CHAPTER 4

Work, Energy, and Power

Chapter Highlights

Work done by a constant force and a variable force; kinetic and potential energies, work-energy theorem, power. Potential energy of a spring, conservation of mechanical energy, conservative and non-conservative forces.

WORK

The term 'work' as understood in everyday life has a different meaning in scientific sense. If a coolie is carrying a load on his head and waiting for the arrival of the train, he is not performing any work in the scientific sense. In the present study, we will take a look into the scientific aspect of this most commonly used term, i.e., work.

Work Done by Constant Force

The physical meaning of the term work is entirely different from the meaning attached to it in everyday life. In everyday life, the term 'work' is considered to be synonym of 'labour', 'toil', effort', etc. In physics, there is a specific way of defining work.

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

For work to be done, following two conditions must be fulfilled.

1. A force must be applied.
2. The applied force must produce a displacement in any direction except perpendicular to the direction of the force.

Suppose a force $\vec{F}$ is applied on a body in such a way that the body suffers a displacement $\vec{S}$ in the direction of the force. Then the work done is given by

$$\boldsymbol{W = FS}$$

However, the displacement does not always take place in the direction of the force. Suppose a constant force $\vec{F}$ applied on a body, produces a displacement $\vec{S}$ in the body in such a way that $\vec{S}$ is inclined to $\vec{F}$ at an angle θ. Now the work done will be given by the dot product of force and displacement.

$$W = \vec{F} \cdot \vec{S}$$

Since work is the dot product of two vectors, it is of scalar quantity.

$$W = FS \cos \theta$$

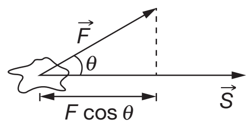
or $$W = (F \cos \theta)S$$

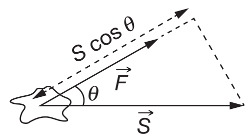
$\therefore$ W = component of force in the direction of displacement × magnitudes of displacement.

So work is the product of the component of force in the direction of displacement and the magnitude of the displacement.

Also, $$W = F\,(S \cos \theta)$$

or work is product of the component of displacement in the direction of the force and the magnitude of the displacement.





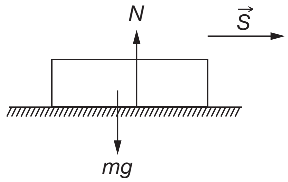
Special Cases

Case I: When $\theta = 90°$, then $W = FS \cos 90° = 0$

So, work done by a force is zero if the body is displaced in a direction perpendicular to the direction of the force.

Examples:

1. Consider a body sliding over a horizontal surface. The work done by the force of gravity and the reaction of the surface will be zero. This is because both the force of gravity and the reaction act normally to the displacement.



The same argument can be applied to a man carrying a load on his head and walking on a railway platform.

2. Consider a body moving in a circle with constant speed. At every point of the circular path, the centripetal force and the displacement are mutually perpendicular (Fig. 4.1). So, the work done by the centripetal force is zero. The same argument can be applied to a satellite moving in a circular orbit. In this case, the gravitational force is always perpendicular to displacement. So, work done by gravitational force is zero.
3. The tension in the string of a simple pendulum is always perpendicular to displacement (Fig. 4.2). So, work done by the tension is zero.

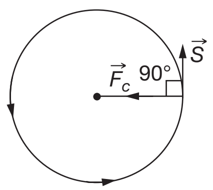


Fig. 4.1

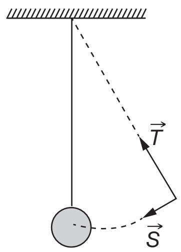


Fig. 4.2

Case II: When $S = 0$, then $W = 0$.

So, work done by a force is zero if the body suffers no displacement on the application of a force.

Example:

A person carrying a load on his head and standing at a given place does no work.

Case III: When $0° \le \theta < 90°$ [Fig. 4.3], then $\cos \theta$ is positive. Therefore,

$W = (FS \cos \theta)$ is positive.

So, work done by a force is said to be positive if the applied force has a component in the direction of the displacement.

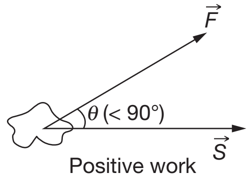


Fig. 4.3

Examples:

1. When a horse pulls a cart, the applied force and the displacement are in the same direction. So, work done by the horse is positive.
2. When a load is lifted, the lifting force and the displacement act in the same direction. So, work done by the lifting force is positive.
3. When a spring is stretched, both the stretching force and the displacement act in the same direction. So, work done by the stretching force is positive.

Case IV: When $90° < \theta \le 180°$ (Fig. 4.4), then $\cos \theta$ is negative.

Therefore, $W = (FS \cos \theta)$ is negative.

So, work done by a force is said to be negative if the applied force has a component in a direction opposite to that of the displacement.

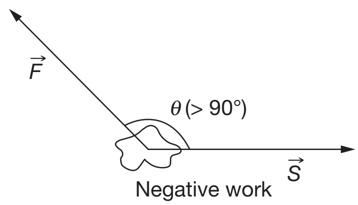


Fig. 4.4

Examples:

1. When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.
2. When a body is dragged along a rough surface, the work done by the frictional force is negative. This is because the frictional force acts in a direction opposite to that of the displacement.
3. When a body is lifted, the work done by the gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upwards direction.

SOLVED EXAMPLE

1. Figure 4.5 shows four situations in which a force acts on a box while the box slides rightward a distance *d* across a frictionless floor. The magnitudes of the forces are identical, their orientations are as shown. Rank the situations according to the work done on the box during the displacement, from most positive to most negative.

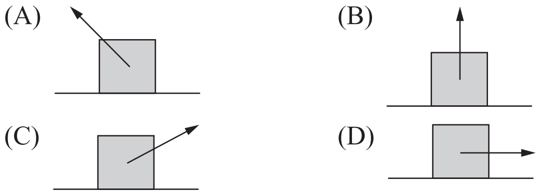


Fig. 4.5

Solution: (D, C, B, A)

Explanation:

In (D), $\theta = 0°$, cos $\theta = 1$ (maximum value). So, work done is maximum.
In (C), θ = 90°, cos θ is positive. Therefore, W is positive.
In (B), $\theta = 90°$, cos θ is zero. W is zero.
In (A), θ is obtuse, cos θ is negative. W is negative.

Work Done by Multiple Forces

If several forces act on a particle, then we can replace $\vec{F}$ in equation $W = \vec{F} \cdot \vec{S}$ by the net force $\Sigma\vec{F}$, where

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$$

$$\therefore \quad W = [\Sigma\vec{F}] \cdot \vec{S} \qquad (1)$$

This gives the work done by the net force during a displacement $\vec{S}$ of the particle.

Equation (1) can be re-written as:

$$W = \vec{F}_1 \cdot \vec{S} + \vec{F}_2 \cdot \vec{S} + \vec{F}_3 \cdot \vec{S} + \ldots$$

or $$W = W_1 + W_2 + W_3 + \ldots$$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

Important Points about Work

1. Work is defined for an interval or displacement. There is no term like instantaneous work similar to instantaneous velocity.
2. For a particular displacement, work done by a force is independent of type of motion, i.e., whether it moves with constant velocity, constant acceleration, retardation, and so on.
3. For a particular displacement, work is independent of time. Work will be same for same displacement whether the time taken is small or large.
4. When several forces act, work done by a force for a particular displacement is independent of other forces.
5. A force is independent from reference frame. Its displacement depends on frame, so work done by a force is frame-dependent; therefore, work done by a force can be different in different reference frame.
6. Effect of work is change in kinetic energy of the particle or system.
7. Work is done by the source or agent that applies the force.

Units of Work

1. In cgs system, the unit of work is erg.
   One erg of work is said to be done when a force of one dyne displaces a body through one centimetre in its own direction.
   $\therefore$ 1 erg = 1 dyne × 1 cm = 1g cm s^{-2} × 1 cm = 1 g cm^2 s^{-2}
   Erg is also called dyne centimetre.
2. In SI, i.e. International System of units, the unit of work is joule (abbreviated as J). It is named after the famous British physicist James Personal Joule (1818–1869).
   One joule of work is said to be done when a force of one Newton displaces a body through one metre in its own direction.
   1 joule = 1 newton × 1 metre = 1 kg ×1 m/s^2 × 1 m = 1 kg m^2 s^{-2}
   Another name for joule is newton metre.

Relation Between Joule and Erg

1 joule = 1 newton × 1 metre
1 joule = 10^5 dyne × 10^2 cm = 10^7 dyne cm
1 joule = 10^7 erg
1 erg = 10^{-7} joule

Dimensions of Work

$$[\text{Work}] = [\text{Force}]\ [\text{Distance}] = [MLT^{-2}]\ [L] = [ML^2T^{-2}]$$

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time,

On the basis of dimensional formula, the unit of work is kg m^2 s^{-2}.

Note that 1 kg m^2 s^{-2} = (1 kg m s^{-2}) m = 1 N/m = 1 J.

SOLVED EXAMPLES

2. There is an elastic ball and a rigid wall. Ball is thrown towards the wall. The work done by the normal reaction exerted by the wall on the ball is

(A) +ve (B) –ve
(C) Zero (D) None of these

Solution: (C)

As the point of application of force does not move, the work done by normal reaction is zero.

3. Work done by the normal reaction when a person climbs up the stairs is

(A) +ve (B) –ve
(C) Zero (D) None of these

Solution: (C)

As the point of application of force does not move, the work done by normal reaction is zero.

4. Work done by kinetic friction force when a person starts running is ________ .

Solution:

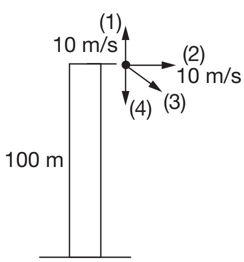
As the point of application of force does not move, the work done by kinetic friction is zero.

Work Done by Various Real Forces

SOLVED EXAMPLES

Work Done by Gravity Force

5. The mass of the particle is 2 kg. It is projected as shown in four different ways with same speed of 10 m/s. Find out the work done by gravity by the time the stone falls on ground.



Solution:

$$W = |\vec{F}||\vec{S}| \cos \theta = 2000 \text{ J in each case.}$$

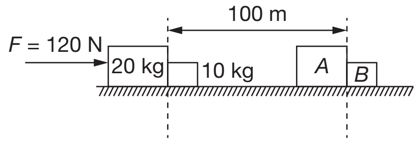
Work Done by Normal Reaction

6. (A) Find work done by force F on A during 100 m displacement.

(B) Find work done by force F on B during 100 m displacement.

(C) Find work done by normal reaction on B and A during the given displacement.

(D) Find out the kinetic energy of blocks A and B finally.



Solution:

(A) $(W_F)_{\text{on } A} = F\Delta S \cos \theta$

$= 120 \times 100 \times \cos 0°$

$= 12000$ J

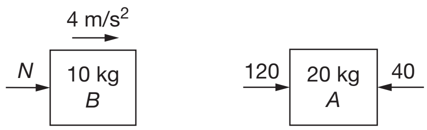
(B) $(W_F)_{\text{on } B} = 0$

$\because$ F does not act on B

(C) $N = 10 \times 4 = 40$ N

$(W_N)_{\text{on } B} = 40 \times 100 \times \cos 0°$

$= 4000$ J



$(W_N)_{\text{on } A} = 40 \times 100 \times \cos 180° = -4000$ J

(D) $v^2 = u^2 + 2as$

$u = 0$

$\therefore v^2 = 2 \times 4 \times 100 \Rightarrow v = 20\sqrt{2}$ m/s

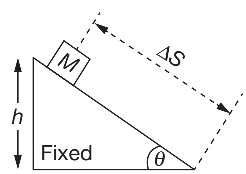
$\therefore \text{KE}_A = \dfrac{1}{2} \times 20 \times 800 = 8000$ J

$\text{KE}_B = \dfrac{1}{2} \times 10 \times 800 = 4000$ J

Work done by normal reaction on system of A and B is zero. That is work done by internal reaction on a rigid system is zero.

Work Done by Other Constant Forces

7. Find out work done by normal reaction and gravity when the time block comes to bottom.



Solution:

$$W_N = 0$$

Since normal reaction is perpendicular to displacement.

$$W_g = \vec{F} \cdot \overrightarrow{\Delta S}$$

$$= mg \cdot \Delta S \cdot \cos(90 - \theta)$$

$$= mg\,\Delta S \sin\theta = mgh$$

8. Find out the speed of the block at the bottom and its kinetic energy.

Solution:

$$v^2 = u^2 + 2as$$

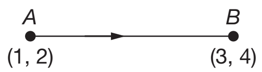
$$v^2 = 0 + 2(g\sin\theta)\frac{h}{\sin\theta}$$

$$v^2 = 2gh$$

$$\Rightarrow \quad v = \sqrt{2gh}$$

$$\text{KE} = \frac{1}{2}mv^2 = mgh$$

9. A force $\vec{F} = 2\hat{i} + 3\hat{j}$ is applied on a particle



Find out the work done by F to move the particle from point A to B.

Solution:

$$W = \vec{F} \cdot \overrightarrow{\Delta S}.$$

$$\Delta S = (3-1)\hat{i} + (4-2)\hat{j}$$

$$= (2\hat{i} + 3\hat{j}) \cdot (2\hat{i} + 2\hat{j})$$

$$= 2 \times 2 + 3 \times 2 = 10 \text{ units}$$

Work Done by a Variable Force

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position. For a variable force, work is calculated for infinitely small displacement, and for this displacement force is assumed to be constant

$$dW = \vec{F} \cdot d\vec{s}$$

The total work done will be sum of infinitely small work

$$W_{A \to B} = \int_A^B \vec{F} \cdot d\vec{s} = \int_A^B (\vec{F}\cos\theta)\,d\vec{s}$$

In terms of rectangular components,

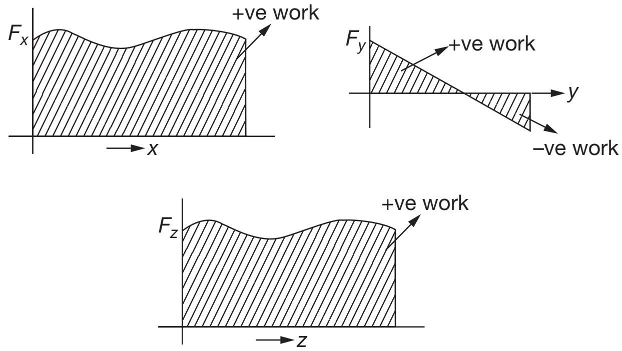
$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\Rightarrow \quad d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$W_{A \to B} = \int_{x_A}^{x_B} F_x\,dx + \int_{y_A}^{y_B} F_y\,dy + \int_{z_A}^{z_B} F_z\,dz$$

Area Under Force-Displacement Curve

Graphically area under the force-displacement is the work done



The work done can be positive or negative as per the area above the x-axis or below the x-axis, respectively.

ENERGY

Energy is defined as internal capacity of doing work. When we say that a body has energy, we mean that it can do work.

Energy appears in many forms such as mechanical, electrical, chemical, thermal (heat), optical (light), acoustical (sound), molecular, atomic, nuclear, etc., and can change from one form to the another.

Kinetic Energy

Kinetic energy is the internal capacity of doing work of the object by virtue of its motion.

Kinetic energy is a scalar property that is associated with state of motion of an object. An aeroplane in straight and level flight has kinetic energy of translation and a rotating wheel on a machine has kinetic energy of rotation. If a particle of mass m is moving with speed v much less than the speed of the light, then the kinetic energy K is given by

$$K = \frac{1}{2}mv^2$$

Important Points for Kinetic Energy

1. As mass m and $v^2\left(\vec{v}\cdot\vec{v}\right)$ are always positive, kinetic energy is always positive scalar, i.e, kinetic energy can never be negative.
2. The kinetic energy depends on the frame of reference,

$K = \dfrac{p^2}{2m}$ and $P = \sqrt{2mK}$; P = linear momentum

The speed v may be acquired by the body in any manner. The kinetic energy of a group of particles or bodies is the sum of the kinetic energies of the individual particles. Consider a system consisting of n particles of masses $m_1, m_2, \ldots, m_n$. Let $\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}$ be their respective velocities. Then, the total kinetic energy E_k of the system is given by

$$E_k = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 + \ldots + \frac{1}{2}m_n{v_n}^2$$

If n is measured in gram and v in cm s^{-1}, then the kinetic energy is measured in erg. If m is measured in kilogram and v in ms^{-1}, then the kinetic energy is measured in joule. It may be noted that the units of kinetic energy are the same as those of work. In fact, this is true of all forms of energy since they are interconvertible.

Typical Kinetic Energies (K)

S.No.	Object	Mass (kg)	Speed (m s^{-1})	K(J)
1	Air molecule	$\approx 10^{-26}$	500	$\approx 10^{-21}$
2	Rain drop at terminal speed	3.5×10^{-5}	9	1.4×10^{-3}
3	Stone dropped from 10 m	1	14	10^2
4	Bullet	5×10^{-5}	200	10^3
5	Running athlete	70	10	3.5×10^3
6	Car	2000	25	6.3×10^5

Relation between Momentum and Kinetic Energy

Consider a body of mass m moving with velocity v. Linear momentum of the body, $p = mv$

Kinetic energy of the body,

$$E_k = \frac{1}{2}mv^2$$

$$\Rightarrow \quad E_k = \frac{1}{2m}(m^2v^2)$$

$$\text{or} \quad E_k = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mE_k}$$

SOLVED EXAMPLES

10. The kinetic energy of a body is increased by 21%. What is the percentage increase in the magnitude of linear momentum of the body?

Solution:

$$E_{k2} = \frac{121}{100}E_{k1} \quad \text{or} \quad \frac{1}{2}m{v_2}^2 = \frac{121}{100}\frac{1}{2}m{v_1}^2$$

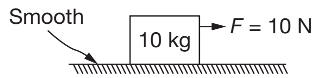
$$\text{or} \quad v_2 = \frac{11}{10}v_1 \quad \text{or} \quad mv_2 = \frac{11}{10}mv_1$$

$$\text{or} \quad p_2 = \frac{11}{10}p_1 \quad \text{or} \quad \frac{p_2}{p_1} - 1 = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\text{or} \quad \frac{p_2 - p_1}{p_1} \times 100 = \frac{1}{10} \times 100 = 10$$

So, the percentage increase in the magnitude of linear momentum is 10%.

11. A force of 10 N is applied on block mass 10 kg for 2 seconds



Find out work done by force F on 10 kg in 3 seconds from starting.

Solution:

$$w = \vec{F}\cdot\overrightarrow{\Delta S}$$

$$\Rightarrow \quad w = \vec{F}\cdot\overrightarrow{\Delta S}\cos 0°$$

$$\Rightarrow \quad w = 10\,\overrightarrow{\Delta S}$$

Now $10 = 10\,a$

$$\therefore \quad a = 1 \text{ m/s}^2$$

$$s = \frac{1}{2}at^2 = \frac{1}{2}\times 1 \times 2^2 = 2 \text{ m}$$

$$w = 10 \times 2 = 20 \text{ J.}$$

12. Find kinetic energy of block after 2 seconds in the above problem?

Solution:

$$V = 0 + at \quad \Rightarrow \quad V = 1 \times 2 = 2 \text{ m/s}$$

$$\therefore \quad \text{KE} = \frac{1}{2}\times 10 \times 2^2 = 20.$$

13. A force $F = 0.5x + 10$ acts on a particle. Here F is in newton and x is in metre. Calculate the work done by the force during the displacement of the particle from $x = 0$ to $x = 2$ metre.

Solution:

Small amount of work done dW in giving a small displacement $\vec{dx}$ is given by

$$dW = \vec{F} \cdot \vec{dx}$$

or $\quad dW = Fdx \cos 0°$

or $\quad dW = Fdx \quad [\because \cos 0° = 1]$

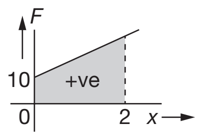
Total work done, $W = \int_{x=0}^{x=2} Fdx = \int_{x=0}^{x=2} (0.5\,x + 10)dx$

$$= \int_{x=0}^{x=2} 0.5x\ dx + \int_{x=0}^{x=2} 10\,dx$$

$$= 0.5\left|\frac{x^2}{2}\right|_{x=0}^{x=2} + 10\left|x\right|_{x=0}^{x=2}$$

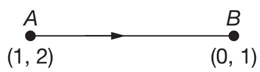
$$= \frac{0.5}{2}\,[2^2 - 0^2] + 10[2 - 0]$$

$$= (1 + 20) = 21 \text{ J.}$$



Work Done by Variable Force

14. A force $\vec{F} = x\hat{i} + 2y\,\hat{j}$ is applied on a particle



Find out work done by F to move the particle from point A to B.

Solution:

$$dw = \vec{F} \cdot \vec{ds}$$

$$dw = (x\hat{i} + 2y\,\hat{j})\,(dx\,\hat{i} + dy\,\hat{i})$$

$$dw = \int_1^0 x\,dx + \int_2^1 2y\,dy$$

$\therefore \quad w = -3.5$ J

15. An object is displaced from position vector $\vec{r_1} = (2\hat{i} + 3\hat{j})\,m$ to $\vec{r_2} = (4\hat{j} + 6\hat{k})\,m$ under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})\,N$. Find the work done by this force.

Solution:

$$W = \int_{\vec{r_i}}^{\vec{r_f}} \vec{F} \cdot \vec{dr} = \int_{\vec{r_1}}^{\vec{r_2}} (3x^2\hat{i} + 2y\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_{\vec{r_1}}^{\vec{r_2}} (3x^2 dx + 2y dy) = [x^3 + y^2]_{(2,\,3)}^{(4,\,6)} = 83 \text{ J}$$

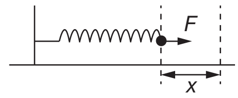
16. An object is displaced from a point A(0, 0, 0) to B(1 m, 1 m, 1 m) under a force $\vec{F} = (y\hat{i} + x\hat{j})\,N$. Find the work done by this force in this process.

Solution:

$$W = 1 \text{ J}$$

Work Done by Spring Force

17. Initially, spring is relaxed. A person starts pulling the spring by applying a variable force F. Find out the work done by F to stretch it slowly to a distance by x.



Solution:

$$\int dW = \int F \cdot ds = \int_0^x K\,x\,dx$$

$$\Rightarrow \quad W = \left(\frac{Kx^2}{2}\right)_0^x = \frac{Kx^2}{2}$$

18. In the above example,

(A) Where has the work gone?
(B) Work done by spring on wall is ________ .
(C) Work done by spring force on man is ________ .

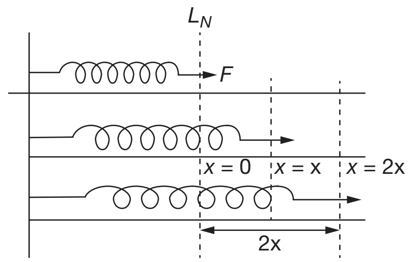
Solution:

(A) It is stored in the form of potential energy in spring.
(B) Zero, as displacement is zero.
(C) $-\dfrac{1}{2}Kx^2$

19. Find out work done by applied force to extend the spring from x to $2x$.

Solution:

Initially, the spring is extended by x



$$W = \vec{F} \cdot \vec{ds}$$

$$W = \int_x^{2x} K\,x\,.dx$$

$$W = \left[\frac{Kx^2}{2}\right]_x^{2x} = \frac{3}{2}\,Kx^2$$

It can also found by difference of PE.

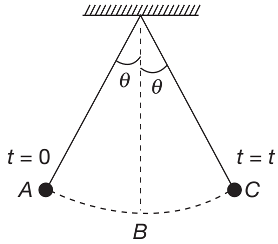
That is, $U_f = \frac{1}{2} K (2x)^2 = 2kx^2$

$$U_i = \frac{1}{2} kx^2$$

$$\Rightarrow \qquad U_f - U_i = \frac{3}{2} kx^2$$

Work Done by Tension

20. Find work done by tension when particle goes from A to B, B to C, and C to A.



Solution:

Zero because $F_T \perp dS$ at all times.

21. In above question, find out work done by gravity from A to B and B to C.

Solution:

$$W_g = \vec{F} \cdot \overrightarrow{\Delta S}$$

$$= mg\, \Delta S \cos \theta$$

$W_g = mg(\ell - \ell \cos \theta)$ for A to B

$W_g = -mg(\ell - \ell \cos \theta)$ for B to C

22. The system is released from rest.

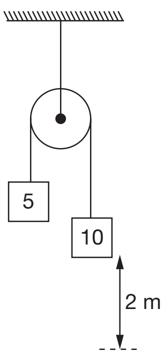
Find

(A) Work done by gravity on 10 kg

(B) Work done by gravity on 5 kg

(C) Work done by tension on 10 kg

(D) Work done by tension on 5 kg



Solution:

(A) $(W_g)_{10\text{ kg}} = 10\text{ g} \times 2$

$$= 200 \text{ J}$$

(B) $(W_g)_{5\text{kg}} = 5\text{ g} \times 2 \times \cos 180°$

$$= -100 \text{ J}$$

(C) $(W_T)_{10\text{ kg}} = \frac{200}{3} \times 2 \times \cos 180°$

$$= \frac{-400}{3} \text{ J}$$

(D) $(W_T)_{5\text{ kg}} = \frac{200}{3} \times 2 \times \cos 0°$

$$= \frac{400}{3}$$

Net work done by tension is zero. Work done by internal tension (i.e. tension acting within system) on the system is always zero if the length remains constant.

23. Find the relation between speed of blocks A and B as shown in Fig. 4.6.

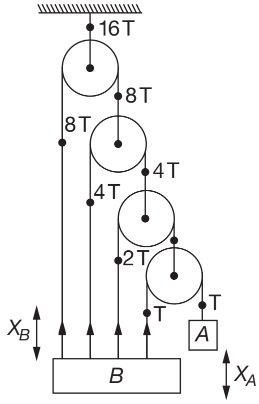


Fig. 4.6

Solution:

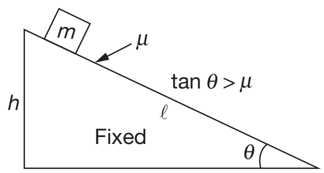
Work done by internal tension is zero.

$\therefore \qquad 15\, T \times X_B - T \times X_A = 0$

$$X_A = 15\, X_B$$

$\therefore \qquad v_A = 15\, v_B$

24. Find work done by normal reaction, gravity, and friction, when block moves from top to the bottom.



Solution:

$$W_N = 0$$

$\therefore \qquad F_N \perp \Delta S$

$$W_g = mg\, \ell \sin \theta$$

$$W_f = -\mu mg \cos \theta \cdot \ell$$

25. What is kinetic energy of block at bottom in the above problem?

Solution:

$$v^2 = u^2 + 2as$$

$$v^2 = 2(g \sin \theta - \mu\, g \cos \theta)\, (\ell)$$

$\therefore \quad KE = \frac{1}{2} m\, 2\,(g \sin\theta - \mu g \cos\theta)\,\ell$

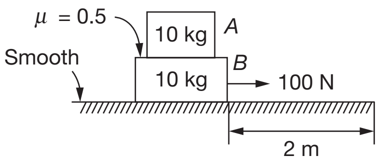
$= mg\,\ell\,(\sin\theta - \mu\cos\theta)$

Work Done by Friction

26. (A) Find out work done by applied force during displacement 2 m.

Ans. $100 \times 2 \times \cos\theta° = 200$ J

(B) Find out work done by frictional force on B by A during the above displacement.



Solution:

$f_{smax} = \mu mg = 0.5 \times 10\,g = 50$ N

Assuming they move together.

$100 = 2a \quad \Rightarrow \quad a = 5\ m/s^2$

Check Friction on A

$f = 10 \times 5 = 50$ N

$f_{reqd} = f_{available}$

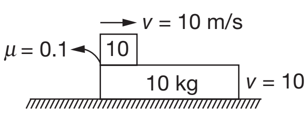
$\therefore$ They move together

Hence $\left.\begin{array}{l}(W_f)_{on\ B} = -100\ J \\ (W_f)_{on\ A} = -100\ J\end{array}\right\}$ Net zero

That is, work done by internal static friction is zero.

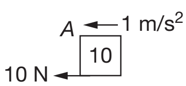
27. (A) Find out the velocity of two blocks when frictional force stops acting.

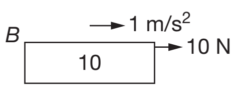
(B) Find out displacement of A and B till velocity becomes equal.



Solution:

(A)





$v_A = 10 - 1t$

$v_B = 1t$

$v_A = v_B$

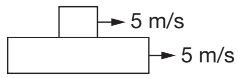
$10 - t = t$

$10 = 2t$

$t = 5$ s

$v_B = v_A = 5$ m/s

Situation becomes



(B) $S_A = 10 \times 5 - \frac{1}{2} \times 1 \times 5^2$

$= 37.5$ m

$S_B = \frac{1}{2} \times 1 \times 5^2 = 12.5$ m

28. In the above question, find work done by kinetic friction on A and B.

Solution:

$(W_{KF})_{on}\ A = 10 \times 37.5 \cos 180°$

$= -375$ J

$(W_{KF})_{on}\ B = 10 \times 12.5 \cos\theta$

$= 125$ J

Work done by KF on system of A and B

$= -375 + 125 = -250$ J

Work done by KF on a system is always negative.

Heat generated $= -(W_{KF})$ on system

(W_{KF}) on system $= -(f_K \times S_{relative})$

$= -10 \times 25 = -250$ J

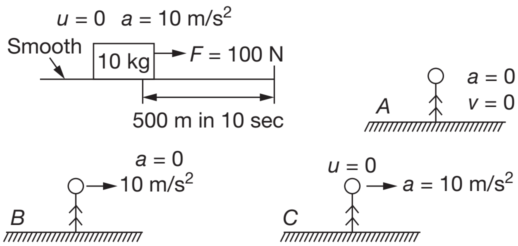
True or False

1. Work done by KF on a body in never zero.
   Ans: False
2. Work done by KF on a system is always negative.
   Ans: True

Kinetic energy of a body frame-dependent as velocity is a frame-dependent quantity. Therefore, pseudo force work has to be considered.

Work Done by Pseudo Force

29. Find out work done by the force F in 10 seconds as observed by A, B, and C.



Solution:

$$(W_F)_{\text{on block w.r.t } A} = 100 \times 500 \text{ J} = 50{,}000 \text{ J}$$

$$(W_F)_{\text{on block w.r.t } B} = 100\,[500 - 10 \times 10] = 40{,}000 \text{ J}$$

$$(W_F)_{\text{on block w.r.t } C} = 100\,[500 - 500] = 0$$

Work Done by Internal Force

$$F_{AB} = -F_{BA}$$

That is, sum of internal forces is zero.

But it is not necessary that work done by internal force be zero. There must be some deformation or reformation between the system to do internal work. In case of a rigid body, work done by internal force is zero.

Work-Energy Theorem

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$$W_C + W_{NC} + W_{PS} = \Delta K$$

Where, W_C is the work done by all the conservative forces.

W_{NC} is the work done by all non-conservative forces.

W_{PS} is the work done by all pseudo forces.

Modified form of Work-Energy Theorem

We know that conservative forces are associated with the concept of potential energy, that is

$$W_C = -\Delta U$$

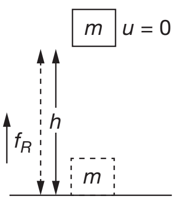
So, work-energy theorem may be modified as

$$W_{NC} + W_{PS} = \Delta K + \Delta U$$

$$W_{NC} + W_{PS} = \Delta E$$

SOLVED EXAMPLES

30. A block is released from a height h above the ground. Its speed is $\sqrt{gh}$ just before reaching the ground.



Find work done by resistive force in above situation.

Solution:

Identify initial and final state and all forces.

$$W_g + W_{\text{air res.}} + W_{\text{int force}} = \Delta K$$

$$mgh + W_{\text{air res.}} + 0 = \frac{1}{2}\text{m}\left(\sqrt{gh}\right)^2 - 0$$

$$W_{\text{air res.}} = -\frac{mgh}{2}.$$

31. A sphere suspended from point O by means a string is released from horizontal position as shown in Fig. 4.7.

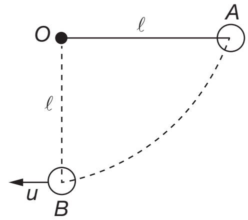


Fig. 4.7

Find its velocity u at lowest position.

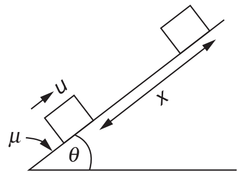
Solution:

$$W_g + W_T = \Delta K$$

$$mg\ell + 0 = \frac{1}{2}\,mu^2 - 0$$

$$u = \sqrt{2g\ell}.$$

32. A block of mass m is projected with initial speed u on a rough inclined surface. Using work-energy theorem, find out x when the block stops moving.



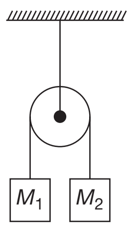
Solution:

$$W_g + W_f + W_N = \Delta K$$

$$-mg\,x\sin\theta - \mu\,mg\cos\theta + 0 = 0 - \frac{1}{2}mu^2$$

$$x = \frac{\mu^2}{2g(\sin\theta + mg\cos\theta)}.$$

33. Using work-energy theorem, find out velocity of the blocks when they move a distance x.



$M_2 > M_1$ and $u = 0$

Solution:

$$(W_{\text{all F}})_{\text{system}} = (\Delta K)_{\text{system}}$$

$$(W_g)_{\text{sys}} + (W_T)_{\text{sys}} = (\Delta K)_{\text{sys}}$$

as $$(W_T)_{\text{sys}} = 0$$

$$M_2gx - M_1gx = \frac{1}{2}(M_1 + M_2)v^2 - 0 \qquad (1)$$

$$v = \sqrt{\frac{2(M_2 - M_1)gx}{M_1 + M_2}}.$$

34. In the above question, find out acceleration of blocks

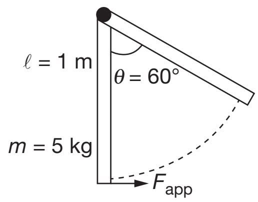
Solution:

Differentiating Equation (1) in above question.

$$(M_2g - M_1g) = \frac{1}{2}(M_1 + M_2)\, 2v\frac{dv}{dx}$$

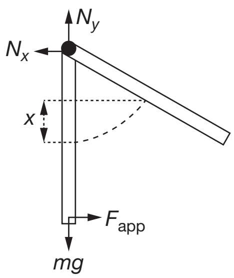
$$\Rightarrow \quad \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g = v\frac{dv}{dx} = a.$$

35. Find the work done by applied force by slowly bringing the rod to the inclined position.



Solution:

$$W_{\text{ALL}} = \Delta K \quad \text{by work-energy theorem}$$



$$x = \frac{\ell}{2} - \frac{\ell}{2}\cos 60° = \frac{1}{4}\text{ m} = 0.25\text{ m}$$

$$W_{N_x} + W_{N_y} + W_g + W_{\text{Fapp}} = \Delta K$$

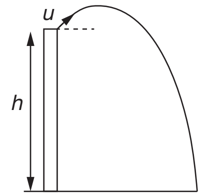
$$\therefore \quad 0 - mg(0.25) + W_{\text{Fapp}} = 0 - 0$$

$\because$ $\Delta K = 0$ when slowly brought

$$\therefore \quad W_{\text{Fapp}} = 5 \times 9.8 \times 0.25$$

It can be seen that $W_{\text{Fapp}} = mgh = 5 \times 9.8 \times 0.25$.

36. A stone is projected with initial velocity u from a building of height h. After some time, the stone falls on the ground. Find out speed of stone just before it strikes the ground.



Solution:

$$W_{\text{all forces}} = \Delta K$$

$$W_g = \Delta K$$

$$mgh = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$v = \sqrt{u^2 + 2gh}.$$

POWER

Power is defined as the time rate of doing work.

When the time taken to complete a given amount of work is important, we measure the power of the agent doing work.

The average power ($\vec{P}$ or p_{av}) delivered by an agent is given by

$$\vec{P} \quad \text{or} \quad p_{av} = \frac{W}{t}$$

where W is the amount of work done in time t.

Power is the ratio of two scalars–work and time. So, power is a scalar quantity. If time taken to complete a given amount of work is more, then power is less. For a short duration dt, if P is the power delivered during this duration, then

$$P = \frac{\vec{F}\cdot d\vec{S}}{dt} = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\cdot\vec{v}$$

This is instantaneous power. It may be +ve, –ve is zero.

By definition of dot product,

$$P = Fv\cos\theta$$

where θ is the smaller angle between $\vec{F}$ and $\vec{v}$.

This P is called instantaneous power if dt is very small.

Power is also the rate at which energy is supplied.

$$\text{Net power} = P_1 + P_2 + P_3 + \ldots$$

$$P_{\text{net}} = \frac{dw_1}{dt} + \frac{dw_2}{dt} \ldots\ldots$$

$$P_{\text{net}} = \left(\frac{dw_1 + dw_2 + \ldots\ldots\ldots}{dt}\right)$$

$$P_{\text{net}} = \frac{dK}{dt} \qquad \because W_{\text{all}} = \Delta K$$

$\therefore$ Rate of change of KE is also power.

SOLVED EXAMPLE

37. A block moves in uniform circular motion because a cord tied to the block is anchored at the centre of a circle. Is the power of the force exerted on the block by the cord position, negative, or zero?

Solution:

Zero

Explanation:

$\vec{F}$ and $\vec{v}$ are perpendicular.

$\therefore$ Power $= \vec{F} \cdot \vec{v} = Fv \cos 90° =$ Zero.

Unit of Power

A unit power is the power of an agent which does unit work in unit time.

The power of an agent is said to be one watt if it does one joule of work in one second.

$$1 \text{ watt} = 1 \text{ joule/section} = 10^7 \text{ erg/second}$$

Also, $$1 \text{ watt} = \frac{1 \text{ newton} \times 1 \text{ metre}}{1 \text{ second}}$$

$$= 1 \text{ N ms}^{-1}.$$

Dimensional Formula of Power

$$[\text{Power}] = \frac{[\text{Work}]}{[\text{Time}]}$$

$$= \frac{[ML^2T^{-2}]}{[T]} = [\text{ML}^2\ \text{T}^{-3}]$$

Power has 1 dimension in mass, 2 dimensions in length and 3 dimensions in time.

S.No.	Human Activity	Power (W)
1	Heart beat	1.2
2	Sleeping	83
3	Sitting	120
4	Riding in a car	140
5	Walking (4.8 km h^{-1})	265
6	Cycling (15 km h^{-1})	410
7	Playing Tennis	440
8	Swimming (breaststroke, 1.6 km h^{-1})	475
9	Skating	535
10	Climbing Stairs (116 steps min^{-1})	685
11	Cycling (21.3 km h^{-1})	700
12	Playing Basketball	800
13	Tube light	40
14	Fan	60

SOLVED EXAMPLES

38. What is the power of external force and friction at the moment shown in Fig. 4.8.

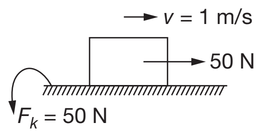


Fig. 4.8

Solution:

$$P_{\text{ext}} = 50 \times 1 = 50 \text{ W}$$

$$P_f = -50 \times 1 = -50 \text{ W}.$$

39. A stone is projected with velocity at an angle θ with horizontal. Find out

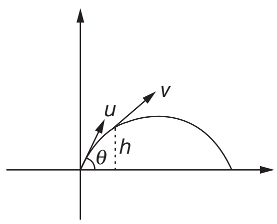
(A) Average power of the gravity during time t.

(B) Instantaneous power due to gravitational force at time t, where t is time of flight.

Solution:

$$P = \frac{w}{T} = -\frac{mgh}{t} = -\frac{mg\left[u\sin\theta t - \frac{1}{2}gt^2\right]}{t}$$

$$P = mg\left[\frac{gt}{2} - u\sin\theta\right]$$



(A) When is average power zero?

$$\frac{gt}{2} = u\sin\theta \Rightarrow t = \frac{2u\sin\theta}{g}, \text{ i.e., time of flight.}$$

(B) Instantaneous power

$$P = \vec{F} \cdot \vec{v}$$

$$= (-mg\,\hat{j})\,[u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}\,)]$$

$$= -mg(u\sin\theta - gt)$$

(C) When is P_{inst} zero?
When F and V are $\perp$, i.e., at $t = \dfrac{u\sin\theta}{g}$, which is at the highest point.

(D) When is P_{inst} positive?
From base to highest point.

(E) When is P_{inst} negative?
From highest point to base.

POTENTIAL ENERGY

Potential energy is the internal capacity of doing work of a system by virtue of its configuration.

In case of conservative force (field), potential energy is equal to negative of work done by the conservative force in shifting the body from some reference position to given position.

Therefore, in case of conservative force,

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r}$$

i.e., $$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = -W$$

Whenever and wherever possible, we take the reference point at ∞ and assume potential energy to be zero there, i.e., if we take $r_1 = \infty$ and $U_1 = 0$, then

$$U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W$$

Important Points for Potential Energy

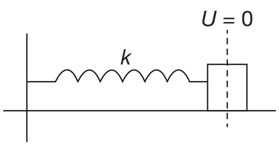
1. Potential energy can be defined only for conservative forces. It has no relevance for non-conservative forces.
2. Potential energy can be positive or negative, depending upon choice of frame of reference.
3. Potential energy depends on frame of reference, but change in potential energy is independent of reference frame.
4. Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.
5. It is a function of position and does not depend on the path.

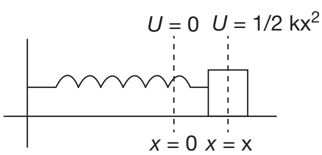
Gravitation Potential Energy

$U = mgh$ for a particle at a height h above reference level.

Potential Energy of Spring

As above, $W_{SPF} = -\Delta U$





$\therefore$
$$W_{SPF} = -\Delta U$$
$$W_{SPF} = U_i - U_F$$
$$W_{SPF} = 0 - \frac{1}{2}kx^2 = -\frac{1}{2}kx^2$$
$$U = \frac{1}{2}Kx^2$$

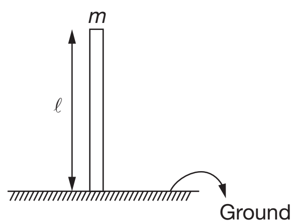
Where x is change in length from its natural length.

NOTE

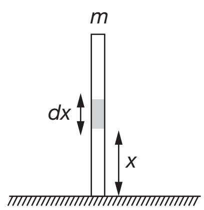
Gravitational PE can be +ve, –ve, or zero, but spring PE will always be +ve.

SOLVED EXAMPLES

40. Calculate potential energy of a uniform vertical rod of mass m and length ℓ assuming ground as a zero potential energy level.



Solution:



$$dU = dm\, gh$$
$$\int_0^U dU = \int_0^{\ell}\left(\frac{m}{\ell}dx\right)gx$$
$$U = \frac{mg\ell}{2}.$$

41. Calculate potential energy of rod shown in Fig. 4.9 assuming ground as zero potential energy level.

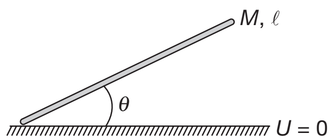
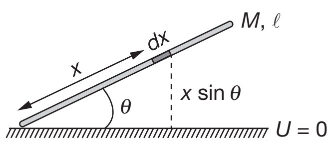


Fig. 4.9

Solution:



$$dU = dm \cdot g \cdot h$$

$$\int dU = \int_0^{\ell} \frac{M}{\ell}\, dx \cdot g \cdot x \sin\theta$$

$$\therefore \qquad U = \frac{Mg\sin\theta\, L}{2}$$

Also, state by COM concept and show that $U = Mg\left(\frac{L}{2}\sin\theta\right)$.

42. A chain of mass m is kept on a hemisphere as shown in Fig. 4.10. Find out potential energy of the chain assuming reference line as a zero potential energy level.

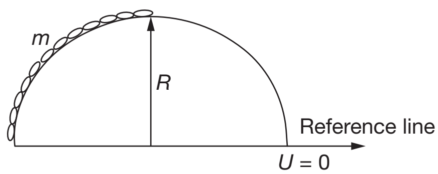
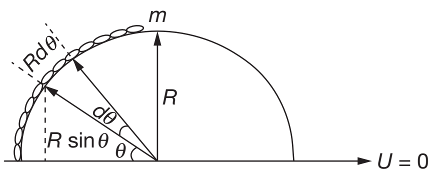


Fig 4.10

Solution:



We know that $\frac{\text{arc}}{\text{Radius}} = \theta$

$\therefore$ Elemental length $= Rd\theta$

$$\therefore \qquad dm = \frac{m}{\pi/2}d\theta = \frac{2m}{\pi}d\theta$$

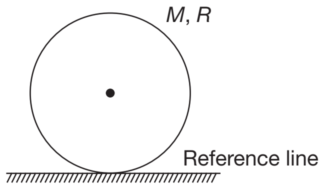
$$\text{Now} \qquad dU = dm\, gh = \left(\frac{2m}{\pi}d\theta\right)(g)(R\sin\theta)$$

$$\therefore \qquad \int_0^U dU = \frac{2m}{\pi} Rg \int_0^{\pi/2} \sin\theta\, d\theta$$

$$U = \frac{2mgR}{\pi}(-\cos\theta)_0^{\pi/2}$$

$\Rightarrow \qquad U = mg\left(\frac{2R}{\pi}\right)$ [Mention that $\left(\frac{2R}{\pi}\right)$ is the height of COM]

43. Find out potential energy of the solid sphere assuming reference line as a zero potential energy level.



Solution:

$$U = Mgh_{cm}$$

For symmetrical body, COM is the geometrical centre of the body.

$$\therefore \qquad U = mgR.$$

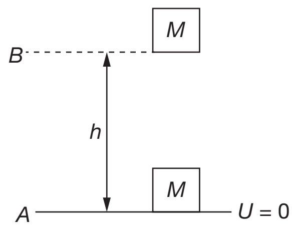
44. Find

(A) W_g from A to B

(B) W_g from B to A

(C) Calculate $U_A - U_B$

(D) Calculate $U_B - U_A$



Solution:

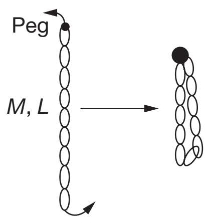
$$(W_g)_{\text{A to B}} = -mgh$$

$$(W_g)_{\text{B to A}} = mgh$$

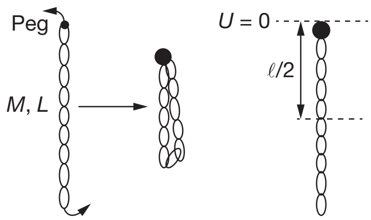
$$U_A - U_B = -mgh$$

$$U_B - U_A = mgh.$$

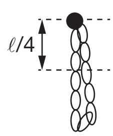
45. Find out work done by external agent to slowly hang the lower end of the chain to the Peg.



Solution:



Initially, $U_i = -Mg\frac{L}{2}$



$$U_f = -Mg\frac{L}{4}$$

$\therefore \quad U_i - U_f = (-Mg\frac{L}{2}) - (-Mg\frac{L}{4}) = Mg\frac{L}{4}$

Now $\quad W_{\text{ALL}} = \Delta K$

$\therefore \quad W_g + W_{\text{applied force}} + W_N = \Delta K$

We have $\quad W_g = U_i - U_f$

and $\quad W_N = 0$

and $\quad \Delta K = 0$

$\therefore \quad W_{\text{applied force}} = \frac{MgL}{4}.$

46. In above example, find out the work done by external agent to slowly hang the middle link to peg.

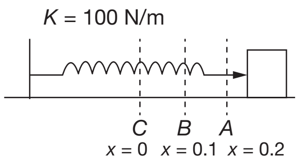
Solution:

$$U_i = \left(-\left(\frac{M}{2}\right)g\frac{L}{8}\right) - \left(\frac{M}{2}g\frac{L}{4}\right)$$

$\Rightarrow \quad U_f = -Mg\frac{L}{2}$

$\therefore \quad W = U_i - U_f.$

47. Find out the work done by spring force from A to B and from B to C. $x = 0$ is position of natural length.



Solution:

$$(W_{\text{spring}})_{A \to B} = U_i - U_f$$

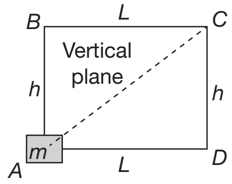
$$= \frac{1}{2}K(0.2)^2 - \frac{1}{2}K(0.1)^2$$

$\therefore \quad (W_{\text{spring}})_{A \to B} = \frac{3}{2}$ J

Similarly, $\quad (W_{\text{spring}})_{BC} = \frac{1}{2}$ J

48. (A) The mass m is moved from A to C along three different paths

(i) ABC (ii) ADC (iii) AC



Find out work done by gravity in the three cases.

(B) The block is moved from A to C along three different paths. Applied force is horizontal. Find work done by friction force in path Fig 4.11

(i) ABC (ii) ADC (iii) AC

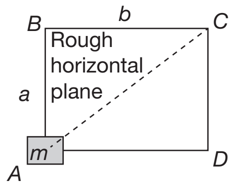


Fig 4.11

Solution:

(A) (i) mgh

(ii) $-mgh$

(iii) $-mgh$

(B) (i) $W_{ABC} = -\mu mg\ (a + b)$

(ii) $W_{ADC} = -\mu mg\ (a + b)$

(iii) $W_{ALC} = -\mu mg\ (\sqrt{a^2 + b^2})$.

CONSERVATIVE FORCES

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.

Consider a body of mass m being raised to a height h vertically upwards as show in Fig. 4.12(a, b, and c). The work done is mgh. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result mgh once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal parts is zero. The work done along the vertical parts add up to mgh. Thus, we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the initial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

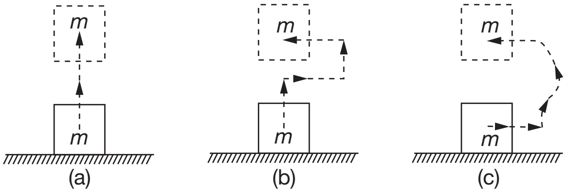


Fig 4.12

Examples of Conservative Forces

1. Gravitational force, not only due to the Earth but also in its general form as given by the universal law of gravitation, is a conservative force.
2. Elastic force in a stretched or compressed spring is a conservative force.
3. Electrostatic force between two electric charges is a conservative force.
4. Magnetic force between two magnetic poles is a conservative force.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and electrostatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of Conservative Forces

1. Work done by or against a conservative force depends only on the initial and final positions of the body.
2. Work done by or against a conservative force does not depend upon the nature of the path between initial and final positions of the body.

> If the work done by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.

3. Work done by or against a conservative force in a round trip is zero.

> If a body moves under the action of a force that does no total work during any round trip, then the force is conservative, otherwise it is non-conservative.
>
> The concept of potential energy exists only in the case of conservative forces.

4. The work done by a conservative force is completely recoverable.

> Complete recoverability is an important aspect of the work of a conservative force.

NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force, etc., are non-conservative forces.

S.No.	Conservative Forces	Non-conservative Forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a system, the kinetic enrgy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable.	Work done in not completely recoverable.

SOLVED EXAMPLES

49. Figure 4.13 shows three paths connecting points *a* and *b*. A single force *F* does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force *F* conservative?

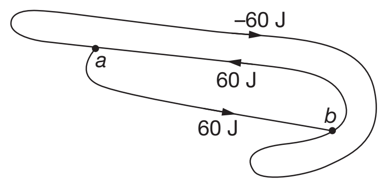


Fig 4.13

Solution:

No

Explanation:

For a conservative force, the work done in a round trip should be zero.

50. Find work done by force $\vec{F} = x\,\hat{i} + y\,\hat{j}$ in displacing a particle from position $A(0, 0)$ to $B(2, 3)$.

Solution:

$$dW = \vec{F}\cdot ds$$
$$= (x\hat{i} + y\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$$
$$W = \int_0^2 xdx + \int_0^3 ydy$$
$$= \left[\frac{x^2}{2}\right]_0^2 + \left[\frac{y^2}{2}\right]_0^3 = \frac{13}{2} \text{ units}$$

All fundamental forces of nature are conservative in nature.

True or False

1. In case of a non-conservative force, work done along two different paths will always be different.
 Ans. False
2. In case of non-conservative force, work done along two different paths may be different.
 Ans. True
3. In case of non-conservative force, work done along all possible paths cannot be same.
 Ans. True

Statement: In case of non-conservative force, work done in a closed loop will not be zero. It is always negative. For example, Kinetic friction, air resistance, viscosity force.

Statement: We can define PE for conservative forces only.

51. Find work done by force $\vec{F} = x\hat{i} + xy\hat{j}$ in displacing a particle from position $O(0, 0)$ to $C(2, 2)$.

Solution:

$$\int dw = \int_0^2 xdx \qquad + \int_0^2 xydy$$

can be found — cannot be found until x is known in terms of y, i.e., until equation of path is known.

52. Find the work done by $\vec{F}$ from O to C for above example if paths are given as in Fig. 4.14.

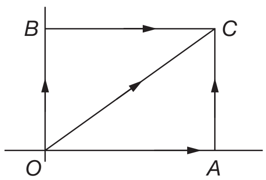


Fig. 4.14

Solution:

(i) $OAC \Rightarrow OA + AC$

for OA $\quad y = 0$

$\therefore \quad dy = 0$

$\therefore \quad \int dw_{OA} = \int_0^2 xdx + 0$

$\therefore \quad W_{OA} = 2$ J

for AC $\quad x = 2 \quad dx = 0$

$$\int dw_{AC} = 0 + 2\int_0^2 ydy$$

$\therefore \quad W_{AC} = 4$ J

$\therefore \quad W_{OAC} = W_{OA} + W_{AC} = 2 + 4 = 6$ J

(ii) $OBC \Rightarrow OB + BC$

for OB $\quad x = 0 \quad dx = 0$

$\therefore \quad W_{OB} = O$

for BC $\quad y = 2 \quad dy = 0$

$\therefore \quad \int dw = \int xdx$

$\therefore \quad W = \left[\frac{x^2}{2}\right]_0^2 = 2$

$\therefore \quad W_{OAC} \neq W_{OBC}$

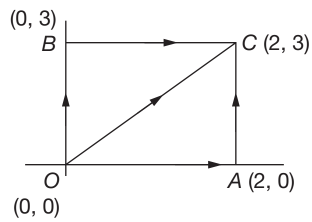
Hence, the force is non-conservative.

(iii) For W_{OC} $\quad dw = xdy + xydx$

for OC $\quad x = y \quad dx = dy$

$$dw = \int_0^2 xdx + \int_0^2 y^2dy$$
$$W = \frac{14}{3} \text{ unit.}$$

53. Find out work done by given force $\vec{F} = y\hat{i} + x\hat{j}$ for given paths and decide whether the force is conservative or non-conservative.



Solution:

(i) $OAC \Rightarrow OA + AC$

for OA $\quad y = 0 \quad dy = 0$

$\therefore \quad dw = 0 \quad W_{OA} = 0$

for AC $\quad x = 2 \quad dx = 0$

$$\int dw = \int_0^3 dy \quad \Rightarrow W = 6 \text{ J}$$

$\Rightarrow \quad W_{OAC} = 6$ units

(ii) $OBC \Rightarrow OB + BC$

for $OB \quad x = 0 \quad dx = 0$

$\therefore \quad dw = 0$

for $BC \quad y = 3 \quad dy = 0$

$$\int dw = \int_0^2 3dx \quad \Rightarrow \quad W = 6 \text{ units}$$

$\Rightarrow \quad W_{OBC} = 6$ units

(iii) OC

for $OC \quad y = \frac{3}{2}x \quad dy = \frac{3}{2}\, dx$

$$\therefore \quad \int dw = \int_0^2 \frac{3}{2} x dx + \int_0^2 \frac{3}{2} x dx$$

$$\Rightarrow \quad \int dw = 3\int_0^2 x^2 dx$$

$\Rightarrow \quad W_{OC} = 6$ units

Above force seems conservative but cannot be confirmed yet unless we can integrate it without the knowledge of path.

Again we had $dw = xdy + ydx$ and $xdy + ydx$ can be written as dxy

$$\therefore \quad \int dw = \int dxy$$

$$W = \int_{0,0}^{2,3} dxy = [xy]_{0,0}^{2,3} = 6 \text{ J}$$

Hence, knowledge of path was not required to integrate the above; so F is conservative.

POTENTIAL ENERGY AND CONSERVATIVE FORCE

$$F_S = -\partial U / \partial_s,$$

That is, the projection of the field force, the vector $\boldsymbol{F}$, at a given point in the direction of the displacement dr equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial / \partial s$ emphasizes the fact of deriving with respect to a definite direction.

So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projection F_x, F_y, and F_z of the vector $\boldsymbol{F}$ on the unit vectors $\boldsymbol{i}, \boldsymbol{j}$, and $\boldsymbol{k}$. Hence, one can readily find the vector itself:

$$F = F_x \boldsymbol{i} + F_u \boldsymbol{j} + F_z \boldsymbol{k}, \quad \text{or}$$

When conservative force does positive work, then potential energy decreases

$$dU = -dw$$

$$dU = -F \cdot ds$$

$$dU = -(F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot (d_x \hat{i} + d_y \hat{j} + d_z \hat{k})$$

$$dU = -F_x dx - F_y dy - F_z dz$$

If y and z are constants, then $dy = 0 \quad dz = 0$

$$dU = -F_x dx$$

$$\therefore \quad F_x = -\frac{dU}{dx} \text{ if } y \text{ and } z \text{ are constants}$$

$$\equiv F_x = \frac{-\partial U}{\partial x}$$

Similarly $\quad F_y = \frac{-\partial u}{\partial y};$

$$F_z = \frac{-\partial u}{\partial z}$$

$$\Rightarrow \quad F = -\left(\frac{\partial U}{\partial x} \boldsymbol{i} + \frac{\partial U}{\partial y} \boldsymbol{j} + \frac{\partial U}{\partial z} \boldsymbol{k} \right).$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇U. We shall use the second, more convenient, designation, where ∇ ('nabla') signifies the symbolic vector or operator

$$\nabla = \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$$

Potential Energy Curve

1. A graph plotted between the potential energy a particle and its displacement from the centre of force field is called potential energy curve.
2. From Fig. 4.15, we can predict the rate of motion of a particle at various positions.

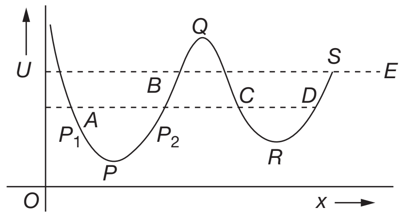


Fig. 4.15

3. Force on the particle is $F_{(x)} = -\frac{dU}{dx}$

Case I: On increasing x, if U increases, force is in (–) ve x-direction, i.e., attraction force.

Case II: On increasing x, if U decreases, force is in (+) ve x-direction, i.e., repulsion force.

SOLVED EXAMPLES

54. $U = \frac{1}{2}kx^2$

Find F associated with this potential energy.

Solution:

$$F_x = \frac{-\partial u}{\partial x} = -kx$$

$$F_y = 0$$

$$F_z = 0.$$

55. $U = x^2 + y^2$

Give the force whose potential energy is given as above.

Solution:

$$F_x = \frac{-\partial u}{\partial x} = -[2x + 0] = -2x$$

$$F_y = \frac{-\partial u}{\partial y} = -(2y + 0) = -2y$$

$$\vec{F} = -2x\hat{i} - 2y\hat{j}$$

56. Find out the potential energy of given force $F = -2x\hat{i} - 2y\hat{j}$.

Solution:

$$dU = -dw$$

$$\int dU = \int -(-2x\hat{i} - 2y\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$\Rightarrow \quad \int dU = \int 2x dx + \int 2y dy$$

$$\therefore \quad U = x^2 + y^2 + C.$$

57. Find out the formula for potential energy of the force $F = y\hat{i} + x\hat{j}$.

Solution:

$$dU = -dw$$

$$dU = -(y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$\int dU = \int -y dx + \int -x dy$$

$$\Rightarrow \quad \int dU = -\int dxy$$

$$\Rightarrow \quad U = -xy + c.$$

58. Find out the force for which potential energy $U = -xy$.

Solution:

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right]$$

$$\Rightarrow \quad \vec{F} = -\left[\frac{\partial(-xy)}{\partial y}\hat{i} + \frac{\partial(-xy)}{\partial y}\hat{j}\right]$$

$$\vec{F} = y\hat{i} + x\hat{j}$$

Hence verifying the previous example.

59. Find out force for which $U = x^2 \sin y + \ell n\, ye^x + \tan z\sqrt{x}$.

Solution:

$$F_x = \frac{-\partial U}{\partial x}\hat{i} = -[2x \sin y + \log e^x + \tan z \frac{1}{2\sqrt{x}}]\hat{i}$$

Similarly, F_y and F_z can be calculated.

60. The potential energy between two atoms in a molecule is given by, $U_{(x)} = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The system is in stable equilibrium when

(A) $x = 0$ (B) $x = \frac{a}{2b}$

(C) $x = \left(\frac{2a}{b}\right)^{1/6}$ (D) $x = \left(\frac{11a}{5b}\right)$

Solution: (C)

Given that, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

We know $F = -\frac{du}{dx}$

$$= (-12)\, ax^{-13} - (-6b)x^{-7} = 0$$

or $\frac{-6b}{x^7} = \frac{12a}{x^{13}}$

or $x^6 = 12a/6b = 2a/b$

or $x = \left(\frac{2a}{b}\right)^{1/6}$

EQUILIBRIUM OF A PARTICLE

Different Positions of a Particle

Position of Equilibrium

If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium $\frac{dU}{dx} = 0$. Points P, Q, R, and S are the states of equilibrium positions.

Types of Equilibrium

1. **Stable Equilibrium:** When a particle is displaced slightly from a position, and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

   Necessary conditions: $-\frac{dU}{dx} = 0,$ and $\frac{d^2U}{dx^2} = +\text{ve}$

2. **Unstable Equilibrium:** When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

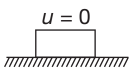
   Condition: $-\frac{dU}{dx} = 0$ potential energy is maximum, i.e. $= \frac{d^2U}{dx^2} = -\text{ve}$

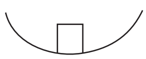
3. **Neutral Equilibrium:** In the neutral equilibrium, potential energy is constant. When a particle is displaced from its position, it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

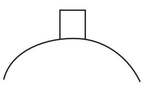
A particle is in equilibrium if the acceleration of the particle is zero. As acceleration is frame-dependent quantity, equilibrium depends on motion of observer also.

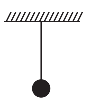
SOLVED EXAMPLES

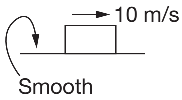
61. Which is not a case of equilibrium state in the following.

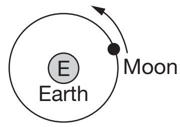
(A)


(B)


(C)


(D)


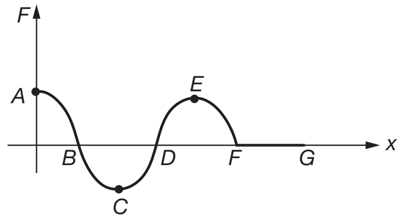
(E)


(F)


Solution: (F)

As moon is always accelerated. It has centripetal acceleration or it is changing its velocity all the time.

62. Find out positions of equilibrium and determine whether they are stable, unstable, or neutral.



Solution:

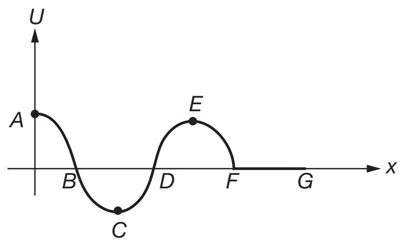
Equilibrium is at *B*, *D*, *F* as force is zero here.

For checking type of equilibrium, displace slightly. We have *B* as stable equilibrium,

D as unstable equilibrium,

and *F* as neutral equilibrium

63. Identify the points of equilibrium and discuss their nature.



Solution:

C, *E*, *F* are points of equilibrium because $F = \frac{-\partial U}{\partial x}$

When slope of *U*-*x* curve is zero, then *F* is zero.

Check stability through slopes at nearby points.

If we move right then slope should be positive for stable equilibrium and vice versa. In short, it is like a hill and plateau.

MECHANICAL ENERGY

Mechanical energy *E* of an object or a system is defined as the sum of kinetic energy *K* and potential energy *U*, i.e.,

$$E = K + U$$

Important Points for Mechanical Energy

1. It is a scalar quantity having dimensions $[ML^2T^{-2}]$ and SI units joule.
2. It depends on frame of reference.
3. A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if $E = 0$ either both PE and KE are zero or PE may be negative and KE may be positive such that KE + PE = 0.
4. As mechanical energy $E = K + U$, i.e., $E - U = K$. Now as K is always positive, $E - U \geq 0$, i.e., for existence of a particle in the field, $E \geq U$.
5. As mechanical energy $E = K + U$ and K is always positive, so, if U is positive E will be positive. However, if potential energy U is negative, E will be positive if $K > |U|$ and E will be negative if $K < |U|$.
   That is, mechanical energy of a body or system can be negative and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around a planet in bound state.

SOLVED EXAMPLES

64. A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm (Fig. 4.16). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring?

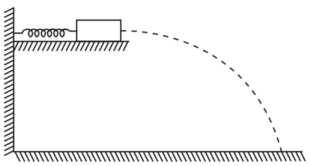


Fig. 4.16

Solution:

When block released, the block moves horizontally with speed v till it leaves the spring.

By energy conservation, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$v^2 = \frac{kx^2}{m} \quad \Rightarrow \quad v = \sqrt{\frac{kx^2}{m}}$$

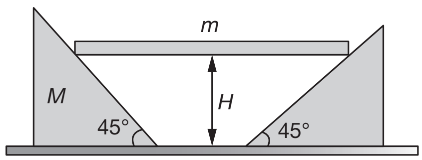
Time of flight $\quad t = \sqrt{\frac{2H}{g}}$

So, horizontal distance travelled from the free end of the spring is $v \times t$

$$= \sqrt{\frac{kx^2}{m}} \times \sqrt{\frac{2H}{g}} = \sqrt{\frac{100 \times (0.05)^2}{0.1}} \times \sqrt{\frac{2 \times 2}{10}} = 1 \text{ m}$$

So, at a horizontal distance of 1 m from the free end of the spring.

65. A rigid body of mass m is held at a height H on two smooth wedges of mass M each of which are themselves at rest on a horizontal frictionless floor. On releasing the body, it moves down pushing aside the wedges. The velocity of recede of the wedges from each other when rigid body is at a height h from the ground is



(A) $\sqrt{\frac{2mg(H-h)}{m+2M}}$ (B) $\sqrt{\frac{2mg(H-h)}{2m+M}}$

(C) $\sqrt{\frac{8mg(H-h)}{m+2M}}$ (D) $\sqrt{\frac{8mg(H-h)}{2m+M}}$

Solution: (C)

CONCEPTS AT A GLANCE

- Work done $W = \vec{F} \cdot \vec{s} = Fs \cos \theta$ if force F is constant

  $W = \int F \cdot ds$ if force is variable

  $W = \Delta KE$ (work-energy theorem)

  $W = \Delta PE$ (for conservative forces)

  $W = \frac{1}{2} kx^2$ in a spring

  $W = \frac{1}{2}$ stress × strain × volume (in elastic bodies)

  $W = \frac{F \cdot x}{2}$, where x is extension produced in a spring or elastic bodies.

  $W = \int P\, dV$, where P is pressure and V is volume.

  $W = \int P \cdot dt$, where P represents power

- Power $P = \frac{dW}{dt} = \frac{dE}{dt}$

  $P = \vec{F} \cdot \vec{v}$ if F is constant

  $P = \int \vec{F} \cdot d\vec{v}$ if F is variable or v is variable.

- Potential energy exists only for conservative forces. Non-conservative forces do not show PE.
- In conservative forces, work done is independent of path followed. It depends only on the initial and final position. Total work done in a round trip is zero.
- PE = $\frac{1}{2} kx^2$ (in a spring) It is only positive

  PE = $\frac{-GM_1M_2}{r}$ (in gravitational fields). It may be positive or negative

  PE = mgh if h is small

  PE = $\frac{q_1q_2}{4\pi \epsilon_0 r}$ (in electric fields). It may be positive or negative.

- If a body is in static or dynamic equilibrium, then $W = 0$.
- If a force is always perpendicular to velocity, then work done by this force is zero.
- Mechanical energy = KE + PE is conserved, if internal forces are conservative and do no work.
- KE + PE is not conserved, if non-conservative forces are present.
- KE = $\frac{p^2}{2m}$, where p is momentum of the body.
- If a lighter and a heavier body have equal KE, then heavier body has more momentum.
- If a lighter and heavier body have equal momentum, then lighter body has more KE.
- Area under power-time graph gives work.
- ΔU = change in PE = $\int F \cdot dr$ for conservative forces.
- If $\frac{dU}{dr} = 0$, body is said to be in equilibrium. Equilibrium is stable if U is minimum; unstable if U is maximum, and neutral if U = constant.
- If $\frac{l}{n}$ th part of the chain hangs then the work done to pull up the hanging chain is $\frac{mgl}{2n^2}$
- If maximum displacement is found in a spring, use work-energy theorem and at maximum displacement, velocity of block will be zero. In steady state, displacement in a spring is to be found using $F = -kx$.
- For a rolling body, KE = $\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$.
- Change in total energy $\Delta E = W_{ext} + W_{int}$ (non-conservative).
- No work will be done when force is perpendicular to velocity. For example, no work is done by centripetal force in a circular motion. No work is done by the magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$ as $\vec{F}$ and $\vec{v}$ are perpendicular.

BRAIN MAP

WORK, ENERGY AND POWER

1. Work done by a constant force $W = \vec{F}\cdot\vec{S} = FS\cos\theta$.

2. Work done by a variable force $= \int \vec{F}\cdot\vec{ds}$

3. If force is along the line of motion, work done can be calculated by measuring the area between *F-S* curve and displacement axis.

4. Work done by conservative force is path independent while that by non-conservative force is path dependent.

5. Power is the rate at which an agent does work.

Average Power, $\overline{P} = \dfrac{\Delta W}{\Delta t}$

Instantaneous Power

$P = \dfrac{dw}{dt} = \vec{F}\cdot\vec{v}$

6. Slope of *W-t* graph gives power of agent.

Area enclosed between *P-t* curve and time axis gives work done by the agent.

7. Kinetic energy $= \dfrac{1}{2}mv^2$

8. Gravitational potential energy $= \pm\, mgh$, we take positive GPE when body is above reference level and negative when the body is below the reference level.

9. Spring's elastic energy

$= \dfrac{1}{2}kx^2$

10. Work energy theorem

$W_{ext} = K_f - K_i = \Delta K$

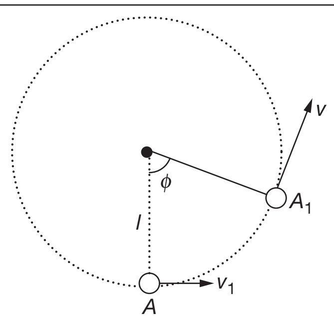
MOTION IN A VERTICAL CIRCLE

(Body suspended with the help of a string)
If body has velocity v_1 at its lowest position A, then at any position A_1

Velocity, $v = \sqrt{v_1^2 - 2gl\,(1 - \cos\phi)}$

Tension in string, $T = \dfrac{mv_1^2}{l} - 2\,mg + 3\,mg\cos\phi$

To complete the circle, $v_1 \geq \sqrt{5gl}$.



EXERCISES

Single Option Correct Type

1. The displacement–time graph of particle is shown in Fig. 4.17.

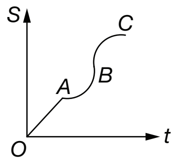


Fig. 4.17

(A) Work done by all the forces in part *OA* is greater than zero
(B) Work done by all the forces in part *AB* is greater than zero
(C) Work done by all the forces in part *BC* is greater than zero
(D) Work done by all the forces in part *AB* is less than zero

2. A small block of mass 0.1 kg is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm as shown in Fig. 4.18. The spring constant is 100 N/m. When released the block moves horizontally till it leaves the spring, it will hit the ground 2 m below the spring.

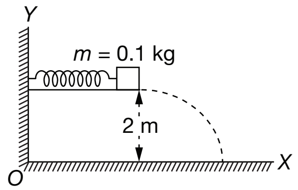


Fig. 4.18

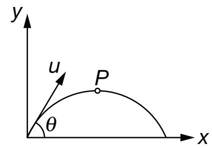
(A) At a horizontal distance of 1 m from free end of the spring.
(B) At a horizontal distance of 2 m from free end of the spring.
(C) Vertically below the edge on which the mass is resting.
(D) At a horizontal distance of $\sqrt{2}$ m from free end of the spring.

3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
(A) Its velocity is constant.
(B) Its acceleration is constant.
(C) Its kinetic energy is constant.
(D) It moves in a straight line.

4. Two identical balls are projected, one vertically up and the other at an angle of 30° with the horizontal, with same initial speed. The potential energy at the highest point is in the ratio
(A) 4 : 3 (B) 3 : 4 (C) 4 : 1 (D) 1 : 4

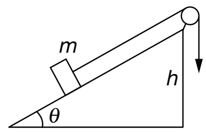
5. The unit of power is
(A) Kilowatt (B) Kilowatt-hour
(C) Dyne (D) Joule

6. A particle projected with an initial velocity u at angle θ from the ground. The work done by gravity during the time it reaches the highest point P is:



(A) $\dfrac{-mu^2 \sin^2\theta}{2}$ (B) $+\dfrac{mu^2 \sin^2\theta}{2}$
(C) 0 (D) $+mu^2 \sin\theta$

7. A block of mass m is slowly pulled up on inclined plane of height h and inclination θ with the top of a string parallel to the incline. Which of the following statement is correct for the block when it moves up from the bottom to the top of the incline?



(A) Work done by the normal reaction force is zero.
(B) Work done by the string on block is *mgh*.
(C) Work done by the gravity is *mgh*.
(D) Work done by the block is $-mgh/2$.

8. A block of mass 2 kg is lifted through a chain. When block moves through 2 m vertically the velocity becomes 4 m/s. Work done by chain force until it moves 2 m is ($g = 10$ ms^{-2})
(A) 40 J (B) 24 J
(C) 56 J (D) None of these

9. A position-dependent force $F = 7 - 2x + 3x^2 N$ acts on a small body of mass 2 kg and displaces it from $x = 0$ to $x = 5$ m. The work done in joule is
(A) 70 J (B) 270 J
(C) 35 J (D) 135 J

10. A car comes to a skidding stop in 15 m. The force on the car due to the road is 1000 N. The work done by road on the car and car on the road, respectively, is
(A) –15 kJ, zero (B) zero, 15 kJ
(C) 15 kJ, zero (D) –15 kJ, 15 kJ

11. A particle is released from rest at origin. It moves under the influence of potential field $U = x^2 - 3x$, where U is in Joule and x is in metre. Kinetic energy at $x = 2$ m will be

(A) 2 J (B) 1 J (C) 1.5 J (D) 0 J

12. The potential energy of a particle of mass m is given by $U = \frac{1}{2}kx^2$ for $x < 0$ and $U = 0$ for $x \geq 0$. If total mechanical energy of the particle is E. Then its speed at $x = \sqrt{\frac{2E}{k}}$ is

(A) Zero (B) $\sqrt{\frac{2E}{m}}$ (C) $\sqrt{\frac{E}{m}}$ (D) $\sqrt{\frac{E}{2m}}$

13. A cricket ball is hit for a six leaving the bat at an angle of 45° to the horizontal with kinetic energy K. At the top position, the kinetic energy of the ball is

(A) Zero (B) K (C) $K/2$ (D) $K/\sqrt{2}$

14. A bullet losses 19% of its kinetic energy when passes through an obstacle. The percentage change in its speed is

(A) Reduced by 10% (B) Reduced by 19%
(C) Reduced by 9.5% (D) Reduced by 11%

15. Two springs A and B ($k_A = 2k_B$) are stretched by applying forces of equal magnitudes at the ends. If the energy stored in A is E, then energy stored in B is

(A) $\frac{E}{2}$ (B) $2E$ (C) E (D) $\frac{E}{4}$

16. A body constrained to move in y-direction is subjected to a force given by $\vec{F} = (-2\vec{i} + 15\vec{j} + 6\vec{k})N$. The work done by this force in moving the body a distance of 10 m along the y-axis is

(A) 20 J (B) 150 J (C) 60 J (D) 190 J

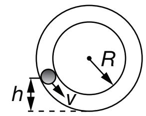
17. A particle of mass 2 kg starts moving in a straight line with an initial velocity of 2 m/s at a constant acceleration of 2 m/s^2. The rate of change of kinetic energy is

(A) Four times the velocity at any moment.
(B) Two times the displacement at any moment.
(C) Four times the rate of change of velocity at any moment.
(D) Constant throughout.

18. A block of mass m = 0.1 kg is released from a height of 4 m on a curved smooth surface. On the horizontal smooth surface, it collides with a spring of force constant 800 N/m. The maximum compression in spring will be (g = 10 m/s^2)

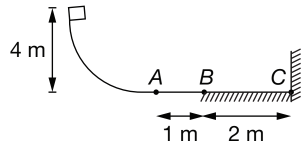
(A) 1 cm (B) 5 cm (C) 10 cm (D) 20 cm

19. With what minimum speed v must a small ball should be pushed inside a smooth vertical tube from a height h so that it may reach the top of the tube? Radius of the tube is R. (Assume radius of cross-section of tube is negligible in comparison to R.)



(A) $\sqrt{2g(h+2R)}$ (B) $\frac{5}{2}R$
(C) $\sqrt{g(5R-2h)}$ (D) $\sqrt{2g(2R-h)}$

20. A block of mass m = 0.1 kg is released from a height of 4 m on a curved smooth surface. On the horizontal surface, path AB is smooth and path BC offers coefficient of friction $\mu = 0.1$. If the impact of block with the vertical wall at C be perfectly elastic, the total distance covered by the block on the horizontal surface before coming to rest will be: (take g = 10 m/s^2)

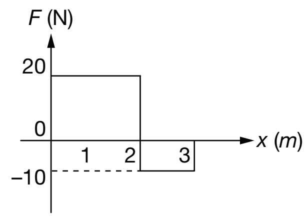


(A) 29 m (B) 49 m (C) 59 m (D) 109 m

21. An ideal spring with spring-constant k is hung from the ceiling and a block of mass m is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is

(A) $\frac{4mg}{k}$ (B) $\frac{2mg}{k}$ (C) $\frac{mg}{k}$ (D) $\frac{mg}{2k}$

22. A variable force F starts acting on a block of mass 5 kg resting on a smooth horizontal surface. F is varying with displacement x as shown in F–x curve. The velocity of body when its displacement is 3 m will be



(A) 2 ms^{-1} (B) $2\sqrt{2}$ ms^{-1}
(C) $2\sqrt{3}$ ms^{-1} (D) 6 ms^{-1}

23. When a body moves in a circle, the work done by the centripetal force is always

(A) > 0 (B) < 0
(C) Zero (D) None of these

24. A force acts on a 3 gram particle such that its position $x = 3t - 4t^2 + t^3$, where x is in metre and t is in second. The work done during first 4s is

(A) 825 mJ (B) 285 mJ
(C) 528 mJ (D) Zero

25. A particle is acted upon by a conservative force $F = (7\hat{i} - 6\hat{j})$ N. The work done by the force when the particle moves from origin (0, 0) to the position (–3 m, 4 m) is given by

(A) 3 J (B) 10 J
(C) –45 J (D) None of these

26. An object of mass 10 kg falls from rest through a vertical distance of 10 m and acquires a velocity of 10 m/s. The work done by the push of air on the object is (g = 10 m/s^2)

(A) 500 J (B) –500 J (C) 250 J (D) –250 J

27. The relationship between force and position is shown in Fig. 4.19 (in one-dimensional case). The work done by the force in displacing a body from x = 1 cm to x = 5 cm is

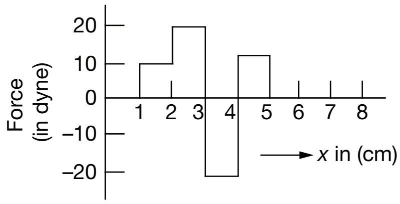
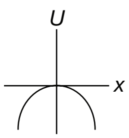
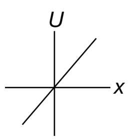


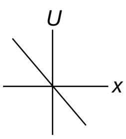
Fig. 4.19

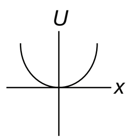
(A) 20 ergs (B) 60 ergs
(C) 70 ergs (D) 700 ergs

28. A particle is acted upon by a force $F = kx$, ($k > 0$), where x is displacement of particle. If potential energy at origin is zero, then the potential energy of the particle varies with x as

(A) 

(B) 

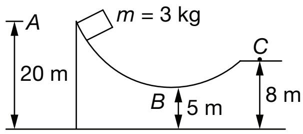
(C) 

(D) 

29. A position-dependent force $F = x^2 - 3$ Newton acts on a small body of mass 2 kg and displaces it from x = 0 to x = 5 m. The work done is

(A) 110 J (B) $\frac{80}{3}$ J
(D) $\frac{95}{2}$ J (D) Zero

30. A block of mass 3 kg slides down a rough curved path from point A as shown. If it stops at C, the work done by friction is (g = 10 ms^{-2})

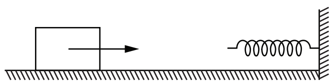


(A) –360 J (B) –240 J (C) –600 J (D) –450 J

31. A block of mass m is placed on an another rough block of mass M and both are moving horizontally with same acceleration a due to a force which is applied on the lower block, then work done by lower block on the upper block in moving a distance s will be

(A) Mas (B) $(m + M)as$
(C) $\frac{M^2}{m}as$ (D) mas

32. A 1 kg block moves towards a light spring with a velocity of 8 m/s. When the spring is compressed by 3 m, its momentum becomes half of the original momentum. Spring constant of the spring is

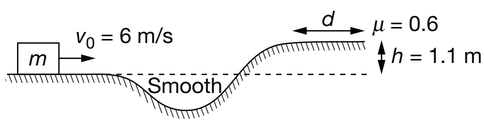


(A) 18/3 N/m (B) 16/3 N/m
(C) 3 N/m (D) 8 N/m

33. A block of mass 2 kg is held over a vertical spring with spring unstretched. Suddenly, if block is left free, maximum compression of spring is [spring constant K = 200 N/m]:

(A) 0.2 m (B) 0.1 m (C) 0.4 m (D) 0.05 m

34. A block m slides with a speed of v_0 = 6 m/s along a track from one level to a higher level as shown. The track is frictionless until the block reaches the higher level, where co-efficient of friction is 0.6. The distance d travelled by block on higher level before stopping is (g = 10 m/s^2)



(A) $\frac{7}{6}$ m (B) $\frac{5}{6}$ m (C) $\frac{29}{6}$ m (D) 3 m

35. In Fig. 4.20 shown here, pulley and spring are ideal. If k is spring constant of spring, the potential energy stored in it is ($m_1 > m_2$)

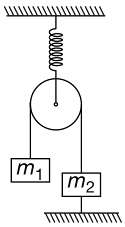


Fig. 4.20

(A) $\frac{2m_1^2g^2}{k}$ (B) $\frac{2m_2^2g^2}{k}$

(C) $\frac{(m_1+m_2)^2g^2}{k}$ (D) $\frac{1}{2}\frac{(m_1-m_2)^2g^2}{k}$

36. Potential energy (in joule) of a particle of mass 1 kg moving in x-y plane is $U = 3x + 4y$, here x and y are in meter. If at time $t = 0$, particle is at rest at point P(6 m, 4 m). Then

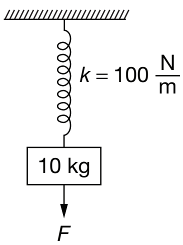
(A) acceleration of particle is $(3\hat{i}+4\hat{j})$ m/s.
(B) time when it crosses y-axis is $t = 1$ s.
(C) speed of particle when it crosses y-axis is 10 m/s.
(D) it crosses y-axis at $y = -8$ m.

37. A locomotive of mass m starts moving so that its velocity varies as $v = \alpha s^{2/3}$, where α is a constant and s is the distance traversed. The total work done by all the forces acting on the locomotive during the first t second after the start of motion is

(A) $\frac{1}{8}m\alpha^4t^2$ (B) $\frac{m\alpha^6t^4}{162}$

(C) $\frac{m\alpha^6t^4}{81}$ (D) $\frac{m\alpha^4t^2}{2}$

38. A vertical spring of force constant 100 N/m is attached with a hanging mass of 10 kg. Now an external force is applied on the mass so that the spring is stretched by additional 2 m. The work done by the force F is (g = 10 m/s^2)



(A) 200 J (B) 400 J (C) 450 J (D) 600 J

39. A body of mass 2 kg is moved from a point A to a point B by an external agent in a conservative force field. If the velocity of the body at the points A and B are 5 m/s and 3m/s, respectively, and the work done by the external agent is –10 J, then the change in potential energy between points A and B is

(A) 6 J (B) 36 J
(C) 16 J (D) None of these

40. A uniform chain has a mass m and length L. It is placed on a frictionless table with length l_0 hanging over the edge. The chain begins to slide down. The speed v with which the chain slides away from the edge is given by

(A) $v = \sqrt{\frac{gl_0}{L}(L+l_0)}$ (B) $v = \sqrt{\frac{gl_0}{L}(L-l_0)}$

(C) $v = \sqrt{\frac{g}{L}(L^2-l_0^2)}$ (D) $v = \sqrt{2g(L-l_0)}$

41. In the adjoining Fig. 4.21, block A is of mass m and block B is of mass 2 m. The spring has a force constant k. All the surfaces are smooth and the system is released from rest with spring unstretched, then

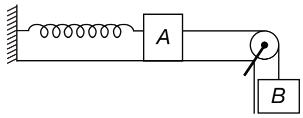


Fig. 4.21

(A) the maximum extension of the spring is $\frac{4mg}{k}$.

(B) the speed of block A when extension in spring is $\frac{2mg}{k}$, is $2g\sqrt{\frac{m}{k}}$.

(C) the net acceleration of block B when the extension in the spring is maximum, is $\frac{g}{2}$.

(D) tension in the thread for extension of $\frac{2mg}{k}$ in spring is mg.

42. A spring is compressed between two toy carts of masses m_1 and m_2. When the toy carts are released, the spring exerts on each toy cart equal and opposite forces for the same time t. If the coefficients of friction μ between the ground and the toy carts are equal, then the displacements of the toy carts are in the ratio

(A) $\frac{s_1}{s_2} = \frac{m_2}{m_1}$ (B) $\frac{s_1}{s_2} = \frac{m_1}{m_2}$

(C) $\frac{s_1}{s_2} = \left(\frac{m_2}{m_1}\right)^2$ (D) $\frac{s_1}{s_2} = \left(\frac{m_1}{m_2}\right)^2$

43. A body of mass m is dropped from a height h on a sand floor. If the body penetrates x m into the sand, the average resistance offered by the sand to the body is

(A) $mg\left(\frac{h}{x}\right)$ (B) $mg\left(1+\frac{h}{x}\right)$

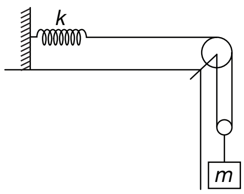
(C) $mgh+mgx$ (D) $mg\left(1-\frac{h}{x}\right)$

44. A block of mass m is pulled by a constant power P placed on a rough horizontal plane. The friction co-efficient between the block and surface varies with its speed v as $\mu=\frac{1}{\sqrt{1+v}}$. The acceleration of the block when its speed is 3 m/s will be

(A) $\frac{P}{3m}-\frac{g}{2}$ (B) $\frac{P}{3m}+\frac{g}{2}$

(C) $\frac{P}{3m}$ (D) $\frac{g}{2}$

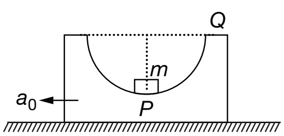
45. A block of mass m is released from rest when the extension in the spring is x_0. The maximum downward displacement of the block is



(A) $\frac{mg}{2k}-x_0$ (B) $\frac{mg}{2k}+x_0$

(C) $\frac{2mg}{k}-x_0$ (D) $\frac{2mg}{k}+x_0$

46. A small block of mass m lying at rest at point P of a wedge having a smooth semi-circular track of radius R. What should be the minimum value of horizontal acceleration a_0 of wedge so that mass can just reach the point Q?



(A) $g/2$ (B) $\sqrt{g}$

(C) g (D) Not possible

47. A bob is suspended from a crane by a cable of length $l = 5$ m. The crane and the bob are moving at a constant speed v_0. The crane is stopped by a bumper and the bob on the cable swings out an angle of 60°. The initial speed v_0 is ($g = 9.8$ m/s^2)

(A) 10 m/s (B) 7 m/s

(C) 4 m/s (D) 2 m/s

48. A particle is moving with kinetic energy E, straight up an inclined plane with angle α, the co-efficient of friction being μ. The work done against friction, up to when the particle comes to rest, is

(A) $\frac{E\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}$ (B) $\frac{E\cos\alpha}{\sin\alpha+\mu\cos\alpha}$

(C) $\frac{E}{\sin\alpha+\mu\cos\alpha}$ (D) $\frac{E}{g(\sin\alpha+\mu\cos\alpha)}$

49. A block of mass m is pulled by a constant power P placed on a rough horizontal plane. The friction co-efficient between the block and surface is μ. The maximum velocity of the block is

(A) $\frac{P}{mg}$ (B) $\frac{P}{\mu mg}$ (C) $\frac{\mu P}{mg}$ (D) Infinite

50. A ball of mass m is attached to a light string of length L and suspended vertically. A constant horizontal force, whose magnitude F equals the weight of the ball is applied. The speed of the ball as it reaches 90° level is,

(A) $\sqrt{gL}$ (B) $\sqrt{2gL}$ (C) $\sqrt{3gL}$ (D) Zero

51. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a position. In the same time t, the work done on the two moving charged particles is

(A) Same as the force law is involved in the two experiments.
(B) Less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.
(C) More in the case of positron, as the positron moves away a larger distance.
(D) Same as the work done by charged particle on the stationary proton.

52. A man squatting on the ground gets straight up and stands. The force of reaction of ground on the man during the process is

(A) Constant and equal to mg in magnitude.
(B) Constant is greater than mg in magnitude.
(C) Variable but always greater than mg.
(D) At first greater than mg and later becomes equal to mg.

53. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200 N and is directly opposed to the motion. The work done by the cycle on the road is

(A) +2000 J (B) −200 J
(C) Zero (D) −20,000 J

54. A body is falling freely under the action of gravity alone in vaccum. Which of the following quantities remain constant during the fall?
(A) Kinetic energy
(B) Potential energy
(C) Total mechanical energy
(D) Total linear momentum

55. During inelastic collision between two bodies, which of the following quantities always remain conserved?
(A) Total kinetic energy
(B) Total mechanical energy
(C) Total linear momentum
(D) Speed of each body

56. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to
(A) $t^{3/4}$ (B) $t^{3/2}$ (C) $t^{1/4}$ (D) $t^{1/2}$

57. Two springs have force constants, K_1 and $K_2 (K_1 > K_2)$ The work done, when both are stretched by the same amount of length will be
(A) Equal
(B) Greater for K_1
(C) Greater for K_2
(D) Given data is incomplete

58. Choose the incorrect statement.
(A) No work is done on moving a block uniformly on a smooth horizontal table.
(B) Work done by earth's gravitational force on moon is zero, considering moon's orbit to be circular.
(C) No work is done by weight lifter holding a 175 kg mass steadily on his shoulder for 30 s.
(D) Work done by frictional force is always negative.

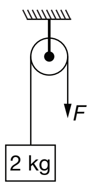
59. A bob is suspended from a crane by a cable of length $l = 5$ m. The crane and the bob are moving at a constant speed v_0. The crane is stopped by a bumper and the bob on the cable swings out an angle of 60°. The initial speed v_0 is ($g = 9.8$ m/s^2)
(A) 10 m/s (B) 7 m/s (C) 4 m/s (D) 2 m/s

60. When a body moves in a circle, the work done by the centripetal force is always
(A) >0 (B) <0
(C) Zero (D) None of these

More than One Option Correct Type

61. A ball of mass m is attached to the lower end of a light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length and comes to rest again after descending through a distance x.
(A) $x = mg/k$
(B) $x = 2mg/k$
(C) The ball will have no acceleration at the position where it was descending through $x/2$.
(D) The ball will have an upward acceleration equal to g at its lowermost position.

62. A block of mass 2 kg is hanging over a smooth and light pulley through a light string. The other end of string is pulled by a constant force F. The kinetic energy of block increases by 16 J in 2s, then



(A) force F may be 24 N.
(B) force F must be 24 N.
(C) potential energy must be increase.
(D) potential energy may be increase.

63. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
(A) its velocity is constant.
(B) its acceleration is constant.
(B) its kinetic energy is constant.
(D) it moves in a circular path.

64. A particle of mass 5 kg moving in the X-Y plane has its potential energy given by $U = (-7x + 24y)$ Joule. The particle is initially at origin and has velocity $\vec{u} = (14.4\hat{i} + 4.2\hat{j})$ m/s
(A) The particle has speed 25 m/s at $t = 4$ s.
(B) The particle has an acceleration 5 m/s^2.
(C) The acceleration of particle is normal at its initial velocity.
(D) None of these.

65. Figure 4.22 shows a massless spring fixed at the bottom end of an inclined of inclination 37° (tan 37° = 3/4). A small block of mass 2 kg start slipping down the incline from a point 4.8 m away from free end of spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance 1 m up the inclined, then (g = 10 m/s^2).

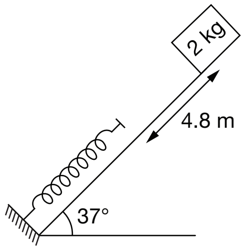


Fig. 4.22

(A) Coefficient of friction between block and inclined is 0.5.
(B) Coefficient of friction between block and inclined is 0.75.
(C) Value of spring constant is 1000 N/m.
(D) Value of spring constant is 2000 N/m.

66. A particle of mass m is attached to a light string of length l, the other end of which is fixed. Initially, the string is kept horizontal and the particle is given an upward velocity v. The particle is just able to complete a circle.

(A) The string becomes slack when the particle reaches its highest point.
(B) The velocity of the particle becomes zero at the highest point.
(C) The kinetic energy of the ball in initial position was $\frac{1}{2}mv^2 = mgl$.
(D) The particle again passes through the initial position.

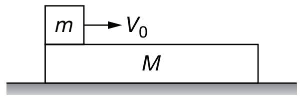
67. A particle of mass 5 kg moving in the X-Y plane has its potential energy given by $U = (-7x + 24y)$ Joule. The particle is initially at origin and has velocity $\vec{u} = (14.4\hat{i} + 4.2\hat{j})$ m/s.

(A) The particle has speed 25 m/s at $t = 4$ s.
(B) The particle has an acceleration $5\ \text{m/s}^2$.
(C) The acceleration of particle is normal to its initial velocity.
(D) None of these.

68. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are:

(A) The spring was initially compressed by a distance x and was finally in its natural length.
(B) It was initially stretched by a distance x and finally was in its natural length.
(C) It was initially in its natural length and finally in a compressed position.
(D) It was initially in its natural length and finally in a stretched position.

69. The co-efficient of friction between the block and plank is μ and its value is such that block becomes stationary with respect to plank before it reaches the other end. Then



(A) the work done by friction on the block is negative.
(B) the work done by friction on the plank is positive.
(C) the net work done by friction is negative.
(D) net work done by the friction is zero.

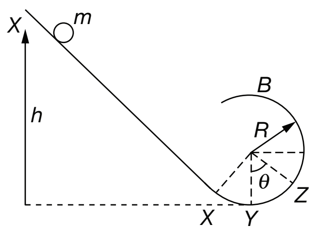
70. Potential energy associated with a conservative force is given by $U = Ax^2$, where A is a constant then

(A) force always tends to accelerate the particle towards origin.
(B) force always tends to accelerate the particle away from origin.
(C) force always tends to accelerate the particle towards the origin if A is positive.
(D) force always tends to accelerate the particle away from origin if A is negative.

Passage Based Questions

Passage 1

A ball of mass m and radius r is allowed to roll on rough inclined plane, which is joined by a circular track of radius R. Friction throughout the track is sufficient to support rolling of the sphere. If h is height of centre of mass of the ball from the bottom of the track.



71. The speed of the ball when it is at an angular position of θ with respect to vertical

(A) $\sqrt{\frac{10g}{7}[h-r+R)+(R-r)\cos\theta}$

(B) $\sqrt{\frac{10g}{7}[(h-R)+(R-r)\cos\theta}$

(C) $\sqrt{\frac{5g}{7}[h-(r+R)+(R-r\cos\theta)}$

(D) $\sqrt{\frac{5g}{7}[h-(R+r)(1-\cos\theta)}$

72. Frictional force acting on the ball as it passes through point *Y* is

(A) Change its direction from left to right.
(B) Changes its direction from right to left.
(C) Does not change its direction.
(D) Not zero at point *Y*.

73. The friction force plays a role in energy transformation as

(A) It converts the gravitational potential energy into rotational KE only.
(B) It converts the gravitational potential energy into translational KE only.
(C) It has no role in energy transformation.
(D) None of these.

Passage 2

A person trying to lose weight, lifts a 10 kg mass 0.5 m, 1000 times daily. Fat supplies 4×10^7 J of energy per kilogram which is converted into potential energy to raise the weight with 20% efficiency rate. The potential energy lost each time the person lowers the mass is dissipated ($g = 10$ m/s^2).

74. How much work does the person do against the gravitational force daily?

(A) 25 kJ (B) 50 kJ (C) 10 kJ (D) 75 kJ

75. How much energy does fat supply each day?

(A) 5×10^4 J (B) 2.5×10^5 J
(C) 8×10^6 J (D) 4×10^6 J

76. How much fat will the person use up in 10 days?

(A) 6.25×10^{-2} kg (B) 12.5×10^{-2} kg
(C) 25×10^{-2} kg (D) 3.125×10^{-2} kg

Passage 3

A train starts from rest at $t = 0$ along a straight track with a constant acceleration of 5 m/s^2. A passenger at rest in train observes a particle of mass 1 kg on the floor with which it has a co-efficient of friction $\mu_s = \mu_k = 0.6$. At $t = 4$ *s*, a horizontal force $F = 13$ N is applied on the particle for 2 s duration. The passenger observes that the particle is now moving in a perpendicular direction of motion of the train. ($g = 10$ m/s^2)

77. The direction in which force ($F = 13$ N) is applied is at

(A) 90° with the direction of motion of the train.
(B) $\cos^{-1}\frac{5}{13}$ with the direction of motion of the train.
(C) $\cos^{-1}\frac{5}{12}$ with the direction of motion of the train.
(D) $\cos^{-1}\frac{12}{13}$ with the direction of motion of the train.

78. The magnitude of acceleration of the particle with respect to the ground at $t = 5$ s is

(A) $\sqrt{61}$ m/s^2 (B) $\sqrt{72}$ m/s^2
(C) $\sqrt{8}$ m/s^2 (D) 6 m/s^2

79. The momentum of the particle at $t = 6$ s with respect to the train is

(A) 12 kg-m/s (B) 10 kg-m/s
(C) 6 kg-m/s (D) 8 kg-m/s

80. The kinetic energy of the particle at $t = 20$ s with respect to the ground is

(A) 5×10^3 J (B) 6×10^3 J
(C) 8×10^3 J (D) 7×10^3 J

Passage 4

We generally ignore the kinetic energy of the moving coil of a spring but consider a spring of mass *m,* equilibrium length *L* and spring constant *k*. Consider a spring, as described above, that has one end fixed and the other end moving with speed *v*. Assume that the speed of points along the length of the spring varies linearly with distance *L* from the fixed end. Assume also that the mass *m* of the spring is distributed uniformly along the length of the spring. Assume further that the force applied by the spring is spring constant times its deformation. In a spring gun, such a spring of mass 0.243 kg and force constant 3200 N/m is compressed 2.50 cm from its unstretched length. When the trigger is pulled, the spring pushes horizontally the ball of mass of 0.053 kg.

81. The speed of small length (dx) at a distance x from fixed end is

(A) $\frac{x}{L}v$ (B) v (C) $\frac{L}{x}v$ (D) xv

82. Kinetic energy of the spring

(A) $\frac{1}{2}mv^2$ (B) $\frac{1}{6}mv^2$ (C) mv^2 (D) $\frac{1}{4}mv^2$

83. Ball's speed when the spring reaches its uncompressed length is

(A) 3.9 m/s (B) 6.1 m/s
(C) 14 m/s (D) 1.62 m/s

Passage 5

A cutting tool under microprocessor control has several forces acting on it. One such force is $\vec{F} = -\alpha xy^2 \hat{j}$, a force in the negative y-direction whose magnitude depends on the position of the tool. The constant α is 2.50 N. Consider the displacement of the tool from the origin to the point $x = 3.00$ m, $y = 3.00$ m.

84. Calculate the magnitude of work done on the tool by $\vec{F}$. If this displacement is along the straight line $y = x$ that connects these two points.

(A) 2.50 J (B) 500 J (C) 50.6 J (D) 2 J

85. Calculate the work done on the tool by $\vec{F}$ of the tool is first moved out along the x-axis to the point $x = 3.00$ m, $y = 0$ and then moved parallel to the y-axis to $x = 3.00$ m, $y = 3.00$ m

(A) 67.5 J (B) 85 J
(C) 102 J (D) 7.5 J

86. What can you predict about $\vec{F}$?

(A) Force is non-conservative.
(B) Force is conservative.
(C) Force is neither conservative nor non-conservative.
(D) Data insufficient to conclude.

Match the Column Type

87. The displacement-time (x–t) graph of a body acted upon by some forces is shown in Fig. 4.23.

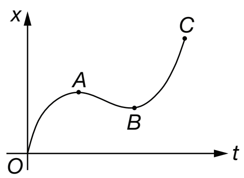
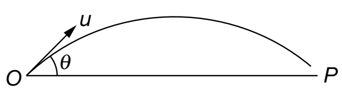


Fig. 4.23

Column-I	Column-II
(A) For OA, the total work done by all the forces together is	(1) Positive
(B) For AB acceleration is	(2) Negative
(C) From O to B velocity is	(3) First positive, then negative
(D) At B acceleration is	(4) First negative, then positive
	(5) Zero

88. A projectile is launched at angle θ to the horizontal from O and it hits the target P on level ground.



Column-I	Column-II
(A) Magnitude of radial acceleration	(1) Increases
(B) Magnitude of tangential acceleration	(2) Can be negative
(C) Power delivered by gravity	(3) First increases, then decrease
(D) Rate of change of speed of projectile with respect to time	(4) First decreases, then increases

89. In the following columns: Column-I some types of potential energies are given and in Column-II some possible values of these potential energies are given. Match the following:

Column-I	Column-II
(A) Electrostatic potential energy	(1) Positive
(B) Gravitational potential energy	(2) Negative
(C) Elastic potential energy	(3) Zero
(D) Magnetic potential energy	(4) Not defined

90. In Fig. 4.24, $m_1 = 8$ kg, $m_2 = 16$ kg, $K = 100$ N/m, $\mu = 0.2$

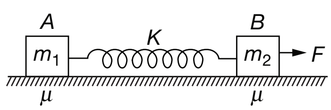


Fig. 4.24

Column-I	Column-II
(A) The minimum value of F (in N) in order to shift the block of mass m_1 is	(1) 12
(B) Negative of work done by friction (in J) on block B till this moment is	(2) 40
(C) Work done by F till this moment	(3) Zero
(D) The minimum value of F in order to shift the block of mass m_2 if it is applied on A.	(4) 32
	(5) 6.4

91. A block of mass 2 kg is released from rest on a smooth inclined plane of inclination 30° and connected by a massless spring of force constant 1000 N/m as shown in Fig. 4.25. Initially, the spring is in its natural length. An external variable force also acts on the block down the inclined plane. Block comes to rest for a moment after travelling a distance of 20 cm along the inclined plane. From initial to this moment, Column-I gives work done by various forces and Column-II gives their values.

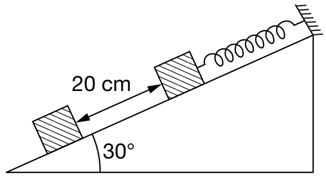


Fig. 4.25

Column-I	Column-II
(A) Work done by gravity	(1) Zero
(B) Work done by spring	(2) 18 J
(C) Work done by external force	(3) –20 J
(D) Work done by normal force	(4) 2 J
	(5) 20 J

Assertion-Reason Type

92. **Assertion:** Work done by a force in a certain interval of time may not depend on initial velocity.

Reason: Work done by a force is frame-dependent.

(A) A (B) B (C) C (D) D

93. **Assertion:** Friction force is a non-conservative force.

Reason: When a body is moved on a rough surface in a closed path, the work done by friction force is zero.

(A) A (B) B (C) C (D) D

94. **Assertion:** Static frictional force may be greater than kinetic frictional force.

Reason: Static frictional force is always equal to $\mu_s N$ (N = normal reaction)

(A) A (B) B (C) C (D) D

95. **Assertion:** For stable equilibrium, force has to be zero and potential energy should be minimum.

Reason: For equilibrium, it is not necessary that force is not zero.

(A) A (B) B (C) C (D) D

96. **Assertion:** Work done by frictional force on a sphere rolling without slipping on an inclined plane is negative.

Reason: Work done by the force F, $W = \int \vec{F} \cdot d\vec{S}$

(A) A (B) B (C) C (D) D

97. **Assertion:** Work done by friction force may be positive.

Reason: Force of friction always opposes relative motion.

(A) A (B) B (C) C (D) D

98. **Assertion:** Work done by spring force is always negative.

Reason: In compression or stretching of a spring from its natural length, work is done on the spring against the restoring force.

(A) A (B) B (C) C (D) D

99. **Assertion:** When force retards the motion of a body, the work done is zero.

Reason: Work done depends on angle between force and displacement.

(A) A (B) B (C) C (D) D

100. **Assertion:** Total mechanical energy of the system can never be greater than potential energy.

Reason: If non-conservative forces perform negative work, then mechanical energy of the system decreases.

(A) A (B) B (C) C (D) D

101. Assertion: A body at rest can possess mechanical energy.

Reason: A body at rest cannot possess kinetic energy with respect to an inertial frame.

(A) A (B) B (C) C (D) D

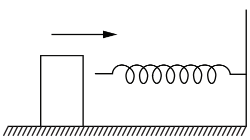
Integer Type

102. A mass $m = 1$ kg moving horizontally with velocity $v_0 = 2$ m/s collides in elastically with a pendulum of same mass. Find the maximum change in potential energy (in Joule) of combined mass.

103. A block of mass 0.5 kg is kept in an elevator moving down with an acceleration 2 m/s^2. Find the magnitude work done (in Joule) by the normal contact force on the block in first second. Initially system is at rest ($g = 10$ m/s^2).

104. State principle of conservation of mechanical energy. A block of mass 2 kg moving with speed 2 m/s compresses a spring through a distance 20 cm before its speed is halved. The value of spring constant is 75 N then the value of N is ?

105. A 1 kg block collides with a horizontal light spring of force constant 2 N/m. The maximum compression in the spring is 4 m. Assuming that co-efficient of kinetic friction between the block and the horizontal surface is 0.25, what is initial speed of the block (approx.)?



106. A truck of mass 2000 kg has a velocity of 8 ms^{-1} when it starts from a point A to descend a slope AB, 200 m long shown in the Fig. 4.26. The truck arrives at B which is 18 m below the level of A with a velocity of 20 ms^{-1}. The resistance in Newton offered is $40x$, find the value of x? (given $g = 10$ ms^{-1})

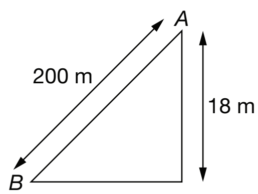
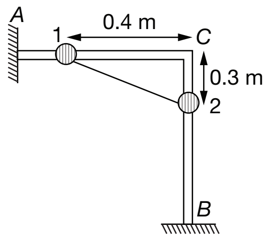


Fig. 4.26

107. Two identical beads of $m = 100$ g are connected by an inextensible massless string that can slide along the two arms AC and BC of a rigid smooth wireframe in a vertical plane. If the system is released from rest, the kinetic energy of the first particle when they have moved by a distance of 0.1 m is $16x \times 10^{-3}$ J. Find the value of x. ($g = 10$ m/s^2)



108. Two blocks A and B of equal mass $m = 1$ kg are lying on a smooth horizontal surface as shown in Fig. 4.27. A spring of force constant $K = 200$ N/m is fixed at one end of block A. Block B collides with another end of the spring with velocity $v_0 = 2$ m/s. What will be the maximum compression of the spring? [in decimeter]

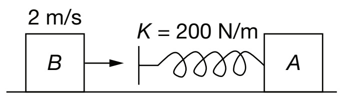


Fig. 4.27

109. One end of an unstretched springs of force constant k_1 is attached to the ceiling of an elevator. A block of mass 1.5 kg is attached to other end. Another spring of force constant k_2 is attached to the bottom of the mass and to the floor of the elevator as shown in Fig. 4.28. At equilibrium, the deformation in both the spring are equal and is 40 cm. If the elevator moves with constant acceleration upward, the additional deformation in both the springs have 8 cm. Find the elevator's acceleration ($g = 10$ ms^{-2})

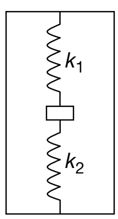
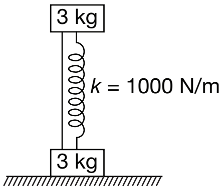


Fig. 4.28

110. A system consists of two identical cubes, each of mass 3 kg, linked together by a compressed weightless spring of force constant 1000 N/m. The cubes are also connected by a thread which is burnt at a certain moment. At what minimum value of initial compression, x_0 (in cm) of the spring will the lower cube bounce up after the thread is burnt through?



111. A homogeneous rope of mass per unit length λ and length l kept on ground and one end of the rope is fixed to ground at O. The left end of the rope (with respect to fixed end) is pulled by an external agent which imparts constant velocity v to it. Find the work done by the external agent (in joule) to place the moving end extremely right with respect to fixed end. Take $\lambda = 1$ kg/m, $v = 1$ ms^{-1} and $l = 1$ m.

Previous Years' Questions

112. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to **[2003]**

(A) $t^{3/4}$ (B) $t^{3/2}$ (C) $t^{1/4}$ (D) $t^{1/2}$

113. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is **[2003]**

(A) 12.50 N/m (B) 18.75 N/m
(C) 25.00 N/m (D) 6.25 N/m

114. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is **[2003]**

(A) 0.2 J (B) 10 J (C) 20 J (D) 0.1 J

115. Consider the following two statements: **[2003]**

A: Linear momentum of a system of particles is zero
B: Kinetic energy of a system of particles is zero. Then

(A) **A** does not imply **B** and **B** does not imply **A**
(B) **A** implies **B** but **B** does not imply **A**
(C) **A** does not imply **B** but **B** implies **A**
(D) **A** implies **B** and **B** implies **A**

116. A force $\vec{F} = \left(5\hat{i} + 3\hat{j} + 2\hat{k}\right)$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = \left(2\hat{i} - \hat{j}\right)m$. The work done on the particle in joules is **[2004]**

(A) + 10 (B) +7 (C) –7 (D) + 13

117. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? **[2004]**

(A) 12 J (B) 3.6 J (C) 7.2 J (D) 1200 J

118. A particle moves in a straight line with retardation proportion to its displacement. Its loss of kinetic energy for any displacement x is proportional to **[2004]**

(A) x (B) e^x (C) x^2 (D) $\log_e x$

119. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle; the motion of the particle takes place in a plane. It follows that **[2004]**

(A) Its kinetic energy is constant.
(B) Its acceleration is constant.
(C) Its velocity is constant.
(D) It moves in a straight line.

120. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is **[2004]**

(A) $\dfrac{mv_1t^2}{t_1}$ (B) $\dfrac{mv_1^2t}{t_1^2}$

(C) $\dfrac{mv_1t}{t_1}$ (D) $\dfrac{mv_1^2t}{t_1}$

121. A body of mass m is accelerated uniformly from rest to a speed v in a time t. The instantaneous power delivered to the body as a function of time is given by **[2005]**

(A) $\dfrac{mv^2}{t^2}t^2$ (B) $\dfrac{mv^2}{t^2}t$

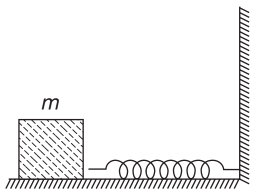
(C) $\dfrac{1}{2}\dfrac{mv^2}{t^2}t^2$ (D) $\dfrac{1}{2}\dfrac{mv^2}{t^2}t$

122. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is **[2005]**

(A) 20 m/s (B) 40 m/s

(C) $10\sqrt{30}$ m/s (D) 10 m/s

123. The block of mass *m* moving on the frictionless horizontal surface collides with the spring of spring constant *k* and compresses it by length *L*. The maximum momentum of the block after collision is **[2005]**



(A) $\frac{kL^2}{2m}$ (B) $\sqrt{mk}L$

(C) $\frac{mL^2}{k}$ (D) Zero

124. The potential energy of a 1 kg particle free to move along the *x*-axis is given by $V(x)=\left(\frac{x^4}{4}-\frac{x^2}{2}\right)$ J. The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is **[2006]**

(A) $\frac{3}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 2

125. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is **[2006]**

(A) –0.5 J (B) –1.25 J

(C) 1.25 J (D) 0.5 J

126. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by **[2007]**

(A) 8.5 cm (B) 5.5 cm

(C) 2.5 cm (D) 11.0 cm

127. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[2008]**

(A) 0. 16 J (B) 1.00 J

(C) 0.67 J (D) 0.34 J

128. An athlete in the Olympic Games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[2008]**

(A) 200 J – 500 J

(B) $2\times10^5\text{ J}-3\times10^5\text{ J}$

(C) $20{,}000\text{ J}-50{,}000\text{ J}$

(D) $2{,}000\text{ J}-5{,}000\text{ J}$

129. The potential energy function for the force between two atoms in a diatomic molecule is approximately given as $U(x)=\frac{a}{x^{12}}-\frac{b}{x^6}$, where *a* and *b* are constants and *x* is the distance between the atoms. If the dissociation energy of the molecule is $D=\left[U(x=\infty)-U_{\text{at equilibrium}}\right]$, *D* is **[2010]**

(A) $\frac{b^2}{2a}$ (B) $\frac{b^2}{12a}$

(C) $\frac{b^2}{4a}$ (D) $\frac{b^2}{6a}$

130. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1: If stretched by the same amount, work done on S_1 will be more than that on S_2

Statement 2: $k_1 < k_2$ **[2012]**

(A) Statement 1 is false, Statement 2 is true.

(B) Statement 1 is true, Statement 2 is false

(C) Statement 1 is true, Statement 2 is true and Statement 2 is the correct explanation for Statement 1

(D) Statement 1 is true, Statement 2 is true and Statement 2 is **not** the correct explanation of Statement 1

131. When a rubber band is stretched by a distance *x*, it exerts a restoring force of magnitude $F = ax + bx^2$, where *a* and *b* are constants. The work done in stretching the unstretched rubber band by *L* is: **[2014]**

(A) aL^2+bL^3 (B) $\frac{1}{2}(aL^2+bL^3)$

(C) $\frac{aL^2}{2}+\frac{bL^3}{3}$ (D) $\frac{1}{2}\left(\frac{2L^2}{2}+\frac{bL^2}{3}\right)$

132. A person trying to lose weight by burning fat lifts a mass of 10 kg up to a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg, which is converted into mechanical energy with a 20% efficiency rate. Take $g = 9.8\,\text{ms}^{-2}$: **[2016]**

(A) 6.45×10^{-3} kg (B) 9.89×10^{-3} kg

(C) 12.89×10^{-3} kg (D) 2.45×10^{-3} kg

133. A point particle of mass *m* moves along the uniformly rough track PQR as shown in the Fig. 4.29. The coefficient of friction, between the particle and the rough track equals. The particle is released, from rest, from the point *P* and it comes to rest at a point *R*. The energies, lost by the ball, over the parts, *PQ* and *QR*, of the track, are equal to each other, and no energy is lost when particle changes direction from *PQ* to *QR*.

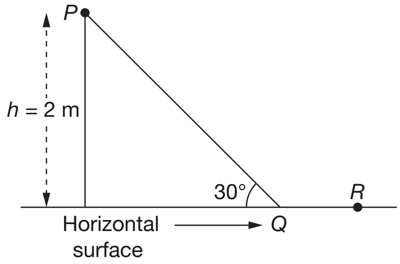


Fig. 4.29

The values of the coefficient of friction and the distance *x* (= *QR*), are, respectively close to: **[2016]**

(A) 0.2 and 3.5 m (B) 0.29 and 3.5 m
(C) 0.29 and 6.5 m (D) 0.2 and 6.5 m

ANSWER KEYS

Single Option Correct Type

1. (B) **2.** (A) **3.** (C) **4.** (C) **5.** (A) **6.** (A) **7.** (A) **8.** (C) **9.** (D) **10.** (A)
11. (A) **12.** (B) **13.** (C) **14.** (A) **15.** (B) **16.** (B) **17.** (A) **18.** (C) **19.** (D) **20.** (C)
21. (B) **22.** (C) **23.** (C) **24.** (C) **25.** (C) **26.** (B) **27.** (A) **28.** (A) **29.** (B) **30.** (A)
31. (D) **32.** (B) **33.** (A) **34.** (A) **35.** (A) **36.** (C) **37.** (B) **38.** (A) **39.** (A) **40.** (C)
41. (A) **42.** (C) **43.** (B) **44.** (A) **45.** (A) **46.** (C) **47.** (B) **48.** (A) **49.** (B) **50.** (D)
51. (C) **52.** (D) **53.** (C) **54.** (C) **55.** (C) **56.** (B) **57.** (B) **58.** (D) **59.** (B) **60.** (C)

More than One Option Correct Type

61. (B), (C) and (D) **62.** (A) and (D) **63.** (B) and (D) **64.** (A), (B) and (C) **65.** (A) and (C)
66. (A) and (D) **67.** (A), (B) and (C) **68.** (A) and (B) **69.** (A), (B) and (C) **70.** (C) and (D)

Passage Based Questions

Passage 1

71. (C) **72.** (A) **73.** (A)

Passage 2

74. (B) **75.** (A) **76.** (A)

Passage 3

77. (B) **78.** (A) **79.** (A) **80.** (A)

Passage 4

81. (A) **82.** (B) **83.** (B)

Passage 5

84. (C) **85.** (A) **86.** (A)

Match the Column Type

87. (A) → 2; (B) → 4; (C) → 3; (D) → 1
88. (A) → 3; (B) → 4; (C) → 1, 2; (D) → 1, 2
89. (A) → (1, 2, 3); (B) → (2, 3); (C) → (1, 3); (D) → (1, 2, 3)
90. (A) → 2; (B) → 1; (C) → 5; (D) → 4
91. (A) → 4, (B) → 3, (C) → 2, (D) → 1

Assertion-Reason Type

92. (B) **93.** (C) **94.** (C) **95.** (C) **96.** (D) **97.** (B) **98.** (D) **99.** (D) **100.** (B) **101.** (C)

Integer Type

102. 1 J **103.** –4 J **104.** 2 **105.** 7 m/s **106.** 3
107. 4 **108.** 1 **109.** 2 **110.** 9 cm **111.** 1 J

Previous Years' Questions

112. (B) **113.** (B) **114.** (D) **115.** (C) **116.** (B) **117.** (B) **118.** (C) **119.** (A) **120.** (B) **121.** (B)
122. (B) **123.** (B) **124.** (A) **125.** (B) **126.** (B) **127.** (C) **128.** (D) **129.** (C) **130.** (A) **131.** (C)
132. (C) **133.** (B)

HINTS AND SOLUTIONS

Single Option Correct Type

1. Work done is positive when slope of the graph increases.
The correct option is (B)

2. $\frac{1}{2}(100)\left(\frac{5}{100}\right)^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{5}{2}}$;

$x = v\sqrt{\frac{2h}{g}} = 1\text{ m}$

The correct option is (A)

3. The correct option is (C)

4. $h_1 = \frac{u^2}{2g}, h_2 = \frac{(u\sin 30°)^2}{2g} = \frac{u^2}{8g}$

$h_1 : h_2 = 4:1$

The correct option is (C)

5. The correct option is (A)

6. Work done $= -mgH_{\max} = -mg\frac{u^2\sin^2\theta}{2g} = \frac{-mu^2\sin^2\theta}{2}$

The correct option is (A)

7. Work done by normal = 0
The correct option is (A)

8. Net work done = change in kinetic energy

$$W - mgh = \frac{1}{2}\times mv^2$$

$$W = 2\times 10\times 2 + \frac{1}{2}\times 2\times 4^2 = 56\text{ J}$$

The correct option is (C)

9. $w = \int_0^5 F\cdot dx = \int_0^5 \left(7-2x+3x^2\right)dx = 135\text{ J}$

The correct option is (D)

10. Work done by road on the car = –1000 × 15 J = – 15 kJ
Work done by car on the road = 0
(∵ no displacement of road)
The correct option is (A)

11. $F = -\frac{\delta U}{\delta x} = -2x+3$

$W = \int_0^2 F\,dx = \int_0^2 (-2x+3)\,dx = [-x^2+3x]_0^2 = 2\text{ J}$

The correct option is (A)

12. Potential energy of particle at $x = \sqrt{\frac{2E}{k}}$ is zero. ($x > 0$)

∴ KE = E

or $\frac{1}{2}mv^2 = E$ or $v = \sqrt{\frac{2E}{m}}$

The correct option is (B)

13. $K' = \frac{1}{2}mu^2\cos^2\theta$

$= \frac{K}{2}$

The correct option is (C)

14. $k = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2k}{m}}$

$v' = \sqrt{\frac{2k'}{m}} = \sqrt{\frac{2\times 0.81k}{m}} = 0.9\,v$

∴ Percentage decrease in speed = 10%
The correct option is (A)

15. $F = k_A x_A = k_B x_B \Rightarrow \frac{x_A}{x_B} = \frac{k_B}{k_A} = \frac{1}{2}$

$$\frac{E_A}{E_B} = \frac{k_A x_A^2}{k_B x_B^2} = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$\Rightarrow \quad E_B = 2E_A = 2E$

The correct option is (B)

16. $\vec{F} = -2\hat{i} + 15\hat{j} + 6\hat{k}$

$\vec{s} = 10\hat{j}$

$W = \vec{F} \cdot \vec{s} = 150$ J

The correct option is (B)

17. Rate of change of kinetic energy = Fv (For constant acceleration)

$= mav = 4v$

The correct option is (A)

18. Using work energy theorem,

$$mgh = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{2mgh}{k}} = 0.1 \text{ m} = 10 \text{ cm}$$

The correct option is (C)

19. For minimum v, velocity of ball at the topmost point will be zero.

By conservation of energy,

$$\frac{1}{2}mv^2 = mg(2R - h)$$

$$v = \sqrt{2g(2R - h)}$$

The correct option is (D)

20. Using work energy theorem,

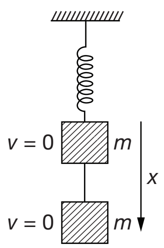
$\mu mgs = mgh$

$\Rightarrow \quad s = 40$ m (where s is the total distance travelled on the rough surface)

The correct option is (C)

21. Let x be the maximum extension of the spring. From conservation of mechanical energy decrease in gravitational potential energy = increase in elastic potential energy

$$\therefore \quad Mgx = \frac{1}{2}kx^2 \quad \text{or} \quad x = \frac{2Mg}{k}$$



The correct option is (B)

22. W = area of $F - x$ curve $= 20 \times 2 - 10 \times 1 = 30$ J

By work-energy theorem $W_{\text{net}} = \Delta \text{KE}$

$$\therefore \quad 30 = \frac{1}{2} \times 5 \times v^2, \; v = 2\sqrt{3} \text{ ms}^{-1}$$

The correct option is (C)

23. The correct option is (C)

24. $x = 3t - 4t^2 + t^3, \; \frac{dx}{dt} = 3 - 8t + 3t^2, \; v(t = 0) = 3$ m/s,

$v(t = 4) = 19$ m/s

$$w_{\text{ext}} = \Delta\text{KE} = k_f - k_i = \frac{1}{2} \times 3 \times 10^{-3}\left(19^2 - 3^2\right) = 528 \text{ mJ}$$

The correct option is (C)

25. $\vec{r} = -3\hat{i} + 4\hat{j}, \; w = \vec{F} \cdot \vec{r} = -21 - 24 = -45$ J

The correct option is (C)

26. By work energy theorem,

$$\frac{1}{2} \times 10\left(10^2 - 0\right) = 10 \times 10 \times 10 + W_{\text{air}}, \; W_{\text{air}} = -500 \text{ J}$$

The correct option is (B)

27. Work done = (10 + 20 + 10) – 20 = 20 ergs

The correct option is (A)

28. $F = kx = -\frac{dU}{dx} \Rightarrow U = -\frac{1}{2}kx^2$

The correct option is (A)

29. $$W = \int_{x_1}^{x_2} F\,dx = \int_0^5 (x^2 - 3)dx = \left[\frac{x^3}{3} - 3x\right]_0^5$$

$$= \frac{125}{3} - 15 = \frac{80}{3} \text{J}$$

The correct option is (B)

30. The correct option is (A)

31. Force on the upper block = ma

Work done = F.S = mas

The correct option is (D)

32. $$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}kx^2, \; P_1^2 - P_2^2 = mkx^2, \; \frac{3}{4}P_1^2 = mkx^2$$

$$\frac{3}{4}m^2v^2 = mkx^2, \; k = \frac{3}{4} \times 1 \times \frac{64}{9} = \frac{16}{3} \text{ N/m}$$

The correct option is (B)

33. Let block stops after compression x of spring. Using work-energy theorem

$$mgx = \frac{1}{2}Kx^2$$

$$x = \frac{2mg}{K} = \frac{2 \times 2 \times 10}{200} = \frac{4}{20} = \frac{2}{10} = 0.2 \text{ m}$$

The correct option is (A)

34. By work-energy theorem, $W_{mg} + W_{\text{friction}} = k_f - k_i$

$$-mgh - \mu mgd = 0 - \frac{1}{2}mv_0^2,$$

$$-10\times1.1-0.6\times10\times d=-\frac{1}{2}\times6^2$$

$$11+6d=18,\ d=\frac{7}{6}\text{ m}$$

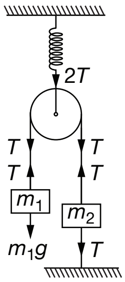
The correct option is (A)

35. $T=m_1g$

$kx=2T=2m_1g$

$$x=\frac{2m_1g}{k}$$

$$\text{Energy stored}=\frac{1}{2}kx^2=\frac{1}{2}k\frac{4m_1^2g^2}{k^2}=\frac{2m_1^2g^2}{k}$$



The correct option is (A)

36. $F_x=-\frac{\partial U}{\partial x}=-3,\ F_y=-\frac{\partial U}{\partial y}=-4,\ \vec{F}=-3\hat{i}-4\hat{j},$

$\vec{a}=-3\hat{i}-4\hat{j}$

For motion along x-axis, when it crosses y-axis

$u_x=0,\ a_x=-3,\ S_x=-6,\ -6-\frac{1}{2}(-3)t^2,\ t=2\text{ s},\ v_x=-6\text{ m/s}$

For motion along y-axis, when it crosses y-axis

$u_y=0,\ a_y=-4,\ t=2\text{ s},\ S_y=\frac{1}{2}(-4)(2)^2=-8,\ y=-8+4$

$=-4\text{ m},\ v_y=-4\times2=-8\text{ m/s}$

$v=\sqrt{v_x^2+v_y^2}=10\text{ m/s}.$

The correct option is (C)

37. $v=\alpha s^{2/3}$

$$\frac{ds}{dt}=\alpha s^{2/3}$$

$$\int_0^s\frac{ds}{s^{2/3}}=\int_0^t\alpha\,dt\ \Rightarrow\ 3s^{1/3}=\alpha t$$

$$s=\frac{\alpha^3}{27}t^3\Rightarrow ds=\frac{\alpha^3}{9}t^2dt$$

$$v=\frac{\alpha^3}{9}t^2$$

$$W=\frac{1}{2}mv^2=\frac{1}{2}m\left(\frac{\alpha^3t^2}{9}\right)^2=\frac{m\alpha^6t^4}{162}$$

The correct option is (B)

38. x = elongation in spring due to mass 10 kg $=\frac{10\times10}{100}=1$ m

$$W_F=\frac{1}{2}\times100\times\left[(3)^2-(1)^2\right]-10\times10\times2=200\text{ J}$$

The correct option is (A)

39. Work done by external agent in moving the body from

A to $B=(U_B+K_B)-(U_A+K_A)$

$$-10=(U_B-U_A)+(K_B-K_A)$$

$$U_B-U_A=-10+\left(\frac{1}{2}\times2\times3^2-\frac{1}{2}\times2\times5^2\right)=6\text{ J}$$

The correct option is (A)

40. $\frac{MgL}{2}-\frac{Ml_0^2}{2L}g=\frac{1}{2}Mv^2$

$\therefore\quad v=\sqrt{\frac{g}{L}\left(L^2-l_0^2\right)}$

The correct option is (C)

41. (A) At maximum extension $V_A=V_B=0$

$$2mgx=\frac{1}{2}kx^2$$

$$x_m=\frac{4mg}{k}$$

(B) $2mg\frac{2mg}{k}=\frac{1}{2}k\left(\frac{2mg}{k}\right)^2+\frac{1}{2}(m+2m)v^2$

$$v=2g\sqrt{\frac{m}{3k}}$$

(C) Net upward force $\Rightarrow\frac{4\,mg}{k}\times k=4\,mg$

net downward force $\Rightarrow 2\,mg$

$$a=\frac{4\,mg-2\,mg}{3\,m}=\frac{2}{3}g$$

(D) for $x=\frac{2mg}{k},\ a=0\qquad\therefore\ T=2\,mg$

The correct option is (A)

42. Minimum stopping distance $=s$

Force of friction $=\mu mg$

Work done against the friction $W=\mu mgs$

Initial kinetic energy of the toy cart $=\left(p^2/2m\right)$

$\therefore\quad \mu mgs=\left(p^2/2m\right)$

$$\frac{s_1}{s_2}=\left(\frac{m_2}{m_1}\right)^2$$

The correct option is (C)

43. If the body strikes the sand floor with a velocity v, then

$$Mgh = \frac{1}{2}Mv^2$$

Let F be the resisting force acting on the body. Then, the resultant force = $F - mg$

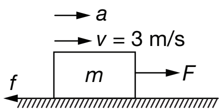
Using work-energy theorem

$$(F - Mg)x = \frac{1}{2}Mv^2$$

or $(F - Mg)x = Mgh \Rightarrow F = Mg\left(1 + \frac{h}{x}\right)$

The correct option is (B)

44. $P = Fv,\ F = \frac{P}{3}$



$$\mu = \frac{1}{\sqrt{3+1}} = \frac{1}{2}$$

$$f = \frac{1}{2}mg$$

$$F - f = ma$$

$$\therefore \quad a = \frac{\frac{P}{3} - \frac{1}{2}mg}{m}$$

$$a = \frac{P}{3m} - \frac{g}{2}$$

The correct option is (A)

45. If block M moves a distance of x, the extension in spring increases by $2x$.

By work-energy theorem,

$$\frac{1}{2}k[x_0^2 - (x_0 + 2x)^2] + Mgx = O$$

$$-\frac{1}{2}k(4x^2 + 4xx_0) + Mgx = O$$

$$\frac{1}{2}k(4x + 4x_0) = Mg$$

$$x + x_0 = \frac{Mg}{2k}$$

$$x = \frac{Mg}{2k} - x_0$$

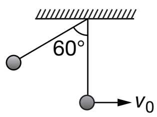
The correct option is (A)

46. On applying work-energy theorem in the frame of wedge.

$ma_0R - mgR = 0 \Rightarrow a_0 = g$

The correct option is (C)

47. By conservation of energy,



$$mgl(1 - \cos 60°) = \frac{1}{2}mv_0^2$$

$$v_0 = 7\,\text{m/s}$$

The correct option is (B)

48. From conservation of energy

$mgs\ \sin\alpha - E = -\ \mu mg\ \cos\alpha \cdot s$

or, $s = \dfrac{E}{mg(\sin\alpha + \mu\cos\alpha)}$

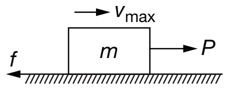
$\therefore$ work done against friction

$$W_f = \mu mg\ \cos\alpha \cdot s = \frac{\mu E\cos\alpha}{\sin\alpha + \mu\cos\alpha}$$

The correct option is (A)

49. $P = Fv$

For maximum velocity,

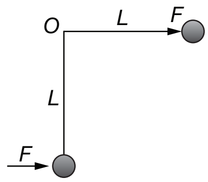


$$F = f = \mu mg$$

$$v_{max} = \frac{P}{\mu mg}$$

The correct option is (B)

50. By work-energy theorem,



$$W_{mg} + W_F = \Delta\text{KE}$$

$$-mgL + FL = K_f - K_i$$

But $F = mg$

$\therefore \quad -mgL + mgL = K_f$

$\Rightarrow \quad K_f = 0$

The correct option is (D)

51. Force between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to a proton.

We know that work done = force × distance. As forces are same in case of proton and positron but distance moved by positron is larger, hence, work done will be more.

The correct option is (C)

52. When the man is squatting on the ground he is tilted somewhat, hence he also has to balance frictional force besides his weight in this case

R = reactional force = friction + mg

$\Rightarrow R > mg$

When the man gets straight up, in that case, friction ≈ 0

$\Rightarrow$ Reactional force $\approx mg$

The correct option is (D)

53. Here, work is done by the frictional force on the cycle and is equal to $200 \times 10 = -2000$ J.

As the road is not moving, work done by the cycle on the road = zero.

The correct option is (C)

54. As the body is falling freely under gravity, the potential energy decreases and kinetic energy increases but total mechanical energy (PE + KE) of the body and earth system will be constant as external force on the system is zero.

The correct option is (C)

55. When we are considering the two bodies as system, the total external force on the system will be zero.

Hence, total linear momentum of the system remains conserved

The correct option is (C)

56. We know that $F \times v = Power$

$\therefore \quad F \times v = c$ where $c =$ constant

$\therefore \quad m\dfrac{dv}{dt} \times v = c \quad \left(\because F = ma = \dfrac{mdv}{dt}\right)$

$\therefore \quad m\int_0^v vdv = c\int_0^t dt \quad \therefore \dfrac{1}{2}mv^2 = ct$

$\therefore \quad v = \sqrt{\dfrac{2c}{m}} \times t^{1/2}$

$\therefore \quad \dfrac{dx}{dt} = \sqrt{\dfrac{2c}{m}} \times t^{1/2}$ where $v = \dfrac{dv}{dt}$

$\therefore \quad \int_0^x dx = \sqrt{\dfrac{2c}{m}} \times \int_0^t t^{1/2}dt$

$$x = \sqrt{\frac{2c}{m}} \times \frac{2t^{3/2}}{3} \Rightarrow x \propto t^{3/2}$$

The correct option is (B)

57. The correct option is (B)

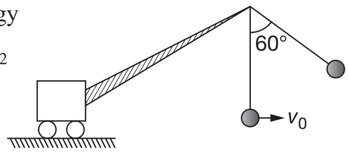
58. Work done by friction may be positive.

The correct option is (D)

59. By conservation of energy

$mgl(1-\cos 60°) = \dfrac{1}{2}mv_0{}^2$

$v_0 = 7$ m/s



The correct option is (B)

60. The correct option is (C)

More than One Option Correct Type

61. $mgx = \dfrac{1}{2}kx^2$ or $x = \dfrac{2mg}{k}$

The correct option is (B), (C) and (D)

62. Acceleration should be equal to 2 m/s^2

So F may be 24 or 16 N

And potential also may increase and decrease

The correct option is (A) and (D)

63. As work done by the force is zero.

The correct option is (B) and (D)

64. $F_x = -\dfrac{dU}{dx} = 7 \Rightarrow a_x = \dfrac{F_x}{m} = \dfrac{7}{5}$

$F_y = -\dfrac{dU}{dx} = -24 \Rightarrow a_y = \dfrac{F_y}{m} = \dfrac{-24}{5}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(\dfrac{7}{5}\right)^2 + \left(\dfrac{24}{5}\right)^2} = 5$ m/s^2

$\vec{a} = \left(\dfrac{7}{5}\right)\hat{i} - \left(\dfrac{24}{5}\right)\hat{j}$ and $\vec{v} = 14.4\hat{i} + 4.2\hat{j}$

$\vec{a}\cdot\vec{v} = \dfrac{7}{5} \times 14.4 - \dfrac{24}{5} \times 4.2 = 0$

Hence $\vec{a}$ is perpendicular to $\vec{v}$.

$V_x = u_x + a_x t \quad V_y = v_y + a_y t$

$V_x = 14.4 + \left(\dfrac{7}{5}\right)^4 \quad V_y = (4.2) - \left(\dfrac{24}{5}\right)^4$

$v = \sqrt{V_x^2 + V_y^2} = 25$ m/s

The correct option is (A), (B) and (C)

65. Work done by friction = loss in potential energy

$$\mu(mg\cos 37°)(6) = mg(4\sin 37°)$$

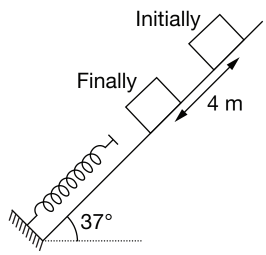
$$\mu = \frac{4}{6}\tan 37° = \frac{1}{2} = 0.5$$

and $\quad \dfrac{1}{2}k(0.2)^2 = -\mu(2g\cos 37°)(5) + 2g(5\sin 37°)$

$\dfrac{1}{2}k(0.04) = -0.5 \times 20 \times \dfrac{4}{5} \times 5 + 20 \times 5 \times \dfrac{3}{5}$

$0.02\,k = 20$

$k = 1000$ N/m



The correct option is (A) and (C)

66. The correct option is (A) and (D)

67. $F_x = -\dfrac{dU}{dx} = 7 \Rightarrow a_x = \dfrac{F_x}{m} = \dfrac{7}{5}$

$F_y = -\dfrac{dU}{dx} = -24 \Rightarrow a_y = \dfrac{F_y}{m} = \dfrac{-24}{5}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(\dfrac{7}{5}\right)^2 + \left(\dfrac{24}{5}\right)^2} = 5$ m/s^2

$$\vec{a}=\left(\frac{7}{5}\right)\hat{i}-\left(\frac{24}{5}\right)\hat{j} \text{ and } \vec{v}=14.4\hat{i}+4.2\hat{j}$$

$$\vec{a}\cdot\vec{v}=\frac{7}{5}\times 14.4-\frac{24}{5}\times 4.2=0$$

Hence $\vec{a}$ is perpendicular to $\vec{v}$.

$$V_x=u_x+a_xt \quad V_y=v_y+a_yt$$

$$V_x=14.4+\left(\frac{7}{5}\right)^4 \quad V_y=(4.2)-\left(\frac{24}{5}\right)^4$$

$$v=\sqrt{V_x^2+V_y^2}=25 \text{ m/s}$$

The correct option is (A), (B) and (C)

68. The correct option is (A) and (B)

69. A block sliding on plank friction will act opposite to velocity of block. Work done by friction on block will be negative. Reaction pairs of friction will act on plank in forward direction, i.e., in direction of displacement, plank friction will do negative work on it.

When block reaches other end, both block and plank will move with common velocity.

$$(M+m)V_C=mv_0$$

$$V_C=\frac{mv_0}{M+m}$$

$$\text{Loss in KE}=\frac{1}{2}mv_0^2-\frac{1}{2}(M+m)\left(\frac{mv_0}{M+m}\right)^2$$

This loss in KE is work against friction so total work by friction is negative.

The correct option is (A), (B) and (C)

70. $u=Ax^2$

$$\therefore \quad F=-\frac{du}{dx}=-\frac{d}{dx}Ax^2=-2Ax$$

$$\text{Acceleration, } a=-\frac{2Ax}{m}$$

For +ve A, a is –ve for the +ve value of x,

i.e., when A is +ve, acceleration of the particle is towards origin and vice versa.

The correct option is (C) and (D)

Passage Based Questions

Passage 1

71. The correct option is (C)

72. The correct option is (A)

73. The correct option is (A)

Passage 2

74. The correct option is (B)

75. The correct option is (A)

76. The correct option is (A)

Passage 3

77. $F\cos\theta=ma$, $13\cos\theta=5$, $\cos\theta=\dfrac{5}{13}$

The correct option is (B)

78. $F\sin\theta-\mu mg=ma_1$

$13\times\dfrac{12}{13}-0.6\times 10=ma_1$ (a_1 = acceleration of the particle with respect to train)

$a_1=6 \text{ m/s}^2$

$a_{\text{net}}=\sqrt{36+25}=\sqrt{61} \text{ m/s}^2$

The correct option is (A)

79. $(\vec{v}_{P/T})_{t=6}=12$ m/s

The correct option is (A)

80. $\text{KE}=\dfrac{1}{2}\times 1\times 100^2=5\times 10^3$ J

The correct option is (A)

Passage 4

81. Speed varies linearly

Speed at a distance x is $\dfrac{xv}{L}$

The correct option is (A)

82. Kinetic energy of the spring

$$\int_0^L \frac{1}{2}\frac{M}{L}dx\left(\frac{xv}{L}\right)^2=\frac{1}{2}\frac{M}{L^3}\frac{L^3}{3}v^2=\frac{1}{6}Mv^2$$

The correct option is (B)

83. $\dfrac{1}{2}\times 3200\times\left(\dfrac{2.5}{100}\right)^2=\dfrac{1}{6}(0.243)v^2+\dfrac{1}{2}(0.053)v^2$

$v=3.9$ m/s

The correct option is (B)

Passage 5

84. $\vec{dr}=dxi+dyj,\ dw=F\cdot dr=-\alpha x y^2\, dy$

On the path $x=y$, so $=-\alpha y^3 dy$,

$$w=\int_{y_1=0}^{y_2=3}-\alpha y^3 dy=-50.6$$

The correct option is (C)

85. $d\omega = -\alpha xy^2 dy$ from $x = 0$ to $x = 3$ $dw = 0$

$w = -2.50 \times 3 = 67.5$ J

The correct option is (A)

86. For these two paths, the same starting and ending point work done is difference.

So forces is non-conservative,

The correct option is (A)

Match the Column Type

87. **(A)** For *OA* net acceleration is negative but displacement is positive, therefore work done is negative.

(B) For *AB*, *x*-*t* curve first is downward parabola up to *A* and then upward parabola, therefore acceleration is first negative, then positive.

(C) From *O* to *A* slope of *x*-*t* curve is positive, from *A* to *B* slope of *x*-*t* curve is negative, therefore velocity is first positive, then negative.

(D) *B* is part of upward parabola, therefore, acceleration is positive.

$\Rightarrow$ (A) $\to$ 2; (B) $\to$ 4; (C) $\to$ 3; (D) $\to$ 1

88. $\Rightarrow$ (A) $\to$ 3; (B) $\to$ 4; (C) $\to$ 1, 2; (D) $\to$ 1, 2

89. $\Rightarrow$ (A) $\to$ (1,2,3); (B) $\to$ (2,3); (C) $\to$ (1,3); (D) $\to$ (1,2,3)

90. To shift block *A*, $Kx = \mu m_1 g$, $x = \frac{16}{100}$ m

Applying work-energy theorem on block *B*,

$$W_F + W_{fr} + W_{sp} = 0,\ Fx - \frac{1}{2}Kx^2 - \mu m_2 gx = 0,\ F = 40 \text{ N}$$

$$W_{fr} \text{ on } B = -\mu m_2 gx = -(0.2)\,(16)\,(10)\left(\frac{16}{100}\right) = -5.12$$

$$W_F = Fx = (40)\left(\frac{16}{100}\right) = 6.4$$

To shift block *B*, $Kx = \mu m_2 g$, $x = \frac{32}{100}\ m$

Applying work-energy theorem on block *A*,

$$W_F + W_{Fr} + W_{sp} = 0$$

$$F = \mu m_1 g + \frac{1}{2}Kx = (0.2)(8)(10) + \frac{1}{2}(100)\left(\frac{32}{100}\right) = 32 \text{ N}$$

$\Rightarrow$ (A) $\to$ 2; (B) $\to$ 1; (C) $\to$ 5; (D) $\to$ 4

91. $w_g = \vec{F}\cdot\vec{S} = (mg\sin\theta)S = 2\times 10\times\frac{1}{2}\times\frac{20}{100} = 2$ J

$$w_s = -\frac{1}{2}kx^2 = -\frac{1}{2}(1000)\left(\frac{20}{100}\right)^2 = -20\text{J}$$

$w_N = 0$

From work energy theorem, $w_g + w_{sp} + w_N + w_{ex} = \Delta k$

or $2 - 20 + 0 + w_{ex} = 0$

$w_{ex} = 18$ J

$\Rightarrow$ (A) $\to$ 4, (B) $\to$ 3, (C) $\to$ 2, (D) $\to$ 1

Integer Type

102. $1\times 2 = (1+1)v \Rightarrow v = 1$ m/s

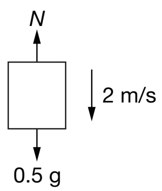
Maximum change $= \frac{1}{2}(2\times 1)(1)^2 = 1$ J

103. $0.5\text{ g} - N = 0.5\times 2$

$N = 4$ N

$d = 0 + \frac{1}{2}\times 2\times 1^2 = 1$ m

$W = \vec{F}\cdot\vec{d} = -4$ J



104. If total work done by all the non-conservative forces on a particle or the system of particles is zero, then total mechanical energy, i.e., total kinetic energy + total potential energy remains constant.

Applying conservation of mechanical energy at initial and final position,

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2,\ k = \frac{3\,mv^2}{4x^2}$$

$$= \frac{3\times 2\times 2^2}{4\times(0.2)^2} = 150 \text{ N/m},\ N = 2$$

105. Let *u* be the speed of the block at the instant of collision.

From work-energy theorem, we have

$\Delta K = W_g + W_N + W_f + W_s$

or, $0 - \frac{1}{2}mu^2 = 0 + 0 + (-f_k.4) - \frac{1}{2}k(4)^2$

or, $\frac{1}{2}mu^2 = 4\mu_k mg + \frac{1}{2}k\times 16$

Solving, we get

$v = 7$ m/s

106. The gain in kinetic energy, loss in potential energy and finally the loss of energy. This loss of energy is equal to the work done in overcoming resistance.

$$\Delta KE = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = 33600 \text{ J}$$

$\Delta PE = 36000$ J, loss $= \Delta$ PE $- \Delta$ KE $= 24000$ J $= R \times 200$

$R = 120$ N

$X = 3$

107. Let m be the mass of the beads.

By energy conservation

$$mgh = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \quad (1)$$

$$\frac{v_1}{v_2} = \frac{4}{3} \quad (2)$$

$$v_1 = \frac{4\sqrt{2}}{5}, \; v_2 = \frac{3\sqrt{2}}{5}$$

$$\text{KE} = \frac{1}{2} \times 100 \times 10^{-3} \times \frac{16 \times 2}{25} = 64 \times 10^{-3} \text{ J}$$

$x = 4$

108. From momentum conservation, $2m = (2m)v_C$

$V_C = 1$ m/s

From mechanical energy conservation,

$$\frac{1}{2}m(2)^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}(2m)V_C^2$$

$x_0 = 10$ cm $= 1$ dm.

109. At equilibrium, $(K_1 + K_2)\,x = mg \Rightarrow K_1 + K_2 = \dfrac{mg}{x}$ (1)

$(x = 40$ cm, $x_1 = 48$ cm)

In case of constant acceleration

$(K_1 + K_2)\,x_1 - \text{mg} = \text{m.a.}$ (2)

from (1) and (2), $a = 2$ ms^{-2}.

110. Let x be the elongation in the spring when it returns back

$$\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + mg\left(x + x_0\right)$$

$$kx = -mg \pm \left(mg - kx_0\right) \quad (1)$$

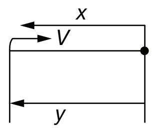
and also lower block will bounce up if $kx \geq mg$ (2)

So $x_0 = \dfrac{3mg}{k} \Rightarrow x_0 = 9$ cm

111. The momentum of the rope at any time is given by

$P = \lambda(y - x)V$

so, $F = \dfrac{dP}{dt} = \lambda V\left(\dfrac{dy}{dt} - \dfrac{dx}{dt}\right)$



Also $y + (y - x) = L$,

$2y - x = L$, $2.\dfrac{dy}{dt} - \dfrac{dx}{dt} = 0$

$\therefore \dfrac{dy}{dt} = \dfrac{1}{2}\dfrac{dx}{dt}$

$$F = \lambda V\left[\left(-\frac{V}{2}\right) - (-V)\right] = \frac{\lambda V^2}{2}$$

$\therefore$ Work done $= F \times 2l = \lambda V^2 \times l = 1 \times 1 \times 1 = 1$ J

Previous Years' Questions

112. We know that $F \times v =$ Power

$\therefore \quad F \times v = c \quad$ where $c =$ constant

$\therefore \quad m\dfrac{dv}{dt} \times v = c \quad \left(\because F = ma = \dfrac{mdv}{dt}\right)$

$\therefore \quad m\displaystyle\int_0^v vdv = c\int_0^t dt \quad \therefore \frac{1}{2}mv^2 = ct$

$\therefore \quad v = \sqrt{\dfrac{2c}{m}} \times t^{1/2}$

$\therefore \quad \dfrac{dx}{dt} = \sqrt{\dfrac{2c}{m}} \times t^{1/2}$ where $v = \dfrac{dv}{dt}$

$\therefore \quad \displaystyle\int_0^x dx = \sqrt{\frac{2c}{m}} \times \int_0^t t^{1/2}dt$

$$x = \sqrt{\frac{2c}{m}} \times \frac{2t^{3/2}}{3} \Rightarrow x \propto t^{3/2}$$

The correct option is (B)

113. $k = 5 \times 10^3$ N/m

$$W = \frac{1}{2}k\left(x_2^2 - x_1^2\right) = \frac{1}{2} \times 5 \times 10^3\left[(0.1)^2 - (0.05)^2\right]$$

$$= \frac{5000}{2} \times 0.15 \times 0.05 = 18.75 \text{ N/m}$$

The correct option is (B)

114. The elastic potential energy

$= \dfrac{1}{2} \times \text{Force} \times \text{extension}$

$= \dfrac{1}{2} \times 200 \times 0.001 = 0.1$ J

The correct option is (D)

115. Kinetic energy of a system of particle is zero only when the speed of each particles is zero. And if speed of each particle is zero, the linear momentum of the system of particle has to be zero.

Also the linear momentum of the system may be zero even when the particles are moving. This is because linear momentum is a vector quantity. In this case, the kinetic energy of the system of particles will not be zero.

$\therefore$ A does not imply B but B implies A.

The correct option is (C)

116. Work done in displacing the particle,

$$W = \vec{F}.\vec{x} = (5\hat{i}+3\hat{j}+2\hat{k}).(2\hat{i}-\hat{j})$$

$$= 10-3 = 7\text{ J}$$

The correct option is (B)

117. Let the surface PE be =

C.M. of hanging part = 0.3 m below the table.

$$U_i = -m'gx = -\frac{4}{2}\times 0.6\times 10\times 0.30$$

$\Delta U = m'gx = 3.6\text{ J} =$ Work done in putting the entire chain on the table.

The correct option is (B)

118. Given: retardation $\propto$ displacement

i.e., $a = -x$

But $a = v\dfrac{dv}{dx}$

$$\therefore \frac{vdv}{dx} = -x \Rightarrow \int_{v_1}^{v_2} v\,dv = -\int_0^x xdx$$

$$\left(v_2^2 - v_1^2\right) = -\frac{x^2}{2}$$

$$\Rightarrow \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}m\left(\frac{-x^2}{2}\right)$$

$\therefore$ Loss in kinetic energy, $\therefore \Delta K \propto x^2$

The correct option is (C)

119. Work done by such force is always zero since force is acting in a direction perpendicular to velocity.

$\therefore$ From work-energy theorem $= \Delta K = 0$

K remains constant.

The correct option is (A)

120. Let acceleration of body be a

$$\therefore v_1 = 0 + at_1 \Rightarrow a = \frac{v_1}{t_1}$$

$$\therefore v = at \Rightarrow v = \frac{v_1 t}{t_1}$$

$$P_{\text{inst}} = \vec{F}.\vec{v} = (m\vec{a}).\vec{v}$$

$$= \left(\frac{mv_1}{t_1}\right)\left(\frac{mv_1}{t_1}\right) = m\left(\frac{v_1}{t_1}\right)^2 t$$

The correct option is (B)

121. $u = 0;\ v = u + aT;\ v = aT$

Instantaneous power $= F\times v = m\cdot a\cdot at = m\cdot a^2\cdot t$

$\therefore$ Instantaneous power $= m\dfrac{v^2}{T^2}t$

The correct option is (B)

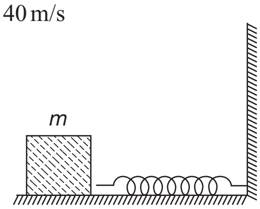
122. Using conservation of energy,

$$m(10\times 100) = m\left(\frac{1}{2}v^2 + 10\times 20\right)$$

or $\quad \dfrac{1}{2}v^2 = 800$ or $v = \sqrt{1600} = 40\text{ m/s}$

The correct option is (B)

123. $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kL^2$



$$\Rightarrow v = \sqrt{\frac{k}{m}}\cdot L$$

Momentum $= m\times v = m\times\sqrt{\dfrac{k}{m}}\cdot L = \sqrt{km}\cdot L$

The correct option is (B)

124. Velocity is maximum when KE is maximum. For minimum PE

$$\frac{dv}{dx} = 0 \Rightarrow x^3 - x = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow \text{Min. PE} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}\text{ J}$$

$$\text{KE}_{(\text{max.})} + \text{PE}_{(\text{min})} = 2(\text{Given})$$

$$\text{KE}_{(\text{max.})} = 2 + \frac{1}{4} = \frac{9}{4}$$

$$\text{KE}_{(\text{max.})} = \frac{1}{2}mv^2_{\text{max.}}$$

$$\Rightarrow \frac{1}{2}\times 1\times v^2_{\text{max.}} = \frac{9}{4} \Rightarrow v_{\text{max.}} = \frac{3}{\sqrt{2}}$$

The correct option is (A)

125. $\text{KE} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\times 0.1\times 25 = 1.25\text{ J}$

$$W = -mgh = \left(\frac{1}{2}mv^2\right) = 1.25\text{ J}$$

$$\left[\because mgh = \frac{1}{2}mv^2 \text{ by energy conservation}\right]$$

The correct option is (B)

126. Let the block compress the spring by x before stopping kinetic energy of the block = (PE of compressed sprit + work done against friction.

$$\frac{1}{2}\times 2\times (4)^2 = \frac{1}{2}\times 10{,}000\times x^2 + 15\times x$$

$$10{,}000\times x^2 + 30x - 32 = 0$$

$$\Rightarrow 5000x^2 + 15 - 16 = 0$$

$$\therefore x = \frac{-15\pm\sqrt{(15)^2 - 4\times(5000)(-16)}}{2\times 5000}$$

$$= 0.05\text{ m} = 5.5\text{ cm}$$

The correct option is (B)

127. Initial kinetic energy of the system

$$\text{KE}_i = \frac{1}{2}mu^2 + \frac{1}{2}M(0)^2 = \frac{1}{2}\times 0.5\times 2\times 2 + 0 = 1\text{ J}$$

For collision, applying conservation of linear momentum

$m \times u = (m + M) \times v$

$\therefore 0.5 \times 2 = (0.5 + 1) \times v \Rightarrow v = \frac{2}{3}$ m/s

Final kinetic energy of the system is

$KE_f = \frac{1}{2}(m + M)v^2 = \frac{1}{2}(0.5 + 1) \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{3}$ J

$\therefore$ Energy loss during collision $= \left(1 - \frac{1}{3}\right)$J $= 0.67$ J

The correct option is (C)

128. The average speed of the athlete

$v = \frac{100}{10} = 10$ m/s

$\therefore KE = \frac{1}{2}mv^2$

If mass is 40 kg then, $KE = \frac{1}{2} \times 40 \times (10)^2 = 2000$ J

If mass is 100 kg then, $KE = \frac{1}{2} \times 100 \times (10)^2 = 5000$

The correct option is (D)

129. At equilibrium, $\frac{dU(x)}{dx} = 0$

$\Rightarrow \frac{-12a}{x^{11}} = \frac{-6b}{x^5} \Rightarrow x = \left(\frac{2a}{6}\right)^{1/6}$

$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$ and $U_{(x=\infty)} =$

$\therefore D = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$

130. $F = kx$

$W = \frac{1}{2}kx^2$

131. $\int_0^w dw = \int_0^L \left(ax + bx^2\right)dx$

$= \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^L$

$= \frac{aL^2}{2} + \frac{bL^3}{3}$

The correct option is (A)

132. 1 kg fat gives $3 \cdot 8 \times 10^7$ J

$W = 10 \times 1 \times 9.8 \times 1000 = 98000$ J

Mass burnt $= \dfrac{98000}{3.8 \times 10^7 \times \frac{20}{100}}$

$= 128947.368 \times 10^{-7}$ kg

$= 12.89 \times 10^{-3}$ kg.

The correct option is (C)

133. From work-energy theorem

$2mg - 2\sqrt{3}\,\mu mg - \mu mgx = 0$

$2 - 2\sqrt{3}\,\mu - \mu x = 0$

$\mu(x + 2\sqrt{3}) = 2$ (1)

From given condition

$2\sqrt{3}\,\mu mg = \mu mgx$

$x = 2\sqrt{3}$

$= 2 \times 1.732$

$= 3.464$

≈ 3.5 m

$\mu = \frac{2}{x + 2\sqrt{3}} = \frac{2}{4\sqrt{3}} = \frac{1}{2\sqrt{3}} = 0.2886$

$\approx .29$

The correct option is (B)