefore\ y_{cm} = \frac{4a + 3\pi a}{4 + \pi}$$

The correct option is (A)

90. $$F_{av} = \frac{\Delta P}{\Delta t} = \frac{15[(6\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} - 2\hat{j})]}{0.1}$$

$$= 150(5\hat{i} + 6\hat{j} + 5\hat{k})$$

The correct option is (C)

91. $v_1 = \left(\dfrac{1-e}{2}\right)u$ and $v_2 = \left(\dfrac{1+e}{2}\right)u$ (v_1 and v_2 are the velocities of the balls after impact)

Also, $k_f = \dfrac{3}{4}k_i \Rightarrow \left(\dfrac{1+e}{2}\right)^2 + \left(\dfrac{1-e}{2}\right)^2 = \dfrac{3}{4}$; $e = \dfrac{1}{\sqrt{2}}$

The correct option is (A)

92. When c.m. moves a distance S, distance covered by the point of application of force F is $2S$, therefore work done = $F(2S)$.

The correct option is (A)

93. $$\frac{1}{2}Ka^2 = \frac{1}{2}mv^2$$

Change in momentum = $2mv$

$$\text{Rate of change of momentum} = \frac{2mv}{\frac{T}{2}} = \frac{4m\sqrt{\frac{K}{m}}a}{2\pi\sqrt{\frac{m}{K}}}$$

$$\therefore \qquad F = \frac{2aK}{\pi}$$

The correct option is (A)

94. Applying conservation of momentum of the system along horizontal direction

$$-4mv + m(v_1 \cos\theta - v) = 0$$

$$v_1 = \frac{5v}{\cos\theta}$$

where v_1 = velocity of particle with respectively to hemi-spherical and $\omega = \dfrac{v_1}{R}$

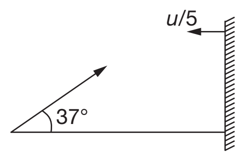
The correct option is (A)

95. Let the ball collides with the wall after time t. Let velocity of ball after collision is v.

$$\frac{-v - \left(-\frac{u}{5}\right)}{-\frac{u}{5} - u\cos 37°} = \frac{1}{4},\quad -v + \frac{u}{5} = -\frac{u}{4},\quad v = \frac{u}{5} + \frac{u}{4} = \frac{9u}{20}$$

$$\text{Also, } (u\cos 37°)t = \frac{9u}{20}(T - t)$$

$$\frac{4ut}{5} = \frac{9u}{20}\left(\frac{2u}{g}\frac{3}{5} - t\right) \Rightarrow t = \frac{54u}{125g}$$



The correct option is (C)

96. $18 = \dfrac{1}{2}a \times (3)^2$; $a = 4 \text{ m/s}^2$; $\dfrac{v_B}{a} = \dfrac{3}{2}$; $v_B = \dfrac{3 \times 4}{2} = 6$ m/s

The correct option is (C)

97. Vertical velocity of bomb after 10 s will be

$$u_y = gt = 100 \text{ m/s}$$

$m_1 : m_2 = 1:5$

$m_1 + m_2 = 12\,\text{kg}$

$\therefore \quad m_1 = 2\,\text{kg}$ and $m_2 = 10\,\text{kg}$

Applying conservation of linear momentum in x-direction,

$mu_x = m_1 v_{x_1} + m_2 v_{x_2}$

$\Rightarrow \quad v_{x_2} = 0$

Applying conservation of linear momentum in y-direction,

$mv_y = m_1 v_{y_1} + m_2 v_{y_2}$, $12 \times 100 = 2 \times 0 + 10 v_{y_2}$

$v_{y_2} = 120\,\text{m/ s}$, $v_2 = \sqrt{v_{x_2}^2 + v_{y_2}^2} = 120$ m/s

The correct option is (C)

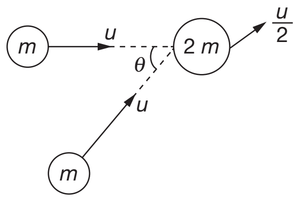
98. Initial momentum $= 2mu\cos\frac{\theta}{2}$

Where θ is angle between initial velocities

By conservation of linear momentum,

$2m \times \frac{u}{2} = 2mu\cos\frac{\theta}{2}$

$\cos\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 120°$



The correct option is (A)

More than One Option Correct Type

99. $a = \dfrac{g\sin\theta}{\left(1 + \dfrac{I}{mR^2}\right)} = \dfrac{g\sin\theta}{\left(1 + \dfrac{2}{5}\right)}$

Acceleration does not depend on the mass and radius in this case.

The correct option is (C) and (D)

100. The correct option is (B) and (C)

101. (A) At maximum extension $V_A = V_B = 0$

$$2mgx = \frac{1}{2}kx^2$$

$$x_m = \frac{4mg}{k}$$

(B) $2mg\dfrac{2mg}{k} = \dfrac{1}{2}k\left(\dfrac{2mg}{k}\right)^2 + \dfrac{1}{2}(m + 2m)v^2$

$$N = 2g\sqrt{\frac{2m}{3k}}$$

(C) Net upward force $\Rightarrow \dfrac{4mg}{k} \times R = 4mg$

Net downward force $\Rightarrow 2\,mg$

$$a = \frac{4mg - 2mg}{3m} = \frac{2}{3}g$$

(D) For $x = \dfrac{2mg}{k}$, $a = 0$

$\therefore \quad T = 2\,mg$

The correct option is (A), (B) and (C)

102. The correct option is (A), (B)and (C)

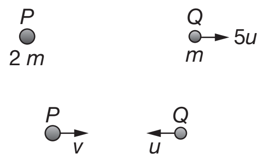
103. The correct option is (A), (B), (C) and (D)

104. $5mu = 2mv - mu$

$v = 3u$

$\frac{1}{2}m(5u)^2 + W = \frac{1}{2}mu^2 + \frac{1}{2} \times 2mv^2$

$W = -3mu^2$



The correct option is (B) and (D)

105. The correct option is (A) and (C)

106. $J\dfrac{l}{4} = \dfrac{ml^2}{12}\omega \Rightarrow \omega = \dfrac{3J}{ml}$, $v_{\text{cm}} = \dfrac{J}{m}$

$v_A = v_{\text{cm}} - \dfrac{l}{3}\omega = 0$, $v_{\text{upper}} = \left|v_{\text{cm}} - \dfrac{l}{2}\omega\right| = \dfrac{J}{2m}$

$v_{\text{lower}} = \left|v_{\text{cm}} + \dfrac{l}{2}\omega\right| = \dfrac{5}{2}\dfrac{J}{m}$

The correct option is (A), (B), (C) and (D)

107. Assume x-axis along OA and y-axis perpendicular to it in the plane of ring.

After 1st collision

Velocity of particle $= \dfrac{v}{\sqrt{2}}$ along x-axis

And velocity of ring $= \dfrac{v}{\sqrt{2}}$ along y-axis

After second collision velocity of ring $= \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$

Velocity of particle = 0

Time to return at point $A = 4 \times \frac{R\sqrt{2}}{v}$

The correct option is (A), (B) and (C)

Passage Based Questions

Passage 1

108. $v_{cm} = \frac{Mv_0 + m(0)}{M+m}$

The correct option is (D)

109. $Mv_0 = Mv_1 + mv_2$

$v_2 - v_1 = v_0$

The correct option is (A)

110. The correct option is (A)

111. Distance $= L + \left(\frac{M-m}{m+M}\right)v_0 \frac{L}{v_0}$

$= \left(\frac{2M}{M+m}\right)L$

The correct option is (C)

Passage 2

112. $v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1+m_2}$, $v_{cm} = \frac{m_2v_0}{m_1+m_2}$

The correct option is (C)

113. Spring will be in maximum elongation condition when both block have same velocity.

From momentum conservation $m_2v_0 = (m_1+m_2)v_e$

$$v_c = \frac{m_2v_0}{m_1+m_2}$$

From ME conservation

$$\frac{1}{2}m_2v_0^2 = \frac{1}{2}(m_1+m_2)v_C^2 + \frac{1}{2}kx^2$$

$$m_2v_0^2 = (m_1+m_2)\frac{m_2^2v_0^2}{(m_1+m_2)} + kx^2$$

$$v_0^2\left[m_2 - \frac{m_2^2}{m_1+m_2}\right] = kx^2$$

$$v_0^2\left(\frac{m_1m_2}{m_1+m_2}\right) = kx^2, \quad x = v_0\left[k\left(\frac{m_1m_2}{m_1+m_2}\right)\right]^{\frac{1}{2}}$$

The correct option is (A)

114. From momentum conservation

$$m_2v_0 = m_2\left(\frac{3v_0}{4}\right)m_1v$$

$$\frac{1}{4}m_2v_0 = m_1v$$

$$v = \frac{m_2}{4m_1}v_0$$

The correct option is (A)

Passage 3

115. As the area of F-t curve and $F_{ave} - t$ curve must be equal.

The correct option is (A)

116. When the separation is $2R$, potential energy is zero and it increases as the separation decreases. The separation between the centre of the two spherical balls fist decreases and then increases.

The correct option is (C)

117. In elastic collision some part of kinetic energy converted into potential energy permanently.

The correct option is (C)

Passage 4

118. For moth, momentum $= 8m(2v) = 16(mv)$ (RHS).

For spider, momentum $= 16(mv)$ (LHS).

∴ The centre of mass of the entire system does not move due to lack of any external force.

∴ Net momentum of the system = 0

Hence, the rod does not move.

The correct option is (D)

119. Displacement of moth $= \frac{12L}{3} \times 2 = 8L$

∴ Point p is at the table supporting B.

The correct option is (B)

120. After spider eats the moth, let the speed of rod be v_0 (RHS).

∴ Speed of spider relative to rod $= \frac{v}{2}$ (LHS)

∴ Speed of spider $= \left(\frac{v}{2} - v_0\right)$ (LHS)

Since, total momentum of system is still zero.

$$24m\left(\frac{v}{2} - v_0\right) = 48mv_0$$

$$\therefore \quad v_0 = \left(\frac{v}{6}\right)$$

The correct option is (C)

121. Time when both insects meet $t_1 = \frac{4L}{v} = 4T = 16$ s

After this the spider moves a distance $8L$ with relative speed $v/2$.

$\therefore$ Further, time required is $t_2 = \frac{8L}{\frac{L}{2T}} = 16T = 64$ s

$\therefore$ Total time = 80 sec

The correct option is (C)

Passage 5

122. Block P will not moves as surface P and Q are smooth.

The correct option is (A)

123. Force of friction between the blocks is zero and that between rough surface and block Q is μ 6 Mg. Work done = (μ 6 Mg) 2s, i.e. $W = 12\ \mu$Mgs.

The correct option is (B)

124. Let v and v' be the velocities of mass M and block Q after collision.

From principle of conservation of momentum

$$Mv_0 = Mv + 4Mv'$$

From principle of conservation of KE

$$\frac{1}{2}Mv_0^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\times 4Mv'^2$$

From above equation, $v' = \frac{2}{5}v_0$ and $v = -\frac{3}{5}v_0$

KE of block $Q = \frac{1}{2}\times 4M \times v'^2$

$$= \frac{1}{2}\times 4M \times \left(\frac{2}{5}\right)^2 = \frac{8}{25}Mv_0^2 .$$

The correct option is (A)

Assertion-Reason Type

125. The correct option is (B)

126. The correct option is (A)

127. The correct option is (A)

128. The correct option is (D)

129. The correct option is (A)

130. The correct option is (D)

131. Assertion is true but reason is false.

A body may not have momentum but may have potential energy by virtue of position (e.g., compressed or stretched spring). But if the body has no kinetic energy, then its velocity is zero and therefore, its momentum is also zero. Also, dimensions of momentum = $[MLT^{-1}]$ and dimensions of energy = $[ML^2T^{-2}]$, i.e., dimensions of momentum is not equal to dimension of energy.

132. Assertion is false but reason is true.

The total momentum of a many particle system can change only when some external forces are applied on the system. So rate of change of momentum is proportional to external forces acting on the system. The total momentum of the whole system remains constant when no external force is acted upon it (according to law of conservation of momentum).

The correct option is (D)

133. The correct option is (D)

134. The correct option is (D)

Match the Column Type

135. (A) – 1, 4; (B) $\to$ 2; (C) $\to$ 3; (D) $\to$ 3, 4

136. (A) By conservation of linear momentum (considering u_1 as positive)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$8u_1 - 6 = 8v_1 + 2v_2$$

$$8v_1 + 2v_2 = 8u_1 - 6 \qquad (1)$$

$$\because \quad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{1}{2}$$

$$\therefore \quad 2(v_2 - v_1) = u_1 - u_2$$

$$2v_2 - 2v_1 = u_1 + 3 \qquad (2)$$

(1) – (2) $\Rightarrow$ $10v_1 = 7u_1 - 9$

$$v_1 = \frac{7u_1 - 9}{10}$$

For $u_1 > \frac{9}{7}$; v_1 has same direction as u_1

$$\Rightarrow \quad v_2 = \frac{u_1 + 3}{2} + v_1 = \frac{u_1 + 3}{2} + \frac{7u_1 - 9}{10} = \frac{12u_1 + 6}{10}$$

$$v_2 = \frac{6u_1 + 3}{5}$$

$\therefore$ u_1 and v_2 always have same direction.

(B) If maximum energy is transferred to m_2, v_1 should be zero.

By conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$8u_1 - 6 = 0 + 2v_2$$

$$v_2 = 4u_1 - 3 \quad (3)$$

Collision is elastic

$$\therefore \quad v_2 - v_1 = u_1 - u_2$$

$$v_2 = u_1 + 3 \quad (4)$$

(3) and (4)

$$\Rightarrow \quad 4u_1 - 3 = u_1 + 3$$

$$\Rightarrow \quad 3u_1 = 6$$

$$u_1 = 2 \text{ m/s}$$

(C) By conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$8 \times 0.5 - 6 = 8v_1$$

$$v_1 = -\frac{1}{4} \text{ m/s}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - v_1}{0.5 + 3} = \frac{1}{14}$$

(D) For perfectly inelastic collision,

$$(\Delta K)_{\text{loss}} = \frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$$

$$= \frac{8 \times 2}{2 \times (8 + 2)}(3 + 3)^2 = 28.8 \text{ J}$$

$\therefore$ During inelastic collision loss of energy will be less than 28.8 J.

136. (A) $\to$ 3, 4; (B) $\to$ 3; (C) $\to$ 1; (D) $\to$ 1, 2, 3, 4

137. In inelastic collision, there is a loss of energy while in elastic collision, there is no loss of energy.

(A) $\to$ 1, 2, 3; (B) $\to$ 1, 2, 4; (C) $\to$ 1, 3; (D) $\to$ 1, 4

138. Kinetic energy of B before collision = 3 × 10 × 0.2 = 6 J

So loss of energy = 6 J

At equilibrium $kx = mg$

$$x = \frac{20}{100} = 0.2 \text{ m}$$

Energy at equilibrium of mass 2 kg

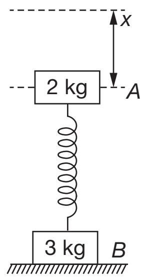
$$= \frac{1}{2} \times 100 \times (0.2)^2 = 2 \text{ J}$$

Work done by gravitational force

$$mg(0.2 + x) = 20 \times (0.2 + 0.2) = 8 \text{ J}$$

At the time of collision spring is in relaxed position, so energy stored in the spring at that moment is zero.

$\therefore$ (A) $\to$ 2; (B) $\to$ 4; (C) $\to$ 3; (D) $\to$ 1



139. $v_{\text{cm}} = \dfrac{3 \times 2 + 6 \times 0}{3 + 6} = \dfrac{2}{3}$ m/s

For maximum deformation, the block has same velocity.

$$3 \times 2 = 3 \times v + 6 \times v, \; v = \frac{2}{3} \text{ m/s}$$

The 6 kg block has maximum velocity when spring is non-deformed.

$$3 \times 2 = 3 \times v_1 + 6 \times v_2, \; \frac{1}{2} \times 3 \times 2^2 = \frac{1}{2} \times 3 \times v_1^2 + \frac{1}{2} 6v_2^2$$

$$\Rightarrow \quad v_2 = \frac{4}{3} \text{ m/s}, \; v_1 = -\frac{2}{3} \text{ m/s}$$

$\therefore$ (A) $\to$ 1, 3 4; (B) $\to$ 1, 2, 3 and 4; (C) $\to$ 1; (D) $\to$ 1, 2

Integer Type

140. $\dfrac{4m_1m_2}{(m_1 + m_2)^2}$ $4\eta = 4$ or $\eta = 1$ i.e. $\dfrac{m_2}{m_1} = 1$

141. Equation of Newton's collision law

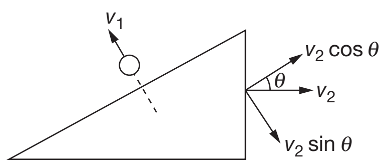
$$\frac{v_1 + v_2 \sin\theta}{v_0}, \; e = \frac{v_1 + \frac{v_2}{2}}{v_0}$$

$$2v_1 + v_2 = 7 \quad (1)$$

From momentum conservation

$$mv \sin 30 = -mv_1 \sin 30 + mv_2$$

$$5 = -\frac{v_1}{2} + 2v_2 \quad (2)$$



After collision

Solving $v_1 = 2$ m/s

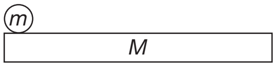
142. For BC part to be maximum, the c.m. of the system is at point C i.e.,

$$H_{max} = \frac{m\frac{L}{2}}{\left(m+\frac{m}{2}\right)} = \frac{L}{3} = 1 \text{ cm}$$

143. $v_{mM} = 4\hat{i} + 3\hat{j}$

$v_m = u\hat{i} + 3\hat{j}$

$v_M = -v\hat{i}$



Here, along horizontal direction $mv_m + Mv_M = 0$

$\Rightarrow \quad v = \frac{um}{M} = u\eta$ and $u + v = 4 \Rightarrow u = \frac{4}{1+\eta}$

To move one meter with respect to ground

$$1 = \frac{2\times\frac{4}{1+\eta}\times 3}{g} \Rightarrow 1+\eta = \frac{24}{g}$$

or $\quad \eta = \frac{14}{10} = 1.4 = 14\times 10^{-1}$

$\therefore \quad x = 14$

144. Acceleration of the system $g\tan\theta$

$\therefore \quad g\tan\theta = \frac{m'g}{m'+m+m} \Rightarrow \frac{3}{4} = \frac{m'}{m'+m+M}$

$\Rightarrow \quad m' = 9$ kg

145. $v_x = 7$ m/s

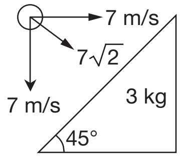
Velocity along y-axis of ball just before collision

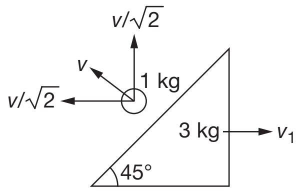
$v = \sqrt{2gy}$, $v = \sqrt{2\times 9.8\times 2.5} = 7$ m/s

as $\quad v_x = v_y$

So it strikes the plane of incline perpendicularly.

Let ball rebounds with velocity v and let v_1 be the velocity of the wedge.





Applying COM. in horizontal direction

$$1\times 7 = -1\times\frac{v}{\sqrt{2}} + 3v_1$$

$$7\sqrt{2} = 3\sqrt{2}v_1 - v \qquad (1)$$

Applying the equation for coefficient of restitution

$$e = 1 = \frac{\frac{v_1}{\sqrt{2}} + v}{7\sqrt{2}}, \quad 7\sqrt{2} = \frac{v_1}{\sqrt{2}} + v$$

$$14 = v_1 + v\sqrt{2} \qquad (2)$$

Solving Equations (1) and (2)

$v_1 = 4$ m/s

Previous Years' Questions

146. Let n be the number of bullets that the man can fire in one second.

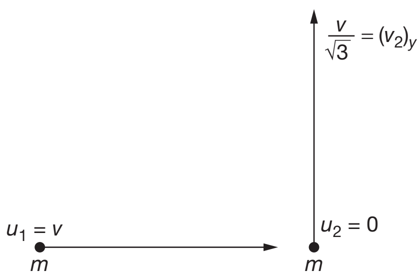
$\therefore$ change in momentum per second $= n\times mv = F$

[m = mass of bullet, v = velocity] ($\because$ F is the force)

$\therefore \frac{F}{mv} = \frac{144\times 1000}{40\times 1200} = 3$

The correct option is (D)

147.



In x-direction: $mv + 0 = m(0) + m(v_2)x$

In y-direction: $0 + 0 = m\left(\frac{v}{\sqrt{3}}\right) + m(v_2)_y$ is

$\Rightarrow (v_2)_y = \frac{v}{\sqrt{3}}$ and $(v_2)_x = v$

$$\therefore \; v_2 = \sqrt{\left(\frac{v}{\sqrt{3}}\right)^2 + v^2}$$

$$\Rightarrow \; v_2 = \sqrt{\frac{v^2}{3} + v^2} = v\sqrt{\frac{4}{3}} = \frac{2v}{\sqrt{3}}$$

in x-direction, $mv = mv_1\cos\theta \qquad (1)$

where v_1 is the velocity of second mass

In y-direction, $0 = \frac{mv}{\sqrt{3}} - mv_1\sin\theta$

or $m_1 v_1 \sin\theta = \dfrac{mv}{\sqrt{3}}$ (2)

Squaring and adding Equations (1) and (2)

$$v_1^2 = v^2 + \frac{v^2}{\sqrt{3}} \Rightarrow v_1 = \frac{2}{\sqrt{3}} v$$

The correct option is (D)

148. Let the velocity and mass of 4 kg piece be v_1 and m_1 and that of 12 kg piece be v_2 and m_2.

Applying conservation of linear momentum

$$m_2 v_1 = m_1 v_1$$

$$\Rightarrow v_1 = \frac{12\times4}{4} = 12 \text{ ms}^{-1}$$

$$\therefore \text{KE}_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2}\times4\times144 = 288\text{ J}$$

The correct option is (B)

149. In completely inelastic collision, all energy is not lost (so, Statement–1 is true) and the principle of conservation of momentum holds good for all kinds of collisions (so, Statement–2 is true). Statement–2 explains Statement–1 correctly because applying the principle of conservation of momentum, we can get the common velocity and hence the kinetic energy of the combined body.

The correct option is (A)

150. In each impulse, x varies from 0 to $2m$ and again from $2m$ to 0 during the time interval of $4s$. We have impulse = change in momentum

$$= m\,|v_2 - v_1| = 2\times0.4\times(1-0) = 0.8 \text{ Ns}$$

The correct option is (B)

151. $\Delta k = \dfrac{1}{2}mv^2 - \dfrac{1}{2}(m+M)\dfrac{m^2v^2}{(m+M)^2}$

$$= \frac{1}{2}mv^2\left[1 - \frac{m}{m+M}\right] = \frac{1}{2}mv^2\left[\frac{M}{m+M}\right]$$

$$\therefore \quad f = \frac{M}{(m+M)}$$

The correct option is (C)

152. Standard result

$$z_0 = \frac{3h}{4}$$

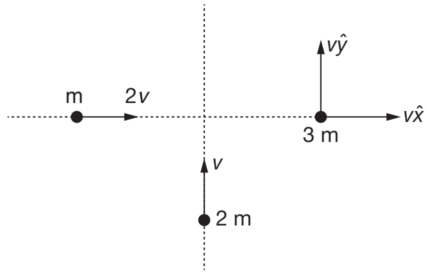
The correct option is (A)

153. $P_{xi} = P_{xf}$

$$2mv = 3mv_x$$

$$\Rightarrow v_x = \frac{2v}{3}$$

$$2mv = 3mv_y$$



$$\Rightarrow v_y = \frac{2v}{3}$$

$$\text{KE}_f = \frac{1}{2}3m\left(v_x^2 + v_y^2\right)$$

$$= \frac{1}{2}3m\left[\frac{4v^2}{9} + \frac{4v^2}{9}\right]$$

$$= \frac{4mv^2}{3}$$

$$\text{KE}_i = \frac{1}{2}m(2v)^2 + \frac{1}{2}2m(v^2)$$

$$= 2mv^2 + mv^2 = 3mv^2$$

$$\%\text{ Loss} = \frac{\text{KE}_i - \text{KE}_f}{\text{KE}_i}\times100 = \frac{3mv^2 - \dfrac{4mv^2}{3}}{3mv^2}\times100$$

$$= \frac{5}{9}\times100 = 55.5\% \simeq 56\%$$

The correct option is (B)