CHAPTER 4

Quadratic Equations and Expressions

Chapter Highlights

Quadratic equation, Common roots, Symmetric function of the roots, Graph of a quadratic expression, Greatest and least values of a quadratic expression, Nature of roots of a quadratic equation with respect to one or two real numbers, Relation between roots and coefficients of a polynomial equation, Formation of a polynomial equation from given roots, Sign of a polynomial expression, Rational algebraic expression

QUADRATIC EQUATION

An algebraic expression of the form: $ax^2 + bx + c$, where $a\ (\neq 0), b, c \in R$ is called a real quadratic expression.

An equation of the form: $ax^2 + bx + c = 0$, where $a\ (\neq 0), b, c \in R$ is called a real quadratic equation.

The numbers a, b, c are called the coefficients of the quadratic equation and the expression $b^2 - 4ac$ is called its discriminant. Discriminant of a quadratic equation is usually denoted by D or Δ.

Roots of the Quadratic Equation

A root of the quadratic equation

$$ax^2 + bx + c = 0 \qquad (1)$$

is a number α (real or complex) such that $a\alpha^2 + b\alpha + c = 0$. The roots of the quadratic Eq. (1) are given by,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nature of Roots of the Quadratic Equation

1. If $D < 0$, then roots α, β are imaginary
2. If $D > 0$, then roots α, β are real and distinct
3. If $D = 0$, then roots α, β are real and equal

TRICK(S) FOR PROBLEM SOLVING

For the quadratic equation $ax^2 + bx + c = 0$

- One root will be reciprocal of the other if $a = c$.
- One root is zero if $c = 0$.
- Roots are equal in magnitude but opposite in sign if $b = 0$.
- Both roots are zero if $b = c = 0$.
- Roots are positive if $D > 0$, a and c are of same sign and b is of opposite sign.
- Roots are of opposite sign if a and c are of opposite sign.
- Roots are negative if $D > 0$ and a, b, c are of the same sign.
- Roots are rational $\Leftrightarrow D$ is a perfect square
- Roots are irrational $\Leftrightarrow D$ is positive but not a perfect square.
- If $a + b + c = 0$, then $1, \frac{c}{a}$ are the roots of the equation
  $$ax^2 + bx + c = 0$$
  and if $a - b + c = 0$, then the roots are -1 and $-\frac{c}{a}$.
- If $ax^2 + bx + c = 0$ is satisfied by more than two values, it is an identity and $a = b = c = 0$ and vice-versa.
- If $ax^2 + bx + c = 0$, where $a, b, c \in R$, has one root $p + iq$, then the other root will be $p - iq$. Hence, the imaginary roots occur in conjugate pair.

- If $ax^2 + bx + c = 0$, where a, b, c are rational, has one root $p+\sqrt{q}$ then the other root will be $p-\sqrt{q}$. Hence, irrational roots occur in conjugate pair if the coefficients are rational.
- The quadratic equation whose roots are reciprocals of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$ (i.e., the coefficients are written in reverse order).
- If $a = 1$, $b, c \in Z$ and the roots are rational numbers, then these roots must be integers.
- The condition that the roots of the equation $ax^2 + bx + c = 0$ may be in the ration $m : n$ is $mnb^2 = ac(m + n)$.
- If sum of roots of $ax^2 + bx + c^3 = 0$ is equal to the sum of their reciprocals, then ab^2, bc^2, ca^2 are in A.P. or $\frac{c}{b}, \frac{b}{a}, \frac{a}{c}$ are in H.P.

SOLVED EXAMPLES

1. If $a + b + c = 0$ and a, b, c are rational, then the roots of the equation $(b + c - a) x^2 + (c + a - b) x + (a + b - c) = 0$ are

(A) rational (B) irrational
(C) imaginary (D) equal

Solution: (A)

We have,

$$D = (c + a - b)^2 - 4 (b + c - a) (a + b - c)$$
$$= (a + b + c - 2b)^2 - 4 (a + b + c - 2a)(a + b + c - 2c)$$
$$= (-2b)^2 - 4 (-2a) (-2c) = 4 (b^2 - 4ac)$$
$$= 4 [(-a - c)^2 - 4ac] = 4 (a - c)^2$$
$$= [2 (a - c)]^2 = \text{perfect square}$$

∴ Roots are rational

2. The number of values of the pair (α, β) for which the equation $\alpha(x+1)^2+\beta(x^2-3x-2)+x+1=0, \forall x\in R$ is

(A) 1 (B) 0
(C) infinite (D) None of these

Solution: (B)

The equation

$$(\alpha+\beta)x^2 + (2\alpha - 3\beta + 1)x + (\alpha + 2\beta + 1) = 0$$

is an identify in x if

$$\alpha+\beta=0 \quad (1)$$

$$2\alpha-3\beta+1=0 \quad (2)$$

and $$\alpha-2\beta+1=0 \quad (3)$$

Solving Eq. (1) and (2), we get $\alpha = -\frac{1}{5}$ and $\beta = \frac{1}{5}$.

But these values do not satisfy Eq. (3). Hence the given equation cannot be an identify for any values of α and β.

3. If $x^2 + lx + m$ is an integer for every integer x, then

(A) l is always an integer but m need not be an integer
(B) m is always in integer but l need not be an integer
(C) l and m, both are always integer
(D) None of these

Solution: (C)

Let $g(x) = x^2 + lx + m$, then $g(x)$ is an integer for every integer x.

Therefore, $g(0) = m \Rightarrow m$ is an integer

Also,

$g(1) = 1 + l + m \Rightarrow l$ is an integer ($\because$ m is integer)

4. The equation $125^x + 45^x = 2.27^x$ has

(A) no solution
(B) one solution
(C) two solutions
(C) more than two solutions

Solution: (B)

The given equation can be written as

$$(5/3)^{3x} + (5/3)^x = 2$$

Putting $(5/3)^x = t$, the equation becomes

$$t^3 + t - 2 = 0$$
$$\Rightarrow \quad t^3 - 1 + (t - 1) = 0$$
$$\Rightarrow \quad (t - 1)(t^2 + t + 1) + (t - 1) = 0$$
$$\Rightarrow \quad (t - 1)(t^2 + t + 2) = 0$$
$$\Rightarrow \quad t = 1$$
$$\text{or} \quad t^2 + t + 2 = 0$$

But $t^2 + t + 2 = 0$ does not have real solutions.

Therefore, $t = 1 \Rightarrow (5/3)^x = 1 \Rightarrow x = 0$.

5. For $a > 0$, the roots of the equation $\log_{ax} a + \log_x a^2 + \log_{a^2x} a^3 = 0$, are given by:

(A) $a^{-4/3}$ (B) $a^{-3/4}$ (C) $a^{1/2}$ (D) a^{-1}

Solution: (A)

We have,

$$\frac{\log_a a}{\log_a a + \log_a x} + \frac{2\log_a a}{\log_a x} + \frac{3\log_a a}{2\log_a a + \log_a x} = 0$$

$\Rightarrow \quad \frac{1}{1+t}+\frac{2}{t}+\frac{3}{2+t}=0 \quad (\text{let } \log_a x = t)$

$\Rightarrow \quad \frac{2t+t^2+2t^2+6t+4+3t^2+3t}{t(1+t)(2+t)}=0$

$\Rightarrow \quad 6t^2+11t+4=0$

$\Rightarrow \quad 6t^2+8t+3t+4=0$

$\Rightarrow \quad (2t+1)(3t+4)=0$

$\Rightarrow \quad t=-\frac{1}{2},-\frac{4}{3}$

$\Rightarrow \quad \log_a x=-\frac{1}{2},-\frac{4}{3}$

$\therefore \quad x=a^{-1/2}, a^{-4/3}$

6. The number of solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$ is

(A) 0 (B) 1 (C) 2 (D) infinite

Solution: (A)

Put $5^x = y$. Then the given equation becomes

$$\sin(e^x) = y+\frac{1}{y} = \left(\sqrt{y}-\frac{1}{\sqrt{y}}\right)^2+2 \quad (\because 5^x > 0)$$

$\Rightarrow \quad \sin(e^x) \geq 2.$

Which is not possible for any real value of x.
Hence, the given equation has no real solution.

7. If $x = 2 + 2^{2/3} + 2^{1/3}$ then the value of $x^3 - 6x^2 + 6x$ is

(A) 3 (B) 2
(C) 1 (D) None of these

Solution: (B)

We have,

$$x-2 = 2^{2/3}+2^{1/3}$$

Cube both sides, we get

$$(x-2)^3 = 2^2+2+3\cdot 2^{2/3}\cdot 2^{1/3}(x-2)$$
$$= 6+6(x-2)$$

or $\quad x^3-6x^2+12x-8=-6+6x.$

$\therefore \quad x^3-6x^2+6x=2.$

8. The values of a, for which the quadratic equation $3x^2 + 2(a^2+1)x + (a^2-3a+2) = 0$ possesses roots of opposite sign, are

(A) $1 < a < 2$ (B) $a \in (2, \infty)$
(C) $1 < a < 3$ (D) None of these

Solution: (A)

Roots are of opposite sign if (a) roots are real and distinct, (b) product is negative.

So, $\quad D = 4(a^2+1)^2 - 12(a^2-3a+2) > 0$

and product of roots $= \frac{a^2-3a+2}{3} < 0$

$\Rightarrow \quad a^2-3a+2=(a-1)(a-2)<0$

$\therefore \quad 1 < a < 2$

Clearly for these values of a, $D > 0$.
Hence, $1 < a < 2$.

9. The number of real solutions of the equation $27^{1/x} + 12^{1/x} = 2 \times 8^{1/x}$ is

(A) one (B) two (C) infinite (D) zero

Solution: (D)

The given equation can be written as

$$\left(\frac{3}{2}\right)^{3/x}+\left(\frac{3}{2}\right)^{1/x}=2.$$

Put $\left(\frac{3}{2}\right)^{1/x}=t$, then the equation becomes

$$t^3+t-2=0 \quad \Rightarrow \quad (t-1)(t^2+t+2)=0.$$

But $t^2+t+2=0$ has no real roots,

$\therefore \quad t=1$

$\Rightarrow \quad \left(\frac{3}{2}\right)^{1/x}=1 \quad \Rightarrow \quad \frac{1}{x}=0$

which is not possible for any value of x.

10. For all real values of x, $\left|\frac{12x}{4x^2+9}\right|$

(A) ≤ 1 (B) ≤ 2 (C) >1 (D) >2

Solution: (A)

Let $\quad \frac{12x}{4x^2+9}=y,$

Now, $\quad 4yx^2-12x+9y=0$

As x is real,

$$D = 144-4\cdot 4y\cdot 9y \geq 0 \quad \Rightarrow \quad 1-y^2 \geq 0$$

$\Rightarrow \quad y^2 \leq 1;$

$\therefore \quad |y| \leq 1.$

Hence, $\quad \left|\frac{12x}{4x^2+9}\right| \leq 1.$

11. If $x^2 - 3x + 2$ be one of the factors of the expression $x^4 - px^2 + q$, then

(A) $p = 4, q = 5$ (B) $p = 5, q = 4$
(C) $p = -5, q = -4$ (D) None of these

Solution: (B)

Since $x^2 - 3x + 2$ is one of the factors of the expression $x^4 - px^2 + q$, therefore, on dividing the expression by factor, remainder = 0 i.e., on dividing $x^4 - px^2 + q$ by $x^2 - 3x + 2$, the remainder

$$(15 - 3p)\,x + (2p + q - 14) = 0$$

On comparing both sides, we get

$$15 - 3p = 0 \quad \text{or} \quad p = 5$$

and $$2p + q - 14 = 0 \quad \text{or} \quad q = 4.$$

12. If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is

(A) 1 (B) 0
(C) 2 (D) None of these

Solution: (A)

The roots of the equation are given by,

$$x = \frac{b \pm \sqrt{b^2 - 4c}}{2}$$

If $$\alpha = \frac{b + \sqrt{b^2 - 4c}}{2}$$

and $$\beta = \frac{b - \sqrt{b^2 - 4c}}{2}$$

Then, $$\alpha - \beta = 1$$

$\Rightarrow$ $$\sqrt{b^2 - 4c} = 1$$

$\Rightarrow$ $$b^2 - 4c = 1.$$

13. If $p\,(q - r)\,x^2 + q\,(r - p)\,x + r\,(p - q) = 0$ has equal roots, then $\dfrac{2}{q} =$

(A) $p + \dfrac{1}{r}$ (B) $\dfrac{1}{p} + r$

(C) $p + r$ (D) $\dfrac{1}{p} + \dfrac{1}{r}$

Solution: (D)

Since $$p\,(q - r) + q\,(r - p) + r\,(p - q) = 0$$

$\therefore$ one root is 1

$\therefore$ other root $= \dfrac{r\,(p - q)}{p\,(q - r)}$.

Since roots are equal

$\therefore$ $$\frac{rp - rq}{pq - pr} = 1$$

$\Rightarrow$ $$rp - rq = pq - pr$$

$\Rightarrow$ $$2rp = q\,(p + r)$$

$\therefore$ $$\frac{2}{q} = \frac{p + r}{pr} = \frac{1}{p} + \frac{1}{r}.$$

14. If $c \neq 0$ and the equation $\dfrac{p}{2x} = \dfrac{a}{x + c} + \dfrac{b}{x - c}$ has two equal roots, then p can be

(A) $(\sqrt{a} - \sqrt{b})^2$ (B) $(\sqrt{a} + \sqrt{b})^2$
(C) $a + b$ (D) $a - b$

Solution: (A, B)

We can write the given equation as

$$\frac{p}{2x} = \frac{(a + b)x + c\,(b - a)}{x^2 - c^2}$$

or $$p\,(x^2 - c^2) = 2\,(a + b)\,x^2 - 2c\,(a - b)\,x$$

or $$(2a + 2b - p)\,x^2 - 2c\,(a - b)\,x + pc^2 = 0$$

For this equation to have equal roots

$$c^2(a - b)^2 - pc^2\,(2a + 2b - p) = 0$$

$\Rightarrow$ $$(a - b)^2 - 2p\,(a + b) + p^2 = 0 \qquad (\because c^2 \neq 0)$$

$\Rightarrow$ $$[p - (a + b)]^2 = (a + b)^2 - (a - b)^2 = 4ab$$

$\Rightarrow$ $$p - (a + b) = \pm\, 2\sqrt{ab}$$

$\Rightarrow$ $$p = a + b \pm 2\sqrt{ab} = (\sqrt{a} \pm \sqrt{b})^2$$

15. If $(7 - 4\sqrt{3})^{x^2 - 4x + 3} + (7 + 4\sqrt{3})^{x^2 - 4x + 3} = 14$, then the value of x is given by

(A) $2, 2 \pm \sqrt{2}$ (B) $2 \pm \sqrt{3}, 3$
(C) $3 \pm \sqrt{2}, 2$ (D) None of these

Solution: (A)

Since $$(7 + 4\sqrt{3})\,(7 - 4\sqrt{3}) = 1,$$

$\therefore$ the given equation becomes

$$y + \frac{1}{y} = 14$$

where $$y = (7 - 4\sqrt{3})^{x^2 - 4x + 3}$$

$\Rightarrow$ $$y^2 - 14y + 1 = 0 \Rightarrow y = 7 \pm 4\sqrt{3}$$

Now $$y = 7 \pm 4\sqrt{3}$$

$\Rightarrow$ $$x^2 - 4x + 3 = -1$$

$\Rightarrow$ $$x = 2, 2$$

Also, $$y = 7 - 4\sqrt{3}$$

$\Rightarrow$ $$x^2 - 4x + 3 = 1 \Rightarrow x = 2 \pm \sqrt{2}$$

Sum and Product of the Roots

If α and β are roots of $ax^2 + bx + c = 0$, then

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Formation of Equation with Given Roots

If α and β are roots of

$$f(x) = ax^2 + bx + c = 0,$$

then $$f(x) = (x - \alpha)(x - \beta) = 0$$

$$= x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e., x^2 – (sum of the roots) x + (product of the roots) = 0.

CAUTION

A quadratic equation with all odd integer coefficients cannot have rational roots.

SOLVED EXAMPLES

16. If r be the ratio of the roots of the equation $ax^2 + bx + c = 0$, then $\frac{(r+1)^2}{r} =$

(A) $\frac{a^2}{bc}$ (B) $\frac{b^2}{ca}$

(C) $\frac{c^2}{ab}$ (D) None of these

Solution: (B)

Given equation is $ax^2 + bx + c = 0$ (1)

Let the root of equation (1) be α and $r\alpha$, then

$$\alpha + r\alpha = \frac{-b}{a} \quad (2)$$

and $$r\alpha^2 = \frac{c}{a} \quad (3)$$

From Eq. (2),

$$\alpha = -\frac{b}{a(r+1)} \quad (4)$$

Putting the value of α in Eq. (3), we get

$$\frac{rb^2}{a^2(r+1)^2} = \frac{c}{a}$$

or, $$\frac{b^2}{ac} = \frac{(r+1)^2}{r}$$

17. If the ratio of the roots of $lx^2 + nx + n = 0$ is $p : q$, then

(A) $\sqrt{\frac{q}{p}} + \sqrt{\frac{p}{q}} + \sqrt{\frac{l}{n}} = 0$

(B) $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

(C) $\sqrt{\frac{q}{p}} + \sqrt{\frac{p}{q}} + \sqrt{\frac{l}{n}} = 1$

(D) $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 1$

Solution: (B)

Let the roots be α and β.

Then $$\alpha + \beta = \frac{-n}{l};\ \alpha\beta = \frac{n}{l};$$

and $$\frac{\alpha}{\beta} = \frac{p}{q}$$

Now, $$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta}$$

$$= \frac{\alpha + \beta + \alpha\beta}{\sqrt{\alpha\beta}} = \frac{-\frac{n}{l} + \frac{n}{l}}{\sqrt{\frac{n}{l}}} = 0$$

18. In a quadratic equation with leading coefficient 1, a student reads the coefficient 16 of x wrongly as 19 and obtain the roots as – 15 and – 4. The correct roots are

(A) 6, 10 (B) –6, –10

(C) –7, –9 (D) None of these

Solution: (B)

Since coefficient of $x = 16$,

∴ sum of roots = –16

Since constant term = (–15) (–4) = 60,

∴ correct answer is –6, –10.

19. If the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ are equal in magnitude but opposite in sign, then their product is

(A) $\frac{1}{2}(a^2 + b^2)$ (B) $\frac{-1}{2}(a^2 + b^2)$

(C) $\frac{1}{2}ab$ (D) $\frac{-1}{2}ab$

Solution: (B)

We have, $$((x + b) + (x + a)c = (x + a)(x + b)$$

$$\Rightarrow \quad x^2 + bx + ax - 2cx + ab - bc - ca = 0$$

Now, let roots be α and β, then

$$\alpha + \beta = 0,\ \alpha\beta = ab - bc - ac$$

$$\alpha + \beta = 0 \Rightarrow b + a = 2c$$

and $\quad \alpha\beta = ab - (b + a)\,c$

$\Rightarrow \quad \alpha\beta = ab - \dfrac{(a+b)^2}{2}$

$\Rightarrow \quad \alpha\beta = \dfrac{1}{2}(-a^2 - b^2)$

$\therefore \quad \alpha\beta = -\dfrac{1}{2}(a^2 + b^2)$

20. If sin θ and cos θ are the roots of the equation $ax^2 + bx + c = 0$, then

(A) $(a - c)^2 = b^2 - c^2$ (B) $(a - c)^2 = b^2 + c^2$
(C) $(a + c)^2 = b^2 - c^2$ (D) $(a + c)^2 = b^2 + c^2$

Solution: (D)

Since sin θ and cos θ are the roots of the equation $ax^2 + bx + c = 0$

$\therefore \quad \sin\theta + \cos\theta = -\dfrac{b}{a}$ and $\sin\theta\cos\theta = \dfrac{c}{a}$

Now $\quad (\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

$\therefore \quad \dfrac{b^2}{a^2} = 1 + \dfrac{2c}{a} = \dfrac{a + 2c}{a}$

$\Rightarrow \quad b^2 = a\,(a + 2c) = a^2 + 2ac$

$\Rightarrow \quad b^2 + c^2 = a^2 + 2ac + c^2 = (a + c)^2$

Hence, $\quad (a + c)^2 = b^2 + c^2$

21. In copying a quadratic equation of the form $x^2 + px + q = 0$, a student wrote the coefficient of x incorrectly and the roots were found to be 3 and 10; another student wrote the same equation but he wrote the constant term incorrectly and thus he found the roots to be 4 and 7. The roots of the correct equation are

(A) 5, 6 (B) 4, 6
(C) 4, 5 (D) None of these

Solution: (A)

In case of the first student, product of the roots = 3 × 10 = q. So the correct value of q is 30.

In case of the second student, sum of the roots = 4 + 7 = $-p$.

So the correct value of p is –11.

$\therefore$ The correct equation is $x^2 - 11x + 30 = 0$

or $\quad (x - 5)(x - 6) = 0;$

$\therefore \quad x = 5, 6.$

$\therefore$ Roots of the correct equation are 5, 6.

22. If α, β are the roots of $x^2 - 2px + q = 0$ and γ, δ are roots of $x^2 - 2rx + s = 0$ and $\alpha, \beta, \gamma, \delta$ are in A.P., then

(A) $p - q = r^2 - s^2$ (B) $s - q = r^2 - p^2$
(C) $r - s = p^2 - q^2$ (D) None of these

Solution: (B)

We have, $\quad \alpha + \beta = 2p;$

$\alpha\beta = q,\ \gamma + \delta = 2r$ and $\gamma\delta = s$

$\because \quad \alpha, \beta, \gamma, \delta$ are in A.P.

$\therefore \quad \beta - \alpha = \delta - \gamma \Rightarrow (\beta - \alpha)^2 = (\delta - \gamma)^2$

$\Rightarrow \quad (\beta + \alpha)^2 - 4\beta\alpha = (\delta + \gamma)^2 - 4\delta\gamma$

$\Rightarrow \quad 4p^2 - 4q = 4r^2 - 4s;$

or $\quad s - q = r^2 - p^2$

23. The rational values of a and b in $ax^2 + bx + 1 = 0$ if $\dfrac{1}{4+\sqrt{3}}$ is a root, are

(A) $a = 13, b = -8$ (B) $a = -13, b = 8$
(C) $a = 13, b = 8$ (D) $a = -13, b = -8$

Solution: (A)

One root $= \dfrac{1}{4+\sqrt{3}} \times \dfrac{4-\sqrt{3}}{4-\sqrt{3}} = \dfrac{4-\sqrt{3}}{13}$

$\therefore \quad$ other root $= \dfrac{4+\sqrt{3}}{13}$

$\therefore$ The quadratic equation is

$$x^2 - \left(\frac{4+\sqrt{3}}{13} + \frac{4-\sqrt{3}}{13}\right)x + \frac{4+\sqrt{3}}{13}\cdot\frac{4-\sqrt{3}}{13} = 0$$

or $\quad 13x^2 - 8x + 1 = 0$

This equation must be identical with $ax^2 + bx + 1 = 0;$

$\therefore \quad a = 13$ and $b = -8.$

24. If a and b are rational and α, β be the roots of $x^2 + 2ax + b = 0$, then the equation with rational coefficients one of whose roots is $\alpha + \beta + \sqrt{\alpha^2 + \beta^2}$ is

(A) $x^2 + 4ax - 2b = 0$ (B) $x^2 + 4ax + 2b = 0$
(C) $x^2 - 4ax + 2b = 0$ (D) $x^2 - 4ax - 2b = 0$

Solution: (B)

Since α, β are roots of $x^2 + 2ax + b = 0$

$$\alpha + \beta = -2a \quad \text{and} \quad \alpha\beta = b$$

Let $\quad y = \alpha + \beta + \sqrt{\alpha^2 + \beta^2}$

$\Rightarrow \quad (y + 2a)^2 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4a^2 - 2b$

$\Rightarrow \quad y^2 + 4ay + 2b = 0$

So, the required equation is $x^2 + 4ax + 2b = 0$.

25. If c, d are the roots of the equation $(x-a)(x-b)-k=0$, then the roots of the equation $(x-c)(x-d)+k=0$ are

(A) c, d (B) a, c (C) b, d (D) a, b

Solution: (D)

We have, $(x-a)(x-b)-k=0$

$$\Rightarrow \quad x^2-(a+b)x+ab-k=0 \qquad (1)$$

Since the roots of Eq. (1) are c and d

$$\therefore \quad c+d=a+b, \qquad (2)$$

$$\text{and} \quad cd=ab-k \qquad (3)$$

Now $(x-c)(x-d)+k=0$

$$\Rightarrow \quad x^2-(c+d)x+cd+k=0$$

$$\Rightarrow \quad x^2-(a+b)x+ab=0$$

[Putting the values of $a+b$ and ab from Eqs (2) and (3)]

$$\Rightarrow \quad (x-a)(x-b)=0 \Rightarrow x=a, b.$$

26. If the roots of the equations $x^2-bx+c=0$ and $x^2-cx+b=0$ differ by the same quantity then $b+c$ is equal to

(A) 4 (B) 1 (C) 0 (D) –4

Solution: (D)

We know that if α, β are roots of the equation

$$Ax^2+Bx+C=0,$$

then

$$\alpha-\beta=\frac{\sqrt{B^2-4AC}}{A}$$

Equating the value of $\alpha-\beta$ from both the given equations, we get

$$\sqrt{b^2-4c}=\sqrt{c^2-4b}$$

$$\Rightarrow \quad b^2-4c=c^2-4b$$

$$\Rightarrow \quad b^2-c^2=-4(b-c)$$

$$\Rightarrow \quad (b-c)(b+c+4)=0$$

$$\Rightarrow \quad b+c=-4 \qquad (\because b \neq c)$$

27. If α, β are non-real roots of $ax^2+bx+c=0$, $(a, b, c \in R)$, then

(A) $\alpha\beta=1$ (B) $\alpha=\beta$

(C) $\overline{\alpha\beta}=1$ (D) $\alpha=\overline{\beta}$

Solution: (D)

Here $b^2-4ac<0$

$$\therefore \quad \alpha=\frac{-b+i\sqrt{4ac-b^2}}{2a}$$

$$\text{and} \quad \beta=\frac{-b-i\sqrt{4ac-b^2}}{2a}$$

$$\therefore \quad \alpha=\overline{\beta}.$$

COMMON ROOTS

One Root Common

If α is a common root of the equations

$$a_1x^2+b_1x+c_1=0 \qquad (1)$$

$$\text{and} \quad a_2x^2+b_2x+c_2=0 \qquad (2)$$

then we have

$$a_1\alpha^2+b_1\alpha+c_1=0$$

$$\text{and} \quad a_2\alpha^2+b_2\alpha+c_2=0$$

These give

$$\frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1} \qquad (a_1b_2-a_2b_1\neq 0).$$

Thus, the required condition for one common root is $(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)=(c_1a_2-c_2a_1)^2$ and the value of the common root is $\alpha=\dfrac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$ or $\dfrac{b_1c_2-b_2c_1}{c_1a_2-c_2a_1}$.

Both Roots Common

If the Eq. (1) and (2) have both roots common, then these equations will be identical. Thus the required condition for both roots common is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

TRICK(S) FOR PROBLEM SOLVING

- To find the common root of two equations, make the coefficient of second degree terms in two equations equal and subtract. The value of x so obtained is the required common root.
- If two quadratic equations with real coefficients have an imaginary root common, then both roots will be common and the two equations will be identical. The required condition is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

- If two quadratic equations have an irrational root common, then both roots will be common and the two equations will be identical. The required condition is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

- If α is a repeated root of the quadratic equation
$$f(x) = ax^2 + bx + c = 0,$$
then α is also a root of the equation $f'(x) = 0$.
- If α is repeated common root of two quadratic equations $f(x) = 0$ and $\phi(x) = 0$, then α is also a common root of the equations $f'(x) = 0$ and $\phi'(x) = 0$.

SOLVED EXAMPLES

28. The value of k so that the equations $x^2 - x - 12 = 0$ and $kx^2 + 10x + 3 = 0$ may have one root in common, is

(A) $\frac{43}{16}$ (B) 3 (C) -3 (D) $\frac{-43}{16}$

Solution: (B, D)

Let α be the common root

Then, $\alpha^2 - \alpha - 12 = 0$ and $k\alpha^2 + 10\alpha + 3 = 0$

Solving the two equations, we get

$$\frac{\alpha^2}{117} = \frac{\alpha}{-12k - 3} = \frac{1}{10 + k}$$

$\Rightarrow$ $(-12k - 3)^2 = 117\,(10 + k)$

$\Rightarrow$ $9\,(4k + 1)^2 = 117\,(10 + k)$

$\Rightarrow$ $(4k + 1)^2 = 13\,(10 + k)$

$\Rightarrow$ $16k^2 + 8k + 1 = 130 + 13k$

$\Rightarrow$ $16k^2 - 5k - 129 = 0$

$\Rightarrow$ $16k^2 - 48k + 43k - 129 = 0$

$\therefore$ $k = 3$ or $k = \frac{-43}{16}$

29. If the equations $ax^2 + bx + c = 0$ and $x^2 + 2x + 3 = 0$ have a common root, then $a : b : c =$

(A) $2 : 4 : 5$ (B) $1 : 3 : 4$
(C) $1 : 2 : 3$ (D) None of these

Solution: (C)

For the equation $x^2 + 2x + 3 = 0$,

Discriminant $= (2)^2 - 4 \cdot 1 \cdot 3 < 0$.

$\therefore$ roots of $x^2 + 2x + 3 = 0$ are imaginary. Since the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$ are given to have a common root, therefore both roots will be common. Hence both the equations are identical.

$\therefore$ $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

i.e. $a : b : c = 1 : 2 : 3$.

30. If $a, b, c \in R$ and the equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ have two roots in common, then

(A) $a = b \neq c$ (B) $a = b = -c$
(C) $a = b = c$ (D) None of these

Solution: (C)

We have, $x^3 + 3x^2 + 3x + 2 = 0$

$\Rightarrow$ $(x + 1)^3 + 1 = 0$

$\Rightarrow$ $(x + 1 + 1)\,[(x + 1)^2 - (x + 1) + 1] = 0$

$\Rightarrow$ $(x + 2)\,(x^2 + x + 1) = 0$

$\Rightarrow$ $x = -2, \frac{-1 \pm \sqrt{3}i}{2}$

$\Rightarrow$ $x = -2, \omega, \omega^2$

Since $a, b, c \in R$, $ax^2 + bx + c = 0$ cannot have one real and one imaginary root. Therefore, two common roots of $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ are ω, ω^2.

Thus, $-\frac{b}{a} = \omega + \omega^2 = -1$

$\Rightarrow$ $a = b$

and $\frac{c}{a} = \omega \cdot \omega^2 = 1 \Rightarrow c = a$

$\Rightarrow$ $a = b = c$

31. If the equations $k\,(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k\,(2x^2 + 1) + px + 4x^2 - 2 = 0$ have both the roots common, then the value of $2r - p$ is

(A) 0 (B) 1
(C) -1 (D) None of these

Solution: (A)

The two equations can be written as

$$x^2\,(6k + 2) + rx + (3k - 1) = 0 \qquad (1)$$

and $$x^2\,(12k + 4) + px + (6k - 2) = 0 \qquad (2)$$

Divide by 2, we get

$$x^2\,(6k + 2) + \frac{p}{2}x + (3k - 1) = 0 \qquad (3)$$

Comparing Eq. (1) and (3), we get

$$r = \frac{p}{2}$$

$\therefore$ $2r - p = 0$

32. If the equations $x^2 - ax + b = 0$ and $x^2 + bx - a = 0$ have a common root, then

(A) $a + b = 1$
(B) $a = b$
(C) $a - b = 2$
(D) $a + b = 0$ or $a - b = 1$

Solution: (D)

Let α be a common root of the given equations.

Then $\alpha^2 - a\alpha + b = 0$ and $\alpha^2 + b\alpha - a = 0$

$\Rightarrow (a + b)\ \alpha - (a + b) = 0$

$\Rightarrow (a + b)\ (\alpha - 1) = 0$

$\Rightarrow a + b = 0$ or $\alpha = 1$

If $\alpha = 1$,

then $1 - a + b = 0 \Rightarrow a - b = 1$.

SYMMETRIC FUNCTION OF THE ROOTS

A function of α and β is said to be a symmetric function if it remains unchanged when α and β are interchanged.

For example, $\alpha^2 + \beta^2 + 2\alpha\beta$ is a symmetric function of α and β whereas $\alpha^2 - \beta^2 + 3\alpha\beta$ is not a symmetric function of α and β.

TRICK(S) FOR PROBLEM SOLVING

In order to find the value of a symmetric function of α and β, express the given function in terms of $\alpha + \beta$ and $\alpha\beta$. The following results may be useful.

- $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta\ (\alpha + \beta)$
- $\alpha^4 + \beta^4 = (\alpha^3 + \beta^3)\ (\alpha + \beta) - \alpha\beta\ (\alpha^2 + \beta^2)$
- $\alpha^5 + \beta^5 = (\alpha^3 + \beta^3)\ (\alpha^2 + \beta^2) - \alpha^2\beta^2\ (\alpha + \beta)$
- $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
- $\alpha^2 - \beta^2 = (\alpha + \beta)\ (\alpha - \beta)$
- $\alpha^3 - \beta^3 = (\alpha - \beta)\ [(\alpha + \beta)^2 - \alpha\beta]$
- $\alpha^4 - \beta^4 = (\alpha + \beta)\ (\alpha - \beta)\ (\alpha^2 + \beta^2)$

SOLVED EXAMPLE

33. If α and β be the roots of the equation $x^2 + px - \dfrac{1}{2p^2} = 0$, where $p \in R$, then the minimum value of $\alpha^4 + \beta^4$ is

(A) $\sqrt{2}$ (B) $2 + \sqrt{2}$

(C) $2 - \sqrt{2}$ (D) $\sqrt{2}$

Solution: (B)

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2) - 2\alpha^2\beta^2$$

$$= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$$

$$= \left(p^2 + \frac{1}{p^2}\right)^2 - \frac{1}{2p^4} = p^4 + \frac{1}{2p^4} + 2$$

$$= \left(p^2 - \frac{1}{\sqrt{2}p^2}\right)^2 + 2 + \sqrt{2} \geq 2 + \sqrt{2}$$

Therefore, minimum value of $\alpha^4 + \beta^4$ is $2 + \sqrt{2}$

GRAPH OF A QUADRATIC EXPRESSION

We have, y or $f(x) = ax^2 + bx + c$ where $a, b, c \in R, a \neq 0$.

1. The shape of the curve $y = f(x)$ is a parabola
2. The axis of the parabola is y-axis (incase $b = 0$) or parallel to y-axis.
3. If $a > 0$, then the parabola opens upwards.
4. If $a < 0$, then the parabola opens downwards
5. For $D > 0$, parabola cuts x-axis in two distinct points

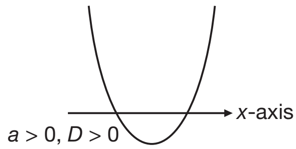


Fig. 4.1(a)

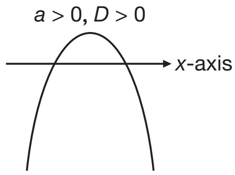


Fig. 4.1(b)

6. For $D = 0$, parabola touches x-axis in one point.

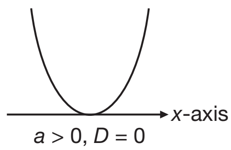


Fig. 4.2(a)

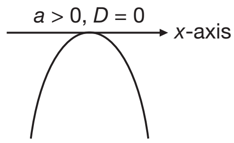


Fig. 4.2(b)

7. For $D < 0$, parabola does not cut x-axis.

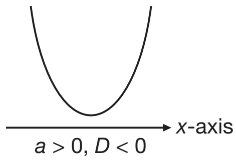


Fig. 4.3(a)

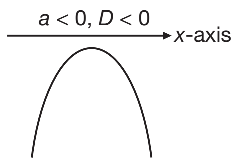


Fig. 4.3(b)

GREATEST AND LEAST VALUES OF A QUADRATIC EXPRESSION

1. If $a > 0$, then the quadratic expression $y = ax^2 + bx + c$ has no greatest value but it has least value

$$\frac{4ac - b^2}{4a} \quad \text{at} \quad x = -\frac{b}{2a}$$

2. If $a < 0$, then the quadratic expression $y = ax^2 + bx + c$ has no least value but it has greatest value

$$\frac{4ac - b^2}{4a} \quad \text{at} \quad x = -\frac{b}{2a}$$

Sign of Quadratic Expression

We have, y or $f(x) = ax^2 + bx + c$ where $a, b, c \in R, a \neq 0$.

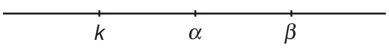
1. If $a > 0$ and $D < 0$, then $f(x) > 0$ for all $x \in R$ *i.e.*, $f(x)$ is positive for all real values of x.
2. If $a < 0$ and $D < 0$, then $f(x) < 0$ for all $x \in R$ *i.e.*, $f(x)$ is negative for all real values of x.
3. If $a > 0$ and $D = 0$, then $f(x) \geq 0$ for all $x \in R$ *i.e.*, $f(x)$ is positive for all real values of x except at vertex, where $f(x) = 0$.
4. If $a < 0$ and $D = 0$, then $f(x) \leq 0$ for all $x \in R$ *i.e.* $f(x)$ is negative for all real values of x except at vertex, where $f(x) = 0$.
5. If $a > 0$ and $D > 0$, let $f(x) = 0$ have two real roots α and $\beta(\alpha < \beta)$, then $f(x) > 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) < 0$ for all $x \in (\alpha, \beta)$.
6. If $a < 0$ and $D > 0$, let $f(x) = 0$ have two real roots α and $\beta(\alpha < \beta)$. Then $f(x) < 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) > 0$ for all $x \in (\alpha, \beta)$.

NATURE OF ROOTS OF A QUADRATIC EQUATION WITH RESPECT TO ONE OR TWO REAL NUMBERS

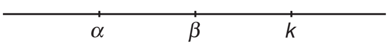
Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R, a \neq 0$. Let α, $\beta(\alpha < \beta)$ be the roots of the corresponding quadratic equation. Let $k, k_1, k_2 \in R$ and $k_1 < k_2$.

Nature of Roots with Respect to One Real Number

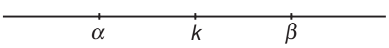
1. If both the roots of $f(x) = 0$ are greater than k, then $D \geq 0$, $a\,f(k) > 0$ and $k < -\dfrac{b}{2a}$



2. If both the roots of $f(x) = 0$ are less than k, then $D \geq 0$, $a\,f(k) > 0$ and $k > -\dfrac{b}{2a}$

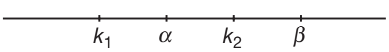


3. If one root is less than k and other is greater than k, then $D > 0$ and $a\,f(k) < 0$

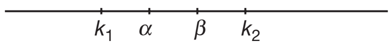


Roots with Respect to Two Real Numbers

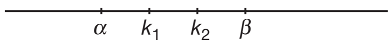
1. If exactly one root of $f(x) = 0$ lies in the interval (k_1, k_2), then $D > 0$ and $f(k_1) \cdot f(k_2) < 0$



2. If both roots of $f(x) = 0$ lie between k_1 and k_2, then $D \geq 0$, $a\,f(k_1) > 0$, $a\,f(k_2) > 0$ and $k_1 < \dfrac{\alpha + \beta}{2} < k_2$



3. If k_1 and k_2 lie between the roots of $f(x) = 0$, then $D \geq 0$, $a\,f(k_1) < 0$ and $a\,f(k_2) < 0$.



TRICK(S) FOR PROBLEM SOLVING

1. Let $f(x) = 0$ be a polynomial equation. Let p and q be two real numbers, $p < q$.
   (a) If $f(p) \cdot f(q) < 0$, then the equation $f(x) = 0$ has odd number of real roots between p and q.
   (b) If $f(p) \cdot f(q) > 0$, then the equation $f(x) = 0$ has either no real root or even number of real roots between p and q.
   (c) If $f(p) = f(q)$, then the equation $f'(x) = 0$ has at least one real root between p and q (This is due to **Rolle's Theorem**)
2. (a) If the coefficients of the polynomial equation $f(x) = 0$ have p changes of signs, then the equation $f(x) = 0$ will have atmost p, positive roots.
   (b) If the coefficients of the polynomial equation $f(-x) = 0$ have q changes of signs, then the equation $f(x) = 0$ will have atmost q, negative roots.
   (c) The polynomial equation $f(x) = 0$ will have atmost $p + q$ real roots where p and q are the changes of signs of coefficients in $f(x)$ and $f(-x)$. (This is due to **Descarte's Rule of signs**)

   For example, consider

   $f(x) = 2x^5 - 6x^4 + 7x^3 - 8x^2 + 5x + 3$

   + − + − + +

   Then, $f(-x) = -2x^5 - 6x^4 - 7x^3 - 8x^2 - 5x + 3$

   − − − − − +

   Clearly, $f(x)$ has 4 changes of signs and $f(-x)$ has only one change of sign, Therefore, the equation $f(x) = 2x^5 - 6x^4 + 7x^3 - 8x^2 + 5x + 3 = 0$ has atmost four positive roots and one negative root. Also, the equation has atmost $(4 + 1) = 5$ real roots.
3. (a) A polynomial equation $f(x) = 0$ has exactly one root equal to α if $f(\alpha) = 0$ and $f'(\alpha) \neq 0$.
   (b) A polynomial equation $f(x) = 0$ has exactly two roots equal to α if $f(\alpha) = 0$, $f'(\alpha) = 0$ and $f''(\alpha) \neq 0$.
   (c) In general, a polynomial equation $f(x) = 0$ has exactly n roots equal to α if

   $$f(\alpha) = f'(\alpha) = f''(\alpha) = \ldots = f^{n-1}(\alpha) = 0$$

   and $f^{n}(\alpha) \neq 0$

RELATION BETWEEN ROOTS AND COEFFICIENTS OF A POLYNOMIAL EQUATION

Let $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$, a_0, $a_1, a_2, \dots, a_n \in R$, $a_0 \neq 0$ be a polynomial equation of degree n, having n roots $\alpha_1, \alpha_2, \dots \alpha_n$. Then,

1. Sum of all roots

$$\sigma_1 = \alpha_1 + \alpha_2 + \dots + \alpha_n$$
$$= \Sigma\alpha_1 = -\frac{a_1}{a_0} = (-1)^1 \frac{a_1}{a_0}$$

2. Sum of the product of two roots

$$\sigma_2 = \alpha_1\alpha_2 + \alpha_1\alpha_3 + \dots$$
$$= \Sigma\alpha_1\alpha_2 = \frac{a_2}{a_0} = (-1)^2 \frac{a_2}{a_0}$$

3. Sum of the product of three roots

$$\sigma_3 = \alpha_1\alpha_2\alpha_3 + \alpha_2\alpha_3\alpha_4 + \dots$$
$$= \Sigma\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0} = (-1)^3 \frac{a_3}{a_0} \text{ and so on.}$$

In general, $\sigma_r = \Sigma\alpha_1\alpha_2 \dots \alpha_r = (-1)^r \dfrac{a_r}{a_0}$.

Particular Cases

Quadratic Equation

If α, β are roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Cubic Equation

If α, β, γ are roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$, then

$$\sigma_1 = \alpha + \beta + \gamma = -\frac{b}{a}$$
$$\sigma_2 = \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$
$$\sigma_3 = \alpha\beta\gamma = -\frac{d}{a}$$

Biquadratic Equation

If $\alpha, \beta, \gamma, \delta$ are roots of the biquadratic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$, then

$$\sigma_1 = \alpha + \beta + \gamma + \delta = -\frac{b}{a}$$
$$\sigma_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$$
$$\sigma_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$$
$$\sigma_4 = \alpha\beta\gamma\delta = \frac{e}{a}$$

FORMATION OF A POLYNOMIAL EQUATION FROM GIVEN ROOTS

If $\alpha_1, \alpha_2, \alpha_3, \dots \alpha_n$ are the roots of a polynomial equation of degree n, then the equation is

$$x^n - \sigma_1x^{n-1} + \sigma_2x^{n-2} - \sigma_3x^{n-3} + \dots + (-1)^n\sigma_n = 0$$

where $\sigma_r = \Sigma\alpha_1\alpha_2 \dots \alpha_r$.

Particular Cases

Quadratic Equation

If α, β are the roots of a quadratic equation, then the equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

Cubic Equation

If α, β, γ are the roots of a cubic equation, then the equation is

$$x^3 - \sigma_1x^2 + \sigma_2x - \sigma_3 = 0$$

or $$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - \alpha\beta\gamma = 0$$

Biquadratic Equation

If $\alpha, \beta, \gamma, \delta$ are the roots of a biquadratic equation, then the equation is

$$x^4 - \sigma_1x^3 + \sigma_2x^2 - \sigma_3x + \sigma_4 = 0$$

or

$$x^4 - (\alpha + \beta + \gamma + \delta)x^3 + (\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)x^2$$
$$- (\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)x + \alpha\beta\gamma\delta = 0$$

SIGN OF A POLYNOMIAL EXPRESSION

Step 1: Factorize the given polynomial expression as

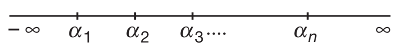
$$f(x) = (x - \alpha_1)^{k_1} (x - \alpha_2)^{k_2} (x - \alpha_3)^{k_3} \dots$$
$$(x - \alpha_{n-1})^{k_{n-1}} (x - \alpha_n)^{k_n}$$

where $k_1, k_2, \dots k_n \in N$ and

$$\alpha_1, \alpha_2, \alpha_3, \dots \alpha_n \in R \ (\alpha_1 < \alpha_2 < \alpha_3 \dots < \alpha_n)$$

Step 2: Plot the points $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ on the real line.

Step 3: Mark plus sign in the interval of the right of the largest of these numbers i.e., on the right of α_n.



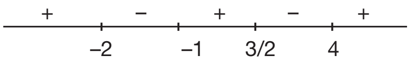
Step 4: If k_n is even, put '+' sign of the left of α_n and if k_n is odd, put '–' sign on the left of α_n.

Step 5: Consider the next interval and put a sign in it using the above rule. Thus, consider all the intervals.

Step 6: The solution of $f(x) > 0$ is the union of all the intervals in which there is a '+' sign and the solution of $f(x) < 0$ is the union of all the intervals in which there is a '–' sign.

For example, consider the polynomial expression

$$f(x) = (x+2)^{40}(x+1)^{31}\left(x-\frac{3}{2}\right)^{58}(x-4)^{37}$$



Thus, $f(x) > 0$ if $x \in (-\infty, -2) \cup \left(-1, \frac{3}{2}\right) \cup (4, \infty)$

and $f(x) < 0$ if $x \in (-2, -1) \cup \left(\frac{3}{2}, 4\right)$.

SOLVED EXAMPLES

34. If the expression $\left(ax - 1 + \frac{1}{x}\right)$ is non-negative for all positive real x, then the minimum value of a must be

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) None of these

Solution: (C)

We have,

$$ax - 1 + \frac{1}{x} \geq 0 \Rightarrow \frac{ax^2 - x + 1}{x} \geq 0$$

$\Rightarrow$ $ax^2 - x + 1 \leq 0$ as $x > 0$

It will hold if $a > 0$ and $D \leq 0$

$\Rightarrow$ $a > 0$ and $1 - 4a \leq 0 \Rightarrow a \geq \frac{1}{4}$

Therefore, the minimum value of a is $\frac{1}{4}$.

35. If α, β are the roots of the equation $x^2 - 3x + a = 0$, $a \in R$ and $\alpha < 1 < \beta$, then a belong to

(A) $\left(-\infty, \frac{9}{4}\right)$ (B) $(-\infty, 2)$

(C) $(2, \infty)$ (D) $\left(\frac{9}{4}, \infty\right)$

Solution: (B, C)

Since 1 lies between the roots of the given equation, therefore, $D > 0$ and $f(1) < 0 \Rightarrow 9 - 4a > 0$ and $1 - 3 + a < 0$

$\Rightarrow$ $a < \frac{9}{4}$ and $a < 2$

$\therefore$ $a < 2$

36. The equation

$$\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1 \text{ has}$$

(A) no solution
(B) one solution
(C) two solutions
(D) more than two solutions

Solution: (D)

Put $\sqrt{x-1} = t \Rightarrow x - 1 = t^2$ or $x = t^2 + 1$,

The given equation reduces to

$$\sqrt{t^2+1+3-4t} + \sqrt{t^2+1+8-6t} = 1$$

where $t \geq 0$.

$\Rightarrow$ $|t-2| + |t-3| = 1$,

where $t \geq 0$. This equation will be satisfied if $2 \leq t \leq 3$.

Therefore, $2 \leq \sqrt{x-1} \leq 3$ or $5 \leq x \leq 10$.

$\therefore$ The given equation is satisfied for all values of x lying in [5, 10].

37. If the roots of the equation

$$x^2 - 2ax + a^2 + a - 3 = 0$$

are real and less than 3, then

(A) $a < 2$ (B) $2 \leq a \leq 3$
(C) $3 < a \leq 4$ (D) $a > 4$

Solution: (A)

We can write the given equation as

$$(x-a)^2 = 3 - a$$

This shows that $a \leq 3$ and $x = a \pm \sqrt{3-a}$

Both the roots of the given equation will be less than 3 if the larger of the two roots is less than 3, that is, if

$$a + \sqrt{3-a} < 3$$

$\Rightarrow$ $\sqrt{3-a} - (3-a) < 0$

$\Rightarrow$ $\sqrt{3-a}\,(1 - \sqrt{3-a}) < 0$

$\Rightarrow$ $a < 3$ and $1 - \sqrt{3-a} < 0$

But $\sqrt{3-a} > 1 \Rightarrow 3 - a > 1$ or $a < 2$

Thus, $a < 3$ and $a < 2 \Rightarrow a < 2$

38. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that min $f(x) >$ max $g(x)$, then the relation between b and c is:

(A) $|c| < |b|\sqrt{2}$ (B) $0 < c < b\sqrt{2}$

(C) $|c| < |b|\,2$ (D) $|c| > |b|\sqrt{2}$

Solution: (D)

$$\min f(x) = -\frac{D}{4a} = -\frac{4b^2 - 8c^2}{4}$$

$$= -(b^2 - 2c^2) \quad \text{(upward parabola)}$$

$$\max g(x) = -\frac{D}{4a} = \frac{4c^2 + 4b^2}{4}$$

$$= b^2 + c^2 \quad \text{(downward parabola)}$$

Now $\quad 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow \quad c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}\,|b|$

39. For the equation $|x^2| + |x| - 6 = 0$, the roots are

(A) real and equal
(B) real with sum 0
(C) real with sum 1
(D) real with product 0

Solution: (B)

For, $\quad x < 0, |x| = -x$

$\therefore$ equation is

$$x^2 - x - 6 = 0 \Rightarrow x = -2, 3$$

$\because x < 0$

$\therefore \quad x = -2$ is the solution

For, $\quad x \geq 0, |x| = x,$

$\therefore$ equation is

$$x^2 + x - 6 = 0 \Rightarrow x = 2, -3$$

$\because x \geq 0$

$\therefore \quad x = 2$ is the solution.

Hence, $x = 2, -2$ are the solutions and their sum is zero.

40. If $a \leq 0$, then the root of the equation

$$x^2 - 2a\,|x - a| - 3a^2 = 0 \text{ is}$$

(A) $(1 - \sqrt{2})a$ (B) $(-1 + \sqrt{6})a$
(C) $(1 + \sqrt{2})a$ (D) $-(1 + \sqrt{6})a$

Solution: (A, B)

If $x - a < 0$, $|x - a| = -(x - a)$
$\therefore$ equation becomes $x^2 + 2a(x - a) - 3a^2 = 0$

$\Rightarrow \quad x^2 + 2ax - 5a^2 = 0$

$\Rightarrow \quad x = -(1 + \sqrt{6})\ a,\ (-1 + \sqrt{6})\,a$

$\because x < a \leq 0$

$\therefore \quad x = (-1 + \sqrt{6})$ a

If $\quad x - a \geq 0$, $|x - a| = x - a$

$\therefore$ the equation becomes $x^2 - 2a(x - a) - 3a^2 = 0$

$\Rightarrow \quad x^2 - 2ax - a^2 = 0$

$\Rightarrow \quad x = (1 + \sqrt{2})\ a,\ (1 - \sqrt{2})\ a$

$\because x \geq a$ and $a \leq 0$

$\therefore \quad x = (1 - \sqrt{2})\ a.$

41. If $f(x) = x - [x]$, $x\ (\neq 0) \in R$, where $[x]$ is the greatest integer less than or equal to x, then the number of solutions of $f(x) + f\left(\frac{1}{x}\right) = 1$ are

(A) 0 (B) 1 (C) infinite (D) 2

Solution: (C)

We have, $\quad f(x) + f\left(\frac{1}{x}\right) = 1$

$\Rightarrow \quad x - [x] + \frac{1}{x} - \left[\frac{1}{x}\right] = 1$

$\Rightarrow \quad x + \frac{1}{x} - 1 = [x] + \left[\frac{1}{x}\right]$

$\Rightarrow \quad \frac{x^2 + 1 - x}{x} = \text{(integer) } k \quad \text{(say)}$

$\Rightarrow \quad x^2 - (k + 1)\,x + 1 = 0$

Since x is real, so $(k + 1)^2 - 4 \geq 0$

$\Rightarrow \quad k^2 + 2k - 3 \geq 0 \quad \Rightarrow \quad (k + 3)(k - 1) \geq 0$

$\Rightarrow \quad k \leq -3 \quad \text{or} \quad k \geq 1$

Therefore, number of solutions is infinite.

42. If $(\log_5 x)^2 + \log_5 x < 2$, then x belongs to the interval

(A) $\left(\frac{1}{25}, 5\right)$ (B) $\left(\frac{1}{5}, \frac{1}{\sqrt{5}}\right)$
(C) $(1, \infty)$ (D) None of these

Solution: (A)

We have, $(\log_5 x)^2 + \log_5 x < 2$
Put $\log_5 x = a$ then $a^2 + a < 2$

$\Rightarrow \quad a^2 + a - 2 < 0$

$\Rightarrow \quad (a + 2)(a - 1) < 0$

$\Rightarrow \quad -2 < a < 1 \quad \text{or} \quad -2 < \log_5 x < 1$

$\therefore \quad 5^{-2} < x < 5$

i.e., $\quad \frac{1}{25} < x < 5$

43. The greatest negative integer satisfying $x^2 - 4x - 77 < 0$ and $x^2 > 4$ is

(A) –4 (B) –7
(C) –6 (D) None of these

Solution: (C)

We have, $x^2 - 4x - 77 < 0$ and $x^2 - 4 > 0$

$\Rightarrow \quad (x + 7)(x - 11) < 0$ and $(x - 2)(x + 2) > 0$

$\Rightarrow \quad -7 < x < 11$ and $x < -2$ or $x > 2$

$\therefore \quad -7 < x < -2$

44. The solution of the inequation $4^{-x + 0.5} - 7.2^{-x} < 4$, $x \in R$, is

(A) $(-2, \infty)$ (B) $(2, \infty)$

(C) $\left(2, \frac{7}{2}\right)$ (D) None of these

Solution: (A)

The given inequation is

$$4^{-x + 0.5} - 7.2^{-x} < 4, x \in R$$

Let $\quad 2^{-x} = t$

$\therefore \quad 2t^2 - 7t < 4$

$\Rightarrow \quad 2t^2 - 7t - 4 < 0$

$\Rightarrow \quad (2t + 1)(t - 4) < 0$

$\Rightarrow \quad \frac{1}{2} - < t < 4$

but $\quad 2^{-x} > 0$

so $\quad 0 < t < 4$

$\Rightarrow \quad 0 < 2^{-x} < 4$

$\Rightarrow \quad -2 < x < \infty$ or $x \in (-2, \infty)$

45. The real values of x for which $3^{72}\left(\frac{1}{3}\right)^x\left(\frac{1}{3}\right)^{\sqrt{x}} > 1$, are

(A) $x \in [0, 64]$ (B) $x \in (0, 64)$

(C) $x \in [0, 64)$ (D) None of these

Solution: (C)

The given inequation is valid only when

$$x \geq 0 \quad (1)$$

The given inequation can be written in the form

$$3^{72 - x - \sqrt{x}} > 1$$

$\Rightarrow \quad 72 - x - \sqrt{x} > 0 \quad (\because 3 > 1)$

$\Rightarrow \quad x + \sqrt{x} - 72 < 0$

$\Rightarrow \quad (\sqrt{x} + 9)(\sqrt{x} - 8) < 0$

But $\quad \sqrt{x} + 9 > 0$ for all $x \geq 0$

$\therefore \quad \sqrt{x} - 8 < 0 \Rightarrow \sqrt{x} < 8$

$\therefore \quad 0 \leq x < 64$ [from (1)].

46. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$ is

(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(1, \frac{5}{2}\right)$

(C) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (D) None of these

Solution: (C)

We have, $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$

$\Rightarrow \quad x^2 - 3x + 2 \leq \frac{3}{4}$

$\Rightarrow \quad x^2 - 3x + \frac{5}{4} \leq 0$

$\Rightarrow \quad 4x^2 - 12x + 5 \leq 0$

$\Rightarrow \quad 4x^2 - 10x - 2x + 5 \leq 0$

$\Rightarrow \quad (2x - 5)(2x - 1) \leq 0$

$\Rightarrow \quad \frac{1}{2} \leq x \leq \frac{5}{2} \quad (1)$

Also, $\quad x^2 - 3x + 2 > 0$

$\Rightarrow \quad (x - 1)(x - 2) > 0$

$\Rightarrow \quad x < 1$ or $x > 2 \quad (2)$

From Eqs (1) and (2), we get

$$x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$$

47. The values of a which make the expression $x^2 - ax + 1 - 2a^2$ always positive for real values of x, are

(A) $-\frac{2}{3} < a < \frac{2}{3}$ (B) $-\frac{2}{3} \leq a \leq \frac{2}{3}$

(C) $a < 1$ (D) $0 < a < \frac{2}{3}$

Solution: (A)

Since the coefficient of x^2 is 1 which is positive, therefore the given expression is positive for all real values of x if $D < 0$.

$\Rightarrow \quad (-a)^2 - 4(1 - 2a^2) < 0$

$\Rightarrow \quad 9a^2 - 4 < 0$

$\Rightarrow \quad (3a + 2)(3a - 2) < 0$

$\Rightarrow \quad -\frac{2}{3} < a < \frac{2}{3}$.

48. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

(A) $a < 2$ (B) $2 \le a \le 3$
(B) $3 \le a \le 4$ (D) $a > 4$

Solution: (A)

Since the roots of the given equation are real

$\therefore \quad B^2 - 4AC \ge 0$

$\Rightarrow \quad 4a^2 - 4(a^2 + a - 3) \ge 0$

$\Rightarrow \quad -a + 3 \ge 0$ or $a \le 3$ (1)

Since the root is less than 3, so $f(3) > 0$

$\Rightarrow \quad 3^2 - 2a(3) + a^2 + a - 3 > 0$

$\Rightarrow \quad a^2 - 5a + 6 > 0$ or $(a - 2)(a - 3) > 0$

$\Rightarrow \quad a < 2$ or $a > 3$ (2)

From Eq. (1) and (2), we have $a < 2$.

49. The value of k for which the number 3 lies between the roots of the equation $x^2 + (1 - 2k)x + (k^2 - k - 2) = 0$ is given by,

(A) $2 < k < 5$ (B) $k < 2$
(C) $2 < k < 3$ (D) $k > 5$

Solution: (A)

Let $\quad f(x) = x^2 + (1 - 2k)x + k^2 - k - 2$

The number 3 lies between the roots of the given equation, if $f(3) < 0$.

Now, $\quad f(3) = 9 + (1 - 2k)3 + k^2 - k - 2$

$= 10 - 7k + k^2 = k^2 - 7k + 10$

Hence $\quad f(3) < 0$

$\Rightarrow \quad k^2 - 7k + 10 < 0$

$\Rightarrow \quad (k - 2)(k - 5) < 0 \Rightarrow 2 < k < 5.$

50. Solution of $2^x + 2^{|x|} \ge 2\sqrt{2}$ is

(A) $(-\infty, \log_2(\sqrt{2}+1))$

(B) $\left[\log_2(\sqrt{2}+1), \infty\right)$

(C) $\left(\frac{1}{2}, \log_2(\sqrt{2}-1)\right)$

(D) $(-\infty, \log_2(\sqrt{2}-1)] \cup \left[\frac{1}{2}, \infty\right)$

Solution: (D)

We have, $\quad 2^x + 2^x \ge 2\sqrt{2} \qquad (x \ge 0)$

$\Rightarrow \quad 2^x \ge \sqrt{2} \Rightarrow x \ge \frac{1}{2}$

and $\quad 2^x + 2^{-x} \ge 2\sqrt{2} \qquad (x < 0)$

$\Rightarrow \quad t + \frac{1}{t} \ge 2\sqrt{2} \qquad$ (where $t = 2^x$)

$\Rightarrow \quad t^2 - 2\sqrt{2}\,t + 1 \ge 0$

$\Rightarrow \quad [t - (\sqrt{2} - 1)][t - (\sqrt{2} + 1)] \ge 0$

$\Rightarrow \quad t \le \sqrt{2} - 1$ or $t \ge \sqrt{2} + 1$

but $\quad t > 0$

$\Rightarrow \quad 0 < 2^x \le \sqrt{2} - 1$ or $2^x \ge \sqrt{2} + 1$

$\Rightarrow \quad -\infty < x \le \log_2(\sqrt{2} - 1)$

or $\quad x \ge \log_2(\sqrt{2} + 1)$

(but not acceptable as $x < 0$)

$\therefore \quad x \in (-\infty, \log_2(\sqrt{2} - 1)] \cup \left[\frac{1}{2}, \infty\right)$

51. If $a < b$, then the solution of $x^2 + (a + b)x + ab < 0$, is given by

(A) $x < -b$ or $x < -a$ (B) $a < x < c$
(B) $x < a$ or $x > b$ (D) $-b < x < -a$

Solution: (D)

$x^2 + (a + b)x + ab < 0$

$\Rightarrow \quad (x + a)(x + b) < 0$

$\Rightarrow \quad x + a < 0, x + b > 0$

or $\quad x + a > 0, x + b < 0$

$\Rightarrow \quad x < -a, x > -b$

or $\quad x > -a, x < -b$

$\Rightarrow \quad -b < x < -a$

or $\quad -a < x < -b$

Since $\quad a < b$

$\therefore \quad -a > -b$

Hence $\quad -b < x < -a.$

52. The conditions that the equation $ax^2 + bx + c = 0$ has both the roots positive is that

(A) a and b are of the same sign
(B) a, b and c are of the same sign
(C) a and c are of the same sign opposite to that of b
(D) b and c are of the same sign opposite to that of a

Solution: (C)

Since both the roots are positive

$\therefore \quad \frac{-b}{a} > 0, \frac{c}{a} > 0$

$\Rightarrow \quad \frac{b}{a} < 0, \frac{c}{a} > 0$

$\therefore$ a and c have same sign opposite to that of b.

53. The smallest value of $x^2 - 3x + 3$ in the interval $\left(-3, \frac{3}{2}\right)$ is

(A) –20 (B) –15

(C) 5 (D) $\frac{3}{4}$

Solution: (D)

We have,
$$x^2 - 3x + 3 = \left(x - \frac{3}{2}\right)^2 + 3 - \frac{9}{4}$$
$$= \left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$$

$\therefore$ smallest value $= \frac{3}{4}$, which lies in the interval $\left(-3, \frac{3}{2}\right)$.

RATIONAL ALGEBRAIC EXPRESSION

An expression of the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$, is known as a rational algebraic expression.

Sign Scheme for a Rational Algebraic Expression in *x*

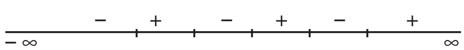
Step 1: Factorise the numerator and denominator of the given rational expression into linear factors. Make the coefficient of x positive in all factors.

Step 2: Find the real values of x by equating all the factors to zero.

Step 3: If n distinct real values of x are obtained then the entire line will be divided into $(n + 1)$ parts.

Step 4: Plot all these points on the number line in order.

Step 5: Start with '+' sign from extreme right and change the sign alternatively in other parts.



CAUTION

If the rational expression in x occurs under modulus sign, then first of all remove the modulus sign and then proceed.

In order to remove the modulus sign, the following results may be useful:

- $|x| = k \Leftrightarrow x = \pm k$
- $|x| < k \Leftrightarrow = -k < x < k$
- $|x| > k \Leftrightarrow = x < -k$ or $x > k$.

To Find the Values of a Rational Expression in x, Where x is Real

TRICK(S) FOR PROBLEM SOLVING

- Put the given rational expression equal to y and form the quadratic equation in x.
- Find the discriminant D of the quadratic equation obtained in step 1.
- Since x is real, therefore, put $D \geq 0$. We get an inequation in y.
- Solve the above inequation for y. The values of y so obtained determine the set of values attained by the given rational expression.

TRICK(S) FOR PROBLEM SOLVING

The general quadratic expression $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ in x and y may be resolved into two linear rational factors if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

or
$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

- If sum of coefficients of a polynomial equation $a_0 + a_1x + a_2x^2 + \ldots + a_n x^n = 0$ is zero, then $x = 1$ is always atleast one root of equation *e.g.*, if $a(b - c)\,x^2 + b(c - a)\,x + c\,(a - b) = 0$, then as $\Sigma a\,(b - c) = 0$, $x = 1$ is atleast one root of this equation.
- Least value of the expression $(x - y)^2 + (y - z)^2 + (z - x)^2$ is 0.
- Sum of real roots of the equation $a_n\,|x|^n + a_{n-1}\,|x|^{n-1} + \ldots + a_0 = 0$ is 0, e.g. if $|x| = 2$ satisfies the equation, then $x = 2$ and $x = -2$ are real roots, their sum is 0.
- Length of latus rectum of parabola $y = ax^2 + bx + c$ is $\frac{1}{|a|}$.

SOLVED EXAMPLES

54. The sum of the real roots of the equation

$$|x - 2|^2 + |x - 2| - 2 = 0 \text{ is}$$

(A) 2 (B) 6 (C) 4 (D) 8

Solution: (C)

Put $|x - 2| = t$.

The given equation becomes

$$t^2 + t - 2 = 0 \quad \text{or} \quad (t + 2)(t - 1) = 0$$

Since $t + 2 = |x - 2| + 2 > 0$

$\therefore$ we get $t - 1 = 0$

$\Rightarrow |x - 2| = 1 \Rightarrow x - 2 = \pm 1$

$\Rightarrow x = 3, 1.$

Thus, the sum of roots is 4.

55. If x is real, the expression $\frac{x^2 + 2x - 11}{x - 3}$ takes all real values except those which lie between a and b, then a and b are

(A) –12, –4 (B) –12, 2 (C) 4, 12 (D) –4, 4

Solution: (C)

Let $y = \frac{x^2 + 2x - 11}{x - 3}$

$\Rightarrow xy - 3y = x^2 + 2x - 11$

$\Rightarrow x^2 + (2 - y)x + (3y - 11) = 0 \quad (D \geq 0)$

$\Rightarrow (2 - y)^2 - 4(3y - 11) \geq 0$

$\Rightarrow 4 + y^2 - 4y - 12y + 44 \geq 0$

$\Rightarrow y^2 - 16y + 48 \geq 0$

$\Rightarrow y^2 - 12y - 4y + 48 \geq 0$

$\Rightarrow (y - 4)(y - 12) \geq 0$

$\Rightarrow y \leq 4 \quad \text{or} \quad y \geq 12$

56. For real values of x, the expression $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ lies between

(A) $-\frac{1}{7}$ and 7 (B) $\frac{1}{7}$ and 7

(C) $\frac{1}{3}$ and 3 (D) None of these

Solution: (B)

Let $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$\Rightarrow yx^2 + 3xy + 4y = x^2 - 3x + 4$

$\Rightarrow (y - 1)x^2 + 3(y + 1)x + 4(y - 1) = 0$

Since x is real,

$\therefore$ discriminant ≥ 0

$\Rightarrow 9(y + 1)^2 - 16(y - 1)^2 \geq 0$

$\Rightarrow 9(y^2 + 2y + 1) - 16(y^2 - 2y + 1) \geq 0$

$\Rightarrow -7y^2 + 50y - 7 \geq 0 \Rightarrow 7y^2 - 50y + 7 \leq 0$

$\Rightarrow (y - 7)\left(y - \frac{1}{7}\right) \leq 0 \Rightarrow \frac{1}{7} \leq y \leq 7.$

57. If x is real, then the maximum value of $3 - 6x - 8x^2$ is

(A) $\frac{17}{8}$ (B) $\frac{33}{8}$

(C) $\frac{21}{8}$ (D) None of these

Solution: (B)

Let $y = 3 - 6x - 8x^2$

then $8x^2 + 6x + y - 3 = 0.$

Since x is real,

$\therefore 6^2 - 4 \cdot 8(y - 3) \geq 0,$

or $36 - 32y + 96 \geq 0$

or $32y \leq 132$

$\therefore y \leq \frac{132}{32}$

or $y \leq \frac{33}{8}$

Hence, maximum value of $y = \frac{33}{8}$.

58. For all real x, the minimum value of $\frac{1 - x + x^2}{1 + x + x^2}$ is

(A) 0 (B) $\frac{1}{3}$ (C) 1 (D) 3

Solution: (B)

Let $z = \frac{1 - x + x^2}{1 + x + x^2}$

$\Rightarrow z + zx + zx^2 = 1 - x + x^2$

$\Rightarrow zx^2 - x^2 + zx + x + z - 1 = 0$

$\Rightarrow x^2(z - 1) + x(z + 1) + (z - 1) = 0$

For real x,

$B^2 - 4AC \geq 0$

$\Rightarrow (z + 1)^2 - 4(z - 1)(z - 1) \geq 0$

$\Rightarrow z^2 + 2z + 1 - 4z^2 + 8z - 4 \geq 0$

$\Rightarrow -3z^2 + 10z - 3 \geq 0$

$\Rightarrow -3z^2 + 9z + z - 3 \geq 0$

$\Rightarrow -3z(z - 3) + 1(z - 3) \geq 0$

$\Rightarrow (z - 3)(-3z + 1) \geq 0$

$\Rightarrow \frac{1}{3} \leq z \leq 3$

$\therefore$ minimum value of $z = \frac{1}{3}$.

59. Given that, for all real x, the expression $\dfrac{x^2-2x+4}{x^2+2x+4}$ lies between $\dfrac{1}{3}$ and 3. The values between which the expression $\dfrac{9\cdot 3^{2x}+6\cdot 3^x+4}{9\cdot 3^{2x}-6\cdot 3^x+4}$ lies are

(A) 0 and 2 (B) –1 and 1

(C) –2 and 0 (D) $\dfrac{1}{3}$ and 3.

Solution: (D)

Given $\dfrac{1}{3}<\dfrac{x^2-2x+4}{x^2+2x+4}<3$ for all $x \in R$.

$\Rightarrow$ $\dfrac{1}{3}<\dfrac{x^2+2x+4}{x^2-2x+4}<3$ for all $x \in R$. (1)

Let $3^{x+1}=y$

Then $y \in R$ for all $x \in R$.

$$\therefore \quad \frac{9\cdot 3^{2x}+6\cdot 3^x+4}{9\cdot 3^{2x}-6\cdot 3^x+4}=\frac{3^{2x+2}+2\cdot 3^{x+1}+4}{3^{2x+2}-2\cdot 3^{x+1}+4}=\frac{y^2+2y+4}{y^2-2y+4}$$

From Eq. (1),

$$\frac{1}{3}<\frac{y^2+2y+4}{y^2-2y+4}<3$$

$$\therefore \quad \frac{1}{3}<\frac{9\cdot 3^{2x}+6\cdot 3^x+4}{9\cdot 3^{2x}-6\cdot 3^x+4}<3.$$

EXERCISES

Single Option Correct Type

1. The roots of the equation $2^{x+2}\cdot 3^{\frac{3x}{x-1}}=9$ are given by

(A) $\log_2\left(\dfrac{2}{3}\right)-2$ (B) 3, – 3

(C) $-2,\ 1-\dfrac{\log 3}{\log 2}$ (D) $1-\log_2 3,\ 2$

2. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2+b\,|x|+c=0$ is

(A) 0 (B) 2

(C) 4 (D) None of these

3. If $x^2-x+1=0$, then value of x^{3n} is

(A) 0 (B) –1 (C) 1 (D) –1, 1

4. The number of negative integral solutions of

$$x^2\cdot 2^{x+1}+2^{|x-3|+2}=x^2\cdot 2^{(|x-3|+4)}+2^{x-1} \text{ is}$$

(A) 4 (B) 2 (C) 1 (D) 0

5. If α and β ($\alpha<\beta$), are the roots of the equation $x^2+bx+c=0$, where $c<0<b$, then

(A) $0<\alpha<\beta$

(B) $\alpha<0<\beta<|\alpha|$

(C) $\alpha<\beta<0$

(D) $\alpha<0<|\alpha|<\beta$

6. If α and β are the roots of $x^2+px+q=0$ and α^4 and β^4 are the roots of $x^2-rx+s=0$, then the equation $x^2-4qx+2q^2-r=0$ has always

(A) two real roots

(B) two positive roots

(C) two negative roots

(D) one positive and one negative root

7. If a, b, c, d and p are distinct real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\le 0$ then a, b, c and d

(A) are in A.P. (B) are in G.P.

(C) are in H.P. (D) satisfy $ab=cd$

8. Let S denotes the set of all values of x for which the equation $2x^2-2(2a+1)x+a(a+1)=0$ has one root less than a and other root greater than a, then S equals

(A) (0, 1) (B) (–1, 0)

(C) (0, 1/2) (D) None of these

9. Let a, b, c be positive real numbers, such that $bx^2+(\sqrt{(a+c)^2+4b^2})x+(a+c)\ge 0,\ \forall\, x\in R$, then a, b, c are in:

(A) G.P. (B) A.P.

(C) H.P. (D) None of these

10. If the ratio of the roots of $x^2+bx+c=0$ and $x^2+qx+r=0$ be the same, then

(A) $r^2c=b^2q$ (B) $r^2b=c^2q$

(C) $rb^2=cq^2$ (D) $rc^2=bq^2$

11. If $0 \leq x < \frac{\pi}{2}$, then the solution of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is given by x equal to

(A) $\frac{\pi}{6}, \frac{\pi}{3}$ (B) $\frac{\pi}{3}, \frac{\pi}{2}$
(C) $\frac{\pi}{6}, \frac{\pi}{2}$ (D) None of these

12. If one of the roots of the equation $x^2 - (p + 1)x + p^2 + p - 8 = 0$ is greater than 2 and the other root is smaller than 2, then p is such that

(A) $-\frac{11}{3} < p < 3$ (B) $-2 < p < 3$
(C) $2 < p < 3$ (D) None of these

13. The common roots of the equations $x^3 + 2x^2 + 2x + 1 = 0$ and $1 + x^{130} + x^{1988} = 0$ are (where ω is a none real cube root of unity)

(A) ω (B) ω^2
(C) -1 (D) None of these

14. If 'x' satisfies $|x^2 - 3x + 2| + |x - 1| = x - 3$, then

(A) $x \in \phi$ (B) $x \in [1,2]$
(C) $x \in [3, \infty)$ (D) $x \in (-\infty, \infty)$

15. The number of solutions (s) of the equation

$\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} \leq 4 - 2x - x^2$ is

(A) one (B) two (C) four (D) infinite

16. If $(a^2 - 1)x^2 + (a - 1)x + a^2 - 4a + 3 = 0$ is an identity in x, then the value of a is

(A) 1 (B) 3 (C) -1 (D) -3

17. Both the roots of the equation $(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$ are always

(A) positive (B) negative
(C) real (D) None of these

18. If α, β are the roots of the equation $x^2 + px + q = 0$ then $\frac{\alpha}{\beta}$ is a root of the equation

(A) $px^2 + (2q - p^2)x + p = 0$
(B) $qx^2 + (p^2 - 2q)x + q = 0$
(C) $qx^2 + (2q - p^2)x + q = 0$
(D) None of these

19. If $a, b, c \in R$ and quadratic equation $x^2 + (a + b)x + c = 0$ has no real roots then

(A) $c(a + b + c) > 0$
(B) $c + c(a + b + c) > 0$
(C) $c + c(a + b - c) > 0$
(D) $c(a + b - c) > 0$

20. If $ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$ has distinct real roots in (1, 2) then a and $5a + 2b + c$ have

(A) same sign (B) opposite sign
(C) not determined (D) None of these

21. If $a < 0$ the positive root of the equation $x^2 - 2a|x - a| - 3a^2 = 0$ is

(A) $a(-1-\sqrt{6})$ (B) $a(-1+\sqrt{6})$
(C) $a(1-\sqrt{2})$ (D) None of these

22. If $px^2 + qx + r = 0$ has no real roots and p, q, r are real such that $p + r > 0$, then

(A) $p - q + r \leq 0$ (B) $p + r \geq q$
(C) $p + r = q$ (D) None of these

23. Given $lx^2 - mx + 5 = 0$ does not have two distinct real roots, the minimum value of $5l + m$ is

(A) 5 (B) -5 (C) 1 (D) -1

24. The set of possible values of λ for which $\lambda^2 - (\lambda^2 - 5\lambda + 5)x + (2\lambda^2 - 3\lambda - 4) = 0$ has roots whose sum and product are both less than 1 is

(A) $\left(-1, \frac{5}{2}\right)$ (B) $(1, 4)$
(C) $\left[1, \frac{5}{2}\right]$ (D) $\left(1, \frac{5}{2}\right)$

25. If 1 lies between the roots of $3x^2 - 3\sin\theta - 2\cos^2\theta = 0$ then

(A) $\frac{-1}{2} < \sin\theta < \frac{1}{2}$ (B) $\frac{-1}{2} < \sin\theta < 0$
(C) $\frac{1}{2} < \sin\theta < 1$ (D) None of these

26. If α, β are the roots of the equation $375x^2 - 25x - 2 = 0$ and $S_n = \alpha^n + \beta^n$, then $\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} S_r$ is

(A) $\frac{7}{12}$ (B) $\frac{1}{12}$
(C) $\frac{35}{12}$ (D) None of these

27. The solution set of $(x)^2 + (x + 1)^2 = 25$, where (x) is the least integer greater than or equal to x, is

(A) (2, 4) (B) $(-5, 4] \cup (2, 3]$
(C) $[-4, -3) \cup [3, 4)$ (D) None of these

28. Number of solutions of $\log_2(9 - 2^x) = 10^{\log_{10}(3-x)}$ is

(A) 1
(B) 2
(C) 3
(D) None of these

29. If $ax^2 + bx + 6 = 0$ does not have two distinct real roots $a \in R$, $b \in R$, then the least value of $3a + b$ is

(A) 4 (B) –1 (C) 1 (D) –2

30. If α, β be the roots of $x^2 + px - q = 0$ and γ, δ be the roots of $x^2 + px + r = 0$, $q + r \neq 0$, then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} =$

(A) 1 (B) q (C) r (D) $q + r$

31. Number of integral solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is

(A) 0 (B) 1 (C) 2 (D) 3

32. If the ratio of the roots of $\lambda x^2 + \mu x + v = 0$ is equal to the ratio of the roots of $x^2 + x + 1 = 0$, then λ, μ, v are in

(A) A.P. (B) G.P.
(C) H.P. (D) None of these

33. If $c < a < b < d$, then roots of the equation $bx^2 + (1 - b(c + d))x + bcd - a = 0$

(A) are real and one lies between c and a
(B) real and distinct in which one lies between a and b
(C) real and distinct in which one lies between c and d
(D) roots are not real

34. If the roots of the equation $ax^2 + bx + c = 0$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then the value of $(a+b+c)^2$ is

(A) $b^2 - 2ac$ (B) $b^2 - 4ac$
(C) $4b^2 - ac$ (D) $2b^2 - ac$

35. If α, β be roots of $x^2 + px + 1 = 0$ and γ, δ be the roots of $x^2 + qx + 1 = 0$, then $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$

(A) $p^2 + q^2$ (B) $p^2 - q^2$
(C) $q^2 - p^2$ (D) None of these

36. If a and b are odd integers then $[x]^2 + a[x] + b = 0$ (where [·] denotes greatest integer function) has

(A) finite number of roots
(B) infinite number of roots
(C) no roots
(D) None of these

37. If $\log_9(x^2 - 5x + 6) > \log_3(x - 4)$, x belongs to

(A) $(-\infty, 4)$
(B) $(4, \infty)$
(C) $(-\infty, -4) \cup (4, \infty)$
(D) no real value of x

38. Let a, b, c be real numbers, $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

(A) $\gamma = \frac{\alpha+\beta}{2}$ (B) $\gamma = \alpha + \frac{\beta}{2}$
(C) $\gamma = \alpha$ (D) $\alpha < \gamma < \beta$

39. Number of solutions of the equation $x^2 - 2 - 2[x] = 0$ ([·] denotes greatest integer function) is

(A) 1 (B) 2
(C) 3 (D) None of these

40. The number of real roots of the equation $2^{\sin^2 x} - 2^{\cos^2 x} = 1$ is

(A) 2 (B) 1
(C) infinite (D) None of these

41. If the absolute value of the difference of roots of the equation $x^2 + px + 1 = 0$ exceeds, $\sqrt{3p}$, then

(A) $p < -1$ or $p > 4$ (B) $p > 4$
(C) $-1 < p < 4$ (D) $0 \le p < 4$

42. If the roots of $x^2 + ax + b = 0$ are c and d, then roots of $x^2 + (2c + a)x + c^2 + ac + b = 0$ are

(A) 1, $d - c$ (B) 0, $d - c$
(C) 1, $c - d$ (D) None of these

43. If the equation $x^2 + 2(k + 1)x + 9k - 5 = 0$ has only negative roots, then

(A) $k \le 0$ (B) $k \ge 0$ (C) $k \ge 6$ (D) $k \le 6$

44. If the product of the roots of the equation $x^2 - 3kx + 2e^{2\ln k} - 1 = 0$ is 7, then for real roots the value of k is equal to

(A) 1 (B) 2 (C) 3 (D) 4

45. The solution set of

$$\left(\frac{3}{5}\right)^x = x - x^2 - 9 \text{ is}$$

(A) {0} (B) {1}
(C) ϕ (D) None of these

46. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(A) infinite number of real roots
(B) no real roots
(C) exactly one real root
(D) exactly four real roots

47. Suppose the cube $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?

(A) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(B) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(C) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(D) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

48. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

(A) 1 (B) 4 (C) 3 (D) 2

49. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is

(A) greater than $4ab$
(B) less than $4ab$
(C) greater than $-4ab$
(D) less than $-4ab$

50. The equation

$$\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1 \text{ has}$$

(A) no solution
(B) one solution
(C) two solutions
(D) more than two solutions

51. If $x, y \in [0, 10]$, then the number of solutions (x, y) of the inequation $3^{\sec^2 x - 1}\sqrt{9y^2 - 6y + 2} \le 1$ is

(A) 2 (B) 4 (C) 6 (D) infinite

52. The equation $(x - n)^m + (x - n^2)^m + (x - n^3)^m + \dots + (x - n^m)^m = 0$ (m is odd positive integer), has

(A) all real roots
(B) one real and $(n - 1)$ imaginary roots
(C) one real and $(m - 1)$ imaginary roots
(D) no real root

53. If $f(x) = x - [x]$, $x\ (\neq 0) \in R$, where $[x]$ is the greatest integer less than or equal to x, then the number of solutions of $f(x) + f\left(\frac{1}{x}\right) = 1$ are

(A) 0 (B) 1 (C) infinite (D) 2

54. If $x^2 - (a + b + c)x + (ab + bc + ca) = 0$ has imaginary roots, where $a, b, c \in R^+$, then $\sqrt{a}, \sqrt{b}, \sqrt{c}$

(A) can be the sides of a triangle
(B) cannot be the sides of a triangle
(C) nothing can be said
(D) None of these

55. If $x_1, x_2, x_3, \dots, x_n$ are the roots of the equation $x^n + ax + b = 0$, then the value of $(x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \dots (x_1 - x_n)$ is equal to

(A) $nx_1^{n-1} + a$
(B) $n(x_1)^{n-1}$
(C) $nx_1 + b$
(D) $nx_1^{n-1} + b$

56. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

(A) $a < 2$
(B) $2 \le a \le 3$
(B) $3 \le a \le 4$
(D) $a > 4$

57. For all real x, the minimum value of $\frac{1 - x + x^2}{1 + x + x^2}$ is

(A) 0 (B) $\frac{1}{3}$ (C) 1 (D) 3

58. Given that, for all real x, the expression $\frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ lies between $\frac{1}{3}$ and 3. The values between which the expression $\frac{9 \cdot 3^{2x} + 6 \cdot 3^x + 4}{9 \cdot 3^{2x} - 6 \cdot 3^x + 4}$ lies are

(A) 0 and 2
(B) –1 and 1
(C) –2 and 0
(D) $\frac{1}{3}$ and 3.

59. The value of k for which the number 3 lies between the roots of the equation $x^2 + (1 - 2k)x + (k^2 - k - 2) = 0$ is given by

(A) $2 < k < 5$
(B) $k < 2$
(C) $2 < k < 3$
(D) $k > 5$

60. The number of negative integral solutions of $x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{(|x-3|+4)} + 2^{x-1}$ is

(A) 4 (B) 2 (C) 1 (D) 0

61. If α and β $(\alpha < \beta)$, are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then

(A) $0 < \alpha < \beta$
(B) $\alpha < 0 < \beta < |\alpha|$
(C) $\alpha < \beta < 0$
(D) $\alpha < 0 < |\alpha| < \beta$

62. If the ratio of the roots of $x^2 + bx + c = 0$ and $x^2 + qx + r = 0$ be the same, then

(A) $r^2c = b^2q$
(B) $r^2b = c^2q$
(C) $rb^2 = cq^2$
(D) $rc^2 = bq^2$

63. The number of solutions of $|[x] - 2x| = 4$, where $[x]$ is the greatest integer $\le x$, is

(A) 2 (B) 4 (C) 1 (D) infinite

64. If α, β are the roots of the equation $x^2 + px + q = 0$ then $\frac{\alpha}{\beta}$ is a root of the equation

(A) $px^2 + (2q - p^2)x + p = 0$
(B) $qx^2 + (p^2 - 2q)x + q = 0$
(C) $qx^2 + (2q - p^2)x + q = 0$
(D) None of these

65. If $ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$ has distinct real roots in (1, 2) then a and $5a + 2b + c$ have

(A) same sign (B) opposite sign
(C) not determined (D) None of these

66. If $a < 0$, the positive root of the equation $x^2 - 2a|x - a| - 3a^2 = 0$ is

(A) $a(-1-\sqrt{6})$ (B) $a(-1+\sqrt{6})$
(C) $a(1-\sqrt{2})$ (D) None of these

67. If $px^2 + qx + r = 0$ has no real roots and p, q, r are real such that $p + r > 0$, then

(A) $p - q + r \leq 0$ (B) $p + r \geq q$
(C) $p + r = q$ (D) None of these

68. Given $lx^2 - mx + 5 = 0$ does not have two distinct real roots, the minimum value of $5l + m$ is

(A) 5 (B) –5 (C) 1 (D) –1

69. If 1 lies between the roots of $3x^2 - 3\sin\theta - 2\cos^2\theta = 0$ then

(A) $\frac{-1}{2} < \sin\theta < \frac{1}{2}$ (B) $\frac{-1}{2} < \sin\theta < 0$
(C) $\frac{1}{2} < \sin\theta < 1$ (D) None of these

70. If α, β are the roots of the equation $375x^2 - 25x - 2 = 0$ and $S_n = \alpha^n + \beta^n$, then $\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} S_r$ is

(A) $\frac{7}{12}$ (B) $\frac{1}{12}$
(C) $\frac{35}{12}$ (D) None of these

71. If $ax^2 + bx + 6 = 0$ does not have two distinct real roots $a \in R$, $b \in R$, then the least value of $3a + b$ is

(A) 4 (B) –1 (C) 1 (D) –2

72. If the ratio of the roots of $\lambda x^2 + \mu x + v = 0$ is equal to the ratio of the roots of $x^2 + x + 1 = 0$, then λ, μ, v are in

(A) A.P. (B) G.P.
(C) H.P. (D) None of these

73. If $c < a < b < d$, then roots of the equation $bx^2 + (1 - b(c + d))x + bcd - a = 0$

(A) are real and one lies between c and a
(B) real and distinct in which one lies between a and b
(C) real and distinct in which one lies between c and d
(D) roots are not real

74. If the roots of $x^2 + ax + b = 0$ are c and d then roots of $x^2 + (2c + a)x + c^2 + ac + b = 0$ are

(A) $1, d - c$ (B) $0, d - c$
(C) $1, c - d$ (D) None of these

75. The solution set of $(x)^2 + (x + 1)^2 = 25$, where (x) is the least integer greater than or equal to x, is

(A) (2, 4)
(B) $(-5, 4] \cup (2, 3]$
(C) $[-4, -3) \cup [3, 4)$
(D) None of these

76. Let S denote the set of all values of S for which the equation $2x^2 - 2(2a + 1)x + a(a + 1) = 0$ has one root less than a and other root greater than a, then S equals

(A) (0, 1) (B) (–1, 0)
(C) $\left(0, \frac{1}{2}\right)$ (D) None of these

77. Solution of $2^x + 2^{|x|} \geq 2\sqrt{2}$ is

(A) $(-\infty, \log_2(\sqrt{2}+1))$
(B) (0, 8)
(C) $\left(\frac{1}{2}, \log_2(\sqrt{2}-1)\right)$
(D) $(-\infty, \log_2(\sqrt{2}-1)] \cup \left[\frac{1}{2}, \infty\right)$

78. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that min. $f(x) >$ max. $g(x)$, then the relation between b and c is

(A) $|c| < |b|\sqrt{2}$ (B) $0 < c < b\sqrt{2}$
(C) $|c| < |b|\sqrt{2}$ (D) $|c| > |b|\sqrt{2}$

79. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real less than 3, then:

(A) $a < 2$ (B) $2 \leq a \leq 3$
(C) $3 < a \leq 4$ (D) $a > 4$

80. The solution set of $\left|\frac{x+1}{x}\right| + |x + 1| = \frac{(x+1)^2}{|x|}$ is

(A) $\{x \mid x \geq 0\}$
(B) $\{x \mid x > 0\} \cup \{-1\}$
(C) $\{-1, 1\}$
(D) $\{x \mid x \geq 1 \text{ or } x \leq -1\}$

81. If α, β are the roots of the equation $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then

(A) $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$
(B) $\frac{b^2 - 2ac}{a^2} = \frac{B^2 - 2AC}{A^2}$
(C) $\frac{b^2 - 8ac}{a^2} = \frac{B^2 - 8AC}{A^2}$
(D) None of these

82. Let a, b, c be real, if $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$ then $1 + \frac{c}{a} + \left|\frac{b}{a}\right|$ is

(A) < 0 (B) > 0
(C) ≤ 0 (D) None of these.

83. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

(A) H.P. (B) G.P.
(C) A.P. (D) None of these

84. If the equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then their other roots satisfy the equation

(A) $x^2 + a(b + c)x + a^2bc = 0$
(B) $x^2 - a(b + c)x + a^2bc = 0$
(C) $x^2 - a(b + c)x - a^2bc = 0$
(D) None of these

85. If $(ax^2 + bx + c)y + a'x^2 + b'x + c' = 0$, then the condition that x may be a rational function of y is

(A) $(ac' - a'c)^2 = (ab' - a'b)(bc' - b'c)$
(B) $(ab' - a'b)^2 = (ac' - a'c)(bc' - b'c)$
(C) $(bc' - b'c)^2 = (ab' - a'b)(ac' - a'c)$
(D) None of these

86. If n and r are positive integers such that $0 < r < n$, then the roots of the quadratic equation ${}^nC_{r-1}x^2 + 2 \cdot {}^nC_r x + {}^nC_{r+1} = 0$ are

(A) real and distinct
(B) rational
(C) rational but not integer
(D) imaginary

87. If the equations $x^2 - px + q = 0$ and $x^2 - ax + b = 0$ have a common root and the other root of the second equation is the reciprocal of the other root of the first, then $(q - b)^2 =$

(A) $aq(p - b)^2$ (B) $bq(p - a)^2$
(C) $bq(p - b)^2$ (D) None of these

88. If the two equations $ax^2 + bx + c = 0$ and $2x^2 - 3x + 4 = 0$ have a common root, then

(A) $6a = 4b = -3c$ (B) $3a = -4b = 3c$
(C) $6a = -4b = 3c$ (D) None of these

89. If a, b, c are rational and $ax^2 + bx + c = 0$ and $3x^2 + x - 5 = 0$ have a common root, then $3a + b + 2c =$

(A) 0 (B) 1
(C) 2 (D) None of these

90. If $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have a common root and $\frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in A.P., then a_1, b_1, c_1 are in

(A) A.P. (B) G.P.
(C) H.P. (D) None of these

91. If x is real, then the minimum value of $\frac{(a + x)(b + x)}{(c + x)}$ $(x > -c)$, for $a > c, b > c$ is

(A) $(\sqrt{a - b} + \sqrt{c - b})^2$
(B) $(\sqrt{a - c} + \sqrt{b - c})^2$
(C) $(\sqrt{a - c} - \sqrt{b - c})^2$
(D) None of these

92. If the ratio of the roots of $a_1x^2 + b_1x + c_1 = 0$ be equal to the ratio of the roots of $a_2x^2 + b_2x + c_2 = 0$, then $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ are in

(A) A.P. (B) G.P.
(C) H.P. (D) None of these

93. If α, β be the roots of the equation $x^2 - px + q = 0$ and $\alpha > 0, \beta > 0$, then the value of $\alpha^{1/4} + \beta^{1/4}$ is $\left(p + 6\sqrt{q} + 4q^{1/4}\sqrt{p + 2\sqrt{q}}\right)^k$, where k is equal to

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

94. If a, b are the roots of the equation $x^2 + px + 1 = 0$ and c, d are the roots of the equation $x^2 + qx + 1 = 0$, then $(a - c)(b - c)(a + d)(b + d) =$

(A) $p^2 - q^2$ (B) $q^2 - p^2$
(C) $p^2 + q^2$ (D) $2(p^2 - q^2)$

95. If $q \neq 0$ and the equation $x^3 + px^2 + q = 0$ has a root of multiplicity 2, then p and q are connected by

(A) $p^2 + 2q = 0$
(B) $p^2 - 2q = 0$
(C) $4p^3 + 27q + 1 = 0$
(D) $4p^3 + 27q = 0$

96. If the roots of the equation $ax^2 + bx + c = 0$, are of the form $\frac{\alpha}{\alpha - 1}$ and $\frac{\alpha + 1}{\alpha}$, then the value of $(a + b + c)^2$ is

(A) $b^2 - 2ac$ (B) $2b^2 - ac$
(C) $b^2 - 4ac$ (D) $4b^2 - 2ac$

97. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in

(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) arithmetico-geometric progression

98. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval
(A) $(-\infty, 4)$ (B) $[4, 5]$
(C) $(5, 6]$ (D) $(6, \infty)$

99. If for real number a, the equation $(a-2)(x-[x])^2 + 2(x-[x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\le x$) has no integral solution and has exactly one solution in (2, 3), then a lies in the interval
(A) $(-1, 2)$ (B) $(0, 1)$
(C) $(-1, 0)$ (D) $(2, 3)$

100. Let a, b, c be distinct positive numbers such that each of the quadratics $ax^2 + bx + c$, $bx^2 + cx + a$ and $cx^2 + ax + b$ is non-negative for all $x \in R$. If $R = \dfrac{a^2 + b^2 + c^2}{ab + bc + ca}$, then
(A) $1 \le R < 4$ (B) $1 < R \le 4$
(C) $1 \le R \le 4$ (D) $1 < R < 4$

101. The set of values of a for which the equation $(x^2 + x + 2)^2 - (a-3)(x^2 + x + 2)(x^2 + x + 1) + (a-4)(x^2 + x + 1)^2 = 0$ has at least one real root is
(A) $\left(5, \dfrac{19}{3}\right)$ (B) $\left[5, \dfrac{19}{3}\right]$
(C) $\left[5, \dfrac{19}{3}\right)$ (D) $\left(5, \dfrac{19}{3}\right]$

102. If all real values of x obtained from the equation $4^x - (a-3)2^x + a - 4 = 0$ are non-positive, then a belongs to
(A) $[4, 5]$ (B) $(4, 5]$
(C) $[4, 5)$ (D) $(4, 5)$

103. Let $f(x) = x^2 + ax + b$ be a quadratic polynomial, where a and b are integers. If for a given integer n, $f(n)f(n+1) = f(m)$ for some integer m, then the value of m is
(A) $n(n+1) + an + b$ (B) $n(n+1) + a + bn$
(C) $n(n+1) + a + b$ (D) None of these

104. If for any real x, we have $-1 \le \dfrac{x^2 + nx - 2}{x^2 - 3x + 4} \le 2$, then n belongs to
(A) $[-\sqrt{40} + 6, -1]$
(B) $[-\sqrt{40} + 6, \sqrt{40} - 6]$
(C) $[-1, \sqrt{40} - 6]$
(D) None of these

105. If $b > a$, then the equation $(x-a)(x-b) - 1 = 0$ has
(A) both roots in $(-\infty, a)$
(B) one root in $(-\infty, a)$ and other in (b, ∞)
(C) both roots in (b, ∞)
(D) both roots in $[a, b]$

106. The quadratic equation
$$\frac{(x+b)(x+c)}{(b-a)(c-a)} + \frac{(x+c)(x+a)}{(c-b)(a-b)} + \frac{(x+a)(x+b)}{(a-c)(b-c)} = 1$$
has
(A) two real and distinct roots
(B) imaginary roots
(C) equal roots
(D) infinite roots

107. The equation $ax^4 - 2x^2 - (a-1) = 0$ will have real and unequal roots if
(A) $a < 0, a \ne 1$ (B) $a > 0, a \ne 1$
(C) $0 < a < 1$ (D) None of these

108. If the equation $x^2 + [a^2 - 5a + b + 4]x + b = 0$ has roots -5 and 1, where $[a]$ denotes the greatest integer less than or equal to a, then the set of values of a is
(A) $\left(\dfrac{5-3\sqrt{5}}{2}, \dfrac{5+3\sqrt{5}}{2}\right)$
(B) $\left(0, \dfrac{5+3\sqrt{5}}{2}\right)$
(C) $\left(-1, \dfrac{5-3\sqrt{5}}{2}\right] \cup \left[\dfrac{5+3\sqrt{5}}{2}, 6\right)$
(D) None of these

109. Let α_1, β be the roots of the equation $x^2 - ax + p = 0$ and γ, δ be the roots of the equation $x^2 - bx + q = 0$. If $\alpha, \beta, \surd, \delta$ are in increasing G.P., then the value of $\dfrac{q+p}{q-p}$ is equal to
(A) $\dfrac{b^2 - a^2}{b^2 + a^2}$ (B) $\dfrac{b^2 + a^2}{b^2 - a^2}$
(C) $\dfrac{b+a}{b-a}$ (D) $\dfrac{b-a}{b+a}$

110. If t_n denotes the nth term of an A.P. and $t_p = \dfrac{1}{q}$ and $t_q = \dfrac{1}{p}$, then which of the following is necessarily a root of the equation $(p + 2q - 3r)x^2 + (q + 2r - 3p)x + (r + 2p - 3q) = 0$
(A) t_p (B) t_q
(C) t_{pq} (D) t_{p+q}

111. If the roots of the equation $4x^2 + 4ax + b = 0$ are real and differ at most by a, then b lies in

(A) $\left(0, \frac{a^2}{2}\right)$ (B) $\left(\frac{a^2}{2}, a^2\right)$

(C) $[0, a^2]$ (D) $(0, a^2)$

112. The roots of the equation $ax^2 + bx + c = 0$, where $a \in R^+$, are two consecutive odd positive integers, then

(A) $|b| \le 4a$ (B) $|b| \ge 4a$

(C) $|b| = 2a$ (D) None of these

113. If a, b, c, d are real numbers, then the number of real roots of the equation $(x^2 + ax - 3b)(x^2 - cx + b)(x^2 - dx + 2b) = 0$ are

(A) 3 (B) 4

(C) 6 (D) at least 2

More than One Option Correct Type

114. If $a \le 0$, then the root of the equation $x^2 - 2a|x - a| - 3a^2 = 0$ is

(A) $(1-\sqrt{2})a$ (B) $(-1+\sqrt{6})a$

(C) $(1+\sqrt{2})a$ (D) $-(1+\sqrt{6})a$

115. If $x^2 - 3x + 2$, be one of the factors of the expression $x^4 - px^2 + q$, then

(A) $p = 5$ (B) $q = 4$

(C) $p = 4$ (D) $q = 5$

116. If $c \ne 0$ and the equation $\frac{p}{2x} = \frac{a}{x+c} + \frac{b}{x-c}$ has two equal roots, then p can be

(A) $(\sqrt{a}-\sqrt{b})^2$ (B) $(\sqrt{a}+\sqrt{b})^2$

(C) $a + b$ (D) $a - b$

117. For $a > 0$, the roots of the equation $\log_{ax} a + \log_x a^2 + \log_{a^2x} a^3 = 0$, are given by

(A) $a^{1/2}$ (B) $a^{-1/2}$ (C) $a^{4/3}$ (D) $a^{-4/3}$

118. Solution of $|x^2 + 4x + 3| + 2x + 5 = 0$ is

(A) 4 (B) –4

(C) $-1-\sqrt{3}$ (D) $1+\sqrt{3}$

119. If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then the roots are

(A) $\frac{-3}{5}$ (B) $\frac{-3}{2}$ (C) 3 (D) $\frac{1}{3}$

120. If the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of x and y, then

(A) $1 \le x \le 3$

(B) $2 \le x \le 3$

(C) $-\frac{1}{3} \le y \le \frac{1}{3}$

(D) $\frac{1}{3} \le x \le 1.$

121. If α, β are the roots of $ax^2 + bx + c = 0$ and α^4, β^4 are the roots of $lx^2 + mx + n = 0$, then the roots of the equation $a^2 l x^2 - 4aclx + 2c^2 l + a^2 m = 0$ are

(A) real (B) imaginary

(C) opposite in sign (D) equal

122. If a, b, c are positive rational numbers such that $a > b > c$ and the quadratic equation $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ has a root in the interval (–1, 0,) then

(A) $c + a < 2b$

(B) both roots of the given equation are rational

(C) the equation $ax^2 + 2bx + c = 0$ has both negative real roots

(D) the equation $cx^2 + 2ax + b = 0$ has both negative real roots

123. If the equation $x^2 + a^2x + b^2 = 0$ has two roots each of which exceeds a number c, then

(A) $a^4 > 4b^2$

(B) $c^2 + a^2c + b^2 > 0$

(C) $-\frac{a^2}{2} > c$

(D) $-\frac{a^2}{2} < c$

124. If $b^2 \ge 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all the roots of the equation will be real if

(A) $b > 0, a < 0, c > 0$

(B) $b < 0, a > 0, c > 0$

(C) $b < 0, a > 0, c > 0$

(D) $b > 0, a < 0, c < 0$

125. If the equation $x^2 + (a - b)x - a - b + 1 = 0$, where $a, b \in$ R, has unequal real roots for all $b \in R$, then

(A) $a < 0$ (B) $a > 0$

(C) $a > 1$ (D) $a < 1$

Passage Based Questions

Passage 1

Let $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ be two quadratic equations such that $a_1a_2 \neq 0$ and $a_1b_2 \neq a_2b_1$.

If the two equations have a common root α, then

$$a_1\alpha^2 + b_1\alpha + c_1 = 0 \text{ and } a_2\alpha^2 + b_2\alpha + c_2 = 0.$$

Eliminating α using cross-multiplication method gives the condition for a common root. Solving the two equations simultaneously, the common root can be obtained.

If the two equations have both roots common, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

126. If $a, b, c \in R$ and the equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ have two roots common, then

(A) $a = b \neq c$ (B) $a = b = -c$
(C) $a = b = c$ (D) None of these

Passage 2

Let k be any point such that $k \in R$ and α, β are the roots of the quadratic equation $f(x) = ax^2 + bx + c = 0$. If k lies outside and is less than both the roots then the equation must have real and distinct roots and the sign of $f(k)$ is same as the sign of 'a'. Also, k is less than the x-coordinate of the vertex of the parabola $y = ax^2 + bx + c$.

If k lies between both the roots, then the sign of $f(k)$ is opposite to the sign of 'a'.

If k lies outside and is greater than both the roots, then the sign of $f(k)$ is same as the sign of 'a'. Also, k is greater than the x-coordinate of the vertex of the parabola $y = ax^2 + bx + c$.

If both the roots of the equation lie between two real numbers k_1 and k_2, then equation must have real and distinct roots and the sign of $f(k_1)$ and $f(k_2)$ is same as the sign of a. Also, the x-coordinate of the vertex of the parabola $y = ax^2 + bx + c$ lies between k_1 and k_2.

127. The values of 'a' for which the roots of the equation $(a + 1)x^2 - 3ax + 4a = 0$ $(a \neq -1)$ to be greater than unity are

(A) $\frac{-16}{7} \leq a < -1$ (B) $-2 < a < -1$
(C) $0 < a < 1$ (D) None of these

128. The values of 'a' so that 6 lies between the roots of the equation $x^2 + 2(a - 3)x + 9 = 0$, are

(A) $a > -\frac{3}{4}$ (B) $a < -\frac{3}{4}$
(C) $a > \frac{3}{4}$ (D) $a < \frac{3}{4}$

129. The value of a for which the equation $(1 - a^2)x^2 + 2ax - 1 = 0$ has roots belonging to $(0,1)$ is

(A) $a > \frac{1+\sqrt{5}}{2}$ (B) $a > 2$
(C) $\frac{1+\sqrt{5}}{2} < a < 2$ (D) $a > \sqrt{2}$

130. The values of a for which each one of the roots of $x^2 - 4ax + 2a^2 - 3a + 5 = 0$ is greater than 2, are

(A) $a \in (1, \infty)$ (B) $a = 1$
(C) $a \in (-\infty, 1)$ (D) $a \in (9/2, \infty)$

Passage 3

The maximum number of positive real roots of a polynomial equation $f(x) = 0$ is the number of changes of signs from positive to negative and negative to positive in $f(x)$. For example, consider the equation $f(x) = x^3 + 6x^2 + 11x - 6 = 0$. The signs of the various terms are:

$$+ + + -$$

Clearly, there is only one change of sign in the given expression. So, the given equation has at most one positive real root.

The maximum number of negative real roots of a polynomial equation $f(x) = 0$ is the number of changes of signs from positive to negative and negative to positive in $f(-x)$. For example, for the equation $f(x) = x^4 + x^3 + x^2 - x - 1 = 0$, there are three changes of signs in $f(-x)$. So, the given equation has atmost three negative real roots.

If $f(x)$ and $f(-x)$ do not have any changes of signs, the equation $f(x) = 0$ has no real roots.

Now, consider the polynomial

$$P_n(x) = 1 + 2x + 3x^2 + \ldots + (n + 1)x^n.$$

131. If n is even, the number of real roots of $P_n(x)$ is

(A) 0 (B) n
(C) 1 (D) None of these

132. If n is odd, the number of real roots of $P_n(x)$ is

(A) 0 (B) n
(C) 1 (D) None of these

133. If $a_1 < a_2 < a_3 < a_4 < a_5 < a_6$, then the equation $(x - a_1)(x - a_3)(x - a_5) + 3(x - a_2)(x - a_4)(x - a_6) = 0$ has

(A) three real roots
(B) a root in $(-\infty, a_1)$
(C) no real root in (a_1, a_2)
(D) no real root in (a_5, a_6)

Match the Column Type

134.

Column-I	Column-II
(I) If the roots of the equation $(a^2 + b^2)x^2 + 2(bc + ad)x + (c^2 + d^2) = 0$ are real, then a^2, bd, c^2 are in	(A) A.P.
(II) If $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ has equal roots, then a, b, c are in	(B) H.P.
(III) If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the reciprocals of their squares, then bc^2, ca^2 and ab^2 are in	(C) G.P.
(IV) If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \le 0$ then a, b, c and d are in	(D) A.G.P.

135.

Column-I	Column-II
(I) If α, β be the roots of $x^2 + px - q = 0$ and γ, δ be the roots of $x^2 + px + r = 0$, $q + r \neq 0$, then $\dfrac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)} =$	(A) 0
(II) The number of solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$ is	(A) 2
(III) If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is	(A) 6
(IV) The minimum value of $\lvert x\rvert + \lvert x + \frac{1}{2}\rvert + \lvert x - 3\rvert + \lvert x - \frac{5}{2}\rvert$ is	(A) 1

Assertion-Reason Type

Instructions: *In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options:*

(A) Assertion(A) is True and Reason(R) is True; Reason(R) is a correct explanation for Assertion(A)
(B) Assertion(A) is True, Reason(R) is True; Reason(R) is not a correct explanation for Assertion(A)
(C) Assertion(A) is True, Reason(R) is False
(D) Assertion(A) is False, Reason(R) is True

136. **Assertion:** If the roots of the equations $x^2 - bx + c = 0$ and $x^2 - cx + b = 0$ differ by the same quantity, then $b + c$ is equal to –4.

Reason: If α, β are the roots of the equation $Ax^2 + Bx + C = 0$, then $\alpha - \beta = \dfrac{\sqrt{B^2 - 4AC}}{A}$

137. **Assertion:** If the equation $x^2 + 2(k + 1)x + 9k - 5 = 0$ has only negative roots, then $k \le 6$

Reason: The equation $f(x) = 0$ will have both roots negative if and only if
(i) Discriminant ≥ 0,
(ii) Sum of roots < 0,
(iii) Product of roots > 0

138. **Assertion:** If the equations $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root, then their other roots will satisfy the equation $x^2 + ax + bc = 0$

Reason: If the equations $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root, then $a + b + c = 0$

Previous Year's Questions

139. If $\alpha \neq \beta$ with $a^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$, then the equation having α/β and β/α as its roots, is **[2002]**

(A) $3x^2 + 19x + 3 = 0$
(B) $3x^2 - 19x + 3 = 0$
(C) $3x^2 - 19x - 3 = 0$
(D) $x^2 - 16x + 1 = 0$

139. The number of real roots of $3^{2x^2-7x+7}=9$ is **[2002]**

(A) Zero (B) 2
(C) 1 (D) 4

140. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$, are in **[2003]**

(A) arithmetic progression.
(B) geometric progression.
(C) harmonic progression.
(D) arithmetic-geometric-progression.

141. The number of real solutions of the equation $x^2-3|x|+2=0$ is **[2003]**

(A) 2 (B) 4 (C) 1 (D) 3

142. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as the other, is **[2003]**

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$
(C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

143. If $(1-p)$ is a root of quadratic equation $x^2+px+(1-p)=0$, then its roots are **[2004]**

(A) 0, 1 (B) –1, 2
(C) 0, –1 (D) –1, 1

144. If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots, then the value of 'q' is **[2004]**

(A) $\frac{49}{4}$ (B) 4
(C) 3 (D) 12

145. If $2a+3b+6c=0$, then at least one root of the equation $ax^2+bx+c=0$ lies in the interval **[2004]**

(A) (0, 1) (B) (1, 2)
(C) (2, 3) (D) (1, 3)

146. The values of α for which the sum of the squares of the roots of the equation $x^2-(a-2)x-a-1=0$ assume the least value is **[2005]**

(A) 1 (B) 0
(C) 3 (D) 2

147. If roots of the equation $x^2-bx+c=0$ be two consecutive integers, then b^2-4c equals **[2005]**

(A) –2 (B) 3
(C) 2 (D) 1

148. If both the roots of the quadratic equation $x^2-2kx+k^2+k-5=0$ are less than 5, then k lies in the interval **[2005]**

(A) (5, 6] (B) (6, ∞)
(C) (–∞, 4) (D) [4, 5]

149. All the values of m for which both roots of the equations $x^2-2mx+m^2-1=0$ are greater than –2 but less than 4, lie in the interval **[2006]**

(A) $-2<m<0$ (B) $m>3$
(C) $-1<m<3$ (D) $1<m<4$

150. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is **[2006]**

(A) 1/4 (B) 41
(C) 1 (D) 17/7

151. If the difference between the roots of the equation $x^2+ax+1=0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**

(A) (–3, 3) (B) (–3, ∞)
(C) (3, ∞) (D) (–∞, –3)

152. The quadratic equations $x^2-6x+a=0$ and $x^2-cx+6=0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[2008]**

(A) 1 (B) 4
(C) 3 (D) 2

153. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is **[2009]**

(A) greater than $4ab$
(B) less than $4ab$
(C) greater than $-4ab$
(D) less than $-4ab$

154. If a and β are the roots of the equation $x^2-x+1=0$, then the value of $\alpha^{2009}+\beta^{2009}=$ **[2010]**

(A) –1 (B) 1
(C) 2 (D) –2

155. The equation $e^{\sin x}-e^{-\sin x}-4=0$, for x real, has **[2012]**

(A) infinite number of roots
(B) no roots
(C) exactly one root
(D) exactly four roots

156. The real number k for which the equation, $2x^3+3x+k=0$ has two distinct real roots in [0, 1] **[2013]**

(A) lies between 2 and 3
(B) lies between −1 and 0
(C) does not exist
(D) lies between 1 and 2

157. If the equations $x^2+2x+3=0$ and $ax^2+bx+c=0, a,b,c \in R$ have a common root, then $a:b:c$ is **[2013]**

(A) 3 : 2 : 1 (B) 1 : 3 : 2
(C) 3 : 1 : 2 (D) 1 : 2 : 3

158. If $a \in R$ and the equation $-3(x-[x])^2+2(x-[x])+a^2=0$ (where $[x]$ denotes the greatest integer $\le x$) has no integral solution, then all possible values of a lie in the interval **[2014]**

(A) $(-1,0)\cup(0,1)$ (B) $(1,2)$
(C) $(-2,-1)$ (D) $(-\infty,-2)\cup(2,\infty)$

159. Let α and β be the roots of equation $x^2-6x-2=0$. If $a_n=\alpha^n-\beta^n$, for $n\ge 1$, then the value of $\dfrac{a_{10}-2a_8}{2a_9}$ is equal to **[2015]**

(A) – 6 (B) 3
(C) – 3 (D) 6

ANSWER KEYS

Single Option Correct Type

1. (C) **2.** (A) **3.** (C) **4.** (D) **5.** (B) **6.** (A) **7.** (B) **8.** (D) **9.** (B) **10.** (C)
11. (A) **12.** (B) **13.** (A, B, C) **14.** (A) **15.** (A) **16.** (A) **17.** (A) **18.** (C) **19.** (B)
20. (A) **21.** (C) **22.** (B) **23.** (D) **24.** (D) **25.** (C) **26.** (B) **27.** (B) **28.** (A) **29.** (C)
30. (A) **31.** (D) **32.** (B) **33.** (C) **34.** (B) **35.** (C) **36.** (C) **37.** (B) **38.** (D) **39.** (A)
40. (C) **41.** (B) **42.** (B) **43.** (C) **44.** (B) **45.** (C) **46.** (B) **47.** (A) **48.** (D) **49.** (C)
50. (D) **51.** (B) **52.** (C) **53.** (C) **54.** (A) **55.** (A) **56.** (A) **57.** (B) **58.** (D) **59.** (A)
60. (D) **61.** (B) **62.** (C) **63.** (B) **64.** (C) **65.** (A) **66.** (C) **67.** (B) **68.** (D) **69.** (C)
70. (B) **71.** (D) **72.** (B) **73.** (C) **74.** (B) **75.** (B) **76.** (D) **77.** (D) **78.** (D) **79.** (A)
80. (B) **81.** (A) **82.** (A) **83.** (C) **84.** (B) **85.** (A) **86.** (A) **87.** (B) **88.** (C) **89.** (A)
90. (B) **91.** (B) **92.** (B) **93.** (D) **94.** (B) **95.** (D) **96.** (C) **97.** (C) **98.** (A) **99.** (C)
100. (D) **101.** (D) **102.** (B) **103.** (A) **104.** (C) **105.** (B) **106.** (D) **107.** (C) **108.** (C) **109.** (B)
110. (C) **111.** (C) **112.** (B) **113.** (D)

More than One Option Correct Type

114. (A) and (B) **115.** (A) and (B) **116.** (A) and (B) **117.** (B) and (D) **118.** (B) and (C)
119. (A), (B) and (C) **120.** (A) and (C) **121.** (A) and (C) **122.** (A), (B), (C) and (D)
123. (A), (B) and (C) **124.** (C) and (D) **125.** (B) and (C)

Passage Based Questions

126. (C) **127.** (A) **128.** (B) **129.** (B) **130.** (D) **131.** (A) **132.** (C) **133.** (A)

Match the Column Type

134. (I) → (C); (II) → (B); (III) → (A); (IV) → (C)
135. (I) → (D); (II) → (A); (III) → (B); (IV) → (C)

Assertion-Reason Type

136. (A) **137.** (D) **138.** (A)

Previous Year's Questions

139. (A) **140.** (B) **141.** (C) **142.** (B) **143.** (A) **144.** (C) **145.** (A) **146.** (A) **147.** (D) **148.** (C)
149. (C) **150.** (C) **151.** (C) **152.** (A) **153.** (D) **154.** (C) **155.** (B) **156.** (B) **157.** (C) **158.** (D)
159. (A) **160.** (B)

HINTS AND SOLUTIONS

Single Option Correct Type

1. We have, $2^{x+2} \cdot 3^{3x/(x-1)} = 9 = 3^2$

$\Rightarrow$ $(x+2)\log 2 + \dfrac{3x}{x-1}\log 3 = 2\log 3$

$\Rightarrow$ $(x+2)\log 2 + \left(\dfrac{3x}{x-1} - 2\right)\log 3 = 0$

$\Rightarrow$ $(x+2)\left(\log 2 + \dfrac{1}{x-1}\log 3\right) = 0$

$\Rightarrow$ $x = -2$ or $x = 1 - \dfrac{\log 3}{\log 2}$.

The correct option is (C)

2. Since a, b, c are all +ve

$\therefore$ $ax^2 + b|x| + c > 0$ for all real x

$\therefore$ $ax^2 + b|x| + c \neq 0$ for any real x

$\therefore$ no real solution is possible.

The correct option is (A)

3. Since $x^2 - x + 1 = 0$

$\therefore$ $(x-1)(x^2 - x + 1) = 0$

$\Rightarrow$ $x^3 - 1 = 0$

$\Rightarrow$ $x^3 = 1$,

$\therefore$ $x^{3n} = 1$

The correct option is (C)

4. The given equation can be written as

$$2^{x+1}\left[x^2 - \frac{1}{4}\right] = 2^{|x-3|}\cdot 4\,[4x^2 - 1]$$

$$= 16\cdot 2^{|x-3|}\left[x^2 - \frac{1}{4}\right]$$

$\Rightarrow$ $2^{x-3} = 2^{|x-3|}$

[$\because x^2 = \dfrac{1}{4}$ does not give negative integral value]

$\Rightarrow$ $x - 3 = \pm(x-3)$

$\Rightarrow$ either $x - 3 = x - 3$ or $x - 3 = -x + 3$

$\Rightarrow$ $2x = 6$ or $x = 3$

$\therefore$ Given equation does not give any negative integral solution.

The correct option is (D)

5. We have $\alpha + \beta = -b$, $\alpha\beta = c$

As $c < 0, b > 0$, we get

$\alpha < 0 < \beta$

Also, $\beta = -b - \alpha < -\alpha = |\alpha|$

Thus, $\alpha < 0 < \beta < |\alpha|$.

The correct option is (B)

6. The discriminant $= 16q^2 - 4(2q^2 - r)$

$= 8q^2 + 4r = 8q^2 + (\alpha^4 + \beta^4) > 0$

$\therefore$ roots are real.

The correct option is (A)

7. We have, $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$

$\Rightarrow$ $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0$

$\Rightarrow$ $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$(a, b, c, d, p \in R)$

$\Rightarrow$ $ap - b = 0, bp - c = 0, cp - d = 0$

$\Rightarrow$ $\dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = p$

$\Rightarrow$ a, b, c, d are in G.P.

The correct option is (B)

8. The required a satisfies the inequality

$2a^2 - 2(2a+1)a + a(a+1) < 0$

$\Rightarrow$ $a(a+1) > 0 \Rightarrow a \in (-\infty, -1) \cup (0, \infty)$

The correct option is (D)

9. $(a+c)^2 + 4b^2 - 4b(a+c) \leq 0$

$\Rightarrow$ $(a - 2b + c)^2 \leq 0$

$\Rightarrow$ $a - 2b + c = 0$

$\Rightarrow$ $2b = a + c$

$\Rightarrow$ a, b, c are in A.P.

The correct option is (B)

10. Let α, β be the roots of equation $x^2 + bx + c = 0$ and α', β' be the roots of the $x^2 + qx + r = 0$. Then,

$\alpha + \beta = -b;\ \alpha\beta = c,\ \alpha' + \beta' = -q,\ \alpha'\beta' = r$.

It is given that $\dfrac{\alpha}{\beta} = \dfrac{\alpha'}{\beta'} \Rightarrow \dfrac{\alpha+\beta}{\alpha-\beta} = \dfrac{\alpha'+\beta'}{\alpha'-\beta'}$

$\Rightarrow$ $\dfrac{(\alpha+\beta)^2}{(\alpha-\beta)^2} = \dfrac{(\alpha'+\beta')^2}{(\alpha'-\beta')^2}$

$\Rightarrow$ $\dfrac{b^2}{b^2 - 4c} = \dfrac{q^2}{q^2 - 4r} \Rightarrow b^2 r = q^2 c$.

The correct option is (C)

11 Let $16^{\sin^2 x} = y$, then $16^{\cos^2 x} = 16^{1-\sin^2 x} = \dfrac{16}{y}$

The given equation becomes

$y + \dfrac{16}{y} = 10 \Rightarrow y^2 - 10y + 16 = 0$ or $y = 2, 8$

Now, $16^{\sin^2 x} = 2 \Rightarrow 2^{4\sin^2 x} = 2^{(1)}$

$\Rightarrow$ $4\sin^2 x = 1$

$\therefore\quad \sin x = \pm\frac{1}{2} \Rightarrow x = \frac{\pi}{6} \qquad \left(\because 0 \le x < \frac{\pi}{2}\right)$

and $16^{\sin^2 x} = 8$

$\Rightarrow\quad 2^{4\sin^2 x} = 2^3$

$\Rightarrow\quad \sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} \qquad \left(\because\ 0 \le x < \frac{\pi}{2}\right)$

The correct option is (A)

12. The given condition is fulfilled if and only if

$f(2) = 4 - 2(p+1) + p^2 + p - 8 < 0$

$\Rightarrow\quad (p-3)(p+2) < 0 \Rightarrow -2 < p < 3$

The correct option is (B)

13. $(x^3 + 1) + 2x(x+1) = 0$

or $\quad (x+1)[x^2 + x + 1] = 0 \Rightarrow x = -1, \omega, \omega^2.$

Of these $x = \omega, \omega^2$ satisfy the equation

$1 + x^{130} + x^{1988} = 0$

The correct option is (A), (B) and (C)

14. We have, $|x^2 - 3x + 2| + |x - 1| = x - 3$

$\Rightarrow\quad x \ge 3 \Rightarrow x^2 - 3x + 2 + x - 1 = x - 3$

$\Rightarrow\quad x^2 - 3x + 4 = 0$

$\Rightarrow\quad \left(x - \frac{3}{2}\right)^2 + 4 - \frac{9}{4} = 0$

$\Rightarrow$ No solution.

The correct option is (A)

15. We have,

$\sqrt{3x^2 + 6x + 7} = \sqrt{3(x+1)^2 + 4} \ge 2$

and $\sqrt{5x^2 + 10x + 14} = \sqrt{5(x+1)^2 + 9} \ge 3$

$\therefore$ L.H.S. ≥ 5

R.H.S. $= 4 - 2x - x^2 = 5 - (x+1)^2 \le 5$

$\therefore$ the equation holds only when

L.H.S. = R.H.S. = 5

$\therefore\quad x = -1.$

The correct option is (A)

16. Equating the coefficients of similar powers of x, we get

$$\left.\begin{aligned} a^2 - 1 = 0 &\Rightarrow a = \pm 1 \\ a - 1 = 0 &\Rightarrow a = 1 \\ a^2 - 4a + 3 = 0 &\Rightarrow a = 1, 3 \end{aligned}\right\}$$

$\therefore$ common value of $a = 1$.

The correct option is (A)

17. The given equation can be written as

$3x^2 - 2x(a + b + c) + bc + ca + ab = 0$

Discriminant $= 4(a + b + c)^2 - 12(bc + ca + ab)$

$= 4(a^2 + b^2 + c^2 - bc - ca - ab)$

$= 2[(b-c)^2 + (c-a)^2 + (a-b)^2] \ge 0$

Hence, the roots are real.

The correct option is (C)

18. Since α, β are roots of the equation $x^2 + px + q = 0$

$\therefore\quad \alpha + \beta = -p$ and $\alpha\beta = q$

Now $q\left(\frac{\alpha}{\beta}\right)^2 + (2q - p^2)\frac{\alpha}{\beta} + q$

$= \frac{1}{\beta^2}[q(\alpha+\beta)^2 - p^2 ab] = \frac{1}{\beta^2}(qp^2 - p^2 q) = 0$

Thus, $\frac{\alpha}{\beta}$ is a root of the equation

$qx^2 + (2q - p^2)x + q = 0$

The correct option is (C)

19. Since $f(x)$ has no real roots, $f(x)$ has same sign for every x

$\therefore\quad f(0) \cdot f(1) > 0$

The correct option is (B)

20. Let x_1 and x_2 be two roots of $ax^2 + bx + c = 0$

$1 < x_1 < 2$ and $1 < x_2 < 2$

Now $a(5a + 2b + c) = a^2\left(5 + 2\frac{b}{a} + \frac{c}{a}\right)$

$= a^2(5 + 2(-1)(x_1 + x_2) + x_1x_2)$

$= a^2[(x_1 - 2)(x_2 - 2) + 1] > 0$

Hence a and $5a + 2b + c$ are of same sign

The correct option is (A)

21. Since $a < 0$, in case of positive root of the equation $x > a$

$\therefore\quad$ The equation is $x^2 - 2a(x - a) - 3a^2 = 0$

$\Rightarrow\quad x^2 - 2ax - a^2 = 0$

Thus, the roots are $\frac{2a \pm \sqrt{4a^2 + 4a^2}}{2} = \frac{2a \pm 2\sqrt{2}a}{2}$

$= a(1 + \sqrt{2})$ or $a(1 - \sqrt{2})$

$\therefore\quad$ the only positive root possible is $a(1 - \sqrt{2})$.

The correct option is (C)

22. Let $\alpha + i\beta, \alpha - i\beta$ be the roots

Then, $\alpha^2 + \beta^2 = \frac{r}{p} > 0$

$\therefore\quad p, r$ must be of the same sign.

Since $p + r > 0$

$\therefore\quad p, r$ are both positive.

If $\quad q < r, p - q + r > 0$

If $\quad q > 0, (p + r)^2 - (p - r)^2 = 4pr \ge q^2$

($\because$ roots are non-real)

$\therefore\quad (p + r)^2 \le q^2 + (p - r)^2 \ge q^2$

$\therefore\quad p + r \ge q$

The correct option is (B)

23. Let $f(x) = lx^2 - mx + 5$

Since $lx^2 - mx + 5 = 0$ does not have two distinct real roots, therefore either $f(x) \ge 0\ \forall\ x \in R$, or $f(x) \le 0\ \forall\ x \in R$

But $f(0) = 5 > 0$

$\therefore\quad f(x) \ge 0\ \forall\ x \in R$

$\therefore\quad f(-5) \ge 0 \Rightarrow 25l + 5m + 5 \ge 0 \Rightarrow 5l + m \ge -1$

Hence, minimum value of $5l + m$ is -1.

The correct option is (D)

24. Let α, β be the roots of the given equation,

then $\alpha + \beta = \lambda^2 - 5\lambda + 5 < 1$

$\Rightarrow \lambda^2 - 5\lambda + 4 < 0$

$\alpha\beta = 2\lambda^2 - 3\lambda - 4 < 1$

$\Rightarrow 2\lambda^2 - 3\lambda - 5 < 0$

$\therefore (\lambda - 4)(\lambda - 1) < 0$ or $1 < \lambda < 4$

and $(2\lambda - 5)(\lambda + 1) < 0$ or $-1 < \lambda < \frac{5}{2}$

$\therefore$ Required set $= \left(-1, \frac{5}{2}\right) \cap (1, 4) = \left(1, \frac{5}{2}\right)$.

The correct option is (D)

25. Since coefficient of $x^2 > 0$ and 1 lies between the roots of $3x^2 - 3\sin\theta - 2\cos^2\theta = 0$

$\therefore f(1) < 0$

$\Rightarrow 3 - 3\sin\theta - 2\cos^2\theta < 0$

$\Rightarrow 1 + 2(1 - \cos^2\theta) - 3\sin\theta < 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta + 1 < 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta - 1) < 0$

$\Rightarrow \frac{1}{2} < \sin\theta < 1$

TRICK(S) FOR PROBLEM SOLVING

If one root is less than k and other is greatter than k, then $D > 0$ and $af(k) < 0$, where

$f(x) = ax^2 + bx + c$, $a, b, c \in R$, $a \neq 0$

The correct option is (C)

26.
$$\sum_{r=1}^{n} S_r = (\alpha + \beta) + (\alpha^2 + \beta^2) + \ldots + (\alpha^n + \beta^n)$$
$$= (\alpha + \alpha^2 + \ldots + \alpha^n) + (\beta + \beta^2 + \ldots + \beta^n)$$
$$\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} S_r = (\alpha + \alpha^2 + \ldots + \infty) + (\beta + \beta^2 + \ldots + \infty)$$
$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta}$$
$$= \frac{\alpha - \alpha\beta + \beta - \alpha\beta}{1 - (\alpha+\beta) + \alpha\beta}$$
$$= \frac{\alpha + \beta - 2\alpha\beta}{1 - (\alpha+\beta) + \alpha\beta}$$
$$= \frac{\frac{25}{375} + \frac{4}{375}}{1 - \frac{25}{375} - \frac{2}{375}} = \frac{29}{348} = \frac{1}{12}$$

The correct option is (B)

27. If $x = n \in Z$, $(x)^2 + (x + 1)^2 = 25$

$\Rightarrow n^2 + (n + 1)^2 = 25$

$\Rightarrow 2n^2 + 2n - 24 = 0$

$\Rightarrow n^2 + n - 12 = 0$

$\Rightarrow n = 3, -4$

$\therefore x = 3, -4$

If $x = n + k$, $n \in Z$, $0 < k < 1$, then

$(x)^2 + (x + 1)^2 = 25$

$\Rightarrow (n + 1)^2 + (n + 2)^2 = 25$

$\Rightarrow 2n^2 + 6n - 20 = 0$

$\Rightarrow n^2 + 3n - 10 = 0$

$\Rightarrow n = 2, -5$

$\therefore x = 2 + k, -5 + k$, where $0 < k < 1$

$\therefore x > 2, x > -5$

$\therefore$ Solution set is $(-5, -4] \cup (2, 3]$

The correct option is (B)

28. $\log_2(9 - 2^x) = 3 - x$

$\Rightarrow 9 - 2^x = 2^{3-x}$

$\Rightarrow 9 - 2^x = \frac{8}{2^x}$

$\Rightarrow 9 \cdot 2^x - 2^{2x} = 8$

$\Rightarrow 2^{2x} - 9 \cdot 2^x + 8 = 0$

$\Rightarrow (2^x - 8)(2^x - 1) = 0$

$\Rightarrow x = 3$ or $x = 0$

But $x = 3$ is not the solution of original equation,

$\therefore x = 0.$

The correct option is (A)

29. Since $ax^2 + bx + 6 = 0$ does not have two distinct real roots.

$\therefore b^2 - 24a \leq 0$

Let $3a + b = y$

$\therefore 3a = y - b$

$\therefore b^2 - 8(y - b) \leq 0$

i.e., $b^2 + 8b - 8y \leq 0$

Since b is real

$\therefore 64 + 32y \geq 0 \Rightarrow y \geq -2$

$\therefore$ Min. value of y i.e., $3a + b = -2$.

The correct option is (D)

30. Here $\alpha + \beta = -p \Rightarrow \alpha + \beta = \gamma + \delta$

$\gamma + \delta = -p$

Now $(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta = \alpha^2 - \alpha(\alpha + \beta) + r$

$= -\alpha\beta + r = -(-q) + r = q + r$

By symmetry $(\beta - \gamma)(\beta - \delta) = q + r$

$\therefore$ Ratio is 1.

The correct option is (A)

31. $\frac{x+2}{x^2+1} - \frac{1}{2} > 0$

$\Rightarrow \frac{-x^2 - 1 + 2x + 4}{2(x^2+1)} > 0$

$\Rightarrow \frac{3 + 2x - x^2}{2(x^2+1)} > 0$

Since denominator is positive

$\therefore 3 + 2x - x^2 > 0$

$\Rightarrow -1 < x < 3$

$\Rightarrow x = 0, 1, 2$

The correct option is (D)

32. Let α, β be the roots of $\lambda x^2 + \mu x + v = 0$

$\therefore \quad \alpha + \beta = -\dfrac{\mu}{\lambda}, \alpha\beta = \dfrac{v}{\lambda}$

$\therefore \quad \dfrac{(\alpha+\beta)^2}{\alpha\beta} = \dfrac{\frac{\mu^2}{\lambda^2}}{\frac{v}{\lambda}} = \dfrac{\mu^2}{\lambda v}$

$\Rightarrow \quad \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} + 2 = \dfrac{\mu^2}{\lambda v}$ (1)

Let γ, δ be the roots of $x^2 + x + 1 = 0$

$\therefore \quad \gamma + \delta = -1, \gamma\delta = 1$

$\therefore \quad \dfrac{(\gamma+\delta)^2}{\gamma\delta} = 1 \Rightarrow \dfrac{\gamma}{\delta} + \dfrac{\delta}{\gamma} + 2 = 1$ (2)

Since $\dfrac{\alpha}{\beta} = \dfrac{\gamma}{\delta}$,

$\therefore$ from Eq. (1) and (2)

$\dfrac{\mu^2}{\lambda v} = 1 \Rightarrow \mu^2 = \lambda v$

$\therefore \lambda, \mu, v$ are in G.P.

The correct option is (B)

33. Given equation can be written as

$bx^2 + x - bcx - bdx + bcd - a = 0$

$\Rightarrow \quad bx(x - c) - bd(x - c) + x - a = 0$

$\Rightarrow \quad b(x - c)(x - d) + (x - a) = 0$

Let $f(x) = b(x - c)(x - d) + (x - a)$

$f(c) = c - a < 0; f(d) = d - a > 0$

Hence, one root of the given equation lies between c and d.

The correct option is (C)

34. We have, $\dfrac{\alpha}{\alpha-1} + \dfrac{\alpha+1}{\alpha} = -\dfrac{b}{a}$ and $\dfrac{\alpha}{\alpha-1} \cdot \dfrac{\alpha+1}{\alpha} = \dfrac{c}{a}$

$\Rightarrow \quad \dfrac{2\alpha^2 - 1}{\alpha^2 - \alpha} = -\dfrac{b}{a}$ and $\alpha = \dfrac{c+a}{c-a}$

$\Rightarrow \quad (c + a)^2 + 4ac = -2b(c + a)$

$\Rightarrow \quad (c + a)^2 + 2b(c + a) + b^2 = b^2 - 4ac$

$\Rightarrow \quad (a + b + c)^2 = b^2 - 4ac$

The correct option is (B)

35. Here $\alpha + \beta = p; a\beta = 1 \Rightarrow a\beta = \gamma\delta$

$\gamma + \delta = q; \gamma\delta = 1$

Now, $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$= [a\beta - \gamma(\alpha + \beta) + \gamma^2][\alpha\beta + \delta(\alpha + \beta) + \delta^2]$

$= [1 + \gamma p + \gamma^2][1 - p\delta + \delta^2]$

$= [(\gamma^2 + 1) + \gamma p][(\delta^2 + 1) - p\delta]$

$= (-q\gamma + \gamma p)(-q\delta - p\delta)$

$= \gamma\delta(q^2 - p^2) = 1(q^2 - p^2)$

The correct option is (C)

36. Let $y = [x]$

$\therefore$ The given equation $y^2 + ay + b = 0$ must have integral roots which is not possible as a and b are odd integers.

$\therefore$ Discriminant can't be perfect square.

The correct option is (C)

37. $\dfrac{\log_e(x^2 - 5x + 6)}{\log_e 9} > \dfrac{\log_e(x-4)}{\log_e 3}$

$\Rightarrow \quad x^2 - 5x + 6 > x^2 - 8x + 16$

$\Rightarrow \quad 3x - 10 > 0 \Rightarrow x > \dfrac{10}{3}$ (1)

Also, $x^2 - 5x + 6 > 0 \Rightarrow x > 3$ or $x < 2$ (2)

and $\quad x - 4 > 0 \Rightarrow x > 4$ (3)

Common solution from Eqs (1), (2) and (3) is $x > 4$

The correct option is (B)

38. Let $f(x) = a^2x^2 + 2bx + 2c = 0$

Given: $a^2\alpha^2 + b\alpha + c = 0$

and $a^2\beta^2 - b\beta - c = 0$

Now, $\quad f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c$

$= (a^2\alpha^2 + b\alpha + c) + (b\alpha + c)$

$= b\alpha + c = -a^2\alpha^2$

$f(\beta) = a^2\beta^2 + 2b\beta + 2c \}$

$= 3b\beta + 3c = 3(b\beta + c)$

$= 3a^2\beta^2$

Since $0 < \alpha < \beta$

$\therefore \quad \alpha, \beta$ are real

$\therefore \quad f(\alpha) < 0, f(\beta) > 0$

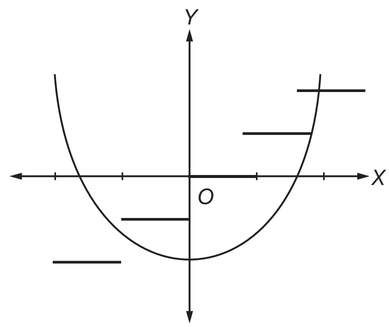
$\therefore \quad f(\gamma) = 0$ where $\alpha < \gamma < \beta$

TRICK(S) FOR PROBLEM SOLVING

Let $f(x) = 0$ be a polynomial equation. Let p and q be two real numbers, $p < q$. If $f(p) \cdot f(q) < 0$, then the equation $f(x) = 0$ has atleast one real roots between p and q.

The correct option is (D)

39. Let us see the graph of $y = x^2 - 2$ and $y = [x]$



If $[x] = -1$

We have $x^2 - 2 + 2 = 0 \Rightarrow x = 0$ not possible

$[x] = 0 \Rightarrow x = \pm\sqrt{2}$ not possible

$[x] = 1 \Rightarrow x = \pm\sqrt{4} = \pm 2$ not possible

$[x] = 2 \Rightarrow x = \pm\sqrt{6}$

$\Rightarrow \; x = \sqrt{6}$ is the only solution.

The correct option is (A)

40. Let $2^{\sin^2 x} = Z \Rightarrow 2^{\cos^2 x} = \dfrac{2}{Z}$

Therefore, the given equation becomes

$Z - \dfrac{2}{Z} - 1 = 0 \Rightarrow Z = 2$ or $Z = -1$

$\Rightarrow \; 2^{\sin^2 x} = 2$ or $2^{\sin^2 x} = -1$ (not possible)

$\Rightarrow \; \sin^2 x = 1 \Rightarrow x = (2n+1)\dfrac{\pi}{2}, n \in 1$

The correct option is (C)

41. Given: $|\alpha - \beta| > \sqrt{3p}$

If α, β are the roots of $x^2 + px + 1 = 0$, then

$\alpha + \beta = -p, \alpha\beta = 1$

$\therefore \; (\alpha - \beta)^2 > 3p$

$\Rightarrow \; (\alpha + \beta)^2 - 4\alpha\beta > 3p$

$\Rightarrow \; (-p)^2 - 4 \cdot 1 > 3p$

$\Rightarrow \; p^2 - 3p - 4 > 0$

$\Rightarrow \; (p-4)(p+1) > 0$

$\Rightarrow \; p > 4, p > -1$ or $p < 4, p < -1$

$\Rightarrow \; p > 4$ or $p < -1$

But p is not –ve (∵ If p is –ve, then $\sqrt{3p}$ is not real)

$\Rightarrow \; p > 4$

The correct option is (B)

42. If $f(x) = x^2 + ax + b$

$f(x+c) = x^2 + (2c+a)x + c^2 + ac + b$

$\therefore$ Roots of the given equation are 0 and $d - c$.

(since roots of $x^2 + ax + b = 0$ are c and d.)

The correct option is (B)

43. Let $f(x) = x^2 + 2(k+1)x + 9k - 5$. Let α, β be the roots of $f(x) = 0$. The equation $f(x) = 0$ will have both negative roots if and only if

(i) Disc. ≥ 0

(ii) $\alpha + \beta < 0$ and

(iii) $f(0) > 0$

Now, discriminant ≥ 0

$\Rightarrow \; 4(k+1)^2 - 36k + 20 \geq 0 \;\Rightarrow\; k^2 - 7k + 6 \geq 0$

$\Rightarrow \; (k-1)(k-6) \geq 0$

$\Rightarrow \; k \leq 1$ or $k \geq 6$ (1)

$(\alpha + \beta) < 0$

$\Rightarrow \; -2(k+1) < 0$

$\Rightarrow \; k + 1 > 0 \Rightarrow k > -1$ (2)

and $\alpha\beta > 0$

$\Rightarrow \; 9k - 5 > 0$

$\Rightarrow \; k > \dfrac{5}{9}$ (3)

From Eqs (1), (2) and (3) we get $k \geq 6$.

The correct option is (C)

44. Product of roots $= 2e^{2 \ln k} - 1 = 7$ (given)

$\Rightarrow \; 2e^{\ln k^2} - 1 = 7$

$\Rightarrow \; 2k^2 - 1 = 7$

$\Rightarrow \; k = \pm 2$

$\Rightarrow \; k = 2$ (Since $\ln k$ is defined for $k > 0$)

The correct option is (B)

45. $x - x^2 - 9 = -(x^2 - x + 9)$

$$= -\left[\left(x - \frac{1}{2}\right)^2 + \frac{35}{4}\right] < 0 \text{ for all } x \in R$$

$\therefore$ no. solution i.e., solution set $= \phi$

$$\left[\because \left(\frac{3}{5}\right)^x > 0 \text{ for all } x \in R\right]$$

The correct option is (C)

46. Let $e^{\sin x} = t$

$\Rightarrow \; t^2 - 4t - 1 = 0$

$\Rightarrow \; t = \dfrac{4 \pm \sqrt{16+4}}{2}$

$\Rightarrow \; t = e^{\sin x} = 2 \pm \sqrt{5}$

$\Rightarrow \; e^{\sin x} = 2 - \sqrt{5}, e^{\sin x} = 2 + \sqrt{5}$

$e^{\sin x} = 2 - \sqrt{5} < 0,$

$\Rightarrow \; \sin x = \ln(2 + \sqrt{5}) > 1$

So both rejected.

Hence no solution

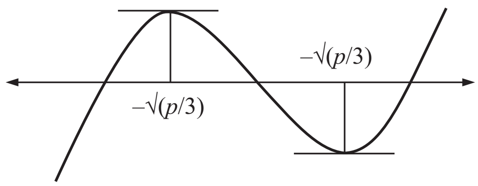
The correct option is (B)

47. Let $f(x) = x^3 - px + q$

Now for maxima/minima

$f'(x) = 0$

$\Rightarrow \; 3x^2 - p = 0$



$\Rightarrow \; x^2 = \dfrac{p}{3}$

$\therefore \; x = \pm\sqrt{\dfrac{p}{3}}$

The correct option is (A)

48. Let α and 4β be roots of $x^2 - 6x + a = 0$ and $\alpha, 3\beta$ be the roots of $x^2 - cx + 6 = 0$, then

$\alpha + 4\beta = 6$ and $4\alpha\beta = a$

$\alpha + 3\beta = c$ and $3\alpha\beta = 6$.

We get $\alpha\beta = 2 \Rightarrow a = 8$

So the first equation is $x^2 - 6x + 8 = 0 \Rightarrow x = 2, 4$

If $\alpha = 2$ and $4\beta = 4$ then $3\beta = 3$

If $\alpha = 4$ and $4\beta = 2$, then $3\beta = 3/2$ (non-integer)

$\therefore$ common root is $x = 2$.

The correct option is (D)

49. $bx^2 + cx + a = 0$

Roots are imaginary $\Rightarrow c^2 - 4ab < 0$

$\Rightarrow c^2 < 4ab \Rightarrow -c^2 > -4ab$

Given expression has minimum value

$= \dfrac{4(3b^2)(2c^2) - 36b^2c}{4(3b^2)}$

$= -\dfrac{12b^2c^2}{12b^2} = -c^2 > -4ab.$

The correct option is (C)

50. Put $\sqrt{x-1} = t \Rightarrow x - 1 = t^2$

or $x = t^2 + 1$, the given equation reduces to

$\sqrt{t^2+1+3-4t} + \sqrt{t^2+1+8-6t}$

$= 1$ where, $t \geq 0$.

$\Rightarrow |t-2| + |t-3| = 1$, where $t \geq 0$. This equation will be satisfied if $2 \leq t \leq 3$.

Therefore, $2 \leq \sqrt{x-1} \leq 3$ or $5 \leq x \leq 10$.

$\therefore$ The given equation is satisfied for all values of x lying in [5, 10].

The correct option is (D)

51. We have, $3^{\sec^2 x - 1}\sqrt{9y^2 - 6y + 2} \leq 1$

$\Rightarrow 3^{\sec^2 x}\sqrt{y^2 - \dfrac{2}{3}y + \dfrac{2}{9}} \leq 1$

$\Rightarrow 3^{\sec^2 x}\sqrt{\left(y - \dfrac{1}{3}\right)^2 + \dfrac{1}{9}} \leq 1$

Now, $\sec^2 x \geq 1 \Rightarrow 3^{\sec 2^x} \geq 3$ and $\sqrt{\left(y - \dfrac{1}{3}\right)^2 + \dfrac{1}{9}} \geq \dfrac{1}{3}$, so we must have $\sec^2 x = 1$ and $y - \dfrac{1}{3} = 0$.

$\Rightarrow x = 0, \pi, 2\pi, 3\pi$ and $y = \dfrac{1}{3}$.

$\therefore$ There are 4 solutions.

The correct option is (B)

52. Let

$f(x) = (x-n)^m + (x-n^2)^m + (x-n^3)^m + \ldots + (x-n^m)^m.$

$\Rightarrow f'(x) = m(x-n)^{m-1} + m(x-n^2)^{m-1} + \ldots + m(x-n^m)^{m-1}$

Since m is odd, $(m-1)$ is even. Therefore, $f'(x) = 0$ has no real root.

$\Rightarrow f(x) = 0$ has one real and $(m-1)$ imaginary roots.

The correct option is (C)

53. We have, $f(x) + f\left(\dfrac{1}{x}\right) = 1$

$\Rightarrow x - [x] + \dfrac{1}{x} - \left[\dfrac{1}{x}\right] = 1$

$\Rightarrow x + \dfrac{1}{x} - 1 = [x] + \left[\dfrac{1}{x}\right]$

$\Rightarrow \dfrac{x^2+1-x}{x}$ = (integer) k (say)

$\Rightarrow x^2 - (k+1)x + 1 = 0$

Since x is real, so $(k+1)^2 - 4 \geq 0$

$\Rightarrow k^2 + 2k - 3 \geq 0 \Rightarrow (k+3)(k-1) \geq 0$

$\Rightarrow k \leq -3$ or $k \geq 1$.

Therefore, number of solutions is infinite.

The correct option is (C)

54. Since the given equation has imaginary roots

$\Rightarrow D < 0$ or $(a+b+c)^2 - 4(ab+bc+ca) < 0$

$\Rightarrow (a^2 + b^2 + c^2 - 2ab - 2bc + 2ac) < 4ac$

$\Rightarrow (a - b + c)^2 < 4ac$

$\Rightarrow -2\sqrt{ac} < a - b + c$

$\Rightarrow \left(a + c + 2\sqrt{ac}\right) > b$

$\Rightarrow \left(\sqrt{a} + \sqrt{c}\right)^2 > b$ or $\sqrt{a} + \sqrt{c} > \sqrt{b}.$

Similarly, $\sqrt{b} + \sqrt{c} > \sqrt{a}$ and $\sqrt{a} + \sqrt{b} > \sqrt{c}.$

Therefore, $\sqrt{a}$, $\sqrt{b}$, $\sqrt{c}$ can be the sides of a triangle.

The correct option is (A)

55. We have,

$x^n + ax + b = (x - x_1)(x - x_2) \ldots (x - x_n)$

$\Rightarrow (x - x_2)(x - x_3) \ldots (x - x_n) = \dfrac{x^n + ax + b}{x - x_1}$

$\Rightarrow (x_1 - x_2)(x_1 - x_3) \ldots (x_1 - x_n) = \lim\limits_{x \to x_1} \dfrac{x^n + ax + b}{x - x_1}$

$= nx_1^{n-1} + a.$

The correct option is (A)

56. Since the roots of the given equation are real

$\therefore B^2 - 4AC \geq 0 \Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$

$\Rightarrow -a + 3 \geq 0$ or $a \leq 3$. (1)

Since the root is less than 3, so $f(3) > 0$

$\Rightarrow 3^2 - 2a(3) + a^2 + a - 3 > 0$

$\Rightarrow a^2 - 5a + 6 > 0$ or $(a-2)(a-3) > 0$

$\Rightarrow a < 2$ or $a > 3$ (2)

From (1) and (2), we have $a < 2$.

The correct option is (A)

57. Let $z = \dfrac{1 - x + x^2}{1 + x + x^2}$

$\Rightarrow z + zx + zx^2 = 1 - x + x^2$

$\Rightarrow zx^2 - x^2 + zx + z + z - 1 = 0$

$\Rightarrow x^2(z-1) + x(z+1) + (z-1) = 0$

For real x, $B^2 - 4AC \geq 0$

$\Rightarrow (z+1)^2 - 4(z-1)(z-1) \geq 0$

$\Rightarrow z^2 + 2z + 1 - 4z^2 + 8z - 4 \geq 0$

$\Rightarrow -3z^2 + 10z - 3 \geq 0 \Rightarrow -3z^2 + 9z + z - 3 - \geq 0$

$\Rightarrow -3z(z-3) + 1(z-3) \geq 0$

$\Rightarrow \ (z-3)(-3z+1) \geq 0 \Rightarrow \frac{1}{3} \leq z \leq 3.$

$\therefore$ minimum value of $z = \frac{1}{3}$.

The correct option is (B)

58. Given, $\frac{1}{3} < \frac{x^2-2x+4}{x^2+2x+4} < 3$ for all $x \in R$.

$\Rightarrow \ \frac{1}{3} < \frac{x^2+2x+4}{x^2-2x+4} < 3$ for all $x \in R$. (1)

Let $3^{x+1} = y$

Then, $y \in R$ for all $x \in R$.

$\therefore \ \frac{9 \cdot 3^{2x} + 6 \cdot 3^x + 4}{9 \cdot 3^{2x} - 6 \cdot 3^x + 4} = \frac{3^{2x+2} + 2 \cdot 3^{x+1} + 4}{3^{2x+2} - 2 \cdot 3^{x+1} + 4}$

$= \frac{y^2+2y+4}{y^2-2y+4}$

From (1), $\frac{1}{3} < \frac{y^2+2y+4}{y^2-2y+4} < 3$

$\therefore \ \frac{1}{3} < \frac{9 \cdot 3^{2x} + 6 \cdot 3^x + 4}{9 \cdot 3^{2x} - 6 \cdot 3^x + 4} < 3.$

The correct option is (D)

59. Let $f(x) = x^2 + (1-2k)x + k^2 - k - 2$

The number 3 lies between the roots of the given equation, if $f(3) < 0$

Now, $f(3) = 9 + (1-2k)3 + k^2 - k - 2$

$= 10 - 7k + k^2 = k^2 - 7k + 10$

Hence, $f(3) < 0 \Rightarrow k^2 - 7k + 10 < 0$

$\Rightarrow \ (k-2)(k-5) < 0 \Rightarrow 2 < k < 5.$

The correct option is (A)

60. The given equation can be written as

$2^{x+1}\left[x^2 - \frac{1}{4}\right] = 2^{|x-3|}[4x^2-1]$

$= 4 \cdot 2^{|x-3|}\left[x^2 - \frac{1}{4}\right]$

$\Rightarrow \ 2^{x-1} = 2^{|x-3|}$

[$\because x^2 = \frac{1}{4}$ does not give negative integral value]

$\Rightarrow \ x - 1 = \pm(x-3)$

$\Rightarrow$ either $x - 1 = x - 3$ or $x - 1 = -x + 3$

$\Rightarrow$ 1 = 3 (not possible) or $2x = 4$

i.e., $x = 2$.

$\therefore$ Given equation does not give any negative integral solution.

The correct option is (D)

61. We have $\alpha + \beta = -b$, $a\beta = c$

As $c < 0$, $b > 0$, we get $\alpha < 0 < \beta$.

Also, $\beta = -b - \alpha < -\alpha = |\alpha|$

Thus, $\alpha < 0 < \beta < |\alpha|$.

The correct option is (B)

62. Let α, β be the roots of equation $x^2 + bx + c = 0$ and α', β' be the roots of the $x^2 + qx + r = 0$. Then,

$\alpha + \beta = -b$; $a\beta = c$, $\alpha' + \beta' = -q$, $\alpha'\beta' = r$.

It is given that $\frac{\alpha}{\beta} = \frac{\alpha'}{\beta'} \Rightarrow \frac{\alpha+\beta}{\alpha-\beta} = \frac{\alpha'+\beta'}{\alpha'-\beta'}$

$\Rightarrow \ \frac{(\alpha+\beta)^2}{(\alpha-\beta)^2} = \frac{(\alpha'+\beta')^2}{(\alpha'-\beta')^2}$

$\Rightarrow \ \frac{b^2}{b^2-4c} = \frac{q^2}{q^2-4r} \Rightarrow b^2 r = q^2 c.$

The correct option is (C)

63. If $x = n \in I$, $|n - 2n| = 4$

$\therefore \ n = \pm 4,$

If $x = n + k$, $n \in I$, $0 < k < 1$ then

$|n - 2(n+k)| = 4$

$\therefore \ |-n - 2k| = 4.$

It is possible if $k = \frac{1}{2}$.

The correct option is (B)

64. Since α, β are roots of the equation $x^2 + px + q = 0$

$\therefore \ \alpha + \beta = -p$ and $a\beta = q$.

Now, $q\left(\frac{\alpha}{\beta}\right)^2 + (2q - p^2)\frac{\alpha}{\beta} + q$

$= \frac{1}{\beta^2}[q(\alpha+\beta)^2 - p^2\alpha\beta] = \frac{1}{\beta^2}[qp^2 - p^2 q] = 0.$

Thus, $\frac{\alpha}{\beta}$ is a root of the equation

$qx^2 + (2q - p^2)x + q = 0.$

The correct option is (C)

65. Let x_1 and x_2 be two roots of $ax^2 + bx + c = 0$

$1 < x_1 < 2$ and $1 < x_2 < 2$

Now, $a(5a + 2b + c) = a^2\left(5 + 2\frac{b}{a} + \frac{c}{a}\right)$

$= a^2(5 + 2(-1)(x_1 + x_2) + x_1 x_2)$

$= a^2\{(x_1 - 2)(x_2 - 2) + 1\} > 0$

Hence, a and $5a + 2b + c$ are of same sign.

The correct option is (A)

66. Since $a < 0$, in case of positive root of the equation $x > a$

$\therefore$ The equation is $x^2 - 2a(x - a) - 3a^2 = 0$

$\Rightarrow \ x^2 - 2ax - a^2 = 0$

Thus, the roots are $\frac{2a \pm \sqrt{4a^2 + 4a^2}}{2} = \frac{2a \pm 2\sqrt{2}a}{2}$

$= a(1+\sqrt{2})$ or $a(1-\sqrt{2})$

$\therefore$ the only positive root possible is $a(1-\sqrt{2})$.

The correct option is (C)

67. Let $\alpha + i\beta$, $\alpha - i\beta$ be the roots

Then, $a^2 + b^2 = \frac{r}{p} > 0$

$\therefore \ p, r$ must be of the same sign.

Since $p + r > 0$ $\therefore$ p, r are both positive.

If $q < 0$, $p - q + r > 0$

If $q > 0$, $(p + r)^2 - (p - r)^2 = 4pr \geq q^2$

[$\because$ roots are non-real]

$\therefore$ $(p + r)^2 \geq q^2 + (p - r)^2 \geq q^2$

$\therefore$ $p + r \geq q$

The correct option is (B)

68. Let $f(x) = lx^2 - mx + 5$

Since $lx^2 - mx + 5 = 0$ does not have two distinct real roots, therefore,

either $f(x) \geq 0\ \forall\ x \in R$, or $f(x) \leq 0\ \forall\ x \in R$

But $f(0) = 5 > 0$

$\therefore$ $f(x) \geq 0\ \forall\ x \in R$

$\therefore$ $f(-5) \geq 0 \Rightarrow 25l + 5m + 5 \geq 0 \Rightarrow 5l + m \geq -1$

Hence, minimum of $5l + m$ is -1.

The correct option is (D)

69. Since coefficient of $x^2 > 0$ and 1 lies between the roots of $3x^2 - 3\sin\theta - 2\cos^2\theta = 0$

$\therefore$ $f(1) < 0$

$\Rightarrow$ $3 - 3\sin\theta - 2\cos^2\theta < 0$

$\Rightarrow$ $1 + 2(1 - \cos^2\theta) - 3\sin\theta < 0$

$\Rightarrow$ $2\sin^2\theta - 3\sin\theta + 1 < 0$

$\Rightarrow$ $(2\sin\theta - 1)(\sin\theta - 1) < 0$

$\Rightarrow$ $\frac{1}{2} < \sin\theta < 1$

The correct option is (C)

70. $\sum_{r=1}^{n} S_r = (\alpha + \beta) + (\alpha^2 + \beta^2) + \ldots + (\alpha^n + \beta^n)$

$= (\alpha + \alpha^2 + \ldots + \alpha^n) + (\beta + \beta^2 + \ldots + \beta^n)$

$$\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} S_r = (\alpha + \alpha^2 + \ldots + \infty) + (\beta + \beta^2 + \ldots + \infty)$$

$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta}$$

$$= \frac{\alpha - \alpha\beta + \beta - \alpha\beta}{1 - (\alpha + \beta) + \alpha\beta}$$

$$= \frac{\alpha + \beta - 2\alpha\beta}{1 - (\alpha + \beta) + \alpha\beta}$$

$$= \frac{\frac{25}{375} + \frac{4}{375}}{1 - \frac{25}{375} - \frac{2}{375}} = \frac{29}{348} = \frac{1}{12}$$

The correct option is (B)

71. Since $ax^2 + bx + 6 = 0$ does not have two distinct real roots

$\therefore$ $b^2 - 24a \leq 0$

Let $3a + b = y$ $\therefore$ $3a = y - b$

$\therefore$ $b^2 - 8(y - b) \leq 0$ i.e., $b^2 + 8b - 8y \leq 0$

Since b is real $\therefore$ $64 + 32y \geq 0 \Rightarrow y \geq -2$

$\therefore$ Min. value of y, i.e., $3a + b = -2$.

The correct option is (D)

72. Let α, β be the roots of $\lambda x^2 + \mu x + v = 0$

$\therefore$ $\alpha + \beta = -\frac{\mu}{\lambda}$, $\alpha\beta = \frac{v}{\lambda}$

$$\therefore \quad \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{\frac{\mu^2}{\lambda^2}}{\frac{v}{\lambda}} = \frac{\mu^2}{\lambda v}$$

$$\Rightarrow \quad \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2 = \frac{\mu^2}{\lambda v} \qquad (1)$$

Let γ, δ be the roots of $x^2 + x + 1 = 0$

$\therefore$ $\gamma + \delta = -1$, $\gamma\delta = 1$

$$\therefore \quad \frac{(\gamma+\delta)^2}{\gamma\delta} = 1 \Rightarrow \frac{\gamma}{\delta} + \frac{\delta}{\gamma} + 2 = 1 \qquad (2)$$

Since $\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$,

$\therefore$ from (1) and (2)

$\frac{\mu^2}{\lambda v} = 1 \Rightarrow \mu^2 = \lambda v$

$\therefore$ λ, μ, v are in G.P.

The correct option is (B)

73. Given equation can be written as

$bx^2 + x - bcx - bdx + bcd - a = 0$;

$\Rightarrow$ $bx(x - c) - bd(x - c) + x - a = 0$

$\Rightarrow$ $b(x - c)(x - d) + (x - a) = 0$

Let $f(x) = b(x - c)(x - d) + (x - a)$

$f(c) = c - a < 0$; $f(d) = d - a > 0$

The correct option is (C)

74. If $f(x) = x^2 + ax + b$

$f(x + c) = x^2 + (2c + a)x + c^2 + ac + b$

$\therefore$ roots of the given equation are 0 and $d - c$.

(since roots of $x^2 + ax + b = 0$ are c and d.)

The correct option is (B)

75. If $x = n \in Z$, $(x)^2 + (x + 1)^2 = 25$

$\Rightarrow$ $n^2 + (n + 1)^2 = 25$

$\Rightarrow$ $2n^2 + 2n - 24 = 0$

$\Rightarrow$ $n^2 + n - 12 = 0$

$\Rightarrow$ $n = 3, -4$

$\therefore$ $x = 3, -4$

If $x = n + k$, $n \in Z$, $0 < k < 1$, then

$(x)^2 + (x + 1)^2 = 25$

$\Rightarrow$ $(n + 1)^2 + (n + 2)^2 = 25$

$\Rightarrow$ $2n^2 + 6n - 20 = 0$

$\Rightarrow$ $n^2 + 3n - 10 = 0$

$\Rightarrow$ $n = 2, -5$

$\therefore$ $x = 2 + k, -5 + k$, where $0 < k < 1$

$\therefore$ $x > 2, x > -5$

$\therefore$ Solution set is $(-5, -4] \cup (2, 3]$

The correct option is (B)

76. The required a satisfies the inequality

$2a^2 - 2(2a + 1)a + a(a + 1) < 0$

$\Rightarrow\ a(a + 1) > 0 \Rightarrow a \in (-\infty, -1) \cup (0, \infty)$

The correct option is (D)

77. We have, $2^x + 2^x \geq 2\sqrt{2}$ $(x \geq 0)$

$\Rightarrow\ 2^x \geq \sqrt{2} \Rightarrow x \geq \dfrac{1}{2}$

and, $2^x + 2^{-x} \geq 2\sqrt{2}$ $(x < 0)$

$\Rightarrow\ t + \dfrac{1}{t} \geq 2\sqrt{2}$ (where $t = 2^x$)

$\Rightarrow\ t^2 - t + 1 \geq 0$

$\Rightarrow (t - (\sqrt{2} - 1))\,(t - (\sqrt{2} + 1)) \geq 0$

$\Rightarrow\ t \leq \sqrt{2} - 1$ or $t \geq \sqrt{2} + 1$ but $t > 0$

$\Rightarrow\ 0 < 2^x \leq \sqrt{2} - 1$ or $2^x \geq \sqrt{2} + 1$

$\Rightarrow\ -\infty < x \leq \log_2(\sqrt{2} - 1)$

or, $x \geq \log_2(\sqrt{2} + 1)$ (but not acceptable as $x < 0$)

$\therefore\ x \in (-\infty, \log_2(\sqrt{2} - 1)] \cup \left[\dfrac{1}{2}, \infty\right)$

The correct option is (D)

78. $\min. f(x) = -\dfrac{D}{4a} = -\dfrac{4b^2 - 8c^2}{4}$

$= -(b^2 - 2c^2)$ (upward parabola)

$\max. g(x) = -\dfrac{D}{4a} = \dfrac{4c^2 + 4b^2}{4}$

$= b^2 + c^2$ (downward parabola)

Now, $2c^2 - b^2 > b^2 + c^2$

$\Rightarrow\ c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}\,|b|$

The correct option is (D)

79. We can write the given equation as

$(x - a)^2 = 3 - a$

This shows that $a \leq 3$ and $x = a \pm \sqrt{3 - a}$

Both the roots of the given equation will be less than 3 if the larger of the two roots is less than 3, that is, if

$a + \sqrt{3 - a} < 3$

$\Rightarrow\ \sqrt{3 - a} - (3 - a) < 0$

$\Rightarrow\ \sqrt{3 - a}\,(1 - \sqrt{3 - a}) < 0$

$\Rightarrow\ a < 3$ and $1 - \sqrt{3 - a} < 0$

But $\sqrt{3 - a} > 1 \Rightarrow 3 - a > 1$ or $a < 2$

Thus, $a < 3$ and $a < 2 \Rightarrow a < 2$

The correct option is (A)

80. $\dfrac{|x + 1|}{|x|} + |x + 1| = \dfrac{|x + 1|^2}{|x|}$

$\Rightarrow\ |x + 1|\left\{\dfrac{1}{|x|} + 1 - \dfrac{|x + 1|}{|x|}\right\} = 0$

$\therefore\ |x + 1| = 0$ or $1 + |x| - |x + 1| = 0$.

$|x + 1| = 0 \Rightarrow x = -1$.

If $x < -1$, $1 + |x| - |x + 1| = 0$

$\Rightarrow\ 1 - x + x + 1 = 0 \Rightarrow 2 = 0$ (absurd)

If $-1 \leq x < 0$, $1 + |x| - |x + 1| = 0$

$\Rightarrow\ 1 - x - (x + 1) = 0 \Rightarrow x = 0$ (not possible)

If $x \geq 0$, $1 + x - (x + 1) = 0 \Rightarrow 0 = 0$

$\Rightarrow$ x can have any value in the interval

$\therefore\ x = -1, x > 0.$ $(\because x \neq 0)$

The correct option is (B)

81. As α, β are roots of $ax^2 + bx + c = 0$, we have

$\alpha + \beta = -b/a,\ \alpha\beta = c/a$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$= \left(-\frac{b}{a}\right)^2 - \frac{4c}{a} = \frac{b^2 - 4ac}{a^2} \quad (1)$$

Now, as $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$,

we have $\alpha + \delta + \beta + \delta = -B/A$, $(\alpha + \delta)(\beta + \delta) = C/A$.

Now, $(\alpha - \beta)^2 = [(\alpha + \delta) - (\beta + \delta)]^2$

$= (\alpha + \delta + \beta + \delta)^2 - 4(\alpha + \delta)(\beta + \delta)$

$$= \frac{B^2}{A^2} - \frac{4C}{A} = \frac{B^2 - 4AC}{A^2} \quad (2)$$

From (1) and (2), we get

$$\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}.$$

The correct option is (A)

82. Given equation is $ax^2 + bx + c = 0$

Since α, β are the roots of the given equation

$\therefore\ \alpha + \beta = \dfrac{-b}{a},\ a\beta = \dfrac{c}{a}.$

Also, since $\alpha < -1, \beta > 1$

$\therefore\ a\beta < -1$

$\Rightarrow\ \dfrac{c}{a} < -1$ or $\dfrac{c}{a} + 1 < 0$

Let $f(x) = ax^2 + bx + c$

As $f(1)\,f(-1) > 0$,

$\therefore\ (a + b + c)(a - b + c) > 0$

or, $(a + c)^2 - b^2 > 0$ or $(a + c)^2 > b^2$

or, $\left(1 + \dfrac{c}{a}\right)^2 > \left(\dfrac{b}{a}\right)^2$

$\Rightarrow\ \left(1 + \dfrac{c}{a}\right) < -\left|\dfrac{b}{a}\right|$ $\left[\because \left(\dfrac{c}{a} + 1\right) < 0\right]$

or, $1 + \dfrac{c}{a} + \left|\dfrac{b}{a}\right| < 0.$

The correct option is (A)

83. Roots of $ax^2 + 2bx + c = 0$ are given by

$$x = \frac{-2b \pm \sqrt{4b^2 - 4ac}}{2a} = \frac{-b}{a}$$

(Since $b^2 = ac$ as a, b, c, are in G.P.)

This is root of $dx^2 + 2ex + f = 0$

$\therefore \quad d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + f = 0$

$\Rightarrow \quad db^2 - 2eba + af = 0$

$\Rightarrow \quad dac - 2eba + af = 0 \qquad (\because b^2 = ac)$

$\Rightarrow \quad 2eb = dc + af$

$\Rightarrow \quad \frac{2e}{b} = \frac{dc+af}{b^2} = \frac{dc+af}{ac} \qquad (\because b^2 = ac)$

$= \frac{d}{a} + \frac{f}{c}$

$\Rightarrow \quad \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

The correct option is (C)

84. Let α, β be the roots of $x^2 + abx + c = 0$ and α, γ be the roots of $x^2 + acx + b = 0$, a being the common root.

$\therefore \quad \alpha + \beta = -ab \qquad (1)$

$\alpha\beta = c \qquad (2)$

$\alpha + \gamma = -ac \qquad (3)$

$\alpha\gamma = b \qquad (4)$

From (1) – (3),

$\beta - \gamma = a(c - b)$

From (2)– (4),

$\alpha(\beta - \gamma) = c - b$

$\therefore \quad \frac{\alpha(\beta-\gamma)}{\beta-\gamma} = \frac{c-b}{a(c-b)}$;

or $\quad \alpha = \frac{1}{a}$.

$\therefore$ From (2) and (4),

$\frac{\beta}{a} = c$, i.e., $\beta = ac$

and, $\frac{\gamma}{a} = b$,

i.e., $\gamma = ab$.

$\therefore$ The quadratic equation whose roots are β, γ is

$x^2 - (\beta + \gamma)x + b\gamma = 0$

or, $x^2 - (ac + ab)x + ac \cdot ab = 0$;

or, $x^2 - a(b + c)x + a^2bc = 0$.

The correct option is (B)

85. The given equation can be written as

$(ay + a')x^2 + (by + b')x + (cy + c') = 0$.

The condition that x may be a rational function of y is,

$(by + b')^2 - 4(ay + a')(cy + c')$ is a perfect square;

that is, $(b^2 - 4ac)y^2 + (2bb' - 4ac' - 4a'c)y + b'^2 - 4a'c'$ is a perfect square.

The corresponding quadratic equation has discriminant = 0

that is, $4(bb' - 2ac' - 2a'c)^2 - 4(b^2 - 4ac)(b'^2 - 4a'c') = 0$;

or, $(ac' + a'c)^2 - 4aa'cc' = abb'c + a'bb'c - a'c'b^2 - acb'^2$

or, $(ac' - a'c)^2 = (ab' - a'b)(bc' - b'c)$.

The correct option is (A)

86. The discriminant of the given equation is

$D = 4[({}^nC_r)^2 - {}^nC_{r-1}\,{}^nC_{r+1}]$

$= 4(a - b)$,

where $a = ({}^nC_r)^2$, $b = {}^nC_{r-1} \cdot {}^nC_{r+1}$

Now, $\frac{a}{b} = \frac{{}^nC_r \cdot {}^nC_r}{{}^nC_{r-1} \cdot {}^nC_{r+1}}$

$= \frac{n!}{r!(n-r)!}\frac{n!}{r!(n-r)!} \cdot \frac{(r-1)!(n-r+1)!}{n!} \cdot \frac{(r+1)! \cdot (n-r-1)!}{n!}$

$= \frac{r+1}{r} \cdot \frac{n-r+1}{n-r} = \left(1+\frac{1}{r}\right)\left(1+\frac{1}{n-r}\right) > 1$

$\therefore \quad a > b \Rightarrow D > 0$

$\Rightarrow$ roots of given equation are real and distinct.

The correct option is (A)

87. Let α and β be the roots of $x^2 - px + q = 0$ and α and $\frac{1}{\beta}$ be the roots of $x^2 - ax + b = 0$.

Then, $\quad \alpha + \beta = p$ and $\alpha\beta = q$.

Also, $\quad \alpha + \frac{1}{\beta} = a$ and $\frac{\alpha}{\beta} = b$.

Now, $\quad (q - b)^2 = \left(\alpha\beta - \frac{\alpha}{\beta}\right)^2 = \alpha^2\left(\beta - \frac{1}{\beta}\right)^2$

$= \frac{\alpha}{\beta} \cdot \beta\alpha\left[(\alpha+\beta) - \left(\alpha + \frac{1}{\beta}\right)\right]^2$

$= bq(p - a)^2$.

The correct option is (B)

88. The roots of $2x^2 - 3x + 4 = 0$ are imaginary, because disc. = $(-3)^2 - 4 \cdot 2 \cdot 4 < 0$. Hence, the common root must be imaginary. But imaginary roots occur in pair. Hence both the roots will be common, i.e., two equations will be identical. So their coefficients will be proportional

i.e., $\frac{a}{2} = \frac{b}{-3} = \frac{c}{4}$,

$\therefore \quad 6a = -4b = 3c$.

The correct option is (C)

89. We have, $3x^2 + x - 5 = 0$.

Its discriminant $= 1 - 4 \cdot 3(-5) = 61$, which is positive but not a perfect square. Hence, both the roots of $3x^2 + x - 5 = 0$ must be irrational as the irrational roots occur in conjugate pair. But one root of $ax^2 + bx + c = 0$ and $3x^2 + x - 5 = 0$ is common. Hence, both the roots of $ax^2 + bx + c = 0$ must also be irrational, that is, both the roots of the given equations are common. Thus, both the equations are the same.

$\therefore \quad \frac{a}{3} = \frac{b}{1} = \frac{c}{-5} = k \quad$ (say)

$\Rightarrow \quad a = 3k;\ b = k,\ c = -5k$.

$\therefore \quad 3a + b + 2c = 9k + k - 10k = 10k - 10k = 0$.

The correct option is (A)

90. Let α be the common root.

Then, $a\alpha^2 + 2b\alpha + c = 0$

and, $a_1\alpha^2 + 2b_1\alpha + c_1 = 0$

By cross-multiplication, we get

$$\frac{\alpha^2}{2(bc_1 - b_1c)} = \frac{\alpha}{ca_1 - ac_1} = \frac{1}{2(ab_1 - ba_1)}$$

$\Rightarrow (ca_1 - ac_1)^2 = 4(bc_1 - b_1c)(ab_1 - ba_1)$ (1)

$\because \frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1}$ are in A.P.,

$\therefore \frac{b}{b_1} - \frac{a}{a_1} = \frac{c}{c_1} - \frac{b}{b_1} = k$ (say)

$\Rightarrow ab_1 - a_1b = -ka_1b_1$ and $bc_1 - b_1c = -kb_1c_1$.

Also, $2k = k + k = \frac{b}{b_1} - \frac{a}{a_1} + \frac{c}{c_1} - \frac{b}{b_1} = \frac{ca_1 - ac_1}{a_1c_1}$

or, $ca_1 - ac_1 = 2ka_1c_1$.

$\therefore$ From (1), $4k^2a_1^2c_1^2 = 4(-ka_1b_1)(-kb_1c_1)$

or, $a_1c_1 = b_1^2$. Hence a_1, b_1, c_1 are in G.P.

The correct option is (B)

91. Let $y = \frac{(a+x)(b+x)}{(c+x)}$

$\Rightarrow x^2 + (a+b)x + ab = cy + xy$

$\Rightarrow x^2 + (a+b-y)x + ab - cy = 0$.

For real x, $B^2 - 4AC \ge 0$

$\Rightarrow (a+b-y)^2 - 4ab + 4cy \ge 0$

$\Rightarrow (a+b)^2 + y^2 - 2(a+b)y - 4ab + 4cy \ge 0$

$\Rightarrow (a-b)^2 + y^2 - 2(a+b-2c)y \ge 0$

$\Rightarrow y^2 - 2(a+b-2c)y + (a-b)^2 \ge 0$

$\Rightarrow [y - (\sqrt{(a-c)} - \sqrt{(b-c)})^2]$

$\times [y - (\sqrt{(a-c)} + \sqrt{(b-c)})^2] \ge 0$

$\therefore y \le (\sqrt{(a-c)} - \sqrt{(b-c)})^2$

or, $y \ge (\sqrt{(a-c)} + \sqrt{(b-c)})^2$.

Hence, the minimum value of y is

$(\sqrt{(a-c)} + \sqrt{(b-c)})^2$

The correct option is (B)

92. Let the ratio of the roots be k. Then, the roots of $a_1x^2 + b_1x + c_1 = 0$ are $\alpha, k\alpha$

and the roots of $a_2x^2 + b_2x + c_2 = 0$ are $\beta, k\beta$.

$\therefore \alpha + k\alpha = \frac{-b_1}{a_1}$ (1)

$\alpha \cdot k\alpha = \frac{c_1}{a_1}$ (2)

$\beta + k\beta = \frac{-b_2}{a_2}$ (3)

$\beta \cdot k\beta = \frac{c_2}{a_2}$. (4)

Dividing (1) by (3), we get

$\frac{\alpha(1+k)}{\beta(1+k)} = \frac{b_1a_2}{a_1b_2}$, or $\frac{\alpha}{\beta} = \frac{b_1a_2}{a_1b_2}$. (5)

Dividing (2) by (4), we get

$\frac{k\alpha^2}{k\beta^2} = \frac{c_1a_2}{a_1c_2}$;

or $\left(\frac{\alpha}{\beta}\right)^2 = \frac{c_1a_2}{a_1c_2}$

or, $\left(\frac{b_1a_2}{a_1b_2}\right)^2 = \frac{c_1a_2}{a_1c_2}$ (Using (5))

$\Rightarrow \left(\frac{b_1}{b_2}\right)^2 = \frac{c_1a_2}{a_1c_2} \times \frac{a_1^2}{a_2^2} = \frac{c_1a_1}{c_2a_2} = \frac{a_1}{a_2} \cdot \frac{c_1}{c_2}$

$\therefore \frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ are in G.P.

The correct option is (B)

93. Since α, β are the roots of the equation

$x^2 - px + q = 0$

$\therefore \alpha + \beta = p$ and $a\beta = q$.

Now, $(\alpha^{1/4} + \beta^{1/4})^4 = [(\alpha^{1/4} + \beta^{1/4})^2]^2$

$= [\alpha^{1/2} + \beta^{1/2} + 2(a\beta)^{1/4}]^2$

$= \left[\sqrt{\alpha + \beta + 2\sqrt{\alpha\beta}} + 2(\alpha\beta)^{1/4}\right]^2$

$= \left[\sqrt{p + 2\sqrt{q}} + 2(q)^{1/4}\right]^2$

$= p + 6\sqrt{q} + 4q^{1/4}\sqrt{p + 2\sqrt{q}}$

$\therefore \alpha^{1/4} + \beta^{1/4} = \left[p + 6\sqrt{q} + 4q^{1/4}\sqrt{p + 2\sqrt{q}}\right]^{1/4}$

The correct option is (D)

94. Since a and b are the roots of the equation

$x^2 + px + 1 = 0$

$\therefore a + b = -p$ (1)

and, $ab = 1$ (2)

Also, since c and d are the roots of the equation

$x^2 + qx + 1 = 0$

$\therefore c + d = -q$ (3)

and, $cd = 1$ (4)

Now, $(a-c)(b-c)(a+d)(b+d)$

$= (ab - bc - ac + c^2)(ab + db + ad + d^2)$

$= [ab - c(b+a) + c^2] \cdot [ab + d(a+b) + d^2]$

$= (1 + cp + c^2)(1 - pd + d^2)$

[Putting the values of $a + b$ and ab]

$= 1 + cp + c^2 - pd - cdp^2 - c^2pd + d^2 + cpd^2 + c^2d^2$

$= 1 + (c^2 + d^2) + c^2d^2 - cdp^2 + p(c-d) + cpd(d-c)$

$= 1 + [(c+d)^2 - 2cd] + c^2d^2 - cdp^2 + p(c-d) + cpd(d-c)$

$= 1 + (q^2 - 2) + 1 - p^2 + p(c-d) + p(d-c)$

[Putting the values of $c + d$ and cd]

$= 2 - 2 + q^2 - p^2 = q2 - p^2$.

The correct option is (B)

95. Let $f(x) = x^3 + px^2 + q = 0$ (1)

Since $f(x) = 0$ has a root of multiplicity 2

$\therefore$ $f(x) = 0$ and $f'(x) = 3x^2 + 2px = 0$ have a common root.

The roots of $3x^2 + 2px = 0$ are $x = 0$ and $x = -2p/3$.

But $x = 0$ is not a root of $f(x) = 0$ $(\because q \neq 0)$

$\therefore$ common root of $f(x) = 0$ and $f'(x) = 0$ is $x = -2p/3$.

$\therefore$ $(-2p/3)^3 + p(-2p/3)^2 + q = 0$

$\Rightarrow$ $4p^3 + 27q = 0$.

The correct option is (D)

96. Since $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$ are roots of the equation

$ax^2 + bx + c = 0$

$\therefore$ $\frac{\alpha}{\alpha-1} + \frac{\alpha+1}{\alpha} = \frac{-b}{a}$

and $\frac{\alpha}{\alpha-1} \cdot \frac{\alpha+1}{\alpha} = \frac{c}{a}$

$\Rightarrow$ $\frac{2\alpha^2-1}{\alpha^2-\alpha} = \frac{-b}{a}$ (1)

and, $\frac{\alpha+1}{\alpha-1} = \frac{c}{a}$

$\Rightarrow$ $a\alpha + a = c\alpha - c$

$\Rightarrow$ $\alpha(c-a) = a + c$ or $\alpha = \frac{c+a}{c-a}$.

$\therefore$ From (1), $\frac{2\left(\frac{c+a}{c-a}\right)^2 - 1}{\left(\frac{c+a}{c-a}\right)^2 - \frac{c+a}{c-a}} = -\frac{b}{a}$

$\Rightarrow$ $\frac{2(c+a)^2 - (c-a)^2}{(c+a)^2 - (c^2-a^2)} = -\frac{b}{a}$

$\Rightarrow$ $\frac{(c+a)^2 + 4ac}{2a^2 + 2ac} = -\frac{b}{a}$

$\Rightarrow$ $(c+a)^2 + 4ac = -2b(a+c)$

$\Rightarrow$ $(c+a)^2 + 2b(a+c) + 4ac = 0$

$\Rightarrow$ $(c+a)^2 + 2b(a+c) + b^2 = b^2 - 4ac$

$\Rightarrow$ $(a+b+c)^2 = b^2 - 4ac$.

The correct option is (C)

97. Given equation is

$ax^2 + bx + c = 0$

Let α, β be the roots of this equation.

then, $\alpha + \beta = -\frac{b}{a}$ and $a\beta = \frac{c}{a}$

Also, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{a^2+\beta^2}{\alpha^2\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$\Rightarrow$ $\alpha + \beta = \left(\frac{\alpha+\beta}{\alpha\beta}\right)^2 - \frac{2}{\alpha\beta}$

$\Rightarrow$ $\left(-\frac{b}{a}\right) = \left(\frac{-b/a}{c/a}\right)^2 - \frac{2}{c/a} \Rightarrow -\frac{b}{a} = \left(\frac{b}{c}\right)^2 - \frac{2a}{c}$

$\Rightarrow$ $\frac{2a}{c} = \left(\frac{b}{c}\right)^2 + \frac{b}{a} \Rightarrow \frac{2a}{c} = \frac{b}{c}\left[\frac{b}{c} + \frac{c}{a}\right]$

$\Rightarrow$ $\frac{2a}{b} = \frac{b}{c} + \frac{c}{a} \Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P.

$\Rightarrow$ $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.

The correct option is (C)

98. Discreminent equals $-4(k-5) \geq 0 \Rightarrow k \leq 5$. The quadratic equation at $x = 5$ must be positive and sum of the roots must be less than 10. These conditions imply $k^2 - 9k + 20 > 0$. So, $k < 4$.

The correct option is (A)

99. Let $y = x - [x]$

$\therefore$ the given equation becomes

$f(y) = (a-2)y^2 + 2y + a^2 = 0$ (1)

Since x is not an integer,

$\therefore$ $y = x - [x] \neq 0$

Then, $a \neq 0$ [$\because$ of (1)]

When $2 < x < 3$, $[x] = 2$

$\Rightarrow$ $0 < x - [x] < 1$ i.e. $0 < y < 1$

Since given equation has exactly one solution in the interval (2, 3)

$\therefore$ (1) has exactly one solution in the interval (0,1)

This is possible if $f(0) f(1) < 0$

[$\because$ otherwise, the equation (1) has either no or two solutions in (0,1)]

$\Rightarrow$ $a^2(a - 2 + 2 + a^2) < 0 \Rightarrow a(a+1) < 0$

$\Rightarrow$ $-1 < a < 0$ i.e., $a \in (-1, 0)$

The correct option is (C)

100. Given, $b^2 \leq 4ac$, $c^2 \leq 4ab$ and $a^2 \leq 4ac$

Equality cannot hold simultaneously

[$\because$ a, b, c are different]

$\therefore$ $a^2 + b^2 + c^2 < 4(ab + bc + ca) \Rightarrow R < 4$

Also, $a^2 + b^2 + c^2 - ab - bc - ca$

$\frac{1}{2}\left[(b-c)^2 + (c-a)^2 + (a-b)^2\right] > 0 = \Rightarrow R > 1$

The correct option is (D)

101. The given equation can be written as

$(z+1)^2 - (a-3)z(z+1) + (a-4)z^2 = 0$

[Putting $x^2 + x + 1 = z$]

$\Rightarrow$ $(1 + 3 - a + a - 4)z^2 + (2 + 3 - a)z + 1 = 0$

$\Rightarrow$ $(5-a)z + 1 = 0$ or $z = \frac{1}{a-5}$

$\Rightarrow$ $x^2 + x + 1 - \frac{1}{a-5} = 0$

$\Rightarrow\ x^2 + x + \dfrac{a-6}{a-5} = 0$

whose roots will be real if discriminant ≥ 0

$\Rightarrow\ 1 - \dfrac{4(a-6)}{a-5} \ge 0 \Rightarrow \dfrac{3a-19}{a-5} \le 0$

$\therefore\ 5 < a \le \dfrac{19}{3}$

The correct option is (D)

102. Given equation can be written as

$(2^x)^2 - (a-4)2^x - 2^x + a - 4 = 0$

$\Rightarrow\ (2^x - 1)(2^x - a + 4) = 0$

$\Rightarrow\ 2^x = 1,\ 2^x = a - 4$

Since $x \le 0$ and $2^x = a - 4$ ($\because$ x is non-positive)

$\therefore\ 0 < a - 4 \le 1$ i.e., $4 < a \le 5$ i.e., $a \in (4, 5]$

The correct option is (B)

103. Let $f(x) = (x - \alpha)(x - \beta)$ (1)

Now, $f(n) f(n+1) = (n - \alpha)(n - \beta)(n + 1 - \alpha)(n + 1 - \beta)$

$= (n - \alpha)(n + 1 - \beta)(n - \beta)(n + 1 - \alpha)$

$= \{n(n+1) - n(\alpha + \beta) - \alpha + \alpha\beta\}\{n(n+1)$

$- n(\alpha + \beta) - \beta + \alpha\beta\}$

$= \{n(n+1) + na + b - \alpha\}\{n(n+1) + na + b - \beta\}$

$= (m - \alpha)(m - \beta)$, where $m = n(n+1) + an + b$

The correct option is (A)

104. We have,

$\dfrac{x^2 + nx - 2}{x^2 - 3x + 4} - 2 \le 0$

$\Rightarrow\ \dfrac{x^2 - (n+6)x + 10}{x^2 - 3x + 4} \ge 0$

$\Rightarrow\ x^2 - (n+6)x + 10 \ge 0$

[$\because x^2 - 3x + 4 > 0\ \forall x \in R$, as its $D < 0$ and $a > 0$]

The above inequality will be true for all real x if its discriminant ≤ 0

i.e., $(n+6)^2 - 40 \le 0$

$\Rightarrow\ -(\sqrt{40} - 6) \le n \le (\sqrt{40} - 6)$ (1)

Also, we have,

$\dfrac{x^2 + nx - 2}{x^2 - 3x + 4} + 1 \ge 0 \Rightarrow \dfrac{2x^2 + (n-3)x + 2}{x^2 - 3x + 4} \ge 0$

$\Rightarrow\ 2x^2 + (n-3)x + 2 \ge 0$

The above inequality will be true for all real x if its discriminant ≤ 0

$\Rightarrow\ (n-3)^2 - 16 \le 0$

$\Rightarrow\ 1 \le n \le 7$ (2)

Drawing the number line for inequalities (1), (2) and taking their intersection we get

$\boldsymbol{n} \in [-1, \sqrt{40} - 6]$

The correct option is (C)

105. Let $f(x) = (x - a)(x - b) - 1$

$\Rightarrow\ f(a) = -1$ and $f(b) = -1$.

Also, the coefficient of $x^2 = 1 > 0$.

Hence, a and b both lie between the roots of the equation $f(x) = 0$

$\therefore$ The equation $(x - a)(x - b) - 1 = 0$ has one root in $(-\infty, a)$ and other in (b, ∞) [$\because b > a$]

The correct option is (B)

106. The given quadratic equation is satisfied by $x = -a$, $x = -b$ and $x = -c$, Hence, the quadratic equation has three roots, which is only possible if it is an identity hence it has infinite roots.

The correct option is (D)

107. Putting $x^2 = y$, the given equation in x reduces to

$ay^2 - 2y - (a - 1) = 0$ (1)

The given biquadratic equation will have four real and distinct roots, if the quadratic equation (1) has two distinct and positive roots. For that, we must have

$D > 0 \Rightarrow a^2 - a + 1 > 0$, which is true $\forall\ a \in R$

Product of roots $> 0 \Rightarrow 0 < a < 1$

sum of roots $> 0 \Rightarrow a > 0$

Hence, the acceptable values of a are $0 < a < 1$.

The correct option is (C)

108. Since –5 and 1 are the roots.

Product of roots $= -5 \times 1 = b \Rightarrow b = -5$

and, sum of roots $= -5 + 1 = -[a^2 - 5a + b + 4]$

$\Rightarrow\ [a^2 - 5a - 1] = 4 \Rightarrow 4 \le a^2 - 5a - 1 < 5$

$\Rightarrow\ a^2 - 5a - 5 \ge 0$ and $a^2 - 5a - 6 < 0$

$\Rightarrow\ a \le \dfrac{5 - \sqrt{45}}{2}$ or $a \ge \dfrac{5 + \sqrt{45}}{2}$ and $-1 < a < 6$

$\Rightarrow\ -1 < a \le \dfrac{5 - 3\sqrt{5}}{2}$ or $\dfrac{5 + 3\sqrt{5}}{2} \le a < 6$

$\Rightarrow\ a \in \left(-1, \dfrac{5 - 3\sqrt{5}}{2}\right] \cup \left[\dfrac{5 + 3\sqrt{5}}{2}, 6\right)$

The correct option is (C)

109. We have,

$\alpha + \beta = \alpha,\ a\beta = p$

and, $\gamma + \delta = b,\ \gamma\delta = q$

If r $(r > 1)$ be the common ratio of the increasing G.P. $\alpha, \beta, \gamma, \delta$ then

$\beta = ar,\ \gamma = ar^2$ and $\delta = ar^3$

The above equations then reduce to

$\alpha(1 + r) = a,\ a^2 r = p$

and, $ar^2(1 + r) = b,\ a^2 r^5 = q$

$\Rightarrow\ r^2 = \dfrac{b}{a}$ and $r^4 = \dfrac{q}{p}$

$\Rightarrow\ \dfrac{q}{p} = \left(\dfrac{b}{a}\right)^2$

Hence, we have $\dfrac{q + p}{q - p} = \dfrac{b^2 + a^2}{b^2 - a^2}$.

The correct option is (B)

110. The sum of the coefficients of the equation = 0

$\therefore$ $x = 1$ is a root of the equation. Let a be the first term and d be the common difference of given A.P.

$$t_p = a + (p-1)d = \frac{1}{q} \quad (1)$$

and, $$t_q = a + (q-1)\,d = \frac{1}{p} \quad (2)$$

Solving (1) and (2), $a = d = \dfrac{1}{pq}$

$\therefore$ $t_{pq} = a + (pq-1)d = 1$

$\therefore$ t_{pq} is the root of the given equation.

The correct option is (C)

111. Let α, β be the roots of the equation $4x^2 + 4ax + b = 0$, then we have,

$\alpha + \beta = -a$ and $\alpha\beta = \dfrac{b}{4}$

According to the given condition,

$|\alpha - \beta| \le a$

$\Rightarrow$ $(\alpha + \beta)^2 - 4\,\alpha\beta \le a^2$

$\Rightarrow$ $a^2 - b^2 \le a^2$

$\Rightarrow$ $b \ge 0$

Also, we have for real roots

$(4a)^2 - 16\,b \ge 0$ i.e., $b \le a^2$

Therefore, $0 \le b \le a^2$

The correct option is (C)

112. Let the roots be α and $\alpha + 2$, where α is an odd positive integer. Then, $a\alpha^2 + b\alpha + c = 0$ (1)

and $a\,(\alpha+2)^2 + b\,(\alpha+2) + c = 0 \Rightarrow a\alpha^2 + b\alpha + c + (4\,a\alpha + 4a + 2b) = 0$

$\Rightarrow$ $2a\,(1+\alpha) + b = 0$ [using (1)]

$\Rightarrow$ $b = -2a\,(1+\alpha)$

$\Rightarrow$ $b^2 = 4a^2\,(1+\alpha)^2 \Rightarrow b^2 \ge 4a^2\,(1+1)^2$

[$\because$ $\alpha \ge 1$ as α is odd positive integer]

$\Rightarrow$ $b^2 \ge 16\,a^2$ or $|b| \ge 4a$

The correct option is (B)

113. The discriminants of the given quadratic equations are, $D_1 = a^2 + 12b$, $D_2 = c^2 - 4b$ and $D_3 = d^2 - 8b$

$\therefore$ $D_1 + D_2 + D_3 = a^2 + c^2 + d^2 \ge 0$

$\Rightarrow$ At least one of D_1, D_2, D_3 is non-negative. Hence, the equation has at least two real roots.

The correct option is (D)

More than One Option Correct Type

114. If $x - a < 0$, $|x - a| = -(x - a)$

$\therefore$ equation becomes $x^2 + 2a\,(x-a) - 3a^2 = 0$

$\Rightarrow$ $x^2 + 2ax - 5a^2 = 0$

$\Rightarrow$ $x = -(1+\sqrt{6})\;a,\;(-1+\sqrt{6})\,a$

$\because$ $x < a \le 0$

$\therefore$ $x = (-1+\sqrt{6})\,a$

If $x - a \ge 0$, $|x - a| = x - a$

$\therefore$ the equation becomes $x^2 - 2a\,(x-a) - 3a^2 = 0$

$\Rightarrow$ $x^2 - 2ax - a^2 = 0$

$\Rightarrow$ $x = (1+\sqrt{2})\,a,\;(1-\sqrt{2})\,a$

$\because$ $x \ge a$ and $a \le 0$

$\therefore$ $x = (1-\sqrt{2})\,a.$

The correct option is (A) and (B)

115. Since $x^2 - 3x + 2 = 0$ is one of the factors of the expression $x^4 - px^2 + q$, therefore, on dividing the expression by factor, remainder = 0, i.e., on dividing $x^4 - px^2 + q$ by $x^2 - 3x + 2$, the remainder

$(15 - 3p)\,x + (2p + q - 14) = 0.$

On comparing both sides, we get

$15 - 3p = 0$ or $p = 5$ and $2p + q - 14 = 0$ or $q = 4$.

The correct option is (A) and (B)

116. We can write given equation as

$$\frac{p}{2x} = \frac{(a+b)x + c(b-a)}{x^2 - c^2}$$

or, $p\,(x^2 - c^2) = 2\,(a+b)\,x^2 - 2c\,(a-b)\,x$

or, $(2a + 2b - p)\,x^2 - 2c\,(a-b)\,x + pc^2 = 0$

For this equation to have equal roots

$c^2\,(a-b)^2 - pc^2\,(2a + 2b - p) = 0$

$\Rightarrow$ $(a-b)^2 - 2p\,(a+b) + p^2 = 0$ [$\because$ $c^2 \ne 0$]

$\Rightarrow$ $[p - (a+b)]^2 = (a+b)^2 - (a-b)^2 = 4ab$

$\Rightarrow$ $p - (a+b) = \pm\,2\sqrt{ab}$

$\Rightarrow$ $p = a + b \pm 2\sqrt{ab} = (\sqrt{a} \pm \sqrt{b})^2$

The correct option is (A) and (B)

117. We have,

$$\frac{\log_a a}{\log_a a + \log_a x} + \frac{2\log_a a}{\log_a x} + \frac{3\log_a a}{2\log_a a + \log_a x} = 0$$

$$\Rightarrow \frac{1}{1+t} + \frac{2}{t} + \frac{3}{2+t} = 0 \quad (\text{let } \log_a x = t)$$

$$\Rightarrow \frac{2t + t^2 + 2t^2 + 6t + 4 + 3t^2 + 3t}{t(1+t)(2+t)} = 0$$

$\Rightarrow$ $6t^2 + 11t + 4 = 0$

$\Rightarrow$ $6t^2 + 8t + 3t + 4 = 0$

$\Rightarrow$ $(2t+1)\,(3t+4) = 0$

$\Rightarrow$ $t = -\dfrac{1}{2}, -\dfrac{4}{3}$

$\Rightarrow$ $\log_a x = -\dfrac{1}{2}, -\dfrac{4}{3}$

$\therefore$ $x = a^{-1/2}, a^{-4/3}$

The correct option is (B) and (D)

118. When $x^2 + 4x + 3 \ge 0$ i.e. $x \ge -1$ or $x \le -3$

Then, $|x^2 + 4x + 3| + 2x + 5 = 0$

$\Rightarrow\ x^2 + 4x + 3 + 2x + 5 = 0$

$\Rightarrow\ x^2 + 6x + 8 = 0$

$\Rightarrow\ (x + 2)(x + 4) = 0$

$\Rightarrow\ x = -2$ or $x = -4$.

Thus, $x = -4$ as $x \in \{x : x \geq -1\} \cup \{x : x \leq -3\}$

When $x^2 + 4x + 3 < 0$ *i.e.* $-3 < x < -1$

Then, $|x^2 + 4x + 3| + 2x + 5 = 0$

$\Rightarrow\ -(x^2 + 4x + 3) + 2x + 5 = 0$

$\Rightarrow\ x^2 + 4x + 3 - 2x - 5 = 0 \Rightarrow x^2 + 2x - 2 = 0$

$$\Rightarrow\ x = \frac{-1 \pm \sqrt{4+8}}{2} = -1 \pm \sqrt{3}$$

$\Rightarrow\ x = -1 - \sqrt{3}$ because $-1 + \sqrt{3}$ does not lie between -3 and -1.

Hence, we have either $x = -4$ or $x = -1 - \sqrt{3}$.

The correct option is (B) and (C)

119. The roots of the equation

$10x^3 - cx^2 - 54x - 27 = 0$ are in H.P.

Putting $x = 1/y$, we get

$27y^3 + 54y^2 + cy - 10 = 0$ has roots in A.P.

Let the roots of the equation in y be

$\alpha - \beta, \alpha, \alpha + \beta$. Then,

$$\text{sum of roots} = \alpha - \beta + \alpha + \alpha + \beta = 3\alpha = \frac{-54}{27} = -2$$

$\therefore\ \alpha = -2/3$

$\because\ \alpha = \dfrac{-2}{3}$ satisfies the equation

$$\therefore\ -27\frac{8}{27} + 54 \cdot \frac{4}{9} - \frac{2c}{3} - 10 = 0$$

$\Rightarrow\ c = 9$.

$$\text{Product of the roots} = (\alpha - \beta)(\alpha)(\alpha + \beta) = \frac{10}{27}$$

$$\text{or,}\quad -\frac{2}{3}\left(\frac{4}{9} - \beta^2\right) = \frac{10}{27}$$

or $b^2 = 1 \Rightarrow \beta = \pm 1$.

$\therefore$ Roots of y-equation are $-\dfrac{5}{3}, -\dfrac{2}{3}, \dfrac{1}{3}$

or, roots of x-equation are $-\dfrac{3}{5}, -\dfrac{3}{2}, 3$.

The correct option is (A), (B) and (C)

120. Given equation is $x^2 + 9y^2 - 4x + 3 = 0$ (1)

or, $x^2 - 4x + 9y^2 + 3 = 0$.

Since x is real, $\therefore (-4)^2 - 4(9y^2 + 3) \geq 0$

$\Rightarrow\ 16 - 4(9y^2 + 3) \geq 0 \Rightarrow 4 - 9y^2 - 3 \geq 0$

$\Rightarrow\ 9y^2 - 1 \leq 0 \Rightarrow (3y - 1)(3y + 1) \leq 0$

$$\Rightarrow\ \frac{-1}{3} \leq y \leq \frac{1}{3}.$$

Equation (1) can also be written as

$9y^2 + 0y + x^2 - 4x + 3 = 0$

Since y is real

$\therefore\ 0^2 - 4.9(x^2 - 4x + 3) \geq 0$

$\Rightarrow\ x^2 - 4x + 3 \leq 0$

$\Rightarrow\ (x - 1)(x - 3) \leq 0$

$\Rightarrow\ 1 \leq x \leq 3$.

The correct option is (A) and (C)

121. We have,

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

$$\text{and,}\quad \alpha^4 + \beta^4 = -\frac{m}{l},\ \alpha^4\beta^4 = \frac{n}{l}$$

The given equation

$a^2 lx^2 - 4aclx + 2c^2 l + a^2 m = 0$

has discriminant

$D = 16a^2c^2l^2 - 4a^2 l(2c^2 l + a^2 m)$

$\quad = 8a^2c^2l^2 - 4a^4 lm$

$$\quad = 4a^4l^2\left(\frac{2c^2}{a^2} - \frac{m}{l}\right) > 0 \quad \left[\frac{-m}{l} = \alpha^4 + \beta^4 > 0\right]$$

Hence, the roots are real.

Also, we have,

$$\text{product of the roots} = \frac{2c^2 l + a^2 m}{a^2 l}$$

$$= \frac{2c^2}{a^2} + \frac{m}{l} = 2\alpha^2\beta^2 - (\alpha^4 + \beta^4)$$

$$= -(\alpha^2 - \beta^2)^2 < 0$$

$\therefore$ The roots are of opposite signs.

The correct option is (A) and (C)

122. Given, $a > b > c$ (1)

The given equation is

$(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ (2)

Since (2) has a root in the interval $(-1, 0)$,

$\therefore\ f(-1) f(0) < 0$

$\Rightarrow\ (2a - b - c)(c + a - 2b) < 0$ (3)

From (1),

$a > b \Rightarrow a - b > 0$ and $a > c \Rightarrow a - c > 0$

$\therefore\ 2a - b - c > 0$ (4)

From (3) and (4), $c + a - 2b < 0$

$\Rightarrow\ c + a < 2b$

The correct option is (A)

Again, sum of the coefficients of the equation $= a + b - 2c + b + c - 2a + c + a - 2b = 0$, $\therefore$ one root is 1 and the other root is $\dfrac{c + a - 2b}{a + b - 2c}$, which is a rational no.

($\because a, b, c$ are rational)

$\therefore$ both the roots of the equation are rational

The correct option is (B)

Since $c + a < 2b$

$\Rightarrow\ 4b^2 > (c + a)^2 = c^2 + a^2 + 2ac$

$\Rightarrow\ 4b^2 - 4ac > c^2 + a^2 - 2ac = (c - a)^2 > 0$

$\therefore$ Discriminant of $ax^2 + 2bx + c > 0$

Also, each of a, b, c is positive,

$\therefore$ the equation $ax^2 + 2bx + c = 0$ has real and negative roots

The correct option is (C)

Similarly, (D) is also correct.

The correct option is (A), (B), (C) and (D)

123. $\because$ Roots are real

$\therefore \quad B^2 - 4AC > 0 \Rightarrow a^4 > 4b^2$

$\Rightarrow$ (A) is correct.

If $f(x) = x^2 + a^2x + b^2$

$\because \quad c$ lies outside the roots.

$\therefore \quad f(c) > 0, \Rightarrow c^2 + a^2c + b^2 > 0$

$\Rightarrow$ (B) is correct.

Further, if α, β are the roots, then $\alpha > c$ and $\beta > c \Rightarrow \alpha + \beta > 2c$

$\Rightarrow \; -a^2 > 2c \Rightarrow \dfrac{-a^2}{2} > c$

$\Rightarrow$ (C) is correct.

The correct option is (A), (B) and (C)

124. Put $x^2 = y$. The given equation becomes

$f(y) = ay^2 + by + c = 0$ (1)

The given equation will have four real roots if (1) has two non-negative roots. This can happen if

$\dfrac{-b}{a} \geq 0,\ af(0) \geq 0,\ b^2 - 4ac \geq 0$

$\Rightarrow \; -ab \geq 0,\ ac \geq 0$ [$\because b^2 - 4ac \geq 0$ is given]

Thus, a and b must have opposite signs whereas $a, -b$ and c must have the same sign.

$\Rightarrow \; a > 0, b < 0, c > 0$ or $a < 0, b > 0, c < 0$.

The correct option is (C) and (D)

125. The quadratic equation

$x^2 + (a - b)x - a - b + 1 = 0$

will have unequal real roots if

$D = (a - b)^2 + 4(a + b - 1) > 0$

$\Rightarrow \; b^2 + (4 - 2a)b + a^2 + 4a - 4 > 0$ (1)

This inequality will hold for all $b \in R$ if and only if discriminant of the quardratic expression on l.h.s. of (1) < 0

$\Rightarrow \; (4 - 2a)^2 - 4(a^2 + 4a - 4) < 0$

$\Rightarrow \; 16 - 16a + 4a^2 - 4a^2 - 16a + 16 < 0$

$\Rightarrow \; 32 - 32a < 0 \Rightarrow a > 1$

The correct option is (B) and (C)

Passage Based Questions

126. We have, $x^3 + 3x^2 + 3x + 2 = 0$

$\Rightarrow \; (x + 1)^3 + 1 = 0$

$\Rightarrow \; (x + 1 + 1)\{(x + 1)^2 - (x + 1) + 1\} = 0$

$\Rightarrow \; (x + 2)(x^2 + x + 1) = 0$

$\Rightarrow \; x = -2, \dfrac{-1 \pm \sqrt{3}i}{2}$

$\Rightarrow \; x = -2, \omega, \omega^2$.

Since $a, b, c \in R$, $ax^2 + bx + c = 0$ cannot have one real and one imaginary root, therefore, two common roots of $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ are ω, ω^2.

Thus, $-\dfrac{b}{a} = \omega + \omega^2 = -1$

$\Rightarrow \; a = b$ and $\dfrac{c}{a} = \omega \cdot \omega^2 = 1$

$\Rightarrow \; c = a$

$\Rightarrow \; a = b = c$.

The correct option is (C)

127. Let $f(x) = (a + 1)x^2 - 3ax + 4a$ and let α, β be the roots of the equation $f(x) = 0$. The equation will have roots greater than 1 iff

(i) Disc. ≥ 0

(ii) $\alpha + \beta > 2$

(iii) $(a + 1)f(1) > 0$

Now, Disc. $\geq 0 \Rightarrow 9a^2 - 16a(a + 1) \geq 0$

$\Rightarrow \; -7a^2 - 16a \geq 0$

$\Rightarrow \; a(7a + 16) \leq 0$

$\Rightarrow \; -\dfrac{16}{7} \leq a \leq -0$ (1)

$\alpha + \beta > 2 \Rightarrow \dfrac{3a}{a + 1} > 0 \Rightarrow \dfrac{3a}{a + 1} - 2 > 0$

$\Rightarrow \; \dfrac{3a - 2a - 2}{a + 1} > 0 \Rightarrow \dfrac{a - 2}{a + 1} > 0$

$\Rightarrow \; a < -1$ or $a > 2$ (2)

and, $(a + 1)f(1) > 0$.

$\Rightarrow \; (a + 1)(a + 1 - 3a + 4a) > 0$

$\Rightarrow \; (a + 1)(2a + 1) > 0$

$\Rightarrow \; a < -1$ or $a > -1/2$ (3)

From (1), (2) and (3), we get:

$\dfrac{-16}{7} \leq a < -1$ i.e., $a \in [-16/7, -1)$

The correct option is (A)

128. Let $f(x) = x^2 + 2(a - 3)x + 9$. If 6 lies between the roots of $f(x) = 0$, then we must have the following:

(i) Disc > 0, and

(ii) $f(6) < 0$ ($\because$ coeff. of x^2 is positive).

Now,

Disc $> 0 \Rightarrow 4(a - 3)^2 - 36 > 0 \Rightarrow (a - 3)^2 - 9 > 0$

$\Rightarrow \; a^2 - 6a > 0$

$\Rightarrow \; a(a - 6) > 0 \Rightarrow a < 0$ or $a > 6$ (1)

and, $f(6) < 0 \Rightarrow 36 + 12(a - 3) + 9 < 0$

$\Rightarrow \quad 12a + 9 < 0 \Rightarrow a < -\dfrac{3}{4}$ (2)

From (1) and (2), we get;

$a < -3/4$ i.e., $a \in (-\infty, -3/4)$.

The correct option is (B)

129. Let $f(x) = (1 - a^2)x^2 + 2ax - 1$.

Then, $f(x) = 0$ has roots between 0 and 1 if

(i) Disc ≥ 0 and

(ii) $(1 - a^2) f(0) > 0$ and $(1 - a^2) f(1) > 0$

Now, Disc $\geq 0 \Rightarrow 4a^2 + 4(1 - a^2) > 0$, which is always true.

$(1 - a^2) f(0) > 0$

$\Rightarrow \quad -(1 - a^2) > 0$

$\Rightarrow \quad a^2 - 1 > 0$

$\Rightarrow \quad a < -1$ or $a > 1$ (1)

and, $\quad (1 - a^2) f(1) > 0$

$\Rightarrow \quad (1 - a^2)(2a - a^2) > 0$

$\Rightarrow \quad a(a - 1)(a + 1)(a - 2) > 0$

$\Rightarrow \quad a < -1$ or $a > 2$ or $0 < a < 1$ (2)

From (1) and (2), we get:

$a < -1$ or $a > 2$.

The correct option is (B)

130. Let $f(x) = x^2 - 4ax + 2a^2 - 3a + 5$. The conditions for both the roots to exceed 2 are

(i) Disc. ≥ 0

(ii) $f(2) > 0$ and

(iii) sum of the roots > 4

Solving these three conditions, we get $a > \dfrac{9}{2}$.

Hence, $a \in \left(\dfrac{9}{2}, \infty\right)$.

The correct option is (D)

131. When $x > 0$, $P_n(x) > 0$ and so $P_n(x) = 0$ can have no positive real roots.

Now, $P_n(x) = 1 + 2x + 3x^2 + \ldots + (n + 1)x^n$

$\Rightarrow \quad xP_n(x) = x + 2x^2 + 3x^3 + \ldots + nx^n + (n + 1)x^{n+1}$

$\Rightarrow \quad (1 - x)P_n(x) = 1 + x + x^2 + \ldots + x^n - (n + 1)x^{n+1}$

$$\Rightarrow \quad P_n(x) = \frac{1 - (n+2)x^{n+1} + (n+1)x^{n+2}}{(1 - x)^2}$$

For negative values of x, $P_n(x)$ will vanish whenever

$f(x) = 1 - (n + 2)x^{n+1} + (n + 1)x^{n+2} = 0$

Now, $\quad f(-x) = 1 - (n + 2)(-1)^{n+1} x^{n+1} + (n + 1)(-1)^{n+2} x^{n+2}$

If n is even, there is no change of sign in this expression and so there is no negative real root of $f(x)$.

The correct option is (A)

132. As discussed in the above problem, if n is odd, there is one change of sign therefore $f(x)$ can have at most one negative real root. In this case

$f(-1) = -2n - 2 < 0, f(0) = 1 > 0$

The correct option is (C)

So, the negative real root lies between –1 and 0.

133. Let $f(x) = (x - a_1)(x - a_3)(x - a_5) + 3(x - a_2)(x - a_4)(x - a_6)$

As $x \to \infty, f(x) \to \infty$

$f(a_1) = 3(a_1 - a_2)(a_1 - a_4)(a_1 - a_6) < 0$

Similarly,

$f(a_2) > 0, f(a_3) > 0, f(a_4) < 0, f(a_5) < 0, f(a_6) > 0$.

Thus, $f(x)$ changes sign in each of two intervals (a_1, a_2), (a_3, a_4) and (a_5, a_6). Since $f(x) = 0$ is a cubic root in x,

$\therefore$ It will have I root in each of the above sub-intervals.

The correct option is (A)

Match the Column Type

134. **(I)** Since the roots of the given equation are real, therefore the discriminant ≥ 0

$\Rightarrow \quad 4(bc + ad)^2 - 4(a^2 + b^2)(c^2 + d^2) \geq 0$

$\Rightarrow \quad b^2c^2 + a^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2 \geq 0$

$\Rightarrow \quad (ac - bd)^2 \leq 0$.

But $(ac - bd)^2$ cannot be negative as it is a square of real number

$\therefore \quad ac - bd = 0$; or $b^2d^2 = a^2c^2$.

Hence, a^2, bd, c^2 are in G.P.

The correct option is (C)

(II) Since the roots are equal,

$\therefore \quad B^2 - 4AC = 0$

$\Rightarrow \quad b^2(c - a)^2 - 4ac(b - c)(a - b) = 0$

$\Rightarrow \quad b^2(c^2 + a^2 - 2ac) - 4ac[ab - ac - b^2 + bc] = 0$

$\Rightarrow \quad b^2(c^2 + a^2 - 2ac + 4ac) + 4a^2c^2 - 4abc(c + a) = 0$

$\Rightarrow \quad [b(c + a)]^2 + (2ac)^2 - 2 \cdot 2ac \cdot b(c + a) = 0$

$\Rightarrow \quad [b(c + a) - 2ac]^2 = 0$

$\Rightarrow \quad b(c + a) = 2ac$

$\Rightarrow \quad b = \dfrac{2ac}{a + c}$

$\therefore \quad b$ is H.M. of a and c, i.e., a, b, c are in H.P.

The correct option is (B)

(III) Let α and β be the roots of the given equation;

then, $\quad \alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

Given, $\quad \alpha + \beta = \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$

$$= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2},$$

$\Rightarrow \quad -\dfrac{b}{a} = \dfrac{\dfrac{b^2}{a^2} - \dfrac{2c}{a}}{\dfrac{c^2}{a^2}} = \dfrac{b^2 - 2ca}{c^2}$

$\Rightarrow \quad 2ca^2 = bc^2 + ab^2$

Hence, bc^2, ca^2 and ab^2 are in A.P.

The correct option is (A)

(IV) We have, $(a^2 + b^2 + c^2)\,p^2 - 2\,(ab + bc + cd)\,p + (b^2 + c^2 + d^2) \le 0$

$\Rightarrow \quad (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$

$\Rightarrow \quad (ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$(\because a, b, c, d, p \in R)$

$\Rightarrow \quad ap - b = 0,\ bp - c = 0,\ cp - d = 0$

$\Rightarrow \quad \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = p$

$\Rightarrow \quad a, b, c, d$ are in G.P.

The correct option is (C)

135. **(I)** We have, $\alpha + \beta = -p$

$\gamma + \delta = -p$

$\Rightarrow \quad \alpha + \beta = \gamma + \delta$

Now, $(\alpha - \gamma)(\alpha - \delta) = a^2 - \alpha(\gamma + \delta) + g\delta$

$= a^2 - \alpha(\alpha + \beta) + r$

$= -a\beta + r = -(-q) + r = q + r$

Similarly, $(\beta - \gamma)(\beta - \delta) = q + r$

$\therefore$ Ratio is 1.

The correct option is (D)

(II) Put $5^x = y$. Then, the given equation becomes

$\sin(e^x) = y + \dfrac{1}{y} = \left(\sqrt{y} - \dfrac{1}{\sqrt{y}}\right)^2 + 2 \qquad [\because 5^x > 0]$

$\Rightarrow$ $\sin(e^x) \ge 2$. which is not possible for any real value of x.

Hence, the given equation has no real solution.

The correct option is (A)

(III) We have,

$x - 2 = 2^{2/3} + 2^{1/3}$. Cube both sides, we get

$(x - 2)^3 = 2^2 + 2 + 3 \cdot 2^{2/3} \cdot 2^{1/3}\,(x - 2)$

$= 6 + 6\,(x - 2)$

or, $x^3 - 6x^2 + 12x - 8 = -6 + 6x$.

$\therefore \quad x^3 - 6x^2 + 6x = 2$.

The correct option is (B)

Assertion-Reasoning Type

136. We know that if α, β are roots of the equation

$Ax^2 + Bx + C = 0$,

then, $\qquad \alpha - \beta = \dfrac{\sqrt{B^2 - 4AC}}{A}$.

Equating the value of $\alpha - \beta$ from both the given equations, we get

$\sqrt{b^2 - 4c} = \sqrt{c^2 - 4b} \Rightarrow b^2 - 4c = c^2 - 4b$

$\Rightarrow \quad b^2 - c^2 = -4\,(b - c) \Rightarrow (b - c)(b + c + 4) = 0$

$\Rightarrow \quad b + c = \mathbf{-4} \qquad (\because b \ne c)$

The correct option is (A)

137. Let $f(x) = x^2 + 2\,(k + 1)\,x + 9k - 5$. Let α, β be the roots of $f(x) = 0$. The equation $f(x) = 0$ will have both negative roots if and only if

(i) Disc. ≥ 0

(ii) $\alpha + \beta < 0$ and

(iii) $f(0) > 0$

Now, discriminant ≥ 0

$\Rightarrow \quad 4\,(k + 1)^2 - 36k + 20 \ge 0$

$\Rightarrow \quad k^2 - 7k + 6 \ge 0$

$\Rightarrow \quad (k - 1)(k - 6) \ge 0$

$\Rightarrow \quad k \le 1$ or $k \ge 6$ (1)

$(\alpha + \beta) < 0 \quad \Rightarrow -2\,(k + 1) < 0$

$\Rightarrow \quad k + 1 > 0 \Rightarrow k > -1$ (2)

and, $\quad a\beta > 0 \Rightarrow 9k - 5 > 0 \Rightarrow k > \dfrac{5}{9}$ (3)

The correct option is (D)

138. Let α, β be the roots of $x^2 + bx + ca = 0$ and α, γ be the roots of $x^2 + cx + ab = 0$, then we have,

$a^2 + b\alpha + ca = 0$ and $a^2 + c\alpha + ab = 0$

Subtracting, we have,

$(b - c)\,\alpha + a(c - b) = 0 \Rightarrow \alpha = a$

Putting $\alpha = a$ in equation $x^2 + bx + ca = 0$, we have

$a^2 + ab + ca = 0$

i.e., $\quad a + b + c = 0$ (1)

Also, we have

$a\beta = ca$ and $\alpha\gamma = ab$

$\Rightarrow \quad \beta = c$ and $\gamma = b$

Now, $\beta + \gamma = b + c$ and $b\gamma = bc$. Hence β, γ will be the roots of the equation

$x^2 - (b + c)\,x + bc = 0$

i.e., $\quad x^2 + ax + bc = 0$. [Using (1)]

The correct option is (A)

Previous Year's Questions

139. Key Idea : The equation having α and β as its roots, is $x^2-(\alpha+\beta)x+a\beta=0$.

Since $a^2=5\alpha-3 \Rightarrow \alpha^2-5\alpha+3=0$

and $\beta^2=5\beta-3 \Rightarrow \beta^2-5\beta+3=0$

The above two equations imply that α and β are the roots of the equation

$x^2-5x+3=0$.

$\alpha+\beta=5$ and $\alpha\beta=3$

Now $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$=\dfrac{25-6}{3}=\dfrac{19}{3}$

and $\dfrac{\alpha}{\beta}.\dfrac{\beta}{\alpha}=1$

Thus, the equation having $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ as its roots is given by

$$x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha}=0$$

$$\Rightarrow x^2-\frac{19}{3}x+1=0$$

$$\Rightarrow 3x^2-19x+1=0$$

The correct option is (A)

140. Key Idea : If the discriminant of $ax^2+bx+c=0$ is positive, then this equation has two real roots.

We have $3^{2x^2-7x+7}=3^2$

$\Rightarrow 2x^2-7x+7=2$

$\Rightarrow 2x^2-7x+5=0$

$D=b^2-4ac$

$\therefore \quad =(-7)^2-4\times 2\times 5$

$=49-40$

$=9$

Since the discriminant is positive, the equation has two real roots.

The correct option is (B)

141. Let α, β be the roots

$$\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$$

$$\alpha+\beta=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$\left(-\frac{b}{a}\right)=\frac{b^2-2ac}{c^2}$$

$$\Rightarrow 2a^2c=b(c^2+ba)$$

$\Rightarrow \dfrac{a}{c},\dfrac{b}{a},\dfrac{c}{b}$ are in HP

The correct option is (C)

142. $x^2-3|x|+2=0$

$\Rightarrow (|x|-1)(|x|-2)=0$, as $|x|^2=x^2$

$\Rightarrow x=\pm 1, \pm 2$.

The correct option is (B)

143. $\beta=2\alpha$

$$3\alpha=\frac{1-3a}{a^2-5a+3}$$

$$2\alpha^2=\frac{2}{a^2-5a+3}$$

So, $\dfrac{(3a-1)^2}{9(a^2-5a+3)^2}=\dfrac{1}{a^2-5a+3}$

$\Rightarrow a=\dfrac{2}{3}$.

The correct option is (A)

144. $(1-p)^2+p(1-p)+(1-p)=0$(since $(1-p)$ is a root of the equation $x^2+px+(1-p)=0$)

$\Rightarrow \quad (1-p)(1-p+p+1)=0$

$\Rightarrow \quad 2(1-p)=0 \Rightarrow (1-p)=0 \Rightarrow p=1$

Now, the sum of roots is $\alpha+\beta=-p$ and the product $\alpha\beta=1-p=0$ (where $\beta=1-p=0$)

$\Rightarrow \quad \alpha+0=-1 \Rightarrow \alpha=-1 \Rightarrow$ Roots are 0, –1

The correct option is (C)

145. Since 4 is one of the root of $x^2+px+12=0$, we have

$16+4p+12=0 \Rightarrow p=-7$

And the equation $x^2+px+q=0$ has equal roots

$\Rightarrow \quad D=49-4q=0 \Rightarrow q=\dfrac{49}{4}$.

The correct option is (A)

146. Let $f'(x)=ax^2+bx+c$, then $f(x)=\dfrac{ax^3}{3}+\dfrac{bx^2}{2}+cx+d$

$\Rightarrow f(x)=\dfrac{1}{6}(2ax^3+3bx^2+6cx+6d)$, Now $f(1)=f(0)=d$,

then according to Rolle's theorem

$\Rightarrow f'(x)=ax^2+bx+c=0$ has at least one root in (0, 1)

The correct option is (A)

147. $x^2-(a-2)x-a-1=0$

Sum of roots, $\alpha+\beta=a-2$

Product, $\alpha\beta=-(a+1)$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=a^2-2a+6=(a-1)^2+5$

$\Rightarrow \quad a=1$

The correct option is (A)

148. Let α, $\alpha+1$ be the roots of the equation, then

$\alpha+(\alpha+1)=b$

$\alpha(\alpha+1)=c$

$\therefore \quad b^2-4c=(2\alpha+1)^2-4\alpha(\alpha+1)=1$

The correct option is (D)

149. $\frac{-b}{a} = 2k < 10 \Rightarrow k < 5$

or, $\frac{c}{a} = k^2 + k - 5 < 25 \Rightarrow (k+6)(k-5) < 0$

$\Rightarrow k < 5$

$\Rightarrow k \in (-\infty, 4)$

The correct option is (C)

150. Equation $x^2 - 2mx + m^2 - 1 = 0$

$\Rightarrow (x - m)^2 - 1 = 0$

$\Rightarrow (x - m + 1)(x - m - 1) = 0$

$\Rightarrow x = m - 1, m + 1$

According to question

$-2 < m - 1, m + 1 < 4$

$\Rightarrow m > -1$ and $m < 3$

$\Rightarrow -1 < m < 3.$

The correct option is (C)

151. Let $y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$, then

$3x^2(y - 1) + 9x(y - 1) + 7y - 17 = 0$

Now, $D \geq 0$ ($\because$ x is real) implies that

$81(y - 1)2 - 4x^3(y - 1)(7y - 17) \geq 0$

$\Rightarrow (y - 1)(y - 41) \leq 0 \Rightarrow 1 \leq y \leq 41$

The correct option is (C)

152. Given equation $x^2 + ax + 1 = 0$

The sum and product of the roots

$\alpha + \beta = -a$

$\alpha\beta = 1$

Now, $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$\Rightarrow \quad |\alpha - \beta| = \sqrt{a^2 - 4}$

$\Rightarrow \quad \sqrt{a^2 - 4} < \sqrt{5}$

$\Rightarrow \quad a^2 - 4 < 5 \Rightarrow a^2 - 9 < 0$

$\Rightarrow \quad a \in (-3, 3)$

The correct option is (A)

153. Let α and 4β be roots of $x^2 - 6x + a = 0$ and α, 3β be the roots of $x^2 - cx + 6 = 0$, then

$\alpha + 4\beta = 6$ and $4\alpha\beta = a$

$\alpha + 3\beta = c$ and $3\alpha\beta = 6$

We get $\alpha\beta = 2 \Rightarrow a = 8$

So the first equation is $x^2 - 6x + 8 = 0 \Rightarrow x = 2, 4$

Now, if $\alpha = 2$ and $4\beta = 4$ then $3\beta = 3$

If $\alpha = 4$ and $4\beta = 2$, then $3\beta = 3/2$ (non-integer)

$\therefore$ common root is $x = 2$.

The correct option is (D)

154. Given equation $bx^2 + cx + a = 0$ has imaginary roots

$\Rightarrow c^2 - 4ab < 0 \Rightarrow c^2 < 4ab \Rightarrow -c^2 > -4ab$

Since $3b^2 > 0$, the expression $3b^2x^2 + 6bcx + 2c^2$ has minimum value.

Minimum value

$$= \frac{4(3b^2)(2c^2) - 36b^2c^2}{4(3b^2)} = -\frac{12b^2c^2}{12b^2} = -c^2 > -4ab.$$

The correct option is (C)

155. $x^2 - x + 1 = 0 \qquad \Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2}$

$x = \frac{1 \pm \sqrt{3}i}{2}$

$\alpha = \frac{1}{2} + i\frac{\sqrt{3}}{2}, \qquad \beta = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

$$\alpha^{2009} + \beta^{2009} = 2\cos 2009\left(\frac{\pi}{3}\right)$$

$$= 2\cos\left[668\pi + \pi + \frac{2\pi}{3}\right] = 2\cos\left(\pi + \frac{2\pi}{3}\right)$$

$$= -2\cos\frac{2\pi}{3} = -2\left(-\frac{1}{2}\right) = 1$$

The correct option is (B)

156. $e^{\sin x} - e^{-\sin x} = 4 \ \Rightarrow e^{\sin x} = t$

$t - \frac{1}{t} - 4$

$t^2 - 4t - 1 - 0 \ \Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2}$

$\Rightarrow t = \frac{4 \pm 2\sqrt{5}}{2} \quad \Rightarrow t = 2 \pm \sqrt{5}$

$e^{\sin x} = 2 \pm \sqrt{5} \quad -1 \leq \sin x \leq 1 \quad \frac{1}{e} \leq e^{\sin x} \leq e$

$e^{\sin x} = 2 + \sqrt{5}$ not possible

$e^{\sin x} = 2 - \sqrt{5}$ not possible

$\therefore$ Hence no solution

The correct option is (B)

157. If $2x^3 + 3x + k = 0$ has 2 distinct real roots in [0, 1], then $f'(x)$ will change sign.

But $f'(x) = 6x^2 + 3 > 0$

So, no value of k exists.

The correct option is (C)

158. Both the roots of the equation $x^2 + 2x + 3 = 0$ are imaginary.

Since, $a, b, c \in R$.

So, if one root is common then both roots are common

Hence, $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

$a : b : c = 1 : 2 : 3.$

The correct option is (D)

159. $a^2 = 3t^2 - 2t$

For non-integral solution

$0 < a^2 < 1$

$a \in (-1,0) \cup (0,1)$.

[**Note:** It is assumed that a real solution of given equation exists.]

The correct option is (A)

160. $x^2 = 6x + 2 \Rightarrow \alpha^2 = 6\alpha + 2$

$\Rightarrow \quad \alpha^{10} = 6\alpha^9 + 2\alpha^8 \quad (1)$

and $\quad \beta^{10} = 6\beta^9 + 2\beta^8 \quad (2)$

Subtract (2) from (1)

$\Rightarrow a_{10} = 6a_9 + 2a_8$

$\Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3.$

The correct option is (B)