CHAPTER

3

Newton's Law of Motion

Chapter Highlights

Force and Inertia, Newton's First Law of Motion, Momentum, Newton's Second Law of Motion, Impulse, Newton's Third Law of Motion, Equilibrium of Concurrent Forces, Static and Kinetic Friction, Laws of Friction, Rolling Friction, Dynamics of Uniform Circular Motion.

FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of Resultant Force

1. It may change only speed.
2. It may change only direction of motion.
3. It may change both the speed and direction of motion.
4. It may change size and shape of a body.

Unit of Force

Newton and $\frac{\text{kg}\cdot\text{m}}{s^2}$ (MKS system)

Dyne and $\frac{\text{g}\cdot\text{cm}}{s^2}$ (CGS system)

$$1 \text{ newton} = 10_5 \text{ dyne}$$

Kilogram Force (kgf)

The force with which earth attracts a 1 kg body towards its centre is called kilogram force, thus

$$kgf = \frac{\text{Force in Newton}}{g}$$

Dimensional Formula of Force

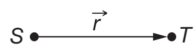
$$[\text{M L T}^{-2}]$$

Fundamental Forces

All the forces observed in nature such as muscular force, tension, reaction, friction, elastic, weight, electric, magnetic, nuclear, etc., can be explained in terms of only following four basic interactions:

Gravitational Force

The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force.



$$\vec{F} = -G\frac{m_1 m_2}{r^3}\vec{r}$$

$\vec{r}$ = position vector of test particle T with respect to source particle S and G = universal gravitational constant

$$= 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2.$$

$\therefore \quad F = mg$

This is the force exerted by earth on any particle of mass m near the earth surface. The value of $g = 9.81 \text{ m/s}^2 \simeq 10 \text{ m/s}^2 \simeq \pi^2 \text{ m/s}^2 \simeq 32 \text{ ft/s}^2$. It is also called acceleration due to gravity near the surface of earth.

Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force

1. These can be attractive or repulsive.
2. These are long range forces.
3. They depend on the nature of medium between the charged particles.
4. All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/molecules.

Nuclear Force

It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus in spite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc., are a result of unbalancing of nuclear forces. It acts within the nucleus that too up to a very small distance.

Weak Force

It acts between any two elementary particles. Under its action, a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.

It has been found that for two protons at a distance of 1 fermi:

$$F_N:F_{EM}:F_W:F_G::1:10^{-2}:10^{-7}:10^{-38}$$

Classification of Forces on the Basis of Contact

Field Force

Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force. Examples

1. Gravitational force
2. Electromagnetic force

Contact Force

Forces which are transmitted between bodies by short-range atomic molecular interactions are called contact forces. When two objects come in contact, they exert contact forces on each other.

Normal Force (N)

It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force.

A table is placed on earth as shown in Fig. 3.1.

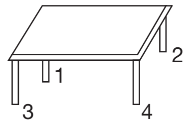


Fig. 3.1

Here the table presses the earth so normal force exerted by four legs of table on earth is as shown in Fig. 3.2.

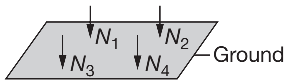
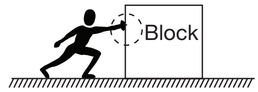
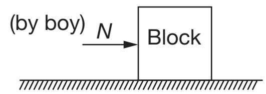


Fig. 3.2

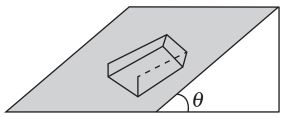
Now a boy pushes a block kept on a frictionless surface.



Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is a normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



Normal force exerted by block on the surface of inclined plane is shown in Fig. 3.3.

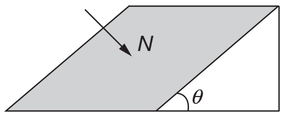


Fig. 3.3

Force acts perpendicular to the surface

Tension

Tension in a string is an electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force F as shown in Fig. 3.4, for calculating the tension at point A we draw FBD of marked portion of the string; here string is massless.

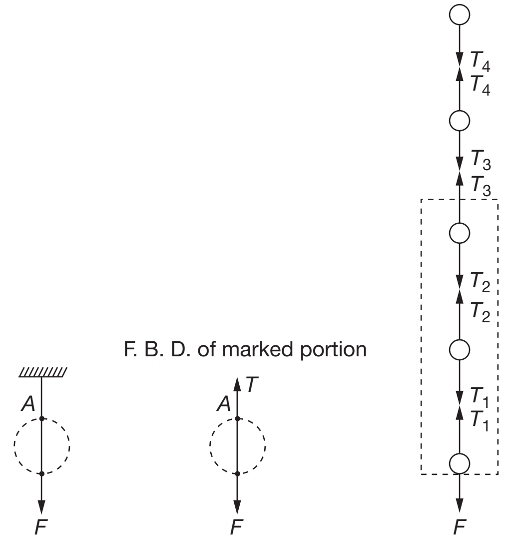


Fig. 3.4

$$\Rightarrow \quad T = F$$

String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown in Fig. 3.5. The attraction force between two segments is equal and opposite due to Newton's third law.

For calculating tension at any segment, we consider two or more than two parts as a system.

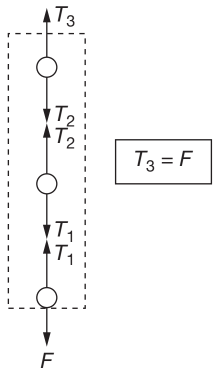


Fig. 3.5

Here interactions between segments are considered as internal forces, so they are not shown in FBD.

Frictional Force

It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact.

FREE BODY DIAGRAM

A FBD consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

Steps for FBD

Step 1: Identify the object or system and isolate it from other objects clearly specifying its boundary.

Step 2: First draw non-contact external force in the diagram. Generally, it is weight.

Step 3: Draw contact forces which acts at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.
In FBD, internal forces are not drawn, only external are drawn.

SOLVED EXAMPLES

1. A block of mass *m* is kept on the ground as shown in Fig. 3.6.

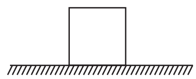
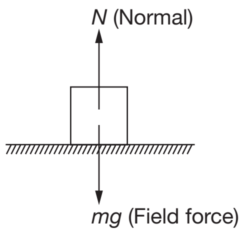


Fig. 3.6

(A) Draw FBD of block.
(B) Are forces acting on block action–reaction pair.
(C) If answer is no, draw action reaction pair.

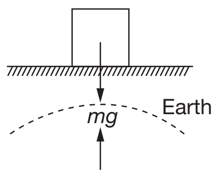
Solution:

(A) FBD of block

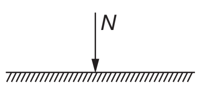


(B) *N* and *Mg* are not action–reaction pair. Although pair acts on different bodies, they are of same nature.

(C) Pair of *mg* of block acts on earth in opposite direction.



Pair of *N* acts on surface



2. Two spheres A and B are placed between two vertical walls as shown in Fig. 3.7. Draw the free body diagrams of both the spheres.

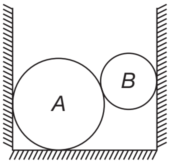
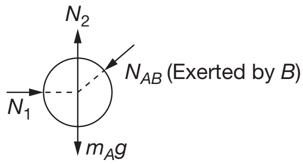


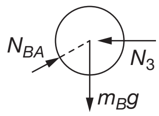
Fig. 3.7

Solution:

FBD of sphere A:



FBD of sphere B:



(Exerted by A)

Notes: Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

3. Three triangular blocks A, B and C of equal masses m are arranged as shown in Fig. 3.8. Draw FBD of blocks A, B and C. Indicate action–reaction pair between A, B and B, C.

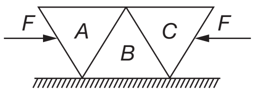
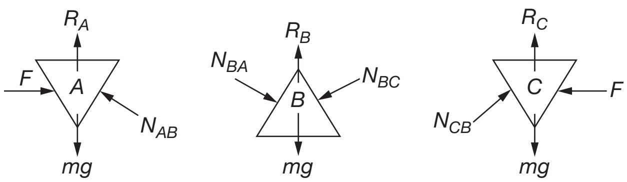


Fig. 3.8

Solution:



NEWTON'S LAWS OF MOTION

First Law of Motion

Each body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Newton's first law is really a statement about reference frames, in that it defines the types of reference frames in which the laws of Newtonian mechanics hold. From this point of view, the first law is expressed as:

If the net force acting on a body is zero, it is possible to find a set of reference frames in which that body has no acceleration.

Newton's first law is sometimes called the law of inertia and the reference frames that it defines are called inertial reference frames.

Newton's law as written by Newton in Latin from an 1803 translation:

> *'Everybody preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon'.*

Examples of this Law

1. A bullet fired on a glass window makes a clean hole through it while a stone breaks the whole of it. The speed of bullet is very high. Due to its large inertia of motion, it cuts a clean hole through the glass. When a stone is thrown, it inertia is much lower so it cannot cut through the glass.
2. A passenger sitting in a bus gets a jerk when the bus starts or stops suddenly.

Second Law of Motion

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Newton's law as written by Newton in Latin from an 1803 translation:

> *'The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.'*

Mathematically,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Or
$$\vec{F} = m\vec{a}$$

where $\vec{p} = m\vec{v}$, $\vec{p}$ = Linear momentum.

In case of two particles having linear momentum $\vec{P}_1$ and $\vec{P}_2$ and moving towards each other under mutual forces, from Newton's second law,

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = \vec{F} = 0$$

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$$

$$\vec{F}_1 + \vec{F}_2 = 0$$

$$\vec{F}_2 = -\vec{F}_1$$

which is Newton's third law.

Important Points about Second Law

1. The second law is obviously consistent with the first law as $F = 0$ Implies $a = 0$.
2. The second law of motion is a vector law. It is actually a combination of three equations, one for each component of the vector:

$$F_x = \frac{dp_x}{dt} = ma_x$$

$$F_y = \frac{dp_y}{dt} = ma_y$$

$$F_z = \frac{dp_z}{dt} = ma_z$$

   This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.
3. The second law of motion given above is strictly applicable to a single point mass. The force F in the law stand for the net external force on the particle and a stands for the acceleration of the particle. Any internal forces in the system are not to be included in F.
4. The second law of motion is a local relation. What this means is that the force F at a point in space (location of the particle) at a certain instant of time is related to a at the same point at the same instant. That is acceleration here and now is determined by the force here and now not by any history of the motion of the particle.

Third Law of Motion

To every action, there is always an equal and opposite reaction. Newton's law as written by Newton in Latin from an 1803 translation:

> *'To every action there is always opposed an equal and opposite reaction: to the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.'*

Important Points about the Third Law

1. The terms 'action' and 'reaction' in the third law mean nothing but 'force'. A simpler and clear way of stating the third law is as follows: Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.
2. The terms 'action' and 'reaction' in the third law may give a wrong impression that action comes before reaction, i.e., action is the cause and reaction the effect. There is no such cause–effect relation implied in the third law. The force on A by B and the force on B by A acts at the same instant. Any one of them may be called action and the other reaction.
3. Action and reaction forces act on different bodies, not on the same body. Thus, if we are considering the motion of any one body (A or B), only one of the two forces is relevant. It is an error to add up the two forces and claim that the net force is zero.
   However, if you are considering the system of two bodies as a whole, F_{AB} (force on A due to B) and F_{BA} (force on B due to A) are internal forces of the system ($A + B$). They add up to give a null force. Internal forces in a body or a system of particles thus gets canceled in pairs. This is an important fact that enables the second law to be applicable to a body or a system of particles.

Applications of Newton's Laws

Case I: When objects are in equilibrium
To solve problems involving objects in equilibrium:

Step 1: Make a sketch of the problem.
Step 2: Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.
Step 3: Choose a convenient coordinate system and resolve all forces into rectangular components along x and y direction.
Step 4: Apply the equations $\sum F_x = 0$ and $\sum F_y = 0$.
Step 5: Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.
Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

Eventually at step 5, you will have enough equations to solve for all unknown quantities.

SOLVED EXAMPLES

4. A 'block' of mass 10 kg is suspended with string as shown in the Fig. 3.9. Find tension in the string ($g = 10$ m/s^2)

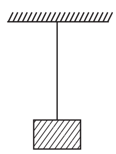
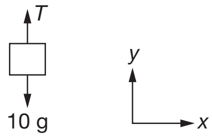


Fig. 3.9

Solution:

FBD of block



$\Sigma F_y = 0$

$T - 10\text{ g} = 0$

$\therefore \quad T = 100$ N.

5. The system shown in Fig. 3.10 is in equilibrium. Find the magnitude of tension in each string; T_1, T_2, T_3 and T_4. ($g = 10$ m/s^{-2})

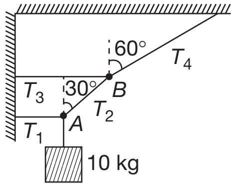
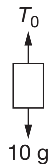


Fig. 3.10

Solution:

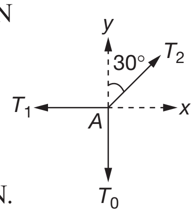
FBD of 10 kg block



$T_0 = 10$ g

$T_0 = 100$ N

FBD of point A



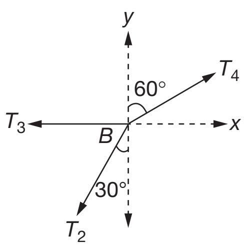
$\Sigma F_y = O \Rightarrow \quad T_2 \cos 30° = \text{T}_0 = 100$ N

$$T_2 = \frac{200}{\sqrt{3}} \text{ N}$$

$\Sigma F_x = O \Rightarrow \quad T_1 = T_2 \sin 30°$

$$= \frac{200}{\sqrt{3}} \cdot \frac{1}{2} = \frac{100}{\sqrt{3}} \text{N.}$$

FBD of point of B



$$\Sigma F_y = O \Rightarrow T_4 \cos 60° = T_2 \cos 30°$$

and $\quad \Sigma F_x = O \Rightarrow T_3 + T_2 \sin 30° = T_4 \sin 60°$

$$\therefore \quad T_3 = \frac{200}{\sqrt{3}} \text{ N, } T_4 = 200 \text{ N.}$$

6. Two blocks are kept in contact as shown in Fig. 3.11. Find:

(A) Forces exerted by surfaces (floor and wall) on blocks.

(B) Contact force between two blocks

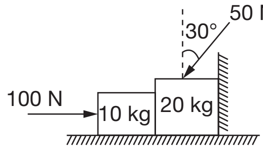
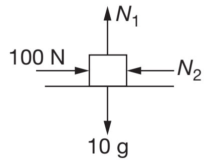


Fig. 3.11

Solution:

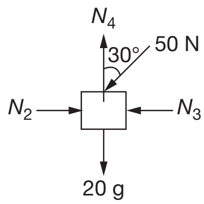
FBD of 10 kg block



$$N_1 = 10 \text{ g} = 100 \text{ N} \qquad (1)$$

$$N_2 = 100 \text{ N} \qquad (2)$$

FBD of 20 kg block



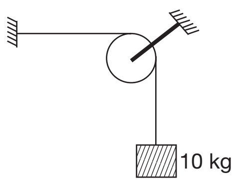
$$N_2 = 50 \sin 30° + N_3$$

$\therefore \quad N_3 = 100 - 25 = 75$ N $\qquad (3)$

and $\quad N_4 = 50 \cos 30° + 20$ g

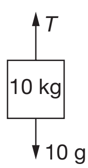
$$N_4 = 243.30 \text{ N.}$$

7. Find magnitude of force exerted by a string on pulley.



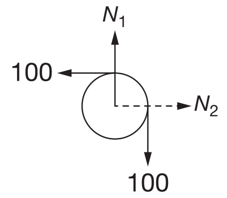
Solution:

FBD of 10 kg block



$$T = 10 \text{ g} = 100 \text{ N}$$

FBD of pulley



Since string is massless, tension in both sides of string is same.

Force exerted by string

$$= \sqrt{(100)^2 + (100)^2}$$

$$= 100\sqrt{2} \text{ N}$$

Note: Since pulley is in equilibrium position, net forces on it is zero.

Hence force exerted by hinge on it is 100 $\sqrt{2}$ N.

Case II. Accelerating Objects

To solve problems involving objects that are in accelerated motion:

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not to include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

Step 3: Choose a convenient coordinate system, show location of coordinate axes explicitly in the free-body diagram and then determine components of forces with reference to these axes and resolve all forces into x and y components.

Step 4: Apply the equations $\sum F_x = m a_x$ and $\sum F_y = m a_y$.

Step 5: Step 4 will give two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to Step 2 and select another object and repeat these steps. Eventually at Step 5 you will have enough equations to solve for all unknown quantities.

SOLVED EXAMPLES

8. A force F is applied horizontally on mass m_1 as shown in Fig. 3.12. Find the contact force between m_1 and m_2.

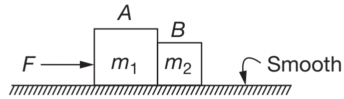
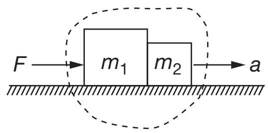


Fig. 3.12

Solution:

Considering both blocks as a system to find the common acceleration.



Common acceleration

$$a = \frac{F}{(m_1 + m_2)} \quad (1)$$

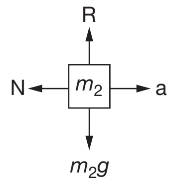
To find the contact force between A and B, we draw FBD of mass m_2.

FBD of mass m_2

$$\Sigma F_x = ma_x$$

$$N = m_2 \cdot a$$

$$N = \frac{m_2 F}{(m_1 + m_2)}.$$



9. The velocity of a particle of mass 2 kg is given by $\vec{v} = at\hat{i} + bt^2\hat{j}$. Find the force acting on the particle.

Solution:

From second law of motion:

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v})$$

$$= 2 \cdot \frac{d}{dt}\,(at\hat{i} + bt^2\hat{j})$$

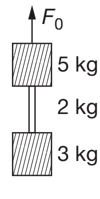
$$\Rightarrow \quad \vec{F} = 2a\hat{i} + 4bt\hat{j}.$$

10. A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s^2 by an external force F_0.

(A) What is F_0?

(B) What is the net force on the rope?

(C) What is the tension at middle point of the rope? ($g = 10$ m/s^2)

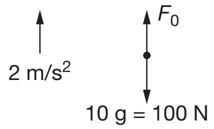


Solution:

For calculating the value of F_0, consider two blocks with the rope as a system.

FBD of whole system

(A)



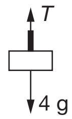
$$F_0 - 100 = 10 \times 2$$

$$F = 120 \text{ N} \quad (1)$$

(B) According to Newton's second law, net force on rope.

$$F = ma$$
$$= (2)(2)$$
$$= 4 \text{ N} \quad (2)$$

(C) For calculating tension at the middle point, we draw FBD of 3 kg block with half of the rope (mass 1 kg) as shown.



$$T - 4\text{ g} = 4.(2);\ T = 48 \text{ N}.$$

11. A block of mass 50 kg is kept on another block of mass 1 kg as shown in Fig. 3.13. A horizontal force of 10 N is applied on the 1 kg block (All surface are smooth). Find ($g = 10$ m/s²)

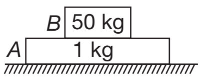
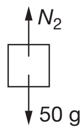


Fig. 3.13

(A) Acceleration of block A and B.
(B) Force exerted by B on A.

Solution:

(A) **FBD of 50 kg**



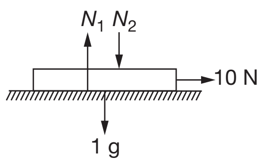
$$N_2 = 50\text{ g} = 500 \text{ N}$$

Along horizontal direction, there is no force $a_B = 0$

(B) **FBD of 1 kg block**

Along horizontal direction

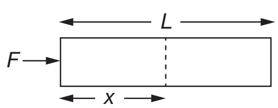
$$10 = 1\, a_A$$
$$a_A = 10 \text{ m/s}^2$$



Along vertical direction

$$\therefore \quad N_1 = N_2 + 1\text{ g}$$
$$= 500 + 10 = 510 \text{ N}.$$

12. A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance x from the end where force is applied.

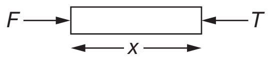


Solution:

Considering rod as a system, we find acceleration of rod

$$a = \frac{F}{m}$$

Now draw FBD of rod having length x as shown below.



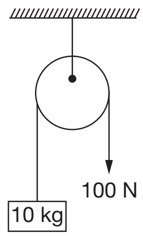
Using Newton's second law

$$F - T = \left(\frac{M}{L}\right) x \cdot a$$

$$T = F - \frac{M}{L} x \cdot \frac{F}{M}$$

$$T = F\left(1 - \frac{x}{L}\right).$$

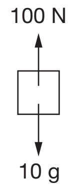
13. One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass ($g = 9.8$ m/s²).



Solution:

Since string is pulled by 100 N force, tension in the string is 100 N.

FBD of 10 kg block



$$100 - 10\text{ g} = 10\, a$$
$$100 - 10 \times 9.8 = 10\, a$$
$$a = 0.2 \text{ m/s}^2.$$

14. Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown Fig. 3.14. If they are released from rest, Find:

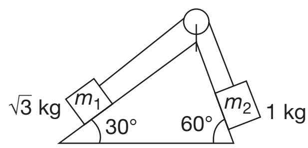
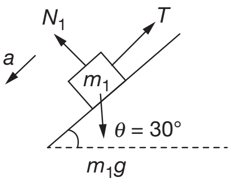


Fig. 3.14

(A) Acceleration of mass m_1 and m_2
(B) Tension in the string
(C) Net force on pulley exerted by string

Solution:

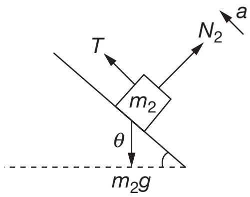
FBD of m_1



$$m_1 g \sin\theta - T = m_1 a$$

$$\frac{\sqrt{3}}{2} g - T = \sqrt{3}\, a \qquad (1)$$

FBD of m_2



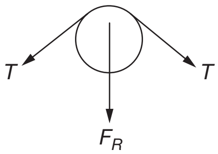
$$T - m_2 g \sin\theta = m_2 a$$

$$T - 1 \cdot \frac{\sqrt{3}}{2} g = 1.a \qquad (2)$$

Adding (1) and (2), we get
$a = 0$
Putting this value in (1), we get

$$T = \frac{\sqrt{3}g}{2},$$

FBD of pulley



$$F_R = \sqrt{2}\,\mathrm{T}$$

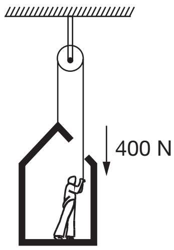
$$F_R = \frac{\sqrt{3}}{2} g.$$

15. A 60 kg painter stands on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.

(A) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.

(B) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1 s?

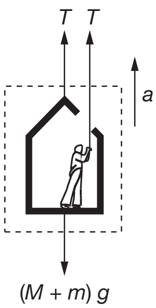
(C) What force should he apply now to maintain the constant speed of 1 m/s?



Solution:

The free body diagram of the painter and the platform as a system can be drawn as shown.

Note that the tension in the string is equal to the force by which he pulls the rope.



(A) Applying Newton's second law,

$$2T - (M + m)g = (M + m)a$$

or $$a = \frac{2T - (M+m)g}{M+m}$$

Here $M = 60$ kg; $m = 15$ kg; $T = 400$ N
$g = 10$ m/s$_2$

$$a = \frac{2(400) - (60+15)(10)}{60+15} = 0.67 \text{ m/s}^2$$

(B) To attain a speed of 1 m/s in one second, the acceleration a must be 1 m/s^2.

Thus, the applied force is

$$F = \frac{1}{2}(M + m)(g + a) = \frac{1}{2}(60 + 15)(10 + 1) =$$
412.5 N

(C) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.
Thus, $2F - (M + m)g = 0$

or $$F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2} = 375 \text{ N}.$$

CONSTRAINED MOTION

String Constraint

When two objects are connected through a string and if the string have the following properties:

1. The length of the string remains constant, i.e. inextensible string.
2. Always remains tight, does not slack.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them.

Steps for String Constraint

Step 1: Identify all the objects and number of strings in the problem.

Step 2: Assume variable to represent the parameters of motion such as displacement, velocity acceleration, etc.

(i) Object which moves along a line can be specified by one variable.

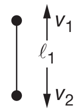
(ii) Objects moving in a plane are specified by two variables.

(iii) Objects moving in 3-D require three variables to represent the motion.

Step 3: Identify a single string and divide it into different linear sections and write in the equation format: $\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5 + \ell_6 = \ell$

Step 4: Differentiate with respect to time

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} + ... = 0$$

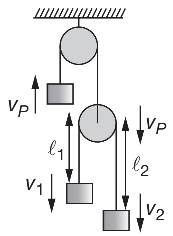


$\dfrac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object so take the velocity of the object along the length of the string

$$\frac{d\ell_1}{dt} = v_1 + v_2$$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+v_1$ represents that upper end is tending to increase the length at rate v_1 and lower end is tending to increase the length at rate v_2.

Step 5: Repeat all above steps for different strings. Let us consider a problem given below



Here $\qquad \ell_1 + \ell_2 = \text{constant}$

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

$$(v_1 - v_P) + (v_P - v_2) = 0$$

$$v_p = \frac{v_1 + v_2}{2}$$

Similarly,

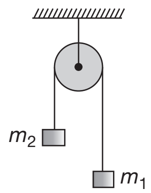
$$a_P = \frac{a_1 + a_2}{2} \quad \text{Remember this result}$$

SOLVED EXAMPLES

16. Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find:

(A) the acceleration of each block

(B) the tension in the string



Solution:

Block m_1 is assumed to be moving downward and block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction. If the values of a_1 and a_2 come out to be positive, then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, the tension on each side of the pulley is same.

The free body diagram of each block is shown in the below Fig. 3.15.

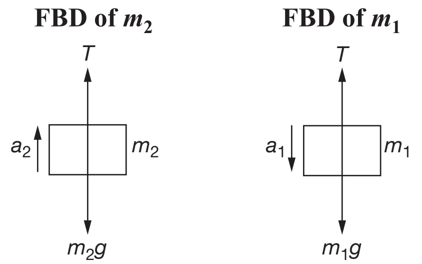


Fig. 3.15

Applying Newton's second law on blocks m_1 and m_2

Block m_1 $\qquad m_1g - T = m_1a \qquad (1)$

Block m_2 $\qquad -m_2g + T = m_2a_2 \qquad (2)$

Number of unknowns: T, a_1 and a_2 (three)

Number of equations: only two

Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

How to determine constraint relation?

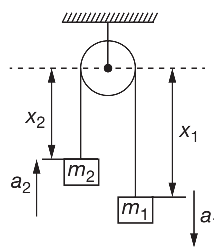
1. Assume the direction of acceleration of each block, e.g., a_1 (downward) and a_2 (upward) in this case.
2. Locate the position of each block from a fixed point (depending on convenience), e.g., centre of the pulley in this case.
3. Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.

Thus, $x_1 + x_2 = \text{constant}$

Differentiating both the sides with respect to time we get $\dfrac{dx_1}{dt} + \dfrac{dx_2}{dt} = 0$

Each term on the left side represents the velocity of the blocks.

Since we have to find a relation between accelerations, we differentiate it once again with respect to time.



Position of each block is locked w. r. t. centre of the pulley

Thus, $$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$$

Since, the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$$\therefore \quad \frac{d^2x_1}{dt^2} = +a_1$$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

$$\therefore \quad \frac{d^2x_2}{dt^2} = -a_2$$

Thus $\quad a_1 - a_2 = 0$

or $\quad a_1 = a_2 = a$ (say) is the required constraint relation.

Substituting $a_1 = a_2 = a$ in Equations (1) and (2) and solving them, we get

(1) $a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g$

(2) $T = \left[\frac{2m_1m_2}{m_1 + m_2}\right] g.$

17. A system of three masses m_1, m_2 and m_3 are shown in the Fig. 3.16.

The pulleys are smooth and massless; the strings are massless and inextensible.

(A) Find the tensions in the strings.

(B) Find the acceleration of each mass.

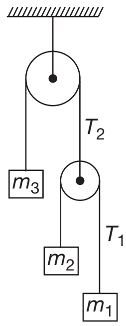
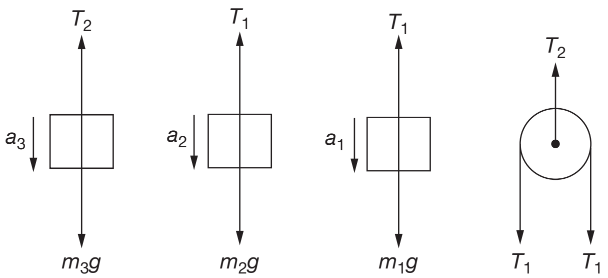


Fig. 3.16

Solution:

All the blocks are assumed to be moving downward and the FBD of each block is shown.

FBD m_3 **FBD m_2** **FBD m_1** **FBD of pulley**



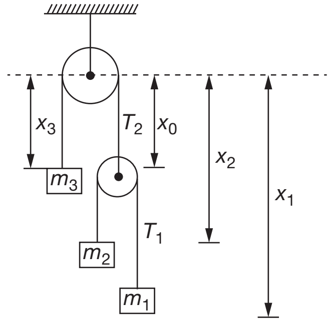
Applying Newton's second law to

Block m_1: $\quad m_1g - T_1 = m_1a_1 \quad (1)$

Block m_2: $\quad m_2g - T_1 = m_2a_2 \quad (2)$

Block m_3: $\quad m_3g - T_2 = m_3a_3 \quad (3)$

Pulley: $\quad T_2 = 2T_1 \quad (4)$



Number of unknowns a_1, a_2, a_3, T_1 and T_2 (Five)

Number of equations: Four

The constraint relation among accelerations can be obtained as follows

For upper string $x_3 + x_0 = c_1$

For lower string $(x_2 - x_0) + (x_1 - x_0) = c_2$

$$x_2 + x_1 - 2x_0 = c_2$$

Eliminating x_0 from the above two relations,

we get $\quad x_1 + x_1 + 2x_3 = 2c_1 + c_2 = \text{constant}.$

Differentiating twice with respect to time,

We get $$\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0$$

or $$a_1 + a_2 + 2a_3 = 0 \quad (5)$$

Solving Equations (1) to (5), we get

(i) $T_1 = \left[\frac{4m_1m_2m_3}{4m_1m_2 + m_3(m_1 + m_2)}\right] g;$

$T_2 = 2T_1$

(ii) $a_1 = \left[\frac{4m_1m_2 + m_1m_3 - 3m_2m_3}{4m_1m_2 + m_3(m_1 + m_2)}\right] g$

$a_2 = \left[\frac{3m_1m_3 - m_2m_3 - 4m_1m_2}{4m_1m_2 + m_3(m_1 + m_2)}\right] g$

$a_3 = \left[\frac{4m_1m_2 - m_3(m_1 + m_2)}{4m_1m_2 + m_3(m_1 + m_2)}\right] g.$

18. The Fig. 3.17 shows one end of a string being pulled down at constant velocity v. Find the velocity of mass m as a function of x.

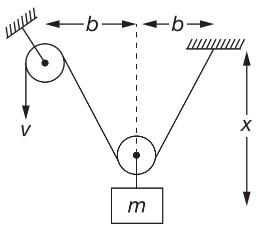
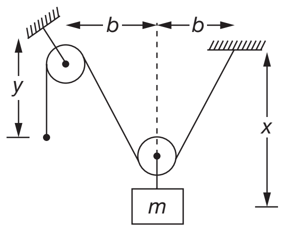


Fig. 3.17

Solution:

Using constraint equation

$$2\sqrt{x^2+b^2}+y = \text{length of string} = \text{constant}$$



Differentiating with respect to time:

$$\frac{2}{2\sqrt{x^2+b^2}}\cdot 2x\left(\frac{dx}{dt}\right)+\left(\frac{dy}{dt}\right)=0$$

$$\Rightarrow \qquad \left(\frac{dy}{dt}\right)=v$$

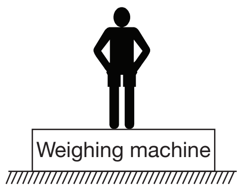
$$\therefore \qquad \left(\frac{dx}{dt}\right)=-\frac{v}{2x}\sqrt{x^2+b^2}.$$

WEIGHING MACHINE

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

SOLVED EXAMPLES

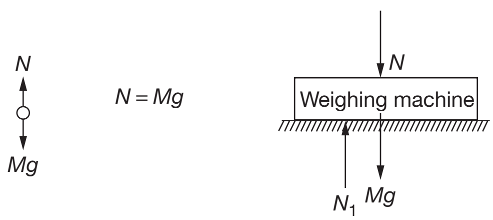
19. A man of mass 60 kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10$ m/s^2).



Solution:

For calculating the reading of weighing machine, we draw FBD of man and machine separately.

FBD of man FBD of weighing machine



Here force exerted by object on upper surface is N

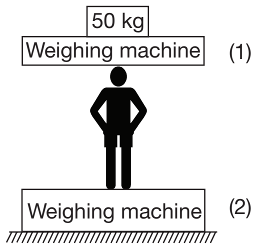
Reading of weighing machine

$$N = Mg$$

$$= 60 \times 10$$

$$N = 600 \text{ N}.$$

20. A man of mass 60 kg is standing on a weighing machine (2) of mass 5 kg placed on ground. Another same weighing machine is placed over man's head. A block of mass 50 kg is put on the weighing machine (1). Calculate the readings of weighing machines (1) and (2).



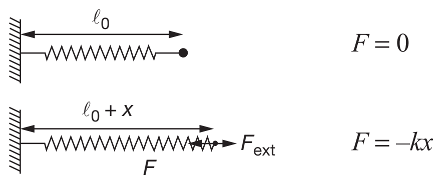
Solution:

500 N, 1150 N.

SPRING FORCE

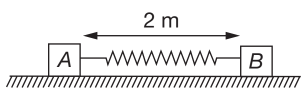
Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1}).

When spring is in its natural length, spring force is zero.



SOLVED EXAMPLES

21. Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m.



Find spring force in following situations:

(A) If block A and B both are displaced by 0.5 m in same direction.

(B) If block A and B both are displaced by 0.5 m in opposite direction.

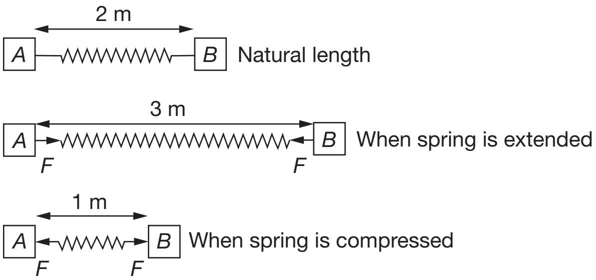
Solution:

(A) Since both blocks are displaced by 0.5 m in same direction, change in length of spring is zero. Hence, spring force is zero.

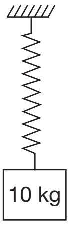
(B) In this case, change in length of spring is 1 m. So spring force is $F = -Kx$

$$= -(200) \cdot (1)$$

$$F = -200 \text{ N.}$$



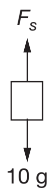
22. Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring. ($g = 10$ m/s^2)



Solution:

In this situation, spring is in extended state so spring force acts in upward direction. Let x be the extension in the spring.

FBD of 10 kg block:



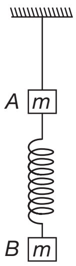
$$F_s = 10 \text{ g}$$

$$\Rightarrow \quad Kx = 100$$

$$\Rightarrow \quad (100)x = (100)$$

$$\Rightarrow \quad x = 1 \text{ m.}$$

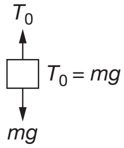
23. Two blocks A and B of same mass m attached with a light spring are suspended by a string as shown in Fig. 3.18. Find the acceleration of block A and B just after the string is cut.



Solution:

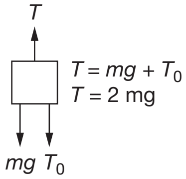
When block A and B are in equilibrium position

FBD of B



(1)

FBD of A

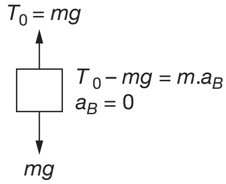


(2)

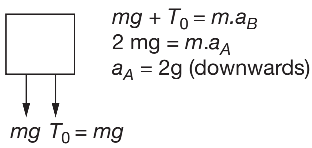
Fig. 3.18

When string is cut, tension T becomes zero. But spring does not change its shape after cutting. So spring force acts on mass B, again draw FBD of blocks A and B as shown.

FBD of B



FBD of A



24. Two blocks A and B of same mass m attached with a light string are suspended by a spring as shown in Fig. 3.19. Find the acceleration of block A and B just after the string is cut.

Solution:

g (upwards), g (downwards).

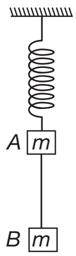


Fig. 3.19

Spring Balance

It does not measure the weight. It measures the force exerted by the object at the hook.

Symbolically, it is represented as shown in Fig. 3.20.

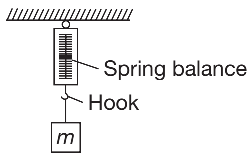


Fig. 3.20

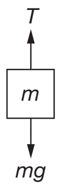
A block of mass m is suspended at hook.

When spring balance is in equilibrium, we draw the FBD of mass m for calculating the reading of balance.

FBD of m.

$$mg - T = 0$$

$$T = mg$$



Magnitude of T gives the reading of spring balance.

SOLVED EXAMPLES

25. A block of mass 20 kg is suspended through two light spring balances as shown in Fig. 3.21.

Calculate the:

1. reading of spring balance (1).
2. reading of spring balance (2).

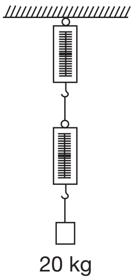
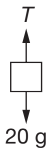


Fig. 3.21

Solution:

For calculating the reading, first we draw FBD of 20 kg block.

FBD of 20 kg.



$$mg - T = 0$$

$$T = 20\text{ g} = 200\text{ N}$$

Since both balances are light, both the scales will read 20 kg.

26. Find the reading of spring balance in the adjoining Fig. 3.22, when pulley and strings are ideal.

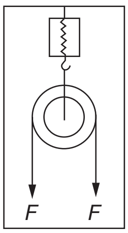


Fig. 3.22

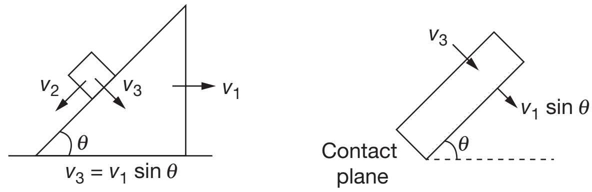
Solution: $2F$.

WEDGE CONSTRAINT

Conditions

1. There is a regular contact between two objects.
2. Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.



In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects are always equal if there is no deformation and they remain in contact.

SOLVED EXAMPLE

27. A rod of mass 2 m moves vertically downward on the surface of wedge of mass as shown in Fig. 3.23. Find the relation between velocity of rod and that of the wedge at any instant.

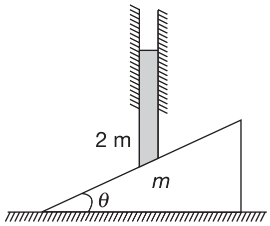
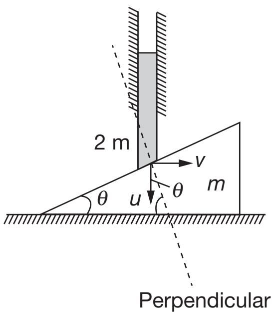


Fig. 3.23

Solution:

Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.



$$u \cos\theta = v \sin\theta$$

$$\frac{u}{v} = \tan\theta$$

$$u = v \tan\theta.$$

NEWTON'S LAW FOR A SYSTEM

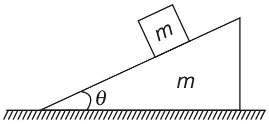
$$\vec{F}_{\text{ext}} = m_1\vec{a_1} + m_2\vec{a_2} + m_3\vec{a_3} + \ldots\ldots$$

$\vec{F}_{\text{ext}}$ = Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and $\vec{a_1}, \vec{a_2}, \vec{a_3}$ are the acceleration of the objects, respectively.

SOLVED EXAMPLES

28. The block of mass m slides on a wedge of mass m which is free to move on the horizontal ground. Find the accelerations of wedge and block (All surfaces are smooth).

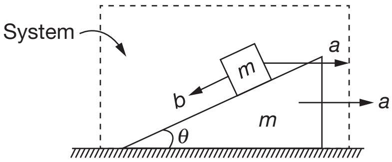


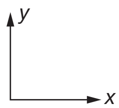
Solution:

Let $a \Rightarrow$ acceleration of wedge

$b \Rightarrow$ acceleration of block with respect to wedge

Taking block and wedge as a system and applying Newton's law in the horizontal direction,

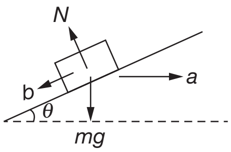




$$F_x = m_1\vec{a}_{1x} + m_2 = \vec{a}_{2x}\ 0$$

$$0 = ma + m(a - b\cos\theta) \qquad (1)$$

here a and b are two unknowns, so for making second equation, we draw FBD of block.



Using Newton's second law along inclined plane

$$mg\sin\theta = m\,(b - a\cos\theta) \qquad (2)$$

Now solving equations (1) and (2), we will get

$$a = \frac{mg\sin\theta\cos\theta}{m(1+\sin^2\theta)} = \frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}$$

and $$b = \frac{2g\sin\theta}{(1+\sin^2\theta)}$$

Thus in vector form:

$$\vec{a}_{\text{wedge}} = a\hat{i} = \left(\frac{g\sin\theta\cos\theta}{1+\sin^2\theta}\right)\hat{i}$$

$$\vec{a}_{\text{block}} = (a - b\cos\theta)\,\hat{i} - b\sin\theta\,\hat{j}$$

$$\vec{a}_{\text{block}} = -\frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}\,\hat{i} - \frac{2g\sin^2\theta}{(1+\sin^2\theta)}\,\hat{j}.$$

29. For the arrangement shown in Fig. 3.24 when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.

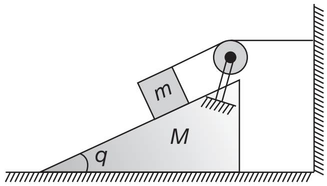
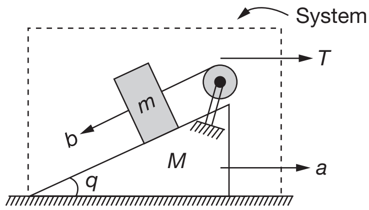


Fig. 3.24

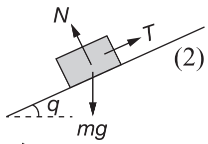
Solution:

Considering block and wedge as a system and using Newton's law for the system along x-direction



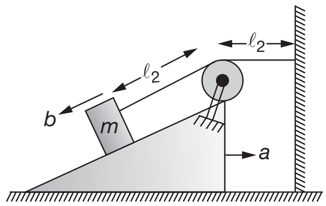
$$T = Ma + m\,(a - b\cos\theta) \qquad (1)$$

FBD of m along the inclined plane



$$mg\sin\theta - T = m\,(b - a\cos\theta) \qquad (2)$$

Using string constraint equation.



$$\ell_1 + \ell_2 = \text{constant}$$

$$\frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_2}{dt^2} = 0$$

$$b - a = 0 \qquad (3)$$

Solving above Equations (1), (2) and (3), we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}.$$

NEWTON'S LAW FOR NON-INERTIAL FRAME

$$\vec{F}_{\text{Real}} + \vec{F}_{\text{Pseudo}} = m\vec{a}$$

Net sum of real and pseudo force is taken in the resultant force.

$\vec{a}$ = Acceleration of the particle in the non-inertial frame

$$\vec{F}_{\text{Pseudo}} = -m\vec{a}_{\text{Frame}}$$

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's third law.

Reference Frame

A frame of reference is basically a coordinate system in which motion of object is analysed. There are two types of reference frames.

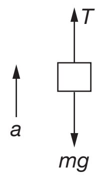
1. **Inertial reference frame:** Frame of reference moving with constant velocity.
2. **Non-inertial reference frame:** A frame of reference moving with non-zero acceleration.

SOLVED EXAMPLES

30. A lift having a simple pendulum attached with its ceiling is moving upward with constant acceleration a. What will be the tension in the string of pendulum with respect to a boy inside the lift and a boy standing on earth when mass of bob of simple pendulum is m?

Solution:

FBD of bob (with respect to ground)



$$T - mg = ma$$

$$T = mg + ma \qquad (1)$$

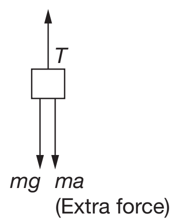
With respect to boy inside the lift, the acceleration of bob is zero.

So the above equation is written as follows:

$$T - mg = m \cdot (0).$$

$$\therefore \qquad T = mg$$

It is incorrect to tell the value of tension in string as mg. For correct results, a free body diagram is made by using Newton's second law.

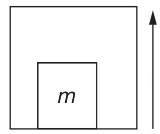


$$T = mg + ma \qquad (2)$$

By using this **extra force**, equations (1) and (2) give the same result. This **extra force** is called **pseudo force**. This **pseudo force** is used when a problem is solved with an accelerating frame (Non-inertial).

> **NOTE**
>
> Magnitude of pseudo force = mass of system × acceleration of frame of reference.
>
> **Direction of force**
>
> Opposite to the direction of acceleration of frame of reference (not in the direction of motion of frame of reference).

31. A box is moving upward with retardation $a < g$, find the direction and magnitude of "pseudo force" acting on block of mass m placed inside the box. Also calculate normal force exerted by surface on block

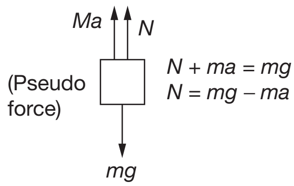


Solution:

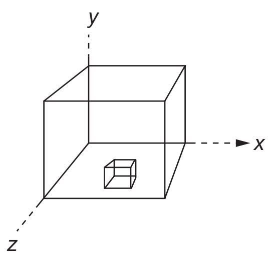
Pseudo force acts opposite to the direction of acceleration of reference frame.

pseudo force = ma in upward direction

FBD of m with respect to box (non-inertial).



32. A block of mass 2 kg is kept at rest on a big box moving with velocity $2\hat{i}$ and having acceleration $-3\hat{i} + 4\hat{j}$ m/s^2. Find the value of 'pseudo force' acting on block with respect to box.



Solution:

$$\vec{F} = -m\vec{a}_{\text{frame}} = -2(-3\hat{i} + 4\hat{j})$$

$$F = 6\hat{i} - 8\hat{j}.$$

33. All surfaces are smooth in the adjoining Fig. 3.25. Find F such that block remains stationary with respect to wedge.

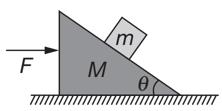


Fig. 3.25

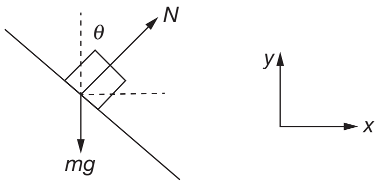
Solution:

Acceleration of (block + wedge) is a = $\dfrac{F}{(M+m)}$

Let us solve the problem by using both frames.

From inertial frame of reference (Ground)

FBD of block with respect to ground (Apply real forces)



with respect to ground, block is moving with an acceleration a.

$\therefore \quad \sum F_y = 0 \Rightarrow N\cos\theta = mg \quad (1)$

and $\quad \sum F_x = ma \Rightarrow N\sin\theta = ma \quad (2)$

From Equations (1) and (2)

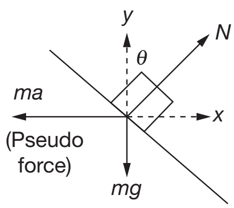
$$a = g\tan\theta$$

$$\therefore \quad F = (M+m)a$$

$$= (M+m)g\tan\theta$$

From non-inertial frame of reference (Wedge)

FBD of block with respect to wedge (real forces + pseudo force)



with respect to wedge, block is stationary

$\therefore \quad \sum F_y = 0 \Rightarrow N\cos\theta = mg \quad (3)$

$\quad \sum F_x = 0 \Rightarrow N\sin\theta = ma \quad (4)$

From Equations (3) and (4), we will get the same result,

i.e. $\quad F = (M+m)\,g\tan\theta.$

FRICTION

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.

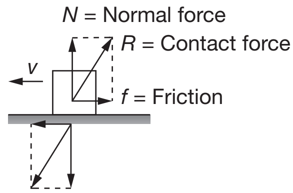


Fig. 3.26

The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (see Fig. 3.26). The perpendicular component is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

Therefore, if R is contact force then

$$R = \sqrt{f^2 + N^2}$$

Reasons for Friction

1. Inter-locking of extended parts of one object into the extended parts of the other object.
2. Bonding between the molecules of the two surfaces or objects in contact.

Friction Force is of Two Types

1. Kinetic
2. Static

Kinetic Friction Force

Kinetic friction exists between two contact surfaces only when there is **relative motion** between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

Direction of Kinetic Friction on an Object

It is opposite to the relative velocity of the object with respect to the other object in contact considered.

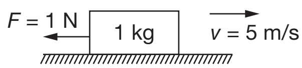
Its direction is not opposite to the force applied; it is opposite to the relative motion of the body considered which is in contact with the other surface.

SOLVED EXAMPLES

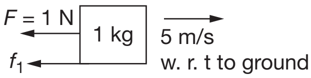
34. Find the direction of kinetic friction force

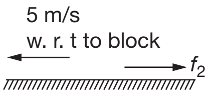
(A) On the block, exerted by the ground.

(B) On the ground, exerted by the block.



Solution:

(A) 

(B) 

where f_1 and f_2 are the friction forces on the block and ground, respectively.

35. The correct relation between magnitude of f_1 and f_2 is

(A) $f_1 > f_2$

(B) $f_2 > f_1$

(C) $f_1 = f_2$

(D) not possible to decide due to insufficient data.

Solution:

By Newton's third law, the above friction forces are action–reaction pair and equal but opposite to each other in direction. Hence (C).

Also note that the direction of kinetic friction has nothing to do with applied force F.

36. All surfaces as shown in the Fig. 3.27 are rough. Draw the friction force on A and B

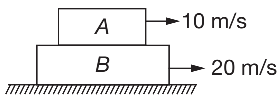


Fig. 3.27

Solution: 

Kinetic friction acts in such a way as to reduce relative motion.

37. Find out the distance travelled by the blocks shown in the Fig. 3.28 before it stops.

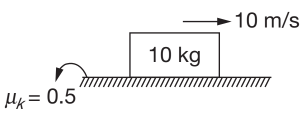


Fig. 3.28

Solution:

$f_x = \mu_k N$

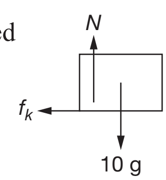
$\mu = \mu_s = \mu_k$ when not mentioned

$f_x = 0.5 \times 100 = 50$ N

$N - 10\text{ g} = 0$

$N = 100$ N

$F = ma$



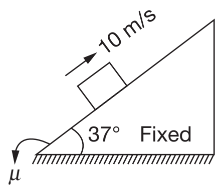
$50 = 10a \Rightarrow a = 5$

$\therefore \quad v_2 = u_2 + 2as$

$0^2 = 10^2 + 2(-5)(5)$

$\therefore \quad S = 10$ m.

38. Find out the distance travelled by the block on incline before it stops. Initial velocity of the block is 10 m/s and co-efficient of friction between the block and incline is $m = 0.5$.



Solution:

$N = mg \cos 37°$

$\therefore \quad mg \sin 37° + \mu N = ma$

$a = 10$ m/s^2 down the incline

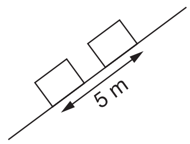
Now $\quad v^2 = u^2 + 2as$

$0 = 10^2 + 2(-10)S$

$\therefore \quad S = 5$ m.

39. Find the time taken in the above case by the block to reach the initial position.

Solution:



$a = g \sin 37° - \mu g \cos 37°$

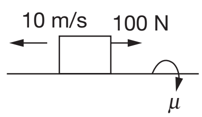
$\therefore \quad a = 2$ m/s^2 down the incline

$\therefore \quad S = ut + \frac{1}{2}at^2$

$\Rightarrow \quad S = \frac{1}{2} \times 2 \times t^2$

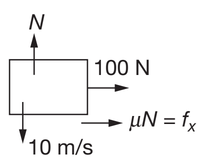
$\therefore \quad t = \sqrt{5}$ s.

40. A block is given a velocity of 10 m/s and a force of 100 N in addition to friction; force is also acting on the block. Find the retardation of the block?



Solution:

As there is relative motion, kinetic friction will act to reduce this relative motion.



$f_k = \mu N = 0.1 \times 10 \times 10 = 10$ N

$100 + 10 = 10a$

$a = \frac{110}{10} = 11$ m/s^2.

41. Find out the acceleration of the block as shown in the Fig. 3.29.

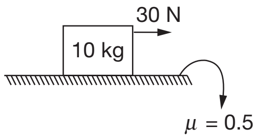


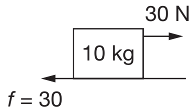
Fig. 3.29

Solution:

$$0 \le f \le f_{max}$$

$$0 \le f_s \le \mu N$$

$\therefore$ $30 - f = 0$



Hence $a = 0$.

Static Friction

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

For example, consider a bed inside a room, when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exist static friction force acting in the opposite direction of the applied force.

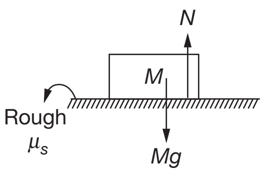
SOLVED EXAMPLES

42. What is value of static friction force on the block?

Solution:

In horizontal direction as acceleration is zero. Therefore, $\Sigma F = 0$.

$\therefore$ $f = 0$



Direction of static friction force:

The static friction force on an object is opposite to its impending motion relative to the surface.

Following steps should be followed in determining the direction of static friction force on an object.

1. Draw the free body diagram with respect to the other object on which it is kept.
2. Include pseudo force also if contact surface is accelerating.
3. Decide the resultant force and the component parallel to the surface of this resultant force.
4. The direction of static friction is opposite to the above component of resultant force.

Note: Here once again the static friction is involved when there is no relative motion between two surfaces.

43. In the following Fig. 3.30, an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction, if the object does not move.

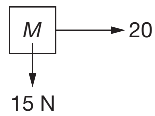
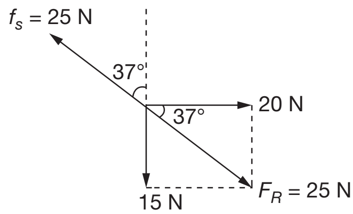


Fig. 3.30

Solution:

In the above problem, we first draw the FBD to find the resultant force.



As the object does not move, this is not a case of limiting friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in above by f_s. Its magnitude is equal to 25 N.

Magnitude of Kinetic and Static Friction

Kinetic Friction

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact. If the surfaces are smooth, μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

Static Friction

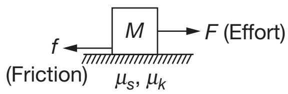
The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. This means it is a variable and self-adjusting force. However, it has a maximum value called limiting friction.

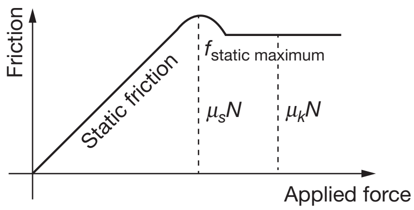
$$f_{max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0 \le f_s \le f_{smax}$$

Here μ_s and μ_k are proportionality constants. μ_s is called coefficient of static friction and μ_k is called coefficient of kinetic friction. They are dimensionless quantities independent of shape and area of contact. It is a property of the two contact surfaces. $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned, then $\mu_s = \mu_k$ can be taken. Value of μ can be from 0 to ∞.



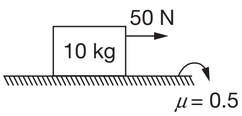


Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and Teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

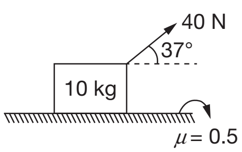
SOLVED EXAMPLES

44. Find acceleration of block. Initially, the block is at rest.

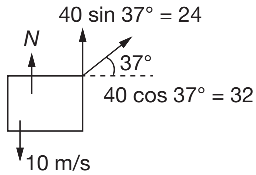


Solution: Zero.

45. Find out acceleration of the block. Initially, the block is at rest.



Solution:



$N + 24 - 100 = 0$

0 for vertical direction

$\therefore \quad N = 76$ N

Now $\quad 0 \le$ of μ_s N

$0 \le f_s \le 76 \times 0.5$

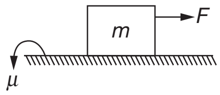
$0 \le f \le 38$ N

$\therefore \quad 32 < 38$

Hence $\quad f = 32$

∴ Acceleration of block is zero.

46. Find out acceleration of the block for different ranges of F.



Solution:

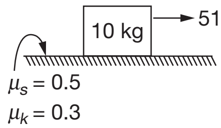
$$0 \le f \le \mu_S \text{N}$$

$$0 \le f \le \mu_S \text{mg}$$

$$a = 0 \quad \text{if} \quad F \le \mu_S \text{mg}$$

$$a = \frac{F - \mu Mg}{M} \quad \text{if} \quad F > \mu \text{Mg}.$$

47. Find out acceleration of the block. Initially, the block is at rest.



Solution:

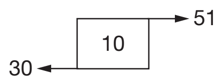
$$0 \le f_s \le \mu_S \text{N}$$

$$0 \le f_s \le 50$$

Now $\quad 51 > 50$

∴ Block will move but if the block starts moving, then kinetic friction is involved.

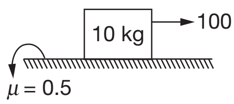
$$K_F = \mu_k N = 0.3 \times 100 = 30 \text{ N}$$



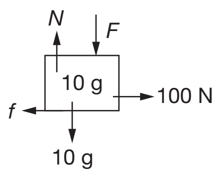
$\therefore \quad 51 - 30 = 10a$

$\therefore \quad a = 2.1 \text{ m/s}^2.$

48. Find out the minimum force that must be applied on the block vertically downwards so that the block doesn't move.



Solution:



$$100 - f_s = 0$$

$$\therefore \quad f_s = 100 \qquad (1)$$

$$F + 10\text{ g} = N \Rightarrow N = 100 + F \qquad (2)$$

Now $\quad 0 \le f_s \le \mu N$

$$0 \le f_s \le \mu N$$

$$100 \le 0.5\text{ N}$$

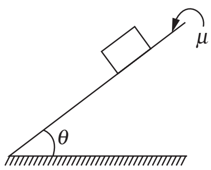
$$100 \le 0.5\text{ N }[100 + F]$$

$$200 \le 100 + F$$

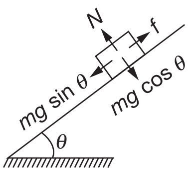
$$F \ge 100\text{ N}$$

$\therefore$ Minimum $\quad F = 100$ N.

49. The angle of inclination is slowly increased. Find out the angle at which the block starts moving.



Solution:



$$0 \le \text{and} \le \mu_s N$$

$$mg\sin\theta > f_{smax}$$

$$mg\sin\theta > \mu N$$

$$mg\sin\theta > \mu\, mg\cos\theta$$

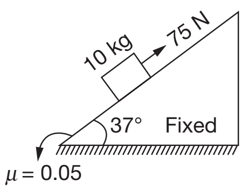
$$\therefore \quad \tan\theta > \mu$$

$$\theta = \tan^{-1}\mu$$

For $\tan\theta \le \mu$ no sliding on inclined plane.

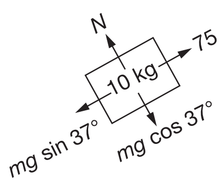
This method is used for finding out the value of μ practically.

50. Find out the acceleration of the block, if the block is initially at rest.

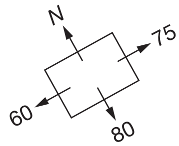


Solution:

FBD of the block excluding friction



$$N = 10\text{ g}\cos 37° = 80\text{ N}$$



Now $\quad 0 \le f_s \le \mu N$

$$0 \le f_s \le 0.5 \times 80$$

$$\therefore \quad f_s \le 40\text{ N}$$

We will put value of f in the last, i.e. in the direction opposite to resultant of other forces. f acts down the incline and its value is = 75 – 60 = 15 N.

51. In the above problem, how much force should be added to 75 N force so that block starts to move up the incline.

Solution:

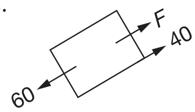
$$\therefore \quad 60 + 40 = 75 + f_s$$

$$\therefore \quad f_s = 25\text{ N.}$$

52. In the above problem, what is the minimum force by which 75 N force should be replaced so that the block does not move.

Solution:

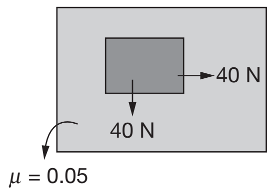
In this case, the block has a tendency to move downwards. Hence, friction acts upwards.



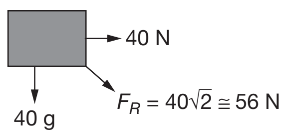
$$\therefore \quad F + 40 = 60$$

$$\therefore \quad F = 20\text{ N.}$$

53. Top view of a block on a table is shown ($g = 10$ m/s^2). Find out the acceleration of the block.



Solution:



Now $\quad f_s \le \mu N$

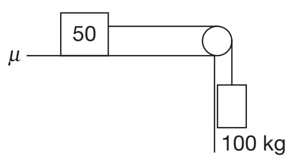
$$\therefore \quad f_s \le 50$$

$$F_R > f_{smax}$$

Hence the block will move.

$$a = \frac{40\sqrt{2} - 50}{10} = \left(4\sqrt{2} - 5\right)\text{ m/s}^2.$$

54. Find minimum μ so that the blocks remain stationary.

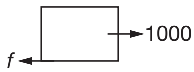


Solution:

$$T = 100\text{ g} = 1000\text{ N}$$

$\therefore$ $f = 1000$ to keep the block stationary

Now $f_{max} = 1000$

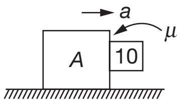


$$\mu N = 1000$$

$$\mu = 2$$

Can μ be greater than 1?
Yes $0 < \mu \le \infty$.

55. Find out minimum acceleration of block A so that the 10 kg block doesn't fall.



Solution:

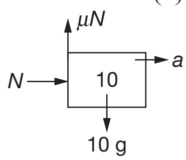
Applying NL in horizontal direction

$$N = 10a \quad (1)$$

Applying NL in vertical direction

$$10\text{ g} = \mu N \quad (2)$$

$$10\text{ g} = \mu 10a$$



from (1) and (2)

$\therefore$ $$a = \frac{g}{\mu} = 20\text{ m/s}^2.$$

56. Find the tension in the string in situation as shown in the Fig. 3.31 below. Forces 120 N and 100 N start acting when the system is at rest and the maximum value of static friction on 10 kg is 90 N and that on 20 kg is 60 N?

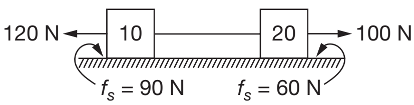
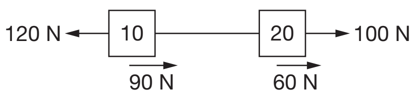


Fig. 3.31

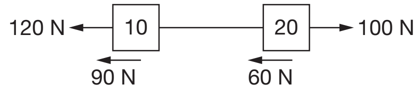
Solution:

(A) Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore, the assumption is not valid.



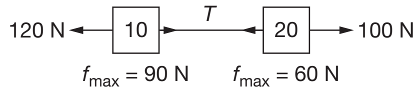
Above assumption is not possible as net force on system comes towards right. Hence, system is not moving towards left.

(B) Similarly, let us assume that system moves towards right.

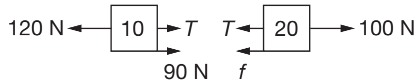


Above assumption is also not possible as net force on the system is towards left in this situation. Hence, assumption is again not valid.

Therefore, it can be concluded that the system is stationary.



Assuming that the 10 kg block reaches limiting friction first using FBDs.



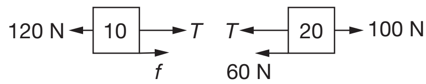
$$120 = T + 90 \Rightarrow T = 30\text{ N}$$

Also $$T + f = 100$$

$\therefore$ $$30 + f = 100$$

$\Rightarrow$ f = 70 N, which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting, we have



$$T + 60 = 100\text{ N}$$

$\Rightarrow$ $$T = 40\text{ N}$$

Also $$f + T = 120\text{ N}$$

$\Rightarrow$ $$f = 80\text{ N}$$

This is acceptable as static friction at this surface should be less than 90 N.

Hence, the tension in the string is $T = 40$ N.

57. In the following Fig. 3.32, force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s.

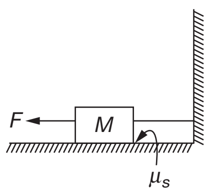
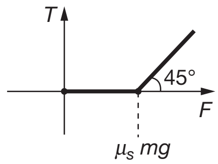


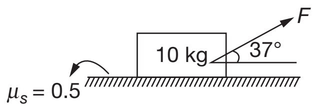
Fig. 3.32

Solution:



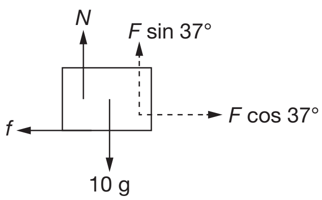
As the external force F is gradually increased from zero, it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s mg$, the further increase in F is transferred to the string.

58. Force F is gradually increased from zero. Determine whether the block will first slide or lift up?



Solution:

There are minimum magnitude of forces required both in horizontal and vertical direction either to slide or lift up the block. The block will first slide on lift up and will depend upon the minimum magnitude of force that is lesser.



For vertical direction to start lifting up

$$F \sin 37° + N - Mg \geq 0.$$

N becomes zero just lifting condition.

$$F_{\text{lift}} \geq \frac{10g}{3/5}$$

$$\therefore \qquad F_{\text{lift}} \geq \frac{500}{3} \text{ N}$$

For horizontal direction to start sliding

$$F \cos 37 \geq \mu_s N$$

$$F \cos 37° > 0.5 \,[10\text{g} - F \sin 37°]$$

$$(\because N = 10 \text{ g} - \text{F} \sin 37°)$$

Hence $$F_{\text{slide}} > \frac{50}{\cos 37° + 0.5 \sin 37°}$$

$$F_{\text{slide}} > \frac{500}{11} \text{ N}$$

$$F_{\text{lift}} > \frac{500}{3} \text{ N.}$$

$$\Rightarrow \qquad F_{\text{slide}} < F_{\text{lift}}$$

Therefore, the block will begin to slide before lifting.

Two block Problems

SOLVED EXAMPLES

59. Find the acceleration of the two blocks. The system is initially at rest and the friction co-efficients are as shown in the Fig. 3.33?

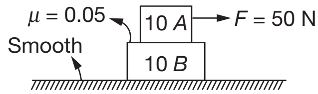


Fig. 3.33

Method of solving

Step 1: Make force diagram.

Step 2: Show static friction force by f because value of friction is not known.

Step 3: Calculate separately for two cases.

Case I: Move together

Step 4: Calculate acceleration.

Step 5: Check value of friction for above case.

Step 6: If required friction is less than available, it means they will move together else move separately.

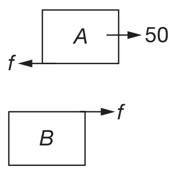
Step 7(a): Above acceleration will be common for both

Case II: Move separately

Step 7(b): If they move separately then kinetic friction is involved, whose value is μN.

Step 8: Calculate acceleration for above case.

Solution:



$$f_{\max} = \mu N$$

$$\therefore \qquad f \leq 50 \text{ N} \quad \text{(available friction)}$$

Move together	**Move separately**
(i) $a = \dfrac{50}{10+10} = 2.5 \text{ m/s}^2$	No need to calculate

(ii) Check friction for B:

$$f = 10 \times 2.5 = 25$$

25 N is required which is less than available friction hence they will move together.

and $$a_A = a_B = 2.5 \text{ m/s}^2.$$

60. Find the acceleration of the two blocks. The system is initially at rest and the friction co-efficients are as shown in the Fig. 3.34?

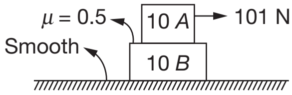
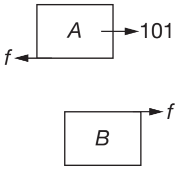


Fig. 3.34

Solution:



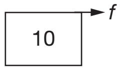
$f_{max} = 50$ N

$\therefore \quad f \leq 50$ N

(A) If they move together

$$a = \frac{101}{20} = 5.05 \text{ m/s}^2$$

(B) Check friction on B

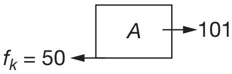


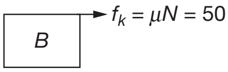
$f = 10 \times 5.05 = 50.5$ (required)

$50.5 > 50$ (therefore, required > available)

Hence they will not move together.

(C) Hence they move separately, kinetic friction is involved.





$$\therefore \quad f \quad \text{or} \quad a_A = \frac{101 - 50}{10} = 5.1 \text{ m/s}^2$$

$$\Rightarrow \quad a_B = \frac{50}{10} = 5 \text{ m/s}^2$$

Also $\quad a_A > a_B$ as force is applied on A.

61. Find the acceleration of the two blocks. The system is initially at rest and the friction co-efficient are as shown in the Fig. 3.35.

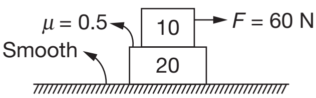


Fig. 3.35

Solution:

Move together	**Move seperately**
$a = \frac{60}{30} = 2$ m/s^2	No need to calculate.

Check friction on 20 kg.

$f = 20 \times 2$

$f = 40$ (which is required)

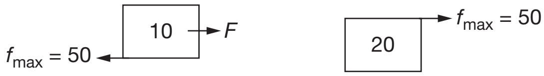
$40 < 50$ (therefore, required < available)

$\therefore$ will move together.

62. In above example, find the maximum F for which two blocks will move together.

Solution:

Observing the critical situation where friction becomes limiting.

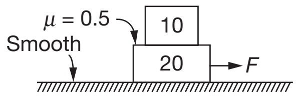


$$\therefore \quad F - f_{max} = 10a \quad (1)$$

$$f_{max} = 20a \quad (2)$$

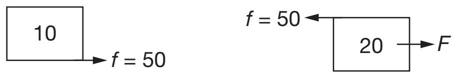
$$\therefore \quad F = 75 \text{ N.}$$

63. Initially, the system is at rest. Find out the minimum value of F for which sliding starts between the two blocks.



Solution:

At just sliding condition, limiting friction is acting.



$$F - 50 = 20a \quad (1)$$

$$f = 10a \quad (2)$$

$$50 = 10a$$

$$\therefore \quad a = 5 \text{ m/s}^2$$

Hence, $\quad F = 50 + 20 \times 5 = 150$ N

$\therefore \quad F_{min} = 150$ N.

64. In the Fig. 3.36 given below, force F is applied horizontally on lower block is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Taking $g = 10$ m/s^2).

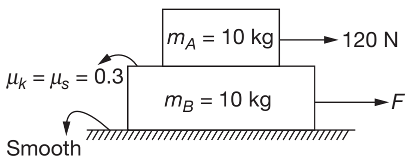


Fig. 3.36

Solution:

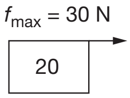
In the above situation, we see that the maximum possible value of friction between the blocks is $\mu_s m_A g = 0.3 \times 10 \times 10 = 30$ N.

Case I: When $F = O$.

Considering that there is no slipping between the blocks, the acceleration of system will be

$$a = \frac{120}{20+10} = 4 \text{ m/s}^2$$

But the maximum acceleration of B can be obtained by the following force diagram.



$$a_B = \frac{30}{20} = 1.5 \text{ m/s}^2$$ (∵ only friction force by block A is responsible for producing acceleration in block B)

Because $4 > 1.5 \text{ m/s}^2$, we can conclude that the blocks do not move together.

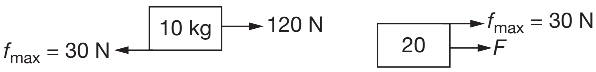
Now drawing the FBD of each block find out individual accelerations.

$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2 \text{ towards right}$$

$$a_B = \frac{30}{20} = 1.5 \text{ m/s}^2 \text{ towards right.}$$

Case II. F is increased from zero till the two blocks just start moving together.

As the two blocks move together, the friction is static in nature and its value is limiting. FBD in this case will be



$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2$$

$$a_B = \frac{F+30}{20} = a_A$$

$$\Rightarrow \quad \frac{F+30}{20} = 9$$

$$\therefore \quad F = 150 \text{ N}$$

Hence when $0 < F < 150$ N, the blocks do not move together and the friction is kinetic. As F increases, acceleration of block B increases from 1.5 m/s^2.

At $F = 150$ N, limiting static friction starts acting and the two blocks start moving together.

Case III: When F is increased above 150 N.

In this scenario, the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case, the FBD is as follows:

$$a_A = a_B = \frac{120-f}{10} = \frac{F+f}{20}$$

∴ when friction force f gets reduced to zero, the above accelerations become

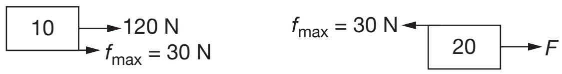
$$a_A = \frac{120}{10} = 12 \text{ m/s}^2$$

$$a_B = \frac{F}{20} = a_A = 12 \text{ m/s}^2$$

$$\therefore \quad F = 240 \text{ N}$$

Hence when $150 \le F \le 240$ N, the static friction force continuously decreases from maximum to zero at $F = 240$ N. The accelerations of the blocks increase from 9 m/s^2 to 12 m/s^2 during the change of force F.

Case IV: When F is increased again from 240 N, the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. FBD at this juncture will be



The blocks move together, therefore

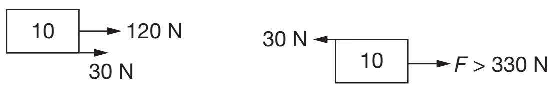
$$a_A = \frac{120+30}{10} = 15 \text{ m/s}^2$$

$$a_B = \frac{F-30}{20} = a_A = 15 \text{ m/s}^2$$

$$\therefore \quad \frac{F-30}{20} = 15 \text{ m/s}^2$$

Hence, $F = 330$ N.

Case V: When F is increased beyond 330 N. In this case, the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).



$$\therefore \quad a_A = 15 \text{ m/s}^2 \text{ which is constant}$$

$$a_B = \frac{F-30}{20} \text{ m/s}^2$$

where $F > 330$ N.

DYNAMICS OF UNIFORM CIRCULAR MOTION

Centripetal Force

If a particle moves on a circular path with a constant speed, its motion is called as a uniform circular motion. In this motion angular speed of the particle is also constant. Linear acceleration in such motion will not have any tangential component, only radial or centripetal acceleration the particle possesses. Therefore in case of uniform circular motion the particle will have acceleration towards the center only and is called as centripetal acceleration having magnitude $\frac{v^2}{R}$ or $\omega^2 R$. The magnitude of acceleration remains constant but its direction changes with time.

If a particle moving on circular path is observed from an inertial frame it has an acceleration $\omega^2 R$ or $\frac{v^2}{R}$ acting towards center. Therefore from Newton's second law of motion, there must be a force acting on the particle towards the center of magnitude $m\omega^2 R$ or $\frac{mv^2}{R}$. This required force for a particle to move on circular path is called as **centripetal force**.

$$\therefore \qquad \text{centripetal force} = \frac{mv^2}{R} \qquad (3.1)$$

The term 'centripetal force' merely a force towards center, it tells nothing about its nature or origin. The centripetal force may be a single force due to a rope, a string, the force of gravity, friction and so forth or it may be resultant of several forces. Centripetal force is not a new kind of force, just as 'upward force' or a 'downward force' is not a new force. Therefore while analyzing motion of particle undergoing circular motion we need not to consider centripetal force as a force, we need to consider only external forces.

SOLVED EXAMPLE

65. A ball of mass 0.5 kg is attached to the end of a cord whose length is 1.50 m. The ball is whirled in a horizontal circle. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed the ball can have before the cord breaks?

Solution:

Because the centripetal force in this case is the force T exerted by the cord on the ball, we have

$$T = m\frac{v^2}{r}$$

Solving for v, we have

$$v = \sqrt{\frac{Tr}{m}}$$

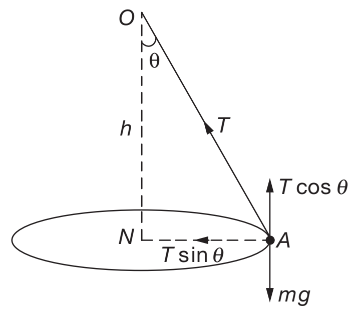
The maximum speed that the ball can have corresponds to the maximum tension. Hence, we find

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\,\text{N})\,(1.50\,\text{m})}{0.500\ \text{kg}}} = 12.2\ \text{m/s}.$$

Some Important Uniform Circular Motions

Conical Pendulum

It consists of a string OA whose upper end O is fixed and a bob is tied at the free end. When the bob is drawn aside and given a horizontal push let it describe a horizontal circle with uniform angular velocity ω in such a way that the string makes an angle θ with vertical. As the string traces the surface of a cone of semi-vertical angle θ it is called conical pendulum. Let T be the tension in string, ℓ be the length and r be the radius of the horizontal circle described. The vertical component of tension balances the weight and the horizontal component supplies the centripetal force.



$$T\cos\theta = mg$$

$$T\sin\theta = mr\omega^2$$

$$\therefore \qquad \tan\theta = \frac{r\omega^2}{g}$$

$$\omega = \sqrt{\frac{g\tan\theta}{r}}$$

$$r = \ell\sin\theta$$

and $$\omega = \frac{2\pi}{T}$$

T being the period i.e., time for one revolution.

$$\therefore \qquad \frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{\ell\sin\theta}}$$

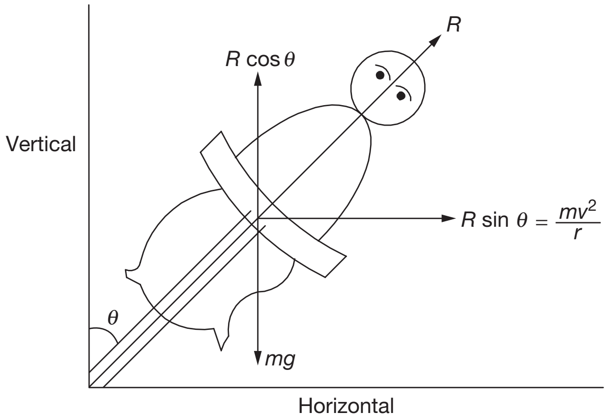
$$T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$$

$$= 2\pi\sqrt{h/g}\text{, where } h = \ell\cos\theta.$$

Motion of a Cyclist on a Circular Path

Let a cyclist moving on a circular path of radius r bend away from the vertical by an angle θ.

R is the normal reaction from the ground. It can be resolved in the horizontal and vertical directions. The components are respectively equal to $R\sin\theta$ and $R\cos\theta$. The vertical component balances his weight mg. The horizontal component $R\sin\theta$ supplies the necessary force for making the circular path.



$$\therefore \quad R\sin\theta = \frac{mv^2}{r}$$

$$R\cos\theta = mg$$

$$\therefore \quad \tan\theta = v^2/rg$$

For less bending of the cyclist, v should be small and r should be great.

Banking of Roads

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction the roads are banked at the turn so that the outer part of the road is somewhat lifted up as compared to the inner part. The surface of the road makes an angle θ with the horizontal throughout the turn. The Fig. 3.37 shows the forces acting on a vehicle when it is moving on the banked road. *ABC* is the section of the road having a slope θ. R is the normal reaction and mg is the weight.

For vertical equilibrium, $R\cos\theta = \text{mg}$

The horizontal components $R\sin\theta$ is the required centripetal force $\frac{mv^2}{r}$

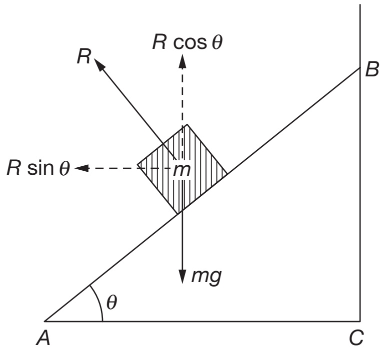


Fig. 3.37

$$R\sin\theta = \frac{mv^2}{r}$$

$$\therefore \quad \tan\theta = v^2/rg$$

Above equation gives the angle of banking required which eliminates the lateral thrust in case of trains on rails or friction in case of road vehicles when rounding a curve.

Overturning and Skidding of Cars

When a car takes a turn round a bend, whether the car tends to skid or topple depends on different factors. Let us consider the case of a car whose wheels are $2a$ metre apart, and whose centre of gravity is h metres above the ground. Let the coefficient of friction between the wheels and the ground be μ.

Figure 3.38 represents the forces on the car.

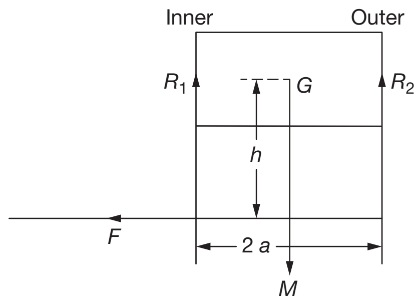


Fig. 3.38

1. The weight Mg of the car acts vertically downwards through the centre of gravity G of the car.
2. The normal reactions of the ground R_1 and R_2 act vertically upwards on the inner and outer wheels respectively.
3. The force of friction F between the wheels and the ground act towards the centre of the circle of which the road forms a part

Let the radius of the circular path be r, and the speed of the car be v.

Considering the vertical forces, since there is no vertical acceleration,

$$R_1 + R_2 = Mg \tag{3.2}$$

The horizontal force F provides the centripetal force for motion in a circle.

Therefore
$$F = \frac{Mv^2}{r} \tag{3.3}$$

Taking moments about G, if there is to be no resultant turning effect about the centre of gravity,

$$Fh + R_1 a = R_2 a \tag{3.4}$$

Conditions for no Skidding

From equation (3.3), it is seen that as the speed increases, the force required to keep the car moving in the circle also increases. However, there is a limit to the frictional force F, because,

$$F_{max} = \mu (R_1 + R_2)$$

Substituting from equation (3.2),

$$F_{max} = \mu Mg$$

Substituting from equation (3.3),

$$\frac{Mv^2}{r} = \mu Mg$$

$$\therefore \quad v^2 = \mu rg$$

or
$$v = \sqrt{\mu rg}$$

This expression gives the maximum speed v with which the car could take the circular path without skidding.

Conditions for no Overturning

From equation (3.4),

$$(R_2 - R_1)a = Fh$$

or
$$R_2 - R_1 = \frac{Fh}{a} = \frac{Mv^2}{r} \cdot \frac{h}{a} \tag{3.5}$$

But
$$R_2 + R_1 = Mg$$

Adding,

$$2R_2 = Mg + \frac{Fh}{a} = Mg + \frac{Mv^2}{r} \cdot \frac{h}{a} \tag{3.6}$$

$$2R_2 = M\left(g + \frac{v^2 h}{ra}\right) \tag{3.7}$$

$$R_2 = \frac{1}{2} M\left(g + \frac{v^2 h}{ra}\right)$$

Substituting for R_2 in equation (3.5),

$$R_2 - R_1 = \frac{1}{2} M\left(g + \frac{v^2 h}{ra}\right) - R_1 = \frac{Mv^2 h}{ra}$$

$$R_1 = \frac{1}{2} M\left(g + \frac{v^2 h}{ra}\right) - \frac{Mv^2 h}{ra}$$

$$= \frac{1}{2} M\left(g + \frac{v^2 h}{ra} - \frac{2v^2 h}{ra}\right)$$

$$= \frac{1}{2} M\left(g - \frac{v^2 h}{ra}\right) \tag{3.8}$$

Equation (3.7) shows that the reaction R_2 is always positive. However, equation (3.8) shows that as the speed v increases, the reaction R_1 decreases, and when $\frac{v^2 h}{ra} = g$, R_1 becomes zero. This means that the inner wheel is no longer in contact with the ground, and the car commences to overturn outwards.

The maximum speed without overturning is given by

$$g = \frac{v^2 h}{ra}$$

$$v = \sqrt{\frac{gra}{h}}$$

The same expression applies also to the case of a train moving on rails in a circular path of radius r. Here $2a$ is the distance between the rails, and h the height of the centre of gravity above the rails.

Centrifugal Force

An observer in a rotating system is another example of a non-inertial observer. Suppose a block of mass m lying on a horizontal, frictionless turntable is connected to a string as in Fig. 3.39. According to an inertial observer, if the block rotates uniformly, it undergoes an acceleration of magnitude v^2/r, where v is its tangential speed. The inertial observer concludes that this centripetal acceleration is provided by the force exerted by the string T, and writes Newton's second law $T = mv^2/r$.

According to a non-inertial observer attached to the turntable, the block is at rest. Therefore, in applying Newton's second law, this observer introduces a fictitious outward force of magnitude mv^2/r. According to the non-inertial observer, this outward force balances the force exerted by the string and therefore $T - mv^2/r = 0$.

In fact, centrifugal force is a sufficient pseudo force only if we are analyzing the particles at rest in a uniformly rotating frame. If we analyze the motion of a particle that

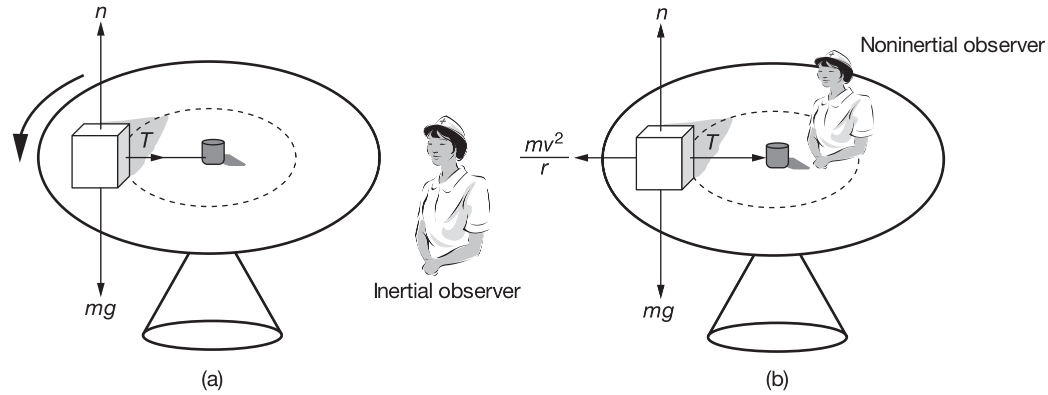


Fig. 3.39

moves in the rotating frame we may have to assume other pseudo forces together with the centrifugal force. Such forces are called **Coriolis forces**. The Coriolis force is perpendicular to the velocity of the particle and also perpendicular to the axis of rotation of the frame. Once again it should be remembered that all these pseudo forces, centrifugal or Coriolis are needed only if the working frame is rotating. If we work from an inertial frame there is no need to apply any pseudo force. There should not be a misconception that centrifugal force acts on a particle because the particle describes a circle.

Therefore when we are working from a frame of reference that is rotating at a constant angular velocity ω with respect to an inertial frame. The dynamics of a particle of mass m kept at a distance r from the axis of rotation we have to assume that a force $m\omega^2 r$ acts radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

You should be careful when using fictitious forces to describe physical phenomena. Remember that fictitious forces are used only in non-inertial frames of references. When solving problems, it is often best to use an inertial frame.

SOLVED EXAMPLES

66. A large mass M and a small mass m hang at the two ends of the string that passes through a smooth tube as shown in Fig. 3.40. The mass m moves around in a circular path, which lies in the horizontal plane. The length of the string from the mass m to the top of the tube is ℓ and θ is the angle this length makes with vertical. What should be the frequency of rotation of mass m so that M remains stationary?

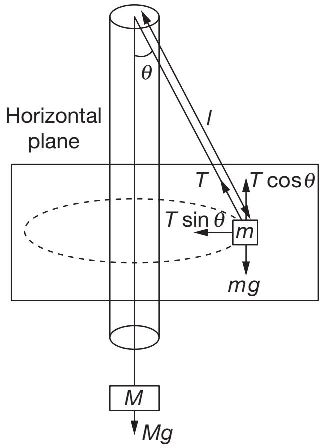


Fig. 3.40

Solution:

The forces acting on mass m and M are shown in Fig. 3.40. When mass M is stationary

$$T = Mg \tag{1}$$

where T is tension in string.

For the smaller mass, the vertical component of tension $T\cos\theta$ balances mg and the horizontal component $T\sin\theta$ supplies the necessary centripetal force.

$$T\cos\theta = mg \tag{2}$$

$$T\sin\theta = mr\omega^2 \tag{3}$$

ω being the angular velocity and r is the radius of horizontal circular path.

From (1) and (3), $Mg\sin\theta = mr\omega^2$

$$\omega = \sqrt{\frac{Mg\sin\theta}{mr}} = \sqrt{\frac{Mg\sin\theta}{ml\sin\theta}} = \sqrt{\frac{Mg}{ml}}$$

$$\text{Frequency of rotation} = \frac{1}{T} = \frac{1}{2\pi/\omega} = \frac{\omega}{2\pi}$$

$$\therefore \quad \text{Frequency} = \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}.$$

67. The 4 kg block in the Fig. 3.41 is attached to the vertical rod by means of two strings. When the system rotates about the axis of the rod, the two strings are extended as indicated in Fig. 3.41. How many revolutions per minute must the system make in order that the tension in upper string is 60 N. What is tension in the lower string?

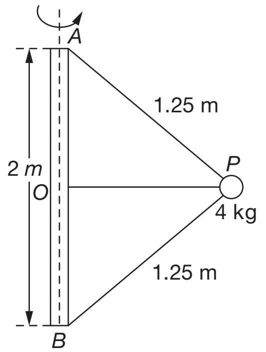


Fig. 3.41

Solution:

The forces acting on block P of mass 4 kg are shown in the Fig. 3.42. If θ is the angle made by strings with vertical, T_1 and T_2 tensions in strings for equilibrium in the vertical direction

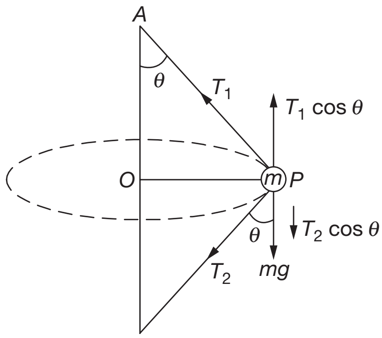


Fig. 3.42

$$T_1\cos\theta = T_2\cos\theta + mg$$

$$(T_1 - T_2)\cos\theta = mg$$

$$\cos\theta = \frac{1}{1.25} = \frac{4}{5}$$

$$\left[\because \cos\theta = \frac{OA}{AP} = \frac{1}{1.25}\right]$$

$$\therefore \quad T_1 - T_2 = \frac{mg}{\cos\theta} = \frac{5\,mg}{4} = \frac{5}{4}\times 4\times 9.8 = 49\text{ N}$$

Given $\quad T_1 = 60$ N

$$T_2 = T_1 - 49 = 60\text{ N} - 49\text{ N} = 11\text{ N}$$

The net horizontal force ($T_1 \sin\theta + T_2 \sin\theta$) provides the necessary centripetal force $m\omega^2 r$.

$$\therefore \quad (T_1 + T_2)\sin\theta = m\omega^2 r$$

$$\Rightarrow \quad \omega^2 = \frac{(T_1 + T_2)\sin\theta}{mr}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-(4/5)^2} = \frac{3}{5}$$

$$r = OP = \sqrt{1.25^2 - 1^2} = 0.75$$

$$\therefore \quad \omega^2 = \frac{(60+11)\frac{3}{5}}{4\times 0.75} = 14.2$$

$$\omega = \sqrt{14.2} = 3.768\text{ rad/s}$$

$$\text{Frequency of revolution} = \frac{\omega}{2\pi} = \frac{3.768}{2\times 3.14}$$

$$= 0.6\text{ rev/s or } 36\text{ rev/min.}$$

68. A metal ring of mass m and radius R is placed on a smooth horizontal table and is set rotating about its own axis in such a way that each part of ring moves with velocity v. Find the tension in the ring.

Solution:

Consider a small part ACB of the ring that subtends an angle $\Delta\theta$ at the centre as shown in Fig. 3.43. Let the tension in the ring be T.

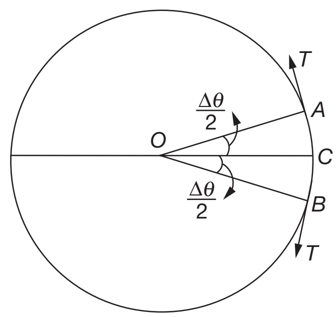


Fig. 3.43

The forces on this elementary portion ACB are

(A) tension T by the part of the ring left to A.
(B) tension T by the part of the ring right to B.
(C) weight $(\Delta m)\,g$.
(D) normal force N by the table.

As the elementary portion ACB moves in a circle of radius R at constant speed v its acceleration towards centre is $\frac{(\Delta m)\,v^2}{R}$.

Resolving the forces along the radius CO

$$T\cos\left(90^\circ-\frac{\Delta\theta}{2}\right)+T\cos\left(90^\circ-\frac{\Delta\theta}{2}\right)=\Delta m\frac{v^2}{R} \quad (1)$$

$$2T\sin\frac{\Delta\theta}{2}=\Delta m\frac{v^2}{R} \quad (2)$$

Length of the part $ACB = R\Delta\theta$. The mass per unit length of the ring is $\frac{m}{2\pi R}$

$\therefore$ mass of this portion ACB, $\Delta m = \frac{R\Delta\theta m}{2\pi R}=\frac{m\Delta\theta}{2\pi}$

Putting this value of Δm in (2),

$$2T\sin\frac{\Delta\theta}{2}=\frac{m\Delta\theta}{2\pi}\frac{v^2}{R}$$

$$\therefore \quad T=\frac{mv^2}{2\pi R}\left(\frac{\frac{\Delta\theta}{2}}{\sin\left(\frac{\Delta\theta}{2}\right)}\right)$$

Since $\left(\frac{\frac{\Delta\theta}{2}}{\sin\left(\Delta\frac{\theta}{2}\right)}\right)$ is equal to 1,

$$T=\frac{mv^2}{2\pi R}.$$

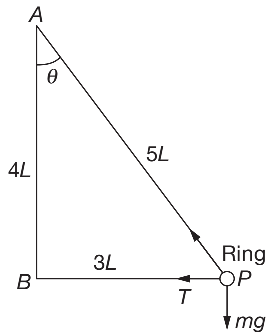
69. A small smooth ring of mass m is threaded on a light inextensible string of length 8L which has its ends fixed at points in the same vertical line at a distance $4L$ apart. The ring describes horizontal circles at constant speed with both parts of the string taut and with the lower portion of the string horizontal. Find the speed of the ring and the tension in the string. The ring is then tied at the midpoint of the string and made to perform horizontal circles at constant speed of $3\sqrt{gL}$. Find the tension in each part of the string.

Solution:

When the string passes through the ring, the tension in the string is the same in both the parts. Also from geometry

$$BP = 3L \quad \text{and} \quad AP = 5L$$

$$T\cos\theta=\frac{4}{5}T=mg \quad (1)$$



$$T+T\sin\theta=T\left(1+\frac{3}{5}\right)=\frac{8}{5}T$$

$$=\frac{mv^2}{BP}=\frac{mv^2}{3L} \quad (2)$$

Dividing (2) by (1),

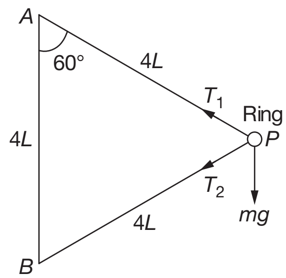
$$\frac{v^2}{3Lg}=2$$

$$v=\sqrt{6Lg}$$

From (1)

$$T=\frac{mg}{4/5}=\frac{5}{4}mg$$

In the second case, ABP is an equilateral triangle.



$$T_1\cos 60^\circ = mg + T_2\cos 60^\circ$$

$$T_1-T_2=\frac{mg}{\cos 60^\circ}=2mg \quad (3)$$

$$T_1\sin 60^\circ+T_2\sin 60^\circ=\frac{mv^2}{r}=\frac{9mgL}{4L\sin 60^\circ}$$

$$T_1+T_2=\frac{9mg}{4\sin^2 60^\circ}=3mg \quad (4)$$

Solving equations (3) and (4),

$$T_1=\frac{5}{2}mg;\ T_2=\frac{1}{2}mg.$$

70. A table with smooth horizontal surface is fixed in a cabin that rotates with angular speed ω in a circular path of radius R. A smooth groove AB of length L ($<< R$) is made on the surface of table as shown in Fig. 3.44.

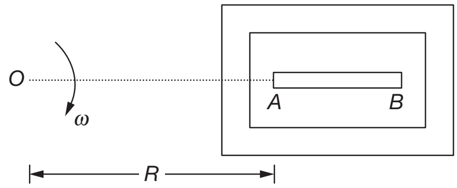


Fig. 3.44

A small particle is kept at the point A in the groove and is released to move, find the time taken by the particle to reach the point B.

Solution:

Let us analyse the motion of particle with respect to table which is moving with cabin with an angular speed of ω. Along AB centrifugal force of magnitude $m\omega^2R$ will act at A on the particle which can be treated as constant from A to B as $L << R$.

$\therefore$ acceleration of particle along AB with respect to cabin $a = \omega^2R$ (constant)

Required time t is given by

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow \quad L = 0 + \frac{1}{2} \times \omega^2Rt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2L}{\omega^2 R}}.$$

Non-uniform Circular Motion

If the speed of the particle moving in a circle is not constant the acceleration has both radial and tangential components. The radial and tangential accelerations are

$$a_r = \omega^2 r = \frac{v^2}{r}$$

$$a_t = \frac{dv}{dt}$$

The magnitude of the resultant acceleration will be

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

If the direction of resultant acceleration makes an angle β with the radius, where then

$$\tan\beta = \frac{dv/dt}{v^2/r}$$

Now as acceleration of particle undergoing non-uniform circular motion is $a = \sqrt{(\omega^2R)^2 + \left(R\frac{d\omega}{dt}\right)^2} = \sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2}$ in the direction $\tan^{-1}\left(\frac{dv/dt}{v^2/r}\right)$ with radius it need resultant force of $m\sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2}$ in the direction of acceleration.

SOLVED EXAMPLE

71. A car goes on a horizontal circular road of radius R, the speed increasing at a rate $\frac{dv}{dt} = a$. The friction coefficient between road and tyre is μ. Find the speed at which the car will skid.

Solution:

Here at any time t, the speed of car becomes v the net acceleration in the plane of road is $\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2}$.

This acceleration is provided by frictional force. At the moment car will slide,

$$M\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2} = \mu Mg$$

$$\Rightarrow \quad v = [R^2(\mu^2g^2 - a^2)]^{1/4}$$

Motion in a vertical circular is a common example of non-uniform circular motion that we will discuss in next lesson of 'Work, Energy and Power', as it needs same idea of energy and its conservation.

CONCEPTS AT A GLANCE

Short Cuts and Points to Note

- Tension is a reaction force produced in a string or rod.
- In a massless string (if not passing over a pulley), tension is equal at each point.
- If pulley is massless and smooth, string is massless and passing over a pulley as shown in Fig. 3.45, then

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2},\ T = \frac{2m_1m_2g}{m_1 + m_2}$$

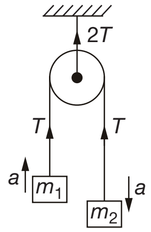


Fig. 3.45

- If the string changes, tension will change. Assuming in Fig. 3.46, pulley is smooth and massless. String is also massless. Then,

$$a = \frac{[(m_1 + m_3) - m_2]g}{m_1 + m_2 + m_3},\ T = \frac{2(m_1 + m_3)m_2g}{m_1 + m_2 + m_3}$$

$$T' = m_3\ (g - a),\ T'' = 2\ T$$

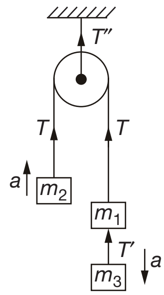


Fig. 3.46

- If the pulley system of Fig. 3.45 moves up with an acceleration a' then,

$$a = \frac{(m_2 - m_1)(g + a')}{m_1 + m_2}$$

and $$T = \frac{2m_1m_2(g + a')}{m_1 + m_2}$$

- If the pulley system shown in Fig. 3.46 moves up with an acceleration a'. Then,

$$a = \frac{[(m_1 + m_3) - m_2](g + a')}{m_1 + m_3 + m_2},$$

$$T = \frac{2(m_1 + m_3)m_2(g + a')}{m_1 + m_2 + m_3},$$

$$T' = m_3\ (g + a' - a)$$

- If $F > 2T$ in Fig. 3.47 is applied on the pulley to move the system upwards.

Then, $$a' = \frac{F - 2T}{m_1 + m_2};\ a = \frac{(m_2 - m_1)(g + a')}{m_1 + m_2}$$

$$T = \frac{2m_1m_2(g + a')}{m_1 + m_2}$$

If $F < 2T$, then $a' = 0$

and $$a = \frac{(m_2 - m_1)(g)}{m_1 + m_2}$$

$$T = \frac{2m_1m_2g}{m_1 + m_2}.$$

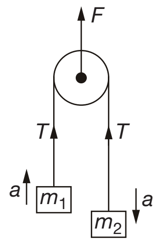


Fig. 3.47

- If the springs are in parallel then their displacements are equal. For example, in Fig. 3.48 (a) and (b), the springs are in parallel, i.e., $k_{eff} = k_1 + k_2$

$+ k_2$

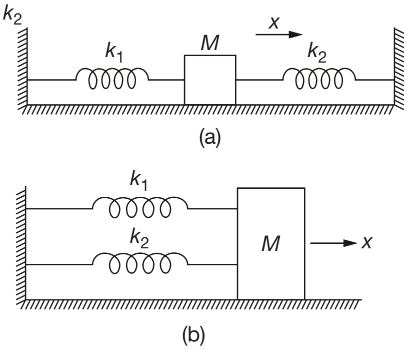


Fig. 3.48

- If the springs are in series, as shown in Fig. 3.49, stretches in spring are unequal and $x = x_1 + x_2$

or $$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}.$$

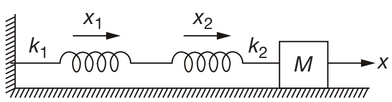


Fig. 3.49

- If the spring is cut $k \propto \frac{1}{l}$. For example, if a spring of spring constant k is cut in the ratio 2 : 3 then shorter spring has $k' = \frac{5k}{2}$ and bigger one has spring constant $k'' = \frac{5k}{3}$.
- In Fig. 3.50, if the block or pulley moves down by x, spring moves down by $2x$. Thus, $T = F' = k\,(2x)$ and $F = 2T = k\,(4x)$.

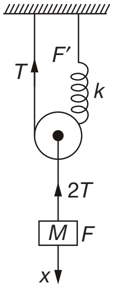


Fig. 3.50

In Fig. 3.51, if the block moves down by x then spring or pulley moves down by $\frac{x}{2}$ · $F = T$, $F'' = 2T = k\left(\frac{x}{2}\right)$.

or $$F = \frac{F''}{2} = k\left(\frac{x}{4}\right).$$

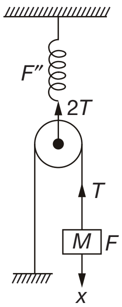


Fig. 3.51

- As shown in Fig. 3.52, if the pulley moves forward by x, then block moves forward by $2x$.

∴ $$a_{block} = 2\,a_{pulley};$$

$$a_{block} = \frac{T}{m} = \frac{F}{2m}$$

$$a_{pulley} = \frac{F}{4m}$$

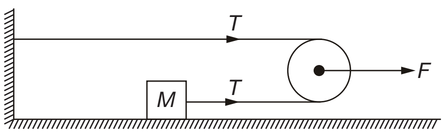


Fig. 3.52

- Since force is a vector, apply vector algebra whenever there are two or more forces.
- Draw free body diagram before you solve the problems. They make the problem very simple.
- If force is applied on the body and body does not move, then friction = force applied and not μN, where N is normal reaction.
- $\mu_s > \mu_k > \mu_R$. Baring few exception, $\mu_s < 1$ and hence $\mu_k < 1$.
- In conservative forces work done depends upon initial and final position. It is independent of the path followed. Net work done in a closed loop equals zero. Gravitational, electrostatic, magnetic forces are conservative. Friction is not conservative.
- If there is no friction, then acceleration down an incline is $a = g\sin\theta$ as shown in Fig. 3.53(a).

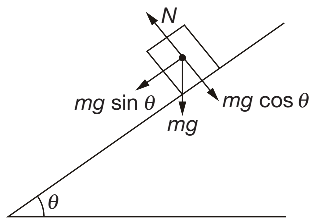


Fig. 3.53(a)

- If there is friction and coefficient of friction between the block and the incline is μ, then

  $a = g\sin\theta - mg\cos\theta$ down the incline

  or $\quad F_{down} = mg\,(\sin\theta - \mu\cos\theta)$

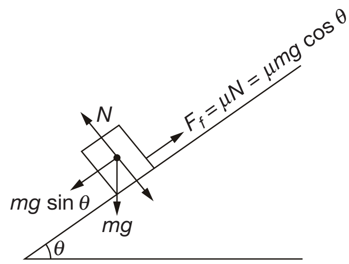


Fig. 3.53(b)

- If the block is to move up the incline with a constant velocity, then $F_{up} = mg\,(\sin\theta + \mu\cos\theta)$ (See Fig. 3.54).

  If it is to move up with an acceleration 'a' also then

  $F_{up} = mg\,(\sin\theta + \mu\cos\theta) + ma$.

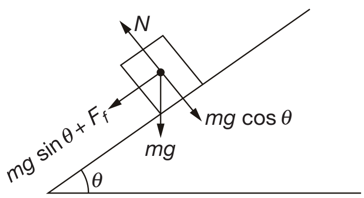


Fig. 3.54

- On a horizontal plane, deceleration due to friction is μg.
- If a lift moves up with an acceleration a, then effective or apparent weight is $m(g + a)$ as ma acting downward is pseudo force to be added to make frame of reference inertial.

  Similarly, if the pulley is moving down with an acceleration a, then apparent weight of the body is $m(g - a)$.
- If the force is a function of distance or velocity then use:

$$\frac{md^2x}{dt^2} = kx, \quad \frac{mdv}{dt} = kx$$

or
$$\frac{mdv}{dt} \cdot \frac{dx}{dt} = kx$$

or
$$\frac{mdv}{dx} \; v = kx.$$

- It is always helpful to choose axis along the incline as x-axis and axis perpendicular to the incline as y-axis.
- Remember frictional force and normal force are always perpendicular and F_f = Force applied if body remains stationary; $F_f = \mu_k N$ if the body is in motion.
- Pulling at an angle decreases the kinetic friction as normal reaction decreases as illustrated in Fig. 3.55.

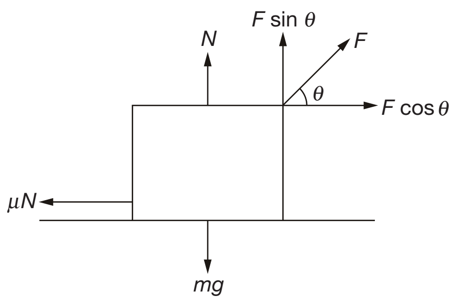


Fig. 3.55

$$N = Mg - F\sin\theta.$$

or,
$$F_f = \mu_k N = \mu_k (Mg - F\sin\theta).$$

- If $\Sigma F_y = 0 = mg - k\,v_y$ then $v_y = \dfrac{mg}{k}$ is terminal velocity as in case of viscosity. $F = 6\pi\eta rv$ (Stoke's law) v is terminal velocity.
- If a body/particle of mass m moves with a linear velocity v along the diameter of a turn table then an extra force is experienced by the body called Coriolis force.

  $F_{coroilis} = 2\,mv\omega$. where ω is angular velocity of the turn table

Caution

- Applying Newton's law without caring about inertial/non-inertial frames.

  In non-inertial frames of reference, first apply pseudo vectors to make the frame of reference inertial, only after applying Newton's laws.
- Considering action and reaction always act on different bodies.

  In case of elastic bodies and springs, action and reaction act on same body. That is, in case of restoring force in a spring or deforming force in elastic bodies, action and reaction act on same body. These forces are therefore called internal forces.
- Considering Newton's third law is always valid.

  In certain cases of electrostatics, Newton's third law fails.
- Assuming friction always acts in a direction opposite to the motion.

  If the friction causes motion, then the friction acts in the direction of motion.
- Considering force constant of a spring does not vary when spring is cut.

  Spring constant $k \propto \dfrac{1}{l}$.
- Assuming friction is always equal to μN.

  If the body is moving, friction = $\mu_k\,N$. If the body is stationary, then friction is equal to force applied.
- Assuming if pulley is massless, then tension in the string on two sides of the pulley is unequal as shown in Fig. 3.56.

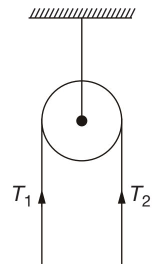


Fig. 3.56

  If pulley is massless and smooth $T_1 = T_2 = T$

  If pulley has mass then only T_1 and T_2 are unequal.
- Not understanding constraints.

  In problems shown in Fig. 3.57, if the pulley moves forward by x, then thread $2x$ is used x below and x above which will be supplied by the block side as other is fixed. Therefore, block will move $2x$. Hence, $a_{block} = 2\,a_{pulley}$

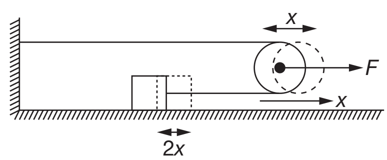


Fig. 3.57

- Considering in equilibrium, body must be at rest.

  In static equilibrium, body is at rest. In dynamic equilibrium, it moves with uniform velocity.
- Assuming there is no tension if the rope is pulled by equal and opposite forces on two ends.

  Tension is equal to either of the force applied.
- Considering impulse always provides acceleration.

  Sharp impulse only provides velocity.

- Considering rough surfaces have more friction.

  In general, it may be true. But polished surfaces may offer more friction. For example, coefficient of friction between glass/wood is 0.23 and glass and glass is 1.0 and between *Cu – Cu* is 1.6.
- Considering horizontal plane as x-axis and therefore normal force N perpendicular to x-axis as shown in Fig. 3.58(a).

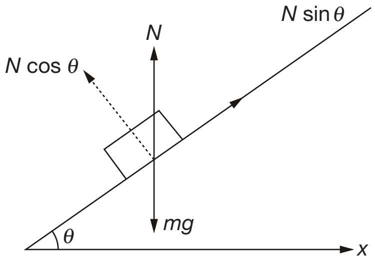


Fig. 3.58(a)

and perpendicular to it as y-axis is more convenient way of solving problems as shown in Fig. 3.58(b).

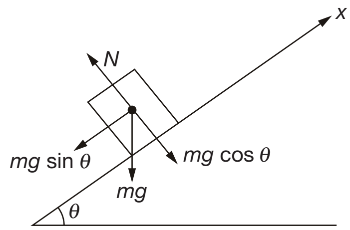


Fig. 3.58(b)

BRAIN MAP I

Law 1. Defines force and inertia
Everybody remains at rest or continues to move with uniform velocity unless an external force is applied to it.

Law 2. Gives relation between force and acceleration
i.e., $\sum \vec{F}_{ext} = M\vec{a}$

Law 3. When a body A exerts a force on another body B, B exerts an equal and opposite force on A. If one of these two forces is considered as action, then other will be reaction.

NEWTON'S LAWS OF MOTION

(Applicable for inertial frame only)

Commonly used assumptions
Rigid body
Inextensible string
Massless string
Massless and frictionless pulley

Commonly used forces
Normal force: Normal to the surfaces of contact and towards the body under consideration.
Weight of body: Equals to Mg and acts vertically downward.
Tension in string: along the string, away from the body under consideration.

Stepwise procedure to solve questions based on motion of connected bodies:
1. Identify the unknown forces and accelerations.
2. Draw FBD of bodies in the system.
3. Resolve forces in the direction of motion and perpendicular to it.
4. Apply $\Sigma\vec{F} = M\vec{a}$ in the direction of motion and $\Sigma\vec{F} = 0$ in the direction of equilibrium.
5. Write constraint relation if required and possible.
6. Solve the equations written in steps 4 and 5 to get the results.

BRAIN MAP 2

NON-INERTIAL FRAME

1. Non-inertial frame
- In this frame, Newton's laws of motion is not applicable.
- To apply Newton's laws of motion in this frame, other than external forces, we need to consider pseudo force(s).

2. In non-interial frame we can write Newton's Law as

$\vec{F}_{ext} + \vec{F}_{Pseudo} = M\vec{a}$

$\vec{F}_{Pseudo} = -M\vec{a}_{frame}$

FRICTION

1. It is a tangential force between the surfaces in contact that opposes relative motion or tendency of the relative motion between the contact surfaces.

2. Frictional force is of three types:
- Static frictional force: Self adjusting force having magnitude less than or equal to $\mu_s N$.
- Limiting frictional force: Maximum value of frictional force having value $\mu_s N$.
- Kinetic frictional force: Having value equal to $\mu_k N$.

3. Angle between the resultant reaction force in limiting friction and normal force is called as angle of friction θ, given by

$\tan\theta = \dfrac{f}{N}$

4. Angle made by an inclined plane with horizontal at which a body starts sliding down itself is called as angle of repose (α) given by, $\alpha = \tan^{-1}(\mu_s)$

CIRCULAR MOTION

1. Angular kinematics relations
- Uniform circular motion, $\Delta\theta = \omega t$
- Uniformly accelerated circular motion,
- $\Delta\theta = \omega_o t + \dfrac{1}{2}\alpha t^2$, $\omega = \omega_o + \alpha t$
- $\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$, $\Delta\theta_n = \omega_o(2n-1)\dfrac{\alpha}{2}$

2. Relation between linear kinematic variable and angular kinematic variable
- Linear velocity, $v = \omega R$
- Tangential acceleration, $a_t = \alpha R$
- Radial or centripetal acceleration, $a_r = \omega^2 R$
- Total acceleration, $a = \sqrt{\alpha^2R^2 + \omega^4R^2}$ at an angle of $\tan^{-1}\left(\dfrac{a_t}{a_r}\right)$ with radius.

3. Centripetal force: $F_r = M\omega^2R = \dfrac{Mv^2}{R}$ towards the center.
- Centrifugal force: $F_r' = M\omega^2R = \dfrac{Mv^2}{R}$, away from the centre (Applicable with respect to a rotating frame).

EXERCISES

Single Option Correct Type

1. With what acceleration a should the box of Fig. 3.59 descend so that the block of mass M exerts a force $Mg/4$ on the floor of the box?

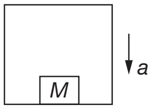


Fig. 3.59

(A) $g/4$ (B) $g/2$ (C) $3g/4$ (D) $4g$

2. A body of mass 2 kg moves vertically downwards with an acceleration a = 19.6 m/s^2. The force acting on the body simultaneously with the force of gravity is (g = 9.8 m/s^2, neglect air resistance)

(A) 19.6 N (B) 19.2 N (C) 59.2 N (D) 58.8 N

3. Two blocks, each having a mass M, rest on frictionless surface as shown in the Fig. 3.60. If the pulleys are light and frictionless, and M on the incline is allowed to move down, then the tension in the string will be

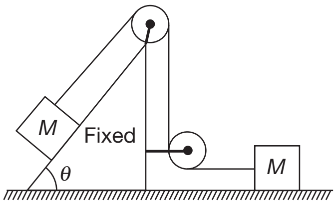


Fig. 3.60

(A) $\frac{2}{3}Mg\sin\theta$ (B) $\frac{3}{2}Mg\sin\theta$

(C) $\frac{Mg\sin\theta}{2}$ (D) $2\,Mg\sin\theta$

4. A girl of mass 50 kg stands on a measuring scale in a lift. At an instant, it is detected that the reading reduces to 40 kg for a while and then returns to original value. It can be said that

(A) The lift was in constant motion upwards
(B) The lift was in constant motion downwards
(C) The lift was suddenly started in downward motion
(D) The lift was suddenly started in upward motion

5. Two blocks of masses 5 kg and 2 kg are connected by a massless string as shown in Fig. 3.61. A vertical force F is applied on the 5 kg block. Find the value of F if tension in the string is 40 N. (g = 10 m/s^2)

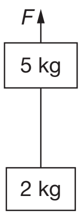


Fig. 3.61

(A) 140 N (B) 70 N
(C) 40 N (D) 100 N

6. The pulleys and strings shown in the Fig. 3.62 are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be:

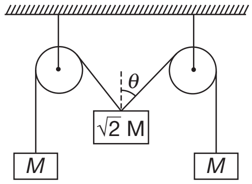


Fig. 3.62

(A) 0° (B) 30° (C) 45° (D) 60°

7. Two masses m and M are connected by a light string passing over a smooth pulley. When set free, m moves up by 1.4 m in 2 s. The ratio $\frac{m}{M}$ is (g = 9.8 ms^{-2})

(A) $\frac{13}{15}$ (B) $\frac{15}{13}$ (C) $\frac{9}{7}$ (D) $\frac{7}{9}$

8. A block of mass m is attached to a massless spring of spring constant K. This system is accelerated upward with acceleration a. The elongation in spring will be

(A) $\frac{mg}{K}$ (B) $\frac{m(g-a)}{K}$

(C) $\frac{m(g+a)}{K}$ (D) $\frac{ma}{K}$

9. The elevator shown in Fig. 3.63 is descending with an acceleration of 2 ms^{-2}. The mass of the block A = 0.5 kg. The force exerted by the block A on the block B is (g = 10 ms^{-2})

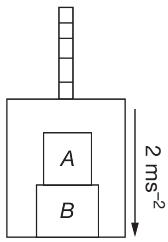


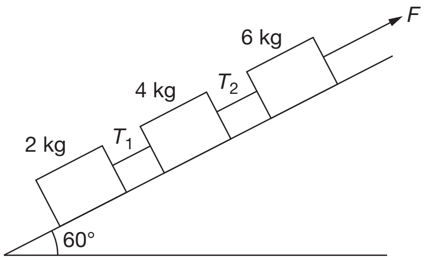
Fig. 3.63

(A) 2 N (B) 4 N (C) 6 N (D) 8 N

10. A body of mass 1.5 kg is thrown vertically upwards with an initial velocity of 40 m/s reaches its highest point after 3 s. The air resistance acting on the body during the ascent is (assuming air resistance to be uniform, g = 10 m/s^2)

(A) 35 N (B) 25 N (C) 15 N (D) 5 N

11. Three blocks of masses 2 kg, 4 kg and 6 kg are connected by string and resting on a frictionless incline of 60° as shown. A force of 120 N is applied upward along the incline to the 6 kg block. If the strings are ideal, the ratio T_1/T_2 will be (g = 10 ms^{-2})



(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 4

12. A block of mass 20 kg is balanced by three strings A, B and C as shown in Fig. 3.64. Ratio of tensions in string A and B (T_A/T_B) is

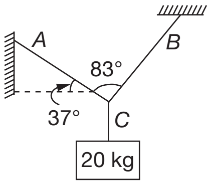


Fig. 3.64

(A) $\frac{5}{8}$ (B) $\frac{5\sqrt{3}}{8}$ (C) $\frac{5}{6}$ (D) $\frac{8}{5}$

13. A metal sphere is hung by a string fixed to a wall. The force acting on the sphere is shown in Fig. 3.65. Which of the following statement is incorrect?

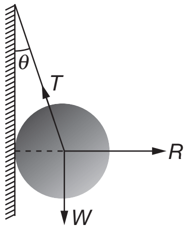


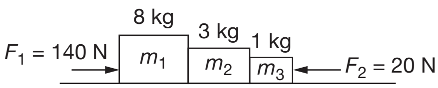
Fig. 3.65

(A) $\vec{R}+\vec{T}+\vec{W}=0$
(B) $T^2 = R^2 + W^2$
(C) $T = R + W$
(D) $R = W\tan\theta$

14. A string of length L and mass M are lying on a horizontal table. A force F is applied at one of its ends. Tension in the string at a distance x from the ends at which force is applied is

(A) Zero (B) F
(C) $F(L-x)/L$ (D) $F(L-x)/M$

15. Three blocks m_1, m_2 and m_3 of masses 8 kg, 3 kg and 1 kg are placed in contact on a smooth surface. Forces F_1 = 140 N and F_2 = 20 N are acting on blocks m_1 and m_3, respectively, as shown. The reaction between blocks m_2 and m_3 is

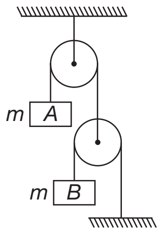


(A) 2.5 N (B) 7.5 N
(C) 22.5 N (D) 30 N

16. A fireman wants to slide down a rope. The breaking load for the rope is $\frac{3}{4}$ th of the weight of the fireman. The acceleration of the fireman to prevent the rope from breaking will be (Acceleration due to gravity is g)

(A) $g/4$ (B) $g/2$ (C) $3g/4$ (D) Zero

17. Two blocks A and B of equal masses m are suspended with ideal pulley and string arrangement as shown. The acceleration of mass B is



(A) $\frac{g}{3}$ (B) $\frac{5g}{3}$
(C) $\frac{2g}{3}$ (D) $\frac{2g}{5}$

18. In the arrangement shown in Fig. 3.66, if the surface is smooth, the acceleration of the block m_2 will be

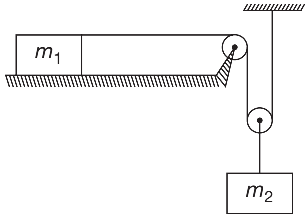
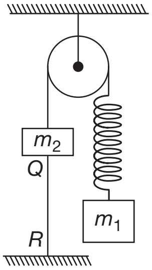


Fig. 3.66

(A) $\frac{m_2 g}{4m_1 + m_2}$ (B) $\frac{2m_2 g}{4m_1 + m_2}$
(C) $\frac{2m_2 g}{m_1 + 4m_2}$ (D) $\frac{2m_1 g}{m_1 + m_2}$

19. In the shown system, $m_1 > m_2$. Thread QR is holding the system. If this thread is cut, then just after cutting.



(A) Acceleration of mass m_1 is zero and that of m_2 is directed upward
(B) Acceleration of mass m_2 is zero and that of m_2 is directed downward
(C) Acceleration of both the blocks will be same
(D) Acceleration of system is given by $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$kg, where k is a spring factor

20. The ratio of T_1 and T_2 is (see Fig. 3.67) (neglect friction)

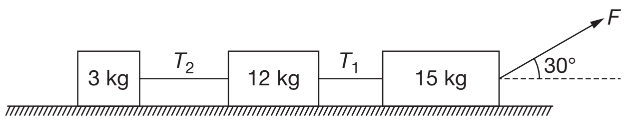


Fig. 3.67

(A) $\sqrt{3}:2$ (B) $1:\sqrt{3}$ (C) 1 : 5 (D) 5 : 1

21. An elevator starts from rest with a constant upward acceleration. It moves 2 m in the first 0.6 second. A passenger in the elevator is holding a 3 kg package by a vertical string. When the elevator is moving, what is the tension in the string?

(A) 4 N (B) 62.7 N (C) 29.4 N (D) 20.6 N

22. In the system shown in Fig. 3.68, m_B = 4 kg and m_A = 2 kg. The pulleys are massless and friction is absent everywhere. The acceleration of block *A* is (g = 10 m/s^2)

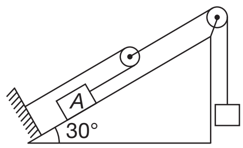


Fig. 3.68

(A) $\frac{10}{3}$ m/s^2 (B) $\frac{20}{3}$ m/s^2

(C) $\frac{5}{2}$ m/s^2 (D) $\frac{5}{3}$ m/s^2

23. Three blocks A, B and C of equal weights of mass 2 kg each are hanging on a string passing over a fixed pulley as shown in Fig. 3.69. What is the tension in the string connected between blocks *B* and *C* ?

(A) Zero (B) 13 N
(C) 3.3 N (D) 19.6 N

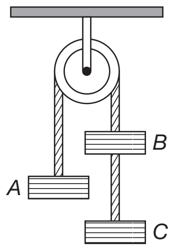
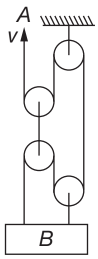


Fig. 3.69

24. In the arrangement shown, end *A* of light inextensible string is pulled up with constant velocity *v*. The velocity of block *B* is



(A) $v/2$ (B) v (C) $v/3$ (D) $3v$

25. In the arrangement shown in Fig. 3.70, thread is inextensible and massless. All the pulleys are also massless. If friction in all pulleys are negligible, then:

(A) Tension in thread is equal to $\frac{mg}{2}$.

(B) Acceleration of pulley *C* is equal to $\frac{g}{2}$(downward).

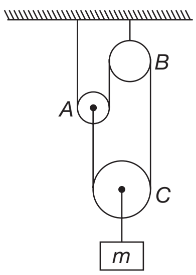
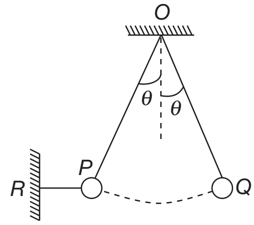


Fig. 3.70

(C) Acceleration of pulley *A* is equal to $\frac{g}{2}$(upward).

(D) Acceleration of block of mass m is equal to g (downward).

26. A ball of mass 1 kg is at rest in position *P* by means of two light strings, *OP* and *RP*. The string *RP* is now cut and the ball swings to position *Q*. If $\theta = 45°$. Find the ratio of tensions in the strings in positions *OP* (when *RP* was not cut) and *OQ* (when *RP* was cut). Taking g = 10 m/s^2.



(A) 1 (B) 2 (C) 3 (D) 1.5

27. Two spheres *A* and *B* are placed between two vertical walls as shown in Fig. 3.71. Friction is absent everywhere. The ratio of N_A to N_B is

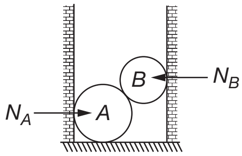
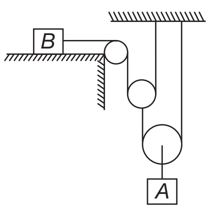


Fig. 3.71

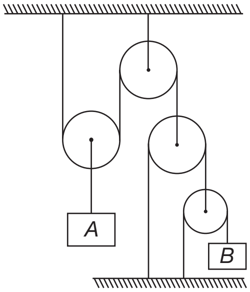
(A) 1
(B) 2
(C) 4
(D) Cannot be determined

28. The block *B* has a mass of 10 kg. The coefficient of friction between block *B* and the surface is μ = 0.5. Determine the acceleration of the block *A* of mass 16 kg. Neglect the mass of the pulleys and cords (Taking g = 10 m/s^2).



(A) Zero (B) 2 m/s^2
(C) 1 m/s^2 (D) None of these

29. Block *A* moves upward with acceleration $\frac{1}{2}$ m/s^2. The acceleration of block *B* in downward direction will be



(A) 2 m/s^2 (B) 3 m/s^2
(C) 4 m/s^2 (D) 6 m/s^2

30. In the Fig. 3.72, the force with which the man should pull the rope to hold the plank in position is F. If weight of the man is 60 kgf, the plank and pulleys have negligible masses, then ($g = 10$ m/s^2)

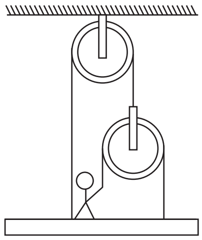
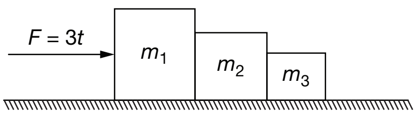


Fig. 3.72

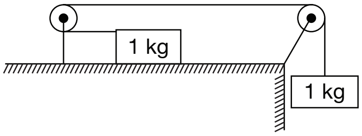
(A) $F = 150$ N
(B) $F = 300$ N
(C) $F = 600$ N
(D) $F = 1200$ N

31. A time-dependent force $F = 3t$ (F in Newton and t in second) acts on three blocks m_1, m_2 and m_3 kept in contact on a rough ground as shown. Co-efficient of friction between blocks and ground is 0.4. If m_1, m_2 and m_3 are 3 kg, 2 kg and 1 kg, respectively, the time after which the blocks started to move is ($g = 10$ ms^{-2})



(A) 4 s (B) 8 s
(C) $\frac{8}{3}$ s (D) $\frac{4}{3}$ s

32. A block of mass 1 kg is placed on a rough horizontal surface connected by a light string passing over two smooth pulleys as shown. Another block of 1 kg is connected to the other end of the string. The acceleration of the system is (co-efficient of friction $\mu = 0.2$)



(A) 0.8 g (B) 0.4 g
(C) 0.5 g (D) Zero

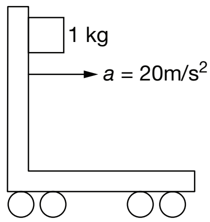
33. A block of 10 kg is pulled by a constant speed on a rough horizontal surface by a force of 19.6 N. The co-efficient of friction is

(A) 0.1 (B) 0.2 (C) 0.3 (D) 0 .4

34. A body of mass m is kept stationary on a rough inclined plane of inclination θ. The magnitude of force acting on the body by the inclined plane is

(A) mg (B) $mg \sin\theta$
(C) $mg \cos\theta$ (D) $mg\sqrt{1+\cos^2\theta}$

35. A block of mass 1 kg just remains in equilibrium with the vertical wall of a cart accelerating uniformly with 20 m/s^2 as shown. The co-efficient of friction between block and wall is ($g = 10$ m/s^2)



(A) 0.1 (B) 0.2 (C) 0.5 (D) 1

36. What acceleration must the cart in Fig. 3.73 have so that the block A will not fall? (μ is co-efficient of friction between cart and block.)

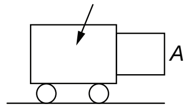
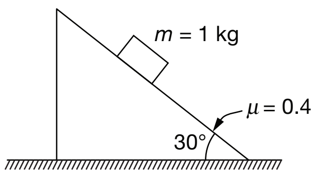


Fig. 3.73

(A) μg (B) $\frac{g}{\mu}$
(C) $\frac{\mu}{g}$ (D) $\mu + g$

37. A block of mass 1 kg is placed on a rough incline as shown. The co-efficient of friction between block and incline is 0.4. The acceleration of block is ($g = 10$ ms^{-2}, $\sqrt{3} = 1.7$)



(A) zero (B) 1.6 ms^{-2}
(C) 6.5 ms^{-2} (D) 5 ms^{-2}

38. A mass m rests on a horizontal surface. The co-efficient of friction between the mass and the surface is μ. If the mass is pulled by a force F as shown in Fig. 3.74, the limiting friction between the mass and the surface will be

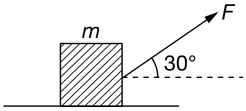


Fig. 3.74

(A) μmg (B) $\mu[mg-(\sqrt{3}/2)F]$
(C) $\mu[mg-(F/2)]$ (D) $\mu[mg+(F/2)]$

39. A wagon of mass M has a block of mass m attached to it as shown in the Fig. 3.75. The co-efficient of friction between the block and wagon is μ. The minimum acceleration of the wagon that holds the block m from falling is

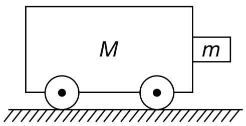
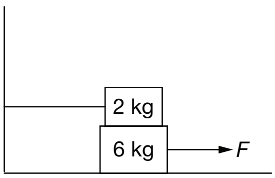


Fig. 3.75

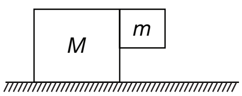
(A) $\frac{g}{\mu}$ (B) $\frac{\mu}{g}$ (C) μg (D) $\frac{M\mu g}{m}$

40. A block of mass 2 kg is resting over another block of mass 6 kg. 2 kg block is connected to one end of a string fixed to a vertical wall as shown. If the co-efficient of friction between the blocks is 0.4, the force required to pull out the 6 kg block with an acceleration of 1.5 m/s^2 will be ($g = 10$ ms^{-2})



(A) 17 N (B) 9 N (C) 8 N (D) 1 N

41. With what minimum acceleration mass M must be moved on frictionless surface so that m remains stick to it as shown. The co-efficient of friction between M and m is μ.



(A) μg (B) $\frac{g}{\mu}$ (C) $\frac{\mu mg}{M+m}$ (D) $\frac{\mu mg}{M}$

42. A block of mass 0.1 kg is held against a wall by applying a horizontal force of 5 N on the block. If the co-efficient of friction between the block and the wall is 0.5, the magnitude of the frictional force acting on the block is

(A) 2.5 N (B) 0.98 N (C) 4.9 N (D) 0.49 N

43. Consider the situation shown in Fig. 3.76, find the tension in string AB considering the pulley and string as frictionless and massless:

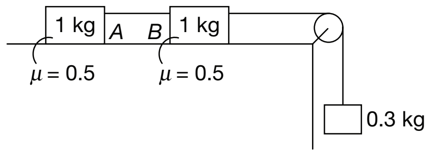


Fig. 3.76

(A) 3 N (B) 2 N (C) 8 N (D) Zero

44. A body of mass 60 kg is dragged with just enough force to start moving on a rough surface with co-efficient of static and kinetic friction 0.5 and 0.4, respectively. On applying the same force, what is the acceleration ($g = 9.8$ m/s^2)?

(A) 0.98 m/s^2 (B) 9.8 m/s^2
(C) .54 m/s^2 (D) 5.292 m/s^2

45. A box of mass 8 kg is placed on a rough inclined plane of inclination θ. Its downward motion can be prevented by applying an upward pull F and it can be made to slide upwards by applying a force $2F$. The co-efficient of friction between the box and the inclined plane is

(A) $\frac{1}{3}\tan\theta$ (B) $3\tan\theta$
(C) $\frac{1}{2}\tan\theta$ (D) $2\tan\theta$

46. A block of mass m on a rough horizontal surface is acted upon by two forces as shown in Fig. 3.77. For equilibrium of block, the co-efficient of friction between block and surface is

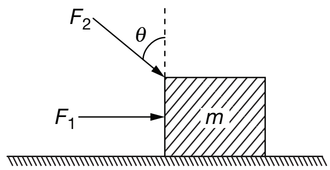


Fig. 3.77

(A) $\frac{F_1+F_2\sin\theta}{mg+F_2\cos\theta}$ (B) $\frac{F_1\cos\theta+F_2}{mg-F_2\sin\theta}$
(C) $\frac{F_1+F_2\cos\theta}{mg+F_2\sin\theta}$ (D) $\frac{F_1\sin\theta-F_2}{mg-F_2\cos\theta}$

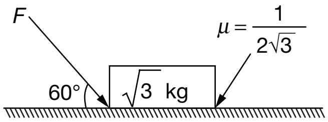
47. A car starts from rest to cover a distance x. The co-efficient of friction between the road and tyres is μ. The minimum time in which the car can cover distance x is proportional to

(A) μ (B) $\frac{1}{\sqrt{\mu}}$ (C) $\sqrt{\mu}$ (D) $\frac{1}{\mu}$

48. A lift is moving downwards with an acceleration equal to acceleration due to gravity. A body of mass M kept on the floor of the lift is pulled horizontally. If the co-efficient of friction is μ, then the frictional resistance offered by the body is

(A) Mg (B) μMg (C) $2\mu Mg$ (D) Zero

49. What is the maximum value of the force F such that the block shown in the arrangement, does not move?



(A) 20 N (B) 10 N (C) 12 N (D) 15 N

50. Consider the system shown in Fig. 3.78. The wall is smooth, but the surface of blocks A and B in contact are rough. The friction on B due to A in equilibrium is

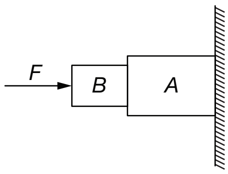


Fig. 3.78

(A) Upward
(B) Downward
(C) Zero
(D) The system cannot remain in equilibrium.

51. Two blocks of masses 4 kg and 2 kg are connected by a heavy string of mass 3 kg and placed on rough horizontal plane. The 2 kg block is pulled with a constant force F as shown in Fig. 3.79. The co-efficient of friction between the blocks and the ground is 0.5. What is the value of F so that tension in the string is constant throughout the motion of the blocks?

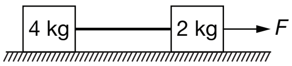


Fig. 3.79

(A) 40 N (B) 30 N (C) 45 N (D) 60 N

52. In the arrangement shown in the Fig. 3.80, there is a friction force between the blocks of masses m and 2 m. Block of mass $2m$ is kept on a smooth horizontal plane. The mass of the suspended block is m. If block A is stationary with respect to block of mass 2 m. The minimum value of co-efficient of friction between m and 2 m is

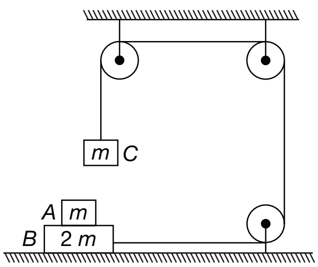


Fig. 3.80

(A) 1 /2 (B) $1/\sqrt{2}$ (C) 1 / 4 (D) 1 /3

53. A block of mass m is placed on the top of another block of mass M as shown in the Fig. 3.81. The co-efficient of friction between them is μ. The maximum acceleration with which the block M may move so that m also moves along with it is

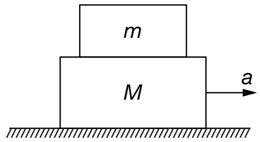


Fig. 3.81

(A) μg (B) g/μ (C) μ^2/g (D) g/μ^2

54. The value of frictional force and acceleration of block of mass 10 kg in the Fig. 3.82 are

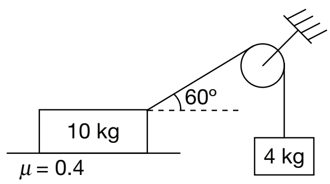


Fig. 3.82

(A) 10 N, 1 m/s^2 (B) 20 N, 2 m/s^2
(C) 20 N, 0 m/s^2 (D) 10 N, 0 m/s^2

55. A block of mass 1 kg start moving at $t = 0$ with speed 2 m/s on rough horizontal surface with co-efficient of friction 0.2. A horizontal force F is applied in the same direction of velocity which varies with time shown in Fig. 3.83(b). Find the speed of particle at t = 3 s ($g = 10\ m/s^2$).

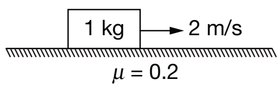


Figure 3.83(a)

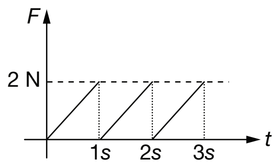
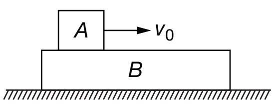


Figure 3.83(b)

(A) 1 m/s (B) Zero
(C) 5 m/s (D) 2 m/s

56. A block A of mass m is placed over a plank B of mass 2 m. Plank B is placed over a smooth horizontal surface. The co-efficient of friction between A and B is $\frac{1}{2}$. Block A is given a velocity v_0 towards right. Acceleration of B relative to A is



(A) $\frac{g}{2}$ (B) g (C) $\frac{3g}{4}$ (D) Zero

57. In the arrangement shown in Fig. 3.84, co-efficient of friction between the two blocks is $\mu = ½$. The force of friction acting between the two blocks is

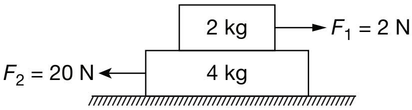


Fig. 3.84

(A) 8 N (B) 10 N (C) 6 N (D) 4 N

58. In the Fig. 3.85, $m_A = 2$kg and $m_B = 4$ kg. For what minimum value of F, A starts slipping over B. ($g = 10$ m/s^2)

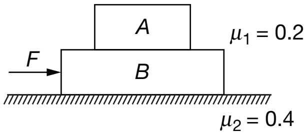


Fig. 3.85

(A) 24 N (B) 36 N (C) 12 N (D) 20 N

59. The upper half of an incline plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the co-efficient of friction for the lower half is given by

(A) 2 tanϕ (B) tanϕ
(C) 2 sinϕ (D) 2 cosϕ

60. The co-efficient of friction between the blocks is 0.4 and that of the lower block and the ground is 0.8. A horizontal force of 110 N is applied on the lower block as shown in Fig. 3.86. The force of friction between the two blocks is

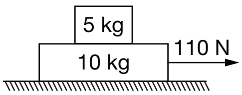


Fig. 3.86

(A) 20 N (B) 15 N (C) 120 N (D) 0 N

61. A pebble of mass 0.05 kg is thrown vertically upwards. The direction and magnitude of the net force on the pebble is given below, choose the incorrect option.

(A) During its upward motion, force is 0.5 N in vertically upward.
(B) During its downward motion, force is 0.5 N in vertically downward.
(C) At the highest point, where it is momentarily at rest, force is 0.5 N in vertically downward.
(D) If the pebble was thrown at an angle of say 45° with the horizontal direction, force is 0.5 N in vertically downward (Ignoring air resistance).

62. The magnitude and direction of the net force acting on a stone of mass 0.1 kg is given below, choose the incorrect statement.

(A) Just after it is dropped from the window of a stationary train, $F = 1.0$ N (Vertically down ward).
(B) Just after it is dropped from the window of a train running at a constant velocity of 36 km/h, $F = 1.0$ N (Vertically downward).
(C) Just after it is dropped from the window of a train accelerating with 1 ms^{-2}, $F = 1.0$ N (Vertically downward).
(D) Just after it is dropped from the window of a train accelerating with 1 ms^{-2}, $F = 2.0$ N (Vertically downward).

63. One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves in a circular motion with speed v, the net force on the particle (directed towards the centre) is:

(A) T (B) $T - \frac{mv^2}{l}$
(C) $T + \frac{mv^2}{l}$ (D) 0

64. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 ms^{-1}. How long does the body take to stop?

(A) 2s (B) 4s (C) 6s (D) 8s

65. A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 ms^{-1} to 3.5 ms^{-1} in 25 s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force?

(A) 0.18 N (B) 0.36 N
(C) 0.9 N (D) None of these

66. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude of the acceleration of the body.

(A) 2 m/s^2 (B) 4 m/s^2
(C) 6 m/s^2 (D) 8 m/s^2

67. The driver of three-wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

(A) 1162.5 N (B) 116.25 N
(C) 1112 N (D) None of these

68. A man of mass 70 kg stands on a weighing scale in a lift which is moving. Choose the correct statement.
(A) Reading of weighing scale is 700 N upwards with a uniform speed of 10 ms^{-1}.
(B) Reading of weighing scale is 700 N downwards with a uniform acceleration of 5 ms^{-2}.
(C) Reading of weighing scale is 700 N upwards with a uniform acceleration of 5 ms^{-2}.
(D) Reading of weighing scale is 700 N if the lift mechanism failed and it fall down freely under gravity.

69. A ball is travelling with uniform translatory motion. This means that
(A) it is at rest.
(B) the path can be a straight line or circular and the ball travels with uniform speed.
(C) all parts of the ball have the same velocity (magnitude and direction) and the velocity is constant.
(D) the centre of the ball moves with constant velocity and the ball spins about its centre uniformly.

70. A metre scale is moving with uniform velocity. This implies
(A) the force acting on the scale is zero, but a torque about the centre of mass can act on the scale.
(B) the force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero.
(C) the total force acting on it need not be zero but the torque on it is zero.
(D) neither the force nor the torque needs to be zero.

71. A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is
(A) frictional force along westward.
(B) muscle force along southward.
(C) frictional force along south-west.
(D) muscle force along south-west.

72. A body with mass 5 kg is acted upon by a force $F = (-3\hat{i} + 4\hat{j})\ N$. If its initial velocity at $t = 0$ is $v = (6\hat{i} - 12\hat{j})\ ms^{-1}$, the time at which it will just have a velocity along the y-axis is
(A) Never (B) 10 s (C) 2 s (D) 15 s

73. A car of mass m starts from rest and acquires a velocity along east, $v = v\hat{i}\ (v > 0)$ in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is
(A) $\frac{mv}{2}$ eastward and is exerted by the car engine.
(B) $\frac{mv}{2}$ eastward and is due to the friction on the tyres exerted by the road.
(C) more than $\frac{mv}{2}$ eastward exerted due to the engine and overcomes the friction of the road.
(D) $\frac{mv}{2}$ exerted by the engine.

74. Two bodies of masses 10 kg and 20 kg, respectively, kept on a smooth, horizontal surface are tied to the ends of a light string. A horizontal force $F = 600$ N is applied to body of mass 10 kg. What is the tension in the string in each case?
(A) 200 N (B) 100 N (C) 400 N (D) 600 N

75. Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the tension the string when the masses are released.
(A) 96 N (B) 80 N
(C) 100 N (D) None of these

76. At a curved path of the road, the roadbed is raised a little on the side away from the center of the curved path. The slope of the roadbed is given by
(A) $\tan^{-1}\frac{v^2 g}{r}$ (B) $\tan^{-1}\frac{rg}{v^2}$
(C) $\tan^{-1}\frac{r}{gv^2}$ (D) $\tan^{-1}\frac{v^2}{rg}$

77. A car of mass m is being driven on a circular path of radius R. In which of the following circumstances it will not slip (μ is coefficient of friction between surface and road)
(A) $\frac{mv^2}{R} \geq \mu mg$ (B) $\frac{mv^2}{R} = 4\mu mg$
(C) $\frac{mv^2}{R} > mg$ (D) None

78. For a particle rotating in a vertical circle with uniform speed, the maximum and minimum tension in the string are in the ratio 5 : 3. If the radius of vertical circle is 2 m, the speed of revolving body is ($g = 10$ m/s^2)
(A) $\sqrt{5}$ m/s (B) $4\sqrt{5}$ m/s
(C) 5 m/s (D) 10 m/s

79. A stone of mass 1.5 kg is tied at the end of 0.5 m long string and whirled in a vertical circular path at constant speed of 2 ms^{-1}. The maximum tension in the string is ($g = 10\ ms^{-2}$)
(A) 27 N (B) 3 N (C) 90 N (D) 15 N

80. A particle of mass 0.1 kg is whirled at the end of a string in a vertical circle of radius 1.0 m at a constant speed of 5 m/s. The tension in the string at the highest point of its path is ($g = 10$ m/s^2)

(A) 0.5 N (B) 1.0 N (C) 1.5 N (D) 2.0 N

81. A motor car is traveling at 60 m/s on a circular road of radius 1200 m. It is increasing its speed at the rate of 4 m/s^2. The acceleration of the car is

(A) 3 ms^{-2} (B) 4 ms^{-2}
(C) 5 ms^{-2} (D) 7 ms^{-2}

82. A 2 kg stone at the end of a string 1 m long is whirled in a vertical circle at a constant speed. The speed of the stone is 4 m/s. The tension in the string will be 52 N, when the stone is

(A) at the top of the circle
(B) at the bottom of the circle
(C) halfway down
(D) none of the above

83. A particle is moving with a constant angular acceleration of 4 rad/s^2 in a circular path. At $t = 0$, particle was at rest. Find the time at which the magnitudes of centripetal acceleration and tangential acceleration are equal.

(A) 1 s (B) 2 s (C) $\frac{1}{2}$ s (D) $\frac{1}{4}$ s

84. A particle is rotating in a circle of radius R with constant angular velocity ω. Its average velocity during t seconds after start of motion is

(A) $\frac{2R}{t}\sin\left(\frac{\omega t}{2}\right)$ (B) $\frac{2R}{t}\cos\left(\frac{\omega t}{2}\right)$

(C) $\frac{R}{t}\sin\left(\frac{\omega t}{2}\right)$ (D) $\frac{R}{t}\cos\left(\frac{\omega t}{2}\right)$

85. A particle is moving along the circular path with a speed v and tangential acceleration is g at an instant. If the radius of the circular path be r, then the net acceleration of the particle at that instant is

(A) $\frac{v^2}{r}+g$ (B) $\frac{v^2}{r^2}+g^2$

(C) $\left[\frac{v^4}{r^2}+g^2\right]^{\frac{1}{2}}$ (D) $\left[\frac{v^2}{r}+g^2\right]^{\frac{1}{2}}$

86. A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length l. The other end of the spring is fixed and it is rotated in horizontal circle with an angular velocity ω, in gravity free space. The increase in length of the spring will be

(A) $\frac{m\omega^2 l}{k}$ (B) $\frac{m\omega^2 l}{k-m\omega^2}$

(C) $\frac{m\omega^2 l}{k+m\omega^2}$ (D) none of these

87. The angular velocity of a wheel increases from 1200 rpm to 4500 rpm in 10 s. The number of revolutions made during this time is

(A) 950 (B) 475 (C) 237.5 (D) 118.75

88. A body moves along a path PQR from P to R shown as a dashed line in Fig. 3.87. When the particle is at Q, its speed is decreasing. The acceleration of the particle at Q is best represented by the vector

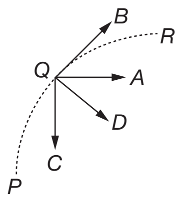


Fig. 3.87

(A) A (B) B (C) C (D) D

89. A constant power is supplied to a rotating disc. Angular velocity (ω) of disc varies with number of rotations (n) made by the disc as

(A) $\omega \propto n^{1/3}$ (B) $\omega \propto n^{3/2}$
(C) $\omega \propto n^{2/3}$ (D) $\omega \propto n^2$

90. A disc is rotating with an angular velocity ω_0. A constant retarding torque is applied on it to stop the disc. The angular velocity becomes $\omega_0/2$ after n rotations. How many more rotations will it make before coming to rest?

(A) n (B) $2n$

(C) $\frac{n}{2}$ (D) $\frac{n}{3}$

91. An insect crawls up a hemispherical surface very slowly (Fig. 3.88). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by

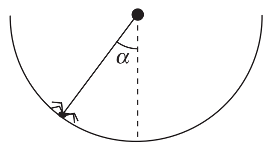


Fig. 3.88

(A) $\cot\alpha = 3$ (B) $\tan\alpha = 3$
(C) $\sec\alpha = 3$ (D) $\mathrm{cosec}\,\alpha = 3$

92. Starting from rest, a particle rotates in a circle of radius $R = \sqrt{2}\ m$ with an angular acceleration $\alpha = (\pi/4)\ \text{rad/s}^2$. The magnitude of average velocity of the particle over the time it rotates a quarter circle is

(A) 1.5 m/s (B) 2 m/s
(C) 1 m/s (D) 1.25 m/s

93. As shown in the Fig. 3.89, a mass m and another mass 10 m are connected with a string. Friction is sufficient to prevent the slipping of 10 m. Mass m is given a velocity u in vertical direction. For complete circular motion of mass m keeping heavier particle stationary, the value of u is

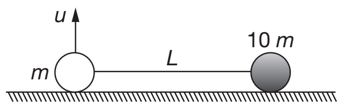


Fig. 3.89

(A) $u > \sqrt{3gL}$

(B) $\sqrt{3gL} < u < \sqrt{5gL}$

(C) $\sqrt{3gL} < u < \sqrt{13gL}$

(D) $\sqrt{11gL} < u < \sqrt{13gL}$

More than One Option Correct Type

94. The blocks B and C in the Fig. 3.90 have mass m each. The strings AB and BC are light, having tensions T_1 and T_2, respectively. The system is in equilibrium with a constant force mg acting on C.

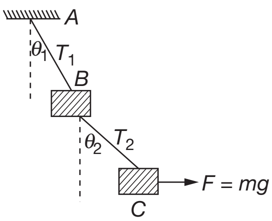


Fig. 3.90

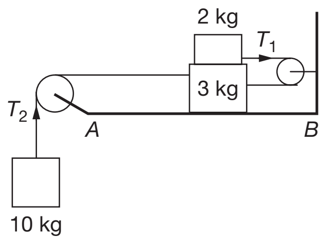
(A) $\tan\theta_1 = 1/2$ (B) $\tan\theta_2 = 1/2$

(C) $T_1 = \sqrt{5}\text{mg}$ (D) $T_2 = \sqrt{2}\text{mg}$

95. Four forces act on a point object. The object will be in equilibrium if

(A) all of them are in the same plane
(B) they are opposite to each other in pair
(C) the sum of x, y and z components of all the force is zero separately
(D) they are from a closed figure of four sides

96. Co-efficient of friction between the two blocks is 0.3, whereas the surface AB is smooth ($g = 10\ \text{ms}^{-2}$).



(A) Acceleration of the system of masses is 5.86 ms^{-2}
(B) Tension T_1 in the string is 17.7 N
(C) Tension T_2 in the string is about 41.4 N
(D) Acceleration of 10 kg mass is 7.55 ms^{-2}

97. In the given Fig. 3.91, pulleys are massless and frictionless, and strings are light and inextensible. A force is applied on pulley A vertically upward. At any time, acceleration of 5 kg is a_1 (upward) and 10 kg is a_2 (upward) then ($g = 10\ \text{m/s}^2$)

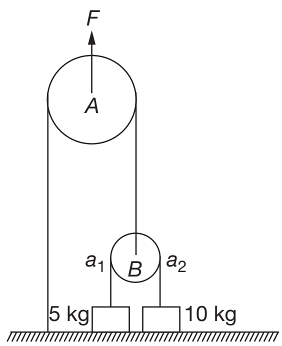
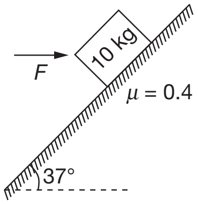


Fig. 3.91

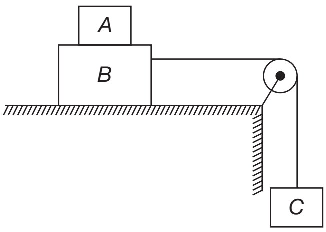
(A) $a_1 = 0$, $a_2 = 0$ if $F = 100$ N
(B) $a_1 = 5\ \text{m/s}^2$ and $a_2 = 0$ if $F = 300$ N
(C) $a_1 = 15\ \text{m/s}^2$ and $a_2 = 2.5\ \text{m/s}^2$ if $F = 500$ N
(D) acceleration of the masses is independent of F

98. A block of mass 10 kg is placed on a rough inclined plane of inclination 37° ($\tan 37° = 3/4$). The co-efficient of friction between block and surface is 0.4. A horizontal force F = 50 N is applied on the block, then ($g = 10\ \text{m/s}^2$)



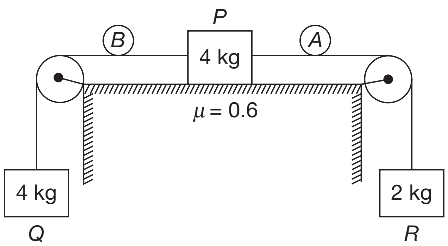
(A) Acceleration of block is zero.
(B) Acceleration of block is 2.4 m/s^2 along the inclined plane.
(C) Frictional force between block and surface is 44 N.
(D) Frictional force between block and surface is 20 N.

99. Two blocks *A* and *B* each of mass *m* is placed as shown, co-efficient of friction between *A* and *B* is 0.5 and surface is smooth. Block *B* is connected to a block *C* of mass *M* with the help of massless string. Then



(A) If $M = 2m$, acceleration of block *A* and *B* is $g/2$.
(B) If $M = 2m$, friction force between *A* and *B* is $\frac{1}{2}$ mg.
(C) Relative motion starts between blocks *A* and *B* if $M > 2m$
(D) For any value of *M* acceleration of blocks *A* and *B* are equal.

100. A block *P* of mass 4 kg is placed on horizontal rough surface with co-efficient of friction $\mu = 0.6$. And two blocks *R* and *Q* of masses 2 kg and 4 kg connected with the help of massless strings *A* and *B*, respectively, passing over frictionless pulleys as shown, then ($g = 10$ m/s^2)



(A) Acceleration of block *P* is zero.
(B) Tension in string *A* is 20 N.
(C) Tension in string *B* is 40 N.
(D) Contact force on block *P* is $20\sqrt{5}$ N.

101. Four identical blocks each of mass *m* are kept on a horizontal frictionless plane in contact with adjacent blocks as shown in Fig. 3.92. A force *F* is applied on the system

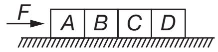
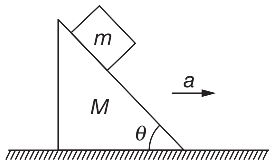


Fig. 3.92

(A) Acceleration of each block is $\frac{F}{4m}$
(B) Net force on the block *C* is $\frac{F}{4}$
(C) Net force on the block *A* is $\frac{3F}{4}$
(D) Force by the block *C* on block *D* is $\frac{F}{8}$

102. A block of mass *m* is placed on a smooth inclined surface of wedge of mass *M* as shown. Wedge is accelerating horizontally with an acceleration *a* such that block is relatively at rest on the inclined surface.



(A) The value of *a* is $g \cot\theta$.
(B) The value of *a* is $g \tan\theta$.
(C) Normal force on the wedge due to the horizontal surface is $(m + M)g$.
(D) Normal force on the surface due to wedge is Mg.

103. Three identical spheres, each of mass *m*, are kept in contact inside a box as shown in Fig. 3.93. If box is moving vertically upward with an acceleration $g/4$, then (neglect friction)

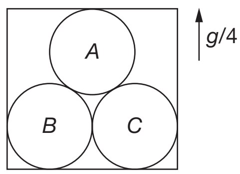


Fig. 3.93

(A) Normal force applied by the spheres on the bottom of the box is $\frac{9}{4}mg$.
(B) Normal force applied by the spheres on the bottom of the box is $\frac{15}{4}mg$.
(C) Normal force between spheres *A* and *B* is $2\sqrt{3}\,mg$.
(D) Normal force between spheres *A* and *B* is $\frac{5mg}{4\sqrt{3}}$.

Passage Based Questions

Passage 1

In the system shown in the Fig. 3.94 (A), (B), (C) and (D), the scales of the springs are calibrated in Newton.

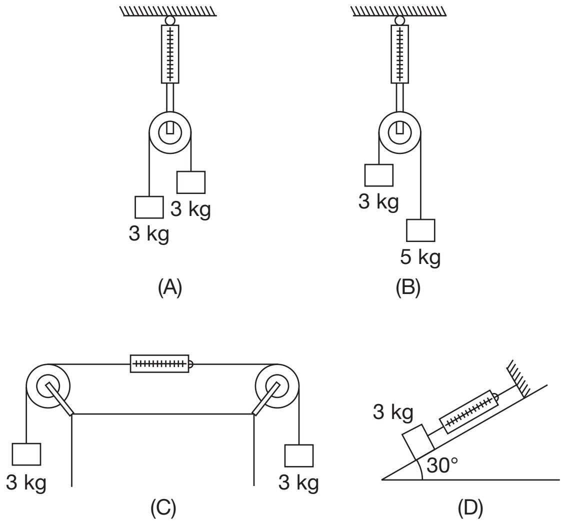


Fig. 3.94

104. Reading of the spring scale in figure (A)
(A) 30 N (B) 45 N (C) 60 N (D) 22.5 N

105. Reading of the spring scale in figure (B)
(A) 90 N (B) 62.5 N (C) 55 N (D) 75 N

106. Reading of the spring scale in figure (C)
(A) 30 N (B) 45 N (C) 15 N (D) 22.5 N

107. Reading of the spring scale in figure (D)
(A) 30 N (B) 45 N (C) 15 N (D) 22.5 N

Passage 2

A smooth ring of mass m can slide on a fixed horizontal rod. A massless string tied to the ring passes over a fixed smooth pulley of mass m and carries a block of mass 2 m as shown in Fig. 3.95. At an instant, the string between ring and pulley makes an angle $\theta = 30°$ with the horizontal.

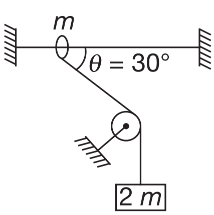


Fig. 3.95

108. Acceleration of block is
(A) $\frac{3}{5}g$ (B) $\frac{g}{3}$
(C) $\frac{2\sqrt{3}}{5}g$ (D) None of these

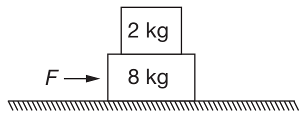
109. Acceleration of ring is
(A) $\frac{3}{5}g$ (B) $\frac{g}{3}$
(C) $\frac{2\sqrt{3}}{5}g$ (D) None of these

110. Normal force due to ring on the rod is
(A) $\frac{4mg}{5}$ (B) mg
(C) $\frac{7mg}{5}$ (D) None of these

111. Reaction force on the pulley due to support is
(A) $\frac{mg}{5}$ (B) $\frac{mg}{5}\sqrt{61}$
(C) $3mg$ (D) None of these

Passage 3

A plank of mass $m_1 = 8$ kg with a block (mass $m_2 = 2$ kg) on it is placed on a horizontal rough floor. A horizontal force F is applied on the plank. The co-efficient of friction for all the contact surfaces is 1/5.



112. If F = 20 N, then acceleration of the block with respect to the floor will be
(A) 2 m/s^2 (B) 3 m/s^2
(C) 4 m/s^2 (D) 0 m/s^2

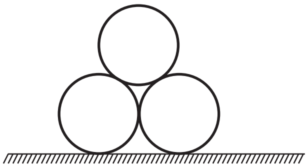
113. If F = 40 N, then acceleration of the block with respect to the floor will be
(A) 2 m/s^2 (B) 3 m/s^2
(C) 4 m/s^2 (D) 0 m/s^2

114. If F = 60 N, then acceleration of the block with respect to the floor will be
(A) 2 m/s^2 (B) 3 m/s^2
(C) 1 m/s^2 (D) 0 m/s^2

115. Frictional force between the plank and the block if F = 80 N is
(A) 4 N (B) 8 N (C) 10 N (D) 6 N

Passage 4

Three identical cylinders of mass M each are stacked as shown. These are sufficient friction to avoid slipping at any contact. There is no compression between two bottom cylinders. The normal reaction and friction force between the top and a bottom cylinder is N and f, respectively.



116. The friction force between ground and a bottom cylinder is

(A) $\frac{N}{2}$ (B) $\frac{\sqrt{3}}{2}N$

(C) $\frac{Mg}{2+\sqrt{3}}$ (D) f

117. The value of normal reaction N is

(A) $\frac{Mg}{2}$ (B) $\frac{Mg}{\sqrt{3}}$

(C) $\frac{Mg}{2+\sqrt{3}}$ (D) Mg

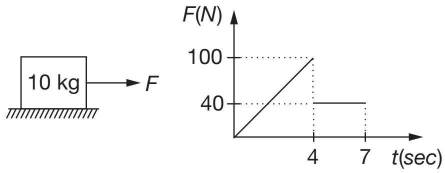
118. The value of friction force f is

(A) $\frac{Mg}{2}$ (B) $\frac{Mg}{2+\sqrt{3}}$

(C) $\frac{Mg}{2(2+\sqrt{3})}$ (D) $\frac{Mg}{\sqrt{3}}$

Passage 5

The 10 kg block is resting on a horizontal surface when the force F is applied to it for 7 s. The variation of force F with time is shown in the graph. The co-efficient of static and kinetic friction are both 0.50. ($g = 10$ m/s^2)



119. The maximum velocity reached by block during motion is

(A) 3 m/s (B) 20 m/s

(C) 5 m/s (D) 16 m/s

120. The velocity of the block at the end of 7 s is

(A) 1 m/s (B) 2 m/s

(C) 2.5 m/s (D) 3.6 m/s

121. The total time t during which the block is in motion

(A) 5 s (B) 5.4 s

(C) 7 s (D) 3 s

122. The displacement of the block during constant retardation before it stops.

(A) 10.5 m (B) 0.4 m

(C) 12.6 m (D) 10.9 m

Match the Column Type

123. Two blocks of mass 5 kg and 3 kg are placed on a smooth floor as shown in Fig. 3.96. There is no friction between the blocks. A horizontal force 15 N is applied.

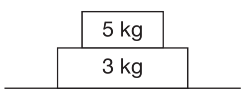
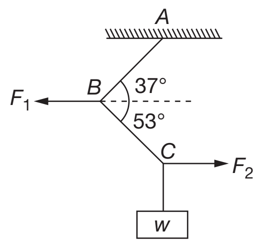


Fig. 3.96

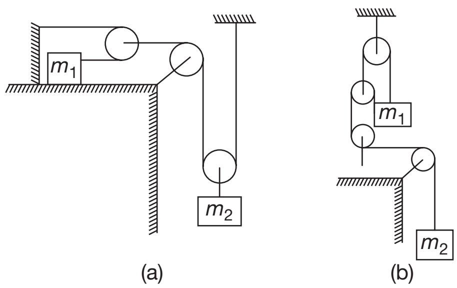
Column-I	Column-II
(A) Acceleration of 3 kg block (in SI unit) if force is applied on 5 kg block.	(1) 50
(B) Acceleration of 3 kg block (in SI unit) if force is applied on 3 kg block.	(2) 0
(C) If force is applied on 3 kg block at an angle of 30° with horizontal direction, force of interaction (in SI unit) between the two blocks is	(3) 42.5
(D) If force is applied on 5 kg block at an angle of 30° with horizontal direction, force of interaction (in SI unit) between the two blocks is	(4) 5
	(5) 10

124. In the arrangement shown, a block of weight w is attached at one end of a massless inextensible string ABC, which is fixed at the roof at point A. Two horizontal forces F_1 and F_2 one applied on the string at points B and C, respectively



Column-I	Column-II
(A) Value of F_1	(1) $5w/4$
(B) Value of F_2	(2) $25w/12$
(C) Tension in the BC part of the string	(3) $5w/3$
(D) Tension in the AB part of the string	(4) $3w/4$
	(5) $7w/4$

125. In each of the following arrangements, let acceleration of mass m_2 be a_2 and acceleration of mass m_1 be a_1 ($m_1 = 1$ kg, $m_2 = 4$ kg).



Match Column-I with Column-II

Column-I	Column-II
(A) $\frac{a_1}{a_2}$ in condition (a)	(1) 1/3
(B) $\frac{a_1}{a_2}$ in condition (b)	(2) 4
(C) Tension in the string (in N) connecting m_2 in (a) will be	(3) 32
(D) Tension in the string (in N) connecting m_2 in (b) will be	(4) 8
	(5) $\frac{160}{37}$

126. Three blocks are arranged as shown in Fig. 3.97. All the strings are massless and inextensible. All pulleys are frictionless and massless. Co-efficient of friction between surfaces and blocks are 0.5. If the blocks were initially at rest, then find (given tan 37° = 3/4)

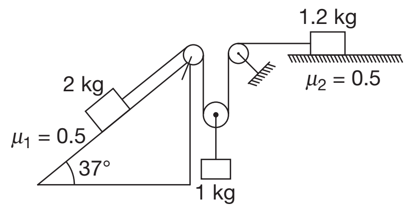


Fig. 3.97

Column-I	Column-II
(A) Tension in the string attached to 2 kg block (in N)	(1) 7
(B) Contact force by the surface on 1.2 kg (in N)	(2) 10
(C) Acceleration of 1.2 kg block (in m/s^2)	(3) 13
(D) Friction force on 2 kg block (in N)	(4) 5
	(5) Zero

Assertion-Reason Type

127. Assertion: Velocity of particle with respect to ground (assuming inertial frame) is $\vec{v}$. If it is observed from any inertial frame, then its velocity is observed to be $\vec{v}$.

Reason: Non-accelerated frame is inertial frame.

(A) A (B) B (C) C (D) D

128. Assertion: A block is at rest on an inclined plane of inclination θ, net contact force on the block is mg.

Reason: If block is at rest, then net force on the block is zero.

(A) A (B) B (C) C (D) D

129. Assertion: Without friction between our feet and the ground, it will not be possible to walk.

Reason: Frictional force is necessary to start motion.

(A) A (B) B (C) C (D) D

130. **Assertion:** A body may have acceleration even if its velocity is zero.

Reason: Acceleration is rate of change of velocity at that instant.

(A) A (B) B (C) C (D) D

131. **Assertion:** Two blocks side by side moving with same acceleration may have contact force between them.

Reason: If external force acting on one of the two blocks causing same acceleration to them, then contact force exists between them.

(A) A (B) B (C) C (D) D

132. **Assertion:** It is easier to pull an object (kept on a rough surface) than to push it.

Reason: Co-efficient of friction is more in case of pushing as compared to pulling.

(A) A (B) B (C) C (D) D

133. **Assertion:** A monkey slides down a vertical rope with constant acceleration ($< g$). The tension force on the monkey is in upward direction.

Reason: In assertion, net force on the monkey is in downward direction.

(A) A (B) B (C) C (D) D

134. **Assertion:** A block of mass m is at rest on a inclined surface of inclination θ. Normal contact force between block and surface is $mg \cos\theta$.

Reason: If block of mass m is at rest on inclined surface of inclination θ. Friction force on the block is $mg \sin\theta$.

(A) A (B) B (C) C (D) D

135. **Assertion:** Value of frictional force as seen from an inertial frame, for a pair of solids may change if it is observed from a non-inertial frame.

Reason: In a non-inertial frame, the effect of pseudo force has to be considered.

(A) A (B) B (C) C (D) D

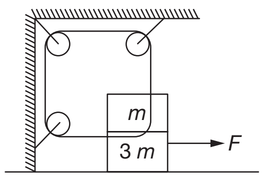
136. **Assertion:** A car accelerates on a horizontal road due to the force exerted by the engine of the car.

Reason: To accelerate a body, force is always needed in the direction of required acceleration.

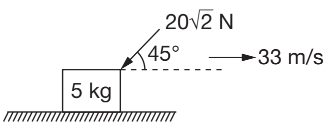
(A) A (B) B (C) C (D) D

Integer Type

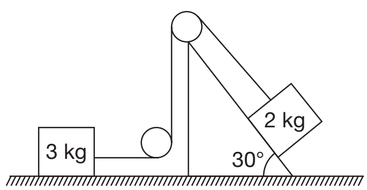
137. In the diagram shown, the co-efficient of friction between the blocks is 0.9. Find the minimum value of force F such that relative motion starts between the blocks if $m = 0.5$ kg (surface is smooth).



138. A block of mass 5 kg is kept on a rough horizontal floor. It's given velocity is 33 m/s towards right. A force of $20\sqrt{2}$ N continuously acts on the block as shown. If the co-efficient of friction between block and floor is 0.5, find the velocity of the block after 5 seconds ($g = 10$ m/s^2).



139. Find the acceleration of each block and tension in the string. If all surfaces are frictionless and wedge is fixed.



140. In the given Fig. 3.98, strings are massless and pulley is frictionless. Find ratio of tension in the strings BC and AB ($g = 10$ m/s^2).

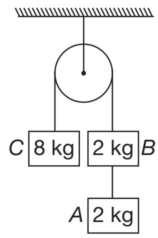


Fig. 3.98

141. In Fig. 3.99 shown, find the acceleration of block

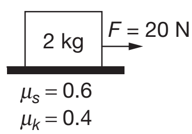
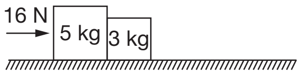


Fig. 3.99

142. Two blocks of masses 5 kg and 3 kg are placed on a smooth horizontal surface. A horizontal force $F = 16$ N is applied on 5 kg as shown. Find normal force between the blocks.



143. For the given Fig. 3.100. Find the tension in the string.

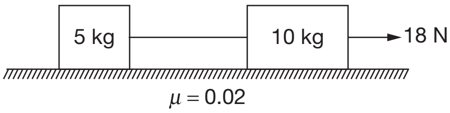
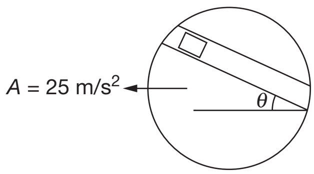


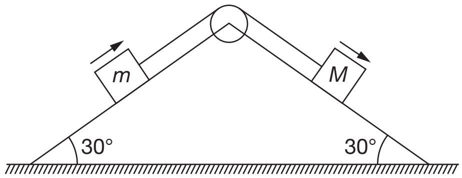
Fig. 3.100

144. A small block of mass m is placed in a groove carved inside a disc. The disc is placed on smooth horizontal surface and pulled with an acceleration of magnitude 25 m/s^2 as shown. Find half of the acceleration of block with respect to the disc in m/s^2?



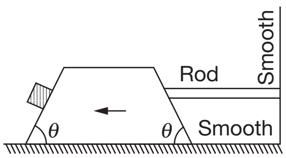
(Given $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $g = 10$ m/s^2 and co-efficient of friction between groove and the block is $\mu = \frac{2}{5}$)

145. Two blocks are connected by a string passing over a pulley as shown. Wedge is fixed. M is sliding down with speed 11 m/s at an instant and suddenly the string breaks. The co-efficient of friction between the blocks and wedge is $\frac{1}{\sqrt{2}}$ for both the blocks. Find the velocity of block M with respect to m after $t = 15$ seconds from the moment the string breaks.



[Taking $g = 10$ ms^{-1} and $\sqrt{\frac{3}{2}} \approx 1.2$ and assuming that the length of inclined is sufficiently long.]

146. In the diagram shown, no relative motion takes place between the wedge and the block placed on it. The rod slides downwards over the wedge and pushes the wedge to move in horizontal direction. The mass of wedge is same as that of the block and is equal to $M = 1$ kg. If $\tan\theta = \frac{1}{\sqrt{3}}$, find the mass of rod.

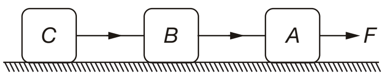


Previous Years' Questions

147. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is **[2002]**

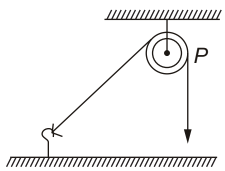
(A) 8 : 1 (B) 9 : 7 (C) 4 : 3 (D) 5 : 3

148. Three identical blocks of masses $m = 2$ kg each are drawn by a force $F = 10.2$ N with an acceleration of 0.6 m/s^2 on a surface, then what is the tension (in N) in the string between B and C, if there is no friction between the surface and the blocks A and B **[2002]**



(A) 9.2 (B) 3.4 (C) 4 (D) 9.8

149. One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 840 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope? **[2002]**

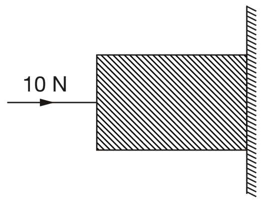


(A) 16 (B) 6
(C) 4 (D) 8

150. A lift is moving down with an acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are, respectively, **[2002]**

(A) g, g (B) $g-a, g, -a$
(C) $g-a, g$ (D) a, g

151. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The co-efficient of friction between the block and wall is 0.2. The weight of the block is **[2003]**



(A) 20 N (B) 50 N (C) 100 N (D) 2 N

152. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is **[2003]**

(A) $\dfrac{Pm}{M+m}$ (B) $\dfrac{Pm}{M-m}$
(C) P (D) $\dfrac{PM}{M+m}$

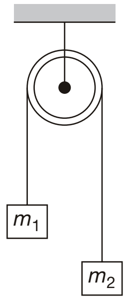
153. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is **[2003]**

(A) Both the scales read M kg each
(B) The scale of the lower one reads M kg and of the upper one zero
(C) The reading of the two scales can be anything but the sum of the readings will be M kg
(D) Both the scales read $M/2$ kg

154. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The co-efficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (Taking $g = 10$ m/s^2) **[2004]**

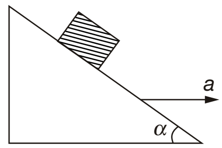
(A) 2.0 (B) 4.0 (C) 1.6 (D) 2.5

155. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when they are free to move? ($g = 9.8$ m/s^2) **[2004]**



(A) 0.2 m/s^2 (B) 9.8 m/s^2
(C) 5 m/s^2 (D) 4.8 m/s^2

156. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary with respect to the incline. Then a is equal to **[2005]**



(A) $g/\tan\alpha$ (B) g cosec α
(C) g (D) $g\tan\alpha$

157. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m. **[2007]**

(A) $\dfrac{mF}{M}$ (B) $\dfrac{(M+m)F}{m}$
(C) $\dfrac{mF}{(m+M)}$ (D) $\dfrac{MF}{(m+M)}$

158. Two fixed frictionless inclined plane making an angle 30° and 60° with the vertical are shown in the Fig. 3.101. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? **[2010]**

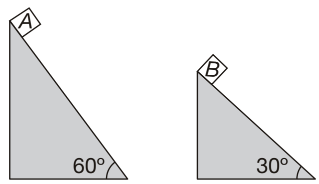


Fig. 3.101

(A) 4.9 ms^{-2} in horizontal direction
(B) 9.8 ms^{-2} in vertical direction
(C) Zero
(D) 4.9 ms^{-2} in vertical direction

159. A block of mass m is placed on a surface with a vertical cross-section given by $y = \frac{x^3}{6}$. If the co-efficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is: **[2014]**

(A) $\frac{1}{6}m$ (B) $\frac{2}{3}m$

(C) $\frac{1}{3}m$ (D) $\frac{1}{2}m$

160. Given in the Fig. 3.102 are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the co-efficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is: **[2015]**

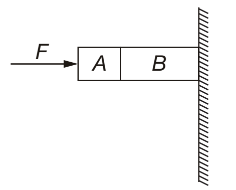


Fig. 3.102

(A) 80 N (B) 120 N (C) 150 N (D) 100 N

ANSWER KEYS

Single Option Correct Type

1. (C)	**2.** (A)	**3.** (C)	**4.** (C)	**5.** (A)	**6.** (C)	**7.** (A)	**8.** (C)	**9.** (B)	**10.** (D)
11. (C)	**12.** (A)	**13.** (C)	**14.** (C)	**15.** (D)	**16.** (A)	**17.** (D)	**18.** (A)	**19.** (A)	**20.** (D)
21. (B)	**22.** (A)	**23.** (B)	**24.** (C)	**25.** (D)	**26.** (B)	**27.** (A)	**28.** (A)	**29.** (C)	**30.** (A)
31. (B)	**32.** (B)	**33.** (B)	**34.** (A)	**35.** (C)	**36.** (B)	**37.** (B)	**38.** (B)	**39.** (A)	**40.** (A)
41. (B)	**42.** (B)	**43.** (D)	**44.** (A)	**45.** (A)	**46.** (A)	**47.** (B)	**48.** (D)	**49.** (A)	**50.** (D)
51. (C)	**52.** (C)	**53.** (A)	**54.** (C)	**55.** (B)	**56.** (C)	**57.** (A)	**58.** (B)	**59.** (A)	**60.** (D)
61. (A)	**62.** (D)	**63.** (A)	**64.** (C)	**65.** (A)	**66.** (A)	**67.** (A)	**68.** (A)	**69.** (C)	**70.** (B)
71. (C)	**72.** (B)	**73.** (B)	**74.** (C)	**75.** (A)	**76.** (D)	**77.** (C)	**78.** (B)	**79.** (A)	**80.** (C)
81. (C)	**82.** (B)	**83.** (C)	**84.** (A)	**85.** (C)	**86.** (B)	**87.** (B)	**88.** (C)	**89.** (A)	**90.** (D)
91. (A)	**92.** (C)	**93.** (C)							

More than One Option Correct Type

94. (A), (C) and (D) **95.** (C) and (D) **96.** (A), (B) and (C) **97.** (A), (B) and (C) **98.** (A) and (D)
99. (A), (B) and (C) **100.** (A), (B), (C) and (D) **101.** (A) and (B) **102.** (B) and (C)
103. (A) and (B)

Passage Based Questions

Passage 1

104. (C) **105.** (D) **106.** (A) **107.** (C)

Passage 2

108. (A) **109.** (C) **110.** (C) **111.** (B)

Passage 3

112. (D) **113.** (A) **114.** (A) **115.** (A)

Passage 4

116. (D) **117.** (A) **118.** (C)

Passage 5

119. (C) **120.** (B) **121.** (B) **122.** (D)

Match the Column Type

123. (A) → 2; (B) → 4; (C) → 1; (D) → 3
124. (A) → 2; (B) → 4; (C) → 1; (D) → 3
125. (A) → 2; (B) → 1; (C) → 3; (D) → 5
126. (A) → 4; (B) → 3; (C) → 5; (D) → 1

Assertion-Reason Type

127. (D) **128.** (A) **129.** (C) **130.** (B) **131.** (C) **132.** (C) **133.** (B) **134.** (D) **135.** (D) **136.** (D)

Integer Type

137. 5 **138.** 0 **139.** 6 N **140.** 2 **141.** 6 **142.** 6 **143.** 6 **144.** 5 **145.** 0 **146.** 3

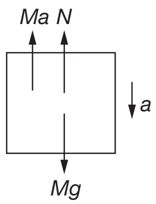
Previous Years' Questions

147. (B) **148.** (B) **149.** (C) **150.** (C) **151.** (D) **152.** (D) **153.** (A) **154.** (A) **155.** (A) **156.** (D)
157. (C) **158.** (D) **159.** (A) **160.** (B)

HINTS AND SOLUTIONS

Single Option Correct Type

1. FBD of block is as shown



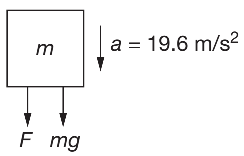
$Mg = N + Ma$

$Mg = \frac{Mg}{4} + Ma$

$a = \frac{3g}{4}$

The correct option is (C)

2.



$F + mg = ma$

$F = m(a - g) = 2\ (19.6 - 9.8) = 19.6$ N

The correct option is (A)

3. $a = \frac{Mg\sin\theta}{2M}$ and $T = Ma$

The correct option is (C)

4. Reading reduces when the lift starts accelerating downwards and then original value is restored as lift moves with constant velocity.

Apparent weight $= m(g \pm a)$, where a is acceleration of lift.

The correct option is (C)

5. $F - 7g = 7a$, $T - 2g = 2a$

On solving, $F = 140$ N

The correct option is (A)

6. For equilibrium of $\sqrt{2}$ M block

$2T\cos\theta = \sqrt{2}Mg$, $T = Mg$, $\cos\theta = \frac{1}{\sqrt{2}}$, $\theta = 45°$

The correct option is (C)

7. $a = \left(\frac{M-m}{M+m}\right)g$, $s = \frac{1}{2}at^2$

$\Rightarrow\ 1.4 = \frac{1}{2}\left(\frac{M-m}{M+m}\right)g(2)^2 \Rightarrow \frac{m}{M} = \frac{13}{15}$

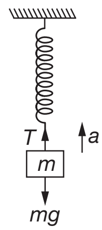
The correct option is (A)

8. $T - mg = ma$

$T = mg + ma$

$Kx = m(g + a)$

$x = \frac{m(g+a)}{K}$



The correct option is (C)

9. $N = m_A(g - a) = 0.5(10 - 2) = 4$ N

The correct option is (B)

10. If retardation of body is a and air resistance is f

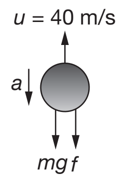
$v = u + at$

$0 = 40 - 3a$

$a = \frac{40}{3}$ m/s^2

$ma = mg + f$

$f = ma - mg = 1.5\left(\frac{40}{3} - 10\right) = 5$ N



The correct option is (D)

11. $m_1 = 2\,\text{kg}$, $m_2 = 4\,\text{kg}$, $m_3 = 6\,\text{kg}$,

$a = \frac{F - (m_1 + m_2 + m_3)g\sin 53°}{m_1 + m_2 + m_3}$

$a = \frac{120 - 12 \times 10 \times 4/5}{12}$, $= \frac{24}{12} = 2\,\text{ms}^{-2}$

$T_1 - m_1 g\sin 53° = m_1 a,$

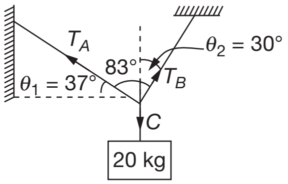
$T_1 = 4 + 20\times\frac{4}{5} = 20\,\text{N}$

$T_2 - (m_1 + m_2)g\sin 53° = (m_1 + m_2)a,$

$T_2 = 12 + 60\times\frac{4}{5} = 60\,\text{N},\ \frac{T_1}{T_2} = \frac{1}{3}$

The correct option is (C)

12.



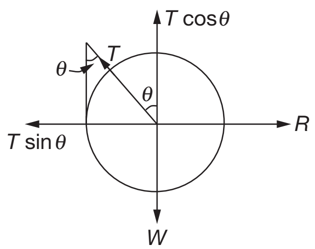
$T_A \cos\theta_1 = T_B \sin\theta_2$

$T_A \cos 37° = T_B \sin 30°$

$T_A\times\frac{4}{5} = T_B\times\frac{1}{2};\ \frac{T_A}{T_B} = \frac{5}{8}$

The correct option is (A)

13.



$T\sin\theta = R$

$T\cos\theta = W$

Solving,

$T^2 = R^2 + W^2$

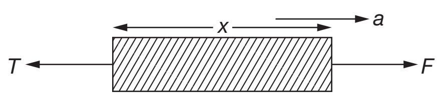
$R = W\tan\theta$

Vertically

$\vec{R} + \vec{T} + \vec{W} = 0$

The correct option is (C)

14. Acceleration $a = \frac{F}{M}$

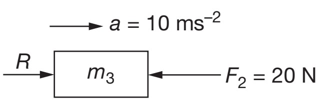


Drawing FBD

$F - T = \frac{M}{L}(x)a \Rightarrow T = F\left(1 - \frac{x}{L}\right)$

The correct option is (C)

15.



$a = \frac{F_1 - F_2}{m_1 + m_2 + m_3} = 10\ \text{ms}^{-2}$

$R - F_2 = m_3 a$

$R = 30\,\text{N}$

The correct option is (D)

16. Let a be the acceleration of the fireman. Then $mg - R = ma$

But, maximum value of $R = \frac{3}{4}\,mg$

$\therefore\quad mg - \frac{3}{4}mg = ma$ or $a = \frac{g}{4}$

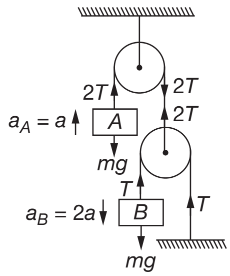
The correct option is (A)

17. $2T - mg = ma$ (1)

$mg - T = 2ma$ (2)

(1) and (2) $\Rightarrow a = \frac{g}{5}$

$\therefore\quad a_B = \frac{2g}{5}$



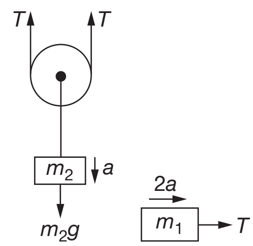
The correct option is (D)

18. $m_2 g - 2T = m_2 a$

or $2T = m_2(g - a)$ (1)

Again, $T = m_1(2a)$

or $2T = 4m_1 a$ (2)



Equating (1) and (2),

$m_2 g - m_2 a = 4m_1 a$

or $(4m_1 + m_2)a = m_2 g$

$a = \frac{m_2 g}{4m_1 + m_2}.$

The correct option is (A)

19. On cutting of string QR, the resultant force m_1 remains zero because its weight $m_1 g$ is balanced by the tension in the spring, but on block m_2 a resultant upward force $(m_1 - m_2)g$

is developed. Thus, block m_1 will have no resultant acceleration, whereas m_2 does have an upward acceleration given by $\frac{(m_1 - m_2)g}{m_2}$.

The correct option is (A)

20. $T_1 = 15A$, $T_2 = 3A$, $\frac{T_1}{T_2} = \frac{5}{1}$

The correct option is (D)

21. $T = m(g + a)$, $T = 62.7$ N

The correct option is (B)

22. If acceleration of block B is a downward, then acceleration of block A is $2a$ upward along the incline.

For block B,

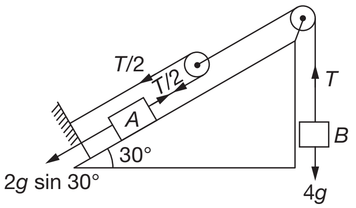
$$4g - T = 4a \quad (1)$$

For block A,

$$\frac{T}{2} - 2g\sin 30° = 2(2a) \Rightarrow T - 2g = 8a \quad (2)$$

From (1) and (2), $a = \frac{g}{6}$

$\therefore$ acceleration of block $A = 2a = \frac{g}{3}$ m/s^2 = $\frac{10}{3}$ m/s^2



The correct option is (A)

23. $a = \frac{40 - 20}{6} = \frac{10}{3}$

Also, $20 - T = 2a$

$\Rightarrow \quad T = \frac{40}{3} = 13.33 \cong 13\,\text{N}$

The correct option is (B)

24. From constraint relation $v_B = \frac{v}{3}$

The correct option is (C)

25. $T = 0$, $a = g$

The correct option is (D)

26. $T_1 = \frac{mg}{\cos\theta}$, $T_2 = mg\cos\theta$

$$\frac{T_1}{T_2} = \sec^2\theta = 2$$

The correct option is (B)

27. $N_A = N_B$

The correct option is (A)

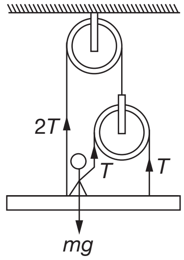
28. Maximum friction force is 50 N, which is greater than 40 N. Block does not move.

The correct option is (A)

29. From constraint relation, $a_B = 8a_A$

The correct option is (C)

30.



$4T = mg$

$\therefore \quad T = \frac{60 \times 10}{4} = 150$ N

The correct option is (A)

31. $f = \mu(m_1 + m_2 + m_3)g = 0.4(3 + 2 + 1) \times 10 = 24$ N

To move the blocks $F \geq f$, $3t \geq 24$, $t \geq 8$s

The correct option is (B)

32. $a = \frac{mg - \mu mg}{2m} = 0.4$ g m/s^2

The correct option is (B)

33. $19.6 = \mu \times 10 \times 9.8$

$\mu = 0.2$

The correct option is (B)

34. The inclined plane exerts a force of $mg\cos\theta$ perpendicular to inclination and $mg\sin\theta$ along inclination.

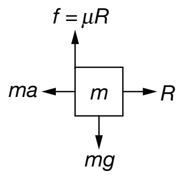
The correct option is (A)

35. $\Sigma F_y = 0$, $R = ma$

$Mg = \mu R = \mu ma$

$\mu = \frac{g}{a} = 0.5$

The correct option is (C)



36. $\mu ma = mg$ or $a = \frac{g}{\mu}$

The correct option is (B)

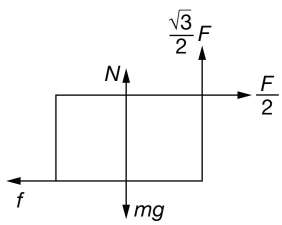
37. $mg\sin\theta = 5$ N,

$f_l = \mu mg\cos\theta = 3.4\,\text{N}$,

$a = \frac{mg\sin\theta - f}{m} = 1.6$ ms^{-2}

The correct option is (B)

38.



$f = \frac{F}{2} = \mu N$

Also, $N = mg - \frac{\sqrt{3}}{2}F$

$\Rightarrow \quad f = \mu\left(mg - \frac{\sqrt{3}}{2}F\right)$

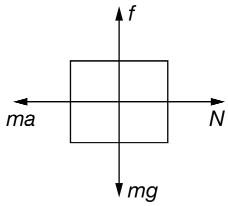
The correct option is (B)

39. $f = \mu N = mg$

Also, $N = ma$

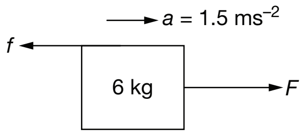
$\Rightarrow \quad \mu ma = mg$

$\Rightarrow \quad a = \frac{g}{\mu}$



The correct option is (A)

40.



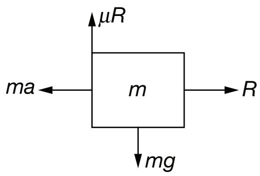
$f = 0.4 \times 2 \times 10 = 8N$

$F - 8 = 6 \times 1.5$

$F = 17N$

The correct option is (A)

41. Drawing FBD of mass m from the frame of M.



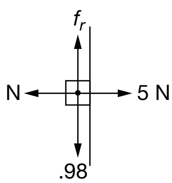
$R = ma$

$mg = \mu R = \mu ma$

$\therefore \quad a = \frac{g}{\mu}$

The correct option is (B)

42. FBD of block is as shown



Maximum frictional force = μN = 0.5 × 5 = 2.5 N

As maximum friction force > frictional force required to avoid motion,

$\therefore \quad f_r = mg = 0.98$ N

The correct option is (B)

43. As weight = 0.3 × 10 = 3 N trying to slide the two block system,

but $\quad f_{max} = 0.5 \times 1 \times 10 + 0.5 \times 1 \times 10 = 10$ N,

Hence the system is in equilibrium and friction of block B is sufficient to balance the weight; hence, tension between A and B is zero.

The correct option is (D)

44. F required to move

$F = 0.5 \times 60 \times 9.8$

$a = \frac{(F - f_x)}{m}$

$a = \frac{0.5 \times 60 \times 9.8 - 0.4 \times 60 \times 9.8}{60} = 0.98 \text{ ms}^{-2}$

The correct option is (A)

45. During downward motion, $F = mg\sin\theta - \mu mg\cos\theta$

During upward motion, $2F = mg\sin\theta + \mu mg\cos\theta$

Solving above two equations, we get $\mu = \frac{1}{3}\tan\theta$

The correct option is (A)

46. $R = mg + F_2\cos\theta, f = \mu R, \ f = \mu[mg + F_2\cos\theta]$

Also, $f = F_1 + F_2\sin\theta$

Equating, $\mu[mg + F_2\cos\theta] = F_1 + F_2\sin\theta$

or $\quad \mu = \frac{F_1 + F_2\sin\theta}{mg + F_2\cos\theta}$

The correct option is (A)

47. $S = \frac{1}{2}\mu g t^2$ or $t \propto \frac{1}{\sqrt{\mu}}$.

The correct option is (B)

48. $R = m(g - a)$ for downward motion of lift

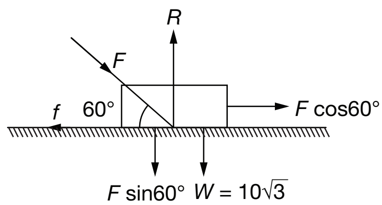
If $a = g$, then $R = 0$

$\therefore \quad F = \mu R = 0$

The correct option is (D)

49. $f = \mu R = \mu(W + F\sin 60°)$

$F\cos 60° = \mu(W + F\sin 60°)$



Substituting $\mu = \frac{1}{2\sqrt{3}}$ and $W = 10\sqrt{3}$, we get $F = 20$ N

The correct option is (A)

50. Surface between wall and A is smooth, so the system will fall with acceleration g.

The correct option is (D)

51. For $a = 0$, tension is constant throughout

$F = \mu \times (4+2+3) \times g = 0.5 \times 9 \times 10 = 45\,\text{N}$

The correct option is (C)

52. $\mu mg = m\left(\frac{mg}{4m}\right) \Rightarrow m = \frac{1}{4}$

The correct option is (C)

53. $a_{\max} = \mu g$

The correct option is (A)

54. Friction is static so $a = 0\ \text{m/s}^2$,

$f = T\cos 60 = 40\cos 60 = 20\,\text{N}$

The correct option is (C)

55. Friction force = 0.2 × 1 × 10 = 2 N

at t = 2s, block comes to rest and then F < friction so speed at t = 3s is zero.

The correct option is (B)

56. Force of friction between the two will be maximum, i.e. μmg

Retardation of A is $a_A = \frac{\mu mg}{m} = \mu g$ and acceleration of B is

$a_B = \frac{\mu mg}{2m} = \frac{\mu g}{2}$

$\therefore$ Acceleration of B relative to A is $a_{BA} = a_A + a_B = \frac{3\mu g}{2}$

Substituting $\mu = \frac{1}{2}$, $a_{BA} = \frac{3g}{4}$

The correct option is (C)

57. Free body diagram of the two bodies are as follows

Let acceleration of both the blocks towards left is a.

Then $a = \frac{f-2}{2} = \frac{20-f}{4}$

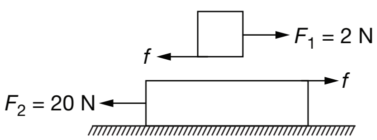
or $2f - 4 = 20 - f$ or $f = 8$ N

Maximum friction between the two blocks can be

$f_{\max} = \mu mg = (0.5)(2)(10) = 10$ N

Now since $f < f_{\max}$

Therefore, friction force between the two blocks is 8 N.



The correct option is (A)

58. Maximum frictional force between A and B could be

$f_1 = \mu_1 mg = (0.2)(2)(10)\,\text{N}$

$f_1 = 4$ N

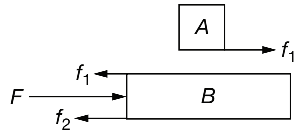
$a = \frac{f_1}{m_A} = \frac{4}{2} = 2\ \text{m/s}^2$

Now, taking ($A + B$) as the system

$(f_2)_{\max} = \mu_2(m_A + m_B)g = 24$ N

$F - 24 = (m_A + m_B)a = 6 \times 2 = 12$

$\therefore \quad F = 36$ N



The correct option is (B)

59. $v^2 = 2g\sin\theta \frac{l}{2}$

Also, $v^2 = -2(g\sin\phi - \mu g\cos\phi)\frac{l}{2}$

$\Rightarrow \quad -g\sin\phi + \mu g\cos\phi = g\sin\phi$

$\Rightarrow \quad \tan\phi = \frac{\mu}{2}$

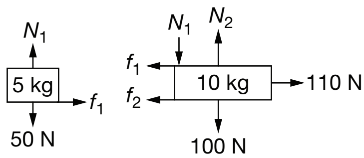
$\Rightarrow \quad \mu = 2\tan\phi$

The correct option is (A)

60. $(f_2)_{\max} = \mu_2 N_2 = 0.8 \times 150 = 120$ N

$\therefore \quad a = 0$

$\therefore \quad f_1 = 0$



The correct option is (D)

61. During upward motion,

Net force acting on pebble (F) = ma

$= 0.05 \times 10\,\text{N}$

$= 0.5$ N

(Vertically downward)

During downward motion,

Net force acting on pebble (F) = ma

$= 0.05 \times 10\,\text{N}$

$= 0.5\,\text{N}$

(Vertically downward)

At the highest point,

Net force acting on pebble (F) = ma

$= 0.05 \times 10\,\text{N}$

$= 0.5\,\text{N}$

(Vertically downward)

The correct option is (A)

62. When stone is dropped from the window of a stationary train, it falls freely under gravity.

Net force acting on stone (F) $= mg = 0.1 \times 10$

$= 1.0\,\text{N}$

(Vertically downward)

Just after the stone is dropped from the window of a train running at a constant velocity, i.e., acceleration of the train is zero. So no force acts on the stone due to the motion it falls freely under gravity. The force acts on it is due to its weight only.

The correct option is (D)

63. When a particle moves in a circle, the required centripetal force is obtained from the tension in the string.

$\therefore$ Net force on the particle directed towards centre

= centripetal force = tension in the string = T

$\therefore$ Correct alternative is T.

The correct option is (A)

64. Retarding force $F = -50$ N

Mass of the body $m = 20$ kg

Initial speed $u = 15$ m/s

Final speed $v = 0$

Time $t = ?$

Force $F = $ ma

or $a = \frac{F}{m} = -\frac{50}{20} = -2.5 \text{ m/s}^2$ (retardation)

Using equation of motion $v = u + at$

$\therefore \quad 0 = 15 + (-2.5)t$

or $t = \frac{15}{2.5} = 6s$

The correct option is (C)

65. Mass of the body $m = 3.0$ kg

Initial speed $u = 2.0$ ms

Time $t = 25$ s

Force $F = ?$

Using the first equation of motion, $v = u + at$

$\therefore \quad 3.5 = 2.0 + a \times 25$

or $a = \frac{3.5 - 2.0}{25} \text{ m/s}^2$

Acceleration $a = \frac{1.5}{25} \text{ m/s}^2$

$\therefore$ Force acting on the body $F = ma$

$= 3.0 \times \frac{1.5}{25} = \frac{4.5}{25} \text{N}$

$= 0.18 \text{N}$

As direction of motion of the body remains unchanged, the direction of force acting on the body is along the direction of motion.

The correct option is (A)

66. Mass of the body $m = 5$ kg

Force acting on body $F_1 = 8$ N

Force perpendicular to force F_1 on the body

$$F_2 = 6 \text{ N}$$

Angle between two forces $\theta = 90°$

Resultant force acting on the body,

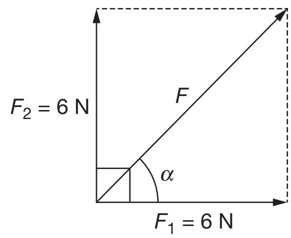
$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(8)^2 + (6)^2 + 2 \times 8 \times 6 \times \cos 90°}$$

$$= \sqrt{64 + 36}$$

$$= 10\,N$$

Acceleration $a = \frac{F}{m} = \frac{10}{5} = 2 \text{ m/s}^2$



The correct option is (A)

67. $= 36 \times \frac{5}{18} \text{m/s} = 10 \text{m/s}$ $\quad \left(\because 1 \text{ km/h} = \frac{5}{18} \text{m/s}\right)$

Final speed of the three-wheeler $v = 0$

Time $t = 4.0$ s

Mass of the three-wheeler $m_1 = 400$ kg

Mass of the driver $m_2 = 65$ kg

Total mass $m = m_1 + m_2 = 400 + 65 = 465$ kg

Using equation of the motion, $v = u + at$

$0 = 10 + a \times 40$

or $a = -\frac{10}{4.0} \text{m/s}^2 = -2.5 \text{m/s}^2$

(Negative sign of acceleration shows that it is retardation)

$\therefore$ Average retarding force acting on the vehicle

$F = ma$

$= 465 \times (-2.5)\text{N} = 1162.5 \text{N}$

The correct option is (A)

68. Given, mass of man (m) = 70 kg

In each case, the weighing scale will read the reaction R, i.e., the apparent weight.

As lift is moving upward with a uniform speed, its acceleration $a = 0$.

$\therefore$ Normal reaction $w = R = mg$

$70 \times 10 \text{N} = 700$ N

w acts vertically downwards and R acts vertically upwards.

$\therefore$ reading on weighing scale $= \frac{700}{10} = 70$ kg

Acceleration of the lift $a = 5 \text{m/s}^2 (\downarrow)$

$\therefore$ normal reaction $R = m(g - a)$

$= 70(10 - 5)\text{N}$

$= 70 \times 5 \text{N} = 350 \text{N}$

$\therefore$ reading on weighing scale $= \frac{350 \text{ N}}{10 \text{ m/s}^2} = 35$ kg

Acceleration of the lift $a = 5 \text{m/s}^2 (\uparrow)$

$\therefore$ normal reacting $R = m(g + a)$

$= 70(10 + 5) = 1050 \text{N}$

$\therefore$ reading on weighing scale $= \dfrac{1050\,\text{N}}{10\ \text{m/s}^2} = 105\,\text{kg}$

Acceleration of the lift when it is falling freely under gravity $a = g(\downarrow)$

$\therefore$ normal reaction $R = m\,(g - a)$

$= m\,(g - g) = 0$

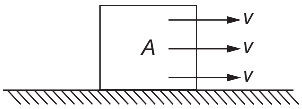
$\therefore$ reading on weighing scale = 0

This is the space of weightlessness.

The correct option is (A)

69. In a uniform translatory motion, all parts of the ball have the same velocity in magnitude and direction and this velocity is constant.

The situation is shown in adjacent diagram where a body A is in uniform translatory motion.



The correct option is (C)

70. To solve this question, we have to apply Newton's second law of motion, in terms of force and change in momentum.

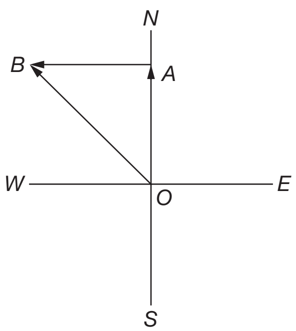
We know that $F = \dfrac{dp}{dt}$

Given that meter scale is moving with uniform velocity, hence, $dp = 0$

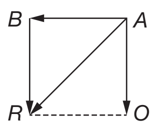
As all part of the scale is moving with uniform velocity and total force is zero, torque will also be zero.

The correct option is (B)

71. Consider the adjacent diagram



Let $OA = p_1$



= initial momentum of player northward

$AB = p_2$ = final momentum of player towards west.

Clearly $OB = OA + AB$

Change in momentum $= p_2 - p_1$

$= AB - OA = AB + (-OA)$

= Clearly resultant AR will be along south-west

The correct option is (C)

72. Given, mass $= m = 5$ kg

Acting force $= F = (-3\hat{i} + 4\hat{j})\,\text{N}$

Initial velocity at $t = 0$, $u = (6\hat{i} - 12\hat{j})$ m/s

Retardation, $\hat{a} = \dfrac{F}{m} = \left(-\dfrac{3\hat{i}}{5} + \dfrac{4\hat{j}}{5}\right)\text{m/s}^2$

As final velocity is along y-axis only, its x-component must be zero.

From $v = u + at$, for x-component only, $0 = 6\hat{i} - \dfrac{3\hat{i}}{5}t$

$t = \dfrac{5\times 6}{3} = 10$ s

The correct option is (B)

73. Given, mass of the car $= m$

As car starts from rest, $u = 0$

Velocity acquired along east $= v\hat{i}$

Duration $= t = 2$ s

We know that $v = u + at$

$\Rightarrow v\hat{i} = 0 + a\times 2$

$\Rightarrow a = \dfrac{v}{2}\hat{i}$

Force, $F = ma = \dfrac{mv}{2}\hat{i}$

Hence, force acting on the car is $\dfrac{mv}{2}$ towards east. As external force on the system is only friction hence, the force $\dfrac{mv}{2}$ is by friction. Hence, force by engine is internal force.

The correct option is (B)

74. Mass of body $B(m_2) = 20\,\text{kg}$

Force applied $(F) = 600$ N

When force is applied on A, then

For body A,

$F - T = m_1 a$

For body B,

$T = m_2 a$

Adding Equations (1) and (2), we get

$F = (m_1 + m_2)a$

or $a = \dfrac{F}{m_1 + m_2}$

$= \dfrac{600}{(10 + 20)} = 20\ \text{m/s}^2$

Substituting value of a in Equation (2), we get

$T = m_2 a = 20 \times 20\,\text{N}$

$= 400$ N

The correct option is (C)

75. Masses connected at the two ends of a light inextensible string are

$m_1 = 8$ kg, $m_2 = 12$ kg

Let T be the tension in the string and masses moves with an acceleration a when masses are released.

For mass m_1

$T = m_1 g = ma$ (1)

For mass m_2

$m_2 g - T = m_2 a$ (2)

Adding Equations (1) and (2), we get

$m_2 g - m_1 a = (m_1 + m_2)a$

$$a = \frac{(m_2 - m_1)}{(m_2 + m_1)} g$$

$$= \frac{12-8}{12+8} \times 10$$

$$= \frac{4}{20} \times 10 = 2 \text{ m/s}^2$$

Substituting value of a in Equation (1), we get

$$T = m_1 g + m_1 a$$

$$= m_1(g + a)$$

$$= 8(10 + 2) = 90 \text{ N}$$

The correct option is (A)

76. The correct option is (D)

77. For no slipping $f_{\text{max}} \geq \frac{mv^2}{R}$.

So $\mu\, mg \geq \frac{mv^2}{R}$

The correct option is (C)

78. $T_{\text{max}} = \frac{mv^2}{r} + mg$

and $T_{\text{min}} = \frac{mv^2}{r} - mg$

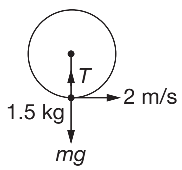
$$\therefore \quad \frac{5}{3} = \frac{\frac{v^2}{2} + 10}{\frac{v^2}{2} - 10}, \quad v = 4\sqrt{5} \text{ m/ s}$$

The correct option is (B)

79. The maximum tension will be at the lowest point.

$$T - mg = \frac{mv^2}{r}$$

$$T = \frac{mv^2}{r} + mg = \frac{1.5 \times 4}{0.5} + 1.5 \times 10 = 27 \text{ N}$$

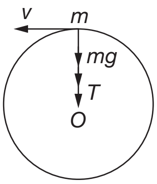


The correct option is (A)

80. $T + mg = \frac{mv^2}{r}$

$$T = \frac{mv^2}{r} - mg = 1.5 \text{ N}$$

The correct option is (C)



81. Tangential acceleration, $a_t = 4$ m/s^2

Radial acceleration, $a_r = \frac{v^2}{r} = \frac{60 \times 60}{1200} = 3 \text{ m/s}^2$,

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{4^2 + 3^2} = 5 \text{ m/s}^2$$

The correct option is (C)

82. $mg = 20$ N and $\frac{mv^2}{r} = \frac{2 \times (4)^2}{1} = 32$ N

It is clear that 52 N tension will be at the bottom of the circle.

Because we know that $T_{\text{Bottom}} = mg + \frac{mv^2}{r}$

The correct option is (B)

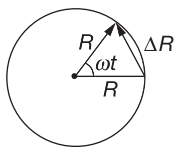
83. $a_t = 4R$

$a_N = \omega^2 R = (\alpha t)^2 R$

$a_t = a_N \quad \Rightarrow \quad 16\, t^2 = 4,\ t = \frac{1}{2}$ s

The correct option is (C)

84.



$$v_{\text{av}} = \frac{|\Delta \vec{R}|}{t} = \frac{\sqrt{R^2 + R^2 - 2R^2 \cos \omega t}}{t} = \frac{2R}{t} \sin\left(\frac{\omega t}{2}\right)$$

The correct option is (A)

85. $a_{\text{net}} = \sqrt{a_R^2 + a_T^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$

The correct option is (C)

86. Let x be the extension in the spring, then

$F = Kx$

or, $m\omega^2 (l + x) = kx$

$$\therefore \quad x = \frac{m\omega^2 l}{k - m\omega^2}$$

The correct option is (B)

87. $\theta = \left(\frac{1200+4500}{2}\right)\times\frac{2\pi}{60}\times 10 = 950\pi$ radian

Number of Revolutions $= \frac{950\pi}{2\pi} = 475$

The correct option is (B)

88. The body will have a centripetal acceleration and a downward tangential acceleration due to retardation.

Thus the resultant will be along C.

The correct option is (C)

89. Since, $P = \tau\omega =$ constant

$\Rightarrow \quad \alpha\omega = c$ (constant)

$\Rightarrow \quad \omega^2 \frac{d\omega}{d\theta} = c$

$\Rightarrow \quad \omega \propto \theta^{1/3}$

$\therefore \quad \omega \propto n^{1/3}$ (as $\theta \propto n$)

The correct option is (A)

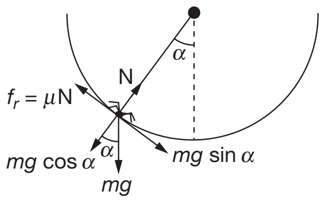
90. Since $\omega^2 - \omega_0^2 = 2\alpha\theta$ where $\theta = 2\pi n$

$\therefore \quad 2\alpha = \frac{\omega_0^2 - \left(\frac{\omega_0}{2}\right)^2}{2\pi n}$ and $0 = \left(\frac{\omega_0^2}{2}\right)^2 - (2\alpha)(2\pi n')$

$\therefore \quad n' = \frac{n}{3}$

The correct option is (D)

91. To avoid slipping $f_r = $ mg sinα at maximum α



$\mu N = mg \sin\alpha$

$\mu mg\cos\alpha = mg\sin\alpha$

$\mu = \tan\alpha$

$\therefore \quad \tan\alpha = \frac{1}{3}$

$\therefore \quad \cot\alpha = 3$

The correct option is (A)

92. Magnitude of average velocity is

$|\vec{v}_{av}| = \left|\frac{\text{displacement}}{\text{time}}\right| = \frac{PQ}{t} = \frac{\sqrt{2}R}{t} = \frac{2}{t}$

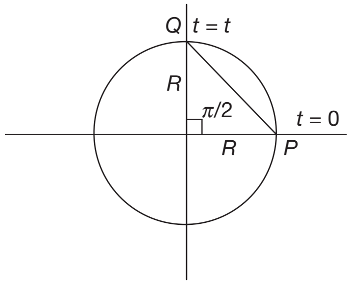
Here t can be found from $\theta = \frac{1}{2}\alpha t^2$

$\Rightarrow \quad t = \sqrt{\frac{2\theta}{\alpha}}$

$t = \sqrt{\frac{2\times\pi/2}{\pi/4}} = 2$ s

$\therefore \quad |\vec{v}_{av}| = \frac{2}{2} = 1$ m/s

The correct option is (C)



93. At the top

$T + mg = \frac{mv^2}{L}$

$T < 10mg$

$v < \sqrt{11gL}$

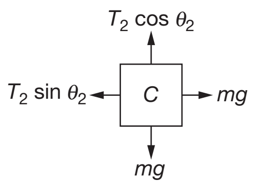
$\sqrt{3gL} < u < \sqrt{13gL}$

The correct option is (C)

More than One Option Correct Type

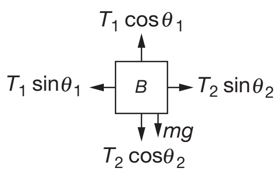
94. $T_2\sin\theta_2 = mg$ (1)

$T_2\cos\theta_2 = mg$ (2)



$T_1\cos\theta_1 = mg + T_2\cos\theta_2$

$T_1\sin\theta_1 = T_2\sin\theta_2$



The correct option is (A), (C) and (D)

95. Resultant force may not be zero for co-planar forces. Hence (a) is not true

Since magnitudes are not equal (b) cannot be true. In (c) and (d), net force is zero

The correct option is (C) and (D)

96. Here $10 - T_2 = 10a$ (1)

$T_2 - T_1 - 0.3 \times 2g = 3a$ (2)

$T_1 - 0.3 \times 2g = 2a$

Summing up $10g - 0.3 \times 4 \times g = 15a$

i.e., $a = 5.86\ \text{ms}^{-2}$

$T_2 = 10 \times 9.8 - 10 \times 5.86\,\text{ms}^{-2} = 41.4\ \text{N}$

$T_1 = 2 \times 5.86 + 0.6 \times 9.8 = 17.7\ \text{N}$

The correct option is (A) (B) and (C)

97. $a_1 > 0$ when $\frac{F}{4} > 50$, $F > 200$

$a_2 > 0$ when $\frac{F}{4} > 100$, $F > 400$

$F = 300$ N

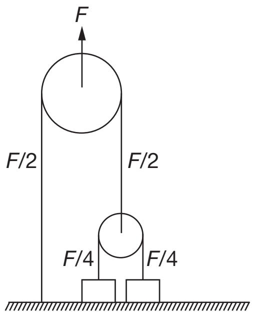
$a_1 = \frac{F/4 - 50}{5} = \frac{300/4 - 50}{5} = 5\ \text{m/s}^2$

$a_2 = 0$

If $F = 500$ N

$a_1 = 15\ \text{m/s}^2$,

$a_2 = 2.5\ \text{m/s}^2$



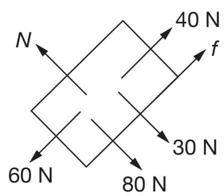
The correct option is (A) (B) and (C)

98. $f_{max} = 0.4 \times 110 = 44$ N

$40 + f = 60$

$f = 20$ N

$a = 0$



The correct option is (A) and (D)

99. Maximum acceleration block $A = \frac{0.5\text{mg}}{m} = \frac{g}{2}$

So, if $M = 2\text{m}$, $a_A = a_B = \frac{2\text{mg}}{4\text{m}} = \frac{g}{2}$ and friction force is $\frac{1}{2}$ mg.

The correct option is (A), (B) and (C)

100. Friction maximum = 24 N

So net applied force on P is less than f_{max}.

Hence, acceleration is zero and $T_A = 20$ N, $T_B = 40$ N

Contact force $= \sqrt{N^2 + (f)^2} = \sqrt{(40)^2 + (20)^2} = 20\sqrt{5}$ N

The correct option is (A) (B) (C) and (D)

101. If acceleration of the system is

$F = 4ma \Rightarrow a = \frac{F}{4\text{m}}$

$\therefore$ since acceleration of each block is $\frac{F}{4\text{m}}$

$\therefore$ net force on each block is $\frac{F}{4}$

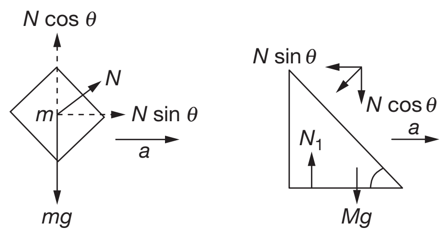
The correct option is (A) and (B)

102. $N\cos\theta = mg$

$N\sin\theta = ma$

$a = g\tan\theta$

$N_1 = Mg + N\cos\theta = Mg + mg$

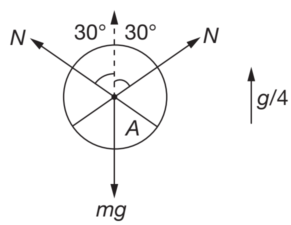


The correct option is (B) and (C)

103. Net upward force on three spheres applied by bottom

$= 3mg + \frac{3}{4}mg = \frac{15mg}{4}$

For sphere A, $N\sqrt{3} = mg + \frac{mg}{4}$, $N = \frac{5mg}{4\sqrt{3}}$



The correct option is (B) and (D)

Passage Based Questions

Passage 1

104. In Figure (a), $T = 3g = 30$ N

$\therefore$ Reading $= 2T = 60$ N

The correct option is (C)

105. $T = \frac{2\times 5\times 3}{(5+3)}g = \frac{150}{4}$N

The correct option is (D)

106. Reading $= T = 30$ N

The correct option is (A)

107. Reading $= mg \sin\theta = 3 \times 10 \times 1/2 = 15$ N

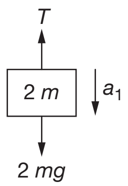
The correct option is (C)

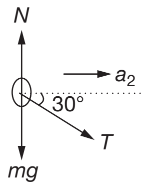
Passage 2

$2\,mg - T = 2ma_1$ (1)

$T\frac{\sqrt{3}}{2} = ma_2$ (2)

$a_2\frac{\sqrt{3}}{2} = a_1$ (3)





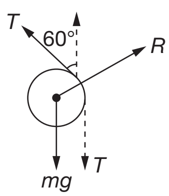
On solving, $a_1 = \frac{3}{5}g$, $a_2 = \frac{2\sqrt{3}}{5}g$ and $T = \frac{4mg}{5}$

and $N = mg + \frac{T}{2} = mg + \frac{2mg}{5} = \frac{7mg}{5}$

$$R = \sqrt{(T+mg)^2 + T^2 + 2(T+mg)T\cdot\cos 120°}$$

$$= \sqrt{\left(\frac{9mg}{5}\right)^2 + \left(\frac{4mg}{5}\right)^2 + 2\left(\frac{9mg}{5}\right)\left(\frac{4mg}{5}\right)\left(-\frac{1}{2}\right)}$$

$$= \sqrt{\left(\frac{mg}{5}\right)^2[81+16-36]} = \frac{mg}{5}\sqrt{61}$$



108. The correct option is (A)

109. The correct option is (C)

110. The correct option is (C)

111. The correct option is (B)

Passage 3

112. The limiting value of frictional force for plank is $= \frac{1}{5}(8+2)(10) = 20$ N

The correct option is (D)

113. The maximum value of friction force for block $= \frac{1}{5}\times 2\times 10 = 4$ N

Common acceleration $(a) = \frac{40-20}{8+2} = 2$ m/s^2

For block, $ma = \mu mg = 4$ N

The correct option is (A)

114. Common acceleration $(a) = \frac{60-20}{8+2} = 4$ m/s^2

For block, $ma > \mu mg$, which is not possible

The correct option is (A)

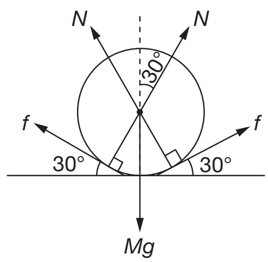
115. Common acceleration $(a) = \frac{80-20}{8+2} = 6$ m/s^2

For block $ma > \mu mg$, which is not possible.

The correct option is (A)

Passage 4

$f_1 = f$ for no rotation of bottom cylinder (1)



FBD of top cylinder:

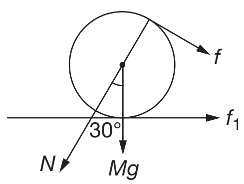
$2N\cos 30 + 2f\sin 30 = Mg$ (2)

FBD of bottom cylinder (left one)

$N\sin 30 = f_1 + f\cos 30$ (3)

Solving (1), (2) and (3)

$N = \frac{Mg}{2}$ $f = \frac{Mg}{2(2+\sqrt{3})}$



116. The correct option is (D)

117. The correct option is (A)

118. The correct option is (C)

Passage-5

$F = 25t,\ 0 \le t \le 4$s $\quad a = \left(\frac{25t-50}{10}\right), 2 < t \le 4$

$F = 40 \quad 4 \le t \le 7\text{s} \quad a' = \dfrac{40-50}{10}, 4 > t \le 7$

$F = 0 \quad t > 7\text{s} \quad a'' = \dfrac{-50}{10}, t > 7$

$v = \int_2^4 (2.5t - 5)\,dt = 5 \text{ m/s} \quad \Rightarrow$ max velocity

$v' = 5 - 1 \times 3 = 2$ m/s, at the end of 7 s

$s_1 = 5 \times 3 - \dfrac{1}{2} \times 1 \times 9 = 10.5 \text{ m}, (4 < t \le 7)$

$s_2 = 2 \times 0.4 - \dfrac{1}{2} \times 5(0.4)^2 = 0.4 \text{ m}, (t > 7)$

$\Rightarrow \quad s = s_1 + s_2 = (10.5 + 0.4)\text{ m} = 10.9 \text{ m}$

119. The correct option is (C)

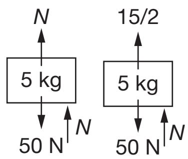
120. The correct option is (B)

121. The correct option is (B)

122. The correct option is (D)

Match the Column Type

123. No force is acting on 3 kg block.



$\dfrac{15}{3} = 5 \text{ m/s}^2$

$N = 50$ N

$N = 50 - 15/2 = 42.5$ N

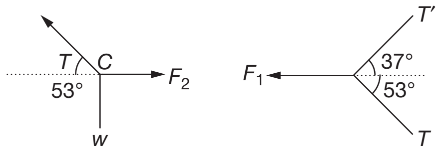
$\Rightarrow$ (A) $\to$ 2, (B) $\to$ 4, (C) $\to$ 1, (D) $\to$ 3

124. $T = \dfrac{w}{\sin(180 - 53°)} = \dfrac{5w}{4}$

$F_2 = T\sin(180 - 37°) = \dfrac{3w}{4}$

and $F_1 = \dfrac{T}{\sin(180 - 37°)} = \dfrac{5T}{3} = \dfrac{25}{12}w$

$\dfrac{T'}{\sin(180° - 53°)} = \dfrac{F_1}{\sin 90°} \Rightarrow T' = \dfrac{25}{12}w \times \dfrac{4}{5} = \dfrac{5w}{3}$



$\Rightarrow$ (A) $\to$ 2, (B) $\to$ 4, (C) $\to$ 1, (D) $\to$ 3

125. In condition (a), by constraint relation $\dfrac{a_1}{a_2} = 4$

In condition (b), by constraint relations $\dfrac{a_1}{a_2} = \dfrac{1}{3}$

In condition (a), tension in string of m_2 is

$$T = \frac{16m_1m_2g}{(16m_1 + m_2)} = 32 \text{ N}$$

In condition (b), tension in string connecting m_2 is

$$T = \frac{4m_1m_2g}{(m_1 + 9m_2)} = \frac{160}{37} \text{ N}$$

$\Rightarrow$ (A) $\to$ 2; (B) $\to$ 1; (C) $\to$ 3; (D) $\to$ 5

126. Friction force between blocks and surfaces is sufficient to prevent motion of blocks.

$T_0 = 1\,g = 10$ N

$2T = T_0$

$\Rightarrow \quad T = 5$ N

$N_2 = 1.2\,g = 12$ N

$f_2 = T = 5 \text{ N} (< \mu_2 N_2 = 6 \text{ N})$

Contact force between 1.2 kg and surface

$= \sqrt{12^2 + 5^2} = 13$ N

$T + f_1 = 2g\sin 37°$

$f_1 = 2 \times 10 \times \dfrac{3}{5} - 5 = 7$ N

$\Rightarrow$ (A) $\to$ 4; (B) $\to$ 3; (C) $\to$ 5; (D) $\to$ 1

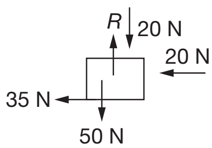
Integer Type

137. There will be relative motion when the block of mass m lifts. So $F = mg = 5$ N.

138. a = retardation of block = 11 m/s^2

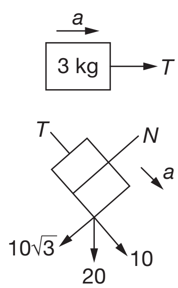
u = initial velocity = 33 m/s

By $v = u - at$



$t = \dfrac{u}{a}$ (time after which its velocity becomes zero) = 3 s

So block will come to rest after 3 s. After that, the applied force is 20 N towards left which is less than the maximum limiting friction. So it will remain at rest after that.



139. $T = 3a$ (1)

$10 - T = 2a$ (2)

From (1) and (2),

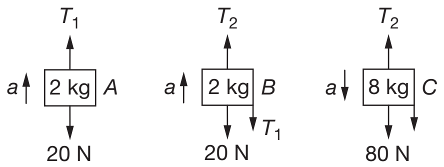
we get $a = 2$ m/s^2 and $T = 6$ N

140. $T_1 - 20 = 2a$ (1)

$T_2 - T_1 - 20 = 2a$ (2)

$80 - T_2 = 8a$ (3)

From (1), (2) and (3)

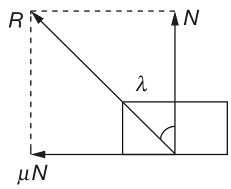


$$a = \frac{40}{12} = \frac{10}{3}\ \text{m/s}^2$$

$$T_1 = \frac{20}{3} + 20 = \frac{80}{3}\ \text{N}$$

$$T_2 = 80 - \frac{80}{3} = \frac{160}{3}\ \text{N}$$

141. Contact force: It is the net force acting between the surfaces of contact.



$$f_{\lim} = \mu_s(2g) = 0.6(20) = (12)\text{N}$$

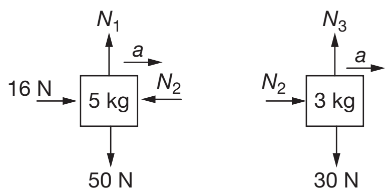
$$f_k = \mu_k(2g) = 0.4(20) = (8)\text{N}$$

$$a = \frac{F - f_k}{m} = \frac{20-8}{2} = (6)\text{m/s}^2$$

142. $16 - N_2 = 5a$ (1)

$N_2 = 3a$ (2)

On solving $a = 2$ m/s^2, $N_2 = 6$ N

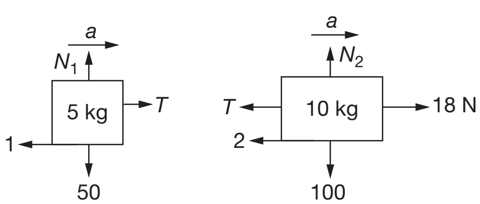


143. Frictional force on 5 kg block = $\mu N_1 = 1$ N

Frictional force on 10 kg block = $\mu N_2 = 2$ N

Now, $18 - T - 2 = 10$ a and $T - 1 = 5a$

$\Rightarrow a = 1$ m/s^2 and $T = 6$ N



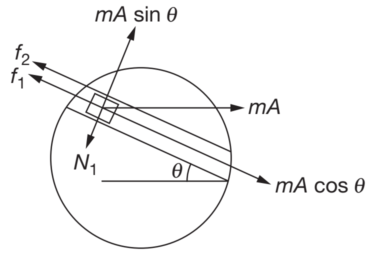
144. $mA\cos\theta - \mu mg - \mu mA\sin\theta = ma$

$$25\times\frac{4}{5} - \frac{2}{5}\times 10 - \frac{2}{5}\times 25\times\frac{3}{5} = a$$

$20 - 4 - 6 = a,$

$20 - 10 = a \Rightarrow a = 10$ m/s^2

$$\frac{a}{2} = 5$$



FBD of block with respect to disc

145. Retardation on m when it is moving up (after string breaks)

$$a = \frac{mg\sin 30° + \mu mg\cos 30°}{m} = \frac{g}{2}\left[1 + \sqrt{\frac{3}{2}}\right] = 11\ \text{ms}^{-2}$$

So it comes to rest at $t = \dfrac{v}{a} = \dfrac{11}{11} = 1$ s

Now since $\mu\, mg\cos\theta - mg\sin\theta > 0$

$\left(\frac{1}{2}\sqrt{\frac{3}{2}}\, m\times 10 - m\times 10\times\frac{1}{2} > 0\right)$ m will remain at rest afterwards.

Retardation on M

$$a = \frac{\mu Mg\cos 30° - Mg\sin 30°}{M} = \frac{g}{2}[1.2 - 1] = 1\ \text{ms}^{-2}$$

It comes to rest at

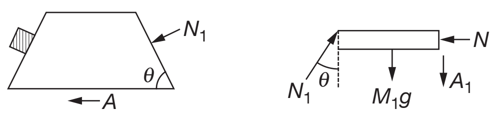
$$t = \frac{v}{a'} = \frac{11}{1} = 11\ \text{s}$$

For M, also limiting friction > $Mg\sin 30°$

Hence it also remains at rest after $t = 11$ s

$\therefore$ Relative velocity at $t = 15$ s $= \vec{v}_1 - \vec{v}_2 = 0$

146. Let M_1 be the mass of the rod.



$M_1 g - N_1\cos\theta = M_1 A_1$ (1)

$N_1\sin\theta = (M + M)A$ (2)

$A = g\tan\theta$ (3)

relation between A_1 and A

$A_1 = A\tan\theta$

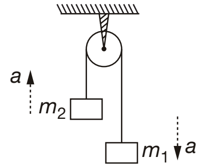
Thus by solving these equations, $M_1 = 3M = 3$ kg

Previous Years' Questions

147. By pulling force method,

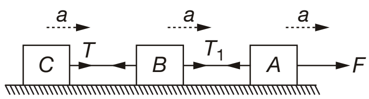
$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{g}{8}; \text{ (given)}$$

$$\Rightarrow \frac{m_1 - m_2}{m_1 + m_2} = \frac{1}{8} \Rightarrow \frac{m_1}{m_2} = \frac{9}{7}$$



The correct option is (B)

148. By pulling force method,



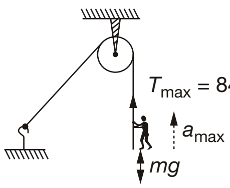
$$a = \frac{F}{m_A + m_B + m_C} = \frac{F}{3m} = \left(\frac{10.2}{6}\right) \text{m/s}^2$$

and $T = m_C(a) = 2 \times \frac{10.2}{6}$

$\therefore \quad T = (3.4)\,\text{N}$

The correct option is (B)

149.



$T - mg = ma$

$\Rightarrow \quad 840 - 600 = 60a$

$\therefore \quad a = 4 \text{ m/s}^2$

The correct option is (C)

150. For the man standing in the lift,

$\vec{a}_{bm} = \vec{a}_b - \vec{a}_m \quad \Rightarrow a_{bm} = (g - a)$

For the man standing on the ground,

$\vec{a}_{bm} = \vec{a}_b - \vec{a}_m \quad \Rightarrow a_{bm} = g - 0 \quad \Rightarrow a_{bm} = g$

The correct option is (C)

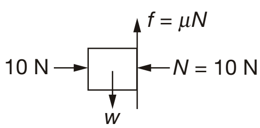
151. For the block to remain stationary with the wall,

$f = w$

$\Rightarrow \quad \mu N = w$

$\Rightarrow \quad (0.2)(10) = w$

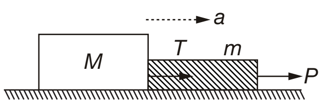
$\therefore \quad w = (2)\,\text{N}$



The correct option is (D)

152. Taking the rope and the block as a system, we get

$$a = \left(\frac{P}{M + m}\right)$$

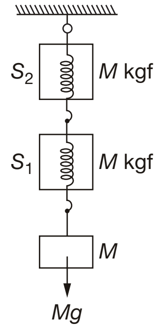


Taking the block as a system, we get $T = Ma$

$$\therefore \quad T = \left(\frac{MP}{M + m}\right)$$

The correct option is (D)

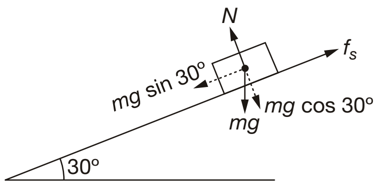
153. Earth pulls the block by a force Mg. The block is turn exerts a force Mg on the spring of spring balance S_1, which therefore shows a reading of M kgf. The spring S_1 is massless and hence it exerts a force of Mg on the spring of spring balance S_2 which leads to the reading of M kgf of spring balance S_2.



The correct option is (A)

154. $mg\sin\theta = f_s$ (For body to be at rest)

$\Rightarrow \quad m \times 10 \times \sin 30° = 10 \Rightarrow m = (2.0)\,\text{kg}$

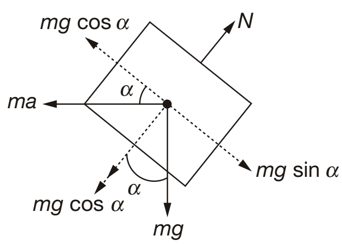


The correct option is (A)

155. $a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g = \frac{(5 - 4.8) \times (9.8)}{(5 + 4.8)} = (0.2) \text{ m/s}^2$

The correct option is (A)

156.



Analysing from inclined surface frame, FBD of block

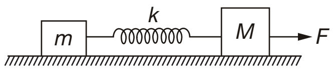
For block to remain stationary,

$mg\sin\alpha = ma\cos\alpha$

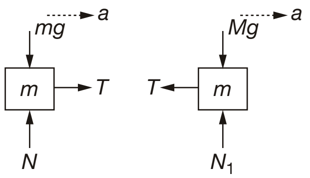
$\therefore \quad a = g\tan\alpha$

The correct option is (D)

157.



FBD



$T = ma$ (1)

where T is force due to spring.

$F - ma = Ma.$ $(\because \quad T = ma)$

$\Rightarrow \quad F = (M + m)a \qquad \Rightarrow a = \left(\frac{F}{M+m}\right)$

Now force acting on the block of mass m is:

$$ma = m\left(\frac{F}{M+m}\right) = \left(\frac{mF}{m+M}\right)$$

The correct option is (C)

158. $mg\sin\theta = ma$

$\Rightarrow \quad a = g\sin\theta$

where a is along inclined plane.

vertical component of acceleration is $(g\sin^2\theta)$.

$\therefore$ relative vertical acceleration of A with respect to B is

$= g(\sin^2 60° - \sin^2 30°) = g/2$

$= 4.9 \text{ m/s}^2$ in vertical direction.

The correct option is (D)

159. $mg\sin\theta = \mu mg\cos\theta$

$\tan\theta = \mu$

$\tan\theta = \frac{1}{2}$ (1)

Also, $y = \frac{x^3}{6}$

$\frac{dy}{dx} = \frac{x^2}{2}$ (2)

$x^2 = 1 \quad$ [using (1)]

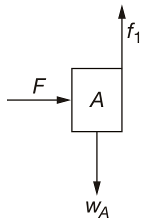
$x = \pm 1$

Now,

$$y_{\max} = \frac{1}{6}m$$

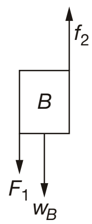
The correct option is (A)

160. FBD of block A,



$f_1 = w_A = 20\,\text{N}$

FBD of block B,



$f_2 = f_1 + w_B$

$= 20 + 100$

$= 120$ N

The correct option is (B)