CHAPTER 2

Kinematics

Chapter Highlights

Frame of reference. Motion in a straight line: Position-time graph, Speed and velocity. Uniform and non-uniform motion, Average speed and instantaneous velocity, Uniformly accelerated motion, Velocity-time, Position-time graphs, Relations for uniformly accelerated motion. Relative velocity, Motion in a plane, Projectile motion, Uniform circular motion.

MOTION IN STRAIGHT LINE

Motion

A body in motion keeps changing its position with respect to its surroundings with the passage of time. If the body does not change its position with respect to time it is said to be at rest.

Frame of Reference

A set of coordinates x, y, z, and t is said to be a frame of reference. Frame of reference may be **inertial** or **non-inertial. Inertial** frame of reference is one which is either fixed or moves with a uniform velocity in the same straight line. **Non-inertial** or **accelerated** frame of reference has an acceleration . Newton's laws are valid only in inertial frame.

One-Dimensional Motion

If the particle changes its position only in one of the x, y, or z directions with respect to time, then the motion is said to be one-dimensional. Since the particle moves along a straight line, the motion may also be termed as linear or rectilinear.

Distance

Distance is the actual path length covered by a moving particle or body in a given time interval.

Displacement

The shortest distance between initial and final position of the particle is called displacement.

Speed

The time rate of change of distance is called speed, that is $v = \frac{dx}{dt}$ unit ms^{-1}.

Velocity

The time rate of change of displacement is called velocity, that is $\vec{v} = \frac{d\vec{x}}{dt}$ unit ms^{-1}.

Acceleration

The time rate of change of velocity is called acceleration, that is $\vec{a} = \frac{d\vec{v}}{dt}$ unit ms^{-2}. Speed, velocity or acceleration may be of four types. Here, we define velocity and others follow.

1. **Uniform Velocity:** If $\frac{dx}{dt}$ = constant throughout the motion and direction of motion does not vary throughout then such a velocity is called uniform velocity.
2. **Variable Velocity:** If $\frac{dx}{dt}$ is not constant but varies at different intervals of time or $\frac{dx}{dt}$ is constant but direction or both vary, then such a velocity is said to be variable.

3. **Instantaneous Speed and Velocity:** The velocity, at a particular instant of time is called instantaneous velocity,

   Instantaneous speed $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

   Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

4. **Average Speed and Average Velocity:** The average speed of a particle in a given interval of time is defined as the ratio of distance travelled to the time taken while average velocity is defined as the ratio of displacement to time taken.

   Thus, if the distance travelled is Δs and displacement of a particle is $\Delta \vec{r}$ in a given time interval Δt, then

   v_{av} = Average speed = $\frac{\Delta s}{\Delta t}$

   $\vec{v}_{av}$ = Average velocity = $\frac{\Delta \vec{r}}{\Delta t}$

   $$\vec{v}_{av} = \frac{\text{Total displacement covered}}{\text{Total time taken}}$$

Average Velocity in Different Cases

1. **Particles covering different displacement in different times:** Assume a particle covers s_1 displacement in t_1 and s_2 in time t_2 and so on then average velocity is

   $$v_{av} \frac{s_1 + s_2 + s_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{s_1 + s_2 + s_3 + ...}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + ...}$$

   Special case if $s_1 = s_2 = s$.

   $$v_{av} \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2} \quad \text{(Harmonic mean)}$$

2. **Bodies moving with different velocity in different intervals of time:** A body moves with velocity v_1 in time t, v_2 in time t_2 and so on then v_{av} is given by

   $$v_{av} = \frac{v_1t_1 + v_2t_2 + ...}{t_1 + t_2 + ...}$$

   Special case if $t_1 = t_2 = t_3 = ... t_n = t$

   Then $v_{av} = \frac{v_1 + v_2 + ... + v_n}{n}$ (Arithmetic mean)

NOTE

If acceleration is uniform and initial velocity is u and final velocity is v, then average velocity will be $\frac{u + v}{2}$.

Average and Instantaneous Acceleration

Average acceleration is defined as the ratio of change in velocity, i.e., $\Delta \vec{v}$ to the time interval Δt in which this change occurs. Hence,

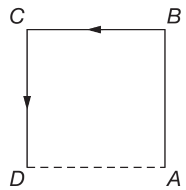
$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

The instantaneous acceleration is defined at a particular instant and is given by

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

SOLVED EXAMPLES

1. A particle moves along the sides AB, BC, CD of a square of side 25 m with a velocity of 15 m/s. Its average velocity is



   (A) 15 m/s (B) 10 m/s
   (C) 7.5 m/s (D) 5 m/s

   Solution: (D)

   $$t = \frac{75}{15} = 5\,\text{s}, \; v_{avg} = \frac{AD}{t} = \frac{25}{5} = 5 \text{ m/s}.$$

2. A person travels along a straight road for the first half time with a velocity v_1 and the second half time with a velocity v_2. Then the mean velocity v is given by

   (A) $v = \frac{v_1 + v_2}{2}$ (B) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

   (C) $v = \sqrt{v_1 v_2}$ (D) $v = \sqrt{\frac{v_2}{v_1}}$

   Solution: (A)

   Displacement in the 1st half time = $v_1 t$

   Displacement in the 2nd half time = $v_2 t$

   ∴ Net displacement = $(v_1 + v_2)\, t$

   ∴ Average velocity = $\frac{(v_1 + v_2)t}{2t} = \frac{v_1 + v_2}{2}$.

3. A particle moves according to the equation $x = 2t^2 - 5t + 6$, find average velocity in the first 3 s and velocity at $t = 3$ s.

   (A) $1\ \text{ms}^{-1}, 7\ \text{ms}^{-1}$ (B) $4\ \text{ms}^{-1}, 3\ \text{ms}^{-1}$
   (C) $2\ \text{ms}^{-1}, 5\ \text{ms}^{-1}$ (D) $3\ \text{ms}^{-1}, 7\ \text{ms}^{-1}$

Solution: (A)

$$x(3) = 2(3)^2 - 5(3) + 6 = 93x(0) = 6$$

$$v_{av} = \frac{x(3)-x(0)}{3-0} = \frac{9-6}{3} = 1 \text{ ms}^{-1}$$

$$\left.\frac{dx}{dt}\right|_{t=3} = 4t - 5 = 4(3) - 5 = 7 \text{ ms}^{-1}.$$

4. If the velocity of a particle is $(10+2t^2)$ m/s, then the average acceleration of the particle between 2 s and 5 s is

(A) 2 m/s^2 (B) 4 m/s^2
(C) 12 m/s^2 (D) 14 m/s^2

Solution: (D)

$$v_1(t = 2\text{ s}) = 10 + 2\times 2^2 = 18 \text{ m/s}$$

$$v_2(t = 5\text{ s}) = 10 + 2\times 5^2 = 60 \text{ m/s}$$

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{42}{3} = 14 \text{ m/s}.$$

5. A particle is moving eastwards with a velocity 5 ms^{-1}. In 10 s, the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is **[2005]**

(A) $\frac{1}{\sqrt{2}}$ ms^{-2} NE (B) $\frac{1}{2}$ ms^{-2} N

(C) Zero (D) $\frac{1}{\sqrt{2}}$ ms^{-2} NW

Solution: (D)

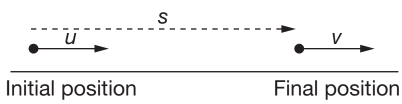
$$a_{av} = \frac{v_f - v_i}{t} = \frac{5i - 5\hat{i}}{10}$$

$$= a = \frac{1}{\sqrt{2}} \text{ ms}^{-2} \text{ NW}.$$

UNIFORMLY ACCELERATED MOTION

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.

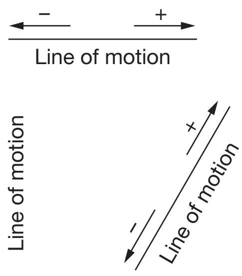
For uniformly accelerated motion along a straight line (x-axis) during a time interval of t seconds, the following important results can be used.



1. $v = u + at$
2. $v^2 = u^2 + 2as$
3. $s = ut + 1/2\, at^2$
4. $s = \left(\frac{v+u}{2}\right)t$
5. $s_n = u + a/2\,(2n - 1)$
6. $x_f = x_i + ut + 1/2\, at^2$
7. u = initial velocity (at the beginning of interval)
   a = acceleration
   v = final velocity (at the end of interval)
   s = displacement $(x_f - x_i)$
   s_n = displacement during the nth s.

DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION

In straight line motion, all the vectors (position, displacement, velocity, and acceleration) will have only one component (along the line of motion), and there will be only two possible directions for each vector.



1. For example, if a particle is moving in a horizontal line (x-axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.
2. For vertical or inclined motion, upward direction can be taken as +ve and downward as –ve.
3. For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg), that is the gravitational pull of the earth. Hence acceleration for this type of motion will always be $a = -g$, i.e., $a = -9.8$ m/s^2 (–ve sign, because the force and acceleration are directed downwards, if we select upward direction as positive).

Equation of motion under gravity (when a body is projected upward with speed u)

1. $v = u - gt$
2. $s = ut - \frac{1}{2}gt^2$
3. $v^2 - u^2 = -2gs$
4. $s_{nth} = u - \frac{g}{2}(2n - 1)$

NOTE

1. If acceleration is in same direction as velocity, then speed of the particle increases.
2. If acceleration is in opposite direction to the velocity then speed decreases, that is the particle slows down. This situation is known as *retardation*.

Reaction Time

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think, and act.

SOLVED EXAMPLES

6. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out the velocity of the particle at the midpoint of its path?

Solution:

Let the total distance be $2x$.

$\therefore$ distance upto midpoint $= x$

Let the velocity at the midpoint be v

and acceleration be a.

From equations of motion

$$v^2 = 10^2 + 2ax \quad (1)$$

$$30^2 = v^2 + 2ax \quad (2)$$

(2) – (1) gives

$$v^2 - 30^2 = 10^2 - v^2$$

$$v^2 = 500$$

$$v = \text{m/s}.$$

7. Mr. Sharma brakes his car with constant acceleration from a velocity of 25 m/s to 15 m/s over a distance of 200 m.

(A) How much time elapses during this interval?

(B) What is the acceleration?

(C) If he has to continue braking with the same constant acceleration, how much longer would it take for him to stop and how much additional distance would he cover?

Solution: (A)

$u = 25$ m/s

$v = 15$ m/s

$s = 200$ m

Using $s = \left(\dfrac{u+v}{2}\right)t$

(B) We can now find the acceleration using $v_x = u_x + a_x t$

$$a = \frac{v-u}{t} = \frac{15-25}{10} = -1 \text{ m/s}^2.$$

The acceleration is negative, which means that the positive velocity is becoming smaller as brakes are applied (as expected).

(C) Now with known acceleration, we can find the total time for the car to go from velocity

$u = 25$ m/s to

$v = 0$. Solving for t, we find

$$t = \frac{v-u}{a} = \frac{0-25}{-1} = 25 \text{ s}.$$

The total distance covered is

$$S = ut + \frac{1}{2}at^2 = 0 + (25)(25) + \frac{1}{2}(-1)(25)^2$$

$$= 625 - 312.5 = 312.5 \text{ m}.$$

Additional distance covered = 312.5 – 200 = 112.5 m.

8. The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. The velocity with which the middle point of the train passes the same point is

(A) $\dfrac{u+v}{2}$ (B) $\dfrac{u^2+v^2}{2}$

(C) $\sqrt{\dfrac{u^2+v^2}{2}}$ (D) $\sqrt{u+v}$

Solution: (C)

$v^2 - u^2 = 2al$ and $v'^2 - u^2 = 2a\dfrac{l}{2} = al$

or $2(v'^2 - u^2) = 2al$

Equating, $2(v'^2 - u^2) = v^2 - u^2$

or $v'^2 = u^2 + \dfrac{v^2-u^2}{2} = \dfrac{v^2+u^2}{2}$

or $v' = \sqrt{\dfrac{v^2+u^2}{2}}$.

9. A body starts from rest with uniform acceleration and remains in motion for n seconds. If its final velocity after n second is v, then its displacement in the last two seconds will be

(A) $\dfrac{2v(n+1)}{n}$ (B) $\dfrac{v(n+1)}{n}$

(C) $\dfrac{v(n-1)}{n}$ (D) $\dfrac{2v(n-1)}{n}$

Solution: (D)

$\because \quad v = 0 + na \Rightarrow a = \dfrac{v}{n}$

$\therefore$ displacement in last two seconds,

$$= S_n - S_{n-2} = \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$$

$$= \frac{a}{2}\left[n^2 - (n-2)^2\right] = \frac{a}{2}[n+(n-2)][n-(n-2)]$$

$$= a(2n-2) = \frac{v}{n}(2n-2) = \frac{2v(n-1)}{n}.$$

10. Two particles start moving from the same point along the same straight line. The first moves with constant velocity v and the second with constant acceleration a. During the time that elapses before the second catches the first, the greatest distance between the particles is

(A) $\dfrac{v^2}{a}$ (B) $\dfrac{v^2}{2a}$

(C) $\dfrac{2v^2}{a}$ (D) $\dfrac{v^2}{4a}$

Solution: (B)

Let x be the distance between the particles after t seconds. Then

$$x = vt - \frac{1}{2}at^2 \qquad (1)$$

For x to be maximum

$$\frac{dx}{dt} = 0 \quad \text{or} \quad t = \frac{v}{a}$$

From (1), we get

$$x = \frac{v^2}{2a}.$$

11. In a car race, car A takes t_0 time less to finish than car B and passes the finishing point with a velocity v_0 more than car B. The cars start from rest and travels with constant accelerations a_1 and a_2. Then the ratio $\dfrac{v_0}{t_0}$ is equal to

(A) $\dfrac{a_1^2}{a_2}$ (B) $\dfrac{a_1 + a_2}{2}$

(C) $\sqrt{a_1 a_2}$ (D) $\dfrac{a_2^2}{a_1}$

Solution: (C)

Let s be the distance travelled by each car.

$$\sqrt{2a_1 s} - \sqrt{2a_2 s} = v_0 \text{ and } \sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}} = t_0$$

$$\therefore \quad \frac{v_0}{t_0} = \frac{\sqrt{a_1} - \sqrt{a_2}}{\dfrac{1}{\sqrt{a_2}} - \dfrac{1}{\sqrt{a_1}}} = \sqrt{a_1 a_2}.$$

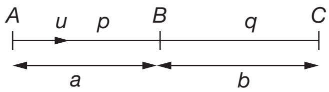
12. A particle moving with uniform retardation along a straight line covers distances a and b in successive intervals in p and q seconds. The acceleration of the particle is

(A) $\dfrac{2(aq-bp)}{q(p+q)}$ (B) $\dfrac{2(bp-aq)}{q(p+q)}$

(C) $\dfrac{2(aq+bp)}{q(p+q)}$ (D) $\dfrac{2(aq-bp)}{q(p-q)}$

Solution: (A)

Let retardation be f and initial velocity be u.



For AB,

$$a = up - \frac{1}{2}fp^2 \qquad (1)$$

For AC,

$$a + b = u(p+q) - \frac{1}{2}f(p+q)^2 \qquad (2)$$

(1) and (2)

$$\Rightarrow \quad a + b = \frac{(a + \frac{1}{2}fp^2)}{p}(p+q) - \frac{1}{2}f(p+q)^2$$

$$\frac{a+b}{p+q} = \frac{a}{p} + \frac{1}{2}fp - \frac{1}{2}fp - \frac{1}{2}fq,$$

$$\frac{1}{2}fq = \frac{a}{p} - \frac{a+b}{p+q} = \frac{ap + aq - ap - bp}{p(p+q)}$$

$$\frac{1}{2}fq = \frac{aq-bp}{(p+q)}, \qquad f = \frac{2(aq-bp)}{q(p+q)}.$$

13. A particle is released from rest from a tower of height 3 h. The ratio of times to fall equal height h i.e., $t_1 : t_2 : t_3$ is

(A) $\sqrt{3} : \sqrt{2} : 1$ (B) 3 : 2 : 1

(C) 9 : 4 : 1 (D) $1 : (\sqrt{2}-1) : (\sqrt{3}-\sqrt{2})$

Solution: (D)

$h = \dfrac{1}{2}gt_1^2$; $2h = \dfrac{1}{2}g(t_1+t_2)^2$ and $3h = \dfrac{1}{2}g(t_1+t_2+t_3)^2$

i.e., $t_1 : (t_1+t_2) : (t_1+t_2+t_3) = 1 : \sqrt{2} : \sqrt{3}$

or $\quad t_1 : t_2 + t_3 = 1 : (\sqrt{2}-1) : (\sqrt{3}-\sqrt{2})$.

14. A particle is projected vertically upwards from a point A on the ground. It takes time t_1 to reach a point B, but it still continues to move up. If it takes further t_2 time to reach the ground from point B. Then height of point B from the ground is

(A) $\frac{1}{2}g(t_1+t_2)^2$ (B) gt_1t_2

(C) $\frac{1}{8}g(t_1+t_2)^2$ (D) $\frac{1}{2}gt_1t_2$

Solution: (D)

Time taken for the particle to reach the highest point is $\frac{t_1+t_2}{2}$.

Therefore, initial vertical velocity of the particle is $u = g\left(\frac{t_1+t_2}{2}\right)$.

Therefore, height of B from the ground is

$$h = ut_1 - \frac{1}{2}gt_1^2 = g\left(\frac{t_1+t_2}{2}\right)t_1 - \frac{1}{2}gt_1^2$$

or $$h = \frac{1}{2}gt_1t_2.$$

15. A ball thrown upward from the top of tower with speed v reaches the ground in t_1 second. If this ball is thrown downward from the top of the same tower with speed v, it reaches the ground in t_2 second. In what time will the ball reach the ground if it is allowed to fall freely under gravity from the top of the tower?

(A) $\frac{t_1+t_2}{2}$ (B) $\frac{t_1-t_2}{2}$

(C) $\sqrt{t_1t_2}$ (D) t_1+t_2

Solution: (C)

$$h = -vt_1 + \frac{1}{2}gt_1^2$$

$$h = vt_2 + \frac{1}{2}gt_2^2$$

and $$h = \frac{1}{2}gt^2$$

Solve to get $$t = \sqrt{t_1t_2}.$$

16. A car starts from rest, moves with an acceleration a and then decelerates at b for sometime to come to rest. If the total time taken is t, the maximum velocity is

(A) $\frac{abt}{a+b}$ (B) $\frac{a^2t}{a+b}$

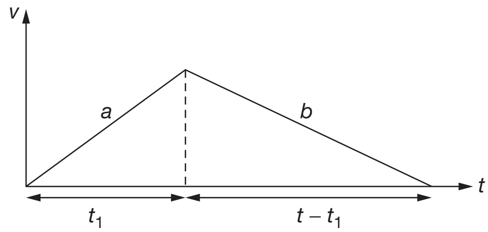
(C) $\frac{at}{a+b}$ (D) $\frac{b^2t}{a+b}$

Solution: (A)

$v = 0 + at_1;\ 0 = at_1 - b\,(t - t_1)$

or $$t_1 = \frac{bt}{a+b}$$

$\therefore$ $$v_{max} = \frac{abt}{a+b}.$$



17. A car starting from rest accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at rate $\frac{f}{2}$ to come to rest. If the total distance covered is 15 s, then **[2005]**

(A) $s = \frac{ft^2}{72}$ (B) $s = \frac{ft^2}{4}$

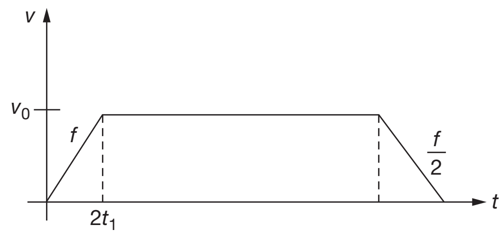
(C) $s = \frac{ft^2}{6}$ (D) $s = \frac{ft^2}{2}$

Solution: (A)

$$s = v_0\, t_1 \text{ and } v_0\, 2t_1 = 2\, s$$

Distance moved with uniform speed $(15 - 3)\, s = 12\, s$

$$v_0 = \sqrt{2sf}\ ;\ 12\, s = v_0\, t$$



$$12\, s = t\sqrt{2sf} \quad \text{or} \quad s = \frac{ft^2}{72}.$$

18. A car starts moving rectilinearly from rest with 5 ms^{-2} for sometime, then uniformly and finally decelerates at 5 ms^{-2} and comes to a stop. The total time of motion equals 25 s. How long does the car move uniformly? Given v_{av} = 72 km/h during motion.

(A) 5 s (B) 10 s

(C) 15 s (D) 20 s

Solution: (C)

Total distance covered = area under v–t graph. From Fig. 2.1

$$20 \times 25 = 5\, t_1^2 + (25 - 2t_1)\, 5t_1$$

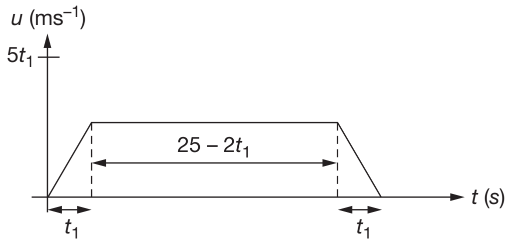


Fig. 2.1

or $\quad 5t_1^2 - 125\, t_1 + 500 = 0$

or $\quad (t_1 - 5)\,(t_1 - 20) = 0$

$\Rightarrow \quad t_1 = 5$ s discard $t_1 = 20$ s.

19. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2}. He reaches the ground with a speed 3 ms^{-1}. At what height did he bail out?

(A) 91 m (B) 182 m
(C) 293 m (D) 111 m

Solution: (C)

$$v^2 = 2gh = 2 \times 10 \times 50\ d$$

$$= 50 + \left[\frac{3^2 - 2\times10\times50}{-2(2)}\right] = 293 \text{ m.}$$

20. When a ball is thrown up vertically with a velocity v_o, it reaches a height h. If one wishes to triple the maximum height then the ball should be thrown with a velocity **[2005]**

(A) $\sqrt{3}\ v_o$ (B) $3\ v_o$
(C) $9\ v_o$ (D) $\frac{3}{2}\ v_o$

Solution: (A)

$$v^2 = 2gh \quad \text{or} \quad v = \sqrt{2gh},$$

i.e., $\quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{h_1}{h_2}}$

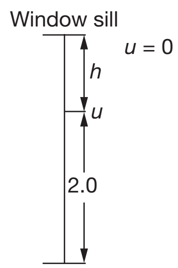
$\therefore \quad v_2 = \sqrt{3}\ v_o.$

21. From the top of a tower, two stones whose masses are in the ratio 1 : 2 are thrown, one straight up with an initial speed u and the second straight down with same speed u. Neglecting air resistance,

Solution: (C)

(A) The heavier stone hits the ground with a higher speed.
(B) The lighter stone hits the ground with a higher speed.
(C) Both the stones will have same speed when they hit the ground.
(D) The speed cannot be determined with the given data.

22. A flowerpot falls off a window sill and falls past the window below. It takes 0.5 s to pass through a 2.0 m high window. Find how high is the window sill from the top of the window?



(A) 10 cm (B) 7.5 cm
(C) 12.5 cm (D) 15 cm

Solution: (C)

$$h = ut + \frac{1}{2}\, at^2$$

or $\quad 2.0 = u\,(.5) + 5\left(\dfrac{1}{4}\right)$

or $\quad u = 1.5$ ms^{-1}.

Using $v^2 - u^2 = 2gh$

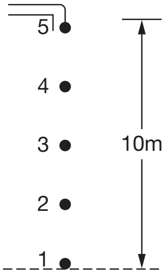
$$h = \frac{1.5^2}{2\times10} = \frac{2.25}{20} = 0.125 \text{ m} = 12.5 \text{ cm.}$$

23. From a tap 10 m high water drops fall at regular intervals. When the first drop reaches the ground, the 5th drop is about to leave the tap. Find the separation between 2nd and 3rd drops.

(A) $\frac{35}{8}$ m (B) $\frac{31}{8}$ m
(C) $\frac{27}{8}$ m (D) None of these

Solution:

$$\frac{1}{2}gt^2 = 10 \quad \text{or} \quad t = \sqrt{2}\ \text{s}$$



Time interval

$$\Delta t = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}} \text{ s.}$$

$$x_2 - x_3 \qquad \frac{1}{2} = g\left[\left(\frac{3}{2\sqrt{2}}\right)^2 - \left(\frac{2}{2\sqrt{2}}\right)^2\right]$$

$$= 5\left[\frac{9}{8} - \frac{1}{2}\right] = \frac{25}{8} \text{ m.}$$

24. When a ball is h metre high from a point O, its velocity is v_o. When it is h m below O, its velocity is $2v$. Find the maximum height from O it will acquire

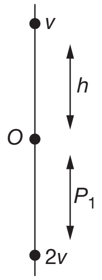


Fig. 2.6

(A) $\frac{2h}{3}$ (B) $\frac{5h}{3}$

(C) $\frac{3h}{2}$ (D) $2h$

Solution: (B)

$$(2v)^2 - v^2 = 2g\,(2h)$$

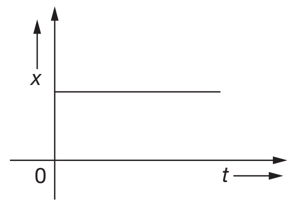
or $$\frac{v^2}{2g} = \frac{2}{3}h;$$

$$h_{\max} = h + \frac{2h}{3} = \frac{5h}{3}.$$

GRAPHICAL REPRESENTATION OF RECTILINEAR MOTION

1. Position vs Time graph

(a) Zero velocity



As position of particle is fix at all the time, so the body is at rest.

Slope; $\frac{dx}{dt} = \tan\theta = \tan 0° = 0$

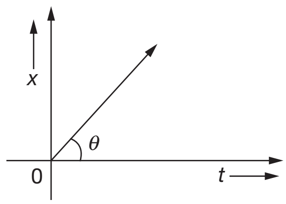
Velocity of particle is zero.

(b) Uniform velocity

Here $\tan\theta$ is constant $\tan\theta = \frac{dx}{dt}$

$\therefore$ $\frac{dx}{dt}$ is constant.

$\therefore$ Velocity of particle is constant.



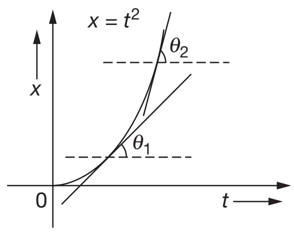
(c) Non-uniform velocity (increasing with time)

In this case;

As time is increasing, θ is also increasing.

$\therefore$ $\frac{dx}{dt} = \tan\theta$ is also increasing

Hence, velocity of particle is increasing.



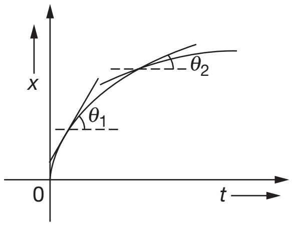
(d) Non-uniform velocity (decreasing with time)

In this case;

As time increases, θ decreases.

$\therefore$ $\frac{dx}{dt} = \tan\theta$ also decreases.

Hence, velocity of particle is decreasing.



2. Velocity vs time graph

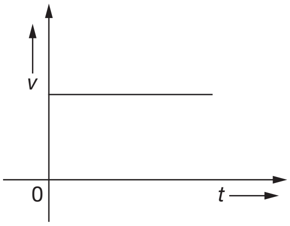
(a) Zero acceleration

Velocity is constant.

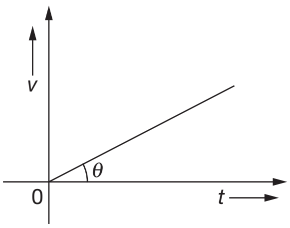
$\tan\theta = 0$

$\therefore \dfrac{dv}{dt} = 0$

Hence, acceleration is zero.



(b) Uniform acceleration



$\tan\theta$ is constant.

$\therefore \dfrac{dv}{dt} = \text{constant}$

Hence, it shows constant acceleration.

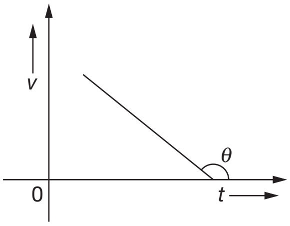
(c) Uniform retardation

Since $\theta > 90°$

$\therefore \tan\theta$ is constant and negative.

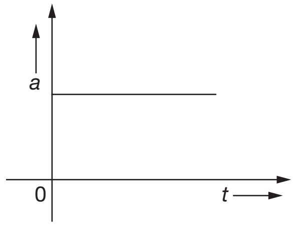
$\therefore \dfrac{dv}{dt} = \text{negative constant}$

Hence, it shows constant retardation.



3. Acceleration vs time graph

(a) Constant acceleration

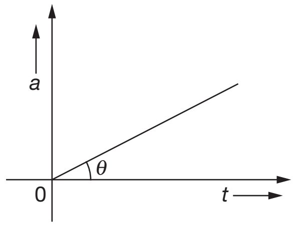


$\tan\theta = 0$

$\therefore \dfrac{da}{dt} = 0$

Hence, acceleration is constant.

(b) Uniformly increasing acceleration



θ is constant.

$0° < \theta < 90° \Rightarrow \tan\theta > 0$

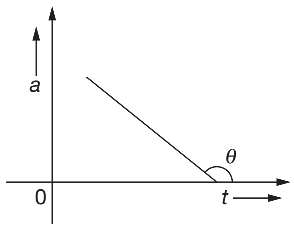
$\therefore \dfrac{da}{dt} = \tan\theta = \text{constant} > 0$

Hence, acceleration is uniformly increasing with time.

(c) Uniformly decreasing acceleration

Since $\theta > 90°$

$\therefore \tan\theta$ is constant and negative.

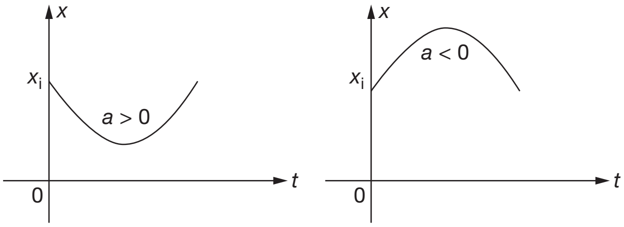


$\therefore \dfrac{da}{dt} = \text{negative constant}$

Hence, acceleration is uniformly decreasing with time

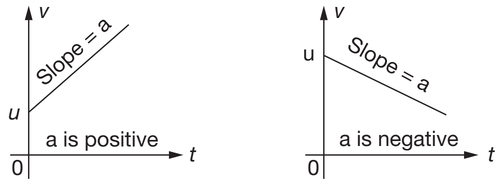
4. Graphs in Uniformly Accelerated Motion ($a \neq 0$)

(a) x is a quadratic polynomial in terms of t. Hence x–t graph is a parabola.



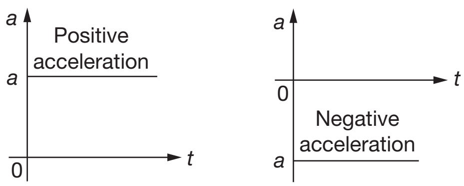
***x–t* graph**

(b) v is a linear polynomial in terms of t. Hence v–t graph is a straight line of slope a.



***v–t* graph**

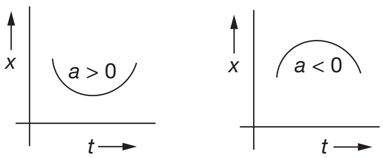
(c) a–t graph is a horizontal line because a is constant.



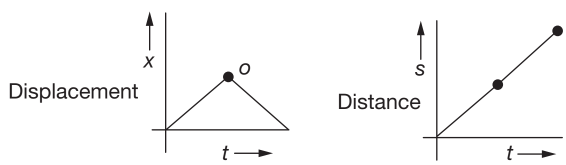
***a–t* graph**

NOTE

1. Displacement(x) – time(t) graph
   (a) Slope gives velocity
   (b) If position time graph is a straight line ⇒ $a = 0$
   (c) If it is a parabola opening upwards ⇒ $a > 0$



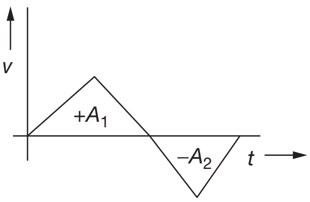
   and if it is a parabola opening downwards ⇒ $a < 0$
   (d) To convert a displacement time graph in to a distance time graph
   Place a plane mirror at O, parallel to time axis and take its reflection



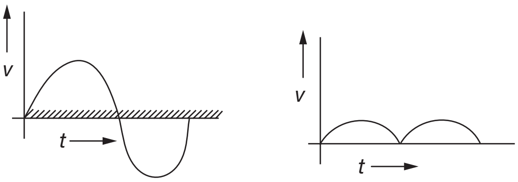
2. v–t graph
   (a) Slope gives acceleration
   (b) Area under v–t curve gives displacement/distance

$$\text{Displacement} = +A_1 - A_2$$

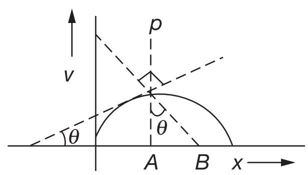
$$\text{Distance} = |A_1| + |A_2|$$



   (c) To convert a velocity time graph into speed time graph.
   Place a plane mirror along time axis

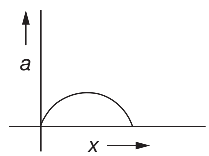


   (d) To get acceleration from v–x graph at a point P, draw tangent and normal at P as shown then sub-normal (AB) will give acceleration.
   The value of subnormal will give acceleration at point P.



3. a–t graph
   (a) Area under a–t graph gives change in velocity
   (b) Area under a–x graph gives

$$\left(\frac{v^2 - u^2}{2}\right)$$



$$\text{Area} = \int a\,dx = \int_u^v \frac{v\,dv}{dx}\cdot dx = \frac{v^2 - u^2}{2}.$$

SOLVED EXAMPLES

25. Time-displacement (t–x) graph of two objects A and B is shown in Fig 2.2. The ratio of their speeds (v_A/v_B) is ($\tan 37° = 3/4$)

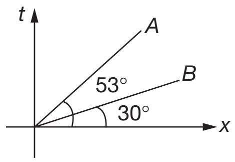


Fig. 2.2

(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{4}{\sqrt{3}}$ (C) $\frac{4}{3\sqrt{3}}$ (D) $\frac{3\sqrt{3}}{4}$

Solution: (A)

$$\frac{v_A}{v_B} = \frac{1/\tan 53°}{1/\tan 30°} = \frac{3/4}{\sqrt{3}} = \frac{\sqrt{3}}{4}.$$

26. The velocity-time curve of a body is shown in Fig. 2.3. The average speed of the body in first seven second is

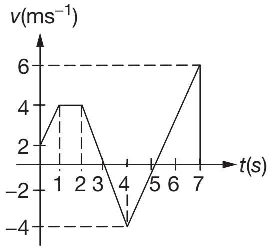


Fig. 2.3

(A) 1 ms^{-1} (B) 2 ms^{-1}

(C) $\frac{11}{7}$ ms^{-1} (D) $\frac{19}{7}$ m/s

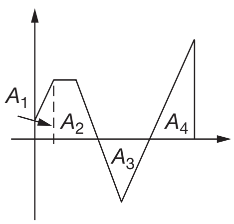
Solution: (D)

$$A_1 = \frac{1}{2}(2+4)\times 1 = 3 \text{ m}$$

$$A_2 = \frac{1}{2}(2+1)\times 4 = 6 \text{ m}$$

$$A_3 = \frac{1}{2}(2\times 4) = 4 \text{ m}$$

$$A_4 = \frac{1}{2}(2\times 6) = 6 \text{ m}$$



Distance travelled in 7 s = $A_1 + A_2 + A_3 + A_4 = 19$ m

$$\text{Average speed} = \frac{19}{7} \text{ m/s.}$$

27. For position-time (x–t) curve as shown in Fig. 2.4, the velocity-time (v–t) curve will be

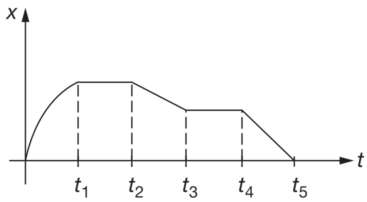
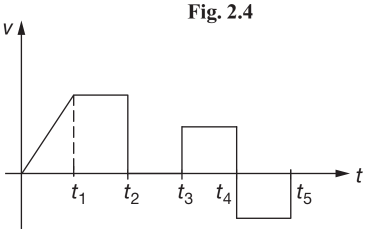
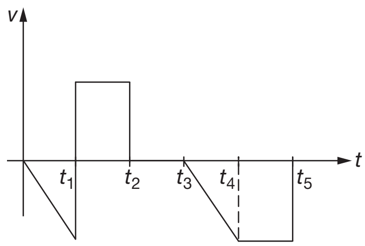


Fig. 2.4

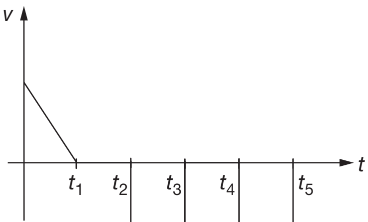
(A)



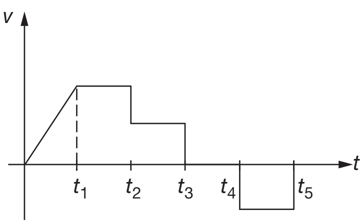
(B)



(C)



(D)



Solution: (C)

$0 - t_1 \rightarrow$ uniformly retarded motion

$t_1 - t_2 \rightarrow$ particle at rest

$t_2 - t_3 \rightarrow$ uniform negative velocity

$t_3 - t_4 \rightarrow$ particle at rest

$t_4 - t_5 \rightarrow$ uniform negative velocity.

28. The acceleration of particle varies with time as shown in Fig. 2.5. If particle start from rest, the velocity of particle after 3 s is

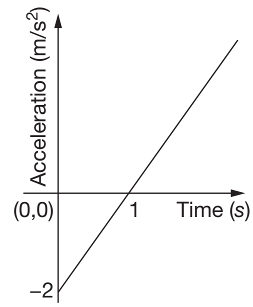


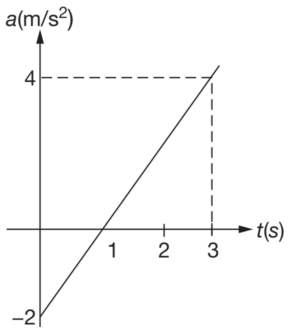
Fig. 2.5

(A) Zero (B) 2 m/s (C) 3 m/s (D) 4 m/s

Solution: (C)

v = area under the curve

$$= \frac{1}{2}\times 2\times 4 - \frac{1}{2}\times 2\times 1 = 3 \text{ m/s.}$$



29. The velocity–displacement graph of a particle is as shown in Fig. 2.6. The acceleration of the particle when displacement is 100 m will be

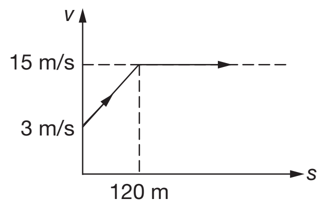


Fig. 2.6

(A) 1.3 m/s^2 (B) 1 m/s^2
(C) 1 m/s^2 (D) 0.13 m/s^2

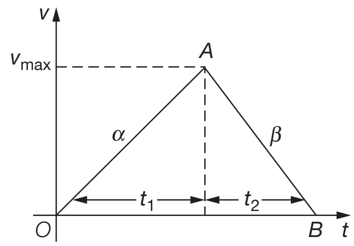
Solution: (A)

The equation of velocity is $v = \frac{1}{10}s + 3$

$$a = v\frac{dv}{ds} = \left(\frac{1}{10}s+3\right)\left(\frac{1}{10}\right),\ a = \frac{1}{100}s + \frac{3}{10}.$$

30. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t. Find the maximum velocity acquired by the car.

Solution:



$$t = t_1 + t_2$$

Slope of OA curve = $\tan\theta = \alpha = \frac{v_{max}}{t_1}$

Slope of AB curve = $\beta = \frac{v_{max}}{t_2}$

$$t = t_1 + t_2$$

$$t = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta}$$

$$v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t.$$

31. The displacement vs time graph of a particle moving along a straight line is shown in the Fig. 2.7. Draw velocity vs time and acceleration vs time graph.

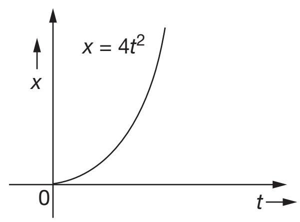


Fig. 2.7

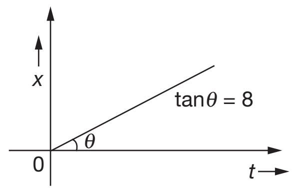
Solution:

$$x = 4t^2$$

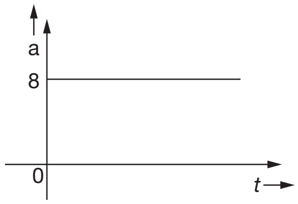
$$v = \frac{dx}{dt} = 8t$$

Hence, velocity–time graph is a straight line having a slope i.e. $\tan\theta = 8$.

$$a = \frac{dv}{dt} = 8$$



Hence, acceleration is constant throughout and is equal to 8.



Motion with Non-Uniform Acceleration (Use of Definite Integrals)

1. $\Delta x = \int_{t_i}^{t_f} v(t)dt$ (displacement in time interval $t = t_i$ to t_f)

   The expression on the right hand side is called the *definite integral* of $v(t)$ between $t = t_i$ and $t = t_f$.
2. Similarly, change in velocity

$$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)dt$$

Solving Problems which Involves Non-uniform Acceleration

1. **Acceleration depending on velocity v or time t:**

   By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t, $\int_{v_0}^{v} dv = \int_0^t a(t)\, dt$. If a is in terms of v, $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$. On integrating, we get a relation between v and t, and then using $\int_{x_0}^{x} dx = \int_0^t v(t)\, dt$, x and t can also be related.
2. **Acceleration depending on velocity v or position x:**

$$a = \frac{dv}{dt} \quad \Rightarrow \quad a = \frac{dv}{dx}\frac{dx}{dt}$$

$$\Rightarrow a = \frac{dx}{dt}\frac{dv}{dx} \quad \Rightarrow \quad a = v\frac{dv}{dx}$$

This is another important expression for acceleration. If a is in terms of x, $\int_{v_0}^{v} v\, dv = \int_{x_0}^{x} a(x)\, dx$. If a is in terms of v, $\int_{v_0}^{v} \frac{v\,dv}{a(v)} = \int_{x_0}^{x} dx$

On integrating, we get a relation between x and v. Using $\int_{x_0}^{x} \frac{dx}{v(x)} = \int_0^t dt$, we can relate x and t.

SOLVED EXAMPLES

32. An object starts from rest at $t = 0$ and accelerates at a rate given by $a = 6t$. What is its

(A) velocity and

(B) displacement at any time t?

Solution:

As acceleration is given as a function of time,

$$\therefore \quad \int_{v(t_0)}^{v(t)} dv = \int_{t_0}^{t} a(t)dt$$

Here $t_0 = 0$ and $v(t_0) = 0$

$$\therefore \quad v(t) = \int_0^t 6tdt = 6\left(\frac{t^2}{2}\right)\Bigg|_0^t = 6\left(\frac{t^2}{2} - 0\right) = 3t^2$$

So, $\quad v(t) = 3t^2$

As $\quad \Delta x = \int_{t_0}^{t} v(t)dt$

$$\therefore \quad \Delta x = \int_0^t 3t^2 dt = 3\left(\frac{t^3}{3}\right)\Bigg|_0^t = 3\left(\frac{t^3}{3} - 0\right) = t^3$$

Hence, velocity $v(t) = 3t^2$ and displacement $\Delta x = t^3$.

33. For a particle moving along x-axis, acceleration is given as $a = x$. Find the position as a function of time? Given that at $t = 0$, $x = 1$, $v = 1$.

Solution:

$$a = x$$

$$\frac{vdv}{dx} = x$$

$$\frac{v^2}{2} = \frac{x^2}{2} + C$$

$t = 0$, $x = 1$ and $v = 1$

$$\therefore \quad C = 0$$

$$v^2 = x^2 \Rightarrow v = x$$

$$\frac{dx}{dt} = x$$

$$\frac{dx}{x} = t$$

$$\ln x = t + C$$

$$0 = 0 + C$$

$$\ln x = t$$

$$\boxed{x = e^t}.$$

34. For a particle moving along x-axis, acceleration is given as $a = v$. Find the position as a function of time? Given that at $t = 0$, $x = 0$, $v = 1$.

Solution:

$$a = v$$
$$\frac{dv}{dt} = v$$
$$\int \frac{dv}{v} = \int dt$$
$$\ln v = t + c$$
$$0 = 0 + c$$
$$v = e^t$$
$$\frac{dx}{dt} = e^t$$
$$\int dx = \int e^t dt$$
$$x = e^t + c$$
$$0 = 1 + c$$
$$x = e^t 1.$$

35. A particle moves according to the law $a = -ky$. Find the velocity as a function of distance y, if v_o is initial velocity.

(A) $v^2 = v_0^2 - ky^2$ (B) $v^2 = v_0^2 - 2ky$
(C) $v^2 = v_0^2 - 2ky^2$ (D) None

Solution: (A)

$$a = \frac{dv}{dt} = \frac{dv}{dy} \cdot \frac{dy}{dt}$$

or $$\int_{v_o}^{v} v dv = \int_0^y -ky dy$$

or $$v_0^2 - v^2 = ky^2.$$

36. A particle moves with a deceleration $\propto \sqrt{v}$. Initial velocity is v_o. Find the time after which it will stop.

(A) $\frac{\sqrt{v_o}}{k}$ (B) $\frac{\sqrt{v_o}}{2k}$
(C) $\frac{2\sqrt{v_o}}{k}$ (D) None of these

Solution: (C)

$$\frac{dv}{dt} = -k\sqrt{v}$$

or $$\int_{v_o}^{0} \frac{dv}{\sqrt{v}} = -k\int dt$$

or $$t = \frac{2\sqrt{v_o}}{k}.$$

37. A particle moves as per the equation $v = a\sqrt{x}$. Find the average velocity in the first s metres of the path.

(A) $\frac{\sqrt{s}}{a}$ (B) $\frac{\sqrt{s}}{2a}$
(C) $\frac{2a}{\sqrt{s}}$ (D) $\frac{2\sqrt{s}}{a}$

Solution: (D)

$$\frac{dx}{dt} = a\sqrt{x}$$

or $$\int_0^s \frac{dx}{\sqrt{x}} = \int a dt$$

or $$t = \frac{2\sqrt{s}}{a}.$$

$$v_{av} = \frac{s}{t} = \frac{2\sqrt{s}}{a}.$$

MOTION IN TWO DIMENSIONS

Projectile Motion

Assume a projectile is projected at an angle θ with horizontal range, with a velocity u from point O as shown in Fig. 2.8. Resolve velocity along x and y-axis. Along y-axis, g acts then maximum height attained.

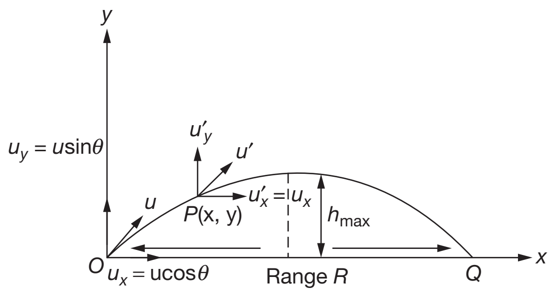


Fig. 2.8 Oblique projectile motion

$$h_{\max} = \frac{u^2 \sin^2 \theta}{2g}.$$

Time of Flight

$$T = \frac{2u \sin \theta}{g}.$$

Horizontal Range

$$R = \frac{u^2 \sin 2\theta}{g}.$$

Note that the range will be same if projected at complement angles, i.e., θ and $(90 - \theta)$ with same velocity.

Maximum Range

$$R_{max} = \frac{u^2}{g} \text{ when } \theta = 45°$$

Trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

or $$y = x\tan\theta\left[1 - \frac{x}{R}\right] \text{ path is parabolic.}$$

Instantaneous velocity = $|v|$

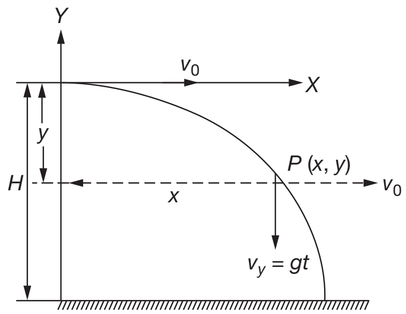
$$= \sqrt{u_x^2 + v_y^2} + \sqrt{u_x^2 + (u_y - gt)^2}$$

$$= \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$$

$$\tan\beta = \frac{u_y - gt}{u_x}.$$

Horizontal Projection from a given Height

1. **Displacement**



(a) Velocity

$$\vec{v} = v_0\hat{i} - gt\hat{j}$$

$$v = \sqrt{v_0^2 + 2gy} \text{ and } \phi = \tan^{-1}\left(\frac{2gy}{v_0}\right)$$

(b) Time of flight

$$T = \sqrt{\frac{2H}{g}}$$

(c) Range

$$R = v_0\sqrt{\frac{2H}{g}}$$

(d) Equation of trajectory

$$y = \frac{gx^2}{2v_0^2}.$$

Range and Time of Flight along an Inclined Plane

Consider an inclined plane of inclination α. Let a projectile be fixed at an angle θ with the horizontal range or at an angle $(\theta - \alpha)$ with respect to inclined plane as shown in Fig. 2.9.

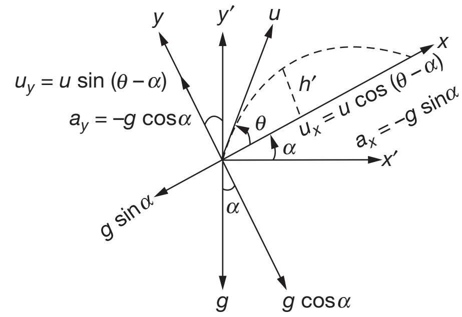


Fig. 2.9 Projectile motion along incline

The time of flight $T' = \dfrac{2u\sin(\theta - \alpha)}{g\cos\alpha}$

Range

$$R' = \frac{2u^2\sin(\theta - \alpha)\cos\theta}{g\cos^2\alpha}$$

$$R = \frac{u^2}{g\cos^2\alpha}[\sin(2\theta - \alpha) - \sin\alpha]$$

Range R' along the inclined is maximum if $2\theta - \alpha = \dfrac{\pi}{2}$ or $\theta - \alpha = \dfrac{\pi}{2} - \theta$. That is, R' is maximum when the direction of projection bisects the angle that the inclined plane makes with Oy' and $R'_{max} = \dfrac{u^2}{g\cos^2\alpha}.[1 - \sin\alpha]$

NOTE

1. In projectile motion along the plane acceleration acts along both x and y axis.
2. To find radius of curvature of a projectile at any point $R = \dfrac{v^2}{a_r}$.

   The velocity v and radial or normal acceleration at that point is used in the above relation.

   If v and a_r cannot be determined, then use

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

SOLVED EXAMPLES

38. A body is projected with a speed of 30 ms^{-1} at an angle of 30° with the vertical. Find the maximum height, time of flight and the horizontal range of the motion. [Take $g = 10$ m/s^2]

Solution:

Here $u = 30$ ms^{-1}, Angle of projection, $\theta = 90 - 30 = 60°$

Maximum height,

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{30^2 \sin^2 60°}{10} = \frac{900}{20} \times \frac{3}{4} = \frac{135}{4} \text{ m}$$

Time of flight,

$$T = \frac{2u \sin\theta}{g} = \frac{2 \times 30 \times \sin 60°}{10} = 3\sqrt{3} \text{ s.}$$

39. A projectile is thrown in the upward direction making an angle of 60° with the horizontal range with a speed of 147 m/s. Find the time after which its inclination with the horizontal is 45°?

Solution:

$$u_x = 147 \times \cos 60° = \frac{147}{2}$$

$$u_y = 147 \times \sin 60° = \frac{147\sqrt{3}}{2}$$

$$v_y = u_y + a_y t = \frac{147\sqrt{3}}{2} - gt$$

$$v_x = u_x = \frac{147}{2}$$

When angle is 45°, $\tan 45° = \frac{v_y}{v_x}$

$\Rightarrow$ $$v_y = v_x$$

$\Rightarrow$ $$\frac{147\sqrt{3}}{2} - gt = \frac{147}{2}$$

$\Rightarrow$ $$\frac{147}{2}(\sqrt{3} - 1) = gt$$

$\Rightarrow$ $$t = \frac{147}{2g}\left(\sqrt{3} - 1\right) \text{ s.}$$

40. A particle is thrown with initial speed u and angle of projection θ. Find the time after which velocity of the projectile becomes perpendicular to the initial velocity.

Solution:

Initial velocity $\vec{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

Velocity after time t is given by

$$\vec{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

when the two velocities are perpendicular their dot product will be zero.

$$\vec{u} \cdot \vec{v} = u^2 \cos^2\theta + u^2 \sin^2\theta - gt(u\sin\theta) = 0$$

$\Rightarrow$ $$t = \frac{u}{g\sin\theta}.$$

41. Large numbers of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread?

Solution:

Maximum distance upto which a bullet can be fired is its maximum range, therefore

$$R_{max} = \frac{v^2}{g} \qquad \text{Maximum area} = \pi(R_{max})^2 = \frac{\pi v^4}{g^2}.$$

42. A projectile can have the same range R for two angles of projection. If t_1 and t_2 are the times of flights in the two cases, then product of the time of flights is proportional to **[2005]**

(A) R^2 (B) $\frac{1}{R^2}$

(C) $\frac{1}{R}$ (D) R

Solution: (D)

$$t_1 = \frac{2u\sin\theta}{g}, \; t_2 = \frac{2u\cos\theta}{g}$$

and $$t_1 t_2 = \frac{2u^2 \sin 2\theta}{g^2} = \frac{2R}{g}.$$

43. A girl angrily throws her engagement ring from the top of a building 12 m high towards her boy friend with an initial horizontal speed of 5 ms^{-1}; the speed with which the ring touches the ground is

(A) 5 ms^{-1} (B) 14.3 ms^{-1}

(C) 1.5 ms^{-1} (D) 16.2 ms^{-1}

Solution: (D)

$$v_y^2 = 2ay = 2 \times 10 \times 12$$

$$v = \sqrt{25 + 240} = 16.2 \text{ ms}^{-1}.$$

44. The radius vector of a point A relative to the origin varies as $r = at\hat{i} + bt^2\hat{j}$, where a and b are positive constants. Find the equation of trajectory.

(A) $y = \frac{b}{a^2} x^2$ (B) $y^2 = \frac{b}{a^2} x$

(C) $y = \frac{a^2}{b} x^2$ (D) None of these

Solution: (A)

$$x = at,\ y = bt^2 \text{ or } y = b\left(\frac{x}{a}\right)^2.$$

45. A man is riding on a flat car moving with 10 ms^{-1}. He attempts to throw a ball through a stationary hoop 5 m above his hand such that the ball moves horizontally through the hoop. He throws the ball with 12 ms^{-1} with respect to himself. Find the horizontal distance from where he throws the ball.

(A) 15 m (B) 14.2 m
(C) 16.7 m (D) 18.2 m

Solution: (C)

$$h_{\max} = 5 = \frac{u_y^2}{2g}$$

$$\therefore \quad u_y = 10$$

$$u_x = \sqrt{12^2 - 10^2} = \sqrt{44}.$$

$$v_x = 10 + \sqrt{44};\ \frac{T}{2} = \frac{u_y}{g} = 1 \text{ s};$$

$$x = v_x;\ \frac{T}{2} = 10 + \sqrt{44} = 16.7 \text{ m}.$$

46. A body standing on a long railroad car throws a ball straight upwards, the car is moving on the horizontal road with an acceleration 1 ms^{-2}. The vertical velocity given is 9.8 ms^{-1}. How far behind the boy will the ball fall on the railroad car?

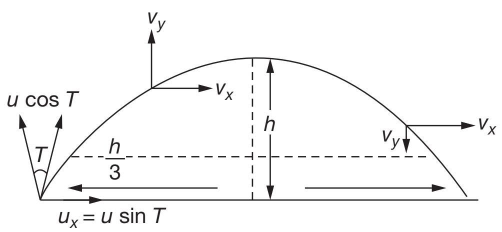
(A) 1 m (B) $\frac{3}{2}$ m
(C) $\frac{7}{4}$ m (D) 2 m

Solution: (D)

$$T = \frac{2u_y}{g} = 2 \times \frac{9.8}{9.8} = 2 \text{ s};$$

$$x = \frac{1}{2}a_x t^2 = \frac{1}{2}(1)(2)^2 = 2 \text{ m}.$$

47. Find the average velocity of a projectile between the instant it crosses one-third the maximum height. It is projected with u making an angle θ with the vertical.



(A) $u \cos\theta$ (B) u
(C) $u \sin\theta$ (D) $u \tan\theta$

Solution: (C)

Note carefully the vertical velocities at the same height are in opposite directions and therefore their average sum = 0. It is horizontal velocity which is uniform and hence $v_{av} = u \sin\theta\ (= u_x)$.

48. A person is standing on a truck moving with 14.7 ms^{-1} on a horizontal road. He throws a ball so that it returns to him when the truck has moved 58.8 m. Find the speed of the ball and angle of projection as seen by a man standing on the road.

(A) 22.5 ms^{-1}, 53° (B) 24.5 ms^{-1}, 53°
(C) 19.6 ms^{-1}, vertical (D) None of these

Solution: (B)

$$T = \frac{58.8}{14.7} = 4 \text{ s}$$

$$T = \frac{2u_y}{g} = 4$$

$$\therefore \quad u_y = 19.6 \text{ ms}^{-1}$$

$$v = \sqrt{14.7^2 + 19.6^2}$$

$$= 24.5 \text{ ms}^{-1}$$

$\tan\beta = \frac{v_y}{v_x} = \frac{19.6}{14.7} = \frac{4}{3}$ or $\beta = 53°$ with respect to horizontal range.

49. In an exhibition, you win a prize if you toss a coin into a small dish placed. The dish is on a sheep 2.1 m away at a height h from the hand. The coin is tossed into the dish if its velocity is 6.4 ms^{-1} at an angle of 60°. Find h.

(A) 1.2 m (B) 1.35 m
(C) 1.5 m (D) 1.65 m

Solution: (C)

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= 2.1 \tan 60 - \frac{9.8(2.1)^2}{2 \times 6.4^2 \times \left(\frac{1}{2}\right)^2}$$

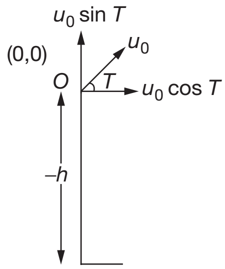
$$= 2.1\sqrt{3} - \frac{4.9 \times 4.4}{10.24}$$

$$= 1.5 \text{ m}.$$

50. A projectile is launched from a height h making an angle θ with the horizontal speed v_o. Find the horizontal distance covered by it before striking the ground.

Solution:

$$-h = v_o \sin \theta\, t - \frac{1}{2} g t^2$$



or $$gt^2 - 2\, v_o \sin\theta\, t - 2h = 0.$$

$$t = \frac{2v_o \sin\theta + \sqrt{4v_o^2 \sin^2\theta + 8gh}}{2g}$$

$$x = \frac{v_o \cos\theta}{2}\left[v_o \sin\theta + \sqrt{v_o^2 \sin^2\theta + 2gh}\right].$$

51. A baseball is projected with a velocity v making an angle θ with the incline of indication α as shown in Fig. 2.10(a). Find the condition that the ball hits the incline at right angle.

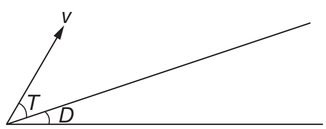


Fig. 2.10(a)

(A) $\cos\theta = \tan\theta$ (B) $\sin\theta = \cos\alpha$
(C) $\tan\theta = \sin\alpha$ (D) $\cos\theta = \cos\alpha$

Solution: (A)

$T = \dfrac{2u_y}{|a_y|} = \dfrac{2v\sin\theta}{g\cos\alpha}$. It will hit vertically the incline if $v_x = 0$.

$$0 = v \cos\theta T - g \sin\alpha T^2$$

or $$v\cos\theta\left(\frac{2v\sin\theta}{g\cos\alpha}\right) - \frac{g\sin\alpha}{2}\left(\frac{2v\sin\theta}{g\cos\alpha}\right)^2 = 0$$

$$\frac{2v^2\sin\theta}{g\cos\alpha}\left[\cos\theta\cos\alpha - \sin\alpha\sin\theta\right] = 0$$

or $$\cos\theta = \tan\alpha.$$

52. At what angle to the horizontal should an object be projected so that the maximum height reached is equal to the horizontal range?

(A) $\tan\theta = 2$ (B) $\tan\theta = 4$
(C) $\tan\theta = 2/3$ (D) $\theta = 3$

Solution: (B)

$$\frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin 2\theta}{g} \Rightarrow \tan\theta = 4.$$

53. A projectile is thrown with an initial velocity of $(x\hat{i} + y\hat{j})$ m/s. If the range of the projectile is double the maximum height reached by it then

(A) $x = 2y$ (B) $y = 2x$
(C) $x = y$ (D) $y = 4x$

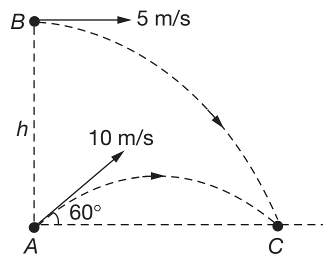
Solution: (B)

$$u\sin\theta = y,\ u\cos\theta = x$$

$$\therefore\quad H = \frac{u^2\sin^2\theta}{2g} = \frac{y^2}{2g},\ R = \frac{u^2(2\sin\theta\cos\theta)}{g} = \frac{2xy}{g}$$

As $R = 2H \Rightarrow \dfrac{2xy}{g} = \dfrac{2y^2}{2g} \Rightarrow y = 2x.$

54. A particle A is projected from the ground with an initial velocity of 10 m/s at an angle at 60° with horizontal. From what height h should another particle B be projected horizontally with velocity 5 m/s so that both the particles collide on ground at point C if both are projected simultaneously (g = 10 m/s^2)



(A) 10 m (B) 30 m (C) 15 m (D) 25 m

Solution: (C)

Horizontal component of velocity A is 10 cos 60° or 5 m/s which is equal to the velocity of B in horizontal direction. They will collide at C if time of flight of both the particles are equal i.e.

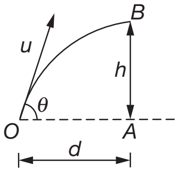
$$t_A = t_B$$

$$\frac{2u\sin\theta}{g} = \sqrt{\frac{2h}{g}}\left(h = \frac{1}{2}gt_B^2\right)$$

or $$h = \frac{2u^2\sin^2\theta}{g}$$

$$\frac{2(10)^2\left(\frac{\sqrt{3}}{2}\right)^2}{10} = 15\text{ m}.$$

55. A stone is to hit a point which is at a distance d away and at a height h above the point from where the stone is projected. The value of initial speed u if the stone is projected at an angle θ will be



(A) $\dfrac{g}{\cos\theta}\sqrt{\dfrac{d}{2(d\tan\theta-h)}}$

(B) $\dfrac{d}{\cos\theta}\sqrt{\dfrac{g}{2(d\tan\theta-h)}}$

(c) $\sqrt{\dfrac{gd^2}{h\cos^2\theta}}$

(D) $\sqrt{\dfrac{gd^2}{(d-h)}}$

Solution: (B)

Equation of trajectory, $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

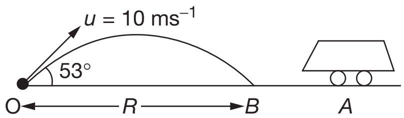
Here $x = d, y = h$

$\therefore \quad h = d\tan\theta - \dfrac{gd^2}{2u^2\cos^2\theta}$

or $\quad u^2 = \dfrac{gd^2}{2(d\tan\theta-h)\cos^2\theta}$

$$u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta-h)}}.$$

56. A boy throws a water-filled balloon at an angle of 53° with a speed of 10 m/s. A car is advancing towards the boy at a constant speed of 5 m/s. If the balloon is to hit the car, how far away should the car be when the balloon is thrown? (g = 10 ms^{-2})



(A) 8 m (B) 9.6 m
(C) 15.6 m (D) 17.6 m

Solution: (D)

$$T = \frac{2u\sin\theta}{g} = \frac{2\times10\times\frac{4}{5}}{10} = \frac{8}{5}\text{ s}$$

$$OB = R = \frac{u^2\sin2\theta}{g} = \frac{100\times2\times\frac{4}{5}\times\frac{3}{5}}{10} = \frac{48}{5}\text{ m}$$

$$AB = \frac{8}{5}\times5 = 8\text{ m}$$

$$OA = OB + AB = \frac{48}{5} + 8 = 17.6\text{ m.}$$

57. A motorcyclist starts from the bottom of a slope of angle 45° to cross the valley PR as shown in the Fig. 2.11. The width of the valley is 90 m and length of the slope is $80\sqrt{2}$ m. The minimum velocity at point O required to clear the valley will be

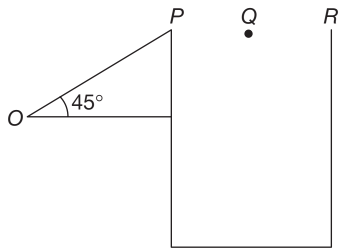


Fig. 2.11

(A) 70 m/s (B) 30 m/s
(C) 50 m/s (D) 100 m/s

Solution: (C)

$$R = \frac{u^2}{g}\sin2\theta = \frac{u^2}{g}$$

Velocity of take-off at P

or $\quad u = \sqrt{Rg} = \sqrt{90\times10} = 30$ m/s

$v = \sqrt{u^2 + 2g\sin\theta S}$ [$v \rightarrow$ velocity at point O]

$$= \sqrt{(30)^2 + 2\times10\times\frac{1}{\sqrt{2}}\times80\sqrt{2}} = 50\text{ m/s.}$$

58. Two particles A and B are projected simultaneously in the directions shown in Fig. 2.12 with velocities v_A = 25 m/s and $v_B = 10\sqrt{3}$ m/s. If they collide in air after 2 s, the angle θ is

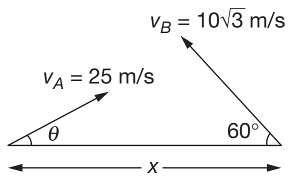
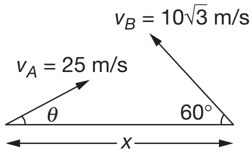


Fig. 2.12

(A) 30° (B) 45° (C) 53° (D) 37°

Solution: (D)

For collision,



$$v_A \sin\theta = v_B \sin 60°$$

$$25\sin\theta = 10\sqrt{3} \times \frac{\sqrt{3}}{2}$$

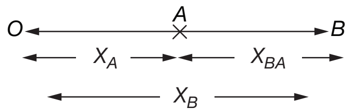
$$\sin\theta = \frac{3}{5}$$

or $\theta = 37°.$

RELATIVE MOTION

Relative Motion of Two Particles

When two particles A and B move along the same straight line, denoting $x_{B/A}$, the relative position coordinate of B with respect to A, we have



$$\vec{x}_B = \vec{x}_A + \vec{x}_{BA} \Rightarrow \vec{x}_{BA} = \vec{x}_B - \vec{x}_A$$

Denoting by v_{BA} and a_{BA} respectively, the relative velocity and the relative acceleration of B with respect to A, we have

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA} \Rightarrow \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA} \Rightarrow \vec{a}_{BA} = \vec{a}_B - \vec{a}_A.$$

Problems on relative velocity can even be solved using vector laws. Use $v_{AB} = v_A - v_B$

or $$v_{AB} = \left(v_{Ax} - v_{Bx}\right)\hat{i} + \left(v_{Ay} - v_{By}\right)\hat{j};$$

$$|v_{AB}| = \sqrt{\left(v_{Ax} - v_{Bx}\right)^2 + \left(v_{Ay} - v_{By}\right)^2};$$

$\tan\beta = \dfrac{v_{Ay} - v_{By}}{v_{Ax} - v_{Bx}}$ with respect to x-direction

$\tan\beta' = \dfrac{v_{Ax} - v_{Bx}}{v_{Ay} - v_{By}}$ with respect to y-direction.

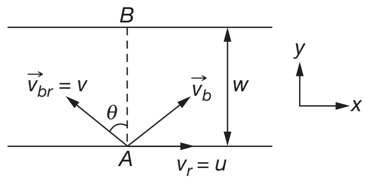
River and Swimmer Problems

In river problems we come across the following three terms:

$\vec{v}_r$ = absolute velocity of river
$= u$

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water
$= v$

Here, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground.



Therefore, $$\vec{v}_b = \vec{v}_r + \vec{v}_{br} = \vec{u} + \vec{v}$$

and $$\vec{v}_{bx} = \vec{v}_{rx} + \vec{v}_{brx} = u - v\sin\theta$$

$$v_{by} = v_{ry} + v_{bry}$$

$$= 0 + v_{br}\cos\theta = v\cos\theta$$

Now, time taken by the swimmer to cross the river is

$$t = \frac{w}{v\cos\theta}$$

Further, displacement along x-axis when he reaches on the outer bank (also called drift) is

$$x = \frac{(u - v\sin\theta)w}{v\cos\theta}$$

1. To cross the river along the shortest path, the swimmer should strike at an obtuse angle to the flow of river so that resultant velocity v is along the normal flow as illustrated in Fig. 2.13. provided $v_{\text{swimmer}} > v_{\text{river}}$

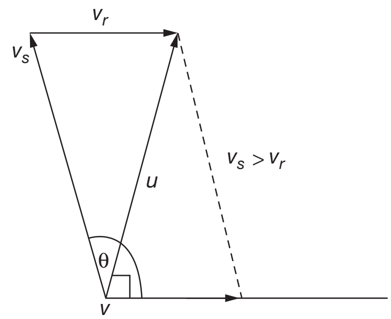


Fig. 2.13

From triangle law $v = \sqrt{v_s^2 - v_r^2}$, where v_s = velocity of swimmer and v_r = velocity or river.

If the width of the river is l then $t = \dfrac{l}{v} = \dfrac{R}{\sqrt{v_s^2 - v_r^2}}$.

2. To cross the river in the shortest time (when $v_{\text{swimmer}} > v_{\text{river}}$), the swimmer should strike at right angle to the flow of the river and $t_{\min} = \dfrac{l}{v_{\text{swimmer}}}$.

SOLVED EXAMPLES

59. The driver of a train A running at 25 ms^{-1} sights a train B on the same track with 15 ms^{-1}. The driver of train A applies brakes to produce a deceleration of 1.0 ms^{-2}. If the trains are 200 m apart, will the trains collide?

(A) Yes (B) No
(C) Collision just avoided (D) None of these

Solution: (C)

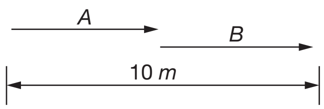
$$v^2 - u^2 = 2as \quad \text{or} \quad s = \frac{25^2 - 15^2}{2\times 1} = 200 \text{ m.}$$

60. Two cars A and B are 5 m long each. Car A is at any instant just behind B. A and B are moving at 54 km/h and 36 km/h, respectively. Find the road distance covered by the car A to overtake B.

(A) 35 m (B) 30 m (C) 32.5 m (D) 27.5 m

Solution: (A)

$v_{AB} = 15 - 10 = 5 \text{ ms}^{-1}$



$$x_{AB} = 10 \text{ m}; \; t = \frac{x_{AB}}{v_{AB}} = 2 \text{ s.}$$

Road distance covered = $v_A\, t$ + length of car A

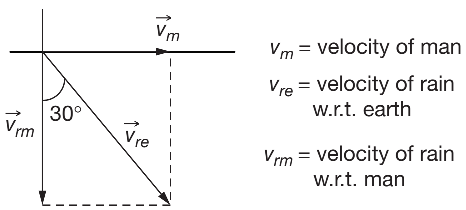
$= 15 \times 2 + 5 = 35$ m.

61. A man holds an umbrella at 30° with the vertical to keep himself dry. He, then, runs at a speed of 10 ms^{-1} and finds the rain drops to be hitting vertically. Speed of the rain drops with respect to the running man and with respect to earth are

(A) $20 \text{ ms}^{-1}, 10 \text{ ms}^{-1}$ (B) $10 \text{ ms}^{-1}, 20\sqrt{3} \text{ ms}^{-1}$
(C) $10\sqrt{3} \text{ ms}^{-1}, 20 \text{ ms}^{-1}$ (D) $20 \text{ ms}^{-1}, 10\sqrt{3} \text{ ms}^{-1}$

Solution: (C)

Velocity of man $|\vec{v}_m| = 10 \text{ ms}^{-1}$



Using $\sin 30° = \dfrac{v_m}{v_{re}}$

or $v_{re} = \dfrac{v_m}{\sin 30} = \dfrac{10}{1/2}$

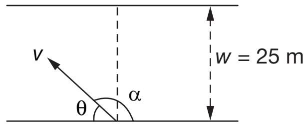
$= 20 \text{ ms}^{-1}$

Again

or $v_{rm} = v_{re} \cos 30$

$= 20 \times \dfrac{\sqrt{3}}{2} = 10\sqrt{3} \text{ ms}^{-1}$.

62. A boat in a speed of 5 m/s in still water crosses the river of width 25 m in 10 s. The boat is heading at an angle of a with downstream, where α is equal to



(A) 150° (B) 120° (C) 90° (D) 60°

Solution: (A)

$$t = \frac{w}{v\sin\theta} \Rightarrow 10 = \frac{25}{5\sin\theta}$$

$$\sin\theta = \frac{1}{2} \Rightarrow$$

$\therefore$ $\alpha = 180° - \theta = 150°$.

63. A swimmer wishes to cross a 800 m wide river flowing at 6 km/hr. His speed with respect to water is 4 km/hr. He crosses the river in shortest possible time. He is drifted downstream on reaching the other bank by a distance of

(A) 800 m (B) 1200 m
(C) $400\sqrt{13}$ m (D) 2000 m

Solution: (B)

For shortest time,

$$t = \frac{w}{v_m} = \frac{0.8}{4} = 0.2 \text{ hr}$$

Drift = $v_r \times t = 6 \times 0.2 = 1.2$ km = 1200 m.

64. A ship moves along the equator to the east with a speed 30 km/h. Southeastern wind blows 60° to the east with 15 kmh^{-1}. Find the wind velocity relative to the ship.

(A) 39.7 kmh^{-1}, $\tan^{-1}\dfrac{1}{5}$ N of W

(B) 23.7 kmh^{-1}, $\tan^{-1}\dfrac{1}{3}$ N of W

(C) 37.5 kmh^{-1}, $\tan^{-1}\dfrac{1}{5}$ N of E

(D) None

Solution: (A)

$$v_{ws} = v_w - v_s$$

$$= (15 \cos 60\,\hat{i} + 15 \sin 60\,\hat{j}) - 30\,\hat{i}$$

$$|v| = \sqrt{(39.5)^2 + (7.5)^2} = 39.7 \text{ kmh}^{-1}$$

$$\tan \beta = \frac{7.5}{37.5} = \frac{1}{5}$$

$$\beta = \tan^{-1}\frac{1}{5} \text{ North of west.}$$

65. A boat moves relative to water with a velocity v and river is flowing with $2v$. At what angle will the boat move with the stream to have minimum drift?

(A) 30° (B) 60° (C) 90° (D) 120°

Solution: (D)

Let boat move at angle θ to the normal as shown in Fig. 2.14 then time to cross the river $= \dfrac{l}{v\cos\theta}$.

Drift $x = (2v - v\sin\theta)\dfrac{l}{v\cos\theta}$ for x to be minimum.

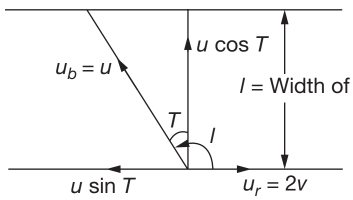


Fig. 2.14

$$\frac{dx}{d\theta} = 0 = 1\,(2\sec\theta\tan\theta - \sec^2\theta)$$

or $\sin\theta = \dfrac{1}{2}$

or $\theta = 30°$ and $\phi = 90 + 30 = 120°$.

66. The compass needle of the airplane shows it is heading north and speedometer indicates a velocity 240 km h^{-1}. Wind is blowing at 100 km h^{-1} to east. Find the velocity of the airplane with respect to the earth.

(A) 260 ms^{-1}, 23° E of N (B) 260 ms^{-1}, 23° W of N
(C) 260 ms^{-1}, 32° E of N (D) None

Solution: (A)

$$v_{AE} = 100\,\hat{i} + 240\,\hat{j}$$

$$v_{AE} = \sqrt{(240)^2 + 100^2} = 260 \text{ ms}^{-1};$$

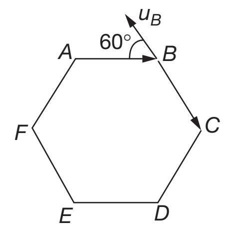
$$\phi = \tan^{-1}\left(\frac{100}{240}\right) = 23° \text{ E of N.}$$

67. Six persons are positioned at the corners of a hexagon of side l. They move at a constant speed v. Each person maintains a direction towards the person at the next corner. When will the persons meet?

(A) $\dfrac{l}{v}$ (B) $\dfrac{2l}{3v}$ (C) $\dfrac{3l}{2v}$ (D) $\dfrac{2l}{v}$

Solution: (D)

$$t = \frac{l}{v_{AB}} = \frac{l}{v_A - v_B \text{ in the direction of A}}$$



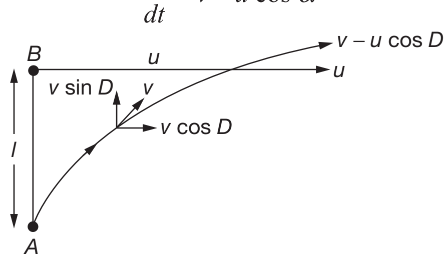
$$= \frac{l}{v - v\cos 60} = \frac{2l}{v}.$$

68. Particle A moves uniformly with velocity v so that vector v is continually aimed at point B which moves rectilinearly with a velocity $u < v$. At $t = 0$, v and u are perpendicular. Find the time when they converge. Assume A and B are separated by l at $t = 0$.

Solution:

A approaches B with a velocity $= v - u\cos\alpha$.

$$\frac{dx}{dt} = v - u\cos\alpha$$



$$\int_0^d dx = \int_0^t (v - u\cos\alpha)\,dt$$

or $\dfrac{l - vt}{u} = \int -\cos\alpha\,dt$

$$ut = \int v\cos\alpha\,dt$$

or $ut = \dfrac{-v(l - vt)}{u}$

or $(v^2 - u^2) = lv$

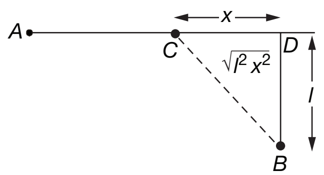
or $t = \dfrac{lv}{v^2 - u^2}$.

69. From point A located on a highway, one has to get by a car as soon as possible to point B located in the field at a distance l from point D. If the car moves n times slower in the field, at what distance x from D one must turn off the highway.

Solution:

Let v be the velocity in the field and nv in the velocity on the highway.

Then $t_1 = \dfrac{AD - x}{nv}$ and $t_2 = \dfrac{\sqrt{l^2 + x^2}}{v}$



For t to be minimum $\dfrac{d}{dx}(t_1 + t_2) = 0$

$$\frac{d}{dt}\left[\frac{1}{v}\left\{\left(\frac{AD-x}{n}\right) - \sqrt{l^2 + x^2}\right\}\right]$$

$$= \frac{1}{n} - \frac{x}{\sqrt{l^2 + x^2}} = 0$$

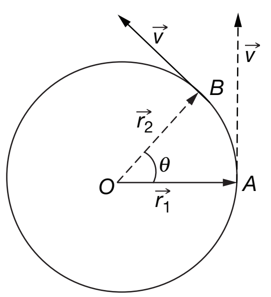
or $\quad l^2 + x^2 = n^2 x^2.$

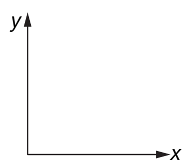
or $\quad x = \dfrac{l}{\sqrt{n^2 - 1}}.$

CIRCULAR MOTION

Uniform Circular Motion

$$\Delta r = S = 2r\sin\frac{\theta}{2}$$





1. Average velocity

$$\left|\vec{V}_{av}\right| = \frac{\left|\Delta\vec{r}\right|}{\Delta t} = \frac{2r\sin(\theta/2)}{t}$$

2. Angular and linear speeds $\omega = \dfrac{\theta}{t}$

   $v = r\theta/t$ and $v = r\omega$

3. Change in velocity

   $$\Delta v = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}.$$

4. Centripetal acceleration

   $$a_r = \frac{2v(\theta/2)}{t} = \frac{v\theta}{t}$$

   Putting $\theta/t = \omega = \dfrac{v}{r}$ we obtain $a_r = \dfrac{v^2}{r} = \omega^2 r$.

NON-UNIFORM CIRCULAR MOTION WITH CONSTANT ANGULAR ACCELERATION

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta.$$

NOTE

Net acceleration in non-uniform circular motion (Fig. 2.15)

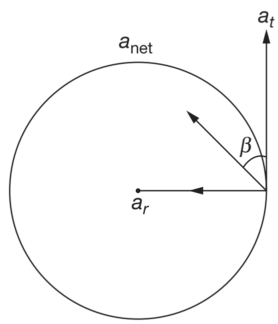


Fig. 2.15

$a_{net} = \sqrt{a_t^2 + a_r^2}$ where, a_t is tangential acceleration and a_r is radial acceleration.

$$\tan\beta = \frac{a_r}{a_t}.$$

SOLVED EXAMPLES

70. If angular displacement of a particle is given by $\theta = a - bt + ct^2$, then find its angular velocity.

Solution:

$$\omega = \frac{d\theta}{dt} = -b + 2ct.$$

71. Is the angular velocity of rotation of hour hand of a watch greater or smaller than the angular velocity of earth's rotation about its own axis?

Solution:

Hour hand completes one rotation in 12 hours while earth completes one rotation in 24 hours. So, angular velocity of hour hand is double the angular velocity of earth. $\left(\omega = \dfrac{2\pi}{T}\right)$.

72. A particle is moving with constant speed in a circular path. Find the ratio of average velocity to its instantaneous velocity when the particle describes an angle $\theta = \dfrac{\pi}{2}$

Solution:

Time taken to describe angle θ,

$$t = \frac{\theta}{\omega} = \frac{\theta R}{v} = \frac{\pi R}{2v}$$

Average velocity

$$= \frac{\text{Total displacement}}{\text{Total time}} = \frac{\sqrt{2}\,R}{\pi R/2v} = \frac{2\sqrt{2}}{\pi}\,v$$

Instantaneous velocity = v

The ratio of average velocity to its instantaneous velocity $= \dfrac{2\sqrt{2}}{\pi}$.

73. A fan is rotating with angular velocity 100 rev/s. Then it is switched off. It takes 5 minutes to stop. (A) Find the total number of revolution made before it stops. (Assume uniform angular retardation.) (B) Find the value of angular retardation (C) Find the average angular velocity during this interval.

Solution:

(A) $\theta = \left(\dfrac{\omega + \omega_0}{2}\right)t = \left(\dfrac{100+0}{2}\right) \times 5 \times 60 = 15{,}000$ revolution.

(B) $\omega = \omega_0 + \alpha t$

$\Rightarrow 0 = 100 - a\,(5 \times 60) \Rightarrow \alpha = \dfrac{1}{3}$ rev./s^2

(C) $\omega_{av} = \dfrac{\text{Total Angle of Rotation}}{\text{Total time taken}} = \dfrac{15000}{50 \times 60}$

$= 50$ rev./s.

74. A fan rotating at $\omega = 100$ rad/s, is switched off. After $2n$ rotation, its angular velocity becomes 50 rad/s. Find the angular velocity of the fan after n rotations.

Solution:

$$\omega^2 = \omega_0^{\ 2} + 2\alpha\theta$$

$$50^2 = (100)^2 + 2\alpha\,(2\pi \cdot 2n) \qquad (1)$$

If angular velocity after n rotation is ω_n

$$\omega_n^{\ 2} = (100)^2 + 2\alpha\,(2\pi \cdot n) \qquad (2)$$

from equation (1) and (2)

$$\frac{50^2 - 100^2}{\omega_n^2 - 100^2} = \frac{2\alpha(2\pi \cdot 2n)}{2\alpha 2\pi n} = 2$$

$$\Rightarrow \quad \omega_n^2 = \frac{50^2 + 100^2}{2}$$

$$\omega = 25\sqrt{10} \text{ rad/s.}$$

75. Find the angular velocity of A with respect to B in the Fig. 2.16 given below:

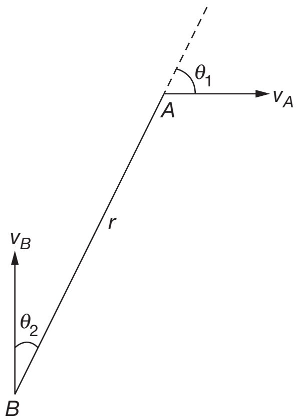
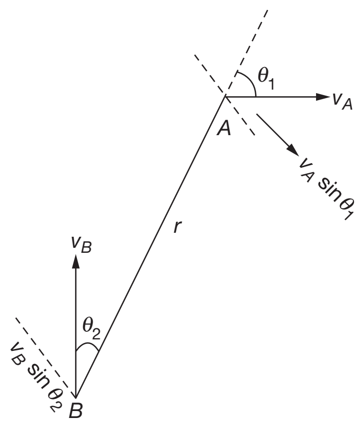


Fig. 2.16

Solution:



$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$$

$$(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$$

$$r_{AB} = r$$

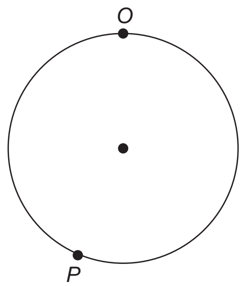
$$\omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}.$$

76. Two runners start simultaneously from the same point on a circular 200 m track in the same direction. Their speeds are 6.2 ms^{-1} and 5.5 ms^{-1}, respectively. How far from the starting point the faster runner will overcome the slower one?

(A) 150 m away from the starting point
(B) 170 m away from the starting point
(C) 120 m away from the starting point
(D) None

Solution: (B)

$$200 = (6.2 - 5.5)\, t \quad \text{or} \quad t = 285.714 \text{ s}$$



$s = (6.2 \times 285.714) = 1770$ m (faster),
$1770 - 8 \times 200 = 170$
Thus, 170 m away from the starting point along the track in the direction of run.

77. A particle moves in the xy plane as $v = a\hat{i} + bx\hat{j}$, where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The particle starts from origin at $t = 0$. Find the radius of curvature of the particle as a function of x.

(A) $\dfrac{a^2 + b^2x^2}{ba}$ (B) $\dfrac{a}{b}\left[1 + \left(\dfrac{bx}{a}\right)^2\right]^{\frac{3}{2}}$

(C) $\dfrac{b}{a}\left[1 + \left(\dfrac{ax}{b}\right)^2\right]^{\frac{3}{2}}$ (D) None of these

Solution: (B)

$$\frac{dv}{dt} = a \text{ or } x = at$$

$$\frac{dy}{dt} = ba\,t$$

or $$y = \frac{bat^2}{2}$$

or $$y = \frac{bx^2}{2a}$$

$$\frac{dy}{dx} = \frac{b}{a}x \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{b}{a}$$

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1 + \left(\frac{b}{a}x\right)^2\right]^{\frac{3}{2}}}{\frac{b}{a}}$$

$$= \frac{a}{b}\left[1 + \left(\frac{b}{a}x\right)^2\right]^{\frac{3}{2}}.$$

CONCEPTS AT A GLANCE

- When a body changes its position with respect to its surrounding with time, it is said to be in motion, otherwise it is at rest.
- $|\text{displacement}| \le$ distance covered .
- Slope of displacement–time graph gives instantaneous velocity.
- Velocity is the ratio of displacement of the body to the time taken, whereas speed is the ratio of the distance travelled by body to the time taken.
- Average speed can never be zero or negative while average velocity can be.
- Area under velocity–time graph gives displacement.
- The slope of velocity–time graph gives acceleration.
- Acceleration is the ratio of the change in velocity to the time taken.
- Acceleration may be zero, negative or positive and has unit of cm/s^2 or m/s^2.
- Area under acceleration–time curve gives change in velocity.
- For uniformly accelerated body, graph of x–t is a parabola.
- If a body starts from rest or falls freely, then $u = 0$.
- If a packet is projected upwards from a height h or released from balloon ascending with a velocity u and t is the time taken for the packet to reach the ground, then $h = -ut + \frac{1}{2}gt^2$.

- In projectile motion the vertical component of velocity decreases and becomes zero at the highest point and then increases. The horizontal component of velocity remains constant.
- When the projectile is fired at an angle θ with the horizontal with velocity u, then
  $H = \dfrac{u^2 \sin^2 \theta}{2g}$ $T = \dfrac{2u \sin\theta}{g}$, and $R = \dfrac{u^2 \sin 2\theta}{g}$,
  where H = maximum height, T = time of flight and R = horizontal range.
- For same velocity of projection, horizontal range of projectile is same for two angles of projection θ and $(90° - \theta)$ with the horizontal.
- In case of circular motion, $\vec{v} = \vec{\omega} \times \vec{r}$.
- Centripetal acceleration, $a_n = \dfrac{v^2}{r} = \omega^2 r$.
- For uniform circular motion, $a_t = 0$ and $a_n = \dfrac{v^2}{r}$.
- For non-uniform circular motion, $a_t = \dfrac{dv}{dt} = r\alpha$; $a_n = \dfrac{v^2}{r}$.

BRAIN MAP

KINEMATICS

1. Relation between kinematic variables for motion in one dimension

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{vdv}{dx}$$

2. Equations of motion in one dimension

- Motion with uniform velocity $S = vt$
- Motion with uniform acceleration,

$$S = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

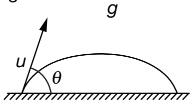
$$S_n = u + (2n - 1)\frac{a}{2}$$

3. Graphical representation of motion

- Slope of tangent to position time graph gives velocity.
- Slope of tangent to $v - t$ curve gives acceleration.
- Area enclosed between $v - t$ curve and time axis between an interval of time gives displacement.
- Slope of tangent to $a - t$ curve gives rate of change of acceleration.
- Area enclosed between $a - t$ curve and time axis between an interval of time gives change in velocity.

4. Projectile on horizontal plane

- Time of flight, $T = \dfrac{2u \sin \theta}{g}$
- Range $R = \dfrac{u^2 \sin 2\theta}{g}$



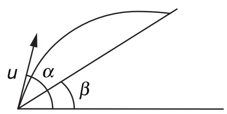
- Maximum Height, $H = \dfrac{u^2 \sin^2\theta}{2g}$
- Equation of trajectory,

$$y = x \tan \theta - \frac{1}{2}\frac{gx^2}{u^2}\sec^2\theta$$

- For maximum range, $\theta = 45°$
- For a given speed and given range, there are two possible angles of projection; θ and $(90° - \theta)$

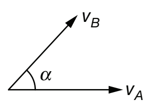
5. Projectile on inclined plane

- Time of flight, $T = \dfrac{2u \sin(\alpha - \beta)}{g \cos \beta}$
- Range, $R = \dfrac{2u^2 \sin(\alpha - \beta) \cos \alpha}{2 \cos^2 \beta}$



6. Relative velocity

- $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$



- $|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos \theta}$
- If relative velocity makes an angle α with v_A then,

$$\tan \alpha = \frac{v_B \sin \theta}{v_A - v_B \cos \theta}$$

EXERCISES

Single Option Correct Type

Motion in One Dimension

1. A car starting from rest is accelerated at constant rate until it attains a constant speed v. It is then retarded at a constant rate until it comes to rest. Considering that the car moves with constant speed for half of the time of total journey, the average speed of the car for the journey is

(A) $\frac{v}{4}$ (B) $\frac{3v}{4}$

(C) $\frac{3v}{2}$ (D) Data insufficient

2. A particle is moving along a circular path of radius 6 m with a uniform speed of $8\ \text{ms}^{-1}$. The average acceleration when the particle completes one half of the revolution is

(A) $\frac{16}{3\pi}\text{ms}^{-2}$ (B) $\frac{32}{3\pi}\text{ms}^{-2}$

(C) $\frac{64}{3\pi}\text{ms}^{-2}$ (D) None of these

3. During an accelerated motion of a particle (initial velocity of particle is zero)

(A) Average velocity of the particle is always less than its final velocity.

(B) Average velocity of the particle is always greater than its final velocity.

(C) Average velocity of the particle may be zero.

(D) Average velocity of the particle is half its final velocity.

Motion Under Uniform Acceleration in Straight Line

4. A particle starts moving from the position of rest under a constant acceleration. It travels a distance x in the first 10 s and distance y in the next 10 s, then

(A) $y = x$ (B) $y = 2x$

(C) $y = 3x$ (D) $y = 4x$

5. The initial velocity of a body moving along a straight line is 7 m/s. It has a uniform acceleration of 4 m/s^2. The distance covered by the body in the 5th second of its motion is

(A) 25 m (B) 35 m

(C) 50 m (D) 85 m

6. The initial velocity of a particle moving along a straight line is 12 ms^{-1} and its retardation is 3 ms^{-2}. The distance moved by the particle in the fourth second of its motion is

(A) 1.5 m (B) 22.5 m

(C) 24 m (D) 72 m

7. A particle has an initial velocity 11 m/s due east and a constant acceleration of 2 m/s^2 due west. The distance covered by the particle in sixth second is

(A) Zero (B) 0.5 m (C) 1 m (D) 2 m

8. A body starts from rest and travels with uniform acceleration such that it covers 8 m during the 2nd second. During the 5th second it would travel

(A) 20 m (B) 24 m (C) 28 m (D) 16 m

9. A bus is beginning to move with an acceleration of 1 m/s^2. A boy who is 48 m behind the bus starts running with constant speed of 10 m/s. The earliest time when the boy can catch the bus is

(A) 8 s (B) 10 s

(C) 12 s (D) 14 s

10. A motor car can be stopped within a distance of s, when it moves with a speed v. If it moves with a speed $4v$, it can be stopped within a distance (assuming constant braking force)

(A) s (B) $4s$ (C) $2s$ (D) $16s$

11. A body is thrown up in a lift with an upward velocity u relative to the lift from its floor and the time of flight is found to be t. The acceleration of the lift will be

(A) $\frac{u - gt}{2}$ (B) $\frac{u + gt}{2}$

(C) $\frac{2u - gt}{t}$ (D) $\frac{u}{t} - g$

12. From a balloon rising vertically upwards at 5 m/s, a stone is thrown up at 10 m/s relative to the balloon. Its velocity with respect to ground after 2 s is (assume $g = 10$ ms^2)

(A) 0 (B) 20 m/s (C) 10 m/s (D) 5 m/s

13. A man running uniformly at 8 m/s is 16 m behind a bus when it starts accelerating at 2 ms^{-2}. Time taken by him to board the bus is

(A) 2 s (B) 3 s (C) 4 s (D) 5 s

14. A particle is moving in positive x-direction with initial velocity of 10 m/s and uniform retardation such that it reaches the initial position after 10 s. The distance traversed by the particle in 6 s is

(A) 24 m (B) 25 m (C) 26 m (D) 27 m

15. A driver applies the brakes on seeing traffic signal 400 m ahead. At the time of applying the brakes the vehicle was moving with 15 ms^{-1} and retarding with 0.3 ms^{-2}. The distance of the vehicle after 1 minute from the traffic light is

(A) 25 m (B) 375 m
(C) 360 m (D) 40 m

16. A body is moving from rest under constant acceleration and let S_1 be the displacement in the first $(p-1)$ s and S_2 be the displacement in the first p s. The displacement in $(p^2-p+1)^{th}$ s will be

(A) S_1+S_2 (B) S_1S_2
(C) S_1-S_2 (D) $\frac{S_1}{S_2}$

17. A bus is moving with a velocity 10 ms^{-1} on a straight road. A motorist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the motorist, with what velocity should the motorist chase the bus?

(A) 50 ms^{-1} (B) 40 ms^{-1}
(C) 30 ms^{-1} (D) 20 ms^{-1}

18. A train starts from station A with uniform acceleration a_1 for some distance and then goes with uniform retardation a_2 for some more distance to come to rest at station B. The distance between A and B is 4 km and the train takes 4 hours to complete this journey. If acceleration and retardation are in km/ hour^2, then

(A) $\frac{a_1}{a_2}=4$ (B) $\frac{1}{a_1}+\frac{1}{a_2}=2$
(C) $a_1a_2=1$ (D) None

Motion Under Gravity

19. A stone is dropped from the top of the tower and reaches the ground in 3 s. Then the height of the tower is (g = 9.8 m/s^2)

(A) 18.6 m (B) 39.2 m
(C) 44.1 m (D) 98 m

20. When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height then the ball should be thrown with velocity

(A) $\sqrt{3}\,v_0$ (B) $3v_0$ (C) $9v_0$ (D) $3/2v_0$

21. A coin is dropped in a lift. It takes time t_1 to reach the floor when lift is stationary. It takes time t_2 when lift is moving up with constant acceleration, then

(A) $t_1>t_2$ (B) $t_2>t_1$
(C) $t_1=t_2$ (D) $t_1>>t_2$

22. A balloon is moving vertically upward with a velocity of 4 m/s. When it is at a height of h, a stone is dropped from it. If it reaches the ground in 4 s, the height of the balloon, when the stone is released, is (g = 9.8 m/s^2)

(A) 62.4 m (B) 42.4 m
(C) 78.4 m (D) 82.2 m

23. One body is dropped, while a second body is thrown downward with an initial velocity of 1 ms^{-1} simultaneously. The separation between these is 1.8 m after a time

(A) 4.5 s (B) 9 s (C) 1.8 s (D) 36 s

24. A ball is dropped from the top of a building. The ball takes 0.5 s to pass the 3 m length of a window some distance from the top of the building. If the velocities of the ball at the top and at the bottom of the window are v_T and v_B respectively, then

(A) $v_T+v_B=12$ ms^{-1}
(B) $v_T-v_B=4.9$ ms^{-1}
(C) $v_Bv_T=1$ ms^{-1}
(D) $\frac{v_B}{v_T}=1$ ms^{-1}

25. A particle is projected vertically upward with a speed of 100 m/s. The distance travelled by the particle in first fifteen seconds is (g = 10 m/s^2)

(A) 375 m (B) 625 m (C) 750 m (D) 500 m

26. A pebble is thrown vertically upwards from a bridge with an initial velocity of 10 ms^{-1}. It strikes water after 5 s. The height of the bridge is (g = 10 m/s^2)

(A) 25 m (B) 50 m (C) 75 m (D) 200 m

27. A ball is projected vertically upwards such that it attains a height of h after 5 s and 9 s of its motion. The speed of projection is (g = 10 ms^{-2})

(A) 20 ms^{-1} (B) 50 ms^{-1} (C) 35 ms^{-1} (D) 70 ms^{-1}

28. A stone is allowed to fall from the top of a tower and cover half the height of the tower in the last second of its journey. The time taken by the stone to reach the foot of the tower is

(A) $(2-\sqrt{2})$ s (B) 4 s
(C) $(2+2\sqrt{2})$ s (D) $(2+\sqrt{2})$ s

29. A ball is dropped from the roof of a tower of height h. The total distance covered by it in the last second of its motion is equal to the distance covered by it in first three seconds. The value of h in meters is ($g = 10$ m/s^2)

(A) 125 (B) 200 (C) 100 (D) 80

30. A ball is thrown vertically upwards from the ground. It crosses a point at the height of 25 m twice at an interval of 4 s. The ball was thrown with the velocity of ($g = 10$ m/s^2)

(A) 20 m/s (B) 25 m/s (C) 30 m/s (D) 35 m/s

31. A balloon rises from rest with a constant acceleration $g/8$. A stone is released from it when it has risen to height h. The time taken by the stone to reach the ground is

(A) $4\sqrt{\frac{h}{g}}$ (B) $2\sqrt{\frac{h}{g}}$ (C) $\sqrt{\frac{2h}{g}}$ (D) $\sqrt{\frac{h}{g}}$

32. Two bodies are projected vertically upwards from one point with the same initial velocities v_0 m/s. The second body is thrown τ s after the first. The two bodies meet after time

(A) $\frac{v_0}{g} - \frac{\tau}{2}$ (B) $\frac{v_0}{g} + \tau$

(C) $\frac{v_0}{g} + \frac{\tau}{2}$ (D) $\frac{v_0}{2g} + \tau$

33. A projectile is fired vertically upwards with an initial velocity u. After an interval of T seconds a second projectile is fired vertically upwards, also with initial velocity u. The correct statement is

(A) They meet at time $t = \frac{u}{g}$

(B) They meet at time $t = \frac{u}{g} + \frac{T}{2}$

(C) They meet at time $t = \frac{u}{g} - \frac{T}{2}$

(D) They never meet

34. A stone is dropped from a height h. Simultaneously, another stone is thrown up from the ground which reaches a height 4 h. The two stones cross each other after time

(A) $\sqrt{\frac{h}{2g}}$ (B) $\sqrt{\frac{h}{8g}}$ (C) $\sqrt{2hg}$ (D) $\sqrt{8hg}$

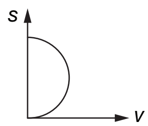
35. An aeroplane is rising vertically with acceleration f. Two stones are dropped from it at an interval of time t. The distance between them at time t' after the second stone is dropped will be

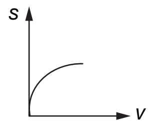
(A) $\frac{1}{2}(g+f)tt'$ (B) $\frac{1}{2}(g+f)(t+2t')t$

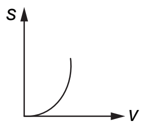
(C) $\frac{1}{2}(g+f)(t-t')^2$ (D) $\frac{1}{2}(g+f)(t+t')^2$

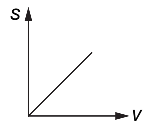
Graph

36. An object is moving with a uniform acceleration which is parallel to its instantaneous direction of motion. The displacement(s) – velocity (v) graph of this object is

(A) 

(B) 

(C) 

(D) 

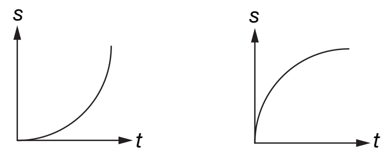
37. Displacement (s) versus time (t) graphs of two particles moving in a straight line along x-axis are shown below. Which of following statement is incorrect.

(A) Particle (i) has accelerated motion

(B) Particle (i) has positive velocity

(C) Particle (ii) has uniform motion

(D) Particle (ii) has a retarded motion



(i) (ii)

38. Position-time curve of a body moving along a straight line is shown in Fig. 2.17. The velocity-time curve for the motion of the particle will be

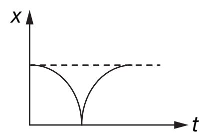
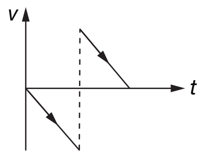
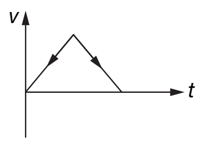
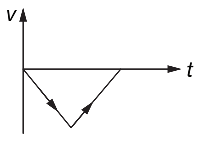
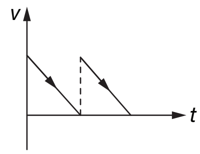


Fig. 2.17

(A) 

(B) 

(C) 

(D) 

39. The velocity-time graph of a particle moving along a straight line is as shown in Fig. 2.18. Calculate the distance covered between $t = 0$ to $t = 10$ s.

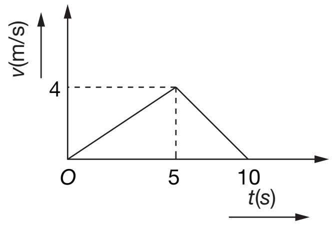
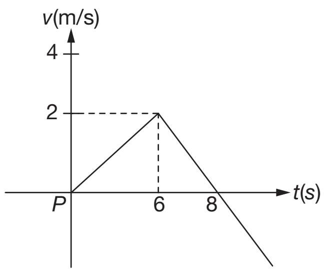


Fig. 2.18

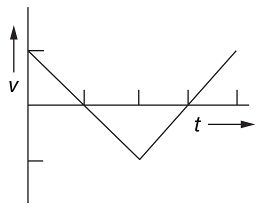
(A) 10 m (B) 20 m (C) 60 m (D) 50 m

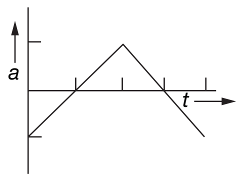
40. The velocity time graph of a particle starting from rest from a point P is shown here. Particle will reach P again, after staring from P in time

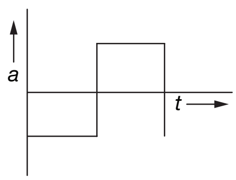


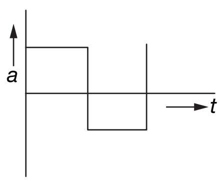
(A) 8 s (B) 10 s (C) 12 s (D) 16 s

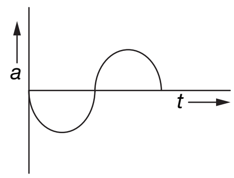
41. The graph given shows the velocity v versus time t for a body. Which of the following graphs shown represents the corresponding acceleration versus time graphs?



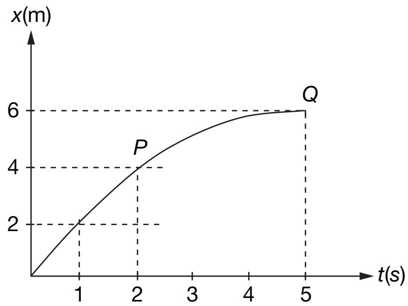
(A)


(B)


(C)


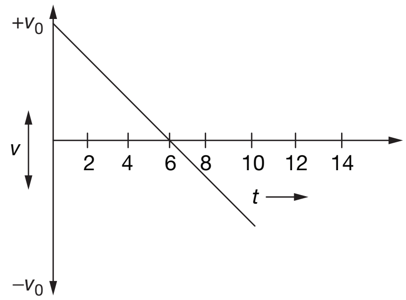
(D)


42. What is the average velocity during time interval $t = 2$ s to $t = 5$ s, in the following position time curve?



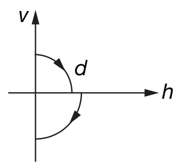
(A) 2 m/s (B) 2/3 m/s
(C) 1.2 m/s (D) 0.4 m/s

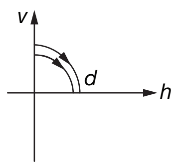
43. Consider the given velocity-time graph. It represents the motion of

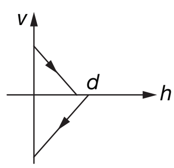


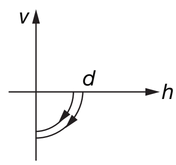
(A) A projectile projected vertically upward, from a point.
(B) An electron in the hydrogen atom.
(C) A bullet fired horizontally from the top of a tower.
(D) An object in the positive direction with decreasing speed.

44. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as

(A)


(B)


(C)


(D)


45. Figure 2.19 shows the acceleration–time graph for a particle in rectilinear motion. The average acceleration in first twenty second is

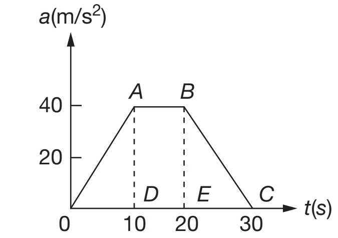


Fig. 2.19

(A) 45 m/s^2 (B) 40 m/s^2
(C) 30 m/s^2 (D) 20 m/s^2

46. Acceleration–time graph of a particle, starting from rest in straight line, is shown in adjacent Fig. 2.20, then

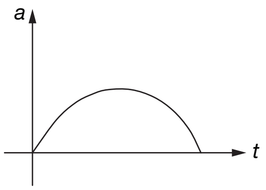


Fig. 2.20

(A) Displacement of particle will first increases then decreases.
(B) Velocity of the particle will first increases then decreases.
(C) Displacement of particle continuously increases.
(D) Speed of the particle first increases in a direction then becomes zero and finally increases in opposite direction.

47. The velocity of a particle that moves in the positive x-direction varies with its position (x) as shown in Fig. 2.21. Its acceleration at $x = 6$ m is

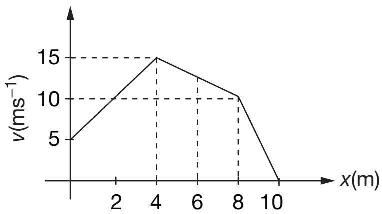


Fig. 2.21

(A) $-\frac{75}{4}$ m/s^2 (B) $-\frac{5}{4}$ m/s^2
(C) $-\frac{25}{2}$ m/s^2 (D) $-\frac{125}{8}$ m/s^2

Variable Acceleration

48. A point moves in a straight line so that its displacement x metre at time t s is given by $x^2 = 1 + t^2$. Its acceleration in m/s^2 at time t s is

(A) $\frac{1}{x^3}$ (B) $\frac{1}{x} - \frac{1}{x^2}$
(C) $\frac{1}{x} - \frac{t^2}{x^3}$ (D) $\frac{-t}{x^2}$

49. The position of a particle as a function of time is $\vec{r} = 4\sin 2\pi t\,\hat{i} + 4\cos 2\pi t\,\hat{j}$ (where t is time in second). Path of this particle will be

(A) an ellipse (B) a hyperbola
(C) a circle (D) any other curved path

50. Acceleration of a particle moving along a straight line is a function of velocity as $a = 2\sqrt{v}$. At $t = 2$ s, its velocity $v = 16\,\text{ms}^{-1}$. Its velocity at $t = 3$ s will be

(A) $20\,\text{ms}^{-1}$ (B) $25\,\text{ms}^{-1}$
(C) $30\,\text{ms}^{-1}$ (D) $22.5\,\text{ms}^{-1}$

51. A particle moving in a straight line has velocity and displacement equation as $v = 4\sqrt{1+s}$, where v is in m/s and s is in m. The initial velocity of the particle is

(A) 4 m/s (B) 16 m/s (C) 2 m/s (D) Zero

52. A body is in rectilinear motion with an acceleration given by $a = 2v^{3/2}$. If particle starts its motion from origin with a velocity of 4 ms^{-1}, the position x of the particle at an instant in terms of v can be given as

(A) $\frac{1}{\sqrt{v}} = \frac{1}{2} - x$ (B) $\sqrt{v} = x + 2$
(C) $\sqrt{v} = x$ (D) $\sqrt{v} = 2x - 1$

53. A particle moves along x-axis as $x = 4(t-2) + a(t-2)^2$. Which of the following is true?

(A) The initial velocity of particle is 4
(B) The acceleration of particle is $2a$
(C) The particle is at origin at $t = 0$
(D) None of these

54. A body starts from origin and moves along x axis such that at any instant, velocity is $v_t = 4t^3 - 2t$ where t is in second and v_t is in ms^{-1}. The acceleration of the particle when it is 2 m from the origin is

(A) 28 ms^{-2} (B) 22 ms^{-2} (C) 12 ms^{-2} (D) 10 ms^{-2}

55. Acceleration of a particle, starting from rest in straight line, changes with time as $a = 6t$ m/s^2. Displacement of the particle at $t = 2$ s, will be

(A) 24 m (B) 8 m (C) 16 m (D) 4 m

56. A particle moves along a straight line such that its displacement s at any time t is given by $s = t^3 - 6t^2 + 3t + 4$ metre. The velocity, when the acceleration is zero, is

(A) –12 ms^{-1} (B) –9 ms^{-1}
(C) 3 ms^{-1} (D) 42 ms^{-1}

57. The velocity of a body depends on time according to the equation $v = 20 + 0.1t^2$. The body is undergoing
(A) Uniform acceleration
(B) Uniform retardation
(C) Non-uniform acceleration
(D) Zero acceleration

58. The position of an object moving along x-axis is given by $x = at^3 + bt + 3$, where x is in metres and t in seconds. If velocity at $t = 1$ s and $t = 4$ s is 0.3 m/s and 27.3 m/s respectively, the value of a and b will be
(A) 0.6 m/s^3, +1.5 m/s (B) 0.6 m/s^3, –1.5 m/s
(C) 1.6 m/s^3, –1.5 m/s (D) None of these

59. The x and y co-ordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$ where x and y are in metre and t in s. The acceleration of the particle at 5 s is
(A) Zero (B) 8 m/s^2
(C) 20 m/s^2 (D) 40 m/s^2

60. The position of a particle is given by $\vec{r} = 3t\hat{i} + \sqrt{3}t^2\hat{j} - 4\hat{k}$, where t is in seconds and $\vec{r}$ is meters. Find out magnitude and direction of velocity $\vec{v}$ with horizontal at $t = \sqrt{3}$ s.
(A) $3\sqrt{5}$ m/s, $\theta = \tan^{-1}(2)$
(B) $3\sqrt{5}$ m/s, $\theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(C) $3\sqrt{2}$ m/s, $\theta = \tan^{-1}(3)$
(D) $3\sqrt{5}$ m/s, $\theta = \tan^{-1}\left(\frac{1}{2}\right)$

61. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as
(A) t^3 (B) t^2 (C) t (D) $t^{1/2}$

62. The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity vector of the particle is
(A) Parallel to the position vector
(B) Perpendicular to the position vector
(C) Directed towards the origin
(D) Directed away from the origin

63. The retardation of a particle moving in a straight line is proportional to its displacement (proportionality constant being unity). Initial velocity of the particle is v_0. Find the total displacement of the particle till it comes to rest.
(A) $\frac{v_0}{2}$ (B) v_0 (C) $\frac{v_0}{3}$ (D) $\frac{v_0^2}{4}$

64. If position (in meter) of a particle moving in straight line is given by $x = t^2 - 2t + 1$ (where t is time in second). The distance travelled by particle in first two second is
(A) Zero (B) 2 m (C) 4 m (D) 3 m

65. A particle moves along the parabolic path $y = ax^2$ in such a way that the x component of the velocity remains constant, say c. The acceleration of the particle is
(A) $ac\hat{k}$ (B) $2ac^2\hat{j}$
(C) $2ac^2\hat{k}$ (D) $a^2c\hat{j}$

66. A car starts from rest from origin an straight line with an acceleration (a) given by the relation $a = \frac{25}{(x+2)^3}$, where a is in m/s^2 and x is in metre. The maximum velocity of the car will be (x is the position of the car)
(A) 2.5 m/s (B) 5 m/s
(C) 10 m/s (D) Infinite

Motion in Two Dimensions

Projectile

67. A projectile is fired horizontally with an initial speed of 20 m/s. Its horizontal speed 3 s later will be
(A) 20 m/s (B) 6.67 m/s
(C) 60 m/s (D) 29.4 m/s

68. A projectile has the same range R for two angles of projection. If T_1 and T_2 be the times of flight in the two cases, then R is
(A) T_1T_2g (B) $\frac{T_1T_2g}{2}$
(C) $\left(T_1^2 + T_1^2\right)g$ (D) $\frac{T_1^2 + T_2^2}{2}g$

69. If the range of a gun which fires a shell with muzzle speed V is R, then the angle of elevation of the gun is
(A) $\cos^{-1}\left(\frac{V^2}{Rg}\right)$ (B) $\cos^{-1}\left(\frac{gR}{V^2}\right)$
(C) $\frac{1}{2}\left(\frac{V^2}{Rg}\right)$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{gR}{V^2}\right)$

70. A bullet is fired with a gun from a tower horizontally with a velocity 400 m/s. At the same time, a stone is dropped from the same tower.
(A) The stone will reach the ground first
(B) The bullet will reach the ground first
(C) Both will reach the ground at the same time
(D) (A) and (B) according to the height of tower

71. A particle is projected with a velocity u, at an angle α, with the horizontal. Time at which its vertical component of velocity becomes half of its net speed at the highest point will be

(A) $\dfrac{u}{2g}$

(B) $\dfrac{u}{2g}(\sin\alpha - \cos\alpha)$

(C) $\dfrac{u}{2g}(2\cos\alpha - \sin\alpha)$

(D) $\dfrac{u}{2g}(2\sin\alpha - \cos\alpha)$

72. A stone is thrown with speed of 20 m/s at an angle of 60° with the ground. Speed of stone when makes an angle of 30° with the horizontal is

(A) 10 m/s (B) $10\sqrt{3}$ m/s

(C) $\dfrac{20}{\sqrt{3}}$ m/s (D) None of these

73. A shell fired from the ground is just able to cross in a horizontal direction the top of a wall 90 m away and 45 m high. The direction of projection of the shell is

(A) 25° (B) 30° (C) 60° (D) 45°

74. Two bodies are projected at angles θ and $(90 - \theta)$ to the horizontal with the same speed. The ratio of their times of flight is

(A) $\sin\theta : 1$ (B) $\cos\theta : 1$

(C) $\sin\theta : \cos\theta$ (D) $\cos\theta : \sin\theta$

75. The velocity of projection of an oblique projectile is $\vec{v} = 3\hat{i} + 2\hat{j}$ (in ms^{-1}). The speed of the projectile at the highest point of the trajectory is

(A) 3 ms^{-1} (B) 2 ms^{-1} (C) 1 ms^{-1} (D) zero

76. A particle is projected with a velocity v such that its range on the horizontal plane is twice the greatest height attained by it. The range of the projectile is (where g is acceleration due to gravity)

(A) $\dfrac{4v^2}{5g}$ (B) $\dfrac{4g}{5v^2}$ (C) $\dfrac{v^2}{g}$ (D) $\dfrac{4v^2}{\sqrt{5}\,g}$

77. A particle is thrown with a speed of 12 m/s at an angle 60° with the horizontal. The time interval between the moments when its speed is 10 m/s is ($g = 10$ m/s^2)

(A) 1.0 s (B) 1.2 s (C) 1.4 s (D) 1.6 s

78. A body is thrown with the velocity v_0 at an angle α with the horizontal. If the body remains in air for 6 s the maximum height reached by the body will be

(A) 9.8 m (B) 19.6 m (C) 20.0 m (D) 44.1 m

79. A projectile's time of flight T is related to the horizontal range R by the equation $gT^2 = 2R$. The angle of projection in degrees is

(A) 30° (B) 45° (C) 60° (D) 90°

80. A particle is projected at an angle α with the horizontal from the foot of an inclined plane making an angle β with horizontal. Which of the following expressions holds good if the particle strikes the inclined plane normally?

(A) $\cot\beta = \tan(\alpha - \beta)$
(B) $\cot\beta = 2\tan(\alpha - \beta)$
(C) $\cot\alpha = \tan(\alpha - \beta)$
(D) $\cot\alpha = 2\tan(\alpha - \beta)$

81. A projectile is thrown horizontally from top of a building of height 10 m with certain speed (u). At the same time another projectile is thrown from ground 10 m away from the building with equal speed (u) on the same vertical plane. If they collide after 2s, then choose the correct options.

(A) The angle of projection for second projectile is 60° and $u = 10$ ms^{-1}
(B) The angle of projection for second projectile is 90° and $u = 5$ ms^{-1}
(C) The angle of projection for second projectile is 60° and $u = 5$ ms^{-1}
(D) The angle of projection for second projectile is 45° and $u = 10$ ms^{-1}

82. A very broad elevator is going up vertically with a constant acceleration of 2 m/s^2. At the instant when its velocity is 4 m/s a ball is projected from the floor of the lift with a speed of 4 m/s relative to the floor at an elevation of 30°. The time taken by the ball to return the floor is ($g = 10$ m/s^2)

(A) $\dfrac{1}{2}$ s (B) $\dfrac{1}{3}$ s (C) $\dfrac{1}{4}$ s (D) 1 s

83. A projectile can have the same range R for two angles of projection. If t_1 and t_2 are the times of flight in the two cases, then

(A) $t_1t_2 \propto R^2$ (B) $t_1t_2 \propto \dfrac{1}{R^2}$

(C) $t_1t_2 \propto R$ (D) $t_1t_2 \propto \dfrac{1}{R}$

84. The equation of projectile is $y = \sqrt{3}x - \dfrac{gx^2}{2}$. The angle of projection is

(A) $\theta = \dfrac{\pi}{6}$ (B) $\theta = \dfrac{\pi}{3}$

(C) $\theta = \dfrac{\pi}{2}$ (D) $\theta = \dfrac{\pi}{12}$

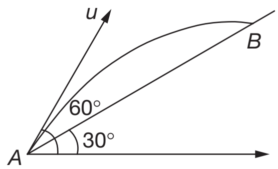
85. If $\vec{r} = bt\hat{i} + ct^2\hat{j}$ where b and c are positive constants, the velocity vector make an angle of 45° with the x and y axes at t equal to

(A) $\frac{b}{2c}$ (B) $\frac{b}{c}$ (C) $\frac{c}{2b}$ (D) $\frac{c}{b}$

86. A body is thrown with the velocity v_o at an angle α with the horizontal. If the body remains in air for the time $t = 4$s, the maximum height reached by the body will be

(A) 9.8 m (B) 19.6 m
(C) 20.0 m (D) 78.4 m

87. Time taken by the projectile to reach form A to B is t. Then the distance AB is equal to



(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}ut$ (D) $2ut$

88. The maximum height of a projectile for two complementary angles of projection is 50 m and 30 m respectively. The initial speed of projectile is

(A) $10\sqrt{34}$ m/s (B) 40 m/s
(C) 20 m/s (D) 10 m/s

89. The angle which the velocity vector of a projectile thrown with a velocity v at an angle θ to the horizontal will make with the horizontal after time t of its being thrown up is

(A) θ (B) $\tan^{-1}(\theta / t)$

(C) $\tan^{-1}\left(\frac{v\cos\theta}{v\sin\theta - gt}\right)$ (D) $\tan^{-1}\left(\frac{v\sin\theta - gt}{v\cos\theta}\right)$

90. A projectile is fired with a velocity u at right angle to a slope, which is inclined at an angle θ with the horizontal. The range of the projectile on the incline is

(A) $\frac{2u^2\sin\theta}{g}$ (B) $\frac{2u^2}{g}\tan\theta\sec\theta$

(C) $\frac{u^2}{g}\sin 2\theta$ (D) $\frac{2u^2}{g}\tan\theta$

91. A particle is projected with a speed of 40 m/s at an angle of 60° with the horizontal. At what height speed of particle becomes half of initial speed ($g = 10$ m/s^2).

(A) 30 m (B) 45 m
(C) 37.5 m (D) 60 m

92. The velocity of a projectile, when it is at the greatest height, is $\sqrt{\frac{2}{5}}$ times its velocity when it is at half of its greatest height. The angle of projection is

(A) 30° (B) 45°
(C) $\tan^{-1}\frac{2}{3}$ (D) 60°

93. The projectiles A and B thrown with velocities v and $\frac{v}{2}$ have the same range. If B is thrown at an angle of 15° to the horizontal, A must have been thrown at an angle

(A) $\sin^{-1}\left(\frac{1}{16}\right)$ (B) $\sin^{-1}\left(\frac{1}{4}\right)$

(C) $2\sin^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

94. A particle is projected with velocity u at an angle of 45° with the horizontal on an inclined plane inclined at an angle α ($\alpha < 45°$) as shown in Fig. 2.22. If particle hits the inclined plane horizontally, then

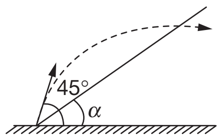
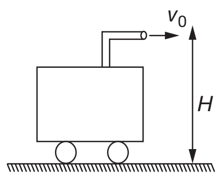


Fig. 2.22

(A) $\tan\alpha = \frac{1}{4}$ (B) $\tan\alpha = 1$

(C) $\tan\alpha = \frac{1}{2}$ (D) $\tan\alpha = \frac{1}{3}$

95. From a canon mounted on a wagon at height H from ground, a shell is fired horizontally with a velocity v_0 with respect to canon. The canon and wagon has combined mass M and can move freely on the horizontal surface. The horizontal distance between shell and canon when the shell touches the ground is



(A) $v_0\sqrt{\frac{2H}{g}}$ (B) $\frac{v_0 m}{M+m}\sqrt{\frac{2H}{g}}$

(C) $\frac{v_0 M}{M+m}\sqrt{\frac{2H}{g}}$ (D) $\frac{v_0 m}{M}\sqrt{\frac{2H}{g}}$

96. Two seconds after projection, a projectile is traveling in a direction inclined at 30° to the horizontal and after one more second, it is traveling horizontally. The initial angle of projection with the horizontal is

(A) 30° (B) 45° (C) $\sin^{-1}\frac{1}{3}$ (D) 60°

97. A body is projected at an angle α with velocity 10 m/s. Its direction of motion makes an angle of $\alpha/2$ from horizontal after t s ($g = 10$ ms^{-2}), where t is.

(A) $\tan\frac{\alpha}{2}$ (B) $\cot\frac{\alpha}{2}$

(C) $\sin\frac{\alpha}{2}$ (D) $\cos\frac{\alpha}{2}$

98. The maximum height attained by a projectile is increased by 5%, keeping the angle of projection constant. The corresponding percentage increase in horizontal range will be

(A) 5% (B) 10% (C) 15% (D) 20%

99. A particle can be projected with a given speed in two possible ways so as to make it pass through a point at a distance r from the point of projection. The product of the times taken to reach this point in the two possible ways is then proportional to

(A) r (B) $\frac{1}{r}$ (C) $\frac{1}{r^2}$ (D) $\frac{1}{r^3}$

100. The equation of motion of a projectile is $y = 12x - \frac{3}{4}x^2$. Given that g = 10 ms^{-2}, what is the range of the projectile?

(A) 12 m (B) 16 m (C) 30 m (D) 36 m

101. A particle is projected from ground with velocity $40\sqrt{2}$ m/s at 45°. At time $t = 2$ s:

(A) Displacement of particle is 100 m
(B) Vertical component of velocity is 30 m/s
(C) Velocity makes an angle of $\tan^{-1}$ (2) with horizontal
(D) Particle is at height of 80 m from ground

102. The equation of trajectory of an oblique projectile is $y = x - \frac{1}{2}x^2$. The time of flight of projectile will be

(A) $\frac{2}{\sqrt{g}}$ (B) $\frac{3}{\sqrt{g}}$ (C) $\frac{4}{\sqrt{g}}$ (D) $\frac{2\sqrt{2}}{\sqrt{g}}$

103. A stone is projected from the ground with velocity 50 m/s at an angle of 30°. It crosses a wall after 3 s. How far beyond the wall the stone will strike the ground ($g = 10$ m/s^2)

(A) 90.2 m (B) 89.6 m (C) 86.6 m (D) 70.2 m

104. A particle is projected upwards with a velocity of 100 m/s at an angle of 37° with the vertical. The time when the particle will move perpendicular to its initial direction is ($g = 10$ m/s^2, tan 53° = 4/3)

(A) 10 s (B) 12.5 s (C) 15 s (D) 16 s

105. A particle is thrown with a speed of 12 m/s at an angle 60° with the horizontal range. The time interval between the moments when its speed is 10 m/s is ($g = 10$ m/s^2)

(A) 1.0 s (B) 1.2 s (C) 1.4 s (D) 1.6 s

106. A particle is thrown horizontally from the top of a tower of height H. The angle made by velocity of particle before hitting the ground is 45° with the horizontal. What is the horizontal range of particle?

(A) H (B) $2H$ (C) $3H$ (D) $4H$

Relative Motion

107. A 150 m long train is moving to north at a speed of 10 m/s. A parrot is flying towards south with a speed of 5 m/s crosses the train. The time taken by the parrot to cross the train would be

(A) 30 s (B) 15 s (C) 8 s (D) 10 s

108. A particle is moving eastwards with a velocity of 4 m/s. In 5 s the velocity changes to 3 m/s northwards. The average acceleration in this time interval is

(A) $\frac{1}{2}$ m/s^2 towards north-east
(B) 1 m/s^2 towards north-west
(C) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-east
(D) $\frac{1}{2}$ m/s^2 towards north-west

109. Rain is falling vertically downwards with a velocity of 3 km/hr. A man walks in the rain with a velocity of 4 km/hr. The raindrops will fall on the man with a velocity of

(A) 1 km/hr (B) 3 km/hr
(C) 4 km/hr (D) 5 km/hr

110. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity v and other with a uniform acceleration a. If α is the angle between the lines of motion of two particles then the least value of magnitude of relative velocity will be at time given by

(A) $\frac{v}{a}\sin\alpha$ (B) $\frac{v}{a}\cos\alpha$

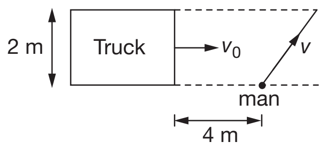
(C) $\frac{v}{a}\tan\alpha$ (D) $\frac{v}{a}\cot\alpha$

111. A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2 m/s. At what angle α with the vertical should the wind screen be placed so that rain drops falling vertically downwards with velocity 6 m/s strike the wind screen perpendicularly?

(A) $\tan^{-1}\left(\frac{1}{3}\right)$ (B) $\tan^{-1}(3)$

(C) $\cos^{-1}(3)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

112. A 2m wide truck is moving with a uniform speed v_0 = 8 m/s along a straight horizontal road. A pedestrian starts to cross the road with a uniform speed v when the truck is 4 m away from him. The minimum value of v so that he can cross the road safely is



(A) 2.62 m/s (B) 4.6 m/s
(C) 3.57 m/s (D) 1.414 m/s

113. Two trains A and B are moving on same track in opposite direction with velocity 25 m/s and 15 m/s respectively. When separation between them becomes 225 m, drivers of both the trains apply brakes producing uniform retardation in train A while retardation of train B increases linearly with time at the rate of 0.3 m/s^3. The minimum retardation of train A to avoid collision will be

(A) 2 m/s^2 (B) 2.5 m/s^2
(C) 2.25 m/s^2 (D) 2.75 m/s^2

114. If rain drops are falling with velocity of 12 m/s at an angle of 30° with the vertical. With what possible speed(s), a man should move in horizontal direction so that rain drops hit him at an angle of 45° with the horizontal.

(A) 18 m/s (B) 6 m/s
(C) Both (A) and (B) (D) None of these

115. A man can row a boat with speed 4 km/hr in still water. If the velocity of water in river is 3 km/hr. The time taken to reach just opposite is (river width = 500 m)

(A) $\frac{500}{\sqrt{7}}$ hr (B) $\frac{1}{2\sqrt{7}}$ hr

(C) 100 hr (D) None

116. A horizontal wind is blowing with a velocity v towards north-east. A man starts running towards north with acceleration a. The time, after which man will feel the wind blowing towards east, is

(A) $\frac{v}{a}$ (B) $\frac{\sqrt{2}v}{a}$ (C) $\frac{v}{\sqrt{2}a}$ (D) $\frac{2v}{a}$

117. A man holds an umbrella at 30° with the vertical to keep himself dry. He, then, runs at a speed of 10 ms^{-1} and finds the rain drops to be hitting vertically. Speed of the rain drops with respect to the running man and with respect to earth are

(A) 20 ms^{-1}, 10 ms^{-1} (B) 10 ms^{-1}, $20\sqrt{3}$ ms^{-1}
(C) $10\sqrt{3}$ ms^{-1}, 20 ms^{-1} (D) 20 ms^{-1}, $10\sqrt{3}$ ms^{-1}

118. A boat which has a speed of 5 m/s in still water crosses the river of width 25 m in 10 s. The boat is heading at an angle of α with downstream, where α is equal to

(A) 150° (B) 120° (C) 90° (D) 60°

119. A swimmer wishes to cross a 800 m wide river flowing at 6 km/hr. His speed with respect to water is 4 km/hr. He crosses the river in shortest possible time. He is drifted downstream on reaching the other bank by a distance of

(A) 800 m (B) 1200 m
(C) $400\sqrt{13}$ m (D) 2000 m

120. A boat, which has a speed of 5 km/h in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in kilometers per hour is

(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

121. A river is flowing from west to east with a speed of 5 m/min. A man can swim in still water with a velocity 10 m/min. In which direction should the man swim, so as to take the shortest possible path to go to the south?

(A) 30° with downstream
(B) 60° with downstream
(C) 120° with downstream
(D) Towards south

122. A boat travels from south bank to north bank of river with a maximum speed of 8 km/h. A river current flows from west to east with a speed of 4 km/h. To arrive at a point opposite to the point of start, the boat should start at an angle

(A) $\tan^{-1}(1/2)$ west of north
(B) $\tan^{-1}(1/2)$ north of west
(C) 30° west of north
(D) 30° north of west

Circular Motion

123. A particle is moving in a circle of radius 1 m with speed varying with time as $v = (2t)$ m/s. In first 2 s

(A) Distance traveled by the particle is 2 m.
(B) Displacement of the particle is (2 sin 2) m.
(C) Average speed of the particle is 1 m/s.
(D) Average velocity of the particle is zero.

124. A road is 5 m wide. Its radius of curvature is $20\sqrt{6}$ m. The outer edge is above the inner edge by a distance of 1 m. This road is most suited for a speed ($g = 10$ ms^{-2})

(A) 10 ms^{-1} (B) $10\sqrt{5}$ ms^{-1}
(C) 100 ms^{-1} (D) $40\sqrt{6}$ ms^{-1}

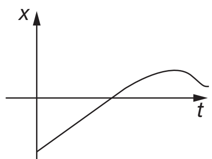
125. A car is moving on a circular path of radius 100 m. Its speed v is changing with time as $v = 2t^2$, where v in ms^{-1} and t in second. The acceleration of car at $t = 5$ s is approximately

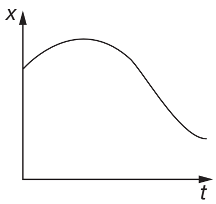
(A) 20 ms^{-1} (B) 25 ms^{-1}
(C) 30 ms^{-1} (D) 32 ms^{-1}

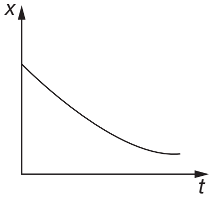
126. A particle is moving on a circular path of radius $\frac{100}{\sqrt{19}}$ m in such a way that magnitude of its velocity varies with time as $v = 2t^2 + t$, where v is velocity in m/s and t is time in s. The acceleration of the particle at $t = 2$ s is

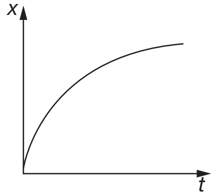
(A) 21 m/s^2 (B) 9 m/s^2
(C) 10 m/s^2 (D) 13.5 m/s^2

127. Among the following four graphs, there is only one graph for which average velocity over the time interval (0, T) can vanish for a suitably chosen T. Which one is it?

(A)


(B)


(C)


(D)


128. A lift is coming from a 8th floor and is just about to reach the 4th floor. Taking ground floor as origin and positive direction upward for all quantities, which one of the following is correct?

(A) $x < 0, v < 0, a > 0$ (B) $x > 0, v < 0, a < 0$
(C) $x > 0, v < 0, a > 0$ (D) $x > 0, v > 0, a < 0$

129. In one-dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$.

(A) The displacement in time T always takes non-negative values.
(B) The displacement x in time T satisfies $-v_0T < x < v_0T$.
(C) The acceleration is always a non-negative number.
(D) The motion has no turning points.

130. A vehicle travels half the distance L with speed V_1 and the other half with speed V_2, then its average speed is

(A) $\frac{V_1 + V_2}{2}$ (B) $\frac{2V_1 + V_2}{V_1 + V_2}$
(C) $\frac{2V_1V_2}{V_1 + V_2}$ (D) $\frac{2V_1V_2}{V_1 + V_2}$

131. The displacement of a particle is given by $x = (t - 2)^2$, where x is in metres and t in seconds. The distance covered by the particle in first 4 s is

(A) 4 m (B) 8 m
(C) 12 m (D) 16 m

132. At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be

(A) $(t_1 + t_2)/2$ (B) $t_1t_2/(t_2 - t_1)$
(C) $t_1t_2/(t_2 + t_1)$ (D) $t_1 - t_2$

133. The horizontal range of a projectile fired at an angle of 15 is 50 m. If it is fired with the same speed at an angle of 45, its range will be

(A) 60 m (B) 71 m
(C) 100 m (D) 141 m

134. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. How long will it take for the drunkard to fall in a pit 13 m away from the start.

(A) 13 s (B) 16 s
(C) 24 s (D) 32 s

135. A car moving along a straight highway with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. How long does it take for the car to stop?

(A) 5.1 s (B) 11.44 s (C) 15.2 s (D) None

136. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m/s^2. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?

(A) 2250 m (B) 1250 m
(C) 1000 m (D) 2000 m

137. On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km. B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

(A) 3 m/s^2 (B) 2 m/s^2
(C) 1 m/s^2 (D) None

138. Two towns A and B connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling at a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion and every 6 min in the opposite direction. What is the period T of the bus service? (in minutes)

(A) 9 (B) 6 (C) 18 (D) None

139. Rain is falling vertically with a speed of 30 ms^{-1}. A woman rides a bicycle at a speed of 10 ms^{-1} in the north to south direction. What is the direction in which she should hold her umbrella?

(A) $\theta = \tan^{-1} 3$ (B) $\theta = \tan \frac{-11}{3}$
(C) $\theta = \tan^{-1} \frac{2}{3}$ (D) $\theta = \tan^{-1} \frac{3}{2}$

140. A man can swim at a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

(A) 250 m (B) 500 m
(C) 750 m (D) 1000 m

141. In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N–E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

(A) $\tan^{-1} \frac{51}{72\sqrt{2}-51}$ (B) $\tan^{-1} \frac{72\sqrt{2}-51}{51}$
(C) $\tan^{-1} 1$ (D) None

142. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall?

(A) 200 m (B) 150 m
(C) 100 m (D) 50 m

143. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

(A) 50 m (B) 100 m
(C) 150 m (D) 200 m

144. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude of acceleration of the stone?

(A) 9.91 m/s^2 (B) 19.82 m/s^2
(C) 31 m/s^2 (D) None

145. The position of a particle is given by

$$r = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k} \text{ m}$$

Where t is in seconds and the coefficients have the proper units for r to be in metres.
What is the magnitude of velocity of the particle $t = 2.0$ s?

(A) $\sqrt{72}$ m/s (B) $\sqrt{41}$ m/s
(C) $\sqrt{11}$ m/s (D) None

146. A particle starts from the origin at $t = 0$ s with a velocity of $10.0\hat{j}$ m/s and moves in the x–y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})$ ms^{-2}
At a time when x-coordinate of the particle 16 m,

(A) 24 m (B) 22 m (C) 20 m (D) None

More than One Option Correct Type

147. A jeep runs around a curve of radius 0.3 km at a constant speed of 60 ms^{-1}. The jeep covers a curve of 60° arc.

(A) Resultant change in velocity of jeep is 60 ms^{-1}.
(B) Instantaneous acceleration of jeep is 12 ms^{-1}.
(C) Average acceleration of jeep is 11.5 ms^{-1}.
(D) Instantaneous and average acceleration are same in this case.

148. Two particles are projected from the same point with same speed u at angles of projection α and β from horizontal strike the horizontal ground. The maximum heights attained by projectiles is h_1 and h_2 respectively, R is the range for both and t_1 and t_2 are their time of flights respectively, then:

(A) $\alpha+\beta=\frac{\pi}{2}$ (B) $R=4\sqrt{h_1h_2}$

(C) $\frac{t_1}{t_2}=\tan\alpha$ (D) $\tan\alpha=\sqrt{h_1/h_2}$

149. Take the z-axis as vertical and xy plane as horizontal. A particle A is projected speed at $4\sqrt{2}$ m/s at an angle 45° to the horizontal in the xz. Particle B is also projected at same instant but with speed 5 m/s at an angle $\tan^{-1}(4/3)$ with horizontal in yz plane, then which of the following statement/s is/are correct? ($g = 10$ m/s^2)

(A) Magnitude of relative velocity of A with respect to B is 5 m/s during motion.
(B) Particle A and B again hit the ground at the same instant.
(C) The separation between A and B when they hit the ground is 4 m.
(D) The path of A with respect to B is straight line.

150. Two projectile are thrown at the same time from two different points. The projectile thrown from the origin has initial velocity $3\hat{i}+3\hat{j}$ with respect to earth. The projectile has initial velocity $a\hat{i}+b\hat{j}$ with respect to earth thrown from the point (10, 5). ($\hat{i}$ is a unit vector along horizontal, $\hat{j}$ along vertical). If the projectile collides after two second, then the

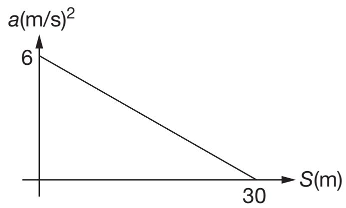
(A) value of a is –2 (B) value of a is $\frac{1}{2}$

(C) value of b is $\frac{1}{2}$ (D) value of b is –2

151. A solid sphere moves at a terminal velocity of 20 m/s in air at a place, where g = 9.8 m/s^2. The sphere is taken in a gravity free hall having air at the same pressure and pushed down at a speed of 20 m/s.

(A) Its initial acceleration will be 9.8 m/s^2 downward.
(B) Its initial acceleration will be 9.8 m/s^2 upward.
(C) The magnitude of acceleration will decrease as the time passes.
(D) It will eventually stop.

152. A train starts from rest at S = 0 and is subjected to acceleration as shown



(A) Change in velocity at the end of 10 m displacement is 50 m/s.
(B) Velocity of the train for S = 10 m is 10 m/s.
(C) The maximum velocity attained by train is not greater than 14 m/s.
(D) The maximum velocity of the train is between 15 m/s and 16 m/s.

Passage Based Questions

Passage 1

The velocity v of a body moving along a straight line is varying with time t as $v=t^2-4t$, where v in m/s and t in seconds.

153. The magnitude of initial acceleration is

(A) Zero (B) 2 m/s^2 (C) 4 m/s^2 (D) 6 m/s^2

154. The magnitude of displacement of particle in first three seconds is

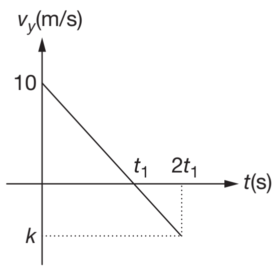
(A) Zero (B) 9 m (C) 18 m (D) 27 m

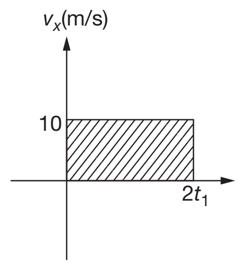
155. Velocity of particle, when its displacement is zero will be

(A) 4 m/s (B) 8 m/s (C) 10 m/s (D) 12 m/s

Passage 2

A projectile is thrown from the origin in x-y plane, where x-axis is along the ground and y-axis is vertically upwards. The vertical velocity and the horizontal velocity vary with respect to time according to the graphs shown. Accelerating due to gravity is g.





156. What is the value of t_1?

(A) $10/g$ (B) $20/g$ (C) $30/g$ (D) None

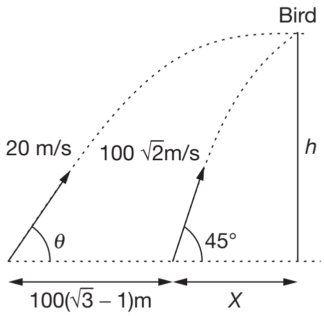
157. What is the value of k?

(A) 10 (B) –10 (C) 20 (D) None

158. What is the initial angle of projection?
(A) 45° (B) 75° (C) 60° (D) None

Passage 3

Two boys (at ground) simultaneously aim their guns at a bird sitting on a tower. The first boy releases his shot with speed of $100\sqrt{2}$ m/s at an angle 45° with the horizontal. The second boy is behind the first boy by a distance $100\left(\sqrt{3}-1\right)$ m and releases his shot with speed 200 m/s. Both the shots hit the bird simultaneously.



159. Angle of projection of the shot fired by the second boy is
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$ (D) None of these

160. After what time shots hit the bird
(A) 1.5 s (B) 2 s
(C) 1 s (D) None of these

161. Horizontal distance of the foot of tower from first boy is
(A) 50 m (B) 75 m
(C) 100 m (D) None of these

Passage 4

A person walks up a stationary escalator in t_1 s. If he is standing on elevator and then the elevator carries him up in t_2 s. The length of elevator is l.

162. Velocity of escalator is
(A) $\frac{l}{t_2}$ (B) $\frac{l}{t_1}$
(C) $\frac{l}{t_1+t_2}$ (D) $\frac{l}{t_1+t_2}$

163. Velocity of man is
(A) $\frac{l}{t_2}$ (B) $\frac{l}{t_1}$
(C) $\frac{l}{t_{1+}+t_2}$ (D) $\frac{l}{t_{1-}+t_2}$

164. If man starts to walk on moving escalator in the direction of motion of escalator, then time taken by the man to move up is
(A) $\frac{t_1t_2}{t_1+t_2}$ (B) $\frac{t_1\,t_2}{t_1-t_2}$
(C) t_1+t_2 (D) t_1-t_2

Passage 5

If a man in a boat rows perpendicular to the banks he is drifted to a distance of 120 m in 10 minutes. If he heads at an angle of α from upstream he crosses the river by shortest path in 12.5 minutes.

165. The speed of the water current is
(A) 20 m/min (B) 15 m/min
(C) 12 m/min (D) 9.6 m/min

166. The velocity of the boat relative to water
(A) 20 m/min (B) 15 m/min
(C) 12 m/min (D) 9.6 m/min

167. The width of river is
(A) 250 m (B) 200 m
(C) 150 m (D) 120 m

Match the Column Type

168. A particle moving along a straight line with uniform acceleration displaces by 13 m and 7 m in second and fifth second of its motion respectively.

Column-I	Column-II
(A) Distance (in m) travelled in first 9 s	(1) 63
(B) Magnitude of acceleration (in ms^{-2}) of particle	(2) 2
(C) Displacement (in m) in first 7 s	(3) 65
(D) Time (in s) when displacement is zero	(4) 16

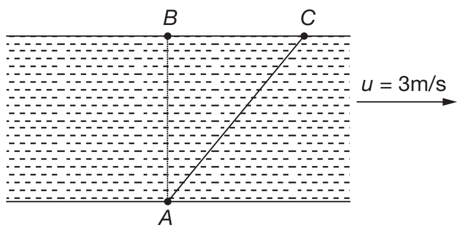
169. At any instant t = 0 a motorbike start from rest in a given direction, a car moving with constant speed overtakes the motorbike at that instant when it is moving with a speed 40 m/s. Motor bike accelerates uniformly till t = 18s and then move with constant speed and overtakes the car at t = 27 s.

Column-I	Column-II
(A) The maximum speed of motorbike in m/s is	(1) 240
(B) The acceleration of motor bike in m/s^2 is	(2) 60
(C) The separation between can and bike at $t = 18$ s in m is	(3) 10/3
(D) The maximum separation between car and bike in m before bike overtake the car is	(4) 180

170. The position (x) of a particle varies with time as $x = t^3 - 3t^2$, where t is time in second.
Match Column-I with Column-II.

Column-I	Column-II
(A) Position of particle is zero at	(1) $t = 3$ s
(B) Velocity of particle is zero at	(2) $t = 2$ s
(C) Magnitude of acceleration of particle is zero at	(3) $t = 5$ s
(D) Magnitude of instantaneous acceleration is equal to 18 m/s^2	(4) $t = 1$ s

171. A man wants to reach point C on the opposite bank of river. He can swim in still water with speed 5 m/s and can walk on ground with 10 m/s. Velocity of river is 3 m/s and given $AB = BC = 500$ m.



Column-I	Column-II
(A) The time to reach point C if he crosses the river by shortest path is	(1) 125 s
(B) The time to reach point C if he crosses the river in shortest time.	(2) 175 s
(C) The time to reach point C if he crosses the river along path AC (approximate value).	(3) 120 s (4) 106 s
(D) Time to reach point B with zero drift	

172. As shown in the Fig. 2.23 there is a particle of mass $\sqrt{3}$ kg, is projected with speed 10 m/s at an angle 30° with horizontal (take $g = 10$ m/s^2) then match the following.

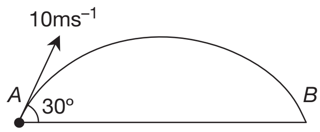


Fig. 2.23

Column-I	Column-II
(A) Average velocity (in m/s) during half of the time of flight is	(1) $\frac{1}{2}$
(B) The time (in s) after which the angle between velocity vector and acceleration vector becomes $\pi/2$	(2) Zero
(C) The time (in s) after which total velocity becomes parallel to the initial horizontal velocity vector	(3) $\frac{\sqrt{325}}{2}$
(D) Work done by gravitational force of body during the total flight	(4) 150

Assertion-Reason Type

173. **Assertion:** Magnitude of displacement is equal to distance whether body changes its direction of motion or not.

Reason: Distance is defined as the actual path length between two points.

(A) A (B) B (C) C (D) D

174. Assertion: The displacement-time graph of a particle moving in a straight line with constant acceleration will be a parabola.

Reason: Acceleration is constant and hence the distance of the particle increases continuously.

(A) A (B) B (C) C (D) D

175. Assertion: Projectile motion is a non-uniform two dimensional motion.

Reason: The acceleration due to gravity becomes the radial acceleration at the point of maximum height.

(A) A (B) B (C) C (D) D

176. Assertion: In a projectile motion, vertical component of the velocity does not remain constant.

Reason: There is no acceleration in vertical direction.

(A) A (B) B (C) C (D) D

177. Assertion: In case of projectile motion, the magnitude of rate of change of velocity is variable (Taking $g = 9.8$ m/s^2)

Reason: In projectile motion, magnitude of velocity first decreases and then increases during the motion

(A) A (B) B (C) C (D) D

178. Assertion: The kinematics equation for uniform acceleration does not apply in the case of uniform circular motion.

Reason: Magnitude and direction of acceleration keeps on changing in case of uniform circular motion.

(A) A (B) B (C) C (D) D

179. Assertion: A particle is projected with speed u at an angle θ with the horizontal. At any time during motion, speed of particle is v at angle α with the vertical then $v \sin\alpha$ is always constant throughout the motion.

Reason: In case of projectile, magnitude of radial acceleration at topmost point is maximum.

(A) A (B) B (C) C (D) D

180. Assertion: A positive acceleration of a body can be associated with a slowing down of the body.

Reason: Acceleration is a vector quantity.

(A) A (B) B (C) C (D) D

181. Assertion: A body started from rest, after time t its speed is $v_0/2$. In the next interval of time t its speed increases to v_0. The body is moving with uniform acceleration.

Reason: In case of uniformly accelerated motion speed must increases equally in equal interval of time.

(A) A (B) B (C) C (D) D

182. Assertion: The shape of the trajectory of the motion of an object is not determined by the acceleration alone.

Reason: The trajectory of the motion of an object is independent of acceleration.

(A) A (B) B (C) C (D) D

183. Assertion: A body is momentarily at rest when it reverses the direction.

Reason: A body cannot have acceleration if its velocity is zero at a given instant of time.

(A) A (B) B (C) C (D) D

184. Assertion: A body can have constant speed in an accelerated motion.

Reason: If a body is accelerating its momentum will always change.

(A) (A) (B) (B) (C) (C) (D) (D)

Integer Type

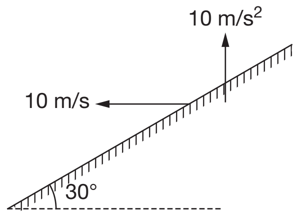
185. A passenger reaches the platform and finds that the second last boggy of the train is passing him. The second last boggy takes 3s to pass the passenger, and the last boggy takes 2s to pass him. If passenger is late by t second from the departure of the train, then the find the value of 2t. (Assume that the train accelerates at constant rate and all the boggies are of equal length.)

186. A ball is projected, so as to just clear two walls, the first of height 12 m at a distance 6 m from point of projection and the second of height 6 m at a distance 12 m from point of projection. Find the half of range (in metre) of projectile.

187. A particle is projected from origin with speed 25 m/s at angle 53° with the horizontal at $t = 0$. Find time of flight.

188. From the top of a wall of height 5 m, a ball is thrown horizontally with speed of 6 m/s. How far from the wall will the ball land?

189. A particle is projected horizontally with velocity 10 m/s from an inclined plane, which is also moving with acceleration 10 m/s^2 vertically upward as shown. The time after which it lands on the plane is $\frac{1}{\sqrt{x}}$ sec. Find the value of x (Taking $g = 10$ m/s^2).



190. A motorboat going downstream overcomes a raft at a point A. After one hour, it turns back and meets the raft again at a distance 6 km from A. Find the velocity of river in (km/hr).

191. A particle moves in a straight line with an acceleration $(12\ s^2)\ ms^{-2}$ where s is the displacement of the particle in metre from O, a fixed point on the line, at time t seconds. The particle has zero velocity when its displacement from O is –2m. Find the velocity (in m/s) of the particle as it passes through O.

192. Rain is falling vertically with a speed of 20 ms^{-1} relative to air. A person is running in the rain with a velocity of 5 ms^{-1} and a wind is also blowing with a speed of 15 ms^{-1} (both towards east). Find the cotangent of the angle with the vertical at which the person should hold his umbrella so that he may not get drenched.

193. A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2 m/s. If the angle of the wind screen with vertical is θ when vertically downward falling raindrops with velocity of 6 m/s strikes the screen perpendicularly. Find tan θ.

194. A particle is to be projected so as to just pass through three equal rings of diameter 4 m and placed in parallel vertical plane at distance 6 m apart symmetrically with there highest points at a height 9 m above the point of projection. If θ is angle of projection then find the value of tan θ.

Previous Years' Questions

195. A ball whose kinetic energy is E is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be **[2002]**

(A) E (B) $E/\sqrt{2}$ (C) $E/2$ (D) Zero

196. From a building, two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then **[2002]**

(A) $v_B > v_A$
(B) $v_A = v_B$
(C) $v_A = v_B$
(D) their velocities depend on their masses

197. A car, moving at a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is **[2003]**

(A) 12 m (B) 18 m (C) 24 m (D) 6 m

198. A boy playing on the roof of a 10 m high building throws a ball at a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?

$$\left[g = 10\ \text{m/s}^2, \sin 30° = \frac{1}{2}, \cos 30° = \frac{\sqrt{3}}{2}\right]$$ **[2003]**

(A) 5.20 m (B) 4.33 m (C) 2.60 m (D) 8.66 m

199. The co-ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by **[2003]**

(A) $3t\sqrt{\alpha^2+\beta^2}$ (B) $3t^2\sqrt{\alpha^2+\beta^2}$
(C) $t^2\sqrt{\alpha^2+\beta^2}$ (D) $\sqrt{\alpha^2+\beta^2}$

200. A ball is released from the top of a tower of height h meters. It takes T s to reach the ground. What is the position of the ball at $\frac{T}{3}$ second? **[2004]**

(A) $\frac{8h}{9}$ metres from the ground
(B) $\frac{7h}{9}$ metres from the ground
(C) $\frac{h}{9}$ metres from the ground
(D) $\frac{17h}{18}$ metres from the ground

201. Which of the following statements is FALSE for a particle moving in a circle with a constant angular speed? **[2004]**

(A) The acceleration vector points to the centre of the circle.
(B) The acceleration vector is tangent to the circle.
(C) The velocity vector is tangent to the circle.
(D) The velocity and acceleration vectors are perpendicular to each other.

202. An automobile travelling at a speed of 60 km/h can brake to stop within a distance of 20 m. If the car is going twice as fast, i.e. 120 km/h, the stopping distance will be **[2004]**

(A) 60 m (B) 40 m
(C) 20 m (D) 80 m

203. A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection? **[2004]**

(A) No (B) Yes, 30°
(C) Yes, 60° (D) Yes, 45°

204. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then **[2005]**

(A) $S = \frac{1}{6} ft^2$ (B) $S = ft$
(C) $S = \frac{1}{4} ft^2$ (D) $S = \frac{1}{72} ft^2$

205. A particle is moving eastwards at a velocity of 5 ms^{-1}. In 10 s the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is **[2005]**

(A) $\frac{1}{2}$ ms^{-2} towards north
(B) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-east
(C) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-west
(D) Zero

206. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is **[2005]**

(A) $2bv^3$ (B) $-2abv^2$
(C) $2av^2$ (D) $-2av^3$

207. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the times of flights in the two cases, then the product of the two times of flights is proportional to **[2005]**

(A) $\frac{1}{R^2}$ (B) R^2 (C) R (D) $\frac{1}{R}$

208. A particle located at $x = 0$ at time $t = 0$ starts moving along with the positve x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[2006]**

(A) t^2 (B) t
(C) $t^{1/2}$ (D) t^3

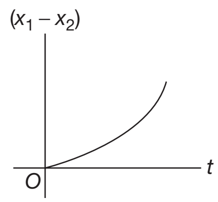
209. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[2007]**

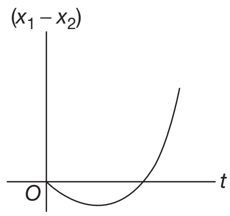
(A) $K/2$ (B) K
(C) Zero (D) $K/4$

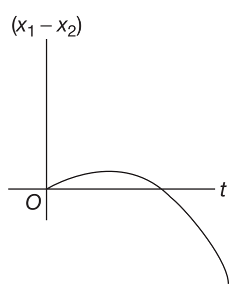
210. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is **[2007]**

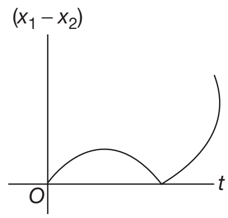
(A) $v_0 + g/2 + f$ (B) $v_0 + 2g + 3f$
(C) $v_0 + g/2 + f/3$ (D) $v_0 + g + f$

211. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant, another body passes through $x = 0$ also moving in the positive x direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t? **[2008]**

(A) 

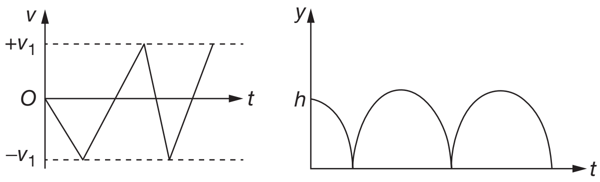
(B) 

(C) 

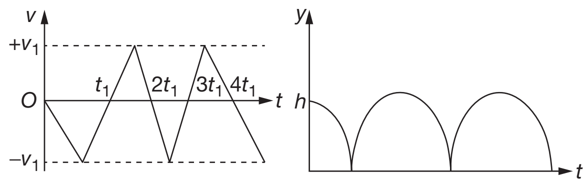
(D) 

212. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be **[2009]**

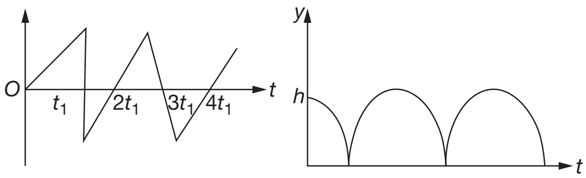
(A)



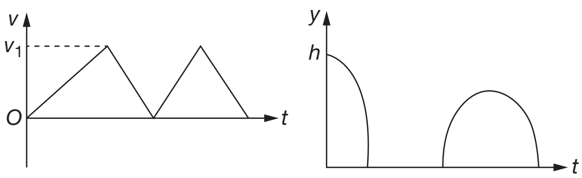
(B)



(C)



(D)



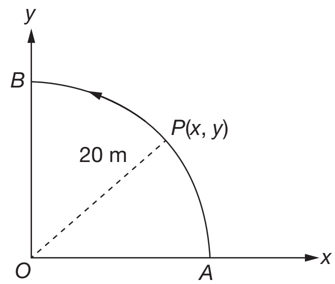
213. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is **[2009]**

(A) $7\sqrt{2}$ units (B) 7 units
(C) 8.5 units (D) 10 units

214. A particle is moving with velocity $\vec{v}=k(y\hat{i}+x\hat{j})$, where k is a constant. The general equation for its path is **[2010]**

(A) $y=x^2+$ constant
(B) $y^2=x+$ constant
(C) $xy=$ constant
(D) $y^2=x^2+$ constant

215. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s=t^3+5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t=2$ s is nearly **[2010]**



(A) 13 m/s^2 (B) 12 m/s^2
(C) 7.2 m/s^2 (D) 14 m/s^2

216. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R,θ) on the circle of radius R is (Here θ is measured from the x-axis) **[2010]**

(A) $-\frac{v^2}{R}\cos\theta\hat{i}+\frac{v^2}{R}\sin\theta\hat{j}$

(B) $-\frac{v^2}{R}\sin\theta\hat{i}+\frac{v^2}{R}\cos\theta\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$

(D) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$

217. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x–y plane as shown in the Fig. 2.24. At a time $t<\frac{v_0\sin\theta}{g}$, the angular momentum of the particle is **[2010]**

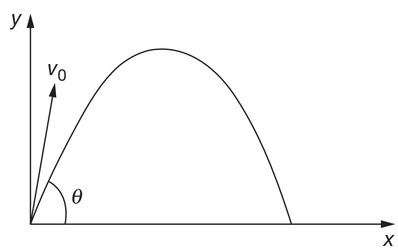


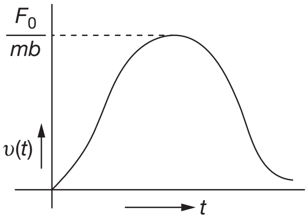
Fig. 2.24

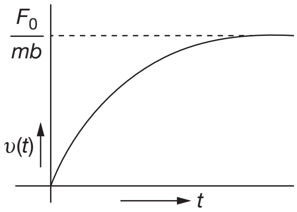
(A) $-mgv_0t^2\cos\theta\hat{j}$ (B) $mg\,v_0t\cos\theta\hat{k}$

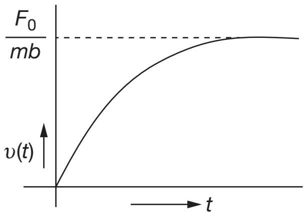
(C) $-\frac{1}{2}mg\,v_0t^2\cos\theta\hat{k}$ (D) $\frac{1}{2}mg\,v_0t^2\cos\theta\hat{i}$

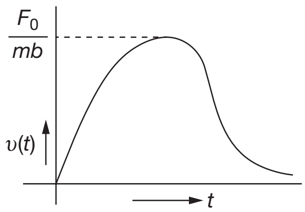
where $\hat{i},\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis, respectively.

218. A particle of mass m is at rest at the origin at time $t=0$. It is subjected to a force $F(t)=F_0e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves? **[2012]**

(A) 

(B) 

(C) 

(D) 

219. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be **[2012]**

(A) $20\sqrt{2}$ m (B) 10 m
(C) $10\sqrt{2}$ m (D) 20 m

220. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10$ m/s^2, the equation of its trajectory is **[2013]**

(A) $y = 2x - 5x^2$ (B) $4y = 2x - 5x^2$
(C) $4y = 2x - 25x^2$ (D) $y = x - 5x^2$

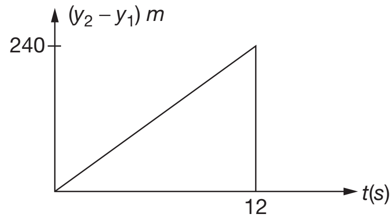
221. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground is n times that taken by it to reach the highest point of its path.

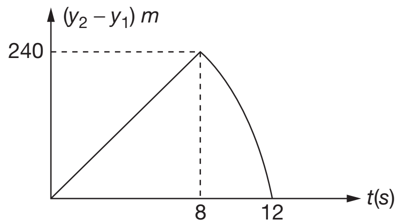
The relation between H, u and n is **[2014]**

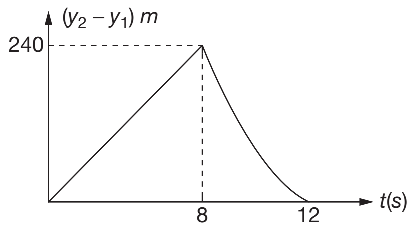
(A) $2gH = n^2u^2$
(B) $gH = (n-2)^2u^2$
(C) $2gH = nu^2(n-2)$
(D) $gH = (n-2)u^2$

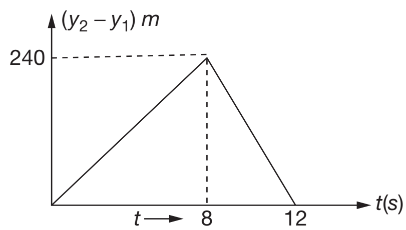
222. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s, respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?

(Assume stones do not rebound after hitting the ground and neglect air resistance, taking $g = 10$ m/s^2)
(The figures are schematic and not drawn to scale) **[2015]**

(A) 

(B) 

(C) 

(D) 

ANSWER KEYS

Single Option Correct Type

Motion in One Dimension

1. (B)	**2.** (C)	**3.** (D)	**4.** (C)	**5.** (A)	**6.** (A)	**7.** (B)	**8.** (B)	**9.** (A)	**10.** (D)
11. (C)	**12.** (D)	**13.** (C)	**14.** (C)	**15.** (A)	**16.** (A)	**17.** (D)	**18.** (B)	**19.** (C)	**20.** (A)
21. (A)	**22.** (A)	**23.** (C)	**24.** (A)	**25.** (B)	**26.** (C)	**27.** (D)	**28.** (D)	**29.** (A)	**30.** (C)
31. (B)	**32.** (C)	**33.** (B)	**34.** (B)	**35.** (B)	**36.** (C)	**37.** (C)	**38.** (A)	**39.** (B)	**40.** (C)
41. (B)	**42.** (B)	**43.** (A)	**44.** (A)	**45.** (C)	**46.** (C)	**47.** (D)	**48.** (C)	**49.** (C)	**50.** (B)
51. (D)	**52.** (B)	**53.** (B)	**54.** (B)	**55.** (B)	**56.** (B)	**57.** (C)	**58.** (B)	**59.** (B)	**60.** (A)
61. (B)	**62.** (B)	**63.** (B)	**64.** (B)	**65.** (B)	**66.** (A)				

Motion in Two Dimensions

67. (A)	**68.** (B)	**69.** (D)	**70.** (C)	**71.** (D)	**72.** (C)	**73.** (D)	**74.** (C)	**75.** (A)	**76.** (A)
77. (D)	**78.** (D)	**79.** (B)	**80.** (B)	**81.** (B)	**82.** (B)	**83.** (C)	**84.** (B)	**85.** (A)	**86.** (B)
87. (A)	**88.** (B)	**89.** (D)	**90.** (B)	**91.** (D)	**92.** (D)	**93.** (D)	**94.** (C)	**95.** (A)	**96.** (D)
97. (A)	**98.** (A)	**99.** (A)	**100.** (B)	**101.** (A)	**102.** (A)	**103.** (C)	**104.** (B)	**105.** (D)	**106.** (B)
107. (D)	**108.** (B)	**109.** (D)	**110.** (B)	**111.** (B)	**112.** (C)	**113.** (B)	**114.** (D)	**115.** (B)	**116.** (C)
117. (C)	**118.** (A)	**119.** (B)	**120.** (B)	**121.** (C)	**122.** (C)	**123.** (B)	**124.** (A)	**125.** (D)	**126.** (C)
127. (B)	**128.** (A)	**129.** (B)	**130.** (C)	**131.** (B)	**132.** (C)	**133.** (C)	**134.** (D)	**135.** (B)	**136.** (B)
137. (C)	**138.** (A)	**139.** (B)	**140.** (C)	**141.** (A)	**142.** (B)	**143.** (A)	**144.** (A)	**145.** (A)	**146.** (A)

More than One Option Correct Type

147. (A), (B) and (C) **148.** (A), (B), (C) and (D) **149.** (A), (B), (C) and (D) **150.** (A) and (C)
151. (B), (C) and (D) **152.** (B) and (C)

Passage Based Questions

Passage 1

153. (C) **154.** (B) **155.** (D)

Passage 2

156. (A) **157.** (B) **158.** (A)

Passage 3

159. (A) **160.** (C) **161.** (C)

Passage 4

162. (A) **163.** (B) **164.** (A)

Passage 5

165. (C) **166.** (A) **167.** (B)

Match the Column Type

168. (A) $\rightarrow$ 3; (B) $\rightarrow$ 2; (C) $\rightarrow$ 1; (D) $\rightarrow$ 4
169. (A) $\rightarrow$ 2; (B) $\rightarrow$ 3; (C) $\rightarrow$ 4; (D) $\rightarrow$ 1
170. (A) $\rightarrow$ 1; (B) $\rightarrow$ 2; (C) $\rightarrow$ 4; (D) $\rightarrow$ 3
171. (A) $\rightarrow$ 2; (B) $\rightarrow$ 3, (C) $\rightarrow$ 4; (D) $\rightarrow$ 1
172. (A) $\rightarrow$ 3; (B) $\rightarrow$ 1; (C) $\rightarrow$ 1; (D) $\rightarrow$ 2

Assertion-Reason Type

173. (D) **174.** (B) **175.** (B) **176.** (C) **177.** (D) **178.** (C) **179.** (B) **180.** (B) **181.** (C) **182.** (C)
183. (B) **184.** (B)

Integer Type

185. 7 **186.** $y = x\tan\theta\left[1-\frac{x}{R}\right]$ **187.** 4 s **188.** 6 meter **189.** $x = 3$

190. 3 km/hr **191.** 8 m/s **192.** 2 **193.** 3 **194.** 2

Previous Years' Questions

195. (C) **196.** (B) **197.** (C) **198.** (D) **199.** (B) **200.** (A) **201.** (B) **202.** (C) **203.** (C) **204.** (D)
205. (C) **206.** (D) **207.** (C) **208.** (A) **209.** (D) **210.** (C) **211.** (B) **212.** (A) **213.** (A) **214.** (D)
215. (D) **216.** (C) **217.** (C) **218.** (C) **219.** (D) **220.** (A) **221.** (C) **222.** (B)

HINTS AND SOLUTIONS

Single Option Correct Type

Motion in One Dimension

1. $v_{\text{avg}} = \dfrac{\frac{1}{2}\times\frac{t}{2}\times v+\frac{t}{2}\times v}{t} = \dfrac{3v}{4}$

The correct option is (B)

2. Change in velocity $\Delta v = 8-(-8) = 16$ m/s

Time taken $\Delta t = \dfrac{\pi r}{v} = \dfrac{\pi\times 6}{8} = \dfrac{3\pi}{4}$

$\therefore$ Average acceleration $= \dfrac{\Delta v}{\Delta t} = \dfrac{16\times 4}{3\pi} = \dfrac{64}{3\pi}$

The correct option is (C)

3. $s = \dfrac{1}{2}at^2$

$v_{\text{avg}} = \dfrac{s}{t} = \dfrac{1}{2}at = \dfrac{1}{2}v$

The correct option is (D)

Motion Under Uniform Acceleration in Straight Line

4. For constant acceleration and equal time interval ratio of distance is 1 : 3.

$\therefore \quad y = 3x$

The correct option is (C)

5. $u = 7$ m/s and $a = 4$ m/s^2

Distance traveled in nth second $= u+\dfrac{a}{2}(2n-1)$

$\therefore$ Distance traveled in 5th second $= 7+\dfrac{4}{2}[2(5)-1] = 25$ m

The correct option is (A)

6. $S_{n\text{th}} = u+\dfrac{1}{2}a(2n-1)$, $S_{4\text{th}} = 12+\dfrac{1}{2}(-3)(2\times 4-1)$,

$S_{4\text{th}} = 1.5$ m

The correct option is (A)

7. Velocity = 0 at $t = 5.5$ s. $S_{6\text{th}}$

= 2 (distance travelled in (5.5)s – distance travelled in 5s) = 0.5 m

The correct option is (B)

8. $8 = 0+a\left(2-\dfrac{1}{2}\right)$ (1)

$S_5 = 0+a\left(5-\dfrac{1}{2}\right)$ (2)

Dividing equation (2) by (1)

We get, $S_5 = 24$ m.

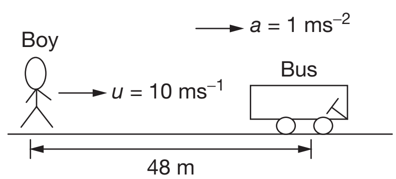
The correct option is (B)

9. Initial velocity of boy with respect to bus = 10 ms^{-1}

Acceleration of boy with respect to bus = –1 ms^{-2}

$s = ut+\dfrac{1}{2}at^2$

$48 = 10t-\dfrac{1}{2}t^2$



$t^2 - 20t + 96 = 0$ s

$t^2 - 12t - 8t + 96 = 0$

$(t-8)(t-12) = 0$

$t = 8\text{s}$ and 12s

The correct option is (A)

10. The stopping distance $S \propto u^2$

The correct option is (D)

11. $S_r = u_r t + \frac{1}{2}a_r t^2$; $0 = ut - \frac{1}{2}(g+a)t^2 \Rightarrow a = \frac{2u - gt}{t}$

The correct option is (C)

12. The correct option is (D)

13. $16 = 8t - \frac{1}{2} \times 2t^2$ (equation relative to bus)

$t = 4\text{s}$

The correct option is (C)

14. $s = ut + \frac{1}{2}at^2 \Rightarrow a = -2 \text{ ms}^{-2}$

Particle comes to rest after 5 seconds

∴ Distance travelled in 6 seconds = magnitude of displacement in 5 seconds + magnitude of displacement in 6th second = 25 + 1 = 26 m

The correct option is (C)

15. The maximum distance covered by the vehicle before coming to rest $= \frac{v^2}{2a} = \frac{(15)^2}{2 \times 0.3} = 375$ m.

The corresponding time = $t = \frac{v}{a} = \frac{15}{0.3} = 50$ s.

Therefore after 50 s, the distance covered by the vehicle = 375 m, from the instant of beginning of braking.

The distance of the vehicle from the traffic signal after one minute = (400 – 375) m = 25 m.

The correct option is (A)

16. $S_1 = \frac{1}{2}a(p-1)^2, S_2 = \frac{1}{2}ap^2$;

$S = \frac{1}{2}a[2(p^2 - p + 1) - 1] = S_1 + S_2$

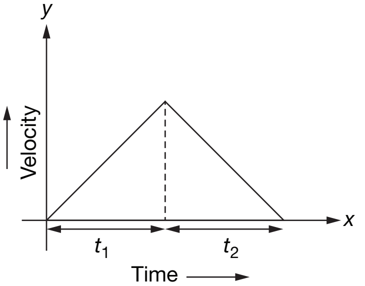
The correct option is (A)

17. Let the velocity of the scooter be v ms^{-1}. Then

$(v-10)100 = 1000$ or $v = 20 \text{ ms}^{-1}$

The correct option is (D)

18.



$a_1 t_1 = a_2 t_2$ (1)

$\frac{1}{2}(t_1 + t_2)a_1 t_1 = 4$ (2)

$t_1 + t_2 = 4$ (3)

$\frac{1}{a_1} + \frac{1}{a_2} = 2$

The correct option is (B)

Motion Under Gravity

19. Let H be the height of tower

∴ $H = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 9 = 44.1\text{m}$

The correct option is (C)

20. $H_{\max} \propto u^2$

∴ $u \propto \sqrt{H_{\max}}$

That is, to triple the maximum height, ball should be thrown with velocity $\sqrt{3}u$.

The correct option is (A)

21. $t_1 > t_2$ because when lift is moving the acceleration coin is more than g hence will take less time.

The correct option is (A)

22. Let height of balloon = h

$h = -ut + \frac{1}{2}gt^2 = -4(4) + \frac{1}{2}(9.8)(16) = 62.4$ m

The correct option is (A)

23. For the dropped body, $h_1 = \frac{1}{2}gt^2$;

For the thrown body, $h_2 = 1 \times t \times \frac{1}{2}gt^2 = t + \frac{1}{2}gt^2$;

$h_2 - h_1 = t$; So, $t = 1.8$ s

The correct option is (C)

24. $\frac{v_T + v_B}{2} = \frac{3}{0.5} = 6$ or $v_T + v_B = 12 \text{ ms}^{-1}$

The correct option is (A)

25. Time to reach at maximum height = $\frac{u}{g}$ = 10 s

Hence, distance = $\frac{u^2}{2g} + \frac{1}{2}g(5)^2$ = 500 + 125 = 625 m

The correct option is (B)

26. $h = ut + \frac{1}{2}gt^2$

Considering downward as positive $u = -10$ m/s, $g = 10$ m/s^2, $t = 5$ s

$h = -10 \times 5 + \frac{1}{2} \times 10 \times 25$, $h = 75$ m

The correct option is (C)

27. $h = ut + \frac{1}{2}gt^2$, $h = ut - 5t^2 \Rightarrow 5t^2 - ut + h = O$

∴ $t_1 + t_2 = \frac{u}{5}$, $u = 5(t_1 + t_2) = 70 \text{ ms}^{-1}$

The correct option is (D)

28. Let the full time of flight be t

$$\frac{H}{2}=\frac{1}{2}g(t-1)^2$$

$$H=\frac{1}{2}gt^2$$

Solving $t=2\pm\sqrt{2}$ because $2-\sqrt{2}<1$

Hence $t=2+\sqrt{2}$ s

The correct option is (D)

29. Let the ball remain in air for n seconds.

Then, $S_n=u+\frac{g}{2}(2n-1)=0+\frac{10}{2}(2n-1)$

$S_n=10n-5$ (1)

The distance covered in first three seconds is also S_n.

Here $S_n=\frac{1}{2}gt^2=\frac{1}{2}(10)(3)^2=45$ (2)

From (1) and (2) $n=5$

$\therefore \quad h=\frac{1}{2}(10)(5)^2=125$ m

The correct option is (A)

30. $h=ut+\frac{1}{2}gt^2 \Rightarrow 25=ut-5t^2$

$5t^2-ut+25=0 \Rightarrow t_1+t_2=\frac{u}{5};\ t_1t_2=5$

$(t_1-t_2)^2=(t_1+t_2)^2-4t_1t_2$

$16=\frac{u^2}{25}-20 \Rightarrow \frac{u}{5}=6 \Rightarrow u=30$ m/s

The correct option is (C)

31. The velocity of balloon at height h, $v=\sqrt{2\left(\frac{g}{8}\right)h}=\sqrt{\frac{gh}{4}}$

When the stone is released from this balloon, it will go upward with velocity $v=\sqrt{\frac{gh}{4}}$

(Same as that of the balloon).

$$h=-\sqrt{\frac{gh}{4}}t+\frac{1}{2}gt^2$$

$$gt^2-\sqrt{gh}\,t-2h=0$$

$$\therefore \quad t=2\sqrt{\frac{h}{g}}$$

The correct option is (B)

32. Height of first body after time t, $h_1=v_0t-\frac{1}{2}gt^2$

Height of second body after time $(t-\tau)$,

$$h_2=v_0(t-\tau)-\frac{1}{2}g(t-\tau)^2$$

If they meet after time t, $h_1=h_2 \quad\Rightarrow\quad \tau=\frac{v_0}{g}+\frac{\tau}{2}$

The correct option is (C)

33. For first projectile, $h_1=ut-\frac{1}{2}gt^2$

For second projectile, $h_2=u(t-T)-\frac{1}{2}g(t-T)^2$

When both meet, i.e., $h_1=h_2$

$$ut-\frac{1}{2}gt^2=u(t-T)-\frac{1}{2}g(t-T)^2$$

$$\Rightarrow\quad uT+\frac{1}{2}gT^2=gtT$$

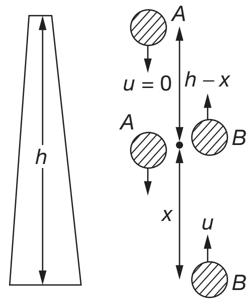
$$\Rightarrow\quad t=\frac{u}{g}+\frac{T}{2}$$

The correct option is (B)

34. If u is the initial speed of the second stone, then

$0=u^2-2g(4h)$

or $u=\sqrt{8gh}$



If they meet at the height x from ground,

For A, $h-x=\frac{1}{2}gt^2$

For B, $x=(\sqrt{8gh})\,t-\frac{1}{2}gt^2$

$\therefore \quad h=\sqrt{8gh}\,t$

or $\quad t=\sqrt{\frac{h}{8g}}$

The correct option is (B)

35. The displacement between first stone and aeroplane after t second $(h_1)=\frac{1}{2}(g+f)t^2$

After time t,

Velocity of aeroplane $=u+ft$

Velocity of first stone $=u-gt$

Where u is velocity of aeroplane when first stone is dropped.

The relative speed of second stone with respect to first stone $=(u+ft)-(u-gt)$

$=(g+f)t$

The relative displacement between first and second stone after time $t'(h_2)$

$=(g+f)tt'$

$$h_1+h_2=\frac{1}{2}(g+f)t^2+(g+f)tt'=\frac{1}{2}(g+f)(t+2t')t$$

The correct option is (B)

Graph

36. $v^2=u^2+2as$, If $u=0$ then $v^2\propto s$

i.e., graph should be parabola symmetric to displacement axis.

The correct option is (C)

37. The correct option is (C)

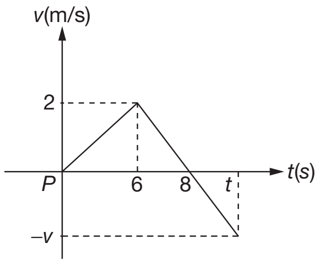
38. This is the situation similar to elastic collision of ball impinging on floor and bouncing back.

The correct option is (A)

39. Distance travelled = Area under the given graph $=\frac{1}{2}\times 10\times 4=20$ m

The correct option is (B)

40.



If after t second particle will reach at P again,

$\therefore$ Area of v–t curve = 0

$$\frac{1}{2}\times 2\times 8-\frac{1}{2}\times(t-8)\times(t-8)\times 1=0$$

$(t-8)^2=16$

$t-8=4$

$t=12$ s

The correct option is (C)

41. Acceleration is negative and constant for first half. It is positive and constant over next half.

The correct option is (B)

42. v_{avg} = slope of line $PQ=\dfrac{6-4}{5-2}=\dfrac{2}{3}$ m/s

The correct option is (B)

43. The correct option is (A)

44. The graph will be parabolic and in downward motion, velocity will be negative and upward motion, velocity will be positive

The correct option is (A)

45. Average acceleration (a) $=\dfrac{\text{Change in velocity}}{\text{Time taken}}$

$\therefore$ Change in velocity = Area of acceleration – time graph

$\therefore$ Average acceleration $=\dfrac{\text{Area OABE}}{20\text{ s}}=\dfrac{600}{20}=30\text{ m/s}^2$

The correct option is (C)

46. Acceleration is positive throughout the motion.

The correct option is (C)

47. At $x=6$ m, $v=12.5$ m/s and $\dfrac{dv}{dx}=-\dfrac{5}{4}$

$$a(x=6\text{ m})=v\frac{dv}{dx}=12.5\times\frac{-5}{4}=-\frac{125}{8}\text{ m/s}^2$$

The correct option is (D)

Variable Acceleration

48. $x^2=1+t^2$

$$2x\frac{dx}{dt}=2t\ \Rightarrow\ \frac{dx}{dt}=\frac{t}{x}\ \therefore\ \frac{d^2x}{dt^2}=\frac{x-t\,dx/dt}{x^2}=\frac{1}{x}-\frac{t^2}{x^3}$$

The correct option is (C)

49. $\vec{r}=4\sin 2\pi t\,\hat{i}+4\cos 2\pi t\,\hat{j}$

$x=4\sin 2\pi t$

$y=4\cos 2\pi t$

$x^2+y^2=16\sin^2 2\pi+16\cos^2 2\pi t$

$x^2+y^2=16$

The correct option is (C)

50. $a=\dfrac{dv}{dt}=2\sqrt{v}$ or $\displaystyle\int\frac{dv}{v^{1/2}}=\int 2dt\Rightarrow 2v^{1/2}=2t+c$

As $v=16\text{ ms}^{-1}$ when $t=2s\Rightarrow c=4$

$\therefore\ v=(t+2)^2$

At $t=3$ s, $v=25\text{ ms}^{-1}$

The correct option is (B)

51. $\dfrac{ds}{dt}=4\sqrt{1+s}$

$$\Rightarrow\ \int_0^s\frac{ds}{\sqrt{1+s}}=\int_0^t 4dt\ \Rightarrow\ 2\sqrt{1+s}=4t$$

$\Rightarrow s=4t^2-1$

$\Rightarrow\ v=8t$ at $t=0, v=0$

The correct option is (D)

52. $a=2v^{3/2}$

$\Rightarrow\ v\dfrac{dv}{dx}=2v^{3/2}$

$$\Rightarrow\ \int_4^v\frac{dv}{2v^{1/2}}=\int_0^x dx$$

$\Rightarrow\ \left[\sqrt{v}\right]_4^v=[x]_0^x$

$\Rightarrow\ \sqrt{v}-2=x$

$\therefore\ \sqrt{v}=x+2$

The correct option is (B)

53. $x=4(t-2)+a(t-2)^2$

At $t=0$, $x=-8+4a=4a-8$, $v=\dfrac{dx}{dt}=4+2a(t-2)$

At $t=0$, $v=4-4a=4(1-a)$

But acceleration, $a=\dfrac{d^2x}{dt^2}=2a$

The correct option is (B)

54. $v_1=4t^3-2t$

$\Rightarrow\ \dfrac{dx_t}{dt}=4t^3-2t$

$\Rightarrow\ \displaystyle\int dx_1=\int 4t^3dt-\int 2t\,dt\ \Rightarrow\ x_t=t^4-t^2$

Since, $x_t=2$ m

$\therefore\ t^4-t^2-2=0\ \Rightarrow\ t=\sqrt{2}\,\text{s}$

$a_t=\dfrac{dv_t}{dt}=12t^2-2=22\text{ ms}^{-2}$

The correct option is (B)

55. $a=6t\ \Rightarrow s=t^3$

The correct option is (B)

56. $s=t^3-6t^2+3t+4$, $v=\dfrac{ds}{dt}=3t^2-12t+3$,

$a=\dfrac{dv}{dt}=6t-12$; a is zero at $t=2$

$v(t=2) = 3\times4 - 12\times2 + 3 = -9$ m/s

The correct option is (B)

57. $a = \dfrac{dv}{dt} = 0.2t$

The correct option is (C)

58. $v = \dfrac{dx}{dt} = 3at^2 + b$ (1)

$v(t = 1\,\text{s}) = 0.3$

$\Rightarrow \quad 3a + b = 0.3$ (1)

$v(t = 4\,\text{s}) = 27.3$

$\Rightarrow \quad 48a + b = 27.3$ (2)

From (1) and (2),

$a = 0.6,\ b = -1.5$

The correct option is (B)

59. $\dfrac{d^2x}{dt^2} = 8 \quad \dfrac{d^2y}{dt^2} = 0$

$\therefore \quad a = 8$

The correct option is (B)

60. $\vec{v} = \dfrac{d\vec{r}}{dt} = 3\hat{i} + 2\sqrt{3}t\,\hat{j}$

$\vec{v}(t = \sqrt{3}\,\text{s}) = 3\hat{i} + 6\hat{j}$

$|\vec{v}| = \sqrt{3^2 + 6^2} = 3\sqrt{5}$ m/s

$\theta = \tan^{-1}\left(\dfrac{6}{3}\right) = \tan^{-1}(2)$

The correct option is (A)

61. $v = \alpha\sqrt{x} \Rightarrow \dfrac{dx}{dt} = \alpha x^{1/2},\ x^{-1/2}dx = \alpha\, dt,$

$\int_0^x x^{-1/2}dx = \alpha\int_0^t dt,\ 2\sqrt{x} = \alpha t,\ x \propto t^2$

The correct option is (B)

62. The velocity vector is given by

$v = \dfrac{dr}{dt} = (-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}$

$\vec{v}\cdot\vec{r} = 0$

The correct option is (B)

63. $a = -s,\ v\dfrac{dv}{ds} = -s,\ \int_{v_0}^{0} v\,dv = -\int_0^s s\,ds,\ \dfrac{v_0^2}{2} = \dfrac{s^2}{2} \Rightarrow s = v_0$

The correct option is (C)

64. $\dfrac{dx}{dt} = 2t - 2 = 0$

$\Rightarrow \quad t = 1$, So, $x_{t=0} = 1$ m, $x_{t=1s} = 0$, $x_{t=2} = 1$ m

Total distance = 2m

The correct option is (C) (B)

65. $y = ax^2,\ \dfrac{dy}{dt} = a(2x)\dfrac{dx}{dt} = 2acx,\ \dfrac{d^2y}{dt^2} = 2ac\dfrac{dx}{dt} = 2ac^2$

$a_y = 2ac^2,\ a_x = 0,\ \vec{a} = a_x\hat{i} + a_y\hat{j}$

$\therefore \quad \vec{a} = 2ac^2\hat{j}$

The correct option is (B)

66. $a = v\dfrac{dv}{dx} = \dfrac{25}{(x+2)^3},\ \dfrac{v^2}{2} = 25\times\left[-\dfrac{1}{2(x+2)^2}\right]_o^x,$

$v^2 = 25\left[\dfrac{1}{4} - \dfrac{1}{(x+2)^2}\right]$

$v = \sqrt{25\left[\dfrac{1}{4} - \dfrac{1}{(x+2)^2}\right]},\ v_{max} = \dfrac{5}{2} = 2.5$ m/s (at $x = \infty$)

The correct option is (A)

Motion in Two Dimensions

Projectile

67. No acceleration in horizontal direction.

So, horizontal speed remains same.

The correct option is (A)

68. $T_1 = \dfrac{2u\sin\theta}{g},\ T_2 = \dfrac{2u\cos\theta}{g}$ and $R = \dfrac{u^2\sin2\theta}{g}$

$\Rightarrow \quad R = \dfrac{T_1T_2g}{2}$

The correct option is (B)

69. $R = \dfrac{V^2\sin2\theta}{g} \Rightarrow \theta = \dfrac{1}{2}\sin^{-1}\left(\dfrac{gR}{V^2}\right)$

The correct option is (D)

70. Since $u_y = 0$, for both particle time will be same.

The correct option is (C)

71. Half of speed at highest point $= \dfrac{u\cos\alpha}{2}$

$\therefore \quad \dfrac{u\cos\alpha}{2} = u\sin\alpha - gt$

$t = \dfrac{u}{2g}(2\sin\alpha - \cos\alpha)$

The correct option is (D)

72. As horizontal component of velocity is same at all the time

$u\cos30° = 20\cos60°$

$u\dfrac{\sqrt{3}}{2} = 20\times\dfrac{1}{2} = 10$

$u = \dfrac{20}{\sqrt{3}}$ ms^{-1}

The correct option is (C)

73. $H = \dfrac{u^2\sin^2\theta}{2g}$ and $R = \dfrac{u^2\sin2\theta}{g},\ \dfrac{45}{180} = \dfrac{1}{4}\tan\theta$

$\Rightarrow \quad \theta = 45°$

The correct option is (D)

74. $T_1 = \dfrac{2v\sin\theta}{g},\ T_2 = \dfrac{2v\sin(90-\theta)}{g}$ or $T_2 = \dfrac{2v\cos\theta}{g}$.

Dividing, $\dfrac{T_1}{T_2} = \dfrac{\sin\theta}{\cos\theta}$.

The correct option is (C)

75. At the highest point, velocity is horizontal.

The correct option is (A)

76. $R = 2H$

$\Rightarrow \quad \cot\theta = \frac{1}{2}$; $\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$

$\therefore$ Range of projectile $R = \frac{2v^2\sin\theta\cos\theta}{g} = \frac{4v^2}{5g}$

The correct option is (A)

77. $v_H = u\cos\theta = 6$, $v_v = \sqrt{v^2 - u^2\cos^2\theta} = 8$

$t_1 = \frac{u\sin\theta - 8}{10}$, $t_2 = \frac{u\sin\theta + 8}{10}$, $t_2 - t_1 = \frac{8\times 2}{10} = 1.6$ s

The correct option is (D)

78. $T = \frac{2u_y}{g}$, $H = \frac{u_y^{\,2}}{2g}$

$\therefore \quad H = \frac{gT^2}{8} = \frac{9.8\times(6)^2}{8} = 44.1$ m

The correct option is (D)

79. $g\left(\frac{2u\sin\alpha}{g}\right)^2 = 2\times\frac{u^2\sin 2\alpha}{g}$

$\Rightarrow \quad \tan\alpha = 1$ or $\alpha = 45°$

80. $\frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{u\cos(\alpha-\beta)}{g\sin\beta}$

$\therefore \quad \tan(\alpha-\beta) = \frac{1}{2}\cot\beta$

The correct option is (B)

81. $u_{1/2} = 2u\cos\theta/2$

$\therefore \quad \tan\frac{\theta}{2} = \frac{10}{10} = 1$, $\theta = 90°$

$2u\cos\frac{\theta}{2}\times 2 = 10\sqrt{2}$

$\therefore \quad u = 5\ \text{ms}^{-1}$

The correct option is (B)

82. $u_x = 4\cos 30° = 2\sqrt{3}$ m/s and $u_y = 4\sin 30° = 2$ m/s

$T = \frac{2u_y}{12} = \frac{u_y}{6} = \frac{2}{6} = \frac{1}{3}s$

The correct option is (B)

83. $t_1 = \frac{2u\sin\theta}{g}, t_2 = \frac{2u\cos\theta}{g}$

$\therefore \quad t_1t_2 = \frac{2u^2\sin 2\theta}{g\times g} = \frac{2}{g}R$

$\therefore \quad t_1t_2 \propto R$

The correct option is (C)

84. We known the equation of trajectory is

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Comparing the equation we get, $\tan\theta = \sqrt{3}$

$\therefore \quad \theta = \frac{\pi}{3}$

The correct option is (B)

85. $\vec{r} = bt\hat{i} + ct^2\hat{j}$, $\vec{v} = b\hat{i} + 2ct\,\hat{j}$, $\tan 45° = \frac{2ct}{b}$, $t = \frac{b}{2c}$

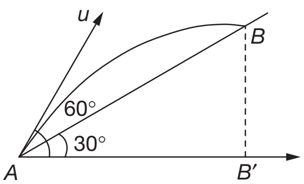
The correct option is (A)

86. $T = \frac{2u_y}{g}$, $\quad H = \frac{u_y^{\,2}}{2g}$

$\therefore \quad H = \frac{gT^2}{8} = \frac{9.8\times(4)^2}{8} = 19.6$ m

The correct option is (B)

87.



$AB' = u\cos 60°\times t = \frac{ut}{2}$

From $\Delta\, ABB'$ $\cos 30° = \frac{AB'}{AB}$

or $AB = \frac{2AB'}{\sqrt{3}} = \frac{2\times ut}{2\times\sqrt{3}} = \frac{ut}{\sqrt{3}}$

The correct option is (A)

88. For angle of projection θ, $H_1 = \frac{u^2\sin^2\theta}{2g}$

For angle of projection $90° - \theta$, $H_2 = \frac{u^2\cos^2\theta}{2g}$

$H_1 + H_2 = \frac{u^2}{2g}$

$u = \sqrt{2g(H_1+H_2)} = 40$ m/s

The correct option is (B)

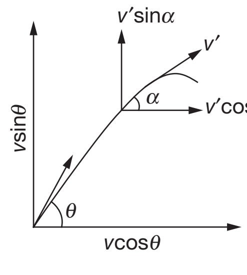
89. According to figure,

$v'\cos\alpha = v\cos\theta$

$v'\sin\alpha = v\sin\theta - gt$

$\therefore \quad \frac{v'\sin\alpha}{v'\cos\alpha} = \frac{v\sin\theta - gt}{v\cos\theta}$

$\therefore \quad \alpha = \tan^{-1}\left(\frac{v\sin\theta - gt}{v\cos\theta}\right)$



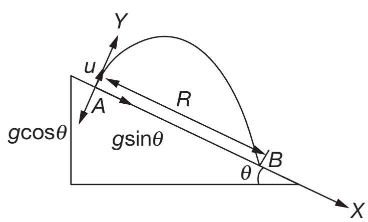
The correct option is (D)

90. Let projectile strikes the plane at B, and its time of flight is T,

$y = u_yt + \frac{1}{2}a_yt^2$

$0 = uT - \frac{g\cos\theta T^2}{2}$

$T = \frac{2u}{g\cos\theta}$



$$R = AB = u_x T + \frac{1}{2}a_x T^2$$

$$R = \frac{1}{2}a_x T^2$$

$$R = \frac{1}{2}g\sin\theta \frac{4u^2}{g^2\cos^2\theta}$$

$$R = \frac{2u^2}{g}\tan\theta\sec\theta$$

The correct option is (B)

91. At maximum height speed becomes half of initial speed,

So, height $= H = \frac{u^2\sin^2\alpha}{2g} = \frac{(40)^2\cdot\sin^2 60^\circ}{2\times 10}$

$$= \frac{1600\times 3/4}{20} = 60 \text{ m}$$

The correct option is (D)

92. At maximum height $v_1 = v\cos\theta$

At half of maximum height $v_2 = \sqrt{v^2\cos^2\theta + (v_y)^2}$

For v_{vertical} $v_y^2 = v^2\sin^2\theta - 2g\frac{H}{2} = v^2\sin^2\theta - g\frac{v^2\sin^2\theta}{2g}$

$$= \frac{v^2\sin^2\theta}{2}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{2}{5}} \Rightarrow \tan\theta = \sqrt{3}\,,\ \theta = 60^\circ$$

The correct option is (D)

93. For projectile A,

$$R = \frac{v^2\sin 2\theta}{g} \quad (1)$$

For projectile B, $R = \frac{v^2\sin(2\times 15)}{4g}$

$$R = \frac{v^2}{8g} \quad (2)$$

From (1) and (2), $\frac{v^2}{8g} = \frac{v^2\sin 2\theta}{g}$

$$\therefore \quad \theta = \frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$$

The correct option is (D)

94. $\tan\alpha = \dfrac{\frac{u^2\sin^2 45^\circ}{2g}}{\frac{u^2\sin 90^\circ}{2g}}$ (if particle hits the inclined plane horizontally)

$$\tan\alpha = \frac{1}{2}$$

The correct option is (C)

95. $x_{\text{rel}} = u_{x_{\text{rel}}}t = v_0\sqrt{\frac{2H}{g}}$

The correct option is (A)

96. $\tan 30^\circ = \frac{u\sin\theta - g(2)}{u\cos\theta}$ $\frac{u\sin\theta}{g} = 3$ and

$\tan 30^\circ = \dfrac{u\sin\theta - \frac{2u\sin\theta}{3}}{u\cos\theta}$, $\frac{1}{\sqrt{3}} = \frac{1}{3}\tan\theta$

$\Rightarrow \theta = 60^\circ$

The correct option is (D)

97. $\tan\frac{\alpha}{2} = \frac{v_y}{v_x}$, $\tan\frac{\alpha}{2} = \frac{u_y - gt}{4x} = \frac{10\sin\alpha - 10t}{10\cos\alpha}$

$t = \sin\alpha - \cos\alpha\tan\frac{\alpha}{2}$, $t = \sin\frac{\alpha}{2}\left[2\cos\frac{\alpha}{2} - \dfrac{\left(2\cos^2\frac{\alpha}{2} - 1\right)}{\cos\frac{\alpha}{2}}\right]$,

$t = \tan\frac{\alpha}{2}$

The correct option is (A)

98. If h be the maximum height attained by the projectile, then

$h = \frac{u^2\sin^2\theta}{2g}$ and $R = \frac{u^2\sin 2\theta}{g}$,

$\frac{R}{h} = \frac{2\sin\theta\cos\theta}{(\sin^2\theta)/2} = 4\cot\theta$

Therefore, $\frac{\Delta R}{R} = \frac{\Delta h}{h}$ (if θ is constant)

$\therefore$ Percentage increase in R = percentage increase in $h = 5\%$

The correct option is (A)

99. $T_1 = \frac{2v\sin\theta}{g}$ and $T_2 = \frac{2v\cos\theta}{g}$, $T_1T_2 = \frac{2(v^2\sin 2\theta)}{g\times g}$

or $T_1T_2 \propto r$

The correct option is (A)

100. $v_{s/g} = 15$ m/sec

$v(t = 2) = 15 - 10\times 2 = -5$ m/s

The correct option is (B)

101. $u_x = 40$ m/s, $u_y = 40$ m/s

At $t = 2$ s.

$v_x = 40$ m/s and $v_y = 40 - 10\times 2 = 20$ m/s

$x = v_x t = 80$ m

$y = u_y t - \frac{1}{2}gt^2 = 60$ m

$\therefore \quad s = \sqrt{x^2 + y^2} = 100$ m

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

The correct option is (A)

102. Compare the given equation $y = x - \frac{1}{2}x^2$ with

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$,

$\Rightarrow \theta = 45^\circ$, $u = \sqrt{2g}$, $T = \frac{2u\sin\theta}{g} = \frac{2}{\sqrt{g}}$

The correct option is (A)

103. Total time of flight $= \dfrac{2u\sin\theta}{g} = \dfrac{2\times 50\times 1}{2\times 10} = 5$ s

Time to cross the wall = 3 s (given)

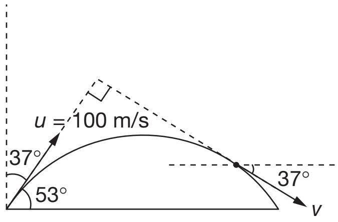
Time in air after crossing the wall = $(5-3) = 2$ s

$\therefore$ Distance traveled beyond the wall

$= (u\cos\theta)t = 50\times\dfrac{\sqrt{3}}{2}\times 2 = 86.6$ m

The correct option is (C)

104



$u\cos 53° = v\cos 37°$

$\Rightarrow\ 100\times\dfrac{3}{5} = v\times\dfrac{4}{5} \Rightarrow v = 75$ m/s

$v_y = -v\sin 37° = -45$ m/s

$u_y = u\sin 53° = 80$ m/s

$v_y = u_y + gt \Rightarrow -45 = 80 - 10t$

$t = 12.5$ s

The correct option is (B)

105. $v_H = u\cos\theta = 6$

$v_v = \sqrt{v^2 - u^2\cos^2\theta} = 8$

$t_1 = \dfrac{u\sin\theta - 8}{10}$

$t_2 = \dfrac{u\sin\theta + 8}{10}$

$t_2 - t_1 = \dfrac{8\times 2}{10} = 1.6$ s

The correct option is (D)

106. $\dfrac{1}{2}gt^2 = H$ (1)

$gt = v_y$ (2)

$v_x = v_y$

Range $= u_x t = v_y t = gt^2 = 2H$

The correct option is (B)

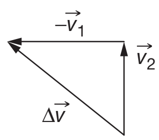
Relative Motion

107. $V_{Pt} = V_P - V_t = 5-(-10) = 15$ m/s

$\therefore\ t = \dfrac{150}{15} = 10s$

The correct option is (D)

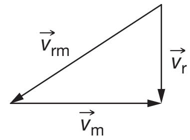
108. $|\Delta\vec{v}| = 5$ m/s



$\vec{a} = 1$ m/s^2 (towards north-west)

The correct option is (B)

109. $v_{rm} = \sqrt{v_r^2 + v_m^2} = 5$ km/hr



The correct option is (D)

110. The correct option is (B)

111. Velocity of rain with respect to car $\vec{v}_{RC} = \vec{v}_R - \vec{v}_C$ should be perpendicular to the wind screen.

From Fig. 2.25,

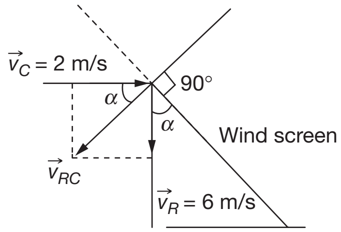


Fig. 2.25

$\tan\alpha = \dfrac{v_r}{v_c} = \dfrac{6}{2}$

$\alpha = \tan^{-1}(3)$

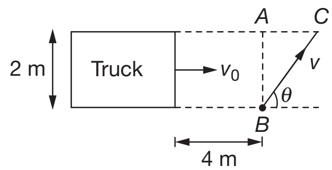
The correct option is (B)

112. For safe crossing, the condition is that the man must cross the road by the time the truck covers the distance $4 + AC$ or $4 + 2\cot\theta$

$\therefore\ \dfrac{4+2\cot\theta}{8} = \dfrac{2/\sin\theta}{v}$

or $v = \dfrac{8}{2\sin\theta + \cos\theta}$ (1)

For minimum v, $\dfrac{dv}{d\theta} = 0$



$\Rightarrow\ \tan\theta = 2$

From equation (1),

$v_{\min} = \dfrac{8}{\sqrt{5}} = 3.57$ m/s

The correct option is (C)

113. For train B,

$-\dfrac{dv}{dt} = 0.3t\ ,\ -\displaystyle\int_{15}^{0} dv = 0.3\int_0^t t\,dt$

$\Rightarrow\ t = 10$ s

In this 10 s, the train B travels a distance of 100 m.

$\therefore$ Train A can travel a distance of 125 m before coming to rest.

$v^2 = u^2 + 2as\ ,\ a = -2.5$ m/s^2

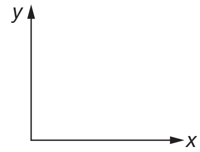
The correct option is (B)

114. The correct option is (D)

115. $t = \dfrac{d}{\sqrt{u_m^2 - u_r^2}} = \dfrac{\frac{1}{2}}{\sqrt{4^2-3^2}} = \dfrac{1}{2\sqrt{7}}$ hr

The correct option is (B)

116. $\vec{v}_w = \frac{v}{\sqrt{2}}\hat{i} + \frac{v}{\sqrt{2}}\hat{j}$



$\vec{v}_m = (a\,t)\,\hat{j}$

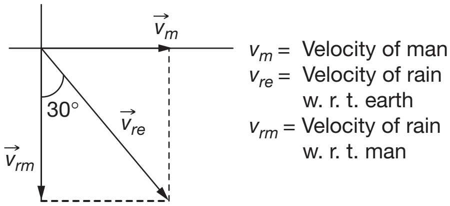
$\vec{v}_{wm} = \frac{v}{\sqrt{2}}\hat{i} + \left(\frac{v}{\sqrt{2}} - at\right)\hat{j}$

It appears due east when, $\frac{v}{\sqrt{2}} - at = 0$

$\therefore \quad t = \frac{v}{\sqrt{2}a}$

The correct option is (C)

117.



Velocity of man $|\vec{v}_m| = 10 \text{ ms}^{-1}$

Using $\sin 30° = \frac{v_m}{v_{re}}$ or $v_{re} = \frac{v_m}{\sin 30} = \frac{10}{1/2}$

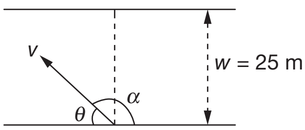
$= 20 \text{ ms}^{-1}$

Again $\cos 30° = \frac{v_{rm}}{v_{re}}$ or $v_{rm} = v_{re}\cos 30$

$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ ms}^{-1}$

The correct option is (C)

118. $t = \frac{w}{v\sin\theta} \Rightarrow 10 = \frac{25}{5\sin\theta}$



$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

$\therefore \quad \alpha = 180° - \theta = 150°$

The correct option is (A)

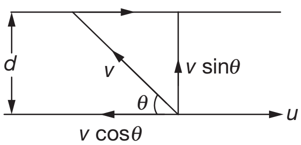
119. For shortest time,

$t = \frac{w}{v_m} = \frac{0.8}{4} = 0.2 \text{ hr}$

Drift $= v_r \times t = 6 \times 0.2 = 1.2 \text{ km} = 1200 \text{ m}$

The correct option is (B)

120. The swimmer must swim as shown



$v = \frac{5}{60}$ km/min

$15 = \frac{1}{v\sin\theta}$

or $v\sin\theta = \frac{1}{15}$

or $\frac{5}{60}\sin\theta = \frac{1}{15}$

$\therefore \quad \sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$ and $u = v\cos\theta = \frac{5}{60} \times \frac{3}{5} = \frac{1}{20}$ km/min = 3 km/h

The correct option is (B)

121. For shortest possible path man should swim at an angle of (90 + θ) with downstream. From the Fig. 2.26,

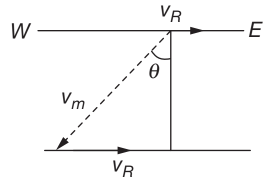


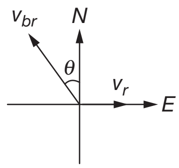
Fig. 2.26

$\sin\theta = \frac{v_r}{v_m} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta = 30°$

The correct option is (A)

122. $v_{br}\sin\theta = v_r$



$\Rightarrow \sin\theta = \frac{4}{8} = \frac{1}{2}$

$\therefore \theta = 30°$ west of north

The correct option is (C)

Circular Motion

123. Distance $= \int_0^2 v\,dt = \int_0^2 2t\,dt = 4$ m

Average speed $= \frac{4}{2} = 2$ m/s

$\omega = \frac{v}{R} = (2t)$ rad/s ,

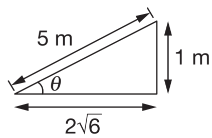
$\theta = \int_0^2 \omega\,dt = 4$ rad

$\therefore$ Displacement $= 2R\sin\frac{\theta}{2} = (2\sin 2)$ m

Average velocity = $\sin 2$ m/s

The correct option is (B)

124. $\tan\theta = \frac{v^2}{rg}$

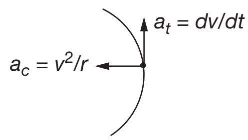


$v = \sqrt{rg\tan\theta}$

$v = \sqrt{20\sqrt{6} \times 10 \times \frac{1}{2\sqrt{6}}} = 10$ m/s

The correct option is (A)

125. $v = 2t^2$, $r = 100$ m



$a_t = \frac{dv}{dt} = 4t$

$a_t\,(t = 5\text{s}) = 20 \text{ ms}^{-2}$

$v\,(t = 5\text{s}) = 50 \text{ ms}^{-1}$

$a_c(t = 5\text{s}) = \frac{v^2}{r} = \frac{50 \times 50}{100} = 25 \text{ ms}^{-2}$

$a = \sqrt{a_c^2 + a_t^2} = \sqrt{1025} \approx 32 \text{ ms}^{-1}$

The correct option is (D)

126. $a_t = \frac{dv}{dt} = 4t + 1 = 9 \text{ m/s}^2$, $a_r = \frac{v^2}{r} = \frac{100\sqrt{19}}{100} = \sqrt{19} \text{ m/s}^2$

$a_{\text{net}} = \sqrt{9^2 + \left(\sqrt{19}\right)^2} = \sqrt{100} = 10 \text{ m/s}^2$

The correct option is (C)

127. If slope of line joining points between $T = 0$ and T is zero, (i.e. parallel to x-axis) then average velocity will be zero. That is possible only in (B)

The correct option is (B)

128. The correct option is (A)

129. The correct option is (B)

130. $$v_{avg} = \frac{2L}{\frac{L}{v_1} + \frac{L}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

The correct option is (C)

131. $x = (t-2)^2$

$x = (t-2)^2$

$x = (t-2)^2$

$t = \frac{4}{2} = 2$

$t = 0, x = 4$

$t = 1, x = 1$

$t = 2, x = 0$

$t = 3, x = 1$

$t = 4, x = 4$

$S = 8$

The correct option is (B)

132. Let length of escalator be L when velocity of girl, is v_1

$$t_1 = \frac{L}{v}$$

Velocity of escalator is v_2

$$t_2 = \frac{L}{v_2}$$

When both are main of

$$t_3 = \frac{L}{v_1 + v_2} = \frac{L}{\frac{L}{t_1} + \frac{L}{t_2}}$$

$$t_3 = \frac{t_1t_2}{t_1 + t_2}$$

The correct option is (C)

133. $$50 = \frac{u^2 \sin 30°}{g}$$

$$R' = \frac{u^2 \sin 90°}{g}$$

$$\frac{R'}{50} = 2$$

$R' = 100$ m

The correct option is (C)

134. Distance covered with 1 step = 1 m

Time taken = 1 s

Time taken to move first 5 m forward = 5 s

Time taken to move 3 m backward = 3 s

Net distance covered = 5 – 3 = 2 m

Net time taken to cover 2 m = 8 s

Drunkard covers 2 m in 8 s

Drunkard covered 4 m in 16 s

Drunkard covered 6 m in 24 s.

The correct option is (D)

135. Initial velocity of the car, $u = 126$ km/h = 35 m/s

Final velocity of the car, $v = 0$

Distance covered by the car before coming to rest, $s = 200$ m

Retardation produced in the car = a

From third equation of motion, a can be calculated as:

$v^2 - u^2 = 2as$

$(0)^2 - (35)^2 = 2 \times a \times 200$

$$a = -\frac{35 \times 35}{2 \times 200} = -3.06 \text{ m/s}^2$$

From first equation of motion, time (t) taken by the car to stop can be obtained as:

$v = u + at$

$$t = \frac{v-u}{a} = \frac{-35}{-3.06} = 11.44 \text{ s}$$

The correct option is (B)

136. For train A,

Initial velocity, $u = 72$ km/h = 20 m/s

Time, $t = 50$ s

Acceleration, $a_1 = 0$ (Since it is moving with a uniform velocity)

From second equation of motion, distance (s_1) covered by train A can be obtained as:

$$s_1 = ut + \frac{1}{2}a_1t^2$$

$$= 2 \times 50 + 0 = 1000 \text{ m}$$

For train B,

Initial velocity, $u = 72$ km/h = 20 m/s

Acceleration, $a = 1$ m/s^2

Time, $t = 50$ s

From second equation of motion, distance (s_{11}) covered by train A can be obtained as:

$$s_1 = ut + \frac{1}{2}a_1t^2$$

$$= 20 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = 2250 \text{ m}$$

Hence, the original distance between the driver of train A and the guard of train B is 2250 – 1000 = 1250 m

The correct option is (B)

137. Velocity of car A, $v_A = 36$ km/h = 10 m/s

Velocity of car B, $v_B = 54$ km/h = 15 m/s

Velocity of car C, $r_C = 54$ km/h = 15 m/s

Relative velocity of car B with respect to car A,

$v_{BA} = v_B - v_A$

= 15 + = 25 m/s

At a certain instance, both cars B and C are at the same distance from car A, i.e.,

$s = 1 \text{ km} = 1000 \text{ m}$

Time taken (t) by car C to cover 1000 m $= \dfrac{1000}{25} = 40$ s

Hence, to avoid an accident, car B must cover the same distance in a maximum of 40 s.

From second equation of motion, minimum acceleration (a) produced by car B can be obtained as:

$$s = ut + \frac{1}{2}at^2$$

$$s = ut + \frac{1}{2}at^2$$

$$a = \frac{1600}{1600} = 1 \text{ m/s}^2$$

The correct option is (C)

138. Let V be the speed of the bus running between towns A and B.

Speed of the cyclist, $v = 20$ km/h

Relative speed of the bus moving in the direction of the cyclist

$= V - v = (V - 20)$ km/h

The bus went past the cyclist every 18 min, i.e., $\dfrac{18}{60}h$ (when he moves in the direction of the bus).

Distance covered by the bus $= (V - 20)\dfrac{18}{60}$ km (1)

Since one bus leaves after every T minutes, the distance travelled by the bus will be equal

to $V \times \dfrac{T}{60}$ (2)

Both equations (1) and (2) are equal

$$(V - 20) \times \frac{18}{60} = \frac{VT}{60} \quad (3)$$

Relative speed of the bus moving in the opposite direction of the cyclist

$= (V + 20)$ km/ h

Time taken by the bus to go past the cyclist $= 6 \text{ min} = \dfrac{6}{60}h$

$$\therefore (V + 20)\frac{6}{60} = \frac{VT}{60}$$

From equations (3) and (4), we get

$$(V + 20)\frac{6}{60} = (V - 20) \times \frac{18}{60}$$

$V + 20 = 3V - 60$

$2V = 80$

$V = 40$ km/h

Substituting the value of V in equation (4), we get

$$(40 + 20) \times \frac{6}{60} = \frac{40T}{60}$$

$$T = \frac{360}{40} = 9 \text{ min}$$

The correct option is (A)

139. The described situation is shown in the given Fig. 2.27.

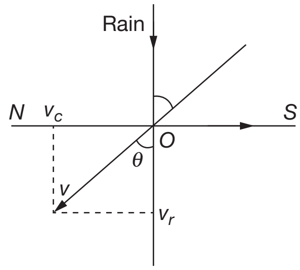


Fig. 2.27

Here,

v_c = Velocity of the cyclist

v_r = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity (v) of the rain with respect to the woman.

$v = v_r + (-v_c)$

$= 30 + (-10) = 20$ m/s

$$\tan\theta = \frac{v_c}{v_r} = \frac{10}{30}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Hence, the woman must hold the umbrella toward the south, at an angle of nearly 18° with the vertical.

The correct option is (B)

140. Speed of the man, $v_m = 4$ km/h

Width of the river = 1 km

Time taken to cross the river $= \dfrac{\text{Width of the river}}{\text{Speed of the river}}$

$$= \frac{1}{4}h = \frac{1}{4} \times 60 = 15 \text{ min}$$

Speed of the river, $v_r = 3$ km/h

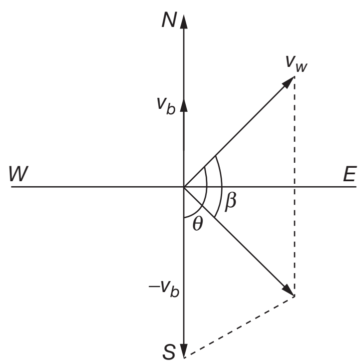
Distance covered with flow of the river $= v_r \times t$

$$= 3 \times \frac{1}{4} = \frac{3}{4} \text{ km}$$

$$= \frac{3}{4} \times 1000 = 750 \text{ m}$$

The correct option is (C)

141.



Velocity of the boat, $v_b = 51$ km/h

Velocity of the wind, $v_w = 72$ km/h

The flag is fluttering in the north-east direction. It shows that the wind is blowing towards the north-east direction. When the ship begins sailing towards the north, the flag will move along the direction of the relative velocity (v_{wb}) of the wind with respect to the boat.

The angle between vw and $(-v_b) = 90° + 45°$

$$\tan\beta = \frac{51\sin(90+45)}{72+51\cos(90+45)}$$

$$= \frac{51\sin 45}{72+51(-\cos 45)} = \frac{51\times\frac{1}{\sqrt{2}}}{72-51\times\frac{1}{\sqrt{2}}}$$

$$= \frac{51}{72\sqrt{2}-51} = \frac{51}{72\times1.414-51} = \frac{51}{50.800}$$

$$\therefore \beta = \tan^{-1}(1.0038) = 45.11°$$

Angle with respect to the east direction $= 45.11° - 45° = 0.11°$

Hence, the flag will flutter almost due east.

142. Speed of the ball, $u = 40$ m/s

Maximum height, $h = 25$ m

In projectile motion, the maximum height reached by a body projected at an angle θ, is given by the relation:

$$h = \frac{u^2\sin^2\theta}{2g}$$

$$25 = \frac{(40)^2\sin^2\theta}{2\times9.8}$$

$\sin^2\theta = 0.30625$

$\sin\theta = 0.5534$

$\therefore \theta = \sin^{-1}(0.5534) = 33.60°$

Horizontal range, $R = \dfrac{u^2\sin 2\theta}{g}$

$$= \frac{(40)^2\times\sin 2\times33.60}{9.8}$$

$$= \frac{1600\times\sin 67.2}{9.8}$$

$$= \frac{1600\times0.922}{9.8} = 150.53 \text{ m}$$

The correct option is (B)

143. Maximum horizontal distance, $R = 100$ m

The cricket will only be able to throw the ball to the maximum horizontal distance when the angle of projection is 45°, i.e., $\theta = 45°$.

The horizontal range for a projection velocity v is given by the relation:

$$R = \frac{u^2\sin 2\theta}{g}$$

$$100 = \frac{u^2}{g}\sin 90°$$

$$\frac{u^2}{g} = 100 \qquad (1)$$

The ball will achieve the maximum height if it is thrown vertically upward. For such motion, the final velocity v is zero at the maximum height H.

Acceleration, $a = -g$

Using the third equation of motion:

$v^2 - u^2 = -2gH$

$$H = \frac{1}{2}\times\frac{u^2}{g} = \frac{1}{2}\times100 = 50 \text{ m}$$

The correct option is (A)

144. Length of the string, l = 80 cm = 0.8 m

Number of revolutions = 14

Time taken = 25 s

$$v = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{14}{25}\text{ Hz}$$

Angular frequency, $\omega = 2\pi v$

$$= 2\times\frac{22}{7}\times\frac{14}{25} = \frac{88}{25}\text{ rad s}^{-1}$$

Centripetal acceleration, $a_c = \omega^2 r$

$$= \left(\frac{88}{25}\right)^2\times0.8$$

$= 9.91$ m/s^2

The direction of centripetal acceleration is always directed along the string, towards the centre, at all points

The correct option is (A)

145. The position of the particle is given by:

$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}$

Velocity $\vec{v}$ of the particle is given as:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k})$$

$\therefore \vec{v} = 3.0\,\hat{i} - 4.0t\,\hat{j}$

At $t = 2.0$ s:

$\vec{v} = 3.0\,\hat{i} - 8.0\,\hat{j}$

The magnitude of velocity is given by:

The correct option is (A)

146. Velocity of the particle, $\vec{v} = 10.0\,\hat{j}$ m/s

Acceleration of the particle $\vec{a} = (8.0\hat{i} + 2.0\hat{j})$

Also,

But, $\vec{a} = \dfrac{d\vec{v}}{dt} = 8.0\,\hat{i} + 2.0\,\hat{j}$

$d\vec{v} = (8.0\,\hat{i} + 2.0\,\hat{j})dt$

Integrating both sides:

$\vec{v}(t) = 8.0t\,\hat{i} + 2.0t\,\hat{j} + \vec{u}$,

where

$\vec{u}$ = Velocity vector of the particle at $t = 0$

$\vec{v}$ = Velocity vector of the particle at time t

But $\vec{v} = \dfrac{d\vec{r}}{dt}$

$d\vec{r} = \vec{v}dt = (8.0t\,\hat{i} + 2.0\,\hat{j} + \vec{u})dt$

Integrating the equations with the conditions at $t = 0$; $r = 0$ and at $t = t$; $r = r$

$$\vec{r} = \vec{u}t + \frac{1}{2}8.0t^2\,\hat{i} + \frac{1}{2}\times 2.0t^2\,\hat{j}$$

$$= \vec{u}t + 4.0t^2\,\hat{i} + r^2\,\hat{j}$$

$$= (10.0\hat{j})t + 4.0t^2\,\hat{i} + t^2\,\hat{j}$$

$$x\hat{i} + y\hat{j} = 4.0t^2\hat{i} + (10t + t^2)\hat{j}$$

Since the motion of the particle is confined to the x–y plane, on equating the coefficients of $\hat{i}$ and $\hat{j}$ we get:

$x = 4t^2$

$$t = \left(\frac{x}{4}\right)^{\frac{1}{2}}$$

And $y = 10t + t^2$

When $x = 16$ m:

$$t = \left(\frac{16}{4}\right)^{\frac{1}{2}} = 2s$$

$\therefore y = 10\times 2 + (2)^2 = 24$ m

The correct option is (A)

More Than One Option Correct Type

147. $\Delta u = (u^2 + u^2 + 2u^2\cos 120°)^{\frac{1}{2}}$

$\Delta u = u = 60\ \text{ms}^{-1}$

Average acceleration $= \dfrac{\Delta u}{t} = \dfrac{u}{\frac{r\theta}{4}} = 11.5\ \text{ms}^{-2}$

Instantaneous acceleration $= \dfrac{u^2}{r} = 12\ \text{ms}^{-2}$

The correct option is (A), (B) and (C)

148. $h_1 = \dfrac{u^2}{2g}\sin^2\alpha$ and $h_2 = \dfrac{u^2}{2g}\cos^2\alpha$

$$R = \frac{u^2}{g}2\sin\alpha\cos\alpha$$

$$R = \frac{u^2}{g}\times 2\frac{\sqrt{2gh_1}}{4}\frac{\sqrt{2gh_2}}{4},\quad R = 4\sqrt{h_1h_2}$$

$$\frac{t_1}{t_2} = \frac{2u\sin\alpha/g}{2u\sin(90-\alpha)/g} = \tan\alpha$$

The correct option is (A), (B), (C) and (D)

149. $\vec{v}_A = 4\hat{i} + 4\hat{k}$ $\qquad \vec{a}_A = -g\hat{k}$

$\vec{v}_B = 3\hat{j} + 4\hat{k}$ $\qquad \vec{a}_B = -g\hat{k}$

$\vec{v}_A - \vec{v}_B = 4\hat{i} - 3\hat{j}$ $\qquad \vec{a}_{AB} = \vec{O}$

$|\vec{v}_{AB}| = 5$ m/s

Time of flight $t_A = \dfrac{2\times 4}{g} = \dfrac{8}{g}$, $t_B = \dfrac{2\times 4}{g} = \dfrac{8}{g}$

Separation when they hit the ground $= 5\times\dfrac{8}{g} = 4$ m

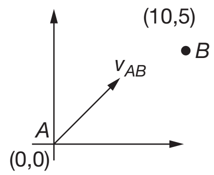
The correct option is (A), (B), (C) and (D)

150. $\vec{v}_{AB} = (3-a)\hat{i} + (3-b)\hat{j}$

$\vec{a}_{AB} = \vec{o}$

$(3-a)\times 2 = 10$ and $(3-b)\times 2 = 5$

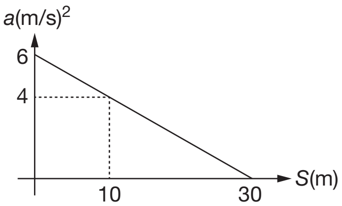
$a = -2$ and $b = \dfrac{1}{2}$



The correct option is (A) and (C)

151. The correct option is (B), (C) and (D)

152.



Area $= \dfrac{1}{2}\times 10\times(6+4) = \dfrac{v^2}{2}$

$v = 10$ m/s

Area upto 30 m $= \dfrac{1}{2}\times 30\times 6 = \dfrac{v^2}{2}$

$v^2 = 180$

$v_{\max} = \sqrt{80} < 14$

The correct option is (B) and (C)

Passage Based Questions

Passage I

153. $a = \frac{dv}{dt} = 2t - 4$

At $t = 0$, $a = -4$ m/s^2

The correct option is (C)

154. $x = \int_0^3 (t^2 - 4t)dt = \left[\frac{t^3}{3} - 2t^2\right]_0^3 = 9 - 18 = -9$ m

The correct option is (B)

155. $x = \frac{t^3}{3} - 2t^2 = 0 \Rightarrow t = 0$ s or 6 s

$v(t=0) = 0$, $v(t = 6\,\text{s}) = 6^2 - 4 \times 6 = 12$ m/s

The correct option is (D)

Passage 2

156. Slope of v_y graph is $-g$

$\therefore \quad -g = \frac{-10}{t_1}$

The correct option is (A)

157. As displacement along y-axis is zero, $k = -v_y = -10$

The correct option is (B)

158. $\tan\alpha = \frac{u_y}{u_x} = 1$

The correct option is (A)

Passage 3

159. $h = 200\sin\theta t - \frac{1}{2}gt^2$ (1)

$h = 100t - \frac{1}{2}gt^2$ (2)

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \pi/6$

The correct option is (A)

160. $200\cos\theta t = 100\left(\sqrt{3} - 1\right) + X$ (3)

$100t = X$ (4)

$200 \times \frac{\sqrt{3}}{2} \times t = 100\left(\sqrt{3} - 1\right) + 100t$

$100\sqrt{3}t - 100\,t = 100\left(\sqrt{3} - 1\right)$

$t = 1$ s

The correct option is (C)

161. $h = 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95$ m

$x = 100$ m

The correct option is (C)

Passage 4

162. Velocity of escalator $= \frac{l}{t_2}$

The correct option is (A)

163. Velocity of man $= \frac{l}{t_1}$

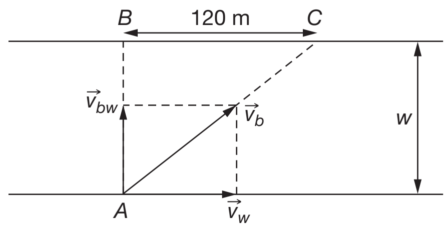
The correct option is (B)

164. When both moves in same direction, $t = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$

The correct option is (A)

Passage 5

165.



Speed of water current $(v_w) = \frac{BC}{t} = \frac{120}{10}$

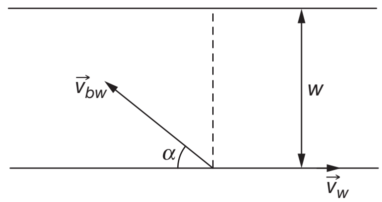
$= 12$ m/min

If width of river is w and speed of boat with respect to water is v_{bw},

$t = \frac{w}{v_{bw}} \Rightarrow w = 10v_{bw}$ (1)

The correct option is (C)

166.



For shortest path

$v_w = v_{bw}\cos\alpha$

$v_{bw}\cos\alpha = 12$ (2)

$t_{min} = \frac{w}{v_{bw}\sin\alpha} = 12.5$ min

$w = 12.5\ v_{bw}\sin\alpha$ (3)

(1) and (3) $\Rightarrow 10v_{bw} = 12.5v_{bw}\sin\alpha$

$\Rightarrow \sin\alpha = \frac{4}{5}$

$\Rightarrow \alpha = 53°$

(2) $\Rightarrow v_{bw} = \frac{12}{\cos\alpha} = 20$ m/min

The correct option is (A)

167. (1) $\Rightarrow w = 10v_{bw} = 200$ m

The correct option is (B)

Match the Column Type

168. $S_{n^{th}} = u + \frac{1}{2}a(2n-1)$

$13 = u + \frac{3}{2}a$ (1)

$7 = u + \frac{9}{2}a$ (2)

(1) – (2)

$\Rightarrow\ 6 = -\frac{6}{2}a$

$\Rightarrow\ a = -2\ \text{m/s}^2$

(1) $\Rightarrow 13 = u - 3$

$u = 16\ \text{ms}^{-2}$

$s = ut + \frac{1}{2}at^2$

$s(t = 7\text{s}) = 16\times 7 - 7^2 = 63$ m

If $s = 0$

$\Rightarrow\ 0 = 16t - t^2$

$\Rightarrow\ t = 16$ s

When $v = 0$, $t = 8$ s

$s\ (t = 8\ \text{s}) = 16 \times 8 - 8^2 = 64$ m

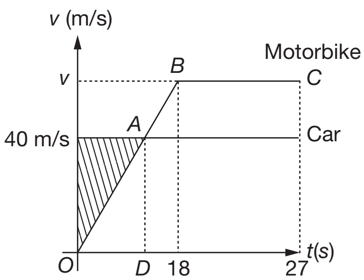
$s\ (t = 9\ \text{s}) = 16 \times 9 - 9^2 = 63$ m

$\therefore$ Distance travelled in 9 s

= displacement in 8 second + displacement in 9th second = 64 + 1 = 65m

$\therefore$ (A) $\rightarrow$ (3); (B) $\rightarrow$ (2); (C) $\rightarrow$ (1); (D) $\rightarrow$ (4)

169.



Let maximum speed of motorbike be = v

$40 \times 27 = \frac{1}{2}(27+9)v$

$v = 60$ m/s

So acceleration of motorbike $= \frac{60}{18} = \frac{10}{3}$

Maximum separation = shaded area

$$= \frac{1}{2}\times 40 \times OD = \frac{1}{2}\times 40 \times 12 = 240$$

Separation at $t = 18$

$$= 240 - \frac{1}{2}\times 20 \times 6 = 180$$

$\therefore$ (A) $\rightarrow$ (2); (B) $\rightarrow$ (3); (C) $\rightarrow$ (4); (D) $\rightarrow$ (1)

170. $x = t^3 - 3t^2$

Position of particle is zero at $t = 0$ s and $t = 3$ s.

$\therefore\ v = 3t^2 - 6t$

Velocity of particle is zero at $t = 0$ s and $t = 2$ s.

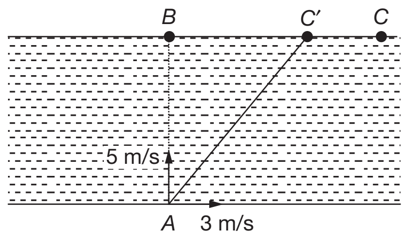
Hence, particle reverses its direction of motion at 2 s.

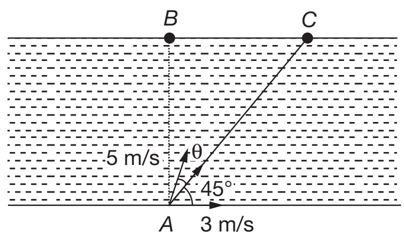
$a = 6t - 6$

Acceleration of particle is zero at $t = 1$ s.

$\therefore$ (A) $\rightarrow$ (1); (B) $\rightarrow$ (2); (C) $\rightarrow$ (4); (D) $\rightarrow$ (3)

171.





Time if he crosses by shortest path

$$= \frac{500}{\sqrt{v^2 - u^2}} + \frac{500}{10} = \frac{500}{4} + 50 = 125 + 50 = 175 \text{ s}$$

Time if he cross river in shortest time

$$= \frac{500}{5} + \frac{C'C}{10} = 100 + \frac{500 - BC'}{10}$$

= 100 + 50 –30 = 120 s

If velocity is along AC then

$5\sin\theta = 3\sin 45°$

$\sin\theta = \frac{3}{5\sqrt{2}}$

$\text{time} = \frac{500}{5\cos(45° - \theta)} \approx 106$ s

$\therefore$ (A) $\rightarrow$ (2); (B) $\rightarrow$ (3); (C) $\rightarrow$ (4); (D) $\rightarrow$ (1)

172. (A) $\rightarrow$ (3); (B) $\rightarrow$ (1); (C) $\rightarrow$ (1); (D) $\rightarrow$ (2)

Integer Type

185. $(n-2)l = \frac{1}{2}at^2$

$(n-1)l = \frac{1}{2}a(t+3)^2$

$nl = \frac{1}{2}a(t+5)^2$

$l = \frac{1}{2}a\left[(t+3)^2 - t^2\right] = \frac{1}{2}a\left[(t+5)^2 - (t+3)^2\right]$

$t^2 + 9 + 6t - t^2 = t^2 + 25 + 10t - t^2 - 9 - 6t$

$9 + 6t = 16 + 4t$

$2t = 7$

186. $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

$12 = 6\tan\theta\left(\frac{R-6}{R}\right)$ (1)

$6 = 12\tan\theta\left(\frac{R-12}{R}\right)$ (2)

From (1) and (2)

$2 = \frac{1}{2}\left(\frac{R-6}{R-12}\right)$, $4(R-12) = R-6$

$4R - 48 = R - 6$, $3R = 42 \Rightarrow R = 14$ m

$\frac{r}{2} = 7$

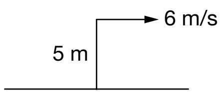
187. $u_x = 15$ m/s, $u_y = 20$ m/s

Time of flight $= \frac{2u_y}{g} = \frac{2\times 20}{10} = 4$ s

188. $u_x = 6$ m/s; $a_x = 0$; $s_x = ?$

$u_y = 0$; $a_y = -g$; $s_y = -5$ m, $t = ?$

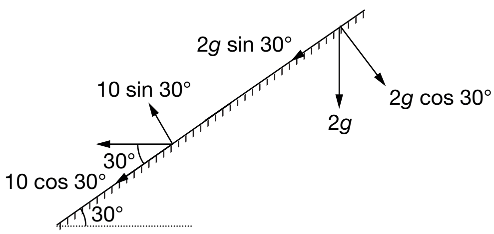
$S_y = u_y t + \frac{1}{2}a_y t^2$



$t = \sqrt{\frac{2\times 5}{10}} = 1$ s

$s_x = u_x t + \frac{1}{2}a_x t^2 = 6\times 1 = 6$ m

189.



Acceleration of ball with respect to plane

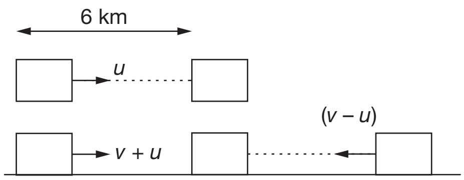
$a_{bp} = a_{bg} - a_{pg} = 2g$ (downward)

$s = ut + \frac{1}{2}at^2$

$0 = (10\sin 30°)t - \frac{1}{2}\times(2g\cos 30°)t^2$

$\Rightarrow t = \frac{1}{\sqrt{3}}$ s

190.



$\frac{6}{u} = 1 + \frac{(v+u)\times 1 - 6}{(v-u)}$

$\frac{6}{u} = \frac{(v-u)+(v+u)-6}{(v-u)}$

$\frac{6}{u} = \frac{2(v-6)}{(v-u)}$

$\Rightarrow 6(v-u) = 2vu - 6u$

$\Rightarrow 6v - 6u = 2vu - 64$

$u = 3$ km/h

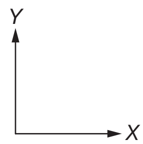
191. $a = \frac{vdv}{ds} = 12\text{ s}^2$

$\int_0^v vdv = 12\int_{-2}^0 s^2 ds$

$\frac{v^2}{2} = 32$

$v = 8$ m/s

192. $\vec{v}_{R/A} = 20(-\hat{j})$



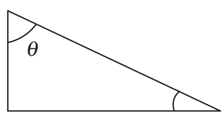
$\vec{v}_P = 5\hat{i}$

$\vec{v}_A = 15\hat{i}$

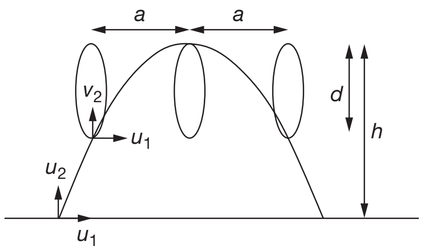
$\vec{v}_{R/P} = 20(-\hat{j}) + 10\hat{i}$

$\cot\theta = 2$

193. $\tan\theta = \frac{6}{2} = 3$



194.



$$2a = u_1 \frac{2v_2}{g} \quad (1)$$

$$v_2 = \sqrt{2gd} \quad (2)$$

$$u_2 = \sqrt{2gh} \quad (3)$$

$$u_1 = \frac{ag}{\sqrt{2gd}}$$

So, $\tan\theta = \frac{u_2}{u_1} = \frac{2\sqrt{hd}}{a} = 2$

Previous Years' Questions

195. K.E. of point of projection, $E = \frac{1}{2}mv^2$

K.E. at highest point of its flight $= \frac{1}{2}m\left(\frac{v}{\sqrt{2}}\right)^2$

$= \frac{E}{2}$

The correct option is (C)

196. Displacement is same in both cases, hence $v_A = v_B$

The correct option is (B)

197. $v^2 = v^2 + 2as$

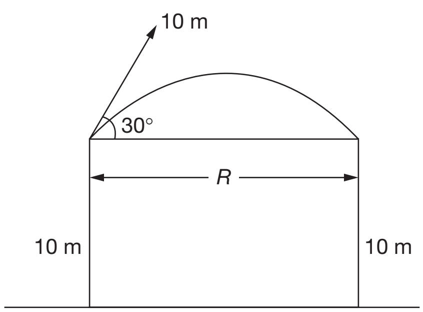
Stopping distance will be

$S = \frac{v^2}{2a}$

$\therefore \quad \frac{S}{6} = \left(\frac{100}{50}\right)^2 = S = 24$ m

The correct option is (C)

198.



$R = \frac{100 + \frac{\sqrt{3}}{2}}{10} \qquad \left(\because \ R = \frac{u^2 \sin 2\theta}{g}\right)$

$= 5\sqrt{3}$

$= 8.66$

The correct option is (D)

199. $x = \alpha t^3,\ y = \beta t^3$

$\Rightarrow v_x = \frac{dx}{dt} = 3\alpha t^2, \quad \Rightarrow \quad v_y = 3\beta t^2$

$\therefore v = \sqrt{v_x^2 + v_y^2} = 3t^2\sqrt{\alpha^2 + \beta^2}$

The correct option is (B)

200. $h = \frac{1}{2}gT^2 \quad (1)$

Height from top after $T/3$ second

$h_1 = \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{h}{9}$

$\therefore \frac{8h}{9}$ metres from the ground.

The correct option is (A)

201. In case of uniform circular motion, only centripetal acceleration exists.

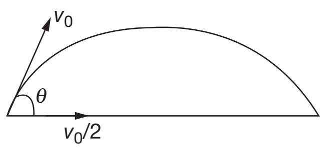
The correct option is (B)

202. Stopping distance $S = \frac{v^2}{2a}$

$\therefore \quad \frac{S}{20} = \left(\frac{120}{60}\right)^2 \Rightarrow S = 80$ m

The correct option is (C)

203.



$\frac{v_0}{2}T = R$

$\Rightarrow \frac{v_0}{2} \times \frac{2u\sin\theta}{g} = \frac{v^2 \sin 2\theta}{g}$

$\Rightarrow \cos\theta = \frac{1}{2}$

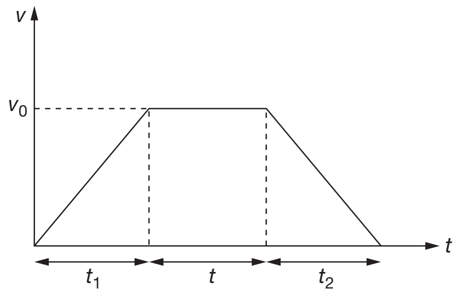
$\therefore \theta = 60°$

The correct option is (C)

204. $\frac{v_0}{t_1} = f$ (1)

$\frac{v_0}{t_2} = \frac{f}{2}$ (2)

$\frac{1}{2}v_0 t_1 = S$ (3)



$\frac{1}{2}(t + t_1 + t_2 + t)v_0 = 15\,S$

$\Rightarrow 2t + \frac{v_0}{f} + \frac{2v_0}{f} = \frac{30S}{v_0}$

$\Rightarrow 2t = \frac{30S}{v_0} - \frac{3v_0}{f} = \frac{15v_0}{f} - \frac{3v_0}{f}$

$\Rightarrow 2t = \frac{12v_0}{f}$

$\Rightarrow t = \frac{6v_0}{f}$

Also, $S = \frac{1}{2}\frac{v_0^2}{f} = \frac{1}{2f}\left(\frac{ft}{6}\right)^2 = \frac{1}{72}ft^2$

The correct option is (D)

205. $a_{av} = \frac{\vec{v}_{fi} - \vec{v}}{\Delta t} = \frac{5\hat{j} - 5\hat{i}}{10} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$

The correct option is (C)

206. $t = ax^2 + bx$

$\Rightarrow 1 = 2axv + bv$

$\Rightarrow v = \frac{1}{2ax + b}$ (1)

Also, $2ax\,v + bc = 1$

$\Rightarrow 2a\left(v^2 + x \cdot a_0\right) + ba_0 = 0$ (Here a_0 is acceleration)

$\Rightarrow a_0(2ax + b) = -2av^2$

$\Rightarrow a_0 = -\frac{2av^2}{2ax + b} = -2av^3$ (from (1))

The correct option is (D)

207. $t_1 = \frac{2u\sin\theta}{g}$

$t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$ [Range will be same for θ and $90 - \theta$]

$\therefore\; t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g} = 2R$

$\therefore\; t_1 t_2 \propto R$

The correct option is (C)

208. $v = \alpha\sqrt{x}$

$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x}$

$\Rightarrow \int_0^x x^{-1/2}dx = \alpha\int_0^t dt$

$\Rightarrow 2\sqrt{x} = \alpha t$

$\Rightarrow x = \frac{\alpha^2}{4}t^2$

$\Rightarrow x\alpha t^2$

The correct option is (A)

209. $K = \frac{1}{2}mv^2$

$K' = \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{K}{4}$

The correct option is (D)

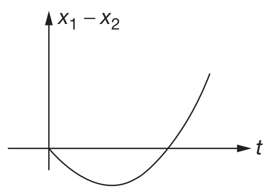
210. $v = v_0 + gt + ft^2$

$\Rightarrow \int_0^x dx = \int_0^1 (v_0 + gt + ft^2)\,dt$

$\Rightarrow x = v_0 + \frac{g}{2} + \frac{f}{3}$

The correct option is (C)

211.



$x_1(t) = \frac{1}{2}at^2$

and $x_2(t) = vt$

$x_1 - x_2 = \frac{a}{2}t^2 - vt$

The correct option is (B)

212. $y = h - \frac{1}{2}gt^2$

and $v = \begin{cases} -gt & t < t_0 \\ gt_0 - gt & t > t_0 \end{cases}$

The correct option is (A)

213. $\vec{v} = \vec{u} + \vec{a}t$

$= (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j})10$

$= 7\hat{i} + 7\hat{j}$

$\therefore\; v = 7\sqrt{2}$ units

The correct option is (A)

214. $\dfrac{dx}{dt} = Ky, \dfrac{dy}{dt} = Kx$

$\because \quad \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{x}{4} \Rightarrow ydy = xdx$

$\Rightarrow y^2 = x^2 + C$

The correct option is (D)

215. $S = t^3 + 5$

$\Rightarrow v = 3t^2$

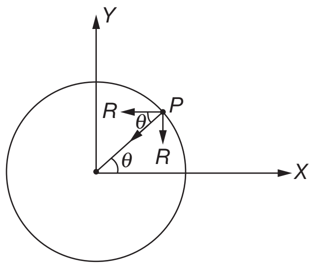
$a_T(T = 2s) = 6 \times 2 = 12 \text{ m/s}^2$

$a_C = \dfrac{v^2}{R} = \dfrac{144}{20} \text{ m/s}^2$

$a_{\text{net}} = \sqrt{a_C^2 + a_T^2} \simeq 14 \text{ m/s}^2$

The correct option is (D)

216.



$\vec{a} = \dfrac{v^2}{R}\cos\theta(-\hat{i}) + \dfrac{v^2}{R}\sin\theta(-\hat{j})$

The correct option is (C)

217. $\vec{L} = \vec{r} \times m\vec{v}$

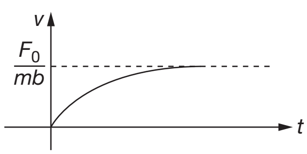
$= m\left[u\cos\theta t\hat{i} + \left(u\sin\theta - \dfrac{1}{2}gt^2\right)\hat{j}\right]$

$\times\left[u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}\right]$

$= -\dfrac{1}{2}mgv_0t^2\cos\theta\hat{k}$

The correct option is (C)

218.



$a = \dfrac{F_0}{m}e^{-bt}$

$\Rightarrow \dfrac{dv}{dt} = \dfrac{F_0}{m}e^{-bt}$

$\Rightarrow \int_0^v dv = \dfrac{F_0}{m}\int_0^t e^{-bt}dt$

$\Rightarrow v = \dfrac{F_0}{m}\left[-\dfrac{e^{-bt}}{b}\right]_0^t = \dfrac{F_0}{bm}\left[1 - e^{-bt}\right]$

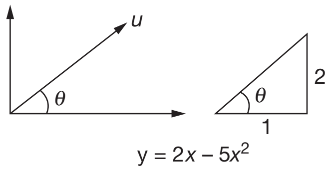
The correct option is (C)

219. $h_{\max} = \dfrac{u^2}{2g} = 10 \text{ m}$

$R_{\max} = \dfrac{u^2}{g} = 2 \times h_{\max} = 20 \text{ m}$

The correct option is (D)

220.



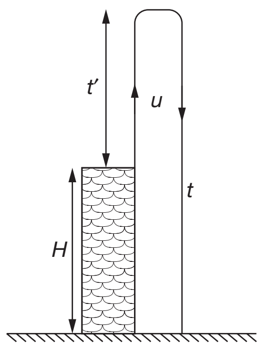
$y = x\tan\theta - \dfrac{1}{2}\dfrac{gx^2}{u^2\cos^2\theta}$

$y = x(2) - \dfrac{1}{2}\dfrac{(10)x^2}{(\sqrt{5})^2.\left(\dfrac{1}{\sqrt{5}}\right)^2}$

$y = 2x - 5x^2$

The correct option is (A)

221.



To reach on ground, time taken is t

$-H = ut - \dfrac{1}{2}gt^2$

$-2H = 2ut - gt^2$

$gt^2 - 2ut - 2H = 0$

$t = \dfrac{2u \pm \sqrt{4u^2 + 8gH}}{2g}$

$t = \dfrac{u + \sqrt{u^2 + 2gH}}{g}$ (1)

For highest point, time taken is t'

$0 = u - gt'$

$t' = \dfrac{u}{g}$ (2)

According to given condition

$t = nt'$

$$\frac{u+\sqrt{u^2+2gH}}{g} = n\frac{u}{g}$$

$$u+\sqrt{u^2+2gH} = nu$$

$$u^2+2gH = (n-1)^2u^2$$

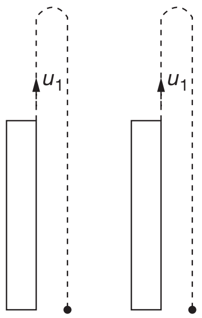
$$(u^2+2gH) = (n^2+1-2n)u^2$$

$$2gH = (n^2-2n)u^2$$

$$2gH = (n-2)nu^2$$

The correct option is (C)

222.



$u_1 = 10$ m/s

$$\vec{s} = \vec{ut} + \frac{1}{2}\vec{at^2}$$

$$-240 = 10t_1 - \frac{1}{2}\times 10\times t_1^2$$

$$t_1^2 - 2t_1 - 48 = 0$$

$$(t_1-8)(t_1+6) = 0$$

$t_1 = 8$ s (Ball 1 reaches the ground in 8 s)

For Ball 2

$$-240 = 40t_2 - \frac{1}{2}\times 10\times t_2^2$$

$$t_2^2 - 8t_2 - 48 = 0$$

$$(t_2-12)(t_2+4) = 0$$

$t_2 = 12$ s (Ball 2 reaches ground in 12 s)

Therefore during the interval $(0 \le t \le 8)$ both balls will have same acceleration ($a_1 = a_2 = -g$). So, the motion of Ball 2 with respect to Ball 1

$$\vec{S}_{\text{rel}} = \vec{u}_{\text{rel}} \times t + \frac{1}{2}\vec{a}_{\text{rel}}\, t^2 (t \le 8)$$

$$(y_2 - y_1) = (40-10)t + \frac{1}{2}\times(0)t^2$$

$$y_2 - y_1 = 30t \quad (\text{for } 0 \le t \le 8)$$

So, the graph of $(y_2 - y_1)$ versus t is a straight line.

After 8 seconds, Ball 1 hits the ground and stops. But Ball 2 continues to fall under gravity. Hence,

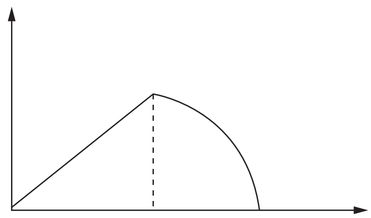
(for $8 \le t \le 12$)

$$(y_2 - y_1) = (u_2 - u_1)t + \frac{1}{2}(a_2 - a_1)t^2$$

$$y_2 - y_1 = 30t - \frac{1}{2}(10)\, t_2$$

$$y_2 - y_1 = 30t - 5t^2$$

This is equation of parabola.



The correct option is (B)