CHAPTER 17

Differential Equations

Chapter Highlights

Differential equation, Linear and non-linear differential equations, Initial value problems, Homogeneous differential equations, Solution by inspection.

DIFFERENTIAL EQUATION

An equation involving an independent variable, a dependent variable and the derivatives of the dependent variable, is called a *differential equation.*

Illustrations

1. $x\dfrac{dy}{dx} + y = x^3$
2. $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 7y = 0$
3. $\left(\dfrac{d^2y}{dx^2}\right) + 4\left(\dfrac{dy}{dx}\right)^3 + 3y = 0$
4. $\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2} = \dfrac{d^3y}{dx^3}$

are some examples of differential equations.

Order of a Differential Equation

The order of highest derivative appearing in a differential equation is called the *order* of the differential equation.

SOLVED EXAMPLES

1. The order of the differential equation whose general solution is given by

$y = c_1 \cos (2x + c_2) - (c_3 + c_4)\ a^{x+c_5} + c_6 \sin (x - c_7)$ is

(A) 3 (B) 4 (C) 5 (D) 2

Solution: (C)

We have,

$$y = c_1 \cos (2x + c_2) - (c_3 + c_4)\ a^{x+c_5} + c_6 \sin (x - c_7)$$

$$= c_1 \cos (2x + c_2) - c_8.\ a^{c_5}.\ a^x + c_6 \sin (x - c_7)$$

where $c_3 + c_4 = c_8$

$\Rightarrow \quad y = c_1 \cos (2x + c_2) - c_9\, a^x + c_6 \sin (x - c_7),$

where $c_9 = c_8.\, a^{c_5}$

Since the above relation contains five arbitrary constants, so the order of the differential equation satisfying it is 5.

2. The order of the differential equation whose general solution is given by $y = (c_1 + c_2) \cos (x + c_3) - c_4\ e^{x+c_5}$ where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

(A) 5 (B) 4 (C) 3 (D) 2

Solution: (C)

We can write $y = A \cos (x + B) - ce^x$

where $\quad A = c_1 + c_2,\ B = c_3$ and $C = c_4 e^{c_5}$.

$$\frac{dy}{dx} = -A \sin (x + B) - Ce^x$$

$$\frac{d^2y}{dx^2} = -A \cos (x + B) - Ce^x$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = -2ce^x$$

$$\Rightarrow \qquad \frac{d^3y}{dx^3} + \frac{dy}{dx} = -2ce^x = \frac{d^2y}{dx^2} + y$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

which is a differential equation of order 3.

Degree of a Differential Equation

The power of the highest order derivative appearing in a differential equation, after it is made free from radicals and fraction, is called the *degree* of the differential equation.

In the above illustrations, differential equation (1) is of first order and first degree, differential equation (2) is of order 3 and degree 1, differential equation (3) is of order 2 and degree 1 and differential equation (4) is of order 3 and degree 2, as after making it free from fractional exponent by squaring, it can be re-written as $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^3y}{dx^3}\right)^2$

SOLVED EXAMPLES

3. The degree of differential equation

$$x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots\ldots\ldots\ldots \text{ is}$$

(A) Three (B) One
(C) Not defined (D) None of these

Solution: (B)

$$x = e^{\left(\frac{dy}{dx}\right)} \Rightarrow \frac{dy}{dx} = \ln x$$

$\therefore$ order = 1 and degree = 1

4. The degree of the differential equation of all tangent lines to the parabola $y^2 = 4ax$ is

(A) 1 (B) 2
(C) 3 (D) None of these

Solution: (B)

The equation of any tangent to the parabola

$$y^2 = 4ax \text{ is } y = mx + \frac{a}{m}, \qquad (1)$$

where m is any arbitrary constant.

Differentiating with respect to x, we get $\frac{dy}{dx} = m$

Substituting the value of m in (1),

we get $\qquad y = x\frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$

$$\Rightarrow \qquad x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + a = 0,$$

which is a differential equation of degree 2.

5. The degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^{3/5} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 5 = 0 \text{ is}$$

(A) 2 (B) 3 (C) 4 (D) 5

Solution: (B)

We have,

$$\left(\frac{d^4y}{dx^4}\right)^{3/5} = 5\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 5$$

$$\Rightarrow \quad \left(\frac{d^4y}{dx^4}\right)^3 = \left(5\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 5\right)^5$$

which is a differential equation of order 4 and degree 3.

6. The order of the differential equation satisfying

$$\sqrt{1-x^4} + \sqrt{1-y^4} = a(x^2 - y^2). \text{ is}$$

(A) 1 (B) 2
(C) 3 (D) None of these

Solution: (A)

Put $x^2 = \sin\alpha$, $y^2 = \sin\beta$.

$\therefore$ Given equation reduces to

$$\cos\alpha + \cos\beta = a(\sin\alpha - \sin\beta)$$

$$\Rightarrow 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = 2a\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\Rightarrow \quad \cot\left(\frac{\alpha-\beta}{2}\right) = a \Rightarrow \alpha - \beta = 2\cot^{-1}a$$

$$\Rightarrow \sin^{-1}x^2 - \sin^{-1}y^2 = 2\cot^{-1}a$$

Differentiating with respect to x, we get

$$\frac{1}{\sqrt{1-x^4}}\cdot 2x - \frac{1}{\sqrt{1-y^4}}\cdot 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{x}{y}\sqrt{\frac{1-y^4}{1-x^4}},$$

which is a differential equation of first order and first degree.

7. The degree of the differential equation

$$\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^4 = 4\log\left(\frac{d^4y}{dx^4}\right) \text{ is}$$

(A) 1 (B) 3
(C) 4 (D) None of these

Solution: (D)

Since, the given differential equation is not a polynomial in differential coefficients, so its degree is not defined.

LINEAR AND NON-LINEAR DIFFERENTIAL EQUATIONS

A differential equation in which the dependent variable and its differential coefficients occur only in the first degree and are not multiplied together is called a linear differential equation.

Illustration 1

$$\frac{dy}{dx} + 2\frac{y}{x} = 3x^2 \text{ and } 3x\log x\frac{dy}{dx} + 4y = 2\log x$$

are linear differential equations of the first order.

Illustration 2

$3\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 6y = e^x$ is a linear differential equation of the second order.

Illustration 3

$$2y\frac{dy}{dx} + 2 = 5x$$

is a non-linear differential equation because y and dy/dx are multiplied together.

Illustration 4

$$\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 5y = 0$$

is a non-linear differential equation because $\frac{dy}{dx}$ occurs in the second degree.

CAUTION

A linear differential equation is always of the first degree but every differential equation of the first degree need not be linear.

Illustration 5

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y^2 = 0$$

is a differential equation of the first degree but not linear.

Solution of a Differential Equation

Any relation between the dependent and independent variables (not involving the derivatives) which, when substituted in the differential equation, reduces it to an identity is called a *solution* of the differential equation.

Consider the differential equation:

$$\frac{dy}{dx} = \cos x \qquad (1)$$

Then, $y = \sin x$ is a solution of (1). We notice that

$$y = \sin x + 1, \; y = \sin x - 3$$

are also solutions of (1).

The solution $y = \sin x + c$, where c is an arbitrary constant, is called the general solution of (1), since every solution of (1) can be obtained from $y = \sin x + c$, for a suitable choice of c.

General Solution

The solution of a differential equation which contains a number of arbitrary constants equal to the order of the differential equation is called the *general solution*. Thus, the general solution of a differential equation of the nth order has n arbitrary constants.

Particular Solution

A solution obtained by giving particular values to arbitrary constants in the general solution is called a *particular solution*.

SOLVED EXAMPLE

8. A solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0 \text{ is}$$

(A) $y = 2$ (B) $y = 2x$
(C) $y = 2x - 4$ (D) $y = 2x^2 - 4$

Solution: (C)

Direct substitution of $\boldsymbol{y = 2x - 4}$ in the equation shows that it is a solution of the given differential equation.

FORMATION OF A DIFFERENTIAL EQUATION

Let $f(x, y, c_1, c_2, \ldots c_n) = 0$ be the solution of a differential equation, where $c_1, c_2, \ldots, c_n$ are n arbitrary constants. If we eliminate these n constants, we obtain the differential equation of the nth order satisfied by the given solution values. Any equation taken together with n relations obtained by differentiating it n times helps us to eliminate the n constants.

Working Rule for Formation of Differential Equations

- Write the given equation.
- Differentiate the given equation with respect to independent variable x as many times as the number of arbitrary constants.
- Eliminate the arbitrary constants with the help of the given equation and the equations obtained by differentiation to get the required differential equation.

SOLVED EXAMPLES

9. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(A) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$ (B) $y^2 = x^2 - 2xy\dfrac{dy}{dx}$

(C) $x^2 = y^2 + xy\dfrac{dy}{dx}$ (D) None of these

Solution: (A)

The equation of circles passing through the origin and having their centres on the x-axis is

$$x^2 + y^2 + 2gx = 0, \quad (1)$$

where g is an arbitrary constant.
Differentiating with respect to x, we get

$$x + y\frac{dy}{dx} + g = 0. \text{ i.e., } g = -\left(x + y\frac{dy}{dx}\right)$$

Putting this value of g in (1), we get

$$x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) = 0,$$

i.e., $$y^2 = x^2 + 2xy\frac{dy}{dx}$$

10. The differential equation of family of parabolas with foci at the origin and axis along the x-axis is

(A) $y\left(\dfrac{dy}{dx}\right)^2 + 2x\dfrac{dy}{dx} - y = 0$

(B) $x\left(\dfrac{dy}{dx}\right)^2 + 2y\dfrac{dy}{dx} - y = 0$

(C) $y\left(\dfrac{dy}{dx}\right)^2 + 2x\dfrac{dy}{dx} + y = 0$

(D) None of these

Solution: (A)

Let the directrix be $x = -2a$ and latus rectum be $4a$. Then, the equation of the parabola is
(distance from focus = distance from directrix),

$$x^2 + y^2 = (2a + x)^2 \text{ or } y^2 = 4a(a + x) \quad (1)$$

Differentiating with respect to x, we get

$$y\frac{dy}{dx} = 2a \text{ or } a = \frac{1}{2}y\frac{dy}{dx}.$$

Putting this value of a in (1), the differential equation is

$$y^2 = 2y\frac{dy}{dx}\left(\frac{y}{2}\frac{dy}{dx} + x\right)$$

or $$y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

11. The differential equation that represents all parabolas each of which has a latus rectum $4a$ and whose axes are parallel to x-axis, is

(A) $a\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 = 0$

(B) $2a\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 = 0$

(C) $2a\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^3 = 0$

(D) None of these

Solution: (B)

Equation of the family of such parabolas is

$$(y - k)^2 = 4a(x - h) \quad (1)$$

where h and k are arbitrary constants.
Differentiating with respect to x, we get

$$(y-k)\frac{dy}{dx}=2a \quad (2)$$

Differentiating again,

$$(y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0 \quad (3)$$

Putting value of $y-k$ from (2) in (3), we get

$$2a\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0,$$

which is the required differential equation.

12. The differential equation satisfied by $ax^2+by^2=1$ is

(A) $xy\,y''+xy'^2+yy'=0$
(B) $xy\,y''+xy'^2-yy'=0$
(C) $xy\,y''-xy'^2+yy'=0$
(D) None of these

Solution: (B)

The given solution is

$$ax^2+by^2=1 \quad (1)$$

Differentiating with respect to x, we get

$$2ax+2byy'=0 \quad (2)$$

Differentiating again, we get

$$2a+2b(y'^2+yy'')=0 \quad (3)$$

From (2), we have

$$\frac{a}{b}=\frac{-yy'}{x}.$$

Also from (3), we have

$$\frac{a}{b}=-(y'^2+yy'')$$

$$\Rightarrow \quad \frac{-yy'}{x}=-(y'^2+yy'')$$

$$\Rightarrow \quad yy'=xy'^2+xyy''$$

$$\Rightarrow \quad x\,yy''+xy'^2-yy'=0$$

which is the required differential equation.

13. The differential equation of the family of general circles is

(A) $y'''(1+y'^2)-3y'y''^2=0$
(B) $y'''(1+y'^2)+3y'y''^2=0$
(C) $y'''(1+y'^2)-3y''y'^2=0$
(D) None of these

Solution: (A)

The equation of the general circle is given by

$$x^2+y^2+2gx+2fy+c=0 \quad (1)$$

Differentiating with respect to x, we get

$$2x+2yy'+2g+2fy'=0 \quad (2)$$

Differentiating again, we get

$$1+y'^2+yy''+fy''=0 \quad (3)$$

Differentiating again, we have

$$2y'y''+yy'''+y'y''+fy'''=0 \quad (4)$$

Eliminating f from (3) and (4), we get

$$y'''(1+yy''+y'^2)-y''(yy'''+3y'y'')=0$$

$$\Rightarrow \quad y'''(1+y'^2)-3y'y''^2=0,$$

which is the required differential equation.

14. The equation of the family of curves which intersect the hyperbola $xy=2$ orthogonally is

(A) $y=\frac{x^3}{6}+c$ (B) $y=\frac{x^2}{4}+c$

(C) $y=-\frac{x^3}{6}+c$ (D) $y=-\frac{x^2}{4}+c$

Solution: (A)

Let $m_1=\frac{dy}{dx}$ for required family of curves at (x, y).

Let $m_2=\frac{dy}{dx}$ for the hyperbola $xy=2$.

Then $\quad m_2=\frac{dy}{dx}=\frac{-2}{x^2}.$

Since the required family of curves is orthogonal to the hyperbola,

$$\therefore \quad m_1\times m_2=-1$$

$$\Rightarrow \frac{dy}{dx}\times\left(\frac{-2}{x^2}\right)=-1 \Rightarrow \frac{dy}{dx}=\frac{x^2}{2}. \Rightarrow dy=\frac{x^2}{2}dx.$$

Integrating, we get $y=\frac{x^3}{6}+c$, which is the required family.

15. The order of the differential equation, of which $xy=ce^x+be^{-x}+x^2$ is a solution, is

(A) 1 (B) 2
(C) 3 (D) None of these

Solution: (B)

We have,

$$xy = ce^{x} + be^{-x} + x^{2} \qquad (1)$$

Differentiating with respect to x, we get

$$x\frac{dy}{dx} + y = ce^{x} - be^{-x} + 2x \qquad (2)$$

Differentiating again,

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = ce^{x} + be^{-x} + 2 = xy - x^2 + 2$$

[Using (1)]

Hence, the required differential equation is

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0$$

The order of this differential equation is 2.

16. The differential equation of all parabolas whose axes are parallel to the axis of y, is

(A) $\frac{d^3y}{dx^3} = 1$ (B) $\frac{d^3y}{dx^3} = -1$

(C) $\frac{d^3y}{dx^3} = 0$ (D) None of these

Solution: (C)

Such parabolas are given by

$$(x - h)^2 = 4a\,(y - k),$$

where h, k a are three arbitrary constants.

Differentiating with respect to x, $(x - h) = 2a\frac{dy}{dx}$.

Differentiating again, $1 = 2a\frac{d^2y}{dx^2}$ i.e., $\frac{d^2y}{dx^2} = \frac{1}{2a}$.

Differentiating once again, $\frac{d^3y}{dx^3} = 0$.

This is the required differential equation.

17. The differential equation of all ellipses centred at the origin is

(A) $xyy_2 - xy_1^2 + yy_1 = 0$

(B) $xyy_2 + xy_1^2 - yy_1 = 0$

(C) $xyy_2 + xy_1^2 + yy_1 = 0$

(D) None of these

Solution: (B)

The general equation of all ellipses centred at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Differentiating with respect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\cdot y_1 = 0 \text{ i.e., } \frac{x}{a^2} + \frac{y}{b^2}y_1 = 0 \qquad (1)$$

Differentiating again, we get

$$\frac{1}{a^2} + \frac{1}{b^2}(y_1^2 + yy_2) = 0 \qquad (2)$$

From (1) and (2), we have

$$\frac{1}{b^2}\left(yy_1 - xy_1^2 - xyy_2\right) = 0$$

$$\Rightarrow \quad xyy_2 + xy_1^2 - yy_1 = 0,$$

which is the required differential equation.

Solution of First Order and First Degree Differential Equations

The following methods may be used to solve first order and first degree differential equations.

Variable Separable Differential Equations

A differential equation of the form

$$f(x) + g(y)\frac{dy}{dx} = 0 \qquad (1)$$

or

$$f(x)\,dx + g(y)\,dy = 0$$

is said to have separated variables.

Integrating Equation (1), we obtain

$$\int f(x)\,dx + \int g(y)\frac{dy}{dx}dx = c,$$

where c is an arbitrary constant.

Hence $\int f(x)\,dx + \int g(y)\,dy = c$ is the solution of Equation (1).

TRICK(S) FOR PROBLEM SOLVING

- There is no need to add arbitrary constants of integration on both sides, since they can be combined to give just one arbitrary constant. Moreover, the general solution of a differential equation of order one must have only one arbitrary constant.
- The constant of integration can be taken as c or log c or $\tan^{-1}c$ and so on depending on the nature of the problem.

INITIAL VALUE PROBLEMS

In many cases instead of finding the general solution, we find a particular solution satisfying a given initial condition, say, the condition that at some point x_0, the solution $y(x)$ has the value y_0. This is expressed as $y(x_0) = y_0$.

A first-order differential equation together with an initial condition is called an *initial value problem.*

SOLVED EXAMPLE

18. Find the particular solution of $\sin\left(\frac{dy}{dx}\right) = a$, given that when $x = 0$, $y = 1$.

Solution:

We have, $\sin\left(\frac{dy}{dx}\right) = a$ or $\frac{dy}{dx} = \sin^{-1}a$

or $dy = \sin^{-1}a\, dx$

Integrating both sides, we get

$$\int dy = \int \sin^{-1} a\, dx$$

$$\Rightarrow \quad y = \sin^{-1} a \int dx$$

$$= \sin^{-1}a \cdot x + c$$

$$\Rightarrow \quad y = x\sin^{-1}a + c$$

When $x = 0$, $y = 1$ $\therefore$ $1 = 0 + c$ or $c = 1$

$\therefore$ The required particular solution is

$$y = x\sin^{-1}a + 1 \text{ or } \frac{y-1}{x} = \sin^{-1}a$$

or $$\sin\left(\frac{y-1}{x}\right) = a$$

Equations Reducible to Variable Separable Form

Sometimes in a given differential equation, the variables are not separable. But, some suitable substitution reduces it to a form in which the variables are separable. For example, the differential equations of the type $\frac{dy}{dx} = f(ax + by + c)$ can be reduced to variable separable form by substituting $ax + by + c = t$. The reduced variable separable form is:

$$\frac{dt}{b\,f(t) + a} = dx$$

Integrating both sides to obtain the solution of this differential equation.

SOLVED EXAMPLES

19. Solve: $(x - y)^2 \frac{dy}{dx} = 1$.

Solution:

We have,

$$(x-y)^2\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{(x-y)^2} \qquad (1)$$

Put $$x - y = z \Rightarrow 1 - \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 - \frac{dz}{dx} \qquad (2)$$

From (1) and (2), we get

$$1 - \frac{dz}{dx} = \frac{1}{(x-y)^2}$$

$$\Rightarrow \quad 1 - \frac{dz}{dx} = \frac{1}{z^2}$$

$$\Rightarrow \quad \frac{dz}{dx} = 1 - \frac{1}{z^2}$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{z^2-1}{z^2}$$

$$\Rightarrow \quad \frac{z^2}{z^2-1}dz = dx$$

Integrating both sides, we get

$$\int \frac{z^2}{z^2-1}dz = \int dx$$

$$\Rightarrow \quad \int\left(1 + \frac{1}{z^2-1}\right)dz = x + c$$

$$\Rightarrow \quad z + \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| = x + c$$

$$\Rightarrow \quad (x-y) + \frac{1}{2}\log\left|\frac{x-y-1}{x-y+1}\right| = x + c.$$

20. The general solution of the differential equation

$\frac{dy}{dx} + y\,g'(x) = g(x).\,g'(x)$,

where $g(x)$ is a given function of x, is

(A) $g(x) + \log[1 + y + g(x)] = c$
(B) $g(x) + \log[1 + y - g(x)] = c$
(C) $g(x) - \log[1 + y - g(x)] = c$
(D) None of these

Solution: (B)

We have,

$$\frac{dy}{dx} = (g(x) - y) \cdot g'(x)$$

Put $\quad g(x) - y = V$

$\Rightarrow \quad g'(x) - \dfrac{dy}{dx} = \dfrac{dV}{dx}$

Hence $\quad g'(x) - \dfrac{dV}{dx} = V \cdot g'(x)$

$\Rightarrow \quad \dfrac{dV}{dx} = (1 - V)\, g'(x)$

$\Rightarrow \quad \dfrac{dV}{1 - V} = g'(x)\, dx$

$\Rightarrow \quad \displaystyle\int \frac{dV}{1 - V} = \int g'(x)\, dx$

$\Rightarrow \quad -\log(1 - V) = g(x) - c$

$\Rightarrow \quad g(x) + \log(1 - V) = c$

$\therefore \quad g(x) + \log[1 + y - g(x)] = c$

21. The solution of the equation $\log \dfrac{dy}{dx} = 9x - 6y + 6$, given that $y = 1$ when $x = 0$, is

(A) $3e^{6y} = 2e^{9x-6} + 6e^6$
(B) $3e^{6y} = 2e^{9x+6} - 6e^6$
(C) $3e^{6y} = 2e^{9x+6} + e^6$
(D) None of these

Solution: (C)

We have,

$$\log \frac{dy}{dx} = 9x - 6y + 6$$

$\Rightarrow \quad \dfrac{dy}{dx} = e^{9x-6y+6} = e^{9x+6} \cdot e^{-6y}$

$\Rightarrow \quad e^{6y}\, dy = e^{9x+6}\, dx$

Integrating, we get $\dfrac{e^{6y}}{6} = \dfrac{e^{9x+6}}{9} + c$

Putting $x = 0$, $y = 1$, we get

$$\frac{e^6}{6} = \frac{e^6}{9} + c \text{ i.e. } c = \frac{e^6}{18}$$

$\therefore$ The solution is $\dfrac{e^{6y}}{6} = \dfrac{e^{9x+6}}{9} + \dfrac{e^6}{18}$

$\Rightarrow \quad 3e^{6y} = 2e^{9x+6} + e^6$

22. The particular solution of $\cos y\, dx + (1 + 2e^{-x}) \sin y\, dy = 0$, when $x = 0$, $y = \dfrac{\pi}{4}$ is

(A) $e^x - 2 = 3\sqrt{2} \cos y$
(B) $e^x + 2 = \sqrt{2} \cos y$
(C) $e^x + 2 = 3\sqrt{2} \cos y$
(D) None of these

Solution: (C)

We have,

$$\cos y\, dx + (1 + 2e^{-x}) \sin y\, dy = 0$$

$\Rightarrow \quad \dfrac{dx}{1 + 2e^{-x}} + \dfrac{\sin y}{\cos y} dy = 0$

$\Rightarrow \quad \displaystyle\int \frac{e^x}{e^x + 2} dx - \int \frac{-\sin y}{\cos y} dy = \log c$

$\Rightarrow \quad \log(e^x + 2) - \log \cos y = \log c$

$\Rightarrow \quad \log\left(\dfrac{e^x + 2}{\cos y}\right) = \log c$

$\Rightarrow \quad e^x + 2 = c \cos y,$

where $\quad 1 + 2 = c \cdot \dfrac{1}{\sqrt{2}}$ i.e. $c = 3\sqrt{2}$

$\therefore$ The solution is $e^x + 2 = 3\sqrt{2} \cos y$

23. The equation of the curve which passes through the point $(2a, a)$ and for which the sum of the cartesian sub tangent and the abscissa is equal to the constant a, is

(A) $y(x - a) = a^2$ (B) $y(x + a) = a^2$
(C) $x(y - a) = a^2$ (D) $x(y + a) = a^2$

Solution: (A)

We have,

Cartesian subtangent + abscissa = constant

$\Rightarrow \quad \dfrac{y}{dy/dx} + x = a$

$\Rightarrow \quad y\dfrac{dx}{dy} + x = a$

$\Rightarrow \quad \dfrac{dy}{y} = \dfrac{dx}{a - x}$

Integrating, we get

$$\log y + \log(x - a) = \log c$$

$\therefore \quad y(x-a) = c$

As the curve passes through the point $(2a, a)$, we have $c = a^2$.

Hence, the required curve is $y(x-a) = a^2$.

24. Solution of the differential equation $x\,dy - y\,dx = 0$ represents

(A) parabola whose vertex is at origin
(B) circle whose centre is at origin
(C) a rectangular hyperbola
(D) straight line passing through origin

Solution: (D)

We have,

$$x\,dy - y\,dx = 0 \Rightarrow \frac{dy}{y} - \frac{dx}{x} = 0$$

Integrating, we get $\log y - \log x = \log c$

$$\Rightarrow \quad \log\frac{y}{x} = \log c \Rightarrow \frac{y}{x} = c \Rightarrow y = cx$$

which is a straight line through the origin.

25. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

(A) Circles (B) Straight lines
(C) Ellipse (D) Parabola

Solution: (D)

We have,

$$2x\frac{dy}{dx} = y + 3 \Rightarrow \frac{2}{y+3}dy = \frac{dx}{x}.$$

Integrating, we get

$$2\log(y+3) = \log x + \log c = \log cx$$

$$\Rightarrow \quad \log(y+3)^2 = \log cx$$

$$\Rightarrow \quad (y+3)^2 = cx$$

which is a parabola.

26. The equation of the curve satisfying the differential equation $y_2(x^2+1) = 4xy_1$, passing through the point $(0, -4)$ and having slope of tangent at $x = 0$ as 4 is

(A) $y = 4\left(\frac{x^5}{5} + x + \frac{2x^3}{3} - 1\right)$

(B) $y = 4\left(\frac{x^5}{5} - x + \frac{2x^3}{3} - 1\right)$

(C) $y = 4\left(\frac{x^5}{5} + x + \frac{2x^3}{3} + 1\right)$

(D) None of these

Solution: (A)

We have,

$$\frac{y_2}{y_1} = 2.\frac{2x}{x^2+1}.$$

Integrating with respect to. x, we get

$$\log y_1 = 2\log(x^2+1) + \log c \Rightarrow y_1 = c(x^2+1)^2$$

Since $\quad y_1(0) = 4, \therefore c = 4$

$\therefore \quad y_1 = 4(x^2+1)^2 \Rightarrow dy = 4(x^4 + 1 + 2x^2)\,dx$

Integrating again, we get $y = 4\left(\frac{x^5}{5} + \frac{2x^2}{3} + x\right) + k.$

Putting $x = 0$, $y = -4$, we get $k = -4$.

$\therefore$ The required equation of curve is

$$y = 4\left(\frac{x^5}{5} + \frac{2x^3}{3} + x - 1\right)$$

27. If $2f(x) = f'(x)$ and $f(0) = 3$, then $f(2)$ equals

(A) $4e^3$ (B) $3e^4$ (C) $2e^3$ (D) $3e^2$

Solution: (B)

We have,

$$2f(x) = f'(x) \Rightarrow \frac{f'(x)}{f(x)} = 2.$$

Integrating, we get

$$\log f(x) = 2x + c_1.$$

$$\Rightarrow \quad f(x) = e^{2x+c_1} = e^{c_1}\cdot e^{2x} = ce^{2x},$$

where $\quad c = e^{c_1}$

Putting $x = 0$, $f(0) = 3$, we get $c = 3$.

$\therefore \quad f(x) = 3e^{2x} \Rightarrow f(2) = 3e^4$

28. The solution of $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represents a circle when

(A) $a = b$ (B) $a = -b$
(C) $a = -2b$ (D) $a = 2b$

Solution: (B)

We have,

$$\frac{dy}{dx} = \frac{ax+g}{by+f} \Rightarrow (by+f)\,dy = (ax+g)\,dx$$

Integrating, we get

$$\frac{by^2}{2} + fy = \frac{ax^2}{2} + gx + c'$$

$$\Rightarrow \quad ax^2 - by^2 + 2gx - 2fy + c = 0.$$

This represents a circle if $a = -b$.

HOMOGENEOUS DIFFERENTIAL EQUATIONS

Homogeneous Function

A function $f(x, y)$ in x and y is called *a homogeneous function* of degree n, if the degree of each term is n.

Illustration 1

$f(x, y) = x^2 + y^2 + xy$ is a homogeneous function of degree 2, since each term is of degree 2.

Illustration 2

$g(x, y) = x^3 + 4x^2y + 7xy^2$ is a homogeneous function of degree 3, since each term is of degree 3.

Illustration 3

$h(x, y) = x^2 + 4x^2y + 7xy^2$ is not a homogeneous function, since first term is of degree 2, while each one of second and third terms is of degree 3.

Homogeneous Differential Equations

A differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$, where $f(x, y)$ as well as $g(x, y)$ is a homogeneous function of same degree in x and y is called a *homogenous differential equation.*

REMEMBER

A function of $\frac{y}{x}$ is always a homogenous function.

Illustration 4

$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$ is a homogenous differential equation, as $(x^2 + y^2)$ as well as xy is a homogenous function of degree 2.

Illustration 5

$\frac{dy}{dx} = x\sin\left(\frac{y}{x}\right)$ is a homogeneous differential equation, as $x\sin\left(\frac{y}{x}\right)$ being a function of $\frac{y}{x}$, is a homogenous function.

Working Rule for Solving a Homogeneous Differential Equation

Let $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ be the given homogeneous differential equation.

- Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$.
- Putting these values in the given equation, we get

$$v + x\frac{dv}{dx} = F(v) \Rightarrow \frac{dv}{[F(v) - v]} = \frac{dx}{x}$$

$$\Rightarrow \int \frac{dv}{F(v) - v} = \log|x| + c.$$

- Replace v by $\frac{y}{x}$ to obtain the required solution.

Equations Reducible to the Homogeneous Form

Type I: Consider a differential equation of the form:

$$\frac{dy}{dx} = \frac{ax + by + c}{Ax + By + C}, \text{ where } \frac{a}{A} \neq \frac{b}{B} \qquad (1)$$

This is clearly non-homogeneous. In order to make it homogeneous, we proceed as follows:

We substitute $x = X + h$ and $y = Y + k$ in Equation (1), where h, k are constants to be determined suitably.

We have $\frac{dx}{dX} = 1$ and $\frac{dy}{dY} = 1$, so that

$$\frac{dy}{dx} = \frac{dy}{dY} \cdot \frac{dY}{dX} \cdot \frac{dX}{dx} = \frac{dY}{dX}$$

Now Equation (1) becomes

$$\frac{dY}{dX} = \frac{aX + bY + (ah + bk + c)}{AX + BY + (Ah + Bk + C)} \qquad (2)$$

Choose h and k so that

$$ah + bk + c = 0,$$

$$Ah + Bk + C = 0.$$

These equations give

$$h = \frac{bC - Bc}{aB - Ab}, k = \frac{Ac - aC}{aB - Ab} \qquad (3)$$

Now Equation (2) becomes

$$\frac{dY}{dX} = \frac{aX + bY}{AX + BY},$$

which being a homogeneous equation can be solved by means of the substitution $Y = vX$.

Type II: Consider a differential equation of the form

$$\frac{dy}{dx} = \frac{ax + by + c}{Ax + By + C},$$

where $$\frac{a}{A} = \frac{b}{B} = k \text{ (say)}$$

Since $aB - Ab = 0$, the above method fails in view of Equation (3).

We have $$\frac{dy}{dx} = \frac{k(Ax + By) + c}{Ax + By + C} \quad (4)$$

Substitute $Ax + By = z$ so that

$$A + B\frac{dy}{dx} = \frac{dz}{dx}$$

Now Equation (4) becomes

$$\frac{dz}{dx} = B.\frac{kz + c}{z + C} + A,$$

which is an equation with variables separable.

SOLVED EXAMPLE

29. Solution of the equation $x\,dy = \left(y + x\frac{f(y/x)}{f'(y/x)}\right)dx$ is

(A) $f\left(\frac{x}{y}\right) = cy$ (B) $f\left(\frac{y}{x}\right) = cx$

(C) $f\left(\frac{y}{x}\right) = cxy$ (D) None of these

Solution: (B)

We have,

$$x\,dy = \left(y + \frac{xf(y/x)}{f'(y/x)}\right)dx$$

$\Rightarrow$ $\frac{dy}{dx} = \frac{y}{x} + \frac{f(y/x)}{f'(y/x)}$ which is homogeneous.

Put $y = Vx$ so that $\frac{dy}{dx} = V + x\frac{dV}{dx}$,

we obtain

$$V + x\frac{dV}{dx} = V + \frac{f(V)}{f'(V)} \Rightarrow \frac{f(V)}{f'(V)}\,dV = \frac{dx}{x}$$

Integrating, we get

$$\log f(V) = \log x + \log c$$

$$\Rightarrow \log f(V) = \log cx \Rightarrow f\left(\frac{y}{x}\right) = cx.$$

Linear Differential Equations

A differential equation of the form

$$\frac{dy}{dx} + Py = Q, \quad (1)$$

where P and Q are functions of x (or constants), is called a *linear differential equation of the first order*.

Working Rule For Solving $\frac{dy}{dx} + Py = Q$

- Find integrating factor (I.F.) = $e^{\int P dx}$.
- The solution of the differential equation is

  y (I.F.) = $\int Q\,(I.F)\,dx + c$,

  where c is constant of integration.

TRICK(S) FOR PROBLEM SOLVING

- Sometimes a first order differential equation which is not expressible as Equation (1) becomes a linear equation of the form

  $\frac{dx}{dy} + Ry = S$,

  where R and S are functions of y alone (or constants). The integrating factor in this case is given by,

  I.F. = $e^{\int R\,dy}$

  The solution of this equation is

  $x \cdot$ (I.F.) = $\int [S \times (\text{I.F.})]\,dy + C$,

  where C is the constant of integration.
- The fact $e^{\log t} = t$ will be frequently used in the solution of linear equations.

Equations Reducible to the Linear Form

Consider a differential equation of the form:

$$\frac{dy}{dx} + Py = Qy^n, \quad (1)$$

where P and Q are functions of x. This equation can be reduced to the linear form as follows:

Dividing both sides of Equation (1) by y^n, we get

$$y^{-n}\frac{dy}{dx} + Py^{-n+1} = Q \quad (2)$$

$$\text{Put } y^{-n+1} = z \Rightarrow (-n+1)\, y^{-n} \frac{dy}{dx} = \frac{dz}{dx}$$

Substituting in Equation (2), we obtain

$$\frac{dz}{dx} + (1-n)\, Pz = (1-n)\, Q,$$

which is now a linear equation with z as the dependent variable.

SOLVED EXAMPLES

30. The general solution of the differential equation $\frac{dy}{dx} = y \tan x - y^2 \sec x$ is

(A) $\tan x = (c + \sec x)\, y$
(B) $\sec y = (c + \tan y)\, x$
(C) $\sec x = (c + \tan x)\, y$
(D) None of these

Solution: (C)

We have

$$\frac{dy}{dx} = y \tan x - y^2 \sec x$$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$$

$$\text{Putting } \frac{1}{y} = V \Rightarrow \frac{-1}{y^2}\frac{dy}{dx} = \frac{dV}{dx},$$

we obtain $\frac{dV}{dx} + \tan x \cdot V = \sec x$, which is linear.

$$\text{I.F.} = e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x.$$

Hence, the solution is

$$V \sec x = \int \sec^2 x\, dx + c \quad \text{or} \quad \frac{1}{y}\sec x = \tan x + c$$

or $\quad \sec x = y\,(c + \tan x)$

31. The general solution of the differential equation $y\,(x^2 y + e^x)\, dx - e^x\, dy = 0$ is

(A) $x^3 y - 3e^x = cy$ (B) $x^3 y + 3e^x = cy$
(C) $y^3 x - 3e^y = cx$ (D) $y^3 x + 3e^y = cx$

Solution: (B)

We have

$$y\,(x^2 y + e^x)\, dx - e^x\, dy = 0$$

$$\Rightarrow \quad e^x \frac{dy}{dx} = x^2 y^2 + y e^x$$

Dividing by $y^2 e^x$, we get

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y} = x^2 e^{-x}$$

$$\text{Put } \frac{1}{y} = V \text{ so that } \frac{-1}{y^2}\frac{dy}{dx} = \frac{dV}{dx}$$

We thus have $\frac{dV}{dx} + V = -x^2 e^{-x}$, which is linear

$$\therefore \quad I.F. = e^{\int 1 dx} = e^x$$

Hence, the solution is

$$V \times e^x = -\int x^2 e^{-x} \cdot e^x\, dx + \frac{c}{3}$$

$$\text{or} \quad \frac{1}{y} e^x = -\frac{x^3}{3} + \frac{c}{3} \text{ or } x^3 y + 3e^x = cy$$

32. The solution of $x^3 \frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying $y\,(1) = 0$, is

(A) $\tan y = (x-2)e^x \log x$
(B) $\sin y = e^x (x-1)\, x^{-4}$
(C) $\tan y = (x-1)\, e^x x^{-3}$
(D) $\sin y = e^x (x-1)\, x^{-3}$

Solution: (B)

We have,

$$\cos y \frac{dy}{dx} + \frac{4}{x}\sin y = \frac{e^x}{x^3}$$

$$\text{Let} \quad \sin y = t \Rightarrow \cos y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\therefore \quad \frac{dt}{dx} + \frac{4}{x} t = \frac{e^x}{x^3}$$

$$\text{I.F.} = e^{\int \frac{4}{x} dx} = e^{4 \log x} = x^4$$

$\therefore$ The solution is

$$t\, x^4 = \int x^4 \cdot \frac{e^x}{x^3}\, dx = xe^x - e^x + c$$

$$\sin y\, x^4 = xe^x - e^x + c$$

$$\because \quad x = 1,\, y = 0 \;\therefore\; c = 0$$

$$\therefore \quad \sin y = e^x (x-1)\, x^{-4}$$

33. Solution of the equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ is

(A) $e^y = e^x - 1 + ce^{-e^x}$ (B) $e^y = e^x - 1 + ce^{e^x}$

(C) $e^x = e^y - 1 + ce^{-e^y}$ (D) None of these

Solution: (A)

We have,

$$\frac{dy}{dx} = e^{x-y}(e^x - e^y)$$

$$\Rightarrow \quad e^y \frac{dy}{dx} + e^x \cdot e^y = e^{2x}$$

Putting $e^y = V$ so that $e^y \frac{dy}{dx} = \frac{dV}{dx}$, we get

$$\frac{dV}{dx} + e^x \cdot V = e^{2x}, \text{ which is linear in } V.$$

$$\text{I.F.} = e^{\int e^x dx} = e^{e^x}$$

So, the solution is

$$V \cdot e^{e^x} = \int e^{e^x} \cdot e^{2x} dx + c$$

$$\Rightarrow \quad e^y \cdot e^{e^x} = \int e^z \cdot z \, dz + c$$

(Putting $e^x = z \Rightarrow e^x dx = dz$)

$$\Rightarrow \quad e^y \cdot e^{e^x} = (z-1)e^z + c = (e^x - 1)e^{e^x} + c$$

$$\Rightarrow \quad e^y = e^x - 1 + ce^{-ex}$$

34. If $g(x)$ is a differential function, then the solution of the differential equation

$dy + [y\,g'(x) - g(x)\,g'(x)]dx = 0$ is

(A) $y = [g(x) - 1] + ce^{-g(x)}$

(B) $ye^{g(x)} = [g(x) + 1]e^{g(x)} + c$

(C) $y = [g(x) - 1]e^{g(x)} + c$

(D) None of these.

Solution: (A)

We have,

$$dy + [yg'(x) - g(x)\,g'(x)]\,dx = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + g'(x) \cdot y = g(x) \cdot g'(x)$$

Thus is a linear differential equation

with $\quad \text{I.F.} = e^{\int g'(x)dx} = e^{g(x)}$

So, the solution is

$$y\,e^{g(x)} = \int g(x) \cdot g'(x)\,e^{g(x)} dx$$

$$\Rightarrow \quad y\,e^{g(x)} = g(x)\,e^{g(x)} - e^{g(x)} + c$$

or $\quad y = [g(x) - 1] + ce^{-g(x)}$

SOLUTION BY INSPECTION

The following derivatives must be remembered as they are very useful in solving some differential equations directly.

1. $d(x + y) = dx + dy$
2. $d(xy) = ydx + xdy$
3. $d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2}$
4. $d\left(\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{x^2}$
5. $d(\log xy) = \frac{y\,dx + x\,dy}{xy}$
6. $d\left(\log \frac{y}{x}\right) = \frac{x\,dy - y\,dx}{xy}$
7. $d(x^m y^n) = x^{m-1} y^{n-1}(my\,dx + nx\,dy)$
8. $d\left(\frac{1}{2}\log(x^2 + y^2)\right) = \frac{x\,dx + y\,dy}{x^2 + y^2}$
9. $d\left(\frac{1}{2}\log\frac{x+y}{x-y}\right) = \frac{x\,dy - y\,dx}{x^2 - y^2}$
10. $d\left(\tan^{-1}\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{x^2 + y^2}$
11. $\frac{d[f(x, y)]^{1-n}}{1-n} = \frac{f'(x, y)}{[f(x, y)]^n}$
12. $d\left(\sqrt{x^2 + y^2}\right) = \frac{x\,dx + y\,dy}{\sqrt{x^2 + y^2}}$

SOLVED EXAMPLES

35. Solution of the equation $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$ is

(A) $y = x\tan\left(\frac{c + x^2 + y^2}{2}\right)$

(B) $x = y\tan\left(\frac{c + x^2 + y^2}{2}\right)$

(C) $y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$

(D) None of these

Solution: (C)

We have,

$$x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0$$

$$\Rightarrow \quad \frac{1}{2}\,d\,(x^2 + y^2) + d\tan^{-1}\left(\frac{y}{x}\right) = 0$$

Integrating, $\frac{1}{2}(x^2 + y^2) + \tan^{-1}\frac{y}{x} = \frac{c}{2}$

$$\Rightarrow \quad x^2 + y^2 + 2\tan^{-1}\frac{y}{x} = c$$

$\therefore\ y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$ is the required solution.

36. The solution of the equation

$y\sin x\,\frac{dy}{dx} = \cos x\left(\sin x - \frac{y^2}{2}\right)$, given $y = 1$ when $x = \frac{\pi}{2}$ is

(A) $y^2 = \sin x$ (B) $y^2 = 2\sin x$
(C) $x^2 = \sin y$ (D) $x^2 = 2\sin y$

Solution: (A)

Put $y^2\sin x = V$.

Then $\quad 2y\frac{dy}{dx}\sin x + y^2\cos x = \frac{dV}{dx}$

So, the given equation becomes $\frac{1}{2}\frac{dV}{dx} = \sin x\cos x$

$$\Rightarrow \quad dV = 2\sin x\cos x\,dx$$

Integrating, we get

$V = \sin^2 x + c$. i.e., $y^2\sin x = \sin^2 x + c$

Putting $y = 1, x = \frac{\pi}{2}$, we get $1 = 1 + c$ i.e. $c = 0$

So, the solution is $y^2\sin x = \sin^2 x$ i.e. $y^2 = \sin x$.

37. Solution of the equation $x\left(\frac{dy}{dx}\right)^2 + (y - x)\frac{dy}{dx} - y = 0$ is

(A) $(x - y + c)(xy - c) = 0$
(B) $(x + y + c)(xy - c) = 0$
(C) $(x - y + c)(2xy - c) = 0$
(D) $(y - x + c)(xy - c) = 0$

Solution: (A)

We have,

$$x + (y - x)\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \left(\frac{dy}{dx} - 1\right)\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 \text{ or } x\frac{dy}{dx} = -y$$

The solution of $\frac{dy}{dx} = 1$ is $y = x + c$

and solution of $x\frac{dy}{dx} = -y$ i.e. $\frac{dy}{y} + \frac{dx}{x} = 0$ is

$$\log(xy) = \log c \text{ i.e. } xy = c$$

Hence, general solution is $(x - y + c)(xy - c) = 0$.

38. Solution of equation $(xy^4 + y)\,dx - x\,dy = 0$ is

(A) $4x^4y^3 + 3x^3 = cy^3$ (B) $3x^3y^4 + 4y^3 = cx^3$
(C) $3x^4y^3 + 4x^3 = cy^3$ (D) None of these

Solution: (C)

We have,

$$xy^4\,dx + y\,dx - x\,dy = 0$$

$$\Rightarrow \quad x\,dx + \frac{y\,dx - x\,dy}{y^4} = 0$$

$$\Rightarrow \quad x^3\,dx + \left(\frac{x}{y}\right)^2 \cdot \frac{y\,dx - x\,dy}{y^2} = 0$$

$$\Rightarrow \quad x^3\,dx + \left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right) = 0$$

Integrating, we get

$$\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 = c'$$

or $\quad 3x^4y^3 + 4x^3 = c\,y^3,$

which is the required solution.

39. The solution of the differential equation $x^2\frac{dy}{dx} - xy = 1 + \cos\frac{y}{x}$ is

(A) $\cos\frac{y}{x} = 1 + \frac{c}{x}$

(B) $x^2 = (c + x^2)\tan\frac{y}{x}$

(C) $\tan\frac{y}{2x} = c - \frac{1}{2x^2}$

(D) $\tan\frac{y}{x} = c + \frac{1}{x}$

Solution: (C)

We have

$$x^2\frac{dy}{dx}-xy=1+\cos\frac{y}{x}=2\cos^2\frac{y}{2x}.$$

$$\Rightarrow \quad \sec^2\left(\frac{y}{2x}\right)\cdot\left(x^2\frac{dy}{dx}-xy\right)=2$$

$$\Rightarrow \quad \frac{1}{2}\sec^2\left(\frac{y}{2x}\right)\cdot\frac{x\frac{dy}{dx}-y}{x^2}=\frac{1}{x^3}$$

$$\Rightarrow \quad \frac{d}{dx}\left(\tan\frac{y}{2x}\right)=\frac{1}{x^3}.$$

Integrating, we get

$$\tan\frac{y}{2x}=c-\frac{1}{2x^2}$$

which is the required solution.

40. Solution of the equation $xdy-[\,y+xy^3\ (1+\log x)]\,dx=0$ is

(A) $\frac{-x^2}{y^2}=\frac{2x^3}{3}\left(\frac{2}{3}+\log x\right)+c$

(B) $\frac{x^2}{y^2}=\frac{2x^3}{3}\left(\frac{2}{3}+\log x\right)+c$

(C) $\frac{-x^2}{y^2}=\frac{x^3}{3}\left(\frac{2}{3}+\log x\right)+c$

(D) None of these

Solution: (A)

We have,

$$x\,dy-y\,dx=xy^3\ (1+\log x)\ dx$$

$$\Rightarrow \quad -\left(\frac{ydx-xdy}{y^2}\right)=xy\ (1+\log x)\ dx$$

$$\Rightarrow \quad -d\left(\frac{x}{y}\right)=xy\ (1+\log x)\ dx$$

$$\Rightarrow \quad -\frac{x}{y}\,d\left(\frac{x}{y}\right)=x^2\ (1+\log x)\ dx$$

Integrating, we get

$$\frac{-\left(\frac{x}{y}\right)^2}{2}=(1+\log x)\,\frac{x^3}{3}-\int\frac{x^3}{3}\cdot\frac{1}{x}\,dx$$

$$\Rightarrow \quad -\frac{x^2}{2y^2}=\frac{x^3}{3}(1+\ \log x)-\frac{x^3}{9}+\frac{c}{2}$$

$$\Rightarrow \quad -\frac{x^2}{y^2}=\frac{2x^3}{3}\left(\frac{2}{3}+\log x\right)+c$$

41. Solution of the differential equation

$$y\ (xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0 \text{ is}$$

(A) $2\log|x|-\log|y|-\frac{1}{xy}=c$

(B) $2\log|x|+\log|y|-\frac{1}{xy}=c$

(C) $2\log|x|-\log|y|+\frac{1}{xy}=c$

(D) None of these

Solution: (A)

We have,

$$(xy^2+2x^2y^3)\ dx+(x^2y-x^3y^2)dy=0$$

$$\Rightarrow \quad xy(\,y\ dx+x\ dy)+x^2y^2(2y\ dx-x\ dy)=0$$

$$\Rightarrow \quad \frac{d(xy)}{x^2y^2}+\left(\frac{2}{x}dx-\frac{1}{y}dy\right)=0 \quad (\text{Dividing by } x^3y^3)$$

On integrating, we get

$$-\frac{1}{xy}+2\log|x|-\log|y|=c$$

42. The general solution of the differential equation $y\ (x^2y+e^x)\ dx-e^xdy=0$ is

(A) $x^3y-3e^x=3cy$ (B) $x^3y+3e^x=3cy$

(C) $xy^3-3e^x=3\ cx$ (D) None of these

Solution: (B)

We have,

$$y(x^2y+e^x)dx-e^xdy=0$$

$$\Rightarrow \quad x^2y^2dx+ye^x\ dx-e^x\ dy=0$$

$$\Rightarrow \quad x^2dx+\frac{ye^x\,dx-e^x\,dy}{y^2}=0$$

$$\Rightarrow \quad x^2dx+\ d\left(\frac{e^x}{y}\right)=0$$

On integrating, we get

$$\frac{x^3}{3}+\frac{e^x}{y}=c \text{ or } x^3y+3e^x=3cy$$

Equations of the Form $\frac{dy}{dx}+P\phi(y)=Q\,f(y)$,

where P and Q are functions of x alone or constants

Step I: Divide both sides by $f(y)$ to get rid of $f(y)$ from the R.H.S., such that

$$\frac{1}{f(y)}\cdot\frac{dy}{dx}+P\cdot\frac{\phi(y)}{f(y)}=Q$$

Step II: Put $\dfrac{\phi(y)}{f(y)} = v$

Step III: If $\dfrac{dv}{dx} = \dfrac{d}{dx}\left[\dfrac{\phi(y)}{f(y)}\right] = k \cdot \dfrac{1}{f(y)}\dfrac{dy}{dx}$, where k is some constant, the equation reduces to the form.

$$\frac{1}{k}\frac{dv}{dx} + Pv = Q$$

or $$\frac{dv}{dx} + kPv = kQ,$$

which is a linear differential equation with v as the dependent variable.

EXERCISES

Single Option Correct Type

1. Solution of equation $\dfrac{dy}{dx} = \dfrac{y\dfrac{d(\phi(x))}{dx} - y^2}{\phi(x)}$ is

(A) $y = \dfrac{\phi(x)+c}{x}$ (B) $y = \dfrac{\phi(x)}{x} + c$

(C) $y = \dfrac{\phi(x)}{x+c}$ (D) $y = \phi(x) + x + c$

2. The family passing through (0, 0) and satisfying the differential equation $\dfrac{y_2}{y_1} = 1$ (where $y_n = \dfrac{d^n y}{dx^n}$) is

(A) $y = k$ (B) $y = kx$

(C) $y = k(e^x + 1)$ (D) $y = k(e^x - 1)$

3. The solution of differential equation

$\sec^2 y \dfrac{dy}{dx} + 2x \tan y = x^3$ is

(A) $\tan y = \dfrac{1}{2}(x^2 - 1) + ce^{-x^2}$

(B) $\tan y = \dfrac{1}{2}(x^2 - 1) + ce^{x^2}$

(C) $\tan y = \dfrac{1}{2}(x^2 + 1) + ce^{-x^2}$

(D) $\tan y = \dfrac{1}{2}(x^2 + 1) + ce^{x^2}$

4. Solution of the differential equation $ydx + (x + x^2y)\,dy = 0$ is

(A) $\log y = Cx$ (B) $-\dfrac{1}{xy} + \log y = C$

(C) $\dfrac{1}{xy} + \log y = C$ (D) $-\dfrac{1}{xy} = C$

5. The family of curves represented by $\dfrac{dy}{dx} = \dfrac{x^2+x+1}{y^2+y+1}$ and the family represented by $\dfrac{dy}{dx} + \dfrac{y^2+y+1}{x^2+x+1} = 0$

(A) Touch each other

(B) Are orthogonal

(C) Are one and the same

(D) None of these

6. If $\phi(x)$ is a differentiable function then the solution of $dy + (y\,\phi'(x) - \phi(x)\,\phi'(x))\,dx = 0$ is

(A) $y = (\phi(x) - 1) + c\,e^{-\phi(x)}$

(B) $y\phi(x) = (\phi(x))^2 + c$

(C) $ye^{\phi(x)} = \phi(x)\,e^{\phi(x)} + c$

(D) $(y - \phi(x)) = (\phi(x))\,e^{-\phi(x)}$

7. Solution of the differential equation

$x = 1 + \left(xy\dfrac{dy}{dx}\right) + \dfrac{(x^2y^2)}{2!}\left(\dfrac{dy}{dx}\right)^2 + \dfrac{(xy)^3}{3!}\left(\dfrac{dy}{dx}\right)^3 + \ldots$ is

(A) $y = \ln(x) + c$ (B) $y = (\ln x)^2 + c$

(C) $y = \pm\sqrt{(\ln x)^2 + c}$ (D) $xy = x^y + k$

8. The orthogonal trajectories of the family of semi-cubical parabola is given by

(A) $x^2 + 3y^2 = c^2$ (B) $3x^2 + y^2 = c^2$

(C) $x + 3y^2 = c^2$ (D) $3y^2 + 2x^2 = c^2$

9. Solution of the differential equation

$2y \sin x \dfrac{dy}{dx} = 2\sin x \cos x - y^2 \cos x$ satisfying $y\left(\dfrac{\pi}{2}\right) = 1$ is given by

(A) $y^2 = \sin x$ (B) $y = \sin^2 x$

(C) $y^2 = \cos x + 1$ (D) $y^2 \sin x = 4\cos^2 x$

10. The solution of the differential equation $y\,dx - x\,dy + (\log x)\,dx = 0$ is

(A) $y = \log x + cx$ (B) $y = 1 + \log x + c$

(C) $y + cx = \log \frac{1}{x}$ (D) None of these

11. Solution of $(x^2 \sin^3 y - y^2 \cos x)\,dx + (x^3 \cos y \sin^2 y - 2y \sin x)\,dy = 0$ is

(A) $\frac{x^3 \sin^3 y}{3} = c$

(B) $x^3 \sin^3 y = y^2 \sin x + c$

(C) $\frac{x^3 \sin^3 y}{3} = y^2 \sin x + c$

(D) None of these

12. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the x-intercept of the normal and passing through (2, 1) is

(A) $x^2 + y^2 - x = 0$ (B) $4x^2 + 2y^2 - 9y = 0$

(C) $2x^2 + 4y^2 - 9x = 0$ (D) $4x^2 + 2y^2 - 9x = 0$

13. The solution of the differential equation

$$\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2} \text{ is}$$

(A) $\frac{y}{4} + \frac{1}{x^2 + y^2} = c$ (B) $\frac{y}{x} - \frac{1}{x^2 + y^2} = c$

(C) $\frac{x}{y} - \frac{1}{x^2 + y^2} = c$ (D) None of these

14. The solution of the differential equation $(x - y)(2dy - dx) = 3dx - 5dy$ is

(A) $2x - y = \log(x - y + z) + c$

(B) $2x + y = \log(x - y + z) + c$

(C) $2y - x = \log(x - y + z) + c$

(D) None of these

15. Solution of $\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \frac{\sqrt{1 - (x^2 + y^2)}}{\sqrt{x^2 + y^2}}$ is

(A) $\sin^{-1}\sqrt{x^2 + y^2} = c$

(B) $\tan^{-1}\frac{y}{x} = c$

(C) $\sin^{-1}\sqrt{x^2 + y^2} = \tan^{-1}\frac{y}{x} = c$

(D) None of these

16. Which of the following does not represent the orthogonal trajectory of the system of curves $\left(\frac{dy}{dx}\right)^2 = \frac{a}{x}$

(A) $9a(y + c)^2 = 4x^3$

(B) $y + c = \frac{-2}{3\sqrt{a}} x^{3/2}$

(C) $y + c = \frac{2}{3\sqrt{a}} x^{3/2}$

(D) All are orthogonal trajectories

17. Solution of $\left(\frac{x + y - 1}{x + y - 2}\right)\frac{dy}{dx} = \left(\frac{x + y + 1}{x + y + 2}\right)$, given that $y = 1$ when $x = 1$, is

(A) $\log\left|\frac{(x - y)^2 - 2}{2}\right| = 2(x + y)$

(B) $\log\left|\frac{(x - y)^2 + 2}{2}\right| = 2(x - y)$

(C) $\log\left|\frac{(x + y)^2 + 2}{2}\right| = 2(x - y)$

(D) None of these

18. Solution of $(y + x\sqrt{xy}\,(x + y) + y)\,dx + (y\sqrt{xy}\,(x + y) - x)\,dy = 0$ is

(A) $x^2 + y^2 = 2\tan^{-1}\sqrt{\frac{y}{x}} + c$

(B) $x^2 + y^2 = 4\tan^{-1}\sqrt{\frac{y}{x}} + c$

(C) $x^2 + y^2 = \tan^{-1}\sqrt{\frac{y}{x}} + c$

(D) None of these

19. A particle starts at the origin and moves along the x-axis in such a way that its velocity at the point $(x, 0)$ is given by the formula $\frac{dx}{dt} = \cos^2 \pi x$. Then the particle never reaches the point on

(A) $x = \frac{1}{4}$ (B) $x = \frac{3}{4}$

(C) $x = \frac{1}{2}$ (D) $x = 1$

20. The equation of the curve for which the cartesian subtangent varies as the reciprocal of the square of the abscissa, is

(A) $x = ce^{y^3/3k}$ (B) $x = ce^{y^2/3k}$

(C) $y = ce^{x^3/3k}$ (D) None of these

21. Solution of $\frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$ is

(A) $x^y = cy^x$ (B) $xy = c$

(C) $(xy)^x = c$ (D) None of these

22. If the solution of $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(A) 2 (B) –2 (C) 3 (D) –4

23. Solution of $\frac{x\,dy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$ is

(A) $x - \tan^{-1}\frac{y}{x} = c$ (B) $\tan^{-1}\frac{y}{x} = c$

(C) $x + \tan^{-1}\frac{y}{x} = c$ (D) None of these

24. If $y = y(x)$ and

$$\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x,\ y(0) = 1, \text{ then } y\left(\frac{\pi}{2}\right) =$$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) 1

25. The equation of the curve passing through the point (1, 1) and having slope $\frac{2ay}{x(y-a)}$ is

(A) $y^a \cdot x^{2a} = e^y$

(B) $y^a \cdot x^{2a} = e^{y-1}$

(C) $y^{2a} \cdot x^a = e^y$

(D) None of these

26. The solution of the differential equation $(x\cos x - \sin x + yx^2)\,dx + x^3 dy = 0$ is equal to

(A) $\frac{\sin x}{x} + xy = c$ (B) $\frac{\sin x}{x} + x = c$

(C) $\frac{\sin x}{x} + y = c$ (D) None of these

27. The solution of the differential equation

$y\,dx - x\,dy + 3x^2y^2e^{x^3}dx = 0$ is

(A) $\frac{x}{y} + e^{x^3} = c$ (B) $\frac{x}{y} - e^{x^3} = c$

(C) $\frac{y}{x} + e^{x^3} = c$ (D) $\frac{y}{x} - e^{x^3} = c$

28. Solution of the differential equation $[y(1 + x^{-1}) + \sin y]\,dx + (x + \log x + x\cos y)\,dy = 0$ is

(A) $xy + y\log x = c$

(B) $xy + x\sin y = c$

(C) $xy + y\log x + x\sin y = c$

(D) None of these

29. The differential equation of the curve for which the normal at every point passes through a fixed point (h, k) is

(A) $y - k = \frac{dx}{dy}(h - x)$ (B) $y - k = \frac{dx}{dy}(x - h)$

(C) $y - k = \frac{dy}{dx}(h - x)$ (D) $y - k = \frac{dy}{dx}(x - h)$

30. Solution of the differential equation $[3xy^2 + x\sin(xy)]\,dy + [y^3 + y\sin(xy)]\,dx = 0$ is

(A) $xy^3 - \cos xy = c$ (B) $xy^3 + \cos xy = c$

(C) $xy^2 - \cos xy = c$ (D) $xy^2 + \sin xy = c$

31. The equation of the curve which passes through the point $(2a, a)$ and for which the sum of the cartesian sub tangent and the abscissa is equal to the constant a, is

(A) $y(x - a) = a^2$ (B) $y(x + a) = a^2$

(C) $x(y - a) = a^2$ (D) $x(y + a) = a^2$

32. Solution of differential equation

$$\frac{dt}{dx} = \frac{t\left(\frac{d\,g(x)}{dx}\right) - t^2}{g(x)} \text{ is}$$

(A) $t = \frac{g(x)+c}{x}$ (B) $\frac{g(x)}{x} + c$

(C) $t = \frac{g(x)}{x+c}$ (D) $t = g(x) + x + c$

33. If the curve $y = f(x)$ passing through the point (1, 2) and satisfies the differential equation $xdy + (y - x^3y^2)\,dx = 0$, then

(A) $xy = \frac{1}{2}$ (B) $x^3y = 2$

(C) $\frac{1}{xy} = 2$ (D) None of these

34. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(A) e^{-kT} (B) $T^2 - \frac{I}{k}$

(C) $I - \frac{kT^2}{2}$ (D) $I - \frac{k(T-t)^2}{2}$

35. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to

(A) –2 (B) 7 (C) 5 (D) 13

36. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population become zero is

(A) 2 ln 18 (B) ln 9
(C) 1/2 ln 18 (D) ln 18

37. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) variable radius and fixed centre
(B) variable radius and variable centre
(C) fixed radius and variable centre on x-axis
(D) fixed radius and variable centre on y-axis

38. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

(A) $y = \ln x + x$ (B) $y = x \ln x + x^2$
(C) $y = xe^{(x-1)}$ (D) $y = x \ln x + x$

39. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

(A) $(x-2)y'^2 = 25 - (y-2)^2$
(B) $(y-2)y'^2 = 25 - (y-2)^2$
(C) $(y-2)^2y'^2 = 25 - (y-2)^2$
(D) $(x-2)^2y'^2 = 25 - (y-2)^2$

40. The differential equation which represents the family of curves $y = c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants is

(A) $y' = y^2$ (B) $y'' = y'y$
(C) $y'' = y'$ (D) $yy'' = (y')^2$

41. The general solution of the differential equation

$$\frac{dy}{dx} + y\,g'(x) = g(x) \cdot g'(x),$$

where $g(x)$ is a given function of x, is

(A) $g(x) + \log[1 + y + g(x)] = C$
(B) $g(x) + \log[1 + y - g(x)] = C$
(C) $g(x) - \log[1 + y - g(x)] = C$
(D) None of these

42. Solution of the equation $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0$ is

(A) $y = x \tan\left(\frac{c + x^2 + y^2}{2}\right)$

(B) $x = y \tan\left(\frac{c + x^2 + y^2}{2}\right)$

(C) $y = x \tan\left(\frac{c - x^2 - y^2}{2}\right)$

(D) None of these

43. Solution of the equation $x\,dy = \left(y + x\frac{f(y/x)}{f'(y/x)}\right)dx$ is

(A) $f\left(\frac{x}{y}\right) = cy$ (B) $f\left(\frac{y}{x}\right) = cx$

(C) $f\left(\frac{y}{x}\right) = cxy$ (D) None of these

44. Solution of the equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ is

(A) $e^y = e^x - 1 + ce^{-e^x}$ (B) $e^y = e^x - 1 + ce^{e^x}$

(C) $e^x = e^y - 1 + ce^{-e^y}$ (D) None of these

45. Solution of the equation $x\left(\frac{dy}{dx}\right)^2 + (y - x)\frac{dy}{dx} - y = 0$ is

(A) $(x - y + c)(xy - c) = 0$
(B) $(x + y + c)(xy - c) = 0$
(C) $(x - y + c)(2xy - c) = 0$
(D) $(y - x + c)(xy - c) = 0$

46. The differential equation of the family of general circles is

(A) $y'''(1 + y'^2) - 3y'y''^2 = 0$
(B) $y'''(1 + y'^2) + 3y'y''^2 = 0$
(C) $y'''(1 + y'^2) - 3y''y'^2 = 0$
(D) None of these

47. The equation of the family of curves which intersect the hyperbola $xy = 2$ orthogonally is

(A) $y = \frac{x^3}{6} + C$ (B) $y = \frac{x^2}{4} + C$

(C) $y = -\frac{x^3}{6} + C$ (D) $y = -\frac{x^2}{4} + C$

48. The solution of the differential equation

$$x^2\frac{dy}{dx} - xy = 1 + \cos\frac{y}{x} \text{ is}$$

(A) $\cos\frac{y}{x} = 1 + \frac{c}{x}$

(B) $x^2 = (c + x^2)\tan\frac{y}{x}$

(C) $\tan\frac{y}{2x} = c - \frac{1}{2x^2}$

(D) $\tan\frac{y}{x} = c + \frac{1}{x}$

49. Solution of the differential equation $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2}\right)$, given that $y = 1$ when $x = 1$, is $k(y-x) + \log\left|\frac{(x+y)^k - k}{k}\right| = 0$, where $k =$

(A) 1 (B) 2 (C) 3 (D) 4

50. Solution of the equation $xdy - [\,y + xy^3\,(1 + \log x)]\,dx = 0$ is

(A) $\frac{-x^2}{y^2} = \frac{2x^3}{3}\left(\frac{2}{3} + \log x\right) + C$

(B) $\frac{x^2}{y^2} = \frac{2x^3}{3}\left(\frac{2}{3} + \log x\right) + C$

(C) $\frac{-x^2}{y^2} = \frac{x^3}{3}\left(\frac{2}{3} + \log x\right) + C$

(D) None of these

51. Solution of equation $\frac{dy}{dx} = \frac{y\frac{d(\phi(x))}{dx} - y^2}{\phi(x)}$ is

(A) $y = \frac{\phi(x) + c}{x}$ (B) $y = \frac{\phi(x)}{x} + c$

(C) $y = \frac{\phi(x)}{x + c}$ (D) $y = \phi(x) + x + c$

52. Solution of the differential equation $ydx + (x + x^2y)\,dy = 0$ is

(A) $\log y = Cx$ (B) $-\frac{1}{xy} + \log y = C$

(C) $\frac{1}{xy} + \log y = C$ (D) $-\frac{1}{xy} = C$

53. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the x-intercept of the normal and passing through (2, 1) is

(A) $x^2 + y^2 - x = 0$ (B) $4x^2 + 2y^2 - 9y = 0$

(C) $2x^2 + 4y^2 - 9x = 0$ (D) $4x^2 + 2y^2 - 9x = 0$

54. Solution of $\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \frac{\sqrt{1-(x^2+y^2)}}{\sqrt{x^2+y^2}}$ is

(A) $\sin^{-1}\sqrt{x^2+y^2} = c$

(B) $\tan^{-1}\frac{y}{x} = c$

(C) $\sin^{-1}\sqrt{x^2+y^2} = \tan^{-1}\frac{y}{x} + c$

(D) None of these

55. The solution of the differential equation $(x\cos x - \sin x + yx^2)\,dx + x^3dy = 0$ is equal to

(A) $\frac{\sin x}{x} + xy = c$ (B) $\frac{\sin x}{x} + x = c$

(C) $\frac{\sin x}{x} + y = c$ (D) None of these

56. Solution of the differential equation $[y\,(1 + x^{-1}) + \sin y]\,dx + (x + \log x + x\cos y)\,dy = 0$ is

(A) $xy + y\log x = c$

(B) $xy + x\sin y = c$

(C) $xy + y\log x + x\sin y = c$

(D) None of these

57. The equation of the curve, passing through (2, 5) and having the area of triangle formed by the x-axis, the ordinate of a point on the curve and the tangent at the point 5 square units, is

(A) $xy = 10$ (B) $x^2 = 10\,y$

(C) $y^2 = 10x$ (D) $xy^{1/2} = 10$

58. If the curve $y = f(x)$ passing through the point (1, 2) and satisfies the differential equation $xdy + (y + x^3y^2)\,dx = 0$, then

(A) $xy = \frac{1}{2}$ (B) $x^3y = 2$

(C) $\frac{1}{xy} = 2$ (D) None of these

59. The solution of the differential equation $\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$ is

(A) $\sqrt{x^2+y^2} = a\cos\left\{c + \tan^{-1}\frac{y}{x}\right\}$

(B) $\sqrt{x^2+y^2} = a\sin\left\{c + \tan^{-1}\frac{y}{x}\right\}$

(C) $\sqrt{x^2+y^2} = a\sin\left\{c + \tan^{-1}\frac{x}{y}\right\}$

(D) None of these

60. The solution of the equation

$(2x \log y)\, dx + \left(\dfrac{x^2}{y} + 3y^2\right) dy = 0$ is

(A) $x^2 \log y + y^3 = c$ (B) $y^3 \log x + x^3 = c$
(C) $x^2 \log y - y^3 = c$ (D) None of these

61. The solution of the equation

$\dfrac{y + \sin x \cos^2(xy)}{\cos^2(xy)} dx + \left(\dfrac{x}{\cos^2(xy)} + \sin y\right) dy = 0$ is

(A) $\tan(xy) + \cos x - \cos y = c$
(B) $\tan(xy) - \cos x - \cos y = c$
(C) $\tan(xy) + \cos x + \cos y = c$
(D) None of these

62. The solution of the equation $\dfrac{dy}{dx} = \dfrac{y}{2y \log y + y - x}$ is

(A) $x = y \log y + \dfrac{c}{y}$ (B) $y = x \log x + \dfrac{c}{x}$

(C) $x = -y \log y + \dfrac{c}{y}$ (D) None of these

63. Solution of the equation $\cos^2 x \dfrac{dy}{dx} - y \tan 2x = \cos^4 x$, when $|x| < \dfrac{\pi}{4}$ and $y\left(\dfrac{\pi}{6}\right) = \dfrac{3\sqrt{3}}{8}$, is

(A) $y = \dfrac{\sin 2x}{2\left(\tan^2 x - 1\right)}$ (B) $y = \dfrac{\sin 2x}{2\left(1 - \tan^2 x\right)}$

(C) $y = \dfrac{\sin 2x}{2(1 + \tan^2 x)}$ (D) None of these

64. The solution of the equation $\dfrac{dy}{dx} + x(x + y) = x^3(x + y)^3 - 1$ is

(A) $(x + y)^{-3} = ce^{x2} + x^2 + 1$
(B) $(x + y)^{-2} = ce^{x2} - x^2 + 1$
(C) $(x + y)^{-2} = ce^{x2} + x^2 + 1$
(D) None of these

65. The solution of the equation $\sin y \dfrac{dy}{dx} = \cos y (1 - x \cos y)$ is

(A) $\sec y = (1 + x) + ce^x$
(B) $\tan y = (1 + x) + ce^x$
(C) $\sec y = (1 + x) + ce^{-x}$
(D) None of these

66. The solution of the differential equation $x(y^2e^{xy} + e^{x/y})dy = y(e^{x/y} - y^2e^{xy})\, dx$ is

(A) $xy = \ln(e^{y/x} + c)$ (B) $xy = \ln(e^{x/y} + c)$

(C) $\dfrac{y}{x} = \ln(e^{xy} + c)$ (D) $\dfrac{x}{y} = \ln(e^{xy} + c)$

67. Solution of the equation

$x\int_0^x y(t)\, dt = (x + 1)\int_0^x ty(t)\, dt,\ x > 0$ is

(A) $y = \dfrac{c}{x^3} e^{-\frac{1}{x}}$ (B) $y = \dfrac{c}{x^3} e^{\frac{1}{x}}$

(C) $y = \dfrac{c}{x} e^{-\frac{1}{x^3}}$ (D) $y = \dfrac{c}{x} e^{\frac{1}{x^3}}$

68. The solution of the differential equation $(1 + \tan y)(dx - dy) + 2xdy = 0$ is

(A) $x(\sin y + \cos y) = \sin y + ce^{-y}$
(B) $x(\sin y - \cos y) = \sin y + ce^{-y}$
(C) $x(\sin y + \cos y) = \cos y + ce^{-y}$
(D) None of these

69. The equation of the curve satisfying the differential equation

$\sqrt{x - y\dfrac{dy}{dx}} = |x^2 - y^2|$

and passing through the point (1, 0) is

(A) $(2x - 1) + \dfrac{1}{x^2 - y^2} = 0$

(B) $(3x - 2) + \dfrac{1}{x^2 - y^2} = 0$

(C) $(2x - 3) + \dfrac{1}{x^2 - y^2} = 0$

(D) None of these

70. Solution of the differential equation $2y \sin x \dfrac{dy}{dx} = 2 \sin x \cos x - y^2 \cos x$ satisfying $y\left(\dfrac{\pi}{2}\right) = 1$ is given by

(A) $y^2 = \sin x$ (B) $y = \sin^2 x$
(C) $y^2 = \cos x + 1$ (D) $y^2 \sin x = 4 \cos^2 x$

71. The solution of the equation $y(2x^2y + e^x)\, dx - (e^x + y^3)\, dy = 0$, if $y(0) = 1$, is

(A) $6e^x - 4x^3y - 3y^3 - 3y = 0$
(B) $6e^x + 4x^3y - 3y^3 - 3y = 0$
(C) $6e^x + 4x^3y + 3y^3 - 3y = 0$
(D) None of these

72. The solution of the equation $ye^{-x/y}\, dx - (xe^{-x/y} + y^3)\, dy = 0$ is

(A) $3e^{-x/y} + y^2 = c$ (B) $2e^{-x/y} + y^2 = c$
(C) $2e^{-x/y} + y^2 = c$ (D) None of these

73. The differential equation corresponding to $y = \sum_{i=1}^{3} c_i e^{m_i x}$, where c_i's are arbitrary constants and m_1, m_2, m_3 are roots of the equation $m^3 - 7m + 6 = 0$, is

(A) $y_3 - 7y_1 + 6y = 0$ (B) $y_3 + 7y_1 + 6y = 0$
(C) $y_3 - 7y_1 - 6y = 0$ (D) None of these

74. If $y = c_1e^{2x} + c_2e^{x} + c_3e^{-x}$ satisfies the differential equation $\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$, then $\frac{a^3+b^3+c^3}{abc}$ is equal to

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

More than One Option Correct Type

75. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding sub-normal is

(A) Linear (B) Homogenous
(C) Exact (D) None of these

76. If $g(x)$ be a function defined on [–1, 1]. If the area of the equilateral triangle with two of its vertices at (0, 0) and $(x, g(x))$ is $\frac{\sqrt{3}}{4}$, then the function is

(A) $g(x) = \pm\sqrt{(1-x^2)}$ (B) $g(x) = -\sqrt{(1-x^2)}$
(C) $g(x) = \sqrt{(1-x^2)}$ (D) $g(x) = \sqrt{(1+x^2)}$

77. For a certain curve $y = f(x)$ satisfying $\frac{d^2y}{dx^2} = 6x - 4$, $f(x)$ has a local minimum value 5 when $x = 1$.

(A) Equation of the curve is $y = x^3 - 2x^2 + x + 5$
(B) $f(x)$ has a local maximum at $x = \frac{1}{3}$
(C) Global maximum value of $f(x)$ is 7
(D) Global minimum value of $f(x)$ is 5

78. The solution of the equation $\frac{dy}{dx} + x = xe^{(n-1)y}$ is

(A) $\frac{1}{(n-1)}\log\left(\frac{e^{(n-1)y}-1}{e^{(n-1)y}}\right) = \frac{x^2}{2} + c$

(B) $e^{(n-1)y} = ce^{(n-1)y + (n-1)\frac{x^2}{2}} + 1$

(C) $\log\left(\frac{e^{(n-1)y}-1}{(n-1)\,e^{(n-1)y}}\right) = n^2 + c$

(D) $e^{(n-1)y} = c\,e^{(n-1)\frac{x^2}{2}+x} + 1$

Passage Based Questions

Passage 1

A differential equation is said to be exact if it can be derived from its primitive by direct differentiation without any further transformation such as elimination, etc. The differential equation

$$(x^2 - ay)dx + (y^2 - ax)dy = 0$$

is exact in as much as it can be derived from its primitive

$$x^3 - 3axy + y^3 = c$$

by direct differentiation.

The necessary and sufficient condition for the differential equation $M\,dx + N\,dy = 0$ to be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

where $\frac{\partial M}{\partial y}$ is the derivative of M with respect to y keeping x as constant and $\frac{\partial N}{\partial x}$ is the derivative of N with respect to x keeping y as constant.

If the equation $M\,dx + N\,dy = 0$ is exact, then it can be integrated as follows:

Firstly, integrate M with respect to x regarding y as constant. Then, integrate with respect to y those of the terms in N which do not involve x. The sum of the two expressions thus obtained equated to a constant is the required solution.

79. The solution of the equation

$$x\,dx + y\,dy = \frac{a^2(x\,dy - y\,dx)}{x^2 - y^2}$$ is

(A) $x^2 + y^2 + 2a^2\tan^{-1}\frac{x}{y} = A$

(B) $x^2 + y^2 - 2a^2\tan^{-1}\frac{x}{y} = A$

(C) $x^2 - y^2 + 2a^2\tan^{-1}\frac{x}{y} = A$

(D) None of these

80. The solution of the equation

$$\left[y\left(1+\frac{1}{x}\right) + \cos y\right]dx + \left[x + \log x - x\sin y\right]dy = 0$$ is

(A) $y(x + \log x) - x\cos y = c$
(B) $y(x + \log x) + x\sin y = c$
(C) $y(x + \log x) + x\cos y = c$
(D) None of these

Passage 2

Some equations which are not exact can be made exact on multiplication by some suitable function known as an *integrating factor*.

The equation

$$xdy - ydx = 0$$

which is not exact becomes so on multiplication by $1/y^2$, for then we

$$\frac{x}{y^2}dy - \frac{1}{y}dx = 0$$

which is easily seen to be exact.

We can solve it either by re-arranging the terms and making them exact differential or by the method of exact equations. We now give some rules for finding integrating factors of differential equation

$$Mdx + Ndy = 0$$

to make it exact.

I. If $Mx + Ny \neq 0$ and the equation is homogeneous, then $\dfrac{1}{Mx + Ny}$ is an I.F.

II. If the equation $Mdx + Ndy = 0$ is not exact but is of the form $f_1(xy)y\,dx + f_2(xy)x\,dy = 0$, then $\dfrac{1}{Mx - Ny}$ is an I.F., provided $Mx - Ny \neq 0$

III. When $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N}$ is a function of x alone, say $f(x)$, then I.F. $= e^{\int f(x)\,dx}$

IV. When $\dfrac{\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}}{M}$ is a function of y alone, say $f(y)$, then I.F. $= e^{\int f(y)\,dy}$

81. The solution of the differential equation $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$ is

(A) $\dfrac{x}{y} - 2\log x + 3\log y = c$

(B) $\dfrac{x}{y} + 2\log x + 3\log y = c$

(C) $\dfrac{x}{y} - 2\log x - 3\log y = c$

(D) None of these

82. The solution of the equation $(xy\sin xy + \cos xy)\,y\,dx + (xy\sin xy - \cos xy)\,x\,dy = 0$ is

(A) $y\sec xy = cx$ (B) $x\sec xy = cy$
(C) $x\operatorname{cosec} xy = cy$ (D) None of these

83. The integrating factor to make the differential equation $\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2y\,dy = 0$ exact is

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{x^2}$

(C) $\dfrac{1}{x^3}$ (D) $\dfrac{1}{x^4}$

84. The integrating factor to make the differential equation $(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dy = 0$ exact is

(A) $\dfrac{1}{y}$ (B) $\dfrac{1}{y^2}$

(C) $\dfrac{1}{y^3}$ (D) $\dfrac{1}{y^4}$

Passage 3

A differential equation of the form $y = px + f(p)$, where $p = \dfrac{dy}{dx}$ is known as Clairaut's equation. We now find the solution of the above equation.

The given equation is

$$y = px + f(p) \qquad (1)$$

It is solvable for y.

Differentiating with respect to x, we get

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + f'(p)\cdot\frac{dp}{dx}$$

$$\Rightarrow \qquad p = p + x\frac{dp}{dx} + f'(p)\cdot\frac{dp}{dx}$$

$$\Rightarrow \qquad x\frac{dp}{dx} + f'(p).\frac{dp}{dx} = 0$$

$$\text{Factorizing } \frac{dp}{dx} + [x + f'(p)] = 0$$

Cancelling the factor $x + f'(p)$ which does not involve $\dfrac{dp}{dx}$, we have $\dfrac{dp}{dx} = 0$

Integrating, $\quad p = c \qquad (2)$

Eliminating p between (1) and (2), the required solution of (1) is

$$y = cx + f(c)$$

85. The solution of the equation $(y - px)(p - 1) = p$ is

(A) $y = cx + \dfrac{c}{c-1}$ (B) $y = cx - \dfrac{c}{c-1}$

(C) $x = cy + \dfrac{c}{c-1}$ (D) None of these

86. The solution of the equation

$$\left(\frac{dy}{dx}\right)^2 (x^2 - a^2) - 2\left(\frac{dy}{dx}\right)xy + y^2 - b^2 = 0 \text{ is}$$

(A) $y = cx + \sqrt{a^2c^2 + b^2}$

(B) $y = cx + \sqrt{b^2c^2 + a^2}$

(C) $y = cx - \sqrt{a^2c^2 + b^2}$

(D) $y = cx - \sqrt{b^2c^2 + a^2}$

87. The solution of the equation

$$(x - a)\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - y = 0 \text{ is}$$

(A) $y = cx + \dfrac{ac^2}{c+1}$ (B) $y = cx - \dfrac{ac^2}{c+1}$

(C) $y = cx - \dfrac{a^2c^2}{c+1}$ (D) None of these

88. The solution of the equation

$p^2x(x - 2) + p(2y - 2xy - x + 2) + y^2 + y = 0$ is

(A) $(y + cx + 2c)(y - cx + 1) = 0$

(B) $(y - cx + 2c)(y + cx + 1) = 0$

(C) $(y - cx + 2c)(y - cx + 1) = 0$

(D) $(y - cx + 2c)(y - cx - 1) = 0$

Passage 4

A curve which cuts every member of a given family of curves according to a given law is called a *trajectory* of the family. We shall consider only the case when each trajectory cuts every member of a given family at a constant angle. The trajectory will be called Orthogonal if the constant angle is a right angle. For example, every line through the origin of coordinates is an orthogonal trajectory of the family of concentric circles with centre at the origin. Let the equation of the given family of curves be

$$f(x, y, c) = 0 \quad (1)$$

Differentiate (1) and eliminate the arbitaray constant c between (1) and the resulting equation. That gives the differential equation of the family (1).

Let it be $F\left(x, y, \dfrac{dy}{dx}\right) = 0$ (2)

Replace $\dfrac{dy}{dx}$ by $-\dfrac{dx}{dy}$.

The differential equation of the orthogonal trajectory is

$$F\left(x, y, -\frac{dx}{dy}\right) = 0 \quad (3)$$

Integrate (3) to get the equation of the required orthogonal trajectory.

89. Orthogonal trajectory of the family of hyperbolas $xy = k^2$ is

(A) $x^2 + y^2 = c$ (B) $x^2 - y^2 = c$

(C) $2x^2 - 2y^2 = c$ (D) None of these

90. The orthogonal trajectory of the family of parabolas $y^2 = 4ax$ is

(A) $2x^2 + y^2 = c$

(B) $x^2 + 2y^2 = c$

(C) $2x^2 - y^2 = c$

(D) None of these

Match the Column Type

91. The differential equation of family of

Column-I	Column-II
I. Circles passing through the origin and having their centres on the x-axis	(A) $xy\dfrac{d^2y}{dx^2} + x\left(\dfrac{dy}{dx}\right)^2 - y\dfrac{dy}{dx} = 0$
II. Parabolas with foci at the origin and axis along the x-axis	(B) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$
III. Parabolas each of which has a latus rectum $4a$ and whose axes are parallel to x-axis	(C) $y\left(\dfrac{dy}{dx}\right)^2 + 2x\left(\dfrac{dy}{dx}\right) - y = 0$
IV. Ellipsescentred at the origin	(D) $2a\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 = 0$

92.

Column-I	Column-II
I. $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$	(A) $x^3y + 3e^x = cy$
II. $y(x^2y + e^x)dx - e^x dy = 0$	(B) $3x^4y^3 + 4x^3 = cy^3$
III. $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$	(C) $y = x\tan(c - x)$
IV. $(xy^4 + y)dx - xdy = 0$	(D) $y = 2x - 4$

93.

Column-I	Column-II
I. $y\,dx - x\,dy + (\log x)\,dx = 0$	(A) $\tan^{-1}\frac{y}{x} + x = c$
II. $(x^2\sin^3 y - y^2\cos x)\,dx + (x^3\cos y\sin^2 y - 2y\sin x)\,dy = 0$	(B) $y = 1 + \log x + c$
III. $\frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$	(C) $x^y = cy^x$
IV. $\frac{x\,dy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$	(D) $\frac{x^3\sin^3 y}{3} = y^2\sin x + c$

Assertion-Reason Type

Instructions: *In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options:*

(A) Assertion(A) is True and Reason(R) is True; Reason(R) is a correct explanation for Assertion(A)
(B) Assertion(A) is True, Reason(R) is True; Reason(R) is not a correct explanation for Assertion(A)
(C) Assertion(A) is True, Reason(R) is False
(D) Assertion(A) is False, Reason(R) is True

94. **Assertion:** The order of the differential equation, of which $xy = ce^x + be^{-x} + x^2$ is a solution, is 2.

Reason: The differential equation is

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0.$$

95. **Assertion:** A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis and the y-axis in points A and B, respectively, such that $\frac{1}{OA} + \frac{1}{OB} = 1$, where O is the origin. The equation of such a curve passing through (5, 4) is $(x - 1)^2 + (y - 1)^2 = 25$.

Reason: $OA = x + y\frac{dy}{dx}$ and $OB = \frac{\left(x + y\frac{dy}{dx}\right)}{\frac{dy}{dx}}$

96. **Assertion:** The solution of the equation $x\sin\theta d\theta + (x^3 - 2x^2\cos\theta + \cos\theta)\,dx = 0$ is $2\cos\theta = x + cx\,e^{-x2}$

Reason: Integrating factor $= \frac{e^{x^2}}{x}$

97. **Assertion:** The differential equation of all straight lines which are at a constant distance p from the origin is $(y - xy_1)^2 = p^2(1 + y_1^2)$

Reason: The general equation of any straight line which is at a constant distance p from the origin is $x\cos\alpha + y\sin\alpha = p$.

Previous Year's Questions

98. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are **[2002]**

(A) $\left(1, \frac{2}{3}\right)$ (B) (3, 1)

(C) (3, 3) (D) (l, 2)

99. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is **[2002]**

(A) $\frac{e^{-2x}}{4}$ (B) $\frac{e^{-2x}}{4} + cx + d$

(C) $\frac{1}{4}e^{-2x} + cx^2 + d$ (D) $\frac{1}{4}e^{-2x} + c + d$

100. The differential equation of all non-vertical lines in a plane is **[2002]**

(A) $\frac{d^2y}{dx^2} = 0$ (B) $\frac{d^2x}{dy^2} = 0$

(C) $\frac{dy}{dx} = 0$ (D) $\frac{dx}{dy} = 0$

101. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively **[2003]**

(A) 2, 1 (B) 1, 2 (C) 3, 2 (D) 2, 3

102. The solution of the differential equation $(1+y^2)(x-e^{2\tan^{-1}y})\frac{dy}{dx}=0$, is **[2003]**

(A) $(x-2)=k\,e^{-\tan^{-1}y}$

(B) $2x\,e^{\tan^{-1}}y=e^{2\tan^{-1}}+k$

(C) $x\,e^{\tan^{-1}y}=\tan^{-1}y+k$

(D) $x\,e^{2\tan^{-1}y}=e^{\tan^{-1}y}+k$

103. The differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant is **[2004]**

(A) $2(x^2-y^2)y'=xy$
(B) $2(x^2+y^2)y'=xy$
(C) $(x^2-y^2)y'=2xy$
(D) $(x^2+y^2)y'=2xy$

104. The solution of the differential equation $y\,dx+(x+x^{2y})\,dy=0$ is **[2004]**

(A) $-\frac{1}{xy}=C$ (B) $-\frac{1}{xy}+\log y=C$

(C) $\frac{1}{xy}+\log y=C$ (D) $\log y=Cx$

105. The differential equation representing the family of curves $y^2=2c\left(x+\sqrt{c}\right)$ where $c>0$, is a parameter, is of order and degree as follows: **[2005]**

(A) order 1, degree 2 (B) order 1, degree 1
(C) order 1, degree 3 (D) order 2, degree 2

106. If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the equation is **[2005]**

(A) $y\log\left(\frac{x}{y}\right)=cx$ (B) $x\log\left(\frac{y}{x}\right)=cy$

(C) $\log\left(\frac{y}{x}\right)=cx$ (D) $\log\left(\frac{x}{y}\right)=cy$

107. The differential equation whose solution is $Ax^2+By^2=1$, where A and B are arbitrary constants is of **[2006]**

(A) second order and second degree
(B) first order and second degree
(C) first order and first degree
(D) second order and first degree

108. The differential equation of all circles passing through the origin and having their centres on the x-axis is **[2007]**

(A) $x^2=y^2+xy\frac{dy}{dx}$ (B) $x^2=y^2+3xy\frac{dy}{dx}$

(C) $y^2=x^2+2xy\frac{dy}{dx}$ (D) $y^2=x^2-2xy\frac{dy}{dx}$

109. The solution of the differential equation $\frac{dy}{dx}=\frac{x+y}{x}$ satisfying the condition $y(1)=1$ is **[2008]**

(A) $y=\ln x+x$ (B) $y=x\ln x+x^2$
(C) $y=xe^{(x-1)}$ (D) $y=x\ln x+x$

110. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y=2$ is **[2008]**

(A) $(x-2)y'^2=25-(y-2)^2$
(B) $(y-2)y'^2=25-(y-2)^2$
(C) $(y-2)^2y'^2=25-(y-2)^2$
(D) $(x-2)^2y'^2=25-(y-2)^2$

111. The differential equation which represents the family of curves $y=c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants is **[2009]**

(A) $y'=y^2$ (B) $y''=y'y$
(C) $yy'=y'$ (D) $yy'=(y')^2$

112. Solution to the differential equation $\cos x\,dy=y(\sin x-y)\,dx$, $0<x<\frac{\pi}{2}$ is **[2010]**

(A) $y\sec x=\tan x+c$ (B) $y\tan x=\sec x+c$
(C) $\tan x=(\sec x+c)y$ (D) $\sec x=(\tan x+c)y$

113. Let l be the purchase value of an equipment and $V(t)$ be the value of equipment after it has been used for t years. The value $V(t)$ depreciates at a rate given by the differential equation $\frac{dV(t)}{dt}=k(T-t)$, where $k>0$ is a constant and T is the total life in years of the equipment. Then, the scrap value $V(T)$ of the equipment is **[2011]**

(A) $l-\frac{kT^2}{2}$ (B) $l-\frac{k(T-t)^2}{2}$

(C) e^{-kT} (D) $T^2-\frac{l}{k}$

114. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt}=0.5p(t)-450$ with initial condition $p(0)=850$, then the value of t for which $p(t)=0$ is **[2012]**

(A) 2 ln 18 (B) ln 9

(C) $\frac{1}{2}\ln 18$ (D) ln 18

115. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[2013]**

(A) 3000 (B) 3500 (C) 4500 (D) 2500

116. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If initially $p(0) = 100$, then $p(t)$ equals **[2014]**

(A) $400 - 300\, e^{t/2}$ (B) $300 - 200\, e^{-t/2}$

(C) $600 - 500\, e^{t/2}$ (D) $400 - 300\, e^{-t/2}$

117. Let $y(x)$ be the solution of the differential equation $(x\log x)\frac{dy}{dx} + y = 2x\log x, (x \geq 1)$. Then $y(e)$ is equal to: **[2015]**

(A) 0 (B) 2

(C) $2e$ (D) e

118. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)dx = x\,dy$. then $f\left(-\frac{1}{2}\right)$ is equal to: **[2016]**

(A) $\frac{4}{5}$ (B) $-\frac{2}{5}$

(C) $-\frac{4}{5}$ (D) $\frac{2}{5}$

ANSWER KEYS

Single Option Correct Type

1. (C)	**2.** (D)	**3.** (A)	**4.** (B)	**5.** (B)	**6.** (A)	**7.** (C)	**8.** (D)	**9.** (A)	**10.** (D)
11. (C)	**12.** (D)	**13.** (B)	**14.** (C)	**15.** (C)	**16.** (D)	**17.** (D)	**18.** (B)	**19.** (C)	**20.** (C)
21. (A)	**22.** (B)	**23.** (C)	**24.** (A)	**25.** (D)	**26.** (A)	**27.** (C)	**28.** (C)	**29.** (A)	**30.** (A)
31. (A)	**32.** (C)	**33.** (B)	**34.** (C)	**35.** (B)	**36.** (A)	**37.** (C)	**38.** (D)	**39.** (C)	**40.** (D)
41. (B)	**42.** (C)	**43.** (B)	**44.** (A)	**45.** (A)	**46.** (A)	**47.** (A)	**48.** (C)	**49.** (B)	**50.** (A)
51. (C)	**52.** (B)	**53.** (D)	**54.** (C)	**55.** (A)	**56.** (C)	**57.** (A)	**58.** (B)	**59.** (B)	**60.** (A)
61. (B)	**62.** (A)	**63.** (B)	**64.** (C)	**65.** (A)	**66.** (B)	**67.** (A)	**68.** (A)	**69.** (C)	**70.** (A)
71. (B)	**72.** (B)	**73.** (A)	**74.** (B)						

More than One Option Correct Type

75. (A) and (B) **76.** (B) and (C) **77.** (A), (B), (C) and (D) **78.** (A) and (B)

Passage Based Questions

79. (A) **80.** (C) **81.** (A) **82.** (B) **83.** (D) **84.** (C) **85.** (A) **86.** (A) and (C) **87.** (B)
88. (C) **89.** (B) **90.** (A)

Match the Column Type

91. I → (B); II → (C); III → (D); IV → (A)

92. I → (C); II → (A); III → (D); IV → (B)

93. I → (B); II → (D); III → (C); IV → (A)

Assertion-Reason Type

94. (A) **95.** (A) **96.** (A) **97.** (A)

Previous Year's Questions

98. (C)	**99.** (B)	**100.** (A)	**101.** (B)	**102.** (B)	**103.** (C)	**104.** (B)	**105.** (C)	**106.** (C)	**107.** (A)
108. (C)	**109.** (D)	**110.** (C)	**111.** (D)	**112.** (D)	**113.** (A)	**114.** (A)	**115.** (B)	**116.** (A)	**117.** (B)
118. (A)									

HINTS AND SOLUTIONS

Single Option Correct Type

1. Given equation can be re-written as

$$\frac{dy}{dx} - y\frac{\phi'(x)}{\phi(x)} = -\frac{y^2}{\phi(x)}$$

$$\Rightarrow \quad -\frac{1}{y^2}\cdot\frac{dy}{dx} + \frac{1}{y}\cdot\frac{\phi'(x)}{\phi(x)} = \frac{1}{\phi(x)} \qquad (1)$$

Putting $\frac{1}{y} = u$, we get $-\frac{1}{y^2}\frac{dy}{dx} = \frac{du}{dx}$

Now (1) becomes $\frac{du}{dx} + u\frac{\phi'(x)}{\phi(x)} = \frac{1}{\phi(x)}$

I.F. $= e^{\int \frac{\phi'(x)}{\phi(x)}dx} = f(x)$.

Multiplying both sides by I.F., we have

$$\frac{d}{dx}[u\phi(x)] = 1 \Rightarrow y = \frac{\phi(x)}{x+c}$$

The correct option is (C)

2. $\frac{y_2}{y_1} = 1 \Rightarrow d(\log y_1) = 1$

$\Rightarrow \quad \log y_1 = x + c$

$\Rightarrow \quad y_1 = ke^x$

$\Rightarrow \quad y = ke^x + B$

Since it passes through (0, 0), $k + B = 0$

Thus, family is $y = k(e^x - 1)$.

The correct option is (D)

3. $\sec^2 y\,\frac{dy}{dx} + 2x\tan y = x^3$

Let $\tan y = v$

$\Rightarrow \quad \sec^2 y\frac{dy}{dx} = \frac{dv}{dx}$

The given equation becomes

$$\frac{dv}{dx} + 2xv = x^3$$

Now, I.F. $= e^{\int 2x dx} = e^{x^2}$

Hence, the solution of the differential equation is

$$v\cdot e^{x^2} = \int x^3\cdot e^{x^2}dx + c$$

Let $x^2 = t \Rightarrow dt = 2x\,dx$

$$\Rightarrow \quad v\cdot e^{x^2} = \frac{1}{2}\int te^t dt + c = \frac{1}{2}e^t(t-1) + c$$

$$\Rightarrow \quad v\cdot e^{x^2} = \frac{1}{2}e^{x^2}(x^2-1) + c$$

$$\therefore \quad \tan y = \frac{1}{2}(x^2-1) + ce^{-x^2}$$

The correct option is (A)

4. $ydx + (x + x^2y)dy = 0$

$$\Rightarrow \quad \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2,$$

$$\Rightarrow \quad x^{-2}\frac{dx}{dy} + x^{-1}\left(\frac{1}{y}\right) = -1$$

Put $\quad x^{-1} = t \Rightarrow -x^{-2}\frac{dx}{dy} = \frac{dt}{dy}$,

get, $\quad -\frac{dt}{dy} + t\left(\frac{1}{y}\right) = -1 \Rightarrow \frac{dt}{dy} - \left(\frac{1}{y}\right)t = 1$

It is linear in t.

Integrating factor $= e^{\int -\frac{1}{y}dy} = e^{-\log y} = y^{-1}$

$\therefore$ Solution is $t(y^{-1}) = \int (y^{-1})dy + c$

$$\Rightarrow \quad \frac{1}{x}\cdot\frac{1}{y} = \log y + c \Rightarrow \log y - \frac{1}{xy} = C$$

The correct option is (B)

5. For the family of curves represented by the first differential equation, the slope of the tangent at any point (x, y) is given by

$$\left(\frac{dy}{dx}\right)_{c_1} = \frac{x^2+x+1}{y^2+y+1}$$

For the family of curves represented by the second differential equation, the slope of the tangent at any point is given by

$$\left(\frac{dy}{dx}\right)_{c_2} = -\frac{y^2+y+1}{x^2+x+1}$$

Since, $\left(\frac{dy}{dx}\right)_{c_1} \times \left(\frac{dy}{dx}\right)_{c_2} = -1$

Hence, the two curves are orthogonal.

The correct option is (B)

6. The given equation can be written in the linear form as follows:

$$\frac{dy}{dx} + yf'(x) = f(x)f'(x)$$

The integrating factor of this equation is $e^{\int \phi'(x)dx} = e^{f(x)}$.

Hence $\frac{d}{dx}(ye^{f(x)}) = f(x)f'(x)\,e^{f(x)}$

Integrating, we have $ye^{\phi(x)} = \int te^t\,dt + c$, (where $t = \phi(x)$)

$= te^t - e^t + c$

Hence, $y = (\phi(x) - 1) + c\,e^{-\phi(x)}$

The correct option is (A)

7. $x = e^{yx\frac{dy}{dx}} \Rightarrow \ln x = xy\dfrac{dy}{dx}$

$\Rightarrow \quad \int y\,dy = \int \frac{1}{x}\ln x\,dx$

$\Rightarrow \quad \frac{y^2}{2} = \frac{(\ln x)^2}{2} + k$

$\Rightarrow \quad y = \pm\sqrt{(\ln x)^2 + c}$

The correct option is (C)

8. Family of semi-cubical parabolas is given by

$ay^2 = x^3$ (1)

a is variable parameter

∴ Differentiating the above semi-cubical equation, we have

$2ay\dfrac{dy}{dx} = 3x^2$ (2)

Eliminating a from (1) and (2)

$\frac{2x^3}{y^3}\cdot y\frac{dy}{dx} = 3x^2 \Rightarrow 2x\frac{dy}{dx} = 3y$ (3)

Now for orthogonal trajectories changing $\frac{dy}{dx} \to -\frac{dx}{dy}$ in (3)

∴ $-2x\dfrac{dx}{dy} = 3y \Rightarrow 3y\,dy + 2x\,dx = 0$

Integrating we have

$\Rightarrow \frac{3y^2}{2} + \frac{x^2}{1} = \frac{c^2}{2} \Rightarrow 3y^2 + 2x^2 = c^2$

The correct option is (D)

9. The given equation can be written as

$2y\sin x\dfrac{dy}{dx} + y^2\cos x = \sin 2x$

$\Rightarrow \frac{d}{dx}(y^2\sin x) = \sin 2x$

On integrating, we get $y^2\sin x = -\frac{1}{2}\cos 2x + c$

Putting $x = \frac{\pi}{2}$ and $y = 1$, we get $c = \frac{1}{2}$.

Hence, $y^2\sin x = \frac{1}{2}(1 - \cos 2x)$

$\Rightarrow y^2\sin x = \sin^2 x$ or $y^2 = \sin x$.

The correct option is (A)

10. Here $y - x\dfrac{dy}{dx} + \log x = 0$

i.e., $x\dfrac{dy}{dx} - y = \log x$

i.e., $\dfrac{dy}{dx} - \dfrac{y}{x} = \dfrac{\log x}{x}$

∴ $e^{\int P dx} = e^{\int -1/x\,dx} = e^{(-\log x)} = e^{\log x^{-1}} = x^{-1}$

∴ solution is $yx^{-1} = \int \frac{\log x}{x}\cdot\frac{1}{x}dx + c$

i.e., $\frac{y}{x} = \int \frac{1}{x^2}\log x\,dx + c = \int e^{-t}\cdot t\,dt + c$

Put $\log x = t$ ∴ $\frac{1}{x}dx = dt$

Also, $x = e^t$

∴ $\frac{y}{x} = t\frac{e^{-t}}{-1} - 1\cdot\frac{e^{-t}}{-1} + c$

$= e^{-t}[-t + 1] + c$

$= e^{-\log x}(-\log x + 1) + c$

$= \frac{1}{x}(1 - \log x) + c$

∴ $y = (1 - \log x) + cx$

The correct option is (D)

11. The given diff. eqn. can be written as

$(\sin y\,dx + x\cos y\,dy)\,x^2\sin^2 y$

$= 2y\sin x\,dy + y^2\cos x\,dx$

$\Rightarrow d(x\sin y)(x\sin y)^2 = d(y^2\sin x)$

Integrating, we get

$\frac{(x\sin y)^3}{3} = y^2\sin x + c$

The correct option is (C)

12. Equation of the normal at (x, y) is $Y - y = -\dfrac{dx}{dy}(X - x)$

∴ x-intercept $= y\dfrac{dy}{dx} + x$ (Putting $Y = 0$)

Given: $y^2 = 2x\left(y\dfrac{dy}{dx} + x\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - 2x^2}{2xy}$. Put $y = vx$. Then

$v + x\frac{dy}{dx} = \frac{v^2 - 2}{2v} \Rightarrow x\frac{dv}{dx} = \frac{-(2 + v^2)}{2v}$

$\Rightarrow \int \frac{2v}{v^2 + 2}dv + \int\frac{dx}{x} = \log k$

$\Rightarrow \log(v^2 + 2) + \log x = \log k$

$\Rightarrow \quad x(v^2 + 2) = k \Rightarrow y^2 + 2x^2 - kx = 0$

If this passes through (2, 1), $i = \frac{9}{2}$

Then, the equation becomes $4x^2 + 2y^{\;2}\ 9x = 0$.

The correct option is (D)

13. Given: $\dfrac{x+y\dfrac{dy}{dx}}{y-x\dfrac{dy}{dx}}=x^2+2y^2+\dfrac{y^4}{x^2}$

$\Rightarrow \dfrac{d(x^2+y^2)}{(x^2+y^2)^2}=2\dfrac{d\left(\dfrac{x}{y}\right)}{\left(\dfrac{x}{y}\right)^2}$

Integrating, we get

$-\dfrac{1}{x^2+y^2}=\dfrac{-1}{x/y}+c\Rightarrow c=\dfrac{y}{x}-\dfrac{1}{x^2+y^2}$

The correct option is (B)

14. The given equation can be written as

$(2x-2y+5)dy=(x-y+3)dx$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{x-y+3}{2(x-y)+5}$

Put $x-y=V\Rightarrow 1-\dfrac{dy}{dx}=\dfrac{dV}{dx}$

Therefore, the given equation becomes

$1-\dfrac{dV}{dx}=\dfrac{V+3}{2V+5}$ or $\dfrac{dV}{dx}=\dfrac{V+2}{2V+5}$

$\Rightarrow dx=\dfrac{2V+5}{V+2}dV$ or $dx=\left(2+\dfrac{1}{V+2}\right)dV$

On integrating, we get

$x=2V+\log(V+2)+c$

$\Rightarrow x=2(x-y)+\log(x-y+2)+c$

Therefore, $2y-x=\log(x-y+2)+c$, is the required solution.

The correct option is (C)

15. Put $x=r\cos\theta, y=r\sin\theta$

$\therefore dx=-r\sin\theta\,d\theta+dr\cos\theta$

$dy=r\cos\theta\,d\theta+dr\sin\theta$

$\therefore$ We have

$$\frac{r\cos\theta(-r\sin\theta d\theta+dr\cos\theta)+r\sin\theta(r\cos\theta d\theta+dr\sin\theta)}{(r\cos\theta)(r\cos\theta d\theta+dr\sin\theta)-r\sin\theta(-r\sin\theta d\theta+dr\cdot\cos\theta)}=\frac{\sqrt{1-r^2}}{\sqrt{r^2}}$$

$$\Rightarrow \frac{-r^2\sin\theta\cos\theta d\theta+r\cos^2\theta dr+r^2\sin\theta\cos\theta d\theta+r\sin^2\theta dr}{r^2\cos^2\theta d\theta+r\cos\theta\sin\theta dr+r^2\sin^2\theta d\theta-r\sin\theta\cos\theta dr}=\frac{\sqrt{1-r^2}}{r}$$

$$\Rightarrow \frac{r\,dr}{r^2d\theta}=\frac{\sqrt{1-r^2}}{r}$$

$$\Rightarrow \frac{dr}{\sqrt{1-r^2}}=d\theta$$

$$\Rightarrow \sin^{-1}r=\theta+c$$

$$\Rightarrow \sin^{-1}\sqrt{x^2+y^2}=\tan^{-1}\frac{y}{x}+c.$$

The correct option is (C)

16. The family of curves which are orthogonal (i.e., intersect at right angles) to a given system of curves is obtained by substituting $-\dfrac{dx}{dy}$ for $\dfrac{dy}{dx}$ in the differential equation of the given system.

The given differential equation is $\left(\dfrac{dy}{dx}\right)^2=\dfrac{a}{x}$

Replacing $\dfrac{dy}{dx}$ by $-\dfrac{dx}{dy}$, we get

$\left(\dfrac{dx}{dy}\right)^2=\dfrac{a}{x}\Rightarrow\left(\dfrac{dy}{dx}\right)^2=\dfrac{x}{a}\Rightarrow\dfrac{dy}{dx}=\pm\sqrt{\dfrac{x}{a}}$

Integrating we get, $y+c=\pm\dfrac{2}{3\sqrt{a}}x^{3/2}$ (1)

$\Rightarrow (y+c)^2=\dfrac{4}{9a}x^3\Rightarrow 9a(y+c)^2=4x^3$ (2)

From (1) and (2), all of the first three given options represent required equations.

The correct option is (D)

17. $\left(\dfrac{x+y-1}{x+y-2}\right)\dfrac{dy}{dx}=\left(\dfrac{x+y+1}{x+y+2}\right)$

Put $x+y=t$

$1+\dfrac{dy}{dx}=\dfrac{dt}{dx}\Rightarrow\dfrac{dt}{dx}-1=\left(\dfrac{t+1}{t+2}\right)\left(\dfrac{t-2}{t-1}\right)$

$\Rightarrow \dfrac{dt}{dx}=\left(\dfrac{t^2-t-2}{t^2+t-2}\right)+1$

On solving, we get $2(y-x)+\log\left(\dfrac{(x+y)^2-2}{2}\right)=0$

The correct option is (D)

18. The given differential equation is

$[y+x\sqrt{xy}(x+y)]\,dx+(y\sqrt{xy}(x+y)-x)\,dy=0$

$\Rightarrow y\,dx-x\,dy+\sqrt{xy}\,(x+y)(x\,dx+y\,dy)=0$

$\Rightarrow x\,dx+y\,dy=\dfrac{x\,dy-y\,dx}{(x+y)\sqrt{xy}}$

$\Rightarrow \dfrac{1}{2}d(x^2+y^2)=2\,d\left(\tan^{-1}\sqrt{\dfrac{y}{x}}\right)$

$\Rightarrow x^2+y^2=4\tan^{-1}\sqrt{\dfrac{y}{x}}+c.$

The correct option is (B)

19. Given: $\frac{dx}{dt} = \cos^2 \pi x$. Differentiate with respect to t,

$$\frac{d^2x}{dt^2} = -2\pi \sin 2\pi x = -\text{ve}$$

$$\because \quad \frac{d^2x}{dt^2} = 0$$

$\Rightarrow \quad 2\pi \sin 2\pi x = 0 \Rightarrow \sin 2\pi x = \sin \pi$

$\Rightarrow 2\pi x = \pi \Rightarrow x = \frac{1}{2}$

The correct option is (C)

20. Given: Cartesian sub-tangent $\propto \frac{1}{\text{square of abscissa}}$

i.e., $\frac{y}{dy/dx} = \frac{k}{x^2}$ or $\frac{dy}{y} = \frac{x^2}{k}dx$

Integrating, $\log y = \frac{x^3}{3k} + \log c$

or $y = ce^{x^3/3k}$

The correct option is (C)

21. $\frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$

$\Rightarrow \quad x(y \log x - x)\frac{dy}{dx} = y(x \log y - y)$

$\Rightarrow \quad \left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \log y - \frac{y}{x}$

$\Rightarrow \quad \frac{y}{x} + \log x \cdot \frac{dy}{dx} = \log y + \frac{x}{y}\frac{dy}{dx}$

$\Rightarrow \quad \frac{d}{dx}(y \log x) = \frac{d}{dx}(x \log y)$

$\Rightarrow \quad y \log y = x \log y + \log c$

$\Rightarrow \quad \log x^y = \log y^x + \log c$

$\Rightarrow \quad x^y = c\, y^x$

The correct option is (A)

22. $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$\Rightarrow \quad (2y+f)dy = (ax+3)dx$

$\Rightarrow \quad \frac{2y^2}{2} + fy = \frac{ax^2}{2} + 3x + c$

For the curve to be circle, $a = -2$ where $f^2 + 9 + 4c > 0$.

The correct option is (B)

23. Given diff. equation can be written as

$$\frac{x\,dy - y\,dx}{x^2+y^2} = -dx \Rightarrow \frac{x\,dy - y\,dx}{x^2\left[1+\left(\frac{y}{x}\right)^2\right]} = -dx$$

$$\Rightarrow \quad \frac{d(y/x)}{1+\left(\frac{y}{x}\right)^2} + dx = 0$$

Integrating, we get

$\tan^{-1}\frac{y}{x} + x = c$

The correct option is (C)

24. $\frac{dy}{dx}\left(\frac{2+\sin x}{1+y}\right) = -\cos x,\ y(0) = 1$

$$\Rightarrow \quad \frac{dy}{(1+y)} = \frac{-\cos x}{2+\sin x}dx$$

Integrating both sides $\Rightarrow \ln(1+y) = -\ln(2+\sin x) + c$ Put $x = 0$ and $y = 1 \Rightarrow \ln(2) = -\ln 2 + c \Rightarrow c = \ln 4$.

Put $x = \frac{\pi}{2}$, $\ln(1+y) = \ln 3 + \ln 4\ \frac{4}{3} = \ln \Rightarrow y = \frac{1}{3}$

The correct option is (A)

25. We have, $\frac{dy}{dx} = \frac{2ay}{x(y-a)}$

$$\Rightarrow \quad \frac{y-a}{y}dy = \frac{2a}{x}dx$$

On integrating both sides, we get

$a \log|y| - y = -2a \log|x| + \text{loc } c$

$\Rightarrow \quad y^a \times x^{2a} = ce^y$

Since, the curve passes through (1, 1), therefore

$1 = ce \Rightarrow c = \frac{1}{e}$

So, the equation of the curve is

$y^a \cdot x^{2a} = e^{y-1}$

The correct option is (D)

26. We have $(x\cos x - \sin x + yx^2)\,dx + x^3 dy = 0$

$$\Rightarrow \quad \frac{x\cos x - \sin x}{x^2}dx + ydx + xdy = 0$$

$$\Rightarrow \quad d\left(\frac{\sin x}{x}\right) + d(xy) = d(c)$$

$$\Rightarrow \quad \frac{\sin x}{x} + xy = c$$

The correct option is (A)

27. Dividing the given equation by y^2, we get

$$\frac{y\,dx - x\,dy}{y^2} = -3x^2 e^{x^3}\,dx \Rightarrow \frac{d}{dx}\left(\frac{x}{y}\right) = -\frac{d}{dx}\left(e^{x^3}\right)$$

On integrating, we get, $\frac{x}{y} = -e^{x^3} + c \Rightarrow \frac{x}{y} + e^{x^3} = c$

The correct option is (A)

28. The given diff. equation can be written as

$$\Rightarrow (y\,dx + x\,dy) + \left(\frac{y}{x}dx + \log x\right)dy$$

$$+ \sin y\,dx + x\cos y\,dy = 0$$

$\Rightarrow d(xy) + d(y\log x) + d(x\sin y) = 0$

Integrating, we get

$xy + y \log x + x \sin y = c.$

The correct option is (C)

29. The equation of the normal at any point (x, y) is given by, $Y - y = -\frac{dx}{dy}(X - x)$.

This passes through (h, k)

$\Rightarrow k - y = -\frac{dx}{dy}(h - x) \Rightarrow (y - k) = (h - x)\frac{dx}{dy}$.

The correct option is (A)

30. $3xy^2dy + y^3dx + \sin(xy)(xdy + ydx) = 0$

or $d(xy^3) + \sin(xy)\, d(xy) = 0$

On integrating, we get

$xy^3 - \cos xy = c$

The correct option is (A)

31. We have

Cartesian subtangent + abscissa = constant

$\Rightarrow \frac{y}{dy/dx} + x = a \Rightarrow y\frac{dy}{dx} + x = a \Rightarrow \frac{dy}{y} = \frac{dx}{a - x}$

Integrating, we get $\log y + \log(x - a) = \log c$

$\therefore \; y(x - a) = c$

As the curve passes through the point $(2a, a)$, we have $c = a^2$

Hence the required curve is $y(x - a) = a^2$.

The correct option is (A)

32. $\frac{dt}{dx} - t\frac{g'(x)}{g(x)} = -\frac{t^2}{g(x)}$

$$\Rightarrow \frac{1}{t^2}\frac{dt}{dx} + \frac{1}{t}\frac{g'(x)}{g(x)} = \frac{1}{g(x)} \qquad (1)$$

Let $\frac{1}{t} = z$

$\therefore \frac{1}{t^2}\frac{dt}{dx} = \frac{dz}{dx}$

Now by (1), we have

$$\frac{dz}{dx} + z\frac{g'(x)}{g(x)} = \frac{1}{g(x)}$$

I.F. $= e^{\int \frac{g'(x)}{g(x)}dx} = g(x)$

Therefore, the solution is $z\,g(x) = x + c \Rightarrow g(x)\frac{1}{t} = x + c$

$$\Rightarrow \qquad t = \frac{g(x)}{x + c}$$

The correct option is (C)

33. $xdy + (y + x^3y^2)\,dx = 0 \Rightarrow xdy + ydx = -x^3y^2dx$

$\Rightarrow \frac{x\,dy + ydx}{x^2y^2} = -x\,dx$

$\Rightarrow \frac{d(xy)}{(xy)^2} = -x\,dx$

Integrating, we get

$$\Rightarrow -\frac{1}{xy} = -\frac{x^2}{2} + c \qquad (1)$$

Using (1, 2) in (1), we get

$-\frac{1}{2} = -\frac{1}{2} + c \Rightarrow c = 0.$

$\therefore \; -\frac{1}{xy} = -\frac{x^2}{2} \Rightarrow y = \frac{2}{x^3}$ or $x^3y = 2$

is the required curve.

The correct option is (B)

34. $\int_I^{V(t)} dV(t) = \int_{t=0}^{T} -k(T - t)dt$

$\Rightarrow V(T) - I = k\left[\frac{(T - t)^2}{2}\right]_0^T$

$\Rightarrow V(T) - I = -k\left(\frac{T^2}{2}\right)$

$\Rightarrow \quad V(T) = I - \frac{kT^2}{2}$

The correct option is (C)

35. We have

$$\frac{dy}{dx} = y + 3$$

$\Rightarrow \frac{1}{y + 3}dy = dx$

$\Rightarrow \ln|(y + 3)| = x + k$, where k is a constant of integration

$\Rightarrow (y + 3) = c\,e^x$

Initially when $x = 0$, $y = 2$

$\Rightarrow c = 5$

Finally the required solution is $y + 3 = 5e^x$

$\Rightarrow y(\ln 2) = 5e^{\ln 2} - 3 = 10 - 3 = 7$

The correct option is (B)

36. $2\frac{dp(t)}{900 - p(t)} = -dt$

$-2\ln[900\; p(t)] = -t + c$

when $t = 0$, $p(0) = 850$

$-2\ln(50) = c$

$\therefore \; 2\ln\left(\frac{50}{900 - p(t)}\right) = -t$

$900 - p(t) = 50\,e^{t/2}$

$p(t) = 900 - 50\,e^{t/2}$

let $p(t_1) = 0$

$0 = 900 - 500\;e^{t_1/2}$

$\therefore \; t_1 = 2\ln 18$

The correct option is (A)

37. $\int \frac{y\,dy}{\sqrt{1-y^2}} = \int dx \Rightarrow -\frac{1}{2}\int \frac{-2y\,dy}{\sqrt{1-y^2}} = x + c$

$\Rightarrow -\sqrt{1-y^2} = x + c \Rightarrow 1 - y^2 = x^2 + c^2 + 2cx$

$\Rightarrow x^2 + y^2 + 2cx + c^2 - 1 = 0$

Which is a circle with centre $(-c, 0)$ and radius 1.

The correct option is (C)

38. $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = 1 + v$$

$$\Rightarrow dv = \frac{dx}{x}$$

$$\therefore v = \log x + c$$

$$\Rightarrow \frac{y}{x} = \log x + c$$

Since, $y(1) = 1$, we have

$$y = x \log x + x$$

The correct option is (D)

39. $(x-h)^2 + (y-2)^2 = 25$ (1)

$$\Rightarrow 2(x-h) + 2(y-2)\frac{dy}{dx} = 0$$

$$\Rightarrow (x-h) = -(y-2)\frac{dy}{dx}$$

Substituting in Equation (1), we have

$$(y-2)^2\left(\frac{dy}{dx}\right)^2 + (y-2)^2 = 25$$

$(y-2)^2 y'^2 = 25 - (y-2)^2$

The correct option is (C)

40. $y = c_1 e^{c_2 x}$ (1)

$y' = c_2 c_1 e^{c_2 x}$

$y' = c_2 y$ (2)

$y'' = c_2 y'$

From Equation (2)

$$c_2 = \frac{y'}{y}$$

So, $y'' = \frac{(y')^2}{y} \Rightarrow yy'' = (y')^2$

The correct option is (D)

41. We have, $\frac{dy}{dx} = (g(x) - y).\, g'(x)$

Put $g(x) - y = V \Rightarrow g'(x) - \frac{dy}{dx} = \frac{dV}{dx}$

Hence, $g'(x) - \frac{dV}{dx} = V \cdot g'(x)$

$$\Rightarrow \frac{dV}{dx} = (1 - V)\, g'(x) \Rightarrow \frac{dV}{1-V} = g'(x)\,dx$$

$$\Rightarrow \int \frac{dV}{1-V} = \int g'(x)\,dx$$

$\Rightarrow -\log(1 - V) = g(x) - C$

$\Rightarrow g(x) + \log(1 - V) = C$

$\therefore g(x) + \log[1 + y - g(x)] = C$

The correct option is (B)

42. We have, $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0$

$$\Rightarrow \frac{1}{2}\, d(x^2 + y^2) + d\tan^{-1}\left(\frac{y}{x}\right) = 0$$

Integrating, $\frac{1}{2}(x^2 + y^2) + \tan^{-1}\frac{y}{x} = \frac{c}{2}$

$$\Rightarrow x^2 + y^2 + 2\tan^{-1}\frac{y}{x} = c$$

$\therefore y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$ is the required solution.

The correct option is (C)

43. We have, $x\,dy = \left(y + \frac{xf(y/x)}{f'(y/x)}\right)dx$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{f(y/x)}{f'(y/x)}$ which is homogeneous.

Put $y = Vx$ so that $\frac{dy}{dx} = V + x\frac{dV}{dx}$,

we obtain,

$$V + x\frac{dV}{dx} = V + \frac{f(V)}{f'(V)} \Rightarrow \frac{f(V)}{f'(V)}\,dV = \frac{dx}{x}$$

Integrating, we get

$\log f(V) = \log x + \log c$

$\Rightarrow \log f(V) = \log cx \Rightarrow f\left(\frac{y}{x}\right) = cx$

The correct option is (B)

44. We have, $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

$$\Rightarrow e^y\frac{dy}{dx} + e^x \cdot e^y = e^{2x}.$$

Putting $e^y = V$ so that $e^y\frac{dy}{dx} = \frac{dV}{dx}$, we get

$\frac{dV}{dx} + e^x \times V = e^{2x}$, which is linear in V.

I.F. $= e^{\int e^x dx} = e^{e^x}$.

So, the solution is

$$V \cdot e^{e^x} = \int e^{e^x} \cdot e^{2x}\,dx + c$$

$\Rightarrow \quad e^y \cdot e^{e^x} = \int e^z \cdot z\, dz + c$

[Putting $e^x = z \Rightarrow e^x\, dx = dz$]

$\Rightarrow \quad e^y \cdot e^{e^x} = (z-1)\, e^z + c = (e^x - 1)\, e^{e^x} + c$

$\Rightarrow \quad e^y = e^x - 1 + ce^{-ex}$

The correct option is (A)

45. We have, $x\left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$

$\Rightarrow \quad \left(\frac{dy}{dx} - 1\right)\left(x\frac{dy}{dx} + y\right) = 0$

$\Rightarrow \quad \frac{dy}{dx} = 1$ or $x\frac{dy}{dx} = -y$

The solution of $\frac{dy}{dx} = 1$ is $y = x + c$

and solution of $x\frac{dy}{dx} = -y$ i.e., $\frac{dy}{y} + \frac{dx}{x} = 0$ is

$\log(xy) = \log c$ i.e., $xy = c$.

Hence, general solution is $(x - y + c)(xy - c) = 0$.

The correct option is (A)

46. The equation of the general circle is given by

$x^2 + y^2 + 2gy + 2fy + c = 0$ (1)

Differentiating with respect to x, we get

$2x + 2yy' + 2g + 2fy' = 0$ (2)

Differentiating again, we get

$1 + y'^2 + yy'' + fy'' = 0$ (3)

Differentiating again, we have

$2y'y'' + yy''' + y'y'' + fy''' = 0$ (4)

Eliminating f from (3) and (4), we get

$y'''(1 + yy'' + y'^2) - y''(yy''' + 3y'y'') = 0$

$\Rightarrow y'''(1 + y'^2) - 3y'y''^2 = 0$, which is the required differential equation.

The correct option is (A)

47. Let $m_1 = \frac{dy}{dx}$ for required family of curves at (x, y).

Let $m_2 = \frac{dy}{dx}$ for the hyperbola $xy = 2$.

Then, $m_2 = \frac{dy}{dx} = \frac{-2}{x^2}$.

Since the required family of curves is orthogonal to the hyperbola, $\therefore m_1 \times m_2 = -1$

$\Rightarrow \quad \frac{dy}{dx} \times \left(\frac{-2}{x^2}\right) = -1 \Rightarrow \frac{dy}{dx} = \frac{x^2}{2}. \Rightarrow dy = \frac{x^2}{2}dx$

Integrating, we get $y = \frac{x^3}{6} + C$, which is the required family.

The correct option is (A)

48. We have, $x^2\frac{dy}{dx} - xy = 1 + \cos\frac{y}{x} = 2\cos^2\frac{y}{2x}$.

$\Rightarrow \quad \sec^2\left(\frac{y}{2x}\right)\cdot\left[x^2\frac{dy}{dx} - xy\right] = 2$

$\Rightarrow \quad \frac{1}{2}\sec^2\left(\frac{y}{2x}\right)\cdot\frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x^3}$

$\Rightarrow \quad \frac{d}{dx}\left(\tan\frac{y}{2x}\right) = \frac{1}{x^3}$.

Integrating, we get $\tan\frac{y}{2x} = c - \frac{1}{2x^2}$,

which is the required solution.

The correct option is (C)

49. Let $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$.

Therefore, the given equation becomes

$\left(\frac{t-1}{t-2}\right)\left(\frac{dt}{dx} - 1\right) = \frac{t+1}{t+2}$

$\Rightarrow \quad \frac{dt}{dx} - 1 = \left(\frac{t+1}{t+2}\right)\left(\frac{t-2}{t-1}\right)$

$\Rightarrow \quad \frac{dt}{dx} = \frac{t^2 - t - 2}{t^2 + t - 2} + 1$

$\Rightarrow \quad \frac{dt}{dx} = \frac{2t^2 - 4}{t^2 + t - 2}$

$\Rightarrow \quad \frac{t^2 + t - 2}{t^2 - 2}dt = 2dx$

$\Rightarrow \quad \left(1 + \frac{t}{t^2 - 2}\right)dt = 2dx$

On integrating, we get

$$t + \frac{1}{2}\log\left|t^2 - 2\right| = 2x + c$$

$\Rightarrow (x + y) + \frac{1}{2}\log\left|(x+y)^2 - 2\right| = 2c$ (1)

Given, $y = 1$, when $x = 1$, therefore $\log 2 = 2c$.

Substituting the value of c in (1), we get

$2(y - x) + \log|(x + y)^2 - 2| = \log 2$

$\Rightarrow \quad 2(y - x) + \log\left|\frac{(x+y)^2 - 2}{2}\right| = 0. \therefore k = 2.$

The correct option is (B)

50. We have, $x\, dy - y\, dx = xy^3(1 + \log x)\, dx$

$\Rightarrow \quad -\left(\frac{ydx - xdy}{y^2}\right) = xy(1 + \log x)\, dx$

$$\Rightarrow \quad -d\left(\frac{x}{y}\right) = xy\,(1 + \log x)\,dx$$

$$\Rightarrow \quad -\frac{x}{y}\,d\left(\frac{x}{y}\right) = x^2\,(1 + \log x)\,dx$$

Integrating, we get

$$\frac{-\left(\frac{x}{y}\right)^2}{2} = (1 + \log x)\,\frac{x^3}{3} - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx$$

$$\Rightarrow \quad -\frac{x^2}{2y^2} = \frac{x^3}{3}(1 + \log x) - \frac{x^3}{9} + \frac{C}{2}$$

$$\Rightarrow \quad -\frac{x^2}{y^2} = \frac{2x^3}{3}\left(\frac{2}{3} + \log x\right) + C$$

The correct option is (A)

51. Given equation can be rewritten as

$$\frac{dy}{dx} - y\frac{\phi'(x)}{\phi(x)} = -\frac{y^2}{\phi(x)}$$

$$\Rightarrow \quad -\frac{1}{y^2}\cdot\frac{dy}{dx} + \frac{1}{y}\cdot\frac{\phi'(x)}{\phi(x)} = \frac{1}{\phi(x)} \qquad (1)$$

Putting $\frac{1}{y} = u$, we get $-\frac{1}{y^2}\frac{dy}{dx} = \frac{du}{dx}$.

Now, (1) becomes $\frac{du}{dx} + u\frac{\phi'(x)}{\phi(x)} = \frac{1}{\phi(x)}$

I.F. = $e^{\int \frac{\phi'(x)}{\phi(x)}} = \phi(x)$. Multiplying both sides by I.F., we have

$$\frac{d}{dx}[u\,\phi(x)] = 1 \Rightarrow y = \frac{\phi(x)}{x + c}$$

The correct option is (C)

52. $ydx + (x + x^2y)dy = 0$

$$\Rightarrow \quad \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2,$$

It is Bernoulli's form. Divide by x^2

$$x^{-2}\frac{dx}{dy} + x^{-1}\left(\frac{1}{y}\right) = -1$$

Put $x^{-1} = t, -x^{-2}\frac{dx}{dy} = \frac{dt}{dy}$

We get, $-\frac{dt}{dy} + t\left(\frac{1}{y}\right) = -1 \Rightarrow \frac{dt}{dy} - \left(\frac{1}{y}\right)t = 1$

It is linear in t.

Integrating factor = $e^{\int -\frac{1}{y}dy} = e^{-\log y} = y^{-1}$

$\therefore$ Solution is $t(y^{-1}) = \int (y^{-1})\,dy + c$

$$\Rightarrow \quad \frac{1}{x}\cdot\frac{1}{y} = \log y + c \Rightarrow \log y - \frac{1}{xy} = C$$

The correct option is (B)

53. Equation of the normal at (x, y) is $Y - y = -\frac{dx}{dy}(X - x)$

$\therefore$ x-intercept $= y\frac{dy}{dx} + x$ (Putting $Y = 0$)

Given, $y^2 = 2x\left(y\frac{dy}{dx} + x\right)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - 2x^2}{2xy}.$$ Putting $y = vx$. Then,

$$v + x\frac{dy}{dx} = \frac{v^2 - 2}{2v} \Rightarrow x\frac{dv}{dx} = \frac{-(2 + v^2)}{2v}$$

$$\Rightarrow \quad \int \frac{2v}{v^2 + 2}dv + \int \frac{dx}{x} = \log K$$

$$\Rightarrow \quad \log(v^2 + 2) + \log x = \log K$$

$$\Rightarrow \quad x\,(v^2 + 2) = K \Rightarrow y^2 + 2x^2 - Kx = 0$$

If this passes through (2, 1), $K = \frac{9}{2}$

Then, the equation becomes $4x^2 + 2y^2 - 9x = 0$.

The correct option is (D)

54. Put $x = r\cos q$, $y = r\sin q$

$\therefore$ $dx = -r\sin q\,dq + dr\cos q$

$dy = r\cos q\,dq + dr\sin q$

$\therefore$ We have

$$\frac{r\cos\theta(-r\sin\theta\,d\theta + dr\cos\theta) + r\sin\theta\,(r\cos\theta\,d\theta + dr\sin\theta)}{(r\cos\theta)\,(r\cos\theta\,d\theta + dr\sin\theta) - r\sin\theta\,(-r\sin\theta\,d\theta + dr\cos\theta)} = \frac{\sqrt{1 - r^2}}{\sqrt{r^2}}$$

$$\Rightarrow \quad \frac{-r^2\sin\theta\cos\theta\,d\theta + r\cos^2\theta\,dr + r^2\sin\theta\cos\theta\,d\theta + r\sin^2\theta\,dr}{r^2\cos^2\theta\,d\theta + r\cos\theta\sin\theta\,dr + r^2\sin^2\theta\,d\theta - r\sin\theta\cos\theta\,dr} = \frac{\sqrt{1 - r^2}}{r}$$

$$\Rightarrow \quad \frac{r\,dr}{r^2 d\theta} = \frac{\sqrt{1 - r^2}}{r} \Rightarrow \frac{dr}{\sqrt{1 - r^2}} = dq$$

$$\Rightarrow \quad \sin^{-1} r = q + c \Rightarrow \sin^{-1}\sqrt{x^2 + y^2} = \tan^{-1}\frac{y}{x} + c.$$

The correct option is (C)

55. We have $(x\cos x - \sin x + yx^2)\,dx + x^3dy = 0$

$$\Rightarrow \quad \frac{x\cos x - \sin x}{x^2}dx + ydx + xdy = 0$$

$$\Rightarrow \quad d\left(\frac{\sin x}{x}\right) + d\,(xy) = d\,(c)$$

$$\Rightarrow \quad \frac{\sin x}{x} + xy = c$$

The correct option is (A)

56. The given differential equation can be written as

$$\Rightarrow (y\,dx + x\,dy) + \left(\frac{y}{x}dx + \log x\right)dy$$

$$+ \sin y\,dx + x\cos y\,dy = 0$$

$\Rightarrow d\,(xy) + d\,(y\log x) + d\,(x\sin y) = 0$

Integrating, we get

$xy + y\log x + x\sin y = c$

The correct option is (C)

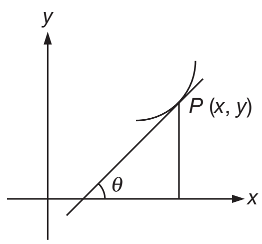
57. $Y - y = \dfrac{dy}{dx}\,(X - x) \Rightarrow 5 = \dfrac{1}{2}y^2\dfrac{dx}{dy}$

$$\Rightarrow\quad 10\,\frac{dy}{y^2} = -\,dx$$

$$\Rightarrow\quad -\frac{10}{y} + x = c$$

Since it passes through (2, 5), therefore, $c = 0$.

Thus, the equation of curve is $xy = 10$



The correct option is (A)

58. $xdy + (y + x^3y^2)\,dx = 0 \quad \Rightarrow xdy + ydx = -x^3y^2dx$

$$\Rightarrow\quad \frac{x\,dy + ydx}{x^2y^2} = -x\,dx \Rightarrow \frac{d\,(xy)}{(xy)^2} = -x\,dx$$

Integrating, we get

$$-\frac{1}{xy} = -\frac{x^2}{2} + c \qquad (1)$$

Using (1, 2) in (1), we get

$$-\frac{1}{2} = -\frac{1}{2} + c \;\Rightarrow c = 0$$

$$\therefore\quad -\frac{1}{xy} = -\frac{x^2}{2} \;\Rightarrow y = \frac{2}{x^3} \text{ or } x^3y = 2$$

is the required curve.

The correct option is (B)

59. Let $x = r\cos\theta$, $y = r\sin\theta$

so that $x^2 + y^2 = r^2$ (1)

and, $\tan q = \dfrac{y}{x}$ (2)

From (1), we have $d(x^2 + y^2) = d(r^2)$

$\Rightarrow\quad x\,dx + y\,dy = rdr$ (3)

From (2), we have $d\left(\dfrac{y}{x}\right) = d(\tan\theta)$

i.e., $\dfrac{x\,dy - y\,dx}{x^2} = \sec^2\theta\,d\theta$

i.e., $x\,dy - y\,dx = x^2\sec^2\theta\,d\theta = r^2\cos^2\theta\sec^2\theta\,d\theta$ (4)

Using (3) and (4) in the given equation, we get;

$$\frac{rdr}{r^2d\theta} = \sqrt{\frac{a^2 - r^2}{r^2}} \text{ i.e., } \frac{dr}{\sqrt{a^2 - r^2}} = d\theta$$

i.e., $\sin^{-1}\left(\dfrac{r}{a}\right) = \theta + c$

or, $r = a\sin(\theta + c)$

or, $\sqrt{x^2 + y^2} = a\sin[c + \tan^{-1}(y/x)]$

The correct option is (B)

60. The given differential equation can be written as

$$(\log y)\,2x\,dx + \frac{x^2}{y}\,dy + 3y^2dy = 0$$

$\Rightarrow\quad (\log y)\,d(x^2) + x^2d(\log y) + d(y^3) = 0$

$\Rightarrow\quad d(x^2\log y) + d(y^3) = 0$

$\Rightarrow\quad x^2\log y + y^3 = c$ (Integrating both sides)

The correct option is (A)

61. The given differential equation can be written as;

$$\frac{y\,dx + x\,dy}{\cos^2(xy)} + \sin x\,dx + \sin y\,dy = 0$$

$\Rightarrow \sec^2(xy)\,d(xy) + \sin x\,dx + \sin y\,dy = 0$

$\Rightarrow \tan(xy) - \cos x - \cos y = c$ (Integrating both sides)

The correct option is (B)

62. The equation can be written as

$$\frac{dx}{dy} = \frac{2y\,ln\,y + y - x}{y} = (2\,ln\,y + 1) - \frac{x}{y}$$

i.e., $\dfrac{dx}{dy} + \dfrac{1}{y}\cdot x = (2\ln y + 1)$

$$\therefore\quad \text{I.F.} = e^{\int P\,dy} = e^{\int \frac{1}{y}dy} = e^{ln\,y} = y$$

$\therefore$ The solution is

$$xy = \int (2\ln y + 1)\cdot y\,dy = y^2\ln y + c$$

$$\Rightarrow\quad x = y\ln y + \frac{c}{y}$$

The correct option is (A)

63. The given equation can be written as;

$$\frac{dy}{dx} - \sec^2 x\cdot\tan 2x\cdot y = \cos^2 x$$

$$\text{I.F.} = e^{\int -\tan 2x\sec^2 x\,dx} = e^{\int \frac{2\tan x}{\tan^2 x - 1}\sec^2 x\,dx}$$

$= e^{\int \frac{dt}{t}}$ where $t = \tan^2 x - 1$

$= e^{\ln|t|} = |t| = |\tan^2 x - 1|$

It is given that $|x| < \pi/4$ and for this region $\tan^2 x < 1$

$\therefore$ I.F. $= 1 - \tan^2 x$

$\therefore$ The solution is;

$$y(1 - \tan^2 x) = \int \cos^2 x \left(1 - \tan^2 x\right) dx$$

$$= \int \left(\cos^2 x - \sin^2 x\right) dx = \int \cos 2x \, dx$$

$$= \frac{\sin 2x}{2} + c$$

Now, when $x = \pi/6$, $y = \frac{3\sqrt{3}}{8}$

$$\therefore \quad \frac{3\sqrt{3}}{8}\left(1 - \frac{1}{3}\right) = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + c \Rightarrow c = 0$$

$$\therefore \quad y = \frac{\sin 2x}{2\left(1 - \tan^2 x\right)}$$

The correct option is (B)

64. The given equation can be written as

$$\left(\frac{dy}{dx} + 1\right) + x(x + y) = x^3(x + y)^3$$

$$\Rightarrow \quad \frac{d(x + y)}{dx} + x\,(x + y) = x^3(x + y)^3$$

$$\Rightarrow \quad (x + y)^3 \frac{d(x + y)}{dx} + x\,(x + y)^{-2} = x^3$$

Let $(x + y)^{-2} = z$ so that $-2\,(x + y)^{-3} \frac{d(x + y)}{dx} = \frac{dz}{dx}$

The given equation reduces to

$$\frac{-1}{2}\frac{dz}{dx} + xz = x^3 \text{ i.e., } \frac{dz}{dx} - 2xz = -2x^3$$

I.F. $= e^{\int -2x\,dx} = e^{-x2}$

$\therefore$ The solution is,

$$z \cdot e^{-x^2} = \int -2x^3 \cdot e^{-x^2} dx = (x^2 + 1)\, e^{-x^2} + c$$

or, $\quad \frac{1}{(x + y)^2} = ce^{x^2} + x^2 + 1$

The correct option is (C)

65. We have, $\sin y \frac{dy}{dx} = \cos y\,(1 - x \cos y)$

$$\Rightarrow \quad \sin y \frac{dy}{dx} - \cos y = -x \cos^2 y$$

Dividing by $\cos^2 y$, we get

$$\tan y \cdot \sec y \cdot \frac{dy}{dx} - \sec y = -x$$

Let $\sec y = v \Rightarrow \sec y \tan y \frac{dy}{dx} = \frac{dv}{dx}$

So, we get

$\frac{dv}{dx} - v = -x$, which is a linear differential equation with $P = -1$, $Q = -x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int -1 dx} = e^{-x}$

The solution is given by

$$ve^{-x} = \int -x\, e^{-x} dx = xe^{-x} + e^{-x} + c$$

$$= e^{-x}(x + 1) + c$$

or, $\quad v = (1 + x) + ce^x$

or, $\quad \sec y = (1 + x) + ce^x$

The correct option is (A)

66. The given equation is

$$xy^2 e^{xy}\, dy + xe^{x/y} dy = ye^{x/y}\, dx - y^3 e^{xy} dx$$

$$\Rightarrow \quad y^2 \cdot e^{xy}(xdy + ydx) = e^{x/y}(ydx - xdy)$$

$$\Rightarrow \quad e^{xy}(xdy + ydx) = e^{x/y}\left(\frac{y\,dx - x\,dy}{y^2}\right)$$

$$\Rightarrow \quad e^{xy} \cdot d(xy) = e^{x/y}(d(x/y))$$

$$\Rightarrow \quad d(e^{xy}) = d(e^{x/y}) \Rightarrow e^{xy} = e^{x/y} + c$$

or, $\quad xy = \ln\,(e^{x/y} + c)$

The correct option is (B)

67. Differentiating the given equation with respect to x, we get

$$xy\,(x) + 1\int_0^x y(t)\, dt = (x + 1)xy\,(x) + 1\int_0^x t\, y(t)\, dt$$

$$\Rightarrow \quad \int_0^x y(t)\, dt = x^2 y\,(x) + \int_0^x t\, y(t)\, dt$$

Differentiating again with respect to x we get,

$$y(x) = x^2 y'(x) = 2xy(x) + xy(x)$$

$$\Rightarrow (1 - 3x)\, y(x) = \frac{x^2 dy(x)}{dx}$$

$$\Rightarrow \quad \frac{(1 - 3x)\,dx}{x^2} = \frac{dy(x)}{y(x)}$$

On Integrating, we get

$$y = \frac{c}{x^3} e^{-1/x}$$

The correct option is (A)

68. We have,

$$(1 + \tan y)\,(dx - dy) + 2xdy = 0$$

$$\Rightarrow \quad (1 + \tan y)\, dx = (1 + \tan y - 2x)\, dy$$

$\Rightarrow \quad \frac{dx}{dy} + \frac{2}{1 + \tan y} x = 1$, which is linear in x

Now, $\int \frac{2}{1 + \tan y} dy = \int \frac{2 \cos y}{\sin y + \cos y} dy$

$$= \int\left(1 + \frac{\cos y - \sin y}{\cos y + \sin y}\right)dy$$

$$= y + \log(\cos y + \sin y)$$

$$\therefore \quad \text{I.F.} = e^{\int \frac{2}{1+\tan y}dy} = e^y \cdot e^{\log(\cos y + \sin y)}$$

$$= (\cos y + \sin y)\, e^y$$

The solution is given by,

$$x \cdot e^y(\cos y + \sin y) = e^y \sin y + c$$

or, $x(\sin y + \cos y) = \sin y + ce^{-y}$

The correct option is (A)

69. We have,

$$x - y\frac{dy}{dx} = (x^2 - y^2)^2$$

$$\Rightarrow \quad \frac{d(x^2 - y^2)}{2(x^2 - y^2)^2} = dx$$

$$\Rightarrow \quad \frac{-1}{(x^2 - y^2)} = 2x + c$$

Since it passes through (1, 0),

$\therefore \quad -1 = 2 + c \Rightarrow c = -3$

Thus, the curve is

$$(2x - 3) + \frac{1}{x^2 - y^2} = 0$$

The correct option is (C)

70. The given equation can be written as

$$2y \sin x \frac{dy}{dx} + y^2 \cos x = \sin 2x$$

$$\frac{d}{dx}\left(y^2 \sin x\right) = \sin 2x$$

On integrating, we get

$$y^2 \sin x = \frac{-1}{2}\cos 2x + c$$

Put $x = \dfrac{\pi}{2}$, $y = 1$, we get $c = \dfrac{-1}{2}$

Hence, the solution is

$$y^2 \sin x = \frac{1}{2}(1 - \cos 2x) = \sin^2 x$$

$\Rightarrow \quad y^2 = \sin x$

The correct option is (A)

71. We have, $ye^x\,dx - e^x\,dy + 2x^2y^2dx - y^3dy = 0$

$$\Rightarrow \quad \frac{ye^x dx - e^x dy}{y^2} + 2x^2dx - y\,dy = 0 \qquad (1)$$

Let $\dfrac{e^x}{y} = t \Rightarrow \dfrac{ye^x - e^x\dfrac{dy}{dx}}{y^2} = \dfrac{dt}{dx}$

$$\Rightarrow \quad \frac{ye^x dx - e^x dy}{y^2} = dt$$

$\therefore$ (1) becomes, $dt + 2x^2dx - y\,dy = 0$

Integrating, we get,

$$t + \frac{2x^3}{3} - \frac{y^2}{2} = c$$

$$\Rightarrow \quad \frac{e^x}{y} + \frac{2}{3}x^3 - \frac{y^2}{2} = 0$$

Put $x = 0$, $y = 1$, we get $c = 1/2$

Hence, the solution is

$6e^x + 4x^3y - 3y^3 - 3y = 0$

The correct option is (B)

72. The given equation can be written as

$(y\,dx - x\,dy)e^{-x/y} - y^3dy = 0$

$$\Rightarrow \quad \frac{y\,dx - x\,dy}{y^2}\,e^{-x/y} = y\,dy$$

$\Rightarrow \quad d(x/y)e^{-x/y} = y\,dy$

On integrating, we get

$$-e^{-x/y} = \frac{y^2}{2} + c$$

or, $2e^{-x/y} + y^2 = c$

The correct option is (B)

73. Given, $y = c_1e^{m1x} + c_2e^{m2x} + c_3e^{m3x}$ (1)

$\Rightarrow \quad y_1 = c_1m_1e^{m1x} + c_2m_2e^{m2x} + c_3m_3e^{m3x}$

$= m_1(y - c_2e^{m2x} - c_3e^{m3x}) + c_2m_2e^{m2x} + c_3m_3e^{m3x}$ [from (1)]

$= m_1y + c_2(m_1 - m_2)e^{m2x} + c_3(m_3 - m_1)e^{m3x}$ (2)

Now,

$y_2 = m_1y_1 + c_2m_2(m_2 - m_1)e^{m2x} + c_3m_3(m_3 - m_1)e^{m3x}$

$= m_1y_1 + m_2[y_1 - m_1y - c_3(m_3 - m_1)e^{m3x}]$

$+ c_3m_3(m_3 - m_1)\,e^{m3x}$ [from (2)]

$= (m_1 + m_2)\,y_1 - m_1m_2y + c_3(m_3 - m_1)(m_3 - m_2)e^{m3x}$ (3)

Further,

$y_3 = (m_1 + m_2)y_2 - m_1m_2y_1 + c_3m_3(m_3 - m_1)(m_3 - m_2)\,e^{m3x}$

$= (m_1 + m_2)y_2 - m_1m_2y_1 + m_3[y_2 - (m_3 - m_1)\,y_1 + m_1m_2y]$ [from (3)]

$= (m_1 + m_2 + m_3)y_2 - (m_1m_2 + m_1m_3 + m_2m_3)\,y_1 + m_1m_2m_3y$

$= 0 \cdot y_2 - (-7)y_1 - 6y \Rightarrow y_3 - 7y_1 + 6y = 0$

The correct option is (A)

74. We have,

$$\frac{dy}{dx} = 2c_1e^{2x} + c_2e^x - c_3e^{-x}$$

$$\frac{d^2y}{dx^2} = 4c_1e^{2x} + c_2e^x - c_3e^{-x}$$

$$\frac{d^3y}{dx^3} = 8c_1e^{2x} + c_2e^x - c_3e^{-x}$$

Substituting in the given differential equation, we get

$8 + 4a + 2b + c = 0$

$1 + a + b + c = 0$

$-1 + a - b + c = 0$

$\Rightarrow a = -2, b = -1, c = 2$

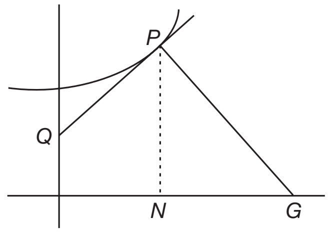
Thus, $\dfrac{a^3 + b^3 + c^3}{abc} = -\dfrac{1}{4}$

The correct option is (B)

More than One Option Correct Type

75. The subnormal is $y\dfrac{dy}{dx}$.

Equation of tangent is



$Y - y = \dfrac{dy}{dx}(X - x)$

Put $X = 0$, $Y = y - x\dfrac{dy}{dx}$

$\therefore$ Initial ordinate is $y - x\dfrac{dy}{dx}$

According to question,

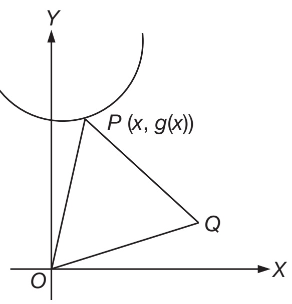
$y\dfrac{dy}{dx} = y - x\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x+y}$, which is homogeneous

Again we can write, $\dfrac{dx}{dy} = \dfrac{x+y}{y} = \dfrac{x}{y} + 1$, which is linear.

The correct option is (A) and (B)

76. Length of the side of

$\Delta OPQ = \sqrt{x^2 + \{g(x)\}^2}$



Now, area of triangle $= \dfrac{\sqrt{3}}{4}(\text{side})^2$

$\therefore \dfrac{\sqrt{3}}{4} = \dfrac{\sqrt{3}}{4}[x^2 + \{g(x)\}^2]$

$\therefore g(x) = \sqrt{1 - x^2}$ or $-\sqrt{1 - x^2}$

The correct option is (B)and (C)

77. Integrating the given equation, we have

$\dfrac{dy}{dx} = 3x^2 - 4x + c$

When $x = 1$, $\dfrac{dy}{dx} = 0$ so that $c = 1$

$\therefore \dfrac{dy}{dx} = 3x^2 - 4x + 1$ (1)

On integrating, we get

$y = x^3 - 2x^2 + x + c_1.$

When $x = 1$, $y = 5$, so that $c_1 = 5$.

$\therefore y = x^3 - 2x^2 + x + 5$

From (1), we get the criticial points $x = \dfrac{1}{3}$, $x = 1$.

At $x = \dfrac{1}{3}$, $\dfrac{d^2y}{dx^2} = -2 < 0$, therefore at $x = \dfrac{1}{3}$, y has a local maximum.

Also, at $x = 1$, $\dfrac{d^2y}{dx^2} = -2 > 0$, therefore at $x = 1$, y has a local minimum.

Also, $f(1) = 5$, $f\left(\dfrac{1}{3}\right) = \dfrac{139}{27}$, $f(0) = 5$ and $f(2) = 7$.

Thus, the global maximum value = 7 and the global minimum value = 5.

The correct option is (A), (B), (C) and (D)

78. We have, $\dfrac{dy}{dx} = x[e^{(n-1)y} - 1]$

$\Rightarrow \dfrac{dy}{e^{(n-1)y} - 1} = x\,dx$

$\Rightarrow \dfrac{1}{n-1}\displaystyle\int \dfrac{(n-1)\,e^{(n-1)y}}{\left(e^{(n-1)y} - 1\right)e^{(n-1)y}}\,dy = \dfrac{x^2}{2} + c$

$\dfrac{1}{n-1}\displaystyle\int \dfrac{dt}{(t-1)t} = \dfrac{x^2}{2} + c \Rightarrow [\text{Put } t = e^{(n-1)y}]$

$\Rightarrow \dfrac{1}{n-1}\displaystyle\int\left(\dfrac{1}{(t-1)} - \dfrac{1}{t}\right)dt = \dfrac{x^2}{2} + c$

$\Rightarrow \dfrac{1}{n-1}\log\dfrac{t-1}{t} = \dfrac{x^2}{2} + c$

$\Rightarrow \dfrac{1}{n-1}\log\left(\dfrac{e^{(n-1)y} - 1}{e^{(n-1)y}}\right) = \dfrac{x^2}{2} + c$

$$\Rightarrow \frac{e^{(n-1)y}-1}{e^{(n-1)y}} = e^{(n-1)\left(\frac{x^2}{2}+c\right)} = e^{(n-1)\frac{x^2}{2}} \cdot e^{(n-1)c}$$

$$\Rightarrow e^{(n-1)y} - 1 = ke^{(n-1)y + (n-1)\frac{x^2}{2}} \quad [k = e^{(n-1)c}]$$

$$\Rightarrow e^{(n-1)y} = 1 + ke^{(n-1)y + (n-1)\frac{x^2}{2}}$$

The correct option is (A) and (B)

Passage Based Questions

Passage 1

79. The given equation is

$$x\,dx + y\,dy = \frac{a^2x}{x^2+y^2}dy - \frac{a^2y}{x^2+y^2}dx$$

$$\text{or,} \quad \left[x + \frac{a^2y}{x^2+y^2}\right]dx + \left[y - \frac{a^2x}{x^2+y^2}\right]dy = 0 \qquad (1)$$

Comparing it with $Mdx + Ndy = 0$

Here, $M = x + \dfrac{a^2y}{x^2+y^2}$; $N = y - \dfrac{a^2x}{x^2+y^2}$

$$\frac{\partial M}{\partial y} = 0 + \frac{a^2(x^2+y^2) - y(2y)}{(x^2+y^2)^2} = \frac{a^2(x^2-y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = 0 - \frac{a^2(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = \frac{a^2(x^2-y^2)}{(x^2+y^2)^2}$$

Since $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ ∴ Equation (1) is exact. Hence, the solution is

$$\int_{y\text{ constant}} M\,dx + \int(\text{terms in } N \text{ not containing } x)\,dy = c$$

$$\text{i.e.,} \quad \int_{y\text{ constant}} \left(x + \frac{a^2y}{x^2+y^2}\right)dx + \int y\,dy = c$$

$$\text{or} \quad \frac{x^2}{2} + a^2y \int_{y\text{constant}} \frac{ax\,dx}{x^2+y^2} + \frac{y^2}{2} = c$$

$$\text{or} \quad \frac{x^2}{2} + a^2y \cdot \frac{1}{y}\tan^{-1}\frac{x}{y} + \frac{y^2}{2} = c$$

$$\text{or} \quad x^2 + y^2 + 2a^2\tan^{-1}\frac{x}{y} = 2c$$

$$\text{or} \quad x^2 + y^2 + 2a^2\tan^{-1}\frac{x}{y} = A$$

The correct option is (A)

80. The given equation is

$$\left[y\left(1+\frac{1}{x}\right) + \cos y\right]dx + [x + \log x - x\sin y]dy = 0 \qquad (1)$$

Comparing it with $M\,dx + N\,dy = 0$

Here, $M = y\left(1+\dfrac{1}{x}\right) + \cos y$, $N = x + \log x - x\sin y$

$$\frac{\partial M}{\partial y} = 1 + \frac{1}{x} - \sin y, \quad \frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$$

Since, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ ∴ equation (1) is exact.

Hence, solution is

$$\text{i.e.,} \quad \int_{y\text{ constant}} M\,dx + \int(\text{terms in } N \text{ not containing } x)\,dy = c$$

$$\text{or,} \quad y\int\left(1+\frac{1}{x}\right)dx + \cos y\int 1\,dx = c$$

or, $y(x + \log x) + x\cos y = c$

The correct option is (C)

Passage 2

81. The given equation is

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)\,dy = 0 \qquad (1)$$

Comparing with $Mdx + Ndy = 0$

Here, $M = x^2y - 2xy^2$; $N = -x^3 + 3x^2y$

$$\frac{\partial M}{\partial y} = x^2 - 4xy \ ; \ \frac{\partial N}{\partial x} = -3x^2 + 6xy$$

Since $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$, Equation (1) is not exact.

But (1) in homogeneous in x and y

$$\therefore \text{I.F.} = \frac{1}{Mx+Ny} = \frac{1}{x^2y - 2x^2y^2 - x^2y + 3x^2y^2} = \frac{1}{x^2y^2}$$

Multiplying (1) by $\dfrac{1}{x^2y^2}$ it becomes

$$\left(\frac{1}{y} - \frac{2}{x}\right)dx - \left(\frac{x}{y^2} - \frac{3}{y}\right)dy = 0 \qquad (2)$$

which is exact $\left[\because \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x} = \dfrac{-1}{y^2}\right]$

Hence, the solution is

$$\int_{y\text{ constant}} \left(\frac{1}{y} - \frac{2}{x}\right)dx - \int \frac{-3}{y}dy = c$$

$$\text{or,} \quad \frac{x}{y} - 2\log x + 3\log y = c$$

The correct option is (A)

82. The given equation is

$(xy \sin xy + \cos xy)\, y\, dx + (xy \sin xy - \cos xy)\, x\, dy = 0$ (1)

It is of the form $f_1(xy)y\, dx + f_2(xy)x\, dy = 0$

Here, $M = (xy \sin xy + \cos xy)\, y$

and, $N = (xy \sin xy - \cos xy)x$

$$\text{I.F.} = \frac{1}{Mx - Ny}$$

$$= \frac{1}{xy\,(xy \sin xy + \cos xy) - xy/xy \sin xy - \cos xy}$$

$$= \frac{1}{2\,xy \cos xy}$$

Multiplying (1) by $\frac{1}{2\,xy \cos xy}$, it becomes

$$\left[\frac{y \sin xy}{2 \cos xy} + \frac{1}{2x}\right]dx + \left[\frac{x \sin xy}{2 \cos xy} - \frac{1}{2y}\right]dy = 0$$

or, $\left(y \tan xy + \frac{1}{x}\right) dx + \left(x \tan xy - \frac{1}{y}\right) dy = 0$

which is exact $\left[\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \tan xy + xy \sec^2 xy\right]$

Hence, the solution is

$$\int_{y \text{ constant}} \left(y \tan xy + \frac{1}{x}\right) dx + \int \frac{-1}{y} dy = a$$

or, $y \cdot \frac{\log \sec xy}{y} + \log x - \log y = c_1$

or, $\log \sec xy + \log \frac{x}{y} = \log c$ or $\log \frac{x}{y} \sec xy = \log c$

or, $\frac{x}{y} \sec xy = c$

or, $x \sec xy = cy$

The correct option is (B)

83. The given equation is

$\left(xy^2 - e^{\frac{1}{x^3}}\right) dx - x^2 y\, dy = 0$ (1)

Here, $M = xy^2 - e^{\frac{1}{x^3}}$, $N = -x^2 y$

$$\frac{\partial M}{\partial y} = 2xy, \quad \frac{\partial N}{\partial x} = -2xy$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2xy - (-2xy)}{-x^2 y} = \frac{4xy}{-x^2 y} = \frac{-4}{x} = f(x)$$

$$\therefore \quad \text{I.F.} = e^{\int f(x)\, dx} = e^{\int \frac{-4}{x} dx} = e^{-4} \log x = e \log x^{-4}$$

$$= x^{-4} = \frac{1}{x^4}$$

The correct option is (D)

84. The given equation is

$(y^4 + 2y)\, dx + (xy^3 + 2y^4 - 4x)dy = 0.$ (1)

Here, $M = y^4 + 2y$; $N = xy^3 + 2y^4 - 4x$

$$\frac{\partial M}{\partial y} = 4y^3 + 2 ; \quad \frac{\partial N}{\partial x} = y^3 - 4$$

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{(y^3 - 4) - (4y^3 + 2)}{y^4 + 2y} = \frac{-3(y^3 + 2)}{y(y^3 + 2} = \frac{-3}{y}$$

$$= f(y)$$

$$\therefore \quad \text{I.F.} = e^{\int f(y)\, dy} = e^{\int \frac{-3}{y} dy} = e^{\log y^{-3}} = y^{-3} = \frac{1}{y^3}$$

The correct option is (C)

Passage 3

85. The equation is

$$y - px = \frac{p}{p - 1}$$

or, $y = px + \frac{p}{p - 1}$

It is of Clairaut's form, hence its solution is

$$y = cx + \frac{c}{c - 1}$$

The correct option is (A)

86. The given equation is

$$p^2(x^2 - a^2) - 2pxy + y^2 - b^2 = 0$$

or, $y^2 - 2pxy + p^2x^2 = a^2p^2 + b^2$

or, $(y - px)^2 = a^2p^2 + b^2$

or, $y - px = \pm \sqrt{a^2 p^2 + b^2}$

or, $y = px \pm \sqrt{a^2 p^2 + b^2}$

Both the component equations are of Clairaut's form

$\therefore$ The solution is

$$y = cx \pm \sqrt{a^2 c^2 + b^2}$$

or, $(y - cx)^2 = a^2c^2 + b^2$

The correct option is (A) and (C)

87. The given equation is

$$(x - a)\, p^2 + px = (1 + p)y$$

or, $(1 + p)y = px(p + 1) - ap^2$

or, $y = px - \frac{ap^2}{p + 1}$

which is of Clairaut's form and hence its solutioin is

$$y = cx - \frac{ac^2}{c + 1}$$

The correct option is (B)

88. The given equation is

$$p^2x^2 - 2p^2x + 2py - 2pxy - px + 2p + y^2 + y = 0$$

or, $(y^2 - 2pxy + p^2x^2) + 2p(y - px) + (y - px) + 2p = 0$

or, $(y-px)^2+(2p+1)(y-px)+2p=0$

or, $(y-px+2p)(y-px+1)=0$

Both the component equations are of Clairaut's form and hence the solution is

$(y-cx+2c)(y-cx+1)=0$

The correct option is (C)

Passage 4

89. The equation of the series of hyperbolas is

$$xy=k^2 \quad (1)$$

(1) Differentiating (1), $y+x\dfrac{dy}{dx}=0$ (2)

[The arbitrary constant k is eliminated]

(2) Replacing $\dfrac{dy}{dx}$ by $-\dfrac{dx}{dy}$, the differential equation of orthogonal trajectories is

$$y-x\frac{dx}{dy}=0$$

or, $xdx-y\,dy=0$ (3)

(3) Integrating (3), $\dfrac{x^2}{2}-\dfrac{y^2}{2}=c_1$

or, $x^2-y^2=2c_1$ or $x^2-y^2=c$

The correct option is (B)

90. The equation of the series of parabolas is

$$y^2=4ax \quad (1)$$

(1) Differentiating (1), $2y\dfrac{dy}{dx}=4a$ or $y\dfrac{dy}{dx}=2a$ (2)

Eliminating a between (1) and (2), we get

$$y^2=2y\cdot\frac{dy}{dx}x \text{ or } y=2x\frac{dy}{dx}$$

(2) Replacing $\dfrac{dy}{dx}$ by $-\dfrac{dx}{dy}$, the differential equation of orthogonal trajectories is

$$y=-2x\frac{dx}{dy}\Rightarrow y\,dy=-2xdx \quad (3)$$

(3) Integrating (3), $\dfrac{y^2}{2}=-x^2+c_1$

or, $2x^2+y^2=2c_1$ or $2x^2+y^2=c$

The correct option is (A)

Match the Column Type

91. **I.** The equation of circles passing through the origin and having their centres on the x-axis is

$$x^2+y^2+2gx=0, \quad (1)$$

where g is an arbitrary constant.

Differentiating with respect to x, we get

$$x+y\frac{dy}{dx}+g=0. \text{ i.e., } g=-\left(x+y\frac{dy}{dx}\right).$$

Putting this value of g in (1), we get

$$x^2+y^2-2x\left(x+y\frac{dy}{dx}\right)=0,$$

i.e., $$y^2=x^2+2xy\frac{dy}{dx}$$

The correct option is (B)

II. Let the directrix be $x=-2a$ and latus rectum be $4a$. Then, the equation of the parabola is

(distance from focus = distance from directrix),

$$x^2+y^2=(2a+x)^2 \text{ or } y^2=4a(a+x) \quad (1)$$

Differentiating with respect to x, we get

$$y\frac{dy}{dx}=2a \text{ or } a=\frac{1}{2}y\frac{dy}{dx}.$$

Putting this value of a in (1), the differential equation is

$$y^2=2y\frac{dy}{dx}\left(\frac{y}{2}\frac{dy}{dx}+x\right)$$

or, $$y\left(\frac{dy}{dx}\right)^2+2x\left(\frac{dy}{dx}\right)-y=0$$

The correct option is (C)

III. Equation of the family of such parabolas is

$$(y-k)^2=4a(x-h) \quad (1)$$

where h and k are arbitrary constants.

Differentiating with respect to x, we get

$$(y-k)\frac{dy}{dx}=2a \quad (2)$$

Differentiating again,

$$(y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0 \quad (3)$$

Putting value of $y-k$ from (2) in (3), we get

$$2a\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0$$

The correct option is (D)

IV. The general equation of all ellipses centred at the origin is $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1.$

Differentiating with respect to x, we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\cdot y_1=0 \text{ i.e., } \frac{x}{a^2}+\frac{y}{b^2}y_1=0. \quad (1)$$

Differentiating again, we get

$$\frac{1}{a^2}+\frac{1}{b^2}\left(y_1^2+yy_2\right)=0 \qquad (2)$$

From (1) and (2), we have

$$\frac{1}{b^2}\left(yy_1-xy_1^2-xyy_2\right)=0$$

$$\Rightarrow \quad xyy_2+xy_1^2-yy_1=0,$$

which is the required differential equation.

The correct option is (A)

92. I. $\dfrac{x\,dy}{x^2+y^2}=\left(\dfrac{y}{x^2+y^2}-1\right)dx$

$$\Rightarrow \frac{x\,dy-y\,dx}{x^2+y^2}=-dx$$

$$\Rightarrow d\left(\tan^{-1}\left(\frac{y}{x}\right)\right)=-dx$$

Integrating both sides, we get

$$\tan^{-1}\frac{y}{x}=-x+c \text{ or } y=x\tan(c-x)$$

The correct option is (C)

II. We have $y(x^2y+e^x)\,dx-e^x dy=0$

$$\Rightarrow e^x\frac{dy}{dx}=x^2y^2+ye^x$$

Dividing by y^2e^x, we get $\dfrac{1}{y^2}\dfrac{dy}{dx}-\dfrac{1}{y}=x^2e^{-x}$

Put $\dfrac{1}{y}=V$ so that $\dfrac{-1}{y^2}\dfrac{dy}{dx}=\dfrac{dV}{dx}$

We thus have $\dfrac{dV}{dx}+V=-x^2e^{-x}$, which is linear

$\therefore$ I.F. $=e^{\int 1dx}=e^x$

Hence, the solution is

$$V\cdot e^x=-\int x^2e^{-x}\cdot e^x\,dx+\frac{c}{3}$$

or, $\dfrac{1}{y}e^x=-\dfrac{x^3}{3}+\dfrac{c}{3}$ or $x^3y+3e^x=cy$

The correct option is (A)

III. Direct substitution of $y=2x-4$ in the equation shows that it is a solution of the given differential equation.

The correct option is (D)

IV. We have, $xy^4\,dx+y\,dx-x\,dy=0$

$$\Rightarrow \quad x\,dx+\frac{y\,dx-x\,dy}{y^4}=0$$

$$\Rightarrow \quad x^3\,dx+\left(\frac{x}{y}\right)^2\frac{y\,dx-x\,dy}{y^2}=0$$

$$\Rightarrow \quad x^3\,dx+\left(\frac{x}{y}\right)^2 d\left(\frac{x}{y}\right)=0$$

Integrating, we get $\dfrac{x^4}{4}+\dfrac{1}{3}\left(\dfrac{x}{y}\right)^3=c'$

The correct option is (B)

93. I. Here, $y-x\dfrac{dy}{dx}+\log x=0$

i.e., $x\dfrac{dy}{dx}-y=\log x$

i.e., $\dfrac{dy}{dx}-\dfrac{y}{x}=\dfrac{\log x}{x}$

$\therefore\ e^{\int P dx}=e^{\int -1/x\,dx}=e^{(-\log x)}=e^{\log x^{-1}}=x^{-1}$

$\therefore$ solution is $yx^{-1}=\displaystyle\int\frac{\log x}{x}\cdot\frac{1}{x}dx+c$

i.e., $\dfrac{y}{x}=\displaystyle\int\frac{1}{x^2}\log x\,dx+c=\int e^{-t}.t\,dt+c$

Put $\log x=t\ \therefore\ \dfrac{1}{x}dx=dt$

Also, $x=ct$

$$=t\frac{e^{-t}}{-1}-1\cdot\frac{e^{-t}}{-1}+c=-e^{-t}[-t+1]+c$$

$$=e^{-\log x}[-\log x+1]+c=\frac{1}{x}(1+\log x)+c$$

$\therefore\ y=(1+\log x)+c$

The correct option is (B)

II. The given differential equation can be written as

$$(\sin y\,dx+x\cos y\,dy)\,x^2\sin^2 y$$
$$=2y\sin x\,dy+y^2\cos x\,dx$$

$\Rightarrow d(x\sin y)(x\sin y)^2=d(y^2\sin x)$

Integrating, we get

$$\frac{(x\sin y)^3}{3}=y^2\sin x+c$$

The correct option is (D)

III. $\dfrac{dy}{dx}=\dfrac{y(x\log y-y)}{x(y\log x-x)}$

$$\Rightarrow \quad x(y\log x-x)\frac{dy}{dx}=y(x\log y-y)$$

$$\Rightarrow \quad \left(\log x-\frac{x}{y}\right)\frac{dy}{dx}=\log y-\frac{y}{x}$$

$$\Rightarrow \quad \frac{y}{x}+\log x\cdot\frac{dy}{dx}=\log y+\frac{x}{y}\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d}{dx}(y\log x)=\frac{d}{dx}(x\log y)$$

$$\Rightarrow \quad y\log y=x\log y+\log c$$

$$\Rightarrow \quad \log x^y=\log y^x+\log c$$

$\Rightarrow x^y = c\,y^x$.

The correct option is (C)

IV. Given differential equation can be written as

$$\frac{x\,dy - y\,dx}{x^2 + y^2} = -\,dx$$

$$\Rightarrow \frac{x\,dy - y\,dx}{x^2\left(1+\left(\frac{y}{x}\right)^2\right)} = -\,dx$$

$$\Rightarrow \frac{d(y/x)}{1+\left(\frac{y}{x}\right)^2} + dx = 0$$

Integrating, we get

$$\tan^{-1}\frac{y}{x} + x = c$$

The correct option is (A)

Assertion-Reason Type

94. We have, $xy = ce^x + be^{-x} + x^2$ (1)

Differentiating with respect to x, we get

$$x\frac{dy}{dx} + y = ce^x - be^{-x} + 2x \quad (2)$$

Differentiating again,

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = ce^x + be^{-x} + 2 = xy - x^2 + 2$$

[Using (1)]

Hence, the required differential equation is

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0.$$

The order of this differential equation is 2.

The correct option is (A)

95. The equation of the normal at (x, y) is,

$$(X - x) + (Y - y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{X}{x + y\frac{dy}{dx}} + \frac{Y}{\frac{\left(x + y\frac{dy}{dx}\right)}{\frac{dy}{dx}}} = 1$$

$$\Rightarrow OA = x + y\frac{dy}{dx},\ OB = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$$

Also, $\frac{1}{OA} + \frac{1}{OB} = 1$

$$\Rightarrow 1 + \frac{dy}{dx} = x + y\frac{dy}{dx} \Rightarrow (y - 1)\frac{dy}{dx} + (x - 1) = 0$$

Integrating, we get

$(y - 1)^2 + (x - 1)^2 = c$

Since the curve passes through (5, 4), $c = 25$

Hence, the curve is

$(x - 1)^2 + (y - 1)^2 = 25$

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The correct option is (A)

96. The given equation can be written as

$$-\sin\theta\frac{d\theta}{dx} + \cos\theta\left(2x - \frac{1}{x}\right) = x^2$$

Put $\cos\theta = u \Rightarrow \sin\theta\frac{d\theta}{dx} = \frac{du}{dx}$

The equation reduces to

$$\frac{du}{dx} + u\left(2x - \frac{1}{x}\right) = x^2$$

$$\text{I.F.} = e^{\int\left(2x - \frac{1}{x}\right)dx = e^{x^2 - \ln x}} = \frac{e^{x^2}}{x}$$

$\therefore$ The solution is

$$u \cdot \frac{e^{x^2}}{x} = \int x\,e^{x^2}dx = \frac{1}{2}\int e^{x^2}d(x^2) = \frac{1}{2}e^{x^2} + c$$

$$\Rightarrow 2\cos\theta = x + cxe^{-x2}$$

The correct option is (A)

97. Any straight line which is at a constant distance p from the origin is

$x\cos\alpha + y\sin\alpha = p$ (1)

Differentiating both sides with respect to 'x', we get

$$\cos\alpha + \sin\alpha\frac{dy}{dx} = 0$$

$$\Rightarrow \tan\alpha = \frac{-1}{y_1} \text{ where } y_1 = \frac{dy}{dx}$$

$$\therefore \sin\alpha = \frac{1}{\sqrt{1 + y_1^2}};\ \cos\alpha = \frac{-y_1}{\sqrt{1 + y_1^2}}$$

Putting in (1), we get

$$x \cdot \frac{-y_1}{\sqrt{1 + y_1^2}} + y\frac{1}{\sqrt{1 + y_1^2}} = p$$

$$\Rightarrow (y - xy_1)^2 = p^2(1 + y_1{}^2)$$

The correct option is (A)

Previous Year's Questions

98. The given differential equation is

$$\left(1+3\frac{dy}{dx}\right)^{2/3}=4\left(\frac{d^3y}{dx^3}\right)$$

Making the above equation free from radical 2/3, the equation can be rewritten as

$$\left(1+3\frac{dy}{dx}\right)^{2}=4\left(\frac{d^3y}{dx^3}\right)^3$$

This shows that the order and degree of given equation are 3 and 3 respectively.

The correct option is (C)

99. The equation $\frac{d^2y}{dx^2}=e^{-2x}$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{-2x}}{-2}+c$$

$$\Rightarrow y=\frac{e^{2x}}{4}+cx+d$$

The correct option is (B)

100. The general equation of all non-vertical lines in a plane is $ax + by = 1$, where $b \neq 0$

$$\therefore a+b\frac{dy}{dx}=0$$

$$\Rightarrow b\frac{d^2y}{dx^2}=0$$

$$\Rightarrow \frac{d^2y}{dx^2}=0$$

which is the desired differential equation.

The correct option is (A)

101. Equation $y^2 = 4a(x-h)$

$$\Rightarrow 2yy'=4a$$

$$\Rightarrow yy'=2a$$

$$\Rightarrow yy''+(y')^2=0$$

The correct option is (B)

102. $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dx}{dy}=0$

$$\Rightarrow (1+y^2)\frac{dx}{dy}+x=e^{\tan^{-1}y}$$

$$\Rightarrow \frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{I.F}=e^{\int \text{pdy}}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

Therefore the required solutions is

$$xe^{\tan^{-1}y}=\int e^{\tan^{-1}y}\cdot\frac{e^{\tan^{-1}y}}{1+y^2}\cdot dy+K_1$$

$$xe^{\tan^{-1}y}=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy+K_1$$

$$xe^{\tan^{-1}y}=\frac{1}{2}e^{2\tan^{-1}y}+K_1$$

$$2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+K$$

The correct option is (B)

103. $x^2+y^2-2ay=0$

Differentiating above equation:

$2x + 2yy' - 2ay' = 0$

$\Rightarrow a=\frac{x+yy'}{y}$ (eliminating a)

$\Rightarrow (x^2-y^2)y'=2xy.$

The correct option is (C)

104. The equation $ydx + xdy + x^2y\,dy = 0$ implies

$$\frac{d(xy)}{x^2y^2}+\frac{1}{y}dy=0 \Rightarrow -\frac{1}{xy}+\log=C.$$

The correct option is (B)

105. $y^2 = 2c(x+\sqrt{c})$ (1)

Differentiating (1) we obtain

$2yy' = 2c \cdot 1$ or $yy' = c$ (2)

$\Rightarrow y^2=2yy'(x+\sqrt{yy'})$ [on putting value of c from (2) in (1)]

On simplifying, we get

$(y-2xy')^2=4yy'^3$ (3)

Hence equation (3) is order 1 and degree 3.

The correct option is (C)

106. $\frac{xdy}{dx}=y(\log y-\log x+1)$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}\left(\log\left(\frac{y}{x}\right)+1\right)$$

Substituting $y = vx$

$$\frac{dy}{dx}=v+\frac{xdv}{dx}$$

$$\Rightarrow v+\frac{xdv}{dx}=v(\log v+1)$$

$$\frac{xdv}{dx}=v\log v$$

$$\Rightarrow \frac{dv}{v\log v}=\frac{dx}{x}k$$

Put $\log v = z$

$$\frac{1}{v}dv=dz$$

$\Rightarrow \frac{dz}{z} = \frac{dx}{x}$

$\ln z = \ln x + \ln c$

$z = cx$

$\log v = cx$

$\log\left(\frac{y}{x}\right) = cx$

The correct option is (C)

107. Given equation $Ax^2 + By^2 = 1$ implies (1)

$Ax + By\frac{dy}{dx} = 0$, and (2)

$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0$ (3)

From (2) and (3)

$$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$$

$$\Rightarrow xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

The correct option is (A)

108. General equation of all such circles is $x^2 + y^2 + 2gx = 0$.

On differentiating, we get

$$2x + 2y\frac{dy}{dx} + 2g = 0$$

Equating the expression for g from above equation in the general equation, we get

$$x^2 + y^2 + \left(-2x - 2y\frac{dy}{dx}\right)x = 0$$

$$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$$

The correct option is (C)

109. Substituting, $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = 1 + v$$

$$\Rightarrow dv = \frac{dx}{x}$$

$\therefore v = \log x + c$

$$\Rightarrow \frac{y}{x} = \log x + c$$

Since, $y(1) = 1$, we have

$y = x\log x + x$

The correct option is (D)

110. $(x - h)^2 + (y - 2)^2 = 25$ (1)

$$\Rightarrow 2(x - h) + 2(y - 2)\frac{dy}{dx} = 0$$

$$\Rightarrow (x - h) = -(y - 2)\frac{dy}{dx}$$

Substitution in (1) implies that

$$(y - 2)^2\left(\frac{dy}{dx}\right)^2 + (y - 2)^2 = 25$$

$(y - 2)^2 y'^2 = 25 - (y - 2)^2$.

The correct option is (C)

111. Given $y = c_1 e^{c_2 x}$ (1)

$\Rightarrow \quad y' = c_2 c_1 e^{c_2 x}$.

So, $\quad y' = c_2 y$ (2)

$\Rightarrow \quad y'' = c_2 y'$

From (2)

$$c_2 = \frac{y'}{y}$$

So, $y'' = \frac{(y')^2}{y} \Rightarrow yy'' = (y')^2$

The correct option is (D)

112. $\cos x\, dy = y(\sin x - y)\, dx$

$$\Rightarrow \frac{dy}{dx} = y\tan x - y^2\sec x$$

$$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$$

Let $\frac{1}{y} = t$

$$\Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow -\frac{dy}{dx} - t\tan x = -\sec x \Rightarrow \frac{dt}{dx} + (\tan x)t = \sec x.$$

$$\text{I.F} = e^{\int \tan x dx} = \sec x$$

Solution is $t(\text{I.F}) = \int(\text{I.F})\sec x\, dx$

$$\Rightarrow \frac{1}{y} = \sec x = \tan x + c$$

The correct option is (D)

113. $\frac{dV}{dt} = -k(T - t) \Rightarrow dV = -k(T - t)dt$

Integrate

$$V = \frac{-k(T - t)^2}{(-2)} + c \Rightarrow V = \frac{k(T - t)^2}{2} + c$$

at $t = 0 \Rightarrow V = I$

$$l = \frac{kT^2}{2} + c \Rightarrow c = l - \frac{kT^2}{2} \Rightarrow c = V(T) = l - \frac{kT^2}{2}$$

The correct option is (A)

114. $\frac{d(p(t))}{dt} = \frac{1}{2}p(t) - 450$

$$\frac{d(p(t))}{dt} = \frac{p(t) - 900}{2}$$

$$2\int \frac{d(p(t))}{p(t) - 900} = \int dt$$

$$2\ln|p(t) - 900| = t + c$$

$t = 0$

$\Rightarrow 2\ln 50 = 0 + c$

$\Rightarrow c = 2\ln 50$

$\therefore 2\ln|p(t) - 900| = t + 2\ln 50$

$p(t) = 0$

$\Rightarrow 2\ln 900 = t + 2\ln 50$

$$t = 2(\ln 900 - \ln 50) = 2\ln\left(\frac{900}{50}\right) = 2\ln 18.$$

The correct option is (A)

115. Given $\int_{2000}^{p} dP = \int_{0}^{25} \left(100 - 12\sqrt{x}\right) dx$

$$\left(P - 2000\right) = 25 \times 100 - \frac{12 \times 2}{3}(25)^{3/2}$$

$$P = 3500.$$

The correct option is (B)

116. $\frac{dp}{dt} = \frac{p - 400}{2}$

$$\frac{dp}{p - 400} = \frac{1}{2}dt$$

$$\ln|p - 400| = \frac{1}{2}t + c$$

at $t = 0, p = 100$

$\ln 300 = c$

$$\ln\left|\frac{p - 400}{300}\right| = \frac{t}{2}$$

$\Rightarrow |p - 400| = 300e^{t/2}$

$\Rightarrow 400 - p = 300e^{t/2}$ (as $p < 400$)

$\Rightarrow p = 400 - 300e^{t/2}$

The correct option is (A)

117. $\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{2x\ln x}{x\ln x}$

$\therefore$ I.F. $= e^{\int \frac{1}{x\ln x}dx} = e^{\ln(\ln x)} = \ln x$

The solution is

$y\ln x = \int 2\ln x dx$

$\Rightarrow y\ln x = 2(x\ln x - x) + c$

For $x = 1, c = 2$

$y\ln x = 2(x\ln x - x + 1)$

Put $x = e \Rightarrow y(e) = 2$.

The correct option is (B)

118. We have $ydx + xy^2dx = xdy$

$$\Rightarrow \frac{xdy - ydx}{y^2} = xdx$$

$$\Rightarrow -d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right)$$

Integrating, we get

$$-\frac{x}{y} = \frac{x^2}{2} + C$$

Since, it passes through (1, –1)

$\therefore\ 1 = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

$$\Rightarrow x^2 + 1 + \frac{2x}{y} = 0 \Rightarrow y = \frac{-2x}{x^2 + 1}$$

$$\therefore\ f\left(-\frac{1}{2}\right) = \frac{4}{5}$$

The correct option is (A)