CHAPTER 9

Binomial Theorem

Chapter Highlights

Binomial Expression, Binomial Theorem, Pascal's triangle, Middle Term in The Binomial Expansion.

BINOMIAL EXPRESSION

An algebraic expression consisting of only two terms is called a binomial expression. For example, expressions such as

$$x + a, \quad 4x + 3y, \quad 2x - \frac{4}{y}$$

are all binomial expressions.

BINOMIAL THEOREM

This theorem gives a formula by which any power of a binomial expression can be expanded. It was first given by Sir Isaac Newton.

Binomial Theorem for Positive Integral Index

If x and y are real numbers, then for all $n \in N$,

$$(x + y)^n = {}^nC_0\, x^n y^0 + {}^nC_1\, x^{n-1} y^1 + {}^nC_2\, x^{n-2} y^2 + \ldots + {}^nC_{n-1}\, x^1 y^{n-1} + {}^nC_n\, x^0 y^n \quad \ldots(1)$$

i.e., $(x + y)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} y^r$

Here ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ are called **binomial coefficients**.

For the sake of convenience, we may denote ${}^nC_r$ by $C_r \cdot {}^nC_r$ may also be denoted as $\binom{n}{r}$.

SPECIAL CASES

1. Replacing y by $-y$ in (1), we get
   $(x - y)^n = {}^nC_0\, x^n y^0 - {}^nC_1\, x^{n-1} y^1 + {}^nC_2\, x^{n-2} y^2 \ldots + (-1)^n\, {}^nC_n\, x^0 y^n \quad \ldots(2)$
2. Replacing x by 1 and y by x, we get
   $(1 + x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \ldots + {}^nC_n\, x^n$
3. Replacing x by 1 and y by $-x$, we get
   $(1 - x)^n = {}^nC_0 - {}^nC_1\, x + {}^nC_2\, x^2 - \ldots + (-1)^n\, {}^nC_n\, x^n$.
4. Adding (1) and (2), we get
   $(x + y)^n + (x - y)^n = 2\,(x^n + {}^nC_2\, x^{n-2} y^2 + {}^nC_4\, x^{n-4} y^4 + \ldots)$
   = 2 (sum of terms at odd places).
   The last term is ${}^nC_n\, y^n$ or ${}^nC_{n-1}\, xy^{n-1}$ according as n is even or odd respectively.
5. Subtracting Eq. (2) from (1), we get
   $(x + y)^n - (x - y)^n = 2\,({}^nC_1\, x^{n-1} y + {}^nC_3\, x^{n-3} y^3 + \ldots)$
   = 2 (sum of terms at even places)
   The last term is ${}^nC_{n-1}\, xy^{n-1}$ or ${}^nC_n\, y^n$ according as n is even or odd respectively.

TRICK(S) FOR PROBLEM SOLVING

- The coefficient of $(r + 1)$th term in the expansion of $(1 + x)^n$ is ${}^nC_r$.
- The coefficient of x^r in the expansion of $(1 + x)^n$ is ${}^nC_r$.

IMPORTANT POINTS

- The positive integer n is called the index of the binomial.
- Number of terms in the expansion of $(x + y)n$ is $n + 1$, i.e., one more than the index n.
- In the expansion of $(x + y)n$, the power of x goes on decreasing by 1 and that of y goes on increasing by 1 so that the sum of powers of x and y in any term is n.

- The binomial coefficients of the terms equidistant from the beginning and the end are equal.
- If n is odd, then number of terms in $(x + a)n + (x - a)n$ and $(x + a)n - (x - a)n$ are equal to $\frac{n+1}{2}$.
- If n is even, then the number of terms in the expansion of $(x + a)n + (x - a)n$ are $\left(\frac{n+2}{2}\right)$ and that in the expansion of $(x + a)n - (x - a)n$ are $\left(\frac{n}{2}\right)$
- $xn + yn$ is divisible by $x + y$ if n is odd as $xn + yn = (x + y)(xn^{-1} - xn^{-2}y + xn^{-3}y^2 - \ldots + yn^{-1})$
- $xn - yn$ is divisible by $x - y$ if n is even as $xn - yn = (x - y)(xn^{-1} + xn^{-2}y + xn^{-3}y^2 + \ldots + yn^{-1})$
- The coefficient of xn^{-1} in the expansion of $(x + 1)(x + 2) \ldots (x + n) = \frac{n(n+1)}{2}$.
- The coefficient of xn^{-1} in the expansion of $(x - 1)(x - 2) \ldots (x - n) = -\frac{n(n+1)}{2}$.

NOTE

- Students are advised to remember the following values:

$${}^nC_0 = 1,\ {}^nC_1 = n,\ {}^nC_2 = \frac{n(n-1)}{2!},$$

$${}^nC_3 = \frac{n(n-1)(n-2)}{3!},$$

$${}^nC_r = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$$

and ${}^nC_n = 1$.

Also, ${}^nC_r = {}^nC_{n-r}$ for $1 \le r \le n$.

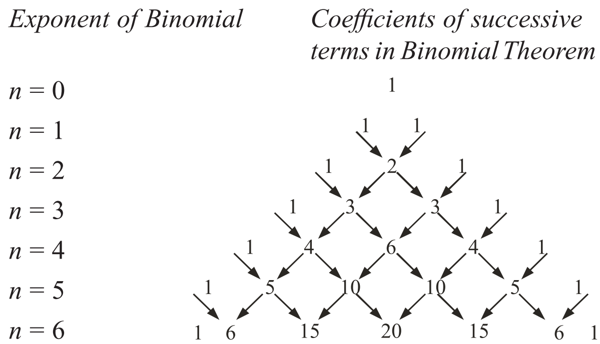
i.e., ${}^nC_0 = {}^nC_n$, ${}^nC_1 = {}^nC_{n-1}$, ${}^nC_2 = {}^nC_{n-2}$, …

i.e., coefficients of terms equidistant from the beginning and end are equal.

- $\sum_{i=0}^{n} ({}^nC_i)^2 = {}^{2n}C_n$
- $\sum_{i=0}^{n} i\,({}^nC_i) = n \cdot 2^{n-1}$
- $\sum_{i=2}^{n} i(i-1) \cdot {}^nC_i = n(n-1)2^{n-2}$
- $\sum_{i=k}^{n} i(i-1)\ldots(i-k+1) \cdot {}^nC_i = n(n-1)\ldots(n-k+1) \cdot 2^{n-k}$
- $\sum_{i=1}^{n} i\left({}^nC_i\right)^2 = n \cdot {}^{2n-1}C_{n-1}$

PASCAL'S TRIANGLE

The coefficients ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, … , ${}^nC_n$ or simply C_0, C_1, C_2 …, C_n are called *binomial coefficients* and they can be evaluated with the help of *Pascal's triangle* as below:



I. Each row starts and ends in 1.
II. The coefficients of an expansion are obtained from the coefficients of the previous expansion.
III. Each coeffcient is the sum of the two coefficients which lie on either side of it in the previous row.

General Term in the Expansion of $(x + y)^n$

In the binomial expansion of $(x + y)^n$, the $(r + 1)$th term from the beginning is usually called the *general term* and it is denoted by T_{r+1}, i.e.,

$$T_{r+1} = {}^nC_r\, x^{n-r} y^r$$

SOLVED EXAMPLES

1. The sum of the coefficients in the expansion of $(1 + 5x - 7x^3)^{3165}$ is

(A) 1 (B) 2^{3165}
(C) 2^{3164} (D) – 1

Solution: (D)

Putting $x = 1$ in $(1 + 5x - 7x^3)^{3165}$, the required sum of coefficients $= (1 + 5 - 7)^{3165} = (-1)^{3165} = -1$.

2. The 8th term of $\left(3x + \frac{2}{3x^2}\right)^{12}$, when expanded in ascending power of x, is

(A) $\frac{228096}{x^3}$ (B) $\frac{228096}{x^9}$
(C) $\frac{328179}{x^9}$ (D) none of these

Solution: (A)

When $\left(3x+\dfrac{2}{3x^2}\right)^{12}$ is expanded, the power of x goes on decreasing as the terms proceed. Hence, it is expanded in descending powers of x. So $\left(\dfrac{2}{3x^2}+3x\right)^{12}$, when expanded, will be in ascending powers of x.

Now, t_8 in $\left(\dfrac{2}{3x^2}+3x\right)^{12} = {}^{12}C_7 \left(\dfrac{2}{3x^2}\right)^{12-7} \cdot (3x)^7$

$$= \frac{12!}{7!\,5!} \cdot \left(\frac{2}{3x^2}\right)^5 \cdot (3x)^7$$

$$= \frac{12\times 11\times 10\times 9\times 8}{5\times 4\times 3\times 2} \cdot \frac{2^5\cdot 3^2}{x^3}$$

$$= \frac{228096}{x^3}$$

3. If A is the sum of the odd terms and B the sum of even terms in the expansion of $(x + a)^n$, then $A^2 - B^2 =$
   (A) $(x^2 + a^2)^n$ (B) $(x^2 - a^2)^n$
   (C) $2(x^2 - a^2)^n$ (D) none of these

Solution: (B)

We have,

$(x + a)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 + {}^nC_3\, x^{n-3} a^3 + \ldots + {}^nC_n\, x^n$

$= ({}^nC_0\, x^n + {}^nC_2\, x^{n-2} a^2 + \ldots) + ({}^nC_1\, x^{n-1} a^1 + {}^nC_3\, x^{n-3} a^3 + \ldots)$

$= A + B$

$(x - a)^n = {}^nC_0\, x^n - {}^nC_1\, x^{n-1} a^1 + {}^nC_2\, x^{n-2} a^2 - {}^nC_3\, x^{n-3} a^3 + \ldots + {}^nC_n (-1)^n a^n$

$= ({}^nC_0\, x^n + {}^nC_2\, x^{n-2} a^2 + \ldots) - ({}^nC_1\, x^{n-1} a^1 + {}^nC_3\, x^{n-3} a^3 + \ldots)$

$= A - B$

$\therefore\ A^2 - B^2 = (A + B)(A - B) = (x + a)^n (x - a)^n = (x^2 - a^2)^n$

4. The 7th term in $\left(\dfrac{1}{y}+y^2\right)^{10}$, when expanded in descending power of y, is
   (a) $\dfrac{210}{y^2}$ (b) $\dfrac{y^2}{210}$
   (c) $210y^2$ (d) none of these

Solution: (C)

When $\left(\dfrac{1}{y}+y^2\right)^{10}$ is expanded, the powers of y go on increasing as the terms proceed. Hence it is expanded in ascending powers of y. So $\left(y^2+\dfrac{1}{y}\right)^{10}$, when expanded, will be in descending powers of y.

Hence, $t_7 = {}^{10}C_6\, (y^2)^4 \left(\dfrac{1}{y}\right)^6 = \dfrac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}\, y^2$

$= 210y^2$

5. The term independent of x in $(1 + x)^m \left(1+\dfrac{1}{x}\right)^n$ is
   (A) ${}^{m+n}C_m$ (B) ${}^{m+n}C_n$
   (C) ${}^{m+n}C_{m-n}$ (D) none of these

Solution: (B)

We have,

$$(1 + x)^m \left(1+\frac{1}{x}\right)^n = (1 + x)^m \left(\frac{x+1}{x}\right)^n = \frac{(1+x)^{m+n}}{x^n} = x^{-n}(1 + x)^{m+n}$$

$\therefore$ Required term independent of x = coefficient of x^0 in $x^{-n}(1 + x)^{m+n}$ = coefficient of x^n in $(1 + x)^{m+n}$ $= {}^{m+n}C_n$

6. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$ is
   (A) ${}^{100}C_{47}$ (B) ${}^{100}C_{53}$
   (C) $-{}^{100}C_{53}$ (D) $-{}^{100}C_{100}$

Solution: (C)

We have, $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$

$= (x - 3)^{100} + {}^{100}C_1 (x - 3)^{99} \cdot 2^1 + {}^{100}C_2 (x - 3)^{98} \cdot 2^2 + \ldots + {}^{100}C_{100}\, 2^{100}$

$= [(x - 3) + 2]^{100} = (x - 1)^{100} = (1 - x)^{100}$

$\therefore$ coefficient of $x^{54} = {}^{100}C_{53} (-1)^{53} = \mathbf{-{}^{100}C_{53}}$

7. The coefficient of x^m in $(1 + x)^m + (1 + x)^{m+1} +, \ldots, + (1 + x)^n$, $m \le n$ is
   (A) ${}^nC_m$ (B) ${}^nC_{m+1}$
   (C) ${}^{n+1}C_{m+1}$ (D) none of these

Solution: (C)

The coefficient of x_m in
$(1 + x)^m + (1 + x)^{m+1} + (1 + x)^{m+2} + \ldots + (1 + x)^n$

$= {}^mC_m + {}^{m+1}C_m + {}^{m+2}C_m + \ldots + {}^nC_m$

$= {}^{m+1}C_{m+1} + {}^{m+1}C_m + {}^{m+2}C_m + \ldots + {}^nC_m$

$(\because\ {}^mC_m = {}^{m+1}C_{m+1} = 1)$

$= {}^{m+2}C_{m+1} + {}^{m+2}C_m + \ldots + {}^nC_m$

$(\because\ {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

$= {}^{m+3}C_{m+1} + \ldots + {}^nC_m = {}^{n+1}C_{m+1}.$

8. The coefficient of x^3 in the expansion of $(1 - x + x^2)^6$ is
   (A) 50 (B) – 50
   (C) 68 (D) none of these

Solution: (B)

$(1 - x + x^2)^6 = [1 - x(1 - x)]^6$
$= {}^6C_0 - {}^6C_1\, x(1 - x) + {}^6C_2\, x^2 (1 - x)^2$
$- {}^6C_3\, x^3 (1 - x)^3 + \ldots$ to 7 terms
$= {}^6C_0 - {}^6C_1\, x(1 - x) + {}^6C_2\, x^2 (1 - 2x + x^2)$
$- {}^6C_3\, x^3 (1 - 3x + 3x^2 - x^3) + \ldots$ to 7 terms

$\therefore$ Coefficient of $x^3 = -2 \cdot {}^6C_2 - {}^6C_3$, (collecting coefficients of x^3 from each term)

$$= -2\frac{6!}{2!\,4!} - \frac{6!}{3!\,3!} = -50$$

9. The value of x in the expression $(x + x^{\log_{10} x})^5$, if the third term in the expansion is 10,00,000, is

(A) 10^{-1} (B) 10^{1}
(C) $10^{-5/2}$ (D) $10^{5/2}$

Solution: (B, C)

Put $\log_{10} x = z$
Then, given expression $= (x + x^z)^5$.
Now, $T_3 = {}^5C_2\, x^3 (x^z)^2 = 10x^{3+2z} = 10^6$
$\therefore \quad x^{3+2z} = 10^5$.
Taking log, we get
$(3 + 2z)\log_{10} x = 5\log_{10} 10$
$\Rightarrow (3 + 2z)\, z = 5$ or $2z^2 + 3z - 5 = 0$
$\Rightarrow (z - 1)(2z + 5) = 0 \Rightarrow z = 1, -\dfrac{5}{2}$
$\therefore \log_{10} x = 1$ or $-\dfrac{5}{2} \quad \therefore \quad x = 10^1$ or $10^{-5/2}$.

10. The value of $\dfrac{{}^nC_1}{2} + \dfrac{{}^nC_3}{4} + \dfrac{{}^nC_5}{6} + \ldots$ is

(A) $\dfrac{2^n - 1}{n}$ (B) $\dfrac{2^n + 1}{n}$
(C) $\dfrac{2^n - 1}{n + 1}$ (D) $\dfrac{2^n + 1}{n + 1}$

Solution: (C)

The rth term of the given expression is

$$T_r = \frac{{}^nC_{2r-1}}{2r}$$

Since $\dfrac{1}{r+1} \cdot {}^nC_r = \dfrac{1}{n+1} \cdot {}^{n+1}C_{r+1}$

$$\therefore \quad T_r = \frac{{}^nC_{2r-1}}{2r} = \frac{1}{n+1} \cdot {}^{n+1}C_{2r}$$

$$\therefore \quad \frac{{}^nC_1}{2} + \frac{{}^nC_3}{4} + \frac{{}^nC_5}{6} + \ldots$$

$$= \frac{1}{n+1}({}^{n+1}C_2 + {}^{n+1}C_4 + \ldots)$$

$$= \frac{1}{n+1}(2^{n+1-1} - {}^{n+1}C_0)$$

$$= \frac{2^n - 1}{n+1}$$

11. The coefficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$ is

(A) 40 (B) 50
(C) −50 (D) 60

Solution: (D)

We have, $(1 + x^2)^5 (1 + x)^4$
$= (1 + {}^5C_1\, x^2 + {}^5C_2\, x^4 + \ldots)(1 + {}^4C_1\, x + {}^4C_2\, x^2 + {}^4C_3\, x^3 + {}^4C_4\, x^4)$
$= (1 + 5x^2 + 10x^4 + \ldots)(1 + 4x + 6x^2 + 4x^3 + x^4)$
The term giving x^5 in the above product is
$(5x^2)(4x^3) + (10x^4)(4x) = (20 + 40)\, x^5 = 60x^5$
Hence, the coefficient is 60.

12. If $(1 + x - 2x^2)^6 = 1 + a_1 x + a_2 x^2 + \ldots + a_{12} x^{12}$, then $a_2 + a_4 + a_6 + \ldots + a_{12} =$

(A) 21 (B) 11
(C) 31 (D) none of these

Solution: (C)

Given
$(1 + x - 2x^2)^6 = 1 + a_1 x + a_2 x^2 + \ldots + a_{12} x^{12}$
Putting $x = 1$, we get
$0 = 1 + a_1 + a_2 + \ldots + a_{12}$ ...(1)
Putting $x = -1$, we get
$64 = 1 - a_1 + a_2 - \ldots + a_{12}$ ...(2)
Adding Eq. (1) and (2), we get
$64 = 2(1 + a_2 + a_4 + \ldots)$
$\therefore \quad a_2 + a_4 + a_6 + \ldots + a_{12} = 31$

13. If 7^{103} is divided by 25, then the remainder is

(A) 20 (B) 16
(C) 18 (D) 15

Solution: (C)

We have, $7^{103} = 7(49)^{51} = 7(50 - 1)^{51}$
$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots - 1)$
$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots) - 7 + 18 - 18$
$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots) - 25 + 18$
$= k + 18$ (say) Q k is divisible by 25,
$\therefore$ remainder is 18.

14. The sum of rational terms in the expansion of $(\sqrt{2} + 3^{1/5})^{10}$ is

(A) 31 (B) 41
(C) 51 (D) none of these

Solution: (B)

$(r + 1)$th term in the given expansion is given by

$t_{r+1} = {}^{10}C_r\, 2^{\frac{10-r}{2}}\, 3^{\frac{r}{5}}$, where $r = 0, 1, 2, ..., 10$

For rational terms

r = a multiple of 5 = 0, 5, 10...(1)

$10 - r$ = a multiple of 2 = 0, 2, 4, 6, 8, 10 ...(2)

From Eq. (1) and (2) possible values of r are : 0 and 10

$\therefore$ sum of rational terms

$= t_1 + t_{11} = {}^{10}C_0 (\sqrt{2})^{10} (3^{1/5})^0 + {}^{10}C_{10} (\sqrt{2})^0 (3^{1/5})^{10}$

$= 2^5 + 3^2 = 32 + 9 = 41$

15. In the expansion of $(x + a)^n$ if the sum of odd terms be P and the sum of even terms be Q, then $4PQ =$

(A) $(x + a)^n - (x - a)^n$ (B) $(x + a)^n + (x - a)^n$

(C) $(x + a)^{2n} - (x - a)^{2n}$ (D) none of these

Solution: (C)

We have,

$(x + a)^n = x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + {}^nC_3 x^{n-3} a^3 + ...$

$= (x^n + {}^nC_2 x^{n-2} a^2 + ...) + ({}^nC_1 x^{n-1} a + {}^nC_3 x^{n-3} a^3 + ...)$

$= P + Q$

$\therefore$ $(x - a)^n = P - Q$, as the terms are alternatively positive and negative.

$\therefore$ $4PQ = (P + Q)^2 - (P - Q)^2 = (x + a)^{2n} - (x - a)^{2n}$

16. If $C_0, C_1, C_2, ..., C_n$ are the coefficients of the expansion of $(1 + x)^n$, then the value of $\sum_0^n \frac{C_k}{k+1}$ is

(A) 0 (B) $\frac{2^n - 1}{n}$

(C) $\frac{2^{n+1} - 1}{n+1}$ (D) none of these

Solution: (C)

Here, $t_{r+1} = \frac{{}^nC_r}{r+1} = \frac{1}{r+1} \cdot {}^nC_r$

$= \frac{1}{n+1} \cdot {}^{n+1}C_{r+1}$

Putting $r = 0, 1, 2, ... n$ and adding, we get $\sum_0^n \frac{C_k}{k+1}$

$= \frac{1}{n+1} ({}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + ... + {}^{n+1}C_{n+1})$

$= \frac{1}{n+1} (2^{n+1} - {}^{n+1}C_0) = \frac{2^{n+1} - 1}{n+1}$

17. The value of

$$\frac{(18^3 + 7^3 + 3 \cdot 18 \cdot 7 \cdot 25)}{3^6 + 6 \cdot 243 \cdot 2 + 15 \cdot 81 \cdot 4 + 20 \cdot 27 \cdot 8 + 15 \cdot 9 \cdot 16 + 6 \cdot 3 \cdot 32 + 64}$$ is

(A) 0 (B) 1

(C) 2 (D) none of these

Solution: (B)

The numerator is of the form

$a^3 + b^3 + 3ab (a + b) = (a + b)^3$

where $a = 18$ and $b = 7$

$\therefore$ Numerator $= (18 + 7)^3 = 25^3$.

For denominator, $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243$

${}^6C_1 = 6, {}^6C_2 = 15, {}^6C_3 = 20$

${}^6C_4 = {}^6C_2 = 15, {}^6C_5 = {}^6C_1 = 6, {}^6C_6 = 1$

$\therefore$ denominator $= 3^6 + {}^6C_1\, 3^5 \cdot 2^1 + {}^6C_2\, 3^4 \cdot 2^2 + {}^6C_3\, 3^3 \cdot 2^3 + {}^6C_4\, 3^2 \cdot 2^4 + {}^6C_5\, 3 \cdot 2^5 + {}^6C_6\, 2^6$

This is clearly the expansion of

$(3 + 2)^6 = 5^6 = (25)^3$

$\therefore$ $\frac{\text{Numerator}}{\text{Denominator}} = \frac{(25)^3}{(25)^3} = 1$

18. Larger of $99^{50} + 100^{50}$ and 101^{50} is

(A) 101^{50} (B) $99^{50} + 100^{50}$

(C) both are equal (D) none of these

Solution: (A)

We have,

$101^{50} = (100 + 1)^{50}$

$= 100^{50} + 50 \cdot 100^{49} + \frac{50 \cdot 49}{1 \cdot 2} \cdot 100^{48} + ...$

and $99^{50} = (100 - 1)^{50}$

$= 100^{50} - 50 \cdot 100^{49} + \frac{50 \cdot 49}{1 \cdot 2} \cdot 100^{48} - ...$

Subtracting, we get

$101^{50} - 99^{50} = 2 (50 \cdot 100^{49} + \frac{50 \cdot 49 \cdot 48}{1 \cdot 2 \cdot 3} \times 100^{47} + ...)$

$= 100^{50} + 2 \cdot \frac{50 \cdot 49 \cdot 48}{1 \cdot 2 \cdot 3} \cdot 100^{47} + ... > 100^{50}$

Hence, $101^{50} > 99^{50} + 100^{50}$.

19. For all $n \in N$, $2^{4n} - 15n - 1$ is divisible by

(A) 225 (B) 125

(C) 325 (D) none of these

Solution: (A)

We have, $2^{4n} = (2^4)^n = (16)^n = (1 + 15)^n$

$\therefore$ $2^{4n} = 1 + {}^nC_1 \cdot 15 + {}^nC_2\, 15^2 + {}^nC_3\, 15^3 + ...$

$\Rightarrow$ $2^{4n} - 1 - 15n = 15^2 ({}^nC_2 + {}^nC_3 \cdot 15 + ...)$

$= 225\, K$, where K is an integer.

Hence, $2^{4n} - 15n - 1$ is divisible by 225.

20. When 5^{99} is divided by 13, the remainder is

(A) 8 (B) 9

(C) 10 (D) none of these

Solution: (A)

We have,

$5^{99} = 5^3 \cdot 5^{96} = (125)(625)^{24}$

$= [13 \times 9 + 8] (1 + 48 \times 13)^{24}$

$= (13 \times 9 + 8) [1 + {}^{24}C_1 \times (48 \times 13) + {}^{24}C_2 (48 \times 13)^2 + ... + (48 \times 13)^{24}]$

= 8 + terms containing powers of 13.
Hence remainder = 8.

21. The last digit of the number $(32)^{32}$ is

(A) 4 (B) 6
(C) 8 (D) none of these

Solution: (B)

$(32)^{32} = (2 + 3 \times 10)^{32}$
$= 2^{32} + 10k$, where $k \in N$
Therefore, last digits in $(32)^{32}$ = last digit in $(2)^{32}$
But $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32$
$\therefore\ 2^{32} = (2^5)^6 \cdot 2^2 = (32)^6 \cdot 4 = (2 + 30)^6 \cdot 4$
$= (2^6 + 10r)\,4,\ r \in N$
Last digit in 2^{32} = last digit in $(2)^6 \cdot 4$ = last digit in $4 \times 4 = $ **6**
$\therefore$ Last digit in $(32)^{32} = 6$.

22. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, then

$$2C_0 + 2^2 \cdot \frac{C_1}{2} + 2^3 \cdot \frac{C_2}{3} + \ldots + 2^{n+1}\frac{C_n}{n+1} =$$

(A) $\frac{3^{n+1}-1}{n+1}$ (B) $\frac{3^n-1}{n}$

(C) $\frac{3^{n+2}-1}{n+2}$ (D) none of these

Solution: (A)

We have,

$$t_{r+1} = 2^{r+1}\frac{{}^nC_r}{r+1} = 2^{r+1} \cdot \frac{1}{n+1} \cdot {}^{n+1}C_{r+1}$$

Putting $r = 0, 1, 2, \ldots, n$ and adding, we get the required sum

$$= \frac{1}{n+1}\left(2 \cdot {}^{n+1}C_1 + 2^2 \cdot {}^{n+1}C_2 + \ldots + 2^{n+1} \cdot {}^{n+1}C_{n+1}\right)$$

$$= \frac{1}{n+1}\left[(1+2)^{n+1} - {}^{n+1}C_0\right] = \frac{3^{n+1}-1}{n+1}.$$

23. For integer n > 1, the digit at units place in the number $\sum_{r=0}^{100} r! + 2^{2^n}$ is

(A) 0 (B) 1
(C) 2 (D) 3

Solution: (A)

Since the digit at units place in each of 5!, 6!, ..., 100! is 0 and 0! + 1! + 2! + 3! + 4! = 34.

therefore the digit at units place in $\sum_{r=0}^{100} r!$ is 4.

Now, $2^{2^n} = 2^{4k}$, $k \in N$ (2^n is a multiple of 4 form $n > 1$)

$\therefore$ The digit at units place in $2^{2^n} = 2^{4k} = (16)^k$ is 6.

Thus, the digit at units place in $\sum_{r=0}^{100} r! + 2^{2^n}$ is 0.

24. When $32^{(32)^{(32)}}$ is divided by 7, the remainder is

(A) 4 (B) 6
(C) 8 (D) none of these

Solution: (A)

$(32)^{32} = (2^5)^{32} = 2^{160} = (3 - 1)^{160}$
$= {}^{160}C_0 3^{160} - {}^{160}C_1 \cdot 3^{159}$
$+ \ldots + {}^{160}C_{159} \cdot 3 + {}^{160}C_{160} \cdot 3^0$
$= 3k + 1$, where $k \in N$
Now, $32^{(32)^{(32)}} = (32)^{3k+1} = (2^5)^{3k+1} = 2^{15k+5}$
$= 2^{3(5k+1)} \cdot 2^2 = (2^3)^{5k+1} \cdot 4$
$= 4(7 + 1)^{5k+1}$
$= 4[{}^{5k+1}C_0 7^{5k+1} + {}^{5k+1}C_1 7^{5k}$
$+ \ldots + {}^{5k+1}C_{5k} 7 + {}^{5k+1}C_{5k+1} \cdot 7^0]$
$= 4(7n + 1)$, where $n \in N$
$= 28n + 4$.
Therefore, when $32^{(32)^{(32)}}$ is divided by 7, the remainder is 4.

25. The number of non zero terms in the expansion of $(1 + 3\sqrt{2}x)^9 + (1 - 3\sqrt{2}x)^9$ is

(A) 9 (B) 0
(C) 5 (D) 10

Solution: (C)

In the expansion of
$(1 + 3\sqrt{2}\,x)^9 + (1 - 3\sqrt{2}\,x)^9$
2nd, 4th, 6th, 8th and 10th terms get cancelled.
$\therefore$ Number of non-zero terms in
$2\,[{}^9C_0 + {}^9C_2\,(3\sqrt{2x})^2 + \ldots + {}^9C_8\,(3\sqrt{2x})^8]$ is **5.**

26. The expression $[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$ is a polynomial of degree

(A) 5 (B) 6
(C) 7 (D) 8

Solution: (C)

$[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$
$= 2\,[{}^5C_0 x^5 + {}^5C_2 x^3 (x^3 - 1) + {}^5C_4 x (x^3 - 1)^2]$
$= 2\,[x^5 + 10x^3 (x^3 - 1) + 5x (x^3 - 1)^2]$
$= 5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x$
which is a polynomial of degree 7.

27. The value of x, for which the 6th term in the expansion of $\left[2^{\log_2\sqrt{(9^{x-1}+7)}} + \frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right]^7$ is 84, is equal to

(A) 4 (B) 3
(C) 2 (D) 1

Solution: (C,D)

The given expression

$$= \left[\sqrt{9^{x-1}+7}+\frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$$

Given, $T_6 = 84$

$$\Rightarrow {}^7C_5\,(\sqrt{9^{x-1}+7})^{7-5}\left(\frac{1}{(3^{x-1}+1)^{1/5}}\right)^5 = 84$$

$$\Rightarrow {}^7C_5\,(9^{x-1}+7)\cdot\frac{1}{(3^{x-1}+1)} = 84$$

$\Rightarrow 9^{x-1} + 7 = 4\,(3^{x-1} + 1)$
$\Rightarrow 3^{2x} - 12\cdot 3^x + 27 = 0$
$\Rightarrow (3^x - 3)\,(3^x - 9) = 0$
$\Rightarrow 3^x = 3, 9 \Rightarrow x = 1, 2$

28. When 3^{37} is divided by 80, the remainder is

(A) 3 (B) 4
(C) 6 (D) none of these

Solution: (A)

We have, $3^{37} = 3^{4.9}\cdot 3 = 3(81)^9 = 3(80 + 1)^9$
$= 3({}^9C_0\cdot 80^9 + {}^9C_1 80^8 + \ldots + {}^9C_9)$

Thus, when 3^{37} is divided by 80, the *m* remainder is 3.

29. If the second term in the expansion $\left(\sqrt[13]{a}+\frac{a}{\sqrt{a^{-1}}}\right)^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is

(A) 8 (B) 12
(C) 4 (D) none of these

Solution: (C)

Given : $T_2 = 14a^{5/2}$

$$\Rightarrow {}^nC_1\,(a^{1/13})^{n-1}\cdot\left(\frac{a}{a^{-1/2}}\right)^1 = 14a^{5/2}$$

$\Rightarrow n\cdot a^{(n-1)/13}\cdot a^{3/2} = 14a^{5/2}$
$\Rightarrow n\cdot a^{(n-1)/13} = 14a \Rightarrow n\cdot a^{(n-14)/13} = 14$
$\Rightarrow n = 14$

$$\therefore \frac{{}^nC_3}{{}^nC_2} = \frac{{}^{14}C_3}{{}^{14}C_2} = 4$$

30. The number of irrational terms in the expansion of $(4^{1/5} + 7^{1/10})^{45}$ is

(A) 40 (B) 5
(C) 41 (D) none of these

Solution: (C)

Total number of terms in the expansion of $(4^{1/5} + 7^{1/10})^{45}$ is 45 + 1, i.e., 46.

The general term in the expansion is

$$T_{r+1} = {}^{45}C_r\cdot 4^{\frac{45-r}{5}}\cdot 7^{\frac{r}{10}}$$

T_{r+1} is rational if r = 0, 10, 20, 30, 40.

∴ Number of rational terms = 5.

∴ Number of irrational terms = 46 – 5 = 41.

31. In the expansion of $(1 + x + x^3 + x^4)^{10}$, the coefficient of x^4 is

(A) ${}^{40}C_4$ (B) ${}^{10}C_4$
(C) 210 (D) 310

Solution: (D)

$(1 + x + x^3 + x^4)^{10} = [(1 + x)\,(1 + x^3)]^{10}$
$= (1 + x)^{10}\,(1 + x^3)^{10}$
$= (1 + {}^{10}C_1\,x + {}^{10}C_2\,x^2 + {}^{10}C_3\,x^3 + {}^{10}C_4\,x^4 \ldots)$
$\times (1 + {}^{10}C_1\,x^3 + {}^{10}C_2\,x^6 \ldots)$

∴ Coefficient of $x^4 = ({}^{10}C_1)\,({}^{10}C_1) + {}^{10}C_4$

$$= 100 + \frac{10\cdot 9\cdot 8\cdot 7}{1\cdot 2\cdot 3\cdot 4} = 100 + 210 = 310$$

32. If $A = {}^{2n}C_0\cdot{}^{2n}C_1 + {}^{2n}C_1\,{}^{2n-1}C_1 + {}^{2n}C_2\,{}^{2n-2}C_1 + \ldots$, then A is

(A) 0 (B) 2^n
(C) $n\,2^{2n}$ (D) 1

Solution: (C)

A = coeff. of x in $[{}^{2n}C_0(1 + x)^{2n}$
$+ {}^{2n}C_1\,(1 + x)^{2n-1} + \ldots)]$
= coeff. of x in $(1 + (1 + x))^{2n}$
= coeff. of x in $(2 + x)^{2n}$

$$= \text{coeff. of } x \text{ in } 2^{2n}\left(1+\frac{x}{2}\right)^{2n} = n\cdot 2^{2n}$$

33. The greatest integer which divides the number $101^{100} - 1$ is

(A) 100 (B) 1000
(C) 10000 (D) 100000

Solution: (C)

By Binomial theorem

$$(1 + x)^n = \left[1+nx+\frac{n(n-1)}{2}\cdot x^2 \ldots + x^n\right]$$

or $$(1 + x)^n - 1 = nx + \frac{n(n-1)}{2}\,x^2 \ldots + x^n$$

If $x = n$, $$(1 + n)^n - 1 = n^2 + \frac{n(n-1)}{2}\,n^2 \ldots n^n$$

$$(1 + n)^n - 1 = n^2\left[1+\frac{n(n-1)}{2}\ldots + n^{n-2}\right]$$

Put $n = 100$,

$$(1+100)^{100} - 1 = (100)^2\left[1+\frac{100(100-1)}{2}\ldots + 100^{98}\right]$$

$$(101)^{100} - 1 = (100)^2\left[1 + \frac{100\times 99}{2}... + 100^{98}\right]$$

Clearly $(101)^{100} - 1$ is divisible by
$(100)^2 = 10000$

34. If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, its coefficient is

(A) ${}^{2n}C_{\frac{4n-p}{3}}$ (B) ${}^{2n}C_{\frac{2n-p}{3}}$

(C) ${}^{2n}C_{\frac{4n-p}{3}}$ (D) none of these

Solution: (A)

Let t_{r+1} contains x^p.

Then, $t_{r+1} = {}^{2n}C_r\,(x^2)^{2n-r}\left(\frac{1}{x}\right)^r = {}^{2n}C_r\,x^{4n-3r}$

$\therefore\quad 4n - 3r = p;\quad$ or $r = \dfrac{4n-p}{3}$

Hence, the coefficient of $x^p = {}^{2n}C_{\frac{4n-p}{3}}$

35. Given positive integers $r > 1$, $n > 2$ and the coefficients of $(3r)$th term and $(r + 2)$th term in the binomial expansion of $(1 + x)^{2n}$ are equal, then $r =$

(A) $\frac{n}{2}$, n even (B) $\frac{n}{2}$

(C) n (D) 1

Solution: (A)

We have, $t_{3r} = {}^{2n}C_{3r-1}\,x^{3r-1}$

and $t_{r+2} = {}^{2n}C_{r+1}\,x^{r+1}$

Given, ${}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow\ 3r - 1 = r + 1$; or $(3r - 1) + (r + 1) = 2n$

$\Rightarrow\ 2r = 2$; or $4r = 2n$

$\Rightarrow\ r = 1$ (impossible); or $r = \frac{n}{2}$.

But r is a positive integer greater than 1. So the value of r **is** $\frac{n}{2}$ **provided** n **is an even integer** (> 2), otherwise r has no value.

36. If the last term in the binomial expansion of $\left(3\sqrt{2} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3.3\sqrt{9}}\right)^{\log_3 8}$, then the 5th term is

(A) $2\cdot{}^{10}C_6$ (B) $4\cdot{}^{10}C_4$

(C) $\frac{1}{2}\,{}^{10}C_6$ (D) ${}^{10}C_6$

Solution: (D)

The last term of $\left(\sqrt[3]{2} - \frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$

$$\Rightarrow\quad {}^nC_n\cdot\left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$$

$$\Rightarrow\quad (-1)^n\cdot\left(\frac{1}{2}\right)^{n/2} = \left(\frac{1}{3^{5/3}}\right)^{\log_3 8} = 3^{-\frac{5}{3}\cdot 3\log_3 2}$$

$$= 2^{-5} = \left(\frac{1}{2}\right)^5$$

$\Rightarrow\quad n = 10$

Therefore, 5th term in $\left(\sqrt[3]{2} - \frac{1}{\sqrt{2}}\right)^{10}$ is

$$T_5 = T_{4+1} = {}^{10}C_4\left(\sqrt[3]{2}\right)^{10-4}\left(-\frac{1}{\sqrt{2}}\right)^4$$

$$= {}^{10}C_4\cdot(4)\frac{1}{4} = {}^{10}C_4 = {}^{10}C_6$$

37. If $(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$, then $a_0 + a_2 + a_4 + ... + a_{2n}$ is equal to

(A) $3n^2 + \frac{1}{2}$ (B) $\frac{1-3^n}{2}$

(C) $\frac{3^n-1}{2}$ (D) $\frac{3^n+1}{2}$

Solution: (D)

Putting $x = -1, 1$ successively in the given equation and adding, we shall get the result.

Method for Finding the Independent Term or Constant Term

Step I Write down the general term in the expansion of $(x + a)^n$ i.e., $(r + 1)$ th term
$\Rightarrow t_{r+1} = {}^nC_r x^{n-r} a^r$

Step II Separate the constants and variables. Also group them separately.

Setp II Since, we need to find the term independent of x in the given binomial expansion, equate to zero the index of x and accordingly we will get the value of r for which there exists a term independent of x in the expansion.

Greatest Term (Numerically) in the Expansion of $(1 + x)^n$

Method 1

1. Let T_r (the rth term) be the greatest term.
2. Find T_{r-1}, T_r, T_{r+1} from the given expansion.
3. Put $\frac{T_r}{T_{r+1}} \geq 1$ and $\frac{T_r}{T_{r-1}} \geq 1$. This will give an inequality from where value or values of r can be obtained.

4. Then, find the rth term T_r which is the greatest term.

Method 2

1. Find the value of $k = \dfrac{(n+1)|x|}{1+|x|}$
2. If k is an integer, then T_k and T_{k+1} are equal and both are greatest terms.
3. If k is not an integer, then $T_{(k)+1}$ is the greatest term, where (k) is the greatest integral part of k.

TRICK(S) FOR PROBLEM SOLVING

To find the greatest term in the expansion of $(x+y)^n$, write $(x+y)^n = x^n\left(1+\dfrac{y}{x}\right)^n$ and then find the greatest term in $\left(1+\dfrac{y}{x}\right)^n$.

SOLVED EXAMPLES

38. The greatest term (numerically) in the expansion of $(2+3x)^9$, when $x = \dfrac{3}{2}$, is

(A) $\dfrac{5\times 3^{11}}{2}$ (B) $\dfrac{5\times 3^{13}}{2}$

(C) $\dfrac{7\times 3^{13}}{2}$ (D) none of these

Solution: (C)

We have,

$$(2+3x)^9 = 2^9\left(1+\frac{3x}{2}\right)^9 = 2^9\left(1+\frac{9}{4}\right)^9 \qquad \left(\because x = \frac{3}{2}\right)$$

$$\therefore\ m = \left|\frac{x(n+1)}{(x+1)}\right|$$

$$= \left|\frac{\left(\frac{9}{4}\right)(9+1)}{\left(\frac{9}{4}\right)+1}\right| = \frac{90}{13} = 6\frac{12}{13} \neq \text{Integer}$$

The greatest term in the expansion is $T_{[m]+1} = T_{6+1} = T_7$

Hence, the greatest term $= 2^9 \cdot T_7$

$$= 2^9 \cdot T_{6+1} = 2^9 \cdot {}^9C_6\left(\frac{9}{4}\right)^6$$

$$= 2^9 \cdot {}^9C_3\left(\frac{9}{4}\right)^6 = 2^9 \cdot \frac{9\cdot 8\cdot 7}{1\cdot 2\cdot 3}\cdot\frac{3^{12}}{2^{12}} = \frac{7\times 3^{13}}{2}$$

39. The greatest term (numerically) in the expansion of $(3-5x)^{11}$ when $x = \dfrac{1}{5}$ is

(A) 55×3^9 (B) 46×3^9

(C) 55×3^6 (D) none of these

Solution: (A)

We have,

$$(3-5x)^{11} = 3^{11}\left(1-\frac{5x}{3}\right)^{11} = 3^{11}\left(1-\frac{1}{3}\right)^{11} \qquad \left(\because x = \frac{1}{5}\right)$$

$$\therefore\ m = \frac{|x|\,(n+1)}{(|x|+1)} \qquad \left(-\frac{1}{3}<0\right)$$

$$= \frac{\left(\left|-\frac{1}{3}\right|\right)(11+1)}{\left(\left|-\frac{1}{3}\right|+1\right)} = 3$$

The greatest terms in the expansion are T_3 and T_4

$\therefore$ Greatest term (when $r = 2$) $= 3^{11}\,|T_{2+1}|$

$$= 3^{11}\left|{}^{11}C_2\left(-\frac{1}{3}\right)^2\right| = 3^{11}\left|\frac{11\cdot 10}{1\cdot 2}\times\frac{1}{9}\right| = 55\times 3^9$$

and greatest term (when $r = 3$) $= 3^{11}\,|T_{3+1}|$

$$= 3^{11}\left|{}^{11}C_3\left(-\frac{1}{3}\right)^3\right| = 3^{11}\left|\frac{11\cdot 10\cdot 9}{1\cdot 2\cdot 3}\times -\frac{1}{27}\right| = \mathbf{55\times 3^9}$$

From above, we see that the values of both greatest terms are equal.

MIDDLE TERM IN THE BINOMIAL EXPANSION

The middle term in the binomial expansion of $(x+y)^n$ depends upon the value of n.

1. If n is even, then there is only one middle term i.e. $\left(\dfrac{n}{2}+1\right)$th term.
2. If n is odd, then there are two middle terms, i.e., $\left(\dfrac{n+1}{2}\right)$th and $\left(\dfrac{n+3}{2}\right)$th terms.

IMPORTANT POINTS

- When there are two middle terms in the expansion, their binomial coefficients are equal.
- Binomial coefficient of the middle term is the greatest binomial coefficient.

SOLVED EXAMPLE

40. The greatest coefficient in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is

(A) $\frac{1\cdot 3\cdot 5...(2n-1)\cdot 2^n}{n!}$ (B) $\frac{2n!}{(n!)^2}$

(C) $\frac{n!}{\left[\left(\frac{n}{2}\right)!\right]^2}$ (D) none of these

Solution: (A, B)

Since the middle term has greatest coefficient,

$\therefore$ greatest coefficient = coefficient of the middle term

$$= {}^{2n}C_n = \frac{(2n)!}{n!n!}$$

$$= \frac{2n(2n-1)(2n-2)(2n-3)...4\cdot 3\cdot 2\cdot 1}{n!n!}$$

$$= \frac{[(2n-1)(2n-3)...3\cdot 1]\,[2n(2n-2)(2n-4)...4\cdot 2]}{n!n!}$$

$$= \frac{[1\cdot 3\cdot 5...(2n-1)]2^n[n(n-1)(n-2)...2\cdot 1]}{n!n!}$$

$$= \frac{1\cdot 3\cdot 5...(2n-1)2^n n!}{n!n!} = \frac{1\cdot 3\cdot 5...(2n-1)2^n}{n!}.$$

*p*th Term from the End in the Binomial Expansion of $(x + y)^n$

pth term from the end in the expansion of $(x + y)^n$ is $(n - p + 2)$th term from the beginning.

Properties of Binomial Coefficients

In the binomial expansion of $(1 + x)^n$, the coefficients ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ... ${}^nC_n$ are denoted by $C_0, C_1, C_2, ... C_n$ respectively.

1. If n is even, then greatest coefficient = ${}^nC_{n/2}$

2. If n is odd, then greatest coefficient is ${}^nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$.

3. $C_0 + C_1 + C_2 + ... + C_n = 2^n$

4. $C_0 + C_2 + C_4 + ... = C_1 + C_3 + C_5 + ... = 2^{n-1}$

5. $C_0 - C_1 + C_2 - C_3 + C_4 - ... + (-1)^n C_n = 0$

6. $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = \frac{(2n)!}{(n!)^2} = {}^{2n}C_n$

7. $C_0^2 - C_1^2 + C_2^2 - C_3^2 + ...$

$$= \begin{cases} 0, \text{ if } n \text{ is odd} \\ (-1)^{n/2}\cdot {}^nc_{n/2}, \text{ if } n \text{ is even} \end{cases}$$

8. $C_0C_1 + C_1C_2 + C_2C_3 + ... + C_{n-1}C_n = {}^{2n}C_{n-1}$

9. $C_0C_r + C_1C_{r+1} + ... C_{n-r}C_n = {}^{2n}C_{n-r}$ or ${}^{2n}C_{n+r}$

10. $C_1 + 2C_2 + 3C_3 + ... + n.C_n = n.2^{n-1}$

11. $C_1 - 2C_2 + 3C_3 - ... = 0$

12. $C_n + 2C_1 + 3C_2 + ... + (n + 1)C_n = (n + 2)2^{n-1}$

Properties of ${}^nCr$

If $0 < r < n$, $n, r \in N$, then

1. $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$

2. $\frac{{}^nC_r}{r+1} = \frac{{}^{n+1}C_{r+1}}{n+1}$

3. ${}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$

4. $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

5. $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{r+1}{n-r}$ **6.** ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$

7. ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$

8. ${}^nC_r = {}^nC_{n-r}$.

9. ${}^nC_r$ is greatest if $r = \begin{cases} n/2 \text{ if } n \text{ is even} \\ \frac{n-1}{2} \text{ or } \frac{n+1}{2} \text{ if } n \text{ is odd} \end{cases}$

Thus, if n is even, the greatest coefficient is ${}^nC_{n/2}$ and if n is odd, the greatest coefficient is ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$, both being equal.

10. The greatest term in $(1 + x)^{2n}$ has the greatest coefficient if $\frac{n}{n+1} < x < \frac{n+1}{n}$.

TRICK(S) FOR PROBLEM SOLVING

1. The number of terms in the expansion of $(x + y + z)^n$, where n is a positive integer, is $\frac{(n+1)(n+2)}{2}$.
2. The number of terms in the expansion of $(x + y + z + w)^n$, where n is a positive integer, is $\frac{(n+1)(n+2)(n+3)}{6}$.
3. Coefficient of $x^{n_1}y^{n_2}z^{n_3}$ in the expansion of $(x + y + z)^n$ is $\frac{n!}{n_1!\,n_2!\,n_2!}$ where $n = n_1 + n_2 + n_3$
4. In the expansion of $(x_1 + x_2 + ... + x_k)^n$, the sum of all the coefficients is obtained by putting all the variables xi equal to 1 and it is equal to kn.
5. Coefficient of xm in $(1 + x^r)^n$ (m, r and $n \in N$) is zero, if m is not an integral multiple of r, e.g., coefficient of x^{1000} in the expansion of $(1 + x^3)4000$ is 0 as 1000 is not an integral multiple of 3.

SOLVED EXAMPLES

41. The value of $\frac{C_0}{1.3} - \frac{C_1}{2.3} + \frac{C_2}{3.3} - \frac{C_3}{4.3} + ... + \frac{(-1)^n C_n}{(n+1)\cdot 3}$ is

(A) $\frac{1}{3(n+1)}$ (B) $\frac{1}{n+1}$

(C) $\frac{3}{n+1}$ (D) none of these

Solution: (A)

We have, $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$

Integrating both sides w.r.t. x form –1 to 0, we get

$$\int_{-1}^{0} (1+x)^n \, dx = \int_{-1}^{0} (C_0 + C_1 x + C_2 x^2 + ...$$

$$+ C_n x^n) dx$$

$$\left.\frac{(1+x)^{n+1}}{n+1}\right|_{-1}^{0} = \left(\cos x + \frac{C_1 x^2}{2} + ... + \frac{C_n x^{n+1}}{n+1}\right)_{-1}^{0}$$

$$\Rightarrow \quad C_0 - \frac{C_1}{2} + \frac{C_2}{3} +, ..., + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$$

$\therefore$ The given expression $= \frac{1}{3}\left(C_0 - \frac{C_1}{2} + \frac{C_2}{3} - ...\right)$

$$= \frac{1}{3} \cdot \frac{1}{n+1} = \frac{1}{3(n+1)}$$

42. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n x^n$, then $C_0C_2 + C_1C_3 + C_2C_4 + ... + C_{n-2} C_n =$

(A) $\frac{(2n)!}{(n!)^2}$ (B) $\frac{(2n)!}{(n-1)!(n+1)!}$

(C) $\frac{(2n)!}{(n-2)!(n+2)!}$ (D) none of these

Solution: (C)

We have,

$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + ...$
$+ C_{n-2} x^{n-2} + C_{n-1} x^{n-1} + C_n x^n$ ...(1)

and $(x + 1)^n = C_0 x^n + C_1 x^{n-1} + C_2 x^{n-2} + C_3 x^{n-3}$
$+ C_4 x^{n-4} + ... + C_{n-2} x^2 + C_{n-1} x + C_n$ ...(2)

Multiplying Eq. (1) and (2) and equating the coefficients of x^{n-2}, we get

$C_0C_2 + C_1C_3 + C_2C_4 + ... + C_{n-2} C_n$
= the coefficient of x^{n-2} in $(1 + x)^{2n}$

$$= {}^{2n}C_{n-2} = \frac{(2n)!}{(n-2)!\,(n+2)!}$$

43. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n x^n$, then $\sum\sum_{0 \le i \le j \le n} (C_i + C_j)^2 =$

(A) $(n - 1) \cdot {}^{2n}C_n + 2^{2n}$ (B) $n \cdot {}^{2n}C_n + 2^{2n}$
(C) $(n + 1) \cdot {}^{2n}C_n + 2^{2n}$ (D) none of these

Solution: (A)

$\sum\sum_{0 \le i \le j \le n} (C_i + C_j)^2 \quad i = 0, 1, 2, ..., (n - 1)$

$j = 1, 2, 3, ..., n$

and $i < j$

$= n\,(C_0^2 + C_1^2 + ... + C_n^2) + 2\,\Sigma\,\Sigma\, C_i C_j \quad 0 \le i < j \le n$

$= n \cdot {}^{2n}C_n + [(C_0 + C_1 + ... + C_n)^2$
$- (C_0^2 + C_1^2 + ... + C_n^2)]$

$= n \cdot {}^{2n}C_n + (2^n)^2 - {}^{2n}C_n = \mathbf{(n - 1) \cdot {}^{2n}C_n + 2^{2n}}$

44. If $(1 + x)^n = C_0 + C_1x + ... + C_nx^n$, then the value of $\sum_{r=0}^{n}\sum_{s=0}^{n}(C_r + C_s)$ is equal to

(A) $(n + 1)2^{n+1}$ (B) $(n + 1)2^n$
(C) $n2^{n+1}$ (C) none of these

Solution: (A)

$$\sum_{r=0}^{n}\sum_{s=0}^{n}(C_r + C_s) = \sum_{r=0}^{n}\sum_{s=0}^{n}C_r + \sum_{r=0}^{n}\sum_{s=0}^{n}C_s$$

$$= \sum_{s=0}^{n}\left(\sum_{r=0}^{n}C_r\right) + \sum_{r=0}^{n}\left(\sum_{s=0}^{n}C_s\right)$$

$$= \sum_{s=0}^{n}2^n + \sum_{r=0}^{n}2^n$$

$= (n + 1)2^n + (n + 1)2^n$
$= (n + 1)2^{n+1}$

45. If $(1 + x)^n = C_0 + C_1 + ... + C_nx^n$, then the value of $\sum\sum_{0 \le r < s \le n}(C_r + C_s)$ is equal to

(A) $n2^n$ (B) 2^{n+1}
(C) $(n - 1)2^n$ (D) none of these

Solution: (C)

$$\sum_{r=0}^{n}\sum_{s=0}^{n}(C_r + C_s)$$

$$= \sum_{r=0}^{n}(C_r + C_r) + 2\sum\sum_{0 \le r < s \le n}(C_r + C_s)$$

$$\Rightarrow \quad (n + 1)2^{n+1} = 2\left(\sum_{r=0}^{n}C_r\right) + 2\sum\sum_{0 \le r < s \le n}(C_r + C_s)$$

$$\Rightarrow \quad (n+1)2^{n+1} = 2\cdot 2^n + 2\sum_{0\le r<s\le n}\sum (C_r + C_s)$$

$$\Rightarrow \quad \sum_{0\le r<s\le n}\sum (C_r + C_s) = (n-1)2^n$$

46. The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$ is

(A) $2^{19} - \frac{1}{2}\cdot {}^{20}C_{10}$ (B) $2^{19} + \frac{1}{2}\cdot {}^{20}C_{10}$

(C) 2^{19} (D) 2^{20}

Solution: (B)

We have, $\sum_{r=0}^{10} {}^{20}C_r = {}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{10}$

But ${}^{20}C_0 + {}^{20}C_1 + ... {}^{20}C_{20} = 2^{20}$

and $\because$ ${}^{20}C_{20} = {}^{20}C_0$, ${}^{20}C_{19} = {}^{20}C_1$

${}^{20}C_{18} = {}^{20}C_2$, ${}^{20}C_{11} = {}^{20}C_9$

$$\therefore \quad \sum_{r=0}^{10} {}^{20}C_r = ({}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{20}) - ({}^{20}C_{11} + {}^{20}C_{12} + ... + {}^{20}C_{20})$$

$$= 2^{20} + {}^{20}C_{10} - ({}^{20}C_{10} + {}^{20}C_9 + ... + {}^{20}C_0)$$

$$\Rightarrow \quad 2\,({}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{10}) = 2^{20} + {}^{20}C_{10}$$

$$\therefore \quad {}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{10} = \mathbf{2^{19} + \frac{1}{2}\,{}^{20}C_{10}}$$

47. ${}^{n+1}C_2 + 2\,[{}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2] =$

(A) $\frac{n(n+1)(2n+1)}{6}$ (B) $\frac{n(n+1)}{2}$

(C) $\frac{n(n-1)(2n-1)}{6}$ (D) none of these

Solution: (A)

We have,

$${}^{n+1}C_2 + 2\,[{}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2]$$
$$= {}^{n+1}C_2 + 2\,[{}^3C_3 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2]$$
$$= {}^{n+1}C_2 + 2\,[{}^4C_3 + {}^4C_2 + ... + {}^nC_2]$$
$$= {}^{n+1}C_2 + 2\,[{}^5C_3 + ... + {}^nC_2]$$
$$= {}^{n+1}C_2 + 2\cdot {}^{n+1}C_3$$
$$= {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3$$
$$= {}^{n+2}C_3 + {}^{n+1}C_3$$
$$= \frac{n(n+1)(n+2)}{6} + \frac{n(n+1)(n-1)}{6}$$
$$= \frac{n(n+1)(2n+1)}{6}$$

48. The coefficient of x in the expansion of $[\sqrt{1+x^2} - x]^{-1}$ in ascending powers of x, when $|x| < 1$, is

(A) 0 (B) $\frac{1}{2}$

(C) $-\frac{1}{2}$ (D) 1

Solution: (D)

We have,

$$(\sqrt{1+x^2} - x)^{-1} = \frac{1}{(\sqrt{1+x^2} - x)} \times \frac{(\sqrt{1+x^2} + x)}{(\sqrt{1+x^2} + x)}$$

$$= \frac{(1+x^2)^{1/2} + x}{1+x^2-x^2} = (1+x^2)^{1/2} + x$$

$\therefore$ coefficient of x in the expansion of $(\sqrt{1+x^2} - x)^{-1}$

= coefficient of x in the exp. of $[(1 + x^2)^{1/2} + x]$

= **1.** ($\because$ coefficient of x in the exp. of $\sqrt{1+x^2}$ is 0)

49. If the expansion of $(1 + x)^{50}$, the sum of coefficients of add powers of x is

(A) 2^{50} (B) 2^{49}

(C) 0 (D) none of these

Solution: (B)

The sum of coefficients of odd powers of x

$= {}^{50}C_1 + {}^{50}C_3 + ... + {}^{50}C_{49}$

$= 2^{50-1} = 249$

50. The value of the sum of the series ${}^{14}C_0\cdot{}^{15}C_1 + {}^{14}C_1\cdot{}^{15}C_2 + {}^{14}C_2\cdot{}^{15}C_3 + ... + {}^{14}C_{14}\cdot{}^{15}C_{15}$ is

(A) ${}^{29}C_{12}$ (B) ${}^{29}C_{10}$

(C) ${}^{29}C_{14}$ (D) ${}^{29}C_{16}$

Solution: (C)

We have,

$(1 + x)^{14} = {}^{14}C_0 + {}^{14}C_1\,x + {}^{14}C_2\,x^2 + ... + {}^{14}C_{14}\,x^{14}$ ...(1)

and $(x + 1)^{15} = {}^{15}C_0\,x^{15} + {}^{15}C_1\,x^{14} + {}^{15}C_2\,x^{13} + {}^{15}C_3\,x^{12} + ... + {}^{15}C_{15}$ ...(2)

Multiplying Eq. (1) and (2) and equating the coefficient of x^{14}, we get

${}^{14}C_0\cdot{}^{15}C_1 + {}^{14}C_1\cdot{}^{15}C_2 + {}^{14}C_2\cdot{}^{15}C_3 + ... + {}^{14}C_{14}\cdot{}^{15}C_{15}$

= the coefficient of x^{14} in $(1 + x)^{29}$ = $\mathbf{{}^{29}C_{14}}$

51. If $(1 + x)^n = C_0 + C_1x + ... + C_nx^n$, then the value of $\sum_{r=0}^{n}\sum_{s=0}^{n} C_rC_s$ is equal to

(A) 2^n (B) 2^{2n}

(C) 2^{4n} (D) none of these

Solution: (B)

$$\sum_{r=0}^{n}\sum_{s=0}^{n} C_rC_s = \sum_{r=0}^{n}\left(C_r\sum_{s=0}^{n} C_s\right) = \sum_{r=0}^{n} 2^n\cdot C_r$$

$$= 2^n\left(\sum_{r=0}^{n} C_r\right) = 2^n \cdot 2^n = 2^{2n}.$$

52. If $(1 + x)^n = C_0 + C_1x + ... + C_nx^n$, then the value of $\sum\sum_{0 \le r < s \le n} C_rC_s$ is equal to

(A) $\frac{1}{2}(2^{2n} - {}^{2n}C_n)$ (B) $\frac{1}{4}(2^{2n} - {}^{2n}C_n)$

(C) $\frac{1}{2}(2^n - {}^{2n}C_n)$ (D) none of these

Solution: (A)

$$\sum_{r=0}^{n}\sum_{s=0}^{n} C_rC_s = \sum_{r=0}^{n} C_r^2 + \sum\sum_{0 \le r < s \le n} C_rC_s$$

$$\Rightarrow\ 2^{2n} = {}^{2n}C_n + 2\sum\sum_{0 \le r < s \le n} C_r\, C_s$$

$$\Rightarrow\ \sum\sum_{0 \le r < s \le n} C_r\, C_s = \frac{1}{2}(2^{2n} - {}^{2n}C_n)$$

53. $7^9 + 9^7$ is divisible by

(A) 16 (B) 24

(C) 64 (D) 72

Solution: (C)

We have,

$$7^9 + 9^7 = (1 + 8)^7 - (1 - 8)^9$$
$$= (1 + {}^7C_1 \cdot 8^1 + {}^7C_2 \cdot 8^2 + ... + {}^7C_7 \cdot 8^7)$$
$$- (1 - {}^9C_1 \cdot 8^1 + {}^9C_2 \cdot 8^2 - ... {}^9C_9 \cdot 8^9)$$
$$= 16 \times 8 + 64\,[({}^7C_2 + ... + {}^7C_7 \cdot 8^5)$$
$$- ({}^9C_2 - ... - {}^9C_9 \cdot 8^7)]$$
$$= 64 \text{ (an integer)}$$

Hence, $7^9 + 9^7$ is divisible by **64.**

EXERCISES

Single Option Correct Type

1. The coefficient of x^{17} in the expansion of $(x - 1)(x - 2)(x - 3) \ldots (x - 18)$ is

(A) $\frac{171}{2}$ (B) 342

(C) – 171 (D) 684

2. The fractional part of $\frac{2^{4n}}{15}$ is

(A) $\frac{2}{15}$ (B) $\frac{1}{15}$

(C) $\frac{4}{15}$ (D) none of these

3. If $\{x\}$ denotes the fractional part of x, then $\left\{\frac{2^{2003}}{17}\right\}$ is

(A) 2/17 (B) 4/17

(C) 8/17 (D) 16/17

4. The sum of the coefficients of all the integral powers of x in the expansion of $\left(1+2\sqrt{x}\right)^{80}$ is

(A) $\frac{1}{2}(3^{80}+1)$ (B) $\frac{1}{2}(3^{80}-1)$

(C) $(3^{80} + 1)$ (D) $(3^{80} – 1)$

5. If $[x]$ denotes the greatest integer less than or equal to x, then $[(6\sqrt{6}+14)^{2n+1}]$

(A) is an even integer (B) is an odd integer

(C) depends on n (D) none of these

6. The two consecutive terms in the expansion of $(3x + 2)^{74}$, whose coefficients are equal, are

(A) 20th and 21st (B) 30th and 31st

(C) 40th and 41st (D) none of these

7. If in the expansion of $\left(2^x + \frac{1}{4^x}\right)^n$, $\frac{T_3}{T_2} = 7$ and the sum of the coefficients of 2nd and 3rd terms is 36, then the value of x is

(A) $-\frac{1}{3}$ (B) $-\frac{1}{2}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

8. The interval in which x must lie so that the numerically greatest term in the expansion of $(1 – x)^{21}$ has the greatest coefficient is, $(x > 0)$.

(A) $\left[\frac{5}{6}, \frac{6}{5}\right]$ (B) $\left(\frac{5}{6}, \frac{6}{5}\right)$

(C) $\left(\frac{4}{5}, \frac{5}{4}\right)$ (D) $\left[\frac{4}{5}, \frac{5}{4}\right]$

9. If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 - \quad \ldots + (-1)^n (n+1)$$

C_n^2], where n is an even positive integer, is

(A) 0 (B) $(-1)^{n/2}(n+1)$
(C) $(-1)^{n/2}(n+2)$ (D) $(-1)^n n$

10. If $(1 + 2x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $a_r =$

(A) $({}^nC_r)^2$ (B) ${}^nC_r \cdot {}^nC_{r+1}$
(C) ${}^{2n}C_r$ (D) ${}^{2n}C_{r+1}$

11. If $\frac{1}{\sqrt{4x+1}}\left\{\left(\frac{1+\sqrt{4x+1}}{2}\right)^n - \left(\frac{1-\sqrt{4x+1}}{2}\right)^n\right\}$

$= a_0 + a_1x + \ldots + a_5x^5$, then n equals

(A) 11 (B) 9
(C) 10 (D) none of these

12. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q} = 0$ if $p < q$) is maximum when m is

(A) 5 (B) 10
(C) 15 (D) 20

13. The number of distinct terms in the expansion of $\left(x^3 + 1 + \frac{1}{x^3}\right)^n$; $x \in R^+$ and $n \in N$ is

(A) $2n$ (B) $3n$
(C) $2n + 1$ (C) $3n + 1$

14. The number of terms with integral coefficients in the expansion of $(17^{1/3} + 35^{1/2}x)^{600}$ is

(A) 100 (B) 50
(C) 150 (D) 101

15. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then

(A) $Re(z) = 0$ (B) $I_m(z) = 0$
(C) $Re(z) > 0, I_m(z) > 0$ (D) $Re(z) > 0, I_m(z) < 0$

16. The greatest value of the term independent of x in the expansion of $(x \sin\alpha + x^{-1}\cos\alpha)^{10}$, $\alpha \in R$, is

(A) $\frac{10!}{2^5}$ (B) $\frac{10!}{(5!)^2}$

(C) $\frac{1}{2^5}\frac{10!}{(5!)^2}$ (D) none of these

17. If coefficient of x^n in $(1 + x)^{101}(1 - x + x^2)^{100}$ is non-zero, then n can not be of the form

(A) $3t + 1$ (B) $3t$
(C) $3t + 2$ (D) $4t + 1$

18. The sum of the last ten coefficients in the expansion of $(1 + x)^{19}$ when expanded in ascending powers of x is

(A) 2^{18} (B) 2^{19}

(C) $2^{18} - {}^{19}C_{10}$ (D) $\frac{1}{2}(2^{19} - 1)$

19. The number of integral terms in the expansion of $(2\sqrt{5} + \sqrt[6]{7})^{642}$ is

(A) 105 (B) 107
(C) 321 (D) 108

20. The number of positive terms in the sequence

$$x_n = \frac{195}{{}^nP_n} - \frac{{}^{(n+3)}P_3}{{}^{(n+1)}P_{n+1}} \text{ is}$$

(A) 14 (B) 11
(C) 12 (D) 13

21. The digit at unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$ is

(A) 0 (B) 1
(C) 2 (D) 3

22. The positive integer which is just greater than $(1 + 0.0001)^{1000}$ is

(A) 3 (B) 4
(C) 5 (D) 2

23. The coefficient of x^n in the polynomial $(x + {}^nC_0)(x + 3\ {}^nC_1)(x + 5\ {}^nC_2) \ldots (x + (2n + 1)\ {}^nC_n)$ is

(A) $n.2^n$ (B) $n.2^{n+1}$
(C) $(n + 1).2^n$ (D) $n.2^{n-1}$

24. The interval in which x (> 0) must be so that the greatest term in the expansion of $(1 + x)^{2n}$ has the greatest coefficient is

(A) $\left(\frac{n-1}{n}, \frac{n}{n-1}\right)$ (B) $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$

(C) $\left(\frac{n}{n+2}, \frac{n+2}{n}\right)$ (D) none of these

25. If n is positive integer and k is a positive integer not exceeding n, then

$\sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2$, where $C_k = {}^nC_k$, is

(A) $\frac{n(n+1)(n+2)}{12}$ (B) $\frac{n(n+1)^2(n+2)}{12}$

(C) $\frac{n(n+1)^2(n+2)}{6}$ (D) none of these

26. If the fourth term in the expansion of $\left(\sqrt{\frac{1}{x^{\log x+1}}}+x^{1/12}\right)^6$ is equal to 200 and $x > 1$, then x is equal to

(A) $10^{\sqrt{2}}$ (B) 10
(C) 10^4 (D) none of these

27. The coefficient of $\lambda^n\mu^n$ in the expansion of $[(1+\lambda)(1+\mu)(\lambda+\mu)]^n$ is

(A) $\sum_{r=0}^{n} C_r^2$ (B) $\sum_{r=0}^{n} C_{r+2}^2$

(C) $\sum_{r=0}^{n} C_{r+3}^2$ (D) $\sum_{r=0}^{n} C_r^3$

28. If $\alpha = 18^3 + 7^3 + 3.\ 18.7.25$, and
$\beta = 3^6 + 6.243.2 + 15.81.4 + 20.27.8 + 15.9.16 + 6.3.32 + 64$
then the value of $\alpha\beta^{-1}$ is
(A) 1 (B) 5
(C) 25 (D) 100

29. If there is a term containing x^{2r} in $\left(x+\frac{1}{x^2}\right)^{n-3}$, then

(A) $n-2r$ is a positive integral multiple of 3
(B) $n-2r$ is even
(C) $n-2r$ is odd
(D) none of these

30. If P_n denotes the product of the binomial coefficients in the expansion of $(1+x)^n$, then $\frac{P_{n+1}}{P_n}$ equals

(A) $\frac{(n+1)^n}{n!}$ (B) $\frac{n^n}{n!}$

(C) $\frac{(n+1)^n}{(n+1)!}$ (C) $\frac{(n+1)^{n+1}}{(n+1)!}$

31. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is

(A) 210 (B) 105
(C) 70 (D) 112

32. The value of $\frac{1}{81^n}-\frac{10}{81^n}\,{}^{2n}C_2+\frac{10^2}{81^n}\,{}^{2n}C_2$
$-\frac{10^3}{81^n}\,{}^{2n}C_3+\ldots+\frac{10^{2n}}{81^n}$ is

(A) 2 (B) 0
(C) 1/2 (D) 1

33. If n is an even integer and a, b, c are distinct, the number of distinct terms in the expansion of $(a+b+c)^n + (a+b-c)^n$ is

(A) $\left(\frac{n}{2}\right)^2$ (B) $\left(\frac{n+1}{2}\right)^2$

(C) $\left(\frac{n+2}{2}\right)^2$ (D) $\left(\frac{n+3}{2}\right)^2$

34. Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is
(A) ${}^{12}C_6+3$ (B) ${}^{12}C_6+1$
(C) ${}^{12}C_6$ (D) ${}^{12}C_6+2$

35. $({}^mC_0+{}^mC_1-{}^mC_2-{}^mC_3)+({}^mC_4+{}^mC_5-{}^mC_6-{}^mC_7)+\ldots = 0$ if and only if for some positive integer k, $m =$
(A) $4k$ (B) $4k+1$
(C) $4k-1$ (D) $4k+2$

36. If the sum of the coefficients in the expansions of $(1+2x)^m$ and $(2+x)^n$ are respectively 6561 and 243, then the position of the point (m, n) with respect to the circle $x^2+y^2-4x-6y-32=0$
(A) is inside the circle
(B) is outside the circle
(C) is on the circle
(D) can not be fixed

37. Let $n(>1)$ be a positive integer. Then largest integer m such that (n^m+1) divides $1+n+n^2+\ldots+n^{255}$ is
(A) 128 (B) 63
(C) 64 (D) 32

38. The coefficient of x^n in the expansion $(2x+3)^n - (2x+3)^{n-1}(5-2x)+(2x+3)^{n-2}(5-2x)^2+\ldots+(-1)^n(5-2x)^n$ is

(A) $\frac{1}{8}2^n$ (B) $(n+1)2^n$

(C) $(n+1)2^{n-3}$ (D) $-(n+1)2^{n-2}$

39. The value of the sum of the series $3^nC_0 - 8^nC_1 + 13^nC_2 - 18^nC_3 + \ldots$ upto $(n+1)$ terms is
(A) 0 (B) 3^n
(B) 5^n (D) none of these

40. The value of $2({}^nC_0)+\frac{3}{2}({}^nC_1)+\frac{4}{3}({}^nC_2)+\frac{5}{4}({}^nC_3)\ldots$is

(A) $\frac{2^n(1-n)-1}{n+1}$ (B) $\frac{2^n(n+3)-1}{n+1}$

(C) $\frac{2^n-1}{n+1}$ (D) $\frac{2^n+2}{n-1}$

41. Which of the following expansions will have term containing x^3 ?

(A) $\left(x^{-\frac{1}{5}} + 2x^{\frac{3}{5}}\right)^{25}$ (B) $\left(x^{\frac{3}{5}} + 2x^{-\frac{1}{5}}\right)^{24}$

(C) $\left(x^{\frac{3}{5}} - 2x^{-\frac{1}{5}}\right)^{23}$ (D) $\left(x^{\frac{3}{5}} + 2x^{-\frac{1}{5}}\right)^{22}$

42. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(A) 132 (B) 144
(C) –132 (D) –144

43. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is

(A) an irrational number
(B) an odd positive integer
(C) an even positive integer
(D) a rational number other than positive integers

44. If $a_i(i = 0, 1, 2, ..., 16)$ be real constants such that for every real value of x, $(1 + x + x^2)^8 = a_0 + a_1x_2 + a_2x^2 ... + a_{16}x^{16}$, then a_5 is equal to

(A) 502 (B) 504
(C) 506 (D) 508

45. Statement-1: $\sum_{r=0}^{n}(r+1)\,{}^nC_r = (n+2)2^{n-1}$

Statement-2: $\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = (1+x)^n + nx(1+x)^{n-1}$

(A) Statement-1 is false, Statement-2 is true
(B) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(D) Statement-1 is true, Statement-2 is false

46. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than equal to $\frac{9}{10}$, then n is greater than

(A) $\frac{1}{\log_{10}^4 - \log_{10}^3}$ (B) $\frac{1}{\log_{10}^4 + \log_{10}^3}$

(C) $\frac{9}{\log_{10}^4 - \log_{10}^3}$ (D) $\frac{4}{\log_{10}^4 - \log_{10}^3}$

47. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is

(A) 0 (B) 2
(C) 7 (D) 8

48. If $C_0, C_1, C_2, ..., C_n$ denote the binomial coefficients in the expansion of $(1 + x)^n$, then

$$\sum_{r=0}^{n}(-1)^r \cdot {}^nC_r \cdot \frac{1+r\log_e 10}{(1+\log_e 10^n)^r} =$$

(A) 2 (B) 1
(C) 0 (D) none of these

49. If $C_0, C_1, C_2, ..., C_n$ are the coefficients of the expansion of $(1 + x)^n$, then the value of $\sum_0^n \frac{C_k}{k+1}$ is

(A) 0 (B) $\frac{2^n - 1}{n}$

(C) $\frac{2^{n+1}-1}{n+1}$ (D) none of these

50. Larger of $99^{50} + 100^{50}$ and 101^{50} is

(A) 101^{50} (B) $99^{50} + 100^{50}$
(C) both are equal (D) none of these

51. The greatest coefficient in the expansion of $(x + y + z + w)^{15}$ is

(A) $\frac{15!}{3!(4!)^3}$ (B) $\frac{15!}{(3!)^3\,4!}$

(C) $\frac{15!}{2!\,(4!)^2}$ (D) none of these

52. The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$ is

(A) $2^{19} - \frac{1}{2} \cdot {}^{20}C_{10}$ (B) $2^{19} + \frac{1}{2} \cdot {}^{20}C_{10}$

(C) 2^{19} (D) 2^{20}

53. ${}^{n+1}C_2 + 2\,[{}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2] =$

(A) $\frac{n(n+1)(2n+1)}{6}$ (B) $\frac{n(n+1)}{2}$

(C) $\frac{n(n-1)(2n-1)}{6}$ (D) none of these

54. If $A = {}^{2n}C_0 \cdot {}^{2n}C_1 + {}^{2n}C_1\,{}^{2n-1}C_1 + {}^{2n}C_2\,{}^{2n-2}C_1 + ...$, then A is

(A) 0 (B) 2^n
(C) $n2^{2n}$ (D) 1

55. The greatest integer which divides the number $101^{100} - 1$ is

(A) 100 (B) 1,000
(C) 10,000 (D) 1,00,000

56. Given positive integers $r > 1$, $n > 2$ and the coefficients of $(3r)$th term and $(r + 2)$th term in the binomial expansion of $(1 + x)^{2n}$ are equal, then $r =$

(A) $\frac{n}{2}$, n even (B) $\frac{n}{2}$
(C) n (D) 1

57. Let n be a positive integer such that

$(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, then $a_r =$

(A) a_{n-r}, $0 \le r \le 2n$ (B) a_{2n}, $0 \le r \le 2n$
(C) a_{2n-r}, $0 \le r \le 2n$ (D) none of these

58. If $\{x\}$ denotes the fractional part of x, then $\left\{\frac{2^{2003}}{17}\right\}$ is

(A) $\frac{2}{17}$ (B) $\frac{4}{17}$
(C) $\frac{8}{17}$ (D) $\frac{16}{17}$

59. If $[x]$ denotes the greatest integer less than or equal to x, then $[(6\sqrt{6}+14)^{2n+1}]$

(A) is an even integer (B) is an odd integer
(C) depends on n (D) none of these

60. If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2 - 2C_1^2 + 3C_2^2 - ... + (-1)^n (n+1)\, C_n^2],$$ where n is an even positive integer, is

(A) 0 (B) $(-1)^{n/2} (n + 1)$
(C) $(-1)^{n/2} (n + 2)$ (D) $(-1)^n n$

61. The sum of the series

$$1+\frac{1}{3^2}+\frac{1.4.1}{1.2.3^4}+\frac{1.4.7}{1.2.3}\frac{1}{3^6}+...$$ is

(A) $\sqrt{\frac{3}{2}}$ (B) $\left(\frac{3}{2}\right)^{\frac{1}{3}}$
(C) $\sqrt{\frac{1}{3}}$ (D) $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

62. If coefficient of x^n in $(1 + x)^{101} (1 - x + x^2)^{100}$ is non-zero, then n cannot be of the form

(A) $3t + 1$ (B) $3t$
(C) $3t + 2$ (D) $4t + 1$

63. The digit at unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$ is

(A) 0 (B) 1
(C) 2 (D) 3

64. The coefficient of x^n in the polynomial $(x + {}^nC_0) (x + 3\,{}^nC_1) (x + 5\,{}^nC_2)... (x + (2n + 1)\,{}^nC_n)$ is

(A) $n.2^n$ (B) $n.2^{n+1}$
(C) $(n+1).2^n$ (D) $n.2^{n-1}$

65. If n is an even integer and a, b, c are distinct, the number of distinct terms in the expansion of $(a + b + c)^n + (a + b - c)^n$ is

(A) $\left(\frac{n}{2}\right)^2$ (B) $\left(\frac{n+1}{2}\right)^2$
(C) $\left(\frac{n+2}{2}\right)^2$ (D) $\left(\frac{n+3}{2}\right)^2$

66. $({}^mC_0 + {}^mC_1 - {}^mC_2 - {}^mC_3) + ({}^mC_4 + {}^mC_5 - {}^mC_6 - {}^mC_7) + ... = 0$ if and only if for some positive integer k, $m =$

(A) $4k$ (B) $4k + 1$
(C) $4k - 1$ (D) $4k + 2$

67. Let $n(> 1)$ be a positive integer. Then, largest integer m such that $(n^m + 1)$ divides $1 + n + n^2 + ... + n^{255}$ is

(A) 128 (B) 63
(C) 64 (D) 32

68. The value of $2({}^nC_0) + \frac{3}{2}({}^nC_1) + \frac{4}{3}({}^nC_2) + \frac{5}{4}({}^nC_3)$...is

(A) $\frac{2^n(1-n)-1}{n+1}$ (B) $\frac{2^n(n+3)-1}{n+1}$
(C) $\frac{2^n-1}{n+1}$ (D) $\frac{2^n+2}{n-1}$

69. If $A = {}^{2n}C_0\,{}^{2n}C_1 + {}^{2n}C_1\,{}^{2n-1}C_1 + {}^{2n}C_2\,{}^{2n-2}C_1 + ...$, then A is

(A) 0 (B) 2^n
(C) $n\,2^{2n}$ (D) 1

70. The coefficient of $\lambda^n\mu^n$ in the expansion of $[(1 + \lambda) (1 + \mu) (\lambda + \mu)]^n$ is

(A) ${}^nC_r^2$ (B) ${}^nC_{r+2}^2$
(C) ${}^nC_{r+3}^2$ (D) ${}^nC_r^3$

71. The sum to $(n + 1)$ terms of the series

$$\frac{C_0}{2}-\frac{C_1}{3}+\frac{C_2}{4}-\frac{C_3}{5}+...$$ is

(A) $\frac{1}{n(n+1)}$ (B) $\frac{1}{n+2}$
(C) $\frac{1}{n+1}$ (D) none of these

72. Let $R = (5\sqrt{5} + 11)^{2n+1}$ and $f = R - [R]$ where [ ] denotes the greatest integer function. Then $Rf =$

(A) 2^{2n+1}
(B) $W2^{4n+1}$
(C) 4^{2n+1}
(D) none of these

73. Let n and k be positive integers such that $n \geq \dfrac{k(k+1)}{2}$. The number of solutions $(x_1, x_2, ..., x_k)$, $x_1 \geq 1$, $x_2 \geq 2$, ..., $x_k \geq k$, all integers, satisfying $x_1 + x_2 + ... + x_k = n$, is

(A) ${}^mC_{k-1}$
(B) ${}^mC_k$
(C) ${}^mC_{k+1}$
(D) none of these

where $m = \dfrac{1}{2}(2n - k^2 + k - 2)$

74. $\sum\limits_{r=0}^{n} {}^nC_r \sin rx \cos(n-r)x =$

(A) $2^{n-1} \sin(n-1)x$
(B) $2^n \sin nx$
(C) $2^{n-1} \sin nx$
(D) none of these

75. ${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + ... + {}^{n+k}C_n =$

(A) ${}^{n+k-1}C_{n+1}$
(B) ${}^{n+k}C_{n+1}$
(C) ${}^{n+k+1}C_{n+1}$
(D) none of these

76. If $S_n = 1 + q + q^2 + q^3 + ... + q^n$ and

$S'_n = 1 + \left(\dfrac{q+1}{2}\right) + \left(\dfrac{q+1}{2}\right)^2 + ... + \left(\dfrac{q+1}{2}\right)^n$, $q \neq 1$, then

${}^{n+1}C_1 + {}^{n+1}C_2 \cdot S_1 + {}^{n+1}C_3 \cdot S_2 + ... + {}^{n+1}C_{n+1} \cdot S_n =$

(A) $2^{n-1} \cdot S'_n$
(B) $2^n \cdot S'_n$
(C) $2^{n+1} \cdot S'_n$
(D) none of these

77. If $(1 + x)^{15} = C_0 + C_1 x + C_2 x^2 + ... + C_{15} x^{15}$, then the value of $C_2 + 2C_3 + 3C_4 + ... + 14C_{15}$ is

(A) 219923
(B) 16789
(C) 219982
(D) none of these

78. If $a_0, a_1, a_2, ..., a_{2n}$ be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, then

$a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... - a_{2n-1}^2 + a_{2n}^2 =$

(A) a_{2n}
(B) a_n
(C) a_0
(D) none of these

79. The coefficient of x^{50} in the expression

$(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ... + 1001x^{1000}$ is

(A) ${}^{1000}C_{50}$
(B) ${}^{1001}C_{50}$
(C) ${}^{1002}C_{50}$
(D) none of these

80. The sum of the series

$$\sum_{r=0}^{n} (-1)^r \cdot {}^nC_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \text{... to } m \text{ terms}\right]$$

is

(A) $\dfrac{1 - \frac{1}{2^{mn}}}{2^m - 1}$
(B) $\dfrac{1 - \frac{1}{2^{mn}}}{2^n - 1}$
(C) $\dfrac{1 - \frac{1}{2^m}}{2^n - 1}$
(D) none of these

81. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + ... + C_n x^n$, then for n even, $C_0^2 - C_1^2 + C_2^2 - ... + (-1)^n C_n^2$ is equal to

(A) 0
(B) $(-1)^{n/2}\, {}^nC_{n/2}$
(C) ${}^nC_{n/2}$
(D) none of these

82. $\sum\limits_{k=0}^{n} \dfrac{{}^nC_k}{(k+1)(k+2)} =$

(A) $\dfrac{2^{n+1} - n - 3}{(n+1)(n+2)}$
(B) $\dfrac{2^{n+2} - n - 3}{(n+1)(n+2)}$
(C) $\dfrac{2^{n+2} - n + 3}{(n+1)(n+2)}$
(D) none of these

83. For all $n \in N$, the integer just above $(\sqrt{3} + 1)^{2n}$ is divisible by

(A) 2^{n+1}
(B) $2^n + 1$
(C) $2^{n+1} + 1$
(D) none of these

84. If $C_0, C_1, C_2, ..., C_n$ be the coefficients in the expansion of $(1 + x)^n$, then

$\dfrac{2^2 \cdot C_0}{1 \cdot 2} + \dfrac{2^3 \cdot C_1}{2 \cdot 3} + ... + \dfrac{2^{n+2} \cdot C_n}{(n+1)(n+2)}$ is equal to

(A) $\dfrac{3^{n+1} - 2n - 5}{(n+1)(n+2)}$
(B) $\dfrac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$
(C) $\dfrac{3^{n+2} + 2n - 5}{(n+1)(n+2)}$
(D) none of these

85. ${}^mC_r + {}^mC_{r-1} \cdot {}^nC_1 + {}^mC_{r-2} \cdot {}^nC_2 + ... + {}^mC_1 \cdot {}^nC_{r-1} + {}^nC_r =$

(A) ${}^{m+n}C_{r-1}$
(B) ${}^{m+n}C_r$
(C) ${}^{m+n}C_{r+1}$
(D) none of these

86. If a, b, c and d are any four consecutive coefficients of any binomial expansion, then $\dfrac{a+b}{a}, \dfrac{b+c}{b}, \dfrac{c+d}{c}$ are in

(A) A.P.
(B) G.P.
(C) H.P.
(D) none of these

87. The last two digits of the number 3^{400} are

(A) 38
(B) 27
(C) 01
(D) none of these

88. The sum $\frac{C_0}{1.2}-\frac{C_1}{2.3}+\frac{C_2}{3.4}-\frac{C_3}{4.5}+\ldots$ to $(n+1)$ terms is

(A) $\frac{1}{(n+2)}$ (B) $\frac{2^n}{(n+2)}$

(C) $\frac{2^n-1}{(n+2)}$ (D) none of these

89. The sum of the series

$(1.2)\ C_2+(2.3)\ C_3+\ldots\ldots+(n-1.n)\ C_n$ is

(A) $n\ (n-1)2^{n-1}$ (B) $n\ (n-1)2^{n-2}$

(C) $n\ (n-1)2^n$ (D) none of these

90. If n is an even positive integer and $k=\frac{3n}{2}$, then

$\sum_{r=1}^{k}(-3)^{r-1}\ {}^{3n}C_{2r-1}=$

(A) 1 (B) –1

(C) 0 (D) none of these

91. The coefficient of x^{301} in the expansion of

$(1+x)^{500}+x\ (1+x)^{499}+x^2\ (1+x)^{498}+\ldots.\ .+x^{500}$ is

(A) ${}^{501}C_{301}$ (B) ${}^{500}C_{301}$

(C) ${}^{501}C_{300}$ (D) none of these

92. The fractional part of $\frac{\left(\sqrt{6}\right)^{2n}}{5}$, $n\in N$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$

(C) $\frac{1}{6}$ (D) none of these

93. The coefficient of x^n in the expansion of $(x+C_0)(x-3C_1)(x+5C_2)\ldots..$ up to $(n+1)$ terms, where $C_r={}^nC_r$ is equal to

(A) 0 (B) 1

(C) –1 (D) none of these

94. The number of irrational terms in the expansion of

$\left(\sqrt[8]{5}+\sqrt[6]{2}\right)^{100}$ is

(A) 96 (B) 97

(C) 98 (D) none of these

95. Let n be an odd natural number greater than 1. Then, the number of zeros at the end of the sum 99^n+1 is

(A) 2 (B) 3

(C) 4 (D) none of these

96. $\sum_{r=0}^{n}\frac{1}{(2r)!(2n-2r)!}=$

(A) $\frac{2^{2n}}{(2n)!}$ (B) $\frac{2^{2n-1}}{(2n)!}$

(C) $\frac{2^{2n+1}}{(2n)!}$ (D) none of these

97. The coefficient of x^n in polynomial

$(x+{}^{2n+1}C_0)\ (x+{}^{2n+1}C_1)(x+{}^{2n+1}C_2)\ldots.(x+{}^{2n+1}C_n)$ is

(A) 2^{2n+1} (B) 2^{2n}

(C) 2^{2n-1} (D) none of these

98. If 7 divides $32^{32^{32}}$, the remainder is

(A) 2 (B) 4

(C) 8 (D) none of these

More than One Option Correct Type

100. If the 4th term in the expansion of $\left(2+\frac{3}{8}x\right)^{10}$ has the maximum numerical value, then the range of values of x is

(A) $-2\le x\le 2$ (B) $-\frac{64}{21}\le x\le -2$

(C) $2\le x\le\frac{64}{21}$ (D) none of these

101. Three consecutive binomial coefficients can never be in

(A) G.P. (B) H.P.

(C) A.P. (D) A.G.P.

102. The value of x, for which the 6th term in the expansion of the binomial $\left[\sqrt{2^{\log(10-3^x)}}+\sqrt[5]{2^{(x-2)\log 3}}\right]^m$ is equal to 21 and it is known that the binomial coefficient of the 2nd, 3rd and 4th terms in the expansion represent respectively the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10), is

(A) 1 (B) 0

(C) 2 (D) none of these

103. ${}^nC_0\ {}^{2n}C_m-{}^nC_1\ {}^{2n-2}C_m+{}^nC_2\ {}^{2n-4}C_m-\ldots..=$

(A) $\binom{n}{m-n}2^{2n-m}$ if $m\ge n$

(B) 0 if $m<n$

(C) $\binom{n}{m-n}2^{2n+m}$ if $m\ge n$

(D) 1 if $m<n$

Passage Based Questions

Passage 1

If x, y are real numbers and $x,y > 0$, then to find the greatest term in the expansion of $(x + y)^n$, we proceed as follows:

Let T_{r+1} and T_r be $(r + 1)$th and rth terms, respectively in the expansion of $(x + y)^n$. Then,

$$\frac{T_{r+1}}{T_r} = \frac{{}^nC_r\, x^{n-r} y^r}{{}^nC_{r-1} x^{n-r+1} y^{r-1}} = \frac{(n-r+1)y}{rx}$$

$$\therefore \quad \frac{T_{r+1}}{T_r} - 1 = \left(\frac{(n+1)y}{x+y} - r\right)\left(\frac{x+y}{rx}\right).$$

Now, two cases arise:

Case I : $\frac{(n+1)y}{x+y}$ is an integer

Let $\frac{(n+1)y}{x+y} = k,\ 0 < k \le n$

Then, $\frac{T_{r+1}}{T_r} - 1 > 0$ for $1 \le r < k$

i.e., $T_r < T_{r+1}$ for $1 \le r < k$, $T_r > T_{r+1}$ for $k + 1 < r \le n$ and $T_{k+1} = T_k$.

Thus, there are two greatest terms, the kth term and the $(k + 1)$th term having equal values.

Case II : $\frac{(n+1)y}{x+y}$ is not an integer.

Let k be its integral part.

Then, $\frac{T_{r+1}}{T_r} - 1 > 0$ for $1 \le r < k$

i.e., $T_r < T_{r+1}$ for $1 \le r < k$, $T_r > T_{r+1}$ for $k + 1 \le r \le n$ and, $T_{k+1} > T_k$.

Thus , there is only one greatest term, the $(k + 1)$th term, where k is the integral part of $\frac{(n+1)y}{x+y}$.

104. The greatest term in the expansion of $(1 + x)^{10}$, when $x = \frac{2}{3}$ is

(A) $210\left(\frac{2}{3}\right)^4$ (B) $6300\left(\frac{2}{3}\right)^3$

(C) $\left(\frac{2}{3}\right)^5$ (D) none of these

105. The numerically greatest term in the expansion of $(3 - 5x)^{15}$, when $x = \frac{1}{5}$ is

(A) 4th term (B) 5th term

(C) 6th term (D) none of these

106. The greatest term in the expansion of $\sqrt{3}\left(1 + \frac{1}{\sqrt{3}}\right)^{20}$ is

(A) $\frac{25840}{9}$ (B) $\frac{24840}{9}$

(C) $\frac{26840}{9}$ (D) none of these

107. If 4th term in the expansion of $\left(2 + \frac{3x}{8}\right)^{10}$ has the greatest numerical value, then x belongs to

(A) $(-\infty, -2] \cup [2, \infty)$ (B) $\left(-\frac{64}{21}, \frac{64}{21}\right)$

(C) $\left(-\frac{64}{21}, -2\right) \cup \left(2, \frac{64}{21}\right)$ (D) none of these

Passage 2

Let a and b be positive integers and b not a perfect square, then for every positive integer n, the number $(a + \sqrt{b})^n$ is irrational. Also,

$$(a+\sqrt{b})^n + (a-\sqrt{b})^n$$
$$= 2\left[{}^nC_0 a^n + {}^nC_2 a^{n-2}(\sqrt{b})^2 + {}^nC_4 a^{n-4}(\sqrt{b})^4 + ...\right]$$

Clearly, R.H.S. is an even integer say E.

Let $(a + \sqrt{b})^n = I + F$, where I is the integral part and F is the fractional part of $(a + \sqrt{b})^n$ i.e., $0 < F < 1$.

Let $\left(a - \sqrt{b}\right)^n = F',\ 0 < F' < 1$

Then, $I + F + F' = E$.

As, I and E are integers, $F + F'$, must be an integer.

But $0 < F + F' < 2 \Rightarrow F + F' = 1 \Rightarrow I = E - 1$.

Thus, integral part of $(a + \sqrt{b})^n$ is $(a + \sqrt{b})^n + (a - \sqrt{b})^n$.

The fractional part of $(a + \sqrt{b})^n$ is $1 - (a - \sqrt{b})^n$

108. Let $R = (5 + 2\sqrt{6})^n$ and f = fractional part of R, then $R(1 - f) =$

(A) 1 (B) -1

(C) 0 (D) none of these

109. $[(3+\sqrt{5})^{2n}]+1$, where $[x]$ denotes the integral part of x, is divisible by

(A) 2^{n-1} (B) 2^n
(C) 2^{n+1} (D) none of these

110. If $n \in N$ such that $(7+4\sqrt{3})^n = I + f$, where $I \in N$ and $0 < f < 1$. Then, the value of $(I+f)(I-f)$ is

(A) 0 (B) 1
(C) 7^{2n} (D) 2^{2n}

Match the Column Type

111.

Column–I	Column–II
(A) If 7^{103} is divided by 25, then the remainder is	1. 8
(B) The sum of rational terms in the expansion of $(\sqrt{2}+3^{1/15})^{10}$ is	2. 225
(C) For all $n \in N$, $2^{4n}-15n-1$ is divisible by	3. 18
(D) When 5^{99} is divided by 13, the remainder is	4. 41

112.

Column–I	Column–II
(A) The number of integral terms in the expansion of $(5^{1/2}+7^{1/8})^{1028}$ is	1. 210
(B) The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is	2. 2520
(C) The coefficient of $x^2y^3z^5$ in the expansion $(x+y+z)^{10}$ is	3. 129
(D) The least remainder when 17^{30} is divided by 5 is	4. 4

Assertion-Reason Type

Instructions: *In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options:*

(A) Assertion(A) is True and Reason(R) is True; Reason(R) is a correct explanation for Assertion(A)
(B) Assertion(A) is True, Reason(R) is True; Reason(R) is not a correct explanation for Assertion(A)
(C) Assertion(A) is True, Reason(R) is False
(D) Assertion(A) is False, Reason(R) is True

113. Assertion: If n is a positive integer and k is a positive integer not exceeding n, then

$$\sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2, \text{ where } C_k = {}^nC_k, \text{ is}$$

$$\frac{n(n+1)^2(n+2)}{12}$$

Reason: $\frac{C_k}{C_{k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n-k+1}{k}$

114. Assertion: If P_n denotes the product of the binomial coefficients in the expansion of

$(1+x)^n$, then $\frac{P_{n+1}}{P_n}$ equals $\frac{(n+1)^n}{n!}$

Reason: ${}^{n+1}C_{r+1} = \frac{n+1}{r+1}\,{}^nC_r$

115. Assertion: The coefficient of x^n in the expansion $(2x+3)^n-(2x+3)^{n-1}(5-2x)+(2x+3)^{n-2}(5-2x)^2+\ldots+(-1)^n(5-2x)^n$ is $(n+1)2^n$

Reason: $a^n + a^{n-1}b + a^{n-2}b^2 + \ldots + b^n$

$$= \frac{a^{n+1}-b^{n+1}}{a-b}$$

116. Assertion: The interval in which $x (x > 0)$ must lie so that the numerically greatest term in the expansion of $(1-x)^{21}$ has the greatest coefficient is, $\left(\frac{5}{6}, \frac{6}{5}\right)$.

Reason: If n is odd, then numerically greatest coefficient in the expansion of $(1-x)^n$ is $\frac{{}^nC_{n-1}}{2}$ or $\frac{{}^nC_{n+1}}{2}$.

117. Assertion: If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also is $\frac{n}{n+2} < x <$

$\frac{n+2}{n}$.

Reason: For even positive integer, the greatest coefficient in the expansion of $(1 + x)^n$ is ${}^nC_{n/2}$.

118. Assertion: Sum of the infinite series

$$1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3}+\ldots\infty \text{ is}$$

$2^{1/3}$.

Reason: $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\infty$, where n is rational..

119. Assertion: The value of $\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+\ldots$ is equal to $\frac{2^n-1}{n+1}$

Reason: ${}^nC_2 + {}^nC_4 + \ldots = 2^{n-1}$

120. Assertion: The value of $\frac{{}^{11}C_0}{1}+\frac{{}^{11}C_1}{2}+\frac{{}^{11}C_2}{3}+\ldots+\frac{{}^{11}C_{11}}{12}$ is $\frac{1}{12}\left(2^{12}-1\right)$

Reason: For $0 \le k \le n$, ${}^nC_k = \frac{n}{k}\cdot{}^{n-1}C_{k-1}$

121. Assertion: If $a_1, a_2, \ldots a_n$ are in A.P. and S_n is the sum of first n terms, then

$$\sum_{k=0}^{n} {}^nC_k S_k = 2^{n-2}(na_1 + S_n)$$

Reason: $\sum_{k=0}^{n} k.{}^nC_k = n2^{n-1}$

and $\sum_{k=0}^{n} k^2.{}^nC_k = n2^{n-1} + n(n-1)2^{n-2}$

Previous Year's Questions

121. The coefficient of x^5 in $(1 + 2x + 3x^2 + \ldots)^{-3/2}$ is:

(A) 21 (B) 25 **[2002]**

(C) 26 (D) none of these

122. If $|x| < 1$, then the coefficient of x^n in expansion of $(1 + x + x^2 + x^3 + \ldots)^2$ is : **[2002]**

(A) n (B) $n-1$

(C) $n+2$ (D) $n+1$

123. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is **[2003]**

(A) 32 (B) 33

(C) 34 (D) 35

124. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1-\alpha x)$ is the same if α equals **[2004]**

(A) $-\frac{5}{3}$ (B) $\frac{3}{5}$

(C) $-\frac{3}{10}$ (D) $\frac{10}{3}$

125. The coefficient of x^n in expansion of $(1 + x)(1 - x)^n$ is

(A) $(n-1)$ (B) $(-1)^n(1-n)$ **[2004]**

(C) $(-1)^{n-1}(n-1)^2$ (D) $(-1)^{n-1}n$

126. If the coefficients of rth, (r + 1)th and (r + 2)th terms in the binomial expansion of (1 + y)m are in A. P., then m and r satisfy the equation **[2005]**

(A) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$

(B) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$

(C) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$

(D) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

127. The value of ${}^{50}C_4 + \sum_{r-1}^{6} {}^{56-r}C_3$ is **[2005]**

(A) ${}^{55}C_4$ (B) ${}^{55}C_3$

(C) ${}^{56}C_3$ (D) ${}^{56}C_4$

128. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax^2-\left(\frac{1}{bx}\right)\right]^{11}$, then a and b satisfy the relation **[2005]**

(A) $a - b = 1$ (B) $a + b = 1$

(B) $\frac{a}{b} = 1$ (D) $ab = 1$

129. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as **[2005]**

(A) $1-\frac{3}{8}x^2$ (B) $3x+\frac{3}{8}x^2$

(C) $-\frac{3}{8}x^2$ (D) $\frac{x}{2}-\frac{3}{8}x^2$

130. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3+\ldots$, then a_n is

(A) $\frac{b^n-a^n}{b-a}$ (B) $\frac{a^n-b^n}{b-a}$ **[2006]**

(a) $\frac{a^{n+1}-b^{n+1}}{b-a}$ (D) $\frac{b^{n+1}-a^{n+1}}{b-a}$

131. For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots$, and $a_1 = a_2 = 10$, then (m, n) is

(A) (20, 45) (B) (35, 20) **[2006]**

(C) (45, 35) (D) (35, 45)

132. In the binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals **[2007]**

(A) $\frac{5}{n-4}$ (B) $\frac{6}{n-5}$

(C) $\frac{n-5}{6}$ (D) $\frac{n-4}{5}$

133. The sum of the series **[2007]**

${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots - \ldots + {}^{20}C_{10}$ is

(A) $-{}^{20}C_{10}$ (B) $\frac{1}{2}\,{}^{20}C_{10}$

(C) 0 (D) $^{2}\quad {}^{0}C_{10}$

134. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[2008]**

(A) $\frac{1}{\log_{10}^4-\log_{10}^3}$ (B) $\frac{1}{\log_{10}^4+\log_{10}^3}$

(C) $\frac{9}{\log_{10}^4-\log_{10}^3}$ (D) $\frac{4}{\log_{10}^4-\log_{10}^3}$

135. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is **[2008]**

(A) 0 (B) 2

(C) 7 (D) 8

136. The coefficient of x^7 in the expansion of the expression $(1-x-x^2+x^3)^6$ is **[2011]**

(A) –132 (B) –144

(c)132 (D) 144

137. If n is a natural number, then $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}$ is

(1) an irrational number **[2012]**

(2) an odd positive integer

(3) an even positive integer

(4) a rational number other than positive integers

138. If x = -1 and x = 2 are extreme points of $f(x)=\alpha\log|x|+\beta x^2+x$, then **[2013]**

(A) $\alpha=-6, \beta=\frac{1}{2}$ (B) $\alpha=-6, \beta=-\frac{1}{2}$

(C) $\alpha=2, \beta=-\frac{1}{2}$ (D) $\alpha=2, \beta=\frac{1}{2}$

139. If the coefficients of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$, in powers of x, are both zero, then (a, b) is equal to **[2014]**

(A) $\left(16, \frac{251}{3}\right)$ (B) $\left(14, \frac{251}{3}\right)$

(C) $\left(14, \frac{272}{3}\right)$ (D) $\left(16, \frac{272}{3}\right)$

140. If $X=\{4^n-3n-1 : n\in N\}$ and $Y=\{9(n-1) : n \in N\}$, where N is the set of natural numbers, then the set $X \cup Y$ is equal to **[2014]**

(A) N (B) Y - X

(C) X (D) Y

141. The sum of the coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is: **[2015]**

(A) $\frac{1}{2}\left(3^{50}\right)$ (B) $\frac{1}{2}\left(3^{50}-1\right)$

(C) $\frac{1}{2}\left(2^{50}+1\right)$ (D) $\frac{1}{2}\left(3^{50}+1\right)$

ANSWER KEYS

Single Option Correct Type

1. (C)	**2.** (A)	**3.** (B)	**4.** (C)	**5.** (D)	**6.** (A)	**7.** (B)	**8.** (C)	**9.** (A)	**10.** (A)
11. (A)	**12.** (B)	**13.** (A)	**14.** (C)	**15.** (C)	**16.** (B)	**17.** (A)	**18.** (C)	**19.** (C)	**20.** (A)
21. (C)	**22.** (B)	**23.** (C)	**24.** (C)	**25.** (B)	**26.** (C)	**27.** (B)	**28.** (C)	**29.** (C)	**30.** (A)
31. (B)	**32.** (C)	**33.** (D)	**34.** (C)	**35.** (B)	**36.** (D)	**37.** (B)	**38.** (C)	**39.** (A)	**40.** (A)
41. (C)	**42.** (C)	**43.** (C)	**44.** (B)	**45.** (C)	**46.** (C)	**47.** (A)	**48.** (C)	**49.** (C)	**50.** (A)
51. (A)	**52.** (B)	**53.** (A)	**54.** (C)	**55.** (C)	**56.** (A)	**57.** (C)	**58.** (C)	**59.** (A)	**60.** (C)
61. (B)	**62.** (C)	**63.** (B)	**64.** (C)	**65.** (C)	**66.** (C)	**67.** (A)	**68.** (B)	**69.** (C)	**70.** (D)
71. (D)	**72.** (C)	**73.** (A)	**74.** (C)	**75.** (C)	**76.** (B)	**77.** (A)	**78.** (B)	**79.** (C)	**80.** (B)
81. (B)	**82.** (B)	**83.** (A)	**84.** (B)	**85.** (B)	**86.** (C)	**87.** (C)	**88.** (A)	**89.** (B)	**90.** (C)
91. (A)	**92.** (B)	**93.** (A)	**94.** (B)	**95.** (A)	**96.** (B)	**97.** (B)	**98.** (B)		

More than One Option Correct Type

99. (A), (B) and (C) **100.** (A), (B), (C) and (D) **101.** (A), (B), (C) and (D) **102**. (A) and (C)

Passage Based Questions

103. (C) **104.** (B) **105.** (D) **106**. (A) **107.** (B) **108.** (B) **109.** (A)

Match the Column Type

110. (A) → 3; (B) → 4; (C) → 2; (D) → 1 **111.** (A) → 3; (B) → 1; (C) → 2; (D) → 4

Assertion-Reason Type

112. (A) **113.** (A) **114.** (A) **115**. (A) **116.** (A) **117.** (D) **118.** (C) **119.** (A) **120**. (A)

Previous Year's Questions

121. (D)	**122.** (D)	**123.** (B)	**124.** (C)	**125.** (B)	**126.** (C)	**127.** (D)	**128**. (D)	**129.** (C)	**130.** (D)
131. (D)	**132.** (D)	**133**. (B)	**134.** (A)	**135.** (B)	**136.** (B)	**137.** (A)	**138**. (C)	**139.** (D)	**140.** (D)
141. (D)									

HINTS AND SOLUTIONS

Single Option Correct Type

1. Coefficient of $x^{17} = -(1 + 2 + 3 + ... + 18)$

$$= -\frac{18}{2}(1 + 18)$$

$$= -9 \times 19 = \mathbf{-171}$$

The correct option is (C)

2. We have, $\frac{2^{4n}}{15} = \frac{16^n}{15} = \frac{(1+15)^n}{15}$

$$= \frac{1 + {}^nC_1 15 + {}^nC_2 15^2 + ... + {}^nC_n 15^n}{15}$$

$$= \frac{1+15k}{15}, \text{ where } k \in N$$

$$= \frac{1}{15} + k$$

$\therefore$ Fractional part of $\frac{2^{4n}}{15}$ is $\frac{1}{15}$.

The correct option is (B)

3. $2^{2003} = (2^4)^{500}. 2^3$

$\Rightarrow$ $2^{2003} = 8(16)^{500}$

$\Rightarrow$ $2^{2003} = 8(17 - 1)^{500}$

$\Rightarrow$ $2^{2003} = 8[(17)^{500} - {}^{500}C_1(17)^{499} + ...$
$- {}^{500}C_{499}(17) + 1]$

$\Rightarrow \quad \dfrac{2^{2003}}{17} = 8k + \dfrac{8}{17}$,

where $k = (17)^{499} - {}^{500}C_1(17)^{498} + ... + {}^{500}C_{499}$

such that k is an integer

$\therefore \quad \left\{\dfrac{2^{2003}}{17}\right\} = \dfrac{8}{17}$

The correct option is (C)

4. The coefficients of the integral powers of x are

${}^{80}C_0, {}^{80}C_2 \cdot 2^2, {}^{80}C_4 \cdot 2^4, ..., {}^{80}C_{80} \cdot 2^{80}$

Now, $(1+2)^{80} = {}^{80}C_0 + {}^{80}C_1 \cdot 2 + {}^{80}C_2 \cdot 2^2$
$+ ... + {}^{80}C_{80} \cdot 2^{80}$ ...(1)

and $(1-2)^{80} = {}^{80}C_0 - {}^{80}C_1 2 + {}^{80}C_2 \cdot 2^2$
$- ... + {}^{80}C_{80} \cdot 2^{80}$ ...(2)

Adding Eq. (1) and (2), we get

$3^{80} + 1 = 2({}^{80}C_0 + {}^{80}C_2 \cdot 2^2 + {}^{80}C_4 \cdot 2^4$
$+ ... + {}^{80}C_{80} \cdot 2^{80})$

$\therefore \quad {}^{80}C_0 + {}^{80}C_2 \cdot 2^2 + {}^{80}C_4 \cdot 2^4 + ... + {}^{80}C_{80} \cdot 2^{80} = \dfrac{1}{2}(3^{80} + 1)$

The correct option is (A)

5. Let $I + f = (6\sqrt{6} + 14)^{2n+1}$

Assuming, $f' = (6\sqrt{6} - 14)^{2n+1}$ ...(1)

Now, $I + f - f' = (6\sqrt{6} + 14)^{2n+1} - (6\sqrt{6} - 14)^{2n+1}$

$\Rightarrow \quad I + f - f' = 2[{}^{2n+1}C_1 (6\sqrt{6})^{2n}\, 14^1$
$+ {}^{2n+1}C_3 (6\sqrt{6})^{2n+2}\, (14)^3 + ...]$

$\Rightarrow \quad I + f - f' = 2$ (Integer) = even ...(2)

Now, $0 \le f < 1$

Also, $0 \le f - f' < 1$

$\therefore \quad 0 \le f - f' < 0 \Rightarrow f - f' = 0$

Substituting respective values in (2), we get

I = even integer

The correct option is (A)

6. Since, $\dfrac{T_3}{T_2} = 7$ (given)

$\Rightarrow \quad \dfrac{{}^nC_2\,(2^x)^{n-2}\,(4^{-x})^2}{{}^nC_1\,(2^x)^{n-1}.(4^{-x})} = 7$

$\Rightarrow \quad \left(\dfrac{n-1}{2}\right) \cdot \dfrac{1}{(2^x)^3} = 7$ ...(1)

Also, ${}^nC_2 + {}^nC_1 = 36$

$\Rightarrow \quad \dfrac{n(n-1)}{2} + n = 36$

$\Rightarrow \quad n^2 + n - 72 = 0$

$\Rightarrow \quad n = 8, -9$

$n = -9$ is not possible as in Eq. (1), $n - 1$ should be positive.

Substituting $n = 8$ in Eq. (1), we get

$2^{3x} = \dfrac{1}{2} = 2^{-1}$

$\Rightarrow \quad 3x = -1 \Rightarrow \quad x = -\dfrac{1}{3}$

The correct option is (A)

7. If n is odd, then numerically greatest coefficient in the expansion of $(1-x)^n$ is $\dfrac{{}^nC_{n-1}}{2}$ or $\dfrac{{}^nC_{n+1}}{2}$.

Therefore in $(1-x)^{21}$, the numerically greatest coefficient is ${}^{21}C_{10}$ or ${}^{21}C_{11}$. So, the numerically greatest term

$= {}^{21}C_{11}x^{11}$ or ${}^{21}C_{10}x^{10}$ and

$|{}^{21}C_{10}x^{10}| > |{}^{21}C_9 x^9|$

$\Rightarrow \quad \dfrac{21!}{10!\,11!} > \dfrac{21!}{9!\,12!}x$ and

$\dfrac{21!}{11!\,10!}x > \dfrac{21!}{9!\,12!}$ ($\because x > 0$)

$\Rightarrow \quad x < \dfrac{6}{5}$ and $x > \dfrac{5}{6} \quad \Rightarrow \quad x \in \left(\dfrac{5}{6}, \dfrac{6}{5}\right)$

The correct option is (B)

8. $C_0^2 - 2C_1^2 + 3C_2^2 - 4C_3^2 + ... + (-1)^n\,(n+1)C_n^2$

$= [C_0^2 - C_1^2 + C_2^2 - C_3^2 + ... + (-1)^n\, C_n^2]$
$- [C_1^2 - 2C_2^2 + 3C_3^2 - ... + (-1)^n\, nC_n^2]$

$= (-1)^{n/2} \dfrac{n!}{\left(\dfrac{n}{2}\right)!\left(\dfrac{n}{2}\right)!} - (-1)^{\frac{n}{2}-1} \dfrac{n}{2}\, {}^nC_{\frac{n}{2}}$

$= (-1)^{n/2} \dfrac{n!}{\left(\dfrac{n}{2}\right)!\left(\dfrac{n}{2}\right)!}\left(1 + \dfrac{n}{2}\right)$

$\therefore \quad \dfrac{2\left(\dfrac{n}{2}\right)!\left(\dfrac{n}{2}\right)!}{n!}\,[C_0^2 - 2C_1^2 + 3C_2^2 - ... + (-1)^n\,(n+1)\,C_n^2]$

$= (-1)^{n/2}(n+2)$

The correct option is (C)

9. We have, $(1 + 2x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$

$\Rightarrow \quad (1+x)^{2n} = \sum_{r=0}^{2n} a_r x^r$

$\Rightarrow \quad \sum_{r=0}^{2n} {}^{2n}C_r\, x^r = \sum_{r=0}^{2n} a_r\, x^r \Rightarrow a_r = 2n\mathbf{C}\mathbf{r}.$

The correct option is (C)

10. Given:

$$\frac{1}{\sqrt{4x+1}}\left[\left(\frac{1+\sqrt{4x+1}}{2}\right)^n-\left(\frac{1-\sqrt{4x+1}}{2}\right)^n\right]$$

$= a_0 + a_1x + a_2x^2 + ... + a_5x^5$

Now, $$\frac{1}{\sqrt{4x+1}}\left[\left(\frac{1}{2}+\frac{\sqrt{4x+1}}{2}\right)^n-\left(\frac{1}{2}-\frac{\sqrt{4x+1}}{2}\right)^n\right]$$

$$=\frac{1}{\sqrt{4x+1}}\left[{}^nC_1\left(\frac{1}{2}\right)^{n-1}\frac{\sqrt{4x+1}}{2}\right.$$

$$\left.+{}^nC_3\left(\frac{1}{3}\right)^{n-3}\left(\frac{\sqrt{4x+1}}{2}\right)^3+...\right]$$

$$={}^nC_1\frac{1}{2^n}+{}^nC_3\frac{1}{2^n}(4x+1)+{}^nC_5\frac{1}{2^n}(4x+1)^2$$

$$+...+{}^nC_r\frac{1}{2^n}(4x+1)^{\frac{r-1}{2}}+...$$

The expansion contains a term x^5 if $\frac{r-1}{2} = 5$ or $r = 11$.

The correct option is (A)

11. $$\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$$

= Coefficient of x^m in $(1 + x)^{10} (1 + x)^{20} = {}^{30}C_m$

m = 15 (for maximum value)

The correct option is (C)

12. $$\left(x^3+1+\frac{1}{x^3}\right)^n=\left[1+\left(x^3+\frac{1}{x^3}\right)\right]^n$$

$$={}^nC_0+{}^nC_1\left(x^3+\frac{1}{x^3}\right)+...+{}^nC_n\left(x^3+\frac{1}{x^3}\right)^n$$

All the terms are distinct with powers $(x^3)^0$, (x^3), $(x^3)^2$, ... $(x^3)^n$, $(x^3)^{-n}$,... $(x^3)^{-1}$. Hence, $(2n+1)$ terms.

The correct option is (C)

13. **(d).** $t_{r+1} = {}^{600}C_r\, 17^{\frac{600-r}{3}}\, 35^{\frac{r}{2}}\, x^r$

As $0 \le r \le 600$ and $\frac{r}{2}$ and $200-\frac{r}{3}$ are integers $\Rightarrow r$ should be a multiple of 6

$\therefore$ r = 0, 6, 12,... 600

The correct option is (D)

14. $$z=2\left[\left(\frac{\sqrt{3}}{2}\right)^5+{}^5C_2\left(\frac{\sqrt{3}}{2}\right)^3\frac{i^2}{4}+{}^5C_4\left(\frac{\sqrt{3}}{2}\right)\frac{i^4}{16}\right]$$

= Purely real number

Hence, $I_m(z) = 0$

The correct option is (B)

15. Term independent of $x = {}^{10}C_5 (\sin\alpha)^5 (\cos\alpha)^5$

$$=\frac{1}{2^5}\ {}^{10}C_5\sin^5 2\alpha$$

Hence, the greatest value $=\frac{1}{2^5}\ \frac{10!}{(5!)^2}$

The correct option is (C)

16 $(1 + x)^{101} (1 - x + x^2)^{100} = (1 + x) (1 + x^3)^{100}$

$= (1 + x) (C_0 + C_1x^3 + C_2x^6 + ... + C_{100}x^{300})$

Clearly in this expression x^λ will be present if $\lambda = 3t$, or $\lambda = 3t + 1$

So, λ can not be of the form $3t + 2$.

The correct option is (A)

17. Sum of last 10 coefficients,

${}^{19}C_{10} + {}^{19}C_{11} + ... + {}^{19}C_{19} = S$(say)

Also, ${}^{19}C_0 + {}^{19}C_1 + ... + {}^{19}C_9 = {}^{19}C_{19} + {}^{19}C_{18} + ... + {}^{19}C_{10}$

$= S \quad \therefore \quad ({}^nC_n = {}^nC_{n-r})$

$$\therefore\quad 2S=\sum_{n=0}^{19}{}^{19}C_n=2^{19}\Rightarrow S=2^{18}$$

The correct option is (A)

18. $(2.5^{1/2} + 7^{1/6})^{642}$ has a general term of the form ${}^{642}C_r (2.5^{1/2})^{642-r} (7^{1/6})^r$

$= {}^{642}C_r\, 2^{642-r}.\ 5^{321-r/2}.\ 7^{r/6}$

and will be rational if only r is a multiple of 2 and 6.

$\therefore$ r must be the LCM of 2 and 6 which is 6.

$\therefore$ r takes the values 0, 6, 12, 18,..., 642.

There are 108 values.

The correct option is (D)

19. $$x_n=\frac{195}{n!}-\frac{(n+3)(n+2)(n+1)}{(n+1)!}>0$$

$\Rightarrow$ $195 > (n + 2) (n + 3)$.

Hence, $n \le 11$

$\therefore$ n can take the values 1, 2, 3,..., 11.

$\therefore$ Number of positive terms = 11

The correct option is (B)

20. We have

$17^{1995} + 11^{1995} - 7^{1995}$

$= (7 + 10)^{1995} + (1 + 10)^{1995} - 7^{1995}$

$= [7^{1995} + {}^{1995}C_1.\ 7^{1994}.\ 10^1 + {}^{1995}C_2.\ 7^{1993}.\ 10^2 + ... + {}^{1995}C_{1995}.\ 10^{1995}] + [{}^{1995}C_0 + {}^{1995}C_1.\ 10^1 + {}^{1995}C_2.\ 10^2 + ... + {}^{1995}C_{1995}.\ 10^{1995}] - 7^{1995}$

$= [{}^{1995}C_1.\ 7^{1994}.\ 10^1 + ... + 10^{1995}] + [{}^{1995}C_1.\ 10^1 + ... + {}^{1995}C_{1995}.\ 10^{1995}] + 1$

= a multiple of 10 + 1.

Thus, the units place digit is 1.

The correct option is (B)

21. $(1 + 0.0001)^{1000}$

$= 1 + 1000 \times 10^{-4} + \dfrac{1000 \times 999}{2} 10^{-8} + {}^{1000}C_3\, 10^{-12} + ...$

$< 1 + \dfrac{1}{10} + \dfrac{1}{100} + \dfrac{1}{1000} + ... = \dfrac{1}{1 - \dfrac{1}{10}} = \dfrac{10}{9}$

So, the integer just greater than the given expression must be 2.

The correct option is (D)

22. Given polynomial is

$(x + {}^nC_0)(x + 3.\, {}^nC_1)(x + 5.\, {}^nC_2) ...(x + (2n + 1).\, {}^nC_n)$

$= x^{n+1} + x^n [{}^nC_0 + 3.\, {}^nC_1 + 5.\, {}^nC_2 +$

$... + (2n + 1).\, {}^nC_n] + x^{n-1} (...) + ...$

$\therefore$ Coefficient of x^n in the expression is

$$\sum_{r=0}^{n} (2r+1)\,{}^nC_r = \sum_{r=0}^{n} 2r\,{}^nC_r + \sum_{r=0}^{n} {}^nC_r$$

$$= 2\sum_{r=0}^{n} \cancel{r}\, \frac{n}{\cancel{r}}\, {}^{n-1}C_{r-1} + 2^n$$

$$= 2n\sum_{r=0}^{n} {}^{n-1}C_{r-1} + 2^n$$

$= 2n.2^{n-1} + 2^n = (n + 1)\, 2^n$

The correct option is (C)

23. Greatest coefficient in the expansion of $(1 + x)^{2n}$ is ${}^{2n}C_n$. We are given ${}^{2n}C_n x^n$ is the greatest term.

$\therefore$ ${}^{2n}C_{n-1} x^{n-1} < {}^{2n}C_n x^n$

and ${}^{2n}C_{n+1} x^{n+1} < {}^{2n}C_n x^n$

$$\Rightarrow \frac{{}^{2n}C_{n-1}}{{}^{2n}C_n} < x < \frac{{}^{2n}C_n}{{}^{2n}C_{n+1}}$$

$$\Rightarrow \frac{(2n)!}{(n-1)!\,(n+1)!}\,\frac{n!\,n!}{(2n)!} < x < \frac{(2n)!\,(n+1)!\,(n-1)!}{n!\,n!\quad (2n)!}$$

$$\Rightarrow \frac{n}{n+1} < x < \frac{n+1}{n}$$

The correct option is (B)

24. We know that $\dfrac{C_k}{C_{k-1}} = \dfrac{{}^nC_k}{{}^nC_{k-1}} = \dfrac{n-k+1}{k}$

$$\therefore \sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2 = \sum_{k=1}^{n} k^3 \left(\frac{n-k+1}{k}\right)^2$$

$$= \sum_{k=1}^{n} k(n-k+1)^2$$

Put $n - k + 1 = p \Rightarrow k = n - p + 1$.

When $k = 1$, $p = n$ and when $k = n$, $p = 1$.

$$\therefore \text{Series} = \sum_{p=1}^{n} (n-p+1)p^2 = \sum_{p=1}^{n} (np^2 - p^3 + p^2)$$

$$= \sum_{p=1}^{n} (n+1)p^2 - \sum_{p=1}^{n} p^3$$

$= (n + 1)\,(1^2 + 2^2 + 3^2 + ... + n^2)$

$- (1^3 + 2^3 + 3^3 + ... + n^3)$

$$= \frac{(n+1)n\,(n+1)\,(2n+1)}{6} - \frac{n^2(n+1)^2}{4}$$

$$= \frac{n(n+1)^2}{2}\left(\frac{2n+1}{3} - \frac{n}{2}\right)$$

$$= \frac{n(n+1)^2\,(n+2)}{12}$$

The correct option is (B)

25. Given, $T_4 = 200$

$$\Rightarrow {}^6C_3 \left(\sqrt{\frac{1}{x^{\log_{10} x+1}}}\right)^3 (x^{1/12})^3 = 200$$

$$\Rightarrow 20.\, x^{\frac{3}{2(\log_{10} x+1)} + \frac{1}{4}} = 200 \Rightarrow x^{\left[\frac{3}{2(\log x+1)} + \frac{1}{4}\right]} = 10$$

$$\Rightarrow \frac{3}{2(\log_{10} x + 1)} + \frac{1}{4} = \log_x 10 = \frac{1}{\log_{10} x}$$

$$\Rightarrow \frac{3}{2(y+1)} + \frac{1}{4} = \frac{1}{y} \text{ where } y = \log_{10} x$$

$\Rightarrow$ $y = -4$ or $y = 1$

$\Rightarrow$ $\log_{10} x = -4$ or $\log_{10} x = 1$

$\Rightarrow$ $x = 10^{-4}$ or 10

$\Rightarrow$ $x = 10$ $\quad (\because x > 1)$

The correct option is (B)

27. General term in $(1 + \lambda)^n (1 + \mu)^n (\lambda + \mu)^n$ is

$t_{p,q,r} = ({}^nC_p \lambda^p)({}^nC_q \mu^q)({}^nC_r \lambda^{n-r} \mu^r)$

$\Rightarrow$ $t_{p,q,r} = {}^nC_p\, {}^nC_q\, {}^nC_r\, \lambda^{p+n-r}\, \mu^{q+r}$

The term contains coefficient of $\lambda^n \mu^n$ if

$p + \cancel{n} - r = \cancel{n}$ and $q + r = n$

$\Rightarrow$ $p = r$ and $q = n - r$

Now, $t_{r,(n-r),r}$ contains coefficient of $\lambda^n \mu^n$

$\Rightarrow$ Coefficient of $\lambda^n \mu^n = {}^nC_r\, {}^nC_{n-r}\, {}^nC_r$

$\therefore$ Coefficient of $\lambda^n \mu^n = ({}^nC_r)^3$

The correct option is (D)

28. The numerator (α) is of the form

$a^3 + b^3 + 3ab\,(a + b) = (a + b)^3$

$\therefore \alpha = (18 + 7)^3 = 25^3$

Also, $\beta = 3^6 + {}^6C_1 3^5 \cdot 2^1 + {}^6C_2 3^4 \cdot 2^2 + {}^6C_3 3^3 \cdot 2^3$

$+ {}^6C_4 3^2 \cdot 2^4 + {}^6C_5 3^3 \cdot 2^5 + {}^6C_6 2^6$

which is an expansion of

$(3 + 2)^6 = 5^6 = (25)^3$

$\therefore \ \dfrac{\alpha}{\beta} = \dfrac{(25)^3}{(25)^3} = 1$

The correct option is (A)

29. General Term in $\left(x + \dfrac{1}{x^2}\right)^{n-3}$ is

$t_{k+1} = {}^{n-3}C_k x^{n-3-k}\left(\dfrac{1}{x^2}\right)^k$

$\Rightarrow \ t_{k+1} = {}^{n-3}C_k x^{n-3(k+1)}$

There is a term containing x^{2r}, if

$n - 3(k+1) = 2r$

$\Rightarrow \ n - 2r = 3(k+1), k \in \mathrm{N}$

$\therefore \ n - 2r$ is a positive integral multiple of 3.

The correct option is (A)

30. Given: $P_n = {}^nC_0\, {}^nC_1\, {}^nC_2 \ldots {}^nC_n$

Now, $\dfrac{P_{n+1}}{P_n} = \dfrac{{}^{n+1}C_0\ {}^{n+1}C_1\ {}^{n+1}C_2 \ldots {}^{n+1}C_{n+1}}{{}^nC_0\ {}^nC_1\ {}^nC_2 \ldots {}^nC_n}$

$\Rightarrow \ \dfrac{P_{n+1}}{P_n} = {}^{n+1}C_0\left(\dfrac{{}^{n+1}C_1}{{}^nC_0}\right)\left(\dfrac{{}^{n+1}C_2}{{}^nC_1}\right)$

$\ldots\left(\dfrac{{}^{n+1}C_n}{{}^nC_n}\right){}^{n+1}C_{n+1}$

Since, ${}^{n+1}C_{r+1} = \dfrac{n+1}{r+1}\,{}^nC_r$

$\Rightarrow \ \dfrac{P_{n+1}}{P_n} = 1\left(\dfrac{n+1}{1}\right)\left(\dfrac{n+1}{2}\right)\ldots\left(\dfrac{n+1}{n}\right)1$

$\therefore \ \dfrac{P_{n+1}}{P_n} = \dfrac{(n+1)^n}{n!}$

The correct option is (A)

31. $\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}}$

$= \dfrac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x^{1/2}\,(x^{1/2} - 1)}$

$= \dfrac{(x^{1/3}+1)\,(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} - \dfrac{x^{1/2} + 1}{x^{1/2}}$

$= x^{1/3} + 1 - 1 - x^{-1/2} = x^{1/3} - x^{-1/2}$

$\Rightarrow \ \left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}}\right)^{10} = (x^{1/3} - x^{-1/2})^{10}$

T_{r+1} for $(x^{1/3} - x^{-1/2})^{10}$ is ${}^{10}C_r(x^{1/3})^{10-r}\,(-1)^r\,(x^{-1/2})^r$

For term independent of x,

$\dfrac{10-r}{3} - \dfrac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0 \Rightarrow r = 4$

Hence, required coefficient $= {}^{10}C_4(-1)^4 = 210$.

The correct option is (A)

32. Given expression $= \dfrac{1}{(81)^n}\,((1-10)^{2n}) = 1$

The correct option is (D)

33. Let $n = 2m,\ m \in N$

$\therefore \ (a + b + c)^n + (a + b - c)^n = [(a + b) + c]^{2m}$

$+ [(a + b) - c]^{2m}$

$= 2[(a + b)^{2m} + {}^{2m}C_2(a + b)^{2m-2}\,c^2 +$

$\ldots + {}^{2m}C_{2m}c^{2m}]$

Therefore, the number of distinct terms in the expansion

$= (2m + 1) + (2m - 1) +$

$\ldots + 3 + 1 = \left(\dfrac{m+1}{2}\right).\ (2m + 1 + 1)$

$= (m + 1)^2 = \left(\dfrac{n}{2} + 1\right)^2 = \left(\dfrac{n+2}{2}\right)^2$

The correct option is (C)

34. Coefficient of t^{24} in $(1 + t^2)^{12}\,(1 + t^{12})\,(1 + t^{24})$

$=$ coefficient of t^{24} in $(1 + {}^{12}C_6\,t^{12} + {}^{12}C_{12}\,t^{24})$

$(1 + t^{12} + t^{24})$

$=$ coefficient of t^{24} in $({}^{12}C_6 + 2)t^{24} = {}^{12}C_6 + 2$

The correct option is (D)

35. Consider

$(\cos\theta - i\sin\theta)^m = {}^mC_0\cos^m\theta - {}^mC_1\cos^{m-1}\theta\ i\sin\theta$

$+ \ldots + {}^mC_m\,(-i\sin\theta)^m$ ...(1)

$(\cos\theta + i\sin\theta)^m = {}^mC_0\cos^m\theta + {}^mC_1\cos^{m-1}\theta\ i\sin\theta$

$+ \ldots + {}^mC_m\,(i\sin\theta)^m$ ...(2)

Adding (1) and (2), we get

$2\cos m\theta = 2[{}^mC_0\cos^m\theta - {}^mC_2\cos^{m-2}\theta\sin^2\theta\ldots]$ ...(3)

Subtracting (1) from (2), we get

$2\,i\sin m\theta = 2i\,[{}^mC_1\cos^{m-1}\theta\sin\theta - {}^mC_3\cos^{m-3}$

$\theta\sin^3\theta\ldots]$ ...(4)

Adding (3) and (4), we get

$\cos m\theta + \sin m\theta = [{}^mC_0\cos^m\theta + {}^m\mathbf{C}_1\cos^{m-1}\theta\sin\theta$

$- {}^mC_2\cos^{m-2}\theta\sin^2\theta - {}^mC_3\cos^{m-3}\theta\sin^3\theta\ldots]$

$\Rightarrow \ \sqrt{2}\,\sin\left(m\theta + \dfrac{\pi}{4}\right) = [{}^mC_0\cos^m\theta + {}^mC_1\cos^{m-1}\theta\sin\theta$

$- {}^mC_2\cos^{m-2}\theta\sin^2\theta - {}^mC_3\cos^{m-3}\theta\sin^3\theta\ldots]$

Putting $\theta = \dfrac{\pi}{4}$, we get

$\sqrt{2}\,\sin\left(\dfrac{(m+1)\pi}{4}\right) = \dfrac{1}{2^{m/2}}\ [({}^mC_0 + {}^mC_1 - {}^mC_2$

$- {}^mC_3) + ({}^mC_4 + {}^mC_5 - {}^mC_6 - {}^mC_7) + \ldots$

$+ ({}^mC_{m-3} + {}^mC_{m-2} - {}^mC_{m-1} - {}^mC_m)]$

Hence, $m + 1 = 4k$, for given quantity to be 0.

$\Rightarrow \quad m = 4k - 1$, where $k \in N$

The correct option is (C)

36. Sum of coefficients in $(1 + 2x)^m = 3^m = 6561 = 3^8$ $\Rightarrow m = 8$ Sum of coefficient on $(2 + x)^n = 3^n = 243 = 3^5$ $\Rightarrow n = 5$

Since $S_1 < 0$, so the point lies inside the circle.

The correct option is (A)

37. We have, $S = 1 + n + n^2 + ... + n^{255}$

$$\Rightarrow \quad S = \frac{1(n^{256}-1)}{n-1} = (n^{128}+1)\,\frac{n^{128}-1}{n-1}$$

$\therefore \quad S = (n^{128} + 1)\,(1 + n + n^2 + ... + n^{127})$

Thus, the largest value of m for which $1 + n + n^2 + ... + n^{255}$ is divisible by $n^m + 1$ is 128.

The correct option is (A)

38. The expansion is a G.P. with $(n + 1)$ terms of the form

$$a^n + a^{n-1}b + a^{n-2}b^2 + ... + b^n = \frac{a^{n+1}-b^{n+1}}{a-b}$$

$$= \frac{(2x+3)^{n+1}-(2x-5)^{n+1}}{8},$$

where $a = 2x + 3$ and $b = 2x - 5$

$\therefore$ Coefficient of $x^n = \frac{1}{8}\,[(n + 1).\,2^n(3) - (n + 1).\,2^n(-5)]$ $= (n + 1)\cdot 2^n$

The correct option is (B)

39. Let S denotes the sum of the series. General term of the series is given by,

$T_r = (-1)^r\,(3 + 5r)\,{}^nC_r$, where $r = 0, 1, 2,..., n$

$$\therefore \quad S = \sum_{r=0}^{n}(-1)^r\,(3+5r)\,{}^nC_r$$

$$\Rightarrow \quad S = 3\sum_{r=0}^{n}(-1)^r\,{}^nC_r + 5\sum_{r=0}^{n}(-1)^r\,r\,{}^nC_r$$

$\Rightarrow \quad S = 3(C_0 - C_1 + C_2 - C_3 + C_4 ...)$ $+ 5(- C_1 + 2C_2 - 3C_3 + 4C_4...)$

$\therefore \quad S = 0 + 0 = 0$

The correct option is (A)

40. $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 ... {}^nC_nx^n$

On integrating between the limits 0 and, we get

$$\frac{(1+x)^{n+1}-1}{n+1} = {}^nC_0x + \frac{{}^nC_1x^2}{2} + \frac{{}^nC_2x^3}{3} + \frac{{}^nC_3x^4}{4} ...$$

Multiplying with x and differentiating, we get

$$\frac{d}{dx}\left[x\left(\frac{(1+x)^{n+1}-1}{n+1}\right)\right]$$

$$= \frac{d}{dx}\left[{}^nC_0x^2 + \frac{{}^nC_1x^3}{2} + \frac{{}^nC_2x^4}{3} + \frac{{}^nC_3x^5}{5} ...\right]$$

$$\Rightarrow \quad \frac{(1+x)^{n+1} + x(n+1)\,(1+x)^n - 1}{n+1}$$

$$= 2\,{}^nC_0x + \frac{3\,{}^nC_1x^2}{2} + \frac{4\,{}^nC_2x^3}{3} ...$$

put $x = 1$, we get

$$\frac{2^{n+1} + (n+1)2^n - 1}{n+1} = 2\,{}^nC_0 + \frac{3}{2}\,{}^nC_1 + \frac{4}{3}\,{}^nC_2 + ...$$

$$= \frac{2^n(n+3)-1}{n+1}$$

The correct option is (B)

41. For option (a)

General term

$= {}^{25}C_r\,(x^{-1/5})^{25-r}\,(2x^{3/5})^r$

There is a term containing x^3 if

$$\frac{-25+r}{5} + \frac{3r}{5} = 3$$

$$\Rightarrow \quad -5 + \frac{4r}{5} = 3$$

$\therefore \quad r = 10$ i.e. an integer

Hence, T_{11} will be the term containing x^3 and it will be ${}^{25}C_{10}2^{10}x^3$.

Similarly, try all the other options, and in none you will have the value of r as an integer, Hence, no other binomial will have the term of x^3.

The correct option is (A)

42. We have $(1 - x - x^2 + x^3)^6 = (1 - x)^6\,(1 - x^2)^6$

Coefficient of x^7 in

$(1 - x - x^2 + x^3)^6 \quad = {}^6C_1 \cdot {}^6C_3 - {}^6C_3 \cdot {}^6C_2 + {}^6C_5 \cdot {}^6C_1$

$= 6 \times 20 - 20 \times 15 + 6 \times 6 = -144$

The correct option is (D)

43. $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$

$$= 2[{}^{2n}C_1\left(\sqrt{3}\right)^{2n-1} + {}^{2n}C_3\left(\sqrt{3}\right)^{2n-3}$$

$$+ {}^{2n}C_5\left(\sqrt{3}\right)^{2n-5} + ...]$$

= which is an irrational number

The correct option is (A)

44. $(1 + x + x^2)^8 = \left(\frac{1-x^3}{1-x}\right)^8 = (1 - x^3)^8(1 - x)^{-8}$

$= (1 - {}^8C_1x^3 + {}^8C_2x^6 - ....)(1 + {}^8C_1x^1 + {}^9C_2x^2 + {}^{10}C_3x^3 + ...)$

a_5 = coefficient of $x^5 = {}^{12}C_5 - {}^8C_1{}^9C_2 = 792 - 288 = 504$

The correct option is (B)

45. $\sum_{r=0}^{n}(r+1)\,{}^nC_r = \sum_{r=0}^{n} r\,{}^nC_r + {}^nC_r$

$= \sum_{r=0}^{n} r\frac{n}{r}\,{}^{n-1}C_{r-1} + \sum_{r=0}^{n}{}^nC_r$

$= n\,2^{n-1} + 2^n = 2^{n-1}(n+2)$

Statement-1 is true

$\sum(r+1)\,{}^nC_r\,x^r = \sum r\,{}^nC_r x^r + \sum {}^nC_r x^r$

$= n\sum_{r=0}^{n}{}^{n-1}C_{r-1}x^r + \sum_{r=0}^{n}{}^nC_r x^r$

$= nx(1+x)^{n-1} + (1+x)^n$

Substituting $x = 1$

$\sum(r+1)^n C_r = n\,2^{n-1} + 2^n$

Hence Statement-2 is also true and is a correct explanation of Statement-1.

The correct option is (B)

46. $1 - q^n \geq \frac{9}{10}$

$\Rightarrow \left(\frac{3}{4}\right)^n \leq \frac{1}{10} \Rightarrow n \geq \log_{\frac{3}{4}} 10$

$\Rightarrow n \geq \frac{1}{\log_{10}^{4} - \log_{10}^{3}}$

The correct option is (A)

47. $8^{2n} - (62)^{2n+1} = (1+63)^n - (63-1)^{2n+1}$

$= (1+63)^n + (1-63)^{2n+1}$

$= (1 + {}^nc_1 63 + {}^nc_2 (63)^2 + ... + (63)^n)$

$+ (1 - {}^{(2n+1)}c_1 63 + {}^{(2n+1)}c_2 (63)^2 + ... + (-1)(63)^{(2n+1)})$

$= 2 + 63({}^n c_1 + {}^nc_2 (63) + ... + (63)^{n-1}$

$- {}^{(2n+1)}c_1 + {}^{(2n+1)}c_2 (63) + ... - (63)^{(2n)})$

$\therefore$ Remainder is 2.

The correct option is (B)

48. Let loge10 = x.

Then, $\sum_{r=0}^{n}(-1)^r \cdot {}^nC_r \cdot \frac{1 + r\log_e 10}{(1+\log_e 10^n)^r}$

$= \sum_{r=0}^{n}(-1)^r\,{}^nC_r \cdot \frac{1+rx}{(1+nx)^r}$

$= \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{1}{1+nx}\right)^r + \sum_{r=0}^{n}(-1)^r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1}\frac{rx}{(1+nx)^r}$

$= \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{1}{1+nx}\right)^r$

$- \frac{nx}{1+nx} \cdot \sum_{r=0}^{n}(-1)^{r-1} \cdot {}^{n-1}C_{r-1}\left(\frac{1}{1+nx}\right)^{r-1}$

$= \left(1 - \frac{1}{1+nx}\right)^n - \frac{nx}{1+nx}\left(1 - \frac{1}{1+nx}\right)^{n-1}$

$= \left(\frac{nx}{1+nx}\right)^n - \left(\frac{nx}{1+nx}\right)^n = 0.$

The correct option is (C)

49. Here, $tr_{+1} = \frac{{}^nC_r}{r+1} = \frac{1}{r+1} \cdot nCr$

$= \frac{1}{n+1} \cdot n^{+1}Cr_{+1}$

Putting $r = 0, 1, 2, ... n$ and adding we get, $\sum_{0}^{n}\frac{C_k}{k+1}$

$= \frac{1}{n+1}\{n^{+1}C_1 + n^{+1}C_2 + n^{+1}C_3 + ... + n^{+1}Cn_{+1}\}$

$= \frac{1}{n+1}\{2n^{+1} - n^{+1}C_0\} = \frac{2^{n+1}-1}{n+1}$

The correct option is (C)

50. We have,

$101^{50} = (100+1)^{50}$

$= 100^{50} + 50 \cdot 100^{49} + \frac{50 \cdot 49}{1 \cdot 2} \cdot 100^{48} + ...$

and, $99^{50} = (100-1)^{50}$

$= 100^{50} - 50 \cdot 100^{49} + \frac{50 \cdot 49}{1 \cdot 2} \cdot 100^{48} - ...$

Subtracting, we get

$101^{50} - 99^{50} = 2\,[50 \cdot 100^{49} + \frac{50 \cdot 49 \cdot 48}{1 \cdot 2 \cdot 3} \times 100^{47} + ...]$

$= 100^{50} + 2 \cdot \frac{50 \cdot 49 \cdot 48}{1 \cdot 2 \cdot 3} \cdot 100^{47} + ... > 100^{50}$

Hence, $10150 > 99^{50} + 100^{50}$.

The correct option is (A)

51. The greatest coefficient is

$= \frac{n!}{(q!)^{k-r}[(q+1)!]^r}$ [Here, $n = 15$, $q = 3$, $r = 3$, $k = 4$]

The correct option is (A)

52. We have, $\sum_{r=0}^{10}{}^{20}C_r = {}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{10}$

But ${}^{20}C_0 + {}^{20}C_1 + ... {}^{20}C_{20} = 2^{20}$

and, $\because {}^{20}C_{20} = {}^{20}C_0$, ${}^{20}C_{19} = {}^{20}C_1$

${}^{20}C_{18} = {}^{20}C_2$ ... and ${}^{20}C_{11} = {}^{20}C_9$

$\therefore \sum_{r=0}^{10}{}^{20}C_r = ({}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{20})$

$- ({}^{20}C_{11} + {}^{20}C_{12} + ... + {}^{20}C_{20})$

$= 2^{20} + {}^{20}C_{10} - ({}^{20}C_{10} + {}^{20}C_9 + ... + {}^{20}C_0)$

$\Rightarrow 2\,[{}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{10}] = 2^{20} + {}^{20}C_{10}$

$\therefore {}^{20}C_0 + {}^{20}C_1 + ... + {}^{20}C_{10} = 219 + \frac{1}{2}\,20C10$

The correct option is (B)

53. We have,

$^{n+1}C_2 + 2\,[^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2]$

$= {}^{n+1}C_2 + 2\,[^3C_3 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2]$

$= {}^{n+1}C_2 + 2\,[^4C_3 + {}^4C_2 + \ldots + {}^nC_2]$

$= {}^{n+1}C_2 + 2\,[^5C_3 + \ldots + {}^nC_2]$

$= {}^{n+1}C_2 + 2\cdot {}^{n+1}C_3$

$= {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3$

$= {}^{n+2}C_3 + {}^{n+1}C_3$

$= \dfrac{n(n+1)(n+2)}{6} + \dfrac{n(n+1)(n-1)}{6}$

$= \dfrac{n(n+1)(2n+1)}{6}$

The correct option is (A)

54. A = coefficient of x in $[^{2n}C_0(1+x)^{2n}$ $+ {}^{2n}C_1(1+x)^{2n-1} + \ldots]$

= coefficient of x in $(1 + (1 + x))^{2n}$

= coefficient of x in $(2 + x)^{2n}$

= coefficient of x in $2^{2n}\left(1+\dfrac{x}{2}\right)^{2n} = n\cdot 2^{2n}$

The correct option is (C)

55. By binomial theorem

$(1+x)^n = \left[1 + nx + \dfrac{n(n-1)}{2}\cdot x^2 \ldots x^n\right]$

or, $(1+x)^n - 1 = nx + \dfrac{n(n-1)}{2}\, x^2 \ldots x^n$

If $x = n$, $(1+n)^n - 1 = n^2 + \dfrac{n(n-1)}{2}\, n^2 \ldots n^n$

$(1+n)^n - 1 = n^2\left[1 + \dfrac{n(n-1)}{2} \ldots n^{n-2}\right]$

Put $n = 100$,

$(1+100)^{100} - 1 = (100)^2\left[1 + \dfrac{100(100-1)}{2} \ldots 100^{98}\right]$

$(101)^{100} - 1 = (100)^2\left[1 + \dfrac{100\times 99}{2} \ldots 100^{98}\right]$

Clearly $(101)^{100} - 1$ is divisible by

$(100)^2 = 10000$

The correct option is (C)

56. We have, $t_{3r} = {}^{2n}C_{3r-1}\, x^{3r-1}$

and, $t_{r+2} = {}^{2n}C_{r+1}\, x^{r+1}$.

Given, ${}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow$ $3r - 1 = r + 1$; or $(3r - 1) + (r + 1) = 2n$

$\Rightarrow$ $2r = 2$; or $4r = 2n$

$\Rightarrow$ $r = 1$ (impossible); or $r = \dfrac{n}{2}$.

But r is a positive integer greater than 1. So, the value of r is $\dfrac{n}{2}$, provided n is an even integer (> 2), otherwise r has no value.

The correct option is (A)

57. We have,

$(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$ ...(1)

Replacing x by $\dfrac{1}{x}$, we get

$\left(1+\dfrac{1}{x}+\dfrac{1}{x^2}\right)^n = \sum_{r=0}^{2n} \dfrac{a_r}{x^r}$

Multiplying both sides by x^{2n}, we get

$(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^{2n-r}$ ...(2)

From (1) and (2), we have

$\sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{2n} a_r x^{2n-r}$

On equating the coefficient of x^{2n-r} on both sides, we get

$a_{2n-r} = a_r$ for $0 \le r \le 2n$.

The correct option is (C)

58. $2^{2003} = (2^4)^{500}.\ 2^3$

$\Rightarrow$ $2^{2003} = 8\,(16)^{500}$

$\Rightarrow$ $2^{2003} = 8\,(17 - 1)^{500}$

$\Rightarrow$ $2^{2003} = 8[(17)^{500} - {}^{500}C_1(17)^{499} + \ldots$ $- {}^{500}C_{499}(17) + 1]$

$\Rightarrow$ $\dfrac{2^{2003}}{17} = 8k + \dfrac{8}{17}$,

where $k = (17)^{499} - {}^{500}C_1(17)^{498} + \ldots + {}^{500}C_{499}$

such that k is an integer

$\therefore$ $\left\{\dfrac{2^{2003}}{17}\right\} = \dfrac{8}{17}$

The correct option is (C)

59. Let $1 + f = (6\sqrt{6}+14)^{2n+1}$

Assuming, $f' = (6\sqrt{6}-14)^{2n+1}$ ...(1)

Now, $I + f - f' = (6\sqrt{6}+14)^{2n+1} - (6\sqrt{6}-14)^{2n+1}$

$\Rightarrow$ $I + f - f' = 2\,\{^{2n+1}C_1(6\sqrt{6})^{2n}\, 14^1$ $+ {}^{2n+1}C_3(6\sqrt{6})^{2n+2}\,(14)^3 + \ldots\}$

$\Rightarrow$ $I + f - f' = 2$ {Integer} = even ...(2)

Now, $0 \le f < 1$

Also, $0 \le f - f' < 1$

$\therefore$ $0 \le f - f' < 0$ {Using (1)}

$\therefore$ $0 \le f - f' < 0 \Rightarrow f - f' = 0$

Substituting respective values in (2), we get

I = even integer

The correct option is (A)

60. $C_0^2 - 2C_1^2 + 3C_2^2 - 4C_3^2 + \ldots$ $+ (-1)^n\,(n+1)C_n^2$

$= [C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^n\, C_n^2]$

$- [C_1^2 - 2C_2^2 + 3C_3^2 - ... + (-1)^n \, nC_n^2]$

$= (-1)^{n/2} \dfrac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!} - (-1)^{\frac{n}{2}-1} \dfrac{n}{2} \, {}^nC_{\frac{n}{2}}$

$= (-1)^{n/2} \dfrac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!} \left(1 + \dfrac{n}{2}\right)$

$\therefore \quad 2\left(\dfrac{n}{2}\right)!\left(\dfrac{n}{2}\right)! [C_0^2 - 2C_1^2 + 3C_2^2 - ...$

$+ (-1)^n (n+1) C_n^2] = (-1)^{n/2}(n+2)$

The correct option is (C)

61. Let $S = 1 + \dfrac{1}{3^2} + \dfrac{1.4}{1.2}\dfrac{1}{3^4} + \dfrac{1.4.7}{1.2.3}\dfrac{1}{3^6} + ...$

$\Rightarrow S = 1 + \left(\dfrac{1}{3}\right)\left(\dfrac{1}{3}\right) + \dfrac{\frac{1}{3}.\frac{4}{3}}{1.2}\left(\dfrac{1}{3}\right)^2 + \dfrac{\frac{1}{3}.\frac{4}{3}.\frac{7}{3}}{1.2.3}\left(\dfrac{1}{3}\right)^3 + ...$

$\Rightarrow S = 1 + \left(\dfrac{1}{3}\right)\left(\dfrac{1}{3}\right) + \dfrac{\frac{1}{3}\left(1 + \frac{1}{3}\right)}{2!}\left(\dfrac{1}{3}\right)^3$

$+ \dfrac{\frac{1}{3}\left(1 + \frac{1}{3}\right)\left(2 + \frac{1}{3}\right)}{3!}\left(\dfrac{1}{3}\right)^3 + ...$

which is an equivalent of

$S = \left(1 - \dfrac{1}{3}\right)^{-\frac{1}{3}} \Rightarrow S = \left(\dfrac{2}{3}\right)^{-\frac{1}{3}} = \left(\mathbf{\dfrac{3}{2}}\right)^{\frac{1}{3}}$

The correct option is (B)

62. $(1 + x)^{101} (1 - x + x^2)^{100} = (1 + x) (1 + x^3)^{100}$

$= (1 + x) (C_0 + C_1x^3 + C_2x^6 + ... + C_{100}x^{300})$

Clearly, in this expression $x\lambda$ will be present if $\lambda = 3$, or $\lambda = 3t + 1$

So, λ cannot be of the form $3t + 2$.

The correct option is (C)

63. We have,

$17^{1995} + 11^{1995} - 7^{1995}$

$= (7 + 10)^{1995} + (1 + 10)^{1995} - 7^{1995}$

$= [7^{1995} + {}^{1995}C_1.\ 7^{1994}.\ 10^1 + {}^{1995}C_2.\ 7^{1993}.\ 10^2 +$

$... + {}^{1995}C_{1995}.\ 10^{1995}] + [{}^{1995}C_0 + {}^{1995}C_1.\ 10^1$

$+ {}^{1995}C_2.\ 10^2 + ...$

$+ {}^{1995}C_{1995}.\ 10^{1995}] - 7^{1995}$

$= [{}^{1995}C_1.\ 7^{1994}.\ 10^1 + ... + 10^{1995}]$

$+ [{}^{1995}C_1.\ 10^1 + ... + {}^{1995}C_{1995}.\ 10^{1995}] + 1$

$=$ a multiple of 10 + 1.

Thus, the unit's place digit is **1.**

The correct option is (B)

64. Given polynomial is

$(x + nC_0) (x + 3.\ nC_1) (x + 5.\ nC_2) ...(x + (2n + 1).\ nCn)$

$= xn^{+1} + xn \{nC_0 + 3.\ nC_1 + 5.\ nC_2 +$

$... + (2n + 1).\ nCn\} + xn^{-1} (...) + ...$

$\therefore$ Coefficient of xn in the expression is

$\sum_{r=0}^{n} (2r+1)^nC_r = \sum_{r=0}^{n} 2r\ {}^nC_r + \sum_{r=0}^{n} {}^nC_r$

$= 2\sum_{r=0}^{n} \not{r} \dfrac{n}{\not{r}}\ {}^{n-1}C_{r-1} + 2^n$

$= 2n\sum_{r=0}^{n} {}^{n-1}C_{r-1} + 2^n$

$= 2n.2n^{-1} + 2n = (n + 1)\ 2n$

The correct option is (C)

65. Let $n = 2m,\ m \in N$

$\therefore \quad (a + b + c)n + (a + b - c)n = [(a + b) + c]^2m$

$+ [(a + b) - c]^2m$

$= 2\{(a + b)^2m + {}^2mC_2(a + b)^2m^{-2}\ c^2 +$

$... + {}^2mC_2mc^2m\}$

Therefore, the number of distinct terms

$= (2m + 1) + (2m - 1) +$

$... + 3 + 1 = \left(\dfrac{m+1}{2}\right).\ (2m + 1 + 1)$

$= (m + 1)^2 = \left(\dfrac{n}{2} + 1\right)^2 = \left(\dfrac{n+2}{2}\right)^2$

The correct option is (C)

66. Consider,

$(\cos\theta - i \sin\theta)m = mC_0 \cos m\theta - mC_1\cos m^{-1}\theta\ i \sin\theta$

$+ ... + mCm\ (- i \sin\theta)m \quad ...(1)$

$(\cos\theta + i \sin\theta)m = mC_0\cos m\theta + mC_1 \cos m^{-1}\theta\ i \sin\theta$

$+ ... + mCm\ (i \sin\theta)m \quad ...(2)$

Adding (1) and (2), we get

$2\cos m\theta = 2[mC_0\cos m\theta - mC_2\cos m^{-2}\theta \sin^2\theta...] \quad ...(3)$

Subtracting (1) from (2), we get

$2\ i \sin m\theta = 2i\,[mC_1\cos m^{-1}\theta \sin\theta - mC_3\cos m^{-3}$

$\theta \sin^3\theta...] \quad ...(4)$

Adding (3) and (4), we get

$\cos m\theta + \sin m\theta = [mC_0\cos m\theta + m\mathbf{C}_1\cos m^{-1}\theta \sin\theta$

$- mC_2\cos m^{-2}\theta \sin^2\theta - mC_3\cos m^{-3}\theta \sin^3\theta...]$

$\Rightarrow \sqrt{2} \sin\left(m\theta + \dfrac{\pi}{4}\right) = [mC_0\cos m\theta + mC_1\cos m^{-1}\theta \sin\theta$

$- mC_2\cos m^{-2}\theta \sin^2\theta - mC_3\cos m^{-3}\theta \sin^3\theta ...]$

Putting $\theta = \dfrac{\pi}{4}$, we get

$\sqrt{2} \sin\left(\dfrac{(m+1)\pi}{4}\right) = \dfrac{1}{2^{m/2}} [(mC_0 + mC_1 - mC_2$

$- mC_3) + (mC_4 + mC_5 - mC_6 - mC_7) + ...$

$+ (mCm_{-3} + mCm_{-2} - mCm_{-1} - mCm)]$

Hence, $m + 1 = 4k$, for given quantity to be 0.

$\Rightarrow$ $m = \mathbf{4k - 1}$, where $k \in N$

The correct option is (C)

67. We have, $S = 1 + n + n^2 + ... + n^{255}$

$$\Rightarrow \quad S = \frac{1(n^{256}-1)}{n-1} = (n^{128}+1)\frac{n^{128}-1}{n-1}$$

$\therefore$ $S = (n^{128} + 1)(1 + n + n^2 + ... + n^{127})$

Thus, the largest value of m for which $1 + n + n^2 + ... + n^{255}$ is divisible by $nm + 1$ is 128.

The correct option is (A)

68. $(1 + x)n = nC_0 + nC_1x + nC_2x^2 + nC_3x^3 ... nCnxn$

On integrating w.r.t x between limits 0 and x, we get

$$\frac{(1+x)^{n+1}-1}{n+1} = {}^nC_0x + \frac{{}^nC_1x^2}{2} + \frac{{}^nC_2x^3}{3} + \frac{{}^nC_3x^4}{4} ...$$

Multiplying with x and differentiating, we get

$$\frac{d}{dx}\left\{x\left(\frac{(1+x)^{n+1}-1}{n+1}\right)\right\} = \frac{d}{dx}\left\{{}^nC_0x^2 + \frac{{}^nC_1x^3}{2} + \frac{{}^nC_2x^4}{3} + \frac{{}^nC_3x^5}{5}...\right\}$$

$$\Rightarrow \frac{(1+x)^{n+1} + x(n+1)(1+x)^n - 1}{n+1} = 2\,{}^nC_0x + \frac{3\,{}^nC_1x^2}{2} + \frac{4\,{}^nC_2x^3}{3}...$$

put $x = 1$, we get

$$\frac{2^{n+1} + (n+1)2^n - 1}{n+1} = 2\,{}^nC_0 + \frac{3}{2}\,{}^nC_1 + \frac{4}{3}\,{}^nC_2 + ...$$

$$= \frac{\mathbf{2^n(n+3)-1}}{\mathbf{n+1}}$$

The correct option is (B)

69. A = coefficient of x in $[{}^2nC_0(1 + x)^2n + {}^2nC_1(1 + x)^2n^{-1} + ...]$

= coefficient of x in $(1 + (1 + x))^2n$

= coefficient of x in $(2 + x)^2n$

= coefficient of x in $2^2n\left(1+\frac{x}{2}\right)^{2n} = n.22n$

The correct option is (C)

70. General term in $(1 + \lambda)n\ (1 + \mu)n\ (\lambda + \mu)n$ is

$tp, q, r = (nCp\lambda p)\ (nCq\mu q)\ (nCr\lambda n\!-\!r\mu r)$

$\Rightarrow$ $tp, q, r = nCp\ nCq\ nCr\ \lambda p\!+\!n\!-\!r\ \mu q\!+\!r$

term contains coefficient of $\lambda n\mu n$ if

$p + \not{n} - r = \not{n}$ and $q + r = n$

$\Rightarrow$ $p = r$ and $q = n - r$

Now, $tr,\ _{(}n\!-\!r_{),}\ r$ contains coefficient of $\lambda n\mu n$

$\Rightarrow$ Coefficient of $\lambda n\mu n = nCr\ nCn\!-\!r\ nCr$

$\therefore$ Coefficient of $\lambda n\mu n$ = $\mathbf{(nCr)3}$

The correct option is (D)

71. Since $(1 - x)n = C_0 - C_1x + C_2x^2 - C_3x^3 + ...$

$\therefore$ $x(1 - x)n = C_0x - C_1x^2 + C_2x^3 - C_3x^4 + ...$

$$\Rightarrow \int_0^1 x(1-x)^n\,dx$$

$$= \int_0^1 (C_0x - C_1x^2 + C_2x^3 - C_3x^4 + ...)\,dx$$

$$= \frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - ... \text{ up to } (n+1) \text{ terms}$$

For L.H.S. put $1 - x = t$, $\therefore$ $dx = -dt$

$$\therefore \text{ L.H.S.} = \frac{1}{(n+1)(n+2)}$$

The correct option is (D)

72. Here, $f = R - [R]$ is the fraction part of R. Thus if I is the integral part of R, then

$R = I + f = (5\sqrt{5} + 11)^2n^{+1}$, and $0 < f < 1$.

Now, $\because$ $5\sqrt{5} - 11 = 18034 < 1$,

$\therefore$ if $f' = (5\sqrt{5} - 12)^2n^{+1}$, then $0 < f' < 1$

Now, $I + f - f' = (5\sqrt{5} + 11)^2n^{+1} - (5\sqrt{5} - 11)^2n^{+1}$

$= 2\,[{}^2n^{+1}C_1\,(5\sqrt{5})^2n \times 11$

$+ {}^2n^{+1}C_3\,(5\sqrt{5})^2n^{-2} \times 11^3 + ...]$ ...(1)

= An even integer.

$\Rightarrow$ $f - f'$ must also be an integer.

$\Rightarrow$ $f - f' = 0$, $\quad\because$ $0 < f < 1, 0 < f' < 1$

$\Rightarrow$ $f = f'$

$\therefore$ $Rf = Rf' = (5\sqrt{5} + 11)^2n^{+1}\,(5\sqrt{5} - 11)^2n^{+1}$

$= (125 - 121)^2n^{+1} = 42n + 1$.

The correct option is (C)

73. The number of solutions of $x_1 + x_2 + ... + xk = n$

= coefficient of tn in $(t + t^2 + t^3 + ...)(t^2 + t^3 + ...)$

$...(tk + tk^{+1} + ...)$

= coefficient of tn in $t^{1+2+...+}k\,(1 + t + t^2 + ...)k$

But, $1 + 2 + ... + k = \frac{1}{2}k(k + 1) = r$ (say)

and, $1 + t + t^2 + ... = \dfrac{1}{1-t}$

Thus, number of required solutions

= coefficient of $tn^{-}r$ in $(1 - t)^{-}k$

= coefficient of $tn^{-}r$ in $(1 + kC_1\,t + k^{+1}C_2\,t^2 + k^{+2}C_3\,t^3 + ...)$

$= k^{+}n^{-}r^{-1}Cn_{-}r$

$= k^{+}n^{-}r^{-1}Ck_{-1} = mCk - 1$

where, $m = k + n - r - 1$

$= k + n - 1 - \frac{1}{2}k(k + 1)$

$= \frac{1}{2}[2k + 2n - 2 - k^2 - k] = \frac{1}{2}(2n - k^2 + k - 2).$

The correct option is (A)

74. We have, $\sum_{r=0}^{n} {}^nC_r \sin rx \cos(n-r)x$

$= \frac{1}{2}[(nC_0 \sin 0x \cos nx + nCn \sin nx \cos 0x)$

$+ (nC_1 \sin x \cos(n-1)x + nCn_{-1} \sin(n-1)x \cdot \cos x)$

$+ (nC_2 \sin 2x \cos(n-2)x + nCn_{-2} \sin(n-2)x \cdot \cos 2x)$

$+ \ldots + (nCn \sin nx \cos 0x + nC_0 \sin 0x \cos nx)]$

$= \frac{1}{2}[nC_0 \sin nx + nC_1 \sin nx + \ldots + nCn \sin nx]$

$= \frac{1}{2}[nC_0 + nC_1 + \ldots + nCn] \sin nx = \frac{2^n \sin nx}{2}$

$\therefore \sum_{r=0}^{n} {}^nC_r \sin rx \cos(n-r)x = 2n - 1 \sin nx.$

The correct option is (C)

75. We have,

$nCn + n^{+1}Cn + n^{+2}Cn + \ldots + n^{+}kCn$

$=$ coeff. of xn in $(1+x)n + (1+x)n^{+1} + \ldots$

$+ (1+x)n^{+}k.$

Now, $(1+x)n + (1+x)n^{+1} + (1+x)n^{+2} + \ldots$

$+ (1+x)n^{+}k$

$= (1+x)n\left[\frac{(1+x)^{k+1} - 1}{x}\right]$

$= \frac{1}{x}(1+x)n^{+}k^{+1} - \frac{1}{x}(1+x)n$

Equating the coefficient of xn, we get

$nC_0 + n^{+1}Cn + n^{+2}Cn + \ldots + n^{+}kCn$

$= n^{+}k^{+1}Cn_{+1} - 0 = n + k + 1Cn + 1$

$\left[\text{There is no term containing } x^n \text{in } \frac{1}{x}(1+x)^n\right].$

The correct option is (C)

76. We have,

$Sn = 1 + q + q^2 + \ldots + qn = \left(\frac{1-q^{n+1}}{1-q}\right) = \left(\frac{1-q^{n+1}}{1-q}\right)$ ...(1)

and, $S'n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots + \left(\frac{q+1}{2}\right)^n$

$= \frac{1 - \left(\frac{q+1}{2}\right)^{n+1}}{1 - \frac{q+1}{2}} = \frac{2^{n+1} - (q+1)^{n+1}}{(1-q)\cdot 2^n}$ ...(2)

Now, $n^{+1}C_1 + n^{+1}C_2 \cdot S_1 + n^{+1}C_3 \cdot S_2 + \ldots$

$+ n^{+1}Cn_{+1} \cdot Sn$

$= n^{+1}C_1\left(\frac{1-q}{1-q}\right) + n^{+1}C_2\left(\frac{1-q^2}{1-q}\right) + n^{+1}C_3\left(\frac{1-q^3}{1-q}\right)$

$+ \ldots + n^{+1}Cn_{+1}\left(\frac{1-q^{n+1}}{1-q}\right)$

$= \frac{1}{1-q}[n^{+1}C_1(1-q) + n^{+1}C_2(1-q^2)$

$+ n^{+1}C_3(1-q^3) + \ldots + n^{+1}Cn_{+1}(1-qn^{+1})]$

$= \frac{1}{1-q}[(n^{+1}C_1 + n^{+1}C_2 + \ldots + n^{+1}Cn_{+1})$

$- (n^{+1}C_1 \cdot q + n^{+1}C_2 \cdot q^2 + \ldots + n^{+1}Cn_{+1} \cdot qn^{+1})]$

$= \frac{1}{1-q}[(2n^{+1} - n^{+1}C_0) - \{(n^{+1}C_0 + n^{+1}C_1 q + n^{+1}C_2 q^2$

$+ \ldots + n^{+1}Cn_{+1} qn^{+1} - n^{+1}C_0\}]$

$= \frac{1}{1-q}[(2n^{+1} - 1) - \{(1+q)n^{+1} - 1\}]$

$= \frac{1}{1-q}[2n^{+1} - (1+q)n^{+1}] = 2n\, S'n.$

The correct option is (B)

77. We have, $(1+x)^{15} = C_0 + C_1 x + C_2 x^2 + \ldots + C_{15} x^{15}$

$\Rightarrow \frac{(1+x)^{15}}{x} = \frac{C_0}{x} + C_1 + C_2 x + C_3 x^2 + \ldots + C_{15} x^{14}$

Differentiating both sides w.r.t. x, we get

$\frac{x \cdot 15(1+x)^{14} - 1 \cdot (1+x)^{15}}{x^2}$

$= -\frac{C_0}{x^2} + C_2 + 2C_3 x + 3C_4 x^2 + \ldots + 14C_{15} x^{13}$

Putting $x = 1$ on both sides, we get

$15 \cdot 2^{14} - 2^{15} = -C_0 + C_2 + 2C_3 + 3C_4 + \ldots + 14C_{15}$

$\Rightarrow 2^{14}(15 - 2) + 1 = C_2 + 2C_3 + 3C_4 + \ldots + 14C_{15}$

$\therefore$ The given series $= 2^{14} \cdot 13 + 1 = 219923.$

The correct option is (A)

78. $(1 + x + x^2)n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 +$

$\ldots + a_2n_{-1} x^2n^{-1} + a_2n\, x^2n$ ...(1)

Replacing x by $\frac{1}{x}$ in (1), we get

$(1 + x + x^2)n = a_0 x^2n + a_1 x^2n^{-1} + a_2 x^2n^{-2} + \ldots$

$+ a_2n_{-1} x + a_2n$

Again, replacing x by $-x$ in (1), we get

$(1 - x + x^2)n = a_0 - a_1 x + a_2 x^2 - a_3 x^3 + \ldots$

$- a_2n_{-1} x^2n^{-1} + a_2n\, x^2n$ ...(2)

Multiplying (1) and (2), we get

$(1 + x^2 + x^4)n = (a_0 x^2n + a_1 x^2n^{-1} + a_2 x^2n^{-2} +$

$\ldots + a_2n_{-1} x + a_2n) \times (a_0 - a_1 x + a_2 x^2 +$

$\ldots - a_2n_{-1} x^2n^{-1} + a_2n\, x^2n)$ ...(3)

[Note that $(1 - x + x^2)(1 + x + x^2) = (1 + x^2)^2 - x^2$

$= 1 + x^2 + x^4$]

Finally, replace x by x^2 in (1), we get

$(1 + x^2 + x^4)n = a_0 + a_1 x^2 + \ldots + an\, x^2n + \ldots + a_2n\, x^4n$ ...(4)

Now, equating the coefficients of x^2n on the right hand sides of (3) and (4), we get

$$a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots - a_{2n-1}^2 + a_{2n}^2 = an.$$

The correct option is (B)

79. Let $S = (1 + x)^{100} + 2x(1 + x)^{999} + 3x^2(1 + x)^{998} + \ldots$
$+ 1000\, x^{999}(1 + x) + 1001 \cdot x^{1000}$

This is an A.G.S. of common ratio $r = \dfrac{x}{1+x}$

$\therefore \left[\dfrac{x}{1+x}\right] S = x(1 + x)^{999} + 2x^2(1 + x)^{998} + \ldots$

$+ 1000 \cdot x^{1000} + \dfrac{1001x^{1001}}{1+x}$

On subtracting, we get

$\left(1-\dfrac{x}{1+x}\right) S = (1 + x)^{1000} + x(1 + x)^{999}$

$+ x^2(1 + x)^{998} + \ldots + x^{1000} - \dfrac{1001x^{1001}}{1+x}$

$\Rightarrow S = [(1 + x)^{1001} + x(1 + x)^{1000} + x^2(1 + x)^{999}$

$+ \ldots + x^{1000}(1 + x)] - 1001\, x^{1001}$

$$= \frac{(1+x)^{1001}\left[1-\left\{\dfrac{x}{1+x}\right\}^{1001}\right]}{1-\dfrac{x}{1+x}} - 1001\, x^{1001}$$

$$= (1 + x)^{1002}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right] - 1001\, x^{1001}$$

$= (1 + x)^{1002} - x^{1001}(1 + x) - 1001\, x^{1001}$

$= (1 + x)^{1002} - x^{1002} - 1002\, x^{1001}$ ...(1)

Now, the coefficient of x^{50} on the R.H.S. of (1)

$= 1002C50$.

The correct option is (C)

80. We have,

$$t_1 = 1\sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{1}{2}\right)^r = \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{1}{2}\right)^r$$

$$= \left(1-\frac{1}{2}\right)^n = \frac{1}{2^n}$$

$$\left[\because \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r x^r = (1-x)^n\right]$$

$$t_2 = \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{3}{2^2}\right)^r = \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{3}{4}\right)^r$$

$$= \left(1-\frac{3}{4}\right)^n = \frac{1}{4^n} = \frac{1}{2^{2n}}$$

$$t_3 = \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{7}{2^3}\right)^r = \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r\left(\frac{7}{8}\right)^r$$

$$= \left(1-\frac{7}{8}\right)^n = \frac{1}{8^n} = \frac{1}{2^{3n}}$$

... ...

... ...

$\therefore$ Required sum

$$= \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \ldots \text{ to } m \text{ terms}$$

$$= \frac{1}{2^n}\left[\frac{1-\left(\dfrac{1}{2^n}\right)^m}{1-\dfrac{1}{2^n}}\right] = \frac{\mathbf{1}-\dfrac{\mathbf{1}}{\mathbf{2}^{mn}}}{\mathbf{2}^n-\mathbf{1}}.$$

The correct option is (B)

81. We have,

$(1 - x)n = nC_0 - nC_1\, x + nC_2\, x^2 - \ldots + (-1)n\; nCn\; xn$,

and, $(x + 1)n = nC_0\, xn + nC_1\, xn^{-1} + nC_2\, xn^{-2} + \ldots + nCn$.

The given series is the coefficient of xn in the product of R.H.S. of the above two.

$\therefore$ Sum of the series = coefficient of xn in $(1 - x)n \cdot (x + 1)n$

= coefficient of xn in $(1 - x^2)n$

= coefficient of xn in

$[nC_0 + nC_1(-x^2) + nC_2(-x^2)^2 + \ldots + nCn(-x^2)n]$

Since n is even, let $n = 2m$. Then,

sum = coefficient of x^2m in

$[{}^2mC_0 + {}^2mC_1(-x^2) + {}^2mC_2(-x^2)^2 + \ldots + {}^2mC_2m(-x^2)^2m]$

$= {}^2mCm(-1)m = nCn/2\,(-1)n/2$.

The correct option is (B)

82. We have,

$$t_{r+1} = \frac{{}^nC_r}{(r+1)(r+2)} = \frac{1}{r+2}\cdot\left[\frac{1}{r+1}{}^nC_r\right]$$

$$= \frac{1}{r+2}\cdot\left[\frac{1}{n+1}{}^{n+1}C_{r+1}\right]$$

$$= \frac{1}{n+1}\cdot\left[\frac{1}{r+2}\cdot{}^{n+1}C_{r+1}\right]$$

$$= \frac{1}{n+1}\cdot\frac{1}{n+2}\cdot n^{+2}Cr_{+2}$$

Putting $r = 0, 1, 2, \ldots, n$ and adding, we get the required sum

$$= \frac{1}{(n+1)(n+2)}[n^{+2}C_2 + n^{+2}C_3 + \ldots + n^{+2}Cn_{+2}]$$

$$= \frac{1}{(n+1)(n+2)}[2n^{+2} - (n^{+2}C_0 + n^{+2}C_1)]$$

$$= \frac{2^{n+2}-n-3}{(n+1)(n+2)}.$$

The correct option is (B)

83. Let $(\sqrt{3} + 1)^2n = p + f$, where p is the integral part and $0 < f < 1$.

$\therefore$ integer just above $(\sqrt{3} + 1)^2n = p + 1$

Now, $(\sqrt{3} + 1)^2n = \{(\sqrt{3} + 1)^2\}n = (4 + 2\sqrt{3})n$

$= 2n\ (2 + \sqrt{3}\ )n$

Thus, $p + f = 2n\ (2 + \sqrt{3}\ )n$

Also, $0 < \sqrt{3} - 1 < 1$

$\therefore\ 0 < (\sqrt{3} - 1)^2 n < 1$

Let $f_1 = (\sqrt{3} - 1)^2 n = (4 - 2\sqrt{3}\ )n = 2n\ (2 - \sqrt{3}\ )n$, then $0 < f_1 < 1$.

Now, $p + f = 2n\ (2 + \sqrt{3}\ )n$

$= 2n\ [2n + nC_1\ 2n^{-1}\ \sqrt{3} + nC_2\ 2n^{-2}\ (\sqrt{3}\ )^2$

$+ ... + nCn\ (\sqrt{3}\ )n]$ ...(1)

$f_1 = 2n\ (2 - \sqrt{3}\ )n$

$= 2n\ [2n - nC_1\ 2n^{-1}\sqrt{3} + nC_2\ 2n^{-2}\ (\sqrt{3}\ )^2$

$+ ... + (-1)n \cdot nCn\ (\sqrt{3}\ )n]$ ...(2)

(1) + (2) $\Rightarrow$

$p + f + f_1 = 2n \cdot 2\ [\cdot\ 2n + nC_2\ 2n^{-2}\ (\sqrt{3}\ )^2 + ...]$

= an even integer ...(A)

$\therefore f + f_1$ = even number $- p$ = an integer ...(B)

Also, $0 < f < 1,\ 0 < f_1 < 1$

$\therefore\ 0 < f + f_1 < 2$ ...(C)

From (B) and (C), $f + f_1 = 1$ ...(D)

From (A), $p + 1 = 2n^{+1}$, an integer.

Hence, integer just above $(\sqrt{3} + 1)^2 n$ i.e., $(p + 1)$ is divisible by $2n + 1$.

The correct option is (A)

84. We have,

$$tr_{+1} = \frac{2^{r+2}\ {}^nC_r}{(r+1)(r+2)} = \frac{2^{r+2}}{r+2} \cdot \frac{1}{r-1}\ nCr$$

$$= \frac{2^{r+2}}{r+2} \cdot \frac{1}{n+1}\ n^{+1}Cr_{+1}$$

$$= \frac{2^{r+2}}{n+1} \cdot \left(\frac{1}{r+2}\ {}^{n+1}C_{r+1}\right)$$

$$= \frac{2^{r+2}}{n+1} \cdot \frac{1}{n+2}\ {}^{n+2}C_{r+2}$$

$$\left[\because \frac{1}{r+1}\ {}^nC_r = \frac{1}{n+1}\ {}^{n+1}C_{r+1}\right]$$

Putting $r = 0, 1, 2, ..., n$ and adding we get,

The given expression

$$= \frac{1}{(n+1)(n+2)}\ \{2^2 \cdot n^{+2}C_2 + 2^3 \cdot n^{+2}C_3 + ...$$

$$+ 2n^{+2} \cdot n^{+2}Cn_{+2}\}$$

$$= \frac{1}{(n+1)(n+2)}\ \{(1 + 2)n^{+2} - n^{+2}C_0 - 2 \cdot n^{+2}C_1\}$$

$$= \frac{3^{n+2} - 2(n+2) - 1}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}.$$

The correct option is (B)

85. The given series

$= mCr \cdot nC_0 + mCr_{-1} \cdot nC_1 + ... + mC_0 \cdot nCr$,

Now, $(1 + x)m = mC_0 + mC_1 \cdot x + mC_2\, x^2 + ...$

$+ mCr\ xr + ... + mCm\ xm$,

and, $(1 + x)n = nC_0 + nC_1\, x + nC_2\, x^2 + ... + nCr\ xr + ... + nCn\ xn$

The given series is the coefficient of xr in the product of R.H.S. of above two.

$\therefore$ Sum of the series = coefficient of xr in $(1 + x)m \cdot (1 + x)n$

= coefficient of xr in $(1 + x)m^+ n$

$= m + nCr$.

The correct option is (B)

86. Let the expansion be that of $(1 + x)n$.

Let a, b, c, d be the $(r + 1)$th, $(r + 2)$th, $(r + 3)$th and $(r + 4)$ th coefficients.

$\therefore\quad a = nCr,\ b = nCr_{+1},\ c = nCr_{+2},\ d = nCr_{+3}$.

$$\text{Now, } \frac{a}{a+b} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} = \frac{{}^nC_r}{{}^{n+1}C_{r+1}}$$

$$= \frac{n!}{r!\,(n-r)!} \times \frac{(r+1)!\,(n-r)!}{(n+1)!} = \frac{r+1}{n+1}$$

$$\text{Similarly, } \frac{b}{b+c} = \frac{(r+1)+1}{n+1} = \frac{r+2}{n+1},$$

$$\frac{c}{c+d} = \frac{(r+2)+1}{n+1} = \frac{r+3}{n+1}.$$

$$\therefore\ \frac{a}{a+b} + \frac{c}{c+d} = \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2r+4}{n+1} = \frac{2(r+2)}{n+1}$$

$$= \frac{2b}{b+c}.$$

$$\Rightarrow\ \frac{a}{a+b},\ \frac{b}{b+c},\ \frac{c}{c+d} \text{ are in A.P.}$$

$$\therefore\ \frac{a+b}{a},\ \frac{b+c}{b},\ \frac{c+d}{c} \text{ are in H.P.}$$

The correct option is (C)

87. $3^{400} = (3^4)^{100} = (81)^{100} = (1 + 80)^{100} = 1 + {}^{100}C_1\ (80) + {}^{100}C_2\ (80)^2 + ... + {}^{100}C_{100}\ (80)^{100}$

= 1 + 8000 + last two digits in each term is 00

$\therefore$ Last two digits = 01

The correct option is (C)

88. Consider $(1 + x)n = C_0 + C_1x + C_2x^2 + C_3x^3 + ...$ ....(1)

Integrating equation (1) w.r. to x, between limits 0 and x, we get

$$\int_0^x (C_0 + C_1x + C_2x^2 + ...)dx = \int_0^x (1+x)^n\, dx$$

$$\Rightarrow \quad C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3} + ... = \frac{(1+x)^{n+1}-1}{n+1} \qquad ...(2)$$

Integrating equation (2), taking limits from – 1 to 0, we get

$$\int_{-1}^{0}\left[C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3} + ...\right]dx = \int_{-1}^{0}\frac{(1+x)^{n+1}-1}{n+1}dx \,. \qquad ...(3)$$

$$\Rightarrow \left[\frac{C_0x^2}{2} + \frac{C_1x^3}{2.3} + \frac{C_2x^4}{3.4} + ...\right]_{-1}^{0} = \left[\frac{(1+x)^{n+2}}{(n+1)(n+2)} - \frac{x}{n+1}\right]_{-1}^{0}$$

$$\Rightarrow \; -\left[\frac{C_0}{1.2} - \frac{C_1}{2.3} + \frac{C_2}{3.4} - ....\right]$$

$$= \frac{1}{(n+1)(n+2)} - \frac{1}{n+1} = -\frac{1}{n+2}$$

$$\therefore \quad \frac{C_0}{1.2} - \frac{C_1}{2.3} + \frac{C_2}{3.4} + ... = \frac{1}{n+2}$$

The correct option is (A)

89. We have,

$(1 + x)n = C_0 + C_1x + C_2x^2 + ...+ Cnxn$ ...(1)

Differentiating equation (1) w.r.t. x, we get

$n\,(1 + x)n^{-1} = C_1 + 2C_2x + 3C_3x^2 + ... + nCnxn^{-1}$...(2)

Differentiating equation (2) w. r. t. x, we get

$n(n – 1)\,(1 + x)n^{-2} = (1.2)C_2 + (2.3)\,C_3x +....$

$....+ n(n –1)Cnxn^{-2}$...(3)

Putting $x = 1$ in equation (3), we have

$(1.2)\,C_2 + (2.3)C_3 +.......+ (n – 1)n\;Cn = n(n– 1)2n –2.$

The correct option is (B)

90. Let $n = 2m,$ then $k = 3m$

$$\therefore \; \sum_{r=1}^{k}(-3)^{r-1}\;{}^{3n}C_{2r-1} = \sum_{r=1}^{3m}(-3)^{r-1}\;{}^{3m}C_{2r-1}$$

$= (–3)^0\;{}^6mC_1 + (–3)^1\;{}^6m\;C_3 + (–3)^2\;{}^6mC_5+....$

$= {}^6mC_1– (\sqrt{3})^2\;{}^{6m}C_3 + (\sqrt{3})^4\;{}^{6m}C_{5-.....}$

$$= \frac{1}{\sqrt{3}}\left[\sqrt{3}^{6m}C_{1-}\left(\sqrt{3}\right)^3\,{}^{6m}C_3 + \left(\sqrt{3}\right)^5\,{}^{6m}C_{5-....}\right]$$

$$= \frac{1}{\sqrt{3}}\text{ imaginary part of }(1+\sqrt{3}\,i)^{6m}$$

$$= \frac{1}{\sqrt{3}}\text{ imaginary part of }2^6m\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right)^{6m}$$

$$= \frac{1}{\sqrt{3}}\text{ imaginary part of }2^6m\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{6m}$$

$$= \frac{1}{\sqrt{3}} \times 2^{6m} \times 0 = \mathbf{0} \qquad (\because \sin 2m\,\pi = 0)$$

The correct option is (C)

91. The given series

$$S = (1+x)^{500}\left[1 + \frac{x}{1+x} + \left(\frac{x}{1+x}\right)^2 + ... + \left(\frac{x}{1+x}\right)^{500}\right]$$

$$= (1+x)^{500} \times \frac{1-\left(\dfrac{x}{1+x}\right)^{501}}{1-\dfrac{x}{1+x}} = (1+x)^{501} - x^{501}$$

Hence, the coefficient of x^{301} in $S = 501C301$.

The correct option is (A)

92. We have,

$(\sqrt{6})^{2n} = 6n = (1 + 5)n$

$= C_0 + C_1.5 + C_2.5^2 + C_3.5^3 +...+ Cn.5n$ [where $Cr = nCr$]

Thus, we have $\dfrac{(\sqrt{6})^{2n}}{5}$

$$= \frac{1}{5} + \left[C_1 + C_2.5 + C_3.5^2 + ... + C_n.5^{n-1}\right] = \frac{1}{5} + \text{integer}$$

whose fractional part is $\dfrac{1}{5}$.

The correct option is (B)

93. We have,

$(x + C_0)\,(x –3C_1)\,(x + 5C_2)$..... up to $(n + 1)$ terms

$= x\,n^{+1} + [C_0 –3C_1 + 5C_2–.....(n + 1)\text{ terms}]xn+....$

Thus, coefficient of $xn = C_0 – 3C_1 + 5C_2–....(n + 1)$ terms

$$= \sum_{r=0}^{n} C_r(-1)^r(2r+1)$$

We have,

$$(1 - x^2)n = \sum_{r=0}^{n} C_r(-1)^r x^{2r}$$

i.e., $x\,(1 – x^2)n = \displaystyle\sum_{r=0}^{n} C_r(-1)^r x^{2r+1}$ [multiplying by x]

i.e., $(1 – x^2)n– 2nx^2\,(1 –x^2)n^{-1} = \displaystyle\sum_{r=0}^{n} C_r(-1)^r(2r+1)x^{2r}$

[differentiating w.r.t.x]

Putting $x = 1$, we have,

$$\sum_{r=0}^{n} C_r(-1)^r(2r+1) = 0.$$

The correct option is (A)

94. $tr_{+1} = {}^{100}\,Cr\,\left(\sqrt[8]{5}\right)^{100-r}\,\left(\sqrt[6]{2}\right)^r$. As 2 and 5 are coprime, tr_{+1} will be rational if $100– r$ is a multiple of 8 and r is a multiple of 6. Also, $0 \le r \le 100$

$\therefore\; r = 0, 6, 12,....96$ ...(1)

$\Rightarrow \quad 100 – r \;\; = 4, 10, 16,.....,100$

But $100 – r$ is to be a multiple of 8, so

$100 – r = 0, 8\; 16, 24,..., 96$ ...(2)

The common terms in (1) and (2) are 16, 40, 64 and 88

$\therefore \quad r = 84, 60, 36, 12$ give rational terms.

$\therefore$ The number of irrational terms = 101 – 4 = 97

The correct option is (B)

95. $1 + 99n = 1 + (100 – 1)n = 1+ \{nC_0100n –nC_1.100n^{-1} +....– nCn\}$ because n is odd

$= 100\{nC_0.100n^{-1} - n\,C_1.100n^{-2} + ... - n\,Cn_{-2}.100 + nCn_{-1}\}$
$= 100 \times$ integer whose unit's place is different from 0.
[$\because\ nCn_{-1} = n$, has odd digit at unit's place]
$\therefore$ There are two zeros at the end of the sum $99n + 1$.
The correct option is (A)

96. Let $S = \sum_{r=0}^{n} \frac{1}{(2r)!(2n-2r)!}$

$= \frac{1}{(2n)!}\sum_{r=0}^{n} \frac{(2n)!}{(2r)!(2n-2r)!} = \frac{1}{(2n)!}\sum_{r=0}^{n} {}^{2n}C_{2r}$

$= \frac{1}{(2n)!}\left({}^{2n}C_0 + {}^{2n}C_2 + {}^{2n}C_4 + ... + {}^{2n}C_{2n}\right)$
Now,
$(1+1)^2n = {}^2nC_0 + {}^2nC_1 + {}^2nC_2 + ...+ {}^2nC_2n$
and, $(1-1)^2n = {}^2nC_0 - {}^2nC_1 + {}^2nC_2 - .....+ {}^2nC_2n$
On adding, we get
$2^2n = 2({}^2nC_0 + {}^2nC_2 + {}^2nC_4 + ...+ {}^2nC_2n)$
$\Rightarrow\ 2^2n^{-1} = {}^2nC_0 + {}^2nC_2 + ...+ {}^2nC_2n$

$\therefore S = \frac{2^{2n-1}}{(2n)!}$.
The correct option is (B)

97. $(x + {}^2n^{+1}C_0)(x + {}^2n^{+1}C_1)(x + {}^2n^{+1}C_2)....(x + {}^2n^{+1}Cn)$
$= xn^{+1} + xn\,({}^2n^{+1}C_0 + {}^2n^{+1}C_1 + {}^2n^{+1}C_2 + ...$
$+ {}^2n^{+1}Cn) + ...$
$\therefore$ Coeffcient of xn (say)
$S = 2n^{+1}C_0 + {}^2n^{+1}C_1 + {}^2n^{+1}C_2 + ... + {}^2n^{+1}Cn$ ...(1)
$\Rightarrow S = {}^2n^{+1}C_2n_{+1} + {}^2n^{+1}C_2n + {}^2n^{+1}C_2n_{-1} + ...\ {}^2n^{+1}Cn_{+1}$
($\because\ nCr = nCn\_r$) ...(2)
On adding (1) and (2), we get
$2S = 2^2n^{+1}\quad \therefore\ S = 22n$
The correct option is (B)

98. $32 = 2^5 \Rightarrow (32)^{32} = (2^5)^{32} = 2^{160} = (3-1)^{160}$
$= 3m + 1, m \in N$
$\therefore (32)^{3232} = (32)^3m^{+1} = 2^{5(3}m^{+1)}$
$= 2^{3(5}m^{+1)}\,2^2 = 4.8^5m^{+1}$
$= 4\,(7+1)^5\,m^{+1} = 4(7n+1), n \in N = 28\,n + 4$
$\therefore$ When 7 divides $(32)^{3232}$, remainder = 4
The correct option is (B)

More than One Option Correct Type

99. SInce T_4 is the numerically greatest term in the expansion of

$2^{10}\left(1+\frac{3x}{16}\right)^{10}$

$\therefore\quad \left|\frac{T_4}{T_3}\right| \geq 1$ and $\left|\frac{T_4}{T_5}\right| \geq 1$

or, $\left|\frac{T_4}{T_3}\right| \geq 1$ and $\left|\frac{T_5}{T_4}\right| \leq 1$

Now, $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r}\cdot x$

Taking $r = 3$ and $r = 4$ and replacing x by $\frac{3x}{16}$ and n by 10, in the above two, we get

$\left|\frac{11-3}{3}\cdot\frac{3x}{16}\right| \geq 1$ and $\left|\frac{11-4}{4}\cdot\frac{3x}{16}\right| \leq 1$

or, $|x| \geq 2$ and $|x| \leq \frac{64}{21}$
If x is positive, then $|x| = x$

$\therefore x \geq 2$ and $x \leq \frac{64}{21}$

$\therefore 2 \leq x \leq \frac{64}{21}$ ...(1)
If x is negative, then $|x| = -x$

$\therefore -x \geq 2$ and $-x \leq \frac{64}{21}$

or, $x \leq -2$ and $x \geq -\frac{64}{21}$

$\therefore -\frac{64}{21} \leq \boldsymbol{x} \leq -2$ ...(2)
(1) and (2) gives the required range.
The correct option is (B, C)

100. Let Tr, Tr_{+1}, Tr_{+2} be in G.P.

$\Rightarrow \frac{T_r}{T_{r+1}}, 1, \frac{T_{r+2}}{T_{r+1}}$ are in G.P.

$\Rightarrow \frac{r}{n-r+1}, 1, \frac{n-r}{r+1}$ are in G.P.

$\Rightarrow 1^2 = \frac{r(n-r)}{(n-r+1)(r+1)}$
$\Rightarrow n\,(r+1) - (r^2 - 1) = nr - r^2$
$\Rightarrow n + 1 = 0$
$\Rightarrow n = -1$, which is not possible.
Again, if Tr, Tr_{+1}, Tr_{+2} are in H.P.

$\Rightarrow \frac{1}{T_r}, \frac{1}{T_{r+1}}, \frac{1}{T_{r+2}}$ are in A.P.

$\Rightarrow \frac{T_{r+1}}{T_r}, 1, \frac{T_{r+1}}{T_{r+2}}$ are in A.P.

$\Rightarrow 2 = \frac{n-r+1}{r} + \frac{r+1}{n-r}$

$\Rightarrow 2r(n-r) = (n-r)^2 + (n-r) + r^2 + r$

$\Rightarrow 2rn - 2r^2 = n^2 - 2nr + r^2 + n - r + r^2 + r$

$\Rightarrow n^2 + 4r^2 - 4nr + n = 0 \Rightarrow (n-2r)^2 + n = 0$

This is not possible as both $(n - 2r)^2$ and n are positive.

The correct option is (A, B)

101. Given, mC_1, mC_2 and mC_3 are the first, third and fifth terms of an A.P., which will also be in A.P. of common difference $2d$.

$\therefore 2 \cdot mC_2 = mC_1 + mC_3$

$\Rightarrow m(m-1) = m + \dfrac{m(m-1)(m-2)}{1\cdot 2\cdot 3}$

$\Rightarrow 6m - 6 = 6 + m^2 - 3m + 2$ [$\because m \neq 0$]

$\Rightarrow m^2 - 9m + 14 = 0$

$\Rightarrow (m-2)(m-7) = 0$

Since 6th term is 21, $m = 2$ is ruled out,

$\therefore$ we have $m = 7$ and

$21 = {}^7C_5 \left[\sqrt{2^{\log(10-3^x)}}\right]^{7-5} \times \left[\sqrt[5]{2^{(x-2)\log 3}}\right]^5$ (given)

$= \dfrac{7\cdot 6}{1\cdot 2}\, 2^{\log(10-3^x)} \cdot 2^{(x-2)\log 3}$

$\Rightarrow 21 = 21\cdot 2^{\log(10-3^x)+\log 3^{x-2}}$

$\therefore 2^{\log[(10-3^x)3^{x-2}} = 1$

$\Rightarrow \log[(10-3x)\cdot 3x^{-2}] = 0$ [$\because 2^0 = 1$]

$\Rightarrow (10-3x)\cdot 3x^{-2} = 1$ ($\because \log 1 = 0$)

$\Rightarrow 3^2x^{-2} - 10\cdot 3x^{-2} + 1 = 0$

$\Rightarrow 3^2x - 10\cdot 3x + 9 = 0 \Rightarrow (3x-1)(3x-9) = 0$

$\therefore 3x - 1 = 0$ which gives $x = 0$

or, $3x = 9 = 3^2$ which gives $x = 2$.

Hence, $x = 0$ or 2.

The correct option is (B, C)

7. The given series can be written as

$S = \sum_{r=0}^{n} {}^nC_r \cdot {}^{2n-2r}C_m (-1)^r$

$= \sum_{r=0}^{n} {}^nC_r (-1)^r \times$ coefficient of xm in $(1+x)^2n^{-2}r$

= coefficient of xm in $\sum_{r=0}^{n} {}^nC_r(-1)^r[(1+x)^2]^{n-r}$

= coefficient of xm in $[(1+x)^2 - 1]n$

= coefficient of xm in $(x^2 + 2x)n$

= coefficient of xm in $xn\,(x+2)n$

= coefficient of xm^-n in $(x+2)n$

$= nCm\_n2n^{-(m^-n)} = \begin{pmatrix} n \\ m-n \end{pmatrix} 2^{2n-m}$ if $m \geq n$ and 0 if $m < n$.

The correct option is (A, B)

Passage Based Questions

101. We have $Tr = {}^{10}Cr_{-1}xr^{-1}$ and $Tr_{+1} = {}^{10}Crxr$.

$\therefore \dfrac{T_{r+1}}{T_r} = \dfrac{{}^{10}C_r x^r}{{}^{10}C_{r-1^{xr-1}}}$

$= \dfrac{{}^{10}C_r}{{}^{10}C_{r-1}}x = \dfrac{10!}{(10-r)!r!} \times \dfrac{(10-r+1)!(r-1)!}{10!}x$

$\Rightarrow \dfrac{T_{r+1}}{T_r} = \dfrac{11-r}{r}.x$

$\Rightarrow \dfrac{T_{r+1}}{T_r} = \left(\dfrac{11-r}{r}\right) \times \dfrac{2}{3}$ [$\because x = 2/3$]

Now,

$\dfrac{T_{r+1}}{T_r} > 1 \Rightarrow \left(\dfrac{11-r}{r}\right) \times \dfrac{2}{3} > 1 \Rightarrow 22 > 5r \Rightarrow r < 4\dfrac{2}{5}$

$\therefore$ (4 + 1)th, i.e., 5th term is the greatest term.

Putting $r = 4$ in Tr_{+1}, we get

$T_5 = {}^{10}C_5 x^4 \Rightarrow T_5 = {}^{10}C_5 \left(\dfrac{2}{3}\right)^4$ [$\because x = 2/3$]

$\Rightarrow T_5 = 210\left(\dfrac{2}{3}\right)^4$

The correct option is (A)

102. We have, $Tr_{+1} = {}^{15}Cr\, 3^{15-}r\,(-5x)r$

and, $Tr = {}^{15}Cr_{-1}\, 3^{15-}r^{+1}\,(-5x)r-{}^1$

$\Rightarrow \dfrac{T_{r+1}}{T_r} = \dfrac{{}^{15}C_r 3^{15-r}(-5x)^r}{{}^{15}C_{r-1}3^{16-r}(-5x)^{r-1}} = \dfrac{15-r+1}{r}\left(\dfrac{-5x}{3}\right)$

$\Rightarrow \dfrac{T_{r+1}}{T_r} = \dfrac{16-r}{r} \times \left(-\dfrac{5}{3} \times \dfrac{1}{5}\right)$, when $x = \dfrac{1}{5}$

$\Rightarrow \dfrac{T_{r+1}}{T_r} = \dfrac{16-r}{r} \times \dfrac{1}{3}$, numerically

[Neglecting minus sign]

Now, $\dfrac{T_{r+1}}{T_r} > 1$ (numerically)

$\Rightarrow \dfrac{16-r}{r} \times \dfrac{1}{3} > 1 \Rightarrow 16 > 4r \Rightarrow r < 4$

Since 4 is an integer, therefore 4th and 5th terms are numerically greatest terms.

The correct option is (A, B)

103. Let $(r + 1)$ th term be the greatest term. Then

$Tr_{+1} = \sqrt{3}\ {}^{20}C_r \left(\dfrac{1}{\sqrt{3}}\right)^r$ and $Tr = \sqrt{3}\ {}^{20}C_{r-1}\left(\dfrac{1}{\sqrt{3}}\right)^{r-1}$

Now, $\dfrac{T_{r+1}}{T_r} = \dfrac{20-r+1}{r}\left(\dfrac{1}{\sqrt{3}}\right)$

$\therefore Tr_{+1} \geq Tr \Rightarrow 20 - r + 1 \geq \sqrt{3}$ r

$\Rightarrow \quad 21 \geq r(\sqrt{3}+1) \Rightarrow r \leq \dfrac{21}{\sqrt{3}+1}$

$\Rightarrow \quad r \leq 7.686 \Rightarrow r = 7.$

Hence, the greatest term is $T_8 = \sqrt{3}^{20}C_7\left(\dfrac{1}{\sqrt{3}}\right)^7$

$= \dfrac{25840}{9}.$

The correct option is (A)

104. Since T_4 is the numerically greatest term

$\therefore \left|\dfrac{T_4}{T_3}\right| \geq 1$ and $\left|\dfrac{T_4}{T_5}\right| \geq 1 \Rightarrow \left|\dfrac{T_4}{T_3}\right| \geq 1$ and $\left|\dfrac{T_5}{T_4}\right| \leq 1$

Now, $T_3 = {}^{10}C_2(2^{10-2})\left(\dfrac{3}{8}x\right)^2$,

$T_4 = {}^{10}C_3(2^{10-3})\left(\dfrac{3x}{8}\right)^3$

and, $T_5 = {}^{10}C_4(2^{10-4})\left(\dfrac{3x}{8}\right)^4$

$$\therefore \quad \frac{T_4}{T_3} = \frac{{}^{10}C_3 \times 2^7 \times \left(\frac{3x}{8}\right)^3}{{}^{10}C_2 \times 2^8 \times \left(\frac{3x}{8}\right)^2} = \frac{x}{2}$$

$$\text{and,} \quad \frac{T_5}{T_4} = \frac{{}^{10}C_4 \times 2^6 \times \left(\frac{3x}{8}\right)^4}{{}^{10}C_3 \times 2^7 \times \left(\frac{3x}{8}\right)^3} = \frac{21x}{64}$$

$\therefore \left|\dfrac{T_4}{T_3}\right| \geq 1$ and $\left|\dfrac{T_5}{T_4}\right| \leq 1 \Rightarrow \left|\dfrac{x}{2}\right| \geq 1$ and $\left|\dfrac{21x}{64}\right| \leq 1$

$\Rightarrow \quad |x| \geq 2$ and $|x| \leq \dfrac{64}{21}$

$\Rightarrow \quad x \in (-\infty, -2] \cup [2, \infty)$ and $x \in \left(-\dfrac{64}{21}, \dfrac{64}{21}\right)$

$\Rightarrow \quad x \in \left(-\dfrac{64}{21}, -2\right) \cup \left(2, \dfrac{64}{21}\right)$

The correct option is (C)

105. (a). Let $(5+2\sqrt{6})n = I + f$ where I and f are the integral and the fractional parts of $(5+2\sqrt{6})n$ respectively.

Let $(5-2\sqrt{6})n = g$ where g is a fraction.

Since $0 < 5-2\sqrt{6} < 1$, therefore $0 < (5-2\sqrt{6})n < 1$ for every positive integer n.

We have,

$I+f+g \quad = (5+2\sqrt{6})n + (5-2\sqrt{6})n$

$= 2[C_0 5^n + C_2(2\sqrt{6})^2.5n^{-2} + ...]$

i.e., $I+f+g \quad = 2k, k \in I^+$ ...(1)

$\therefore f+g = 2k-1 =$ an integer ...(2)

Since $0 < f, g < 1$, therefore we have

$0 < f+g < 2$ ....(3)

Thus, using (2) and (3), we have

$f+g = 1$

[since the only integral value in (0,2) is 1]

i.e., $g = 1-f$ ....(4)

Therefore, we have,

$R(1-f) = Rg = (5+2\sqrt{6})n.(5-2\sqrt{6})n$

$= \left(5^2 - \left(2\sqrt{6}\right)^2\right)^n = 1.$

The correct option is (A)

106. Let $(3+\sqrt{5})^2 n = I + f$ where I and f are the integral and the fractional parts of $(3+\sqrt{5})^2 n$ respectively.

Let $(3+\sqrt{5})^2 n = g$, where g is a fraction.

Since $0 < 3-\sqrt{5} < 1$, therefore

$0 < (3-\sqrt{5})^2 n < 1$ for every positive integer n.

We have,

$I+f+g = (3+\sqrt{5})^2 n + (3-\sqrt{5})^2 n$

$= (14+6\sqrt{5})n + (14-6\sqrt{5})n$

$= 2n\,[(7+3\sqrt{5})n + (7-3\sqrt{5})n\,]$

$= 2n.\,2[C_0.\,7n + C_2\,(7)n^{-2}(3\sqrt{5})^2 + ...]$

i.e., $I+f+g \quad = (2n^{+1})\,k, k \in I^+$ ...(1)

i.e., $f+g = (2n^{+1})\,k - I =$ an integer ...(2)

Since $0 < f, g < 1$, therefore we have

$0 < f+g < 2$ ....(3)

Thus, using (2) and (3), we have

$f+g = 1$ ...(4)

[since the only integral value in (0,2) is1]

Putting (4) in equation (1), we have

$I+1 \quad = (2n^{+1})k$

$\therefore$ $[(3+\sqrt{5})^2 n\,] + 1$ is divisible by $2n+1$.

The correct option is (C)

107. Let $g = (7-4\sqrt{3})n$. Then, $0 < g < 1$ as $0 < 7-4\sqrt{3} < 1$

Now, $I+f+g = (7+4\sqrt{3})n + (7-4\sqrt{3})n$

$= 2\,(nC_0\,7n + nC_2\;7n^{-2}\,(4\sqrt{3})^2 + ...)$

$=$ an integer

$\Rightarrow \quad f+g = I \quad \Rightarrow g = I-f$

Thus, $(I+f)\,(I-f) = (I+f)\,g = (7+4\sqrt{3})n\,(7-4\sqrt{3})n =$ **1.**

The correct option is (B)

Match the Column Type

108. I We have, $7^{103} = 7(49)^{51} = 7(50-1)^{51}$
$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots - 1)$
$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots) - 7 + 18 - 18$
$= 7(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots) - 25 + 18$
$= k + 18$ (say) $\because$ k is divisible by 25,
$\therefore$ remainder is 18.
The correct option is (C)

109. II $(r+1)$th term in the given expansion is given by
$t_{r+1} = {}^{10}C_r\, 2^{\frac{10-r}{2}}\, 3^{\frac{r}{5}}$, where $r = 0, 1, 2, \ldots, 10$
For rational terms
r = a multiple of 5 = 0, 5, 10 ...(1)
$10 - r$ = a multiple of 2 = 0, 2, 4, 6, 8, 10 ...(2)
From (1) and (2), possible values of r are : 0 and 10
$\therefore$ sum of rational terms
$= t_1 + t_{11} = {}^{10}C_0 (\sqrt{2})^{10} (3^{1/5})^0 + {}^{10}C_{10} (\sqrt{2})^0 (3^{1/5})^{10}$
$= 2^5 + 3^2 = 32 + 9 = 41.$
The correct option is (D)

110. III We have, $2^4 n = (2^4)n = (16)n = (1+15)n$
$\therefore 2^4 n = 1 + nC_1 \cdot 15 + nC_2\, 15^2 + nC_3\, 15^3 + \ldots$
$\Rightarrow 2^4 n - 1 - 15n = 15^2 [nC_2 + nC_3 \cdot 15 + \ldots]$
$= 225\,k$, where k is an integer.
Hence, $2^4 n - 15n - 1$ is divisible by 225.
The correct option is (B)

111. IV We have,
$5^{99} = 5^3 \cdot 5^{96} = (125)(625)^{24}$
$= [13 \times 9 + 8](1 + 48 \times 13)^{24}$
$= (13 \times 9 + 8)[1 + {}^{24}C_1 \times (48 \times 13)$
$+ {}^{24}C_2 (48 \times 13)^2 + \ldots + (48 \times 13)^{24}]$
= 8 + terms containing powers of 13.
Hence, remainder = 8.
The correct option is (A)

112. I $Tr_{+1} = {}^{1028}Cr\,(5^{1/2})^{1028-r}.\,(7^{1/8})r$
$= {}^{1028}Cr\; 5^{514}.\, 5^{-r/2}.\, 7^{r/8}$
Tr_{+1} will be integral if both $\frac{r}{2}$ and $\frac{r}{8}$ are integers where r lies between 0 to 1028. It will be so if r is a multiple of 8 between 0 to 1028,
i.e., 0, 8, 16, 32,..., 1024
Now, $Tn = 1024$, $a = 0$, $d = 8$
$\therefore 1024 = 0 + (n-1)8 \Rightarrow 1024 = 8(n-1)$
$\Rightarrow n - 1 = \frac{1024}{8} = 128 \Rightarrow n = 129$
The correct option is (C)

113. II $\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}$
$= \frac{(x^{1/3})^3 + 1^3 x}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x^{1/2}(x^{1/2} - 1)}$
$= \frac{(x^{1/3}+1)(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} - \frac{x^{1/2}+1}{x^{1/2}}$
$= x^{1/3} + 1 - 1 - x^{-1/2} = x^{1/3} - x^{-1/2}$
$\Rightarrow \left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10} = (x^{1/3} - x^{-1/2})^{10}$
Tr_{+1} for $(x^{1/3} - x^{-1/2})^{10}$ is ${}^{10}Cr(x^{1/3})^{10-}r\,(-1)r\,(x^{-1/2})r$
For term independent of x,
$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0 \Rightarrow r = 4$
Hence, required coefficient $= {}^{10}C_4(-1)^4 = 210.$
The correct option is (A)

114. III Coefficient of $x^{n_1} y^{n_2} z^{n_3}$ in the expansion of $(x+y+z)^{10}$ is
$\frac{(n_1+n_2+n_3)!}{n_1!\, n_2!\, n_3!} = \frac{10!}{2!\,3!\,5!} = 2520$
The correct option is (B)

115. IV $17 = 2 \pmod 5$
$(17)^5 = (2)^5 \pmod 5 = 2 \pmod 5$
$\Rightarrow (17^5)^6 = (2)^6 \pmod 5 \Rightarrow (17)^{30} = 4 \pmod 5$
The correct option is (D)

Assertion-Reason Type

116. We know that $\frac{C_k}{C_{k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n-k+1}{k}$
$\therefore \sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2 = \sum_{k=1}^{n} k^3 \left(\frac{n-k+1}{k}\right)^2$
$= \sum_{k=1}^{n} k(n-k+1)^2$
Put $n - k + 1 = p \Rightarrow k = n - p + 1.$
When $k = 1$, $p = n$ and when $k = n$, $p = 1$.
$\therefore$ Series $= \sum_{p=1}^{+n} (n-p+1)p^2 = \sum_{p=1}^{n} (np^2 - p^3 + p^2)$
$= \sum_{p=1}^{n} (n+1)p^2 - \sum_{p=1}^{n} p^3$
$= (n+1)[1^2 + 2^2 + 3^2 + \ldots + n^2]$
$- [1^3 + 2^3 + 3^3 + \ldots + n^3]$

$$= \frac{(n+1)n(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4}$$

$$= \frac{n(n+1)^2}{2}\left[\frac{2n+1}{3} - \frac{n}{2}\right] = \frac{n(n+1)^2(n+2)}{12}$$

The correct option is (A)

117. Given: $Pn = nC_0\, nC_1\, nC_2 \ldots nCn$

Now, $\frac{P_{n+1}}{P_n} = \frac{{}^{n+1}C_0\, {}^{n+1}C_1\, {}^{n+1}C_2 \ldots {}^{n-1}C_n}{{}^nC_0\, {}^nC_1\, {}^nC_2 \ldots {}^nC_n}$

$$\Rightarrow \frac{P_{n+1}}{P_n} = {}^{n+1}C_0\left(\frac{{}^{n+1}C_1}{{}^nC_0}\right)\left(\frac{{}^{n+1}C_2}{{}^nC_1}\right)$$

$$\ldots\left(\frac{{}^{n+1}C_n}{{}^nC_n}\right){}^{n+1}C_{n+1}$$

Since, $n^{+1}Cr_{+1} = \frac{n+1}{r+1}\, {}^nC_r$

$$\Rightarrow \frac{P_{n+1}}{P_n} = 1\left(\frac{n+1}{1}\right)\left(\frac{n+1}{2}\right)\ldots\left(\frac{n+1}{n}\right)1$$

$$\therefore \frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n!}$$

The correct option is (A)

118. The expansion is a G.P. with $(n + 1)$ terms of the form

$$an + an^{-1}b + an^{-2}b^2 + \ldots + bn = \frac{a^{n+1} - b^{n+1}}{a - b}$$

$$= \frac{(2x+3)^{n+1} - (2x-5)^{n+1}}{8},$$

where $a = 2x + 3$ and $b = 2x - 5$

$\therefore$ Coefficient of $xn = \frac{1}{8}\,[(n + 1).\, 2n(3) - (n + 1).\, 2n(-5)] = (n + 1).\, 2n$

The correct option is (A)

119. If n is odd, then numerically greatest coefficient in the expansion of $(1 - x)n$ is $\frac{{}^nC_{n-1}}{2}$ or $\frac{{}^nC_{n+1}}{2}$.

Therefore, in $(1 - x)^{21}$, the numerically greatest coefficient is ${}^{21}C_{10}$ or ${}^{21}C_{11}$. So, the numerically greatest term

$= {}^{21}C_{11}x^{11}$ or ${}^{21}C_{10}x^{10}$ and

$|{}^{21}C_{10}x^{10}| > |{}^{21}C_9 x^9|$

$$\Rightarrow \frac{21!}{10!11!} > \frac{21!}{9!12!}x \text{ and}$$

$$\frac{21!}{11!10!}x > \frac{21!}{9!12!} \quad (\because x > 0)$$

$$\Rightarrow x < \frac{6}{5} \text{ and } x > \frac{5}{6} \Rightarrow x \in \left(\frac{5}{6}, \frac{6}{5}\right)$$

The correct option is (A)

120. Since n is even, therefore the greatest coefficient is ${}^nC_{n/2}$.

$\therefore$ The greatest term $= {}^nC_{n/2}\, xn^{/2}$

$$\therefore \quad {}^nC_{n/2}\, xn^{/2} > {}^nC_{\frac{n}{2}-1}\, x^{\frac{n}{2}-1}$$

and, ${}^nC_{\frac{n}{2}}\, x^{\frac{n}{2}} > {}^nC_{\frac{n}{2}+1}\, x^{\frac{n}{2}+1}$

$$\Rightarrow \frac{n - \frac{n}{2} + 1}{\frac{n}{2}}\, x > 1 \text{ and } \frac{n - \left(\frac{n}{2}+1\right) + 1}{\frac{n}{2}+1}\, x < 1$$

$$\Rightarrow x > \frac{\frac{n}{2}}{\frac{n}{2}+1} \text{ and } x < \frac{\frac{n}{2}+1}{\frac{n}{2}}$$

$$\Rightarrow x > \frac{n}{n+2} \text{ and } x < \frac{n+2}{n} \;\therefore\; \frac{n}{n+2} < x < \frac{n+2}{n}.$$

The correct option is (A)

121. Given series is

$$S = 1 + \frac{2}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2} + \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3} + \ldots\infty$$

and we know that

$$(1 + x)n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\infty$$

Comparing these two, we get

$$nx = \frac{2}{3}\cdot\frac{1}{2} \qquad \ldots(1)$$

$$\text{and, } \frac{n(n-1)}{2\cdot 1}x^2 = \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2} \qquad \ldots(2)$$

Now, divide (2) by square of (1), we get

$$\Rightarrow \frac{\frac{n(n-1)}{2\cdot 1}x^2}{n^2x^2} = \frac{\frac{2}{3}\times\frac{5}{6}\times\frac{1}{4}}{\frac{2}{3}\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}}$$

$$\Rightarrow \frac{n-1}{2n} = \frac{5}{4} \Rightarrow \frac{n-1}{n} = \frac{5}{2}$$

$\Rightarrow 5n = 2n - 2$

$\Rightarrow 3n = -2 \Rightarrow n = -\frac{2}{3}$

putting value of n in (1), we get

$$-\frac{2}{3}x = \frac{2}{3} \times \frac{1}{2}$$

$$\Rightarrow x = -\frac{2}{3} \times \frac{1}{2} \times \frac{3}{2} = -\frac{1}{2}$$

$\therefore$ Sum of given series

$$= \left(1 - \frac{1}{2}\right)^{2/3} = \left(\frac{1}{2}\right)^{-2/3} = (2)^{2/3} = (4)^{1/3}.$$

The correct option is (D)

122 We have, $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \ldots$

$$= \frac{n}{2} + \frac{n(n-1)(n-2)}{3!\cdot 4} + \frac{n(n-1)(n-2)(n-3)}{5!\cdot 6} + \ldots$$

$= \frac{1}{n+1}\left[\frac{(n+1)(n)}{2!} + \frac{(n+1)(n)(n-1)(n-2)}{4!} + \frac{(n+1)(n)(n-1)(n-2)(n-3)}{6!} + ...\right]$

$= \frac{1}{n+1} [n^{+1}C_2 + n^{+1}C_4 + ... + ...]$

$= \frac{1}{n+1} [2n - 1] = \frac{2^n - 1}{n+1}$

$[\because nC_0 + nC_2 + nC_4 + ... = 2n^{-1},$

$\therefore nC_2 + nC_4 + ... = 2n^{-1} - 1].$

The correct option is (C)

123. Using $nCk = \frac{n}{k} . n^{-1}Ck_{-1}$, for $0 \le k \le 11$,

$\frac{^{11}C_k}{k+1} = \frac{^{12}C_{k+1}}{12}$

$\therefore$ the given expression $= \frac{1}{12}\sum_{k=0}^{11} {^{12}C_{k+1}}$

$= \frac{1}{12}\left[\sum_{k=0}^{12} {^{12}C_k} - {^{12}C_0}\right] = \frac{1}{12}(2^{12} - 1)$

The correct option is (A)

124. Let d be the common difference of the A.P., then we have,

$\sum_{k=0}^{n} {^nC_k S_k} = \sum_{k=0}^{n} {^nC_k}.\frac{k}{2}[2a_1 + (k-1)d]$

$= \left(a_1 - \frac{d}{2}\right)\sum_{k=0}^{n} k.{^nC_k} + \frac{d}{2}\sum_{k=0}^{n} k^2.{^nC_k}$

$= \left(a_1 - \frac{d}{2}\right)n2^{n-1} + \frac{d}{2}[n2^{n-1} + n(n-1)2^{n-2}]$

$= a_1n2n^{-1} + dn(n-1)2n^{-3}$

$= a_1n2n^{-1} + n(an - a_1)2n^{-3}$ $[\because an - a_1 = (n-1)d]$

$= n2n^{-3}[4a_1 + an - a_1] = n2n^{-3}(2a_1 + a_1 + an)$

$= 2n^{-2}\left[na_1 + \frac{n(a_1 + a_n)}{2}\right] = 2n^{-2}(na_1 + Sn).$

The correct option is (A)

Previous Year's Questions

121. $\because (1 + 2x + 3x^2 + ...)^{-3/2} = [(1-x)^{-2}]^{-3/2}$

$= (1-x)^3$

Now, coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{-3/2}$ = coefficient of x^5 in $(1 - x)^3 = 0$

The correct option is (D)

122. $\because$ $(1 + x + x^2 + x^3 + ...)^2 = [(1-x)^{-1}]^2$

$= (1-x)^{-2}$

Now, coefficient of x^n in $(1 + x + x^2 + ...)^2$ = coefficient of x^n in $(1 - x)^3 = 0$

$= {^{n+2-1}C_{2-1}} = {^{n+1}C_1}$

$= n + 1$

The correct option is (D)

123. General term $= {^{256}C_r}(\sqrt{3})^{256-r}[(5)^{1/8}]^r$

For integral terms, r should be 8k And then k assumes values from 0 to 32.Hence, (B) is the correct answer.

The correct option is (B)

124. Coefficient of Middle term in $(1 + \alpha x)^4 = t_3 = {^4C_2}\cdot \alpha^2$ Coefficient of Middle term in $(1 - \alpha x)^6 = t_4 = {^6C_3}(-\alpha)^3$

Given that ${^4C_2}\alpha^2 = -{^6C_3}.\alpha^3$

$\Rightarrow -6 = 20\alpha$

$\Rightarrow \alpha = \frac{-3}{10}$

The correct option is (C)

125. Coefficient of x^n in the expansion of $(1 + x)(1 - x)^n = (1 + x)({^nC_0} - {^nC_1}x + \ldots\ldots + (-1)^{n-1}\,{^nC_{n-1}}\,x^{n-1} + (-1)^n\,{^nC_n}\,x^n)$ is $(-1)^n\,{^nC_n} + (-1)^{n-1}\,{^nC_{n-1}} = (-1)^n(1 - n)$.

The correct option is (B)

126. Given that ${^mC_{r-1}}, {^mC_r}, {^mC_{r+1}}$ are in A.P.

$\Rightarrow 2\,{^mC_r} = {^mC_{r-1}} + {^mC_{r+1}}$

$\Rightarrow 2 = \frac{^mC_{r-1}}{^mC_r} + \frac{^mC_{r+1}}{^mC_r}$

$= \frac{r}{m-r+1} + \frac{m-r}{r+1}$

$\Rightarrow m^2 - m(4r + 1) + 4r^2 - 2 = 0.$

The correct option is (C)

127. The expression ${^{50}C_4} + \sum_{r-1}^{6} {^{56-r}C_3}$

$= {^{50}C_4} + \left[{^{55}C_3} + {^{54}C_3} + {^{53}C_3} + {^{52}C_3} + {^{51}C_3} + {^{50}C_3}\right]$

$= ({^{50}C_4} + {^{50}C_3}) + {^{51}C_3} + {^{52}C_3} + {^{53}C_3} + {^{54}C_3} + {^{55}C_3}$

$= \left({^{51}C_4} + {^{51}C_3}\right) + {^{52}C_3} + {^{54}C_3} + {^{55}C_3}$

$= {^{55}C_4} + {^{55}C_3} = {^{56}C_4}.$

The correct option is (D)

128. T_{r+1} in the expansion ∵

$$\left[ax^2+\frac{1}{bx}\right]^{11} = {}^{11}C_r\left(ax^2\right)^{11-r}\left(\frac{1}{bx}\right)^r$$

$= {}^{11}C_r\,(a)^{11-r}\,(b)^{-r}\,(x)^{22-2r-r}$

$\Rightarrow 22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ coefficient of $x^7 = {}^{11}C_5\,(a)^6\,(b)^{-5}$ ……(1)

Also, T_{r+1} in the expansion

$$\left[ax-\frac{1}{bx^2}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$$

$= {}^{11}C_r\,a^{11-r}\,(-1)^r \times (b)^{-r}\,(x)^{-2r}\,(x)^{11-r}$

Now $11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

$\therefore$ coefficient of $x^{-7} = {}^{11}C_6\,a^5 \times 1 \times (b)^{-6}$

$\Rightarrow {}^{11}C_5\,(a)^6\,(b)^{-5} = {}^{11}C_6\,a^5\,(b)^{-6}$

$\Rightarrow ab = 1$

The correct option is (D)

129. $$(1-x)^{1/2}\left[1+\frac{3}{2}x+\frac{3}{2}\left(\frac{3}{2}-1\right)x^2-1-3\left(\frac{1}{2}x\right)-3(2)\ \frac{1}{2}\right.$$

$$\left.-3(2)\left(\frac{1}{2}x\right)^2\right]$$

$$=(1-x)^{1/2}\left[-\frac{3}{8}x^2\right]=-\frac{3}{8}x^2$$ (because the higher powers of x are neglected)

The correct option is (C)

130. We have

$(1 - ax)^{-1}\,(1 - bx)^{-1} = (1 + ax + a^2x^2 + \ldots\ldots)\,(1 + bx + b^2x^2 + \ldots.)$

$\therefore$ Coefficient of $x^n = b^n + ab^{n-1} + a^2b^{n-2} + \ldots. + a^{n-1}b + a^n = \dfrac{b^{n+1}-a^{n+1}}{b-a}$

$$\therefore a_n=\frac{b^{n+1}-a^{n+1}}{b-a}$$

The correct option is (D)

131. We have

$(1 - y)^m(1 + y)^n = [1 - {}^mC_1y + {}^mC_2y^2 - \ldots.][1 + {}^nC_1y + {}^nC_2y^2 + \ldots]$

$$=1+(n-m)+\left\{\frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn\right\}y^2+\ldots.$$

$$\therefore a_1=n-m=10 \text{ and } a_2=\frac{m^2+n^2-m-n-2mn}{2}=10$$

So, $n - m = 10$ and $(m - n)^2 - (m + n) = 20 \Rightarrow m + n = 80$

$\therefore m = 35, n = 45$

The correct option is (D)

132 Since the sum of 5th and 6th terms is zero, we have

${}^nC_4\,a^{n-4}(-b)^4 + {}^nC_5\,a^{n-5}\,(-b)^5 = 0 \Rightarrow \left(\dfrac{a}{b}\right)=\dfrac{n-5+1}{5}$

The correct option is (D)

133. We have

$(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1x + \ldots + {}^{20}C_{10}x^{10} + \ldots + {}^{20}C_{20}x^{20}$

Put x = 1,

$0 = {}^{20}C_0 - {}^{20}C_1 + \ldots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \ldots + {}^{20}C_{20}$

$0 = 2\,({}^{20}C_0 - {}^{20}C_1 + \ldots - {}^{20}C_9) + {}^{20}C_{10}$

$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + \ldots + {}^{20}C_{10} = \dfrac{1}{2}\ {}^{20}C_{10}.$

The correct option is (B)

134. $$1-q^n\geq\frac{9}{10}$$

$$\Rightarrow\left(\frac{3}{4}\right)^n\leq\frac{1}{10}$$

$$\Rightarrow n\geq-\log_{\frac{3}{4}}10$$

$$\Rightarrow n\geq\frac{1}{\log_{10}^{4}-\log_{10}^{3}}$$

The correct option is (A)

135. We can write $8^{2n} - (62)^{2n+1}$

$= (1 + 63)^n - (63 - 1)^{2n+1}$

$= (1 + 63)^n + (1 - 63)^{2n+1}$

$= (1 + {}^nc_1 63 + {}^nc_2(63)^2 + \ldots. + (63)^n) + (1 - {}^{(2n+1)}c_1\,63 + {}^{(2n+1)}c_2\,(63)^2 + \ldots + (-1)\,(63)^{(2n+1)})$

$= 2 + 63({}^nc_1 + {}^nc_2(63) + \ldots. + (63)^{n-1} - {}^{(2n+1)}c_1 + {}^{(2n+1)}c_2(63) + \ldots.\ (63)^{(2n)})$

$\therefore$ Reminder is 2

The correct option is (B)

136. $$\left[1-x-x^2(1-x)\right]^6=(1-x)^6(1-x^2)^6$$

$$=\begin{bmatrix}{}^6C_0-{}^6C_1x+{}^6C_2\times^2-{}^6C_3x^3+{}^6C_4x^4\\-{}^6C_5x^5+{}^6C_6x^6\end{bmatrix}$$

$$\times\left[{}^6C_0-{}^6C_1x^2+{}^6C_2x^4-{}^6C_3x^6+\ldots.\right]$$

Coefficient of

$$x^7={}^6C_1{}^6C_3-{}^6C_3{}^6C_2+{}^6C_5{}^6C_1$$

$$=120-300+36=-144$$

The correct option is (B)

137. $$(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}=\left[\left(\sqrt{3}+1\right)^2\right]^n$$

$$-\left[\left(\sqrt{3}-1\right)^2\right]^n=(4+2\sqrt{3})^n-(4-2\sqrt{3})^n$$

$$=2^n\left[\left(2+\sqrt{3}\right)^n-(2-\sqrt{3})^n\right]$$

$$= 2^n \begin{Bmatrix} \left[{}^nC_0 2^n + {}^nC_1 2^{n-1}\sqrt{3} + {}^nC_2 2^{n-2} 3 + ...\right] \\ -\left[{}^nC_0 2^{n-n} C_1 2^{n-1}\sqrt{3} + {}^nC_2 2^{n-2} 3 - ....\right] \end{Bmatrix}$$

$$= 2^{n+1}\left[{}^nC_1 2^{n-1}\sqrt{3} + {}^nC_3 2^{n-3} 3\sqrt{3} + ...\right] = 2^{n+1}\sqrt{3}$$

(some integer)

which is irrational

The correct option is (A)

138. $f'(x) = \frac{\alpha}{x} + 2\beta x + 1 \; 2\beta x^2 + x + \alpha = 0$ has roots -1 and 2

The correct option is (C)

139. $1(1-2x)^{18} + ax(1-2x)^{18} + bx^2(1-2x)^{18}$

Coefficient of

$x^3 : (-2)^{3\,18}C_3 + a(-2)^{2\,18}C_2 + b(-2)^{18}C_1 = 0$

$\frac{4\times(17\times16)}{(3\times2)} - 2a\cdot\frac{17}{2} + b = 0 \;....(i)$

Coefficient of

$x^4 : (-2)4^{18}C_4 + a(-2)^{3\,18}C_3 + b(-2)^{2\,18}C_2 = 0$

$(4\times20) - 2a\cdot\frac{16}{3} + b = 0 \;....(ii)$

From equation (i) and (ii), we get

$4\left(\frac{17\times8}{3} - 20\right) + 2a\left(\frac{16}{3} - \frac{17}{2}\right) = 0$

$4\left(\frac{17\times8-60}{3}\right) + \frac{2a(-19)}{6} = 0$

$a = \frac{4\times76\times6}{3\times2\times19}$

$\Rightarrow a = 16$

$\Rightarrow b = \frac{2\times16\times16}{3} - 80 = \frac{272}{3}$

The correct option is (D)

140. Set X contains elements of the form

$4^n - 3n - 1 = (1+3)^n - 3n - 1$

$= 3^n + {}^nC_{n-1}3^{n-1}.....{}^nC_2 3^2$

$= 9(3^{n-2} + {}^nC_{n-1}3^{n-1}... + {}^nC_2)$

Set X has natural numbers which are multiples of 9 (not all)

Set Y has all multiples of 9 $X \cup Y = Y$

The correct option is (D)

141. $t_{r+1} = {}^{50}C_r \cdot (1)^{50-r} \cdot \left(-2x^{1/2}\right)^r$

$= {}^{50}C_r \cdot 2^r \cdot x^{r/2}(-1)^r$

$\Rightarrow r =$ an even integer.

$\Rightarrow$ Sum of coefficient

$\sum_{r=0}^{25} {}^{50}C_{2r} \cdot 2^{2r} = \frac{1}{2}\left((1+2)^{50} + (1-2)^{50}\right) = \frac{1}{2}\left(3^{50}+1\right)$

The correct option is (D)