CHAPTER 7

Permutations and Combinations

Chapter Highlights

Factorial notation, Fundamental principles of counting, Permutation, Key results on permutations, Combination, Key results on combination, Derangement, Exponent of prime p in n!, Number of divisors

FACTORIAL NOTATION

We often come across products of the form $1 \cdot 2$, $1 \cdot 2 \cdot 3$, $1 \cdot 2 \cdot 3 \cdot 4$, …

Instead of writing all the factors of such a product in full, it is convenient to use a special notation. We write

$$1! = 1,$$
$$2! = 1 \cdot 2,$$
$$3! = 1 \cdot 2 \cdot 3,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$n! = 1 \cdot 2 \cdot 3 \ldots n.$$

"$n!$" denotes the product of the first n natural numbers. We read '$n!$' as 'n factorial'. $n!$ is also written as '$\lfloor \underline{n}$' and read as 'factorial n'. It is easy to see that

$1! = 1$, $2! = 1 \cdot 2 = 2$, $3! = 1 \cdot 2 \cdot 3 = 6$, $4! = 1 \cdot 2 \cdot 3 \cdot 4. = 24$, and so on.

IMPORTANT POINTS

- We know that

  $$n! = n\ (n-1)\ (n-2)\ (n-3)\ \ldots\ 3 \cdot 2 \cdot 1$$
  $$= n\ (n-1)! = n\ (n-1)\ (n-2)!$$
  $$= n\ (n-1)\ (n-2)\ (n-3)! \text{ and so on.}$$

  Thus, if $m, n \in N$ and $m > n$, then $m!$ can be expressed in terms of $n!$

  For example

  $$8! = 8 \cdot 7 \cdot 6!$$
  $$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!$$

  Also, $m! = n!$ if and only if $m = n$
- Putting $n = 1$ in $n! = n\ (n-1)!$, we have

  $$1! = 1 \cdot 0!$$
  $$\therefore \quad 0! = 1$$
- $(2n)! = 2^n \cdot n!\ [1 \cdot 3 \cdot 5 \ldots (2n-1)]$

CAUTION

- The factorial is defined only for whole numbers.

  We do not define the factorial of proper fractions or negative integers.
- $(2n)! \neq 2(n)!$
- $(m+n)! \neq m! + n!$
- $(mn)! \neq m!\ n!$

FUNDAMENTAL PRINCIPLES OF COUNTING

Multiplication Principle

If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the two operations in succession can be performed in $m \times n$ different ways.

Illustration

Anu wishes to buy a birthday card for the brother Manu and send it by post. Five different types of cards are available at the card-shop, and four different types of postage stamps are available at the post-office. In how many ways can she choose the card and the stamp?

Solution

She can choose the card in five ways. For each choice of the card she has four choices for the stamp. Therefore, there are 5×4 ways, i.e., 20 ways of choosing the card and the stamp.

Addition Principle

If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways. Then either of the two operations can be performed in $(m + n)$ ways.

NOTE

The above two principles can be extended for any finite number of operations.

Illustration

Suppose there are 5 gates in a stadium, 2 on one side and 3 on the other. Sohan has to go out of the stadium. He can go out from any one of the 5 gates. Thus, the number of ways in which he can go out is 5. Hence, the work of going out through the gates on one side will be done in 2 ways and the work of going out through the gates on other side will be done in 3 ways. The work of going out will be done when the Sohan goes out from side I or side II. Thus, the work of going out can be done in $(2 + 3) = 5$ ways.

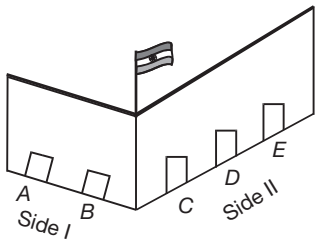


Fig. 7.1

NOTE

Addition theorem of counting is also true for more than two works.

SOLVED EXAMPLES

1. In a certain test, a_i students gave wrong answers to at least i questions where $i = 1, 2, 3, \ldots, k$. No student gave more than k wrong answers. The total number of wrong answers given is

(A) $a_1 + a_2 + \ldots + a_k$
(B) $a_1 + a_2 + \ldots + a_{k-1}$
(C) $a_1 + a_2 + \ldots + a_{k+1}$
(D) None of these

Solution: (A)

Total number of wrong answers

$$= 1 \cdot (a_1 - a_2) + 2 \cdot (a_2 - a_3) +, \ldots,$$
$$+ (k-1)(a_{k-1} - a_k) + ka_k$$
$$= a_1 + a_2 + a_3 +, \ldots, + a_k$$

2. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is

(A) $\frac{17}{7}$ (B) $\frac{1}{4}$ (C) 41 (D) 1

Solution: (C)

The word BAC cannot be spelt if the m selected coupons do not contain atleast one of A, B and C.

Number of ways of selecting m coupons which are A or $B = 2^m$.

This also includes the case when all the m coupons are A or all are B.

Number of ways of selecting m coupons which are B or $C = 2^m$.

This also includes the case when all the m coupons are B or all are C.

Number of ways of selecting m coupons which are C or $A = 2^m$.

This also includes the case when all the m coupons are C or all are A.

Number of ways of selecting m coupons when all are $A = 1^m$.

Number of ways of selecting m coupons when all are $B = 1^m$.

Number of ways of selecting m coupons when all are $C = 1^m$.

$\therefore$ Required number $= 2^m + 2^m + 2^m - (1^m + 1^m + 1^m)$
$= 3 \cdot 2^m - 3 \cdot 1^m = 3(2^m - 1)$

3. There are 4 candidates for the post of a lecturer in Mathematics and one is to be selected by votes of 5 men. The number of ways in which the votes can be given is

(A) 1048 (B) 1024
(C) 1072 (D) None of these

Solution: (B)

Each man can vote for any one of the 4 candidates and this can be done in 4 ways.

Similar is the case with every other man.

($\because$ Repetition is allowed)

Hence 5 men can vote in 4^5 i.e., 1024 ways.

4. There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is

(A) 10^2 (B) 18
(C) 2^{10} (D) 1023

Solution: (D)

Each bulb has two choices, either switched on or off

∴ Required number of ways

$$= 2^{10} - 1 = 1024 - 1 = 1023$$

(Since in one way when all are switched off, the hall will not be illuminated.)

5. A telegraph has 5 arms and each arm is capable of 4 distinct positions, including the position of rest. The total number of signals that can be made is

(A) 473 (B) 1023
(C) 1173 (D) None of these

Solution: (B)

Each arm can be set in 4 ways.

∴ Five arms can be set in $4 \times 4 \times 4 \times 4 \times 4$ ways.

But this includes the way when all the arms are in the position of rest, when no signal is sent.

Hence, required number of signals

$$= 4^5 - 1 = 1024 - 1 = 1023$$

6. There are stalls for 10 animals in a ship. The number of ways the shipload can be made if there are cows, calves and horses to be transported, animals of each kind being not less than 10, is

(A) 59049 (B) 49049
(C) 69049 (D) None of these

Solution: (A)

Each stall can be filled in 3 ways as there are three types of animals (animals of each category being not less than 10).

Shipload, i.e., filling up of 10 stalls, can be made in

$$3 \times 3 \times \ldots \text{ up to 10 times} = 3^{10} = 59049$$

7. The number of ways in which two 10-paise, two 20-paise, three 25-paise and one 50-paise coins can be distributed among 8 children so that each child gets only one coin, is

(A) 1720 (B) 1680
(C) 1570 (D) None of these

Solution: (B)

Total number of coins = 2 + 2 + 3 + 1 = 8.

2 coins are 10 paise, 2 are 20 paise, 3 are 25 paise and 1 is of 50 paise.

∴ Required no. of ways

$$= \frac{8!}{2! \times 2! \times 3! \times 1!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1 \times 3 \times 2 \times 1 \times 1}$$

$$= 8 \times 7 \times 6 \times 5 = 1680$$

8. The number of ways in which the letters of the word BALLOON can be arranged so that two L's do not come together, is

(A) 700 (B) 800
(C) 900 (D) None of these

Solution: (C)

There are in all seven letters in the word BALLOON in which *L* occurs 2 times and *O* occurs 2 times.

∴ The number of arrangements of the seven letters of the word $= \frac{7!}{2! \text{ x } 2!} = 1260.$

If two L's always come together, taking them as one letter, we have to arrange 6 letters in which *O* occurs 2 times.

∴ The no. of arrangements in which the two L's come together

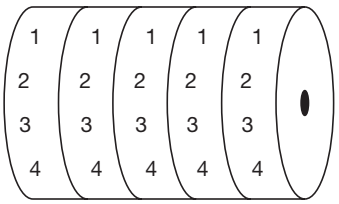
$$= \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360.$$

Hence, the required no. of ways in which the two L's do not come together = 1260 – 360 = 900.

9. A Letter lock contains 5 rings each marked with four different letters. The number of all possible unsuccessful attempts to open the lock is

(A) 625 (B) 1024 (C) 624 (C) 1023

Solution: (C)



Number of options on 1st Ring = 4
Number of options on 2nd Ring = 4
Number of options on 3rd Ring = 4
Number of options on 4th Ring = 4
Number of options on 5th Ring = 4
∴ Total number of options/arrangements

$$= 4 \times 4 \times 4 \times 4 \times 4 = 1024$$

10. We have $(n + 1)$ white balls and $(n + 1)$ black balls. In each set the balls are numbered from 1 to $(n + 1)$. If these balls are to be arranged in a row so that two consecutive balls are of different colours, then the number of these arrangements is

(A) $(2n + 2)!$ (B) $2 \times (2n + 2)$
(C) $2 \times (n + 1)!$ (D) $2\,[(n + 1)!]^2$

Solution: (A)

Between $(n + 1)$ white balls there are $(n + 2)$ gaps in which $(n + 1)$ black balls are to arranged. Number of required arrangements

$$= (n + 1)!\ (n + 1)! = [(n + 1)!]^2$$

Now between $(n + 1)$ black balls $(n + 1)$ white balls are to be filled in number of ways $= (n + 1)!\ (n + 1)!$

$\therefore$ Required number of ways $= 2[(n + 1)!]^2$

11. Number of points having position vector $a\hat{i} + b\hat{j} + c\hat{k}$, where $a, b, c \in \{1, 2, 3, 4, 5\}$ such that $2^a + 3^b + 5^c$ is divisible by 4 is

(A) 140 (B) 70
(C) 100 (D) None of these

Solution: (A)

$$4m = 2^a + 3^b + 5^c = 2^a + (4 - 1)^b + (4 + 1)^c$$
$$4m = 4k + 2^a + (-1)^b + (1)^c$$

$\therefore$ $a = 1$, $b =$ even, $c =$ any number

$a \neq 1$, $b =$ odd, $c =$ any number

$\therefore$ Required number $= 1 \times 2 \times 5 \times 4 \times 3 \times 5 = 70$

12. A family consists of grandfather, 5 sons and daughters and 8 grandchildren. They are to be seated in a row for dinner. The grandchildren wish to occupy the 4 seats at each end and the grandfather refuses to have a grand child on either side of him. The number of ways in which the family can be made to sit is

(A) 11360 (B) 11520
(C) 21530 (D) None of these

Solution: (B)

The total number of seats

= 1 grand father + 5 sons and daughters
+ 8 grand children
= 14

The grand children wish to occupy the 4 seats on either side of the table = 4! ways

= 24 ways

and grand father can occupy a seat in $(5 - 1)$ ways = 4 ways
(Since 4 gaps between 5 sons and daughters)
and the remaining seats can be occupied in 5! ways

= 120 ways (5 seats for sons and daughters)

Hence, required number of ways

$$= 24 \times 4 \times 120 = 11520$$

13. In a test a_i students gave wrong answers to atleast i questions, where $i = 1, 2, 3, 4, \ldots, k$. No one gave wrong answer to greater than k, then the total number of given wrong answers is

(A) $a_1 + 2a_2 + 3a_3 + \ldots + ka_k$
(B) $a_1 + a_2 + a_3 + \ldots + a_k$
(C) Zero
(D) $a_1a_2 + a_2a_3 + \ldots + a_{k-1}a_k$

Solution: (B)

Total number of wrong answers

$$= 1\ (a_1 - a_2) + 2\ (a_2 - a_3) +, \ldots,$$
$$+ (k - 1)\ (a_{k-1} - a_k) + ka_k$$

$\therefore$ Required number of ways $= a_1 + a_2 +, \ldots, + a_k$

PERMUTATION

Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a *permutation*.

IMPORTANT POINTS

Permutation of things means arrangement of things. The word arrangement is used if order of things is taken into account. Thus, if order of different things changes, then their arrangement also changes.

Notations

Let r and n be positive integers such that $1 \leq r \leq n$. Then, the number of permutations of n different things, taken r at a time, is denoted by the symbol ${}^nP_r$ or $P(n, r)$.

KEY RESULTS ON PERMUTATIONS

1. ${}^nP_r = \dfrac{n!}{(n - r)!} = n(n - 1)\ (n - 2)\ \ldots\ [n - (r - 1)]$, $0 \leq r \leq n$.
2. Number of permutations of n different things taken all at a time is: ${}^nP_n = n!$.
3. The number of permutations of n things, taken all at a time, out of which p are alike and are of one type, q are alike and are of second type and rest are all different is $\dfrac{n!}{p!\,q!}$.

4. The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.
5. **Permutations under Restrictions**
   - **(a)** Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is
   $$r \cdot {}^{n-1}P_{r-1}$$
   - **(b)** Number of permutations of n different things, taken r at a time, when s particular things are to be always included in each arrangement, is
   $$s!\,[r-(s-1)] \cdot {}^{n-s}P_{r-s}$$
   - **(c)** Number of permutations of n different things, taken r at a time, when a particular thing is never taken in each arrangement, is
   $${}^{n-1}P_r$$
   - **(d)** Number of permutations of n different things, taken all at a time, when m specified things always come together, is $m! \times (n-m+1)!$.
   - **(e)** Number of permutations of n different things, taken all at a time, when m specified things never come together, is $n! - m! \times (n-m+1)!$.
6. **Circular Permutations**
   - **(a)** Number of circular arrangements (permutations) of n different things is: $(n-1)!$.
   - **(b)** Number of circular arrangements (permutations) of n different things when clockwise and anti-clockwise arrangements are not different, i.e., when observation can be made from both sides is: $\frac{1}{2}(n-1)!$.
   - **(c)** Number of circular permutations of n different things, taken r at a time, when clockwise, and anti-clockwise orders are taken as different, is $= \frac{{}^nP_r}{r}$.
   - **(d)** Number of circular permutations of n different things, taken r at a time, when clockwise and anti-clockwise orders are not different, is $= \frac{{}^nP_r}{2r}$.

TRICK(S) FOR PROBLEM SOLVING

Numbers Divisible by 2, 3, 4, 5, 8, 9, 25

- All those numbers having their last digit as an even number (i.e., 0, 2, 4, 6 or 8) are divisible by 2.
- All those numbers the sum of whose digits is divisible by 3 are themselves divisible by 3.
- All those numbers whose last two digits are divisible by 4 are themselves divisible by 4.
- All those numbers whose last digit is either 0 or 5 are divisible by 5.
- All those numbers whose last three digit number is divisible by 8 are divisible by 8.
- All those numbers the sum of whose digits is divisible by 9 are themselves divisible by 9.
- All those numbers whose last two digits are divisible by 25 are themselves divisible by 25.

SOLVED EXAMPLES

14. The number of positive terms in the sequence
$$x_n = \frac{195}{4\,{}^nP_n} - \frac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}},\ n \in N \text{ is}$$
(A) 2 (B) 3
(C) 4 (D) None of these

Solution: (C)

We have,
$$x_n = \frac{195}{4 \cdot {}^np_n} - \frac{{}^{n+3}p_3}{{}^{n+1}p_{n+1}}$$
$$= \frac{195}{4 \cdot n!} - \frac{(n+3)(n+2)(n+1)}{(n+1)!}$$
$$= \frac{195}{4 \cdot n!} - \frac{(n+3)(n+2)}{n!}$$
$$= \frac{195 - 4n^2 - 20n - 24}{4 \cdot n!}$$
$$= \frac{171 - 4n^2 - 20n}{4 \cdot n!}$$

x_n is positive.

$$\therefore \quad \frac{171 - 4n^2 - 20n}{4 \cdot n!} > 0$$
$$\Rightarrow \quad 4n^2 + 20n - 171 < 0$$

which is true for $n = 1, 2, 3, 4$.

Hence, the given sequence has 4 positive terms.

15. The number of ways in which the letters of the word "STRANGE" can be arranged so that the vowels may appear in the odd places, is
(A) 1440
(B) 1470
(C) 1370
(D) None of these

Solution: (A)

There are 5 consonants and 2 vowels in the word STRANGE. Out of 7 places for the 7 letters, 4 places are odd and 3 places are even.

Two vowels can be arranged in 4 odd places in P (4, 2) ways = 12 ways and then 5 consonants can be arranged in the remaining 5 places in $P(5, 5)$ ways

$$= 5 \times 4 \times 5 \times 3 \times 2 \times 1 = 120 \text{ ways.}$$

Hence, the required number of ways

$$= P(4, 2) \times P(5, 5) = 12 \times 120 = 1440$$

16. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]\,dx$ is equal to

(A) $(\pi/4) - 1$ (B) $\pi^4/32$

(C) $(\pi^4/32) + \pi/2$ (D) $\pi/2$

Solution: (A)

Any number between 1 and 1000000 must be of less than seven digits. Therefore, it must be of the form

$$a_1\, a_2\, a_3\, a_4\, a_5\, a_6$$

where $a_1, a_2, a_3, a_4, a_5, a_6 \in \{0, 1, 2, \ldots, 9\}$

According to question, sum of the digits = 18

Thus, $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 18$

where $0 \le a_i \le 9$, $i = 1, 2, 3, \ldots, 9$.

Required number

$$= \text{coefficient of } x^{18} \text{ in } (1 + x + x^2 + \ldots + x^9)^6$$

$$= \text{coefficient of } x^{18} \text{ in } \left(\frac{1-x^{10}}{1-x}\right)^6$$

$$= \text{coeff. of } x^{18} \text{ in } [(1-x^{10})^6 (1-x)^{-6}]$$

$$= \text{coeff. of } x^{18} \text{ in } [(1 - {}^6C_1 x^{10})(1-x)^{-6}]$$

(leaving terms containing powers of x greater than 18)

$$= \text{coeff. of } x^{18} \text{ in } (1-x)^{-6} - {}^6C_1.$$

coeff. of x^8 in $(1-x)^{-6}$

$$= {}^{6+18-1}C_{18} - 6 \cdot {}^{6+8-1}C_{18} = {}^{23}C_5 - 6 \cdot {}^{13}C_8$$

$$= \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{120} - 6\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{120}$$

$$= 33649 - 7722 = 25927$$

17. The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is

(A) 432 (B) 108 (C) 36 (D) 18

Solution: (B)

The total number of numbers that can be formed with the digits 3, 4, 5, 6 taken all at a time

$$= P(4, 4) = 4! = 24.$$

Each of the digits 3, 4, 5, 6 occurs in 3!

$$= 3 \times 2 = 6 \text{ times in unit's place.}$$

$\therefore$ Sum of the digits in the unit's place of all the numbers

$$= (3 + 4 + 5 + 6) \times 6 = 18 \times 6 = 108$$

18. Three boys and three girls are to be seated around a table, in a circle. Among them, the boy X does not want any girl neighbour and the girl Y does not want any boy neighbour. The number of such arrangements possible is

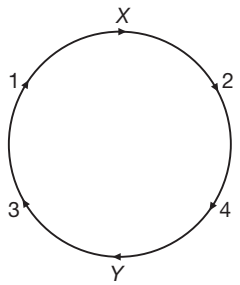
(A) 4 (B) 6

(C) 8 (D) None of these

Solution: (A)

As shown in figure, 1, 2 and X are the three boys and 3, 4 and Y are three girls, Boy X will have neighbours as boys 1 and 2 and the girl Y will have neighbors as girls 3 and 4.

1 and 2 can be arranged in P (2, 2) ways



$$= 2! = 2 \times 1 = 2 \text{ ways.}$$

Also, 3 and 4 can be arranged in P (2, 2) ways

$$= 2! = 2 \times 1 = 2 \text{ ways.}$$

Hence, required number of permutations

$$= 2 \times 2 = 4$$

19. The number of divisors of 9600 including 1 and 9600 are

(A) 60 (B) 58 (C) 48 (D) 46

Solution: (C)

$\because$ $9600 = 2^7 \times 3 \times 5^2$

$\therefore$ No. of divisors $= (7 + 1) \times (1 + 1) \times (2 + 1)$

$$= 8 \times 2 \times 3 = 48$$

TRICK(S) FOR PROBLEM SOLVING

If $N = p_1^{a_1} \cdot p_2^{a_2} \ldots p_k^{a_k}$, where $p_1, p_2, \ldots, p_k$ are different primes and $a_1, a_2, \ldots, a_k$ are natural numbers, then the total number of divisors of N including, and N is $(a_1 + 1)\ (a_2 + 1)\ \ldots\ (a_k + 1)$.

20. There are six teachers. Out of them two are primary teachers, two are middle teachers and two secondary teachers. They are to stand in a row, so as the primary teachers, middle teachers and secondary teachers are always in a set. The number of ways in which they can do so, is

(A) 34 (B) 48
(C) 52 (D) None of these

Solution: (B)

There are 2 primary teachers. They can stand in a row in $P(2, 2) = 2!$

$$= 2 \times 1 \text{ ways} = 2 \text{ ways}$$

There are 2 middle teachers. They can stand in a row in $P(2, 2) = 2! = 2 \times 1$ ways $= 2$ ways.

There are 2 secondary teachers. They can stand in a row in $P(2, 2) = 2!$

$$= 2 \times 1 \text{ ways} = 2 \text{ ways}$$

These three sets can be arranged in themselves in

$$= 3! = 3 \times 2 \times 1 = 6 \text{ ways}$$

Hence, the required number of ways

$$= 2 \times 2 \times 2 \times 6 = 48$$

21. A teaparty is arranged for 16 people along two sides of a large table with 8 chairs on each side. Four men want to sit on one particular side and two on the other side. The number of ways in which they can be seated is

(A) $\frac{6!\,8!\,10!}{4!\,6!}$ (B) $\frac{8!\,8!\,10!}{4!\,6!}$
(C) $\frac{8!\,8!\,6!}{6!\,4!}$ (D) None of these

Solution: (B)

There are 8 chairs on each side of the table. Let the sides be represented by A and B. Let four persons sit on side A, then number of ways of arranging 4 persons on 8 chairs on side $A = {}^8P_4$ and then two persons sit on side B. The number of ways of arranging 2 persons on 8 chairs on side $B = {}^8P_2$ and the remaining 10 persons can be arranged in remaining 10 chairs in 10! ways.

Hence the total number of ways in which the persons can be arranged

$$= {}^8P_4 \times {}^8P_2 \times 10! = \frac{8!\,8!\,10!}{4!\,6!}$$

22. The maximum number of points into which 4 circles and 4 straight lines intersect is

(A) 26 (B) 50
(C) 56 (D) 72

Solution: (B)

Four lines intersect each other in ${}^4C_2 = 6$ points and 4 circles intersect in ${}^4P_2 = 12$ points. Each line cuts 4 circles into 8.

$\therefore$ 4 lines cut four circles into 32 points.

$\therefore$ Required number $= 6 + 12 + 32 = 50$.

23. There are n seats round a table numbered 1, 2, 3, …, n. The number of ways in which $m(\le n)$ persons can take seats is

(A) ${}^nP_m$ (B) ${}^nC_m \times (m-1)!$
(C) $\frac{1}{2}.{}^nP_m$ (D) ${}^{n-1}P_m$

Solution: (A)

Since the seats are numbered,

$\therefore$ the arrangement is not circular.

$\therefore$ the required number of ways

= the number of arrangements of n things taken m at a time

$$= {}^nP_m$$

COMBINATION

Each of the different groups or selections which can be made by taking some or all of a number of things (irrespective of order) is called a *combination.*

IMPORTANT POINTS

Combination of things means selection of things. Obviously, in selection of things order of things has no importance. Thus, with the change of order of things selection of things does not change.

Notations

The number of combinations of n different things taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$. Thus,

$${}^nC_r = \frac{n!}{r!\,(n-r)!} = \frac{{}^nP_r}{r!} \qquad (0 \le r \le n)$$

$$= \frac{n\,(n-1)\,(n-2)\ldots(n-r+1)}{r\,(r-1)\,(r-2)\ldots 3\cdot 2\cdot 1}$$

If $r > n$,

then ${}^nC_r = 0$.

NOTE

Selecting things without any order in called combination and arrangement of things in some order is called permutation.

KEY RESULTS ON COMBINATION

1. ${}^nC_r = {}^nC_{n-r}$, $0 \le r \le n$

2. ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = n$

3. If ${}^nC_x = {}^nC_y$ then either $x = y$ or $x + y = n$.

4. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, $1 \le r \le n$

5. $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$

6. $n \cdot {}^{n-1}C_{r-1} = = (n - r + 1)\, {}^nC_{r-1}$

7. $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$, $1 \le r \le n$

8. If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.

9. If n is odd then the greatest value of ${}^nC_r$ is

$${}^nC_{\frac{n+1}{2}} \quad \text{or} \quad {}^nC_{\frac{n-1}{2}}$$

10. ${}^nC_r = \dfrac{r \text{ decreasing numbers starting with } n}{r \text{ increasing numbers starting with 1}}$

$$= \frac{n\,(n-1)\,(n-2)\,....(n-r+1)}{1 \cdot 2 \cdot 3.....r}$$

11. ${}^nP_r = r!\, {}^nC_r = n\,(n-1)\,(n-2)\,\ldots\,(n-r+1)$.

12. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$.

13. ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$.

14. Number of combinations of n different things taken r at a time

(a) when p particular things are always included = ${}^{n-p}C_{r-p}$.

(b) when p particular things are never included = ${}^{n-p}C_r$.

(c) when p particular things are not together in any selection = ${}^nC_r - {}^{n-p}C_{r-p}$.

15. (a) Number of selections of r consecutive things out of n things in a row = $n - r + 1$.

(b) Number of selections of r consecutive things out of n things along a circle

$$\begin{cases} n, & \text{when} \quad r < n \\ 1, & \text{when} \quad r = n \end{cases}$$

16. (a) Number of selections of zero or more things out of n different things

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

(b) Number of combinations of n different things selecting at least one of them is

$${}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1$$

(c) Number of selections of r things ($r \le n$) out of n identical things is 1.

(d) Number of selections of zero or more things out of n identical things = $n + 1$.

(e) Number of selections of one or more things out of n identical things = n.

(f) If out of $(p + q + r + t)$ things, p are alike of one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of selections is

$$(p + 1)\,(q + 1)\,(r + 1)\,2^t - 1$$

(g) The number of ways of selecting some or all out of $p + q + r$ items where p are alike of one kind, q are alike of second kind and rest are alike of third kind is $[(p + 1)\,(q + 1)\,(r + 1)] - 1$.

Division into Groups

1. (a) Number of ways of dividing $m + n$ different things in two groups containing m and n things respectively ($m \ne n$) is

$${}^{m+n}C_m = \frac{(m+n)!}{m!\,n!}$$

(b) Number of ways of dividing $m + n + p$ different things in three groups containing m, n and p things respectively ($m \ne n \ne p$) is $\dfrac{(m+n+p)!}{m!\,n!\,p!}$, if the order of the groups is not important and $\dfrac{(m+n+p)!}{m!\,n!\,p!} \times 3!$, if the order of the groups is important.

(c) Number of ways of dividing $2m$ different things in two groups, each containing m things and the order of the groups is not important, is $\dfrac{(2n)!}{2!\,(n!)^2}$, of the groups is not important and $\dfrac{(m+n+p)!}{m!\,n!\,p!} \times 3!$, if the order of the groups is important.

(d) Number of ways of dividing $2m$ different things in two groups, each containing m things and the order of the groups is important, is $\dfrac{(2n)!}{(n!)^2}$.

(e) Number of ways of dividing $3m$ different things in three groups, each containing m things and the order of the groups is not important, is $\dfrac{(3m)!}{3!\,(m!)^3}$.

(f) Number of ways of dividing $3m$ diferent things in three groups, each containing m things and the order of the groups is important, is $\dfrac{(3m)!}{(m!)^3}$.

2. (a) Number of ways of dividing n identical things into r groups, if blank groups are allowed is

$${}^{n+r-1}C_{r-1}$$

(b) Number of ways of dividing n identical things into r groups, if blank groups are not allowed is ${}^{n-1}C_{r-1}$.

(c) Number of ways of dividing n identical things into r groups such that no group contains less than m things and more than k $(m < k)$ things is coefficient of x^n in the expansion of

$$(x^m + x^{m+1} + \ldots + x^k)^r$$

3. The number of ways of selecting r things out of n things of which p are alike and are of one kind, q are alike and are of second, s are alike and are of third kind and so on, is
= coefficient of x^r in
$[(1 + x + x^2 + \ldots + x^p)\,(1 + x + x^2 + \ldots + x^q) \times (1 + x + x^2 + \ldots + x^s)\ \ldots]$

4. The number of ways of selecting r things out of n things of which p are a like and are of one kind, q are alike and are of second kind and rest $(n - p - q)$ things are all different is
= coefficient of x^r in
$[(1 + x + x^2 + \ldots + x^p)\,(1 + x + x^2 + \ldots + x^q) \times (1 + x)^{n-p-q}]$

5. The number of ways of selecting r things out of n things of which p are alike and are of one kind, q are alike and are of second kind, s are alike and are of third kind when each thing is taken at least once
= coefficient of x^{r-3} in
$[(1 + x + x^2 + \ldots + x^{p-1})\,(1 + x + x^2 + \ldots + x^{q-1}) \times (1 + x + x^2 + \ldots + x^{s-1})\ \ldots]$

6. The number of ways in which r identical things can be distributed among n persons when each person can get zero or more things
= coefficient of x^r in $(1 + x + x^2 + \ldots + x^r)^n$
= coefficient of x^r in $(1 - x)^{-n} = {}^{n+r-1}C_r$

7. The number of non-negative integral solutions of the equation $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n+r-1}C_r$.

8. The number of terms in the expansion of

$$(a_1 + a_2 + a_3 + \ldots + a_n)^r \text{ is } {}^{n+r-1}C_r$$

SOLVED EXAMPLES

24. ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3 =$

(A) ${}^{52}C_4$
(B) ${}^{51}C_4$
(C) ${}^{52}C_3$
(D) None of these

Solution: (A)

We have,

$${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$$
$$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$
$$= ({}^{47}C_4 + {}^{47}C_3) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$$
$$= {}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$$
$$= {}^{49}C_4 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 = {}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3$$
$$= {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$$

25. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7 =$

(A) 8 (B) 0
(C) 6 (D) None of these

Solution: (B)

We have,

$${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$$
$$= ({}^{15}C_8 + {}^{15}C_9) - ({}^{15}C_6 + {}^{15}C_7)$$
$$= {}^{16}C_9 - {}^{16}C_7 \qquad (\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$$
$$= {}^{16}C_9 - {}^{16}C_9 \qquad (\because {}^nC_r = {}^nC_{n-r})$$
$$= 0$$

26. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$
(C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

Solution: (D)

$${}^nC_r + 2{}^nC_{r-1} + {}^nC_{r-2} = ({}^nC_r + {}^nC_{r-1}) + ({}^nC_{r-1} + {}^nC_{r-2})$$
$$= {}^{n+1}C_r + {}^{n+1}C_{r-1}$$
$$= {}^{n+2}C_r = \binom{n+2}{r}$$

27. A gentleman invites 13 guests to a dinner and places 8 of them at one table and remaining 5 at the other, the tables being round. The number of ways he can arrange the guests is

(A) $\frac{11!}{40}$ (B) 9!
(C) $\frac{12!}{40}$ (D) $\frac{13!}{40}$

Solution: (D)

The number of ways in which 13 guests may be divided into groups of 8 and 5 = $^{13}C_5 = \dfrac{13!}{5!\,8!}$.

Now, corresponding to one such group, the 8 guests may be seated at one *round table* in (8 – 1)! i.e., 7! ways and the five guests at the other table in (5 – 1)! i.e., 4! ways.

But each way of arranging the first group of 8 persons can be associated with each way of arranging the second group of 5, therefore, the two processes can be performed together in $7! \times 4!$ ways.

Hence required number of arrangements

$$= \frac{13!}{5!\,8!} \times 7! \times 4! = \frac{13!}{5 \cdot 4!\,8 \cdot 7!} \times 7! \times 4! = \frac{13!}{40}$$

28. If there are 12 persons in a party, and if each of them shakes hands with each other, then number of handshakes happen in the party is

(A) 66 (B) 48
(C) 72 (D) None of these

Solution: (A)

Total number of handshakes

= The number of ways of selecting 2 persons from among 12 persons

$$= {}^{12}C_2 = \frac{12 \text{ x } 11}{2 \text{ x } 1} = 66$$

29. The number of ways in which a committee of 5 can be chosen from 10 candidates so as to exclude the youngest if it includes the oldest, is

(A) 196 (B) 178
(C) 202 (D) None of these

Solution: (A)

There are two different ways of forming the committee

(i) oldest may be included
(ii) oldest may be excluded

(i) If oldest is included, then youngest has to be excluded and we are to select 4 candidates out of 8. This can be done in

$${}^8C_4 = \frac{8!}{4!\,4!} = \frac{8 \text{ x } 7 \text{ x } 6 \text{ x } 5}{4 \text{ x } 3 \text{ x } 2} = 70 \text{ ways}$$

(ii) If oldest is excluded, then we are to select 5 candidates from 9 which can be done in

$${}^9C_5 = \frac{9!}{5!\,4!} = \frac{9 \text{ x } 8 \text{ x } 7 \text{ x } 6}{4 \text{ x } 3 \text{ x } 2 \text{ x } 1} = 126 \text{ ways}$$

Hence, the total number of ways in which committee can be formed

$$= 126 + 70 = 196$$

30. $\displaystyle\int_0^{\pi} x\, f(\sin x)\, dx$ is equal to

(A) $\pi \displaystyle\int_0^{\pi/2} f(\cos x)\, dx$ (B) $\pi \displaystyle\int_0^{\pi} f(\cos x)\, dx$

(C) $\pi \displaystyle\int_0^{\pi} f(\sin x)\, dx$ (D) $\dfrac{\pi}{2} \displaystyle\int_0^{\pi/2} f(\sin x)\, dx$

Solution: (C)

Let w be a non-negative integer such that

$$3x + y + z + w = 30$$

Let $a = x - 1,\ b = y - 1,\ c = z - 1,\ d = w,$

then

$$3a + b + c + d = 25, \text{ where } a, b, c, d \geq 0 \qquad (1)$$

Clearly, $0 \leq a \leq 8$. If $a = k$, then

$$b + c + d = 25 - 3. \qquad (2)$$

Number of non-negative integral solutions of Eq. (2)

$$= {}^{n+r-1}C_r$$

$$= {}^{3+25-3k-1}C_{25-3k}$$

$$= {}^{27-3k}C_{25-3k} = {}^{27-3k}C_2$$

$$= \frac{(27-3k)(26-3k)}{2}$$

$$= \frac{3}{2}(3k^2 - 53k - 234)$$

$\therefore$ Required number

$$= \frac{3}{2}\sum_{k=0}^{8}(3k^2 - 53k + 234)$$

$$= \frac{3}{2}\left(3\frac{8 \times 9 \times 17}{6} - 53\frac{8 \times 9}{2} + 234 \times 9\right) = 1215$$

31. The number of different ways in which 8 persons can stand in a raw so that between two particular persons A and B there are always two persons, is

(A) 60 (5!)
(B) $4! \times 5!$
(C) 15 (4!) × 5!
(D) None of these

Solution: (A)

The number of selections of 4 persons including A, B considering these four as a group, the number = $^6C_2$ of arrangement with the other four = 5!.

But in each group of four, number of arrangements = 2! × 2! therefore, the required number of ways = $^6C_2 \times 5! \times 2! \times 2!$.

32. There are 10 points in a plane of which no three points are coffinear but 4 points are concyclic. The number of different circles that can be drawn through atleast 3 of these points is

(A) 110 (B) 112 (C) 116 (D) 117

Solution: (D)

Since a unique circle can be drawn through three points, therefore a selection of three points results in a circle. So, the maximum number of circles using 10 points is ${}^{10}C_3$. Now, out of these 10 points 4 are concyclic, hence ${}^4C_3$ circles are actually single circle.

$\therefore$ Required number of circles $= {}^{10}C_3 - {}^4C_3 + 1 = 117$.

33. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed from a meeting consisting of 8 ladies and 7 gentlemen, if Mr X refuses to serve in a committee if Mr Y is a member is

(A) 1960 (B) 1540
(C) 3240 (D) None of these

Solution: (B)

3 ladies out of 8 can be selected in ${}^8C_3$ ways and 4 gentlemen out of 7 in ${}^7C_4$ ways.

Now each way of selecting 3 ladies is associated with each way of selecting 4 gentlemen.

Hence, the required number of ways

$$= {}^8C_3 \times {}^7C_4 = 56 \times 35 = 1960$$

We now find the number of committees of 3 ladies and 4 gentlemen in which both Mrs X and Mr Y are members. In this case, we can select 2 other ladies from the remaining 7 in ${}^7C_2$ ways and 3 other gentlemen from the remaining 6 in ${}^6C_3$ ways.

$\therefore$ The number of ways in which both Mrs X and Mr Y are always included $= {}^7C_2 \times {}^8C_3 = 21 \times 20 = 420$.

Hence, the required number of committes in which Mrs X and Mr Y do not serve together = 1960 – 420 = 1540.

34. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is

(A) ${}^{16}C_{11}$ (B) ${}^{16}C_5$ (C) ${}^{16}C_9$ (D) ${}^{20}C_9$

Solution: (C)

Out of 22 players, 2 are to be included and 4 are to be excluded. We have to select a team of 11 players. So the remaining 9 players are to be selected from the remaining 16 players. This can be done in ${}^{16}C_9$ ways.

35. A candidate is required to answer 7 questions out of 12 questions which are divided into two groups each containing 6 questions. He is not permitted to attempt more than 5 questions from each group. The number of ways in which he can choose the 7 questions is

(A) 780 (B) 640
(C) 820 (D) None of these

Solution: (A)

A candidate can attempt 5 questions from group I and 2 from group II or 4 from group I and 3 from group II or 3 from group I and 4 from group III or 2 from group I and 5 from group II. This can be done in

$${}^6C_5 \times {}^6C_2 + {}^6C_4 \times {}^6C_3 + {}^6C_3 \times {}^6C_4 + {}^6C_2 \times {}^6C_5$$
$$= 6 \times 15 + 15 \times 20 + 20 \times 15 + 15 \times 6$$
$$= 90 + 300 + 300 + 90 = 780$$

36. A boy has 3 Library tickets and 8 books of his interest in the library. Out of these 8, he does not want to borrow Chemistry part II, unless Chemistry part I is also borrowed. The number of ways in which he can choose the three books to be borrowed is

(A) 41 (B) 32
(C) 51 (D) None of these

Solution: (A)

The following are the different possibilities in which three books can be borrowed:

(i) When Chemistry part II is selected, then Chemistry part I is also borrowed and the third book is selected from the remaining 6 books.

(ii) When Chemistry part II is not selected, in this case he has to select the three books from the remaining 7 books.

First choice can be made in ${}^6C_1 = 6$ ways.

Second choice can be made in

$${}^7C_3 = \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35 \text{ ways}$$

Total number of ways in which he can choose the three books to be borrowed = 6 + 35 = 41.

37. The number of words that can be formed from the letters *a, b, c, d, e, f,* taken 3 at a time, each word containing at least one vowel is

(A) 96
(B) 84
(C) 106
(D) None of these

Solution: (A)

The total number of words

$$= ({}^2C_1 \times {}^4C_2 + {}^2C_2 \times {}^4C_1)\, 3!$$
$$= (12 + 4) \times 6 = 96.$$

38. In an examination a candidate has to pass in each of the papers to be successful. It the total number of ways to fail is 63, how many papers are there in the examination?

(A) 6 (B) 8
(C) 14 (D) None of these

Solution: (A)

Let the number of papers be n.

$\therefore$ Total number of ways to fail or pass

$$^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

But there is only one way to pass, i.e., when he fails in none.

$\therefore$ Total number of ways to fail $= 2^n - 1$

$\therefore$ From question, $2^n - 1 = 63$;

$\therefore$ $2^n = 64 = 2^6$

$\therefore$ $n = 6$.

39. The total number of selections from 4 boys and 3 girls if each selection has to contain at least one boy is

(A) 106 (B) 120
(C) 240 (D) None of these

Solution: (B)

Number of selections of at least one boy from 4 boys

$$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1$$

Number of selections of any number of girls from 3 girls

$$= {}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3$$

$\therefore$ Required number of selections of at least one boy from 4 boys and 3 girls $= (2^4 - 1)2^3 = 15 \times 8 = 120$.

40. A boat is to be manned by eight men of whom 2 can only row on bow side and 1 can only row on stroke side; the number of ways in which the crew can be arranged is

(A) 4360 (B) 5760
(C) 5930 (D) None of these

Solution: (B)

First we have to select 2 men for bow side and 3 for stroke side.

$\therefore$ The number of selections of the crew for two sides

$$= {}^5C_2 \times {}^3C_3$$

For each selection, there are 4 persons each on both sides who can be arranged in $4! \times 4!$ ways.

$\therefore$ Required number of arrangements

$$= {}^5C_2 \times {}^3C_3 \times 4! \times 4! = \frac{5 \text{ x } 4}{2} \text{ x } 1 \text{ x } 24 \text{ x } 24 = 5760$$

41. The number of ways in which a mixed doubles game in tennis can be arranged from 5 married couples, if no husband and wife play in the same game, is

(A) 46 (B) 54
(C) 60 (D) None of these

Solution: (C)

Let the sides of the game be A and B. Given 5 married couples, i.e., 5 husbands and 5 wives. Now, 2 husbands for two sides A and B can be selected out of $5 = {}^5C_2 = 10$ ways.

After choosing the two husbands their wives are to be excluded (since no husband and wife play in the same game). So we are to choose 2 wives out of remaining $5 - 2 = 3$ wives, i.e., ${}^3C_2 = 3$ ways.

Again two wives can interchange their sides A and B in $2! = 2$ ways.

Therefore, the required number of ways $= 10 \times 3 \times 2 = 60$.

42. The number of seven letter words that can be formed by using the letters of the word SUCCESS so that the two C are together but no two S are together, is

(A) 24 (B) 36
(C) 54 (D) None of these

Solution: (A)

Considering CC as single object, U, CC, E can be arranged in 3! ways

$$\times U \times CC \times E \times$$

Now the three S are to be placed in the four available places.

Hence required number of ways $= 3! \cdot {}^4C_3 = 24$.

43. Four boys picked up 30 mangoes. The number of ways in which they can divide them if all mangoes be identical, is

(A) 5456 (B) 3456
(C) 5462 (D) None of these

Solution: (A)

Clearly, 30 mangoes can be distributed among 4 boys such that each boy can receive any number of mangoes.

Hence, total number of ways $= {}^{30+4-1}C_{4-1} = {}^{33}C_3$

$$= \frac{33.32.31}{1.2.3} = 5456$$

TRICK(S) FOR PROBLEM SOLVING

Number of ways of dividing n idential things into r groups, if blank groups are allowed is

$$^{n+r-1}C_{r-1}$$

44. The number of ways of choosing 10 balls from infinite white, red, blue and green balls is

(A) 70 (B) 84 (C) 286 (D) 86

Solution: (C)

$$\text{Required ways} = \left\{ \begin{array}{l} \text{Coefficient of } x^{10} \\ \text{in } (1 + x + x^2 + ...)^4 \end{array} \right\}$$

$$= \text{Coefficient of } x^{10} \text{ in } \left(\frac{1}{1-x}\right)^4$$

$$= \text{Coefficient of } x^{10} \text{ in } (1-x)^{-4}$$

$$= \text{Coefficient of } x^{10} \text{ in } \left(1 + 4x + \frac{5.4}{2!}x^2 + \frac{4.5.6}{3!}x^3 + \ldots\right)$$

$$= \text{Coefficient of } x^{10} \text{ in } (1 + {}^4C_1 x + {}^5C_2 x^2 + {}^6C_3 x^3 + {}^7C_4 x^4 + {}^8C_5 x^5 + {}^9C_6 x^6 + \ldots {}^{13}C_{10} x^{10})$$

$$\therefore \text{ Required number of ways} = {}^{13}C_{10} = \frac{13.12.11}{3.2.1} = 286$$

45. In a class tournament where the participants were to play one game with another, two class players fell ill, having played 3 games each. If the total number of games played is 84, the number of participants at the beginning was

(A) 22 (B) 15
(C) 17 (D) None of these

Solution: (B)

Suppose the two players did not play at all so that the remaining $(n-2)$ players played ${}^{n-2}C_2$ matches. Since these two players played 3 matches each, hence the total number of matches is

$${}^{n-1}C_2 + 3 + 3 = 84 \quad \text{(given)}$$

or $$\frac{(n-2)(n-3)}{1.2} = 78 \text{ or } n^2 - 5n + 6 = 156$$

or $$n^2 - 5n - 150 = 0 \text{ or } (n-15)(n+10) = 0$$

$$\therefore \qquad n = 15 \ (n \neq -10)$$

46. An examination paper, which is divided into two groups consisting of 3 and 4 questions respectively carries the note: It is not necessary to answer all the questions. One question atleast should be answered from each group. The number of ways can an examinee select the questions is

(A) 22
(B) 105
(C) ${}^3P_3 \times {}^4P_4$
(D) ${}^3C_3 \times {}^4C_4$

Solution: (B)

Number of ways in which at least one question can be selected out of 3 are

$${}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3 - 1 = 7$$

Number of ways in which at least one question can be selected out of 4 are

$${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 = 15$$

$\therefore$ Total number of ways $= 7 \times 15 = 105$

47. Given 5 different green dyes, 4 different blue dyes and 3 different red dyes, the number of combinations of dyes that can be chosen by taking atleast one green and one blue dye is

(A) 248 (B) 120
(C) 3720 (D) 465

Solution: (A)

Number of ways of selecting at least one green dye out of 5 different green dyes

$${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 - 1$$

Also, at least one blue dye can be selected out of 4 blue dyes in

$${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1$$

Again, 3 different red dyes can be selected in

$${}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3 \text{ ways}$$

$\therefore$ Required ways $= (2^5 - 1)(2^4 - 1)(2^3) = 3720$

48. Out of 18 points in a plane no three are in the same straight line except five points which are collinear. The number of straight lines that can be formed joining them is

(A) 143 (B) 144
(C) 153 (D) None of these

Solution: (B)

The number of st. lines

$$= {}^{18}C_2 - ({}^5C_2 - 1) = 144$$

DERANGEMENT

Rearrangement of objects such that no one goes to its original place is called derangement.

If 'n' things are arranged in a row, the number of ways in which they can be deranged so that none of them occupies its original place is

$$n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!}\right) = n! \sum_{r=0}^{n} (-1)^r \frac{1}{r!}$$

For example, if 4 letters are taken out of 4 different envelopes, then the number of ways in which they can be reinserted in the envelopes so that no letter goes in to its original envelope

$$= 4!\left(1-1+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$$

$$= 24\left(1-1+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right) = 9$$

Some Useful Results for Geometrical Problems

1. If n distinct points are given in the plane such that no three of which are collinear, then the number of line segments formed $= {}^nC_2$.
   If m of these points are collinear ($m \geq 3$), then the number of line segments is $({}^nC_2 - {}^mC_2) + 1$.
2. The number of diagonals in an n-sided closed polygon $= {}^nC_2 - n$.
3. If n distinct points are given in the plane such that no three of which are collinear, then the number of triangles formed $= {}^nC_3$.
   If m of these points are collinear ($m \geq 3$), then the number of triangles formed $= {}^nC_3 - {}^mC_3$.
4. If n distinct points are given on the circumference of a circle, then
   (a) Number of st. lines $= {}^nC_2$
   (b) Number of triangles $= {}^nC_3$
   (c) Number of quadrilaterals $= {}^nC_4$ and so on
5. **Number of Rectangles and Squares:**
   (a) Number of rectangles of any size in a square of size $n \times n$ is $\sum_{r=1}^{n} r^3$ and number of squares of any size is $\sum_{r=1}^{n} r^2$.
   (b) Number of rectangles of any size in a rectangle of size $n \times p$ $(n < p)$ is $\frac{np}{4}(n+1)(p+1)$ and number of squares of any size is $\sum_{r=1}^{n}(n+1-r)(p+1-r)$.
6. If m parallel lines in a plane are intersected by a family of other n parallel lines. Then, total number of parallelograms so formed is

$$m_{C_2} \times n_{C_2}$$

i.e., $$\frac{mn(m-1)(n-1)}{4}$$

SOLVED EXAMPLES

49. The number of diagonals in a polygon of n sides is

(A) $\frac{n(n-3)}{2}$ (B) $\frac{n(n-1)}{2}$

(C) $\frac{(n-1)(n-2)}{2}$ (D) None of these

Solution: (A)

The number of diagonals + number of sides = number of selections of two vertices from n vertices

$\therefore$ the number of diagonals. $= {}^nC_2 - n$

$$= \frac{n(n-1)}{2} - n = \frac{n^2-n-2n}{2} = \frac{n(n-3)}{2}.$$

50. If each of 10 points on a straight line be joined to each of 10 points on a parallel line then the total number of triangles that can be formed with the given points as vertices, is

(A) 860 (B) 900

(C) 920 (D) None of these

Solution: (B)

A triangle is formed for each selection of 2 points from one line and 1 point from the other line.

$\therefore$ The number of triangles

$$= {}^{10}C_2 \times {}^{10}C_1 + {}^{10}C_1 \times {}^{10}C_2$$

$$= \frac{10\times 9}{2}\times 10 + 10 \times \frac{10\times 9}{2} = 900$$

EXPONENT OF PRIME *p* IN *n*!

Exponent of a prime p in $n!$ is denoted by $E_p(n!)$ and is given by,

$$E_p(n!) = \left(\frac{n}{p}\right) + \left(\frac{n}{p^2}\right) + \left(\frac{n}{p^3}\right) + ... + \left(\frac{n}{p^k}\right),$$

where $p^k < n < p^{k+1}$

and $\left(\frac{n}{p}\right)$ denotes the greatest integer less than or equal to $\frac{n}{p}$.

For example, exponent of 3 in (100)! is

$$E_3(100!) = \left(\frac{100}{3}\right) + \left(\frac{100}{3^2}\right) + \left(\frac{100}{3^3}\right) + \left(\frac{100}{3^4}\right)$$

$$= 33 + 11 + 3 + 1 = 48$$

NUMBER OF DIVISORS

Let $N = p_1^{a_1}.p_2^{a_2}.p_3^{a_3}...p_k^{a_k}$ where $p_1, p_2, p_3, \ldots p_k$ are different primes and $a_1, a_2, a_3, \ldots, a_k$ are natural numbers then:

1. The total number of divisors of N including 1 and N is $= (a_1 + 1)(a_2 + 1)(a_3 + 1) \ldots (a_k + 1)$.
2. The total number of divisors of N excluding 1 and N is $= (a_1 + 1)(a_2 + 1)(a_3 + 1) \ldots (a_k + 1) - 2$.
3. The total number of divisors of N excluding 1 or N is $= (a_1 + 1)(a_2 + 1)(a_3 + 1) \ldots (a_k + 1) - 1$.
4. The susm of these divisors is
$= (p_1^0 + p_1^1 + p_1^2 + ... + p_1^a)$
$(p_2^0 + p_2^1 + p_2^2 + ... + p_2^{a_2})$
$(p_k^0 + p_k^1 + p_k^2 + ... + p_k^{a_k})$
5. The number of ways in which N can be resolved as a product of two factors is
$\begin{cases} \frac{1}{2}(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1), \text{ If } N \text{ is not a perfect square} \\ \frac{1}{2}[(\alpha_1+1)(\alpha_2+1)\ldots(\alpha_k+1)+1], \text{ If } N \text{ is a perfect square} \end{cases}$
6. The number of ways in which a composite number N can be resolved into two factors which are relatively prime (or co-prime) to each other is equal to 2^{n-1}, where n is the number of different factors in N.

EXERCISES

Single Option Correct Type

1. Let y be an element of the set $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and x_1, x_2, x_3 be integers such that $x_1x_2x_3 = y$, then the number of positive integral solutions of $x_1x_2x_3 = y$ is
(A) 64 (B) 27
(C) 81 (D) None of these

2. If m = number of distinct rational numbers $\frac{p}{q} \in (0, 1)$ such that $p, q \in \{1, 2, 3, 4, 5\}$ and n = number of mappings from $\{1, 2, 3\}$ onto $\{1, 2\}$, then $m - n$ is
(A) 1 (B) –1
(C) 0 (D) None of these

3. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary then the rank of the word RANDOM is
(A) 614 (B) 615 (C) 613 (D) 616

4. If eight persons are to address a meeting then the number of ways in which a specified speaker is to speak before another specified speaker, is
(A) 40320 (B) 2520
(C) 20160 (D) None of these

5. The number of permutations of letters a, b, c, d, e, f, g so that neither the pattern beg nor cad appears is
(A) $\frac{7!}{3!3!}$ (B) $\frac{7!}{2!3!3!}$
(C) 4806 (D) None of these

6. The number of ways of selecting 10 balls from the unlimited number of red, green, white and yellow balls, if selection must include 2 red and 3 yellow balls, is
(A) 36 (B) 56
(C) 112 (D) None of these

7. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2\}$. Then, the number of onto functions from A to B is:
(A) 8 (B) 14
(C) 12 (D) None of these

8. Given five line segments of lengths 2, 3, 4, 5, 6 units. Then the number of triangles that can be formed by joining these lines is
(A) ${}^5C_3 - 3$ (B) ${}^5C_3 - 1$
(C) ${}^5C_3$ (D) ${}^5C_3 - 2$

9. If a represents the number of permutations of $(x + 2)$ things taken together, b represents the number of permutations of 11 things taken together out of x things, and c represents the number of permutations of $(x - 11)$ things taken together so that $a = 182bc$, then $x =$
(A) 15 (B) 12 (C) 10 (D) 18

10. How many different nine digit numbers can be formed from the number 22 33 55 8 88 by rearranging its digits so that the odd digits occupy even positions?
(A) 16 (B) 36 (C) 60 (D) 180

11. For a game in which two partners play against two other partners, six persons are available. If every possible pair must play with every other possible pair, then the total number of games played is
(A) 90 (B) 45 (C) 30 (D) 60

12. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is
(A) 216 (B) 600 (C) 240 (D) 3125

13. A box contains two white balls, three black balls and four red balls. The number of ways in which three balls can be drawn from the box if atleast one black ball is to be included in the draw, is
(A) 32 (B) 64
(C) 128 (D) None of these

14. The sum of all the numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time is
(A) 66666 (B) 84844
(C) 93324 (D) None of these

15. If the number of ways in which n different things can be distributed among n persons so that at least one person does not get any thing is 232. Then n is equal to
(A) 3 (B) 4
(C) 5 (D) None of these

16. Every body in a room shakes hands with every body else. The total number of hand shakes is 66. The total number of persons in the room is
(A) 11 (B) 12 (C) 13 (D) 14

17. ${}^{m}C_{r+1} + = \sum_{k=m}^{n} {}^{k}C_{r} =$
(A) ${}^{n}C_{r+1}$ (B) ${}^{n+1}C_{r+1}$
(C) ${}^{n}C_{r}$ (D) None of these

18. Two straight lines intersect at a point O. Points A_1, A_2, …, A_n are taken on one line and points B_1, B_2, …, B_n on the other. If the point O is not to be used, the number of triangles that can be drawn using these points as vertices, is
(A) $n\,(n-1)$ (B) $n\,(n-1)^2$
(C) $n^2\,(n-1)$ (D) $n^2\,(n-1)^2$

19. If the letters of the word MOTHER are written in all possible orders and these words are written out as in a dictionary, then the rank of the word MOTHER is
(A) 240 (B) 261 (C) 308 (D) 309

20. The number of divisors a number 38808 can have, excluding 1 and the number itself is
(A) 70 (B) 72
(C) 71 (D) None of these

21. The number of positive integral solutions of $15 < x_1 + x_2 + x_3 \le 20$, is equal to
(A) 785 (B) 685
(C) 1150 (D) None of these

22. The number of different 7 digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is
(A) ${}^{7}P_2 2^5$ (B) ${}^{7}C_2\, 2^5$
(C) ${}^{7}C_2 5^2$ (D) None of these

23. The tensdigit of $1! + 2! + 3! + \ldots + 49!$ is
(A) 1 (B) 2 (C) 3 (D) 4

24. Let S be the set of all functions from the set A to the set A. If $n(A) = k$ then $n(S)$ is
(A) $k!$ (B) k^k (C) $2^k - 1$ (d) 2^k

25. There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum umber of triangles with vertices at these points is
(A) ${}^{3p}C_3$ (B) $p^2\,(p-1)$
(C) $p^2\,(4p-1)$ (D) $p^2\,(4p-3)$

26. The number of ways in which thirty five apples can be distributed among 3 boys so that each can have any number of apples, is
(A) 1332 (B) 666
(C) 333 (D) None of these

27. The number of non-negative solutions of $x_1 + x_2 + x_3 +, \ldots, + x_n \le n$ (where n is positive integer) is
(A) ${}^{2n}C_n - 1$ (B) ${}^{2n-1}C_n - 1$
(C) ${}^{2n+1}C_{n-1}$ (D) ${}^{2n-1}C_{n-1} - 1$

28. Eleven animals of a circus have to be placed in eleven cages one in each cage. If 4 of the cages are too small for 6 of the animals, then the number of ways of caging the animals is
(A) 304800 (B) 504800
(C) 604800 (D) None of these

29. If n is even and
${}^{n}C_0 < {}^{n}C_1 < {}^{n}C_2 < \ldots < {}^{n}C_r > {}^{n}C_{r+1} > \ldots > {}^{n}C_n$
then $r =$
(A) $\frac{n}{2}$ (B) $\frac{n-1}{2}$
(C) $\frac{n-2}{2}$ (C) $\frac{n+2}{2}$

30. In a network of railways, a small island has 15 stations. The number of different types of tickets to be printed for each class, if every station must have tickets for other station, is

(A) 230 (B) 210
(C) 340 (D) None of these

31. The number of ordered pairs (m, n), $m, n \in \{1, 2,\ldots, 50\}$ such that $6^n + 9^m$ is a multiple of 5 is

(A) 6250 (B) 1250
(C) 1875 (D) None of these

32. A set contains $(2n + 1)$ elements. The number of subsets of the set which contain at most n elements is

(A) 2^n (B) 2^{n+1}
(C) 2^{2n-1} (D) 2^{2n}

33. There are n concurrent lines and another line parallel to one of them. The number of different triangles that will be formed by the $(n + 1)$ lines, is

(A) $\frac{(n-1)n}{2}$ (B) $\frac{(n-1)(n-2)}{2}$

(C) $\frac{n(n+1)}{2}$ (D) $\frac{(n+1)(n+2)}{2}$

34. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is

(A) 6 (B) 7 (C) 8 (D) 9

35. If all permutations of the letters of the word *AGAIN* are arranged as in dictionary, the forty ninth word is

(A) *NAAGI* (B) *NAGAI*
(C) NAAIG (D) *NAIAG*

36. The number of ways of choosing n objects out of $(3n + 1)$ objects of which n are identical and $(2n + 1)$ are distinct, is

(A) 2^{2n} (B) 2^{2n+1}
(C) $2^{2n} - 1$ (D) None of these

37. In a group of boys, two boys are brothers and in this group 6 more boys are there. In how many ways they can sit if the brothers are not to sit along with each other

(A) 4820 (B) 1410
(C) 2830 (D) None of these

38. If 20% of three subsets (i.e., subsets containing exactly three elements) of the set $A = \{a_1, a_2,\ldots, a_n\}$ contain a_1, then the value of n is

(A) 15 (B) 16 (C) 17 (C) 18

39. The number of two digit numbers which are of the form xy with $y < x$ are given by

(A) 45 (B) 55
(C) 17 (D) None of these

40. A crocodile is known to have not more than 68 teeth. The total number of crocodiles with different set of teeth is

(A) 68 (B) 68! (C) 16^{17} (D) 68^{68}

41. For $x \in R$, let $[x]$ denotes the greatest integer $\le x$, then the value of

$$\left(-\frac{1}{3}\right)+\left(-\frac{1}{3}-\frac{1}{100}\right)+\left(-\frac{1}{3}-\frac{2}{100}\right)$$

$$+,\ldots,+\left(-\frac{1}{3}-\frac{99}{100}\right) \text{ is}$$

(A) –100 (B) – 123 (C) –135 (D) –153

42. The total number of ways in which a beggar can be given at least one rupee from four 25 p. coins, three 50 p. coins and 2 one rupee coins is

(A) 54 (B) 53 (C) 51 (D) 48

43. A student is allowed to select atmost n books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select books is 63, then $n =$

(A) 4 (B) 3 (C) 7 (D) 8

44. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

(A) 16 (B) 36 (C) 60 (D) 180

45. In a certain test there are n questions. In this test 2^k students gave wrong answers to at least $(n - k)$ questions, where $k = 0, 1, 2,\ldots, n$. If the total number of wrong answers is 4095, then value of n is

(A) 11 (B) 12 (C) 13 (D) 15

46. The number of permutations of the letters *a, b, c, d* such that *b* does not follow *a, c* does not follow *b*, and *d* does not follow *c*, is

(A) 12 (B) 14 (C) 13 (D) 11

47. If $S = \sum_{r=0}^{m} {}^{n+r}C_k$, then

(A) $S + {}^{n}C_{k+1} = {}^{n+m}C_{k+1}$

(B) $S + {}^{n}C_{k+1} = {}^{n+m+1}C_{k+1}$

(C) $S + {}^{n}C_{k} = {}^{n+m}C_{k}$

(D) None of these

48. The number of ways of dividing 15 men and 15 women into 15 couples, each consisting of a man and a woman, is

(A) 1240 (B) 1840
(C) 1820 (D) 2005

49. Suman writes letters to his five friends. The number of ways can be letters be placed in the envelopes so that atleast two of them are in the wrong envelopes are

(A) 119 (B) 120
(C) 125 (D) None of these

50. **Statement 1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^{9}C_{3}$.

Statement 2: The number of ways of choosing any 3 places from 9 different places is ${}^{9}C_{3}$.

(A) Statement 1 is false, Statement 2 is true
(B) Statement 1 is true, Statement 2, is true; Statement 2 is the correct explanation for Statement 1
(C) Statement 1 is true; Statement 2 is true; Statement 2 is **not** as correct explanation for Statement 1
(D) Statement 1 is true, Statement 2 is false

51. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

(A) 880 (B) 629 (C) 630 (D) 879

52. The number of 4-digit numbers with distinct digits is

(A) 504 (B) 4536
(C) 4634 (D) 5040

53. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement 1: The number of different ways the child can buy the six ice-creams is ${}^{10}C_{5}$.

Statement 2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(A) Statement 1 is false, Statement 2 is true
(B) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1
(C) Statement 1 is true, Statement 2 is true; Statement 2 is **not** a correct explanation for Statement 1
(D) Statement 1 is true, Statement 2 is false

54. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(A) $8 \cdot {}^{6}C_{4} \cdot {}^{7}C_{4}$ (B) $6 \cdot 7 \cdot {}^{8}C_{4}$
(C) $6 \cdot 8 \cdot {}^{7}C_{4}$ (D) $7 \cdot {}^{6}C_{4} \cdot {}^{8}C_{4}$

55. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then the number of such arrangements is

(A) less than 500
(B) at least 500 but less than 750
(C) at least 750 but less than 1000
(D) at least 1000

56. One ticket is selected at random from 50 tickets numbered 00, 01, 02, …, 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals

(A) $\frac{1}{14}$ (B) $\frac{1}{7}$ (C) $\frac{5}{14}$ (D) $\frac{1}{50}$

57. In a certain test, a_i students gave wrong answers to at least i questions where $i = 1, 2, 3, \ldots, k$. No student gave more than k wrong answers. The total number of wrong answers given is

(A) $a_1 + a_2 + \ldots + a_k$
(B) $a_1 + a_2 + \ldots + a_{k-1}$
(C) $a_1 + a_2 + \ldots + a_{k+1}$
(D) None of these

58. A gentleman invites 13 guests to a dinner and places 8 of them at one table and remaining 5 at the other, the tables being round. The number of ways he can arrange the guests is

(A) $\frac{11!}{40}$ (B) 9! (C) $\frac{12!}{40}$ (D) $\frac{13!}{40}$

59. There are stalls for 10 animals in a ship. The number of ways the shipload can be made if there are cows, calves and horses to be transported, animals of each kind being not less than 10, is

(A) 59049 (B) 49049
(C) 69049 (D) None of these

60. In an examination a candidate has to pass in each of the papers to be successful. If the total number of ways to fail is 63, how many papers are there in the examination?

(A) 6 (B) 8
(C) 14 (D) None of these

61. If A denotes the property that two elements of $A = \{1, 5, 9, 13 \ldots, 1093\}$ add up to 1094, then the maximum number of elements in A can be

(A) 126 (B) 136 (C) 137 (D) 138

62. $\sum_{0 \le i \le j \le 10} \sum {}^{10}C_{j} \; {}^{j}C_{i}$ is equal to

(A) 3^{10} (B) $3^{10} - 1$ (C) 2^{10} (D) $2^{10} - 1$

63. If eight persons are to address a meeting then the number of ways in which a specified speaker is to speak before another specified speaker, is
(A) 40320 (B) 2520
(C) 20160 (D) None of these

64. The number of permutations of letters *a, b, c, d, e, f, g* so that neither the pattern beg nor cad appears is
(A) $\frac{7!}{3!\,3!}$ (B) $\frac{7!}{2!\,3!\,3!}$
(C) 4806 (D) None of these

65. The sum of all the numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time is
(A) 66666 (B) 84844
(C) 93324 (D) None of these

66. Two straight lines intersect at a point O. Points $A_1, A_2, \ldots, A_n$ are taken on one line and points $B_1, B_2, \ldots, B_n$ on the other. If the point O is not to be used, the number of triangles that can be drawn using these points as vertices, is
(A) $n\,(n-1)$ (B) $n\,(n-1)^2$
(C) $n^2\,(n-1)$ (D) $n^2\,(n-1)^2$

67. If the letters of the word MOTHER are written in all possible orders and these words are written out as in a dictionary, then the rank of the word MOTHER is
(A) 240 (B) 261 (C) 308 (D) 309

68. The number of divisors a number 38808 can have, excluding 1 and the number itself is
(A) 70 (B) 72
(C) 71 (D) None of these

69. The number of different 7 digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is
(A) ${}^7P_2 2^2$ (B) ${}^7C_2\, 2^5$
(C) ${}^7C_2 5^5$ (D) None of these

70. There are 10 points in a plane of which no three points are collinear and 4 points are concyclic. The number of different circles that can be drawn through at least 3 of these points is
(A) 116 (B) 120
(C) 117 (D) None of these

71. A set contains $(2n + 1)$ elements. The number of subsets of the set which contains at most n elements is
(A) 2^n (B) 2^{n+1} (C) 2^{2n-1} (D) 2^{2n}

72. If all permutations of the letters of the word *AGAIN* are arranged as in dictionary, the forty-ninth word is
(A) *NAAGI* (B) *NAGAI*
(C) *NAAIG* (D) *NAIAG*

73. The number of ways of choosing n objects out of $(3n + 1)$ objects of which n are identical and $(2n + 1)$ are distinct, is
(A) 2^{2n} (B) 2^{2n+1}
(C) $2^{2n} - 1$ (D) None of these

74. If 20% of three subsets (i.e., subsets containing exactly three elements) of the set $A = \{a_1, a_2, \ldots, a_n\}$ contain a_1, then the value of n is
(A) 15 (B) 16 (C) 17 (C) 18

75. For $x \in R$, let $[x]$ denotes the greatest integer $\le x$, then the value of
$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]$$
$$+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right] \text{ is}$$
(A) –100 (B) –123 (C) –135 (D) –153

76. The total number of ways in which a beggar can be given at least one rupee from four 25 p. coins, three 50 p. coins and 2 one rupee coins is
(A) 54 (B) 53 (C) 51 (D) 48

77. In a certain test there are n questions. In this test 2^k students gave wrong answers to at least $(n - k)$ questions, where $k = 0, 1, 2, \ldots, n$. If the total number of wrong answers is 4095, then value of n is
(A) 11 (B) 12 (C) 13 (D) 15

78. The number of permutations of the letters *a, b, c, d* such that *b* does not follow *a, c* does not follow *b*, and *d* does not follow *c*, is
(A) 12 (B) 14 (C) 13 (D) 11

79. If $S = \sum_{r=0}^{m} {}^{n+r}C_k$, then
(A) $S + {}^nC_{k+1} = {}^{n+m}C_{k+1}$
(B) $S + {}^nC_{k+1} = {}^{n+m+1}C_{k+1}$
(C) $S + {}^nC_k = {}^{n+m}C_k$
(D) None of these

80. If the number of ways in which n different things can be distributed among n persons so that at least one person does not get any thing is 232. Then, n is equal to
(A) 3 (B) 4
(C) 5 (D) None of these

81. If m = number of distinct rational numbers $\frac{p}{q} \in (0, 1)$ such that $p, q \in \{1, 2, 3, 4, 5\}$ and n = number of mappings from $\{1, 2, 3\}$ onto $\{1, 2\}$, then $m - n$ is
(A) 1 (B) –1
(C) 0 (D) None of these

82. Let y be an element of the set $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and x_1, x_2, x_3 be integers such that $x_1x_2x_3 = y$, then the number of positive integral solutions of $x_1x_2x_3 = y$ is
(A) 64 (B) 27
(C) 81 (D) None of these

83. Given 5 different green dyes, 4 different blue dyes and 3 different red dyes, the number of combinations of dyes that can be chosen by taking at least one green and one blue dye is
(A) 248 (B) 120 (C) 3720 (D) 465

84. Number of points having position vector $a\hat{i} + b\hat{j} + c\hat{k}$, where $a, b, c \in \{1, 2, 3, 4, 5\}$ such that $2^a + 3^b + 5^c$ is divisible by 4 is
(A) 140 (B) 70
(C) 100 (D) None of these

85. ${}^{2n}C_r (0 \le r \le 2n)$ is greatest when r is equal to
(A) $\frac{n}{2}$ (B) $\frac{n+1}{2}$
(C) $r = n$ (D) None of these

86. The number of even numbers greater than 100 that can be formed by the digits 0, 1, 2, 3 (no digit being repeated) is
(A) 20 (B) 30
(C) 40 (D) None of these

87. The number of positive numbers less than 1000 and divisible by 5 (no digit being repeated) is
(A) 150 (B) 154
(C) 166 (D) None of these

88. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. The number of telephone numbers having all six digits distinct is
(A) 8400 (B) 7200
(C) 9200 (D) None of these

89. The total number of ways of selecting five letters from the letters of the word INDEPENDENT, is
(A) 4200 (B) 3320
(C) 3840 (D) None of these

90. The sum of five digit numbers which can be formed with the digits 3, 4, 5, 6, 7 using each digit only once in each arrangement, is
(A) 5666600 (B) 6666600
(C) 7666600 (D) None of these

91. The sum of all the numbers that can be formed by writing all the digits 3, 2, 3, 4 only once is
(A) 39996 (B) 49996
(C) 57776 (D) None of these

92. The sum of all numbers greater than 10000 formed by using the digits 1, 3, 5, 7, 9, no digit being repeated in any number, is
(A) 4666600 (B) 5666600
(C) 6666600 (D) None of these

93. The sum of all numbers greater than 1000 formed by using the digits 0, 1, 2, 3, no digit being repeated in any number, is
(A) 38664 (B) 48664
(C) 58664 (D) None of these

94. The number of four digit numbers that can be formed from the digits 0, 1, 2, 3, 4, 5 with at least one digit repeated is
(A) 420 (B) 560
(C) 780 (D) None of these

95. The number of odd numbers lying between 40000 and 70000 that can be made from the digits 0, 1, 2, 4, 5, 7 if digits can be repeated in the same number is
(A) 864 (B) 932
(C) 766 (D) None of these

96. A table has provision for 7 seats, 4 being on one side facing the window and 3 being on the opposite side. The number of ways in which 7 people can be seated at the table if 2 people, X and Y, must sit on the same side, is
(A) 3260 (B) 2160
(C) 3350 (D) None of these

97. There are four oranges, five apples and six mangoes in a fruit basket. The number of ways in which a person can make a selection of fruits among the fruits in the basket, is
(A) 210 (B) 330
(C) 209 (D) None of these

98. The number of zeros at the end of 100! is
(A) 36 (B) 18
(C) 24 (D) None of these

99. The largest integer n such that 33! is divisible by 2^n is
(A) 30 (B) 31
(C) 32 (D) None of these

100. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + 4x_4 = 20$ is
(A) 436 (B) 536
(C) 602 (D) None of these

101. The product of r consecutive positive integers is divisible by
(A) $r!$ (B) $(r-1)!$
(C) $(r+1)!$ (D) None of these

102. The number of ordered triplets of positive integers which are solutions of the equation $x + y + z = 100$ is
(A) 5081 (B) 6005
(C) 4851 (D) None of these

103. The number of words that can be formed, with the letters of the work 'Pataliputra' without changing the relative order of the vowels and consonants, is
(A) 3600 (B) 4200
(C) 3680 (D) None of these

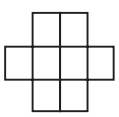
104. On a new year day every student of a class sends a card to every other student. The postman delivers 600 cards. The number of students in the class are
(A) 42 (B) 34
(C) 25 (D) None of these

105. For any positive integers m, n (with $n \geq m$), let and $\binom{n}{m} = {}^nC_m$, then $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m} =$
(A) $\binom{n+1}{m}$ (B) $\binom{n+1}{m+1}$
(C) $\binom{n}{m+1}$ (D) None of these

106. The number of 7 digit numbers the sum of whose digits is even, is
(A) 35×10^5 (B) 45×10^5
(C) 50×10^5 (D) None of these

107. The number of ways of choosing m coupons out of an unlimited number of coupons bearing the letters A, B and C so that they cannot be used to spell the word BAC, is
(A) $3(2^m - 1)$ (B) $3(2^{m-1} - 1)$
(C) $3(2^m + 1)$ (D) None of these

108. Six X's have to be placed in squares of the figure given below, such that each row contains at least one X. The number of different ways in which this can be done is



(A) 26 (B) 28
(C) 18 (D) None of these

109. The number of ways in which 16 identical things can be distributed among 4 persons if each person gets at least 3 things, is
(A) 33 (B) 35
(C) 38 (D) None of these

110. The number of ways in which 30 marks can be alloted to 8 questions if each question carries at least 2 marks, is
(A) 115280 (B) 117280
(C) 116280 (D) None of these

111. In an examination the maximum marks for each of the three papers are 50 each. Maximum marks for the fourth paper are 100. The number of ways in which the candidate can score 60% marks in aggregate is
(A) 110256 (B) 110456
(C) 110556 (D) None of these

112. The number of integers between 1 and 1000000 that have the sum of the digits 18, is
(A) 25927 (B) 25827
(C) 24927 (D) None of these

113. The number of non-negative integral solutions to the system of equations $x + y + z + u + t = 20$ and $x + y + z = 5$ is
(A) 336 (B) 346
(C) 246 (D) None of these

114. The number of positive integral solutions of the inequality $3x + y + z \leq 30$, is
(A) 1115 (B) 1215
(C) 1315 (D) None of these

115. In a city no person has identical set of teeth and there is no person without a tooth. Also, no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, then the maximum population of the city is
(A) 2^{32} (B) $2^{32} - 1$
(C) $2^{32} + 1$ (D) None of these

116. Eleven scientists are working on a secret project. They wish to lock up the documents in a cabinet such that cabinet can be opened if six or more scientists are present. Then, the smallest number of locks needed is
(A) 460 (B) 461
(C) 462 (D) None of these

117. The number of numbers greater than 10^6 that can be formed using the digits of the number 2334203, if all the digits of the given number must be used, is

(A) 360
(B) 420
(C) 260
(D) None of these

118. If 'n'is an integer between 0 and 21, then the minimum value of n! $(21 - n)!$ is
(A) 9! 2! (B) 10! 11!
(C) 20! (D) 21!

119. In how many ways can 20 oranges be given to four children if each child should get at least one orange?
(A) 869 (B) 969
(C) 973 (D) None of these

120. The total number of 5-digit numbers of different digits in which the digit in the middle is the largest is
(A) $\sum_{n=4}^{9}\left({}^{n}P_4 - {}^{n-1}P_3\right)$ (B) $\sum_{n=4}^{9}{}^{n}P_4$
(C) $\sum_{n=4}^{9}{}^{n-1}P_3$ (D) None of these

121. A train is going from Delhi to Indore, stops at nine intermediate stations. Six persons enter the train during the journey with six different tickets. The number of different sets of tickets possessed by them is
(A) ${}^{50}C_6$ (B) ${}^{54}C_6$
(C) ${}^{45}C_6$ (D) None of these

122. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is
(A) 5 (B) 6 (C) 7 (D) 8

123. If a, b, c are three natural numbers in A.P. such that $a + b + c = 21$, then the possible number of values of a, b, c is
(A) 13 (B) 14 (C) 15 (D) 16

124. The number of ways in which a mixed doubles game can be arranged from amongst n couples such that no husband and wife play in the same game, is
(A) ${}^{n}P_4$ (B) ${}^{n}C_4$ (C) $\frac{1}{2}{}^{n}P_4$ (D) $\frac{1}{2}{}^{n}C_4$

More than One Option Correct Type

125. There are 10 points in a plane, no three of which are in the same straight line excepting 4, which are collinear. Then, number of
(A) straight lines formed by joining them is 40
(B) triangles formed by joining them is 116
(C) straight lines formed by joining them is 45
(D) triangles formed by joining them is 120

126. If N is the number of positive integral solutions of $x_1 x_2 x_3 x_4 = 770$. Then,
(A) N is divisible by 4 distinct primes
(B) N is a perfect square
(C) N is a perfect 4th power
(D) N is a perfect 8th power

127. Let $E = \left[\frac{1}{3}+\frac{1}{50}\right]+\left[\frac{1}{3}+\frac{2}{50}\right]+\ldots+$ up to 50 terms, then
(A) E is divisible by exactly 2 primes
(B) E is prime
(C) $E \geq 30$
(D) $E \leq 35$

128. The number of ways in which three numbers in A.P. can be selected from 1, 2, 3, …, n is
(A) $\frac{n(n-2)}{4}$, when n is even
(B) $\frac{1}{4}(n-1)^2$, when n is odd
(C) $\frac{n(n-2)}{2}$, when n is even
(D) None of these

129. Let N denotes, the greatest number of points in which n straight lines and n circles intersect. Then,
(A) $n/(N - {}^{m}C_2)$
(B) $m/(N - {}^{n}P_2)$
(C) $N - {}^{m}C_2$ is an even integer
(D) $N - {}^{m}C_2 - {}^{n}P_2$ is an even integer

130. If $n < p < 2n$ and p is prime and $N = {}^{2n}C_n$, then
(A) p/N (B) p does not divide N
(C) p^2/N (D) p^2 does not divide N

131. If n objects are arranged in a row, the number of ways of selecting three of these objects so that no two adjacent objects are selected, is
(A) ${}^{n-2}C_3$ (B) ${}^{n-2}C_{n-5}$
(C) ${}^{n-3}C_2$ (D) ${}^{n-3}C_{n-5}$

132. If ${}^{n}C_{r-1} = (k^2 - 8)\,({}^{n+1}C_r)$, then k belongs to
(A) $[-3, -2\sqrt{2}\,]$ (B) $[-3, -2\sqrt{2})$
(C) $[2\sqrt{2}, 3]$ (D) $(2\sqrt{2}, 3]$

133. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \le n$ (where n is a positive integer) is
(A) ${}^{n+4}C_n$ (B) ${}^{n+4}C_4$
(C) ${}^{n+3}C_3$ (D) ${}^{n+3}C_n$

Passage Based Questions

Passage 1

There are five different boxes and seven different balls. All the seven balls are to be distributed in the five boxes placed in a row so that any box can receive any number of balls.

134. In how many ways can these balls be distributed so that no box is empty?
(A) 71 (B) 16800
(C) 1775 (D) None of these

135. Suppose, all the balls are identical, then in how many ways can all these balls be distributed into these boxes?
(A) 110 (B) 220 (C) 330 (D) 1440

136. In how many ways can these balls be distributed so that box 2 and box 4 contain only 1 and 2 balls, respectively?
(A) 5522 (B) 8505
(C) 2305 (D) None of these

137. In how many ways can these balls be distributed into these boxes if ball 2 can be put into either box 2 or box 4?
(A) 12360 (B) 31250 (C) 13490 (D) 31526

138. In how many ways can these balls be distributed such that no box is empty and ball 2 and ball 4 cannot be put in the same box?
(A) 1200 (B) 15000
(C) 3800 (D) None of these

Passage 2

There are three pots and four coins. All these coins are to be distributed into these pots where any pot can contain any number of coins.

139. In how many ways all these coins can be distributed if all coins are identical but all pots are different?
(A) 15 (B) 16 (C) 17 (D) 81

140. In how many ways all these coins can be distributed if all coins are different but all pots are identical?
(A) 14 (B) 21
(C) 27 (D) None of these

141. In how many ways all these coins can be distributed such that no pot is empty if all coins are different but all pots are identical?
(A) 16 (B) 6 (C) 42 (D) 21

142. In how many ways all these coins can be distributed such that no pot is empty if all coins are identical but all pots are different?
(A) 6 (B) 3 (C) 9 (D) 27

143. In how many ways all these coins can be distributed if all coins are identical and two pots are also identical?
(A) 1 (B) 10 (C) 9 (D) 11

144. In how many ways all these coins can be distributed if out of 4 coins 2 coins are identical and all pots are different?
(A) 45 (B) 27
(C) 54 (D) None of these

Passage 3

In the World Cup, the tournament is arranged as per the following rules:

In the beginning, 16 teams are taken and divided into 2 groups of 8 teams each. Teams of each group play a match against each other in the same group. From each group, 4 top teams qualify for the next round.

In the next round, two teams play each other in each group and the losing team goes out of the tournament. Then, four winning teams play for semifinal round and finally there is one final. The rules of the tournament are such that every match can result only in a win or a loss and not in a tie.

145. The total number of matches played in the tournament is
(A) 51 (B) 64 (C) 63 (D) 52

146. The maximum number of matches that a team going out of the tournament in the first round can win is
(A) 1 (B) 2 (C) 3 (D) 4

147. The minimum number of matches that a team must win in order to qualify for the second round is
(A) 4 (B) 5 (C) 6 (D) 7

148. Which of the following statements about a team which has already qualified for the second round is true?

To win the world Cup

(A) it has to win exactly 14 matches
(B) it has to win exactly 5 matches
(C) it has to win exactly 4 matches
(D) it has to win exactly 3 matches

Match the Column Type

149.

Column-I	Column-II
(I) The sum of the digits in the unit's place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is…	(A) 286
(II) The number of ways in which a committee of 5 can be chosen from 10 candidates so as to exclude the youngest if it includes the oldest, is…	(B) 108
(III) The number of divisors of 9600 including 1 and 9600 are…	(C) 196
(IV) The number of ways of choosing 10 balls from infinite white, red, blue and green balls is…	(D) 48

150.

Column-I	Column-II
(I) The number of diagonals in a polygon of n sides is…	(A) ${}^{2n}C_n - 1$
(II) The number of non-negative solutions of $x_1 + x_2 + x_3 + \ldots + x_n \le n$ (where n is a positive integer) is…	(B) $\frac{(n-1)(n-2)}{2}$
(III) If n is even and ${}^nC_0 < {}^nC_1 < {}^nC_2 < \ldots < {}^nC_r > {}^nC_{r+1} > \ldots > {}^nC_n$ then $r = \ldots$	(C) $\frac{n}{2}$
(IV) There are n concurrent lines and another line parallel to one of them. The number of different triangles that will be formed by the $(n+1)$ lines, is…	(D) $\frac{n(n-3)}{2}$

151.

Column-I	Column-II
(I) ${}^{n-2}C_r + 2.{}^{n-2}C_{r-1} + {}^{n-2}C_{r-2} = \ldots$	(A) $n+3$
(II) ${}^mC_{r+1} + \sum_{k=m}^{n} {}^kC_r = \ldots$	(B) ${}^{n+1}C_{r+1}$
(III) If ${}^nC_{n-r} + 3 \cdot {}^nC_{n-r+1} + 3 \cdot {}^nC_{n-r+2} + {}^nC_{n-r+3} = {}^xC_r$, then $x = \ldots$	(C) $\frac{n(n^r-1)}{n-1}$
(IV) The total number of permutations of n different things taken not more than r at a time, when each thing may be repeated any number of times is…	(D) ${}^nC_r$

Assertion-Reason Type

Instructions: *In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options:*

(A) Assertion(A) is True and Reason(R) is True; Reason(R) is a correct explanation for Assertion(A)
(B) Assertion(A) is True, Reason(R) is True; Reason(R) is not a correct explanation for Assertion(A)
(C) Assertion(A) is True, Reason(R) is False
(D) Assertion(A) is False, Reason(R) is True

152. **Assertion:** In an examination consisting of 9 papers, a candidate has to pass in more papers than the numbers of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is 256.

Reason: ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

Previous Year's Questions

153. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is **[2003]**

(A) 140 (B) 196
(C) 280 (D) 346

154. If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals **[2003]**

(A) ${}^{n+2}C_r$ (B) ${}^{n+2}C_{r+1}$
(C) ${}^{n+1}C_r$ (D) ${}^{n+1}C_{r+1}$

155. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order? **[2004]**

(A) 120 (B) 480
(C) 360 (D) 240

156. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[2004]**

(A) 5 (B) ${}^8C_3$
(C) 3^8 (D) 21

157. If the letters of word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number **[2005]**

(A) 601 (B) 600
(C) 603 (D) 602

158. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote is **[2006]**

(A) 5040 (B) 6210
(C) 385 (D) 1110

159. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent? **[2008]**

(A) $8 \cdot {}^6C_4 \cdot {}^7C_4$ (B) $6 \cdot 7 \cdot {}^8C_4$
(C) $6 \cdot 8 \cdot {}^7C_4$ (D) $7 \cdot {}^6C_4 \cdot {}^8C_4$

160. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then the number of such arrangements is **[2009]**

(A) less than 500
(B) at least 500 but less than 750
(C) at least 750 but less than 1000
(D) at least 1000

161. There are two urns. Urn *I* has 3 distinct red balls and Urn *II* has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[2010]**

(A) 36 (B) 66
(C) 108 (D) 3

162. **Statement-1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$ **[2011]**

Statement-2: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is false.
(C) Statement-1 is false, Statement-2 is true.
(D) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

163. Assuming the balls to be identical except for difference in colors, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is **[2012]**

(A) 880 (B) 629
(C) 630 (D) 879

164. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is **[2013]**

(A) 5 (B) 10
(C) 8 (D) 7

165. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is: **[2015]**

(A) 192 (B) 120
(C) 72 (D) 216

166. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is: **[2016]**

(A) 58^{th} (B) 46^{th}
(C) 59^{th} (D) 52^{nd}

ANSWER KEYS

Single Option Correct Type

1. (A)	**2.** (D)	**3.** (A)	**4.** (C)	**5.** (C)	**6.** (B)	**7.** (B)	**8.** (A)	**9.** (B)	**10.** (C)
11. (B)	**12.** (A)	**13.** (B)	**14.** (C)	**15.** (B)	**16.** (B)	**17.** (B)	**18.** (C)	**19.** (D)	**20.** (A)
21. (B)	**22.** (B)	**23.** (A)	**24.** (B)	**25.** (D)	**26.** (C)	**27.** (A)	**28.** (C)	**29.** (A)	**30.** (B)
31. (B)	**32.** (D)	**33.** (B)	**34.** (B)	**35.** (A)	**36.** (A)	**37.** (D)	**38.** (A)	**39.** (A)	**40.** (C)
41. (C)	**42.** (A)	**43.** (B)	**44.** (C)	**45.** (B)	**46.** (D)	**47.** (B)	**48.** (A)	**49.** (B)	**50.** (C)
51. (D)	**52.** (B)	**53.** (A)	**54.** (D)	**55.** (D)	**56.** (A)	**57.** (A)	**58.** (D)	**59.** (A)	**60.** (A)
61. (C)	**62.** (A)	**63.** (C)	**64.** (C)	**65.** (C)	**66.** (C)	**67.** (D)	**68.** (A)	**69.** (B)	**70.** (C)
71. (D)	**72.** (A)	**73.** (A)	**74.** (A)	**75.** (C)	**76.** (A)	**77.** (B)	**78.** (D)	**79.** (B)	**80.** (B)
81. (D)	**82.** (A)	**83.** (C)	**84.** (B)	**85.** (C)	**86.** (A)	**87.** (B)	**88.** (A)	**89.** (B)	**90.** (B)
91. (A)	**92.** (C)	**93.** (A)	**94.** (C)	**95.** (A)	**96.** (B)	**97.** (C)	**98.** (C)	**99.** (B)	**100.** (B)
101. (A)	**102.** (C)	**103.** (A)	**104.** (C)	**105.** (B)	**106.** (B)	**107.** (A)	**108.** (A)	**109.** (B)	**110.** (C)
111. (C)	**112.** (A)	**113.** (A)	**114.** (B)	**115.** (B)	**116.** (C)	**117.** (A)	**118.** (B)	**119.** (B)	**120.** (A)
121. (C)	**122.** (C)	**123.** (A)	**124.** (C)						

More than One Option Correct Type

125. (A) and (B) **126.** (B), (C) and (D) **127.** (B) and (D) **128.** (A) and (B) **129.** (A), (C) and (D)
130. (A) and (D) **131.** (A) and (B) **132.** (B) and (D) **133.** (A) and (B)

Passage Based Questions

134. (B)	**135.** (C)	**136.** (B)	**137.** (B)	**138.** (B)	**139.** (A)	**140.** (A)	**141.** (B)	**142.** (B)	**143.** (C)
144. (C)	**145.** (C)	**146.** (C)	**147.** (A)	**148.** (D)					

Match the Column Type

149. (I) → (B); (II) → (C); (III) → (D); (IV) → (A)
150. (I) → (D); (II) → (A); (III) → (C); (IV) → (B)
151. (I) → (D); (II) → (B); (III) → (A); (IV) → (C)

Assertion-Reason Type

152. (A)

Previous Year's Questions

153. (B)	**154.** (B)	**155.** (C)	**156.** (D)	**157.** (A)	**158.** (C)	**159.** (D)	**160.** (D)	**161.** (C)	**162.** (D)
163. (D)	**164.** (A)	**165.** (A)	**166.** (A)						

HINTS AND SOLUTIONS

Single Option Correct Type

1. Number of solutions of the given equation is the same as the number of solutions of the equation

$x_1x_2x_3x_4 = 30 = 2 \times 3 \times 5$

Here, x_4 is there because if $x_1x_2x_3 = 15$, then $x_4 = 2$ and if $x_1x_2x_3 = 5$, then $x_4 = 6$ etc.

x_4 is in fact a dummy variable.

Each of 2, 3 and 5 will be a factor of exactly one of x_1, x_2, x_3, x_4 in 4 ways.

$\therefore$ Required number $= 4^3 = 64$

The correct option is (D)

2. Now, $n = 2^3 - 2 = 6$

Also, $m = 4 + 3 + 2 + 1 - 1 = 9 \left(\text{as } \frac{2}{4} = \frac{1}{2}\right)$

$\therefore \quad m - n = 3$

The correct option is (D)

3. *A D M N O R* (in order)

Number of words beginning with

A	_	_	_	_	_	=	5!
D	_	_	_	_	_	=	5!
M	_	_	_	_	_	=	5!
N	_	_	_	_	_	=	5!
O	_	_	_	_	_	=	5!
R	A	D	_	_	_	=	3!
R	A	M	_	_	_	=	3!
R	A	N	D	M	O	=	1
R	A	N	D	O	M	=	1

$\therefore$ Rank of word RANDOM = 614

The correct option is (D)

4. Let *A*, *B* be the corresponding speakers. Without any restriction the eight persons can be arranged among themselves in 8! ways, but the number of ways in which *A* speaks before *B* and the number of ways in which *B* speaks before *A* make up 8!. Also number of ways in which *A* speaks before *B* is exactly same as the number of ways in which *B* speaks before *A*.

$\therefore$ the required number of ways $= \frac{1}{2}(8!) = 20160.$

The correct option is (C)

5. Total number of permutations = 7!

Let *A* be the property that '*beg*' occurs.

B be the property that *cad* occurs.

Number of permutations with $A = 5!$

= that of with *B* and $n(A \cap B) = 3!$

$\therefore \quad n(A \cup B) = 5! + 5! - 3! = 234$

$\therefore$ Required number = 7! – 234 = 4806

The correct option is (C)

6. Number of 10 balls selections

= coefficient of x^{10} in $(x^2 + x^3 + \ldots)(1 + x + x^2 + \ldots)(1 + x + x^2 + \ldots)(x^3 + x^4 + \ldots)$

= coefficient of x^5 in $(1 + x + x^2 + \ldots)^4$

= coefficient of x^5 in $(1 - x)^{-4}$

$= {}^{(4+5-1)}C_5 = {}^8C_5 = 56$ ways.

The correct option is (B)

7. Since each element of *A* can be associated with elements of 8 in two ways, therefore the total number of functions from *A* to *B* is $2 \times 2 \times 2 \times 2 = 16$. Out of these functions, the functions which are not onto are $f(x) = 1, \forall\, x \in A$ and $f(x) = 2\ \forall\ x \in A$. Thus, the number of onto functions = 16 – 2 = 14.

The correct option is (B)

8. We know that in any trinagle the sum of two sides is always greater than the third side.

$\therefore$ The triangle will not be formed if we select segments of lengths (2, 3, 5), (2, 3, 6) or (2, 4, 6).

Hence number of triangles formed $= {}^5C_3 - 3$.

The correct option is (A)

9. $a = {}^{x+2}P_{x+2},\ b = {}^xP_{11},\ C = {}^{x-11}P_{x-11}$

$\Rightarrow \quad a = (x + 2)!,\ b = \frac{x!}{(x-11)!},\ c = (x - 11)!$

Now, $a = 182\ bc$

$\Rightarrow \quad (x + 2)! = 182 \frac{x!}{(x-11)!}(x - 11)!$

$\Rightarrow \quad (x + 2)(x + 1)x! = 182x!$

$\Rightarrow \quad (x + 2)(x + 1) = 182 = 14 \times 13$

$\Rightarrow \quad x + 1 = 13$

$\therefore \quad x = 12$

The correct option is (B)

10. The four digits 3, 3, 5, 5, can be arranged at four even places in $\frac{4!}{2!\,2!} = 6$ ways and the remaining digits viz., 2, 2, 8, 8, 8 can be arranged at the five odd places in $\frac{5!}{2!\,3!} = 10$ ways.

Thus, the number of possible arrangements is (6) (10) = 60.

The correct option is (C)

11. For one game four persons are required. This can be done in ${}^6C_4 = 15$ ways. Once a set of four persons are selected, number of games possible will be $\frac{{}^4C_2}{2} = 3$ games.

$\therefore$ Total number of possible games $= 3 \times 15 = 45$.

The correct option is (B)

12. We know that a number is divisible by 3 if the sum of its digits is divisible by 3.

Now the sum of the digits 1, 2, 3, 4 and 5 is 15, which is divisible by 3.

$\therefore$ All the five digit numbers formed by the digits 1, 2, 3, 4, 5 are divisible by 3 and their number = 5! = 120.

When we include 0, the four other digits whose sum is divisible by 3 are 1, 2, 4 and 5.

$\therefore$ The number of numbers in this case

$$= 4 \times 4! = 4 \times 24 = 96.$$

Hence the required number of numbers

$$= 120 + 96 = 216$$

The correct option is (A)

13. The number of ways of selecting 3 balls out of total 9 (2 white, 3 black, 4 red balls) is ${}^9C_3$

i.e., $$\frac{9 \times 8 \times 7}{6} = 84$$

The number of ways of selecting 3 balls out of non-black six balls is ${}^6C_3$

i.e., $$\frac{6 \times 8 \times 4}{3 \times 2 \times 1} = 20$$

Therefore, the number of ways of selecting 3 balls out of 9 balls so as to include atleast one black ball = 84 – 20 = 64.

The correct option is (B)

14. The total number of numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time = ${}^4P_4 = 4! = 24$. Consider the digit in the unit's place in all these numbers. Each of the digits 2, 3, 4, 5 occurs in 3! = 6 times in the unit's place

$\therefore$ total for the digits in the unit's place

$$= (2 + 3 + 4 + 5)\, 6 = 84$$

Since each of the digits 2, 3, 4, 5 occurs 6 times in any one of the remaining places

$\therefore$ the required total

$$= 84\,(1 + 10 + 10^2 + 10^3) = 84\,(1111) = 93324.$$

The correct option is (C)

15. Total number of ways = n^n

Number of ways so that each person gets at least one thing (here exactly one thing) = $n!$

Given, $n^n - n! = 232$

$1^1 - 1! = 0$, $2^2 - 2! = 2$, $3^3 - 3! = 21$, $4^4 - 4! = 232$

$\therefore$ $n = 4$

The correct option is (C)

16. Let total number of persons be n.

Since two persons shake hands,

so number of ways of shaking hands = ${}^nC_2$.

According to question, we have ${}^nC_2 = 66$

or $$\frac{n!}{(n-2)!\,2!} = 66 \quad \text{or} \quad \frac{n\,(n-1)(n-2)!}{(n-2)!\,2 \times 1} = 66$$

or $n^2 - n = 132$ or $n^2 - n - 132 = 0$

or $n^2 - 12n + 11n - 132 = 0$

or $n\,(n-12) + 11\,(n-12) = 0 \quad \Rightarrow \quad n = -11$ or 12

But, $n = -11$ is not possible.

$\therefore$ $n = 12$.

The correct option is (B)

17. ${}^mC_{r+1} + \sum_{k=m}^{n} {}^kC_r$

$= {}^mC_{r+1} + {}^mC_r + {}^{m+1}C_r + \ldots + {}^{n-1}C_r + {}^nC_r$

$= {}^{m+1}C_{r+1} + {}^{m+1}C_r + \ldots + {}^{n-1}C_r + {}^nC_r$

$= {}^{m+2}C_{r+1} + {}^{m+2}C_r + \ldots + {}^{n-1}C_r + {}^nC_r$

$\vdots \qquad \vdots \quad \vdots \qquad \vdots$

$= {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}$

The correct option is (B)

18. No. of triangles = ${}^{2n}C_3 - {}^nC_3 - {}^nC_3$

$$= \frac{2n\,(2n-1)\,(2n-2)}{6} - \frac{2n\,(n-1)\,(n-2)}{6}$$

$$= \frac{1}{3}\, n\,(n-1)\,(3n) = n^2\,(n-1)$$

The correct option is (C)

19. *E H M O R T* (alphabetical order)

Number of words beginning with

E	_	_	_	_	_	= 5!
H	_	_	_	_	_	= 5!
M	*E*	_	_	_	_	= 4!
M	*H*	_	_	_	_	= 4!
M	*O*	*E*	_	_	_	= 3!
M	*O*	*H*	_	_	_	= 3!
M	*O*	*R*	_	_	_	= 3!
M	*O*	*T*	*E*	_	_	= 2!
M	*O*	*T*	*H*	*E*	*R*	= 1!

$\therefore$ Rank of word MOTHER = 309

The correct option is (D)

20. Factorizing the given number, we have

$$38808 = 2^3 \cdot 3^2 \cdot 7^2 \cdot 11$$

Therefore the total number of divisors

$$= (3+1)\,(2+1)\,(1+1) - 1 = 71.$$

But this includes the division by the number itself.

Hence, the required number of divisors

$$= 71 - 1 = 70$$

TRICK(S) FOR PROBLEM SOLVING

Let $N = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \ldots p_k^{a_k}$ where $p_1, p_2, p_3 \ldots p_k$ are different primes and $a_1, a_2, \ldots, a_k$ are natural numbers then total number of divisors of N excluding 1 and N is $(a_1 + 1)\,(a_2 + 1) \ldots (a_k + 1) - 2$

The correct option is (A)

21. We have, $15 < x_1 + x_2 + x_3 \le 20$

$\Rightarrow \quad x_1 + x_2 + x_3 = 16 + r$, $r = 0, 1, 2, 3, 4$.

Now, number of positive integral solutions of $x_1 + x_2 + x_3 = 16 + r$ is ${}^{16+r-1}C_{3-1} = {}^{15+r}C_2$

Thus, total number of solutions

$$= \sum_{r=0}^{4} {}^{15+r}C_2 \quad {}^{15}C_2 + {}^{16}C_2 + {}^{17}C_2 + {}^{18}C_2 + {}^{19}C_2$$

$${}^{20}C_3 - {}^{15}C_3 = 685$$

TRICK(S) FOR PROBLEM SOLVING

The total number of positive integral solutions of the equation $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n-1}C_{r-1}$

The correct option is (B)

22. Other than 2, remaining five places are to be filled by by 1 and 3

$\therefore$ number of ways for five places

$$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

For 2, selecting 2 places out of $7 = {}^7C_2$

$\therefore$ Required no. of ways $= {}^7C_2 \cdot 2^5$.

The correct option is (B)

23. We know that $1! + 2! + 3! + 4! = 33$.

Also, $5! = 120$, $6! = 720$, $7! = 5040$, $8! = 40320$ and

$$9! = 362880.$$

Thus, tens digit of $1! + 2! + \ldots + 9!$ is 1.

Also, note that $n!$ is divisible by 100 for all $n \geq 10$. Therefore, the tens digit of $1! + 2! + \ldots + 49!$ is 1.

The correct option is (A)

24. Each element of the set A can be given the image in the set A in k ways.

$\therefore$ the required number of functions, i.e.,

$n(S) = k \times k \times \ldots (k \text{ times}) = k^k$.

The correct option is (B)

25. $p_{C_1} \times p_{C_1} \times p_{C_1} + p_{C_2} \times 2p_{C_1} \times 3_{C_1}$

$$= p^2(4p - 3)$$

The correct option is (D)

26. Number of ways $= {}^{37}C_2 = \dfrac{37!}{35! \times 2!} = 666$ ways

For the distribution equation.

$$x_1 + x_2 + x_3 + \ldots + x_r = n$$

The number of ways in which n things can be distributed among r in such a way each can receive none, one or more or all of n items are ${}^{n+r-1}C_{r-1}$

The correct option is (C)

27. In general, we know that

For the distribution equation

$x_1 + x_2 + x_3 + \ldots + x_n \leq n$

Let required ways $= W$

$$\Rightarrow \quad W = \left\{\begin{matrix}\text{No. of ways of}\\ \text{distributing}\\ \text{1 item}\end{matrix}\right\} + \left\{\begin{matrix}\text{No. of ways of}\\ \text{distributing}\\ \text{2 items}\end{matrix}\right\}$$

$$+ \ldots + \left\{\begin{matrix}\text{No. of Ways of}\\ \text{distributing}\\ n \text{ items}\end{matrix}\right\}$$

$$= {}^{1+n-1}C_{n-1} + {}^{2+n-1}C_{n-1} + \ldots + {}^{n+n-1}C_{n-1}$$

$$= {}^nC_{n-1} + {}^{n+1}C_{n-1} + \ldots + {}^{2n-1}C_{n-1}$$

$$= ({}^nC_{n-1} + {}^nC_n) + {}^{n+1}C_{n-1} + \ldots + {}^{2n-1}C_{n-1} - {}^nC_n$$

$$= \{({}^{n+1}C_n + {}^{n+1}C_{n-1}) + \ldots + {}^{2n-1}C_{n-1}\} - {}^nC_n$$

- - - -

34. Distinct n digit numbers which can be formed using digits 2, 5 and 7 are 3^n. We have to find n so that

$3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$

$\Rightarrow n-2 \geq 5$ or $n \geq 7$

The correct option is (B)

So, the least value of n is 7.

35. Starting with the letter A and arranging the other four letters, there are 4! = 24 words. These are the first 24 words. Then, starting with G and arranging A, A, I and N in different ways, there are $\frac{4!}{2!1!1!} = 12$ words, the 37th word start with I and there are 12 words starting with I. Total goes upto 80 words. Now, 49th word is *NAAGI*.

The correct option is (A)

36. If we choose k $(0 \leq k \leq n)$ identical objects, then we must choose $(n-k)$ distinct objects. This can be done in ${}^{2n+1}C_{n-k}$ ways. Thus, the required number of ways

$$= \sum_{k=0}^{n} {}^{2n+1}C_{n-k} = {}^{2n+1}C_n + {}^{2n+1}C_{n-1} + \ldots + {}^{2n+1}C_0$$

$= 2^{2n}$

The correct option is (A)

37. 6 boys can sit in 6! ways

$$\times B_1 \times B_2 \times B_3 \times B_4 \times B_5 \times B_6 \times$$

Now, two brothers can sit in any of the 7 cross marked (×) places

Therefore, required number of ways $= 6! \times {}^7C_2 \times 2!$

$= 30240$

The correct option is (D)

38. The number of subsets of A containing exactly three elements is ${}^nC_3$ whereas the number of three subsets of A that contain a_1 is ${}^{n-1}C_2$. We are given,

$${}^{n-1}C_2 = \frac{20}{100}\ ({}^nC_3)$$

$$\Rightarrow \frac{(n-1)(n-2)}{2} = \frac{1}{5}\frac{n(n-1)(n-2)}{6}$$

$\Rightarrow n = 15$

The correct option is (A)

39. When zero is in the place of y then there are nine numbers and when 1 is in the place of y then there are 8 numbers and so on. Hence, required number is $\sum_{n=1}^{9} n = 45$.

The correct option is (A)

40. The required number is equal to the number of all the subsets of a 68 elements set i.e.,

$${}^{68}C_0 + {}^{68}C_1 + {}^{68}C_2 + \ldots + {}^{68}C_{68} = 2^{68} = 16^{17}$$

The correct option is (C)

41. For $0 \leq r \leq 66$, $0 \leq \frac{r}{100} < \frac{2}{3}$

$$\Rightarrow -\frac{2}{3} < -\frac{r}{100} \leq 0$$

$$\Rightarrow -\frac{1}{3} - \frac{2}{3} < -\frac{1}{3} - \frac{r}{100} \leq -\frac{1}{3}$$

$$\therefore \left(-\frac{1}{3} - \frac{r}{100}\right) = -1 \text{ for } 0 \leq r \leq 66$$

Also, for $67 \leq r \leq 100$, $\frac{67}{100} \leq \frac{r}{100} \leq 1$

$$\Rightarrow -1 \leq -\frac{r}{100} \leq -\frac{67}{100}$$

$$\Rightarrow -\frac{1}{3} - 1 \leq -\frac{1}{3} - \frac{r}{100} \leq -\frac{1}{3} - \frac{67}{100}$$

$$\therefore \left(-\frac{1}{3} - \frac{r}{100}\right) = -2 \text{ for } 67 \leq r \leq 100$$

Hence, $\sum_{r=0}^{100}\left(-\frac{1}{3} - \frac{r}{100}\right) = 67(-1) + 2(-34) = -135$

The correct option is (C)

42. The following four cases are there:

(A) At least one one rupee coin and any number of other coins $= 2 \times 4 \times 5 = 40$ ways.

(B) At least two 50 p. coins and any number of 25 p. coins $= 2 \times 5 = 10$ ways.

(C) One 50 p. coin and at least two 25 p. coins $= 1 \times 3 = 3$ ways.

(D) Four 25 p. coins in one way only $\therefore$ total = 54.

The correct option is (A)

43. The number of ways in which at most n books can be selected out of a collection of $(2n+1)$ books is

$${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots {}^{2n+1}C_n$$

$$\Rightarrow \frac{1}{2}[({}^{2n+1}C_1 + {}^{2n+1}C_1) + ({}^{2n+1}C_2 + {}^{2n+1}C_2) + \ldots + ({}^{2n+1}C_n + {}^{2n+1}C_n)] = 63$$

$$\Rightarrow ({}^{2n+1}C_1 + {}^{2n+1}C_{2n}) + ({}^{2n+1}C_2 + {}^{2n+1}C_{2n-1}) + \ldots + ({}^{2n+1}C_n + {}^{2n+1}C_{n+1}) = 126$$

$$\Rightarrow {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_{2n} = 126$$

$\Rightarrow 2^{2n+1} - 2 = 126$

$\Rightarrow 2^{2n+1} = 128 = 2^7$

$\Rightarrow 2n + 1 = 7$

$\Rightarrow n = 3$

The correct option is (B)

44. The four digits 3, 3, 5, 5 can be arranged at four even places in $\frac{4!}{2!2!} = 6$ ways and the remaining digits viz., 2, 2, 8, 8, 8 can be arranged at the five odd places in $\frac{5!}{2!3!} = 10$ ways. Thus, the number of possible arrangements is (6) (10) = 60.

The correct option is (C)

45. The number of students answering at least r questions incorrectly is 2^{n-r}.

$\therefore$ The number of students answering exactly $r (1 \leq r \leq n-1)$ questions incorrectly is $2^{n-r} - 2^{n-(r+1)}$.

Also, the number of students answering all questions wrongly is $2^0 = 1$.

Thus, the total number of wrong answers is

$1\,(2^{n-1} - 2^{n-2}) + 2\,(2^{n-2} - 2^{n-3}) + 3\,(2^{n-3} - 2^{n-4}) + \ldots$

$+ (n-1)\,(2^1 - 2^0) + n\,(2^0)$

$= 2^{n-1} + 2^{n-2} + \ldots + 2^0 = 2^n - 1$

Now, $2^n - 1 = 4095 \Rightarrow 2^n = 4096 = 2^{12} \Rightarrow n = 12$

The correct option is (B)

46.

d

c

| a | | | |

$\longrightarrow$ 2 ways.

d

a

| b | a | d | c |

$\longrightarrow$ 3 ways.

$d \quad a \quad c$

$b \quad c \quad a$

a

| c | | | |

$\longrightarrow$ 3 ways.

| d | | | |

$\longrightarrow$ 3 ways.

$\therefore$ Total number of ways = 11.

The correct option is (D)

47. $S = \sum_{r=0}^{m} {}^{n+r}C_k$

$= {}^{n}C_k + {}^{n+1}C_k + {}^{n+2}C_k + \ldots + {}^{n+m}C_k$

$= {}^{n}C_{k+1} + {}^{n}C_k + {}^{n+1}C_k + {}^{n+2}C_k + \ldots + {}^{n+m}C_k - {}^{n}C_{k+1}$

$= {}^{n+1}C_{k+1} + {}^{n+1}C_k + {}^{n+2}C_k + \ldots + {}^{n+m}C_k - {}^{n}C_{k+1}$

$\therefore \quad S = {}^{n+m+1}C_{k+1} - {}^{n}C_{k+1}$

The correct option is (B)

48. The number of ways of choosing first couple is $({}^{15}C_1)$ $({}^{15}C_1) = 15^2$. The number of ways of choosing 2nd couple is $({}^{14}C_1)$ $({}^{14}C_1) = 14^2$, and so on. Thus, the number of ways of choosing the couples is

$15^2 + 14^2 + 13^2 + \ldots + 2^2 + 1^2$

$= \dfrac{15 \times (15+1)\,[2(15)+1]}{6} = 1240$

TRICK(S) FOR PROBLEM SOLVING

Number of ways of dividing n identical things into r groups, if blank groups are allowed is ${}^{n+r-1}C_{r-1}$.

The correct option is (A)

49. Required number of ways

$= \sum_{r=2}^{5} {}^{5}C_{5-r}\, r! \left[1 - \dfrac{1}{1!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \ldots + \dfrac{(-1)^r}{r!}\right]$

$= \sum_{r=2}^{5} \dfrac{5!}{(5-r)!} \left[\dfrac{1}{2!} - \dfrac{1}{3!} + \ldots + \dfrac{(-1)^r}{r!}\right]$

$= \dfrac{5!}{3!}\left(\dfrac{1}{2!}\right) + \dfrac{5!}{2!}\left(\dfrac{1}{2!} - \dfrac{1}{3!}\right) + \dfrac{5!}{1!}\left(\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!}\right)$

$+ \dfrac{5!}{0!}\left(\dfrac{1}{2!} - \dfrac{1}{3!} + \dfrac{1}{4!} - \dfrac{1}{5!}\right)$

$= 10 + 20 + (60 - 20 + 5) + (60 - 20 + 5 - 1)$

$= 10 + 20 + 45 + 44 = 119.$

TRICK(S) FOR PROBLEM SOLVING

If $r\,(0 \le r \le n)$ objects occurpy the places assigned to them and none of the remaining objects occupies its original places, then the number of such ways is

$$= {}^{n}C_r \cdot (n-r)! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^{n-r} \frac{1}{(n-r)!}\right]$$

The correct option is (B)

50. Number of ways to distribute 10 identical balls in four distinct boxes such that no box remains empty $= {}^{10-1}C_{4-1} = {}^{9}C_3$

Number of ways to select 3 different places from 9 places $= {}^{9}C_3$.

Clearly statement-2 is not a correct explanation of statement-1.

The correct option is (C)

51. $(10+1)(9+1)(7+1) - 1 = 11.10.8 - 1 = 879$

The correct option is (D)

52. Number of ways to fill thousand's place = 9

Number of ways to fill hundred's place = 9

Number of ways to fill ten's place = 8

Number of ways to fill unit place = 7

$\therefore \quad 9 \times 9 \times 8 \times 7 = 4536$

The correct option is (B)

53. $x_1 + x_2 + x_3 + x_4 + x_5 = 6$

${}^{5+6-1}C_{5-1} = {}^{10}C_4$

The correct option is (A)

54. Other than S, seven letters M, I, I, I, P, P, I can be arranged in $\dfrac{7!}{2!\,4!} = 7 \cdot 5 \cdot 3.$

Now four S can be placed in 8 spaces in ${}^{8}C_4$ ways.

Desired number of ways $= 7 \cdot 5 \cdot 3 \cdot {}^{8}C_4 = 7 \cdot {}^{6}C_4 \cdot {}^{8}C_4$.

The correct option is (D)

55. Four novels can be selected from 6 novels in ${}^{6}C_4$ ways. One dictionary can be selected from 3 dictionaries in ${}^{3}C_1$ ways. As the dictionary selected is fixed in the middle, the remaining 4 novels can be arranged in 4! ways.

$\therefore$ The required number of ways of arrangement

$= {}^{6}C_4 \times {}^{3}C_1 \times 4! = 1080$

The correct option is (D)

56. $S = \{00, 01, 02, \ldots 49\}$

Let A be the event that sum of the digits on the selected ticket is 8 then

$A = \{08, 17, 26, 35, 44\}$

Let B be the event that the product of the digits is zero

$B = \{00, 01, 02, 03, \ldots 09, 10, 20, 30, 40\}$

$A \cap B = \{8\}$

Required probability $= P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{\frac{1}{50}}{\frac{14}{50}} = \dfrac{1}{14}$

The correct option is (A)

57. Total number of wrong answers

$= 1 \cdot (a_1 - a_2) + 2.(a_2 - a_3) + \ldots + (k-1)(a_{k-1} - a_k) + ka_k$

$= a_1 + a_2 + a_3 + \ldots + a_k$

The correct option is (A)

58. The number of ways in which 13 guests may be divided into groups of 8 and $5 = {}^{13}C_5 = \dfrac{13!}{5!\,8!}$

Now, corresponding to one such group, the 8 guests may be seated at one *round table* in $(8-1)!$ i.e., 7! ways and the five guests at the other table in $(5-1)!$ i.e., 4! ways.

But each way of arranging the first group of 8 persons can be associated with each way of arranging the second group of 5, therefore, the two processes can be performed together in $7! \times 4!$ ways.

Hence, required number of arrangements

$= \dfrac{13!}{5!\,8!} \times 7! \times 4! = \dfrac{13!}{5 \cdot 4!\,8 \cdot 7!} \times 7! \times 4! = \dfrac{13!}{40}$

The correct option is (D)

59. Each stall can be filled in 3 ways as there are three types of animals (animals of each category being not less than 10).

Shipload, i.e., filling up of 10 stalls, can be made in

$3 \times 3 \times \ldots$ up to 10 times $= 3^{10} = 59049$.

The correct option is (A)

60. Let the number of papers be n.

$\therefore$ Total number of ways to fail or pass

${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$.

But there is only one way to pass, i.e., when he fails in none.

$\therefore$ Total number of ways to fail $= 2^n - 1$.

$\therefore$ From question, $2^n - 1 = 63$;

$\therefore$ $2^n = 64 = 2^6$

$\therefore$ $n = 6$.

The correct option is (A)

61. Since the elements in A are in A.P., therefore, the number of elements in $A = \dfrac{1093 - 1}{4} + 1 = 274$.

Since, sum of equidistant terms in A.P. is equal to the sum of first and last term $= 1 + 1093 = 1094$.

$\Rightarrow$ maximum number of elements in A, that add up to 1094

$= \dfrac{274}{2} = 137$.

The correct option is (C)

62. $\displaystyle\sum_{0 \le i \le j \le 10} \sum {}^{10}C_j \; {}^jC_i$

$= {}^{10}C_0 \times {}^0C_0 + {}^{10}C_1 ({}^1C_0 + {}^1C_1) + {}^{10}C_2 ({}^2C_0 + {}^2C_1 + {}^2C_2)$

$+ {}^{10}C_3 ({}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3) + \ldots$

$+ {}^{10}C_{10}({}^{10}C_0 + {}^{10}C_1 + \ldots + {}^{10}C_{10})$

$= {}^{10}C_0 \cdot 2^0 + {}^{10}C_1 \cdot 2^1 + {}^{10}C_2 \cdot 2^2 + \ldots + {}^{10}C_{10} \cdot 2^{10}$

$= (1+2)^{10} = 3^{10}$.

The correct option is (A)

63. Let A, B be the corresponding speakers. Without any restriction the eight persons can be arranged among themselves in 8! ways, but the number of ways in which A speaks before B and the number of ways in which B speaks before A make up 8!. Also, number of ways in which A speaks before B is exactly same as the number of ways in which B speaks before A.

$\therefore$ the required number of ways $= \dfrac{1}{2}(8!) = 20160$.

The correct option is (C)

64. Total number of permutations = 7!

Let A be the property that '*beg*' occurs.

B be the property that *cad* occurs.

Number of permutations with $A = 5!$

= that of with B and $n(A \cap B) = 3!$

$\therefore$ $n(A \cup B) = 5! + 5! - 3! = 234$

$\therefore$ Required number $= 7! - 234 = 4806$

The correct option is (C)

65. The total number of numbers that can be formed with the digits 2, 3, 4, 5 taken all at a time $= {}^4P_4 = 4! = 24$. Consider the digit in the unit's place in all these numbers. Each of the digits 2, 3, 4, 5 occurs in $3! = 6$ times in the unit's place

$\therefore$ total for the digits in the unit's place

$= (2 + 3 + 4 + 5)\,6 = 84$

Since each of the digits 2, 3, 4, 5 occurs 6 times in any one of the remaining places, therefore, the required total

$= 84\,(1 + 10 + 10^2 + 10^3) = 84\,(1111) = 93324$.

The correct option is (C)

66. Number of triangles $= {}^{2n}C_3 - {}^nC_3 - {}^nC_3$

$= \dfrac{2n\,(2n-1)\,(2n-2)}{6} - \dfrac{2n\,(n-1)\,(n-2)}{6}$

$= \dfrac{1}{3}\,n\,(n-1)\,(3n) = n^2\,(n-1)$.

The correct option is (C)

67. *E H M O R T* (alphabetical order)

Number of words beginning with

E	_	_	_	_	_	$= 5!$
H	_	_	_	_	_	$= 5!$
M	*E*	_	_	_	_	$= 4!$
M	*H*	_	_	_	_	$= 4!$
M	*O*	*E*	_	_	_	$= 3!$
M	*O*	*H*	_	_	_	$= 3!$

M	*O*	*R*	_	_	_	= 3!
M	*O*	*T*	*E*	_	_	= 2!
M	*O*	*T*	*H*	*E*	*R*	= 1!

$\therefore$ Rank of word MOTHER = 309

The correct option is (D)

68. Factorizing the given number, we have

$38808 = 2^3 \cdot 3^2 \cdot 7^2 \cdot 11$

Therefore, the total number of divisors

$= (3+1)(2+1)(1+1) - 1 = 71.$

But this includes the division by the number itself.

Hence, the required number of divisors

$= 71 - 1 = 70.$

The correct option is (A)

69. Other than 2, remaining five places are to be filled by 1 and 3

$\therefore$ number of ways for five places

$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

For 2, selecting 2 places out of $7 = {}^7C_2$

$\therefore$ Required number of ways $= {}^7C_2 \cdot 2^5$.

The correct option is (B)

70. The required number of circles

$= ({}^{10}C_3 - {}^4C_3) + 1 = 117.$

The correct option is (C)

71. Number of subsets

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n = N$ (say)

$\therefore \quad N = 2^{2n+1} - N$

$\Rightarrow \quad 2N = 2^{2n+1}$

$\Rightarrow \quad N = 2^{2n}.$

The correct option is (D)

72. Starting with the letter *A* and arranging the other four letters, there are 4! = 24 words. These are the first 24 words. Then, starting with *G* and arranging *A*, *A*, *I* and *N* in different ways, there are $\frac{4!}{2!1!1!} = 12$ words, the 37th word starts with *I* and there are 12 words starting with *I*. Total goes up to 48 words. Now, 49th word is **NAAGI**.

The correct option is (A)

73. If we choose k $(0 \le k \le n)$ identical objects, then we must choose $(n-k)$ distinct objects. This can be done in ${}^{2n+1}C_{n-k}$ ways. Thus, the required number of ways

$$= \sum_{k=0}^{n} {}^{2n+1}C_{n-k} = {}^{2n+1}C_n + {}^{2n+1}C_{n-1} + \ldots + {}^{2n+1}C_0$$

$= 2^{2n}.$

The correct option is (A)

74. The number of subsets of A containing exactly three elements is ${}^nC_3$ whereas the number of three subsets of A that contain a_1 is ${}^{n-1}C_2$. We are given,

$${}^{n-1}C_2 = \frac{20}{100}\ ({}^nC_3)$$

$$\Rightarrow \quad \frac{(n-1)(n-2)}{2} = \frac{1}{5}\,\frac{n(n-1)(n-2)}{6}$$

$\Rightarrow \quad n = 15$

The correct option is (A)

75. For $0 \le r \le 66$, $0 \le \frac{r}{100} < \frac{2}{3}$

$$\Rightarrow \quad -\frac{2}{3} < -\frac{r}{100} \le 0$$

$$\Rightarrow \quad -\frac{1}{3} - \frac{2}{3} < -\frac{1}{3} - \frac{r}{100} \le -\frac{1}{3}$$

$$\therefore \quad \left[-\frac{1}{3} - \frac{r}{100}\right] = -1 \text{ for } 0 \le r \le 66$$

Also, for $67 \le r \le 100$, $\frac{67}{100} \le \frac{r}{100} \le 1$

$$\Rightarrow \quad -1 \le -\frac{r}{100} \le -\frac{67}{100}$$

$$\Rightarrow \quad -\frac{1}{3} - 1 \le -\frac{1}{3} - \frac{r}{100} \le -\frac{1}{3} - \frac{67}{100}$$

$$\therefore \quad \left[-\frac{1}{3} - \frac{r}{100}\right] = -2 \text{ for } 67 \le r \le 100$$

Hence, $\sum_{r=0}^{100}\left[-\frac{1}{3} - \frac{r}{100}\right] = 67(-1) + 2(-34) = -135.$

The correct option is (C)

76. The following four cases are there

(A) At least one one rupee coin and any number of other coins is $2 \times 4 \times 5 = 40$ ways.

(B) At least two 50 p. coins and any number of 25 p. coins = $2 \times 5 = 10$ ways.

(C) One 50 p. coin + at least two 25 p. coins = $1 \times 3 = 3$ ways.

(D) Four 25 p. coins in one way only $\therefore$ total = 54.

The correct option is (A)

77. The number of students answering at least r questions incorrectly is 2^{n-r}.

$\therefore$ The number of students answering exactly

r $(1 \le r \le n-1)$ questions incorrectly is $2^{n-r} - 2^{n-(r+1)}$.

Also, the number of students answering all questions wrongly is $2^0 = 1$.

Thus, the total number of wrong answers is

$1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + 3(2^{n-3} - 2^{n-4}) + \ldots$

$+ (n-1)(2^1 - 2^0) + n(2^0)$

$= 2^{n-1} + 2^{n-2} + \ldots + 2^0 = 2^n - 1.$

Now, $2^n - 1 = 4095$

$\Rightarrow \quad 2^n = 4096 = 2^{12}$

$\Rightarrow \quad n = 12.$

The correct option is (B)

78.

d

c

a				$\longrightarrow$ 2 ways.

d

a

b	a	d	c	$\longrightarrow$ 3 ways.

d a c

b c a

a

c				$\longrightarrow$ 3 ways.

d				$\longrightarrow$ 3 ways.

$\therefore$ Total number of ways = 11.

The correct option is (D)

79. $S = \sum_{r=0}^{m} {}^{n+r}C_k$

$= {}^{n}C_k + {}^{n+1}C_k + {}^{n+2}C_k + \ldots + {}^{n+m}C_k$

$= {}^{n}C_{k+1} + {}^{n}C_k + {}^{n+1}C_k + {}^{n+2}C_k + \ldots + {}^{n+m}C_k - {}^{n}C_{k+1}$

$= {}^{n+1}C_{k+1} + {}^{n+1}C_k + {}^{n+2}C_k + \ldots + {}^{n+m}C_k - {}^{n}C_{k+1}$

$\therefore \quad S = {}^{n+m+1}C_{k+1} - {}^{n}C_{k+1}$

The correct option is (B)

80. Total number of ways = n^n

Number of ways so that each person gets at least one thing (here, exactly one thing) = $n!$

Given, $n^n - n! = 232$

$1^1 - 1! = 0$, $2^2 - 2! = 2$, $3^3 - 3! = 21$, $4^4 - 4! = 232$

$\therefore \quad n = 4$

The correct option is (B)

81. Now, $n = 2^3 - 2 = 6$

Also, $m = 4 + 3 + 2 + 1 - 1 = 9 \left(\text{as } \frac{2}{4} = \frac{1}{2}\right)$

$\therefore \quad m - n = 3$

The correct option is (D)

82. Number of solutions of the given equation is the same as the number of solutions of the equation

$x_1x_2x_3x_4 = 30 = 2 \times 3 \times 5$

Here, x_4 is there because if $x_1x_2x_3 = 15$, then $x_4 = 2$ and if $x_1x_2x_3 = 5$, then $x_4 = 6$, etc.

x_4 is in fact a dummy variable.

Each of 2, 3 and 5 will be a factor of exactly one of x_1, x_2, x_3, x_4 in 4 ways.

$\therefore \quad$ Required number = $4^3 = 64$

The correct option is (A)

83. Number of ways of selecting at least one green dye out of 5 different green dyes

${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 - 1$

Also, at least one blue dye can be selected out of 4 blue dyes in

${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1$

Again, 3 different red dyes can be selected in

${}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3$ ways

$\therefore$ Required ways = $(2^5 - 1)(2^4 - 1)(2^3) = 3720$

The correct option is (C)

84. $4m = 2^a + 3^b + 5^c = 2^a + (4-1)^b + (4+1)^c$

$4m = 4k + 2^a + (-1)^b + (1)^c$

$\therefore \quad a = 1$, $b =$ even, $c =$ any number

$a \neq 1$, $b =$ odd, $c =$ any number

$\therefore \quad$ Required number = $1 \times 2 \times 5 \times 4 \times 3 \times 5 = 70$

The correct option is (B)

85. We have, $\frac{{}^{2n}C_r}{{}^{2n}C_{r-1}} = \frac{2n-r+1}{r}$ (1)

For ${}^{2n}C_r$ to be greatest $\frac{2n-r+1}{r} \geq 1$

$\Rightarrow \quad 2n - r + 1 \geq r \Rightarrow 2r \leq 2n + 1$

$\Rightarrow \quad r \leq n + \frac{1}{2}$ (2)

From (1), $\frac{{}^{2n}C_{r+1}}{{}^{2n}C_r} = \frac{2n-(r+1)+1}{r+1} = \frac{2n-r}{r+1}$.

For ${}^{2n}C_r$ to be greatest $\frac{2n-r}{r+1} \leq 1$

$\Rightarrow \quad 2n - r \leq r + 1 \Rightarrow 2r \geq 2n - 1$

$\Rightarrow \quad r \geq n - \frac{1}{2}$ (3)

From (2) and (3), we get $n - \frac{1}{2} \leq r \leq n + \frac{1}{2}$

$\Rightarrow \quad r = n$ (since r is a positive integer)

Hence, ${}^{2n}C_r$ is greatest when $r = n$.

The correct option is (C)

86. The numbers are of three or four digits.

To find the number of even numbers of three digits—

The unit's place must be filled by 0 or 2.

$\therefore$ The number of even numbers of three digits (having or not having 0 in hundred's place) = ${}^2P_1 \times {}^3P_2$.

But the number of even numbers of three digits having 0 in hundred's place = ${}^2P_1$ ($\because$ unit's place is naturally filled by 2 and ten's place by one of 1 and 3).

$\therefore \quad$ The number of even numbers of three digits

$= {}^2P_1 \times {}^3P_2 - {}^2P_1$. (1)

Similarly, the number of even numbers of four digits

$= {}^2P_1 \times {}^3P_3 - {}^2P_2$. (2)

Adding (1) and (2), we get the number of even numbers greater than 100

$= ({}^2P_1 \times {}^3P_2 - {}^2P_1) + ({}^2P_1 \times {}^3P_3 - {}^2P_2)$

$= 2 \times 3 \times 2 - 2 + 2 \times 3 \times 2 \times 1 - 2 \times 1 = 20.$

The correct option is (A)

87. Here, the available digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The numbers can be of one, two or three digits and in each of them unit's place must have 0 or 5 as they must be divisible by 5.

The number of numbers of one digit = 1

($\because$ 5 is the only number).

The number of numbers of two digits divisible by 5 = number of all the numbers of two digits divisible by 5 – number of numbers of two digits divisible by 5 and having 0 in ten's place = ${}^2P_1 \times {}^9P_1 - 1$,

($\because$ unit's place can be filled by either 0 or 5 in first category and only by 5 in the second category)

$= 2 \times 9 - 1 = 17.$

The number of numbers of three digits divisible by 5 = number of all the numbers of three digits divisible by 5 = number of numbers of three digits divisible by 5 and having 0 in hundred's place

$= {}^2P_1 \times {}^9P_2 - {}^8P_1 \times 1 = 2 \times 9 \times 8 - 8 = 136.$

$\therefore$ Required number of numbers

$= 1 + 17 + 136 = 154.$

The correct option is (B)

88. Suppose, the first two digits are 41, remaining four digits are to be chosen from 0, 2, 3, 5, 6, 7, 8, 9 so as to make all the digits distinct.

$\therefore$ Remaining four can be chosen in $P(8, 4)$ ways.

We have 8 letters and they are to be arranged at 4 places.

First place can be arranged in 8 different ways.

Second place can be arranged in 7 different ways.

Third place can be arranged in 6 different ways.

Fourth place can be arranged in 5 different ways.

Number of such telephone numbers

$= 8 \times 7 \times 6 \times 5 = 1680.$

Now, considering the all given first two digits (41 or 42 or 46 or 64), total number of the telephone numbers

$= 1680 \times 5 = 8400.$

The correct option is (A)

89. There are 11 letters in the word INDEPENDENT.

3N, 3E, 2D, I, P, T (6 types)

Different possibilities of choosing 5 letters are

(A) **All different**

Number of ways $= {}^6C_5 \times 5! = 6 \times 120 = 720.$

(B) **2 alike, 3 different**

Number of ways $= {}^3C_1 \times {}^5C_3 \times \frac{5!}{2!} = 3 \times 10 \times 60 = 1800$

(C) **3 alike, 2 different**

Number of ways $= {}^2C_1 \times {}^5C_2 \times \frac{5!}{3!}$

$= 2 \times 10 \times 20 = 400$

(D) **2 alike, 2 alike, 1 different**

$= {}^3C_2 \times {}^4C_1 \times \frac{5!}{2!\,2!} = 3 \times 4 \times 30 = 360$

(E) **3 alike, 2 alike** $= {}^2C_1 \times {}^2C_1 \times \frac{5!}{3!\,2!}$

$= 2 \times 2 \times 10 = 40$

Hence, total number of ways

$= 720 + 1800 + 400 + 360 + 40 = 3320.$

The correct option is (B)

90. The number of numbers of five digits, having 3 in unit's place $= {}^4P_4$ (number of arrangements of the remaining four digits)

$= 4! = 24.$

Similar is the case for other digits, i.e., each digit occurs 24 times in the unit's place.

Hence, the sum of the digits in unit's place

$= 24 \times 3 + 24 \times 4 + 24 \times 5 + 24 \times 6 + 24 \times 7$

$= 24\,(3 + 4 + 5 + 6 + 7) = 24 \times 25 = 600$

Similarly, the sum of the digits in each of the other places = 600.

$\therefore$ The required sum

$= 600 \times 10000 + 600 \times 1000 + 600 \times 100 + 600 \times 10 + 600 \times 1$

$= 600\,(10000 + 1000 + 100 + 10 + 1)$

$= 600 \times 11111 = 6666600.$

The correct option is (B)

91. The number of numbers having 2 in unit's place $= \frac{3!}{2!}$

(because the other three places are to be filled by 3, 3, 4 of which two are identical).

The number of numbers having 3 in unit's place = 3!, (because the other three places are to be filled by 2, 3, 4).

The number of numbers having 4 in unit's place $= \frac{3!}{2!}$,

(because the other three places are to be filled by 3, 2, 3 of which two are identical)

$\therefore$ Sum of the digits in unit's place

$= \frac{3!}{2!} \times 2 + 3! \times 3 + \frac{3!}{2!} \times 4 = 6 + 18 + 12 = 36.$

Similarly, the sum of the digits in other places is 36 each.

$\therefore$ Required sum $= 36 \times 1000 + 36 \times 100 + 36 \times 10 + 36 \times 1$

$= 36\,(1000 + 100 + 10 + 1) = 36 \times 1111 = 39996.$

The correct option is (A)

92. All the numbers are of five digits.

The number of numbers having 1 in unit's place = 4! = 24

($\because$ the other four places are to be filled by 3, 5, 7, 9).

Similarly, the number of numbers having 3, 5, 7, or 9 in unit's place is 24 in each case.

$\therefore$ The sum of the digits in unit's place for all the numbers

$= 24 \times 1 + 24 \times 3 + 24 \times 5 + 24 \times 7 + 24 \times 9$

$= 24\,(1 + 3 + 5 + 7 + 9) = 24 \times 25 = 600.$

Similarly, sum of the digits in other places = 600 in every case.

$\therefore$ The required sum of all the numbers that can be made

$= 600 \times 10000 + 600 \times 1000 + 600 \times 100 + 600 \times 10 + 600 \times 1$

$= 600\,(10000 + 1000 + 100 + 10 + 1) = 600 \times 11111$

$= 6666600.$

The correct option is (C)

93. All the numbers are of four digits and they do not have 0 in thousand's place.

The number of numbers having 0 in unit's place = 3!

($\because$ the other three places are to be filled by 1, 2 or 3).

The number of numbers having 1 in unit's place = ${}^2P_1 \times {}^2P_2$

($\because$ thousand's place can be filled by one of 2, 3 and the remaining two places can be filled by the remaining two digits).

Similarly, the number of numbers having 2 or 3 in unit's place

$= {}^2P_1 \times {}^2P_2$ in each case.

Thus, the sum of the digits in unit's place for all the numbers

$= 3! \times 0 + {}^2P_1 \times {}^2P_2 \times 1 + {}^2P_1 \times {}^2P_2 \times 2 + {}^2P_1 \times {}^2P_2 \times 3$

$= 4 + 8 + 12 = 24.$

Similarly, the sum of the digits in ten's and hundred's places = 24 each.

Now, the thousand's place can have only 1 or 2 or 3.

Number of numbers having 1 in thousand's place = 3!, (for the other three places will be filled by 0, 2, 3).

Similarly, the number of numbers having 2 or 3 in thousands's place is 3! in each case.

$\therefore$ Sum of the digits in the thousands's place for all the numbers $= 3! \times 1 + 3! \times 2 + 3! \times 3 = 6 + 12 + 18 = 36.$

$\therefore$ Required sum of all the numbers

$= 36 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1$

$= 36000 + 2400 + 240 + 24 = 38664.$

The correct option is (A)

94. The thousand's place cannot be filled by *O*. So the number of ways to fill the thousands's place = 5.

The remaining three places can be filled by six digits in 6^3 ways, as digits can be repeated.

$\therefore$ The number of four digit numbers

$= 5 \times 6^3 = 1080.$ (1)

Now, the number of numbers of four digits that do not contain any repeated digit = ${}^5P_1 \times {}^5P_3$ {$\because$ thousand's place is to be filled by one of 1, 2, 3, 4, 5 and the remaining three places are to be filled by three of the remaining five digits including 0}. But (1) contains numbers which contain no repeated digits as well as those which contain at least one repeated digit.

$\therefore$ The number of four digit numbers which contain at least one repeated digit = $1080 - {}^5P_1 \times {}^5P_3$

$= 1080 - 5 \times 5 \times 4 \times 3 = 780.$

The correct option is (C)

95. The numbers are of five digits having 4 or 5 in ten thousand's place and 1 or 5 in unit's place and the remaining digits are any of the given six digits.

$\therefore$ Number of ways to fill ten thousand's place = ${}^2P_1 = 2$

Number of ways to fill unit's place = ${}^2P_1 = 2$

Number of ways to fill other three places = 6^3.

$\therefore$ Required number of numbers = $2 \times 2 \times 6^3 = 864.$

The correct option is (A)

96. When *X* and *Y* sit on the side facing the window—

First, we have to select 2 persons for the side facing the window and 3 for the opposite side from the remaining 5. The number of selections for this is equal to ${}^5C_2 \times {}^3C_3$.

For each of the selections, the number of arrangements $= 4! \times 3!.$

$\therefore$ The number of ways to sit when *X*, *Y* are on the side facing the window = ${}^5C_2 \times {}^3C_3 \times 4! \times 3!.$

When *X* and *Y* sit on the other side

As before, the number of ways to sit

$= {}^5C_1 \times {}^4C_4 \times 4! \times 3!$

$\therefore$ Required number of ways

$= {}^5C_2 \times {}^3C_3 \times 4! \times 3! + {}^5C_1 \times {}^4C_4 \times 4! \times 3!$

$= \frac{5 \times 4}{2} \times 1 \times 24 \times 6 + 5 \times 1 \times 24 \times 6 = 2160.$

The correct option is (B)

97. In selection of fruits, fruits of the same type are treated as identical.

Number of selections of any number of oranges from 4 oranges = 4 + 1.

Number of selections of any number of apples from 5 apples = 5 + 1.

Number of selections of any number of mangoes from 6 mangoes = 6 + 1.

$\therefore$ Number of selections of any number of fruits from the basket = $(4 + 1)\,(5 + 1)\,(6 + 1) = 5 \times 6 \times 7 = 210.$

But in one of these selections, there is one which contains no fruits. Excluding this selection, the required number of selections = 210 – 1 = 209.

The correct option is (C)

98. In terms of prime factors 100! can be written as $2^a \cdot 3^b \cdot 5^c \cdot 7^d \ldots$

Now, $E_2\,(100!)$

$= \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$

$= 50 + 25 + 12 + 6 + 3 + 1 = 97$

and, $E_5\,(100!) = \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20 + 4 = 24$

$100! = 2^{97} \cdot 3^b \cdot 5^{24} \cdot 7^d \ldots$

$= 2^{73} \cdot 3^b \cdot (2 \times 5)^{24} \cdot 7^d \ldots$

$= 2^{73} \cdot 3^b \cdot (10)^{24} \cdot 7^d \ldots$

Hence, number of zeros at the end of 100! is 24.

The correct option is (C)

99. In terms of prime factors 33! can be written as $2^a \cdot 3^b 5^c \cdot 7^d$ …

Now, E_2 (33!)

$= \left[\frac{33}{2}\right] + \left[\frac{33}{2^2}\right] + \left[\frac{33}{2^3}\right] + \left[\frac{33}{2^4}\right] + \left[\frac{33}{2^5}\right]$

$= 16 + 8 + 4 + 2 + 1 = 31.$

Hence, the exponent of 2 in 33! is 31.

$\therefore$ Largest value of n is 31.

The correct option is (B)

100. Number of non-negative integral solutions of the given equation

= coefficient of x^{20} in

$(1-x)^{-1}\,(1-x)^{-1}\,(1-x)^{-1}\,(1-x^4)^{-1}$.

= coefficient of x^{20} in $(1-x)^{-3}\,(1-x^4)^{-1}$

= coefficient of x^{20} in $(1 + {}^3C_1 x + {}^4C_2 x^2 + {}^5C_3 x^3 + {}^6C_4 x^4 + \ldots$

$+ {}^{10}C_8 x^8 \ldots + {}^{14}C_{12} x^{12} + \ldots + {}^{18}C_{16} x^{16} + \ldots + {}^{22}C_{20} x^{20} + \ldots)$

$\times (1 + x^4 + x^8 + x^{12} + x^{16} + x^{20} + \ldots)$

$= 1 + {}^6C_4 + {}^{10}C_8 + {}^{14}C_{12} + {}^{18}C_{16} + {}^{22}C_{20}$

$= 1 + {}^6C_2 + {}^{10}C_2 + {}^{14}C_2 + {}^{18}C_2 + {}^{22}C_2$

$= 1 + \left(\frac{6.5}{1.2}\right) + \left(\frac{10.9}{1.2}\right) + \left(\frac{14.13}{1.2}\right) + \left(\frac{18.17}{1.2}\right) + \left(\frac{22.21}{1.2}\right)$

$= 1 + 15 + 45 + 91 + 153 + 231 = 536.$

The correct option is (B)

101. Let r consecutive positive integers be

$m, m+1, m+2, \ldots (m+r-1)$, where $m \in N$

$\therefore$ Product $= m\,(m+1)\,(m+2)\ldots(m+r-1)$

$$= \frac{(m-1)!\,m\,(m+1)\,(m+2)\ldots(m+r-1)}{(m-1)!}$$

$$= \frac{(m+r-1)!}{(m-1)!} = r!.\frac{(m+r-1)!}{r!(m-1)!}$$

$$= r!.\,{}^{m+r-1}C_r$$

which is divisible by $r!$ ($\because {}^{m+r-1}C_r$ is a natural number)

The correct option is (A)

102. The number of triplets of positive integers which are solutions of $x + y + z = 100$

= Coefficient of x^{100} in $(x + x^2 + x^3 + \ldots)^3$

= Coefficient of x^{100} in $x^3\,(1-x)^{-3}$

= Coefficient of x^{100} in

$$x^3\left[1 + 3x + 6x^2 + \ldots + \frac{(n+1)\,(n+2)}{2}x^n + \ldots\right]$$

$$= \frac{(97+1)\,(97+2)}{2} = 49 \times 99 = 4851.$$

The correct option is (C)

103. There are eleven letters in the word 'Pataliputra' and there are two *p*'s, two *r*'s, three *a*'s and four other different letters.

Number of consonants = 6, number of vowels = 5.

Since relative order of the vowels and consonants remains unchanged, therefore, vowels will occupy only vowel's place and consonants will occupy only consonants's place.

Now, 6 consonants can be arranged among themselves in $\frac{6!}{2!2!}$ ways [since there are two *p*'s and two *t*'s] and five vowels can be arranged among themselves in $\frac{5!}{3!}$ ways, since *a* occurs thrice.

$\therefore$ Required number $= \frac{6!}{2!2!} \cdot \frac{5!}{3!} = 3600.$

The correct option is (A)

104. Let n be the number of students.

Now, number of ways in which two students can be selected out of n students is ${}^nC_2$.

$\therefore$ number of pairs of students $= {}^nC_2$.

But for each pair of students, number of cards sent is 2 (since if there are two students A and B, A will send a card to B and B will send a card to A).

$\therefore$ For ${}^nC_2$ pairs, number of cards sent $= 2 \cdot {}^nC_2$.

According to the question, $2 \cdot {}^nC_2 = 600$

or, $2 \cdot \frac{n(n-1)}{2!} = 600$

or, $n^2 - n - 600 = 0$

or, $(n-25)\,(n+24) = 0$

$\therefore$ $n = 25, -24$

But $n \neq -24$

$\therefore$ $n = 25.$

The correct option is (C)

105. $\binom{m}{m} + \binom{m+1}{m} + \binom{m+2}{m} + \ldots + \binom{n}{m}$

$= \binom{m+1}{m+1} + \binom{m+1}{m} + \binom{m+2}{m} + \ldots + \binom{n}{m}$

$= \binom{m+2}{m+1} + \binom{m+2}{m} + \ldots + \binom{n}{m}$

$= \binom{m+3}{m+1} + \binom{m+3}{m} + \ldots + \binom{n}{m}$

$\vdots \qquad \vdots \qquad \vdots$

$= \binom{n}{m+1} + \binom{n}{m} = \binom{n+1}{m+1}$

Thus, $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m} = \binom{n+1}{m+1}.$

The correct option is (B)

106. A number of the seven digits will be of the form.

$ax_1x_2x_3x_4x_5\,0$, $ax_1x_2x_3x_4x_5\,1$, $ax_1x_2x_3x_4x_5\,2$,

$ax_1x_2x_3x_4x_5\,3$, … $ax_1x_2x_3x_4x_5\,9$

where $a \in \{1, 2, 3, \ldots 9\}$

and $x_1, x_2, x_3, x_4, x_5 \in \{0, 1, 2, 3, \ldots 9\}$.

Since sum of the digits should be even, therefore,

if $a + x_1 + x_2 + x_3 + x_4 + x_5$ is an even number, then the digit at units place must be 0, 2, 4, 6, 8 and if $a + x_1 + x_2 + x_3 + x_4 + x_5$ is an odd number, then the digit at units place will be 1, 3, 5, 7, 9.

$\therefore$ Required number $= 9 \times 10 \times 10 \times 10 \times 10 \times 5$

$= 9 \times 10^5 \times 5 = 45 \times 10^5$.

The correct option is (B)

107. The word *BAC* cannot be spelt if the m selected coupons do not contain atleast one of A, B and C.

Number of ways of selecting m coupons which are A or $B = 2^m$.

This also includes the case when all the m coupons are A or all are B.

Number of ways of selecting m coupons which are B or $C = 2^m$.

This also includes the case when all the m coupons are B or all are C.

Number of ways of selecting m coupons which are C or $A = 2^m$.

This also includes the case when all the m coupons are C or all are A.

Number of ways of selecting m coupons when all are $A = 1^m$.

Number of ways of selecting m coupons when all are $B = 1^m$.

Number of ways of selecting m coupons when all are $C = 1^m$.

$\therefore$ Required number $= 2^m + 2^m + 2^m - (1^m + 1^m + 1^m)$

$= 3 \cdot 2^m - 3 \cdot 1^m = 3\,(2^m - 1)$.

The correct option is (A)

108. **First Method:** Number of cross marks in

1st row	2nd row	3rd row	Number of ways
	(Number of selections of squares)		
1	3	2	${}^2C_1 \times {}^4C_3 \times {}^2C_2 = 8$
1	4	1	${}^2C_1 \times {}^4C_4 \times {}^2C_1 = 4$
2	2	2	${}^2C_2 \times {}^4C_2 \times {}^2C_2 = 6$
2	3	1	${}^2C_2 \times {}^4C_3 \times {}^2C_1 = 8$

$\therefore$ Required number $= 8 + 4 + 6 + 8 = 26$

Second Method:

Number of ways of putting six cross marks in 8 squares when there is no restriction $= {}^8C_6 = 28$.

Number of ways when the six cross marks are put in first and second rows $= {}^6C_6 = 1$.

Number of ways when the six cross marks are put in second and third rows $= {}^6C_6 = 1$.

$\therefore$ Required number $= 28 - 2 = 26$.

The correct option is (A)

109. Required number = coefficient of x^{16} in $(x^3 + x^4 + \ldots + x^7)^4$

= coefficient of x^{16} in $x^{12}\,(1 + x + \ldots + x^4)$

= coefficient of x^4 in $\left(\dfrac{1 - x^5}{1 - x}\right)^4$

= coefficient of x^4 in $[(1 - x^5)^4\,(1 - x)^{-4}]$

= coefficient of x^4 in $[(1 - 4x^5 + 6x^{10} - \ldots)\,(1 - x)^{-4}]$

= coefficient of x^4 in $(1 - x)^{-4}$

$= {}^7C_4 = \dfrac{4 \cdot 5 \cdot 6 \cdot 7}{4!} = 35$

The correct option is (B)

110. Required number = coefficient of x^{30} in $(x^2 + x^3 + \ldots + x^{16})^8$

= coefficient of x^{30} in $x^{16}\,(1 + x + \ldots + x^{14})^8$

= coefficient of x^{14} in $\left(\dfrac{1 - x^{15}}{1 - x}\right)^8$

= coefficient of x^{14} in $(1 - x)^{-8} = {}^{21}C_{14} = = 116280$.

The correct option is (C)

111. The candidate must score 150 marks.

$\therefore$ Required number

= coefficient of x^{150} in $(1 + x + \ldots + x^{50})^3\,(1 + x + \ldots + x^{100})$

= coefficient of x^{150} in $\left(\dfrac{1 - x^{51}}{1 - x}\right)^3 \dfrac{1 - x^{101}}{1 - x}$

= coefficient of x^{150} in $(1 - x^{51})^3\,(1 - x^{101})\,(1 - x)^{-4}$

= coefficient of x^{150} in $(1 - 3x^{51} + 3x^{102} - x^{153})$

$(1 - x^{101})\,(1 - x)^{-4}$

[leaving terms containing powers of x greater than 150]

= coefficient of x^{150} in $(1 - x)^{-4}$ – 3. coefficient of x^{99} in $(1 - x)^{-4}$ + 3 coefficient of x^{48} in $(1 - x)^{-4}$ coefficient of x^{49} in $(1 - x)^{-4}$

$= {}^{153}C_{150} - 3 \cdot {}^{102}C_{99} + 3 \cdot {}^{51}C_{48} - {}^{52}C_{49}$.

$= \dfrac{153 \cdot 152 \cdot 151}{6} - 3 \cdot \dfrac{102 \cdot 101 \cdot 100}{6}$

$+ 3 \cdot \dfrac{51 \cdot 50 \cdot 49}{6} - \dfrac{52 \cdot 51 \cdot 50}{6}$

= 110556

The correct option is (C)

112. Any number between 1 and 1000000 must be of less than seven digits. Therefore, it must be of the form

$a_1\,a_2\,a_3\,a_4\,a_5\,a_6$

where $a_1, a_2, a_3, a_4, a_5, a_6 \in \{0, 1, 2, \ldots, 9\}$

According to question, sum of the digits = 18

Thus, $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 18$

where $0 \le a_i \le 9$, $i = 1, 2, 3, \ldots, 9$.

Required number

= coefficient of x^{18} in $(1 + x + x^2 + \ldots + x^9)^6$

= coefficient of x^{18} in $\left(\dfrac{1 - x^{10}}{1 - x}\right)^6$

= coefficient of x^{18} in $[(1-x^{10})^6 (1-x)^{-6}]$

= coefficient of x^{18} in $[(1 - {}^6C_1 x^{10})(1-x)^{-6}]$

[leaving terms containing powers of x greater than 18]

= coefficient of x^{18} in $(1-x)^{-6} - {}^6C_1$. coefficient of x^8 in $(1-x)^{-6}$

$= {}^{6+18-1}C_{18} - 6 \cdot {}^{6+8-1}C_{18} = {}^{23}C_5 - 6 \cdot {}^{13}C_8$

$= \dfrac{23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{120} - 6 \cdot \dfrac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{120}$

$= 33649 - 7722 = 25927$

The correct option is (A)

113. Given, $x + y + z + u + t = 20$ (1)

$x + y + z = 5$ (2)

Given, system of equations is equivalent to

$x + y + z = 5$ (3)

and, $u + t = 15$ (4)

Number of non-negative integral solutions of equation (3)

$= {}^{n+r-1}C_r = {}^{3+5-1}C_5 = {}^7C_5$,

Number of non-negative integral solutions of equation (4)

$= {}^{n+r-1}C_r = {}^{2+15-1}C_{15} = {}^{16}C_{15}$

$\therefore$ Required number $= {}^7C_5 \times {}^{16}C_{15} = 336$.

The correct option is (A)

114. Let w be a non-negative integer such that

$3x + y + z + w = 30$

Let $a = x - 1$, $b = y - 1$, $c = z - 1$, $d = w$, then

$3a + b + c + d = 25$, where $a, b, c, d \geq 0$ (1)

Clearly, $0 \leq a \leq 8$. If $a = k$, then

$b + c + d = 25 - 3k$ (2)

Number of non-negative integral solutions of equation (2)

$= {}^{n+r-1}C_r = {}^{3+25-3k-1}C_{25-3k}$

$= {}^{27-3k}C_{25-3k} = {}^{27-3k}C_2 = \dfrac{(27-3k)(26-3k)}{2}$

$= \dfrac{3}{2}(3k^2 - 53k - 234)$

$\therefore$ Required number $= \dfrac{3}{2}\sum_{k=0}^{8}(3k^2 - 53k + 234)$

$= \dfrac{3}{2}\left[3 \cdot \dfrac{8 \times 9 \times 17}{6} - 53\dfrac{8 \times 9}{2} + 234 \times 9\right] = 1215.$

The correct option is (B)

115. In the mouth, there are 32 places for the teeth. For each place, two choices are possible, i.e., either tooth or not.

Choices for 1 teeth = 2

Similarly choices for 2 teeth = $2 \times 2 = 2^2$

Choices for 3 teeth = $2 \times 2 \times 2 = 2^3$

……..

……..

Hence, choices for 32 teeth = 2^{32}

But this includes one case where there is no teeth.

$\therefore$ Required number of ways = $2^{32} - 1$.

The correct option is (B)

116. For any group of five scientists there must be at least one lock they can not open. Moreover, for any two different groups of five scientists, there must be two locks they cannot open, because if both groups cannot open the same lock, there is a group of six scientists among these two groups who will not be able to open the cabinet. Thus, at least

${}^{11}C_6 = \dfrac{11 \times 10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4 \times 5} = 11 \times 3 \times 2 \times 7$

= 462 locks are needed.

The correct option is (C)

117. The total number of permutations of the digits of given number is

$\dfrac{7!}{2!3!} = 420$

But when 0 occupies the first position, the number becomes a six-digit number which are not acceptable, since such a number will become less then 10^6.

The total number of such six-digit numbers is

$= \dfrac{6!}{2!3!} = 60$

Hence, the required number is 420 – 60 = 360.

The correct option is (A)

118. $n!\,(21-n)! = 21!\dfrac{n!(21-n)!}{21!} = 21!\dfrac{1}{{}^{21}C_n} = \dfrac{21!}{{}^{21}C_n}$

For minimum value, ${}^{21}C_n$ is maximum.

Maximum value of ${}^{21}C_n = {}^{21}C_{\frac{21-1}{2}} = {}^{21}C_{10}$

$\therefore$ Minimum value $= \dfrac{21!}{{}^{21}C_{10}} = \dfrac{21!}{21!} \times 10!11! = 10!11!$

The correct option is (B)

119. The maximum number of oranges that a child can get = 20 – 3 = 17.

Thus, the problem is equivalent to finding the number of integral solutions to the equation

$x_1 + x_2 + x_3 + x_4 = 20$

where $1 \leq x_1, x_2, x_3, x_4 \leq 17$, and x_1, x_2, x_3, x_4 denote the number of oranges given to the four children.

Hence, the required number of ways is

= coefficient of x^{20} in $(x + x^2 + x^3 + \ldots + x^{17})^4$

= coefficient of x^{16} in $(1 + x + x^2 + \ldots + x^{16})^4$

= coefficient of x^{16} in $(1 - x^{17})^4 (1-x)^{-4}$

= coefficient of x^{16} in $(1-x)^{-4}$

$= {}^{16+4-1}C_{16}$

$= {}^{19}C_3 = 969$.

The correct option is (B)

120. The smallest number, which can occur in the middle is 4. The number of numbers with 4 in the middle = ${}^4P_4 - {}^3P_3$

(∴ The other four places are to be filled by 0, 1, 2 and 3, and a number can not begin with 0)

Similarly, the number of numbers with 5 in the middle = ${}^5P_4 - {}^4P_3$, etc.

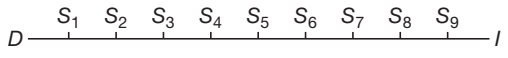
∴ The required number of numbers

$= ({}^4P_4 - {}^3P_3) + ({}^5P_4 - {}^4P_3) + ({}^6P_4 - {}^5P_3) + \ldots + ({}^9P_4 - {}^8P_3)$

$= \sum_{n=4}^{9} ({}^nP_4 - {}^{n-1}P_3)$

The correct option is (A)

121. Let *D* denotes Delhi and *I* denotes Indore.



For S_1, 9 different tickets are available, one for each of the remaining 9 stations. Similarly, for S_2, 8 different tickets are available and so on.

Therefore, total number of 9 different tickets

$= 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$

Thus, the six different tickets must be any six of these 45 and there are clearly as many different sets of 6 tickets as they are combinations of 45 things taken 6 at a time.

Hence, the required number = ${}^{45}C_6$.

The correct option is (C)

122. Using 2, 5 and 7 with repetition each place of *n* digit number can be chosen in 3 ways. Hence, total number of *n*- digit numbers = $3 \times 3 \times 3 \ldots n$ times $= 3^n$.

According to given condition $3^n \geq 900 \Rightarrow 3^{n-2} \geq 100$.

∴ $n - 2 \geq 5 \Rightarrow n \geq 7$.

The correct option is (C)

123. Let $b = a + d$ and $c = a + 2d$.

Given: $a + a + d + a + 2d = 21 \Rightarrow a + d = 7$

∴ $a + c = 14$ and $b = 7$

Now, the number of positive integral solutions of $a + c = 14$ is equal to coefficient of x^{14} in $(x + x^2 + x^3 + \ldots)^2$ = Coefficient of x^{12} in $(1 + x + x^2 \ldots)^2 = {}^{13}C_{12} = 13$.

The correct option is (A)

124. A mixed doubles game involves two males and two females. Two males can be chosen from *n* males in ${}^nC_2$ ways. Having chosen two males, now 2 females are to be chosen from $(n-2)$ females leaving the wives of the two already chosen males. This can be done in ${}^{n-2}C_2$ ways.

Hence, the required number of ways

$= {}^nC_2 . {}^{n-2}C_2 . 2$

[for every choice of 2 males and 2 females, they can be paired in 2 ways].

$= \frac{n(n-1)}{2} . \frac{(n-2)(n-3)}{2} . 2$

$= \frac{n(n-1)(n-2)(n-3)}{2} = \frac{1}{2} . {}^nP_4$

The correct option is (C)

More than One Option Correct Type

125. We know that join of any two points gives a line.

The number of lines obtained from 10 points, no three of which are collinear $= {}^{10}C_2 = \frac{10 \times 9}{2 \times 1} = 45$

Lines obtained from 4 points $= {}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$

∴ Number of lines lost due to 4 collinear points

$= 6 - 1 = 5$

∴ Required number of lines $= 45 - 5 = 40$.

Also, We know that any triangle can be obtained by joining any three points not in the same straight line.

∴ Number of triangles obtained from 10 points, no three of which are collinear

$= {}^{10}C_2 = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120.$

Triangle obtained from 4 points $= {}^4C_3 = 4$

∴ Number of triangles lost due to 4 collinear points = 4.

∴ Required number of triangles $= 120 - 4 = 116$.

The correct option is (A) and (B)

126. We have $770 = 2 \times 5 \times 7 \times 11$. We can assign 2 to x_1 or x_2 or x_3 or x_4. That is 2 can be assigned in 4 ways.

Similarly, each of 5, 7 or 11 can be assigned in 4 ways. Thus, the number of positive integral solutions of

$x_1 x_2 x_3 x_4 = 770$ is $4^4 = 2^8 = 256$.

The correct option is (B), (C) and (D)

127. For $1 \leq x \leq 33$, $\frac{1}{3} < \frac{1}{3} + \frac{x}{50} < 1$ therefore,

$\left[\frac{1}{3} + \frac{x}{50}\right] = 0$ for $1 \leq x \leq 33$.

For $34 \leq x \leq 50$, $1 < \frac{1}{3} + \frac{x}{50} < \frac{4}{3}$

$\Rightarrow \left[\frac{1}{3} + \frac{x}{50}\right] = 1$ for $34 \leq x \leq 50$

Thus, $E = 17$.

The correct option is (B) and (D)

128. Given numbers are 1, 2, 3, … *n*.

Let the three selected numbers in A.P. be *a, b, c,* then

$b = \frac{a + c}{2}$ or $a + c = 2b$ (1)

From (1) it is clear that $a + c$ should be an even integer. This is possible only when both a and c are odd or both are even.

Case I. When n is even. Let $n = 2m$

The number of odd numbers = m

and number of even numbers = m

$\therefore$ number of selections of a and c from m odd numbers = ${}^{m}C_2$

Number of selections of a and c from m even numbers = ${}^{m}C_2$

$\therefore$ Number of ways in this case = $2 \cdot {}^{m}C_2 = m\,(m-1)$

$$= \frac{n}{2}\left(\frac{n}{2}-1\right) = \frac{n\,(n-2)}{4}.$$

Case II. When n is odd. Let $n = 2m + 1$

Then, number of odd numbers = $m + 1$

and number of even numbers = m

$\therefore$ Required number in this case = ${}^{m+1}C_2 + {}^{m}C_2$

$$= \frac{(m+1)m}{2} + \frac{m(m-1)}{2} = m^2 = \left(\frac{n-1}{2}\right)^2$$

$$= \frac{1}{4}(n-1)^2.$$

The correct option is (A) and (B)

129. Two straight lines can intersect in at most one point. Therefore, maximum number of points of intersection of m lines = ${}^{m}C_2 \times 1 = {}^{m}C_2$

Again, two circles can intersect in at most two points.

$\therefore$ Maximum number of points of intersection of n circles = ${}^{n}C_2 \times 2 = 2{}^{n}C_2$

A line can intersect a circle in at most 2 points.

$\therefore$ Maximum number of points of intersection of m lines and n circles = $(mn)2 = 2mn$

$\therefore$ $N = {}^{m}C_2 + 2\,{}^{n}C_2 + 2mn$

$= {}^{m}C_2 + {}^{n}P_2 + 2mn$

(A) $N - {}^{m}C_2 = {}^{n}P_2 + 2mn$

$= n(n-1) + 2mn = n\,(n - 1 + 2m)$

$\therefore$ $n \mid (N - {}^{m}C_2)$.

(B) $N - {}^{n}P_2 = {}^{m}C_2 + 2mn$

$$= \frac{m(m-1)}{2} + 2mn$$

$$= m\left[\frac{m-1}{2} + 2n\right]$$

If m is even, then $(m - 1)$ is not divisible by 2.

$\therefore$ in general m does not divide $(N - nP_2)$.

(C) $N - {}^{m}C_2 = {}^{n}P_2 + 2mn$

$= 2{}^{n}C_2 + 2mn$

$= 2({}^{n}C_2 + mn)$

$= N - {}^{m}C_2$ is an even integer.

(D) $N - {}^{m}C_2 - {}^{n}P_2 = 2mn$ is an even integer.

The correct option is (A), (C) and (D)

130. $N = \dfrac{2n!}{n!n!} = \dfrac{(n+1)(n+2)\ldots(2n)}{n!}$

$\therefore$ $n!N = (n + 1)\,(n + 2)\,\ldots\,(2n)$

Now, $p/(n + 1)\,(n + 2)\,\ldots\,(2n)$ $(\because\ n < p < 2n)$

$\therefore$ $p/n!\ N$

$\therefore$ p/N. $(\because\ n < p < 2n \Rightarrow p$ does not divide $n!)$

If possible, let p^2/N.

$\Rightarrow$ $p^2/n!\ N$

$\Rightarrow$ $p^2/(n + 1)\,\ldots\,(p - 1)p\,(p + 1)\,\ldots\,(2n)$

$\Rightarrow$ $p/(n + 1)\,\ldots\,(p - 1)\,(p + 1)\,\ldots\,(2n)$

This is impossible.

$\therefore$ p^2 does not divide N.

The correct option is (A) and (D)

131. Let A, B,C be the objects selected and x_1, x_2, x_3, x_4 be the number of objects on either side of the selected objects as shown below

$x_1\ (A)\ x_2\ (B)\ x_3\ (C)\ x_4$

We have,

$x_1 + x_2 + x_3 + x_4 = n - 3$

where $x_1, x_4 \geq 0$ and $x_2, x_3 \geq 1$.

Hence, the required number of ways

= coefficient of x^{n-3} in $(1 + x + x^2 + \ldots)^2\,(x + x^2 + x^3 + \ldots)^2$

= coefficient of x^{n-5} in $(1 + x + x^2 + \ldots)^4$

= coefficient of x^{n-5} in $(1 - x)^{-4}$

$= {}^{n-5+4-1}C_{n-5} = {}^{n-2}C_{n-5} = {}^{n-2}C_3$.

The correct option is (A) and (B)

132. We have, $r - 1 \geq 0$, $r \leq n + 1$.

$\therefore$ $1 \leq r \leq n + 1$

$\Rightarrow$ $\dfrac{1}{n+1} \leq \dfrac{r}{n+1} \leq 1.$

Also, $k^2 - 8 = \dfrac{n!}{(r-1)!(n-r+1)!} \cdot \dfrac{r!(n+1-r)!}{(n+1)!}$

$= \dfrac{r}{n+1}$

Thus, $\dfrac{1}{n+1} \geq k^2 - 8 \leq 1$

$\Rightarrow$ $8 < \dfrac{1}{n+1} + 8 \leq k^2 \leq 9$

$\Rightarrow$ $8 < k^2 \leq 9$

$\Rightarrow$ $-3 \leq k < -2\sqrt{2}$

or $2\sqrt{2} < k \leq 3$

$\Rightarrow$ $k \in [-3, -2\sqrt{2})$ or $k \in (2\sqrt{2}, 3]$

The correct option is (B) and (D)

133. Let x_5 be such that $x_1 + x_2 + x_3 + x_4 + x_5 = n$. We now seek the non-negative integral solution of $x_1 + x_2 + x_3 + x_4 + x_5 = n$. The number of required solutions = ${}^{n+4}C_n = {}^{n+4}C_4$.

The correct option is (A) and (B)

Passage Based Questions

134. There can be two cases

(1) 1, 1, 1, 1, 3 (2) 1, 1, 1, 2, 2

Case 1: A box (in which 3 balls can be put) is selected in ${}^5C_1$ ways.

Now, 3 balls can be selected in ${}^7C_3$ ways and remaining 4 balls can be arranged in 4! ways.

Hence, total number of ways = ${}^5C_1 \times {}^7C_3 \times 4!$

Case 2: 2 boxes (in which 2–2 balls can be put) are selected in ${}^5C_2$ ways.

Now, any two balls can be selected in ${}^7C_2$ ways and for the other selected box, 2 balls can be selected from the remaining 5 balls in ${}^5C_2$ ways and the remaining balls can be distributed in 3! ways.

Hence, the total number of ways = ${}^5C_2 \times ({}^7C_2 \times {}^5C_2) \times 3!$

$\therefore$ The required number of ways

$= ({}^5C_1 \times {}^7C_3 \times 4!) + ({}^5C_2 \times {}^7C_2 \times {}^5C_2 \times 3!) = 16800$

The correct option is (B)

135. Required number of ways $= {}^{7+5-1}C_{5-1}$

$= {}^{11}C_4 = 330$

The correct option is (C)

136. One ball can be selected from the box 2 in ${}^7C_1$ ways and two balls can be selected out of remaining 6 balls from the box 4 in ${}^6C_2$ ways and the remaining 4 balls can be distributed in 3 boxes in 3^4 ways.

Hence, the required number of ways

$= {}^7C_1 \times {}^6C_2 \times 3^4 = 8505$

The correct option is (B)

137. Ball 2 can be distributed between 2 boxes (viz., box 2 and box 4) in 2 ways.

Now, the remaining 6 balls can be distributed in any of the 5 boxes in 5^6 ways.

Hence, the required number of ways = 2×5^6 ways

= 31250 ways

The correct option is (B)

138. Required number of ways

= Total number of ways – number of ways in which ball 2 and ball 4 are together.

Here, total number of ways = 16800.

Now, consider ball 2 and ball 4 are together and this arrangement is assumed to be a single ball.

Thus, we have 6 balls to be distributed into 5 boxes, which can be done as 1, 1, 1,2 in ${}^5C_1 \times {}^6C_2 \times 4!$ ways = 1800.

${}^5C_1 \rightarrow$ Number of ways of selecting one box in which two balls are kept together.

${}^6C_2 \rightarrow$ Number of ways of selecting 2 balls out of 6 balls.

$4! \rightarrow$ Number of ways of distribution of remaining 4 balls in remaining 4 boxes.

Hence, the required number of ways = 16800–1800 = 1500.

The correct option is (B)

139. Required number of ways $= {}^{4+3-1}C_{3-1}$

$= {}^6C_2 = 15$

The correct option is (A)

140. Since pots are identical, there will be 4 cases (4, 0, 0), (3, 1, 0), (2, 2, 0) and (2, 1, 1) but since all coins are different hence selection of coins matters.

Therefore, for the first case number of selections = ${}^4C_4 = 1$

For the second case number of selections = ${}^4C_3 \times {}^1C_1 = 4$

For the third case number of selections

$$= \frac{{}^4C_2 \times {}^2C_2}{2!} = 3$$

For the fourth case number of selections

$$= \frac{{}^4C_2 \times {}^2C_1 \times {}^1C_1}{2!} = 6$$

Hence, the total number of distributions

$= 1 + 4 + 3 + 6 = 14$

The correct option is (A)

141. Since no box is empty and all pots are identical, so the possible case is (1, 1, 2). But since all the coins are different, the 2 balls can be selected in ${}^4C_2$ ways and rest can be put in $\frac{{}^2C_1 \times {}^1C_1}{2!}$ ways.

Hence, the required number of distribution

$$= \frac{{}^4C_2 \times {}^2C_1 \times {}^1C_1}{2!} = 6$$

The correct option is (B)

142. Since no pot is empty and all coins are identical the possible case is (1, 1, 2). But since all three pots are different hence, a pot (which contains 2 coins together) can be selected in ${}^3C_1$ ways.

Hence, the required number of distribution

$= {}^3C_1 \times 1 = 3.$

The correct option is (B)

143. The possible arrangements are as follows:

(4, 0, 0) $\rightarrow$ Can be done in 2 ways, i.e., 4 balls can be put in either one of the two identical pots or can be put in different pot.

(3, 1, 0) $\rightarrow$ Can be done in 3 ways, either distinct pot can be filled up with 3 balls, or 1 ball or remained empty.

(2, 2, 0) $\rightarrow$ Can be done in 2 ways, either distinct pot can be filled by 2 balls or remained empty

(2, 1, 1) $\rightarrow$ Can be done in 2 ways either distinct pot can be filled up with 1 ball or 2 ball.

Hence, the required number of ways = 2 + 3 + 2 + 2 = 9

The correct option is (C)

144. The possible arrangements are as follows:

(4, 0, 0) → Can be done in 3 ways, let the 4 coins be *A*, *A*, *B* and *C* then all these 4 coins as a single packet of coins can be arranged in 3 different pots in 3 ways

(3, 1, 0) → Can be done in 18 ways, out of *A*, *B* and *C* we can select one coin in 3 ways i.e., either *A* or *B* or *C*.

Now, we can arrange this selected coin in any 3 pots in 3 ways and the remaining 3 coins as a single packet of coins can be arranged in remaining 2 pots in 2 ways.

Hence, the required number of ways = 3 × 3 × 2 = 18

(2, 2, 0) → Can be done in 12 ways

There are two possible cases:

(i) (*A*, *A*), (*B*, *C*) (ii) (*A*, *B*), (*A*, *C*)

In each of two cases we assume that there are two packets of coins which can be arranged in 3 × 2 = 6 ways.

Since there are two cases, hence the total number of required ways = 6 × 2 = 12

(2, 1, 1) → Can be done in 21 ways

There are 4 possible cases:

(i) (*A*, *A*), (*B*), (*C*) (ii) (*A*, *B*), (*A*), (*C*)

(iii) (*A*, *C*), (*A*), (*B*) (iv) (*B*, *C*), (*A*), (*A*)

For the first 3 cases in each case all the 3 packets of coins can be arranged in 3! ways.

Hence, the number of arrangements = 3 × 3! = 18

Now, in the fourth (iv) case two coins are identical so the third packet of coins can be arranged in 3 pots in 3 ways.

Hence, the total number of arrangements = 18 + 3 = 21

Thus, the required number of ways

= 3 + 18 + 12 + 21 = 54 ways

The correct option is (C)

145. In the first round, total number of matches

$$= 2 \times {}^{8}C_{2} = 2 \times \frac{8 \times 7}{1 \times 2} = 56$$

In the second round, total number of matches

= 2 (2) = 4 [In each group say I–III, II–IV]

Number of semi-finals = 2

Then, final = 1

Total number of matches = 56 + 4 + 2 + 1 = 63

The correct option is (C)

146. Each team will play 7 matches and so any team can win any number.

If matches between 0 to 7 i.e., (0, 1, 2, 3, 4, 5, 6, 7)

Four teams will be selected (7, 6, 5, 4)

Thus, team which wins only **3** matches will be out of the first round.

The correct option is (C)

147. Clearly, minimum number of matches that a team must win in order to qualify for second round is 4.

The correct option is (A)

148. In second round, it has to win one match.

Then, one in semi-final and one final.

∴ total number of matches = 3

The correct option is (D)

Match the Column Type

149. **(I)** The total number of numbers that can be formed with the digits 3, 4, 5, 6 taken all at a time

= *P* (4, 4) = 4! = 24.

Each of the digits 3, 4, 5, 6 occurs in 3!

= 3 × 2 = 6 times in unit's place.

∴ Sum of the digits in the unit's place of all the numbers

= (3 + 4 + 5 + 6) × 6 = 18 × 6 = 108.

The correct option is (B)

(II) There are two different ways of forming the committee

(i) oldest may be included

(ii) oldest may be excluded

(i) If oldest is included, then youngest has to be excluded and we are to select 4 candidates out of 8. This can be done in

$${}^{8}C_{4} = \frac{8!}{4!\,4!} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2} = 70 \text{ ways.}$$

(ii) If oldest is excluded, then we are to select 5 candidates from 9 which can be done in

$${}^{9}C_{5} = \frac{9!}{5!\,4!} = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126 \text{ ways}$$

Hence, the total number of ways in which committee can be formed

= 126 + 70 = 196.

The correct option is (C)

(III) ∵ $9600 = 2^7 \times 3 \times 5^2$

∴ Number of divisors = (7 + 1) × (1 + 1) × (2 + 1)

= 8 × 2 × 3 = 48.

The correct option is (D)

(IV) Required ways = $\left\{ \begin{matrix} \text{Coefficient of } x^{10} \\ \text{in } (1 + x + x^2 + \ldots)^4 \end{matrix} \right\}$

= Coefficient of x^{10} in $\left(\frac{1}{1-x}\right)^4$

$= \text{Coefficient of } x^{10} \text{ in } (1-x)^{-4}$

$= \text{Coefficient of } x^{10} \text{ in} \left(1 + 4x + \frac{5.4}{2!}x^2 + \frac{4.5.6}{3!}x^3 + \ldots\right)$

$= \text{Coefficient of } x^{10} \text{ in } (1 + {}^4C_1 x + {}^5C_2 x^2 + {}^6C_3 x^3$

$+ {}^7C_4 x^4 + {}^8C_5 x^5 + {}^9C_6 x^6 + \ldots {}^{13}C_{10} x^{10})$

$\therefore$ Required ways $= {}^{13}C_{10} = \frac{13.12.11}{3.2.1} = 286$

The correct option is (A)

150. **(I)** The number of diagonals + number of sides = number of selections of two vertices from n vertices

$\therefore$ the number of diagonals $= {}^nC_2 - n$

$= \frac{n(n-1)}{2} - n = \frac{n^2 - n - 2n}{2} = \frac{n(n-3)}{2}$

The correct option is (D)

(II) In general, we know that for the distribution equation

$x_1 + x_2 + x_3 + \ldots + x_r = n$

The number of ways in which n things can be distributed among r in such a way each can receive none, one or more or all of n items are ${}^{n+r-1}C_{r-1}$

$\therefore$ For the distribution equation

$x_1 + x_2 + x_3 + \ldots + x_n \le n$

let required ways $= W$

$\Rightarrow W = \left\{\begin{matrix}\text{Number of ways of}\\ \text{distributing}\\ \text{1 item}\end{matrix}\right\}$

$+ \left\{\begin{matrix}\text{Number of ways of}\\ \text{distributing}\\ \text{2 items}\end{matrix}\right\} + \ldots + \left\{\begin{matrix}\text{Number of ways of}\\ \text{distributing}\\ n \text{ items}\end{matrix}\right\}$

$= {}^{1+n-1}C_{n-1} + {}^{2+n-1}C_{n-1} + \ldots + {}^{n+n-1}C_{n-1}$

$= {}^nC_{n-1} + {}^{n+1}C_{n-1} + \ldots + 2n - 1C_{n-1}$

$= ({}^nC_{n-1} + {}^nC_n) + {}^{n+1}C_{n-1} + \ldots + {}^{2n-1}C_{n-1}$

$= \{({}^{n+1}C_n + {}^{n+1}C_{n-1}) + \ldots + {}^{2n-1}C_{n-1}\} - {}^nC_n$

- - - -

Assertion-Reason Type

152. The candidate is unsucessful if he fails in 9 or 8 or 7 or 6 or 5 papers.

$\therefore$ The number of ways to be unsucessful

$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$

$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$

$= \frac{1}{2}({}^9C_0 + {}^9C_1 + \ldots + {}^9C_9)$

$= \frac{1}{2} \times 2^9 = 2^8 = 256.$

Previous Year's Questions

153. Number of choices is equal to ${}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$

$= 140 + 56.$

The correct option is (B)

154. ${}^nC_{r+1} + {}^nC_{r-1} + {}^nC_r + {}^nC_r$

$= {}^{n+1}C_{r+1} + {}^{n+1}C_r$

$= {}^{n+2}C_{r+1}.$

The correct option is (B)

155. Since in half the arrangement A will be before E and other half E will be before A.

Therefore total number of ways $= \frac{6!}{2} = 360.$

The correct option is (C)

156. Number of balls = 8

And, number of boxes = 3

Hence number of ways $= {}^7C_2 = 21.$

The correct option is (D)

157. Alphabetical order for the name SACHIN is

A, C, H, I, N, S

No. of words starting with A = 5!

No. of words starting with C = 5!

No. of words starting with H = 5!

No. of words starting with I =5!

No. of words starting with N =5!

SACHIN – 1

So the sum is 601.

The correct option is (A)

158. Total number of ways $= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210$

$= 385$

The correct option is (C)

159. Other than S, seven letters M, I, I, I, P, P, I can be arranged in $\frac{7!}{2!4!} = 7 \cdot 5 \cdot 3.$

Now four S can be placed in 8 spaces in ${}^8C_4$ ways.

Hence, desired number of ways $= 7 \cdot 5 \cdot 3 \cdot {}^8C_4 = 7 \cdot {}^6C_4 \cdot {}^8C_4.$

The correct option is (D)

160. Since, 4 novels can be selected from 6 novels in ${}^6C_4$ ways and 1 dictionary can be selected from 3 dictionaries in ${}^3C_1$ ways. As the dictionary selected is fixed in the middle, the remaining 4 novels can be arranged in 4!ways.

$\therefore$ The required number of ways of arrangement $= {}^6C_4 \times {}^3C_1 \times 4! = 1080$

The correct option is (D)

161. Total number of ways $= {}^3C_2 \times {}^9C_2$

$= 3 \times \frac{9 \times 8}{2} = 3 \times 36 = 108$

The correct option is (C)

162. ${}^{(n-1)}C_{(r-1)} = {}^{(10-1)}C_{(4-1)} = {}^9C_3$

Statement 1 is correct

Statement 2 is also correct

From 9 we can select 3 in ${}^9C_3$ways. It is correct explanation.

The correct option is (D)

163. Number of ways of selecting one or more balls from 10 white, 9 green, and 7 black balls

$= (10 + 1)(9 + 1)(7 + 1) - 1 = 11*10*8 - 1 = 879.$

The correct option is (D)

164. Since ${}^{n+1}C_3 - {}^nC_3 = 10 \Rightarrow {}^nC_2 = 10 \Rightarrow n = 5.$

The correct option is (A)

165. Four digit numbers, which start from 6, 7, $8 = 3 \times 4 \times 3 \times 2 = 72$

Five digit numbers = 5! = 120

$\therefore$ Total number of integers = 192.

The correct option is (A)

166. Total number of words which can be formed using all the letters of the word 'SMALL'

$= \frac{5!}{2!} = 60$

Now, 60^{th} word is $\rightarrow$ SMLLA

59^{th} word is $\rightarrow$ SMLAL

58^{th} word is $\rightarrow$ SMALL

The correct option is (A)