

CHAPTER

1

Unit and Dimension

Chapter Highlights

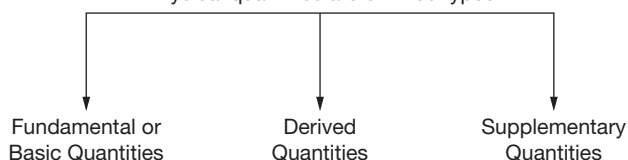
Physics, technology and society, SI units, Fundamental and derived units. Least count, Accuracy and precision of measuring instruments, Errors in measurement, Significant figures. Dimensions of Physical quantities, Dimensional analysis and its applications. Basic Mathematics.

PHYSICAL QUANTITIES

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class X we have studied many physical quantities.

For example, length, velocity, acceleration, force, time, pressure, mass, density etc.

Physical quantities are of three types



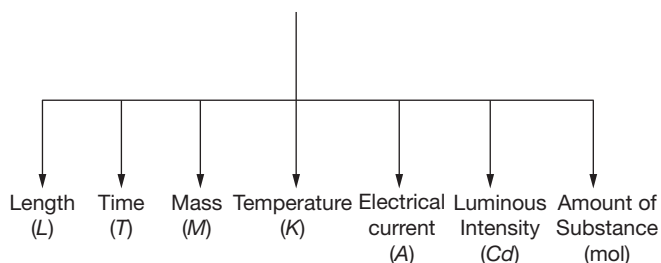
Fundamental (Basic) Quantities

These are the elementary quantities which covers the entire span of physics.

Any other quantities can be derived from these.

All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance, time and velocity cannot be chosen as basic quantities as $v = \frac{d}{t}$). An International Organization named

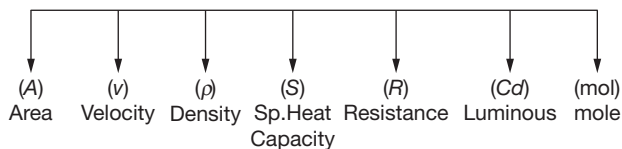
CGPM: General Conference on weight and measures chose seven physical quantities as basic or fundamental.



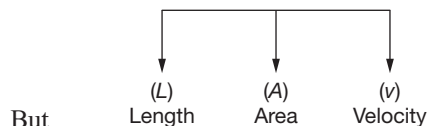
These are the elementary quantities (in our planet) that's why chosen as basic quantities.

In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.

i.e.,



Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities)



cannot be used as basic quantities as

Area = (Length)² so they are not independent.

Derived Quantities

Physical quantities which can be expressed in terms of basic quantities (M, L, T, \dots) are called derived quantities.

i.e., Momentum

$$P = mv$$

$$= (m) \frac{\text{displacement}}{\text{time}} = \frac{ML}{T} M^1 L^1 T^{-1}$$

Here $[M^1 L^1 T^{-1}]$ is called dimensional formula of momentum, and we can say that momentum has

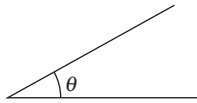
- 1 Dimension in M (mass)
- 1 Dimension in L (meter)
- and -1 Dimension in T (time)

The representation of any quantity in terms of basic quantities (M, L, T, \dots) is called dimensional formula and in the representation, the powers of the basic quantities are called dimensions.

Supplementary Quantities

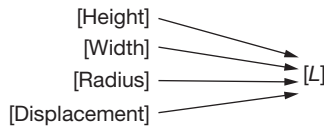
Besides seven fundamental quantities two supplementary quantities are also defined. They are

- Plane angle (The angle between two lines)
- Solid angle



DIMENSIONS

- Height, width, radius, displacement etc. are a kind of length. So we can say that their dimension is $[L]$



here [Height] can be read as “Dimension of Height”

- Area = Length \times Width
- $[Area] = [Length] \times [Width]$
- $= [L] \times [L]$
- $= [L^2]$

For circle

$$Area = \pi r^2$$

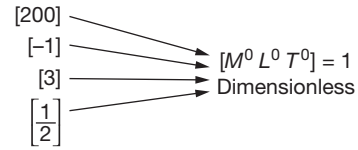
$$[Area] = [\pi] [r^2]$$

$$= [1] [L^2]$$

$$= [L^2]$$

Here π is not a kind of length or mass or time so π shouldn't affect the dimension of Area.

Hence its dimension should be 1 ($M^0 L^0 T^0$) and we can say that it is dimensionless. From similar logic we can say that all the numbers are dimensionless.



- [Volume] = [Length] \times [Height]
- $= L \times L \times L$
- $= [L^3]$

For sphere

$$Volume = \frac{4}{3} \pi r^3$$

$$[Volume] = \left[\frac{4}{3} \pi \right] [r^3]$$

$$= (1) [L^3] = [L^3]$$

So dimension of volume will be always $[L^3]$ whether it is volume of a cuboid or volume of sphere.

Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

- Density = $\frac{\text{mass}}{\text{volume}}$
- $[Density] = \frac{[mass]}{[volume]} = \frac{M}{L^3} = [M^1 L^{-3}]$
- Velocity (v) = $\frac{\text{displacement}}{\text{time}}$
- $[v] = \frac{[displacement]}{[time]} = \frac{L}{T} = [M^0 L^1 T^{-1}]$
- Acceleration (a) = $\frac{dv}{dt}$
- $[a] = \frac{dv \rightarrow \text{kind of velocity}}{dt \rightarrow \text{kind of time}} = \frac{LT^{-1}}{T} = LT^{-2}$
- Momentum (P) = mV
- $[P] = [M] [V]$
- $= [M] [LT^{-1}]$
- $= [M^1 L^1 T^{-1}]$
- Force (F) = ma
- $[F] = [m] [a]$
- $= [M] [LT^{-2}]$
- $= [M^1 L^1 T^{-2}]$
- Work or Energy = force \times displacement
- $[Work] = [force] [displacement]$
- $= [M^1 L^1 T^{-2}] [L]$
- $= [M^1 L^2 T^{-2}]$

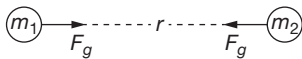
- Power = $\frac{\text{work}}{\text{time}}$
 $[\text{Power}] = \frac{[\text{work}]}{[\text{time}]} = \frac{M^1 L^2 T^{-2}}{T} = [M^1 L^2 T^{-3}]$
- Pressure = $\frac{\text{Force}}{\text{Area}}$
 $[\text{Pressure}] = \frac{[\text{Force}]}{[\text{Area}]} = \frac{M^1 L^1 T^{-2}}{L^2} = M^1 L^{-1} T^{-2}$

Dimensions of Angular Quantities

- Angle (θ)
 (Angular displacement) $\theta = \frac{\text{Arc}}{\text{radius}}$
 $[\theta] = \frac{[\text{Arc}]}{[\text{radius}]} = \frac{L}{L} = [M^0 L^0 T^0]$ (Dimensionless)
- Angular velocity (ω) = $\frac{\theta}{t}$
 $[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0 L^0 T^{-1}]$
- Angular acceleration (α) = $\frac{d\omega}{dt}$
 $[\alpha] = \frac{[d\omega]}{[dt]} = \frac{M^0 L^0 T^{-1}}{T} = [M^0 L^0 T^{-2}]$
- Torque = Force \times Arm length
 $[\text{Torque}] = [\text{force}] \times [\text{arm length}]$
 $= [M^1 L^1 T^{-2}] \times [L] = [M^1 L^2 T^{-2}]$

Dimensions of Physical Constants

Gravitational Constant



If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is,

$$\text{Gravitational force } F_g = \frac{G m_1 m_2}{r^2}$$

where G is a constant called Gravitational constant

$$[F_g] = \frac{[G][m_1][m_2]}{[r^2]}$$

$$[M^1 L^1 T^{-2}] = \frac{[G][M][M]}{[L^2]}$$

$$[G] = M^{-1} L^3 T^{-2}$$

Specific Heat Capacity

To increase the temperature of a body by ΔT , Heat required is $Q = ms \Delta T$

Here, s is called specific heat capacity.

$$[Q] = [m] [s] [\Delta T]$$

Here Q is heat: A kind of energy so $[Q] = M^1 L^2 T^{-2}$

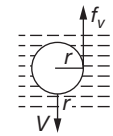
$$[M^1 L^2 T^{-2}] = [M] [S] [K]$$

$$[S] = [M^0 L^2 T^{-2} K^{-1}]$$

Coefficient of Viscosity

If any spherical ball of radius r moves with velocity v in a viscous Liquid, then viscous force acting on it is given by

$$F_v = 6\pi\eta r v$$



Here η is coefficient of viscosity

$$[F_v] = [6\pi] [\eta] [r] [v]$$

$$M^1 L^1 T^{-2} = (1) [\eta] [L] [L T^{-1}]$$

$$[\eta] = M^1 L^{-1} T^{-1}$$

Planck's Constant

If light of frequency ν is falling, energy of a photon is given by

$$E = h \nu$$

Here h = Planck's constant

$$[E] = [h] [\nu]$$

$$\nu = \text{frequency} = \frac{1}{\text{Time Period}}$$

$$\Rightarrow [E] = \frac{1}{[\text{Time Period}]} = \left[\frac{1}{T} \right]$$

$$\text{so } M^1 L^2 T^{-2} = [h] [T^{-1}]$$

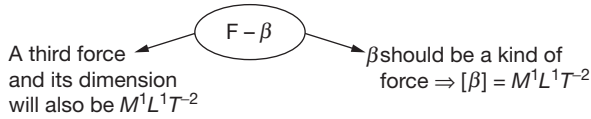
$$[h] = M^1 L^2 T^{-1}$$

Some Special Features of Dimensions

Suppose in any formula, $(L + \alpha)$ term is coming (where L is length). As length can be added only with a length, so α should also be a kind of length.

$$\text{So } [\alpha] = [L]$$

Similarly, consider a term $(F - \beta)$ where F is force. A force can be added or subtracted with a force only and give rise to a third force. So β should be a kind of force and its result $(F - \beta)$ should also be a kind of force.



Rule No. 1: One quantity can be added / subtracted with a similar quantity only and give rise to the similar quantity.

SOLVED EXAMPLES

1. $\frac{\alpha}{t^2} = Fv + \frac{\beta}{x^2}$

Find dimension formula for $[\alpha]$ and $[\beta]$
(here t = time, F = force, v = velocity, x = distance)

Solution:

Since $[Fv] = M^1 L^2 T^{-3}$,

so $\left[\frac{\beta}{x^2}\right]$ should also be $M^1 L^2 T^{-3}$

$$\frac{[\beta]}{[x^2]} = M^1 L^2 T^{-3}$$

$$[\beta] = M^1 L^4 T^{-3}$$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1 L^2 T^{-3}$

so $\frac{[\alpha]}{[t^2]} = M^1 L^2 T^{-3}$

$$[\alpha] = M^1 L^2 T^{-1}$$

2. For n moles of gas, Van der Waal's equation is

$$\left(P - \frac{a}{V^2}\right)(V - b) = nRT$$

Find the dimensions of a and b , where P is gas pressure, v = volume of gas T = temperature of gas

Solution:

$$\left(P - \frac{a}{V^2}\right) \quad (V - b) = nRT$$

↑

should be a kind of pressure

↑

should be a kind of volume

So $\frac{[a]}{[V^2]} = M^1 L^{-1} T^{-2}$

So $[b] = L^3$

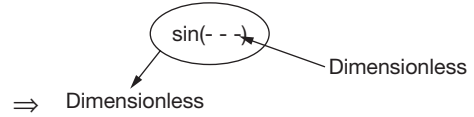
$$\frac{[a]}{[L^3]^2} = M^{-1} L^{-1} T^{-2}$$

$\Rightarrow [a] = M^1 L^5 T^{-2}$.

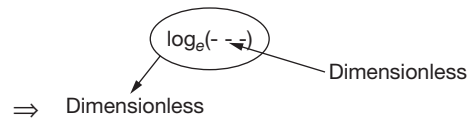
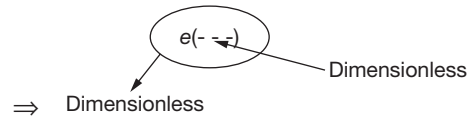
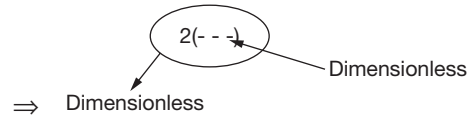
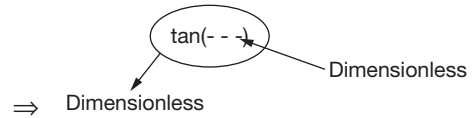
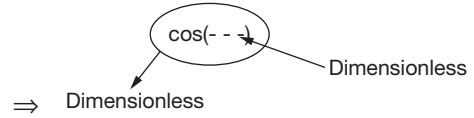
Rule No. 2: Consider a term $\sin(\theta)$

Here θ is dimensionless and $\sin\theta \left(\frac{\text{Perpendicular}}{\text{Hypoteneous}}\right)$ is also dimensionless.

Whatever comes in $\sin(\dots)$ is dimensionless and entire $[\sin(\dots)]$ is also dimensionless.



Similarly:

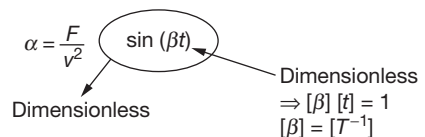


3. $\alpha = \frac{F}{v^2} \sin(\beta t)$

(here v = velocity, F = force, t = time)

Find the dimension of α and β

Solution:



So $[\alpha] = \frac{[F]}{[v^2]}$
 $= \frac{[M^1 L^1 T^{-2}]}{[L^1 T^{-1}]^2} = M^1 L^{-1} T^0$.

4. $\alpha = \frac{Fv^2}{\beta^2} \log_e \left(\frac{2\pi\beta}{v^2} \right)$ where F = force, v = velocity

Find the dimensions of α and β .

Solution:

$$\alpha = \frac{Fv^2}{\beta^2} \log_e \frac{2\pi\beta}{v^2}$$

Dimensionless
Dimensionless

$$\Rightarrow [\alpha] = \frac{[F][v^2]}{[\beta^2]}$$

$$\Rightarrow \frac{[2\pi][\beta]}{[v^2]} = 1$$

$$\Rightarrow \frac{[1][\beta]}{L^2 T^{-2}} = 1$$

$$\Rightarrow [\beta] = L^2 T^{-2}$$

$$\Rightarrow [\alpha] = \frac{[M^1 L^1 T^{-2}][L^2 T^{-2}]}{[L^2 T^{-2}]^2}$$

$$\Rightarrow [\alpha] = M^1 L^{-1} T^0.$$

Uses of Dimensions

(1) To Check the Correctness of the Formula

If the dimensions of the L.H.S and R.H.S are same, then we can say that this equation is at least dimensionally correct. So this equation may be correct.

But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct.

So it cannot be correct.

ei → A formula is given centrifugal force

$$F_c = \frac{mv^2}{r} \quad (\text{where } m = \text{mass, } v = \text{velocity, } r = \text{radius})$$

we have to check whether it is correct or not.

Dimension of L.H.S is

$$[F] = [M^1 L^1 T^{-2}]$$

Dimension of R.H.S is

$$\frac{[m][v^2]}{[r]} = \frac{[M][LT^{-1}]^2}{[L]} = [M^1 L^1 T^{-2}]$$

So this equation is at least dimensional correct.

⇒ We can say that this equation may be correct.

SOLVED EXAMPLES

5. Check whether this equation may be correct or not
Pressure

$$P_r = \frac{3Fv^2}{\pi^2 t^2 x} \quad (\text{where } F = \text{force, } v = \text{velocity, } t = \text{time, } x = \text{distance})$$

Solution:

$$\text{Dimension of L.H.S} = [P_r] = M^1 L^{-1} T^{-2}$$

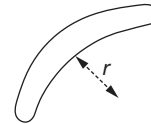
$$\text{Dimension of R.H.S} = \frac{[3][F][v^2]}{[\pi][t^2][x]}$$

$$= \frac{[M^1 L^1 T^{-2}][L^2 T^{-2}]}{[T^2][L]} = M^1 L^2 T^{-6}$$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.

Sometimes a question is asked which is beyond our syllabus, then certainly it must be the question of dimensional analyses.

6. A Boomerang has mass m surface Area A , radius of curvature of lower surface = r and it is moving with velocity v in air of density ρ . The resistive force on it should be



- (A) $\frac{2\rho v A}{r^2} \log\left(\frac{\rho m}{\pi A r}\right)$ (B) $\frac{2\rho v^2 A}{r} \log\left(\frac{\rho A}{\pi m}\right)$
- (C) $\frac{2\rho v^2 A}{r} \log\left(\frac{\rho A r}{\pi m}\right)$ (D) $\frac{2\rho v^2 A}{r^2} \log\left(\frac{\rho A r}{\pi m}\right)$

Solution:

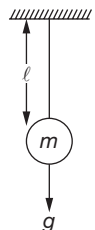
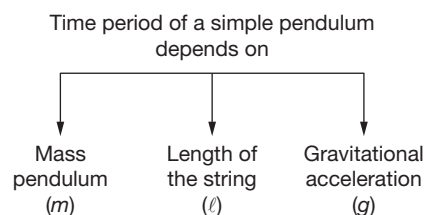
Only C is dimensionally correct.

(2) We can Derive a New Formula Roughly

If a quantity depends on many parameters, we can estimate, to what extent, the quantity depends on the given parameters!

SOLVED EXAMPLES

7. Derive the formula of time period of simple pendulum if



Solution:

So we can say that expression of T should be in this form

$$T = (\text{Some Number}) (m)^a (\ell)^b (g)^c$$

$$M^0 L^0 T^1 = (1) [M^1]^a [L^1]^b [L^1 T^{-2}]^c$$

$$M^0 L^0 T^1 = M^a L^{b+c} T^{-2c}$$

Comparing the powers of M, L and T ,

get $a = 0, b + c = 0, -2c = 1$

so $a = 0, b = \frac{1}{2}, c = \frac{1}{2}$

so $T = (\text{Some Number}) M^0 L^{1/2} g^{-1/2}$

$$T = (\text{Some Number}) \sqrt{\frac{\ell}{g}}$$

The quantity ‘‘Some number’’ can be found experimentally. Measure the length of a pendulum and oscillate it, find its time period by stopwatch.

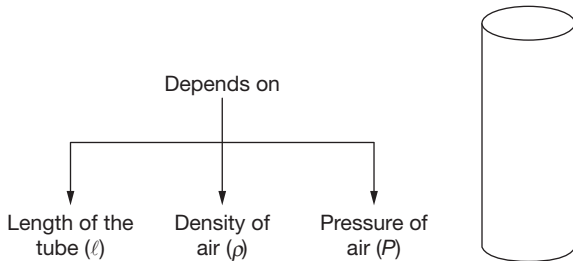
Suppose for $\ell = 1\text{m}$,

we get $T = 2 \text{ sec. so}$

$$2 = (\text{Some Number}) \sqrt{\frac{1}{9.8}}$$

\Rightarrow ‘‘Some number’’ = $6.28 \approx 2\pi$.

8. Derive the formula of natural frequency in a closed pipe if it



Solution:

So we can say that

$$f = (\text{Some Number}) (\ell)^a (\rho)^b (P)^c$$

Equating dimensions of both the sides get

$$a = -1,$$

$$b = -1/2,$$

$$c = 1/2$$

So $f = (\text{some number}) \frac{1}{\ell} \sqrt{\frac{P}{\rho}}$

(3) We can Express any Quantity in Terms of the given Basic Quantities

SOLVED EXAMPLE

9. If velocity (v), force (F) and time (T) are chosen as fundamental quantities, express (i) mass and (ii) energy in terms of v, F and T

Solution:

Let

$$M = (\text{Some Number}) (v)^a (F)^b (T)^c$$

Equating dimensions of both the sides

$$M^1 L^0 T^0 = (1) [L^1 T^{-1}]^a [M^1 L^1 T^{-2}]^b [T^1]^c$$

$$M^1 L^0 T^0 = M^b L^{a+b} T^{-a-2b+c}$$

get $a = -1, b = 1, c = 1$

$$M = (\text{Some Number}) (v^{-1} F^1 T^1)$$

$\Rightarrow [M] = [v^{-1} F^1 T^1]$

Similarly we can also express energy in terms of v, F, T

Let $[E] = [\text{Some Number}] [v]^a [F]^b [T]^c$

$\Rightarrow [MLT^{-2}] = [M^b L^a T^0] [L^1 T^{-1}]^a [MLT^{-2}]^b [T]^c$

$\Rightarrow [M^1 L^1 T^{-2}] = [M^b L^{a-2b+c} T^{-a-2b+c}]$

$\Rightarrow 1 = b; 1 = a - 2b + c; -2 = -a - 2b + c$

get $a = 1; b = 1; c = 1$

$\therefore E = (\text{Some Number}) v^1 F^1 T^1$

or $[E] = [v^1][F^1][T^1]$.

(4) To Find Out Unit of a Physical Quantity

SOLVED EXAMPLE

10. [Force] = $[M^1 L^1 T^{-2}]$

Solution:

As unit of M is kilogram (kg), unit of L is meter (m) and unit of T is second (s)

so unit force can be written as = $(\text{kg})^1 (\text{m})^1 (\text{s})^{-2}$ = kg m/s in MKS system.

In CGS system, unit of force can be written as = $(\text{g})^1 (\text{cm})^1 (\text{s})^{-1}$ = g cm/s.

Limitations of Dimensional Analysis

From Dimensional analysis we get

$$T = (\text{Some Number}) \sqrt{\frac{\ell}{g}}$$

so the expression of T can be

$$T = 2 \sqrt{\frac{\ell}{g}}$$

or

$$T = 50 \sqrt{\frac{\ell}{g}}$$

or

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

$$T = \sqrt{\frac{\ell}{g}} \sin (\dots)$$

or

$$T = \sqrt{\frac{\ell}{g}} \log (\dots)$$

or

$$T = \sqrt{\frac{\ell}{g}} + (t_0)$$

1. Dimensional analysis doesn't give information about the "Some Number" The dimensional constant.
2. This method is useful only when a physical quantity depends on other quantities by multiplication and power relations.

$$\text{(i.e., } f = x^a y^b z^c \text{)}$$

3. It fails if a physical quantity depends on sum or difference of two quantities

$$\text{(i.e. } f = x + y - z \text{)}$$

i.e., we cannot get the relation

$$S = ut + \frac{1}{2} at^2 \text{ from dimensional analysis.}$$

4. This method will not work if a quantity depends on another quantity as sine or cosine, logarithmic or exponential relation. The method works only if the dependence is by power functions.

We equate the powers of M , L and T hence we get only three equations. So we can have only three variables (only three dependent quantities)

So dimensional analysis will work only if the quantity depends only on three parameters, not more than that.

SOLVED EXAMPLE

11. Can Pressure (P), density (ρ) and velocity (v) be taken as fundamental quantities?

Solution:

P , ρ and v are not independent, they can be related as $P = \rho v^2$, so they cannot be taken as fundamental variables.

To check whether the P , ρ , and v are dependent or not, we can also use the following mathematical method:

$$[P] = [M^1 L^{-1} T^{-2}]$$

$$[\rho] = [M^1 L^{-3} T^0]$$

$$[v] = [M^0 L^1 T^{-1}]$$

Check the determinate of their powers:

$$\begin{vmatrix} 1 & -1 & -2 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 1(3) - (-1)(-1) - 2(1) = 0,$$

So these three terms are dependent.

UNITS

Unit

Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic standard called unit.

SI units

In 1971, an international Organization "CGPM" (General Conference on weight and Measure) decided the standard units, which are internationally accepted. These units are called SI units (International system of units)

SI Units of Basic Quantities

Base Quantity	SI Units		
	Name	Symbol	Definition
Length	meter	m	The meter is the length of the path travelled by light in vacuum during a time interval of $1/299,792,458$ of a second (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)

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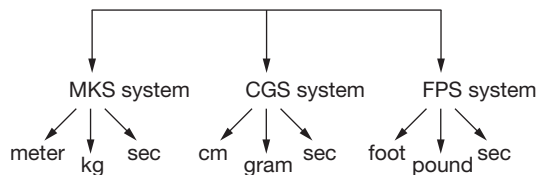
Base Quantity	SI Units		
	Name	Symbol	Definition
Time	second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per metre of length (1948)
Thermodynamic Temperature	kelvin	K	The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	candela	Cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/1683 watt per steradian (1979).

Two Supplementary Units were also Defined

- Plane angle – Unit = radian (rad)
- Solid angle – Unit = Steradian (sr)

Other Classification

If a quality involves only length, then mass and time (quantities in mechanics), and its unit can be written in MKS, CGS or FPS system.



- **For MKS system:** In this system Length, mass and time are expressed in meter, kg and sec. respectively. It comes under SI system.
- **For CGS system:** In this system, length, mass and time are expressed in cm, gram and sec. respectively.
- **For FPS system:** In this system, length, mass and time are measured in foot, pound and sec. respectively.

SI Units of Derived Quantities

- Velocity = $\frac{\text{displacement}}{\text{time}}$ → meter → second
So unit of velocity will be m/s
- Acceleration = $\frac{\text{change in velocity}}{\text{time}} = \frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}^2}$
- Momentum = mv
so unit of momentum will be = (kg) (m/s) = kg m/s

- Force = ma
Unit will be = (kg) × (m/s²) = kg m/s² called newton (N)
- Work = FS
Unit = (N) × (m) = N m called joule (J)
- Power = $\frac{\text{work}}{\text{time}}$
Unit = J/s called watt (w)

Units of Some Physical Constants

- Unit of “Universal Gravitational Constant” (G)

$$F = \frac{G(m_1)(m_2)}{r^2}$$

$$\Rightarrow \frac{\text{kg} \times \text{m}}{\text{s}^2} = \frac{G(\text{kg})(\text{kg})}{\text{m}^2}$$
 so unit of $G = \frac{\text{m}^3}{\text{kg s}^2}$
- Unit of specific heat capacity (S)
 $Q = ms \Delta T$
 $J = (\text{kg}) (S) (K)$
 Units of S = J/kg K
- Unit of μ_0
 Force per unit length between two long parallel wires
 is $\frac{F}{\ell} = \frac{\mu_0}{4\pi} \frac{i_1 i_2}{r^2}$
 $\frac{N}{m} = \frac{\mu_0}{(1)} \frac{(A) (A)}{(m^2)}$
 Unit of $\mu_0 = \frac{N \cdot m}{A^2}$

SI Prefix

Suppose distance between kota to Jaipur is 3000 m. so

$$d = 3000 \text{ m} = 3 \times \underset{\substack{\downarrow \\ \text{kilo(k)}}}{1000} \text{ m}$$

$$= 3 \text{ km (here } k \text{ is the prefix used for } 1000 (10^3))$$

Suppose thickness of a wire is 0.05 m

$$d = 0.05 \text{ m} = 5 \times \underset{\substack{\downarrow \\ \text{centi(c)}}}{10^{-2}} \text{ m}$$

$$= 5 \text{ cm (here } c \text{ is the prefix used for } (10^{-2}))$$

Similarly, the magnitude of physical quantities is over a wide range. So in order to express the very large magnitude as well as very small magnitude more compactly, "CGPM" recommended some standard prefixes for certain power of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^9	giga	G	10^{-6}	micro	μ
10^6	mega	M	10^{-9}	nano	n
10^3	kilo	k	10^{-12}	pico	p
10^2	hecto	h	10^{-15}	femto	f
10^1	deca	da	10^{-18}	atto	a

SOLVED EXAMPLES

12. Convert all in meters (m):

- (A) $5 \mu\text{m}$ (B) 3 km (C) 20 mm
(D) 73 pm (E) 7.5 nm

Solution:

- (A) $5 \mu\text{m} = 5 \times 10^{-6} \text{ m}$
(B) $3 \text{ km} = 3 \times 10^3 \text{ m}$
(C) $20 \text{ mm} = 20 \times 10^{-3} \text{ m}$
(D) $73 \text{ pm} = 73 \times 10^{-12} \text{ m}$
(E) $7.5 \text{ nm} = 7.5 \times 10^{-9} \text{ m}$

13. $F = 5 \text{ N}$ convert it into CGS system

Solution:

$$F = 5 \frac{\text{kg} \times \text{m}}{\text{s}^2}$$

$$= (5) \frac{(10^3 \text{ g})(100 \text{ cm})}{\text{s}^2}$$

$$= 5 \times 10^5 \frac{\text{g cm}}{\text{s}^2} \text{ (in CGS system)}$$

This unit $\left(\frac{\text{g cm}}{\text{s}^2}\right)$ is also called dyne.

14. $G = 6.67 \times 10^{-11} \frac{\text{kg m}^3}{\text{s}^2}$ convert it into CGS system.

Solution:

$$\begin{aligned} G &= 6.67 \times 10^{-11} \frac{\text{kg m}^3}{\text{s}^2} \\ &= (6.67 \times 10^{-11}) (1000 \text{ g}) \frac{(100 \text{ cm})^3}{\text{s}^2} \\ &= 6.67 \times 10^{-2} \frac{\text{g cm}^3}{\text{s}^2}. \end{aligned}$$

15. $\rho = 2 \text{ g/cm}^3$ convert it into MKS system

Solution:

$$\begin{aligned} \rho &= 2 \text{ g/cm}^3 \\ &= (2) \frac{10^{-3} \text{ kg}}{(10^{-2} \text{ m})^3} \\ &= 2 \times 10^3 \frac{\text{kg}}{\text{m}^3}. \end{aligned}$$

16. $v = 90 \text{ km/hour}$ convert it into m/s

Solution:

$$\begin{aligned} v &= 90 \text{ km/hour} \\ &= (90) \frac{(1000 \text{ m})}{(60 \times 60 \text{ s})} \\ v &= (90) \left(\frac{1000}{3600}\right) \frac{\text{m}}{\text{s}} \\ v &= 90 \times \frac{5}{18} \frac{\text{m}}{\text{s}} \\ v &= 25 \text{ m/s}. \end{aligned}$$

**POINT TO REMEMBER**

To convert $\frac{\text{km}}{\text{hour}}$ into $\frac{\text{m}}{\text{s}}$, multiply by $\frac{5}{18}$.

1.10 Chapter 1

17. Convert 7 pm into μm

Solution:

Let $7 \text{ pm} = (x) \mu\text{m}$
 $7 \times (10^{-12}) \text{ m} = (x) \times 10^{-6} \text{ m}$

get $x = 7 \times 10^{-6}$

So $7 \text{ pm} = (7 \times 10^{-6}) \mu\text{m}$

Some SI units of derived quantities are named after the scientist, who has contributed in that field a lot.

SI Derived Units, Named After the Scientist

S.N.	Physical Quantity	SI Units			
		Unit name	Symbol of the unit	Expression in terms of other units	Expression in terms of base units
1.	Frequency $\left(f = \frac{1}{T}\right)$	Hertz	Hz	$\frac{\text{Oscillation}}{\text{s}}$	s^{-1}
2.	Force $(F = ma)$	Newton	N	-	kg m/s^2
3.	Energy, Work, Heat $(W = Fs)$	Joule	J	Nm	$\text{kg m}^2/\text{s}^2$
4.	Pressure, stress $\left(P = \frac{F}{A}\right)$	Pascal	Pa	N/m^2	kg/m s^2
5.	Power, $\left(\text{Power} = \frac{W}{t}\right)$	Watt	W	J/s	$\text{kg m}^2/\text{s}^3$
6.	Electric charge $(q = it)$	Coulomb	C	-	A s
7.	Electric Potential Emf. $\left(V = \frac{U}{q}\right)$	Volt	V	J/C	$\text{kg m}^3/\text{s}^3 \text{ A}$
8.	Capacitance $\left(C = \frac{q}{v}\right)$	Farad	F	C/V	$\text{A s}^4 \text{ kg}^{-1} \text{ m}^{-2}$
9.	Electrical Resistance $(V = i R)$	Ohm	Ω	V/A	$\text{kg m}^2 \text{ s}^{-3} \text{ A}^{-2}$
10.	Electrical Conductance $\left(C = \frac{1}{R} = \frac{i}{V}\right)$	Siemens (mho)	S, Ω	A/V	$\text{kg}^{-1} \text{ m}^{-2} \text{ s}^3 \text{ A}^2$
11.	Magnetic field	Tesla	T	Wb/m^2	$\text{kg s}^{-2} \text{ A}^{-1}$
12.	Magnetic flux	Weber	Wb	V s or J/A	$\text{kg m}^2 \text{ s}^{-2} \text{ A}^{-1}$
13.	Inductance	Henry	H	Wb/A	$\text{kg m}^2 \text{ s}^{-2} \text{ A}^{-2}$
14.	Activity of radioactive material	Becquerel	Bq	$\frac{\text{Disintegration}}{\text{second}}$	s^{-1}

Some SI Units Expressed in Terms of the Special Names and also in Terms of Base Units

Physical Quantity	SI Units	
	In terms of special names	In terms of base units
Torque ($\tau = Fr$)	N m	kg m ² /s ²
Dynamic Viscosity ($F_v = qA \frac{dv}{dr}$)	Poiseiulles (P ℓ) or Pa s	kg/m s
Impulse (J = F Δ t)	N s	kg m/s
Modulus of elasticity ($Y = \frac{\text{stress}}{\text{strain}}$)	N/m ²	kg/m s ²
Surface Tension Constant ($T = \frac{F}{\ell}$)	N/m or J/m ²	kg/s ²
Specific Heat capacity (Q = ms Δ T)	J/kg K (old unit s $\frac{\text{call}}{\text{g. } ^\circ\text{C}}$)	m ² s ⁻² K ⁻¹
Thermal conductivity ($\frac{dQ}{dt} = kA \frac{dT}{dr}$)	W/m K	m kg s ⁻³ K ⁻¹
Electric field Intensity $E = \frac{F}{q}$	V/m or N/C	m kg s ⁻³ A ⁻¹
Gas constant (R) (PV = nRT) or molar Heat Capacity ($C = \frac{Q}{M\Delta T}$)	J/K mol	m ² kg s ⁻² k ⁻¹ mol ⁻¹

Change of Numerical Value with the Change of Unit

Suppose we have

$$\ell = 7 \text{ cm} \xrightarrow{\text{If we convert it into meters, we get}} \\ = \frac{7}{100} \text{ m}$$

we can say that if the unit is increased to 100 times (cm → m), the numerical value became

$$\frac{1}{100} \text{ times } \left(7 \rightarrow \frac{7}{100} \right)$$

So we can say

$$\text{Numerical value} \propto \frac{1}{\text{unit}}$$

we can also tell if in a formal way like the following

Magnitude of a physical quantity = (Its Numerical value) (unit)

$$= (n) (u)$$

Magnitude of a physical quantity always remains constant and it won't change if we express it in some other unit.

So

$$\begin{array}{c} (n) \quad (u) = \text{Constant} \\ \swarrow \quad \searrow \\ n \propto \frac{1}{u} \quad n_1 u_1 = n_2 u_2 \end{array}$$

or

$$\text{Numerical value} \propto \frac{1}{\text{unit}}$$

SOLVED EXAMPLES

18. If unit of length is doubled, the numerical value of area will be ...

Solution:

As unit of length is doubled, unit of area will become four times. So the numerical value of area will become one fourth. Because numerical value $\propto \frac{1}{\text{unit}}$.

19. Force acting on a particle is 5N. If unit of length and time is doubled and unit of mass is halved than the numerical value of the force in the new unit will be.

Solution:

$$\text{Force} = 5 \frac{\text{kg} \times \text{m}}{\text{s}^2}$$

If unit of length and time are doubled and the unit of mass is halved.

$$\text{Then the unit of force will be } \left(\frac{\frac{1}{2} \times 2}{(2)^2} \right) = \frac{1}{4} \text{ times}$$

Hence the numerical value of the force will be 4 times.

SIGNIFICANT FIGURES

Suppose you want to measure the length of a sheet of paper with an ordinary scale. Place the zero mark of the scale exactly at one end of the sheet and read the mark at other end. You may obtain a doubtful digit. It means that exact reading is not possible. It can be understood from the Fig. 1.1 shown below.

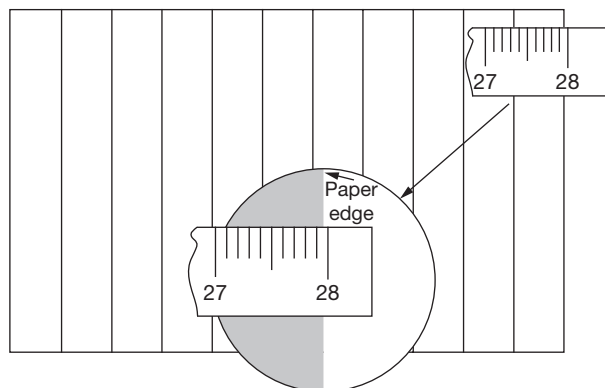


Fig. 1.1

The end of the sheet lies between 27.9 and 28.0 cm. Then you can estimate the distance between 27.9 cm and end of the sheet in this way. You mentally divide the 1 small division into 10 equal parts and guess on which part the edge is falling. You may note down the reading as 27.96 cm. In this reading the digits 2, 7 and 9 are certain but 6 is doubtful. All these digits including doubtful digit are called significant digits. The rightmost or doubtful digit is called the least significant digit and the leftmost digit is called the most significant digit. We can define in this way the reliable digits plus the 1st uncertain digit which are called significant digits or significant figure. So we can say, significant figures indicate the precision of measurement and it depends on the least count of the measuring instrument.

The Rules for Determining the Number of Significant Figures are as follows

1. All the non-zero digits are significant.
Example: 156, 78 contains five significant figures.
2. All the zeros between two non-zero digits are significant no matter where the decimal point is
Example: 108.006 contain six significant figures.
3. If the number is less than 1, the zeros on the right of decimal point but to the left of 1st non-zero digit are not significant.
Example: In 0.002308 the under lined zeros are not significant.
4. All the zeros to the right of the last non-zero digit (trailing zeros) in a number without a decimal point are not significant, unless they come from experiment. Thus 123 m = 12300 cm = 123000 mm has three significant figures. The trailing zeros are not significant. But if these are obtained from a measurement, they are significant.
5. The trailing zeros in a number with a decimal point are significant.
The number 3.500 or 0.06900 have four significant figures.

Now note that choice of change of different units does not change the number of significant digits or figures in measurement.

1. the length 2.308 cm has four significant figures, but in different units, the same value can be written as 0.02308 m or 23.08 mm. All these number have the same number of significant figures. It shows that location of decimal point does not matter in determining the number of significant figures.
When there are zeros at the right end of the number, then there may be some confusion.
2. If length is 500 mm and we don't know least count of the measuring instrument, then we can't be sure that last digits (zeros) are significant or not.

Scientific Notation

If the scale had marking only at each meter, then the digit 5 can be obtained by eye approximation. So only 5 is significant figure, but if the markings are at each centimetre, then only 5,0 of the reading will be significant. If the scale used have marking in millimetres, all three digits 5,00 are significant. To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement in scientific notation, every number should express as $a \times 10^b$ where a is between 1 and 10 and b is any +ve or -ve power of 10, and decimal is placed after the first digit.

Now the confusion mention above can be removed.

$$4.700 \text{ m} = 4.700 \times 10^2 \text{ cm} = 4.700 \times 10^3 \text{ mm}$$

Here power of 10 is irrelevant to the determination of significant figures.

Significant Figure in Algebraic Operation

To know the number of significant figures after an algebraic operation (Addition, subtraction, multiplication and division) certain rules can be followed which are as follows.

1. In multiplication or division, the number of significant digits in the final result should be equal to the number of significant digits in the quantity, which has the minimum number of significant digits.
Example: If mass of an object measured is 4.237 gm (four significant figures) and its volume is measured to be 2.51 (three significant figure) cm^3 , then its density

$$= \frac{\text{mass}}{\text{volume}} = \frac{4.237}{2.51} = 1.6486 \text{ gm/cm}^3$$
 But it should be up to three significant digits. Density = 1.65 gm/cm^3
2. In addition or subtraction the final result should retain as many decimal places as are there in the number with the least decimal place.

SOLVED EXAMPLE

20. Suppose we have to find out the sum of the numbers 436.32 gm, 227.2 g and 0.301 gm by arithmetic addition.

Solution:

$$\begin{array}{r}
 436.32 \\
 227.2 \\
 0.301 \\
 \hline
 663.821
 \end{array}$$

But the least precise measurement (227.2) gm is correct to only one decimal place. So final should be rounded off to one decimal place.

So sum will be 663.8 gm

In subtraction also we follow the similar rule.

Rounding Off the Uncertain Digit (Least Significant Digit)

The least significant digit is rounded according to the rules given below.

1. If the digit next to the least significant (Uncertain) digit is more than 5, the digit to be rounded is increased by 1.
2. If the digit next to the rounded one is less than 5, the digit to be rounded is left unchanged.
3. If the digit next to the rounded one is 5, then the digit to be rounded is increased by 1 if it is odd and left unchanged if it is even.

The insignificant digits are dropped from the result if they appear after the decimal point. Zero replaces them if they appear to the left of the decimal point.

SOLVED EXAMPLES

21. Suppose we have to round off three significant digits to 15462.

Solution:

In 15462, third significant digit is 4. This digit is to be rounded. The digit next to it is 6, which is greater than 5. The third digit should therefore be increased by 1. The digits to be dropped should be replaced by zeros, because they appear to the left of decimal point thus 15462 becomes 15500 on rounding to three significant figures.

22. Round off the following numbers to three significant digits

(A) 14.745 (B) 14.750 (C) 14.650×10^{12}

Solution:

- (A) The third significant digit in 14.745 is 7. The number next to it is less than 5. So 14.745 becomes 14.7 on rounding to three significant digits.
- (B) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.
- (C) 14.650×10^{12} will become 14.6×10^{12} because the digit to be rounded is even and the digit next to it is 5.

ERROR ANALYSIS IN MEASUREMENTS

Measurement is an important aspect of physics. Whenever we want to know about a physical quantity, we take its measurement first of all.

Instruments used in measurement are called measuring instruments.

Error

The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, also has an error.

Various Cause of Errors in Measurement**Least Count Error**

The least count error is the error associated with the resolution of the instrument. Least count may not be sufficiently small. The maximum possible error is equal to the least count.

Instrumental Error

This is due to faulty calibration or change in conditions (e.g. thermal expansion of a measuring scale). An instrument may also have zero error. A correction has to be applied.

Random Error

This is also called chance error. It makes to give different results for same measurements taken repeatedly.

Accidental Error

This error gives too high or too low results. Measurements involving this error are not included in calculations.

Systematic Error

The systematic errors are those errors that tend to be in one direction, either positive or negative. Errors due to air buoyancy in weighing and radiation loss in calorimetry are systematic errors. They can be eliminated by manipulation. Some of the sources of systematic errors are:

1. Instrumental Error
2. Imperfection in experimental technique or procedure
3. Personal errors

Absolute Error, Relative Error and Percentage Error

Mean of Observations

Suppose the observations of a physical quantity are $X_1, X_2, X_3 \dots X_n$.

Arithmetic mean of these observations X_{mean} is defined as

$$X_{\text{mean}} = (X_1 + X_2 + X_3 + \dots + X_n)/n$$

or
$$X_{\text{mean}} = \frac{1}{n} \sum_{i=1}^n x_i$$

The arithmetic mean of these observations is taken as best possible value of the quantity under the given conditions of measurement.

Absolute Error

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement.

(Arithmetic mean is taken as true value of number of observations.)

$$\Delta X_1 = X_{\text{mean}} - X_1,$$

$$\Delta X_2 = X_{\text{mean}} - X_2,$$

.....
.....

$$\Delta X_n = X_{\text{mean}} - X_n$$

The absolute error may be positive or negative.

Absolute Mean Error

The average of the mod of errors is called the average or mean error.

$$\Delta X_{\text{mean}} = (|\Delta X_1| + |\Delta X_2| + |\Delta X_3| \dots |\Delta X_n|)/n$$

Relative Error

The relative error is the ratio of the mean absolute error ΔX_{mean} to the mean value X_{mean} of the quantity measured.

$$\text{Relative error} = \Delta X_{\text{mean}} / X_{\text{mean}}$$

Percentage Error

When the relative error is expressed in percent, it is called as percentage error

$$\text{Thus, percentage error} = (\Delta X_{\text{mean}} / X_{\text{mean}}) \times 100\%$$

Combination of Errors

When we perform some experiment, different observations are used to get a result using algebraic operations like addition, subtraction, multiplication, division etc.

Now we need to calculate the errors in combination of various mathematical operations.

Errors in Sum or Difference

Let $X = A \pm B$

Further, let ΔA is the absolute error in the measurement of A , ΔB the absolute error in the measurement of B and ΔX is the absolute error in the measurement of x .

Then,
$$\begin{aligned} X + \Delta X &= (A \pm \Delta A) \pm (B \pm \Delta B) \\ &= (A \pm B) \pm (\Delta A \pm \Delta B) \\ &= X \pm (\Delta A \pm \Delta B) \end{aligned}$$

or
$$\Delta X = \pm \Delta A \pm \Delta B$$

The four possible values of ΔX

$$(\Delta A + \Delta B), (\Delta A - \Delta B), (-\Delta A - \Delta B) \text{ and } (-\Delta A + \Delta B)$$

Therefore, the maximum absolute error in X is,

$$\Delta X = \pm (\Delta A + \Delta B)$$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in individual quantities.

SOLVED EXAMPLE

23. The original length of a wire is (153.7 ± 0.6) cm. It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.

Solution:

$$\text{Elongation } (l) = 155.3 - 153.7 = 1.6 \text{ cm}$$

$$\Delta l = \pm (\Delta l_1 + \Delta l_2)$$

$$= \pm (0.6 + 0.2) = \pm 0.8 \text{ cm}$$

$$\therefore \text{Elongation} = (1.6 \pm 0.8) \text{ cm.}$$

Errors in Product

Let,
$$X = AB$$

Then $(X \pm \Delta X) = (A \pm \Delta A)(B \pm \Delta B)$

or $X \left(1 \pm \frac{\Delta X}{X}\right) = AB \left(1 \pm \frac{\Delta A}{A}\right) \left(1 \pm \frac{\Delta B}{B}\right)$

or $1 \pm \frac{\Delta X}{X} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$ (as $X = AB$)

or $\pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$

Here, $\left(\frac{\Delta A}{A} \cdot \frac{\Delta B}{B}\right)$ is a small quantity, so can be neglected.

Hence $\pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$

Possible values of $\frac{\Delta X}{X}$ are

$$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right), \left(\frac{\Delta A}{A} - \frac{\Delta B}{B}\right), \left(-\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \text{ and } \left(-\frac{\Delta A}{A} - \frac{\Delta B}{B}\right).$$

Hence, maximum possible value of $\frac{\Delta X}{X} = \pm \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Therefore maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

SOLVED EXAMPLE

24. The measures of the lengths of a rectangle are $l = (3.00 \pm 0.01)$ cm and breadth $b = (2.00 \pm 0.02)$ cm. What is the area of the rectangle?

Solution:

$$\text{Area} = lb = 3 \times 2 = 6 \text{ cm}^2$$

$$\text{Error} = \pm 6 \left(\frac{0.01}{3} + \frac{0.02}{2} \right) = \pm 0.08 \text{ cm}^2$$

$$\text{Area} = (6.00 \pm 0.08) \text{ cm}^2.$$

Errors in Division

Let, $X = \frac{A}{B}$

Then, $X \pm \Delta X = \frac{A \pm \Delta A}{B \pm \Delta B}$

or $X \left(1 \pm \frac{\Delta X}{X}\right) = \frac{A \left(1 \pm \frac{\Delta A}{A}\right)}{B \left(1 \pm \frac{\Delta B}{B}\right)}$

or $\left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A}\right) \left(1 \pm \frac{\Delta B}{B}\right)^{-1}$ (as $X = \frac{A}{B}$)

As $\frac{\Delta B}{B} \ll 1$, so expanding binomially, we get

$$\left(1 \pm \frac{\Delta X}{X}\right) = \left(1 \pm \frac{\Delta A}{A}\right) \left(1 \pm \frac{\Delta B}{B}\right)$$

or $1 \pm \frac{\Delta X}{X} = 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$

Here, $\frac{\Delta A}{A} \cdot \frac{\Delta B}{B}$ is small quantity, so can be neglected.

Therefore, $\pm \frac{\Delta X}{X} = \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B}$

Possible values of $\frac{\Delta X}{X}$ are

$$\left(\frac{\Delta A}{A} - \frac{\Delta B}{B}\right), \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right), \left(-\frac{\Delta A}{A} - \frac{\Delta B}{B}\right) \text{ and } \left(-\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$$

Therefore, the maximum value of $\frac{\Delta X}{X} = \pm \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

or, the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

SOLVED EXAMPLES

25. The change in the velocity of a body is (12.5 ± 0.2) m/s in a time (5.0 ± 0.3) s. Find the average acceleration of the body within error limits.

Solution:

Here

$$v = (12.5 \pm 0.2) \text{ m/s}; \quad t = (5.0 \pm 0.3) \text{ s}$$

$$\therefore a = \frac{12.5}{5.0} = 2.5 \text{ m/s}^2$$

Also

$$\frac{\Delta a}{a} = \pm \left(\frac{\Delta v}{v} + \frac{\Delta t}{t} \right) = \pm \left(\frac{0.2}{12.5} + \frac{0.3}{5.0} \right) = \pm 0.08$$

$$\Delta a = \pm (0.08 \times 2.5) = \pm 0.2$$

$$a = (2.5 \pm 0.2) \text{ m/s}^2.$$

Errors in Quantity Raised to Some Power

Let $X = \frac{A^n}{B^m}$

Then $\ln(X) = n \ln(A) - m \ln(B)$

Differentiating both sides, we get $\frac{dX}{X} = n \cdot \frac{dA}{A} - m \frac{dB}{B}$

In terms of fractional error we may write

$$\pm \frac{\Delta X}{X} = \pm n \frac{\Delta A}{A} \mp m \frac{\Delta B}{B}$$

Therefore maximum value of $\frac{\Delta X}{X} = \pm \left(n \frac{\Delta A}{A} + m \frac{\Delta B}{B} \right)$

26. The time period of oscillation of simple pendulum is given by $t = 2\pi \sqrt{l/g}$. What is the accuracy in the determination of g if 10 cm length is known to 1 mm accuracy and 0.5 s time period is measured from time of 100 oscillations with a watch of 1 s resolution?

Solution:

Here $\frac{\Delta l}{l} = \frac{0.1}{10}$

$\Delta t = 1$ s, and time of 100 oscillations $t = 100 \times 0.5 = 50$ s

$$\therefore \frac{\Delta t}{t} = \frac{1}{50}$$

From

$$t = 2\pi \sqrt{\frac{l}{g}}$$

$$\Rightarrow t^2 = 4\pi^2 \frac{l}{g}$$

$$\Rightarrow g = 4\pi^2 \frac{l}{t^2}$$

$$\therefore \frac{\Delta g}{g} = \pm \left(\frac{\Delta l}{l} + 2 \frac{\Delta t}{t} \right)$$

$$\therefore \text{error, } \frac{\Delta g}{g} \times 100 = \pm \left(\frac{0.1}{10} + \frac{2 \times 1}{50} \right) \times 100 = \pm 5\%$$

Operation	Formula	Maximum	Maximum	Maximum
		Absolute Error	Relative Error	Percentage Error
Sum	$A + B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A + B}$	$\left(\frac{\Delta A + \Delta B}{A + B} \right) \times 100$
Difference	$A - B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A - B}$	$\left(\frac{\Delta A + \Delta B}{A - B} \right) \times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right) \times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A + A\Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right) \times 100$
Power	A^n	$nA^{n-1} \Delta A$	$n \frac{\Delta A}{A}$	$n \frac{\Delta A}{A} \times 100$

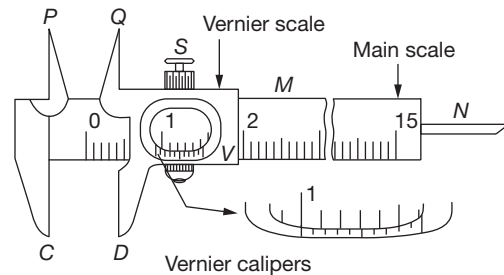
EXPERIMENTS

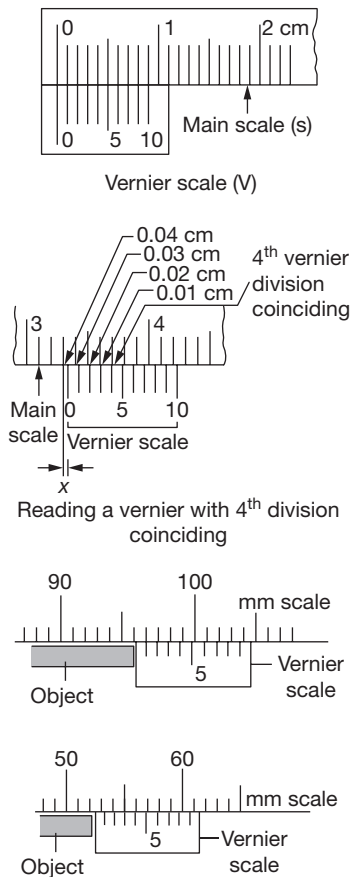
Measurement of Length

The simplest method measuring the length of a straight line is by means of a meter scale. But there exists some limitation in the accuracy of the result:

- The dividing lines have a finite thickness.
- Naked eye cannot correctly estimate less than 0.5 mm
For greater accuracy devices like
 - Vernier calipers
 - Micrometer scales (screw gauge) are used.

Vernier Calipers





It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the vernier scale is shorter than the divisions of the main scale.

Least Count of Vernier Calipers

The least count or vernier constant ($V.C$) is the minimum value of correct estimation of length without eye estimation. If N division of vernier coincides with $(N-1)$ division of main scale, then

$$N(VS) = (N-1) m/s$$

$$\Rightarrow 1VS = \frac{N-1}{N} m/s$$

$$\text{Vernier constant} = 1 m/s - 1 vs = \left(1 - \frac{N-1}{N}\right) m/s = \frac{1 m/s}{N}$$

which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

Length as Measured by Vernier Calipers

The formula for measuring the length is $L =$ main scale reading + least count of vernier scale \times vernier scale division coinciding with a main scale division

Main scale reading is given by the zeroth division of the vernier scale as shown in the Fig. 1.2.

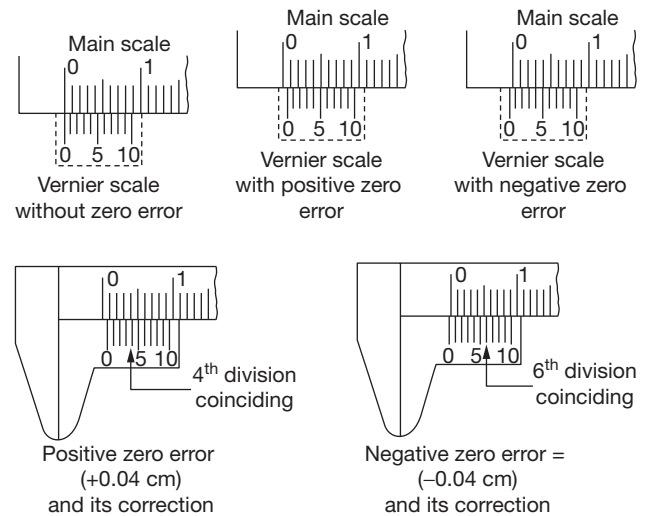


Fig. 1.2

Zero error

If the zero marking of main scale and vernier calipers do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument.

If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

The zero error is always subtracted from the reading to get the corrected value.

If the zero error is positive, its value is calculated as we take any normal reading. If the zero error negative (the zero of vernier scale lies to the left of the zero of main scale),

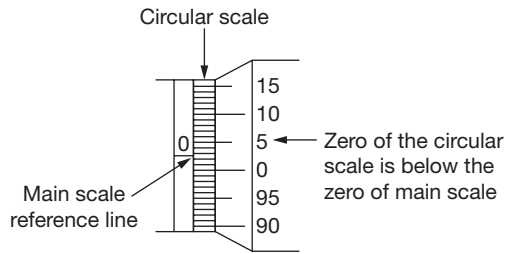
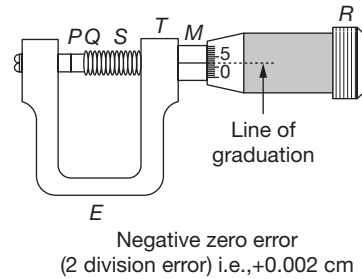
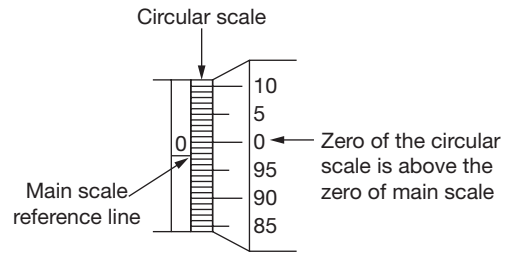
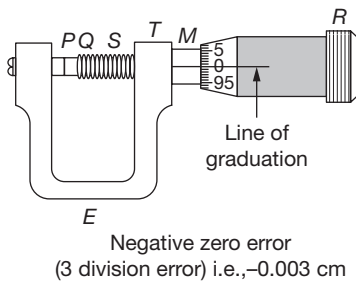
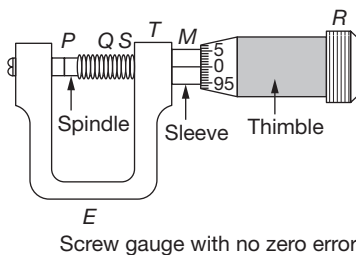
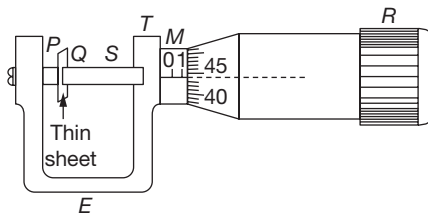
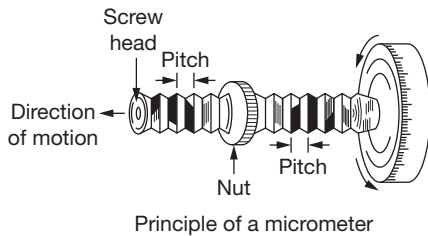
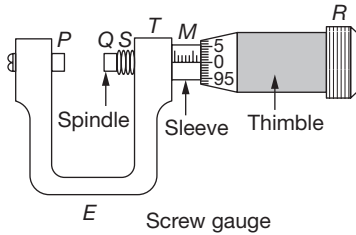
$$\text{Negative zero error} = -[\text{Total number of } vsd - vsd \text{ coinciding}] \times L.C.$$

Do not try to read the main scale at the point where the lines match best. This has no meaning. Read from the vernier scale instead. Sometimes it is difficult to tell whether the best match of lines is for vernier marks 9, 0, or 1. Make your best estimate, but realize that the final result including the vernier must round off to the result you would choose if there was no vernier. If the mark is close to 3.20 on the main scale, but the vernier reading is 9, the length is 3.19 cm. If the mark is close to 3.2 on the main scale and the vernier is 1, the length is 3.21 cm.

Screw Gauge (or Micrometer Screw)

In general vernier callipers can measure accurately upto 0.01 cm and for greater accuracy micrometer screw devices

e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by tuning it axially. The instrument is provided with two scales:



1. The main scale or pitch scale M graduated along the axis of the screw.
2. The cap-scale or head scale H round the edge of the screw head.

Constants of the Screw Gauge

1. **Pitch:** The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for 10 rotation of cap = 5 mm, then pitch = 0.5 mm
2. **Least count:** In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration:, if the total cap division is 100, then least count = $0.5 \text{ mm}/100 = 0.005 \text{ mm}$
3. **Measurement of length by screw gauge:** $L = n \times \text{pitch} + f \times \text{least count}$, where n = main scale reading and f = caps scale reading

Zero Error: In a perfect instrument the zero of the heat scale coincides with the line of graduation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies below the line of graduation and versa. The corresponding corrections will be just opposite.

BASIC MATHEMATICS

Algebra

Quadratic Equation

$ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c \in R$ is called a quadratic equation with real coefficients.

$$\text{Discriminant, } D = b^2 - 4ac$$

If $D > 0 \Rightarrow$ Two roots are real and unequal

If $D = 0 \Rightarrow$ Two roots are real and equal

If $D < 0 \Rightarrow$ Two roots are complex conjugate of each other.

Roots are given by

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

They are denoted by α and β .

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

i.e., $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Determinant

Determinant is a square arrangement of numbers.

They are represented by Δ or D .

SOLVED EXAMPLE

27. $\begin{vmatrix} 4 & 3 \\ 7 & 5 \end{vmatrix}$ is a determinant of order 2 as there are two rows and two columns.

$$\begin{vmatrix} 4 & -1 & 2 \\ 6 & 8 & 3 \\ 1 & 4 & 0 \end{vmatrix} \text{ is a determinant of order 3.}$$

Solution:

Determinants can be simplified to a single number.

The method to simplify a second order determinant is as follows

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

For 3rd order determinant:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - hf) - b(di - gf) + c(dh - ge)$$

A determinant can be expanded along any row or any column. We have to adhere to a sign scheme.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

For example if the same determinant is expanded along the 2nd column, we get

$$-b(di - gf) + e(ai - gc) - h(af - dc)$$

$$\therefore \Delta_2 = \begin{vmatrix} 4 & 3 \\ 7 & 5 \end{vmatrix} = (4)(5) - (7)(3) = 20 - 21 = -1$$

$$\begin{aligned} \therefore \Delta_3 &= \begin{vmatrix} 4 & -1 & 2 \\ 6 & 8 & 3 \\ 1 & 4 & 0 \end{vmatrix} \\ &= 2[(6)(4) - (1)(8)] - 3[(4)(4) - (1)(-1)] \\ &\quad + 0[(4)(8) - (6)(-1)] \end{aligned}$$

(expanding along last column)

$$= 2(16) - 3(17) = 32 - 51 = -19.$$

Exponential Factor

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

is called the exponential factor.

Its value is approximately 2.718 ...

It is an irrational number.

It is derived from the exponential series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

Logarithms

The equation $a^b = c$ in logarithmic notation is written as

$$\log_a c = b \cdot (a > 0 \cdot a \neq 1 \Rightarrow c > 0)$$

When $a = 10$, it is called common logarithm.

When $a = e$, it is called natural logarithm. (Also written as \ln)

We have the following results and rules:

1. $\log_a 1 = 0$
2. $\log_a a = 1$

3. $\log_a(cd) = \log_a c + \log_a d$

4. $\log_a\left(\frac{c}{d}\right) = \log_a c - \log_a d$

5. $\log_{a^\alpha} c^\gamma = \frac{\gamma}{\alpha} \log_a c$

6. $\log_a c = \frac{1}{\log_c a}$

7. $\log_a c = \frac{\log_k c}{\log_k a}$ (base change formula)

8. $a^{\log_a c} = c$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Sequences and Series

Arithmetic Sequence or Progression

$$a, a + d, a + 2d, a + 3d, \dots$$

(d is the common difference)

General term, $t_k = a + (k-1)d$ (k^{th} term)

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Arithmetic mean of two numbers a and b is defined

as $A = \frac{a+b}{2}$

Geometric Progression

$$a, ar, ar^2, ar^3, \dots$$

(r is the common ratio)

General term, $t_k = ar^{k-1}$ (k^{th} term)

Sum to n term, $S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1 = na, r = 1.$

Sum to infinite number of terms,

$$S_\infty = \frac{a}{1-r} \text{ when } |r| < 1$$

Geometric Mean of two numbers a and b is defined as

$$G = \sqrt{ab}.$$

Harmonic Progression

$$\frac{1}{a}, \frac{1}{a+b}, \frac{1}{a+2d}, \frac{1}{a+3d}, \dots$$

General term, $t_k = \frac{1}{a+(k-1)d}$ (k^{th} term)

Harmonic Mean of two numbers a and b is defined

as $H = \frac{2ab}{a+b}$

Series of Natural Numbers

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Algebraic Identities

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
4. $(a + b)^2 - (a - b)^2 = 4ab$
5. $(a + b)^2 = (a - b)^2 + 4ab$
6. $(a - b)^2 = (a + b)^2 - 4ab$

Trigonometry

1. Degree and Radian measure of angle
 π radians = 180 degrees i.e, $\pi = 180^\circ$.
2. Angle = $\frac{\text{Arc}}{\text{Radius}}$ i.e., $\theta = \frac{s}{r}$. In this formula θ is in radians.
3. Signs of trigonometric ratios
All t -ratios are positive in 1st quadrant.
Only sine and cosecant are positive in 2nd quadrant.
Only tangent and cotangent are positive in 3rd quadrant.
Only cosine and secant are positive 4th quadrant.
4. $\sin^2 x + \cos^2 x = 1$
 $\sec^2 x - \tan^2 x = 1$
 $\text{cosec}^2 x - \cot^2 x = 1$
5. $-1 \leq \sin x \leq 1$
 $-1 \leq \cos x \leq 1$
 $\text{cosec } x \geq 1 \text{ or } \text{cosec } x \leq -1$
 $\sec x \geq 1 \text{ or } \sec x \leq -1$
6. $\sec x$ and $\tan x$ are not defined for $x = (2n+1)\frac{\pi}{2}, n \in Z$.
 $\text{cosec } x$ and $\tan x$ are not defined for $x = n\pi, n \in Z$.
7. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
 $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
 $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \cdot \tan B}$
8. $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$
 $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$
 $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$
 $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$
9. $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$
 $\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos C - \cos D = 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$$

$$10. \sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos 2A = \begin{cases} 1 - 2 \sin^2 A \\ \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A} \\ 2 \cos^2 A - 1 \end{cases}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2} \text{ and } \cos^2 A = \frac{1 + \cos 2A}{2}$$

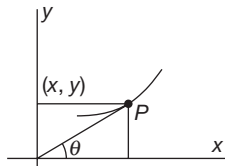
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Calculus

This is the most important tool in Mathematics and it has large number of applications in Physics and Chemistry. We shall here understand some basic concepts of Calculus. It is basically the study of functions and operations performed over them. It is split into two major divisions called as differential calculus and integral calculus.

Differential Calculus (dy/dx as Rate Measurer)

The derivative or differential coefficient of a function is the limit to which the ratio of the small increment in the function to the corresponding small increment in the variable (on which it depends) tends to, when the small increment in the variables approaches zero.



$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \text{Instantaneous change of } y \text{ with respect to } x$$

It is also equal to the slope of the tangent drawn at the point

where dy/dx is to be calculated

$$\frac{dy}{dx} = \tan \theta$$

Formula for calculating $\frac{dy}{dx}$ due to different functions

$$y = x^n; \frac{dy}{dx} = nx^{n-1}$$

SOLVED EXAMPLES

$$28. \text{ Find } \frac{d}{dx}(x^2)$$

Solution: $2x$

$$29. \text{ Find } \frac{d}{dx}(\sqrt{x})$$

$$\text{Solution: } \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$30. \text{ Find } \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$\text{Solution: } \frac{dx^{-2}}{dx} = -2x^{-3} = \frac{-2}{x^3}$$

Standard Differentiation Formulas

$$1. \frac{d}{dx}(x) = 1$$

$$2. y = \sin x; \frac{dy}{dx} = \cos x$$

$$3. y = \cos x; \frac{dy}{dx} = -\sin x$$

$$4. y = \tan x; \frac{dy}{dx} = \sec^2 x$$

$$5. y = \cot x; \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

$$6. y = \sec x; \frac{dy}{dx} = \sec x \tan x$$

$$7. y = \operatorname{cosec} x; \frac{dy}{dx} = -\operatorname{cosec} x \cot x$$

$$8. y = e^x; \frac{dy}{dx} = e^x$$

$$9. y = \log_e x; \frac{dy}{dx} = \frac{1}{x}$$

Rules

If $y = k f(x)$, where k is a constant

$$\text{Then, } \frac{dy}{dx} = k \frac{d}{dx}[f(x)]$$

SOLVED EXAMPLES

$$31. \text{ If } y = 3 \sin x \text{ find } \frac{dy}{dx}$$

Solution:

$$\frac{dy}{dx} = 3 \cos x$$

If $y = \text{constant}$

Then, $\frac{dy}{dx} = 0$

If $y = u \pm v \pm w$, where u, v and w are functions of x

Then, $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$.

32. If $y = e^x - \cos x + \log x$ find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = e^x + \sin x + \frac{1}{x}$$

33. If $y = e^x - \tan x + \log_e x$ find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = e^x - \sec^2 x + \frac{1}{x}$$

If $y = u \cdot v$

Then, $\frac{dy}{dx} = \frac{u dv}{dx} + \frac{v du}{dx}$

34. If $y = e^x \cos x$ find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = -e^x \sin x + e^x \cos x$$

If $y = u/v$

Then, $\frac{dy}{dx} = \frac{v du - u dv}{v^2}$.

35. If $y = \frac{\log x}{\tan x}$ find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \frac{(\tan x) \frac{1}{x} - \log x (\sec^2 x)}{\tan^2 x}$$

Examples Explaining Different Methods of Differentiation

Function of Function - Chain Rule

If $y = f(x)$ and x is a function of some other variable z .

Then $\frac{dy}{dx}$ can be written as the product of two derivatives.

$$\frac{dy}{dx} = \frac{dy}{dz} \frac{dz}{dx}$$

SOLVED EXAMPLES

36. If $y = \cos\{\log(x^n)\}$, Find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \frac{-\sin[\log x^n]}{x^n} \times n x^{n-1}$$

37. If $y = \sqrt{\sin(\log x)}$, find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dx} = \frac{1 \cos(\log x)}{2\sqrt{\sin \log x}} \times \frac{1}{x}$$

Parametric Form

$$x = f(t); \quad y = g(t)$$

where, t is a parameter

Then, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

SOLVED EXAMPLE

38. $\left. \begin{array}{l} y = a \sin t \\ x = a \cos t \end{array} \right\}$ where t is parameter

Find $\frac{dy}{dx}$

Solution:

$$\frac{dy}{dt} = a \cos t$$

$$\frac{dx}{dt} = -a \sin t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\cot(t)$$

Double Differentiation or Second Derivative

The second derivative of y with respect to x is defined as the derivative of the function $\frac{dy}{dx}$ (or the derivative of the

derivative). It is usually written $\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$.

SOLVED EXAMPLE

39. If $x = t^3 + t^2 + t + 1$, find $\frac{dx}{dt}$ and $\frac{d^2 x}{dt^2}$

Solution:

$$\frac{dx}{dt} = 3t^2 + 2t + 1; \quad \frac{d^2 x}{dt^2} = 6t + 2$$

Application of Differentiation in Physics

1. Instantaneous velocity $v = \frac{dx}{dt}$
2. Instantaneous acceleration $a = \frac{dv}{dt}$ also $a = v \frac{dv}{dx}$
3. Instantaneous force $F = \frac{dp}{dt}$ also $F = v \frac{dp}{dx} = \frac{p}{m} \frac{dv}{dx}$
4. Instantaneous power $P = \frac{dw}{dt}$ also $P = v \frac{dw}{dx}$
5. Instantaneous current $i = \frac{dq}{dt}$
6. $F_v = -\eta A \frac{dv}{dx}$; (where F_v = viscous force)
7. $F = -\frac{dU}{dx}$; (where, F = conservative force)



NOTE

$\frac{dy}{dx}$ = rate of change of y with respect to x , called gradient of y .

SOLVED EXAMPLES

40. The position of a particle travelling along x -axis is given by equation $x = \frac{t^3}{3} - \frac{3t^2}{2} + 2t$ (m). Here t is in seconds. Find
- (A) Initial velocity of particle.
 - (B) Find acceleration of the particle when it at rest.
 - (C) Find the velocity of the particle is when it is at equilibrium.

Solution:

(A) $v = \frac{dx}{dt} = t^2 - 3t + 2$ where $t = 0$, $v = 2$ m/s

(B) Rest means $v = 0$

$$t^2 - 3t + 2 = 0$$

$$\begin{array}{cc} & \swarrow \quad \searrow \\ t = 1\text{s} & & t = 2\text{s} \end{array}$$

Now,

$$a = \frac{dv}{dt} = 2t - 3$$

$$\begin{array}{cc} & \swarrow \quad \searrow \\ \text{at } t = 1\text{s} & & \text{at } t = 2\text{s} \\ a = -1 \text{ m/s}^2 & & a = 1 \text{ m/s}^2 \end{array}$$

(C) $F = 0 \Rightarrow a = 0$
 $2t - 3 = 0 \quad t = \frac{3}{2}$

$$v_{t=3/2} = \left(\frac{3}{2}\right)^2 - \frac{3 \times 3}{2} + 2 = \frac{9}{4} - \frac{9}{2} + 2 = \frac{-1}{4} \text{ m/s.}$$

41. The PE of a particle as a function of its position is given by $U = ax^2 - bx$. Find the minimum PE (Minimum PE is at equilibrium)

Solution:

$$F = \frac{-dU}{dx} \Rightarrow -2ax + b = 0 \Rightarrow x = \frac{+b}{2a}$$

At this position, PE is given by

$$U = a \times \left(\frac{b}{2a}\right)^2 - b \left(\frac{b}{2a}\right) = \left(a \times \frac{b^2}{4a^2}\right) - \frac{b^2}{2a}$$

$$U_{\min} = \frac{b^2}{4a} - \frac{b^2}{2a} = \frac{-b^2}{4a}.$$

42. The radius of a circle is increasing at a rate of 1 cm/s. Find the rate of increase of its area, when its radius is 10 cm.

Solution:

$$\frac{dA}{dt} = ?$$

$$\frac{dr}{dt} = 1 \text{ cm/s} \quad (\text{at } r = 10 \text{ cm})$$

Now,

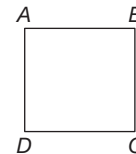
$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} \quad (\text{at } r = 10)$$

$$= \pi \times 2 \times 10 \times 1 = \pi \times 20 = 20\pi \text{ cm}^2/\text{s.}$$

43. Area of square shown increase at the rate of $100 \text{ cm}^2/\text{s}$. Find the velocity of separation of



(A) A and B

(B) A and C

when length of the its side becomes 20 cm.

Solution:

$$\frac{dA}{dt} = 100 \text{ cm}^2/\text{s}$$

$$A = \ell^2 \Rightarrow 100 = 2\ell \times \frac{d\ell}{dt} \Rightarrow \frac{d\ell}{dt} = 2.5 \text{ cm/s}$$

$$\therefore v_{AB} = 2.5 \text{ cm/s}$$

Now, For A and C

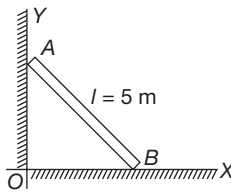


$$y = \sqrt{2} \ell \Rightarrow \frac{dy}{dt} = \sqrt{2} \frac{d\ell}{dt}$$

$$= \sqrt{2} \times 2.5$$

$$v_{AC} = 3.6 \text{ cm/s.}$$

44. A rod AB of length 5 m starts slipping such that B moves horizontally and A moves vertically. When $OB = 3$ m, the velocity of A is 3 m/s. Find the velocity of B at that moment. Also find the relation between speed of A and the speed of B , when the rod makes an angle θ with horizontal



Solution:

$$v_x = \frac{dx}{dt} = \text{speed of } B$$

and $v_y = \frac{dy}{dt} = \text{speed of } A$

$$x^2 + y^2 = \ell^2 \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

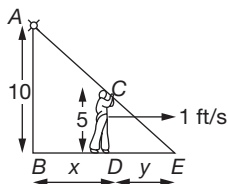
$$\Rightarrow xv_x + yv_y = 0$$

$$v_x = \frac{-y}{x} v_y = \frac{-4}{3} \times (-3)$$

$$v_x = 4 \text{ m/s}$$

$$v_x = \frac{-y}{x} v_y \text{ also means } v_x = (\tan \theta) v_y.$$

45. Man of height 5 ft is walking away from a lamp post of length 10 ft, find the rate at which length of his shadow will increase, if the speed of walking is 1 ft/s.



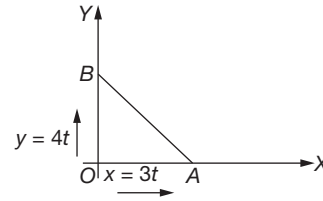
Solution:

Here, $\frac{y}{x+y} = \frac{5}{10} \Rightarrow 2y = x+y$ or $x = y$

$$\frac{dx}{dt} = \frac{dy}{dt} = 1 \text{ ft/s.}$$

46. Two cars A and B start from O simultaneously along OX and OY , with constant speeds 3 m/s and 4 m/s. Find their velocity of separation. (given $OX \perp OY$).

Solution:



$$S = 5t$$

$$v = \frac{dS}{dt} = 5 \text{ m/s.}$$

47. In the previous problem, both A and B start simultaneously from a point O . A with a constant speed of 2 m/s and B in perpendicular direction from rest but with a constant acceleration of 2 m/s². Find their velocity of separation at $t = 2$ s.

Solution:

$$S = \sqrt{(t^2)^2 + (2t)^2} = \sqrt{t^4 + 4t^2}$$

$$v = \frac{dS}{dt} = \frac{4t^3 + 8t}{2\sqrt{t^4 + 4t^2}}$$

At $t = 2$

$$\frac{dS}{dt} = 3\sqrt{2} \text{ m/s}$$

Maximum and Minimum

Suppose a quantity y depends on another quantity x in a manner shown in Fig. 1.3. It becomes maximum at x_1 and minimum at x_2 . At these points the tangent to the curve is parallel to the x -axis and hence its slope is zero i.e., $\tan \theta = 0$. But the slope of curve $y-x$ equals the rate of change of y with respect to x is $\frac{dy}{dx}$. Thus, at a maximum or minimum thus value of y , $\frac{dy}{dx} = 0$.

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. Thus $\frac{dy}{dx}$ decrease at a maximum and hence

rate of change of $\frac{dy}{dx}$ is negative, at a maximum i.e.,
 $\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0$ at a maximum.

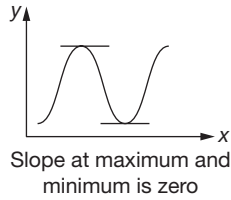


Fig. 1.3

The quantity $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ is the rate of change of the slope.

It is written as $\frac{d^2y}{dx^2}$. Thus the condition of a maximum is

$\frac{dy}{dx} = 0$; and $\frac{d^2y}{dx^2} < 0$. Similarly, at a minimum the slope changes from negative to positive. The slope increases at such a point and hence $\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$. The condition of a minimum is

$$\frac{dy}{dx} = 0$$

and

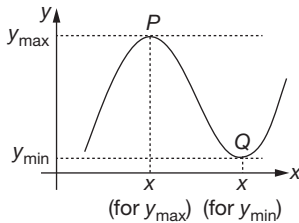
$$\frac{d^2y}{dx^2} > 0$$

at

$$P; \frac{dy}{dx} = 0; \frac{d^2y}{dx^2} = -ve$$

at

$$Q; \frac{dy}{dx} = 0; \frac{d^2y}{dx^2} = +ve$$



Quite often it is well understood from the physical situation whether the quantity is a maximum or a minimum. The test on $\frac{d^2y}{dx^2}$ may then be omitted.

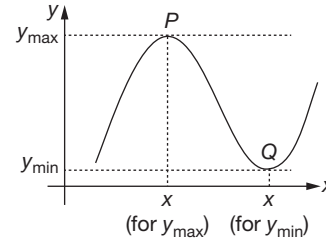
SOLVED EXAMPLES

48. If $y = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + 1$, find maximum and minimum value of y and x corresponding to y (maximum) and y (minimum)

Solution:

$$\text{For } y_{\max} \text{ or } y_{\min} \frac{dy}{dx} = 0 \Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow x = 1 \text{ or } x = 2$$



Now,

$$\frac{d^2y}{dx^2} = 2x - 3$$

$$\frac{d^2y}{dx^2} = -ve \quad \text{and} \quad \frac{d^2y}{dx^2} = +ve$$

(at $x = 1$) (at $x = 2$)

$\therefore y$ is max at $x = 1$

and y is min at $x = 2$

$$y_{\max} = \frac{1}{3} - \frac{3 \times 1^2}{2} + 2 \times 1 + 1 = \frac{11}{6}$$

$$y_{\min} = \frac{2}{3} - \frac{3 \times 2^2}{2} + 2 \times 2 + 1 = \frac{5}{3}$$

49. Find y (maximum) and y (minimum) for the function.

$$y = \frac{x^2}{2} - x$$

Solution:

$$\frac{dy}{dx} = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$$

Now,

$$d^2y/dx^2 = 1 > 0 \quad \therefore \text{(only minimum exists)}$$

$$y_{\min} = \frac{1}{2} - 1 = \frac{1-2}{2} = -\frac{1}{2}$$

Integration

If $y = F(x)$

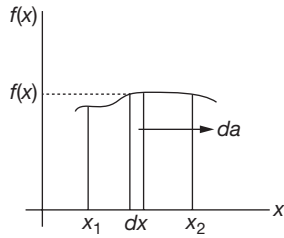
and

$$\frac{dy}{dx} = f(x)$$

$y_2 - y_1 = \int_{x_1}^{x_2} dy = \int_{x_1}^{x_2} f(x) dx = \text{total change in } y \text{ as } x \text{ changes from } x_1 \text{ to } x_2$

$$= [F(x)]_{x_1}^{x_2} = F(x_2) - F(x_1)$$

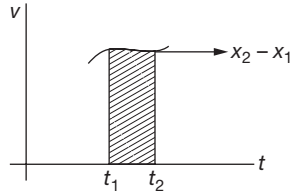
Geometrical meaning of integration



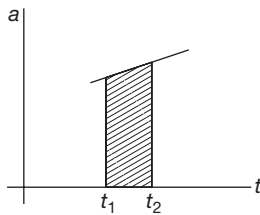
Area under $f(x)$ vs. x -graph from x_1 to x_2 $A = \int_{x_1}^{x_2} f(x)dx$

Application of Integration Physics

1. $\because v = \frac{dx}{dt}$
 $\Rightarrow x_2 - x_1 = \int_{t_1}^{t_2} v dt$



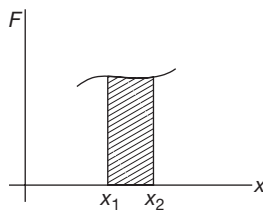
2. $a = \frac{dv}{dt} \Rightarrow v_2 - v_1 = \int_{t_1}^{t_2} a dt$



3. $F = \frac{dP}{dt}$
 $\Rightarrow p_2 - p_1 = J = \int_{t_1}^{t_2} F dt$

4. $W = \int_{t_1}^{t_2} P dt$

5. $W = \int_{t_1}^{t_2} P dt$



$W = U_1 - U_2 = \int_{x_1}^{x_2} F dx$

6. $i = \frac{dq}{dt} \Rightarrow q_2 - q_1 = \int_{t_1}^{t_2} i dt$

Formulae in Integration

$\int x^n dx = \frac{x^{n+1}}{n+1} + C ; x \neq -1$

$\int \cos x \cdot dx = \sin x + C$

$\int \sin x \cdot dx = -\cos x + C$

$\int \sec^2 x \cdot dx = \tan x + C$

$\int \operatorname{cosec}^2 x \cdot dx = -\cot x + C$

$\int \sec x \tan x dx = \sec x + C$

$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$

$\int e^x dx = e^x + C$

$\int \frac{1}{x} dx = \log_e x + C$

In the above formulae c is called constant of integration

Rules for Integration

$\int (U \pm V \pm W) dx = \int U dx \pm \int V dx \pm \int W dx$

$\int n f(x) dx = n \int f(x) dx$

(where n = constant)

SOLVED EXAMPLES

50. Find $\int_1^2 (3x^2 + 2x + 1) dx$

Solution:

$\left[\frac{3x^{2+1}}{3} + \frac{2x^{1+1}}{2} + x \right]_1^2$
 $= [x^3 + x^2 + x]_1^2 = (8 + 4 + 2) - (1 + 1 + 1) = 11$

51. Find $\int_0^{\pi/4} \sin x dx$

Solution: $[-\cos x]_0^{\pi/4} = -\left[\frac{1}{\sqrt{2}} - 1 \right] = 1 - \frac{1}{\sqrt{2}}$

52. $\int \sec(2x + 3) \tan(2x + 3) dx$

Solution: $= \frac{\sec(2x + 3)}{2} + C$

53. The instantaneous power delivered by an engine is given by

$P = (2t + 1)(t + 1) W$

Find work done by the engine between

$t = 1 \text{ s}$ and $t = 2 \text{ s}$.

Solution:

$$\begin{aligned} W &= \int_1^2 P dt = \int_1^2 (2t+1)(t+1) dt \\ &= \int_1^2 (2t^2 + 2t + t + 1) dt = \int_1^2 (2t^2 + 3t + 1) dt \\ \Rightarrow & \left[\frac{2}{3}t^3 + \frac{3}{2}t^2 + t \right]_1^2 \\ &= \left(\frac{2}{3} \times 8 + \frac{3}{2} \times 4 + 2 \right) - \left(\frac{2}{3} + \frac{3}{2} + 1 \right) = \frac{61}{6} \text{ J.} \end{aligned}$$

54. Force experienced by a particle in terms of its position $F = 3x^2 - 2x$. Calculate the work done in displacing the particle from $x = 1$ to $x = 2$.

Solution:

$$W = \int_1^2 F dx = \int_1^2 (3x^2 - 2x) dx = (x^3 - x^2) \Big|_1^2 = 4 \text{ J.}$$

55. Acceleration of a particle travelling along x -axis is given by

$$a = (2t + 1) \text{ m/s}^2$$

Find its velocity at $t = 2$ s if velocity at $t = 1$ s is 5 m/s

Solution:

$$\begin{aligned} v_{t=2} - v_{t=1} &= \int_1^2 a dt \\ v_{t=2} - 5 &= \int_1^2 (2t + 1) dt \Rightarrow v_{t=2} - 5 = \left[t^2 + t \right]_1^2 \\ \Rightarrow v_{t=2} - 5 &= 6 - 2 \\ \therefore v_{t=2} &= 9 \text{ m/s} \end{aligned}$$

56. In the above problem, find the position at $t = 3$ s, if initially position is $x = 10$ m.

Solution:

$$\begin{aligned} v_{t=t} - v_{t=1} &= \int_1^t (2t + 1) dt \\ v_t - 5 &= t^2 + t \Big|_1^t \\ v_t - 5 &= t^2 + t - 2 \\ v_t &= t^2 + t + 3 \\ x_2 - x_1 &= \int_{t_1}^{t_2} v dt \\ \Rightarrow x_{t=3} - 10 &= \int_0^3 (t^2 + t + 3) dt \\ \Rightarrow x_{t=3} - 10 &\Rightarrow \left[\frac{t^3}{3} + \frac{t^2}{2} + 3t \right]_0^3 \end{aligned}$$

$$x_{t=3} - 10 = 9 + \frac{9}{2} + 9$$

$$\Rightarrow x_{t=3} = \frac{45}{2} + 10$$

$$\Rightarrow x_{t=3} = \frac{65}{2} \text{ m.}$$

Co-ordinate Geometry

- The distance d between two points having coordinates (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The equation of a straight line as shown in Fig. 1.4.
 $y = mx + b$

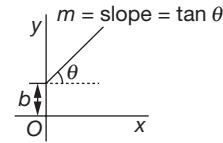


Fig. 1.4

Where b is the y -intercept and m is the slope of the line.

- The equation of a circle or radius R centred at the origin is $x^2 + y^2 = R^2$.
- The equation of an ellipse having the origin at its centre as shown in Fig. 1.5.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Fig. 1.5

where a is the length of the semi-major axis (the longer one) and b is the length of the semi-minor axis (the shorter one).

- The equation of a parabola the vertex of which is at $y = b$ as shown in Fig. 1.6 is $y = ax^2 + b$

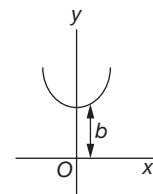


Fig. 1.6

6. The equation of a rectangular hyperbola as shown in Fig. 1.7 $xy = \text{constant}$

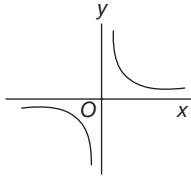


Fig. 1.7

USE OF VECTOR ANALYSIS

Suppose a block of mass M is placed on a smooth horizontal surface. There are two forces F_1 and F_2 acting on the block as shown in the Fig. 1.8.

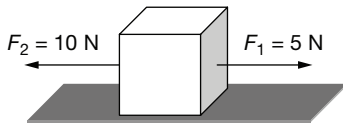


Fig. 1.8

Now the question is in which direction will the block move? And what will be the net force on the block?

You can answer it easily. The block will move towards left and net force will be $(10 - 5) = 5 \text{ N}$ towards left.

Now think of the situation when these forces are neither in the same direction nor opposite to each-other.

Suppose F_1 and F_2 are perpendicular to each other acting on the same block as shown in Fig. 1.9.

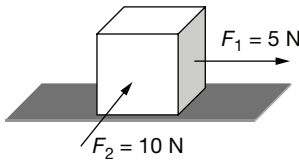


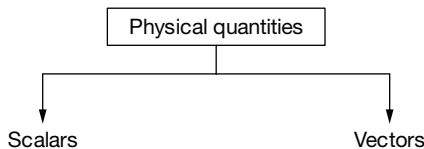
Fig. 1.9

We repeat the same question. In which direction will the block move and what will be the net force? It will be difficult to answer. Why? Since, you do not know about vectors.

Similar problems will be faced in other physical relations. We will now discuss vectors in detail.

SCALARS AND VECTORS

In the last lesson we have already discussed about physical quantities. All physical quantities have been categorised into two parts.



Scalars

Scalars are those physical quantities, which have only magnitude but no direction.

Examples: Density, time, temperature, energy, mass, distance, speed etc.

Vectors

Vectors are those physical quantities, which have both magnitude and direction and obey the vector law of addition.

Examples: Displacement, velocity, acceleration, force etc.

A vector must obey the vector law of addition otherwise it will not be a vector although having both magnitude as well as direction.

Example: Current has both magnitude and direction, but it is not a vector. It is a scalar quantity because it does not obey the vector law of addition, which we will learn in this lesson.

Representation of Vector

Geometrical Method

Geometrically a vector is represented by the directed line segment i.e., by a line to which a direction has been assigned with an arrow-head in the direction of the vector and whose length is proportional to the magnitude of the vector.

To represent a vector geometrically, a line is drawn parallel to the direction of the vector and put an arrow on the line along the direction of the vector. Now this directed line segment, namely, OP as shown in Fig. 1.10 represents the vector in magnitude and direction. It is written as \vec{OP} . O is called the 'initial point' of the vector and P , the 'terminal point' of it. The vector \vec{OP} is also written as \vec{r} i.e., we also write $\vec{r} = \vec{OP}$.

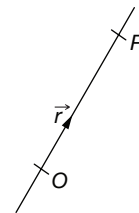


Fig. 1.10

To represent a physical quantity in a vector form, we put an arrow above the symbol of the physical quantity. For example, velocity is denoted by v but in vector form it is represented as \vec{v} which is read as velocity vector.

Magnitude of vector is called absolute value indicated by $|\vec{v}|$ (modulus of velocity vector)

Analytical Method

In this method vector is represented in terms of unit vector (\hat{i} , \hat{j} and \hat{k}), which we will see in details later on.

Different Types of Vector

Like Vectors

Two or more than two Vectors having the same sense of direction are called like vectors or otherwise it is called unlike vectors.

Example: Let body A is moving toward east and another body B is also moving in the same direction towards east, then these two velocity vectors are called like vectors.

Collinear/Parallel Vectors

Vectors having the same line of action or having lines of action parallel to the same line are called collinear vectors.

They may have the same sense or opposite sense of direction.

Coplanar Vectors

Vectors are said to be coplanar if they lie in the same plane or they are parallel to the same plane, otherwise they are said to be non-coplanar vectors.

Zero Vector/Null vector

Vectors having zero magnitude are called zero or null vectors. It is denoted by $\vec{0}$.

In case of a zero vector, its initial and the final point coincide and its direction is indeterminate.

In this case initial and terminal points coincide.

$$\vec{OO}, \vec{AA} \text{ are zero vectors}$$

Examples: The velocity vector of a stationary particle and the acceleration vector of an object moving with uniform velocity is zero vector.

Unit Vector

A unit vector is a vector of unit magnitude and points in a particular direction. It is used to specify the direction only. Unit vector is represented by putting a cap (^) over the quantity.

The unit vector in the direction of \vec{F} is denoted by \hat{F} and defined by

$$\hat{F} = \frac{\vec{F}}{|\vec{F}|}, \Rightarrow \vec{F} = |\vec{F}| \hat{F} \quad (1)$$

To represent any value of force in this direction, we can use this unit vector. Like λ is magnitude of force and we multiply λ with \hat{F} , it will give force along the direction of F .

$$\vec{F} = \lambda \hat{F}$$

Negative of a Vector

The vector whose magnitude is same as that of \vec{a} but the direction is opposite to that of \vec{a} is called the negative of \vec{a} and is written as $-\vec{a}$.



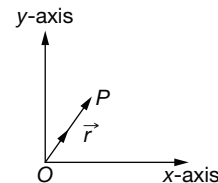
Position Vectors

Position vector represents the position of an object in a plane with respect to a fixed-point that is origin of a coordinate system.

Let O be the origin and P be any point then \vec{OP} is called the position vector of P with respect to the origin O .

It can be represented by a single letter \vec{r} .

$\vec{OP} = \vec{r}$. The length of the vector \vec{r} represents the magnitude of vector and its direction is the direction in which P lies as seen from O .



Equal Vectors

Two vectors are said to be equal if

1. their magnitudes are equal
2. they are parallel and
3. they have the same sense of direction. They needn't have the same initial point.

In the Fig. 1.11 shown length of AB equal to length of CD and AB and CD have same sense of direction.

$$\therefore \vec{AB} = \vec{CD}$$

So here \vec{AB} and \vec{CD} are two equal vectors.

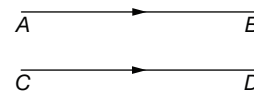


Fig. 1.11

Properties of Vectors

1. **Vectors can be bound:** The vectors in which point of application and direction both are fixed.



Such type of vector's position is fixed and it is called bound vectors.

- Vectors can be sliding:** In sliding the point of application is shifted along the original line of action without any change in magnitude and direction.

Shifted position of \vec{OA} is shown in the Fig. 1.12.

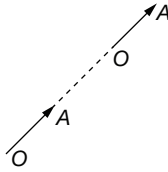


Fig. 1.12

- Vectors can be moved freely:** In free movement of vector its point of application can be changed without any change in magnitude and direction of the vector and is always parallel to the original line of action.

As \vec{OA} has been shifted in a new position as shown in Fig. 1.13.

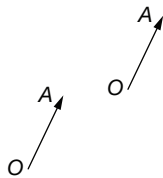
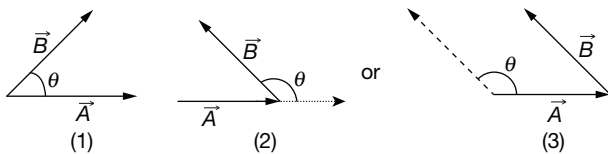


Fig. 1.13

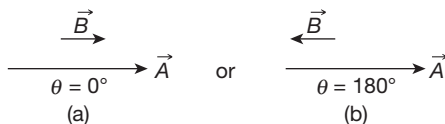
It is worth noting that in this lesson that all vectors have been used as free vector.

- Angle between two vectors means smaller of the two angles between the vectors when they are placed tail to tail by displacing either of the vectors parallel to itself (i.e., $0 \leq \theta \leq \pi$).



Here θ represents the angle between \vec{A} and \vec{B} .

- Angle between collinear vectors is always zero or 180° .

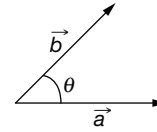


ADDITION OF VECTORS

Graphical Method

Triangle Law of Vector Addition

Let us consider two vectors \vec{a} and \vec{b} as shown.



Now to get the sum of these two vectors ($\vec{a} + \vec{b}$), shift any two vectors parallel to itself until the tail of one vector is at the head of another vector (using the sliding and free vector properties).

Here we place the vector \vec{b} in such a way that its tail touch the head of vector \vec{a} , which is shown in Fig. 1.14.

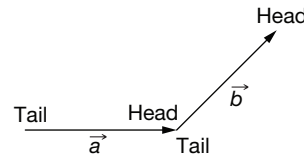
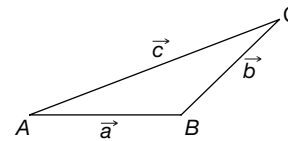


Fig. 1.14

Let $\vec{AB} = \vec{a}$ and $\vec{BC} = \vec{b}$

Then the line joining the tail of \vec{a} and head of vector \vec{b} , \vec{AC} gives the sum of vector \vec{a} and \vec{b}

Let $\vec{AC} = \vec{c}$



$$\vec{a} + \vec{b} + \vec{c} \Rightarrow \vec{AB} + \vec{BC} = \vec{AC}$$

This is known as the triangle law of vector addition.

We can define in this way. "If two vectors are represented by the two sides of a triangle taken in order, then their resultant or vector sum is represented by the third side of the triangle taken in opposite order".

Parallelogram Law of Vector Addition

Let there are two vectors \vec{P} and \vec{Q} in such a way that they have common point initial and different direction as shown in Fig. 1.15.

Let us complete a parallelogram with \vec{P} and \vec{Q} as its adjacent sides.

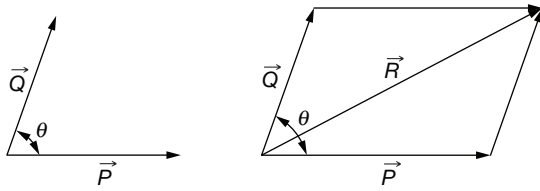


Fig. 1.15

Then, the diagonal of parallelogram passing through their common point will represent in magnitude and direction the resultant of the vectors \vec{P} and \vec{Q} .

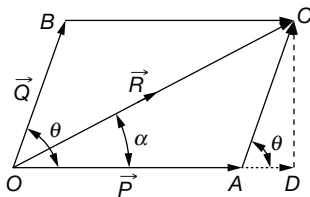
$$\vec{R} = \vec{P} + \vec{Q}$$

Analytical Method

Let the two vectors \vec{Q} and \vec{Q} be represented in magnitude and direction by \vec{OA} and \vec{OB} respectively.

Proof: considering OA and OB as two adjacent sides, parallelogram $OACB$ is constructed.

We drop a perpendicular CD on OD produced.
In right angled triangle ACD .



$$\cos \theta = \frac{AD}{AC}$$

$$\Rightarrow AD = AC \cos \theta$$

$$\text{or, } AD = Q \cos \theta$$

Also,

$$CD = Q \sin \theta \sqrt{37}$$

Now in right angle triangle ODC

$$OC^2 = OD^2 + DC^2$$

$$|R|^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$

$$= P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Let α be the angle, which the resultant \vec{R} makes with \vec{P} .

$$\tan \alpha = \frac{DC}{OD} = \frac{Q \sin \theta}{P + Q \cos \theta}, \quad \alpha = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)$$

Some Special Case

Case I: When \vec{P} and \vec{Q} are in same direction, it means $\theta = 0^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos 0^\circ}$$

$$R = \sqrt{(P+Q)^2} = P + Q$$

In this case resultant is maximum

$$\tan \alpha = \frac{Q \sin 0^\circ}{P + Q \cos 0^\circ} \Rightarrow \alpha = 0$$

Case II: When \vec{P} and \vec{Q} are perpendicular to each other, it means $\theta = 90^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos 90^\circ}$$

$$R = \sqrt{P^2 + Q^2}$$

$$\tan \alpha = \frac{Q}{P} \Rightarrow \alpha = \tan^{-1} \left(\frac{Q}{P} \right)$$

Case III: When \vec{P} and \vec{Q} are in opposite direction, it means $\theta = \pi$

$$\begin{aligned} R &= \sqrt{P^2 + Q^2 + 2PQ \cos \pi} = \sqrt{P^2 + Q^2 - 2PQ} \\ &= \sqrt{(P-Q)^2} \\ &= (P-Q) \end{aligned}$$

In this case resultant will be minimum.

$$\tan \alpha = \frac{Q \sin \pi}{P + Q \cos \pi} = 0$$

Case IV: When two vectors are of same magnitude ($|\vec{P}| = |\vec{Q}|$)

$$R = \sqrt{P^2 + P^2 + 2P^2 \cos \theta}$$

$$= \sqrt{2P^2(1 + \cos \theta)}$$

$$= \sqrt{2P^2(2 \cos^2 \frac{\theta}{2})}$$

$$= 2P \cos \frac{\theta}{2}$$

$$\tan \alpha = \frac{P \sin \theta}{P + P \cos \theta} = \frac{\sin \theta}{2 \cos^2 \frac{\theta}{2}} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}$$

$$\tan \alpha = \tan \frac{\theta}{2}$$

$$\Rightarrow \alpha = \theta/2$$

If two vectors are of equal magnitude then the resultant of two equal vectors bisects the angle between them.

Properties of Vector Addition

It Obeys Commutative Law

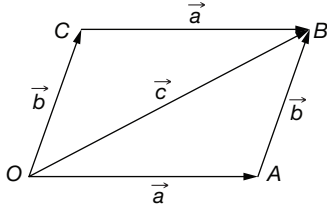
If \vec{a} and \vec{b} are any two vectors,

$$\text{then } \vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Proof: Let $\vec{OA} = \vec{CB} = \vec{a}$

$$\vec{AB} = \vec{OC} = \vec{b}$$

$$\vec{OB} = \vec{C}$$



In $\triangle OAB$, $\vec{OA} + \vec{AB} = \vec{OB}$ (from triangle law of vector addition).

$$\Rightarrow \vec{a} + \vec{b} = \vec{c} \tag{1.1}$$

In $\triangle OCB$, $\vec{OC} + \vec{CB} = \vec{OB}$

$$\Rightarrow \vec{b} + \vec{a} = \vec{c} \tag{1.2}$$

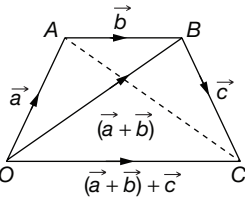
from Equations (1.1) and (1.2)

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

It Obeys Associative Law

If \vec{a} , \vec{b} , and \vec{c} are any three vectors then

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$



Proof: Let $\vec{OA} = \vec{a}$

$$\vec{AB} = \vec{b}, \quad \vec{BC} = \vec{c}$$

In $\triangle OAB$, $\vec{OB} = \vec{OA} + \vec{AB} = \vec{a} + \vec{b}$

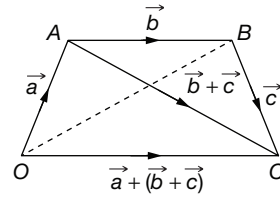
In $\triangle OBC$, $\vec{OC} = \vec{OB} + \vec{BC} = (\vec{a} + \vec{b}) + \vec{c}$ (1.3)

In $\triangle ABC$, $\vec{AC} = \vec{AB} + \vec{BC} = \vec{b} + \vec{c}$

In $\triangle OAC$, $\vec{OC} = \vec{OA} + \vec{AC} = \vec{a} + (\vec{b} + \vec{c})$ (1.4)

from equation (1.3) and (1.4)

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$



SUBTRACTION OF VECTORS

Subtraction of vector can be defined in terms of addition of two vectors.

$$\vec{P} - \vec{Q} = \vec{P} + (-\vec{Q})$$

Let \vec{P} and \vec{Q} are at an angle θ as shown in the Fig. 1.16.

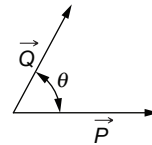
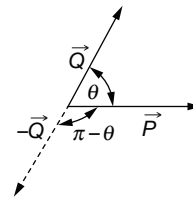


Fig. 1.16

To get $(\vec{P} - \vec{Q})$, first we will draw a vector $-\vec{Q}$ as shown below. Then angle between \vec{P} and $-\vec{Q}$ will be $(\pi - \theta)$.



$$|\vec{P} + (-\vec{Q})| = \sqrt{P^2 + Q^2 + 2PQ \cos(\pi - \theta)}$$

$$|\vec{P} - \vec{Q}| = \sqrt{P^2 + Q^2 - 2PQ \cos \theta}$$

Subtraction is not commutative, i.e.,

$$\vec{P} - \vec{Q} \neq \vec{Q} - \vec{P}$$

SOLVED EXAMPLES

57. There are two vectors having magnitude 3 units and 4 units respectively
- (A) What should be the resultant if angle between them is 60°
- (B) What should be the angle between them if the magnitude of resultant is
- (i) 1 unit (ii) 5 unit?

Solution:

- (A) $|\vec{a}| = 3$ units, $|\vec{b}| = 4$ units and $\theta = 60^\circ$

$$\vec{R} = \vec{a} + \vec{b}$$

$$\begin{aligned} R &= \sqrt{a^2 + b^2 + 2ab \cos \theta} \\ &= \sqrt{9 + 16 + 2 \cdot 3 \cdot 4 \cos 60^\circ} \\ &= \sqrt{25 + 12} = \sqrt{37} \text{ units} \end{aligned}$$

- (B) (i) $|R| = \sqrt{a^2 + b^2 + 2 \cdot 3 \cdot 4 \cos \theta}$

$$\begin{aligned} 1 &= \sqrt{9 + 16 + 2 \cdot 3 \cdot 4 \cos \theta} \\ 1 &= 25 + 24 \cos \theta \end{aligned}$$

$$\cos \theta = \frac{-24}{24} = -1 = \cos \pi$$

$$\Rightarrow \theta = \pi$$

- (ii) $(5)^2 = \sqrt{25 + 24 \cos \theta}$

$$25 = 25 + 24 \cos \theta$$

$$0 = 24 \cos \theta$$

$$\Rightarrow \cos \theta = 0$$

$$\therefore \theta = \pi/2$$

58. What is the property of two vectors \vec{A} and \vec{B} , if

(A) $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

(B) $\vec{A} + \vec{B} = \vec{A} - \vec{B}$?

Solution:

- (A) We know that

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

and according to given problem

$$\sqrt{A^2 + B^2 + 2AB \cos \theta} = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

which on solving gives $\cos \theta = 0$

$$\Rightarrow \theta = \pi/2$$

i.e., the vectors \vec{A} and \vec{B} are perpendicular to each other

(B) given that $\vec{A} + \vec{B} = \vec{A} - \vec{B}$

$$\text{i.e., } 2\vec{B} = 0 \text{ or } \vec{B} = 0$$

$$\text{i.e., } \vec{B} \text{ is a null vector.}$$

RESOLUTION OF VECTORS

It is the process of splitting a single vector into two or more vectors in different directions which together produce the same effect as is produced by the single vector alone. The vectors into which the given single vector is split are called component of vectors. In fact, the resolution of a vector is just opposite to composition of vectors.

Let there is force acting on a block, which is on a frictionless surface, at an angle θ with the horizontal as shown in Fig. 1.17.

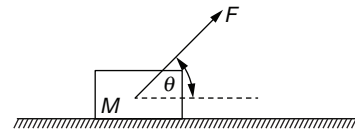


Fig. 1.17

Now due to this force it will move towards right and go some distance.

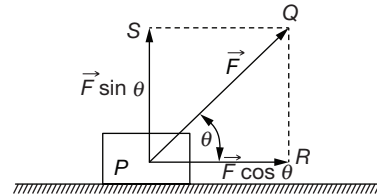
We want to know the force, which is cause of rightward motion.

It can be known by the components of the force F . The cause of horizontal motion is horizontal component which will be $F \cos \theta$.

We can understand these things in this way

Let force F is acting from P to Q

We want to know the force in the direction PR and PS .



In right angle ΔPQR

$$\cos \theta = \frac{PR}{PQ} = \frac{F_{PR}}{F_{PQ}}$$

$$\Rightarrow PR = PQ \cos \theta$$

$$\therefore \vec{F}_{PR} = \vec{F} \cos \theta$$

So, horizontal component = $F \cos \theta$

In right angle ΔPQR

$$\sin \theta = \frac{RQ}{PQ} = \frac{F_{RQ}}{F_{PQ}}$$

$$\vec{F}_{RQ} = \vec{F} \sin \theta$$

$$\vec{F}_{PS} = \vec{F}_{RQ} = \vec{F} \sin \theta$$

So, vertical component = $F \sin \theta$

In this way we resolve the vector in two perpendicular directions.

We can remember it in this way, towards θ the component will be a factor of $\cos \theta$ and other component perpendicular to it and will be a factor of $\sin \theta$.

SOLVED EXAMPLE

59. A mass of 2 kg lies on a plane making an angle 30° to the horizontal. Resolve its weight along and perpendicular to the plane. Assume $g = 10 \text{ ms}^{-2}$.

Solution:

As shown in the Fig. 1.18, the component of weight along the plane = $mg \sin \theta$

$$= 2 \times 10 \times \sin 30 = 10 \text{ N.}$$

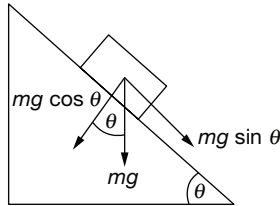


Fig. 1.18

The component of weight perpendicular to plane

$$\begin{aligned} &= mg \cos \theta \ 30^\circ \\ &= 2 \times 10 \times \sqrt{3} / 2 = 17.3 \text{ N.} \end{aligned}$$

Rectangular Components of a Vector in a Plane

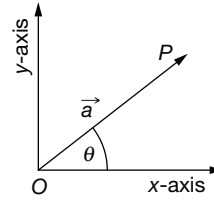
When a vector is resolved along the two axes of a rectangular co-ordinate system (i.e., x and y axis), the components of the vector are called rectangular components of a vector.

Let there is a vector \vec{a} from O to P at an angle θ from the x -axis.

$$\text{Component of } \vec{a} \text{ along } x\text{-axis } (\vec{a}_x) = \vec{a} \cos \theta$$

$$\text{Component of } \vec{a} \text{ along } y\text{-axis } (\vec{a}_y) = \vec{a} \sin \theta$$

These two components are called rectangular components of the vector \vec{a} .



Representation of Rectangular Components

These components are represented in terms of unit vector.

Unit vector along x , and y -axis are represented by \hat{i} and \hat{j} respectively as shown in Fig. 1.19.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

these unit vectors are perpendicular to each other.

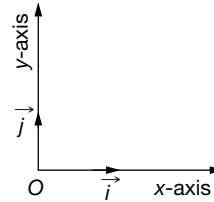


Fig. 1.19

Rectangular components of vector \vec{a} in terms of unit vector can be written as

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

$$\vec{a} = a \cos \theta \hat{i} + a \sin \theta \hat{j}.$$

Rectangular Components of a Vector in Three Dimensions

Components, which we get on resolving a vector, lie in space along three mutually perpendicular directions (i.e., x , y and z axes) are called rectangular components or orthogonal components.

The vector is called non-coplanar (three-dimensional) vector

Let there be a non-coplanar vector \vec{P} from O to A as shown in Fig. 1.20. Taking O as origin and a rectangular parallelepiped with its three edges along the three rectangular axes i.e., x , y and z axes, is constructed.

Here \vec{P} represent the diagonal of the parallelepiped whose intercepts along these axes are \vec{P}_x, \vec{P}_y and \vec{P}_z respectively which are three orthogonal components of \vec{P} .

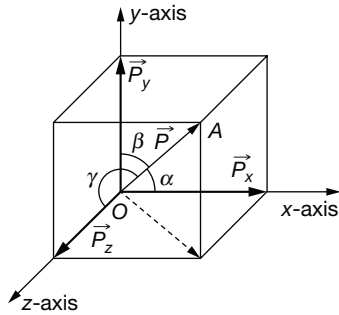


Fig. 1.20

Let α , β and γ are the angles between \vec{P} and x , y and z -axis, respectively.

$$\vec{P}_x = \vec{P} \cos \alpha$$

$$\vec{P}_y = \vec{P} \cos \beta$$

$$\vec{P}_z = \vec{P} \cos \gamma$$

$$P^2 = P_x^2 + P_y^2 + P_z^2$$

or,
$$P = \sqrt{P_x^2 + P_y^2 + P_z^2} \quad (1.5)$$

we can get these components in this way

$$\cos \alpha = \frac{P_x}{P} \Rightarrow P_x = P \cos \alpha$$

$$\cos \beta = \frac{P_y}{P} \Rightarrow P_y = P \cos \beta$$

$$\cos \gamma = \frac{P_z}{P} \Rightarrow P_z = P \cos \gamma$$

Here $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are called the direction cosines of the vector \vec{P} .

Putting the values of P_x , P_y and P_z in (1.5), we get

$$P^2 = P^2 \cos^2 \alpha + P^2 \cos^2 \beta + P^2 \cos^2 \gamma$$

or,
$$P^2 = P^2 (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma)$$

or,
$$(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 1$$

It means that the sum of the squares of the direction cosines of a vector is always unity.

Representation of Rectangular Components or Orthogonal Components

These components are represented in terms of unit vector.

Unit vectors along x , y , and z -axis are represented by \hat{i} , \hat{j} and \hat{k} respectively as shown in Fig. 1.21.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

these unit vectors are perpendicular to each other.

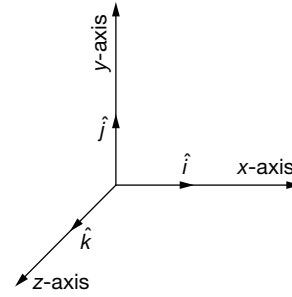


Fig. 1.21

Orthogonal components in terms of unit vector can be written as

$$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

$$\vec{P} = P \cos \alpha \hat{i} + P \cos \beta \hat{j} + P \cos \gamma \hat{k}$$

$$|\vec{P}| = \sqrt{(P_x)^2 + (P_y)^2 + (P_z)^2}$$

SOLVED EXAMPLE

60. If $\vec{P} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, find

- (A) $|\vec{P}|$ and
(B) the direction of cosines of the vector \vec{P} .

Solution:

(A)
$$P = \sqrt{P_x^2 + P_y^2 + P_z^2}$$

$$= \sqrt{(2)^2 + (4)^2 + (-5)^2} = \sqrt{45}$$

(B)
$$\cos \alpha = \frac{P_x}{P} = \frac{2}{\sqrt{45}}$$

$$\cos \beta = \frac{P_y}{P} = \frac{4}{\sqrt{45}}$$

$$\cos \gamma = \frac{P_z}{P} = \frac{-5}{\sqrt{45}}$$

ADDITION OF VECTOR BY ANALYTICAL METHOD

In this method, vectors are added when it is represented in terms of unit vector. At first coplanar vectors are resolved in two components and non-coplanar vectors are resolved in two three components, which are perpendicular to each other, and all components in one direction are added together.

There are two co-planer vectors \vec{P} and \vec{Q} making angles α and β respectively with x -axis as shown in the Fig. 1.22.

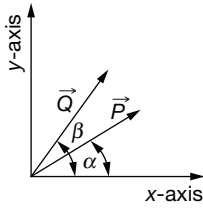


Fig. 1.22

$$\begin{aligned}\vec{P} &= P_x \hat{i} + P_y \hat{j} \\ &= P \cos \alpha \hat{i} + P \sin \alpha \hat{j}\end{aligned}$$

$$\begin{aligned}\vec{Q} &= Q_x \hat{i} + Q_y \hat{j} \\ &= Q \cos \beta \hat{i} + Q \sin \beta \hat{j}\end{aligned}$$

$$\begin{aligned}\vec{P} + \vec{Q} &= (P_x \hat{i} + P_y \hat{j}) + (Q_x \hat{i} + Q_y \hat{j}) \\ &= (P_x + Q_x) \hat{i} + (P_y + Q_y) \hat{j}\end{aligned}$$

$$|\vec{P} + \vec{Q}| = \sqrt{(P_x + Q_x)^2 + (P_y + Q_y)^2}$$

Let \vec{R} be their sum

$$\therefore \vec{R} = (P_x + Q_x) \hat{i} + (P_y + Q_y) \hat{j}$$

$$\vec{R} = R_x \hat{i} + R_y \hat{j} \text{ where}$$

$$R_x = (P_x + Q_x) \text{ and } R_y = (P_y + Q_y)$$

This method can be used in addition and subtraction for any number vectors.

SOLVED EXAMPLES

61. If $\vec{p} = 2\hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} + \hat{k}$, then calculate the magnitude of

(A) $\vec{P} + \vec{Q}$ (B) $\vec{P} - \vec{Q}$ (C) $2\vec{P} + \vec{Q}$

Solution:

$$\begin{aligned}\text{(A) } \vec{P} + \vec{Q} &= (2\hat{i} + 3\hat{j} - 2\hat{k}) + (4\hat{i} - 2\hat{j} + \hat{k}) \\ &= 6\hat{i} + \hat{j} - \hat{k}\end{aligned}$$

$$\text{Magnitude} = \sqrt{36 + 1 + 1} = \sqrt{38}$$

$$\begin{aligned}\text{(B) } \vec{P} - \vec{Q} &= (2\hat{i} + 3\hat{j} - 2\hat{k}) - (4\hat{i} - 2\hat{j} + \hat{k}) \\ &= 2\hat{i} + 3\hat{j} - 2\hat{k} - 4\hat{i} + 2\hat{j} - \hat{k} \\ &= -2\hat{i} + 5\hat{j} - 3\hat{k}\end{aligned}$$

$$\text{Magnitude} = \sqrt{4 + 25 + 9} = \sqrt{38}$$

$$\begin{aligned}\text{(C) } 2\vec{P} + \vec{Q} &= (4\hat{i} + 6\hat{j} - 4\hat{k}) + (4\hat{i} - 2\hat{j} + \hat{k}) \\ &= 8\hat{i} + 4\hat{j} - 3\hat{k}\end{aligned}$$

$$\text{Magnitude} = \sqrt{64 + 16 + 9} = \sqrt{89}$$

62. If the position vectors of P and Q be respectively $(\hat{i} + 3\hat{j} - 7\hat{k})$ and $(5\hat{i} - 2\hat{j} + 4\hat{k})$, find \vec{PQ}

Solution:

Let O be the origin

$$\text{Given } \vec{OP} = \hat{i} + 3\hat{j} - 7\hat{k}$$

$$\vec{OQ} = 5\hat{i} - 2\hat{j} + 4\hat{k}$$

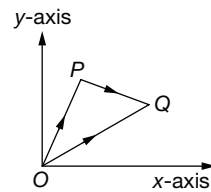
By triangle law of vector addition,

$$\vec{OP} + \vec{PQ} = \vec{OQ}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= (5\hat{i} - 2\hat{j} + 4\hat{k}) - (\hat{i} + 3\hat{j} - 7\hat{k})$$

$$= (4\hat{i} - 5\hat{j} + 11\hat{k})$$



63. Find a unit vector in the direction of the resultant of vectors $\hat{i} + 2\hat{j} + 3\hat{k}$, $-\hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j}$

Solution:

Let \vec{a} be the resultant of given vectors.

$$\begin{aligned}\text{Then } \vec{a} &= (\hat{i} + 2\hat{j} + 3\hat{k}) + (-\hat{i} + 2\hat{j} + \hat{k}) + (3\hat{i} + \hat{j}) \\ &= 3\hat{i} + 5\hat{j} + 4\hat{k}\end{aligned}$$

$$|\vec{a}| = \sqrt{(3)^2 + (5)^2 + (4)^2}$$

$$= \sqrt{9 + 25 + 16} = \sqrt{50} = 5\sqrt{2}$$

Now unit vector along

$$\vec{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{5}{5\sqrt{2}}\hat{j} + \frac{4}{5\sqrt{2}}\hat{k}.$$

MULTIPLICATION OF VECTORS

Multiplication of Vector by Scalar

When any vector is multiplied by a scalar quantity like with any real number, it is just multiplied like an algebraic product.

Example: Let there be a vector \vec{V} and a scalar quantity λ . After multiplication product will be $\vec{V}' = \lambda\vec{V}$

If λ is +ve, then \vec{V}' and \vec{V} will have same direction.

If λ is -ve then \vec{V}' and \vec{V} will have opposite direction.

Multiplication of Vectors

1. Scalar Product (Dot Product)
2. Vector Product (Cross Product)

Scalar Product/Dot Product

Scalar product of two vectors is defined as the product of the magnitude of two vectors with cosine of smaller angle between them.

It is always a scalar, so it is called as scalar product.

Thus if \vec{A} and \vec{B} are two vectors having angle θ between them, then their scalar (Dot) product written as $\vec{A} \cdot \vec{B}$ and read as \vec{A} dot \vec{B} is defined as

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Geometrical Meaning

Let $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$ as shown in Fig. 1.23 and

$$\angle AOB = \theta$$

From B, we drop a perpendicular BL on OA.

$$\cos \theta = \frac{OL}{OB}$$

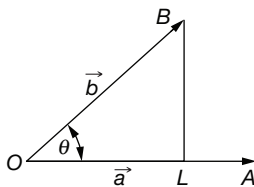


Fig. 1.23

$$OL = OB \cos \theta$$

By definition

$$\begin{aligned} \vec{a} \cdot \vec{b} &= ab \cos \theta \\ &= (OA) \cdot (OB) \cos \theta \\ &= (OA) \cdot (OL) \end{aligned}$$

$$\vec{a} \cdot \vec{b} = (\text{Mod of } \vec{a}) (\text{Projection of } \vec{b} \text{ on } \vec{a})$$

Properties

1. It is always a scalar and it will be positive if angle between them is acute, negative if angle between them is obtuse and zero if angle between them is 90°
2. It obeys commutative law $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
3. It obeys distributive law $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
4. By definition $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

where θ is angle between two vectors.

5. Scalar (Dot) product of two mutually perpendicular vectors is zero i.e.,
 $(\vec{A} \cdot \vec{B}) = AB \cos 90^\circ = 0$
6. Scalar (Dot) product will be maximum when $\theta = 0^\circ$ i.e. vectors are parallel to each other.

$$(\vec{A} \cdot \vec{B})_{\max} = |A| |B|$$

7. If \vec{a} and \vec{b} are unit vectors then

$$|\vec{a}| = |\vec{b}| = 1 \quad \text{and} \quad \vec{a} \cdot \vec{b} = 1 \cdot 1 \cdot \cos \theta = \cos \theta$$

8. Dot product of unit vectors $\hat{i}, \hat{j}, \hat{k}$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

9. Square of a vector

$$\vec{a} \cdot \vec{a} = |a| |a| \cos \theta = a^2$$

10. If the two vectors \vec{A} and \vec{B} , in terms of their rectangular components, are

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$, then,

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad [\text{using (9)}]$$

Example of dot product

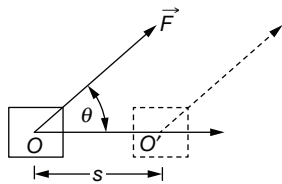
Work Work is the dot product of force and displacement vector.

Let \vec{F} be the force acting on block at angle θ with the horizontal and the block is displaced by this force from O to O' .

Let
$$\vec{OO} = \vec{S}$$

$$\text{Work} = \vec{F} \cdot \vec{S}$$

$$= |\vec{F}| |\vec{S}| \cos \theta$$



SOLVED EXAMPLES

64. Find the angle between two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{k}$.

Solution:

$$A = |\vec{A}| = \sqrt{(2)^2 + (1)^2 + (-1)^2} = \sqrt{6}$$

$$B = |\vec{B}| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{k}) \\ &= (2)(1) + (-1)(-1) = 3 \end{aligned}$$

$$\text{Now, } \cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{3}{\sqrt{6} \cdot \sqrt{2}} = \frac{3}{\sqrt{12}} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ.$$

65. Prove that the vectors $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ are mutually perpendicular.

Solution:

$$\vec{A} \cdot \vec{B} = (2\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$AB \cos \theta = (2)(1) + (-3)(1) + (1)(1) = 0$$

$$AB \cos \theta = 0 \text{ (as } A \neq 0, B \neq 0)$$

$$\therefore \theta = 90^\circ$$

or, the vectors \vec{A} and \vec{B} are mutually perpendicular.

Cross Product/Vector Product

Cross Product/Vector Product of two vectors is defined as product of the magnitude of two vectors with sine of smaller angle between them.

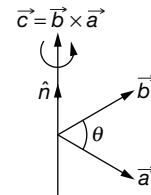
If \vec{A} and \vec{B} are two vectors, then their vector product is written as $\vec{A} \times \vec{B}$ and is read as \vec{A} cross \vec{B} . It is defined as,

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \hat{n}$$

and its direction is perpendicular to the plane of the vectors and is in the sense of advancement of a right handed screw rotated from first vector to the second vector through the smaller angle between them.

$$\vec{C} = |\vec{A}| |\vec{B}| \sin \theta \hat{n}$$

where \hat{n} is unit vector along the \vec{C}



Geometrical Meaning

Let $OACB$ be a parallelogram.

Let O be the origin

$$\text{Let } \vec{OA} = \vec{a} \text{ and } \vec{OB} = \vec{b}$$

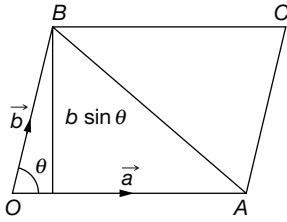
$$\text{Let } \angle AOB = \theta$$

Area of parallelogram $OACB$

$$= 2 \text{ area of triangle } OAB$$

$$= 2 \left(\frac{ab \sin \theta}{2} \right)$$

$$= ab \sin \theta = |\vec{a} \times \vec{b}|$$



Thus, $\vec{a} \times \vec{b}$ is a vector whose modulus is the area of the parallelogram formed by the two vectors as the adjacent sides and direction is perpendicular to both \vec{a} and \vec{b} .

Properties

1. Cross product of two vectors is not commutative

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

2. cross product is not associative

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

3. cross product obey distributive law

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

4. Angle θ should be in the interval $[0^\circ, \pi]$

5. If $\theta = 0$ or π it means two vectors are collinear.

$$\vec{a} \times \vec{b} = 0$$

and conversely, if $\vec{a} \times \vec{b} = 0$ then the vector \vec{a} and \vec{b} are parallel provided \vec{a} and \vec{b} are non-zero vectors.

This may be regarded as a test to decide whether the given two vectors are parallel or not.

6. If $\theta = 90^\circ$, then

$$\vec{a} \times \vec{b} = |a| |b| \sin 90^\circ = |a| |b| \hat{n}$$

7. The vector product of any vector with itself is $\vec{0}$

$$\vec{a} \times \vec{a} = \vec{0}$$

8. If $\vec{a} \times \vec{b} = 0$, then

$$\vec{a} = 0 \text{ or } \vec{b} = 0 \text{ or } \vec{a} \parallel \vec{b}$$

9. If \vec{a} and \vec{b} are unit vectors, then $\vec{a} \times \vec{b} = \sin \theta \hat{n}$

10. Cross product of unit vectors \hat{i} , \hat{j} and \hat{k}

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = \hat{k} = -\hat{j} \times \hat{i}$$

$$\hat{j} \times \hat{k} = \hat{i} = -\hat{k} \times \hat{j}$$

$$\hat{k} \times \hat{i} = \hat{j} = -\hat{i} \times \hat{k}$$

$$\hat{i} \times \hat{k} = -\hat{j} = \hat{j} \times \hat{i}$$

These results can be remembered easily with the help of following method

Around a circle placed \hat{i} , \hat{j} and \hat{k} in anticlockwise direction as shown in Fig. 1.24.

Now cross product of any two unit vectors will give third unit vectors and it will be +ve if on this circle given two vectors are in anticlockwise direction otherwise it will be -ve

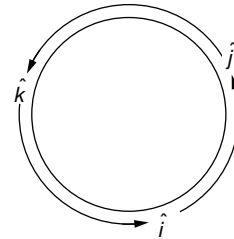


Fig. 1.24

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}, \quad \hat{i} \times \hat{k} = -\hat{j}$$

Vector Product in Terms of Component

$$\text{Let, } \vec{A} = a_1 \hat{i} + b_1 \hat{j} + c_1 \hat{k}$$

$$\vec{B} = a_2 \hat{i} + b_2 \hat{j} + c_2 \hat{k}$$

$$\vec{A} \times \vec{B} = (a_1 \hat{i} + b_1 \hat{j} + c_1 \hat{k}) \times (a_2 \hat{i} + b_2 \hat{j} + c_2 \hat{k})$$

It can be found by the determinant method

$$\begin{aligned} \text{i.e., } \vec{A} \times \vec{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \\ &= \hat{i}(b_1 c_2 - b_2 c_1) - \hat{j}(a_1 c_2 - a_2 c_1) \\ &\quad + \hat{k}(a_1 b_2 - a_2 b_1) \end{aligned}$$

Application and Example

Moment of force is the cross product of position vector of the point of application and the force.

$$\text{Torque } (\tau) = \vec{r} \times \vec{F}$$

When \vec{r} is position vector of point of application.

\vec{F} is force vector.

SOLVED EXAMPLES

66. Find a unit vector perpendicular to $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} - \hat{k}$ both

Solution:

As we have read,

$\vec{C} = \vec{A} \times \vec{B}$ is a vector perpendicular to both \vec{A} and \vec{B} .

Hence, a unit vector \hat{n} perpendicular to \vec{A} and \vec{B} can be written as

$$\hat{n} = \frac{\vec{C}}{C} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\begin{aligned} \text{Here, } \vec{A} \times \vec{B} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & -1 & 1 \end{vmatrix} \\ &= \hat{i}(3+1) + \hat{j}(1-2) + \hat{k}(-2-3) \\ &= 4\hat{i} - \hat{j} - 5\hat{k} \end{aligned}$$

$$\text{Further, } |\vec{A} \times \vec{B}| = \sqrt{(4)^2 + (-1)^2 + (-5)^2} = \sqrt{42}$$

\therefore The desired unit vector is

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \quad \text{or, } \hat{n} = \frac{1}{\sqrt{42}}(4\hat{i} - \hat{j} - 5\hat{k}).$$

67. Let a force \vec{F} be acting on a body free to rotate about a point O and let \vec{r} the position vector of any point P on the line of action of the force. Then torque ($\vec{\tau}$) of this force about point O is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Given, $\vec{F} = (2\hat{i} + 3\hat{j} - \hat{k})$ N and $\vec{r} = (\hat{i} - \hat{j} + 6\hat{k})$ m

Find the torque of this force.

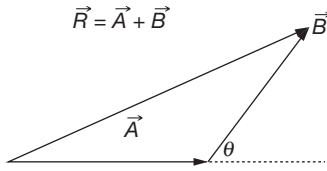
Solution:

$$\begin{aligned} \vec{\tau} = \vec{r} \times \vec{F} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 6 \\ 2 & 3 & -1 \end{vmatrix} \\ &= \hat{i}(1-18) + \hat{j}(12+1) + \hat{k}(3+2) \end{aligned}$$

or, $\vec{\tau} = (-17\hat{i} + 13\hat{j} + 5\hat{k})$ N-m.

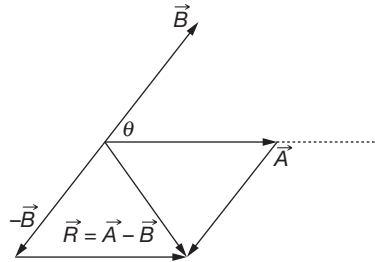
BRAIN MAP
Addition of two vectors

$$\vec{R} = \vec{A} + \vec{B}$$



$$|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$$

$$\tan\alpha = \frac{|\vec{B}|\sin\theta}{|\vec{A}| + |\vec{B}|\cos\theta}$$

Subtraction of two vectors


$$|\vec{A} - \vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta}$$

Multiplication of two vectors
Scalar multiplication

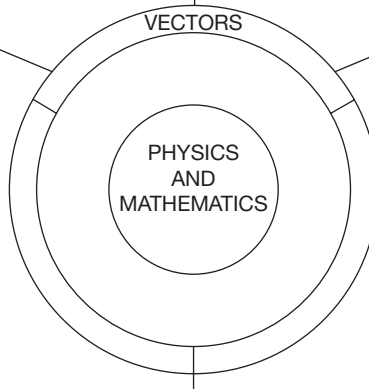
$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

Where θ is the angle between two vectors.

Vector multiplication

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta\hat{n}$$

Where \hat{n} is a unit vector perpendicular to \vec{A} and \vec{B} .



Operation	Formula	Maximum Absolute error	Maximum Relative error	Maximum Percentage error
Sum	$A + B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A + B}$	$\left(\frac{\Delta A + \Delta B}{A + B}\right) \times 100$
Difference	$A - B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A - B}$	$\left(\frac{\Delta A + \Delta B}{A - B}\right) \times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A + A\Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A} \times 100$

EXERCISES

Single Option Correct Type

- If we change unit of a physical quantity then
 - its dimension changes.
 - its dimension remain same.
 - it may change or may not change.
 - its magnitude changes.
- The least count of a stop watch is $1/5$ s. The time of 20 oscillations of a pendulum is measured to be 25 s. The minimum percentage error in the measurement of time will be
 - 0.1%
 - 0.8%
 - 1.8%
 - 8%
- Energy due to position of a particle is given by, $U = \frac{\alpha\sqrt{y}}{y+\beta}$, where α and β are constants, y is distance. The dimensions of $(\alpha \times \beta)$ are
 - $[M^0LT^0]$
 - $[M^{1/2}L^{3/2}T^{-2}]$
 - $[M^0L^{-7/2}T^0]$
 - $[ML^{7/2}T^{-2}]$
- A wire is of mass (0.3 ± 0.003) gm. The radius is (0.5 ± 0.005) mm and length is (6.0 ± 0.06) cm then % error in density is
 - 3
 - 4
 - 6
 - 2
- The equation of state of some gases can be expressed as $\left(P + \frac{a}{V^2}\right)(V - b) = RT$. Here, P is the pressure, V the volume, T the absolute temperature, and a, b, R are constants. The dimensions of a are
 - $[ML^5T^{-2}]$
 - $[ML^{-1}T^{-2}]$
 - $[M^0L^3T^0]$
 - $[M^0L^6T^0]$
- We have error in the measurement of length, radius, mass and current of a wire are 2%, 3%, 2% and 1% then error in its density will be
 - 11%
 - 8%
 - 10%
 - 7%
- A physical quantity x depends on qualities y and z as follows: $x = Ay + B \tan Cz$, where A, B and C are constants. Which of the following do not have the same dimensions?
 - x and B
 - C and z^{-1}
 - y and B/A
 - x and A
- The force F is given in terms of time t and displacement x by the equation $F = A \cos Bx + C \sin Dt$. The dimensions of $\frac{D}{B}$ are
 - $M^0L^0T^0$
 - $M^0L^{-1}T^{-0}$
 - $M^0L^0T^{-1}$
 - $M^0L^1T^{-1}$
- The diameter of a wire is measured with a screw gauge having 50 divisions on circular scale and by one complete rotation of circular scale, main scale moves 0.5 mm. If reading of screw gauge is 0.250 cm. The minimum percentage error in the reading will be
 - 0.4
 - 0.8
 - 4
 - 5
- The length, width and thickness of a block are (100.0 ± 0.1) cm, (10.00 ± 0.01) cm and (1.000 ± 0.001) cm respectively. The maximum possible error in its volume will be
 - ± 0.111 cm³
 - ± 0.012 cm³
 - $+ 0.03$ cm³
 - None of these
- The dimensional formula of magnetic flux is
 - $[ML^2T^{-2}A^{-1}]$
 - $[ML^0T^{-2}A^{-2}]$
 - $[M^0L^{-2}T^{-2}A^{-2}]$
 - $[ML^2T^{-1}A^3]$
- Error in the measurement of radius of sphere is 2%. Then error in the calculation of volume will be
 - 1%
 - 5%
 - 3%
 - 6%
- The velocity of water waves may depend on their wavelength λ , the density of water ρ and the acceleration due to gravity g . The method of dimensions gives the relation between these quantities as
 - $v^2 \propto \lambda g^{-1} \rho^{-1}$
 - $v^2 \propto g \lambda$
 - $v^2 \propto \lambda g \rho$
 - $v^2 \propto g^{-1} \lambda^2$
- A dust particle oscillates in air with a time period which depends on atmospheric pressure P , density of air d and energy of the particle E , then time period is proportional to
 - $P^{-\frac{5}{6}} d^{\frac{1}{2}} E^{\frac{1}{3}}$
 - $P^{\frac{1}{2}} d^3 E^{-2}$
 - $P^{-\frac{1}{3}} d^{\frac{1}{2}} E^2$
 - $P^{-2} d^{-\frac{1}{2}} E^{-3}$
- Which of the following group have different dimension?
 - Potential difference, EMF, voltage
 - Pressure, stress, Young's modulus
 - Heat, energy, work-done
 - Dipole moment, electric-flux, electric field

16. If a copper wire is stretched to make its radius decrease by 0.1%, then the percentage increase in resistance is approximately
 (A) 0.1 % (B) 0.2 %
 (C) 0.4% (D) 0.8%
17. Two resistances of 400 Ω and 800 Ω connected in series with a 6 volt battery of negligible internal resistance. A voltmeter of resistance 10,000 Ω is used to measure the potential difference across 400 Ω . The error in the measurement of potential difference in volts approximately is
 (A) 0.01 (B) 0.02 (C) 0.04 (D) 0.05
18. If there is a positive error of 50 % in the measurement of velocity of a body, then the error in the measurement of kinetic energy is
 (A) 25 % (B) 50 %
 (C) 100 % (D) 125 %
19. The pitch of a screw gauge is 1 mm and there are 100 divisions on its circular scale. When nothing is put in between its jaws, the zero of the circular scale lies 4 divisions below the reference line. When a steel wire is placed between the jaws, two main scale divisions are clearly visible and 67 divisions on the circular scale are observed. The diameter of the wire is
 (A) 2.71 mm (B) 2.67 mm
 (C) 2.63 mm (D) 2.65 mm
20. Charge on the capacitor is given by $Q = I\alpha e^{-\frac{t}{\Delta V \epsilon_0 \beta}}$, where α and β are constant, t = time, I = current, ΔV = Potential difference then, dimension of $\frac{\beta}{\alpha}$ is same as dimension of
 (A) $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (B) $\mu_0 \epsilon_0$ (C) $\frac{\mu_0}{\epsilon_0}$ (D) $\frac{1}{\mu_0 \epsilon_0}$
21. If the speed of light (c), acceleration due to gravity (g) and pressure (p) are taken as fundamental units, the dimensional formula of gravitational constant (G) will be
 (A) $c^0 g p^{-3}$ (B) $c^2 g^3 p^{-2}$
 (C) $c^0 g^2 p^{-1}$ (D) $c^2 g^2 p^{-2}$
22. The dimensions of $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ are
 (A) $[I^0 M^0 L^1 T^{-1}]$ (B) $[I^2 M^4 L^{-3} T^1]$
 (C) $[I^{-2} T^{-2} L^1 M^1]$ (D) $[I^0 M^0 L^{-1} T^1]$
23. What are the dimensions of electrical permittivity?
 (A) $ML^{-2} T^{-2} Q^{-2}$ (B) $M^{-1} L^2 T^{-3} Q^{-1}$
 (C) $M^{-1} L^{-3} T^2 Q^2$ (D) $M^{-1} L^3 T^{-2} Q^{-2}$
24. The torque of force $\vec{F} = (2\hat{i} - 3\hat{j} + 4\hat{k})$ newton acting at the point $\vec{r} = (3\hat{i} + 2\hat{j} + 3\hat{k})$ metre about origin is (in N-m)
 (A) $6\hat{i} - 6\hat{j} + 12\hat{k}$ (B) $17\hat{i} - 6\hat{j} - 13\hat{k}$
 (C) $-6\hat{i} + 6\hat{j} - 12\hat{k}$ (D) $-17\hat{i} + 6\hat{j} + 13\hat{k}$
25. If a unit vector is represented by $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ the value of c is
 (A) 1 (B) $\sqrt{0.11}$
 (C) $\sqrt{0.01}$ (D) 0.39
26. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces
 (A) are not equal to each other in magnitude.
 (B) cannot be predicted.
 (C) are equal to each other.
 (D) are equal to each other in magnitude.
27. Two vectors \vec{A} and \vec{B} are such that $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$. If θ is the angle between positive direction of \vec{A} and \vec{B} then the correct statement is
 (A) $\theta = \pi$ (B) $\theta = \frac{2\pi}{3}$
 (C) $\theta = 0$ (D) $\theta = \frac{\pi}{2}$
28. Given that $\vec{P} + \vec{Q} + \vec{R} = 0$. Two out of the three vectors are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of the other two. Which of the following can be the angles between these vectors?
 (A) $90^\circ, 135^\circ, 135^\circ$ (B) $45^\circ, 45^\circ, 90^\circ$
 (C) $30^\circ, 60^\circ, 90^\circ$ (D) $45^\circ, 90^\circ, 135^\circ$
29. The angle between $\vec{P} + \vec{Q}$ and $\vec{P} - \vec{Q}$ will be
 (A) 90° (B) Between 0° and 180°
 (C) 180° only (D) None of these
30. If the vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other, then the positive value of a is
 (A) Zero (B) 1 (C) 2 (D) 3
31. The angles which the vector $\vec{A} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ makes with the co-ordinate axes are
 (A) $\cos^{-1} \frac{3}{7}, \cos^{-1} \frac{6}{7}$ and $\cos^{-1} \frac{2}{7}$
 (B) $\cos^{-1} \frac{4}{7}, \cos^{-1} \frac{5}{7}$ and $\cos^{-1} \frac{3}{7}$
 (C) $\cos^{-1} \frac{3}{7}, \cos^{-1} \frac{4}{7}$ and $\cos^{-1} \frac{1}{7}$
 (D) None of the above

32. Unit vector parallel to the resultant of vectors $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 8\hat{i} + 8\hat{j}$ will be

- (A) $\frac{24\hat{i} + 5\hat{j}}{13}$ (B) $\frac{12\hat{i} + 5\hat{j}}{13}$
 (C) $\frac{6\hat{i} + 5\hat{j}}{13}$ (D) None of these

33. The value of n so that vectors $2\hat{i} + 3\hat{j} - 2\hat{k}$, $5\hat{i} + n\hat{j} + \hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$ may be coplanar, will be

- (A) 18 (B) 28 (C) 9 (D) 36

34. If \vec{a} and \vec{b} are two vectors then the value of $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ is

- (A) $2(\vec{b} \times \vec{a})$ (B) $-2(\vec{b} \times \vec{a})$
 (C) $\vec{b} \times \vec{a}$ (D) $\vec{a} \times \vec{b}$

35. The component of vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the vector $\hat{i} + \hat{j}$ is

- (A) $\frac{5}{\sqrt{2}}$ (B) $10\sqrt{2}$
 (C) $5\sqrt{2}$ (D) 5

36. Three vectors satisfy the relation $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$, then \vec{A} is parallel to

- (A) \vec{C} (B) \vec{B} (C) $\vec{B} \times \vec{C}$ (D) $\vec{B} \cdot \vec{C}$

37. If $|\vec{A} \times \vec{B}| = \sqrt{3} \vec{A} \cdot \vec{B}$, then the value of $|\vec{A} + \vec{B}|$ is

- (A) $(A^2 + B^2 + AB)^{1/2}$
 (B) $\left(A^2 + B^2 + \frac{AB}{\sqrt{3}}\right)^{1/2}$
 (C) $(A + B)$
 (D) $(A^2 + B^2 + \sqrt{3} AB)^{1/2}$

38. If the angle between the vectors \vec{A} and \vec{B} is θ , the value of the product $(\vec{B} \times \vec{A}) \cdot \vec{A}$ is equal to

- (A) $BA^2 \cos \theta$ (B) $BA^2 \sin \theta$
 (C) $BA^2 \sin \theta \cos \theta$ (D) Zero

39. The (x, y, z) co-ordinates of two points A and B are given respectively as $(0, 3, -1)$ and $(-2, 6, 4)$. The displacement vector from A to B is given by

- (A) $-2\hat{i} + 6\hat{j} + 4\hat{k}$ (B) $-2\hat{i} + 3\hat{j} + 3\hat{k}$
 (C) $-2\hat{i} + 3\hat{j} + 5\hat{k}$ (D) $2\hat{i} - 3\hat{j} - 5\hat{k}$

40. Calculate the area of the triangle determined by the two vectors $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = -3\hat{i} + 7\hat{j}$.

- (A) $\frac{33}{2}$ sq. unit (B) $\frac{35}{2}$ sq. unit
 (C) $\frac{45}{2}$ sq. unit (D) 15 sq. unit

41. Determine that vector which when added to the resultant of $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives a unit vector along the y -direction.

- (A) $5\hat{i} + 2\hat{j} - 4\hat{k}$ (B) $-5\hat{i} + 2\hat{j} - 4\hat{k}$
 (C) $5\hat{i} - 2\hat{j} - 4\hat{k}$ (D) None of these

42. The vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a triangle, then the triangle is

- (A) right angled triangle (B) lateral triangle
 (C) isolated triangle (D) None of these

43. From Fig. 1.24, the correct relation is

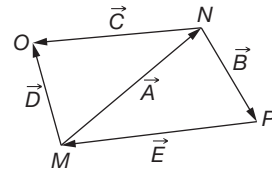


Fig. 1.24

- (A) $\vec{A} + \vec{B} + \vec{E} = \vec{0}$ (B) $\vec{C} - \vec{D} = -\vec{A}$
 (C) $\vec{B} + \vec{E} - \vec{C} = -\vec{D}$ (D) All of the above

44. The ratio of maximum and minimum magnitudes of the resultant of two vectors \vec{a} and \vec{b} is 3 : 1. Now $|\vec{a}|$ is equal to

- (A) $|\vec{b}|$ (B) $2|\vec{b}|$
 (C) $3|\vec{b}|$ (D) $4|\vec{b}|$

45. The projection of a vector $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k}$ on the x - y plane has magnitude

- (A) 3 (B) 4 (C) $\sqrt{14}$ (D) $\sqrt{10}$

46. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between \vec{A} and \vec{B} is

- (A) 120° (B) 60° (C) 90° (D) 0°

47. Given $|\vec{A}_1| = 2, |\vec{A}_2| = 3$ and $|\vec{A}_1 + \vec{A}_2| = 3$. Find the value of $(\vec{A}_1 + 2\vec{A}_2) \cdot (3\vec{A}_1 - 4\vec{A}_2)$

- (A) -64 (B) 60 (C) -60 (D) 64

48. In going from one city to another, a car travels 75 km north, 60 km north-west and 20 km east. The magnitude of displacement between the two cities is (take $1/\sqrt{2} = 0.7$)

- (A) 170 km (B) 137 km
 (C) 119 km (D) 140 km

49. Given $\vec{A} = 2\hat{i} + p\hat{j} + q\hat{k}$ and $\vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$. If $\vec{A} \parallel \vec{B}$, then the values of p and q are, respectively,

- (A) $\frac{14}{5}$ and $\frac{6}{5}$ (B) $\frac{14}{3}$ and $\frac{6}{5}$
 (C) $\frac{6}{5}$ and $\frac{1}{3}$ (D) $\frac{3}{4}$ and $\frac{1}{4}$

50. If $\vec{b} = 3\hat{i} + 4\hat{j}$ and $\vec{a} = \hat{i} - \hat{j}$, the vector having the same magnitude as that of \vec{b} and parallel to \vec{a} is

- (A) $\frac{5}{\sqrt{2}}(\hat{i} - \hat{j})$ (B) $\frac{5}{\sqrt{2}}(\hat{i} + \hat{j})$
 (C) $5(\hat{i} - \hat{j})$ (D) $5(\hat{i} + \hat{j})$

51. Mark the correct statement

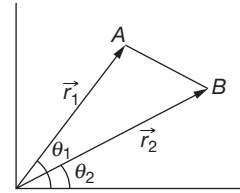
- (A) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$
 (B) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$
 (C) $|\vec{a} - \vec{b}| \geq |\vec{a}| + |\vec{b}|$
 (D) All of the above

52. Two forces $\vec{F}_2 = 500 \text{ N}$ due east and $\vec{F}_1 = 250 \text{ N}$ due north have their common initial point. $\vec{F}_2 - \vec{F}_1$ is

- (A) $250\sqrt{5} \text{ N}$, $\tan^{-1}(2)$ W of N
 (B) 250 N , $\tan^{-1}(2)$ W of N
 (C) Zero
 (D) 750 N , $\tan^{-1}(3/4)$ N of W

53. In a two-dimensional motion of a particle, the particle moves from point A , with position vector \vec{r}_1 , to point B , with position vector \vec{r}_2 . If the magnitudes of these vectors are, respectively, $r_1 = 3$ and $r_2 = 4$ and the angles they make with the x -axis are $\theta_1 = 75^\circ$ and $\theta_2 = 15^\circ$, respectively, then find the magnitude of the displacement vector.

- (A) 15 (B) $\sqrt{13}$
 (C) 17 (D) $\sqrt{15}$



More than One Option Correct Type

54. Which of the following having same dimension?

- (A) Electric flux, electric field, electric dipole moment
 (B) Pressure, stress, young's modulus
 (C) Electromotive force, potential difference, electric voltage
 (D) Heat, potential energy, work done.

55. The ratio of the dimension of Planck's constant and that of moment of inertia is the dimension of

- (A) Time (B) Frequency
 (C) Angular momentum (D) Velocity

56. Out of the following pairs, which one does not have the same dimensions?

- (A) Angular momentum and Planck's constant.
 (B) Impulse and momentum.
 (C) Moment of inertia and moment of force.
 (D) Work and torque.

57. Parsec is the unit of

- (A) Time (B) Distance
 (C) Frequency (D) Angular acceleration

58. (A) Specific gravity of a fluid is a dimensionless quantity (R). It is the ratio of density of fluid to density of water.

- (A) Both A and R are correct and R is the correct explanation of A
 (B) Both A and R are correct but R is not the correct explanation of A
 (C) A is correct but R is false
 (D) Both A and R are false

59. A force F is given by $F = at + bt^2$ where t is time, what are the dimension of a and b ?

- (A) $[MLT^{-1}, MLT^0]$, (B) $[MLT^{-3}, ML^2T^4]$
 (C) $[MLT^{-4}, MLT^{-1}]$ (D) $[ML^{-3}T, MLT^{-4}]$

60. Which of the following is not a unit of Young's modulus?

- (A) Nm^{-1} (B) Nm^{-2}
 (C) Dyne cm^{-2} (D) Mega pascal

61. If M is mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constant is

- (A) $\frac{R^2}{M}$ (B) $\frac{M}{R^2}$
- (C) MR^2 (D) $\frac{M}{R}$
62. A hypothetical experiment conducted to find Young's modulus $Y = \frac{\cos\theta T^x \tau}{l^3}$ where τ is torque and l is length then find x .
(A) 0 (B) 1 (C) 2 (D) 3
63. Find the dimensions of $\frac{B^2}{\mu_0}$
(A) ML^2T^{-2} (B) $ML^{-1}T^{-1}$
(C) $ML^{-2}T^{-2}$ (D) $ML^{-1}T^{-2}$
64. While printing a book a printer made certain mistakes in the following relation. Find the correct relation.
(A) $y = A \sin \omega\theta$ (B) $y = A \sin (\omega x + \theta)$
(C) $y = A \sin (\omega t + \theta)x$ (D) $y = \frac{A}{x} \sin \omega t + \theta$
65. The velocity of surface waves depends upon surface tension, coefficient of viscosity and density. The relation is
(A) $\frac{s^2}{\rho\eta}$ (B) $\frac{s}{\eta}$
(C) $\frac{\eta\rho}{s^2}$ (D) $\frac{\rho}{\eta}$
66. Which of the following pair has different dimensions?
(A) Electric pressure, energy density
(B) Intensity, $\epsilon_0 E_0^2 c$
(C) Reynold's number and time constant
(D) Work, Torque
67. A chocolate cookie is a circular disk of diameter 8.50 ± 0.02 cm and thickness 0.050 ± 0.005 cm. The average volume in cm^3 is
(A) 2.83 ± 0.3 (B) 2.38 ± 0.27
(C) 11.35 ± 1.2 (D) 9.31 ± 1.12
68. The fastest commercial airline service is 1450 mi/h. Find the speed in kmh^{-1} and ms^{-1} .
(A) 1938 kmh^{-1} , 618.3 ms^{-1}
(B) 2030 kmh^{-1} , 623.1 ms^{-1}
(C) 2334 kmh^{-1} , 647.5 ms^{-1}
(D) None
69. Two capacitors $C_1 = 5.2 \mu\text{F}$ and $0.1 \mu\text{F}$ and $c_2 = 12.2 \mu\text{F}$ are joined (i) In series (ii) In parallel. Find the net capacitance in these two cases.
(A) 2.8 %, 1.23 % (B) 3.6 %, 1.31 %
(C) 3.4 %, 1.3 % (D) 3.9 %, 1.15 %
70. V^{-1} stands for
(A) electric flux. (B) electric pressure.
(C) electric field density. (D) capacitance.
71. A spherometer has 20 threads per cm. Its circular scale has 100 divisions. Find the least count of spherometer.
(A) $5 \mu\text{m}$ (B) $50 \mu\text{m}$
(C) $0.5 \mu\text{m}$ (D) $0.5 \mu\text{m}$
72. Which of the following are dimensionally correct?
(A) $h = \frac{2T \cos \theta}{\rho r g}$ (B) $v = \sqrt{\frac{p}{\rho}}$
(C) $\frac{dv}{dt} = \frac{\pi p r^4 t}{\delta \eta l}$ (D) $T = \sqrt{\frac{mgl}{I}}$
73. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a} \sin^{-1} \frac{a}{x}$
(A) is dimensionally correct.
(B) dimensionally incorrect.
(C) such mathematical relations cannot be tested.
(D) cannot say.
74. In CH_4 molecule, there are four C – H bonds. If two adjacent bonds are in $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} - \hat{k}$ direction, then find the angle between these bonds.
(A) $\sin^{-1}\left(\frac{-1}{3}\right)$ (B) $\cos^{-1}\left(\frac{1}{3}\right)$
(C) $\sin^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{-1}{3}\right)$
75. Two vectors \vec{A} and \vec{B} have magnitude 3 each. $\vec{A} \times \vec{B} = -5\hat{k} + 2\hat{i}$. Find angle between A and B
(A) $\cos^{-1} \frac{\sqrt{29}}{9}$ (B) $\tan^{-1}\left(\frac{-5}{2}\right)$
(C) $\sin^{-1}\left(\frac{2}{5}\right)$ (D) $\sin^{-1}\left(\frac{\sqrt{29}}{9}\right)$
76. A particle moving eastwards with 5 m/s^{-1} . In 10 s the velocity changes to 5 m/s^{-1} northwards. The average acceleration in this time is
(A) $\frac{1}{\sqrt{2}} \text{ m/s}^{-2}$ towards Northeast
(B) $\frac{1}{2} \text{ m/s}^{-2}$ towards North
(C) $\frac{1}{\sqrt{2}} \text{ m/s}^{-2}$ towards Northwest
(D) Zero

77. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$ then the value of α is
 (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 1 (D) -1
78. If the angle between the vectors \vec{A} and \vec{B} is θ , the value of the product $(\vec{B} \times \vec{A}) \cdot \vec{A}$ equals
 (A) $BA^2 \sin \theta$ (B) $BA^2 \cos \theta \sin \theta$
 (C) $BA^2 \cos \theta$ (D) Zero
79. A river is flowing from W to E with a speed 5 m/min. A man can swim in still waters at a velocity 10 m/min. In which direction should a man swim to take the shortest path to reach the south bank?
 (A) 30° East of South (B) 60° East of North
 (C) South (D) 30° West of North
80. Electrons in a TV tube move horizontally South to North. Vertical component of earth's magnetic field points down. The electron is deflected towards
 (A) West (B) No deflection
 (C) East (D) North to South
81. If $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{C}$, then
 (A) $\vec{A} = \vec{C}$ always
 (B) $\vec{A} \neq \vec{C}$ always
 (C) \vec{A} may not be equal to \vec{C}
 (D) None of these
82. $A = 3\hat{i} + 4\hat{j}$ find a vector perpendicular to \vec{A} in the plane of \vec{A}
 (A) \hat{k} (B) $-3\hat{i} - 4\hat{j}$
 (C) $4\hat{j} - 3\hat{i}$ (D) $4\hat{i} - 3\hat{j}$
83. Find a vector \vec{x} which is perpendicular to both \vec{A} and \vec{B} but has magnitude equal to that of \vec{B} .
Rule: Inter change coeff. of \hat{i} and \hat{j} and change sign of one of the vectors.
 $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} - 2\hat{k}$
 (A) $\frac{1}{\sqrt{10}}(\hat{i} + 10\hat{j} + 17\hat{k})$
 (B) $\frac{1}{\sqrt{10}}(\hat{i} - 10\hat{j} + 17\hat{k})$
 (C) $\sqrt{\frac{29}{390}}(\hat{i} - 10\hat{j} + 17\hat{k})$
 (D) $\sqrt{\frac{29}{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$
84. Rain is falling vertically with 3 ms^{-1} and a man is moving due North with 4 ms^{-1} . In which direction he should hold the umbrella to protect himself from rains?
 (A) 37° North of vertical
 (B) 37° South of vertical
 (C) 53° North of vertical
 (D) 53° South of vertical
85. A man is moving on his bike with 54 kmh^{-1} . He takes a u-turn in 10 s and continues to move with the same velocity. Find average acceleration during this time.
 (A) 3.0 ms^{-2} (B) 1.5 ms^{-2}
 (C) 0 (D) -1.5 ms^{-2}
86. A man starts from O moves 500 m turns by 60° and moves 500 m again turns by 60° and moves 500 m and so on. Find the displacement after (i) 5th turn, (ii) 3rd turn
 (A) 500 m, 1000 m (B) 500 m, $500\sqrt{3}$ m
 (C) 1000 m, $500\sqrt{3}$ m (D) None of these
87. If $\vec{B} = \lambda \vec{A}$ then $\frac{\vec{B}}{\vec{A}} = \dots$
 (A) λ (B) $\frac{1}{\lambda}$
 (C) $\frac{\lambda}{2}$ (D) Indeterminate
88. The acceleration of a particle as seen from two frames S_1 and S_2 has equal magnitude 5 ms^{-2} .
 (A) The frames must be at rest with respect to each other.
 (B) The frames may be moving with respect to each other but neither should be accelerated with respect to the other.
 (C) The acceleration of frame S_2 with respect to S_1 be 0 or 10 ms^{-2} .
 (D) The acceleration of S_2 with respect to S_1 lies between 0 and 10 ms^{-2} .
89. A man running on a horizontal road at 8 ms^{-1} finds rain falling vertically. If he increases his speed to 12 ms^{-1} , he finds that drops make 30° angle with the vertical. Find velocity of rain with respect to the road.
 (A) $4\sqrt{7} \text{ ms}^{-1}$ (B) $8\sqrt{2} \text{ ms}^{-1}$
 (C) $7\sqrt{3} \text{ ms}^{-1}$ (D) 8 ms^{-1}
90. In Fig. 1.25 (A) shown find the velocity of block m if both the rope ends are pulled with a velocity v .
 (A) $2v \cos \theta$ (B) $\frac{v}{\cos \theta}$
 (C) $\frac{v}{2 \cos \theta}$ (D) $\frac{2v}{\cos \theta}$

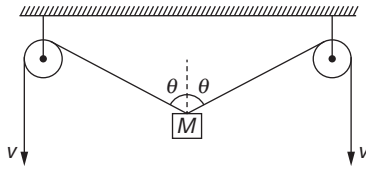


Fig. 1.25

91. Which of the following cannot be in equilibrium?
 (A) 10N, 10N, 5N (B) 5N, 7N, 9N
 (C) 8N, 4N, 13N (D) 9N, 6N, 5N
92. $\vec{A} = 3\hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{B} = 6\hat{i} - \hat{j} + 3\hat{k}$. Find a vector parallel to \vec{A} whose magnitude is equal to that of \vec{B} .
 (A) $\sqrt{\frac{46}{29}}(3\hat{i} + 4\hat{j} + 2\hat{k})$ (B) $\sqrt{\frac{46}{29}}(6\hat{i} - \hat{j} + 3\hat{k})$
 (C) $\sqrt{\frac{29}{46}}(3\hat{i} + 4\hat{j} + 2\hat{k})$ (D) None
93. \vec{a} , \vec{b} , \vec{c} are three coplanar vectors. Find the vector sum.
 $\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$, $\vec{c} = -3\hat{j}$
 (A) $\sqrt{5}$, 297° (B) $\sqrt{5}$, 63°
 (C) $\sqrt{3}$, 297° (D) $\sqrt{3}$, 63°
94. A block of mass m is connected to three springs, each of spring constant k as shown in Fig. 1.26. The block is pulled by x in the direction of C . Find resultant spring constant.

- (A) k (B) $2k$ (C) $3k$ (D) $\frac{3k}{2}$

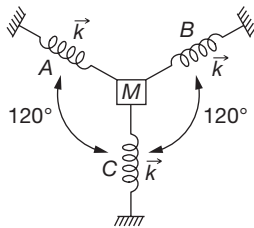


Fig. 1.26

95. A particle moves in the $x - y$ plane under the action of a force \vec{F} such that the value of its linear momentum p at any instant is $p = 2(\cos t \hat{i} + \sin t \hat{j})$. The angle θ between F and p is
 (A) 60° (B) 45° (C) 30° (D) 90°
96. Consider a collection of large number of particles, each moving with a speed v . The direction of velocity is randomly distributed in the collection. The magnitude

of the relative velocity between a pair of particles averaged over all the pairs in the collection

- (A) v (B) $\frac{2v}{\pi}$
 (C) $\frac{\pi v}{4}$ (D) $\frac{4v}{\pi}$

97. A steamer is moving due east with 36 km/h. To a man in the steamer the wind appears to blow at 18 km/h due north. Find the velocity of the wind.
 (A) $5\sqrt{5} \text{ ms}^{-1} \tan^{-1} \frac{1}{2}$ North of East
 (B) $5 \text{ ms}^{-1} \tan^{-1} 2$ North of East
 (C) $5\sqrt{5} \text{ ms}^{-1} \tan^{-1} 2$ North of East
 (D) $5 \text{ ms}^{-1} \tan^{-1} \frac{1}{2}$ North of East
98. The position vector of a particle is $\vec{r} = a[\cos \omega t \hat{i} + \sin \omega t \hat{j}]$. The velocity of the particle is
 (A) parallel to position vector.
 (B) directed towards origin.
 (C) directed away from origin.
 (D) perpendicular to position vector.
99. A force $6\hat{i} + 3\hat{j} + \hat{k}$ Newton displaces a particles from $A(0, 3, 2)$ to $B(5, 1, 6)$. Find the work done.
 (A) 10 J (B) 22 J
 (C) 32 J (D) 41J
100. Wind is blowing NE with $18\sqrt{2} \text{ km h}^{-1}$ and steamer is heading due west with 18 km h^{-1} . In which direction is the flag on the mast fluttering?
 (A) North West (B) North
 (C) South West (D) South
101. The resultant of two forces equal in magnitude is equal to either of two vectors in magnitude. Find the angle between the forces.
 (A) 60° (B) 45° (C) 90° (D) 120°
102. A man goes 100 m north then 100 m east and then 20 m north and then $100\sqrt{2} \text{ m}$ south-West. Find the displacement.
 (A) 20 m West (B) 20 m East
 (C) 20 m North (D) 20 m South
103. If force, length and time are fundamental quantities, then find the dimensions of density is
 (A) $FL^{-4}T^{-2}$ (B) $FL^{-4}T^2$
 (C) $FL^{-1}T^{-2}$ (D) $FL^{-1}T^2$

Previous Years' Questions

104. Dimensions of $\frac{1}{\mu_0 \epsilon_0}$, where symbols have their usual meaning are [2003]
 (A) $[L^{-1}T]$ (B) $[L^2T^2]$
 (C) $[L^2T^{-2}]$ (D) $[LT^{-1}]$
105. The physical quantities not having same dimensions are [2003]
 (A) torque and work.
 (B) momentum and Planck's constant.
 (C) stress and Young's modulus.
 (D) speed and $(\mu_0 \epsilon_0)^{-1/2}$.
106. Which one of the following represents the correct dimensions of the coefficient of viscosity? [2004]
 (A) $[ML^{-1}T^{-2}]$ (B) $[MLT^{-1}]$
 (C) $[ML^{-1}T^{-1}]$ (D) $[ML^{-2}T^{-2}]$
107. Which of the following units denotes the dimensions $[ML^2/Q^2]$, where Q denotes the electric charge? [2006]
 (A) Wb/m^2 (B) Henry (H)
 (C) H/m^2 (D) Weber (Wb)
108. The dimensions of magnetic field in M , L , T and C (Coulomb) is given as [2008]
 (A) $[MLT^{-1}C^{-1}]$ (B) $[MT^2C^{-2}]$
 (C) $[MT^{-1}C^{-1}]$ (D) $[MT^{-2}C^{-1}]$
109. The respective number of significant figure for the numbers 23.023, 0.0003 and 2.1×10^{-3} are [2010]
 (A) 4, 4, 2 (B) 5, 1, 2
 (C) 5, 1, 5 (D) 5, 5, 2
110. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is [2012]
 (A) 6% (B) Zero (C) 1% (D) 3%
111. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If $M = \text{mass}$, $L = \text{length}$, $T = \text{time}$ and $A = \text{electric current}$, then [2013]
 (A) $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$
 (B) $[\epsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$
 (C) $[\epsilon_0] = [M^{-1}L^2T^{-1}A]$
 (D) $[\epsilon_0] = [M^{-1}L^{-3}T^2A]$
112. The current voltage relation of diode is given by $I = (\epsilon - 1000 V/T - 1) \text{ mA}$, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm 0.01 V$ while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? [2014]
 (A) 0.2 mA (B) 0.252 mA
 (C) 0.5 mA (D) 0.05 mA
113. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is [2015]
 (A) 3% (B) 1% (C) 5% (D) 2%
114. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s, and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be: [2016]
 (A) $92 \pm 5.0 \text{ s}$ (B) $92 \pm 1.8 \text{ s}$
 (C) $92 \pm 3 \text{ s}$ (D) $92 \pm 2 \text{ s}$
115. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? [2016]
 (A) 0.80 mm (B) 0.70 mm
 (C) 0.50 mm (D) 0.75 mm

ANSWER KEYS

Single Option Correct Type

1. (B) 2. (B) 3. (D) 4. (B) 5. (A) 6. (C) 7. (D) 8. (D) 9. (A) 10. (D)
 11. (A) 12. (D) 13. (B) 14. (A) 15. (D) 16. (C) 17. (C) 18. (D) 19. (C) 20. (A)
 21. (C) 22. (A) 23. (C) 24. (B) 25. (B) 26. (D) 27. (D) 28. (A) 29. (B) 30. (D)
 31. (A) 32. (B) 33. (A) 34. (A) 35. (A) 36. (C) 37. (A) 38. (D) 39. (C) 40. (A)
 41. (B) 42. (A) 43. (C) 44. (B) 45. (D) 46. (A) 47. (A) 48. (C) 49. (A) 50. (A)
 51. (B) 52. (A) 53. (B)

More than One Option Correct Type

54. (A) 55. (B) 56. (C) 57. (B) 58. (A) 59. (D) 60. (A) 61. (B) 62. (A) 63. (D)
 64. (C) 65. (B) 66. (C) 67. (A) 68. (C) 69. (D) 70. (D) 71. (A) 72. (A), (B) and (C)
 73. (B) 74. (D) 75. (D) 76. (C) 77. (B) 78. (D) 79. (D) 80. (C) 81. (C)
 82. (D) and (A) 83. (D) 84. (C) 85. (A) 86. (A) 87. (D) 88. (D) 89. (A)
 90. (B) 91. (C) 92. (A) 93. (A) 94. (C) 95. (D) 96. (D) 97. (A) 98. (D)
 99. (C) 100. (D) 101. (D) 102. (C) 103. (B)

Previous Years' Questions

104. (C) 105. (B) 106. (C) 107. (B) 108. (C) 109. (B) 110. (A) 111. (A) 112. (A) 113. (A)
 114. (D) 115. (A)

HINTS AND SOLUTIONS

Single Option Correct Type

1. The correct option is (B)

$$2. \frac{\Delta T}{T} \times 100 = \frac{1}{25} \times 100 = 0.8\%$$

The correct option is (B)

3. $[\beta] = L$

$$ML^2T^{-2} = \frac{\alpha[L]^{1/2}}{[L]}$$

$$\alpha = [M][L^{5/2}][T^{-2}]$$

The correct option is (D)

$$4. \% \text{ error in density} = \left[2 \left(\frac{0.005}{0.5} \right) + \frac{0.003}{0.3} + \frac{0.06}{6} \right] \times 100 = 4$$

The correct option is (B)

5. Dimension of $\frac{a}{\nu^2}$ must be that of pressure

$$\frac{MLT^{-2}}{L^2} = \frac{a}{L^6}$$

$$a = ML^5T^{-2}$$

The correct option is (A)

$$6. \frac{\Delta d}{d} \times 100 = \frac{2\Delta r}{r} \times 100 + \frac{\Delta l}{l} \times 100 + \frac{\Delta m}{m} \times 100$$

$$= 2 \times 3 + 2 + 2 = 10\%$$

The correct option is (C)

7. $x = Ay + B \tan Cz$

From the dimensional homogeneity

$$[x] = [Ay] = [B] \Rightarrow \left[\frac{x}{A} \right] = [y] = \left[\frac{B}{A} \right]$$

$$[Cz] = [M^0L^0T^0] = \text{Dimension less}$$

x and B ; C and Z^{-1} ; y and $\frac{B}{A}$ have the same dimension but x and A have the different dimensions.

The correct option is (D)

8. $F = A \cos Bx + C \sin Dt$

$$[D] = [T^{-1}]; [B] = [L^{-1}]$$

$$\left[\frac{D}{B}\right] = \left[\frac{T^{-1}}{L^{-1}}\right] = [M^0 L^1 T^{-1}]$$

The correct option is (D)

9. Least count of screw gauge = $\frac{0.5}{50} = 0.01$ mm
 Percentage error in diameter = $\frac{0.01}{2.50} \times 100 = 0.4\%$

The correct option is (A)

10. $\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t}$ and $V = lbt$

The correct option is (D)

11. $\varepsilon = -\frac{d\phi}{dt} \Rightarrow [\phi] = [\varepsilon][t] = \left[\frac{ML^2T^{-2}}{AT}\right][T] = [ML^2T^{-2}A^{-1}]$

The correct option is (A)

12. The correct option is (D)

13. $[LT^{-1}] = [L]^x [ML^{-3}]^y [LT^{-2}]^z$

Solving, we get, $x = \frac{1}{2}$, $y = 0$ and $z = \frac{1}{2}$

$$v \propto \sqrt{g\lambda}$$

The correct option is (B)

14. $T = kp^x d^y E^z$

$$[T] = [ML^{-1}T^{-2}]^x [ML^{-3}]^y [ML^2T^{-2}]^z$$

$$x + y + z = 0 \quad (1)$$

$$-x - 3y + 2z = 0 \quad (2)$$

$$-2x - 2z = 1 \quad (3)$$

$$x + z = -\frac{1}{2} \quad (4)$$

$$-y = -\frac{1}{2} \quad (5)$$

$$\Rightarrow y = \frac{1}{2}$$

by Equation (2) $-x - \frac{3}{2} + 2z = 0$

$$-x + 2z = \frac{3}{2} \quad (6)$$

Adding (4) and (6)

$$3z = 1, z = \frac{1}{3}$$

$$\Rightarrow x = -\frac{1}{2} - \frac{1}{3} = -\frac{5}{6}$$

The correct option is (A)

15. The correct option is (D)

16. In case of stretching $R \propto \frac{1}{r^4}$

$$\therefore \frac{\Delta R}{R} \times 100 = -4 \frac{\Delta r}{r} \times 100 = 0.4\%$$

The correct option is (C)

17. $I = \frac{6}{400 + 800} = 5 \times 10^{-3}$ A

$$\therefore \text{Voltage drop across } 400 \Omega = 5 \times 10^{-3} \times 400 = 2 \text{ V}$$

Because of the presence of the voltmeter having resistance $G = 10,000 \Omega$ in parallel with 400Ω , the effective resistance is

$$\frac{400 \times 10,000}{10,400} = \frac{10,000}{26} \Omega$$

$$\therefore \text{Voltage measured} = \frac{10,000}{26} \times 5 \times 10^{-3} = \frac{50}{26} \text{ V}$$

$$\therefore \text{Relative error in the measurement} = \frac{1}{26} = 0.04 \text{ v}$$

The correct option is (C)

18. Kinetic energy $E = \frac{1}{2}mv^2$

$$\therefore \frac{\Delta E}{E} \times 100 = \frac{v'^2 - v^2}{v^2} \times 100 = [(1.5)^2 - 1] \times 100$$

$$\therefore \frac{\Delta E}{E} \times 100 = 125\%$$

The correct option is (D)

19. $p = 1$ mm, $N = 100$

$$\text{Least count, } C = \frac{P}{N} = \frac{1 \text{ mm}}{100} = 0.01 \text{ mm}$$

The instrument has a positive zero error $e = +NC = +4 \times 0.01 = +0.04$ mm

(1)

Main scale reading is $2 \times (1 \text{ mm}) = 2$ mm

(2)

Circular scale reading is $67 (0.01) = 0.67$ mm

(3)

$$\therefore \text{observed reading is } R_0 = 2 + 0.67 = 2.67 \text{ mm}$$

(4)

$$\text{So true reading} = R_0 - e = 2.63 \text{ mm}$$

The correct option is (C)

20. The correct option is (A)

21. $G = c^a g^b p^c; M^{-1}L^3T^{-2} = [LT^{-1}]^a [LT^{-2}]^b [ML^{-1}T^{-2}]^c;$

$$a = 0, b = 2 \text{ and } c = -1$$

The correct option is (C)

(5)

22. It is the speed of light in vacuum, hence the dimensions are LT^{-1}

The correct option is (A)

23. Permittivity $\varepsilon = \frac{(\text{charge})^2}{\text{force} \times (\text{distance})^2}$

$$= \frac{Q^2}{MLT^{-2} \times L^2} = M^{-1}L^{-3}T^2Q^2$$

The correct option is (C)

24. The correct option is (B)

25. The correct option is (B)

26. The correct option is (D)

27. The correct option is (D)

28. The correct option is (A)
 29. The correct option is (B)
 30. The correct option is (D)
 31. The correct option is (A)
 32. The correct option is (B)
 33. The correct option is (A)
 34. The correct option is (A)
 35. The correct option is (A)
 36. The correct option is (C)
 37. The correct option is (A)
 38. The correct option is (D)
 39. The correct option is (C)
 40. The correct option is (A)
 41. The correct option is (B)
 42. $\vec{A} \cdot \vec{B} = 3 + 6 + 5 = 14$
 $\vec{A} \cdot \vec{C} = 6 - 2 - 4 = 0$

So \vec{A} and \vec{C} Perpendicular
 \therefore the triangle is right angle triangular

- The correct option is (A)
 43. The correct option is (C)
 44. The correct option is (B)
 45. The correct option is (D)
 46. The correct option is (A)
 47. The correct option is (A)
 48. The correct option is (C)
 49. The correct option is (A)
 50. The correct option is (A)
 51. The correct option is (B)
 52. The correct option is (A)
 53. The correct option is (B)

More than One Option Correct Type

54. Electric flux = Electric field \times area, dipole moment = charge \times length.

The correct option is (A)

55. $hf = I\alpha$ or $\frac{h}{I} = \frac{\alpha}{f} = \frac{T}{T^2} = \frac{1}{T}$

The correct option is (B)

56. MOI $I = Mr^2 = [ML^2]$, moment of force = torque
 $= I\alpha = r \times F = ML^2T^{-2}$

The correct option is (C)

57. astronomical unit of distance

The correct option is (B)

58. The correct option is (A)

59. $a = \frac{F}{t} = MLT^{-3}$, $b = \frac{F}{t^2} = MLT^{-4}$

The correct option is (D)

60. The correct option is (A)

61. $g = \frac{GM}{R^2}$ or $\frac{g}{G} = \frac{M}{R^2}$

The correct option is (B)

62. $Y = ML^{-1}T^{-2} = \frac{T^x(ML^2T^{-2})}{L^3}$

$= ML^{-1}T^{-2}T^x \therefore x = 0$

The correct option is (A)

63. $\frac{B^2}{\mu_0} = \text{energy density}$

$= \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$

The correct option is (D)

64. The correct option is (C)

65. $[M^0LT^{-1}] = [MT^{-2}]^a [ML^{-1}T^{-1}]^b [ML^{-3}]^c$
 $= [M^{a+b+c} L^{-b-3c} T^{-2a-b-3c}]$

on solving $a = 1$, $b = -1$, $c = 0$, $v = \frac{s}{\eta}$

The correct option is (B)

66. The correct option is (C)

67. $V = \pi r^2 l \frac{dV}{V} = \frac{2dr}{r} + \frac{dl}{l}$

$V = \frac{22}{7} \times \frac{8.5}{2} \times \frac{8.5}{2} \times 0.5$

$dV = 2.83 \left(\frac{2 \times .02}{8.5} + \frac{.005}{.050} \right) = 2.83 \left(\frac{1}{21} + \frac{1}{10} \right) = 0.296 \text{ cm}^3$

The correct option is (A)

68. $1450 \times 1.61 = 2334 \text{ kmh}^{-1}$ 1 mile = 1.61 km

$2334 \times \frac{5}{18} \text{ ms}^{-1} = 647.5 \text{ ms}^{-1}$

The correct option is (C)

69. In parallel $c = c_1 + c_2$ and

$\frac{\Delta c}{c} \times 100 = \frac{\Delta c_1 + \Delta c_2}{c_1 + c_2} \times 100 = \frac{0.2 \times 100}{17.4}$

In series $c = \frac{c_1 c_2}{c_1 + c_2}$ and

$\frac{\Delta c}{c} \times 100 = \left(\frac{\Delta c_1}{c_1} + \frac{\Delta c_2}{c_2} + \frac{\Delta c_1 + \Delta c_2}{c_1 + c_2} \right) 100$

$= \left(\frac{0.1}{5.2} + \frac{0.1}{12.2} \right) \times 100 + 1.15 = 3.9\%$

The correct option is (D)

70. $c = q/v$

The correct option is (D)

71. Least count = $\frac{\text{Pitch}}{\text{Number of division on circular scale}}$
 $= \frac{0.1}{2} = 0.5 \times 10^{-3} \text{ cm}$

The correct option is (A)

72. The correct option is (A), (B) and (C)

73. The correct option is (B)

74. $\cos \theta = \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j} - \hat{k})}{\sqrt{3} \cdot \sqrt{3}} = \frac{-1}{3}$ or

$$\theta = \cos^{-1}\left(\frac{-1}{3}\right)$$

The correct option is (D)

75. $\sin \theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|} = \frac{\sqrt{5^2 + 2^2}}{9}$
 $= \frac{\sqrt{29}}{9}$ or $\sin^{-1}\left(\frac{\sqrt{29}}{9}\right)$

The correct option is (D)

76. $a_{av} = \frac{v_f - v_i}{t} = \frac{5\hat{j} - 5\hat{i}}{10}$ or $|a_{av}| = \frac{1}{\sqrt{2}}$ North West.

The correct option is (C)

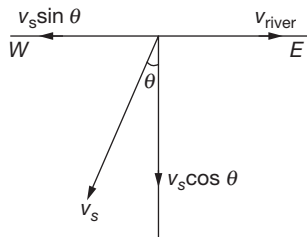
77. $(2\hat{i} + 3\hat{j} + 8\hat{k}) \cdot (4\hat{j} - 4\hat{i} + \alpha\hat{k}) = 0$ or
 $-8 + 12 + 8\alpha = 0$ $\alpha = \frac{-1}{2}$

The correct option is (B)

78. The correct option is (A) and (D)

79. $v_s \sin \theta = v_{\text{river}}$ or

$$\sin \theta = \frac{1}{2} \quad \theta = 30^\circ$$



The correct option is (D)

80. $F = -q(\vec{v} \times \vec{B}) = -e(\hat{j} \times -\hat{k}) = e\hat{i}$

The correct option is (C)

81. The correct option is (C)

82. Is also perpendicular to \vec{A} but not in the same plane. Check using $\vec{A} \cdot \vec{B} = 0$

The correct option is (D) and (A)

83. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix}$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{\hat{i} + 10\hat{j} + 17\hat{k}}{\sqrt{390}}$$

$$\vec{x} = |\vec{B}| \hat{n} = \frac{\sqrt{29}(\hat{i} + 10\hat{j} + 17\hat{k})}{\sqrt{390}}$$

The correct option is (D)

84. $V_{rm} = V_r - V_m = -3\hat{k} - 4\hat{j}$;

$$\tan \beta = \frac{4}{3} \Rightarrow \beta = 53^\circ \text{ North of vertical}$$

The correct option is (C)

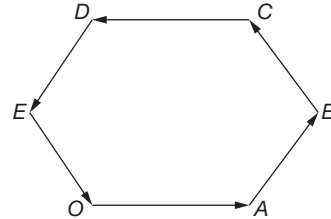
85. $a_{av} = \frac{-15\hat{i} - (15\hat{i})}{10} = -3\hat{i} = 3 \text{ ms}^{-2}$

The correct option is (A)

86. A regular hexagon will be formed if we continue.

After 5th turn displacement = $OE = 500 \text{ m}$

After 3rd turn displacement = $OC = 1000 \text{ m}$ (OC is diameter of the circle circumscribing regular hexagon).



The correct option is (A)

87. \therefore vector division is not allowed.

The correct option is (D)

88. use Parallelogram law.

The correct option is (D)

89. $V_{rm} = (V_{rx} - V_m)\hat{i} + V_{ry}\hat{j}$

Case I: $\tan 90 = \frac{V_{ry}}{V_{rx} - V_m} = \frac{V_{ry}}{V_{rx} - 8}$ or $V_{rx} = 8 \text{ ms}^{-1}$

Case II: $\tan 30 = \frac{V_{rx} - V_m}{V_{ry}} = \frac{8 - 12}{V_{ry}}$ or $V_{ry} = -4\sqrt{3} \text{ ms}^{-1}$

$$V_r = 8\hat{i} - 4\sqrt{3}\hat{j} = 4\sqrt{2^2 + 3} = 4\sqrt{7} \text{ ms}^{-1}$$

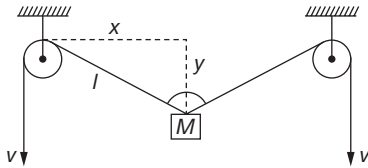
$$\tan \theta = \frac{V_{ry}}{V_{rx}} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \text{ or}$$

$$\theta = \tan^{-1} \frac{\sqrt{3}}{2} \text{ with respect to road (horizontally).}$$

The correct option is (A)

90. $l^2 = x^2 + y^2$ or $2l \frac{dl}{dt} = 0 + 2y \frac{dy}{dt}$ ($\because x$ is constant its derivative is zero)

$$\frac{dy}{dt} = \frac{dl/dt}{y/l} = \frac{v}{\cos \theta}$$



The correct option is (B)

91. as $13N > 8 + 4$ [$\because R_{\max} = A + B$]

The correct option is (C)

92. $\vec{x} = \hat{A} |B| = \frac{(3\hat{i} + 4\hat{j} + 2\hat{k})\sqrt{36+1+9}}{\sqrt{9+16+4}}$

$$= \sqrt{\frac{46}{29}} (3\hat{i} + 4\hat{j} + 2\hat{k})$$

The correct option is (A)

93. $\vec{R} = \vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j}$

$$|R| = \sqrt{5} \text{ and } \tan \theta = -2 \text{ or } \theta = 297^\circ$$

The correct option is (A)

94. $F_{\text{net}} = -(kx + kx \cos 60 + kx \cos 60)$

$$= -2kx \therefore k_{\text{eq}} = 2k$$

The correct option is (C)

95. $F = \frac{dp}{dt} = 2(-\sin t \hat{i} + \cos t \hat{j})$

$$\vec{F} \cdot \vec{p} = 4(-\sin t \hat{i} + \cos t \hat{j}) \cdot (\cos t \hat{i} + \sin t \hat{j}) = 0$$

$$\therefore \theta = 90^\circ$$

The correct option is (D)

96. $\vec{v}_{\text{rel}} = \vec{v}_A - \vec{v}_B$ or

$$|v_{AB}| = \sqrt{v^2 + v^2 - 2v^2 \cos \theta}$$

$$= 2v \sin \frac{\theta}{2}$$

$$(v_{\text{rel}})_{\text{average}} = \frac{\int_0^{2\pi} v_{\text{rel}} d\theta}{\int_0^{2\pi} d\theta} = \frac{1}{2\pi} \int_0^{2\pi} 2v \sin \frac{\theta}{2} d\theta$$

$$= \frac{2v}{\pi} \left[-\cos \left(\frac{\theta}{2} \right) \right]_0^{2\pi} = \frac{4v}{\pi}$$

The correct option is (D)

97. $v_{ws} = v_w - v_s \Rightarrow (v_{wx} \hat{i} + v_{wy} \hat{j}) - 10 \hat{i} = 5 \hat{j}$

or $(v_{wx} \hat{i} + v_{wy} \hat{j}) = 5 \hat{j} + 10 \hat{i}$ or

$$|v_w| = 5\sqrt{5} \text{ and } \tan \theta = \frac{1}{2} \text{ or } \theta = \tan^{-1} \frac{1}{2}$$

i.e. wind is blowing at $5\sqrt{5} \text{ ms}^{-1} \tan^{-1} \frac{1}{2}$ North of East.

The correct option is (A)

98. $\vec{v} = \frac{d\vec{r}}{dt} = a\omega [-\sin \omega t \hat{i} + \cos \omega t \hat{j}]$ and $\vec{v} \cdot \vec{r} = 0$

The correct option is (D)

99. $\vec{d} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

$$W = \vec{F} \cdot \vec{d} = (6\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} - 2\hat{j} + 4\hat{k}) = 32 \text{ J.}$$

The correct option is (C)

100. $\vec{v}_{\text{Res}} = \vec{v}_{\text{steamer}} + \vec{v}_{\text{wind}} = -5\hat{i} + 5\hat{i} + 5\hat{j} = 5\hat{j}$. The flag will flutter in a direction opposite to the direction of motion.

The correct option is (D)

101. $F = \sqrt{F^2 + F^2 + 2F^2 \cos \theta}$ or

$$\cos \theta = \frac{-1}{2}; \theta = 120^\circ$$

The correct option is (D)

102. $d = 100\hat{j} + 100\hat{i} + 20\hat{j} + (-100\hat{i} - 100\hat{j}) = 20\hat{j}$

The correct option is (C)

103. $F = MLT^{-2}$ density $= ML^{-3} = FL^{-4}T^2$

The correct option is (B)

Previous Years' Questions

104. Velocity of light in vacuum $= \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

or $[LT^{-1}] = \left[\frac{1}{\sqrt{\mu_0 \epsilon_0}} \right]$

or $[L^2T^{-2}] = \left[\frac{1}{\mu_0 \epsilon_0} \right]$

\therefore Dimensions of $\frac{1}{\mu_0 \epsilon_0} = [L^2T^{-2}]$

The correct option is (C)

105. [Momentum] = $[MLT^{-1}]$

[Planck's constant] = $[ML^2T^{-1}]$

Momentum and Planck's constant do not have same dimensions.

The correct option is (B)

106. Viscous force $F = 6\pi\eta rv$

$$\therefore \eta = \frac{F}{6\pi rv} \text{ or } [\eta] = \frac{[F]}{[r][v]}$$

$$\text{or } [\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]} \text{ or } [\eta] = [ML^{-1}T^{-1}]$$

The correct option is (C)

107. $[ML^2Q^{-2}] = [ML^2A^{-2}T^{-2}]$

$$[Wb] = [ML^2T^{-2}A^{-1}]$$

$$\left[\frac{Wb}{m^2}\right] = [MT^{-2}A^{-1}]$$

$$[\text{henry}] = [ML^2T^{-2}A^{-2}]$$

$$\left[\frac{H}{m^2}\right] = [MT^{-2}A^{-2}]$$

Obviously henry (H) has dimensions $\frac{ML^2}{Q^2}$

The correct option is (B)

108. Lorentz force $= |\vec{F}| = |q\vec{v} \times \vec{B}|$

$$\therefore [B] = \frac{[F]}{[q][v]} = \frac{MLT^{-2}}{C \times LT^{-1}} = \frac{MLT^{-2}}{CLT^{-1}} = [MT^{-1}C^{-1}]$$

The correct option is (C)

109. According to the rules

23.023 has 5 significant figures

0.0003 has 1 significant figure

2.1×10^{-3} has 2 significant figures

The correct option is (B)

110. $R = \frac{V}{I} \therefore \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

The percentage error in R is

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 = 3\% + 3\% = 6\%$$

The correct option is (A)

111. Unit of ϵ_0 is $C^2m^{-2}N^{-1}$

$$[\epsilon_0] = [AT]^2 [L]^{-2} [MLT^{-2}]^{-1}$$

$$= [A^2 T^2 L^{-2} M^{-1} L^{-1} T^2]$$

$$= [M^{-1} L^{-3} T^4 A^2]$$

The correct option is (A)

112. $I = \left(e^{\frac{1000V}{T}-1} \right) mA$

$$dI = \frac{1000}{T} e^{\frac{1000V}{T}} dV$$

$$dI = \frac{1000}{T} [I + 1] dV = \frac{1000}{300} [5 + 1] \times 0.01 = 0.2 \text{ mA}$$

The correct option is (A)

113. $g \propto LT^2$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + 2 \times \frac{\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{0.1}{20} + 2 \times \frac{1}{90}$$

$$= \frac{\Delta g}{g} \times 100 = 0.5 + 2.22 = 2.72$$

$$\approx 3\%$$

The correct option is (A)

114. $a_{\text{mean}} = 92$

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4|}{4}$$

$$= 1.5$$

Since least count of the clock is 1 s,

$$\therefore \text{Mean time} = 92 \pm 2 \text{ s}$$

The correct option is (D)

115. Least count = $\frac{0.5}{50} \text{ mm} = \frac{1}{100} \text{ mm}$

Zero error = -0.05 mm

Thickness (d) =

$$0.5 \text{ mm} + 25 \times \frac{1}{100} \text{ mm} - (-0.05) \text{ mm} = 0.8 \text{ mm}$$

The correct option is (A)