CHAPTER 24

PROBABILITY

Chapter Highlights

Remarks, Types of Events, Probability of Occurrence of an Event, Some Important Results on Probability, Conditional Probability, Multiplication Theorem of Probability, Law of Total Probability, Baye's Theorem, Probability Distribution, Binomial Distribution, Mathematical Expectation, Poisson Distribution.

Experiment An operation which results in some well defined outcome is called an experiment.

Random Experiment An experiment whose outcome cannot be predicted with certainty is called a random experiment.

For example, tossing of a fair coin or throwing an unbiased die or drawing a card from a well shuffled pack of 52 cards is a random experiment.

Sample Space The set of all possible outcomes of a random experiment is called the sample space, and it is usually denoted by S.

For example, if we toss a coin, there are two possible outcomes, a head (H) or a tail (T). So, the sample space in this experiment is given by $S = \{H, T\}$.

Event A subset of the sample space S is called an Event.

REMARKS

(i) Sample space S plays the same role as the universal set for all problems related to the particular experiment.

(ii) φ is also a subset of S which is called an **impossible event.**

(iii) S is also a subset of S which called a **sure event.**

TYPES OF EVENTS

Simple Event An event having only a single sample point is called a simple event.

For example, when a coin is tossed, and we denote

$E_1 = \{H\}$ = the event of occurrence of head and

$E_2 = \{T\}$ = the event of occurrence of tail

Then, E_1 and E_2 are simple events in the sample space $S = \{H, T\}$.

Mixed Event A subset of the sample space S which contains more than one element is called a mixed event.

For example, when a coin is tossed, and we denote

$E = \{H, T\}$ = the event of occurrence of a head or a tail.

Then, E is a mixed event in the sample space $S = \{H, T\}$.

Equally Likely Events A set of events is said to be equally likely if none of them is expected to occur in preference to the other.

For example, when a fair coin is tossed, then occurrence of head or tail are equally likely events and there is no reason to expect a 'head' or a 'tail' in preference to the other.

Exhaustive Events A set of events is said to be exhaustive when a random experiment always results in the occurrence of atleast one of them.

For example, if a die is thrown, then the events

$A_1 = \{1, 2\}$ and $A_2 = \{2, 3, 4\}$

are not exhaustive as we can get 5 as outcome of the experiment which is not the member of any of the events A_1 and A_2. But, if we consider the events $E_1 = \{1, 2, 3\}$ and $E_2 = \{2, 4, 5, 6\}$, then the set of events E_1, E_2 is exhaustive.

Mutually Exclusive Events A set of events is said to be mutally exclusive if occurrence of one of them precludes the occurrence of any of the remaining events.

In other words, events $E_1, E_2, \ldots, E_n$ are mutually exclusive if and only if

$$E_i \cap E_j = \varphi \text{ for } i \neq j$$

For example, when a coin is tossed, the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events.

Independent Events Two events are said to be independent, if the occurence of one does not depend on the occurrence of the other.

For example, when a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.

CAUTION

If two events A and B are mutually exclusive, they would be strongly dependent as the occurrence of one precludes the occurrence of other.

Complement of an Event The complement of an event E, denoted by $\bar{E}$ or E' or E^c, is the set of all sample points of the space other than the sample points in E.

For example, when a die is thrown, we get the sample space

$S = \{1, 2, 3, 4, 5, 6\}$

If $E = \{1, 2, 3, 4\}$, then $\bar{E} = \{5, 6\}$

Note that $E \cup \bar{E} = S$

Mutually Exclusive and Exhaustive Events A set of events $E_1, E_2, ..., E_n$ of a sample space S form a mutually exclusive and exhaustive system of events, if

(i) $E_i \cap E_j = \varphi$ for $i \neq j$

(ii) $E_1 \cup E_2 \cup ... \cup E_n = S$

For example, when a die is thrown, we get the sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let $E_1 = \{1, 3, 5\}$ = the event of occurrence of an odd number and

$E_2 = \{2, 4, 6\}$ = the event of occurrence of an even number.

Then, $E_1 \cup E_2 = S$ and $E_1 \cap E_2 = \varphi$

NOTE

- Independent events are always taken from different experiments, while mutually exclusive events are taken from a single experiment.
- Independent events can happen together while mutually exclusive events cannot happen together.
- Independent events are connected by the word "and" but mutually exclusive events are connected by the word "or".

PROBABILITY OF OCCURRENCE OF AN EVENT

Let S be a sample space. Then the probability of occurrence of an event A is denoted by $P(A)$ and is defined as

$$P(A) = \frac{n(A)}{n(S)} = \frac{\text{Number of elements in } A}{\text{Number of elements in } S}$$

$$= \frac{\text{Number of cases favourable to event } A}{\text{Total number of cases}}$$

SOLVED EXAMPLES

1. A car is parked among n cars standing in row, but not at either end. On his return, the owner find that exactly r of the n places are still occupied. The probability that both the places neighbouring has car are empty is

(A) $\frac{{}^{n-r}C_2}{{}^{n-1}C_2}$ (B) $\frac{(n-r)(n-r-1)}{(n+1)(n+2)}$

(C) $\frac{(r-2)!}{(n-2)!}$ (D) none of these

Solution: (A)

Total number of selection of places for $(r-1)$ cars (except the owner's car) out of $(n-1)$ places

$$= {}^{n-1}C_{r-1} = \frac{(n-1)!}{(r-1)!\,(n-r)!}$$

If neighbouring place are empty, then $(r-1)$ cars must be parked in $(n-3)$ places

So, the favourable cases $= {}^{n-3}C_{r-1} = \frac{(n-3)!}{(r-1)!\,(n-r-2)!}$

$\therefore$ Required probability

$$= \frac{(n-3)!}{(r-1)!\,(n-r-2)!} \times \frac{(r-1)!\,(n-r)!}{(n-1)!}$$

$$= \frac{(n-1)(n-r-1)}{(n-1)(n-2)} = \frac{{}^{n-r}C_2}{{}^{n-1}C_2}$$

2. There are 100 students in a collage class of which 36 are boys studying statistics and 13 girls not studying statistics. If there are 55 girls in all, the probability that a boy picked up at random is not studying statistics, is

(A) $\frac{3}{5}$ (B) $\frac{2}{5}$

(C) $\frac{1}{5}$ (D) none of these

Solution: (C)

Number of students = 100

Number of girls = 55

∴ Number of boys = 100 – 55 = 45

Out of 45 boys 36 boys are studying Statistics.

∴ Number of boys not studying Statistics = 45 – 36 = 9

∴ Probability that a boy picked up at random is not studyng Statistics = $\frac{9}{45} = \frac{1}{5}$.

3. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that value of the determinant chosen is positive is

(A) $\frac{16}{81}$ (B) $\frac{7}{16}$

(C) $\frac{3}{16}$ (D) none of these

Solution: (C)

Since each element of the determinant can be placed in two ways 0 or 1, total number of ways = 2^4 = 16.

Since value of the determinant is + ve, so we have only 3 cases:

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

Hence, the required probability = $\frac{3}{16}$.

4. An unbiased dice with faces 1, 2, 3, 4, 5 and 6 is round 4 times. Out of four face values obtained the probability that the minimum face value obtained the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is

(A) $\frac{16}{81}$ (B) $\frac{1}{81}$

(C) $\frac{80}{81}$ (D) $\frac{65}{81}$

Solution: (A)

In a single throw the favourable points are 2, 3, 4 and 5 whose number is 4.

∴ All possible outcomes are 6

∴ P = Probability that in a single throw the minimum face value is not less than 2 and the maximum face value is not greater than 5 = $\frac{4}{6} = \frac{2}{3}$

Since the dice is rolled four times and all the four throws are independent events therefore the required probability

$$= \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

5. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral equals

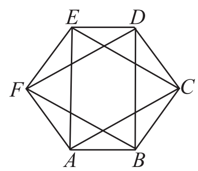
(A) $\frac{1}{2}$ (B) $\frac{1}{5}$

(C) $\frac{1}{10}$ (D) $\frac{1}{20}$

Solution: (C)

3 vertices out of 6 can be chosen in $^6C_3$ ways

$$= \frac{6!}{3!\,3!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$



Only 2 equilateral triangles can be formed, ΔAEC and ΔBDF.

∴ Favourable ways = 2

Required probability = $\frac{2}{20} = \frac{1}{10}$

6. Five persons entered the lift cabin on the ground floor of an 8 floor house. Suppose that each of them independently and with equal probability, can leave the cabin at any floor beginning with the first. The probability of all five persons leaving at different floors, is

(A) $\frac{^8P_5}{7^4}$ (B) $\frac{^9P_5}{7^6}$

(C) $\frac{^7P_5}{7^5}$ (D) none of these

Solution: (C)

Besides the ground floor, there are seven floors. The total number of ways in which each of the five persons can leave cabin at any of the 7 floors = 7^5.

And the favourable number of ways, that is, the number of ways in which 5 persons leave at different floors is $^7P_5$.

∴ The required probability = $^7P_5/7^5$.

7. A student is given a true-false exam with 10 questions. If he gets 8 or more correct answers, he passes the exam. Given that he guesses at the answer to each question, the probability that he passes the exam, is

(A) $\frac{6}{128}$ (B) $\frac{9}{128}$

(C) $\frac{7}{128}$ (D) none of these

Solution: (C)

n = total no. of ways = $2^{10} = 1024$.
Since each answer can be true or false.
and m = favourable number of ways

$= {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = 45 + 10 + 1 = 56.$

since to pass the exam, he must give 8 or 9 or 10 true answers,

Hence, $p = \dfrac{m}{n} = \dfrac{56}{1024} = \dfrac{7}{128}$

8. In a multiple choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks all the correct answers. The candidate decides to tick answers at random. If he is allowed upto three chanes to answer the question, the probability that he will get marks in the question, is

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{5}$

(C) $\dfrac{1}{3}$ (D) none of these

Solution: (B)

The total number of ways of ticking the answers in any one attempt = $2^4 - 1 = 15$.
It is reasonable to assume that in order to derive maximum benefit, the three solutions which he will submit must be all different.

$\therefore$ n = total no. of ways = ${}^{15}C_3$

m = the no. of ways in which the correct solution is excluded = ${}^{14}C_3$.

Hence, the required probability

$$= 1 - \frac{{}^{14}C_3}{{}^{15}C_3} = 1 - \frac{4}{5} = \frac{1}{5}$$

9. Four tickets marked 00, 01, 10 and 11 respectively are placed in a bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on the ticket is 15, is

(A) $\dfrac{3}{1024}$ (B) $\dfrac{5}{1024}$

(C) $\dfrac{7}{1024}$ (D) none of these

Solution: (B)

Let S be the sample space and E be the required event.
Now, $n(S)$ = total number of cases = $4^5 = 1024$.
and $n(E)$ = coefficient of x^{15} in $(x^0 + x + x^{10} + x^{11})^5$

= coefficient of x^{15} in $[(1 + x)^5 (1 + x^{10})^5]$

= coefficient of x^{15} in

$(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)$

$\times (1 + 5x^{10} + 10x^{20} + \ldots) = 5$

$\therefore$ Required probability, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{5}{1024}$.

10. An ordinary cube has four blank faces, one face marked 2, another marked 3. Then the probability of obtaining a total of exactly 12 in 5 throws is

(A) $\dfrac{5}{1296}$ (B) $\dfrac{5}{1944}$

(C) $\dfrac{5}{2592}$ (D) none of these

Solution: (C)

n = Total number of ways = 6^5
To find the favourable no. of ways, a total of 12 in 5 throws can be obtained in the following two ways only:

(i) One blank and four 3's.

or (ii) Three 2's and two 3's.

The no. of ways in case (i) = ${}^5C_1 = 5$
and the no. of ways in case (ii) = ${}^5C_2 = 10$.
$\therefore$ m = the favourable no. of ways.

$= 5 + 10 = 15$

Hence the required probability = $\dfrac{15}{6^5} = \dfrac{5}{2592}$.

11. A person draws a card from a pack of playing cards, replaces it and shuffles the pack. He continues doing this until he shows a spade. The chance that he will fail the first two times is

(A) $\dfrac{9}{64}$ (B) $\dfrac{1}{64}$

(C) $\dfrac{1}{16}$ (D) $\dfrac{9}{16}$

Solution: (A)

Required probability

$$= \frac{{}^{39}C_1}{{}^{52}C_1} \times \frac{{}^{39}C_1}{{}^{52}C_1} \times \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64}$$

12. The sum of two postive quantities is equal to $2n$. The probability that their product is not less than $\dfrac{3}{4}$ times their greatest product is

(A) $\dfrac{3}{4}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{1}{4}$ (D) none of these

Solution: (B)

Let one of the quantities be x. Thus the other is $2n - x$. Then product will be greatest when they are equal i.e., each is n in which case the product is n^2. By the given condition

$$x(2n-x) \geq \frac{3}{4}n^2$$

$$\Rightarrow \; 8nx - 4x^2 \geq 3n^2 \Rightarrow 4x^2 - 8nx + 3n^2 \leq 0$$

$$\Rightarrow \; (2x-3n)(2x-n) \leq 0 \Rightarrow \frac{n}{2} \leq x \leq \frac{3}{2}\,n.$$

$\therefore$ favourable number of cases $= \dfrac{3n}{2} - \dfrac{n}{2} = n.$

Hence, required probability $= \dfrac{n}{2n} = \dfrac{1}{2}.$

13. Four digit numbers are formed using each of the digits 1, 2,...,8 only once. One number from these is picked up at random. The probability that the selected number contains unity is

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$

(C) $\frac{1}{4}$ (D) none of these

Solution: (A)

Required probability

$$= \frac{{}^7C_3}{{}^8C_4} = \frac{\dfrac{7\times6\times5}{1\times2\times3}}{\dfrac{8\times7\times6\times5}{1\times2\times3\times4}} = \frac{4}{8} = \frac{1}{2}$$

14. A book contains 1,000 pages. A page is chosen at random. The probabilities that the sum of the digits of the marked number on the page is equal to 9, is

(A) $\frac{23}{500}$ (B) $\frac{11}{200}$

(C) $\frac{7}{100}$ (D) none of these

Solution: (B)

The number of numbers whose sum is 9 is

One digit number = 1

Two digit numbers = 9

Three digit numbers $= 9+8+7+, ..., +1 = \dfrac{9\times10}{2} = 45$

$\therefore$ Required probability $= \dfrac{55}{1000} = \dfrac{11}{200}.$

15. A binary number is made up of 8 digits. Suppose that the probability of an incorrect digit appearing is p and that of errors in different digits are independent of each other. The the probability of forming an incorrect number, is

(A) $\frac{p}{8}$ (B) p^8

(C) $(1-p)^8$ (D) $1-(1-p)^8$

Solution: (D)

Probability for an incorrect digit $= p$

$\therefore$ Probability for a correct digit $= 1-p$

$\therefore$ Probability for 8 correct digits $= (1-p)^8$

Hence, required probability $= 1-(1-p)^8$.

16. An elevator starts with m passengers and stops at n floors ($m \leq n$). The probability that no two passengers alight at the same floor is

(A) $\frac{{}^nP_m}{m^n}$ (B) $\frac{{}^nP_m}{n^m}$

(C) $\frac{{}^nC_m}{m^n}$ (D) $\frac{{}^nC_m}{n^m}$

Solution: (B)

Since a person can alight at any one of n floors. Therefore, the number of ways in which m passengers can alight at n floors is $\underbrace{n\times n\times n\times\ldots\times n}_{m\,-\,\text{times}} = n^m$.

The number of ways in which all passengers can alight at different floors is ${}^nC_m \times m! = {}^nP_m$.

Hence, required probability $= \dfrac{{}^nP_m}{n^m}$.

17. Four tickets marked 00, 01, 10, 11 respectively are placed in a bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on tickets thus drawn is 23, is

(A) $\frac{25}{256}$ (B) $\frac{100}{256}$

(C) $\frac{231}{256}$ (D) none of these

Solution: (A)

The total number of ways in which 4 tickets can be drawn 5 times $= 4^5 = 1024$.

Favourable number of ways of getting a sum of 23

$= $ coeff. of x^{23} in $(x^{00} + x^{01} + x^{10} + x^{11})^5$

$= $ coeff. of x^{23} in $[(1+x)(1+x^{10})]^5$

$= $ coeff. of x^{23} in $(1+x)^5(1+x^{10})^5$

$= $ coeff. of x^{23} in

$(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)$

$\times(1 + 5x^{10} + 10x^{20} + 10x^{30} + \ldots) = 100$

$\therefore$ required probability $= \dfrac{100}{1024} = \dfrac{25}{256}$.

18. 5 grils and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15. The probability that end seats are occupied by the girls and between any two girls odd number of boys sit, is

(A) $\dfrac{20 \times 10! \times 5!}{15!}$ (B) $\dfrac{20 \times 10!}{15!}$

(C) $\dfrac{20 \times 5!}{15!}$ (D) none of these

Solution: (A)

There are four gaps in between the girls where the boys can sit. Let the number of boys in these gaps be $2a + 1, 2b + 1, 2c + 1, 2d + 1 = 0$, then

$2a + 1 + 2b + 1 + 2c + 1 + 2d + 1 = 10$

or $a + b + c + d = 3$

The number of solutions of above equation

$=$ coefficient of x^3 in $(1 - x)^{-4} = {}^6C_3 = 20$

Thus boys and girls can sit in $20 \times 10! \times 5!$ ways

Total ways $= 15!$

Hence, the required probability $= \dfrac{20 \times 10! \times 5!}{15!}$.

19. The probability that the 13th day of a randomly chosen month is a Friday, is

(A) $\dfrac{1}{12}$ (B) $\dfrac{1}{7}$

(C) $\dfrac{1}{84}$ (D) none of these

Solution: (C)

Any month out of 12 months, can be chosen with probability $= \dfrac{1}{12}$.

There are 7 possible ways in which the month can start and it will be a Friday on 13th day if the first day of the month is Sunday. Its probability $= \dfrac{1}{7}$.

Hence, the required probability $= \dfrac{1}{12} \times \dfrac{1}{7} = \dfrac{1}{84}$.

20. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $(n + 1)$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then n is equal to

(A) 10 (B) 11

(C) 12 (D) 13

Solution: (A)

Both heads appear on n coins and head and a tail appear on $(n + 1)$ coins so

$$P(\text{head}) = \frac{{}^nC_1}{{}^{2n+1}C_1} \cdot 1 + \frac{{}^{n+1}C_1}{{}^{2n+1}C_1} \cdot \frac{1}{2} = \frac{31}{42}$$

$$\Rightarrow \frac{n}{2n+1} + \frac{n+1}{2(2n+1)} = \frac{31}{42}$$

$\Rightarrow 2n + n + 1 = (2n + 1)(31/21)$

$\Rightarrow 63n + 21 = 62n + 31 \Rightarrow n = 10$

21. Two players A and B toss a fair coin cyclically in the following order $A, A, B, A, A, B, \ldots$ till a head shows. Let $\alpha(\beta)$ denote the probability that $A(B)$ gets the head first. Then

(A) $\alpha = 6/7$ (B) $\alpha = 5/7$

(C) $\beta = 1/7$ (D) $\beta = 2/7$

Solution: (A, C)

P(A wins the game)

$= P(H \text{ or } TH \text{ or } TTTH \text{ or } TTTTH \text{ or } TTTTTTH \text{ or } \ldots)$

$= P(H) + P(TH) + P(TTTH) + P(TTTTH) + \ldots$

$$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^7} + \ldots$$

$$= \left(\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \ldots\right) + \left(\frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^8} + \ldots\right)$$

$$= \frac{1/2}{1 - 1/8} + \frac{1/4}{1 - 1/8} = \frac{4}{7} + \frac{2}{7} = \frac{6}{7}$$

$\therefore \beta = 1 - \alpha = 1 - 6/7 = 1/7$

22. The value of C for which $P(X = k) = Ck^2$ can serve as the probability function of a random variable X that takes values 0, 1, 2, 3, 4 is

(A) 1/30 (B) 1/10

(C) 1/3 (D) 1/15

Solution: (A)

$$\sum_{k=0}^{4} P(X = k) = 1 \Rightarrow \sum_{k=0}^{4} Ck^2 = 1$$

$$\Rightarrow C(1^2 + 2^2 + 3^2 + 4^2) = 1 \Rightarrow C = \frac{1}{30}$$

For example, when a coin is tossed, we get the sample space

$S = \{H, T\}$

If E = event of occurrence of a head $= \{H\}$.

then, $P(A)$ = Probability of occurrence of head $= \dfrac{n(A)}{n(S)}$

$= \dfrac{1}{2}$.

Mathematical Expectation of an Event

If a trial is conducted n times, then the total number of trials favourable to an event is approximately $n \cdot P(E)$.

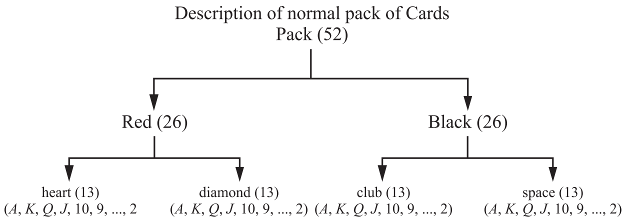


Fig. 24.1

This product is called *mathematical expectation* of the event E.

Heart, Diamond, Club, Spade are four colours or suits of a pack. King cards, Queen cards and Jack cards are called face cards. Thus, there are 12 face cards in a normal pack.

Odds in Favour of an Event and Odds against an Event

If the number of ways in which an event can occur be m and the number of ways in which it does not occur be n, then

(i) odds in favour of the event = $\dfrac{m}{n}$

(ii) odds against the event = $\dfrac{n}{m}$

If odds in favour of an event are $a:b$, then the probability of the occurrence of that event is $\dfrac{a}{a+b}$ and the probability of the non-occurrence of that event is $\dfrac{a}{a+b}$.

23. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cubes of the odds against the second. The chances of happening of each event are

(A) $\dfrac{1}{9}, \dfrac{1}{3}$ (B) $\dfrac{1}{6}, \dfrac{1}{9}$

(C) $\dfrac{1}{3}, \dfrac{1}{6}$ (D) none of these

Solution: (A)

Let the chance of the second event be p. Then the chance of the first event is p^2.

$\therefore$ Odds against the first event are as $1 - p^2 : p^2$
and odds against the second event are $1 - p : p$
Hence, according to the condition given in the question, we have

$$\frac{1-p^2}{p^2} = \left(\frac{1-p}{p}\right)^3$$

or $$\frac{(1-p)(1+p)}{p^2} = \frac{(1-p)^3}{p^3}$$

or $p(p+1) = (1-p)^2$
or $p^2 + p = p^2 - 2p + 1$ or $3p = 1$

$\therefore$ $p = \dfrac{1}{3}$ and $p^2 = \dfrac{1}{9}$

Hence, the probability of the first event = $\dfrac{1}{9}$

TRICK(S) FOR PROBLEM SOLVING

Verbal description of Event	*Equivalent set theoretic Notation*
1. Both A and B occur	(i) $A \cap B$
2. atleast one of A and B occurs	(ii) $A \cup B$
3. A occurs but not B	(iii) $A - B$ or $A \cap \overline{B}$
4. Neither A nor B occurs	(iv) $\overline{A} \cap \overline{B}$
5. Only A occurs	(v) $A \cap \overline{B} \cap \overline{C}$
6. Both A and B, but not C occur	(vi) $A \cap B \cap \overline{C}$
7. All the three events occur	(vii) $A \cap B \cap C$
8. At least one occurs	(viii) $A \cup B \cup C$
9. At least two occur	(ix) $(A \cap B) \cup (B \cap C) \cup (A \cap C)$

Verbal description of Event	*Equivalent set theoretic Notation*
10. One and no more occurs	(x) $(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C) \cup (\bar{A} \cap \bar{B} \cap C)$
11. Exactly two of A, B and C occur	(xi) $(A \cap B \cap \bar{C}) \cup (\bar{A} \cap B \cap C) \cup (A \cap \bar{B} \cap C) \cup (A \cap \bar{B} \cap C)$
12. None occurs	(xii) $\bar{A} \cap \bar{B} \cap \bar{C} = \overline{A \cup B \cup C}$ ▷
13. Not more than two occur	(xiii) $(A \cap B) \cup (B \cap C) \cup (A \cap C) - (A \cap B \cap C)$
14. Exactly one of A and B occurs	(xiv) $(A \cap \bar{B}) \cup (\bar{A} \cap B)$

SOME IMPORTANT RESULTS ON PROBABILITY

1. $0 \le P(A) \le 1$, i.e., the probability of occurrence of an event is a number lying between 0 and 1.
2. $P(\varphi) = 0$, i.e., probability of occurrence of an impossible event is 0.
3. $P(S) = 1$, i.e., probability of occurrence of a sure event is 1.
4. $P(\bar{A}) = 1 - P(A)$
5. If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
6. If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$
7. If A, B, C are any three events, then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
8. If A, B, C are mutually exclusive events, then $A \cap B = \varphi, B \cap C = \varphi, C \cap A = \varphi, A \cap B \cap C = \varphi$ and hence, $P(A \cap B) = 0$, $P(B \cap C) = 0$, $P(C \cap A) = 0$, $P(A \cap B \cap C) = 0$
   $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
9. $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$
10. $P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$
11. $P(A) = P(A \cap B) + P(A \cap \bar{B})$
12. $P(B) = P(B \cap A) + P(B \cap \bar{A})$
13. If $A_1, A_2, \ldots$, An are independent events, then
    $P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2), \ldots, P(A_n)$
14. If $A_1, A_2, \ldots, A_n$ are mutually exclusive events, then $P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$
15. If $A_1, A_2, \ldots, A_n$ are exhaustive events, then $P(A_1 \cup A_2 \cup \ldots \cup A_n) = 1$
16. If $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive events, then
    $P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n) = 1$
17. If $A_1, A_2, \ldots, A_n$ are n events, then
    (a) $P(A_1 \cup A_2 \cup \ldots \cup A_n) \le P(A_1) + P(A_2) +, \ldots, + P(A_n)$
    (b) $P(A_1 \cap A_2 \cap \ldots \cap A_n) \ge 1 - P(\bar{A}_1) - P(\bar{A}_2), \ldots, - P(\bar{A}_n)$

IMPORTANT POINTS

If A and B are independent events, then

(i) A and $\bar{B}$ are independent events.
(ii) $\bar{A}$ and B are independent events.
(iii) $\bar{A}$ and $\bar{B}$ are independent events.
(iv) $P(A \cup B) = 1 - P(\bar{A}) \cdot P(\bar{B})$.

SOLVED EXAMPLES

24. If two events A and B are such that $P(\bar{A}) = 0.3$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$, then $P(B/A \cup \bar{B})$ is equal to

(A) $\frac{5}{8}$ (B) $\frac{7}{8}$

(C) $\frac{3}{8}$ (D) $\frac{1}{2}$

Solution: (C)

We have, $P(A \cup \bar{B}) = P(A) + P(\bar{B}) + P(A \cap \bar{B})$

$= [1 - P(\bar{A})] + [1 - P(B)] - [P(A) - P(A \cup B)]$

$= (1 - 0.3) + (1 - 0.5) - (0.7 - 0.3) = 0.8$

$\therefore \quad P(B/A \cup \bar{B}) = \dfrac{P(B \cap (A \cup \bar{B})}{P(A \cup \bar{B})}$

$= \dfrac{P((B \cap A) \cup (B \cap \bar{B})}{P(A \cup \bar{B})}$

$= \dfrac{P(A \cap B)}{P(A \cup \bar{B})} = \dfrac{0.3}{0.8} = \dfrac{3}{8}$

25. One of the two events must happen. Given that the chance of one is two-third of the other, the odds in favour of the other are

(A) 3 : 5 (B) 2 : 5

(C) 3 : 2 (D) none of these

Solution: (C)

Let the given events be E_1 and E_2

Then $P(E_1) = \dfrac{2}{3}\, P(E_2)$

The events E_1 and E_2 are exhaustive

$\therefore \quad P(E_1 \cup E_2) = 1$

Since E_1 and E_2 are mutually exclusive

$P(E_1) + P(E_2) = 1$

$\Rightarrow \quad \dfrac{2}{3}\, P(E_2) + P(E_1) = 1 \Rightarrow \dfrac{5}{3}\, P(E_2) = 1 \Rightarrow P(E_2) = \dfrac{3}{5}$

$\therefore \quad P(E_2) = \dfrac{3}{5}$ and $P(E_1) = \dfrac{2}{3} \times \dfrac{3}{5} = \dfrac{2}{5}$

Since $P(E_2) = \dfrac{3}{5}$, odds in favour of E_2 are 3 : 5 – 3 *i.e.* 3 : 2.

26. A student takes his examination in four subjects α, β, γ, δ. He estimates his chance of passing in α as $\dfrac{4}{5}$, in β as $\dfrac{3}{4}$, in γ as $\dfrac{5}{6}$ and in δ as $\dfrac{2}{3}$. To qualify he must pass in α and at least two other subjects. The Probability that he qualifies is

(A) $\dfrac{34}{90}$ (B) $\dfrac{61}{90}$

(C) $\dfrac{53}{90}$ (D) none of these

Solution: (B)

$P(\alpha) = \dfrac{4}{5},\ P(\bar{\alpha}) = 1 - \dfrac{4}{5} = \dfrac{1}{5}$

$P(\beta) = \dfrac{3}{4},\ P(\bar{\beta}) = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

$P(\gamma) = \dfrac{5}{6},\ P(\bar{g}) = 1 - \dfrac{5}{6} = \dfrac{1}{6}$

$P(\delta) = \dfrac{2}{3},\ P(\bar{\delta}) = 1 - \dfrac{2}{3} = \dfrac{1}{3}$

Different possibilities to qualify are

(i) passes in α, β, γ and fails in δ

(ii) passes in α, β, δ and fails in γ

(iii) passes in α, γ, δ and fails in β

(iv) passes in all the four sujbects α, β, γ and δ

These are mutually exclusive possibilities.

$\therefore$ Required probability

$= \left(\dfrac{4}{5} \times \dfrac{3}{4} \times \dfrac{5}{6} \times \dfrac{1}{3}\right) + \left(\dfrac{4}{5} \times \dfrac{3}{4} \times \dfrac{2}{3} \times \dfrac{1}{6}\right)$

$+ \left(\dfrac{4}{5} \times \dfrac{5}{6} \times \dfrac{2}{3} \times \dfrac{1}{4}\right) + \left(\dfrac{4}{5} \times \dfrac{3}{4} \times \dfrac{5}{6} \times \dfrac{2}{3}\right)$

$= \dfrac{1}{6} + \dfrac{1}{15} + \dfrac{1}{9} + \dfrac{1}{3} = \dfrac{15 + 6 + 10 + 30}{90} = \dfrac{61}{90}$

27. Seven chits are numbered 1 to 7. Four are drawn one by one with replacements. The probability that the least number on any selected chit is 5, is

(A) $\left(\dfrac{2}{7}\right)^4$ (B) $4 \cdot \left(\dfrac{2}{7}\right)^4$

(C) $\left(\dfrac{3}{7}\right)^3$ (D) none of these

Solution: (C)

P(5 or 6 or 7) in one draw $= \dfrac{3}{7}$

$\therefore$ Probability that in each of 3 draws, the chit bears 5 or

6 or 7 $= \left(\dfrac{3}{7}\right)^3$

28. For any two independent events E_1 and E_2, $P\{(E_1 \cup E_2) \cap (\bar{E}_1) \cap (\bar{E}_2)\}$ is

(A) $< \dfrac{1}{4}$ (B) $> \dfrac{1}{4}$

(C) $\geq \dfrac{1}{2}$ (D) none of these

Solution: (A)

Since $\bar{E}_1 \cap \bar{E}_2 = \overline{E_1 \cup E_2}$

and $(E_1 \cup E_2) \cap (\overline{E_1 \cup E_2}) = \varphi$

$\therefore \quad P[(E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2)] = P(\varphi) = 0 < \dfrac{1}{4}$

29. The probabilites of three events A, B and C are $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$, then

(A) $0.2 \leq P(B \cap C) \leq 0.35$
(B) $0.5 \leq P(B \cap C) \leq 0.85$
(C) $0.1 \leq P(B \cap C) \leq 0.35$
(D) none of these

Solution: (A)

$P(A \cap B) = P(A) + P(B) - P(A \cup C)$
$= 0.6 + 0.4 - 0.8 = 0.2$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C)$

$\Rightarrow P(B \cap C) = 1.2 - P(A \cup B \cup C)$ ...(1)

$\because 0.85 \leq P(A \cup B \cup C) \leq 1$

$\therefore$ (1) $\Rightarrow P(B \cap C) \leq 1.2 - 0.85$

and $P(B \cap C) \geq 1.2 - 1 \Rightarrow 0.2 \leq P(B \cap C) \leq 0.35$

30. A man is known to speak truth 3 out of 4 times. He throws a dice and reports that it is six. The probability that it is actually six is

(A) $\frac{3}{8}$ (B) $\frac{1}{5}$

(C) $\frac{3}{5}$ (D) none of these

Solution: (A)

Let A denotes the event that a six occurs and B the event that the man reports that it is a six.
Then the probability that it is actually a six is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Now $P(A \cap B) = \frac{1}{6} \times \frac{3}{4} = \frac{3}{24}$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$$= \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4} = \frac{8}{24}$$

Hence $P(A/B) = \frac{3/24}{8/24} = \frac{3}{8}$

31. The odds in favour of standing first of three students appearing in an examination are 1 : 2, 2 : 5 and 1 : 7 respectively. The probability that either of them will stand first, is

(A) $\frac{125}{168}$ (B) $\frac{75}{168}$

(C) $\frac{32}{168}$ (D) $\frac{4}{168}$

Solution: (A)

Let the three student be P, Q and R
Let A, B, C denote the events of stading first of the three students P, Q, R respectively,
Given, odds in favour of $A = 1:2$
odds in favour of $B = 2:5$
and odds in favour of $C = 1:7$

$$\therefore P(A) = \frac{1}{1+2} = \frac{1}{3}, P(B) = \frac{2}{2+5} = \frac{2}{7},$$

$$P(C) = \frac{1}{1+7} = \frac{1}{8}$$

Since events A, B, C are mutually exclusive

$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$$= \frac{1}{3} + \frac{2}{7} + \frac{1}{8} = \frac{56+48+21}{168} = \frac{125}{168}$$

32. A, B, C are three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(AB) = 0.08$, $P(AC) = 0.28$, $P(ABC) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then

(A) $0.23 \leq P(BC) \leq 0.32$
(B) $0.41 \leq P(BC) \leq 0.52$
(C) $0.23 \leq P(BC) \leq 0.48$
(D) none of these

Solution: (C)

Let $P(BC) = x$

Since $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$

$\therefore = 0.3 + 0.4 + 0.8 - 0.08 - x - 0.28 + 0.09 = 1.23 - x$

But given $P(A \cup B \cup C) \geq 0.75$ and $P(A \cup B \cup C) \leq 1$

$\therefore 0.75 \leq 1.23 - x \leq 1 \Rightarrow -0.75 \geq -1.23 + x \geq -1$

or $1.23 - 0.75 \geq x \geq 1.23 - 1$ or $0.23 \leq x \leq 0.48$

33. A die is loaded so that the probability of face i is proportional to i, $i = 1, 2, \ldots, 6$. The probability of an even number occuring when the die is rolled, is

(A) $\frac{2}{7}$ (B) $\frac{4}{7}$

(C) $\frac{3}{7}$ (D) none of these

Solution: (B)

Since the probability of the faces are proportional to the numbers on them, we can take the probabilities of faces, 1, 2,... 6 as k, $2k$, ..., $6k$, respectively.
Since one of the faces must occur, we have

$$k + 2k + 3k + 4k + 5k + 6k = 1$$

or $k = \frac{1}{21}$.

$\therefore$ The probability of an even number

$= 2k + 4k + 6k = 12k = 12 \times \frac{1}{21} = \frac{4}{7}$

and the probability of the second event $= \frac{1}{3}$.

34. In the game odd man out, each of $m \geq 2$ person tosses a coin to determine who will buy refreshments for the entire group. The odd man out is the one with a different outcome from the rest. The probability that there is a loser in any game is

(A) $\frac{m-1}{2^{m-1}}$ (B) $\frac{m}{2^{m-1}}$

(C) $m/2^m$ (D) none of these

Solution: (B)

Let S = the sample space, then $n(S) = 2^m$

Let E_1 = the event that one persons gets a head whereas each of the remaining $(m - 1)$ persons get a tail

and E_2 = the event that one persons gets a tail whereas each of the remaining $(m - 1)$ persons get a head.

The required event $E = E_1 \cup E_2$

$\therefore\ n(E) = n(E_1) + n(E_2)$

$= {}^mC_1{}^{m-1}C_{m-1}1 \cdot 1^{m-1} + {}^mC_1{}^{m-1}C_{m-1}1.1^{m-1}$

$= m + m = 2m$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{2m}{2^m} = \frac{m}{2^{m-1}}$.

35. Probbility is 0.45 that a dealer will sell at least 20 television sets during a day, and the probability is 0.74 that he will sell less than 24 televisions. The probability that he will sell 20, 21, 22, or 23 televisions during the day, is

(A) 0.19 (B) 0.32

(C) 0.21 (D) none of these

Solution: (A)

Let A be the event that the sale is at least 20 televisions, i.e., 20, 21, 22, ... and B be the event that sale is less than 24, i.e., 0, 1, 2, 3, ..., 23. Then $A \cap B$ will denote the sale of 20, 21, 22 and 23 televisions. We are given $P(A) = 0.45$ and $P(B) = 0.74$.

It is required to find $P(A \cap B)$.

Also $P(A \cup B) = P$(sale of 0, 1, 2, 3, ..., 20, 21, 22, 23, televisions) $= P(S) = 1$.

From addition rule, required probability is

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.45 + 0.74 - 1 = 0.19$

36. An investment consultant predicts that the odds against the price of a certain stock will go up during the next week are 2 : 1 and the odds in favour of the price remaining the same are 1 : 3. The probability that the price of the stock will go down during the next week, is

(A) $\frac{4}{12}$ (B) $\frac{5}{12}$

(C) $\frac{7}{12}$ (D) none of these

Solution: (B)

Let A denote the event 'stock price will go up', and B be the event 'stock price will remain same'.

Then $P(A) = 1/3$ and $P(B) = 1/4$

$\therefore$ P(Stock price will either go up or remain same) is

$$P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

Hence probability that stock price will go down is given by,

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - (7/12) = 5/12$

37. An *MBA* applies for a job in two firms X and Y. The probability of his being selected in firim X is 0.7 and being rejected at Y is 0.5. The probability of at least one of his applications being rejected is 0.6. The probability that he will be selected in one of the firms, is

(A) 0.6 (B) 0.4

(C) 0.8 (D) none of these

Solution: (C)

Let A and B denote the events that the person is selected in firms X and Y respectively. Then in the usual notations, we are given:

$P(A) = 0.7 \Rightarrow P(\bar{A}) = 1 - 0.7 = 0.3$

$P(\bar{B}) = 0.5 \Rightarrow P(B) = 1 - 0.5 = 0.5$

and $P(\bar{A} \cup \bar{B}) = 0.6$

The probability that the persons will be selected in one of the two firms X or Y is given by

$P(A \cup B) = 1 - P(\bar{A} \cap \bar{B})$

$= 1 - [P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cup \bar{B})]$

$= 1 - (0.3 + 0.5 - 0.6) = 0.8.$

38. The probabilities of three events A, B and C are $P(A) = 0.6$, $P(B) = 0.4$, $P(C) = 0.5$, also $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cup B \cup C) \geq 0.85$, $P(A \cap B \cap C) = 0.2$ and $P(B \cap C) = p_1$. Then

(A) $p_1 \geq 0.35$, (B) $p_1 \leq 0.2$

(C) $0.2 \leq p_1 \leq 0.35$ (D) none of these

Solution: (C)

Given $P(A) = 0.6$, $P(B) = 0.4$, $P(C) = 0.5$,
$P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$
$P(A \cup B \cup C) = 0.2$
and $0.85 \le P(A \cup B \cup C) \le 1$
Now $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.2$
$P(A \cup B \cup C) = 0.6 + 0.4 + 0.5 - 0.2 - 0.3 - p_1 + 0.2$
$\Rightarrow p_1 = 1.2 - P(A \cup B \cup C)$
Since $0.85 \le P(A \cup B \cup C) \le 1$
we have $0.2 \le p_1 \le 0.35$

39. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and
$P(A \cup B) = P(A) + P(B) - P(A) P(B)$, then
(A) $P(B \cap A') = P(B) - P(A)$
(B) $P(A' \cup B') = P(A') + P(B')$
(C) $P(A \cup B)' = P(A')P(B')$
(D) $P(A/B) = P(A)$.

Solution: (C, D)

$P(A \cup B) = P(A) + P(B) - P(A) P(B)$
$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A) P(B)$
$\Rightarrow P(A \cap B) = P(A) P(B)$
$\therefore$ A and B are independent events.
$\Rightarrow P(B \cap A') = P(B) P(A') \ne P(B) - P(A)$
and $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B) = 1 - P(A) P(B)$
$\ne 1 - P(A) + 1 - P(B) = P(A') + P(B')$
Also $P[(A \cup B)'] = P(A' \cap B') = P(A') P(B')$
Since, A and B are independent $P(A|B) = P(A)$.

40. A and B are two events. Odds against A are 2 : 1. Odds in favour of $A \cup B$ are 3 : 1. If $x \le P(B) \le y$, then the ordered pair (x, y) is

(A) $\left(\frac{5}{12}, \frac{3}{4}\right)$ (B) $\left(\frac{2}{3}, \frac{3}{4}\right)$
(C) $\left(\frac{1}{3}, \frac{3}{4}\right)$ (D) none of these

Solution: (A)

We have, $P(A) = \frac{1}{3}$ and $P(A \cap B) = \frac{3}{4}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{4} = \frac{1}{3} + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{12} = P(B) - P(A \cap B)$

$\Rightarrow P(B) = \frac{5}{12} + P(A \cap B) \Rightarrow P(B) \ge \frac{5}{12}$ ...(1)

Again, $P(B) = \frac{5}{12} + P(A \cap B)$

$\Rightarrow P(B) \le 5/12 + P(A)$ $[\because P(A \cap B) \le P(A)]$

$\Rightarrow P(B) \le \frac{5}{2} + \frac{1}{3} = \frac{3}{4}$ ...(2)

From (1) and (2), we obtain $5/12 \le P(B) \le 3/4$
Hence, $x = 5/12$ and $y = 3/4$.

41. Let A, B and C be three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$, If $P(A \cup B \cup C) \ge 0.75$, then
(A) $0.23 \le P(B \cap C) \le 0.48$
(B) $0.23 \le P(B \cap C) \le 0.75$
(C) $0.48 \le P(B \cap C) \le 0.75$
(D) none of these

Solution: (A)

Since $P(A \cup B \cup C) \ge 0.75$, therefore
$0.75 \le P(A \cup B \cup C) \le 1$
$\Rightarrow 0.75 \le P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C)) - P(A \cap C) + P(A \cap B \cap C) \le 1$
$\Rightarrow 0.75 \le 0.3 + 0.4 + 0.8 - 0.08$
$- P(B \cap C) - 0.28 + 0.09 \le 1$
$\Rightarrow 0.75 \le 1.23 - P(B \cap C)) \le 1$
$\Rightarrow -0.48 \le -P(B \cap C) \le -0.23$
$\Rightarrow 0.23 \le P(B \cap C) \le 0.48$

Mutual Independence and Pairwise Independence

Three events A, B, C are said to be mutually independent if $P(A \cap B) = P(A) \cdot P(B)$, $P(A \cap C) = P(A) \cdot P(C)$, $P(B \cap C) = P(B) \cdot P(C)$ and $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

These events would be pairwise independent if, $P(A \cap B) = P(A) \cdot P(B)$, $P(B \cap C) = P(B) \cdot P(C)$ and $P(A \cap C) = P(A) \cdot P(C)$.

Thus mutually independent events are pairwise independent but the converse may not be true.

SOLVED EXAMPLES

42. A man alternately tosses a coin and throws a dice beginning with the coin. The probability that he gets a head before he gets 5 or 6 in the dice is

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$
(C) $\frac{1}{3}$ (D) none of these

Solution: (A)

Probability of getting head $= \frac{1}{2}$ and probability of throwing 5 or 6 with a dice $= \frac{2}{6} = \frac{1}{3}$.

He starts with a coin and alternatively tosses the coin and throws the dice and he will win if he gets a head before he gets 5 or 6. Hence

Probability

$$= \frac{1}{2} + \left(\frac{1}{2}\cdot\frac{2}{3}\right)\cdot\frac{1}{2} + \left(\frac{1}{2}\cdot\frac{2}{3}\right)\cdot\left(\frac{1}{2}\cdot\frac{2}{3}\right)\times\frac{1}{2} + \ldots$$

$$= \frac{1}{2}\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \ldots\right] = \frac{1}{2}\cdot\frac{1}{1-\frac{1}{3}} = \frac{1}{3}\times\frac{3}{2} = \frac{3}{4}$$

43. A bag contains 10 mangoes out of which 4 are rotten, two mangoes are taken out together. If one of them is found to be good, the probability that other is also good, is

(A) $\frac{1}{3}$ (B) $\frac{8}{15}$

(C) $\frac{5}{18}$ (D) $\frac{2}{3}$

Solution: (C)

Number of ways of selecting 2 good mangoes

$= {}^6C_2 = 15$

The number of ways that at least one of the two selected mangoes is to be good

$= {}^6C_1 \times {}^9C_1 = 6 \times 9 = 54$

Required probability $= \frac{15}{54} = \frac{5}{18}$.

44. Ram and Shyam throw with one dice for a prize of Rs 88 which is to be won by the player who throws 1 first. If Ram starts, then mathematical expectation for Shyam is

(A) Rs 32 (B) Rs 40

(C) Rs 48 (D) none of these

Solution: (B)

Probability of winning of Shyam

$$= \frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \ldots$$

$$= \frac{5}{36}\left[1 + \frac{25}{36} + \left(\frac{25}{36}\right)^2 + \ldots\right]$$

$$= \frac{5}{36}\cdot\frac{1}{1-25/36} = \frac{5}{36}\times\frac{36}{11} = \frac{5}{11}$$

Mathematical expectation for Shyam $= \text{Rs } \frac{5}{11} \times 88$

= Rs 40

45. An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses is needed is

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$

(C) $\frac{1}{5}$ (D) $\frac{2}{3}$

Solution: (B)

p = probability of successs (S) $= \frac{2}{6} = \frac{1}{3}$

q = probability of failure (F) $= 1 - \frac{1}{3} = \frac{2}{3}$

Probability that the success occurs in even number of tosses

$= P(FS) + P(FFFS) + P(FFFFFS) + \ldots$

$$= qp + q^3p + q^5p +, \ldots, = \frac{qp}{1-q^2}$$

$$= \frac{2/3\times 1/3}{1-(2/3)^2} = \frac{2/9}{1-4/9} = \frac{2}{9}\times\frac{9}{5} = \frac{2}{5}$$

46. If A and B are such events that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A/B})$ is equal to

(A) $1 - P(A/B)$ (B) $1 - (P(\bar{A}/B)$

(C) $\frac{1 - P(A\cup B)}{P(\bar{B})}$ (D) $\frac{P(\bar{A})}{P(\bar{B})}$

Solution: (C)

$$P(\bar{A}/\bar{B}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$$

$$= \frac{P(\overline{A\cup B})}{P(\bar{B})} = \frac{1-P(A\cup B)}{P(\bar{B})}$$

47. A bag contains four tickets with numbers 112, 121, 211, 222. One ticket is drawn at random from the bag. Let E_i (i = 1, 2, 3) denote the event that i-th digit on the drawn ticket is 2. Then

(A) E_1, E_2, E_3 are pair wise independent

(B) E_1, $\bar{E}_2$ are independent

(C) $\bar{E}_2$ and $\bar{E}_3$ are not independent

(D) E_1, E_2, E_3 are mutually independent.

Solution: (A, B)

$P(E_1) = 2/4 = 1/2$, $P(E_2) = 2/4 = 1/2$.

Similarly $P(E_3) = 1/2$

$P(E_1E_2) = 1/4$, $P(E_1E_3) = 1/4$

$P(E_2E_3) = 1/4$, $P(E_1E_2E_3) = 1/4$

$\therefore$ $P(E_1E_2) = P(E_1)\cdot P(E_2)$

$P(E_2E_3) = P(E_2)\cdot P(E_3)$

$P(E_1E_3) = P(E_1)\cdot P(E_3)$.

$\therefore$ E_1, E_2, E_3 are pair-wise independent, so (*a*) is correct.

Also $P(E_1\bar{E}_2) = 1/4 = P(E_1)\times P(\bar{E}_2)$

So E_1 and $\bar{E}_2$ are independent, so (*b*) is also true.

$P(\bar{E}_2\bar{E}_3) = 1/4 = P(\bar{E}_2)\times P(\bar{E}_3)$

$\therefore$ $\bar{E}_2$ and $\bar{E}_3$ are independent, i.e., (*c*) is not correct.

Finally $P(E_1E_2E_3) = 1/4 \neq P(E_1)\cdot P(E_2)\cdot P(E_3)$

So, E_1, E_2, E_3, are not mutually independent.

46. A person draws a card from a pack, replaces it, shuffles the pack, again draws a card, replaces it and draws again. This he does until he draws a heart. The prob-ability that he will have to make at least four draws is

(A) $\dfrac{27}{256}$ (B) $\dfrac{175}{256}$

(C) $\dfrac{27}{64}$ (D) none of these

Solution: (C)

Probability of drawing a heart $= \dfrac{13}{52} = \dfrac{1}{4}$

P(He required at least 4 draws for heart)

$$= \left(\frac{3}{4}\right)^3\cdot\frac{1}{4} + \left(\frac{3}{4}\right)^4\cdot\frac{1}{4} + \left(\frac{3}{4}\right)^5\cdot\frac{1}{4} + \ldots$$

$$= \frac{\left(\frac{3}{4}\right)^3\cdot\frac{1}{4}}{1-\left(\frac{3}{4}\right)} = \frac{27}{64}$$

TRICK(S) FOR PROBLEM SOLVING

- **Probability regarding *n* letters and their envelopes**

  If *n* letters corresponding to *n* envelopes are placed in the envelopes at random, then

  (i) Probability that all letters are in rights envelopes = $1/n!$.

  (ii) Probability that all letters are not in right envelopes $= 1 - \dfrac{1}{n!}$.

  (iii) Probability that no letter is in right envelopes $= \dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \ldots + (-1)^n\dfrac{1}{n!}$

  (iv) Probability that exactly *r* letters are in right envelopes $= \dfrac{1}{r!}\left[\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \ldots + (-1)^{n-r}\dfrac{1}{(n-r)!}\right]$

CONDITIONAL PROBABILITY

If *A* and *B* are any two events, then $P(A/B)$ denotes the conditional probability of occurrence of event *A*, when *B* has already occurred.

MULTIPLICATION THEOREM OF PROBABILITY

If *A* and *B* are any two events, then

$P(A/B) = \dfrac{P(A\cap B)}{P(B)}$ or $P(A \cap B) = P(B)\cdot P(A/B)$

Important Results

1. $P(B/A) = \dfrac{P(A\cap B)}{P(A)}$, or $P(A\cap B) = P(A)\cdot P(B/A)$.
2. If *A* and *B* are independent events, then $P(B/A) = P(B)$.
3. If *A* and *B* are two events such that $B \neq \varphi$, then $P(A/B) + P(\bar{A}/B) = 1$.
4. If A and B are two events such that $A \neq \varphi$, then $P(B) = P(A)\cdot P(B/A) + P(\bar{A})\cdot P(B/\bar{A})$

LAW OF TOTAL PROBABILITY

Let *S* be the sample space and let $E_1, E_2, \ldots, E_n$ be *n* mutually exclusive and exhaustive events associated with a random experiment. If *A* is any event which occurs with E_1 or E_2 or...E_n, then

$$P(A) = P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) +, \ldots, + P(E_n)\cdot P(A/E_n)$$

SOLVED EXAMPLES

47. If $\bar{E}$ and $\bar{F}$ are the complementary events of events *E* and *F* respectively and if $0 < P(F) < 1$, then

(A) $P(E|F) + P(\bar{E}|F) = 1$

(B) $P(E|F) + P(E|\bar{F}) = 1$

(C) $P(\bar{E}|F) + P(E|\bar{F}) = 1$

(D) $P(E|\bar{F}) + P(\bar{E}|\bar{F}) = 1$

Solution: (A)

$P(E/F) + P(\bar{E}|F)$

$$= \frac{P(E\cap F)}{P(F)} + \frac{P(\bar{E}\cap F)}{P(F)} = \frac{P(E\cap F) + P(\bar{E}\cap F)}{P(F)}$$

$$= \frac{P[(E\cap F)\cup(\bar{E}\cap F)]}{P(F)}$$

($\because$ $E\cap F$ and $\bar{E}\cap F$ are disjoint)

$$= \frac{P[(E \cup \bar{E}) \cap F]}{P(F)} = \frac{P(S \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1.$$

48. The probability that certain electronic conponent fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for one year is 0.99. The probability that a new component will last for one year is

(A) 0.891 (B) 0.692
(C) 0.92 (D) none of these

Solution: (A)

Given: probability that electronic component fails when first used = 0.10 i.e., $P(F) = 0.10$

$\therefore$ $P(F') = 1 - P(F) = 0.90$

and let $P(Y)$ = Probability of new component to last for one year

Obviously the two events are mutually exclusive and exhaustive.

$\therefore$ $P(Y/F) = 0$ and $P(Y/F') = 0.99$

$$\therefore P(Y) = P(F) \cdot P\left(\frac{Y}{F}\right) + P(F') \cdot P\left(\frac{Y}{F'}\right)$$

$= 0.10 \times 0 + 0.90 \times 0.99 = 0 + (0.9)(0.99) = 0.891.$

49. Three groups A, B, C are contesting for position on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3, 0.2 respectively. If the group A wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for group B and C are 0.6 and 0.5 respectively. The probability that the new product will be introduced, is

(A) 0.52 (B) 0.63
(C) 0.74 (D) none of these

Solution: (B)

Given $P(A) = 0.5$, $P(B) = 0.3$ and $P(C) = 0.2$

$\therefore$ $P(A) + P(B) + P(C) = 1$

So the events A, B, C are exhaustive.

If $P(E)$ = Probability of introducing a new product, then as given

$P(E/A) = 0.7$, $P(E/B) = 0.6$ and $P(E/C) = 0.5$

$$\therefore P(E) = P(A) \cdot P\left(\frac{E}{A}\right) + P(B) \cdot P\left(\frac{E}{B}\right) + P(C) \cdot P\left(\frac{E}{C}\right)$$

$= 0.5 \times 0.7 + 0.3 \times 0.6 + 0.2 \times 0.5$

$= 0.35 + 0.18 + 0.10 = 0.63$

50. If $E = E_1 E_2 E_3 E_4 E_5$ and $P(E_1) = \frac{95}{100}$, $P(E_2/E_1) = \frac{94}{99}$, $P(E_3/E_1E_2) = \frac{93}{98}$, $P(E_4/E_1E_2E_3) = \frac{92}{97}$ and $P(E_5/E_1E_2E_3E_4) = \frac{91}{96}$, then $P(E) =$

(A) $\frac{91 \cdot 92 \cdot 93 \cdot 94}{97 \cdot 98 \cdot 99 \cdot 100}$ (B) $\frac{91 \cdot 92 \cdot 93 \cdot 94 \cdot 95}{96 \cdot 97 \cdot 98 \cdot 99 \cdot 100}$

(C) $\frac{94 \cdot 95 \cdot 96}{98 \cdot 99 \cdot 100}$ (D) none of these

Solution: (B)

$\because$ $P(E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5)$ or $P(E_1 E_2 E_3 E_4 E_5)$
$= P(E_1) \cdot P(E_2/E_1) \cdot P(E_3/E_1E_2) \cdot P(E_4/E_1E_2E_3)$
$\cdot P(E_5/E_1E_2E_3E_4)$

$$\therefore P(E) = \frac{95}{100} \times \frac{94}{99} \times \frac{93}{98} \times \frac{92}{97} \times \frac{91}{96}$$

$$= \frac{91 \bullet 92 \bullet 93 \bullet 94 \bullet 95}{96 \bullet 97 \bullet 98 \bullet 99 \bullet 100} \quad (\because E = E_1 E_2 E_3 E_4 E_5)$$

51. A certain player, say X, is known to win with probability 0.3 if the track is fast and 0.4 if the track is slow. For Monday, there is a 0.7 probability of a fast track and 0.3 probability of a slow track. The probability that player X will win a Monday, is

(A) 0.22 (B) 0.11
(C) 0.33 (D) none of these

Solution: (C)

Let us define the events:

W: Player wins on Monday

F: Track is fast

S: Track is slow

We have given: $P(F) = 0.7$, $P(S) = 0.3$, $P(W/F) = 0.3$, $P(W/F) = 0.3$, $P(W/S) = 0.4$.

$\therefore$ P(player X will win on Monday)
$= P(W) = P(W \cap F) + P(W \cap S)$
$= P(F) \times P(W/F) + P(S) \times P(W/S)$
$= 0.7 \times 0.3 + 0.3 \times 0.4 = 0.33$

52. If two events A and B are such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then $P(B/A \cup B')$ equals

(A) 3/4 (B) 5/6
(C) 1/4 (D) 3/7.

Solution: (C)

$$P(B|A \cup B') = \frac{P[B \cap (A \cup B')]}{P(A \cup B')}$$

$$= \frac{P[(B \cap A) \cup (B \cap B')]}{P(A) + P(B') - P(A \cap B')}$$

But, $P[(B \cap A) \cup (B \cap B') = P(B \cap A)$
$= P(A) - P(A \cap B') = 0.2$

and $P(A) + P(B') - P(A \cap B') = 0.7 + 0.6 - 0.5 = 0.8$

$\therefore \quad P(B|A \cup B') = \frac{0.2}{0.8} = \frac{1}{4}$

53. For two events A and B if $P(A) = P(A|B) = 1/4$ and $P(B|A) = 1/2$, then

(A) A and B are mutually exclusive
(B) A and B are independent
(C) A is subevent to B
(D) $P(A'|B) = 3/4$

Solution: (D, B)

$P(A) = P(A|B) = P(A \cap B)/P(B)$

$\Rightarrow \quad P(A \cap B) = P(A)\,P(B)$

Thus, A and B are independent.

Also, $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{3}{4}$

BAYE'S THEOREM

Consider any event 'A' of the sample space 'S'. This event would have occurred due to the different causes (or due to the occurrence of any of the events $E_1, E_2, \ldots E_n$).

Now let us assume that event A is found to have occurred and we have to find the probability that it has occurred due to the occurrence of cause, say, E_i. That means, we are interested in finding $P(E_i/A)$. These types of problems are solved with the help of Baye's theorem.

Statement Let S be the sample space and let $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ... or E_n, then

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)},\ i = 1, 2, \ldots, n$$

The probability $P(E_i)$ and $P(E_i/A)$ are known as priori and posteriori probabilities, respectively.

NOTE

$P(A/E_i)$ gives the contribution of E_i in the occurrence of A.

PROBABILITY DISTRIBUTION

Random Variable A random variable is a real valued function whose domain is the sample space of a random experiment.

A random variable is usually denoted by the capital letters $X, Y, Z, \ldots$, etc.

(a) Discrete Random Variable A random variable which can take only finite or countably infinite number of values is called a discrete random variable.

(b) Continuous Random Variable A random variable which can take any value between two given limits is called a continuous random variable.

Probability Distribution of a Random Variable

If the values of a random variable together with the corresponding probabilites are given, then this description is called a probability distribution of the random variable.

For example, if two coins are tossed, then the probability of 0, 1 and 2 heads occurred is given by,

$$P(X = 0) = P(TT) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(X = 1) = P(HT) + P(TH) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

$$P(X = 2) = P(HH) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Thus, the probability distribution of number of heads, when two coins are tossed is given as:

X:	0	1	2
P(X):	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

BINOMIAL DISTRIBUTION

Consider a random experiment and an event E associated with it.

Let p = Probability of occurrence of event E in one trial.

and $q = 1 - p$ = Probability of non-occurrence of event E in one trial.

If X denotes the number of successes in n trials of the random experiment, then the probability of occurrence of events E (also called success) is given by

Then $P(X = r)$ = Probability of r successes

$$= {}^nC_r\, p^r\, q^{n-r}$$

TRICK(S) FOR PROBLEM SOLVING

- Probability of at most 'r' successes in n trials

$$= \sum_{n=0}^{r} {}^nC_4 p^r q^{n-r}$$

- Probability of at least 'r' successes in n trials

$\sum_{n=r}^{n} {}^nC_4\, p^r q^{n-r}$

- Probability of having first success at the r^{th} trial $= p \cdot q^{r-1}$

SOLVED EXAMPLES

54. A fair coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then the value of n is

(A) 14 (B) 12
(C) 24 (D) 36

Solution: (A)

We have, $P(X = 6) = P(X = 8)$

$$\Rightarrow \quad {}^nC_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{n-6} = {}^nC_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{n-8}$$

$$\Rightarrow \quad {}^nC_6\left(\frac{1}{2}\right)^n = {}^nC_8\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad {}^nC_6 = {}^nC_8 = {}^nC_{n-8} \quad \Rightarrow \quad 6 = n - 8 \quad \text{or} \quad n = 14$$

55. In order to get atleast once a head with probability ≥ 0.9, the number of times a coin needs to be tossed is

(A) 3 (B) 4
(C) 5 (D) none of these

Solution: (B)

Probability of getting atleast one head in n tosses.

$$= 1 - \left(\frac{1}{2}\right)^n \geq 0.9$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n \leq 0.1 \Rightarrow 2^n \geq 10 \Rightarrow n \geq 4$$

Hence, least value of $n = 4$.

56. If X and Y are the independent random variables $B\left(5, \frac{1}{2}\right)$ and $B\left(7, \frac{1}{2}\right)$, then $P(X + Y \geq 1) =$

(A) $\frac{4095}{4096}$ (B) $\frac{309}{4096}$

(C) $\frac{4032}{4096}$ (D) none of these

Solution: (A)

For random variable X, $n = 5$, $p = \frac{1}{2}$ and for random variable Y, $n = 7$, $p = \frac{1}{2}$

Now $P(X + Y \geq 1) = 1 - P(X + Y < 1) = 1 - P(X + Y = 0)$

$= 1 - P(X = 0, Y = 0)$

$= 1 - P(X = 0)\, P(Y = 0)$

($\because$ X and Y are independent)

$$= 1 - {}^5C_0\left(\frac{1}{2}\right)^5 \cdot {}^7C_0\left(\frac{1}{2}\right)^7$$

$$= 1 - \left(\frac{1}{2}\right)^{12} = \frac{4095}{4096}$$

57. A fair coin is tossed 99 times. If X is the number of times heads occurs $P(X = r)$ is maximum when r, is

(A) 49 (B) 50
(C) 51 (D) none of these

Solution: (A, B)

Putting $n = 99$ and $p = 1/2$, we have $(n + 1)\, p = (100)\left(\frac{1}{2}\right) = 50$, and $(n + 1)\, p - 1 = 49$

For maximum value of $P(X = r)$,

$(n + 1)\, p - 1 \leq r \leq (n + 1)\, p$

$\Rightarrow \quad 49 \leq r \leq 50 \quad \therefore \quad r = 49$ and 50

Hence the maximum value of $P(X = r)$ occurs at $r =$ 50 and 49.

58. Suppose X follows a binomial distribution with parameters n and p, where $0 < p < 1$. If $P(X = r)/P(X = n - r)$ is independent of n and r then p is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) none of these

Solution: (B)

$$\frac{P(X = r)}{P(X = n - r)} = \frac{{}^nC_r\, p^r (1 - p)^{n-r}}{{}^nC_{n-r}\, p^{n-r} (1 - p)^r}$$

$$= \frac{(1 - p)^{n-2r}}{p^{n-2r}} = \left(\frac{1}{p} - 1\right)^{n-2r}$$

For $\left(\frac{1}{p} - 1\right)^{n-2r}$ to be independent of n and r, $\frac{1}{p} - 1 = 1$

$$\Rightarrow \quad \frac{1}{p} = 2 \Rightarrow p = \frac{1}{2}.$$

59. Numbers are selected at random one at a time, from the numbers 00, 01, 02,...,99 with replacement. An event E occurs if and only if the product of the two digits of a selected number is 18. If four numbers are selected, then the probability that E occurs at least 3 times, is

(A) $\frac{97}{390625}$ (B) $\frac{68}{390625}$

(C) $\frac{72}{390625}$ (D) none of these

Solution: (A)

Out of the numbers 00, 01, 02, ..., 99, those numbers the product of whose digits is 18 are 29, 36, 63, 92 *i.e*, only 4.

$p = P(E) = \frac{4}{100} = \frac{1}{25}, q = P(\bar{E}) = 1 - \frac{1}{25} = \frac{24}{25}$

Let X be the random variable, showing the number of times E occurs in 4 selectios.

Then $P(E$ occurs at least 3 times$) = P(X = 3$ or $X = 4)$

$= P(X = 3) + P(X = 4)$

$= {}^4C_3\, p^3\, q^1 + {}^4C_4 p^4 q^0 = 4p^3q + p^4$

$= 4 \times \left(\frac{1}{25}\right)^3 \times \frac{24}{25} + \left(\frac{1}{25}\right)^4 = \frac{97}{390625}$

60. The mean and variance of a binomial variable X are 2 and 1 respectively. The probability that X takes values greater than 1, is

(A) $\frac{5}{16}$ (B) $\frac{9}{16}$

(C) $\frac{11}{16}$ (D) none of these

Solution: (C)

Given: mean $np = 2$ ...(1)

and Variance $npq = 1$ ...(2)

Dividing (2) by (1), then $q = \frac{1}{2}$

$\therefore\ p = 1 - q = \frac{1}{2}$

From (1), $n \times \frac{1}{2} = 2, \quad \therefore \quad n = 4$

The binomial distribution is $\left(\frac{1}{2} + \frac{1}{2}\right)^4$

Now, $P(X > 1) = P(X = 2) + P(X = 3) + P(X = 4)$

$= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^4C_3 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 + {}^4C_4 \left(\frac{1}{2}\right)^4$

$= \frac{6+4+1}{16} = \frac{11}{16}$

61. The probability that a man aged x years will die in a year is p. The probability that out of n men $A_1, A_2, A_3, \ldots, A_n$, each aged x, A_1 will die and be first to die, is

(A) $\frac{1}{n^2}$ (B) $1 - (1 - p)^n$

(C) $\frac{1}{n^2}(1 - (1 - p)^n)$ (D) $\frac{1}{n}(1 - (1 - p)^n)$.

Solution: (D)

The probability that a man does not die in a year $= 1 - p$

$\therefore$ Probability that none of the n men dies in a year $= (1 - p)^n$.

$\therefore$ The probability that at least one man dies in a year $= 1 - (1 - p)^n$.

Since every man can die first, the chance that A_1, will die first is $\frac{1}{n}$. Hence, the probability that A_1, will die within a year and he will be first to die $= \frac{1}{n}[1 - (1 - p)^n]$.

62. A box contains 24 balls of which 12 are black and 12 are white. The balls are drawn form the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is

(A) $\frac{5}{64}$ (B) $\frac{27}{32}$

(C) $\frac{5}{32}$ (D) $\frac{11}{32}$

Solution: (C)

Required probability

= Probability of drawing $3W$ and $3B$ balls in 6 drawn and drawing a white ball in 7th draw

$= {}^6C_3 \left(\frac{12}{24}\right)^3 \left(\frac{12}{24}\right)^3 \left(\frac{12}{24}\right) = \frac{5}{32}$

63. Suppose X follows a binomial distribution with parameters n and p, where $0 < p < 1$. If $P(x = r)/P(X = n - r)$ is independent of n and r, then

(A) $p = \frac{1}{2}$ (B) $p = \frac{1}{3}$

(C) $p = \frac{1}{4}$ (D) none of these

Solution: (A)

We have $\frac{P(X = r)}{P(X = n - r)}$

$= \frac{{}^nc_r\, p^r (1-p)^{n-r}}{{}^nc_{n-r}\, p^{n-r} (1-p)^r} = \frac{(1-p)^{n-2r}}{p^{n-2r}}$

$= \left(\frac{1-p}{p}\right)^{n-2r} = \left(\frac{1}{p} - 1\right)^{n-2r}$

and $\frac{1}{p} - 1 > 0$. $\therefore$ the ratio will be independent of n and r if $\frac{1}{p} - 1 = 1$ or $p = \frac{1}{2}$.

64. A contest consists of predicting the results win, draw or defeat of 7 football matches. A sent his entry by predicting at random. The probability that his entry will contain exactly 4 correct predictions is

(A) $8/3^7$ (B) $16/3^7$
(C) $280/3^7$ (D) $560/3^7$

Solution: (C)

P(correct prediction) = 1/3

P(wrong prediction) = 2/3

P(exactly 4 right predictions) $= {}^7C_4(1/3)^4 \cdot (2/3)^3$
$= 280/3^7$

65. If X follows a binomial distribution with parameters $n = 8$ and $p = 1/2$, then $P(|X-4| \le 2) =$

(A) $\frac{119}{128}$ (B) $\frac{116}{128}$
(C) $\frac{29}{128}$ (D) none of these

Solution: (A)

We have

$P(|X-4| \le 2) = P(-2 \le X-4 \le 2) = P(2 \le X \le 6)$
$= 1 - [P(X=0) + P(X=1) + P(X=7) + P(X=8)]$

$$= 1 - \left[{}^8C_0\left(\frac{1}{2}\right)^8 + {}^8C_1\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8\right]$$

$$= 1 - \left(\frac{1}{2}\right)^8 (1+8+8+1) = 1 - \frac{18}{2^8} = \frac{119}{128}$$

Important Results

1. The mean, variance and standard deviation of a binomial distribution are np, npq and $\sqrt{npq}$ respectively.
2. Mode of binomial distribution is that value of r for which $P(X=r)$ is maximum. In other words, $(n+1)p - 1 \le r \le (n+1)p$.

66. If the mean of a binomial distribution is 25, then its standard deviation lies in the interval given below:

(A) [0, 5) (B) (0, 5]
(C) [0, 25) (D) (0, 25]

Solution: (A)

We have, $np = 25$. Now, $0 \le p < 1$ and $0 \le q \le 1$

$\Rightarrow$ $0 \le npq \le np \Rightarrow 0 \le \sqrt{npq} \le \sqrt{np}$

$\Rightarrow$ $0 \le$ S.D. ≤ 5. But $p \ne 0$, therefore $0 \le$ S.D. < 5

$\Rightarrow$ S.D. $\in [0, 5)$

MATHEMATICAL EXPECTATION

Let X be discrete random variable which assumes the values $x_1, x_2, \ldots, x_n$ with corresponding probabilities $p_1, p_2, \ldots p_n$. Then, the expected value of X, denoted by $E(X)$ is defined as

$$E(X) = \sum_{i=1}^{n} x_i\, p_i, \text{ where } \sum_{i=1}^{n} p_i = 1$$

POISSON DISTRIBUTION

The poisson distribution Let X be a discrete random variable which can take on the values 0, 1, 2,... such that the probability function of X is given by

$$f(x) = P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots$$

where λ is a given positive constant. This distribution is called Poisson distribution and a random variable having this distribution is said to be poisson distributed.

EXERCISES

Single Option Correct Type

1. One hundred identical coins, each with probability p of showing up heads, are tossed. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, the value of p is

(A) 1/2 (B) 49/101
(C) 50/101 (D) 51/101

2. If A and B are two events such that $P(A \cup B) \ge \frac{3}{4}$ and $\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$, then

(A) $P(A) + P(B) \le \frac{11}{8}$ (B) $P(A) \cdot P(B) \le \frac{3}{8}$

(C) $P(A) + P(B) \ge \frac{7}{8}$ (D) none of these

3. A point is selected at random from the interior of a circle. The probability that the point is closer to the centre than the boundary of the circle is

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) none of these

4. From a box containing 20 tickets of value 1 to 20, four tickets are drawn one by one. After each draw, the ticket is replaced. The proability that the largest value of tickets drawn is 15 is

(A) $\left(\frac{3}{4}\right)^4$ (B) $\frac{27}{320}$

(C) $\frac{27}{1280}$ (D) none of these

5. If the integers m and n are chosen at random between 1 and 100 then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is

(A) $\frac{1}{5}$ (B) $\frac{1}{7}$

(C) $\frac{1}{4}$ (D) $\frac{1}{49}$

6. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing is

(A) $\frac{1}{9}$ (B) $\frac{36}{37}$

(C) $\frac{1}{37}$ (D) $\frac{47}{40}$

7. A person draws a card from a pack of 52 playing cards, replaces it and shuffles the pack. He continues doing this until be draws a spade, the chance that he will fail in the first two draws is

(A) $\frac{1}{16}$ (B) $\frac{9}{16}$

(C) $\frac{9}{64}$ (D) $\frac{1}{64}$

8. For two events A and B if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

(A) A is subevent of B

(B) A and B are mutually exclusive

(C) A and B are independent and $P\left(\frac{A'}{B}\right) = \frac{3}{4}$

(D) none of the above

9. A and B throw a dice. The probability that A's throw is not greater than B's is

(A) $\frac{5}{12}$ (B) $\frac{7}{12}$

(C) $\frac{1}{6}$ (D) $\frac{1}{2}$

10. A six faced die is so biased that it is twice likely to show an even number as compared to an odd number when thrown. The die is thorwn twice. The probability that the sum of the two numbers is even is

(A) $\frac{5}{9}$ (B) $\frac{4}{9}$

(C) $\frac{1}{3}$ (D) none of these

11. n biscuits are distributed among N boys at random. The probability that particular boy gets r ($< n$) biscuits is

(A) ${}^nC_r \left(\frac{1}{N}\right)^r \left(\frac{N-1}{N}\right)^{n-r}$ (B) ${}^nC_r \left(\frac{1}{N}\right)^r$

(C) ${}^nC_r$ (D) $\frac{r}{n}$

12. Cards are drawn from a pack of 52 cards one by one. The probability that exactly 10 cards will be drawn before the first ace is

(A) $\frac{451}{884}$ (B) $\frac{241}{1456}$

(C) $\frac{164}{4165}$ (D) none of these

13. A student appears for test I, II and III. The student is successful if he passes either in test I and II or test I and III. The probability of the student passing in tests I, II and III are p, q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(A) $p = 1, q = \frac{1}{2}$ (B) $p = 1, q = 0$

(C) $p = q = 1\frac{1}{2}$ (D) $p = q = 1$.

14. A sum of money is rounded off to the nearest rupee. The probability that round off error is at least ten paise is

(A) $\frac{81}{100}$ (B) $\frac{82}{101}$

(C) $\frac{19}{100}$ (D) $\frac{19}{101}$

15. Sixteen players $S_1, S_2, \ldots S_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assuming that all the palyers are to equal strength. The probability that the players S_1 is among the eight winners is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) none of these

16. Three numbers are chosen at random without replcement from {1, 2,...,10}. The probability that the minimum of the chosen numbers is 3, or their maximum is 7, is

(A) $\frac{7}{40}$ (B) $\frac{5}{40}$

(C) $\frac{11}{40}$ (D) none of these

17. There are four machines and it is known that exactly two of them are faulty. They are tested one by one, in a random order till both the faulty machines are identifed. Then the probability that only two tests are needed is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

18. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(AB^c) = 0.5$, then $P[B \backslash (A \cup B^c)] =$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) none of these

19. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shufled pack of eleven cards numbered 2, 3, 4,...,12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8, is

(A) $\frac{193}{792}$ (B) $\frac{164}{792}$

(C) $\frac{231}{792}$ (D) none of these

20. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probabiity that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is,

(A) $\frac{16}{81}$ (B) $\frac{1}{81}$

(C) $\frac{80}{81}$ (D) $\frac{65}{81}$

21. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02,...,99 with replacement. An even E occurs if and only if the product of the two digits of selected number is 18. If four numbers are selected, the probability that the event E occurs at least 3 times, is

(A) $\frac{99}{(25)^4}$ (B) $\frac{86}{(25)^4}$

(C) $\frac{74}{(25)^4}$ (D) $\frac{97}{(25)^4}$

22. These are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is

(A) $\frac{5}{8}$ (B) $\frac{3}{8}$

(C) $\frac{1}{8}$ (D) none of these

23. The altitude through A of ΔABC meets BC at D and the circumscribed circle at E. If $D \equiv (2, 3)$, $E \equiv (5, 5)$ and the ordinate of the orthocentre being a natural number. The probability that the orthocentre lies on the lines $y = 1, y = 2, y = 3,..., y = 10$ is

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{3}{5}$ (D) none of these

24. Two small squares on a chess board are chosen at random. Probability that they have a common side is

(A) 1/3 (B) 1/9
(C) 1/18 (D) none of these

25. Three winning tickets are drawn from an urn of 100 tickets. The probability of winning for a person who buys 4 tickets is

(A) $\frac{7144}{8085}$ (B) $\frac{941}{8085}$
(C) $\frac{6321}{8085}$ (D) none of these

26. A five digit number is selected at random. Then the probability that the digits in the odd places are odd and in the even places are even (no digit being repeated) is

(A) $\frac{9}{10}$ (B) $\frac{1}{10}$
(C) $\frac{1}{25}$ (D) $\frac{1}{75}$

27. The probabilities of four cricketers *A, B, C* and *D* scoring more than 50 runs in a match are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{10}$. It is known that exactly two of the players scored more than 50 runs in a particular match. The probability that these players were *A* and *B* is

(A) $\frac{27}{65}$ (B) $\frac{5}{6}$
(C) $\frac{1}{6}$ (D) none of these

28. The numbers 1, 2, 3,..., n are arranged in a random order. The probability that the digits 1, 2, 3,...k ($k < n$) appear as neighbours in that order is

(A) $\frac{1}{n!}$ (B) $\frac{k!}{n!}$
(C) $\frac{(n-k)!}{n!}$ (D) $\frac{(n-k+1)!}{n!}$

29. If $a \in [-5, 30]$, then the probability that the graph of the function $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x-axis is

(A) $\frac{27}{35}$ (B) $\frac{8}{25}$
(C) $\frac{8}{35}$ (D) $\frac{17}{25}$

30. Fifteen coupons are numbered 1, 2, 3,... 15. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on the selected coupon is 9, is

(A) $\left(\frac{9}{16}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$
(C) $\left(\frac{3}{5}\right)^7$ (D) none of these

31. A bag contains m white and 3 black balls. Balls are drawn one by one without replacement till all the black balls are drawn. The probability that this procedure for drawing balls will come to an end at the rth draw is

(A) $\frac{(r-1)(r-2)}{(m+1)(m+2)(m+3)}$

(B) $\frac{3(r-1)(r-2)}{(m+1)(m+2)(m+3)}$

(C) $\frac{2(r-1)(r-2)}{(m+1)(m+2)(m+3)}$

(D) none of these

32. Suppose $n(\geq 3)$ persons are sitting in a row. Two of them are selected at random. The probability that they are not together is

(A) $1-\frac{2}{n}$ (B) $\frac{2}{n-1}$
(C) $1-\frac{1}{n}$ (D) none of these

33. A fair die is tossed eight times. Probability that on the eighth throw a third six is observed is

(A) ${}^8C_3 \frac{5^5}{6^8}$ (B) $\frac{{}^7C_2 . 5^5}{6^8}$
(C) $\frac{{}^7C_2 . 5^5}{6^7}$ (D) none of these

34. A natural number x is chosen at random from the first one hundred natural numbers. The probability that $\frac{(x-20)(x-40)}{x-30} < 0$ is

(A) $\frac{1}{50}$ (B) $\frac{9}{50}$
(C) $\frac{3}{25}$ (D) $\frac{7}{25}$

35. Four whole numbers taken at random are multiplied together. What is the chance that the last digit in the product is 1, 3, 7 or 9?

(A) $\frac{16}{625}$ (B) $\frac{1}{210}$

(C) $\frac{8}{125}$ (D) $\frac{4}{25}$

36. Four five-rupee coins, 3 two-rupee coins and 2 one-rupee coins are stacked together in a column at random. The probability that the coins of the same denomination are consecutive is

(A) $\frac{13}{9!}$ (B) $\frac{1}{210}$

(C) $\frac{1}{35}$ (D) none of these

37. Numbers 1, 2, 3,..., $2n$ ($n \in N$) are printed on $2n$ cards. The probability of drawing a number n is proportional to r. Then the probability of drawing an even number in one draw is

(A) $\frac{n+2}{n+3}$ (B) $\frac{n+1}{n+3}$

(C) $\frac{1}{2}$ (D) $\frac{n+1}{2n+1}$

38. Three natural numbers are taken at random from the set $A = \{x : 1 \le x \le 100, x \in N\}$. The probabiulity that the $A.M.$ of the numbers taken is 25 is

(A) $\frac{{}^{77}C_2}{{}^{100}C_3}$ (B) $\frac{{}^{25}C_2}{{}^{100}C_3}$

(C) $\frac{{}^{74}C_2}{{}^{100}C_3}$ (D) none of these

39. $2n$ boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is

(A) $\frac{n}{2n-1}$ (B) $\frac{n-1}{2n-1}$

(C) $\frac{2n-1}{4n^2}$ (D) none of these

40. Consider a set 'P' containing n elements. A subset 'A' of 'P' is drawn and there after set 'P' is reconstructed. Now one more subset 'B' of 'P' is drawn. Probability of drawing sets A and B so that $A \cap B$ has exactly one element is

(A) $(3/4)^n . n$ (B) $n. (3/4)^{n-1}$
(C) $n. (3/4)^n$ (D) none of these

41. A car is parked by an owner amongst 25 cars in a row, not at either end. On his return he finds that exactly 15 places are still occupied. The probability that both the neighbouring places are empty is

(A) $\frac{91}{276}$ (B) $\frac{15}{184}$

(C) $\frac{15}{92}$ (D) none of these

42. In a corner hexagon two diagonals are drawn at random. The probability that diagonals intersect at an interior point of the hexagon is

(A) $\frac{5}{12}$ (B) $\frac{7}{12}$

(C) $\frac{2}{5}$ (D) none of these

43. Six different balls are put in three different boxes, no box being empty. The probability of putting balls in the boxes in equal numbers is,

(A) 3/10 (B) 1/6
(C) 1/5 (D) none of these

44. At a railway staition a passenger leaves his luggage in a locker which is opened by dialling a three digit code (say 253, 009, 325 etc.). The passenger chooses the code, closes the locker and leaves for the town. A strange man, who does not know the code, tries to open the locker by dialling three digits at random. The probability that the locker opens after k trials is (Here $k < 1000$)

(A) $\frac{k}{100}$ (B) $\frac{k}{1000}$

(C) $\frac{1000 \quad k}{1000}$ (D) none of these.

45. The decimal parts of the logarithms of two numbers taken at random are found to six places. Probability that second can be subtracted first one without borrowing is

(A) $\left(\frac{9}{20}\right)^6$ (B) $\frac{1}{2^6}$

(C) $\left(\frac{11}{20}\right)^6$ (D) none of these

46. 10% of a certain population suffer from a serious disease. A person suspected of the disease is given two independent tests. Each test makes a correct diagnosis 90% of the time. The probability that the person really has the illness given that both tests are positive is

(A) 0.5 (B) 0.9
(C) 0.6 (D) none of these

47. An ordinary cube has four blank faces, one face marked 2 and another marked 3. Then the probability of obtaining 9 in 5 throws is

(A) $\frac{31}{7776}$ (B) $\frac{5}{2592}$

(C) $\frac{5}{1944}$ (D) $\frac{5}{1296}$

48. There are n persons ($n \geq 3$), among whom are A and B, who are made to stand in a row in random order. Probability that there is exactly one person between A and B is

(A) $\frac{n-2}{n(n-1)}$ (B) $\frac{2(n-2)}{n(n-1)}$

(C) $2/n$ (D) none os these

49. $x_1, x_2, x_3, \ldots x_{50}$ are fifty real numbers such that $x_r < x_{r+1}$ for $r = 1, 2, 3, \ldots, 49$. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number is

(A) $\frac{{}^{20}C_2 \times {}^{30}C_2}{{}^{50}C_5}$ (B) $\frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$

(C) $\frac{{}^{19}C_2 \times {}^{31}C_3}{{}^{50}C_5}$ (D) none of these

50. An artillery target may be either at point I with the probability $\frac{8}{9}$ or at the point II with probability $\frac{1}{9}$. We have 21 shells each of which can be fired either at point I or II. Each shell may hit the target independently of the other shell with probability $\frac{1}{2}$. The number of shells which must be fired at point I to hit the target with maximum probability is

(A) 9 (B) 10

(C) 11 (D) 12

51. The probablity that the birthdays of six different people will fall in exactly two calendar months is

(A) $\frac{1}{6}$ (B) ${}^{12}C_2 \times \frac{2^6}{12^6}$

(C) ${}^{12}C_2 \times \frac{2^6 - 1}{12^6}$ (D) $\frac{341}{12^5}$

52. The probability that the length of a randomly chosen chord of a circle lies between $\frac{2}{3}$ and $\frac{5}{6}$ of its diameter is

(A) $\frac{5}{6}$ (B) $\frac{1}{16}$

(C) $\frac{1}{4}$ (D) $\frac{5}{12}$

53. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $n + 1$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is $\frac{31}{42}$, then n is equal to

(A) 10 (B) 11

(C) 12 (D) 13

54. One mapping is selected at random from all mappings of the set $S = \{1, 2, 3, \ldots n\}$ into itself. The probability that it is one one is $\frac{3}{32}$. Then the value of n is

(A) 3 (B) 4

(C) 5 (D) 6

55. Suppose n people are asked a question successively in a random order and exactly 3 of the n people know that answer. If $n > 6$, the probability that the first four of those asked do not know the answer is

(A) $\frac{{}^{n-4}C_4}{{}^{n}C_4}$ (B) $\frac{{}^{n-3}C_4}{{}^{n}C_4}$

(C) $\frac{1}{{}^{n}C_4}$ (D) none of these

56. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

(A) $\frac{5}{17}$ (B) $\frac{12}{17}$

(C) $\frac{17}{30}$ (D) $\frac{3}{5}$

57. A digit is selected from each of the following two sets:

$I \;= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$II = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

The probability that the product of the digits so chosen is positive is

(A) $\frac{4}{5}$ (B) $\frac{81}{100}$

(C) $\frac{91}{100}$ (D) none of these

58. A square is inscribed in a circle. If p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square then

(A) $p_1 = p_2$

(B) $p_1 > p_2$ and $p_1^2 - p_2^2 < \frac{1}{3}$

(C) $p_1 < p_2$

(D) none of these

59. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is

(A) $\frac{16}{81}$ (B) $\frac{1}{81}$

(C) $\frac{80}{81}$ (D) $\frac{65}{81}$

60. In a game "odd man out" each of $m \geq 2$ persons, tosses a coin to determine who will buy refreshments for the entire group. The odd man out is the one with a different outcome from the rest. The probability that there is a loser in any game is

(A) $\frac{1}{2^{m-1}}$ (B) $\frac{m-1}{2^{m-1}}$

(C) $\frac{m}{2^{m-1}}$ (D) none of these

61. If $A_1, A_2,..., A_n$ are n independent events such that $P(A_i) = \frac{1}{i+1}$, $i = 1, 2,..., n$. The probability that none of the n events occurs is

(A) $\frac{n}{n+1}$ (B) $\frac{1}{n+1}$

(C) $\frac{n}{(n+1)(n+2)}$ (D) none of these

62. In a multiple-choice question, there are four alternative answers of which one or more than one is correct. A candidate will get marks on the questions only if he ticks all the correct answers. The candidate decides to tick all the ocrrect answers. The candidate decides to tick answers at random. If he is allowed up to three chances to answer the questions, the probability that he will get marks on it is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

63. A box contains n pairs of shoes and $2r$ shoes are selected. ($r < n$). The probability that there is exactly one pair is

(A) $\frac{{}^{n-1}C_{r-1}}{{}^{2n}C_{2r}}$ (B) $\frac{n.{}^{n-1}C_{r-1}}{{}^{2n}C_{2r}}$

(C) $\frac{(n.{}^{n-1}C_{r-1})2^{r-1}}{{}^{2n}C_{2r}}$ (D) none of these

64. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to 31/32, then p lies in the interval

(A) $\left(\frac{11}{12}, 1\right]$ (B) $\left(\frac{1}{2}, \frac{3}{4}\right]$

(C) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (D) $\left[0, \frac{1}{2}\right]$

65. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(A) $P(C \mid D) = \frac{P(D)}{P(C)}$ (B) $P(C \mid D) = P(C)$

(C) $P(C \mid D) \geq P(C)$ (D) $P(C|D) < P(C)$

66. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8]. The probability that their minimum is 3, given that their maximum is 6, is:

(A) 3/8 (B) 1/5

(C) 1/4 (D) 2/5

67. A class consists of 80 students, 25 of them are girls. If 10 of the students are rich and 20 of the students are fair complexioned, then the probability of selecting a fair complexioned rich girl from the class (assuming three traits as independent) is

(A) 1/10 (B) 1/32

(C) 5/512 (D) 7/512

68. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$. Then $P(B)$ is

(A) 1/6 (B) 1/3

(C) 2/3 (D) 1/2

69. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. then $P(A \cup B)$ is

(A) 3/5 (B) 0

(C) 1 (D) 2/5

70. A bag contains $n + 1$ coins. It is known that one of these coins shows heads on both sides, whereas the other coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $\frac{7}{12}$, then the value of n is

(A) 5 (B) 4
(C) 3 (D) 2

71. The probabilites of three events A, B and C are $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$, then

(A) $0.2 \leq P(B \cap C) \leq 0.35$
(B) $0.5 \leq P(B \cap C) \leq 0.85$
(C) $0.1 \leq P(B \cap C) \leq 0.35$
(D) none of these

72. If $\bar{E}$ and $\bar{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then

(A) $P(E|F) + P(\bar{E}|F) = 1$
(B) $P(E|F) + P(E|\bar{F}) = 1$
(C) $P(\bar{E}|F) + P(E|\bar{F}) = 1$
(D) $P(E|\bar{F}) + P(\bar{E}|\bar{F}) = 1$

73. The probability that certain electronic component fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for one year is 0.99. The probability that a new component will last for one year is

(A) 0.891 (B) 0.692
(C) 0.92 (D) none of these

74. Four tickets marked 00, 01, 10 and 11, respectively, are placed in a bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on the ticket is 15 is

(A) $\frac{3}{1024}$ (B) $\frac{5}{1024}$
(C) $\frac{7}{1024}$ (D) none of these

75. If X and Y are the independent random variables for $B\left(5, \frac{1}{2}\right)$ and $B\left(7, \frac{1}{2}\right)$, then $P(X + Y \geq 1) =$

(A) $\frac{4095}{4096}$ (B) $\frac{309}{4096}$
(C) $\frac{4032}{4096}$ (D) none of these

76. The sum of two positive quantities is equal to $2n$. The probability that their product is not less than $\frac{3}{4}$ times their greatest product is

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) none of these

77. A book contains 1,000 pages. A page is chosen at random. The probabilities that the sum of the digits of the marked number on the page is equal to 9 is

(A) $\frac{23}{500}$ (B) $\frac{11}{200}$
(C) $\frac{7}{100}$ (D) none of these

78. The probability that a man aged x years will die in a year is p. The probability that out of n men $A_1, A_2, A_3, \ldots A_n$, each aged x, A_1 will die and be first to die is

(A) $\frac{1}{n^2}$ (B) $1 - (1 - p)^n$
(C) $\frac{1}{n^2}(1 - (1 - p)^n)$ (D) $\frac{1}{n}(1 - (1 - p)^n)$.

79. If two events A and B are such that $P(A') = 0.3$, $P(B) = 0.4$ and $P(A \cap B') = 0.5$, then $P(B/A \cup B')$ equals

(A) $\frac{3}{4}$ (B) $\frac{5}{6}$
(C) $\frac{1}{4}$ (D) $\frac{3}{7}$

80. An elevator starts with m passengers and stops at n floors ($m \leq n$). The probability that no two passengers alight at the same floor is

(A) $\frac{^nP_m}{m^n}$ (B) $\frac{^nP_m}{n^m}$
(C) $\frac{^nC_m}{m^n}$ (D) $\frac{^nC_m}{n^m}$

81. If the integers m and n are chosen at random between 1 and 100 then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is

(A) $\frac{1}{5}$ (B) $\frac{1}{7}$
(C) $\frac{1}{4}$ (D) $\frac{1}{49}$

82. n biscuits are distributed among N boys at random. The probability that particular boy gets r ($< n$) biscuits is

(A) $^nC_r\left(\frac{1}{N}\right)^r\left(\frac{N-1}{N}\right)^{n-r}$ (B) $^nC_r\left(\frac{1}{N}\right)^r$
(C) ^nC_r (D) $\frac{r}{n}$

83. Three numbers are chosen at random without replcement from {1, 2,...,10}. The probability that the minimum of the chosen numbers is 3, or their maximum is 7 is

(A) $\frac{7}{40}$ (B) $\frac{5}{40}$

(C) $\frac{11}{40}$ (D) none of these

84. There are four balls of different colours and four boxes of colours, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed such that a ball does not go to a box of its own colour is

(A) $\frac{5}{8}$ (B) $\frac{3}{8}$

(C) $\frac{1}{8}$ (D) none of these

85. The altitude through A of ΔABC meets BC at D and the circumscribed circle at E. If $D \equiv (2, 3)$, $E \equiv (5, 5)$ and the ordinate of the orthocentre being a natural number. The probability that the orthocentre lies on the lines $y = 1, y = 2, y = 3,..., y = 10$ is

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{3}{5}$ (D) none of these

86. If $a \in [-5, 30]$, then the probability that the graph of the function $y = x^2 + 2(a + 4)x - 5a + 64$ is strictly above the x-axis is

(A) $\frac{27}{35}$ (B) $\frac{8}{25}$

(C) $\frac{8}{35}$ (D) $\frac{17}{25}$

87. Fifteen coupons are numbered 1, 2, 3,... 15. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on the selected coupon is 9 is

(A) $\left(\frac{9}{16}\right)^6$ (B) $\left(\frac{8}{15}\right)^7$

(C) $\left(\frac{3}{5}\right)^7$ (D) none of these

88. Consider a set 'P' containing n elements. A subset 'A' of 'P' is drawn and there after set 'P' is reconstructed. Now, one more subset 'B' of 'P' is drawn. Probability of drawing sets A and B so that $A \cap B$ has exactly one element is

(A) $(3/4)^n \cdot n$ (B) $n \cdot (3/4)^{n-1}$

(C) $n \cdot (3/4)^n$ (D) none of these

89. At a railway station a passenger leaves his luggage in a locker which is opened by dialling a three-digit code (say, 253, 009, 325 ...). The passenger chooses the code, closes the locker and leaves for the town. A strange man, who does not know the code, tries to open the locker by dialling three digits at random. The probability that the locker opens after k trials is (Here, $k < 1,000$)

(A) $\frac{k}{100}$ (B) $\frac{k}{1000}$

(C) $\frac{1,000 - k}{1,000}$ (D) none of these

90. The decimal parts of the logarithms of two numbers taken at random are found to six places. Probability that second can be subtracted first one without borrowing is

(A) $\left(\frac{9}{20}\right)^6$ (B) $\frac{1}{2^6}$

(C) $\left(\frac{11}{20}\right)^6$ (D) none of these

91. 10% of a certain population suffer from a serious disease. A person suspected of the disease is given two independent tests. Each test makes a correct diagnosis 90% of the time. The probability that the person really has the illness given that both tests are positive is

(A) 0.5 (B) 0.9

(C) 0.6 (D) none of these

92. The probablity that the birthdays of six different people will fall in exactly two calendar months is

(A) $\frac{1}{6}$ (B) ${}^{12}C_2 \times \frac{2^6}{12^6}$

(C) ${}^{12}C_2 \times \frac{2^6 - 2}{12^6}$ (D) $\frac{341}{12^5}$

93. The probability that the length of a randomly chosen chord of a circle lies between $\frac{2}{3}$ and $\frac{5}{6}$ of its diameter is

(A) $\frac{5}{6}$ (B) $\frac{1}{16}$

(C) $\frac{1}{4}$ (D) $\frac{5}{12}$

94. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that it came from LONDON is

(A) $\frac{5}{17}$ (B) $\frac{12}{17}$

(C) $\frac{17}{30}$ (D) $\frac{3}{5}$

95. In a multiple choice question, there are four alternative answers of which one or more than one is correct. A candidate will get marks on the questions only if he ticks all the correct answers. The candidate decides to tick all the correct answers. The candidate decides to tick answers at random. If he is allowed up to three chances to answer the questions, the probability that he will get marks on it is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{5}$.

96. A box contains n pairs of shoes and $2r$ shoes are selected. $(r < n)$. The probability that there is exactly one pair is

(A) $\frac{{}^{n-1}C_{r-1}}{{}^{2n}C_{2r}}$ (B) $\frac{n.{}^{n-1}C_{r-1}}{{}^{2n}C_{2r}}$

(C) $\frac{(n.{}^{n-1}C_{r-1})2^{r-1}}{{}^{2n}C_{2r}}$ (D) none of these

97. 5 girls and 10 boys sit at random in a row having 15 chairs numbered as 1 to 15. The probability that end seats are occupied by the girls and between any two girls odd number of boys sit is

(A) $\frac{20 \times 10! \times 5!}{15!}$ (B) $\frac{20 \times 10!}{15!}$

(C) $\frac{20 \times 5!}{15!}$ (D) none of these

98. Four tickets marked 00, 01, 10, 11, respectively are placed in a bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on tickets thus drawn is 23 is

(A) $\frac{25}{256}$ (B) $\frac{100}{256}$

(C) $\frac{231}{256}$ (D) none of these

99. Three natural numbers are taken at random from the set $A = \{x : 1 \le x \le 100, x \in N\}$. The probability that the $A.M.$ of the numbers taken is 25 is

(A) $\frac{{}^{77}C_2}{{}^{100}C_3}$ (B) $\frac{{}^{25}C_2}{{}^{100}C_3}$

(C) $\frac{{}^{74}C_2}{{}^{100}C_3}$ (D) none of these

100. If p and q are chosen randomly from the set (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) with replacement then the probability that the roots of the equation $x^2 + px + q = 0$ are real, is

(A) 0.62 (B) 0.32

(C) 0.44 (D) none of these

101. Let A, B, C be three events. If the probability of occurring exactly one event out of A and B is $1 - a$, out of B and C is $1 - 2a$, out of C and A is $1 - a$ and that of occurring three events simultaneously is a^2, then the probability that at least one out of A, B, C will occur, is

(A) $< \frac{1}{2}$ (B) $> \frac{1}{3}$

(C) $> \frac{1}{2}$ (D) $< \frac{1}{3}$

102. Fifteen persons, among whom are A and B, sit down at random at a round table. The probability that there are 4 persons between A and B is

(A) $\frac{1}{3}$ (B) $\frac{1}{7}$

(C) $\frac{1}{5}$ (D) none of these

103. A bag contains n white and n red balls. Pairs of balls are drawn without replcement until the bag is empty. The probability that each pair consists of one white and one red ball is

(A) $\frac{2^{n-1}}{{}^{2n}C_n}$ (B) $\frac{2^{n-1}}{{}^{2n}C_{n-1}}$

(C) $\frac{2^n}{{}^{2n}C_n}$ (D) none of these

104. A bag contains a white and b black balls. Two players A and B alternately draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and and wins the game. If A begins the game and the probability of A winning the game is three times that of B, then $a : b =$

(A) 2 : 1 (B) 3 : 1

(C) 3 : 2 (D) none of these

105. If X and Y are independent binomial variates $B\left(5, \frac{1}{2}\right)$ and $B\left(7, \frac{1}{2}\right)$ then the value of $P(X + Y = 3)$ is

(A) $\frac{55}{1024}$ (B) $\frac{44}{1024}$

(C) $\frac{33}{1024}$ (D) none of these

106. Plant I of *XYZ* manufacturing organization employs 5 production and 3 maintenance foremen, another plant II of same organization employs 4 production and 5 maintenance foremen. From any one of these plants, a single selection of two foremen is made. The probability that one of them would be production and the other maintenance foreman is

(A) $\frac{275}{504}$ (B) $\frac{263}{504}$

(C) $\frac{301}{504}$ (D) $\frac{362}{504}$

107. In a certain recruitment test there are multiple choice questions. There are 4 possible answers to each question and of which one is correct. An intelligent student knows 90% of the answer while a weak student knows only 20%. If an intelligent student gets the correct answer, then the probability that he was guessing is

(A) $\frac{1}{37}$ (B) $\frac{36}{37}$

(C) $\frac{14}{37}$ (D) none of these

108. X folows a binomial distribution with parameters n and p, and Y follows a binomial with parameters m and p. Then, if X and Y are independent,
$P(X = r | X + Y = r + s) =$

(A) $({}^{m}C_{r})({}^{n}C_{s})/{}^{m+n}C_{r+s}$ (B) $3\,{}^{m}C_{r+s}/{}^{m+n}C_{r+s}$

(C) $2\,({}^{m}C_{r})({}^{n}C_{s})/{}^{m+n}C_{r+s}$ (D) none of these

109. Three numbers are selected at random without replacement from the set of numbers $\{1, 2, ..., N\}$. The conditional probability that the third number lies between the first two, if the first number is known to be smaller than the second is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) none of these

110. Four positive integers are taken at random and are multiplied together. Then, the probability that the product ends in an odd digit other than 5 is

(A) $\frac{3}{5}$ (B) $\frac{609}{625}$

(C) $\frac{16}{625}$ (D) $\frac{2}{5}$

111. If four whole numbers taken at random are multiplied together, the chance that the last digit in the product is 1, 3, 7, or 9 is

(A) $\frac{4}{625}$ (B) $\frac{18}{625}$

(C) $\frac{16}{625}$ (D) none of these

112. A pack of playing cards was found to contain only 51 cards. If the first 13 cards, which are examined, are all red, the probability that the missing card is black is

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$

(C) $\frac{2}{9}$ (D) none of these

113. A ten-digit number is formed using the digits from zero to nine, every digit being used exactly once. The probability that the number is divisible by 4 is

(A) $\frac{16}{81}$ (B) $\frac{20}{81}$

(C) $\frac{32}{81}$ (D) none of these

114. Each coefficient of the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. The probability that the equation has non-real complex roots is

(A) $\frac{173}{216}$ (B) $\frac{43}{216}$

(C) $\frac{54}{216}$ (D) none of these

115. A set A contains n elements. A subset P of A is chosen at random and the set A is reconstructed by replacing the elements of P. Another subset Q of A is now chosen at random. The probability that PUQ contains exactly r elements, $1 \le r \le n$ is

(A) $\frac{{}^{n}C_{r}3^{r}}{4^{n}}$ (B) $\frac{{}^{n}C_{r}4^{r}}{3^{n}}$

(C) $\frac{3^n}{4^n}$ (D) none of these

116. A person throws two dice, one the common cube and the other a regular tetrahedron, the number on the lowest face being taken in the case of tetrahedron. The probability that the sum of the numbers appearing on the dice is 6 is

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{6}$ (D) none of these

117. A box contains 2 fifty paise coins, 5 twenty-five paise coins and a certain number $N (\geq 2)$ of ten and five paise coins. Five coins are taken out of the box at random. The probability that the total value of these coins is less than one rupee and fifty paise is

(A) $\frac{10(N+2)}{N+7_{C_5}}$ (B) $1-\frac{10(N+2)}{N+7_{C_5}}$

(C) $1-\frac{5(N+2)}{N+7_{C_5}}$ (D) none of these

118. A and B play a game of tennis. The situation of the game is as follows: if one scores two consecutive points after a deuce, he wins. If loss of a point is followed by win of a point, it is deuce. The probability of a server to win a point is $\frac{2}{3}$. The game is at deuce and A is serving. Probability that A will win the match is (serves are changed after each game)

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) none of these

119. If the papers of 4 students can be checked by any one of the seven teachers, then the probability that all the four papers are chcked by exactly two teachers is

(A) $\frac{2}{49}$ (B) $\frac{3}{49}$

(C) $\frac{6}{49}$ (D) none of these

More than One Option Correct Type

120. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$, then

(A) $P(B \cap A') = P(B) - P(A)$

(B) $P(A' \cup B') = P(A') + P(B')$

(C) $P(A \cup B)' = P(A')P(B')$

(D) $P(A/B) = P(A)$

121. A bag contains four tickets with numbers 112, 121, 211, 222. One ticket is drawn at random from the bag. Let E_i $(i = 1, 2, 3)$ denote the event that ith digit on the drawn ticket is 2. Then,

(A) E_1, E_2, E_3 are pair-wise independent

(B) E_1, $\bar{E}_2$ are independent

(C) $\bar{E}_2$ and $\bar{E}_3$ are not independent

(D) E_1, E_2, E_3 are mutually independent

122. A and B are two events. Odds against A are 2 : 1. Odds in favour of $A \cup B$ are 3 : 1. If $x \leq P(B) \leq y$, then

(A) $x = \frac{5}{12}$ (B) $x = \frac{3}{4}$

(C) $y = \frac{5}{12}$ (D) $y = \frac{3}{4}$

123. If A and B are two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$, then

(A) $P(A) + P(B) \leq \frac{11}{8}$ (B) $P(A)\cdot P(B) \leq \frac{3}{8}$

(C) $P(A) + P(B) \geq \frac{7}{8}$ (D) none of these

124. A student appears for test I, II and III. The student is successful if he passes either in test I and II or test I and III. The probability of the student passing in tests I, II and III are p, q and $\frac{1}{2}$, respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(A) $p = 1$ (B) $p = 0$

(C) $q = 1$ (D) $q = 0$

125. If A and B are any two events, the probability that exactly one of them occurs is

(A) $P(A) + P(B) - 2P(A \cap B)$

(B) $P(\bar{A}) + P(\bar{B}) - 2P(\bar{A} \cap \bar{B})$

(C) $P(A \cap (\bar{B})) + P((\bar{A}) \cap B)$

(D) $P(A) + P(B) - P(A \cup B)$

126. The probability that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Which of the following relations are true?

(A) $p + m + c = 19/20$ (B) $p + m + c = 27/20$
(C) $pmc = 1/10$ (D) $pmc = 1/4$

127. If A and B are two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, then

(A) $P(A \cup B) \geq \frac{2}{3}$

(B) $P(A \cap B') \leq \frac{1}{3}$

(C) $1/6 \leq P(A \cap B) \leq \frac{1}{2}$

(D) $1/6 \leq P(A' \cap B) \leq \frac{1}{2}$

128. A coin has probability p of showing head when tossed. It is tossed n times. Let P_n denote the probability that no two (or more) consecutive heads occurs, then

(A) $P_1 = 1$
(B) $P_2 = 1 - p^2$
(C) $P_n = (1-p)P_{n-1} + p(1-p)P_{n-2}$ for all $n \geq 3$
(D) all of these

129. Let X be a random variable which takes values 0, 1, 2, 3,....and $P(X = r) = pq^r$, where $0 < p < 1$, $q = 1 - p$ and $r = 0, 1, 2, 3, ...$Then,

(A) $P(X \geq a) = q^a$
(B) $P(X \geq a + b \mid X \geq a) = P(X \geq b)$
(C) $P(X = a + b \mid X \geq a) = P(X \geq b)$
(D) $P(X \geq a + b \mid X \geq b) = P(X \geq a)$

130. If A and B are two events such that $P(A) \neq 0$ and $P(A) > 1$, then

(A) $P(B/A) \geq 1 - \frac{P(B')}{P(A)}$

(B) $P(B'/A') \geq 1 - \frac{P(B)}{P(A')}$

(C) $P(A' \cup B'/A) = 1 - P(A \cap B/A)$

(D) none of these

131. If $P(A) = \frac{2}{5}$ and $P(B) = \frac{4}{5}$, then

(A) $P(A \cup B) \geq \frac{4}{5}$ (B) $\frac{1}{5} \leq P(A \cap B) \leq \frac{2}{5}$

(C) $\frac{1}{4} \leq P(A/B) \leq \frac{1}{2}$ (D) $P(A \cap B') \leq \frac{1}{5}$

132. If A and B are two events, then the probability that at most one of A, B occurs is

(A) $1 - P(A \cap B)$
(B) $P(A') + P(B') - P(A' \cap B')$
(C) $P(A') + P(B') + P(A \cup B)$
(D) none of these

Passage Based Questions

Passage 1

Twelve players $S_1, S_2, ..., S_{12}$ play in a chess tournament. They are divided into six pairs at random. From each pair a winner is decided. It is assumed that all players are of equal strength. The probability that

133. Both S_1 and S_2 are among the six winners is

(A) $\frac{12}{33}$ (B) $\frac{13}{33}$

(C) $\frac{14}{33}$ (D) none of these

134. Exactly one of S_1 *and* S_2 is among the six winners is

(A) $\frac{6}{11}$ (B) $\frac{5}{11}$

(C) $\frac{4}{11}$ (D) none of these

135. At least one of S_1 and S_2 is among the six winners is

(A) $\frac{31}{33}$ (B) $\frac{32}{33}$

(C) $\frac{10}{11}$ (D) none of these

Passage 2

A person draws a card from a pack of 52 cards. He replaces the card, shuffles the pack and again draws a card. He replaces it and draws again. This he does until he draws a heart.

Passage 3

A shopkeeper inspected some items in a box containing a, b and c number of items of type P, Q and R, respectively. He picked up an item randomly and put it back into the box if he found that the item selected was regularly sold, he also added the equal number of items of the same type as those previously in the box otherwise he removed all the items of that type.

Passage 4

Two persons A and B are playing with 5 dice. Player A throws 3 dice and player B throws 2 dice. The trials go on simultaneously and successively until 6 shows on at least one of the dice.

136. $P(A)$: The probability that player A and not B first obtains 6 is

(A) $\dfrac{(6^2-5^2)5^3}{6^3-5^5}$ (B) $\dfrac{(6^3-5^3)5^2}{6^5-5^5}$

(C) $\dfrac{(6^3-5^3)(6^2-5^2)}{6^5-5^5}$ (D) none of these

137. $P(B)$: The probability that players B and not A first obtains 6 is

(A) $\dfrac{(6^2-5^2)5^3}{6^5-5^5}$ (B) $\dfrac{(6^3-5^3)5^2}{6^5-5^5}$

(C) $\dfrac{(6^3-5^3)6^2-5^2}{6^5-5^5}$ (D) none of these

138. $P(C)$: The probability that players A and B obtain a 6 simultaneously is

(A) $\dfrac{(6^3-5^3)5^2}{6^5-5^5}$ (B) $\dfrac{(6^2-5^2)5^3}{6^5-5^5}$

(C) $\dfrac{(6^3-5^3)(6^2-5^2)}{6^5-5^5}$ (D) none of these

Match the Column Type

139.

I. An ordinary cube has four blank faces, one face marked 2, another marked 3. Then, the probability of obtaining a total of exactly 12 in 5 throws is	(A) $\frac{11}{16}$
II. A person draws a card from a pack of playing cards, replaces it and shuffles the pack. He continues doing this until he shows a spade. The chance that he will fail the first two times is	(B) $\frac{5}{13}$
III. A is known to tell the truth in 5 cases out of 6 and he states that a white ball was drawn from a bag containing 8 black and 1 white ball. The probability that the white ball was drawn is	(C) $\frac{5}{2592}$

140.

I. The probability that the 13th day of a randomly chosen month is a Friday is	(A) $\frac{16}{625}$
II. A natural number x is chosen at random from the first one hundred natural numbers. The probability that $\dfrac{(x-20)(x-40)}{x-30}<0$ is	(B) $\frac{1}{84}$
III. Four whole numbers taken at random are multiplied together. What is the chance that the last digit in the product is 1, 3, 7 or 9, is	(C) $\frac{1}{210}$
IV. 4 five-rupee coins, 3 two-rupee coins and 2 one-rupee coins are stacked together in a column at random. The probability that the coins of the same denomination are consecutive is	(D) $\frac{9}{50}$

Assertion-Reason Type

Instructions: *In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options:*

(A) Assertion(A) is True and Reason(R) is True; Reason(R) is a correct explanation for Assertion(A)
(B) Assertion(A) is True, Reason(R) is True; Reason(R) is not a correct explanation for Assertion(A)
(C) Assertion(A) is True, Reason(R) is False
(D) Assertion(A) is False, Reason(R) is True

141. Assertion: At the college entrance examination each candidate is admitted or rejected according to whether he has passed or failed the tests. Of the candidates who are really capable, 80% pass the test and of the incapable, 25% pass the test. Given that 40% of the candidates are really capable, then the proportion of capable college students is about 68%

Reason: $P(A/B) = \dfrac{P(A)\,P(B/A)}{P(A)\,P(B/A) + P(A^c)\,P(B/A^c)}$

142. Assertion: A set X contains n elements. Two subsets A and B of X are chosen at random. The probability that A and B have same number of elements is $\dfrac{1\cdot 3\cdot 5\ldots(2n-1)}{2^n(n!)}$

Reason: $({}^nC_0)^2 + ({}^nC_1)^2 + \ldots + ({}^nC_n)^2 = \dfrac{2^n[1\cdot 3\cdot 5\ldots(2n-1)]}{n!}$

143. Assertion: A bag contains $n + 1$ coins. It is known that one of these coins has a head on both sides while the other coins are fair. One coin is selected at random and tossed. If head turns up, the probability that the selected coin was fair, is $\dfrac{n}{n+2}$

Reason: If an event A occurs with two mutually exclusive and exhanstive events E_1 and E_2, then $P(E_i/A) = \dfrac{P(E_i)P(A/E_i)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}, i = 1, 2.$

144. Assertion: A player tosses a coin and scores 1 point for a head and 2 points for a tail. He plays on until his score reaches or passes n. If P_n denotes the probability of getting a score of exactly n, then

$$P_n + \frac{1}{2}P_{n-1} = P_2 + \frac{1}{2}P_1.$$

Reason: $P_n = \dfrac{1}{2}(P_{n-1} + P_{n-2}), n \geq 3$

145. Assertion: A throws $n + 1$ coins and B throws n coins. The chance that A throws greater number of heads than B is $\dfrac{1}{2}$

Reason: $\sum\limits_{0<k<m\leq n}\sum {}^{n+1}C_m\,{}^nC_k = \dfrac{1}{2}.2^{2n+1}$

Previous Year's Questions

146. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. **[2002]**
Probability that the problem is solved, is:

(A) 3/4 (B) 1/2
(C) 2/3 (D) 1/3

147. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial, is: **[2002]**

(A) $\frac{1}{25}$ (B) $\frac{24}{25}$
(C) $\frac{2}{25}$ (D) none of these

148. If A and B are two mutually exclusive events, then: **[2002]**

(A) $P(A) < P(\bar{B})$ (B) $P(A) > P(\bar{B})$
(C) $P(A) < P(B)$ (D) none of these

149. The probability of India winning a test match against West-Indies is 1/2 assuming independence from match to match. The probability that in a match series India's second win occurs at the third test is : **[2002]**

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$
(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

150. A biased coin with probability p, $0 < p < 1$, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even, is 2/5, then p equals : **[2002]**

(A) 1/3 (B) 2/3
(C) 2/5 (D) 3/5

151. A fair die is tossed eight times. The probability that a third six is observed on the eight throw, is: **[2002]**

(A) $\frac{^7C_2 \times 5^5}{6^7}$ (B) $\frac{^7C_2 \times 5^5}{6^8}$

(C) $\frac{^7C_2 \times 5^5}{6^6}$ (D) none of these

152. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is **[2003]**

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

153. The mean and variance of a random variable having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is **[2003]**

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

154. The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is **[2004]**

(A) $\frac{3}{20}$ (B) $\frac{1}{5}$

(C) $\frac{7}{20}$ (D) $\frac{4}{5}$

155. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, the probability $P(E \cup F)$ is **[2004]**

(A) 0.87 (B) 0.77
(C) 0.35 (D) 0.50

156. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is **[2004]**

(A) $\frac{37}{256}$ (B) $\frac{219}{256}$

(C) $\frac{128}{256}$ (D) $\frac{28}{256}$

157. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is **[2005]**

(A) $\frac{2}{9}$ (B) $\frac{1}{9}$

(C) $\frac{8}{9}$ (D) $\frac{7}{9}$

158. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are **[2005]**

(A) equally likely and mutually exclusive
(B) equally likely but not independent
(C) independent but not equally likely
(D) mutually exclusive and independent

159. At a telephone enquiry system the number of phone cells regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is **[2006]**

(A) $\frac{6}{5^e}$ (B) $\frac{5}{6}$

(C) $\frac{6}{55}$ (D) $\frac{6}{e^5}$

160. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is **[2007]**

(A) 1/729 (B) 8/9
(C) 8/729 (D) 8/243

161. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is **[2007]**

(A) 0.06 (B) 0.14
(C) 0.2 (D) 0.7

162. It is given that the events A and B are such that P (A) $= \frac{1}{4}$ and P $\left(\frac{A}{B}\right) = \frac{1}{2}$.P$\left(\frac{B}{A}\right) = \frac{2}{3}$ Then P (B) is **[2008]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

163. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then P (A ∪ B) is **[2008]**

(A) $\frac{3}{5}$ (B) 0

(C) 1 (D) $\frac{2}{5}$

164. One ticket is selected at random from 50 tickets numbered 00, 01, 02, …., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals **[2009]**

(A) $\frac{1}{14}$ (B) $\frac{1}{7}$

(C) $\frac{5}{14}$ (D) $\frac{1}{50}$

165. An urn contains nine balls of which three are red, four are blue and two are green. The experiment is to draw three balls at random without replacement from the urn. The probability that the three balls have different color is **[2010]**

(A) $\frac{2}{7}$ (B) $\frac{1}{21}$

(C) $\frac{2}{23}$ (D) $\frac{1}{3}$

166. Consider 5 independent Bernoulli's trials each with probability of failure 1-*p*. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then *p* lies in the interval **[2011]**

(A) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (B) $\left[0, \frac{1}{2}\right]$

(C) $\left[\frac{11}{12}, 1\right]$ (D) $\left[\frac{1}{2}, \frac{3}{4}\right]$

167. If C and D are two events satisfying $C \subset D$ with $P(D) \neq 0$, then the correct statement among the following is **[2011]**

(A) $P(C \mid D) \geq P(C)$ (B) $P(C \mid D) < P(C)$

(C) $P(C \mid D) = \frac{P(D)}{P(C)}$ (D) $P(C \mid D) = P(C)$

168. Three numbers are chosen at random without replacement from first eight natural numbers. The probability that their minimum is 3, given that their maximum is 6, is **[2012]**

(A) $\frac{3}{8}$ (B) $\frac{1}{5}$

(C) $\frac{1}{4}$ (D) $\frac{2}{5}$

169. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is **[2013]**

(A) $\frac{13}{3^5}$ (B) $\frac{11}{3^5}$

(C) $\frac{10}{3^5}$ (D) $\frac{17}{3^5}$

170. Let A and B be two events such that $p\left(\overline{A \cup B}\right) = \frac{1}{6}, p\left(A \cap B\right) = \frac{1}{4}$ and $p\left(\overline{A}\right) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then, the events A and B are **[2014]**

(A) mutually exclusive and independent
(B) equally likely but not independent
(C) independent but not equally likely
(D) independent and equally likely

171 Let two fair six-faced dice A and B be thrown simultaneously If E, is the event that die A shows up four, E2 is the event that die B shows up two and E3 is the event that the sum of numbers on both dice is odd, then which of the following statements is **NOT true** ?

(A) E,, E2 and E3 are independent. **[2016]**
(B) E, and E2 are independent.
(C) E2 and E3 are independent.
(D) E, and E3 are independent.

ANSWER KEYS

Single Option Correct Type

1. (D)	**2.** (A,C)	**3.** (C)	**4.** (B)	**5.** (A)	**6.** (C)	**7.** (B)	**8.** (C)	**9.** (B)	**10.** (A)
11. (A)	**12.** (C)	**13.** (B)	**14.** (A)	**15.** (A)	**16.** (C)	**17.** (B)	**18.** (C)	**19.** (A)	**20.** (A)
21. (D)	**22.** (B)	**23.** (C)	**24.** (C)	**25.** (B)	**26.** (D)	**27.** (A)	**28.** (D)	**29.** (C)	**30.** (D)
31. (B)	**32.** (A)	**33.** (B)	**34.** (B)	**35.** (A)	**36.** (B)	**37.** (D)	**38.** (C)	**39.** (A)	**40.** (C)
41. (C)	**42.** (A)	**43.** (B)	**44.** (B)	**45.** (C)	**46.** (B)	**47.** (D)	**48.** (B)	**49.** (B)	**50.** (D)
51. (D)	**52.** (C)	**53.** (A)	**54.** (B)	**55.** (B)	**56.** (B)	**57.** (B)	**58.** (B)	**59.** (A)	**60.** (C)
61. (B)	**62.** (D)	**63.** (C)	**64.** (D)	**65.** (C)	**66.** (B)	**67.** (C)	**68.** (B)	**69.** (C)	**70.** (A)
71. (A)	**72.** (A)	**73.** (A)	**74.** (B)	**75.** (A)	**76.** (B)	**77.** (B)	**78.** (D)	**79.** (C)	**80.** (B)
81. (A)	**82.** (A)	**83.** (C)	**84.** (B)	**85.** (C)	**86.** (C)	**87.** (D)	**88.** (C)	**89.** (B)	**90.** (C)
91. (B)	**92.** (C)	**93.** (C)	**94.** (B)	**95.** (D)	**96.** (C)	**97.** (A)	**98.** (A)	**99.** (C)	**100.** (A)
101. (C)	**102.** (B)	**103.** (C)	**104.** (A)	**105.** (A)	**106.** (A)	**107.** (A)	**108.** (A)	**109.** (B)	**110.** (C)
111. (C)	**112.** (A)	**113.** (B)	**114.** (A)	**115.** (A)	**116.** (C)	**117.** (B)	**118.** (C)	**119.** (C)	

More than One Option Correct Type

120. (C), and (D)	**121.** (A) and (B)	**122.** (A), and (D)	**123**. (A), and (C)
124. (A), and (D)	**125.** (A), (B) and (C)	**126.** (B), and (C)	
127. (A), (B), (C) and (D)	**128.** (A), (B), (C) and (D)	**129.** (A), (B), (C) and (D)	
130. (A), (B) and (C)	**131.** (A), (B), (C) and (D)	**132.** (A), and (B)	

Passage Based Questions

133. (C) **134.** (A) **135.** (B) 136. (B) 137. (A) **138.** (C)

Match the Column Type

139. (A) → 4; (B) → 1; (C) → 3;

140. (A) → 2; (B) → 4; (C) → 1; (D) → 3

Assertion-Reason Type

140. (A) **142.** (A) **143**. (A) **144.** (A) **145.** (A)

Previous Year's Questions

146. (A)	**147.** (B)	**148.** (A)	**149.** (B)	**150.** (A)	**151.** (B)	**152.** (B)	**153**. (B)	**154.** (C)	**155.** (B)
156. (D)	**157.** (B)	**158.** (C)	**159.** (D)	**160.** (D)	**161.** ( )	**122.** (B)	**163**. (C)	**164.** (A)	**165.** (A)
166. (B)	**167.** (A)	**168.** (B)	**169.** (B)	**170.** (C)	**171.** (A)				

HINTS AND SOLUTIONS

Single Option Correct Type

1. Let $X \sim B(100, p)$ be the number of coins showing heads and let $q = 1 - p$. Then, since $P(X = 51) = P(X = 50)$, we have ${}^{100}C_{51}(p^{51})(q^{49}) = {}^{100}C_{50}(p^{50})(q^{50})$

$$\Rightarrow \frac{p}{q} = \left(\frac{100!}{50!\,50!}\right)\left(\frac{51!\,49!}{100!}\right)$$

$$\Rightarrow \frac{p}{1-p} = \frac{51}{50} \Rightarrow 50p = 51 - 51p \Rightarrow p = \frac{51}{101}$$

This correct option is (D)

2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore 1 \geq P(A) + P(B) - P(A \cap B) \geq \frac{3}{4}$$

As the minimum value of $P(A \cap B) = \dfrac{1}{8}$,

we get $P(A) + P(B) - \dfrac{1}{8} \geq \dfrac{3}{4}$

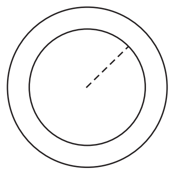
$\Rightarrow\ P(A) + P(B) \geq \dfrac{1}{8} + \dfrac{3}{4} = \dfrac{7}{8}$

As the maximum value of $P(A \cap B) = \dfrac{3}{8}$, we get

$1 \geq P(A) + P(B) - \dfrac{3}{8} \Rightarrow P(A) + P(B) \leq 1 + \dfrac{3}{8} = \dfrac{11}{8}$.

This correct option is (A,C)

3. $n(S)$ = the area of the circle of radius r.



$n(E)$= the area of the circle of radius $\dfrac{r}{2}$.

$\therefore$ the required probability

$$= \frac{n(E)}{n(S)} = \frac{\pi\cdot\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$$

This correct option is (C)

4 The probability of drawing a number less than or equal to 15 in a draw $= \dfrac{15}{20} = \dfrac{3}{4}$.

The probability of drawing the ticket of value 15 in a draw = $\dfrac{1}{20}$.

$\therefore$ the required probability $= {}^4C_1\cdot\dfrac{1}{20}\cdot\left(\dfrac{3}{4}\right)^3 = \dfrac{27}{320}$.

This correct option is (B)

5. We know 7^k, $k \in N$, has 1, 3, 9, 7 at the units place for $k = 4p$, $4p - 1$, $4p - 2$, $4p - 3$ respectively, where $p = 1, 2, 3, ...$

Clearly, $7^m + 7^n$ will be divisible by 5 if 7^m has 3 or 7 in the unit place and 7^n has 7 or 3 in the units place or 7^m has 1 or 9 in the units place and 7^n has 9 or 1 in the units place.

$\therefore$ For any choice of m, n the digit in the units place of $7^m + 7^n$ is 2, 4, 6, 0 or 8. It is divisible by 5 only when this digit is 0.

$\therefore$ the required probability $= \dfrac{1}{5}$.

This correct option is (A)

6. We define the following events

A_1 : He knows the answer

A_1 : He does not know the answer

E : He gets the correct answer

Thus, $P(A_1) = \dfrac{9}{10}$, $P(A_2) = 1 - \dfrac{9}{10} = \dfrac{1}{10}$

$P(E/A_1) = 1$, $P(E/A_2) = \dfrac{1}{4}$

$\therefore$ required probability $= P(A_2/E)$

$$= \frac{P(A_2)\cdot P(E/A_2)}{P(A_1)\,P(E/A_1) + P(A_2)\,P(E/A_2)}$$

$$= \frac{\frac{1}{10}\cdot\frac{1}{4}}{\frac{9}{10}\cdot 1 + \frac{1}{10}\cdot\frac{1}{4}} = \frac{1}{36+1} = \frac{1}{37}$$

This correct option is (C)

7. The required probability

$$= \frac{39}{52}\times\frac{39}{52} = \frac{3}{4}\times\frac{3}{4} = \frac{9}{16}$$

This correct option is (B)

8. $P(A) = P(A/B) = P(A \cap B)/P(B)$

$\Rightarrow\ P(A \cap B) = P(A)\,P(B)$

Thus, A and B are independent

Also, $P(A'/B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)}$

$= 1 - P(A/B) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

This correct option is (C)

9. If B throws 1, then A can throw only 1, if B throws 2, then A can throw 1 and 2 and so on

$\therefore$ the required probability

$$= \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{2}{6} + \frac{1}{6}\cdot\frac{3}{6} + \frac{1}{6}\cdot\frac{4}{6} + \frac{1}{6}\cdot\frac{5}{6} + \frac{1}{6}\cdot\frac{6}{6}$$

$$= \frac{1}{6\times 6}\,(1+2+3+4+5+6) = \frac{6\times 7}{6\times 6\times 2} = \frac{7}{12}$$

This correct option is (B)

10. Probability that the sum is even = 1 – Probability that the sum is odd.

$$= 1 - \left(\frac{1}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{1}{3}\right) = 1 - \frac{4}{9} = \frac{5}{9}$$

This correct option is (A)

11. It is a case of Bernoullian trials with number of trials n and probability, $= \dfrac{1}{N}$ (a success in one trial)

$\therefore\ P(r \text{ successes}) = {}^nc_r\, q^{n-1}\, p^r = {}^nc_r\,(1-p)^{n-r}\, p^r$

$$= {}^nc_r\left(1 - \frac{1}{N}\right)^{n-r}\left(\frac{1}{N}\right)^r$$

$$= {}^nc_r\left(\frac{1}{N}\right)^r\left(\frac{N-1}{N}\right)^{n-r}$$

This correct option is (A)

12. P(first 10 draws are non ace cards) (11th draw is an ace card)

$$= \frac{^{48}C_{10}}{^{52}C_{10}} \times \frac{4}{42} = \frac{164}{4165}$$

This correct option is (C)

13. The student will be successful in three cases.

Now $P(I, II \text{ not } III) + P(I, III, \text{not } II) + P(I, II, III)$

$$= p \cdot q \cdot \left(1 - \frac{1}{2}\right) + p\frac{1}{2}(1-q) + p \cdot q \frac{1}{2}$$

$$= \frac{1}{2}pq + \frac{1}{2}p - \frac{1}{2}pq + \frac{1}{2}pq$$

$$= \frac{1}{2}p(1+q) = \frac{1}{2} \text{ (given)} \Rightarrow p(1+q) = 1$$

This condition will be satisfied when $p = 1$, $q = 0$.

This correct option is (B)

14. The sample space is

$\{-0.50, -0.49, -0.48, \ldots -0.01, 0.00, 0.01, \ldots 0.49\}$

Let A be the event that the round off error is at least 10 paise. Then A^c is the event that the round off error is at most a paise. Sample space for A is given by

$\{-0.09, -0.08, \ldots -0.01, 0.00, 0.01, \ldots 0.09\}$

$$\therefore \; P(A^c) = \frac{19}{100} \qquad \Rightarrow 1 - P(A) = \frac{19}{100}$$

$$\Rightarrow P(A) = 1 - \frac{19}{100} = \frac{81}{100}$$

This correct option is (A)

15. Since all player are of equal strength, with whom S_1 is paired, will have no bearing on the probability of his winning.

The number of ways of choosing 8 winners out of 16 : $^{16}C_8$.

The number of ways of choosing S_1 and 7 other winners out of 15 is $^{15}C_7$.

$$\therefore \; \text{Probability that } S_1 \text{ will win} = \frac{^{15}C_7}{^{16}C_8} = \frac{15!}{7!8!}\,\frac{8!8!}{16!} = \frac{1}{2}$$

This correct option is (A)

16. Let $A(B)$ denote the event that minimum (maximum) number on the chosen tickets is 3 (7).

We have

$P(A) = P$(choosing 3 and two other numbers from 4 to 10)

$$= \frac{^{7}C_2}{^{10}C_3} = \frac{7\times 6\times 3}{10\times 9\times 8} = \frac{7}{40}$$

$P(B) = P$(choosing 7 and choosing two other numbers from 1 to 6)

$$= \frac{^{6}C_2}{^{10}C_3} = \frac{6\times 5\times 3}{10\times 9\times 7} = \frac{1}{8}$$

$P(A \cap B) = P$(choosing 3 and 7 and one other number from 4 to 6)

$$= \frac{3}{^{10}C_3} = \frac{3\times 3\times 2}{10\times 9\times 8} = \frac{1}{40}$$

$\therefore \; P(A \cup B) = P(A) + P(B) - P(A \cap B) = 11/40$

This correct option is (C)

17. The probability that only two tests are needed = (probability that the first machine tested is faulty) × (probability that the second machine tested is faulty given the first machine tested is faulty)

$$= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$$

This correct option is (B)

18. $P(A) = 1 - P(A^c) = 1 - 0.3 = 0.7$, $P(B) = 0.4$

We know that $P(A) = P(A \cap B) + P(A \cap B^c)$

$\Rightarrow 0.7 = P(A \cap B) + 0.5$

or $P(A \cap B) = 0.7 - 0.5 = 0.2$

$P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$

$= 0.7 + (1 - 0.4) - 0.5 = 0.8$

$$\text{Next } P[B \,|\, P(A \cup B^c)] = \frac{P[B \cap (A \cup B^c)]}{P[A \cup B^c]}$$

From set theory

$B \cap (A \cup B^c) = (B \cap A) \cup (B \cap B^c)$

$= (B \cap A) \cup \varphi = B \cap A$

$$\therefore \; \text{Required Probability} = \frac{P(A \cap B)}{P(A \cup B^c)} = \frac{0.2}{0.8} = \frac{1}{4}.$$

This correct option is (B)

19. The possible outcomes on the pair of dice when the sum of the numbers on them is 7 are (1, 6), (2, 5) (3, 4), (4, 3), (5, 2), (6, 1), the probability for which is $\frac{6}{36}$.

The possible outcomes on the dice when the sum is 8 are (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) the probability for which is $\frac{5}{36}$

$\therefore$ The probability that on the coin head comes and on the dice the sum is 7 or 8

$$= \frac{1}{2}\left(\frac{6}{36} + \frac{5}{36}\right) = \frac{11}{72}$$

The probability that from the pack of 11 cards when one card is chosen then number on it is 7 or 8 is $\frac{2}{11}$

$\therefore$ The probability that tail comes on the coin and then on the chosen card number appearing is 7 or 8

$$= \frac{1}{2} \times \frac{2}{11} = \frac{1}{11}$$

Thus, the required probability

$$= \frac{11}{72} + \frac{1}{11} = \frac{121+72}{11\times 72} = \frac{193}{792}$$

This correct option is (A)

20. Probability that the outcome of a single throw of a die is any one of 2, 3, 4 and 5 is equal to $\frac{4}{6} = \frac{2}{3}$.

If the die is rolled four times the required probability is equal to $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$.

This correct option is (A)

21. Out of the numbers 00, 01, 02, ..., 99, those numbers the product of whose digits is 18 are 29, 36, 63, 92.

$\therefore\ P(E) = \frac{4}{100} = \frac{1}{25}$

$P(\bar{E}) = \frac{24}{25}$

The probability that in selecting from numbers the event E occurs at least three times.

$= {}^4C_1\left(\frac{1}{25}\right)^3 \frac{24}{25} + {}^4C_4\left(\frac{1}{25}\right)^4$

$= \frac{4 \times 24}{25^4} + \frac{1}{25^4} = \frac{97}{(25)^4}$

This correct option is (D)

22. The exhaustive cases are 4! = 24

The favourable cases are $4\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$

= 12 – 4 + 1 = 9

$\therefore$ the required probability $= \frac{9}{24} = \frac{3}{8}$.

This correct option is (B)

23. Let the orthocentre be (x, y)

$\therefore\ (x-2)^2 + (y-3)^2 = (5-2)^2 + (5-3)^2$

$\Rightarrow x^2 + y^2 - 4x - 6y = 0$

$\Rightarrow x^2 - 4x + (y-3)^2 - 9 = 0$

$\Rightarrow x = 2 \pm \sqrt{13 - (y-3)^2}$

$\therefore$ y can take the values 1, 2, 3, 4, 5, 6

$\therefore$ reqd. prob. $= \frac{6}{10} = \frac{3}{5}$

This correct option is (C)

24. There are 64 small squares on a chess board.

$\Rightarrow$ Total number of ways to choose two squares

$= {}^{64}C_2 = 32.63$

For favourable ways we must choose two consecutive small squares for any row or any column

$\Rightarrow$ Number of favourable ways = (7.8)2

$\Rightarrow$ Required probability $= \frac{7.8.2}{32.63} = \frac{1}{18}$

This correct option is (C)

25. The required probability $= 1 - \frac{{}^{96}C_3}{{}^{100}C_3}$

$= 1 - \frac{96 \times 95 \times 94}{100 \times 99 \times 98}$

$= 1 - \frac{7144}{8085} = \frac{941}{8085}$

This correct option is (B)

26. The odd places can be filled up in 5 × 4 × 3 ways and the even places in 5 × 4 ways. $\therefore$ the favourable number of ways $= 5 \times 4 \times 3 \times 5 \times 4 = 5^2.2^4.3$

The number of five digit numbers $= 9 \times 10^4$

Hence, the required probability $= \frac{5^2 \times 2^4 \times 3}{9 \times 10^4} = \frac{1}{75}$

This correct option is (D)

27. Let E_1 be the event that exactly two players scored more then 50 runs then $P(E_1) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} \times \frac{9}{10}$

$+ \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \times \frac{9}{10} + \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{10}$

$+ \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{9}{10} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} \times \frac{1}{10} = \frac{65}{240}$

Let E_2 be the event that A and B scored more than 50 runs, then $P(E_1 \cap E_2) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} \times \frac{9}{10} = \frac{27}{240}$

$\therefore$ Required probability $= P(E_2/E_1)$

$= \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{27}{65}$

This correct option is (A)

28. Exhaustive number of cases $= n!$

Assuming the set of numbers $\{1, 2, 3, ..., k\}$ as one the favourable cases $= (n - k + 1)!$

$\therefore$ Probability $= \frac{(n-k+1)!}{n!}$

This correct option is (D)

29. The total length of the interval in which a lies = 30 – (– 5) = 35

If the graph of $y = x^2 + 2(a+4)x - 5a + 64$ is entirely above the x-axis, the discriminant of the above quadratic expression must be negative.

$\therefore\ 4(a+4)^2 + 4(5a - 64) < 0$

$\Rightarrow a^2 + 13a - 48 < 0 \Rightarrow (a+16)(a-3) < 0$

$\Rightarrow -16 < a < 3$

But $a \in [-5, 30]$ $\therefore$ $-5 < a < 3$ for the event to happen. The length of this interval = 3 – (– 5) = 8.

Hence, the required probability $= \frac{8}{35}$.

This correct option is (C)

30. Total ways = 15^7

For favourable ways, we must have 7 coupons numbered from 1 to 9 so that '9' is selected at least once. Thus, total number of favourable ways are, $9^7 - 8^7$

$\Rightarrow$ Required probability $= \dfrac{9^7 - 8^7}{15^7}$

This correct option is (D)

31. If the procedure for drawing balls has to come to an end at the rth draw, all but one black ball must be drawn in the first $(r-1)$ draws.

$\therefore$ The prob. of the reqd. event

$$= \frac{({}^3C_2)({}^mC_{r-3})}{{}^{m+3}C_{r-1}} \cdot \frac{1}{m-r+4}$$

$$= 3\frac{m!}{(r-3)!\,(m-r+3)!} \cdot \frac{(r-1)!\,(m-r+4)}{(m+3)!} \cdot \frac{1}{m-r+4}$$

$$= 3\frac{(r-1)(r-2)}{(m+3)(m+2)(m+1)}.$$

This correct option is (D)

32. The total number of ways of chooosing 2 persons out of n is ${}^nC_2$.

After selecting two persons when the remaining $n-2$ persons sit in a row $(n-1)$ places are created between them in which 2 persons can be arranged in ${}^{n-1}C_2$. 2! ways.

So, required probability is

$$= \frac{{}^{n-1}C_2 \,.\, 2!}{{}^nC_2} = \frac{n-2}{n} = 1 - \frac{2}{n}$$

This correct option is (A)

33. Third six occurs on 8th trial. It means that in first 7 trials we must have exactly 2 sixes and 8th trial must result in a six.

$\therefore$ Required probability

$= {}^7C_2 . (1/6)^2 . (5/6)^5 . (1/6) = \dfrac{{}^7C_2 . 5^5}{6^7}$

This correct option is (B)

34. $\dfrac{(x-20)(x-40)}{x-30}$

$$= \frac{(x-20)(x-30)(x-40)}{(x-30)^2}$$

< 0 for $x = 1, 2, ..., 9, 21, 22, ..., 29$, i.e. 18 natural numbers.

$\therefore$ Reqd. prob. $= \dfrac{18}{100} = \dfrac{9}{50}$

This correct option is (B)

35. If the product of the four numbers ends in one of the digits 1, 3, 7, or 9, each number should have the last digit as one of these 4 digits.

$\therefore$ the number of favourable cases $= 4^4$

Total number of all possible cases $= 10^4$

Hence, the required probability

$$= \frac{4^4}{10^4} = \frac{2^4}{5^4} = \frac{16}{625}$$

This correct option is (A)

36. $n(S) = \dfrac{9!}{4!\,3!\,2!}$ and $n(E) = 3!$

$$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{3!}{\frac{9!}{4!\,3!\,2!}}$$

$$= \frac{3!\,4!\,3!\,2!}{9!}$$

$$= \frac{6.6.2}{9.8.7.6.5} = \frac{1}{210}$$

This correct option is (B)

37. If $P(r)$ is the probability that the number r is drawn in one draw, it is given that $P(r) = kr$, where k is a constant.

Further $P(1) + P(2) + ... + P(2n) = 1$

$\Rightarrow k(1 + 2 + 3 + ... + 2n) = 1 \Rightarrow k = \dfrac{1}{n(2n+1)}$

Hence, the required probability

$= P(2) + P(4) + P(6) + ... + P(2n)$

$$= 2k(1 + 2 + ... + n) = \frac{2}{n(2n+1)} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2n+1}$$

This correct option is (D)

38. $n(S) = {}^{100}C_3$

Since the A.M. of three numbers is 25

$\therefore$ Their sum = 75

$\therefore$ $n(E)$ = the no. of integral sol. of

$x_1 + x_2 + x_3 = 75$

where $x_1 \geq 1, x_2 \geq, x_3 \geq 1$.

= co-eff. of x^{75} in $(x + x^2 + x^3 + ...)^3$

= coeff. of x^{72} in $(1 + x + x^2 + ...)^3$

= coeff. of x^{72} in $\left(\dfrac{1}{1-x}\right)^3$

= coeff. of x^{72} in $(1-x)^{-3} = {}^{74}C_{72} = {}^{74}C_2$

$\therefore$ $P(E) = \dfrac{{}^{74}C_2}{{}^{100}C_3}$

This correct option is (C)

39. Number of ways of forming two groups $= \dfrac{(2n)!}{n!\,n!}$

Leaving two tallest boys, we can divide $2n-2$ boys into two groups in $\dfrac{(2n-2)!}{(n-1)!\,(n-1)!}$. But the two tallest boys can be in any of the groups, each in different. So favourable number of cases

$$= \frac{2(2n-2)!}{(n-1)!\,(n-1)!}$$

This correct option is (A)

40. Let x_i be any element of set P, we have following possibilities;

(i) $x_i \in A, x_i \in B$; (ii) $x_i \in A, x_i \notin B$;

(iii) $x_i \notin A, x_i \in B$; (iv) $x_i \notin A, x_i \notin B$

Clearly, the element $x_i \in A \cap B$ if and only if it belongs to A and B both. Thus, out of these 4 ways only first way is favourable. Now the element that we want to be in the intersection can be chosen in 'n' different ways. Hence required probability is $n.\,(3/4)^n$.

This correct option is (C)

41. Exahaustive number of cases = ${}^{24}C_{14}$

Favourable cases = ${}^{22}C_{14}$

This correct option is (C)

42. $$n(S) = \begin{pmatrix}\text{total number of selections of} \\ \text{two diagonals which intersect} \\ \text{at an interior point}\end{pmatrix}$$

$$\Rightarrow \; n(S) = \begin{pmatrix}\text{the number of} \\ \text{selections of} \\ \text{four vertices}\end{pmatrix} = {}^6C_4 = 15$$

$\therefore$ The required probability $= \dfrac{15}{36} = \dfrac{5}{12}$

This correct option is (A)

43. Total number of ways to distribute the balls so that no box is empty are [(1, 1, 4), (2, 1, 3), (2, 2, 2)]

$$= \frac{3!}{2!}[({}^6C_1 \,.\, {}^5C_1 \,.\, {}^4C_4)] + 3!$$

$$({}^6C_2 \,.\, {}^4C_1 \,.\, {}^3C_3) + ({}^6C_2 \,.\, {}^4C_2 \,.\, {}^2C_2)]$$

$= 90 + 6.60 + 90 = 540$

$\therefore$ Required probability $= \dfrac{90}{540} = \dfrac{1}{6}$.

This correct option is (B)

44. $P(A) = (0.1)(0.1)(0.1) = .001$

If k trials are made, then it is natural to assume that the unsuccessful combinations are not repeated.

Now $P(B) = 1 - P(\overline{B})$

$$= 1 - \frac{999}{1000}\cdot\frac{998}{999}\cdots\frac{1000-k+1}{1000-k+2}\cdot\frac{1000-k}{1000-k+1}$$

$$= 1 - \frac{1000-k}{1000} = \frac{k}{1000}$$

This correct option is (B)

45. $P_1 = x_1x_2x_3 \ldots x_6$

$P_2 = y_1y_2 \ldots y_6$

Any digit after decimal can be 0, 1, 2,..., 9 P_2 can be subtracted from P_1 without borrowing if $x_i \geq y_i$

Let $y_i = \lambda \; (0 \leq \lambda \leq 9) \Rightarrow x_i = \lambda, \lambda + 1, \ldots, 9$

$\Rightarrow$ After choosing y_i, x_i can be chosen in $(10 - \lambda)$ ways.

$\Rightarrow$ Required probability

$$= \left(\frac{\sum_{\lambda=0}^{9}(10-\lambda)}{100}\right) = \left(\frac{11}{20}\right)^6$$

This correct option is (C)

46. Let A be the event that a person has the disease and let B_i be the event that ith test is + ve $(i = 1, 2)$. Then

$P(A) = 0.1, \; P(B_i/A) = 0.9$

$P(B_i/\overline{A}) = 1 - P(B_i/A) = 1 - 0.9 = 0.1$

Let $B = B_1 \cap B_2$

$\therefore \; P(B/A) = P(B_1/A)\; P(B_2/A) = 0.81$

$P(B/\overline{A}) = (0.1)\,(0.1)$

$$\therefore \; P(A/B) = \frac{P(A).\,P(B/A)}{P(A).\,P(B/A) + P(\overline{A}).\,P(B/\overline{A})}$$

$$= \frac{(0.1)(0.81)}{(0.1)(0.81) + (0.9)(0.1)(0.1)}$$

$$= \frac{0.081}{0.081 + .009} = \frac{81}{90} = \frac{9}{10} = 0.9$$

This correct option is (B)

47. A total of 9 can be obtained in the following mutually exclusive ways:

(I) 2 occurs in 3 throws out of 5 and

3 occurs in one out of the remaining 2 throws. The number of such cases is ${}^5C_3.{}^2C_1$

(II) 3 occurs three times out of 5 throws. The number of such cases is ${}^5C_3$.

So, required probability = P(I) + P(II)

$$= \frac{{}^5C_3 \,.\, {}^2C_1}{6^5} + \frac{{}^5C_3}{6^5} = \frac{5}{1296}$$

This correct option is (D)

48. Person that must stand between A and B can be chosen in $(n-2)$ ways. Now number of ways in which x persons can be made to stand so that there is exactly one person in between A and B is equal to $(n-2).2.(n-2)!$

Also, total number of ways in which persons can be made to stand = $n!$

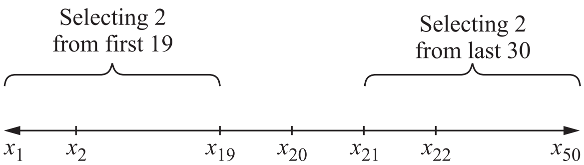
$\therefore$ Required probability

$$= \frac{2(n-2).(n-2)!}{n!} = \frac{2(n-2)}{n(n-1)}$$

This correct option is (B)

49. $n(S) = {}^{50}C_5$, $n(E) = {}^{19}C_2 \times {}^{30}C_2$

$$\therefore\ P(E) = \frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$$



This correct option is (B)

50. Let A denote the event that the target is hit when x shells are fired at point I.

Let E_1, E_2 denote the events hitting I and II restively.

$$\therefore\ P(E_1) = \frac{8}{9},\ P(E_2) = \frac{1}{9}$$

$$\text{Now } P(A/E_1) = 1 - \left(\frac{1}{2}\right)^x \text{ and } P(A/E_2) = 1 - \left(\frac{1}{2}\right)^{21-x}$$

$$\text{Now } P(A) = \frac{8}{9}\left[1 - \left(\frac{1}{2}\right)^x\right] + \frac{1}{9}\left[1 - \left(\frac{1}{2}\right)^{21-x}\right]$$

$$\therefore\ \frac{d\,P(A)}{dx} = \frac{8}{9}\left[\left(\frac{1}{2}\right)^x \log 2\right] + \frac{1}{9}\left[-\left(\frac{1}{2}\right)^{21-x} \log 2\right]$$

For Max. prob. $\frac{d\,P(A)}{dx} = 0$

$\therefore\ x = 12$ ($\because\ 2^{3-x} = 2^{x-21} \Rightarrow 3 - x = x - 21$

$\Rightarrow 2x = 24 \Rightarrow x = 12$)

Since $d^2 \frac{P(A)}{dx} < 0$ for $x = 12$

$\therefore\ P(A)$ is max. when $x = 12$

This correct option is (D)

51. Generally any one birthday can fall in one of the 12 months.

For six different people, the total number of ways is 12^6. Coming to the favourable ways: first any two months can be chosen in ${}^{12}C_2$ ways.

The six birthdays can fall in these 2 months in 2^6 ways. Of these the two cases when all the six birthdays fall in one month are to be removed.

$$\therefore\ \text{required probability} = \frac{{}^{12}C_2\,(2^6 - 2)}{12^6}$$

$$= {}^{12}C_2\,\frac{2(2^5 - 1)}{12^6}$$

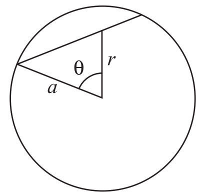
This correct option is (D)

52. If l is the length of the chord, r, the distance of the mid-point of the chord from the centre of the circle and a radius of the given circle, then $r = a\cos\theta$, $l = 2a\sin\theta$.

$$\text{Given: } \frac{2}{3}\,2a < 2a\sin\theta < \frac{5}{6}\,2a \Rightarrow \frac{\sqrt{11}}{6}\,a < a\cos\theta < \frac{\sqrt{5}}{3}a$$

$$\Rightarrow \frac{\sqrt{11}}{6}a < r < \frac{\sqrt{5}}{3}a$$

$\therefore$ The given condition is satisfied if the mid-point of the chord lies within the region between the concentric circles of radius $\frac{\sqrt{11}}{6}$ and $\frac{\sqrt{5}}{3}$.



Hence, the probability

$$= \frac{\text{The area of the circular annulus}}{\text{Area of the given circle}}$$

$$= \left(\frac{5}{9} - \frac{11}{36}\right) = \frac{1}{4}$$

This correct option is (C)

53. Let A_1 denote the event that a coin having heads on both sides is chosen, and A_2 denote the event that a fair coin is chosen.

Let E denote the event that head occurs. Then

$$P(A_1) = \frac{n}{2n+1} \Rightarrow P(A_2) = \frac{n+1}{2n+1},$$

Probability of occurrence of event E, if unfair coin was selected is $P\left(\frac{E}{A_1}\right) = 1$

Probability of occurrence of event E, if fair coin was selected is $P\left(\frac{E}{A_2}\right) = \frac{1}{2}$

$\because\ P(E) = P(A_1 \cap E) + P(A_2 \cap E)$

$$\therefore\ P(E) = P(A_1)\,P\left(\frac{E}{A_1}\right) + P(A_2)\,P\left(\frac{E}{A_2}\right)$$

$$\Rightarrow \frac{31}{42} = \frac{n}{2n+1}.1 + \frac{n+1}{2n+1}.\frac{1}{2} \Rightarrow \frac{31}{42} = \frac{3n+1}{2(2n+1)}$$

$\Rightarrow\ 54n + 62 = 126n + 42 \Rightarrow 2n = 20$

$\therefore n = 10$

This correct option is (A)

54. $$\frac{n!}{n^n} = \frac{3}{32} \Rightarrow \frac{n!}{n^n} = \frac{8 \times 3}{8 \times 32} = \frac{4!}{4^4}$$

This correct option is (B)

55. Out of n people, 3 know the answer and $(n - 3)$ do not know the answer.

The first four persons will not know the answer if they are chosen out of the $(n - 3)$ people who do not know the answer.

Thus, the prob. of the reqd. event is $\frac{{}^{n-3}C_4}{{}^{n}C_4}$.

This correct option is (B)

56. We define the following events:

A_1 : Selecting a pair of consecutive letters from the word **LONDON**

A_2 : Selecting a pair of consecutive letters from the word **CLION**

E : Selecting a pair of letters **ON**'

Then $P(A_1 \cap E) = \frac{2}{5}$, as there are 5 pairs of consecutive letters out of which 2 are **ON**.

$P(A_2 \cap E) = \frac{1}{6}$, as there are 6 pairs of consecutive letters of which one is **ON**.

So, required probability $= P\left(\frac{A_1}{E}\right)$

$$\Rightarrow P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}}$$

$$\therefore P\left(\frac{A_1}{E}\right) = \frac{12}{17}$$

This correct option is (B)

57. Total ways $= 10_{C_1} \times 10_{C_1} = 100$

Favourable cases $= 9_{C_1} \times 9_{C_1} = 81$

$\therefore$ Required probability $= \frac{81}{100}$

This correct option is (B)

58. If a is the radius of the circle, the area of the inscribed square $= 2a^2$ and

$p_1 = \frac{2a^2}{\pi a^2} = \frac{2}{\pi}$, $p_2 = 1 - p_1 = \frac{\pi - 2}{\pi}$

$\pi < 4$ and so $\pi - 2 < 2$ which gives $p_1 > p_2$.

$p_1^2 - p_2^2 = (p_1 + p_2)(p_1 - p_2)$

$= \frac{4 - \pi}{\pi} < \frac{1}{3}$ as $3 < \pi < 4$

This correct option is (B)

59. Let p = prob. of getting face value not less than 2 and not more than 5 in a single throw of die $= \frac{4}{6} = \frac{2}{3}$.

n = no. of times dice is rolled.

x = no. of times we get a number not less than 2 and not more than 5.

Then, $X \sim B(n, p)$

$\therefore$ reqd. prob. $= P(X = 4) = {}^4C_4\, p^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

This correct option is (A)

60. Let A denote the event that there is an odd man out in a game. The total number of possible cases $= 2m$

A person is odd man out if he is alone in getting a head or a tail. The number of ways in which there is exactly one tail (head) and the rest are heads (tails) is ${}^mC_1 = m$. Then the number of favourable ways is $m + m = 2^m$.

$$\therefore P(A) = \frac{2m}{2^m} = \frac{m}{2^{m-1}}$$

This correct option is (C)

61. The required probability $= P(\overline{A}_1 \cap \overline{A}_2 \cap \ldots \cap \overline{A}_n)$

$\Rightarrow P(E) = P(\overline{A}_1)\, P(\overline{A}_2) \ldots P(\overline{A}_n)$

$$\therefore P(E) = \frac{1}{\not{2}} \cdot \frac{\not{2}}{\not{3}} \cdot \frac{\not{3}}{\not{4}} \ldots \frac{\not{n}}{n+1} = \frac{1}{n+1}$$

This correct option is (B)

62. The total no. of ways of ticking one or more alternatives out of 4 is

${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 15$

Out of these 15 combinations, only one is correct. The prob. of ticking the alternatives correct at the first trial $= \frac{1}{15}$, that at the second trial is $\frac{14}{15} \cdot \frac{1}{14} = \frac{1}{15}$ and that at the third trial $=$

$\frac{14}{15} \cdot \frac{13}{14} \cdot \frac{1}{13} = \frac{1}{15}$.

Then, the prob. that the candidate will get marks on the question if he is allowed three trials is

$= \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5}$

This correct option is (D)

63. The box contains $2n$ shoes. We can choose $2r$ shoes out of $2n$ shoes in ${}^{2n}C_{2r}$ ways. We can choose one complete pair out of n pairs in ${}^nC_1$ ways. Now we have to avoid a complete pair. While choosing $(2r - 2)$ shoes out of remaining $(n - 1)$ pairs of shoes, we first choose $(r - 1)$ pairs out of $(n - 1)$ pairs. This can be done in ${}^{n-1}C_{r-1}$ ways. From each of these $(r - 1)$ pairs, choose $(r - 1)$ single (Unmatching) shoes from each pair. This can be done in 2^{r-1} ways. Thus the no. of favourable ways is

$({}^nC_1)\, ({}^{n-1}C_{r-1})^{2r-1}$

Hence, the prob. of the reqd. event

$$= \frac{n\,({}^{n-1}C_{r-1}).2^{r-1}}{{}^{2n}C_{2r}}$$

This correct option is (C)

64. Probability of at least one failure

$= 1 - p(\text{no failure}) \geq \frac{31}{32}$

$\Rightarrow 1 - p^5 \geq \frac{31}{32}$

$\Rightarrow p^5 \leq \frac{1}{32}$

$\Rightarrow p \leq \frac{1}{2}$

Also $p \geq 0$

Hence $p \in \left[0, \frac{1}{2}\right]$

This correct option is (D)

65. We have

$C \subset D$

$\Rightarrow C \cap D \quad = C$

$$\Rightarrow P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \geq P(C)$$

as $0 < P(D) \leq 1$

This correct option is (C)

66. Let event (Given: {1, 2, 3, ..., 8})

A : Maximum of three numbers is 6.

B : Minimum of the numbers is 3

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$$

This correct option is (B)

67. P(fair complexioned rich girl)

$= P$(fair) P(rich) P(girl)

$$= \frac{20}{80} \times \frac{10}{80} \times \frac{25}{80} = \frac{5}{512}$$

This correct option is (C)

68. $\frac{P(A \cap B)}{P(B)} = \frac{1}{2}, \frac{P(A \cap B)}{P(A)} = \frac{2}{3}$

Hence $\frac{P(A)}{P(B)} = \frac{3}{4}$ (But $P(A) = 1/4$)

$\Rightarrow P(B) = \frac{1}{2}$

This correct option is (B)

69. **(c).** $A = \{4, 5, 6\}, B = \{1, 2, 3, 4\}$

Obviously $P(A \cup B) = 1$

70.. Let E_1: a coin with two heads is selected

E_2: a fair coin is selected

A: the toss results in heads.

Then, $P(E_1) = \frac{1}{n+1}$, $P(E_2) = \frac{n}{n+1}$, $P(A/E_1) = 1$

and, $P(A/E_2) = \frac{1}{2}$

$\therefore \quad P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$

$$\Rightarrow \frac{7}{12} = \frac{1}{n+1} \cdot 1 + \frac{n}{n+1} \cdot \frac{1}{2}$$

$\Rightarrow 12 + 6n = 7n + 7 \Rightarrow n = \mathbf{5}.$

This correct option is (A)

71. $P(A \cap B) \quad = P(A) + P(B) - P(A \cup B)$

$= 0.6 + 0.4 - 0.8 = 0.2$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap C)$

$+ P(A \cap B \cap C) - P(A \cap B) - P(B \cap C)$

$\Rightarrow P(B \cap C) = 1.2 - P(A \cup B \cup C)$ ...(1)

$\because 0.85 \leq P(A \cup B \cup C) \leq 1$

$\therefore$ (1) $\Rightarrow P(B \cap C) \leq 1.2 - 0.85$

and $P(B \cap C) \geq 1.2 - 1 \Rightarrow \mathbf{0.2} \leq \mathrm{P(B \cap C)} \leq \mathbf{0.35.}$

This correct option is (A)

72. $P(E/F) + P(\bar{E}\,|F)$

$$= \frac{P(E \cap F)}{P(F)} + \frac{P(\bar{E} \cap F)}{P(F)} = \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)}$$

$$= \frac{P\{(E \cap F) \cup (\bar{E} \cap F)\}}{P(F)}$$

[$\because E \cap F$ and $\bar{E} \cap F$ are disjoint]

$$= \frac{P\{(E \cup \bar{E}) \cap F\}}{P(F)} = \frac{P(S \cap F)}{P(F)} = \frac{P(F)}{P(F)} = \mathbf{1.}$$

This correct option is (A)

73. Given: probability that electronic component fails when first used = 0.10, i.e., $P(F) = 0.10$

$\therefore \; P(F') = 1 - P(F) = 0.90$

and let $P(Y)$ = probability of new component to last for one year

Obviously, the two events are mutually exclusive and exhaustive.

$\therefore \; P(Y/F) = 0$ and $P(Y/F') = 0.99$

$$\therefore \; P(Y) = P(F) \cdot P\left(\frac{Y}{F}\right) + P(F') \cdot P\left(\frac{Y}{F'}\right)$$

$= 0.10 \times 0 + 0.90 \times 0.99 = 0 + (0.9)(0.99) = \mathbf{0.891.}$

This correct option is (A)

74. Let S be the sample space and E be the required event.

Now, $n(S)$ = total number of cases = $4^5 = 1024$.

and, $n(E)$ = coefficient of x^{15} in $(x^0 + x + x^{10} + x^{11})^5$

= coefficient of x^{15} in $[(1 + x)^5 (1 + x^{10})^5]$

= coefficient of x^{15} in

$(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)$

$\times (1 + 5x^{10} + 10x^{20} + \ldots) = 5$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{5}{1024}.$

This correct option is (B)

75. For random variable X, $n = 5$, $p = \frac{1}{2}$ and for random variable Y, $n = 7$, $p = \frac{1}{2}$

Now, $P(X + Y \geq 1) = 1 - P(X + Y < 1) = 1 - P(X + Y = 0)$

$= 1 - P(X = 0, Y = 0)$

$= 1 - P(X = 0)\,P(Y = 0)$

[$\because X$ and Y are independent]

$= 1 - {}^5C_0 \left(\frac{1}{2}\right)^5 \cdot {}^7C_0 \left(\frac{1}{2}\right)^7$

$= 1 - \left(\frac{1}{2}\right)^{12} = \frac{4095}{4096}.$

This correct option is (A)

76. Let one of the quantities be x. Thus, the other is $2n - x$. Then, product will be greatest when they are equal, i.e., each is n in which case the product is n^2. By the given condition

$x(2n - x) \geq \frac{3}{4}n^2$

$\Rightarrow 8nx - 4x^2 \geq 3n^2 \Rightarrow 4x^2 - 8nx + 3n^2 \leq 0$

$\Rightarrow (2x - 3n)(2x - n) \leq 0 \Rightarrow \frac{n}{2} \leq x \leq \frac{3}{2}n.$

$\therefore$ favourable number of cases $= \frac{3n}{2} - \frac{n}{2} = n.$

Hence, required probability $= \frac{n}{2n} = \mathbf{\frac{1}{2}}.$

This correct option is (B)

77. The number of numbers whose sum is 9 is

One-digit number = 1

Two-digit numbers = 9

Three-digit numbers = 9 + 8 + 7 + ... + 1 = $\frac{9 \times 10}{2} = 45$

$\therefore$ Required probability $= \frac{55}{1000} = \frac{11}{200}.$

This correct option is (B)

78. The probability that a man does not die in a year $= 1 - p$

$\therefore$ Probability that none of the n men die in a year $= (1 - p)^n$

$\therefore$ The probability that at least one man dies in a year $= 1 - (1 - p)^n$

Since every man can die first, the chance that A_1, will die first is $\frac{1}{n}$. Hence, the probability that A_1, will die within a year and he will be first to die $= \frac{1}{n}$ **[1 – (1 – p)^n].**

This correct option is (A)

79. $P(B | A \cup B') = \frac{P[B \cap (A \cap B')]}{P(A \cup B')}$

$= \frac{P[(B \cap A) \cup (B \cap B')]}{P(A) + P(B') - P(A \cap B')}$

But $P[(B \cap A) \cup (B \cap B')$ $= P(B \cap A)$

$= P(A) - P(A \cap B') = 0.2$

and, $P(A) + P(B') - P(A \cap B') = 0.7 + 0.6 - 0.5 = 0.8$

$\therefore P(B | A \cup B') = \frac{0.2}{0.8} = \frac{1}{4}.$

This correct option is (C)

80. A person can alight at any one of n floors. Therefore, the number of ways in which m passengers can alight at n floors is $\underbrace{n \times n \times n \times ... \times n}_{m - \text{times}} = n^m$.

The number of ways in which all passengers can alight at different floors is ${}^nC_m \times m! = {}^nP_m$.

Hence, required probability $= \frac{{}^nP_m}{n^m}.$

This correct option is (B)

81. We know 7^k, $k \in N$, has 1, 3, 9, 7 at the unit's place for $k = 4p, 4p - 1, 4p - 2, 4p - 3$ respectively, where $p = 1, 2, 3, ...$

Clearly, $7^m + 7^n$ will be divisible by 5 if 7^m has 3 or 7 in the unit's place and 7^n has 7 or 3 in the unit's place or 7^m has 1 or 9 in the unit's place and 7^n has 9 or 1 in the unit's place.

$\therefore$ For any choice of m, n the digit in the unit's place of $7^m + 7^n$ is 2, 4, 6, 0 or 8. It is divisible by 5 only when this digit is 0.

$\therefore$ the required probability $= \frac{1}{5}.$

This correct option is (A)

82. It is a case of Bernoullian trials with number of trials n and probability $= \frac{1}{N}$ (a success in one trial)

$\therefore P(r \text{ successes}) = {}^nC_r\, q^{n-1}\, p^r = {}^nC_r\, (1 - p)^{n-r}\, p^r$

$= {}^nC_r \left(1 - \frac{1}{N}\right)^{n-r} \left(\frac{1}{N}\right)^r$

$= {}^nC_r \left(\frac{1}{N}\right)^r \left(\frac{N-1}{N}\right)^{n-r}.$

This correct option is (A)

83. Let $A(B)$ denote the event that minimum (maximum) number on the chosen tickets is 3 (7).

We have,

$P(A) = P$(choosing 3 and two other numbers from 4 to 10)

$= \frac{{}^7C_2}{{}^{10}C_3} = \frac{7 \times 6 \times 3}{10 \times 9 \times 8} = \frac{7}{40}$

$P(B) = P$(choosing 7 and choosing two other numbers from 1 to 6)

$= \frac{{}^6C_2}{{}^{10}C_3} = \frac{6 \times 5 \times 3}{10 \times 9 \times 7} = \frac{1}{8}$

$P(A \cap B) = P$(choosing 3 and 7 and one other number from 4 to 6)

$= \frac{3}{{}^{10}C_3} = \frac{3 \times 3 \times 2}{10 \times 9 \times 8} = \frac{1}{40}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) =$ **11/40.**

This correct option is (C)

84. The exhaustive cases are 4! = 24

The favourable cases are $4!\left[\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right]$

= 12 – 4 + 1 = 9

$\therefore$ the required probability = $\frac{9}{24} = \frac{3}{8}$.

This correct option is (B)

85. Let the orthocentre be (x, y)

$\therefore\ (x-2)^2 + (y-3)^2 = (5-2)^2 + (5-3)^2$

$\Rightarrow x^2 + y^2 - 4x - 6y = 0$

$\Rightarrow x^2 - 4x + (y-3)^2 - 9 = 0$

$\Rightarrow x = 2 \pm \sqrt{13-(y-3)^2}$

$\therefore$ y can take the values 1, 2, 3, 4, 5, 6

$\therefore$ Required probability = $\frac{6}{10} = \frac{3}{5}$.

This correct option is (C)

86. The total length of the interval in which a lies = 30 – (– 5) = 35

If the graph of $y = x^2 + 2(a+4)x - 5a + 64$ is entirely above the x-axis, the discriminant of the above quadratic expression must be negative.

$\therefore\ 4(a+4)^2 + 4(5a - 64) < 0$

$\Rightarrow a^2 + 13a - 48 < 0 \Rightarrow (a+16)(a-3) < 0$

$\Rightarrow -16 < a < 3$

But $a \in [-5, 30]$ $\therefore$ $-5 < a < 3$ for the event to happen. The length of this interval = 3 – (– 5) = 8.

Hence, the required probability = $\frac{8}{35}$.

This correct option is (C)

87. Total ways = 15^7

For favourable ways, we must have 7 coupons numbered from 1 to 9 so that '9' is selected at least once. Thus, total number of favourable ways are, $9^7 - 8^7$

$\Rightarrow$ Required probability = $\frac{9^7-8^7}{15^7}$.

This correct option is (D)

88. Let x_i be any element of set P, we have following possibilities:

(i) $x_i \in A, x_i \in B$; (ii) $x_i \in A, x_i \notin B$;

(iii) $x_i \notin A, x_i \in B$; (iv) $x_i \notin A, x_i \notin B$

Clearly, the element $x_i \in A \cap B$ if and only if it belongs to A and B both. Thus, out of these 4 ways only first way is favourable. Now, the element that we want to be in the intersection can be chosen in 'n' different ways. Hence, required probability is $n.\left(\frac{3}{4}\right)^n$.

This correct option is (C)

89. $P(A) = (0.1)(0.1)(0.1) = .001$

If k trials are made, then it is natural to assume that the unsuccessful combinations are not repeated.

Now, $P(B) = 1 - P(\bar{B})$

$= 1 - \frac{999}{1000}.\frac{998}{999}\cdots\frac{1000-k+1}{1000-k+2}.\frac{1000-k}{1000-k+1}$

$= 1 - \frac{1000-k}{1000} = \frac{k}{1000}$.

This correct option is (B)

90. $P_1 = x_1x_2x_3 \ldots x_6$

$P_2 = y_1y_2 \ldots y_6$

Any digit after decimal can be 0, 1, 2,..., 9. P_2 can be subtracted from P_1 without borrowing if $x_i \geq y_i$

Let $y_i = \lambda\ (0 \leq \lambda \leq 9) \Rightarrow x_i = \lambda, \lambda+1, \ldots, 9$

$\Rightarrow$ After choosing y_i, x_i can be chosen in $(10-\lambda)$ ways.

$\Rightarrow$ Required probability

$$= \left(\frac{\sum_{\lambda=0}^{9}(10-\lambda)}{100}\right)^6 = \left(\frac{11}{20}\right)^6$$

This correct option is (C)

91. Let A be the event that a person has the disease and let B_i be the event that ith test is positive $(i = 1, 2)$. Then,

$P(A) = 0.1,\ P(B_i/A) = 0.9$

$P(B_i/\bar{A}) = 1 - P(B_i/A) = 1 - 0.9 = 0.1$

Let $B = B_1 \cap B_2$

$\therefore\ P(B_i/A) = P(B_1/A)\ P(B_2/A) = 0.81$

$P(B/\bar{A}) = (0.1)(0.1)$

$\therefore\ P(A/B) = \frac{P(A).P(B/A)}{P(A).P(B/A) + P(\bar{A}).P(B/\bar{A})}$

$= \frac{(0.1)(0.81)}{(0.1)(0.81) + (0.9)(0.1)(0.1)}$

$= \frac{0.081}{0.081 + .009} = \frac{81}{90} = \frac{9}{10} = \mathbf{0.9}$

This correct option is (B)

92. Generally, any one birthday can fall in one of the 12 months.

For six different people, the total number of ways is 12^6. Coming to the favourable ways: first any two months can be chosen in ${}^{12}C_2$ ways.

The six birthdays can fall in these 2 months in 2^6 ways of these the two cases when all the six birthdays fall in one month are to be removed.

$\therefore$ required probability = $\frac{{}^{12}C_2\,(2^6-2)}{12^6}$

This correct option is (C)

93. If l is the length of the chord, r, the distance of the mid-point of the chord from the centre of the circle and a radius of the given circle, then $r = a\cos\theta$, $l = 2a\sin\theta$.

Given: $\frac{2}{3}\, 2a < 2a\sin\theta < \frac{2}{3}\, 2a \Rightarrow \frac{5}{6}\, a < a\cos\theta < \frac{\sqrt{5}}{3} a$

$\Rightarrow \frac{\sqrt{11}}{6} a < r < \frac{\sqrt{5}}{3} a$.

$\therefore$ The given condition is satisfied if the mid-point of the chord lies within the region between the concentric circles of radius $\frac{\sqrt{11}}{6}$ and $\frac{\sqrt{5}}{3}$.

Hence, the probability

$= \frac{\text{The area of the circular annulus}}{\text{Area of the given circle}}$

$= \left(\frac{5}{9} - \frac{11}{36}\right) = \frac{1}{4}$

This correct option is (C)

94. We define the following events:

A_1 : Selecting a pair of consecutive letters from the word **LONDON**

A_2 : Selecting a pair of consecutive letters from the word **CLIftON**

E : Selecting a pair of letters **ON**'

Then, $P(A_1 \cap E) = \frac{2}{5}$, as there are 5 pairs of consecutive letters out of which 2 are **ON**.

$P(A_2 \cap E) = \frac{1}{6}$, as there are 6 pairs of consecutive letters of which one is **ON**.

So, required probability $= P\left(\frac{A_1}{E}\right)$

$\Rightarrow P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}}$

$\therefore P\left(\frac{A_1}{E}\right) = \mathbf{\frac{12}{17}}$

This correct option is (B)

95. The total number of ways of ticking one or more alternatives out of 4 is

${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 15$

Out of these 15 combinations, only one is correct. The probability of ticking the alternatives correct at the first trial $= \frac{1}{15}$, that at the second trial is $\frac{14}{15}.\frac{1}{14} = \frac{1}{15}$ and that at the third trial $= \frac{14}{15}.\frac{13}{14}.\frac{1}{13} = \frac{1}{15}$.

Then, the probability that the candidate will get marks on the question if he is allowed three trials is

$= \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5}$

This correct option is (D)

96. The box contains $2n$ shoes. We can choose $2r$ shoes out of $2n$ shoes in ${}^{2n}C_{2r}$ ways. We can choose one complete pair out of n pairs in ${}^{n}C_1$ ways. Now, we have to avoid a complete pair. While choosing $(2r - 2)$ shoes out of remaining $(n - 1)$ pairs of shoes, we first choose $(r - 1)$ pairs out of $(n - 1)$ pairs. This can be done in ${}^{n-1}C_{r-1}$ ways. From each of these $(r - 1)$ pairs, choose $(r - 1)$ single (unmatching) shoes from each pair. This can be done in 2^{r-1} ways. Thus, the number of favourable ways is

$({}^{n}C_1)\, ({}^{n-1}C_{r-1})2^{r-1}$

Hence, the probability of the required event

$= \frac{n\,({}^{n-1}C_{r-1}).2^{r-1}}{{}^{2n}C_{2r}}$

This correct option is (C)

97. There are four gaps in between the girls where the boys can sit. Let the number of boys in these gaps be $2a + 1, 2b + 1, 2c + 1, 2d + 1$, then

$2a + 1 + 2b + 1 + 2c + 1 + 2d + 1 = 10$

or, $a + b + c + d = 3$

The number of solutions of above equation

= coefficient of x^3 in $(1 - x)^{-4} = {}^6C_3 = 20$

Thus, boys and girls can sit in $20 \times 10! \times 5!$ ways

Total ways = 15!

Hence, the required probability $= \frac{20 \times 10! \times 5!}{15!}$.

This correct option is (A)

98. The total number of ways in which 4 tickets can be drawn 5 times $= 4^5 = 1024$.

Favourable number of ways of getting a sum of 23

= coefficient of x^{23} in $(x^{00} + x^{01} + x^{10} + x^{11})^5$

= coefficient of x^{23} in $[(1 + x)(1 + x^{10})]^5$

= coefficient of x^{23} in $(1 + x)^5 (1 + x^{10})^5$

= coefficient of x^{23} in

$(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)$

$\times (1 + 5x^{10} + 10x^{20} + 10x^{30} + ...) = 100$

$\therefore$ required probability $= \frac{100}{1024} = \frac{25}{256}$.

This correct option is (A)

99. $n(S) = {}^{100}C_3$

Since the A.M. of three numbers is 25

$\therefore$ Their sum = 75

$\therefore$ $n(E)$ = the number of integral solutions of

$x_1 + x_2 + x_3 = 75$

where $x_1 \geq 1, x_2 \geq 1, x_3 \geq 1$.

= coefficient of x^{75} in $(x + x^2 + x^3 + ...)^3$

= coefficient of x^{72} in $(1 + x + x^2 + ...)^3$

$= \text{ coefficient of } x^{72} \text{ in } \left(\frac{1}{1-x}\right)^3$

$= \text{ coefficient of } x^{72} \text{ in } (1-x)^{-3}$

$= {}^{74}C_{72} = {}^{74}C_2$

$\therefore\ P(E) = \frac{{}^{74}C_2}{{}^{100}C_3}$

This correct option is (C)

100. Roots of $x^2 + px + q = 0$, will be real if

$(p)^2 - 4 \times 1 \times q \geq 0$ i.e., $p^2 \geq 4q$

Different ways for the selection of p and q are

p	q	p	q
1	—	6	1, 2, 3, ..., 8, 9
2	1	7	1, 2, 3, ... , 8, 9, 10
3	1, 2	8	1, 2, 3, ... , 8, 9, 10
4	1, 2, 3, 4	9	1, 2, 3, ... , 8, 9, 10
5	1, 2, 3, 4, 5, 6	10	1, 2, 3, ... , 8, 9, 10.

$\therefore$ Favourable ways in which p and q can be selected = 62.

Total number of ways in which p and q can be selected

$= 10 \times 10 = 100$

$\therefore$ Required probability

$= \frac{62}{100} = \mathbf{0.62.}$

This correct option is (A)

101. Given: $P(A) + P(B) - 2P(A \cap B) = 1 - a$ ...(1)

$P(B) + P(C) - 2P(B \cap C) = 1 - 2a,$ ...(2)

$P(C) + P(A) - 2P(C \cap A) = 1 - a,$ ...(3)

and, $P(A \cap B \cap C) = a^2$ ...(4)

$\therefore\ P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$- P(A \cap B) - P(B \cap C)$

$- P(C \cap A) + P(A \cap B \cap C)$

$= \frac{1}{2}\ \{P(A) + P(B) - 2P(A \cap B) + P(B)$

$+ P(C) - 2P(B \cap C) + P(C)$

$+ P(A) - 2P(C \cap A) + P(A \cap B \cap C)\}$

$= \frac{1}{2}\ \{1 - a + 1 - 2a + 1 - a\} + a^2$

{from (1), (2) (3) and (4)}

$= \frac{3}{2} - 2a + a^2 = (a-1)^2 + \frac{1}{2} > \frac{1}{2}.$

This correct option is (C)

102. Let S be the sample space and E be the event the exactly 4 persons sit between A and B.

Then, $n(S)$ = total number of ways in which 15 persons can sit at a round table = 14!

Total number of persons includng A and B = 15.

Number of persons excluding A and B = 13

Out of these 13 persons 4 can be selected in ${}^{13}C_4$ ways. Regarding A and B and four elected pesons between them as one person, we have only (9 + 1) = 10 persons.

These 10 persons can be arranged along a circle in 9! ways. A and B can be aranged among themseleves in 2! ways and 4 person between A and B can be arranged among themseleves in 4! ways.

$\therefore\ n(E)$ = total number of ways in which 15 persons (A and B included) can sit at a round table so that exactly 4 persons sit between A and B

$= {}^{13}C_4\ 9!\ 2!\ 4!$

Now, required probability,

$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{13}C_4 \cdot 9!\,2!\,4!}{14!} = \frac{13!\,9!\,2!\,4!}{4!\,9!\,14!} = \frac{2}{14} = \frac{1}{7}.$

This correct option is (B)

103. Here, random experiment is; selecting n pairs of balls from a bag containig n white and n red balls.

Let S = the sample space

and E = the event that each of the n pairs of balls drawn consists of one white and one red ball.

Now, $n(S) = ({}^{2n}C_2)\,({}^{2n-2}C_2)\,({}^{2n-4}C_4) \ldots ({}^{4}C_2)({}^{2}C_2)$

$= \frac{2n!}{(2n-2)!\,2!} \cdot \frac{(2n-2)!}{(2n-4)!\,2!} \cdot \frac{(2n-4)!}{(2n-6)!\,2!} \cdots \frac{4!}{2!\,2!} \cdot 1$

$= \frac{(2n)!}{2^n}$

and, $n(E) = ({}^{n}C_1 \cdot {}^{n}C_1) \cdot ({}^{n-1}C_1 \cdot {}^{n-1}C_1) \ldots$

$({}^{2}C_1 \cdot {}^{2}C_1) \cdot ({}^{1}C_1 \cdot {}^{1}C_1)$

$= n^2 \cdot (n-1)^2 \ldots 2^2 \cdot 1^2 = (n!)^2$

$\therefore$ Required probability,

$P(E) = \frac{n(\text{E})}{n(\text{S})} = \frac{(n!)^2\ \ 2^n}{(2n)!} = \left(\frac{2^n}{{}^{2n}C_n}\right).$

This correct option is (C)

104. Bag contains a white and b black balls

Let E_i = the event of drawing a white ball in the ith draw

Then E'_i = the event of drawing a black ball in the ith draw.

Let E = the event that A wins the game

Then, E' = the event that B wins the game.

Since after each draw the ball drawn is replaced

$\therefore\ P(E_i) = \frac{a}{a+b}$ and $E'_i = \frac{b}{a+b}$

Now, $P(E) = P(E_1) + P(E'_1 E'_2 E_3)$

$+ P(E'_1 E'_2 E'_3 E'_4 E_5) + \ldots$ to ∞

$= P(E_1) + P(E'_1)\,P(E'_2) \cdot P(E_3) + P(E'_1) \cdot P(E'_2)$

$\cdot P(E'_3)\,P(E'_4) \cdot P(E_5) + \ldots$ to ∞.

$= \frac{a}{a+b} + \frac{b^2}{(a+b)^2} \bullet \frac{a}{(a+b)} + \frac{b^4}{(a+b)^4} \bullet \frac{a}{a+b} + \ldots$

to ∞.

$$= \left(\frac{a}{a+b}\right) \cdot \left[1 + \left(\frac{b}{a+b}\right)^2 + \left(\frac{b}{a+b}\right)^4 + \ldots \text{ to } \infty\right]$$

$$= \left(\frac{a}{a+b}\right) \cdot \frac{1}{\left[1 - \left(\frac{b}{a+b}\right)^2\right]} = \left[\frac{a(a+b)}{a^2 + 2ab}\right] = \frac{a+b}{a+2b}$$

$$\therefore\ P(E') = 1 - \frac{a+b}{a+2b} = \frac{b}{(a+2b)}$$

According to question, $P(E) = 3P(E')$

$$\therefore\ \frac{a+b}{a+2b} = \frac{3b}{a+2b} \text{ or } a + b = 3b \text{ or } a = 2b$$

Hence, $a : b =$ **2 : 1.**

This correct option is (A)

105. We have,

$P(X + Y = 3) = P(X = 0, Y = 3) + P(X = 1, Y = 2)$
$+ P(X = 2, Y = 1) + P(X = 3, Y = 0)$
$= P(X = 0) P(Y = 3) + P(X = 1) P(Y = 2)$
$+ P(X = 2) P(Y = 1) + P(X = 3) P(Y = 0)$
[$\because$ X and Y are independent]

$$= {}^5C_0 \left(\frac{1}{2}\right)^5 \cdot {}^7C_3 \left(\frac{1}{2}\right)^7 + {}^5C_1 \left(\frac{1}{2}\right)^5 {}^7C_2 \left(\frac{1}{2}\right)^7$$

$$+ {}^5C_2 \left(\frac{1}{2}\right)^5 {}^7C_1 \left(\frac{1}{2}\right)^7 + {}^5C_3 \left(\frac{1}{2}\right)^5 {}^7C_0 \left(\frac{1}{2}\right)^7.$$

$$= \left(\frac{1}{2}\right)^{12} [(1)(35) + (5)(21) + (10)(7) + (10)(1)]$$

$$= \frac{220}{2^{12}} = \frac{55}{1024}.$$

This correct option is (A)

106. Let us define the following events:

A_1: Plant I is selected

A_2: Plant II is selected

B: In a selection of 2 persons, one is production and the other is maintenance foreman.

The required event of selecting one production and one maintenance foreman in a selection of two persons can materialize in the following mutually exclusive ways

(*i*) $A_1 \cap B$ happens, (*ii*) $A_2 \cap B$ happens

Hence, by additions rule of probabiity, the required probability p is given by

$P = P(\text{i}) + P(\text{ii}) = P(A_1 \cap B) + P(A_2 \cap B)$
$= P(A_1) P(B \backslash A_1) + P(A_2) P(B \backslash A_2)$...(1)

Since there are two plants, the selection of each being equally likely, we have

$$P(A_1) = P(A_2) = \frac{1}{2}.$$

$P(B \setminus A_1)$ = probability of selection of one production and one maintenance foreman in a selection of two foremen from the first plant

$$= \frac{{}^5C_1 \times {}^3C_1}{{}^8C_2} = \frac{5 \times 3 \times 2!}{8 \times 7} = \frac{15}{28}$$

Similarly, $P(B \backslash A_2) = \dfrac{{}^4C_1 \times {}^5C_1}{{}^9C_2} = \dfrac{4 \times 5 \times 2!}{9 \times 8} = \dfrac{5}{9}$

Substituting in (1), we get

$$p = \frac{1}{2} \times \frac{15}{28} + \frac{1}{2} \times \frac{5}{9} = \frac{275}{504}.$$

This correct option is (A)

107. Let E_1 be an event of knowing the correct answer, E_2 be an event of not knowing the correct answer, and A be an event of getting correct answer with guessing.

Now, in the usual notations

$$P(A/E_1) = 1, P(A/E_2) = \frac{1}{4} = 0.25$$

$\therefore$ For an intelligent student

$P(E_1) = .90, P(E_2) = 1 - .90 = .10$

$\therefore$ Probability of getting correct answer when he was guessing is

$$P(E_2/A) = \frac{P(E_2)\, P(A/E_2)}{P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2)}$$

$$= \frac{0.10 \times 0.25}{0.90 \times 1 + 0.10 \times 0.25} = \frac{0.025}{0.925} = \frac{1}{37}.$$

This correct option is (A)

108. We have, $P(X = r \mid X + Y = r + s)$

$$= \frac{P[(X = r) \cap (X + Y = r + s)]}{P(X + Y = r + s)}$$

$$= \frac{P[(X = r) \cap (Y = s)]}{P(X + Y = r + s)} = \frac{P(X = r)\, P(Y = s)}{P(X + Y = r + s)}$$

$$\text{and, } P(X + Y = r + s) = \sum_{k=0}^{r+s} P[(X = k) \cap (Y = r + s - k)]$$

$$= \sum_{k=0}^{r+s} ({}^nC_k\, p^k\, q^{n-k})({}^mC_{r+s-k}\, p^{r+s-k}\, q^{m-r-s+k})$$

$$= p^{r+s} q^{m+n-r-s} \sum_{k=0}^{r+s} ({}^nC_k)({}^mC_{r+s-k}).$$

Now, the last sum is the expression for the number of ways of choosing $r + s$ persons out of n men and m women, which is ${}^{m+n}C_{r+s}$. Therefore,

$P(X + Y = r + s) = {}^{m+n}C_{r+s}\, p^{r+s} q^{m+n-r-s}$

so that $P(X = r \mid X + Y = r + s)$

$$= \frac{({}^mC_r\, p^r\, q^{n-r})({}^nC_s\, p^s\, q^{m-s})}{{}^{m+n}C_{r+s}\, p^{r+s} q^{m+n-r-s}} = \frac{({}^mC_r)({}^nC_s)}{{}^{m+n}C_{r+s}}.$$

This correct option is (A)

109. The number of ways of choosing three numbers out of N is ${}^NC_3$. If these are a_1, a_2 and a_3, they must satisfy exactly one of the following inequalities for a successful outcome:

$a_1 < a_2 < a_3$, $a_1 < a_3 < a_2$, $a_2 < a_1 < a_3$,
$a_2 < a_3 < a_1$, $a_3 < a_1 < a_2$, $a_3 < a_2 < a_1$.

Thus, the number of ways of arranging the three numbers in a given order is $({}^NC_3)$ (6), and there are 3 ways in which the first number is less than the second. Now, if A denotes the event: the first number is less than the second number and B the event: the third number lies between the first and the second, we need to find $P(B/A)$. Since

$$P(B \cap A) = \frac{{}^NC_3}{{}^NC_3(6)} = \frac{1}{6} \text{ and } P(A) = \frac{{}^NC_3(3)}{{}^NC_3(6)} = \frac{1}{2},$$

we get, $P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{1/6}{1/2} = \dfrac{1}{3}$.

This correct option is (B)

110. In any number the last digit can be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. We want that the last digit in the product is an odd digit other than 5, i.e., it is any one of the digits 1, 3, 7, 9. This means that the product is not divisible 2 or 5. The probability that a number is divisible by 2 or 5 is $\dfrac{6}{10}$

[∵ in that case the last digit can be one of 0, 2, 4, 5, 6, 8]

∴ The probability that the number is not divisible by 2 or 5 $= 1 - \dfrac{6}{10} = \dfrac{4}{10} = \dfrac{2}{5}$

In order that the product is not divisible by 2 or 5, none of the constituent numbers should be divisible by 2 or 5 and its probability $= \left(\dfrac{2}{5}\right)^4 = \dfrac{16}{625}$.

This correct option is (C)

111. There are 10 digits 0, 1, 2, ... 9 any of which can occur in any number at the last place, i.e. at the unit place.

It is obvious that if the last digit in any of the four numbers is 0, 2, 4, 5, 6, 8 then the product of any of such four numbers will not give a number having its last digit as 1, 3, 7, 9. Hence, it is necessary that the last digit in each of the four numbers must be any of the four digits 1, 3, 7, 9.

Thus, for each of the four numbers, the numbers of ways for the last digit = 10 and favourable number of ways for the last digit = 4.

∴ the probability that the last digit is any of the four numbers 1, 3, 7, 9 is = 4/10 = 2/5

Hence, the required probability that the last digit in each of the four numbers is 1, 3, 7, 9 so that the last digit in their product is 1, 3, 7, 9 = $(2/5)^4 = \dfrac{16}{625}$.

This correct option is (C)

112. Let A_1 be the event that black card is lost, A_2 be the event that red card is lost and let A denote occurrence of first 13 cards which are examined are found to be all red.

Then, we have to find $P(A_1/A)$.

$P(A_1) = P(A_2) = 1/2$

Also, $P(A/A_1) = {}^{26}C_{13}/{}^{51}C_{13}$

and, $P(A/A_2) = {}^{25}C_{13}/{}^{51}C_{13}$

Then, by Baye's Rule,

$$P(A_1/A) = \frac{P(A_1)\cdot P(A/A_1)}{P(A_1)\cdot P(A/A_1) + P(A_2)\cdot P(A/A_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{{}^{26}C_{13}}{{}^{51}C_{13}}}{\frac{1}{2}\cdot\frac{{}^{26}C_{13}}{{}^{51}C_{13}} + \frac{1}{2}\cdot\frac{{}^{25}C_{13}}{{}^{51}C_{13}}} = \frac{{}^{26}C_{13}}{{}^{26}C_{13} + {}^{25}C_{13}}$$

$$= \frac{2}{2+1} = \frac{2}{3}.$$

This correct option is (A)

113. Let n = total number of ways = (10)! – (9)!

To find the favourable number of ways. We observe that a number is divisible by 4 if the last two-digit number is divisible by 4. Hence, the last two digits can be 20, 40, 60, 80, 12, 32, 52, 72, 92, 04, 24, 64, 84, 16, 36, 56, 76, 96, 08, 28, 48, 68. Corresponding to each of 20, 40, 60, 80, 04, 08, the remaining 8 places can be filled up in (8)! ways so that the number of ways in this case = 6 × 8 !

And corresponding to remaining 16 possibilities, the number of ways

= 16(8 ! – 7 !)

Hence, m = favourable number of ways

= 6 × 8 ! + 16(8 ! – 7 !) = 22 × 8 ! – 16 × 7 !

∴ The required probability

$$= \frac{m}{n} = \frac{22 \times 8! - 16 \times 7!}{10! - 9!}$$

$$= \frac{22 \times 8 - 16}{10 \times 9 \times 8 - 9 \times 8} = \frac{160}{648} = \frac{20}{81}.$$

This correct option is (B)

114. Let A denote the event that $ax^2 + bx + c = 0$ has non- real complex roots. Then, $\overline{A}$ is the event that $ax^2 + bx + c = 0$ has real roots. We have,

$P(\overline{A}) = 1 - P(A)$

$ax^2 + bx + c = 0$ has real roots if and only if $b^2 - 4ac \geq 0$ or $b^2 \geq 4ac$. Total number of cases = 6 × 6 × 6 = 216.

We now proceed to find favourable cases. Since maximum value of b is 6, $b^2 \leq 36$. Therefore, $4ac \leq 36$ or $ac \leq 9$

ac	**4*ac***	**(*a*, *c*)**	*b*	**No. of cases**
1	4	(1,1)	2, 3, 4, 5, 6,	5
2	8	(1, 2), (2, 1)	3, 4, 5, 6,	8
3	12	(1, 3), (3, 1)	4, 5, 6	6
4	16	(1, 4), (2, 2) (4, 1)	4, 5, 6	9
5	20	(1, 5), (5, 1)	5, 6	4
6	24	(1, 6), (2, 3), (3, 2), (6, 1)	5, 6	8
7	28	—	—	0
8	32	(4, 2), (2, 4)	6	2
9	36	(3, 3)	6	1

Thus, total number of cases

$5 + 8 + 6 + 9 + 4 + 8 + 2 + 1 = 43$

Thus, $P(\overline{A}) = \dfrac{43}{216}$

Hence, $P(A) = 1 - \dfrac{43}{216} = \dfrac{173}{216}$.

This correct option is (A)

115. Let $A = \{a_1, a_2, a_3, ...a_n\}$. For any $a_i \in A$, $1 \le i \le n$, there are four possibilities

(1) $a_i \in P$ and $a_i \in Q$ (2) $a_i \in P$ and $a_i \notin Q$

(3) $a_i \notin P$ and $a_i \in Q$ (4) $a_i \notin P$ and $a_i \notin Q$

$\therefore$ Total number of cases $= 4 \times 4 \times ..... \times 4$ (n times) $= 4^n$

Further, out of above four possibilities, first three satisfy $a_i \in P \cup Q$

So, the number of cases, when exactly r elements of A belong to $P \cup Q = {}^nC_r(3)^r$

Required probability $= \dfrac{{}^nC_r(3)^r}{4^n}$

This correct option is (A)

116. Let S be the sample space. Then,

$S = \{1, 2, 3, 4\} \times \{1, 2, 3, 4, 5, 6\}$

$\therefore$ $n(S) = 24$

If F be the event that the sum of numbers on dice is 6, then $n(F)$ = coefficient of x^6 in

$(x^1 + x^2 + x^3 + x^4) \times (x^1 + x^2 + x^3 + x^4 + x^5 + x^6)$

$= 1 + 1 + 1 + 1 = 4$

$\therefore$ Required probability $= \dfrac{n(F)}{n(S)} = \dfrac{4}{24} = \dfrac{1}{6}$

This correct option is (C)

117. The total number of coins in the box is $N + 7$. The total value of the selected coins will be greater than or equal to one rupee and fifty paise if the selected coins are

– 1 fifty paise coin, 4 twenty-five paise coins

– 2 fifty paise coins, 3 twenty-five paise coins

–2 fifty paise coins, 2 twenty-five paise coins, and 1 coin out of the N five paise and ten paise coins.

If E = total value of the 5 selected coins is greater than or equal to one rupee and fifty paise, then

$n(E) = {}^2C_1 \cdot {}^5C_4 \cdot {}^NC_0 + {}^2C_2 \cdot {}^5C_3 \cdot {}^NC_0 + {}^2C_2 \cdot {}^5C_2 \cdot {}^NC_1$

$= 10(N + 2)$

Hence, required probability

$= 1 - P(E) = 1 - \dfrac{n(E)}{n(S)} = \mathbf{1} - \dfrac{10(N+2)}{{}^{N+7}C_5}$

This correct option is (B)

118. Let us assume that A wins after n deuces, $n = 0, 1, 2, 3,....$

The game is a deuce if A wins his serve, then B wins his serve or A loses his serve, then B also loses his serve.

$\therefore$ Probability of a deuce $= \dfrac{2}{3}\cdot\dfrac{2}{3} + \dfrac{1}{3}\cdot\dfrac{1}{3} = \dfrac{5}{9}$

So, probability that A wins the game after n deuces

$= \left(\dfrac{5}{9}\right)^n \cdot \dfrac{2}{3}\cdot\dfrac{1}{3}$ [After nth deuce, A serves and wins, then B serves and loses]

$\therefore$ Required probability of A winning the game

$= \displaystyle\sum_{n=0}^{\infty}\left(\frac{5}{9}\right)^n \cdot \frac{2}{3}\cdot\frac{1}{3} = \frac{1}{1-\frac{5}{9}}\cdot\frac{2}{9} = \mathbf{\frac{1}{2}}$

This correct option is (C)

119. The total number of ways in which papers of 4 students can be checked by seven teachers is 7^4.

The number of ways of choosing two teachers out of 7 is ${}^7C_2$. The number of ways in which they can check four papers is 2^4. But this includes two ways in which all the papers will be checked by a single teacher.

$\therefore$ the number of ways in which 4 papers can be checked by exactly two teachers is $2^4 - 2 = 14$.

$\therefore$ the number of favourable ways $= ({}^7C_2)(14) = (21)(14)$

$\therefore$ required probability $= \dfrac{(21)(14)}{7^4} = \dfrac{3 \times 2}{49} = \mathbf{\dfrac{6}{49}}$

This correct option is (C)

More than One Option Correct Type

120. $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)\,P(B)$

$\Rightarrow P(A \cap B) = P(A)\,P(B)$

$\therefore$ A and B are independent events.

$\Rightarrow P(B \cap A') = P(B)\,P(A') \ne P(B) - P(A)$

and, $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B) = 1 - P(A)\,P(B)$

$\ne 1 - P(A) + 1 - P(B) = P(A') + P(B')$.

Also $P[(A \cup B)'] = P(A' \cap B') = \mathbf{P(A')\,P(B')}$

Since A and B are independent $\boldsymbol{P(A/B) = P(A)}$.
This correct option is (C, D)

121. $P(E_1) = 2/4 = 1/2$, $P(E_2) = 2/4 = 1/2$.
Similarly, $P(E_3) = 1/2$
$P(E_1E_2) = 1/4$, $P(E_1E_3) = 1/4$
$P(E_2E_3) = 1/4$, $P(E_1E_2E_3) = 1/4$
$\therefore\ P(E_1E_2) = P(E_1)\cdot P(E_2)$
$P(E_2E_3) = P(E_2)\cdot P(E_3)$
$P(E_1E_3) = P(E_1)\cdot P(E_3)$.
$\therefore\ E_1, E_2, E_3$ are pair-wise independent, so (a) is correct.
Also, $P(E_1\bar{E}_2) = 1/4 = P(E_1)\cdot P(\bar{E}_2)$
So E_1 and $\bar{E}_2$ are independent, so (b) is also true.
$P(\bar{E}_2\bar{E}_3) = 1/4 = P(\bar{E}_2)\cdot P(\bar{E}_3)$
$\therefore$ $\boldsymbol{\bar{E}_2}$ **and** $\boldsymbol{\bar{E}_3}$ **are independent,** i.e., (c) is not correct.
Finally $P(E_1E_2E_3) = 1/4 \neq P(E_1)\cdot P(E_2)\cdot P(E_3)$.
So, ***E*1, *E*2, *E*3, are not mutually independent.**
This correct option is (A, B)

122. We have, $P(A) = \frac{1}{3}$ and $P(A\cup B) = \frac{3}{4}$
$\therefore\ P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$\Rightarrow \frac{3}{4} = \frac{1}{3} + P(B) - P(A\cap B)$
$\Rightarrow \frac{5}{12} = P(B) - P(A\cap B)$
$\Rightarrow P(B) = \frac{5}{12} + P(A\cap B) \Rightarrow P(B) \geq \frac{5}{12}$ ...(i)
Again, $P(B) = \frac{5}{12} + P(A\cap B)$
$\Rightarrow P(B) \leq 5/12 + P(A)$ $[\because\ P(A\cap B) \leq P(A)]$
$\Rightarrow\ P(B) \leq \frac{5}{2} + \frac{1}{3} = \frac{3}{4}$ ...(ii)
From (i) and (ii), we obtain $5/12 \leq P(B) \leq 3/4$
Hence, x = **5/12** and $y = \boldsymbol{\frac{3}{4}}$.
This correct option is (A, D)

123. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$\therefore\ 1 \geq P(A) + P(B) - P(A\cap B) \geq \frac{3}{4}$
As the minimum value of $P(A\cap B) = \frac{1}{8}$,
we get, $P(A) + P(B) - \frac{1}{8} \geq \frac{3}{4}$
$\Rightarrow \boldsymbol{P(A) + P(B) \geq \frac{1}{8} + \frac{3}{4} = \frac{7}{8}}$
As the maximum value of $P(A\cap B) = \frac{3}{8}$, we get
$1 \geq P(A) + P(B) - \frac{3}{8} \Rightarrow \boldsymbol{P(A) + P(B) \leq 1 + \frac{3}{8} = \frac{11}{8}}$.
This correct option is (A, C)

124. The student will be successful in three cases.
Now, $P(I, II, \text{not } III) + P(I, III, \text{not } II) + P(I, II, III)$
$= p\cdot q\cdot\left(1-\frac{1}{2}\right) + p\frac{1}{2}(1-q) + p\cdot q\frac{1}{2}$
$= \frac{1}{2}pq + \frac{1}{2}p - \frac{1}{2}pq + \frac{1}{2}pq$
$= \frac{1}{2}p(1+q) = \frac{1}{2}p(1+q)$ (given) $\Rightarrow p(1+q) = 1$
This condition will be satisfied when $\boldsymbol{p = 1, q = 0}$.

125. P(exactly one of A and B occur)
$= P\{(A\cap\bar{B})\ or\ (\bar{A}\cap B)\} = P(A\cap\bar{B}) + P(\bar{A}\cap B)$
$= \boldsymbol{P(A) - P(A\cap B) + P(B) - P(A\cap B)}$
$= P(A\cup B) - P(A\cap B)$
$= 1 - P(\bar{A}\cap\bar{B}) - 1 + P(\bar{A}\cup\bar{B})$
$= P(\bar{A}) + P(\bar{B}) - 2P(\bar{A}\cap\bar{B})$.
This correct option is (A, D)

126. Let A, B and C, respectively denote the events that the student passes in Mathematics, Physics and Chemistry.
We are given
$P(A) = m$, $P(B) = p$ and $P(C) = c$.
and, $0.75 = P(A\cup B\cup C) = 1 - P(A'\cap B'\cap C')$
$= 1 - P(A')\,P(B')\,P(C')$
$0.75 = 1 - (1-m)(1-p)(1-c)$
$\Rightarrow (1-m)(1-p)(1-c) = 0.25$ ...(1)
Also, $0.50 = P$(passing at least two subjects)
$\Rightarrow .50 = pm(1-c) + pc(1-m) + cm(1-p) + pcm$
and, $0.4 = $ P(exactly two subjects)
$0.4 = pm(1-c) + pc(1-m) + cm(1-p)$ ...(3)
Thus, $\boldsymbol{pcm = 0.1 = 1/10}$ ...(4)
From (3), we get $pm + pc + cm - 3pmc = 0.4$
$\Rightarrow pm + pc + cm = 0.7$
From (1), we get
$1 - (m + p + c) + (pm + pc + cm) - pmc = 0.25$
$\Rightarrow 1 - (m + p + c) + 0.7 - 0.1 = 0.25$
$\Rightarrow \boldsymbol{p + m + c = 1.35 = \frac{27}{20}}$.
This correct option is (B, C)

127. We have,
$P(A\cup B) \geq \max\ \{P(A), P(b)\}$ = **2/3**
Next, $P(A\cap B) = P(A) + P(B) - P(A\cup B) \geq P(A) + P(B) - 1 = 1/6$
and, $P(A\cap B)) \leq \min\{P(A), P(B)\} = 1/2$

$\Rightarrow$ **1/6 ≤ $P(A \cap B)$ ≤ 1/2**

Also, $P(A \cap B') = P(A) - P(A \cap B) \le 1/2 - 1/6 =$ **1/3**

Lastly, $P(A' \cap B) = P(B) - P(A \cap B)$

Hence, $2/3 - 1/2 \le P(A' \cap B) \le 2/3 - 1/6$

$\Rightarrow$ **1/6 ≤ $P(A' \cap B)$ ≤ 1/2.**

This correct option is (A, B, C and D)

128. When $n = 1$, then the two possible outcomes, viz., H and T satisfy the condition that no two (or more) consecutive heads occur. Thus, $P_1 =$ **1.**

When $n = 2$, the possible outcomes are HH, HT, TH and TT. Out of these, first outcome, viz., HH does not satisfy the condition that no two (or more) consecutive heads occur. Thus,

$P_2 = 1 - P(\text{HH}) = 1 - pp =$ **$1 - p2$.**

For $n \ge 3$, if the last outcome is T, then the probability that first $(n - 1)$ tosses do not contain two consecutive (or more) heads is $P^n{}_{-1}$ and if the last outcome is H, then $(n - 1)th$ outcome must be T and the probability that first $(n - 2)$ tosses do not contain two (or more) consecutive heads is $P^n{}_{-2}$. Hence,

$P^n = P^n{}_{-1}\, P$(nth toss results in a tail)

$+ P^n{}_{-2}$ [P((n – 1)th toss results in tail and nth toss results in head]

$= P^n{}_{-1}(1 - p) + P^n{}_{-2}(1 - p)P$

$= \mathbf{(1 - p)\, P_n{-1} + p(1 - p)\, P_n{-2}.}$

This correct option is (A, B, C and D)

129. We have, $P(X \ge a) = \sum_{r=a}^{\infty} pq^r = \frac{pq^a}{1-q} = \boldsymbol{q^a}$

Next, $P(X \ge a + b | X \ge a) = \frac{P[(X \ge a+b) \cap (X \ge a)]}{P(X \ge a)}$

$= \frac{P(X \ge a+b)}{P(X \ge a)} = \frac{q^{a+b}}{q^a} = q^b = P(X \ge b)$

Similarly, $P(X \ge a + b | X \ge b) =$ **$P(X \ge a)$**

Again, $P(X = a + b | X \ge a) = \frac{P[(X = a+b) \cap (X \ge a)]}{P(X \ge a)}$

$= \frac{P(X = a+b)}{P(X \ge a)} = \frac{pq^{a+b}}{q^a} = pq^b = P(X = b)$

This correct option is (A, B, C and D)

130. We have,

(A) $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$\ge \frac{1 - P(A') - P(B')}{P(A)}$

$= \frac{P(A) - P(B')}{P(A)} = \mathbf{1 - \frac{P(B')}{P(A)}}$

(B) $P(B'|A') = \frac{P(A' \cap B')}{P(A')}$

$\ge \frac{1 - P[(A')'] - P[(B')']}{P(A')}$

$= \frac{1 - P(A) - P(B)}{P(A')}$

$= \frac{P(A') - P(B)}{P(A')} = \mathbf{1 - \frac{P(B)}{P(A')}}$

(C) $P(A' \cup B'|A) = \frac{P[(A \cap B)' \cap A]}{P(A)}$

$= \frac{P[S - (A \cap B) \cap A]}{P(A)}$

$= \frac{P(S \cap A - (A \cap B))]}{P(A)}$

$= \frac{P[A - A \cap B]}{P(A)}$

$= \frac{P(A) - P(A \cap B)}{P(A)}$

$= 1 - \frac{P(A \cap B)}{P(A)}$

This correct option is (A, B, and C)

131. We have,

$P(A) = 2/5$ and $P(B) = 4/5$

(a) Since $A, B \subseteq A \cup B$, therefore

$P(A \cup B) \ge \max\{P(A), P(B)\} =$ **4/5.**

(b) Since $A \cap B \subseteq A, B$, therefore

$P(A \cap B) \le \min\{P(A), P(B)\} =$ **2/5**

Also, $P(A \cap B) \ge 1 - P(A') - P(B')$

$= P(A) + P(B) - 1 = \frac{2}{5} + \frac{4}{5} - 1 = \frac{1}{5}$

$\therefore$ **1/5 ≤ $P(A \cap B)$ ≤ $\frac{2}{5}$.**

(c) From (b) above, we have

$\frac{1/5}{4/5} \le \frac{P(A \cap B)}{P(A)} \le \frac{2/5}{4/5}$

i.e., **1/4 ≤ $P(A|B)$ ≤ 1/2.**

(d) $P(A \cap B') = P(A) - P(A \cap B)$

$= \frac{2}{5} - P(A \cap B)$

$\le \mathbf{\frac{2}{5} - \frac{1}{5}}$ $\left[\text{From (b), } P(A \cap B) \ge \frac{1}{5}\right]$

This correct option is (A, B, C and D)

132. P (at most one of A, B occurs)

$= P(A' \cap B) + P(A \cap B') + P(A' \cap B')$

$= [P(B) - P(A \cap B)] + [P(A) - P(A \cap B)]$
$+ 1 - P(A \cup B)$
$= P(A \cup B) - P(A \cap B) + 1 - P(A \cup B) = \mathbf{1 - P(A \cap B)}$
Simplifying option (b), we have
$\mathbf{P(A') + P(B') - P(A' \cap B')}$
$= [1 - P(A)] + [1 - P(B)] - [1 - P(A \cup B)]$
$= 1 - [P(A) + P(B) - P(A \cup B)] = 1 - P(A \cap B)$.

Simplifying option (c), we have
$P(A') + P(B') + P(A \cup B)$
$= P(A') + P(B') - [1 - P(A \cup B)] + 1$
$= P(A') + P(B') - P(A' \cap B') + 1 = \mathbf{1 - P(A \cap B) + 1.}$
This correct option is (A, B)

Passage Based Questions

133. The sample space S = {choosing six players as winners out of 12 players}.
Then, $n(S) = {}^{12}C_6$.
If $A = \{S_1$ and S_2 are among the six winners} then
$n(A)$ = ways of choosing S_1, S_2, and 4 others from 10 players $= {}^{10}C_4$.
Hence, $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{{}^{10}C_4}{{}^{12}C_6} = \dfrac{10.9.8.7}{12.11.10.9} = \mathbf{\dfrac{14}{33}}$.
This correct option is (C)

134. Let B = {exactly one of S_1 and S_2 is among of the six winners}.
Case I: (S_1 and S_2 are paired)
Let $E_1 = \{S_1$ and S_2 are paired}. Since S_1 can be paired with any of the remaining 11 players, therefore $P(E_1) = \dfrac{1}{11}$.
In this case, it is certain that exactly one of S_1 and S_2 will winner, so that $P(B \mid E_1) = 1$.
Case II: (S_1 and S_2 are not paired)
Let $E_2 = \{S_1$ and S_2 are not paired}, then $P(E_2) = \dfrac{10}{11}$.
In this case, we have
$P(B \mid E_2) = P(S_1 \cap S'_2) + P(S'_1 \cap S_2)$
$= P(S_1)P(S'_2) + P(S'_1)P(S_2)$
$= \dfrac{1}{2}.\dfrac{1}{2} + \dfrac{1}{2}.\dfrac{1}{2} = \dfrac{1}{2}$
[Since all players are of equal strength, therefore, $P(S_1) = 1/2$ and $P(S'_1) = 1/2$]
$\therefore\ P(B) = P(E_1)P(B \mid E_1) + P(E_2)P(B \mid E_2)$
$= \dfrac{1}{11}.1 + \dfrac{10}{11}.\dfrac{1}{2} = \mathbf{\dfrac{6}{11}}$.
This correct option is (A)

135. P (exactly one of S_1 and S_2 *is* among the six winners)
$= P$ (at least one of S_1 and S_2 is among the six winner)
$- P$ (both S_1 and S_2 are among the six winners)
$\Rightarrow P$ (at least one of S_1 and S_2 is among the six winners)
$= \dfrac{14}{33} + \dfrac{6}{11} = \mathbf{\dfrac{32}{33}}$.
This correct option is (B)

136. In one throw with 3 dice the chance of getting at least one six
$= 1 - P(\text{no } 6) = 1 - \left(\dfrac{5}{6}\right)^3 = p$, say
In one throw with 2 dice the chance of getting at least one six
$= 1 - \left(\dfrac{5}{6}\right)^2 = q$, say.
Now, A occurs if in the first 'twin throw'. A throws 6 but B does not or if A fails then B must also fail and A must get 6 in the second twin throw and so on. Thus,
$P(A) = p(1-q) + (1-p)(1-q)p(1-q) + (1-p)(1-q)$
$(1-p)(1-q)p(1-q) + \ldots$
$= p(1-q)[1 + t + t^2 + \ldots]$, where $t = (1-p)(1-q)$
$= \dfrac{p(1-q)}{1-t} = \dfrac{p(1-q)}{1-(1-p)(1-q)}$
$= \dfrac{\left(1-\left(\dfrac{5}{6}\right)^3\right)\left(\dfrac{5}{6}\right)^2}{1-\left(\dfrac{5}{6}\right)^3\left(\dfrac{5}{6}\right)^2} = \mathbf{\dfrac{(6^3-5^3)5^2}{6^2-5^5}}$
This correct option is (B)

137. On interchanging p and q in the above problem, we get,
$P(B) = \dfrac{q(1-p)}{1-(1-p)(1-q)} = \mathbf{\dfrac{(6^2-5^2)5^3}{6^5-5^5}}$
This correct option is (A)

138. $P(C) = pq + (1-p)(1-q)pq + (1-p)^2(1-q)^2pq + \ldots$
$= pq(1 + (1-p)(1-q) + (1-p)^2(1-q)^2 + \ldots)$
$= \dfrac{pq}{1-(1-p)(1-q)} = \mathbf{\dfrac{(6^3-5^3)(6^2-5^2)}{6^5-5^5}}$
This correct option is (C)

Match the Column Type

139.

II. n = Total number of ways = 6^5

To find the favourable number of ways, a total of 12 in 5 throws can be obtained in the following two ways only

(i) One blank and four 3's.

or, (ii) Three 2's and two 3's.

The number of ways in case (i) = ${}^5C_1 = 5$

and the number of ways in case (ii) = ${}^5C_2 = 10$.

$\therefore$ m = the favourable number of ways.

= 5 + 10 = 15.

Hence, the required probability = $\frac{15}{6^5} = \mathbf{\frac{5}{2592}}$.

This correct option is (D)

III. Required probability

$$= \frac{{}^{39}C_1}{{}^{52}C_1} \times \frac{{}^{39}C_1}{{}^{52}C_1} \times \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \mathbf{\frac{9}{64}}.$$

This correct option is (A)

IV. Let W denote the event that A draws a white ball and T the event that A speaks truth. In the usual notations, we are given that $P(W) = \frac{1}{9}$, $P(T \backslash W) = \frac{5}{6}$ so that $P(\bar{W}) = 1 - \frac{1}{9} = \frac{8}{9}$

and $P(T \backslash \bar{W}) = 1 - \frac{5}{6} = \frac{1}{6}$.

Using Baye's theorem required probability is given by

$$P(W \backslash T) = \frac{P(W \cap T)}{P(T)}$$

$$= \frac{P(W) \times P(T \backslash W)}{P(W) \times P(T \backslash W) + P(\bar{W}) \times P(T \backslash \bar{W})}$$

$$= \frac{(1\backslash 9) \times (5\backslash 6)}{(1\backslash 9) \times (5\backslash 6) + (8\backslash 9) \times (1\backslash 6)} = \mathbf{\frac{5}{13}}.$$

This correct option is (C)

140. I (b). Any month out of 12 months, can be chosen with probability = $\frac{1}{12}$.

There are 7 possible ways in which the month can start and it will be a Friday on 13th day if the first day of the month is Sunday. Its probability = $\frac{1}{7}$.

Hence, the required probability = $\frac{1}{12} \times \frac{1}{7} = \mathbf{\frac{1}{84}}$.

This correct option is (B)

II. $\frac{(x-20)(x-40)}{x-30}$

$$= \frac{(x-20)(x-30)(x-40)}{(x-30)^2}$$

< 0 for x = 1, 2,..., 9, 21, 22,..., 29, i.e., 18 natural numbers.

$\therefore$ Required probability = $\frac{18}{100} = \mathbf{\frac{9}{50}}$

This correct option is (D)

III. If the product of the four numbers ends in one of the digits 1, 3, 7, or 9, each number should have the last digit as one of these 4 digits.

$\therefore$ the number of favourable cases = 4^4

Total number of all possible cases = 10^4

Hence, the required probability

$$= \frac{4^4}{10^4} = \frac{2^4}{5^4} = \mathbf{\frac{16}{625}}.$$

This correct option is (A)

IV (c). $n(S) = \frac{9!}{4!\,3!\,2!}$ and $n(E) = 3!$

$$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{3!}{\frac{9!}{4!\,3!\,2!}}$$

$$= \frac{3!\,4!\,3!\,2!}{9!}$$

$$= \frac{6 \,.\, 6 \,.\, 2}{9 \,.\, 8 \,.\, 7 \,.\, 6 \,.\, 5} = \mathbf{\frac{1}{210}}$$

This correct option is (C)

Assertion-Reason Type

141. Let A be the event that a really able candidate passes the test and let B be the event that any candidate passes this test. Then, we have,

$P(B/A) = 0.8$, $P(B/A^c) = 0.25$

$P(A) = 0.4$, $P(A^c) = 1 - 0.4 = 0.6$

By Baye's formula,

$$P(A/B) = \frac{P(A)\,P(B/A)}{P(A)\,P(B/A) + P(A^c)\,P(B/A^c)}$$

$$= \frac{0.32}{0.32 + 0.15} = \frac{32}{47} = 68\%.$$

This correct option is (A)

142. We know that the number of subsets that can be formed of X, is 2^n.

$\therefore\ n(S) = 2^n . 2^n = 2^{2n}$

Now, the number of subsets of X which contain exactly r elements, is ${}^nC_r$. Thus, two subsets both containing r elements can be chosen in

${}^nC_r . {}^nC_r = ({}^nC_r)^2$ ways.

$\therefore$ If E = choosing two subsets both containing same number of elements, then

$n(E) = ({}^nC_0)^2 + ({}^nC_1)^2 + ... + ({}^nC_n)^2$

$$= {}^{2n}C_n = \frac{1.2.3.4.......(2n-1)(2n)}{n!n!}$$

$$= \frac{[1.3.5.......(2n-1)][2.4.6....(2n)]}{(n!)^2}$$

$$= \frac{[1.3.5.......(2n-1)]2^n[1.2.3....n]}{(n!)^2}$$

$$= \frac{2^n[1.3.5.......(2n-1)]}{n!}$$

Hence, the required probability is

$$P(E) = \frac{n(A)}{n(S)} = \frac{1.3.5...(2n-1)}{2^n(n!)}$$

This correct option is (A)

143. Let E = {selected coin is fair}
and A = {head turns up}.
Thus, $P(A) = P(E)\,P(A/E) + P(\bar{E})\,P(A/\bar{E})$

$$= \left(\frac{n}{n+1}\right)\frac{1}{2} + \left(\frac{1}{n+1}\right)1 = \frac{n+2}{2(n+1)}$$

$$\text{Hence, } P(E/A) = \frac{P(E)P(A/E)}{P(A)} = \frac{\frac{n}{2(n+1)}}{\frac{n+2}{2(n+1)}} = \frac{n}{n+2}.$$

This correct option is (A)

144. A score of n can be reached in the following two mutually exclusive ways

– throwing a head when the score is $(n-1)$.
– throwing a tail when the score is $(n-2)$.
Hence, we have for $n \geq 3$.

$P_n = P_{n-1}P(H) + P_{n-2}P(T)$

$$= \frac{1}{2}(P_{n-1} + P_{n-2}) \qquad ...(1)$$

Now, from (1), we have

$$P_n + \frac{1}{2}P_{n-1} = P_{n-1} + \frac{1}{2}P_{n-2}$$

$$= P_{n-2} + \frac{1}{2}P_{n-3}$$

$$= .... = P_2 + \frac{1}{2}P_1$$

This correct option is (A)

145. **(a).** Let $P(m, k)$ be the probability that A throws m heads and B throws k heads. Then,

$$P(m, k) = {}^{n+1}C_m\left(\frac{1}{2}\right)^m\left(\frac{1}{2}\right)^{n+1-m} . {}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

$$= {}^{n+1}C_m . {}^nC_k\left(\frac{1}{2}\right)^{2n+1}$$

$$\text{and, required probability} = \sum_{0<k<m\leq n+1}\sum \frac{{}^{n+1}C_m\,{}^nC_k}{2^{2n+1}}$$

$$\text{Now, } \sum_{0<k<m\leq n}\sum {}^{n+1}C_m\,{}^nC_k$$

$$= \frac{1}{2}\left({}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + ... + {}^{n+1}C_{n+1}\right)$$

$$\left({}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n\right)$$

$$= \frac{1}{2}.2^{n+1}2^n = \frac{1}{2}.2^{n+1}.$$

Thus, the required probability $= \frac{1}{2}$

This correct option is (A)

Previous Year's Questions

146. Probabilities of solving the problem by A, B and C are given to be $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively.

$\therefore$ Probability that the problem is not solved

$$= \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$$

$$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{1}{4}$$

Hence, the probability that the problem is solved

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

147. The total number of ways in which numbers can be chosen $= 25 \times 25 = 625$

The number of ways in which cither players can choose same numbers = 25

$\therefore$ Probability that they win a prize $= \frac{25}{625} = \frac{1}{25}$

Thus, the probability that they will not win a prize = $1 - \frac{1}{25}$

$= \frac{24}{25}$.

148. $\therefore$ A and B are two mutually exclusive events

$\therefore A \cap B = \phi \Rightarrow A \subseteq \overline{B}$ and $B \subseteq \overline{A}$

$\Rightarrow P(A) \le P(\overline{B})$ and $P(B) \le P(\overline{A})$

Note : Two events A and B are said to be mutually exclusive events if $A \cup B = \varphi$.

149. Required probability

$= P(A_1 A_2' A_3) + P(A_1' A_2 A_3)$

$= P(A_1)P(A_2')P(A_3) + P(A_1')P(A_2)P(A_3)$ (A' is the compliment of A)

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.

150. Let $q = 1 - p$. Since, head appears first time in an even throw 2 or 4 or 6....

$\therefore \frac{2}{5} = qp + q^3 p + q^5 p + ...$

The RHS is a geometric series

$\Rightarrow \frac{2}{5} = \frac{qp}{1-q^2}$

$\Rightarrow \frac{2}{5} = \frac{(1-p)p}{1-(1-p^2)}$

$\Rightarrow \frac{2}{5} = \frac{1-p}{2-p}$

$\Rightarrow 4 - 2p = 5 - 5p$

$\Rightarrow 3p = 1$

$\Rightarrow p = \frac{1}{3}$

151. Required probability $= {}^7C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 \times \frac{1}{6}$

$= \frac{{}^7C_2 \times 5^5}{6^8}$

152. Select 2 out of 5

$= \frac{2}{5}$

Hence, (D) is the correct answer.

153. The mean and variance respectively are given by *np and npq*.

So, $np = 4$

$npq = 2$

$\Rightarrow q = \frac{1}{2}, \ p = \frac{1}{2}$

$\Rightarrow$ n = 8

Hence, (A) is the correct answer.

154. $E1$: denote the event that A speaks truth

$E2$: denote the event that B speaks truth

Now, the probability that both contradicts each other = $P(E1 \cap \overline{E}2) + P(\overline{E}1 \cap E2) = \frac{4}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{3}{4} = \frac{7}{20}$

155 Applying addition theorem,

$P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.62 + 0.50 - 0.35 = 0.77$

156 Given that the mean $n\,p = 4$ and the variance $n\,p\,q = 2$

$\Rightarrow q = 1/2 \Rightarrow p = 1/2, n = 8$

$\Rightarrow p(x = 2) = {}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = \frac{28}{256}$

157. For a particular house being selected

Probability $= \frac{1}{3}$

Probability that all the persons apply for the same house =

$\left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right)3 = \frac{1}{9}$.

158. Given $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$

$\Rightarrow P(A \cup B) = 5/6$ and $P(A) = 3/4$

Also $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = 5/6 - 3/4 + 1/4 = 1/3$

$P(A)\,P(B) = (3/4)(1/3) = 1/4 = P(A \cap B)$

Hence A and B are independent but not equally likely.

159. Sol: Using Poisson distribution

$P(X = r) = \frac{e^{-m} m^r}{r!}$

$\therefore P(X \le 1) = P(X = 0) + P(X = 1)$

$= e{-5} + 5 \times e{-5} = \frac{6}{e^5}$

160. Sol: Probability of getting score 9 in a single throw $= \frac{4}{36} = \frac{1}{9}$

So, the probability of getting score 9 exactly twice =

${}^3C_2 \times \left(\frac{1}{9}\right)^2 \times \frac{8}{9} = \frac{8}{243}$

161. Sol: The required probability

= 0.7 × 0.2 + (0.7) (0.8) (0.7) (0.2) + (0.7) (0.8) (0.7) (0.8) (0.7) (0.2) + ….

= 0.14 [1 + (0.56) + (0.56)2 + …]

= 0.14 $\left[\frac{1}{1-0.56}\right]=\frac{0.14}{0.44}=\frac{7}{22}$

162. Given conditions imply

$$\frac{P(A\cap B)}{P(B)}=\frac{1}{2},\frac{P(A\cap B)}{P(A)}=\frac{2}{3}$$

Hence, $\frac{P(A)}{P(B)}=\frac{3}{4}$. (But P(A) = 1/4)

⇒ P(B) = $\frac{1}{3}$

163. A = {4, 5, 6}, B = {1, 2, 3, 4}.

Obviously P (A ∪ B) = 1.

164. Let S = {00, 01, 02, …., 49 }

Let A be the event that sum of the digits on the selected ticket is 8 then

A = {08, 17, 26, 35, 44 }

Let B be the event that the product of the digits is zero

B = {00, 01, 02, 03, …., 09, 10, 20, 30, 40 }

A ∩ B = {8}

So the required probability = P(A/B)

$$=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{50}}{\frac{14}{50}}=\frac{1}{14}$$

165. $n(S) = 9C3$

$n(E) = 3C1 \times 4C1 \times 2C1$

$$\text{Probability}=\frac{3\times4\times2}{^9C_3}=\frac{24\times3!}{9!}=\frac{24\times6}{9\times8\times7}=\frac{2}{7}$$

166. $n = 5$

Success = p

Failure = q

P (at least one failure) $\geq\frac{31}{32}$

$1 - P$ (no failure) $\geq\frac{31}{32}$

$$1-P(x=5)\geq\frac{31}{32}$$

$$1-{}^5C_5p^5\geq\frac{31}{32}$$

$$-P^5\geq-\frac{1}{32}$$

$$P^5\leq\frac{1}{32}$$

$$P\leq\frac{1}{2}$$

$$P\in\left[0,\frac{1}{2}\right]$$

167. $C\cap D=C\Rightarrow P(C\cap D)=P(C)$

$$\Rightarrow P\left(\frac{C}{D}\right)=\frac{P(C\cap D)}{P(D)}\geq P(C)$$

168. Sol. Let A be the event that maximum is 6.

B be event that minimum is 3

$P(A)=\frac{^5C_2}{^8C_3}$ (the numbers < 6 are 5)

$P(B)=\frac{^5C_2}{^8C_3}$ (the numbers > 3 are 5)

$$P\left(A\cap B\right)=\frac{^2C_1}{^8C_3}$$

Required probability is

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{^2C_1}{^5C_2}=\frac{2}{10}=\frac{1}{5}$$

169. P (giving correct answer) = 1/3

$$\therefore {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1+{}^5C_5\left(\frac{1}{3}\right)^5$$

$$=\frac{5\times2}{(3)^5}+\frac{1}{(3)^5}=\frac{11}{3^5}.$$

170. $p\left(\overline{A\cup B}\right)=\frac{1}{6}$

$$p(A\cup B)=\frac{5}{6}, P(A)=\frac{3}{4}$$

$$p(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{5}{6}$$

$$P(B)=\frac{5}{6}-\frac{3}{4}+\frac{1}{4}=\frac{1}{3}.$$

$$p(A\cap B)=P(A)\cdot P(B)$$

$$\frac{1}{4}=\frac{3}{4}\times\frac{1}{3}.$$

171. $E_1 \rightarrow$ A shows up 4

$E_2 \rightarrow$ B shows up 2

$E_3 \rightarrow$ Sum is odd (i.e. even + odd or odd + even)

$P(E_1) = \frac{6}{6.6} = \frac{1}{6}$

$P(E_2) = \frac{6}{6.6} = \frac{1}{6}$

$P(E_3) = \frac{3\times3\times2}{6.6} = \frac{1}{2}$

$P(E_1 \cap E_2) = \frac{1}{6.6} = P(E_1) . P(E_2)$

$\Rightarrow E_1$ & E_2 are independent

$P(E_1 \cap E_3) = \frac{1.3}{6.6} = P(E_1) . P(E_3)$

$\Rightarrow E_1$ & E_2 are independent

$P(E_2 \cap E_3) = \frac{1.3}{6.6} = \frac{1}{12} = P(E_2) . P(E_3)$

$\Rightarrow E_2$ & E_3 are independent

$P(E_1 \cap E_2 \cap E_3) = 0$ ie imposible event