

Sequence and Series

Chapter Highlights

Sequence, Series, Progressions, Arithmetic progression (A.P.), Sum of n terms of an A.P., Properties of A.P., Arithmetic mean (A.M.), Geometric progression (G.P.), Geometric mean (G.M.), Some special sequences, Arithmetico-geometric progression (A.G.P.), Method for finding sum of a.G. Series

SEQUENCE

A succession of numbers a_1, a_2, \dots, a_n formed according to some definite rule is called a *sequence*.

A sequence is a function whose domain is the set N of natural numbers and range a subset of real numbers or complex numbers.

A sequence whose range is a subset of real numbers is called a **real sequence**. Since we shall be dealing with real sequences only, we shall use the term sequence to denote a real sequence.

Notation

The different terms of a sequence are usually denoted by a_1, a_2, a_3, \dots or by t_1, t_2, t_3, \dots . The subscript (always a natural number) denotes the position of the term in the sequence. The term at the n th place of a sequence, i.e., t_n is called the **general term of the sequence**.



NOTE

A sequence is said to be **finite or infinite according** as it has finite or infinite number of terms.

Illustrations

- 1, 4, 7, 10, ... 19. In this sequence each term is obtained by adding 3 to the previous term.
- 2, -4, 8, -16, ... In this sequence each term is obtained by multiplying the preceding term by -2.
- 2, 3, 5, 7, 11, 13, ... This is the sequence of prime numbers.

Note that sequence (1) is a finite sequence whereas others are infinite sequences.

SERIES

A series is obtained by adding or subtracting the terms of a sequence.

A series is **finite or infinite** according as the number of terms in the corresponding sequence is finite or infinite.

PROGRESSIONS

If the terms of a sequence follow certain pattern, then the sequence is called a progression. Following are the three special types of progressions:

1. Arithmetic Progression (A.P.)
2. Geometric Progression (G.P.)
3. Harmonic Progression (H.P.)

ARITHMETIC PROGRESSION (A.P.)

A sequence whose terms increase or decrease by a fixed number is called an arithmetic progression. The fixed number is called the **common difference** of the A.P.

In an A.P., the first term is usually denoted by a , the common difference by d and the n th term by t_n . Obviously

$$d = t_n - t_{n-1}$$

Thus, an A.P. can be written as

$$a, a + d, a + 2d, \dots, a + (n-1)d, \dots$$

For example,

1. 1, 3, 5, 7, 9, ...

$$\begin{aligned} \text{Since, 2nd term} - \text{1st term} &= \text{3rd term} - \text{2nd term} \\ &= \text{4th term} - \text{3rd term} \\ &= \dots = 2, \end{aligned}$$

the sequence 1, 3, 5, 7, ... are in A.P. whose first term is 1 and common difference is 2.

2. 5, 3, 1, -1, -3, -5, -7, ... are in A.P. whose first term is 5 and common difference is -2.

The n th term of an Arithmetic Progression

If a is the first term and d is the common difference of an A.P., then its n th term t_n is given by

$$t_n = a + (n - 1)d$$

TRICK(S) FOR PROBLEM SOLVING

To find whether the series is an A.P.

Step I: Obtain a_n (the n^{th} term of the sequence).

Step II: Replace n by $n - 1$ in a_n to get a_{n-1} .

Step III: Calculate $a_n - a_{n-1}$.

If $a_n - a_{n-1}$ is independent of n , the given sequence is an A.P. otherwise it is not an A.P.

$\therefore t_n = An + B$ represents the n^{th} term of an A.P. with common difference A .



IMPORTANT POINTS

- If an A.P. has n terms, then the n th term is called the last term of A.P. and it is denoted by l . That is $l = a + (n - 1)d$
- Three numbers a, b, c are in A.P. if and only if $b - a = c - b$, i.e., if and only if $a + c = 2b$.
- If a is the first term and d the common difference of an A.P. having m terms, then n th term from the end is $(m - n + 1)$ th term from the beginning. Thus, n th term from the end $= a + (m - n)d$.
- Any three numbers in an A.P. can be taken as $a - d, a, a + d$. Any four numbers in an A.P. can be taken as $a - 3d, a - d, a + d, a + 3d$. Similarly 5 numbers in A.P. can be taken as $a - 2d, a - d, a, a + d, a + 2d$.

SOLVED EXAMPLES

1. The number of terms common to two A.P.s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is

- (A) 21 (B) 28
(C) 14 (D) None of these

Solution: (C)

By inspection, first common term to both the series is 23. Second common term = 51.

Third = 79 and so on. These numbers form an A.P. 23, 51, 79, ...

$$\text{Since, } T_{15} = 23 + 14(28) = 23 + 392 = 415 > 407$$

$$\text{and } T_{14} = 23 + 13(28) = 387 < 407$$

\therefore number of common terms = 14.

2. The number of numbers lying between 100 and 500 that are divisible by 7 but not by 21 is
(A) 57 (B) 19
(C) 38 (D) None of these

Solution: (C)

The numbers between 100 and 500 that are divisible by 7 are 105, 112, 119, 126, 133, 140, 147, ..., 483, 490, 497.

Let such numbers be n .

$$\text{Then, } 497 = 105 + (n - 1) \times 7; \text{ or } n = 57.$$

The numbers between 100 and 500 that are divisible by 21 are 105, 126, 147, ..., 483.

Let such numbers be m .

$$\text{Then } 483 = 105 + (m - 1) \times 21; \text{ or } m = 19.$$

$$\text{Hence, the required number} = n - m = 57 - 19 = 38.$$

3. In the series 3, 7, 11, 15, ... and 2, 5, 8, ... each continued to 100 terms, the number of terms that are identical is
(A) 21 (B) 27
(C) 25 (D) None of these

Solution: (C)

Let the n th term of the first series = the m th term of the second series.

$$\therefore 3 + (n - 1) \times 4 = 2 + (m - 1) \times 3,$$

$$\text{or } 4n = 3m \text{ or } \frac{n}{3} = \frac{m}{4} = k \text{ (say)}$$

$$\therefore n = 3k \text{ and } m = 4k$$

As each series is continued to 100 terms,

$$n = 3k \leq 100 \text{ and } m = 4k \leq 100$$

\therefore Possible values of k are 1, 2, 3, ..., 25 and corresponding to each value of k we get one identical term. Hence there are 25 identical terms.

4. If $5^{1+x} + 5^{1-x}$, $\frac{a}{2}$ and $25^x + 25^{-x}$ are three consecutive terms of an A.P., then the values of a are given by
- (A) $a \geq 12$ (B) $a > 12$
 (C) $a < 12$ (D) $a \leq 12$

Solution: (A)

Since $5^{1+x} + 5^{1-x}$, $\frac{a}{2}$, $25^x + 25^{-x}$ are in A.P., we have

$$2 \frac{a}{2} = 5^{1+x} + 5^{1-x} + 25^x + 25^{-x}$$

Now put $5^x = t$ so that $t > 0$, we then have

$$a = 5t + \frac{5}{t} + t^2 + \frac{1}{t^2} = \left(t^2 + \frac{1}{t^2}\right) + 5\left(t + \frac{1}{t}\right)$$

or

$$a = \left(t - \frac{1}{t}\right)^2 + 2 + 5\left[\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 2\right]$$

$$= \left(t - \frac{1}{t}\right)^2 + 5\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 12 \geq 12.$$

Thus, values of a are given by the inequality $a \geq 12$.

5. If $1, \log_y x, \log_x y, -15 \log_x z$ are in A.P., then
- (A) $x = y^{-3}$ (B) $y = z^{-2}$
 (C) $x = z^3$ (D) None of these

Solution: (C)

Let d be the common difference of the A.P.

Then, $\log_y x = 1 + d \Rightarrow x = y^{1+d}$

$$\log_x y = 1 + 2d \Rightarrow y = x^{1+2d}$$

and $-15 \log_x z = 1 + 3d \Rightarrow z = x^{-(1+3d)/15}$

$$\therefore x = y^{1+d} = z^{(1+2d)(1+d)}$$

$$= x^{-(1+d)(1+2d)(1+3d)/15}$$

$$\Rightarrow (1+d)(1+2d)(1+3d) = -15$$

$$\Rightarrow 6d^3 + 11d^2 + 6d + 16 = 0$$

$$\Rightarrow (d+2)(6d^2 - d + 8) = 0 \Rightarrow d = -2$$

$$\therefore x = y^{1+d} = y^{-1}, y = z^{1+2d} = z^{-3} \text{ and } x = (z^{-3})^{-1} = z^3.$$

6. If three positive real numbers a, b, c are in A.P. such that $abc = 4$, then the minimum possible value of b is
- (A) $2^{2/3}$ (B) $2^{1/3}$
 (C) $2^{5/3}$ (D) None of these

Solution: (A)

Let d be the common difference of the A.P., then $4 = abc = (b-d)b(b+d) = b(b^2 - d^2)$

$$\Rightarrow b^3 = 4 + bd^2 \geq 4 \quad (\because b > 0, d^2 \geq 0)$$

$$\Rightarrow b \geq 2^{2/3}$$

Thus, the minimum possible value of b is $2^{2/3}$.

7. There are four numbers of which the first three are in G.P. whose common ratio is $\frac{1}{2}$ and the last three are in A.P. If the last number is two less than the first, then the four members are
- (A) $3, 1, \frac{1}{3}, -\frac{1}{3}$ (B) $2, 1, \frac{1}{2}, 0$
 (C) $4, 1, \frac{1}{4}, -\frac{1}{2}$ (D) None of these

Solution: (B)

The numbers can be taken as $a, \frac{a}{2}, \frac{a}{4}, a-2$.

By question, $\frac{a}{2}, \frac{a}{4}, a-2$ are in A.P.

$$\therefore 2 \cdot \frac{a}{4} = \frac{a}{2} + a - 2; \text{ or } a = 2.$$

Hence, the numbers are $2, 1, \frac{1}{2}, 0$.

SUM OF n TERMS OF AN A.P.

The sum of n terms of an A.P. with first term ' a ' and common difference ' d ' is given by

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

TRICK(S) FOR PROBLEM SOLVING

- If S_n is the sum of n terms of an A.P. whose first term is ' a ' and last term is l , then

$$S_n = \frac{n}{2} (a + l)$$

- If common difference d , number of terms n and the last term l , are given then

$$S_n = \frac{n}{2} [2l - (n-1)d]$$

- $t_n = S_n - S_{n-1}$.
- The sum of an A.P. consisting of odd number of terms = n (middle term), where n is number of terms.

PROPERTIES OF A.P.

- If $a_1, a_2, a_3, \dots, a_n$ are in A.P., then
 - $a_1 + k, a_2 + k, \dots, a_n + k$ are also in A.P.
 - $a_1 - k, a_2 - k, \dots, a_n - k$ are also in A.P.
 - ka_1, ka_2, \dots, ka_n are also in A.P.
 - $\frac{a_1}{k}, \frac{a_2}{k}, \dots, \frac{a_n}{k}, k \neq 0$ are also in A.P.
- If a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots are two A.P.s, then
 - $a_1 + b_1, a_2 + b_2, a_3 + b_3, \dots$ are also in A.P.
 - $a_1 - b_1, a_2 - b_2, a_3 - b_3, \dots$ are also in A.P.
 - $a_1 b_1, a_2 b_2, a_3 b_3, \dots$ are also in A.P.
 - $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots$ may not be in A.P.
- If $a_1, a_2, a_3, \dots, a_n$ are in A.P., then
 - $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$
 - $a_r = \frac{a_{r-k} + a_{r+k}}{2}, 0 \leq k \leq n - r.$

In other words, in an A.P., the sum of two terms equidistant from the beginning and end is a constant and is equal to the sum of first term and last term

- If n th term of a sequence is a linear expression in n then the sequence is an A.P.
- If the sum of first n terms of a sequence is a quadratic expression in n , then the sequence is an A.P.
- If $a_1, a_2, a_3, a_4, \dots, a_n$ are in A.P., then terms taken at regular intervals from this A.P. are also in A.P. e.g., $a_1, a_4, a_7, a_{10}, \dots$ also form an A.P.

SOLVED EXAMPLES

- If $\frac{1^4}{13} + \frac{2^4}{35} + \frac{3^4}{57} + \dots + \frac{n^4}{(2n-1)(2n+1)} = \frac{1}{48} f(n) + \frac{n}{16(2n+1)}$, then $f(n)$ is equal to
 - $n(4n^2 + 3n + 2)$
 - $n(4n^2 + 6n + 5)$
 - $n(4n^2 + 5n + 6)$
 - None of these

Solution: (B)

We have, $t_n = \frac{n^4}{(2n-1)(2n+1)}$

$$= \frac{n^2}{4} + \frac{1}{16} + \frac{1}{16(4n^2-1)}$$

$$= \frac{4n^2+1}{16} + \frac{1}{32} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

$$\begin{aligned} \therefore S_n &= \sum_{n=1}^n t_n \\ &= \sum_{n=1}^n \frac{4n^2+1}{16} + \frac{1}{32} \sum_{n=1}^n \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right) \\ &= \frac{1}{4} \frac{n(n+1)(2n+1)}{6} + \frac{1}{16} n + \frac{1}{32} \left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{2n-1} - \frac{1}{2n+1} \right) \\ &= \frac{n}{48} (4n^2 + 6n + 5) + \frac{1}{32} \left(1 - \frac{1}{2n+1} \right) \\ &= \frac{n(4n^2 + 6n + 5)}{48} + \frac{n}{16(2n+1)} \\ \therefore f(n) &= n(4n^2 + 6n + 5) \end{aligned}$$

- The maximum sum of the series

$$20 + 19 \frac{1}{3} + 18 \frac{2}{3} + 18 + \dots \text{ is}$$

- 310
- 290
- 320
- None of these

Solution: (A)

The given series is arithmetic whose first term = 20, common difference = -.

As the common difference is negative, the terms will become negative after some stage. So the sum is maximum if only positive terms are added.

Now

$$t_n = 20 + (n-1) \left(-\frac{2}{3} \right) \geq 0 \text{ if } 60 - 2(n-1) \geq 0;$$

$$\text{or } 62 \geq 2n \text{ or } 31 \geq n$$

\therefore The first 31 terms are non-negative.

$$\begin{aligned} \therefore \text{Maximum sum} &= S_{31} = \frac{31}{2} \left[2 \times 20 + (31-1) \left(-\frac{2}{3} \right) \right] \\ &= \frac{31}{2} (40 - 20) = 310 \end{aligned}$$

- The sum to n terms of the sequence

$\log a, \log ar, \log ar^2, \dots$ is

- $\frac{n}{2} \log a^2 r^{n-1}$
- $n \log a^2 r^{n-1}$
- $\frac{3n}{2} \log a^2 r^{n-1}$
- None of these

Solution: (A)

The given sequence can be expressed as

$$\log a, (\log a + \log r), (\log a + 2 \log r) \dots$$

which is clearly an A.P. whose first term is $\log a$ and common difference is $\log r$.

$$\text{The } n\text{th term} = \log a + (n-1) \log r$$

$$\text{Since sum to } n \text{ terms} = \frac{n}{2} (a_1 + a_n)$$

$$\begin{aligned} \therefore S_n &= \frac{n}{2} [\log a + \log a + (n-1) \log r] \\ &= \frac{n}{2} [2 \log a + (n-1) \log r] = \frac{n}{2} \log a^2 r^{n-1} \end{aligned}$$

11. Let S_n denotes the sum of n terms of an A.P. whose first term is a . If the common difference $d = S_n - k S_{n-1} + S_{n-2}$ then $k =$

- (A) 1 (B) 2
(C) 3 (D) None of these

Solution: (B)

$$\text{We have, } a_n = S_n - S_{n-1} \quad (1)$$

$$\text{and } a_{n-1} = S_{n-1} - S_{n-2} \quad (2)$$

$$\begin{aligned} \therefore d &= a_n - a_{n-1} \\ &= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2}) \\ &\quad \text{[From (1) and (2)]} \\ &= S_n - 2S_{n-1} + S_{n-2}. \end{aligned}$$

12. The sum of positive terms of the series

$$10 + 9 \frac{4}{7} + 9 \frac{1}{7} + \dots \text{ is}$$

- (A) $\frac{352}{7}$ (B) $\frac{437}{7}$
(C) $\frac{852}{7}$ (D) None of these

Solution: (C)

$$\text{Here, } a = 10, d = -\frac{3}{7}.$$

$$\text{Then, } t_n = 10 + (n-1) \left(-\frac{3}{7}\right)$$

$$t_n \text{ is positive if } 10 + (n-1) \left(-\frac{3}{7}\right) \geq 0;$$

$$\text{or } 70 - 3(n-1) \geq 0 \text{ or } 73 \geq 3n; \text{ or } 24 \frac{1}{3} \geq n$$

\therefore First 24 terms are positive.

\therefore Sum of the positive terms

$$= S_{24} = \frac{24}{2} \left(2 \times 10 + 23 \times \frac{-3}{7}\right)$$

$$= 12 \left(20 - \frac{69}{7}\right) = \frac{852}{7}$$

13. The minimum number of terms from the beginning of the series $20 + 22 \frac{2}{3} + 25 \frac{1}{3} + \dots$, so that the sum may exceed 1568, is

- (A) 25 (B) 27
(C) 28 (D) 29

Solution: (D)

It is in A.P. for which $a = 20, d = 2 \frac{2}{3} = \frac{8}{3}$

$$\text{Now, } S_n > 1568$$

$$\Rightarrow \frac{n}{2} \left[40 + (n-1) \frac{8}{3}\right] > 1568$$

$$\Rightarrow \frac{n}{2} \times \frac{112 + 8n}{3} > 1568$$

$$\Rightarrow n^2 + 14n > \frac{6}{8} \times 1568 = 1176$$

$$\Rightarrow n^2 + 14n - 1176 > 0,$$

$$\text{or } (n+42)(n-28) > 0$$

As n is positive, $n-28 > 0$ i.e., $n > 28$

\therefore Minimum value of $n = 29$.

14. If the first, second and last terms of an A.P. are a, b and $2a$ respectively, then its sum is

- (A) $\frac{ab}{2(b-a)}$ (B) $\frac{ab}{b-a}$
(C) $\frac{3ab}{2(b-a)}$ (D) None of these

Solution: (C)

Here $a_1 = a$ and $a_2 = b$

\therefore Common difference $d = a_2 - a_1 = b - a$

Let n be the number of terms in the series

$$\therefore a_n = 2a = a + (n-1)d$$

$$\text{or } (n-1)d = a \text{ or } (n-1)(b-a) = a$$

$$\therefore n-1 = \frac{a}{b-a}$$

$$\text{or } n = \frac{a}{b-a} + 1 = \frac{a+b-a}{b-a} = \frac{b}{b-a}$$

$$\therefore \text{Sum} = \frac{n}{2} (a_1 + a_n) = \frac{b}{2(b-a)} (a + 2a) = \frac{3ab}{2(b-a)}.$$

15. If S_1 is the sum of an arithmetic series of 'n' odd number of terms and S_2 , the sum of the terms of the series in odd places, then $\frac{S_1}{S_2} =$

(A) $\frac{2n}{n+1}$ (B) $\frac{n}{n+1}$
 (C) $\frac{n+1}{2n}$ (D) $\frac{n+1}{n}$

Solution: (A)

Let the odd number of terms of an arithmetic series be

$$a, a + d, a + 2d, a + 3d, a + 4d, \dots, a + (n - 1)d$$

Then,

$$S_1 = \frac{n}{2} \{2a + (n - 1)d\}$$

$$S_2 = a + (a + 2d) + (a + 4d) + \dots \text{ to } \frac{n+1}{2} \text{ terms}$$

$$= \frac{n+1}{2 \times 2} \left[2a + \left(\frac{n+1}{2} - 1 \right) \times 2d \right]$$

$$= \frac{n+1}{4} (2a + (n - 1)d)$$

$$\therefore \frac{S_1}{S_2} = \frac{2n}{n+1}$$

16. A club consists of members whose ages are in A.P., the common difference being 3 months. If the youngest member of the club is just 7 years old and the sum of the ages of all the members is 250 years, then the number of members in the club are
- (A) 15 (B) 25
 (C) 20 (D) 30

Solution: (B)

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Here $a = 1$ st term = 7 years, $d = 3$ months = $\frac{1}{4}$ year,

$$S_n = 250$$

$$\therefore 250 = \frac{n}{2} \left[2 \times 7 + (n - 1) \times \frac{1}{4} \right]$$

$$\Rightarrow 250 = \frac{n}{2} \left(\frac{n + 55}{4} \right)$$

$$\Rightarrow 2000 = n^2 + 55n$$

$$\Rightarrow n^2 + 55n - 2000 = 0$$

$$\Rightarrow (n - 25)(n + 80) = 0 \Rightarrow n = 25.$$

\therefore Number of members in the club = 25.

17. If a is the first term, d the common difference and S_k the sum to k terms of an A.P., then for $\frac{S_{kx}}{S_x}$ to be independent of x
- (A) $a = 2d$ (B) $a = d$
 (C) $2a = d$ (D) None of these

Solution: (C)

$$\begin{aligned} \text{We have, } \frac{S_{kx}}{S_x} &= \frac{\frac{kx}{2} [2a + (kx - 1)d]}{\frac{x}{2} [2a + (x - 1)d]} \\ &= \frac{k[(2a - d) + kxd]}{(2a - d) + xd} \end{aligned}$$

For $\frac{S_{kx}}{S_x}$ to be independent of x , $2a - d = 0$ or $2a = d$.

18. The sum of n terms of m A.P.s are $S_1, S_2, S_3, \dots, S_m$. If the first term and common difference are 1, 2, 3, ..., m respectively, then $S_1 + S_2 + S_3 + \dots + S_m =$

(A) $\frac{1}{4} mn(m + 1)(n + 1)$

(B) $\frac{1}{2} mn(m + 1)(n + 1)$

(C) $mn(m + 1)(n + 1)$

(D) None of these

Solution: (A)

$$\text{We have, } S_1 = (n/2) [2 \cdot 1 + (n - 1) \cdot 1]$$

$$S_2 = (n/2) [2 \cdot 2 + (n - 1) \cdot 2]$$

$$S_m = (n/2) [2 \cdot m + (n - 1) \cdot m]$$

$$\therefore S_1 + S_2 + \dots + S_m$$

$$= n(1 + 2 + 3 \dots + m) + \frac{n(n-1)}{2} \times (1 + 2 + \dots + m)$$

$$= \frac{m(m+1)}{2} \left(n + \frac{n^2 - n}{2} \right)$$

$$= \frac{m(m+1)}{2} \cdot \frac{n(n+1)}{2} = \frac{1}{4} mn(m + 1)(n + 1).$$

19. If the first, second and the last terms of an A.P. are a, b, c respectively, then the sum is

(A) $\frac{(a+b)(a+c-2b)}{2(b-a)}$

(B) $\frac{(b+c)(a+b-2c)}{2(b-a)}$

(C) $\frac{(a+c)(b+c-2a)}{2(b-a)}$

(D) None of these

Solution: (C)

We have, first term = a , $\therefore T_1 = a$

Second term = b , $\therefore T_2 = b$

Then common difference $d = T_2 - T_1 = b - a$

Also, last term = c .

$$\Rightarrow c = a + (n-1)d \Rightarrow n = \frac{c-a+d}{d}$$

$$\Rightarrow n = \frac{(b+c-2a)}{(b-a)} \quad (\because d = b - a)$$

$$\therefore \text{Sum of } n \text{ terms } S_n = \frac{n}{2}(a+l) = \frac{(b+c-2a)(a+c)}{2(b-a)}$$

20. Four different integers form an increasing A.P. If one of these numbers is equal to the sum of the squares of the other three numbers, then the numbers are

- (A) $-2, -1, 0, 1$ (B) $0, 1, 2, 3$
 (C) $-1, 0, 1, 2$ (D) None of these

Solution: (C)

Let the numbers be $a-d, a, a+d, a+2d$ where $a, d \in Z$ and $d > 0$

Given: $(a-d)^2 + a^2 + (a+d)^2 = a+2d$

$$\Rightarrow 2d^2 - 2d + 3a^2 - a = 0$$

$$\therefore d = \frac{1}{2} \left[1 \pm \sqrt{(1+2a-6a^2)} \right]$$

Since d is positive integer,

$$\therefore 1 + 2a - 6a^2 > 0$$

$$\Rightarrow \left(\frac{1-\sqrt{7}}{6} \right) < a < \left(\frac{1+\sqrt{7}}{6} \right)$$

Since a is an integer,

$$\therefore a = 0,$$

then $d = \frac{1}{2} [1 \pm 1] = 1$ or 0 . Since $d > 0$,

$$\therefore d = 1.$$

Hence, the numbers are $-1, 0, 1, 2$.

21. The sum of all two digit numbers which when divided by 4, yield unity as remainder, is

- (A) 1100 (B) 1200
 (C) 1210 (D) None of these

Solution: (C)

The first two digit number which when divided by 4 leaves remainder 1 is $4 \cdot 3 + 1 = 13$ and last is $4 \cdot 24 + 1 = 97$.

Thus, we have to find the sum

$$13 + 17 + 21 + \dots + 97$$

which is an A.P.

$$\therefore 97 = 13 + (n-1) \cdot 4 \Rightarrow n = 22.$$

and $S_n = \frac{n}{2} [a+l] = 11 \times [13 + 97]$
 $= 11 \times 110 = 1210.$

22. If there are $(2n+1)$ terms in A.P., then the ratio of the sum of odd terms and the sum of even terms is

- (A) $n : (n+1)$ (B) $(n+1) : n$
 (C) $(n-1) : n$ (D) None of these

Solution: (B)

Let the A.P. containing $(2n+1)$ terms be

$$a, a+d, a+2d, a+3d, a+4d, a+5d, \dots, a+2nd.$$

The sum of odd terms of this A.P.

$$= a + (a+2d) + (a+4d) + \dots \text{ to } (n+1) \text{ terms}$$

$$= \frac{n+1}{2} [2a + (n+1-1) \times 2d] = (n+1)(a+nd)$$

The sum of even terms of this A.P.

$$= (a+d) + (a+3d) + (a+5d) + \dots \text{ to } n \text{ terms}$$

$$= \frac{n}{2} [2(a+d) + (n-1) \times 2d] = n(a+nd)$$

Hence, the required ratio = $\frac{n+1}{n}$.

ARITHMETIC MEAN (A.M.)

Single Arithmetic Mean

A number 'A' is said to be the single A.M. between two given numbers a and b provided a, A, b are in A.P.

For example, since 2, 4, 6 are in A.P., therefore, 4 is the single A.M. between 2 and 6.

n-Arithmetic Means

The numbers A_1, A_2, \dots, A_n are said to be the n arithmetic means between two given numbers a and b provided

$$a, A_1, A_2, \dots, A_n, b \text{ are in A.P.}$$

For example, since 2, 4, 6, 8, 10, 12 are in A.P., therefore, 4, 6, 8, 10 are the four arithmetic means between 2 and 12.

Inserting Single A.M. between Two given Numbers

Let a and b be two given numbers and A be the A.M. between them. Then, a, A, b are in A.P. Thus,

$$A - a = b - A \text{ or } 2A = a + b, \text{ or } A = \frac{a+b}{2}.$$

Inserting n -Arithmetic Means between Two given Numbers

Let A_1, A_2, \dots, A_n be the n arithmetic means between two given numbers a and b . Then $a, A_1, A_2, \dots, A_n, b$ are in A.P. Now, $b = (n+2)$ th term of A.P.

$$= a + (n+2-1)d = a + (n+1)d$$

or $d = \frac{b-a}{n+1}$, where d is common difference of A.P.

and $A_1 = a + d = a + \left(\frac{b-a}{n+1}\right)$,

$$A_2 = a + 2d = a + 2\left(\frac{b-a}{n+1}\right),$$

$\vdots \quad \vdots \quad \vdots$

$$A_n = a + nd = a + n\left(\frac{b-a}{n+1}\right)$$



REMEMBER

The sum of n arithmetic means between two given numbers is n times the single A.M. between them, i.e. if a and b are two given numbers and A_1, A_2, \dots, A_n are n arithmetic means between them, then

$$A_1 + A_2 + \dots + A_n = n \left(\frac{a+b}{2}\right)$$

TRICKS(S) FOR PROBLEM SOLVING

- Sum to n terms of the series of the form

$$\frac{1}{t_1 t_2 \dots t_k} + \frac{1}{t_2 t_3 \dots t_{k+1}} + \dots + \frac{1}{t_n t_{n+1} \dots t_{n+k-1}} \text{ is}$$

$$S_n = \frac{1}{(k-1)(t_2 - t_1)} \left(\frac{1}{t_1 t_2 \dots t_{k-1}} - \frac{1}{t_{n+1} t_{n+2} \dots t_{n+k-1}} \right)$$

- Sum to n terms of the series of the form

$$(t_1 t_2 \dots t_k) + (t_2 t_3 \dots t_{k+1}) + \dots + (t_n t_{n+1} \dots t_{n+k-1}) \text{ is}$$

$$S_n = \frac{1}{(k+1)(t_2 - t_1)} (t_n t_{n+1} \dots t_{n+k} - t_0 t_1 t_2 \dots t_k)$$

SOLVED EXAMPLES

23. Between two numbers whose sum is $2\frac{1}{6}$, an even number of arithmetic means are inserted. If the sum of these means exceeds their number by unity, then the number of means are

- (A) 12 (B) 10
(C) 8 (D) None of these

Solution: (A)

Let $2n$ arithmetic means be $A_1, A_2, A_3, \dots, A_{2n}$ between a and b .

$$\text{Then, } A_1 + A_2 + A_3 + \dots + A_{2n} = \frac{a+b}{2} \times 2n$$

$$= \frac{13}{6} \times 2n = \frac{13n}{6}$$

Given: $A_1 + A_2 + A_3 + \dots + A_{2n} = 2n + 1$;

$$\therefore 2n + 1 = \frac{13n}{6}; \text{ or } 12n + 6 = 13n;$$

$$\therefore n = 6.$$

$$\therefore \text{The number of means} = 2n = 2 \times 6 = 12.$$

24. If a, b, c are in A.P. and p is the A.M. between a and b and q is the A.M. between b and c , then

- (A) a is the A.M. between p and q
(B) b is the A.M. between p and q
(C) c is the A.M. between p and q
(D) None of these

Solution: (B)

$\therefore a, b, c$ are in A.P.,

$$\therefore 2b = a + c \quad (1)$$

$\therefore p$ is the A.M. between a and b

$$\therefore p = \frac{a+b}{2} \quad (2)$$

$\therefore q$ is the A.M. between b and c

$$\therefore q = \frac{b+c}{2} \quad (3)$$

Adding Eq. (2) and (3)

$$p + q = \frac{a+b}{2} + \frac{b+c}{2} = \frac{a+c+2b}{2}$$

$$= \frac{2b+2b}{2} = 2b$$

[Using (1)]

$$\therefore 2b = p + q \text{ or } b = \frac{p+q}{2}$$

Hence, b is the A.M. between p and q .

GEOMETRIC PROGRESSION (G.P.)

A sequence (finite or infinite) of non-zero numbers in which every term, except the first one, bears a constant ratio with its preceding term, is called a *geometric progression*.

The constant ratio, also called the *common ratio* of the G.P., is usually denoted by r .

For example, in the sequence, 1, 2, 4, 8, ...,

$$\frac{2}{1} = \frac{4}{2} = \frac{8}{4} = \dots = 2, \text{ which is a constant.}$$

Thus, the sequence is a G.P. whose first term is 1 and the common ratio is 2.

n th Term of a G.P.

If a is the first term and r is the common ratio of a G.P., then its n th term t_n is given by

$$t_n = ar^{n-1}$$



IMPORTANT POINTS

- If a is the first term and r is the common ratio of a G.P., then the G.P. can be written as $a, ar, ar^2, \dots, ar^{n-1}, \dots$ ($a \neq 0$)
- If a is the first term and r is the common ratio of a finite G.P. consisting of m terms, then the n th term from the end is given by ar^{m-n} .
- The n th term from the end of a G.P. with last term l and common ratio r is $\frac{l}{r^{(n-1)}}$.
- Three numbers in G.P. can be taken as $\frac{a}{r}, a, ar$;
Four numbers in G.P. can be taken as

$$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3;$$

Five numbers in G.P. can be taken as

$$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$$

- Three numbers a, b, c are in G.P. if and only if

$$\frac{b}{a} = \frac{c}{b} \text{ i.e. if and only if } b^2 = ac.$$

SOLVED EXAMPLES

25. If a, b, c are respectively the x th, y th and z th terms of a G.P., then

$$(y-z) \log a + (z-x) \log b + (x-y) \log c =$$

- (A) 1 (B) -1
(C) 0 (D) None of these

Solution: (C)

Let A be the first term and R , the common ratio of G.P.

$$\text{Then } a = a_x = AR^{x-1}, b = a_y = AR^{y-1}$$

$$\text{and } c = a_z = AR^{z-1}$$

$$\therefore (y-z) \log a + (z-x) \log b + (x-y) \log c$$

$$= \log a^{y-z} + \log b^{z-x} + \log c^{x-y}$$

$$= \log (a^{y-z} \cdot b^{z-x} \cdot c^{x-y})$$

$$= \log [(AR^{x-1})^{y-z} \times (AR^{y-1})^{z-x} \cdot (AR^{z-1})^{x-y}]$$

$$= \log [A^{y-z+z-x+x-y} \cdot R^{(x-1)(y-z)+(y-1)(z-x)+(z-1)(x-y)}]$$

$$= \log (A^0 \times R^0) = \log 1 = 0.$$

26. If p, q, r are in A.P. and x, y, z are in G.P., then

$$x^{q-r} \cdot y^{r-p} \cdot z^{p-q} =$$

- (A) 1 (B) 2
(C) -1 (D) None of these

Solution: (A)

Let d be the common difference of A.P. and $R (\neq 0)$, the common ratio of G.P., then

$$q = p + d, r = p + 2d$$

$$\text{and } y = xR, z = xR^2$$

$$\text{so that } q - r = -d, r - p = 2d, p - q = -d$$

$$\begin{aligned} \therefore x^{q-r} \cdot y^{r-p} \cdot z^{p-q} &= x^{-d} \cdot (xR)^{2d} \cdot (xR^2)^{-d} \\ &= (x^{-d} \cdot x^{2d} \cdot x^{-d}) (R^{2d} \times R^{-2d}) \\ &= (x^{-d+2d-d}) \cdot (R^{2d-2d}) \\ &= x^0 \cdot R^0 = 1 \times 1 = 1. \end{aligned}$$

27. If, in a G.P., the $(p+q)$ th term is a and the $(p-q)$ th term is b , then p th term is

- (A) $(ab)^{1/3}$ (B) $(ab)^{1/2}$
(C) $(ab)^{1/4}$ (D) None of these

Solution: (B)

Let the G.P. be x, xy, xy^2, \dots

$$\text{Then } t_{p+q} = xy^{p+q-1} = a \quad (1)$$

$$\text{and } t_{p-q} = xy^{p-q-1} = b \quad (2)$$

Dividing Eqs (1) by (2),

$$y^{2q} = \frac{a}{b};$$

$$\therefore y = \left(\frac{a}{b}\right)^{\frac{1}{2q}}$$

From Eq. (1),

$$x = a \left(\frac{b}{a}\right)^{\frac{p+q-1}{2q}}$$

$$\begin{aligned} \therefore t_p = xy^{p-1} &= a \cdot \left(\frac{b}{a}\right)^{\frac{p+q-1}{2q}} \left(\frac{a}{b}\right)^{\frac{p-1}{2q}} \\ &= \frac{1-p+q-1+p-1}{a} \frac{p+q-1}{2q} \times \frac{p-1}{b} \frac{p-1}{2q} \\ &= a^{\frac{1}{2}} b^{\frac{1}{2}} = \sqrt{ab} \end{aligned}$$

28. In a set of four numbers the first three are in G.P. and the last three are in A.P. with a common difference 6. If the first number is same as the fourth, the four numbers are
- (A) 3, 9, 15, 21 (B) 1, 7, 13, 19
(C) 8, -4, 2, 8 (D) None of these

Solution: (C)

Let the last three numbers in A.P. be $b, b+6, b+12$ and the first number be a .

Hence the four numbers are $a, b, b+6, b+12$

$$\text{Given, } a = b + 12 \quad (1)$$

and $a, b, b+6$ are in G.P. i.e., $b^2 = a(b+6)$

$$\text{or } b^2 = (b+12)(b+6) \quad (\because a = b+12)$$

$$\text{or } 18b = -72 \therefore b = -4,$$

From Eq. (1),

$$a = -4 + 12 = 8.$$

Hence, the four numbers are 8, -4, 2 and 8.

29. If the first term of an infinite G.P. is 1 and each term is twice the sum of the succeeding terms, then the common ratio is
- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{3}{4}$ (D) None of these

Solution: (A)

Let the G.P. be $1, r, r^2, r^3, \dots$

$$\text{Given: } t_n = 2(t_{n+1} + t_{n+2} + t_{n+3} + \dots \text{ to } \infty)$$

$$\text{or } t_n = 2 \cdot \frac{t_{n+1}}{1-r}, \quad (\because \text{common ratio} = r)$$

$$\therefore = \frac{t_{n+1}}{t_n}; \text{ or } \frac{1-r}{2} = r$$

$$\text{or } 1-r = 2r; \therefore r = \frac{1}{3}.$$

30. The three numbers a, b, c between 2 and 18 are such that their sum is 25; the numbers 2, a, b are consecutive terms of an A.P. and the numbers $b, c, 18$ are consecutive terms of a G.P. The three numbers are
- (A) 3, 8, 14 (B) 2, 9, 14
(C) 5, 8, 12 (D) None of these

Solution: (C)

$$\text{We have, } a + b + c = 25 \quad (1)$$

$$\because 2, a, b \text{ are in A.P., } \therefore 2a = 2 + b \quad (2)$$

$$\because b, c, 18 \text{ are in G.P., } \therefore c^2 = 18b \quad (3)$$

From Eq. (1) and (2),

$$3b + 2c = 48; \text{ or } 3b = 48 - 2c$$

\therefore From Eq. (3),

$$c^2 = 6(48 - 2c) = 288 - 12c$$

$$\text{or } c^2 + 12c - 288 = 0;$$

$$\text{or } c^2 + 24c - 12c - 288 = 0$$

$$\text{or } (c+24)(c-12) = 0;$$

$$\therefore c = 12, \text{ as } c \neq -24.$$

\therefore From Eq. (3), $b = 8$ and from (2), $a = 5$.

31. Let a_n be the n th term of the G.P. of positive numbers.

Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then

the common ratio is

$$(A) \frac{\alpha}{\beta} \quad (B) \frac{\beta}{\alpha}$$

$$(C) \sqrt{\frac{\alpha}{\beta}} \quad (D) \sqrt{\frac{\beta}{\alpha}}$$

Solution: (A)

Let a be the first term and r , the common ratio of the given G.P. Then

$$\alpha = \sum_{n=1}^{100} a_{2n} \Rightarrow \alpha = a_2 + a_4 + \dots + a_{200}$$

$$\Rightarrow \alpha = ar + ar^3 + \dots + ar^{199}$$

$$\Rightarrow \alpha = ar(1 + r^2 + r^4 + \dots + r^{198}) \quad (1)$$

and $\beta = \sum_{n=1}^{100} a_{2n-1} \Rightarrow \beta = a_1 + a_3 + \dots + a_{199}$

$$\Rightarrow \beta = a + ar^2 + \dots + ar^{198}$$

$$\Rightarrow \beta = a(1 + r^2 + \dots + r^{198}) \quad (2)$$

From Eqs (1) and (2), we get

$$\frac{\alpha}{\beta} = r.$$

32. If a, b, c are in G.P., then $a^2 b^2 c^2 \left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} \right) =$

- (A) $a + b + c$ (B) $ab + ac + bc$
 (C) $a^3 + b^3 + c^3$ (D) None of these

Solution: (C)

Since a, b, c are in G.P.

$$\therefore \frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac \quad (1)$$

Now, $a^2 b^2 c^2 \left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} \right)$

$$= \frac{b^2 c^2}{a} + \frac{a^2 c^2}{b} + \frac{a^2 b^2}{c}$$

$$= \frac{ac \cdot c^2}{a} + \frac{(b^2)^2}{b} + \frac{a^2 \cdot ac}{c} \quad [\text{Using (1)}]$$

$$= a^3 + b^3 + c^3.$$

Sum of n terms of a G.P.

The sum of first n terms of a G.P. with first term a and common ratio $r (\neq 1)$ is given by,

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$$

TRICK(S) FOR PROBLEM SOLVING

- When $r = 1$,
 $S_n = a + a + a + \dots$ upto n terms $= na$.
- If l is the last term of the G.P., then

$$S_n = \frac{l r - a}{r - 1} \quad r \neq 1$$

Sum of an Infinite G.P. when $|r| < 1$

The sum of an infinite G.P. with first term a and common ratio r is

$$S_\infty = \frac{a}{1 - r}; \text{ when } |r| < 1, \text{ i.e., } -1 < r < 1.$$



CAUTION

If $r \geq 1$, then S_∞ does not exist.

SOLVED EXAMPLES

33. The sum S_n to n terms of the series

$$\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$$
 is equal to

- (A) $2^n - n - 1$ (B) $1 - 2^{-n}$
 (C) $2^{-n} + n - 1$ (D) $2^n - 1$

Solution: (C)

We have,

$$S_n = \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{2^3}\right) + \dots + \left(1 - \frac{1}{2^n}\right)$$

$$= n - \left(\frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^n}\right)$$

$$= n - \frac{\frac{1}{2} \left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}} = n - 1 + 2^{-n}$$

35. The minimum number of terms of the series

$$1 + 3 + 9 + 27 + \dots$$

so that the sum may exceed 1000, is

- (A) 7 (B) 5
 (C) 3 (D) None of these

Solution: (A)

Let, the sum of n terms exceeds 1000.

$$\text{Then } \frac{1(1 - 3^n)}{1 - 3} > 1000; \text{ or } \frac{3^n - 1}{2} > 1000$$

$$\text{or } 3^n > 2001; \text{ but } 3^6 = 729 \text{ and } 3^7 = 2187;$$

$$\therefore n > 6; \text{ but } n \text{ is a positive integer;}$$

$$\therefore n = 7, 8, 9, \dots$$

$$\therefore \text{The minimum number of terms} = 7.$$

36. The sum of series $\frac{x}{1-x^2} + \frac{x^2}{1-x^4} + \frac{x^4}{1-x^8} + \dots$ to infinite terms, if $|x| < 1$ is

- (A) $\frac{1}{1-x}$ (B) $\frac{x}{1-x}$
 (C) $\frac{1}{1+x}$ (D) $\frac{x}{1+x}$

Solution: (B)

We have,
$$t_n = \frac{x^{2^{n-1}}}{1-x^{2^{n-1}}} = \frac{1+x^{2^{n-1}}-1}{(1+x^{2^{n-1}})(1-x^{2^{n-1}})}$$

$$= \frac{1}{1-x^{2^{n-1}}} - \frac{1}{1-x^{2^n}}$$

Therefore,
$$S_n = \sum_{n=1}^n t_n$$

$$= \left[\left(\frac{1}{1-x} - \frac{1}{1-x^2} \right) + \left(\frac{1}{1-x^2} - \frac{1}{1-x^4} \right) \right.$$

$$\left. + \dots + \left(\frac{1}{1-x^{2^{n-1}}} - \frac{1}{1-x^{2^n}} \right) \right]$$

$$= \frac{1}{1-x} - \frac{1}{1-x^{2^n}}$$

∴ The sum to infinite terms

$$= \lim_{n \rightarrow \infty} S_n = \frac{1}{1-x} - 1$$

$$\left(\because \lim_{n \rightarrow \infty} x^{2^n} = 0, \text{ as } |x| < 1 \right)$$

$$= \frac{x}{1-x}$$

37. The sum to n terms of the series

$$\frac{3}{1.2} \cdot \frac{1}{2} + \frac{4}{2.3} \left(\frac{1}{2} \right)^2 + \frac{5}{3.4} \left(\frac{1}{2} \right)^3 + \dots$$
 is

- (A) $1 - \frac{1}{(n+1)2^n}$ (B) $1 - \frac{1}{(n+1)2^{n-1}}$
 (C) $1 - \frac{1}{(n-1)2^{n-1}}$ (D) None of these

Solution: (A)

We have,
$$t_n = \frac{n+2}{n(n+1)} \cdot \left(\frac{1}{2} \right)^n = \frac{2(n+1)-n}{n(n+1)} \left(\frac{1}{2} \right)^n$$

$$= \frac{1}{n} \left(\frac{1}{2} \right)^{n-1} - \frac{1}{n+1} \left(\frac{1}{2} \right)^n$$

∴
$$S_n = \sum_{n=1}^n t_n$$

$$= \left[\frac{1}{1} \left(\frac{1}{2} \right) - \frac{1}{2} \left(\frac{1}{2} \right)^1 \right] + \left[\frac{1}{2} \left(\frac{1}{2} \right)^1 - \frac{1}{3} \left(\frac{1}{2} \right)^2 \right]$$

$$+ \dots + \left[\frac{1}{n} \left(\frac{1}{2} \right)^{n-1} - \frac{1}{n+1} \left(\frac{1}{2} \right)^n \right]$$

$$= 1 - \frac{1}{(n+1)2^n}$$

38. If $(1-y)(1+2x+4x^2+8x^3+16x^4+32x^5) = 1-y^6$, ($y \neq 1$), then a value of y/x is

- (A) $\frac{1}{2}$ (B) 2
 (C) $\frac{1}{4}$ (D) 4

Solution: (B)

We can rewrite the given expression as

$$(1-y) \frac{[1-(2x)^6]}{1-2x} = 1-y^6,$$

one of the possible values of y is clearly $2x$. Therefore, one of the possible values of y/x is 2.

39. The value of $(0.2)^{\log_{\sqrt{5}} \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \right)}$ is

- (A) 1 (B) 2
 (C) $\frac{1}{2}$ (D) 4

Solution: (D)

We have,
$$\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \frac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2} = 2^{-1}.$$

∴
$$(0.2)^{\log_{\sqrt{5}} \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \right)} = \left(\frac{1}{5} \right)^{\log_{\sqrt{5}} 2^{-1}}$$

$$= (5^{-1})^{\frac{-1}{1/2} \log_5 2}$$

$$= 5^{2 \log_5 2} = 5^{\log_5 (2^2)} = 2^2 = 4$$

40. $i - 2 - 3i + 4 \dots$ to 100 terms =

- (A) $50(1 - i)$ (B) $25i$
 (C) $25(1 + i)$ (D) $100(1 - i)$

Solution: (A)

$$S = i + 2i^2 + 3i^3 + \dots + 100i^{100}$$

$$S \cdot i = i^2 + 2i^3 + \dots + 99i^{100} + 100i^{101}$$

$$\therefore S(1 - i) = (i + i^2 + i^3 + \dots + 100 \text{ terms}) - 100i^{101}$$

$$= \frac{i(1 - i^{100})}{1 - i} - 100i = \frac{i(1 - 1)}{1 - i} - 100i = -100i$$

$$\therefore S = \frac{-100i}{1 - i} = \frac{-100i(1 + i)}{2} = 50(1 - i)$$

41. The largest value of the positive integer k for which $n^k + 1$ divides $1 + n + n^2 + \dots + n^{127}$ is divisible by

- (A) 8 (B) 16
 (C) 32 (D) 64

Solution: (D)

We have,

$$1 + n + n^2 + \dots + n^{127} = \frac{n^{128} - 1}{n - 1}$$

$$= \frac{(n^{64} - 1)(n^{64} + 1)}{n - 1}$$

$$= (1 + n + n^2 + \dots + n^{63})(n^{64} + 1)$$

$\therefore k = 64$ which is divisible by 8, 16, 32 and 64.

42. If $A = 1 + r^a + r^{2a} + r^{3a} + \dots \infty$ and

If $B = 1 + r^b + r^{2b} + r^{3b} + \dots \infty$, then $\frac{a}{b}$ is equal to

- (A) $\log_B A$ (B) $\log_{1-B}(1 - A)$
 (C) $\log_{\frac{B-1}{B}} \left(\frac{A-1}{A} \right)$ (D) None of these

Solution: (C)

$$A = \frac{1}{1 - r^a} \Rightarrow 1 - r^a = \frac{1}{A} \Rightarrow r^a = 1 - \frac{1}{A} = \frac{A-1}{A}$$

$$B = \frac{1}{1 - r^b} \Rightarrow 1 - r^b = \frac{1}{B} \Rightarrow r^b = 1 - \frac{1}{B} = \frac{B-1}{B}$$

$$\therefore a \log r = \log \left(\frac{A-1}{A} \right) \text{ and } b \log r = \log \left(\frac{B-1}{B} \right)$$

$$\therefore \frac{a}{b} = \frac{\log \left(\frac{A-1}{A} \right)}{\log \left(\frac{B-1}{B} \right)}$$

$$= \log_{\frac{B-1}{B}} \left(\frac{A-1}{A} \right)$$

43. Sum of the series : $(1 + x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + \dots$ upto n terms is

- (A) $\frac{1}{1-x} \left[n - \frac{x^2(1-x^n)}{1-x} \right]$
 (B) $\frac{1}{1-x} \left[n - \frac{x^3(1-x^n)}{1-x} \right]$
 (C) $\frac{1}{1-x} \left[n - \frac{x(1-x^n)}{1-x} \right]$
 (D) None of these

Solution: (A)

We have,

$$(1 + x) + (1 + x + x^2) + (1 + x + x^2 + x^3) + \dots \text{ upto } n \text{ terms}$$

$$= \frac{1-x^2}{1-x} + \frac{1-x^3}{1-x} + \frac{1-x^4}{1-x} + \dots \text{ to } n \text{ terms}$$

$$= \frac{1}{1-x} [(1 + 1 + 1 + \dots n \text{ terms}) - (x^2 + x^3 + x^4 + \dots \text{ to } n \text{ terms})]$$

$$= \frac{1}{1-x} \left[n - \frac{x^2(1-x^n)}{1-x} \right]$$

Properties of G.P.

1. If a_1, a_2, a_3, \dots are in G.P., then

- (a) a_1k, a_2k, a_3k, \dots are also in G.P.
 (b) $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \dots$ are also in G.P.
 (c) $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots$ are also in G.P.
 (d) $a_1^k, a_2^k, a_3^k, \dots$ are also in G.P.

2. If a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots are two G.P.s, then

- (a) $a_1b_1, a_2b_2, a_3b_3, \dots$ are also in G.P.
 (b) $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots$ are also in G.P.
 (c) $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3, \dots$ may not be in G.P.

3. If $a_1, a_2, a_3, \dots, a_n$ are in G.P., then
 (a) $a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2} = \dots$
 (b) $a_r = \sqrt{a_{r-k} a_{r+k}}, 0 \leq k \leq n - r$.
4. If a_1, a_2, a_3, \dots is a G.P. of positive terms, then $\log a_1, \log a_2, \log a_3, \dots$ is also an A.P. and vice-versa.
5. If $a_1, a_2, a_3, \dots, a_{n-2}, a_{n-1}, a_n$ are in G.P., then $a_1 a_n = (a_2 a_{n-1}) = (a_3 a_{n-2}) = \dots = a_r a_{n-(r-1)}$
 In a G.P., the product of two terms equidistant from the beginning and end is a constant and is equal to the product of first term and last term.
6. If first term of a G.P. of n terms is a and last term is l , then the product of all terms of the G.P. is $(al)^{n/2}$.
7. If there be n quantities in G.P. whose common ratio is r and S_m denotes the sum of the first m terms, then the sum of their product taken two by two is $\frac{r}{r+1} S_n S_{n-1}$.
8. If $a^{x_1}, a^{x_2}, a^{x_3}, \dots, a^{x_n}$ are in G.P., then $x_1, x_2, x_3, \dots, x_n$ will be in A.P.
9. Product of a G.P.

Case I: If number of terms is odd, then Product of terms = (middle term)^{No. of terms}

Case II: If number of terms is even, then Product of terms = (Geometric mean of middle terms)^{No. of terms}



REMEMBER

Equal non zero numbers are in G.P.

SOLVED EXAMPLE

44. If the sum of three numbers in G.P. is 63 and the product of the first and the second term is $\frac{3}{4}$ of the third term, then the numbers are
 (A) 3, 12, 48 (B) 4, 12, 36
 (C) 2, 10, 50 (D) None of these

Solution: (A)

Let the three numbers be a, ar, ar^2

Given $a + ar + ar^2 = 63$ (1)

and $a \cdot ar = \frac{3}{4} \cdot ar^2$ or $a = \frac{3}{4} r$ (2)

Putting in (1), $\frac{3}{4} r + \frac{3}{4} r \cdot r + \frac{3}{4} r \cdot r^2 = 63$

or $r^3 + r^2 + r - 84 = 0$

or $(r - 4)(r^2 + 5r + 21) = 0;$

$\therefore r = 4, \frac{-5 \pm \sqrt{25 - 84}}{2}$

\therefore Real value of r is 4. Putting this value in (2),

$a = \frac{3}{4} \times 4 = 3.$

\therefore The three numbers are, $3, 3 \times 4, 3 \times 4^2$,
 i.e., 3, 12, 48.

GEOMETRIC MEAN (G.M.)

Single Geometric Mean

A number G is said to be the *single geometric mean* between two given numbers a and b if a, G, b are in G.P.

For example, since 2, 4, 8 are in G.P., therefore 4 is the G.M. between 2 and 8.

n-Geometric Means

The numbers G_1, G_2, \dots, G_n are said to be the *n geometric means* between two given positive numbers a and b if $a, G_1, G_2, \dots, G_n, b$ are in G.P.

For example, since 1, 2, 4, 8, 16 are in G.P., therefore 2, 4, 8 are three geometric means between 1 and 16.

Inserting Single G.M. between Two given Numbers

Let a and b be two given positive numbers and G be the G.M. between them. Then a, G, b are in G.P. Thus

$\frac{G}{a} = \frac{b}{G}$ or $G^2 = ab$, or $G = \sqrt{ab}$ ($\because G > 0$)

Inserting n-Geometric Means between Two given Numbers

Let $G_1, G_2, G_3, \dots, G_n$ be the *n geometric means* between two given numbers a and b . Then, $a, G_1, G_2, G_3, \dots, G_n, b$ are in G.P.

Now, $b = (n + 2)$ th term of G.P.

$= ar^{n+1}$, where r is the common ratio

or $r^{n+1} = \frac{b}{a}$ or $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

and $G_1 = ar = a \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$$G_2 = ar^2 = a \left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$\vdots \quad \quad \quad \vdots$$

$$G_n = ar^n = a \left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

TRICK(S) FOR PROBLEM SOLVING

- The product of n geometric means between two given numbers is n th power of the single G.M. between them i.e. if a and b are two given numbers and G_1, G_2, \dots, G_n are n geometric means between them, then

$$G_1 G_2 G_3 \dots G_n = (\sqrt[n]{ab})^n.$$

- If A and G are respectively arithmetic and geometric means between two positive numbers a and b then

(A) $A > G$

- (B) the quadratic equation having a, b as its roots is

$$x^2 - 2Ax + G^2 = 0$$

- (C) the two positive numbers are $A \pm \sqrt{A^2 - G^2}$.

- If number of terms of any A.P./G.P. is odd, then A.M./G.M. of first and last terms is middle term of series.
- If number of terms of any A.P./G.P. is even, then A.M./G.M. of middle two terms is A.M./G.M./H.M. of first and last terms respectively.
- If $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P. are in G.P., then p, q, r are in A.P.
- If a, b, c are in A.P. as well as in G.P. then $a = b = c$.
- If a, b, c are in A.P., then x^a, x^b, x^c will be in G.P. ($x \neq \pm 1$)
- Value of recurring decimal

- If $R = 0.bbb \dots = 0.\dot{b}$, then $R = \frac{b}{10^1 - 1}$

For example, $0.\dot{5} = \frac{5}{10^1 - 1} = \frac{5}{9}$

- If $R = 0.ab ab ab \dots = 0.\dot{ab}$, then $R = \frac{ab}{10^2 - 1}$

For example, $0.\dot{37} = \frac{37}{10^2 - 1} = \frac{37}{99}$

- If $R = 0.abc abc abc \dots = 0.\dot{abc}$ then $R = \frac{abc}{10^3 - 1}$ and so on.

For example, $0.\dot{234} = \frac{234}{10^3 - 1} = \frac{234}{999} = \frac{26}{111}$

- If $R = 0.X\dot{Y}$ and x denotes the number of digits in X and y denotes the number of digits in Y , then

$$R = \frac{XY - X}{10^{x+y} - 10^x}$$

For example, if $R = 0.43\dot{6}2$, then

$$R = \frac{4362 - 43}{10^4 - 10^2} = \frac{4319}{9900}$$

SOLVED EXAMPLES

45. If two geometric means g_1 and g_2 and one arithmetic mean A be inserted between two numbers, then

$$\frac{g_1^2}{g_2} + \frac{g_2^2}{g_1} =$$

- (A) $4A$ (B) $3A$
(C) $2A$ (D) A

Solution: (C)

Let the two numbers be a and b .

$$\therefore A = \frac{a+b}{2} \text{ or } 2A = a+b \tag{1}$$

Again, a, g_1, g_2, b are in G.P.

$$\therefore \frac{g_1}{a} = \frac{g_2}{g_1} = \frac{b}{g_2}$$

Now $\frac{g_1}{a} = \frac{g_2}{g_1} \Rightarrow \frac{g_1^2}{a} = g_2$

and $\frac{g_2}{g_1} = \frac{b}{g_2} \Rightarrow \frac{g_2^2}{g_1} = b$

$$\therefore a+b = \frac{g_1^2}{g_2} + \frac{g_2^2}{g_1} \tag{2}$$

\therefore From Eqs (1) and (2), we get

$$2A = \frac{g_1^2}{g_2} + \frac{g_2^2}{g_1}$$

46. Let $a = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$. Then

- (A) $5^5 \geq a$ (B) $3^5 \geq 5!$
(C) $5^5 \geq 6a$ (D) None of these

Solution: (A, B)

Since A.M. > G.M.

$$\Rightarrow \frac{1+2+3+4+5}{5} \geq \sqrt[5]{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}$$

$$\Rightarrow 3 \geq \sqrt[5]{a} \Rightarrow 3^5 \geq a = 5!$$

Also, $5^5 \geq 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = a$.

47. If a, b, c are positive then the minimum value of $a^{\log b - \log c} + b^{\log c - \log a} + c^{\log a - \log b}$ is

- (A) 3 (B) 1
(C) 9 (D) 16

Solution: (A)

Since A.M. \geq G.M.

$$\therefore \frac{a^{\log b - \log c} + b^{\log c - \log a} + c^{\log a - \log b}}{3} \geq \sqrt[3]{a^{\log b - \log c} \cdot b^{\log c - \log a} \cdot c^{\log a - \log b}} \quad (1)$$

Let $x = a^{\log b - \log c} \cdot b^{\log c - \log a} \cdot c^{\log a - \log b}$

$$\Rightarrow \log x = (\log b - \log c) \log a + (\log c - \log a) \log b + (\log a - \log b) \log c$$

$$= 0 \Rightarrow x = 1.$$

\therefore From (1),
 $a^{\log b - \log c} + b^{\log c - \log a} + c^{\log a - \log b} \geq 3.$

SOME SPECIAL SEQUENCES

1. The sum of first n natural numbers

$$\Sigma n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}.$$

2. The sum of squares of first n natural numbers

$$\Sigma n^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. The sum of cubes of the first n natural numbers

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2.$$

SOLVED EXAMPLES

48. The sum of all natural numbers less than 200, that are divisible neither by 3 nor by 5, is

- (A) 10730 (B) 10732
(C) 15375 (D) None of these

Solution: (B)

The required sum

$$\begin{aligned} &= (1 + 2 + 3 + \dots + 199) - (3 + 6 + 9 + \dots + 198) \\ &\quad - (5 + 10 + 15 + \dots + 195) + (15 + 30 + 45 + \dots + 195) \\ &= \frac{199}{2}(1 + 199) - \frac{66}{2}(3 + 198) - \frac{39}{2}(5 + 195) \\ &\quad + \frac{13}{2}(15 + 195) \\ &= 199 \times 100 - 33 \times 201 - 39 \times 100 + 13 \times 105 \\ &= 10732 \end{aligned}$$

49. Sum to 20 terms of the series $1.3^2 + 2.5^2 + 3.7^2 + \dots$ is
(A) 178090 (B) 168090
(C) 188090 (D) None of these

Solution : (C)

We have,

$$\begin{aligned} t_n &= [n\text{th term of } 1, 2, 3, \dots] \times [n\text{th term of } 3, 5, 7, \dots]^2 \\ &= n(2n+1)^2 = 4n^3 + 4n^2 + n. \\ \therefore S_n &= \Sigma t_n = 4 \Sigma n^3 + 4 \Sigma n^2 + \Sigma n \\ &= 4 \cdot \left[\frac{n(n+1)}{2} \right]^2 + 4 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\ &= n^2(n+1)^2 + \frac{2}{3}n(n+1)(2n+1) + \frac{1}{2}n(n+1); \\ \therefore S_{20} &= 20^2 \cdot 21^2 + \frac{2}{3} \times 20 \cdot 21 \cdot 41 + \frac{1}{2} \cdot 20 \cdot 21 = 188090 \end{aligned}$$

50. Number of terms in the sequence 1, 3, 6, 10, 15, ..., 5050 is

- (A) 50 (B) 75
(C) 100 (D) 125

Solution: (C)

Let $S = 1 + 3 + 6 + 10 + 15 + \dots + t_n$ (1)

then $S = 1 + 3 + 6 + 10 + \dots + t_{n-1} + t_n$ (2)

(1) - (2) $\Rightarrow 0 = (1 + 2 + 3 + 4 + \dots \text{ to } n \text{ terms}) - t_n$

$$\Rightarrow t_n = \frac{n(n+1)}{2}$$

Given, $5050 = \frac{n(n+1)}{2} \Rightarrow n^2 + n - 10100 = 0$

$$\begin{aligned} \Rightarrow n &= \frac{-1 \pm \sqrt{1+40400}}{2} \\ &= \frac{-1 \pm \sqrt{40401}}{2} \\ &= \frac{-1 \pm 201}{2} = -101, 100 \end{aligned}$$

$$\therefore n = 100. \quad (\because n \text{ is a positive integer})$$

$$51. \sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j 1 =$$

- (A) $\frac{n(n+1)(2n+1)}{6}$ (B) $\frac{n(n+1)}{2}$
 (C) $\left(\frac{n(n+1)}{2}\right)^2$ (D) $\frac{n(n+1)(n+2)}{6}$

Solution: (D)

We have,

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^i \sum_{k=1}^j 1 &= \sum_{i=1}^n \sum_{j=1}^i j \\ &= \sum_{i=1}^n \frac{i(i+1)}{2} = \frac{1}{2} \left[\sum_{i=1}^n i^2 + \sum_{i=1}^n i \right] \\ &= \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right] \\ &= \frac{n(n+1)(n+2)}{6} \end{aligned}$$

52. The sum of the products of the $2n$ numbers $\pm 1, \pm 2, \pm 3, \dots, \pm 2n$ taking two at a time is

- (A) $-\frac{n(n+1)}{2}$ (B) $\frac{n(n+1)(2n+1)}{6}$
 (C) $-\frac{n(n+1)(2n+1)}{6}$ (D) None of these

Solution: (C)

$$\begin{aligned} \text{We have, } (1-1+2-2+3-3+\dots+n-n)^2 \\ = 1^2+1^2+2^2+2^2+\dots+n^2+n^2+2S, \end{aligned}$$

where S is the required sum

$$\begin{aligned} \Rightarrow 0 &= 2(1^2+2^2+\dots+n^2)+2S \\ \Rightarrow S &= -(1^2+2^2+\dots+n^2) = -\frac{n(n+1)(2n+1)}{6} \end{aligned}$$

TRICK(S) FOR PROBLEM SOLVING

If n th term of a sequence is

$$T_n = an^3 + bn^2 + cn + d,$$

then the sum of n terms is given by,

$$S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d,$$

which can be evaluated using the above results.

SOLVED EXAMPLES

53. The sum of the series $\frac{9}{5^2 \cdot 2 \cdot 1} + \frac{13}{5^3 \cdot 3 \cdot 2} + \frac{17}{5^4 \cdot 4 \cdot 3} + \dots$ to infinite terms, is

- (A) $\frac{2}{5}$ (B) $\frac{1}{5}$
 (C) 1 (D) None of these

Solution: (B)

The general term of the series is

$$t_r = \frac{4r+1}{5^r \cdot r(r-1)}, \text{ where } r \geq 2$$

$$= \frac{5r - (r-1)}{5^r \cdot r(r-1)} = \frac{1}{5^{r-1}(r-1)} - \frac{1}{5^r \cdot r}$$

$$\begin{aligned} \therefore \sum_{r=2}^{\infty} t_r &= \left(\frac{1}{5^1 \cdot 1} - \frac{1}{5^2 \cdot 2} \right) + \left(\frac{1}{5^2 \cdot 2} - \frac{1}{5^3 \cdot 3} \right) + \left(\frac{1}{5^3 \cdot 3} - \frac{1}{5^4 \cdot 4} \right) \\ &\quad + \dots \text{ to infinity} \\ &= \frac{1}{5} \quad (\because \text{terms tend to zero as } n \rightarrow \infty) \end{aligned}$$

54. For any odd integer $n \geq 1$,
 $n^3 - (n-1)^3 + \dots + (-1)^{n-1} 1^3 =$

- (A) $\frac{1}{2}(n-1)^2(2n-1)$
 (B) $\frac{1}{4}(n-1)^2(2n-1)$
 (C) $\frac{1}{2}(n+1)^2(2n-1)$
 (D) $\frac{1}{4}(n+1)^2(2n-1)$

Solution: (D)

Since n is an odd integer $(-1)^{n-1} = 1$ and $n-1, n-3, n-5$ etc., are even integers. We have

$$\begin{aligned}
 & n^3 - (n-1)^3 + (n-2)^3 - (n-3)^3 + \dots + (-1)^{n-1} 1^3 \\
 &= n^3 + (n-1)^3 + (n-2)^3 + \dots + 1^3 \\
 &\quad - 2[(n-1)^3 + (n-3)^3 + \dots + 2^3] \\
 &= n^3 + (n-1)^3 + (n-2)^3 + \dots + 1^3 \\
 &\quad - 2 \times 2^3 \left[\left(\frac{n-1}{2} \right)^3 + \left(\frac{n-3}{2} \right)^3 + \dots + 1^3 \right] \\
 &\quad [\because n-1, n-3 \text{ are even integers}] \\
 &= \left[\frac{n(n+1)}{2} \right]^2 - 16 \left[\frac{1}{2} \left(\frac{n-1}{2} \right) \left(\frac{n-1}{2} + 1 \right) \right]^2 \\
 &= \frac{1}{4} n^2(n+1)^2 - 16 \frac{(n-1)^2(n+1)^2}{16 \times 4} \\
 &= \frac{1}{4} (n+1)^2 [n^2 - (n-1)^2] = \frac{1}{4} (n+1)^2(2n-1).
 \end{aligned}$$

ARITHMETICO-GEOMETRIC PROGRESSION (A.G.P.)

If $a_1, a_2, a_3, \dots, a_n, \dots$ is an A.P. and $b_1, b_2, \dots, b_n, \dots$ is G.P. then the sequence $a_1b_1, a_2b_2, a_3b_3, \dots, a_nb_n, \dots$ is said to be an arithmetico-geometric sequence.

Thus, the general form of an arithmetico geometric sequence is $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \dots$

n^{th} term of A.G.P.

From the symmetry we obtain that the n^{th} term of this sequence is $[a + (n-1)d]r^{n-1}$.

Also, let $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \dots$ be an arithmetico-geometric sequence.

Then, $a + (a+d)r + (a+2d)r^2 + (a+3d)r^3 + \dots$ is an arithmetico-geometric series.

Sum of A.G.P.

- Sum to n terms:** The sum of n terms of an arithmetico-geometric sequence $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \dots$ is given by,

$$S_n = \begin{cases} \frac{a}{1-r} + dr \frac{(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}, & \text{when } r \neq 1 \\ \frac{n}{2}[2a+(n-1)d], & \text{when } r = 1 \end{cases}$$

- Sum to infinite terms:** Let $|r| < 1$. Then $r^n, r^{n-1} \rightarrow 0$ as $n \rightarrow \infty$ and it can also be shown that $n \cdot r^n \rightarrow 0$ as $n \rightarrow \infty$. So, we obtain that $S_n \rightarrow \frac{a}{1-r} + \frac{dr}{(1-r)^2}$, as $n \rightarrow \infty$.

In other words, when $|r| < 1$ the sum to infinity of an arithmetico-geometric series is $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$.

METHOD FOR FINDING SUM OF A.G. SERIES

Method of Differences

Suppose a_1, a_2, a_3, \dots is a sequence such that the sequence $a_2 - a_1, a_3 - a_2, \dots$ is either an A.P. or a G.P. The n^{th} term ' a_n ' of this sequence is obtained as follows:

$$S = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n$$

$$S = a_1 + a_2 + \dots + a_{n-1} + a_n$$

$$\Rightarrow a_n = a_1 + [a_2 - a_1] + [a_3 - a_2] + \dots + [a_n - a_{n-1}]$$

Since the terms within the brackets are either in an A.P. or in a G.P., we can find the value of a_n , the n^{th} term, we can now find the sum of the n terms of the sequence as

$$S = \sum_{k=1}^n a_k$$

SOLVED EXAMPLES

- Find the sum to n terms of the series:

- (A) $2 + 5 + 10 + 17 + \dots$
 (B) $3 + 5 + 9 + 17 + \dots$

Solution

- (A) Here, the difference in consecutive terms are 3, 5, 7, ... which are in A.P.

$$\text{Let } S_n = 2 + 5 + 10 + 17 + \dots + t_{n-1} + t_n$$

Shifting every term one place to the right

$$S_n = 2 + 5 + 10 + \dots + t_{n-1} + t_n$$

Subtracting, we get $0 = (2 + 3 + 5 + 7 + \dots \text{ to } n \text{ terms}) - t_n$

$$\Rightarrow t_n = 2 + [3 + 5 + 7 + \dots \text{ to } (n-1) \text{ terms}]$$

$$= 2 + \frac{n-1}{2} [2 \times 3 + (n-2) \times 2]$$

$$= 2 + \frac{n-1}{2} (2n+2) = 2 + (n-1)(n+1)$$

$$= 2 + (n^2 - 1) = n^2 + 1.$$

Putting $n = 1, 2, 3, \dots, n$ and adding, we get

$$S_n = \sum_{k=1}^n k^2 + n = \frac{n(n+1)(2n+1)}{6} + n$$

$$= \frac{n}{6} (2n^2 + 3n + 1 + 6) = \frac{n}{6} (2n^2 + 3n + 7)$$

- (B) Here the differences of consecutive terms are 2, 4, 8, ... which are in G.P.

Let $S_n = 3 + 5 + 9 + 17 + \dots + t_{n-1} + t_n$

Shifting every term one place to the right

$$S_n = 3 + 5 + 9 + \dots + t_{n-1} + t_n$$

Subtracting, we get $0 = (3 + 2 + 4 + 8 + \dots \text{ to } n \text{ terms})$

$$\begin{aligned} -t_n \\ \Rightarrow t_n = 3 + [2 + 4 + 8 + \dots \text{ to } (n-1) \text{ terms}] \end{aligned}$$

$$= 3 + \frac{2(2^{n-1}-1)}{2-1} = 3 + 2^n - 2$$

$$= 1 + 2^n$$

Putting $n = 1, 2, 3, \dots, n$, we get

$$t_1 = 1 + 2^1$$

$$t_2 = 1 + 2^2$$

$$t_3 = 1 + 2^3$$

\vdots

$$t_n = 1 + 2^n$$

Adding column-wise, we get

$$S_n = n + (2 + 2^2 + 2^3 + \dots + 2^n)$$

$$= n + \frac{2(2^n - 1)}{2 - 1} = 2^{n+1} + n - 2$$

56. Sum to infinity of the series

$$\frac{2}{3} - \frac{5}{6} + \frac{2}{3} - \frac{11}{24} + \dots \text{ is}$$

- (A) $\frac{4}{9}$ (B) $\frac{1}{3}$
 (C) $\frac{2}{9}$ (D) None of these

Solution: (C)

$$\text{Let } S = \frac{2}{3} - \frac{5}{6} + \frac{2}{3} - \frac{11}{24} + \dots \text{ to } \infty \quad (1)$$

Multiplying both sides by $-\frac{1}{2}$, the common ratio of G.P.

$$-\frac{1}{2}S = -\frac{2}{6} + \frac{5}{12} - \frac{8}{24} + \dots \text{ to } \infty \quad (2)$$

Subtracting Eqs (2) from (1), we have

$$\begin{aligned} \frac{3}{2}S &= \frac{2}{3} - \frac{3}{6} + \frac{3}{12} - \frac{3}{24} + \dots \text{ to } \infty \\ &= \frac{2}{3} - \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \dots \text{ to } \infty \right) \\ &= \frac{2}{3} - \frac{\frac{1}{2}}{1 - \left(-\frac{1}{2}\right)} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \end{aligned}$$

$$\therefore S = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

57. The sum of first n terms of the series

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! + \dots \text{ is}$$

- (A) $(n+1)! - 1$ (B) $n! - 1$
 (C) $(n-1)! - 1$ (D) None of these

Solution: (A)

$$\text{Let } S_n = 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! + \dots + n \cdot n!$$

$$\begin{aligned} \Rightarrow S_n &= (2-1)1! + (3-1)2! + (4-1)3! \\ &\quad + (5-1)4! + \dots + [(n+1)-1]n! \\ &= (2 \cdot 1! - 1!) + (3 \cdot 2! - 2!) + (4 \cdot 3! - 3!) \\ &\quad + (5 \cdot 4! - 4!) + \dots + [(n+1)n! - n!] \\ &= (2! - 1!) + (3! - 2!) + (4! - 3!) + (5! - 4!) \\ &\quad + \dots + [(n+1)! - n!] \\ &= (n+1)! - 1! = (n+1)! - 1. \end{aligned}$$

58. If a, b, c are digits, then the rational number represented by $0 \cdot cababab \dots$ is

- (A) $\frac{99c+ab}{990}$ (B) $\frac{99c+10a+b}{99}$
 (C) $\frac{99c+10a+b}{990}$ (D) None of these

Solution: (C)

$$\text{Let } R = 0 \cdot cababab \dots$$

$$\Rightarrow 10^2 R = ca \cdot bababa \dots$$

$$\text{and } 10^4 R = caba \cdot baba \dots$$

$$\Rightarrow (10^4 - 10^2)R = caba - ca$$

$$\Rightarrow R = \frac{caba - ca}{9900} = \frac{1000c + 100a + 10b + a - 10c - a}{9900}$$

$$= \frac{99c + 10a + b}{990}$$

59. The sum of first n terms of the series

$$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 5 \cdot 6^2 + \dots \text{ is } \frac{n(n+1)^2}{2}$$

when n is even. When n is odd the sum is

- (A) $\frac{n^2(n+1)}{2}$ (B) $\frac{n(n+1)^2}{2}$
 (C) $\left[\frac{n(n+1)}{2} \right]^2$ (D) $\frac{n(n+1)}{2}$

Solution: (A)

When n is odd, last term will be n^2 , \therefore then the sum is

$$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots + 2(n-1)^2 + n^2$$

$$= \frac{(n-1)n^2}{2} + n^2 \left[\text{Replacing } n \text{ by } n-1 \text{ in } \frac{n(n+1)^2}{2} \right]$$

$$= \frac{n^3 - n^2 + 2n^2}{2} = \frac{n^3 + n^2}{2} = \frac{n^2(n+1)}{2}$$

60. The sum of the series

$1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + 5 \cdot 2^4 + \dots + 100 \cdot 2^{99}$ is

(A) $99 \cdot 2^{100} + 1$ (B) $100 \cdot 2^{100}$
 (C) $99 \cdot 2^{100}$ (D) $99 \cdot 2^{100} + 1$

Solution: (D)

Let $S = 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + 5 \cdot 2^4 + \dots + 100 \cdot 2^{99}$

$$\therefore 2S = 1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + 99 \cdot 2^{99} + 100 \cdot 2^{100}$$

Subtracting, we get

$$-S = 1 + 1 \cdot 2 + 1 \cdot 2^2 + \dots + 1 \cdot 2^{99} - 100 \cdot 2^{100}$$

$$= (1 + 2 + 2^2 + \dots + 2^{99}) - 100 \cdot 2^{100}$$

$$= \frac{1(2^{100} - 1)}{2 - 1} - 100 \cdot 2^{100} = 2^{100} - 1 - 100 \cdot 2^{100}$$

$$\therefore S = 100 \cdot 2^{100} - 2^{100} + 1 = 99 \cdot 2^{100} + 1.$$

EXERCISES

Single Option Correct Type

- If a, b, c are positive numbers in A.P. such that their product is 64, then the minimum value of b
 (A) = 2 (B) = 4
 (C) = 1 (D) Does not exist
- If three successive terms of a G.P. with common ratio $r (r > 1)$ form the sides of a ΔABC and $[r]$ denotes greatest integer function, then $[r] + [-r] =$
 (A) 0 (B) 1
 (C) -1 (D) None of these
- If $\sum_{j=1}^{21} a_j = 693$, where a_1, a_2, \dots, a_{21} , are in A.P., then $\sum_{i=0}^{10} a_{2i+1}$ is
 (A) 361 (B) 396
 (C) 363 (D) data insufficient
- Number of increasing geometrical progression(s) with first term unity, such that any three consecutive terms, on doubling the middle become an A.P. is
 (A) 0 (B) 1
 (C) 2 (D) infinity
- If a_1, a_2, a_3 (with $a_1 > 0$) are in G.P. with common ratio r , then the value of r for which the inequality $9a_1 + 5a_3 > 14a_2$ holds, cannot be in the interval

- (A) $\left[1, \frac{9}{2}\right]$ (B) $(-\infty, 0)$
 (C) $\left[\frac{5}{9}, 1\right]$ (D) $\left[1, \frac{9}{5}\right]$
- Let $S_n (1 \leq n \leq 9)$ denotes the sum of n terms of series $1 + 22 + 333 + \dots + 999999999$, then for $2 \leq n \leq 9$
 (A) $S_n - S_{n-1} = \frac{1}{9} (10^n - n^2 + n)$
 (B) $S_n = \frac{1}{9} (10^n - n^2 + 2n - 2)$
 (C) $9(S_n - S_{n-1}) = n(10^n - 1)$
 (D) None of these
 - If $\log_{\sqrt{5}} x + \log_{5^{1/3}} x + \log_{5^{1/4}} x + \dots$ upto 7 terms = 35, then x is equal to
 (A) 5 (B) 25
 (C) 125 (D) None of these
 - If $\sum_{n=1}^{\infty} x^{n-1} = a$ and $\sum_{n=1}^{\infty} y^{n-1} = b$ where $|x|, |y| < 1$, then $\sum_{n=1}^{\infty} (xy)^{n-1} =$
 (A) ab (B) $\frac{a+b-1}{ab}$
 (C) $\frac{1}{1-ab}$ (D) $\frac{ab}{a+b-1}$

9. Let $p, q, r \in R^+$ and $27pqr \geq (p+q+r)^3$ and $3p+4q+5r=12$ then $p^3+q^4+r^5$ is equal to
 (A) 3 (B) 6
 (C) 2 (D) None of these
10. The sum of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$ to n terms is
 (A) $\frac{n(n^2+1)}{n^2+n+1}$ (B) $\frac{n(n+1)}{2(n^2+n+1)}$
 (C) $\frac{n(n^2-1)}{2(n^2+n+1)}$ (D) None of these
11. a, b, c are three distinct real numbers, which are in G.P. and $a+b+c=xb$. Then
 (A) $x < -1$ or $x > 3$ (B) $-1 < x < 3$
 (C) $-1 < x < 2$ (D) $0 < x < 1$
12. The sum of the first hundred terms of an A.P. is x and the sum of the hundred terms starting from the third term is y . Then the common difference is
 (A) $\frac{y-x}{2}$ (B) $\frac{y-x}{50}$
 (C) $\frac{y-x}{100}$ (D) $\frac{y-x}{200}$
13. If $\lambda = \sum_{i=1}^{\infty} \frac{1}{i^4}$, then $\sum_{i=1}^{\infty} \frac{1}{(2i-1)^4}$ is
 (A) $\frac{14}{15}\lambda$ (B) $\frac{\lambda}{2}$
 (C) $\frac{16}{15}\lambda$ (D) $\frac{15}{16}\lambda$
14. The sum of all possible products of the first n natural numbers taken two at a time is
 (A) $\frac{1}{2}[\Sigma n^2 - \Sigma n]$ (B) $\frac{1}{2}[(\Sigma n)^2 - \Sigma n]$
 (C) $\frac{1}{2}[\Sigma n^2 - \Sigma(n+1)]$ (D) $\frac{1}{2}[(\Sigma n)^2 - \Sigma n^2]$
15. The minimum value of $8^{\sin x/8} + 8^{\cos x/8}$ is
 (A) $2^{\frac{1}{3+\sqrt{2}/\sqrt{2}}}$ (B) $2^{\frac{3+\sqrt{2}}{\sqrt{2}}}$
 (C) $2^{\frac{1}{3+\sqrt{2}/\sqrt{2}}}$ (D) $2^{\frac{3-\sqrt{2}}{\sqrt{2}}}$
16. If $\log_{2^{1/2}} a + \log_{2^{1/4}} a + \log_{2^{1/8}} a + \log_{2^{1/16}} a + \dots$ upto 20 terms is 840, then a is equal to
 (A) 2 (B) 1
 (C) 4 (D) $\sqrt{2}$
17. Sum to n terms of the series $\frac{1}{(1+x)(1+2x)} + \frac{1}{(1+2x)(1+3x)} + \dots$ is
 (A) $\frac{nx}{(1+x)(1+nx)}$ (B) $\frac{n}{(1+x)[1+(n+1)x]}$
 (C) $\frac{x}{(1+x)(1+(n-1)x)}$ (D) None of these
18. If a, b, c are distinct positive real numbers and $a^2+b^2+c^2=1$, then $ab+bc+ca$ is
 (A) less than 1 (B) equal to 1
 (C) greater than 1 (D) any real number
19. The value of $(n-2)^2 + (n-4)^2 + (n-6)^2 + \dots$ to n terms is
 (A) $\frac{n}{3}(n^2+2)$ (B) $\frac{n}{2}(n^2+3)$
 (C) $\frac{n}{3}(n^2-2)$ (D) $\frac{n}{2}(n^2-3)$
20. The sum to infinity of the series $1 + 2\left(1-\frac{1}{n}\right) + 3\left(1-\frac{1}{n}\right)^2 + \dots$ where $n \in N$, is given by
 (A) $n(n-1)$ (B) $n\left(1-\frac{1}{n}\right)^2$
 (C) n^2 (D) $\left(\frac{n-1}{n}\right)^2$
21. a_1, a_2, a_3, \dots are in A.P. with common difference not a multiple of 3. Then, maximum number of consecutive terms so that all the terms are prime numbers is
 (A) 2 (B) 3
 (C) 5 (D) infinite
22. The coefficient of x^{49} in the product $(x-1)(x-3)\dots(x-99)$ is
 (A) -99^2 (B) 1
 (C) -2500 (D) None of these
23. If x, y, z are three real numbers of the same sign then the value of $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ lies in the interval
 (A) $[2, \infty)$ (B) $[3, \infty)$
 (C) $(3, \infty)$ (D) $(-\infty, 3)$
24. In a G.P. of alternating positive and negative terms, any term is the A.M. of the next two terms. Then the common ratio is

- (A) -1 (B) -3
 (C) -2 (D) $\frac{-1}{2}$
25. If $A = 1 + r^a + r^{2a} + r^{3a} + \dots$ as and $B = 1 + r^b + r^{2b} + r^{3b} + \dots$ as, then $\frac{a}{b}$ is equal to
 (A) \log_B^A (B) $\log_{1-B}^{(1-A)}$
 (C) $\log_{\frac{B-1}{B}} \left(\frac{A-1}{A} \right)$ (D) None of these
26. If the sum of n terms of an A.P. is $cn(n-1)$, where $c \neq 0$, then sum of the squares of these terms is
 (A) $c^2 n^2 (n+1)^2$
 (B) $\frac{2}{3} c^2 n(n-1)(2n-1)$
 (C) $\frac{2c^2}{3} n(n+1)(2n+1)$
 (D) None of these
27. If in an A.P., $S_n = p \cdot n^2$ and $S_m = p \cdot m^2$ where S_r denotes the sum of r terms of the A.P., then S_p is equal to
 (A) $\frac{1}{2} p^3$ (B) mnp
 (C) p^3 (D) $(m+n)p^2$
28. If b_1, b_2 and b_3 ($b_1 > 0$) are three successive terms of a G.P. with common ratio r , the value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds, is given by
 (A) $r > 3$ (B) $r < 1$
 (C) $r = 2.5$ (D) $r = 1.7$
29. If p, q, r are positive and are in A.P., the roots of quadratic equation $px^2 + qx + r = 0$ are all real for
 (A) $\left| \frac{r}{p} - 7 \right| \geq 4\sqrt{3}$ (B) $\left| \frac{p}{r} - 7 \right| \geq 4\sqrt{3}$
 (C) all p and r (D) no p and r
30. The sum to n terms of the series $\frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \dots$ is
 (A) $n - \frac{(3^n - 2^n)}{2^n}$ (B) $n - \frac{2(3^n - 2^n)}{3^n}$
 (C) $2^n - 1$ (D) $3^n - 1$
31. Sum to n terms of the series $\frac{1}{5!} + \frac{1!}{6!} + \frac{2!}{7!} + \frac{3!}{8!} + \dots$ is
 (A) $\frac{2}{5!} - \frac{1}{(n+1)!}$ (B) $\frac{1}{4} \left(\frac{1}{4!} - \frac{n!}{(n+4)!} \right)$
 (C) $\frac{1}{4} \left(\frac{1}{3!} - \frac{3!}{(n+2)!} \right)$ (D) None of these
32. If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$ then a, b, c, d are in
 (A) A.P. (B) G.P.
 (C) H.P. (D) $ab = cd$
33. If $a + b + c = 3$ and $a > 0, b > 0, c > 0$, then the greatest value of $a^2 b^3 c^2$ is
 (A) $\frac{3^{10} \cdot 2^4}{7^7}$ (B) $\frac{3^9 \cdot 2^4}{7^7}$
 (C) $\frac{3^8 \cdot 2^4}{7^7}$ (D) None of these
34. If $\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix} = 0$ and $\alpha \neq \frac{1}{2}$, then
 (A) a, b, c are in A.P. (B) a, b, c are in G.P.
 (C) a, b, c are in H.P. (D) None of these
35. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is
 (A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$
 (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$
36. If a_1, a_2, \dots, a_n are in A.P. with common difference $d \neq 0$, then sum of the series $\sin d [\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \dots + \sec a_{n-1} \sec a_n]$ is
 (A) $\tan a_n - \tan a_1$ (B) $\cot a_n - \cot a_1$
 (C) $\sec a_n - \sec a_1$ (D) $\operatorname{cosec} a_n - \operatorname{cosec} a_1$
37. The first and last term of an A.P. are a and l respectively. If S is the sum of all the terms of the A.P. and the common difference is $\frac{l^2 - a^2}{k - (l + a)}$, then k is equal to
 (A) S (B) $2S$
 (C) $3S$ (D) None of these
38. If a, b, c, d are in G.P., then
 $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) =$
 (A) $(ab + ac + bc)^2$ (B) $(ac + cd + ad)^2$
 (C) $(ab + bc + cd)^2$ (D) None of these

39. If one geometric mean G and two arithmetic means p and q be inserted between two numbers, then G^2 is equal to
 (A) $(3p - q)(3q - p)$ (B) $(2p - q)(2q - p)$
 (C) $(4p - q)(4q - p)$ (D) None of these
40. The product of n positive integers is 1, then their sum is a positive integer, that is
 (A) equal to 1 (B) equal to $n + n^2$
 (C) divisible by n (D) never less than n
41. A man saves ₹200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹40 more than the saving of immediately previous months. His total saving from the start of service will be ₹11040 after
 (A) 21 months (B) 18 months
 (C) 19 months (D) 20 months
42. **Statement-1:** The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \dots + (361 + 380 + 400)$ is 8000.
Statement-2: $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$, for any natural number n .
 (A) Statement-1 is false, Statement-2 is true.
 (B) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1
 (C) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
 (D) Statement-1 is true, statement-2 is false.
43. If 100 times the 100th term of an AP with non-zero common difference equals the 50 times its 50th term, then the 150th term of this AP is
 (A) -150
 (B) 150 times its 50th term
 (C) 150
 (D) zero
44. If the sum of first n terms of two A.P.'s are in the ratio $3n + 8 : 7n + 15$, then the ratio of their 12th terms is
 (A) 8 : 7 (B) 7 : 16
 (C) 74 : 169 (D) 13 : 47
45. The sum of n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$ is
 (A) $2^n - n - \frac{1}{2}$ (B) $1 - 2^{-n}$
 (C) $n + 2^{-n} - 1$ (D) $\frac{1}{2}(2^n - 1)$
46. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is
 (A) -4 (B) -12
 (C) 12 (D) 4
47. The sum to the infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^2} + \frac{14}{3^4} + \dots$ is
 (A) 2 (B) 3
 (C) 4 (D) 6
48. The sum of positive terms of the series $10 + 9\frac{4}{7} + 9\frac{1}{7} + \dots$ is
 (A) $\frac{352}{7}$ (B) $\frac{437}{7}$
 (C) $\frac{852}{7}$ (D) None of these
49. The sum of the products of the $2n$ numbers $\pm 1, \pm 2, \pm 3, \dots, \pm n$ taking two at a time is
 (A) $\frac{n(n+1)}{2}$ (B) $-\frac{n(n+1)}{2}$
 (C) $\frac{n(n+1)(2n+1)}{6}$ (D) $-\frac{n(n+1)(2n+1)}{6}$
50. If a is the first term, d the common difference and S_k the sum to k terms of an A.P., then for $\frac{S_{kx}}{S_x}$ to be independent of x
 (A) $a = 2d$ (B) $a = d$
 (C) $2a = d$ (D) None of these
51. Given that α, γ are roots of the equation $Ax^2 - 4x + 1 = 0$ and β, δ are roots of the equation $Bx^2 - 6x + 1 = 0$. If α, β, γ and δ are in H.P., then
 (A) $A = 5$ (B) $A = -3$
 (C) $B = 8$ (D) $B = -8$
52. The sum of n terms of m A.P.s are $S_1, S_2, S_3, \dots, S_m$. If the first term and common difference are $1, 2, 3, \dots, m$ respectively, then $S_1 + S_2 + S_3 + \dots + S_m =$
 (A) $\frac{1}{4} mn(m+1)(n+1)$
 (B) $\frac{1}{2} mn(m+1)(n+1)$
 (C) $mn(m+1)(n+1)$
 (D) None of these

53. If three positive numbers a, b, c are in H.P., then $a^n + c^n$
- (A) $> 2b^n$ (B) $= 2b^n$
 (C) $< 2b^n$ (D) $> b^n$
54. The sum of first n terms of the series $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! + \dots$ is
- (A) $(n+1)! - 1$ (B) $n! - 1$
 (C) $(n-1)! - 1$ (D) None of these
55. If a, b, c are digits, then the rational number represented by $0 \cdot cababab \dots$ is
- (A) $\frac{99c+ab}{990}$ (B) $\frac{99c+10a+b}{99}$
 (C) $\frac{99c+10a+b}{990}$ (D) None of these
56. The sum of first n terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 5 \cdot 6^2 + \dots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd, the sum is
- (A) $\frac{n^2(n+1)}{2}$ (B) $\frac{n(n+1)^2}{2}$
 (C) $\left[\frac{n(n+1)}{2}\right]^2$ (D) $\frac{n(n+1)}{2}$
57. The sum of the series $1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + 5 \cdot 2^4 + \dots + 100 \cdot 2^{99}$ is
- (A) $99 \cdot 2^{100} + 1$ (B) $100 \cdot 2^{100}$
 (C) $99 \cdot 2^{100}$ (D) $99 \cdot 2^{100} + 1$
58. Four different integers form an increasing A.P. If one of these numbers is equal to the sum of the squares of the other three numbers, then the numbers are
- (A) $-2, -1, 0, 1$ (B) $0, 1, 2, 3$
 (C) $-1, 0, 1, 2$ (D) None of these
59. If three successive terms of a G.P. with common ratio $r (r > 1)$ form the sides of a $\triangle ABC$ and $[r]$ denotes greatest integer function, then $[r] + [-r] =$
- (A) 0 (B) 1
 (C) -1 (D) None of these
60. Let $S_n (1 \leq n \leq 9)$ denotes the sum of n terms of series $1 + 22 + 333 + \dots + 999999999$, then for $2 \leq n \leq 9$
- (A) $S_n - S_{n-1} = \frac{1}{9} (10^n - n^2 + n)$
 (B) $S_n = \frac{1}{9} (10^n - n^2 + 2n - 2)$
 (C) $9(S_n - S_{n-1}) = n(10^n - 1)$
 (D) None of these
61. a, b, c are three distinct real numbers, which are in G.P. and $a + b + c = xb$. Then,
- (A) $x < -1$ or $x > 3$ (B) $-1 < x < 3$
 (C) $-1 < x < 2$ (D) $0 < x < 1$
62. If a_1, a_2, a_3, a_4 are in H.P., then $\frac{1}{a_1 a_4} \sum_{r=1}^3 a_r a_{r+1}$ is a root of
- (A) $x^2 + 2x + 15 = 0$ (B) $x^2 + 2x - 15 = 0$
 (C) $x^2 - 6x - 8 = 0$ (D) $x^2 - 9x + 20 = 0$
63. The sum to n terms of the series $\frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \dots$ is
- (A) $n - \frac{(3^n - 2^n)}{2^n}$ (B) $n - \frac{2(3^n - 2^n)}{3^n}$
 (C) $2^n - 1$ (D) $3^n - 1$
64. If $a + b + c = 3$ and $a > 0, b > 0, c > 0$, then the greatest value of $a^2 b^3 c^2$ is
- (A) $\frac{3^{10} \cdot 2^4}{7^7}$ (B) $\frac{3^9 \cdot 2^4}{7^7}$
 (C) $\frac{3^8 \cdot 2^4}{7^7}$ (D) None of these
65. Let the harmonic mean and the geometric mean of two positive numbers be in the ratio $4 : 5$. The two numbers are in the ratio
- (A) $1 : 1$ (B) $2 : 1$ (C) $3 : 1$ (D) $4 : 1$
66. The first and last term of an A.P. are a and l , respectively. If S is the sum of all the terms of the A.P. and the common difference is $\frac{l^2 - a^2}{k - (l + a)}$, then k is equal to
- (A) S (B) $2S$
 (C) $3S$ (D) None of these
67. If a_1, a_2, \dots, a_n are in A.P. with common difference $d \neq 0$, then sum of the series $\sin d [\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \dots + \sec a_{n-1} \sec a_n]$ is
- (A) $\tan a_n - \tan a_1$
 (B) $\cot a_n - \cot a_1$
 (C) $\sec a_n - \sec a_1$
 (D) $\operatorname{cosec} a_n - \operatorname{cosec} a_1$
68. Sum to n terms of the series $\frac{1}{5!} + \frac{1!}{6!} + \frac{2!}{7!} + \frac{3!}{8!} + \dots$ is
- (A) $\frac{2}{5!} - \frac{1}{(n+1)!}$ (B) $\frac{1}{4} \left(\frac{1}{4!} - \frac{n!}{(n+4)!} \right)$
 (C) $\frac{1}{4} \left(\frac{1}{3!} - \frac{3!}{(n+2)!} \right)$ (D) None of these

69. If $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences given by $a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$ for all $n \in N$. Then, $a_1 a_2 a_3 \dots a_n$ is equal to
- (A) $x - y$ (B) $\frac{x+y}{b_n}$
 (C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$
70. For any odd integer $n \geq 1$, $n^3 - (n-1)^3 + \dots + (-1)^{n-1} 1^3 =$
- (A) $\frac{1}{2}(n-1)^2(2n-1)$
 (B) $\frac{1}{4}(n-1)^2(2n-1)$
 (C) $\frac{1}{2}(n+1)^2(2n-1)$
 (D) $\frac{1}{4}(n+1)^2(2n-1)$
71. For a positive integer n , let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n)-1}$. Then
- (A) $a(100) \leq 100$ (B) $a(100) > 100$
 (C) $a(200) \leq 100$ (D) $a(200) > 100$
72. Let α, β, γ be the roots of the equation $3x^3 - x^2 - 3x + 1 = 0$. If α, β, γ are in H.P. then $|\alpha - \gamma| =$
- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$
 (C) $\frac{4}{3}$ (D) None of these
73. Suppose $a, b > 0$ and x_1, x_2, x_3 ($x_1 > x_2 > x_3$) are roots of $\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}$ and $x_1 - x_2 - x_3 = c$, then a, b, c are in
- (A) A.P. (B) G.P.
 (C) H.P. (D) None of these
74. The coefficient of x^n in the product $(1-x)(1-2x)(1-2^2 \cdot x)(1-2^3 \cdot x) \dots (1-2^n \cdot x)$ is equal to
- (A) $(1-2^{n+1}) 2^{\frac{n(n-1)}{2}}$
 (B) $(2^{n+1} - 1) \cdot 2^{\frac{n(n-1)}{2}}$
 (C) $(1-2^n) 2^{\frac{n(n-1)}{2}}$
 (D) None of these
75. If $0.272727\dots, x$ and $0.727272\dots$ are in H.P., then x must be
- (A) rational (B) integer
 (C) irrational (D) None of these
76. If $a_1 = 0$ and $a_1, a_2, a_3, \dots, a_n$ are real numbers such that $|a_i| = |a_{i-1} + 1|$ for all i then the A.M. of the numbers a_1, a_2, \dots, a_n has value x where
- (A) $x \leq -\frac{1}{2}$ (B) $x \geq -\frac{1}{2}$
 (C) $x < -\frac{1}{2}$ (D) None of these
77. If $a_1, a_2, a_3, \dots, a_n$ are in H.P., then $\frac{a_1}{a_2 + a_3 + \dots + a_n}, \frac{a_2}{a_1 + a_3 + \dots + a_n}, \dots, \frac{a_n}{a_1 + a_2 + \dots + a_{n-1}}$ are in
- (A) A.P. (B) G.P.
 (C) H.P. (D) None of these
78. The consecutive numbers of a three digit number form a G.P. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order and if we increase the second digit of the required number by 2, the resulting number forms an A.P. The number is
- (A) 139 (B) 193
 (C) 931 (D) None of these
79. The largest term of the sequence $\frac{1}{503}, \frac{4}{524}, \frac{9}{581}, \frac{16}{692}, \dots$ is
- (A) $\frac{16}{692}$ (B) $\frac{4}{524}$
 (C) $\frac{49}{1529}$ (D) None of these
80. The coefficient of x^{99} and x^{98} in the polynomial $(x-1)(x-2)(x-3) \dots (x-100)$ are
- (A) -5050 and 12482075
 (B) -4050 and 12582075
 (C) -5050 and 12582075
 (D) None of these
81. The three successive terms of a G.P. will form the sides of a triangle if the common ratio r satisfies the inequality

- (A) $\frac{\sqrt{3}-1}{2} < r < \frac{\sqrt{3}+1}{2}$
 (B) $\frac{\sqrt{5}-1}{2} < r < \frac{\sqrt{5}+1}{2}$
 (C) $\frac{\sqrt{2}-1}{2} < r < \frac{\sqrt{2}+1}{2}$
 (D) None of these
82. If the sides of a right angled triangle are in G.P., then the cosine of the greater acute angle is
 (A) $\frac{1}{1+\sqrt{5}}$ (B) $\frac{1}{1-\sqrt{5}}$
 (C) $\frac{1+\sqrt{5}}{2}$ (D) None of these
83. Sum to n terms of the series $2 + 5 + 14 + 41 + \dots$ is
 (A) $\frac{n}{2} + \frac{1}{4}(3^n - 1)$ (B) $\frac{n}{2} + \frac{3}{4}(3^n - 1)$
 (C) $\frac{n}{2} + \frac{1}{2}(3^n - 1)$ (D) None of these
84. If the p th, q th and r th terms of both an A.P. and a G.P. be respectively a , b and c , then
 (A) $a^c \cdot c^b \cdot b^a = a^c \cdot b^c \cdot a^b$
 (B) $a^{b-1} \cdot b^{c+1} \cdot c^{a-1} = a^{c-1} \cdot b^{a-1} \cdot c^{b+1}$
 (C) $a^b \cdot b^c \cdot c^a = a^c \cdot b^a \cdot c^b$
 (D) None of these
85. If, in a G.P. of $3n$ terms, S_1 denotes the sum of the first n terms, S_2 the sum of the second block of n terms and S_3 the sum of the last n terms, then S_1, S_2, S_3 are in
 (A) A.P. (B) G.P.
 (C) H.P. (D) None of these
86. In a geometric series, the first term is a and common ratio is r . If S_n denotes the sum of n terms and $U_n = \sum_{n=1}^n S_n$, then $rS_n + (1-r)u_n =$
 (A) na (B) $(n-1)a$
 (C) $(n+1)a$ (D) None of these
87. In a Δabc , if $\cot A, \cot B, \cot C$ are in A.P. then a^2, b^2, c^2 are in
 (A) A.P. (B) G.P.
 (C) H.P. (D) A.G. P.
88. If $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$ up to $\infty = \frac{\pi^4}{90}$, then the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$ up to ∞ is
 (A) $\frac{\pi^4}{45}$ (B) $\frac{\pi^4}{96}$
 (C) $\frac{\pi^4}{124}$ (D) None of these
89. If the $(m+1)$ th, $(n+1)$ th and $(r+1)$ th terms of an A.P. are in G.P. and m, n, r are in H.P., then the ratio of the first term of the A.P. to its common difference is
 (A) $\frac{n}{3}$ (B) $-\frac{n}{3}$
 (C) $\frac{n}{2}$ (D) $-\frac{n}{2}$
90. Let there be n numbers in G.P. whose common ratio is r and S_m denotes the sum of their first m terms. The sum of their products taken two at a time is $k S_n S_{n-1}$ where $k =$
 (A) $\frac{r-1}{r}$ (B) $\frac{r-1}{r+1}$
 (C) $\frac{r}{r+1}$ (D) None of these
91. If a, b, c, d are distinct integers in A.P. such that $d = a^2 + b^2 + c^2$, then $a + b + c + d =$
 (A) 2 (B) 1
 (C) 0 (D) None of these
92. If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$, then the value of $1 + \frac{3}{2} + \frac{5}{3} + \dots + \frac{2n-1}{n}$ is
 (A) $n - H_n$ (B) $2n - H_n$
 (C) $(n-1) - H_n$ (D) $n - 2H_n$
93. If a_m be the m th term of an A.P., then $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{2n-1}^2 - a_{2n}^2 =$
 (A) $\frac{n-1}{2n-1}(a_1^2 - a_{2n}^2)$ (B) $\frac{n}{2n-1}(a_{2n}^2 - a_1^2)$
 (C) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (D) None of these
94. If $a_{n+1} = \frac{1}{1-a_n}$ for $n \geq 1$ and $a_3 = a_1$, then $(a_{2001})^{2001} =$
 (A) 1 (B) -1
 (C) 0 (D) None of these
95. If a, b, c are positive numbers in G.P. and $\log\left(\frac{5c}{a}\right), \log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in A.P. then a, b, c

- (A) form the sides of an equilateral triangle
- (B) form the sides of an isosceles triangle
- (C) form the sides of a right angled triangle
- (D) can not form the sides of a triangle

96. If a, b, c are in G.P. and $\log a - \log 2b, \log 2b - \log 3c$ and $\log 3c - \log a$ are in A.P., then a, b, c are the sides of a triangle which is

- (A) right angled
- (B) acute angled
- (C) obtuse angled
- (D) None of these

97. In a sequence of $4n + 1$ terms, the first $2n + 1$ terms are in A.P. having common difference 2 and the last $2n + 1$ terms are in G.P. having common ratio $\frac{1}{2}$. If the middle term of the A.P. is equal to the middle term of the G.P. then the middle term of the sequence is

- (A) $\frac{n \cdot 2^{n+1}}{2^n + 1}$
- (B) $\frac{n \cdot 2^{n+1}}{2^n - 1}$
- (C) $\frac{n \cdot 2^n}{2^n - 1}$
- (D) None of these

98. If S_1, S_2 and S_3 denote the sums up to $n > 1$ terms of three sequences in A.P. whose first terms are unity and common differences are in H.P. then $n =$

- (A) $\frac{2S_3S_1 + S_1S_2 + S_2S_3}{S_1 - 2S_2 + S_3}$
- (B) $\frac{2S_3S_1 - S_1S_2 - S_2S_3}{S_1 + 2S_2 + S_3}$
- (C) $\frac{2S_3S_1 - S_1S_2 - S_2S_3}{S_1 - 2S_2 + S_3}$
- (D) None of these

99. Sum to n terms of the series $1^3 + 3 \cdot 2^3 + 3^3 + 3 \cdot 4^3 + 5^3 + \dots$, where n is even, is

- (A) $\frac{n^2(n^2 - 3n + 1)}{2}$
- (B) $\frac{n^2(n^2 + 3n + 1)}{2}$
- (C) $\frac{n^2(n^2 + 3n + 1)}{4}$
- (D) None of these

100. Let a be a fixed real number such that

$$\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz}$$

If p, q, r are in A.P. then x, y, z are in

- (A) A.P.
- (B) G.P.
- (C) H. P
- (D) None of these

101. If $|a| < 1$ and $|b| < 1$, then the sum of the series $1 + (1+a)b + (1+a+a^2)b^2 + (1+a+a^2+a^3)b^3 + \dots \infty$ is equal to

- (A) $\frac{1}{(1-b)(1-ab)}$
- (B) $\frac{1}{(1-a)(1-ab)}$
- (C) $\frac{1}{(1-a)(1-b)}$
- (D) None of these

102. If $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences given by $a_n = x^{2^n} + y^{2^n}$ and $b_n = x^{2^n} - y^{2^n} \forall n \in N$, then the value of $a_1 a_2 a_3 \dots a_n$ is

- (A) $\frac{x+y}{b_n}$
- (B) $\frac{x-y}{b_n}$
- (C) $\frac{x^2+y^2}{b_n}$
- (D) $\frac{x^2-y^2}{b_n}$

103. The sixth term of an A.P. is equal to 2. The value of the common difference of the A.P. which makes the product $a_1 a_4 a_5$ greatest, is

- (A) $\frac{8}{5}$
- (B) $\frac{2}{3}$
- (C) $\frac{3}{5}$
- (D) $\frac{3}{4}$

104. If the natural numbers are written as

1
2 3
4 5 6
7 8 9 10
.....
.....

Then, the sum of the terms of the n th row is

- (A) $\frac{n(n^2 - 1)}{2}$
- (B) $\frac{n(n^2 + 1)}{4}$
- (C) $\frac{n(n^2 + 1)}{2}$
- (D) None of these

More than One Option Correct Type

- 105.** The H.M. of two numbers is 4. If their A.M. A and G.M. G satisfy the relation $2A + G^2 = 27$, then the numbers are
 (A) 1 (B) 2
 (C) 3 (D) 6
- 106.** If the first and the $(2n - 1)$ th terms of an A.P., G.P. and H.P. are equal and their n th terms are a, b, c respectively, then
 (A) $a = b = c$ (B) $a \geq b \geq c$
 (C) $a + c = b$ (D) $ac - b^2 = 0$
- 107.** The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + bx + \gamma = 0$ are in A.P. The intervals in which β and γ lie are
 (A) $\beta \in \left(-\infty, \frac{1}{3}\right]$ (B) $\beta \in \left[-\frac{1}{27}, \infty\right)$
 (C) $\gamma \in \left(-\infty, \frac{1}{3}\right]$ (D) $\gamma \in \left[-\frac{1}{27}, \infty\right)$
- 108.** If a, b, c are in A.P. and a^2, b^2, c^2 are in H.P. then
 (A) $a = b = c$
 (B) $-\frac{a}{2}, b, c$ are in G.P.
 (C) $-\frac{c}{2}, b, a$ are in G.P.
 (D) $-\frac{a}{2}, b, c$ are in H.P.
- 109.** If the G.M. between a and b be twice the H.M., then $\frac{a}{b}$ is equal to
 (A) $\frac{2 + \sqrt{3}}{2 - \sqrt{3}}$ (B) $\frac{2 - \sqrt{3}}{2 + \sqrt{3}}$
 (C) $\frac{4 + \sqrt{3}}{4 - \sqrt{3}}$ (D) $\frac{4 - \sqrt{3}}{4 + \sqrt{3}}$
- 110.** If a, b, c are in G.P. and x is the A.M. between a and b, y the A.M. between b and c , then
 (A) $\frac{a}{x} + \frac{c}{y} = 1$ (B) $\frac{a}{x} + \frac{c}{y} = 2$
 (C) $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$ (D) None of these
- 111.** The solution of the equations $\log x + \log x^{1/2} + \log x^{1/4} + \dots = y$ and $\frac{1 + 3 + 5 + \dots + (2y - 1)}{4 + 7 + 10 + \dots + (3y + 1)} = \frac{20}{7 \log x}$ is
 (A) $x = 10^5, 10^{-5/7}$
 (B) $y = 10, -\frac{10}{7}$
 (C) $x = 10, -\frac{10}{7}$
 (D) $y = 10^5, 10^{-5/7}$
- 112.** The sum of first ten terms of an A.P. is equal to 155 and the sum of first two terms of a G.P. is 9. If the first term of the A.P. is equal to the common ratio of the G.P. and the first term of the G.P. is equal to the common difference of the A.P., then
 (A) first term of the G.P. is $\frac{2}{3}, 3$
 (B) first term of the A.P. is $\frac{2}{3}, 3$
 (C) Common ratio of the G.P. is $\frac{25}{2}, 2$
 (D) Common difference of the A.P. is $\frac{2}{3}, 3$
- 113.** Let $(1 + x^2)^2 (1 + x)^n = \sum_{k=0}^{n+4} a_k x^k$. If a_1, a_2, a_3 , are in A.P., then n is equal to
 (A) 1 (B) 2
 (C) 3 (D) 4
- 114.** If a, b, c are non-zero real numbers such that $3(a^2 + b^2 + c^2 + 1) = 2(a + b + c + ab + bc + ca)$, then, a, b, c are in
 (A) A.P. (B) G.P.
 (C) H.P. (D) all equal
- 115.** Let $t_n = \underbrace{1, 1, \dots, 1}_{n \text{ times}}$, then
 (A) t_{912} is not prime
 (B) t_{951} is not prime
 (C) t_{480} is not prime
 (D) t_{91} is not prime

Passage Based Questions

Passage 1

In finding the summation of a series, sometimes it is possible to express the n th term t_n as a difference of two terms. Suppose, we want the sum of the series $t_1 + t_2 + \dots + t_n$. We try to express $t_n = x_{n+1} - x_n$ hence, $t_1 + t_2 + \dots + t_n = (x_2 - x_1) + (x_3 - x_2) + \dots + (x_{n+1} - x_n) = x_{n+1} - x_1$. Thus, $S_n = x_{n+1} - x_1$.

At times, we may have to do partial fractions by trial by writing the numerator as a difference of extreme terms.

Suppose, we want to find the sum of the series

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots \text{ up to } n \text{ terms.}$$

Then,
$$t_n = \frac{1}{n(n+1)(n+2)} = \frac{1}{2} \left[\frac{(n+2) - n}{n(n+1)(n+2)} \right]$$

$$= \frac{1}{2} \left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right]$$

We easily get
$$S_n = \frac{1}{2} \left[\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)} \right]$$

116. Sum to n terms of the series

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots \text{ is}$$

- (A) $\frac{n}{n+1}$ (B) $\frac{2n}{n+1}$
 (C) $\frac{n}{n-1}$ (D) None of these

117. Sum to infinite terms of the series

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots \text{ is}$$

- (A) 1 (B) 2
 (C) 4 (D) None of these

118. Sum to n terms of the series $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots$ is

- (A) $\frac{n}{2n+1}$ (B) $\frac{n}{2n-1}$
 (C) $\frac{n-1}{2n+1}$ (D) None of these

119 Sum to infinite terms of the series

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots \text{ is}$$

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$
 (C) $\frac{1}{2}$ (D) None of these

Passage 2

A general arithmetic progression is $a, a+d, a+2d, \dots$ and a general geometric progression is a, ar, ar^2, \dots , then the sequence $a, (a+d)r, (a+2d)r^2, \dots$ is called an arithmetico-geometric progression (A. G.P.).

Note that each term of the A.G.P. is the product of the corresponding terms of the A.P. $a, a+d, a+2d, \dots$ and the G.P. $1, r, r^2, \dots$. The n th term of the A.G.P. is $[a + (n-1)d]r^{n-1}$.

Sum to n terms of A.G.P.

$$\text{Let } S_n = a + (a+d)r + (a+2d)r^2 + \dots$$

$$+ (a + \overline{n-2}d)r^{n-2} + (a + \overline{n-1}d)r^{n-1} \quad (1)$$

$$\Rightarrow rS_n = ar + (a+d)r^2 + \dots + (a + \overline{n-1}d)r^n \quad (2)$$

Subtracting (2) from (1), we get

$$(1-r)S_n = a + dr + dr^2 + \dots + dr^{n-1} - (a + \overline{n-1}d)r^n$$

$$= a + (dr + dr^2 + \dots + \overline{n-1} \text{ terms}) - (a + \overline{n-1}d)r^n$$

$$= a + \frac{dr(1-r^{n-1})}{1-r} - (a + \overline{n-1}d)r^n$$

$$\therefore S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{(a + \overline{n-1}d)r^n}{1-r}$$

$$\therefore \text{Sum to infinite terms} = S = \lim_{n \rightarrow \infty} S_n = \frac{a}{1-r} + \frac{dr}{(1-r)^2},$$

where $|r| < 1$.

120. The sum to infinity of the series

$$1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \dots \text{ is}$$

- (A) 4 (B) 6
 (C) 8 (D) None of these

121. If the sum to infinity of the series

$$3 + 5r + 7r^2 + \dots \text{ is } \frac{49}{9}, \text{ then } r \text{ is equal to}$$

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$
 (C) $\frac{1}{2}$ (D) None of these

122. If the sum to infinity of the series

$$3 + (3 + d)\frac{1}{4} + (3 + 2d)\frac{1}{4^2} + \dots \text{ is } \frac{44}{9}, \text{ then } d =$$

- (A) 1 (B) 2
 (C) 4 (D) None of these

123. $3^{1/3} \cdot 9^{1/9} \cdot 27^{1/27} \cdot 81^{1/81} \dots$ upto $\infty =$

- (A) $\sqrt{27}$ (B) $\sqrt[3]{27}$
 (C) $\sqrt[4]{27}$ (D) None of these

Passage 3

We know that arithmetic mean of the positive numbers lie between them. Suppose, $S_1 > S_2 > 0$ and $S_{n+1} = \frac{1}{2}(S_n + S_{n-1})$

We can easily conclude that S_3 lies between S_2 and S_1 and we can write $S_2 < S_3 < S_1$.

124. The sequence $\{S_{2n}\}$ is
 (A) increasing (B) decreasing
 (C) non-monotonic (D) unbounded

125. The sequence $\{S_{2n+1}\}$ is
 (A) increasing (B) decreasing
 (C) non-monotonic (D) unbounded

126. $S_{2n+1} - S_{2n}$ must be equal to

- (A) $\left(\frac{1}{2}\right)^{2n-1}$ (B) $\left(\frac{1}{2}\right)^{2n-1} (S_1 - S_2)$
 (C) $\left(\frac{1}{2}\right)^{2n-1} (S_2 - S_1)$ (D) Zero

127. If $S_1 > S_2$, then $\lim_{n \rightarrow \infty} S_n$ must be equal to

- (A) $S_1 - S_2$ (B) $S_1 + 2S_2$
 (C) $\frac{S_1 + 2S_2}{3}$ (D) None of these

Match the Column Type

128.

Column-I	Column-II
I. Let S_n denotes the sum of n terms of an A.P. whose first term is a . If the common difference $d = S_n - k S_{n-1} + S_{n-2}$, then $k =$	(A) 29
II. The minimum number of terms from the beginning of the series $20 + 22\frac{2}{3} + 25\frac{1}{3} + \dots$, so that the sum may exceed 1568, is	(B) 4
III. If $5^{1+x} + 5^{1-x}, \frac{a}{2}$ and $25^x + 25^{-x}$ are three consecutive terms of an A.P., then $a \geq k$, where $k =$	(C) 2
IV. If $\log_{2^{12}} a + \log_{2^{14}} a + \log_{2^{16}} a + \log_{2^{18}} a + \dots$ upto 20 terms is 840, then a is equal to...	(D) 12

129.

Column-I	Column-II
I. If the first term of an infinite G.P. is 1 and each term is twice the sum of the succeeding terms, then the common ratio is	(A) $\frac{2}{9}$
II. Sum to infinity of the series $\frac{2}{3} - \frac{5}{6} + \frac{2}{3} - \frac{11}{24} + \dots$ is	(B) $\frac{3}{2}$

$$\text{III. } \lim_{n \rightarrow \infty} (1 + 3^{-1})(1 + 3^{-2})(1 + 3^{-4})(1 + 3^{-8}) \dots (1 + 3^{-2^n}) = \quad (\text{C}) \quad 1$$

$$\text{IV. If } \sum_{k=1}^n \left(\sum_{m=1}^k m^2 \right) = an^4 + bn^3 + cn^2 + dn + e, \text{ then } a + b + c + d + e = \quad (\text{D}) \quad \frac{1}{3}$$

130.

Column-I

Column-II

- I. If a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P., then a, c, e are in (A) A.P.
- II. If $2(y - a)$ is the H.M. between $y - x, y - z$ then $x - a, y - a, z - a$ are in (B) G.P.
- III. If three numbers are in H.P., then the numbers obtained by subtracting half of the middle number from each of them are in (C) H.P.
- IV. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, if $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in (D) A.G.P.

Assertion-Reason Type

Instructions: In the following questions an Assertion (A) is given followed by a Reason (R). Mark your responses from the following options:

- (A) Assertion(A) is True and Reason(R) is True; Reason(R) is a correct explanation for Assertion(A)
- (B) Assertion(A) is True, Reason(R) is True; Reason(R) is not a correct explanation for Assertion(A)
- (C) Assertion(A) is True, Reason(R) is False
- (D) Assertion(A) is False, Reason(R) is True
- 131. Assertion:** Between two numbers whose sum is, $2\frac{1}{6}$ an even number of arithmetic means are inserted. If the sum of these means exceeds their number by unity, then the number of means are 12
- Reason:** If a and b are two given numbers and A_1, A_2, \dots, A_n are n arithmetic means between them, then $A_1 + A_2, \dots, A_n = n \left(\frac{a+b}{2} \right)$
- 132. Assertion:** If a, b, c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is less than 1.
- Reason:** A.M. > G.M. for unequal numbers
- 133. Assertion:** If $a, b, c, d \in R^+$ and a, b, c, d are in H.P., then $b + c > a + d$

Reason: H.M. > A.M. for unequal numbers

134. Assertion: The sum of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$$

to n terms is $\frac{n(n+1)}{2(n^2+n+1)}$

Reason: The n th term of the above series is

$$T_n = \frac{1}{2} \left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)} \right]$$

135. Assertion: The value of $x + y + z$ is 15 if a, x, y, z, b are in A.P., while the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is $\frac{5}{3}$ if a, x, y, z, b are in H.P. The values of a and b are 9, 1 respectively.

Reason: The sum of n A.M.s between two quantities is equal to n times their single mean.

136. Assertion: For every natural number

$$n, (n!)^3 < n^n \left(\frac{n+1}{2} \right)^{2n}$$

Reason: A.M > G.M. for n distinct positive quantities

Previous Year's Questions

137. If $1, \log_3 \sqrt{(3^{1-x} + 2)}, \log_3 (4 \cdot 3x - 1)$ are in AP, then x equals: [2002]
 (A) $\log_3 4$ (B) $1 - \log_3 4$
 (C) $1 - \log_3 3$ (D) $\log_4 3$
138. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \dots \infty$ is: [2002]
 (A) 1 (B) 2
 (C) $3/2$ (D) 4
139. Fifth term of a GP is 2, then the product of its 9 terms is: [2002]
 (A) 256 (B) 512
 (C) 1024 (D) None of these
140. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} \cdot T_n = 21$, then n equals: [2002]
 (A) 5 (B) 7
 (C) 6 (D) 4
141. The sum of the series $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \dots$ upto ∞ is equal to [2003]
 (A) $2 \log_e 2$ (B) $\log_2 2 - 1$
 (C) $\log_e 2$ (D) $\log_e \left(\frac{4}{e} \right)$
142. If $f: R \rightarrow R$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^n f(r)$ is [2003]
 (A) $\frac{7n}{2}$ (B) $\frac{7(n+1)}{2}$
 (C) $7n(n+1)$ (D) $\frac{7n(n+1)}{2}$
143. If $S_n = \sum_{r=0}^n \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^n \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to [2004]
 (A) $\frac{1}{2}n$ (B) $\frac{1}{2}n - 1$
 (C) $n - 1$ (D) $\frac{2n-1}{2}$
144. Let T_r be the r th term of an A.P. whose first term is a and common difference is d . If for some positive integers $m, n, m \neq n, T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$, equals [2004]
 (A) 0 (B) 1
 (C) $\frac{1}{mn}$ (D) $\frac{1}{m} + \frac{1}{n}$
145. The sum of the first n terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is [2004]
 (A) $\frac{3n(n+1)}{2}$ (B) $\frac{n^2(n+1)}{2}$
 (C) $\frac{n(n+1)^2}{4}$ (D) $\left[\frac{n(n+1)}{2} \right]^2$
146. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P. and $|a| < 1, |b| < 1, |c| < 1$, then x, y, z are in [2005]
 (A) G.P.
 (B) A.P.
 (C) Arithmetic – Geometric Progression
 (D) H.P.
147. Let a_1, a_2, a_3, \dots be terms of an A.P. If $\frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}, p \neq q$, then $\frac{a_6}{a_{21}}$ equals [2006]
 (A) $\frac{41}{11}$ (B) $\frac{7}{2}$
 (C) $\frac{2}{7}$ (D) $\frac{11}{41}$
148. If a_1, a_2, \dots, a_n are in H.P., then the expression $a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n$ is equal to [2006]
 (A) $n(a_1 - a_n)$ (B) $(n-1)(a_1 - a_n)$
 (C) $na_1 a_n$ (D) $(n-1)a_1 a_n$
149. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals [2007]

- (A) $\frac{1}{2}(1-\sqrt{5})$ (B) $\frac{1}{2}\sqrt{5}$
- (C) $\sqrt{5}$ (D) $\frac{1}{2}(\sqrt{5}-1)$
- 150.** If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p+q)$ is [2007]
- (A) 2 (B) $1/2$
- (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$
- 151.** The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is [2008]
- (A) -4 (B) -12
- (C) 12 (D) 4
- 152.** The sum to the infinity of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$ is [2009]
- (A) 2 (B) 3
- (C) 4 (D) 6
- 153.** A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \dots = a_{10} = 150$ and a_{10}, a_{11}, \dots are in A.P. with common difference -2 , then the time taken by him to count all notes is [2010]
- (A) 34 minutes (B) 125 minutes
- (C) 135 minutes (D) 24 minutes
- 154.** A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediate preceding month. His total saving from the start of service will be Rs. 11040 after [2011]
- (A) 19 months (B) 20 months
- (C) 21 months (D) 18 months
- 155. Statement 1:** The sum of the series $1 + (1+2+4) + (4+6+9) + (9+12+16) + \dots + (361+380+400)$ is 8000.
- Statement 2:** $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$ for any natural number n . [2012]
- (A) Statement 1 is false, statement 2 is true
- (B) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1
- (C) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1
- (D) Statement 1 is true, statement 2 is false
- 156.** If 100 times the 100^{th} term of an Arithmetic Progression with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this A.P. is [2012]
- (A) -150
- (B) 150 times its 50^{th} term
- (C) 150
- (D) zero
- 157.** The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ... is [2013]
- (A) $\frac{7}{9}(99-10^{-20})$ (B) $\frac{7}{81}(179+10^{-20})$
- (C) $\frac{7}{9}(99+10^{-20})$ (D) $\frac{7}{81}(179-10^{-20})$
- 158.** Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is [2014]
- (A) $\frac{\sqrt{61}}{9}$ (B) $\frac{2\sqrt{17}}{9}$
- (C) $\frac{\sqrt{34}}{9}$ (D) $\frac{2\sqrt{13}}{9}$
- 159.** Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is [2014]
- (A) $\sqrt{2} + \sqrt{3}$ (B) $3 + \sqrt{2}$
- (C) $2 - \sqrt{3}$ (D) $2 + \sqrt{3}$
- 160.** If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9 = k(10)^9$ then k is equal to [2014]
- (A) $\frac{121}{10}$ (B) $\frac{441}{100}$
- (C) 100 (D) 110
- 161.** The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \dots$ is: [2015]
- (A) 96 (B) 142
- (C) 192 (D) 71

162. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is

[2016]

- (A) $\frac{7}{4}$ (B) $\frac{8}{5}$
 (C) $\frac{4}{3}$ (D) 1

163. If the sum of the first terms of the series

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots, \text{ is } \frac{16}{5}m,$$

then m is equal to

[2016]

- (A) 99 (B) 102
 (C) 101 (D) 100

ANSWER KEYS

Single Option Correct Type

1. (B) 2. (C) 3. (C) 4. (B) 5. (D) 6. (C) 7. (A) 8. (D) 9. (A) 10. (B)
 11. (A) 12. (D) 13. (D) 14. (D) 15. (B) 16. (C) 17. (B) 18. (A) 19. (A) 20. (C)
 21. (B) 22. (C) 23. (B) 24. (C) 25. (C) 26. (B) 27. (C) 28. (A, B) 29. (B) 30. (B)
 31. (B) 32. (B) 33. (A) 34. (B) 35. (D) 36. (A) 37. (B) 38. (C) 39. (B) 40. (D)
 41. (A) 42. (B) 43. (D) 44. (B) 45. (C) 46. (B) 47. (B) 48. (C) 49. (D) 50. (C)
 51. (C) 52. (A) 53. (A) 54. (A) 55. (C) 56. (A) 57. (D) 58. (C) 59. (C) 60. (C)
 61. (A) 62. (B) 63. (B) 64. (A) 65. (D) 66. (B) 67. (A) 68. (B) 69. (C) 70. (D)
 71. (D) 72. (C) 73. (C) 74. (A) 75. (A) 76. (B) 77. (C) 78. (C) 79. (C) 80. (C)
 81. (B) 82. (A) 83. (B) 84. (C) 85. (B) 86. (A) 87. (A) 88. (B) 89. (D) 90. (C)
 91. (A) 92. (B) 93. (C) 94. (B) 95. (D) 96. (C) 97. (B) 98. (C) 99. (B) 100. (C)
 101. (A) 102. (B) 103. (A) 104. (C)

More than One Option Correct Type

105. (C, D) 106. (B, D) 107. (A, D) 108. (A, B, C) 109. (A, B)
 110. (B, C) 111. (A, B) 112. (A, C, D) 113. (B, C, D) 114. (A, B, C, D)
 115. (A, B, C, D)

Passage Based Questions

116. (B) 117. (B) 118. (A) 119. (C) 120. (B) 121. (A) 122. (B) 123. (C) 124. (A) 125. (B)
 126. (B) 127. (C)

Match the Column Type

128. I → (C), II → (A), III → (D), IV → (B) 129. I → (D), II → (A), III → (B), IV → (C)
 130. I → (B), II → (B), III → (B), IV → (A)

Assertion-Reason Type

131. (A) 132. (A) 133. (A) 134. (A) 135. (A) 136. (A)

Previous Year's Questions

137. (B) 138. (B) 139. (B) 140. (B) 141. (D) 142. (D) 143. (A) 144. (A) 145. (D) 146. (D)
 147. (D) 148. (D) 149. (D) 150. (D) 151. (B) 152. (B) 153. (A) 154. (C) 155. (B) 156. (D)
 157. (B) 158. (D) 159. (D) 160. (C) 161. (A) 162. (C) 163. (C)

HINTS AND SOLUTIONS

Single Option Correct Type

1. Given $a + c = 2b$

$$\text{Also, } \frac{a+b+c}{3} \geq \sqrt[3]{abc} = \sqrt[3]{64} = 4$$

$$\Rightarrow \frac{3b}{3} \geq 4$$

$$\Rightarrow b \geq 4$$

$$\therefore \text{Minimum } b = 4$$

The correct option is (B)

2. Let sides of triangle be a, ar, ar^2 .

Since $r > 1$, $\therefore ar^2$ is greatest side

$$\therefore a + ar > ar^2 \Rightarrow r^2 - r - 1 < 0$$

$$\Rightarrow \frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2} \Rightarrow 1 < r < \frac{1+\sqrt{5}}{2}$$

$$\therefore (r) = 1.$$

$$\text{Also, } -\frac{1+\sqrt{5}}{2} < -r < -1$$

$$\therefore (-r) = -2$$

$$\therefore (r) + (-r) = 1 - 2 = -1$$

The correct option is (C)

$$3. \sum_{j=1}^{21} a_j = 693 = \frac{21}{2} (a_1 + a_{21})$$

$$\therefore a_1 + a_{21} = 66$$

$$\text{Now, } a_{11} = \text{A.M.} = \frac{693}{21} = 33$$

$$\text{Also, } a_2 + a_{20} = a_3 + a_{19} = \dots = a_9 + a_{13} = a_{10} + a_{12}$$

$$\therefore \sum_{i=0}^{10} a_{2i+1} = 5 \times (a_1 + a_{21}) + a_{11}$$

$$= 5 \times 66 + 33 = 363$$

The correct option is (C)

4. Let a, ar, ar^2 be any three consecutive terms, then according to the given condition, $a, 2ar, ar^2$ are in A.P.

$$\text{i.e. } 4ar = a + ar^2$$

$$\text{or } r = 2 \pm \sqrt{3}$$

Since, given G.P. is increasing, $r = 2 + \sqrt{3}$. Hence, there is only one such G.P.

The correct option is (B)

5. Given $a_1 > 0$ and a_1, a_2, a_3 are in G.P.

$$\text{such that } a_2 = a_1 r \text{ and } a_3 = a_1 r^2$$

$$\therefore 9a_1 + 5a_3 > 14a_2$$

$$\Rightarrow 9a_1 + 5a_1 r^2 > 14a_1 r$$

$$\text{and since } a_1 > 0, \text{ we get } 9 + 5r^2 > 14r$$

$$\Rightarrow 5r^2 - 14r + 9 > 0 \Rightarrow (5r - 9)(r - 1) > 0$$

$$\Rightarrow r \notin \left[1, \frac{9}{5}\right]$$

The correct option is (D)

$$6. S_n = \frac{1}{9}(9) + \frac{2}{9}(99) + \frac{3}{9}(999) + \dots$$

$$= \frac{1}{9} [10 + 2 \cdot 10^2 + 3 \cdot 10^3 + \dots] - \frac{1}{9} [1 + 2 + 3 + \dots]$$

$$= \frac{1}{9} S - \frac{1}{9} \frac{n(n+1)}{2}$$

$$S = 10 + 2 \cdot 10^2 + 3 \cdot 10^3 + \dots + n \cdot 10^n$$

$$\Rightarrow 10S = 10^2 + 2 \cdot 10^3 + \dots + (n-1)10^n + n \cdot 10^{n+1}$$

$$-9S = (10 + 10^2 + 10^3 + \dots + 10^n) - n \cdot 10^{n+1}$$

$$\Rightarrow S = \frac{n}{9} 10^{n+1} - \frac{10^{n+1} - 1}{81}$$

$$\therefore S_n = \frac{n}{81} 10^{n+1} - \frac{10^{n+1} - 1}{9 \cdot 9} - \frac{1}{9} \frac{n(n+1)}{2}$$

$$\Rightarrow 9 \cdot S_n = \frac{(9n-1)10^{n+1}}{81} + \frac{1}{81} - \frac{n(n+1)}{2}$$

$$\therefore 9(S_n - S_{n-1}) = \frac{10^n}{81} [10(9n-1) - (9n-10)] - n$$

$$= n(10^n - 1)$$

The correct option is (C)

7. $\log_{\sqrt{5}} x + \log_{5^{1/3}} x + \log_{5^{1/4}} x + \dots$ upto 7 terms

$$= \frac{\log x}{\log \sqrt{5}} + \frac{\log x}{\log 5^{1/3}} + \frac{\log x}{\log 5^{1/4}} + \dots \text{ upto 7 terms}$$

$$= \frac{\log x}{\log 5} [2 + 3 + 4 + \dots + 8]$$

$$= \frac{\log x}{\log 5} \left[\frac{7}{2}(2+8) \right] = \frac{\log x}{\log 5} (35) = 35 \text{ (given)}$$

$$\therefore \frac{\log x}{\log 5} = 1$$

$$\Rightarrow \log x = \log 5$$

$$\Rightarrow x = 5$$

The correct option is (A)

$$8. \text{ Given } a = \sum_{n=1}^{\infty} x^{n-1} = 1 + x + x^2 + \dots = \frac{1}{1-x}$$

$$\Rightarrow x = \frac{a-1}{a}$$

Similarly $y = \frac{b-1}{b} \therefore \sum_{n=1}^{\infty} (xy)^{n-1} = 1 + (xy) + (xy)^2 + \dots$

$$= \frac{1}{1-xy} = \frac{1}{1 - \left(\frac{a-1}{a}\right)\left(\frac{b-1}{b}\right)}$$

$$= \frac{ab}{ab - (ab - (a+b) + 1)} = \frac{ab}{a+b-1}$$

The correct option is (D)

9. $27pqr \geq (p+q+r)^3$

$$\Rightarrow (pqr)^{1/3} \geq \frac{p+q+r}{3} \Rightarrow p=q=r$$

Also, $3p+4q+5r=12$

$$\Rightarrow p=q=r=1$$

The correct option is (A)

10. Let T_n be the n^{th} term of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$$

Then, $T_n = \frac{n}{1+n^2+n^4} = \frac{n}{(1+n^2)^2 - n^2}$

$$= \frac{n}{(n^2+n+1)(n^2-n+1)}$$

$$= \frac{1}{2} \left(\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1} \right)$$

$$= \frac{1}{2} \left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)} \right]$$

Now

$$\sum_{r=1}^n T_r = \frac{1}{2} \left[\frac{1}{1} - \frac{1}{1+1.2} \right] + \frac{1}{2} \left[\frac{1}{1+1.2} - \frac{1}{1+2.3} \right]$$

$$+ \frac{1}{2} \left[\frac{1}{1+2.3} - \frac{1}{1+3.4} \right] + \dots$$

$$+ \frac{1}{2} \left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)} \right]$$

$$= \frac{1}{2} \left[1 - \frac{1}{1+n(n+1)} \right]$$

$$= \frac{n(n+1)}{2(n^2+n+1)}$$

Trick: Checking for $n=1, 2$. $S_1 = \frac{1}{3}$ and $S_2 = \frac{3}{7}$ which are given by (b).

The correct option is (B)

11. $a+b+c=xb$

Divide by b ,

$$\frac{a}{b} + 1 + \frac{c}{b} = x$$

$$\Rightarrow \frac{1}{r} + 1 + r = x, r \text{ is the common ratio of the G.P.}$$

$$\Rightarrow r^2 + r(1-x) + 1 = 0. \text{ Since } r \text{ is real, therefore discriminant} > 0$$

$$\Rightarrow (1-x)^2 - 4 > 0 \Rightarrow x^2 - 2x + 1 - 4 > 0$$

$$\Rightarrow x^2 - 2x - 3 > 0 \Rightarrow (x+1)(x-3) > 0$$

$$\Rightarrow x < -1 \text{ or } x > 3$$

The correct option is (A)

12. Given: $\frac{100}{2} [2a+99d] = x$

and $\frac{100}{2} [2(a+2d)+99d] = y$

On subtraction, $200d = y - x$

$$\Rightarrow d = \frac{y-x}{200}$$

The correct option is (D)

13. Given: $\dots \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} = \lambda$

$$\therefore \sum_{i=1}^{\infty} \frac{1}{(2i-1)^4} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

$$= \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots - \left[\frac{1}{2^4} + \frac{1}{4^4} + \dots \right]$$

$$= \lambda - \frac{1}{2^4} \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \right]$$

$$= \lambda - \frac{1}{16} \lambda = \frac{15}{16} \lambda$$

The correct option is (D)

14. Let $A = 1.2 + 1.3 + \dots + 2.3 + 2.4 + \dots + (n-1) \cdot n$

Now, $(1+2+3+\dots+n)^2 - (1^2+2^2+3^2+\dots+n^2) = 2A$

$$\Rightarrow A = \frac{1}{2} \left[\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6} \right]$$

$$= \frac{1}{2} [(\Sigma n)^2 - \Sigma n^2]$$

The correct option is (D)

15. A.M. \geq G.M.

$$\Rightarrow \frac{2^{3\sin x/8} + 2^{3\cos x/8}}{2} \geq \sqrt{2^{3\sqrt{2}\cos\left(\frac{x}{8}\frac{\pi}{4}\right)}}$$

Now maximum of $\sqrt{2^{3\sqrt{2}\cos\left(\frac{x}{8}\frac{\pi}{4}\right)}} = \sqrt{2^{3\sqrt{2}.1}} = 2^{3\sqrt{2}}$

$$\text{So, A.M.} \geq 2^{3\sqrt{2}} \Rightarrow 8^{\frac{\sin x}{8}} + 8^{\frac{\cos x}{8}} \geq 2^{\left(\frac{3}{\sqrt{2}}+1\right)}$$

The correct option is (B)

16. $\log_2 a + \log_2 a + \log_2 a + \log_2 a + \dots$
 $= 2 \log_2 a + 4 \log_2 a + 6 \log_2 a + \dots + 40 \log_2 a$
 $= \log_2 a [2 + 4 + 6 + \dots + 40]$
 $= \frac{20}{2} (2 + 40) \log_2 a$
 $= 420 \log_2 a = 840$ (Given)
 $\Rightarrow \log_2 a = 2 \Rightarrow a = 4$
 The correct option is (C)

17. If t_r denotes the r th term of the series, then

$$x t_r = \frac{x}{(1+rx)(1+(r+1)x)}$$

$$= \frac{1}{1+rx} - \frac{1}{1+(r+1)x}$$

$$\Rightarrow x \sum_{r=1}^n t_r = \sum_{r=1}^n \left[\frac{1}{1+rx} - \frac{1}{1+(r+1)x} \right]$$

$$= \frac{1}{1+x} - \frac{1}{1+(n+1)x}$$

$$= \frac{nx}{(1+x)(1+(n+1)x)}$$

$$\Rightarrow \sum_{r=1}^n t_r = \frac{n}{(1+x)[1+(n+1)x]}$$

The correct option is (B)

18. Since a and b are unequal,

$$\frac{a^2 + b^2}{2} > \sqrt{a^2 b^2} \quad [\text{A.M.} > \text{G.M. for unequal numbers}]$$

$$\Rightarrow a^2 + b^2 > 2ab$$

Similarly, $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$

Hence, $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$

$$\Rightarrow ab + bc + ca < 1$$

The correct option is (A)

19. The given series = $\sum_{r=1}^n (n-2r)^2$

$$= \sum_1^n (n^2 - 4nr + 4r^2)$$

$$= n \cdot n^2 - 4n \cdot \frac{n(n+1)}{2} + 4 \cdot \frac{n}{6} (n+1)(2n+1)$$

$$= \frac{n}{3} (n^2 + 2)$$

The correct option is (A)

20. Let $S = 1 + 2 \left(1 - \frac{1}{n}\right) + 3 \left(1 - \frac{1}{n}\right)^2 + \dots$ (1)

$$\therefore \left(1 - \frac{1}{n}\right) S = \left(1 - \frac{1}{n}\right) + 2 \left(1 - \frac{1}{n}\right)^2 + \dots$$
 (2)

(1) and (2)

$$\Rightarrow \frac{S}{n} = 1 + \left(1 - \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right)^2 + \dots \infty$$

$$= \frac{1}{1 - \left(1 - \frac{1}{n}\right)} = n$$

$$\Rightarrow S = n^2$$

The correct option is (C)

21. In this type of A.P. it can be easily shown that exactly one out of any 3 consecutive terms will be multiple of 3. So at most 3 consecutive terms can be prime numbers.

The correct option is (B)

22. The coefficient of $x^{49} = -[1 + 3 + 5 + \dots + 99]$
 $= -2500$

The correct option is (C)

23. A.M. \geq G.M.

$$\Rightarrow \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x} \right) / 3 \geq \left(\frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{x} \right)^{1/3}$$

$$\Rightarrow \left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x} \right) \geq 3$$

The correct option is (B)

24. Let the G.P. be $a - ar + ar^2 - ar^3 + \dots$ with common ratio $= -r$

By the given condition

$$a = \frac{ar^2 - ar}{2} \Rightarrow 2a = ar^2 - ar$$

$$\Rightarrow 2 = r^2 - r$$

$$\Rightarrow r^2 - r - 2 = 0$$

$$\Rightarrow (r-2)(r+1) = 0$$

$$\Rightarrow r = 2, -1$$

\therefore Common ratio = -2 or 1

Hence, common ratio = -2 (\because common ratio is $-ve$)

The correct option is (C)

25. $A = \frac{1}{1-r^a} \Rightarrow 1-r^a = \frac{1}{A} \Rightarrow r^a = 1 - \frac{1}{A} = \frac{A-1}{A}$

$$B = \frac{1}{1-r^b} \Rightarrow 1-r^b = \frac{1}{B} \Rightarrow r^b = 1 - \frac{1}{B} = \frac{B-1}{B}$$

$$\therefore a \log r = \log \left(\frac{A-1}{A} \right) \text{ and } b \log r = \log \left(\frac{B-1}{B} \right)$$

$$\therefore \frac{a}{b} = \frac{\log \left(\frac{A-1}{A} \right)}{\log \left(\frac{B-1}{B} \right)} = \log_{\frac{B-1}{B}} \left(\frac{A-1}{A} \right)$$

The correct option is (C)

26. If t_r be the r th term of the A.P., then

$$\begin{aligned} t_r &= S_r - S_{r-1} \\ &= cr(r-1) - c(r-1)(r-2) \\ &= c(r-1)(r-r+2) = 2c(r-1) \end{aligned}$$

We have, $t_1^2 + t_2^2 + \dots + t_n^2$

$$\begin{aligned} &= 4c^2[0^2 + 1^2 + 2^2 + \dots + (n-1)^2] \\ &= 4c^2 \frac{(n-1)n(2n-1)}{6} \end{aligned}$$

$$= \frac{2}{3} c^2 n(n-1)(2n-1)$$

The correct option is (B)

27. $S_n = \frac{n}{2} [2a + (n-1)d] = pn^2$ (1)

$$S_m = \frac{m}{2} [2a + (m-1)d] = pm^2$$

$$\Rightarrow \frac{2a + (n-1)d}{2a + (m-1)d} = \frac{n}{m}$$

$$\Rightarrow 2am + (n-1)md = 2an + n(m-1)d$$

$$\Rightarrow 2a(m-n) + d(mn - m - nm + n) = 0$$

$$\Rightarrow 2a(m-n) + d(n-m) = 0$$

$$\Rightarrow 2a - d = 0$$

$$\therefore d = 2a$$

$$\therefore (1) \text{ gives, } 2a + (n-1)2a = 2pn$$

$$\Rightarrow 2an = 2pn \Rightarrow a = p \therefore d = 2p$$

Now, $S_p = \frac{p}{2} \cdot [2a + (p-1)d]$

$$= \frac{p}{2} \cdot [2p + (p-1)2p]$$

$$= \frac{p}{2} \cdot [2p + 2p^2 - 2p]$$

$$= \frac{p}{2} \cdot 2p^2 = p^3$$

The correct option is (C)

28. We have, $b_3 > 4b_2 - 3b_1$

$$\Rightarrow b_1 r^2 > 4b_1 r - 3b_1$$

$$\Rightarrow r^2 > 4r - 3 \quad (\because b_1 > 0)$$

$$\Rightarrow r^2 - 4r + 3 > 0 \Rightarrow (r-3)(r-1) > 0$$

$$\Rightarrow r > 3 \text{ or } r < 1$$

The correct option is (A, B)

29. Since p, q, r are in A.P. so $2q = p + r$. The roots of the equation $px^2 + qx + r = 0$ are real if and only if

$$q^2 - 4pr \geq 0 \Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0$$

$$\Rightarrow p^2 + r^2 - 4pr \geq 0 \Rightarrow \frac{p^2}{r^2} - 14 \frac{p}{r} + 1 \geq 0$$

$$\Rightarrow \left(\frac{p}{r} - 7\right)^2 - 48 \geq 0 \Rightarrow \left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}$$

The correct option is (B)

30. $\frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \dots$ to n terms

$$= \left(1 - \frac{2}{3}\right) + \left(1 - \frac{4}{9}\right) + \left(1 - \frac{8}{27}\right) + \left(1 - \frac{16}{81}\right) + \dots$$

$$= n - \frac{2}{3} \left[1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \dots \text{ to } n \text{ terms}\right]$$

$$= n - \frac{2}{3} \cdot \frac{1 - \left(\frac{2}{3}\right)^n}{1 - \frac{2}{3}} = n - \frac{2}{3^n} (3^n - 2^n)$$

The correct option is (B)

31. We have, $t_r = \frac{(r-1)!}{(r+4)!}$ and $t_{r+1} = \frac{r!}{(r+5)!}$

$$\text{Now, } rt_r - (r+5)t_{r+1} = \frac{r!}{(r+4)!} - \frac{r!}{(r+4)!} = 0$$

$$\Rightarrow rt_r - (r+1)t_{r+1} = 4t_{r+1}$$

$$\Rightarrow 4 \sum_{r=1}^{n-1} t_{r+1} = \sum_{r=1}^{n-1} [rt_r - (r+1)t_{r+1}]$$

$$\Rightarrow 4(t_2 + t_3 + \dots + t_n) = 1t_1 - nt_n$$

$$\Rightarrow 4(t_1 + t_2 + \dots + t_n) = 5t_1 - nt_n = 5 \left(\frac{0!}{5!}\right) - \frac{n(n-1)!}{(n+4)!}$$

$$= \frac{1}{4!} - \frac{n!}{(n+4)!}$$

$$\Rightarrow t_1 + t_2 + \dots + t_n = \frac{1}{4} \left[\frac{1}{4!} - \frac{n!}{(n+4)!} \right]$$

The correct option is (B)

32. We have, $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$

$$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0$$

Therefore,

$$\Rightarrow \begin{array}{l} ap - b = 0 \\ b = ap \end{array} \left| \begin{array}{l} bp - c = 0 \\ c = bp \end{array} \right| \Rightarrow d = cp$$

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} = p$$

∴ a, b, c, d are in G.P.
The correct option is (B)

33. Taking A.M. and G.M. of 7 numbers

$$\frac{a}{2}, \frac{a}{3}, \frac{b}{3}, \frac{b}{2}, \frac{c}{2}, \frac{c}{3}, \frac{c}{2}, \text{ we get}$$

$$\frac{2 \cdot \frac{a}{2} + 3 \cdot \frac{b}{3} + 2 \cdot \frac{c}{2}}{7} \geq \left[\left(\frac{a}{2} \right)^2 \left(\frac{b}{3} \right)^3 \left(\frac{c}{2} \right)^2 \right]^{\frac{1}{7}}$$

$$\Rightarrow \frac{3}{7} \geq \left(\frac{a^2 b^3 c^2}{2^2 3^3 2^2} \right)^{\frac{1}{7}} \Rightarrow \frac{3^7}{7^7} \geq \frac{a^2 b^3 c^2}{2^2 \cdot 3^3 \cdot 2^2}$$

$$\Rightarrow a^2 b^3 c^2 \leq \frac{3^{10} \cdot 2^4}{7^7}$$

$$\therefore \text{greatest value of } a^2 b^3 c^2 = \frac{3^{10} \cdot 2^4}{7^7}$$

The correct option is (A)

34. Expanding along R_3 , we get

$$2 \begin{vmatrix} b & a\alpha - b \\ c & b\alpha - c \end{vmatrix} - \begin{vmatrix} a & a\alpha - b \\ b & b\alpha - c \end{vmatrix} + 0 = 0$$

$$\Rightarrow 2(b^2\alpha - bc - a\alpha + bc) - (ab\alpha - ac - ab\alpha + b^2) = 0$$

$$\Rightarrow 2\alpha(b^2 - ac) - (b^2 - ac) = 0$$

$$\text{or } (b^2 - ac)(2\alpha - 1) = 0$$

$$\text{or } b^2 - ac = 0 \quad [\because (2\alpha - 1) \neq 0]$$

$$\therefore b^2 = ac$$

∴ a, b, c are in G.P.

The correct option is (B)

35. Let $a = b - d$ and $c = b + d$,

$$\text{then } a + b + c = \frac{3}{2} \Rightarrow \beta = \frac{1}{2}$$

$$\text{Therefore, the number are } \frac{1}{2} - d, \frac{1}{2}, \frac{1}{2} + d$$

$$(d > 0 \text{ as } a < b < c)$$

$$\text{Now } a^2, b^2, c^2 \text{ are in G.P. } \Rightarrow (b^2)^2 = a^2 c^2$$

$$\Rightarrow \left(\frac{1}{2} \right)^4 = \left(\frac{1}{2} - d \right)^2 \left(\frac{1}{2} + d \right)^2$$

$$\Rightarrow \frac{1}{16} = \left(\frac{1}{4} - d^2 \right)^2 \Rightarrow \frac{1}{4} - d^2 = \pm \frac{1}{4} \Rightarrow d = \pm \frac{1}{\sqrt{2}}$$

$$\therefore d = \frac{1}{\sqrt{2}} \quad (\because d > 0)$$

$$\therefore a = b - d = \frac{1}{2} - \frac{1}{\sqrt{2}}$$

The correct option is (D)

36. As $a_1, a_2, a_3, \dots, a_{n-1}, a_n$ are in A.P.,

$$d = a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1}$$

$$\sin d [\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \dots + \sec a_{n-1} \sec a_n]$$

$$= \frac{\sin(a_2 - a_1)}{\cos a_1 \cos a_2} + \frac{\sin(a_3 - a_2)}{\cos a_2 \cos a_3} + \dots + \frac{\sin(a_n - a_{n-1})}{\cos a_{n-1} \cos a_n}$$

$$= (\tan a_2 - \tan a_1) + (\tan a_3 - \tan a_2) + \dots + (\tan a_n - \tan a_{n-1})$$

$$= \tan a_n - \tan a_1$$

The correct option is (A)

37. We have, $S = \frac{n}{2} (a + l) \Rightarrow \frac{2S}{a + l} = n$ (1)

$$\text{Also, } l = a + (n - 1)d \Rightarrow d = \frac{l - a}{n - 1}$$

$$= \frac{l - a}{\frac{2S}{a + l} - 1} \quad [\text{Using (1)}]$$

$$= \frac{l^2 - a^2}{2S - (l + a)}$$

$$\therefore k = 2S.$$

The correct option is (B)

38. As a, b, c, d are in G.P., therefore

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r \text{ (say)}$$

$$\Rightarrow b = ar, c = br = ar \cdot r = ar^2,$$

$$d = cr = ar^2 \cdot r = ar^3. (a^2 + b^2 + c^2) (b^2 + c^2 + d^2)$$

$$= (a^2 + a^2 r^2 + a^2 r^4) (a^2 r^2 + a^2 r^4 + a^2 r^6)$$

$$= a^4 r^2 (1 + r^2 + r^4) (1 + r^2 + r^4)$$

$$= (a^2 r + a^2 r^3 + a^2 r^5)^2$$

$$= (a \cdot ar + ar \cdot ar^2 + ar^2 \cdot ar^3)^2$$

$$= (ab + bc + cd)^2.$$

The correct option is (C)

39. Let the two numbers be a and b , then

$$G = \sqrt{ab} \text{ or } G^2 = ab$$

Also, p and q are two A.M.s between a and b .

$$\therefore a, p, q, b \text{ are in A.P.}$$

$$\therefore p - a = q - p \text{ and } q - p = b - q$$

$$\therefore a = 2p - q \text{ and } b = 2q - p$$

$$\therefore G^2 = ab = (2p - q)(2q - p).$$

The correct option is (B)

40. Given $x_1 \cdot x_2 \dots x_n = 1$

Since A.M. \geq G.M.

$$\therefore \left(\frac{x_1 + x_2 + \dots + x_n}{n} \right) \geq (x_1 \cdot x_2 \dots x_n)^{1/n} = (1)^{1/n} = 1$$

$$\Rightarrow x_1 + x_2 + \dots + x_n \geq n.$$

Hence $x_1 + x_2 + \dots + x_n$ can never be less than n .

The correct option is (D)

41. Total savings = $200 + 200 + 200 + 240 + 280 + \dots$ to n months = 11040

$$\Rightarrow 400 + \frac{n-2}{2}(400 + (n-3) \cdot 40) = 11040$$

$$\Rightarrow (n-2)(140 + 20n) = 10640$$

$$\Rightarrow 20n^2 + 100n - 280 = 10640$$

$$\Rightarrow n^2 + 5n - 546 = 0$$

$$\Rightarrow (n-21)(n+26) = 0$$

$$\Rightarrow n = 21 \text{ as } n \neq -26$$

The correct option is (A)

42. $T_n = (n-1)^2 + (n-1)n + n^2 = \frac{((n-1)^3 - n^3)}{(n-1) - n}$

$$= n^3 - (n-1)^3$$

$$T_1 = 1^3 - 0^3$$

$$T_2 = 2^3 - 1^3$$

\vdots

$$T_{20} = 20^3 - 19^3$$

$$S_{20} = 20^3 - 0^3 = 8000$$

The correct option is (B)

43. $100(a + 99d) = 50(a + 49d)$

$$2a + 198d = a + 49d$$

$$a + 149d = 0$$

$$T_{150} = a + 149d = 0$$

The correct option is (D)

44. $\frac{s_n}{s'_n} = \frac{3n+8}{7n+15} \Rightarrow \frac{\frac{n}{2}(2a+(n-1)d)}{\frac{n}{2}(2a'+(n-1)d')} = \frac{3n+8}{7n+15}$

$$\Rightarrow \frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{3n+8}{7n+15} = \frac{a+11d}{a'+11d'} = \frac{3(23)+8}{7(23)+15}$$

$$= \frac{77}{176} = \frac{7}{16}$$

The correct option is (B)

45. $S_n = \frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \dots$ upto n terms

$$\begin{aligned} \Rightarrow S_n &= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \dots \\ &= n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right) = n - \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} \\ &= n - 1 + \frac{1}{2^n} = n + 2^{-n} - 1 \end{aligned}$$

The correct option is (C)

46. Let a, ar, ar^2, \dots

$$a + ar = 12 \tag{1}$$

$$ar^2 + ar^3 = 48 \tag{2}$$

dividing Eq. (2) by (1), we have

$$\frac{ar^2(1+r)}{a(r+1)} = 4$$

$$\Rightarrow r^2 = 4 \text{ if } r \neq -1$$

$$\therefore r = -2$$

Also, $a = -12$ [using (1)].

The correct option is (B)

47. Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \dots$ (1)

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots$$
 (2)

From (1) and (2),

$$S\left(1 - \frac{1}{3}\right) = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots$$

$$\frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots\right)$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2}\left(\frac{1}{1 - \frac{1}{3}}\right)$$

$$= \frac{4}{3} + \frac{4}{3^2} \cdot \frac{3}{2} = \frac{4}{3} + \frac{2}{3} = \frac{6}{3}$$

$$\Rightarrow \frac{2}{3}S = \frac{6}{3}$$

$$\Rightarrow S = 3$$

The correct option is (B)

48. Here, $a = 10, d = -\frac{3}{7}$

Then, $t_n = 10 + (n-1) \cdot \left(-\frac{3}{7}\right)$

t_n is positive if $10 + (n-1) \left(-\frac{3}{7} \right) \geq 0$;

or, $70 - 3(n-1) \geq 0$ or $73 \geq 3n$; or $24 \frac{1}{3} \geq n$

\therefore First 24 terms are positive.

\therefore Sum of the positive terms

$$\begin{aligned} = S_{24} &= \frac{24}{2} \left[2 \times 10 + 23 \times \frac{-3}{7} \right] \\ &= 12 \left[20 - \frac{69}{7} \right] = \frac{852}{7}. \end{aligned}$$

The correct option is (C)

49. We have,

$$\begin{aligned} &(1 - 1 + 2 - 2 + 3 - 3 + \dots + n - n)^2 \\ &= 1^2 + 1^2 + 2^2 + 2^2 + \dots + n^2 + n^2 + 25, \end{aligned}$$

where S is the required sum.

$$\Rightarrow o = 2(1^2 + 2^2 + \dots + n^2) + 2S$$

$$\Rightarrow S = -(1^2 + 2^2 + \dots + n^2) = -\frac{n(n+1)(2n+1)}{6}$$

The correct option is (D)

50. We have,
$$\frac{S_{kx}}{S_x} = \frac{\frac{kx}{2} [2a + (kx-1)d]}{\frac{x}{2} [2a + (x-1)d]}$$

$$= \frac{k[(2a-d) + kxd]}{(2a-d) + xd}$$

For $\frac{S_{kx}}{S_x}$ to be independent of x , $2a - d = 0$ or $2a = d$.

The correct option is (C)

51. α, β, γ and are in H.P.

$$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta} \text{ are in A.P.}$$

Let d be the common difference of the A.P.

Since, α, γ are roots of $Ax^2 - 4x + 1 = 0$

$$\therefore \frac{\alpha + \gamma}{\alpha\gamma} = \frac{4/A}{1/A} = 4 \text{ or } \frac{1}{\alpha} + \frac{1}{\gamma} = 4$$

$$\Rightarrow \frac{1}{\alpha} + \frac{1}{\alpha} + 2d = 4 \text{ or } \frac{1}{\alpha} + d = 2 \quad (1)$$

Also, β, δ are roots of $Bx^2 - 6x + 1 = 0$

$$\therefore \frac{\beta + \delta}{\beta\delta} = \frac{1}{\beta} + \frac{1}{\delta} = \frac{6/B}{1/B} = 6 \text{ or } \frac{1}{\alpha} + d + \frac{1}{\alpha} + 3d = 6$$

$$\Rightarrow \frac{1}{\alpha} + 2d = 3 \quad (2)$$

Solving (1) and (2), we get $\frac{1}{\alpha} = 1$ and $d = 1$.

$$\therefore \frac{1}{\alpha} = 1, \frac{1}{\beta} = 2, \frac{1}{\gamma} = 3 \text{ and } \frac{1}{\delta} = 4.$$

Since, $\frac{1}{\alpha\gamma} = A \Rightarrow A = 3$. Also, $\frac{1}{\beta\delta} = B \Rightarrow B = 8$.

The correct option is (C)

52. We have,
$$\begin{aligned} S_1 &= (n/2) [2 \cdot 1 + (n-1) \cdot 1] \\ S_2 &= (n/2) [2 \cdot 2 + (n-1) \cdot 2] \\ S_m &= (n/2) [2 \cdot m + (n-1) \cdot m] \\ \therefore S_1 + S_2 + \dots + S_m &= n(1 + 2 + 3 + \dots + m) + \frac{n(n-1)}{2} \times (1 + 2 + \dots + m) \\ &= \frac{m(m+1)}{2} \left[n + \frac{n^2 - n}{2} \right] \\ &= \frac{m(m+1)}{2} \cdot \frac{n(n+1)}{2} = \frac{1}{4} mn(m+1)(n+1). \end{aligned}$$

The correct option is (A)

53. For the positive numbers a, c we have harmonic mean $H = b$ $\{ \because a, b, c \text{ are in H.P.} \}$ and geometric mean $G = \sqrt{ac}$.
But $G > H$; $\therefore \sqrt{ac} > b$. (1)

For the positive numbers a^n, c^n , we have

$$\text{geometric mean} = \sqrt{a^n c^n}$$

$$\text{arithmetic mean} = \frac{a^n + c^n}{2};$$

$$\therefore \text{A.M.} > \text{G.M.}, \therefore \frac{a^n + c^n}{2} > \sqrt{a^n c^n} \quad (2)$$

From (1) and (2), we get

$$\frac{a^n + c^n}{2} > (\sqrt{ac})^n > b^n, \therefore a^n + c^n > 2b^n.$$

The correct option is (A)

54. Let
$$\begin{aligned} S_n &= 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! + \dots + n \cdot n! \\ \Rightarrow S_n &= (2-1)1! + (3-1)2! + (4-1)3! \\ &\quad + (5-1)4! + \dots + [(n+1)-1]n! \\ &= (2 \cdot 1! - 1!) + (3 \cdot 2! - 2!) + (4 \cdot 3! - 3!) \\ &\quad + (5 \cdot 4! - 4!) + \dots + [(n+1)n! - n!] \\ &= (2! - 1!) + (3! - 2!) + (4! - 3!) + (5! - 4!) \\ &\quad + \dots + [(n+1)! - n!] \\ &= (n+1)! - 1! = (n+1)! - 1 \end{aligned}$$

The correct option is (A)

55. Let $R = 0 \cdot cababab \dots$
 $\Rightarrow 10^2 R = ca \cdot bababa \dots$
and, $10^4 R = caba \cdot baba \dots$

$$\begin{aligned} \Rightarrow (10^4 - 10^2)R &= caba - ca \\ \Rightarrow R &= \frac{caba - ca}{9900} = \frac{1000c + 100a + 10b + a - 10c - a}{9900} \\ &= \frac{99c + 10a + b}{990} \end{aligned}$$

The correct option is (C)

56. When n is odd, last term will be n^2 , \therefore then the sum is

$$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots + 2(n-1)^2 + n^2$$

$$= \frac{(n-1)n^2}{2} + n^2 \left[\text{Replacing } n \text{ by } n-1 \text{ in } \frac{n(n+1)^2}{2} \right]$$

$$= \frac{n^3 - n^2 + 2n^2}{2} = \frac{n^3 + n^2}{2} = \frac{n^2(n+1)}{2}$$

The correct option is (A)

57. Let $S = 1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 100 \cdot 2^{99}$
 $\therefore 2S = 1 \cdot 2 + 2 \times 2^2 + 3 \times 2^3 + \dots + 99 \cdot 2^{99} + 100 \cdot 2^{100}$
 Subtracting, we get

$$\begin{aligned} -S &= 1 + 1 \cdot 2 + 1 \cdot 2^2 + \dots + 1 \cdot 2^{99} - 100 \cdot 2^{100} \\ &= (1 + 2 + 2^2 + \dots + 2^{99}) - 100 \cdot 2^{100} \\ &= \frac{1(2^{100} - 1)}{2 - 1} - 100 \cdot 2^{100} = 2^{100} - 1 - 100 \cdot 2^{100} \end{aligned}$$

$$\therefore S = 100 \cdot 2^{100} - 2^{100} + 1 = 99 \cdot 2^{100} + 1.$$

The correct option is (D)

58. Let the number be $a - d, a, a + d, a + 2d$
 where $a, d \in Z$ and $d > 0$
 Given: $(a - d)^2 + a^2 + (a + d)^2 = a + 2d$
 $\Rightarrow 2d^2 - 2d + 3a^2 - a = 0$

$$\therefore d = \frac{1}{2} \left[1 \pm \sqrt{(1 + 2a - 6a^2)} \right]$$

Since d is positive integer, $\therefore 1 + 2a - 6a^2 > 0$

$$\Rightarrow \left(\frac{1 - \sqrt{7}}{6} \right) < a < \left(\frac{1 + \sqrt{7}}{6} \right)$$

Since a is an integer,

$$\therefore a = 0,$$

$$\text{then } d = \frac{1}{2} [1 \pm 1] = 1 \text{ or } 0. \text{ Since } d > 0, \therefore d = 1.$$

Hence, the numbers are $-1, 0, 1, 2$.

The correct option is (C)

59. Let sides of triangle be a, ar, ar^2 .
 Since $r > 1$, $\therefore ar^2$ is greatest side
 $\therefore a + ar > ar^2 \Rightarrow r^2 - r - 1 < 0$

$$\Rightarrow \frac{1 - \sqrt{5}}{2} < r < \frac{1 + \sqrt{5}}{2} \Rightarrow 1 < r < \frac{1 + \sqrt{5}}{2}$$

$$\therefore [r] = 1. \text{ Also, } -\frac{1 + \sqrt{5}}{2} < -r < -1 \quad \therefore [-r] = -2$$

$$[r] + [-r] = 1 - 2 = -1$$

The correct option is (C)

60. $S_n = \frac{1}{9}(9) + \frac{2}{9}(99) + \frac{3}{9}(999) + \dots$
 $= \frac{1}{9} [10 + 2 \cdot 10^2 + 3 \cdot 10^3 + \dots] - \frac{1}{9} [1 + 2 + 3 + \dots]$

$$= \frac{1}{9} S - \frac{1}{9} \frac{n(n+1)}{2}$$

$$S = 10 + 2 \cdot 10^2 + 3 \cdot 10^3 + \dots + n \cdot 10^n$$

$$\Rightarrow 10S = 10^2 + 2 \cdot 10^3 + \dots + (n-1)10^n + n \cdot 10^{n+1}$$

$$-9S = (10 + 10^2 + 10^3 + \dots + 10^n) - n \cdot 10^{n+1}$$

$$\Rightarrow S = \frac{n}{9} 10^{n+1} - \frac{10^{n+1} - 1}{81}$$

$$\therefore S_n = \frac{n}{81} 10^{n+1} - \frac{10^{n+1} - 1}{9 \cdot 81} - \frac{1}{9} \frac{(n+1)}{2}$$

$$\Rightarrow 9 \cdot S_n = \frac{(9n-1)10^{n+1}}{81} + \frac{1}{81} - \frac{n(n+1)}{2}$$

$$\therefore 9(S_n - S_{n-1}) = \frac{10^n}{81} \{10(9n-1) - (9n-10)\} - n$$

$$= n(10^n - 1)$$

The correct option is (C)

61. $a + b + c = xb$

Divide by b ,

$$\frac{a}{b} + 1 + \frac{c}{b} = x$$

$$\Rightarrow \frac{1}{r} + 1 + r = x, r \text{ is the common ratio of the G.P.}$$

$$\Rightarrow r^2 + r(1-x) + 1 = 0. \text{ Since } r \text{ is real, therefore, discriminant} > 0$$

$$\Rightarrow (1-x)^2 - 4 > 0 \Rightarrow x^2 - 2x + 1 - 4 > 0$$

$$\Rightarrow x^2 - 2x - 3 > 0 \Rightarrow (x+1)(x-3) > 0$$

$$\Rightarrow x < -1 \text{ or } x > 3$$

The correct option is (A)

62. $\sum_1^3 a_r a_{r+1} = a_1 a_2 + a_2 a_3 + a_3 a_4 = 3a_1 a_4$

Since a_1, a_2, a_3, a_4 are in H.P. $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}$ are in A.P.

$$\frac{1}{a_2} - \frac{1}{a_1} = d \Rightarrow a_1 - a_2 = da_1 a_2 \quad (1)$$

$$\text{Similarly, } a_2 - a_3 = d a_2 a_3 \quad (2)$$

$$a_3 - a_4 = d a_3 a_4 \quad (3)$$

On adding (1), (2) and (3), we get

$$a_1 - a_4 = d[a_1 a_2 + a_2 a_3 + a_3 a_4]$$

$$\therefore a_1a_2 + a_2a_3 + a_3a_4 = \frac{a_1 - a_4}{d}$$

$$\Rightarrow \frac{1}{a_4} = \frac{1}{a_1} + 3d \Rightarrow \frac{a_1 - a_4}{d} = 3a_1a_4$$

$$\Rightarrow a_1a_2 + a_2a_3 + a_3a_4 = 3a_1a_4$$

\therefore Given expression = 3. It is a root of $x^2 + 2x - 15 = 0$

The correct option is (B)

$$\begin{aligned} 63. \quad & \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \dots \text{ to } n \text{ terms} \\ & = \left(1 - \frac{2}{3}\right) + \left(1 - \frac{4}{9}\right) + \left(1 - \frac{8}{27}\right) + \left(1 - \frac{16}{81}\right) + \dots \\ & = n - \frac{2}{3} \left[1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \dots \text{ to } n \text{ terms} \right] \\ & = n - \frac{2}{3} \cdot \frac{1 - \left(\frac{2}{3}\right)^n}{1 - \frac{2}{3}} = n - \frac{2}{3^n} (3^n - 2^n) \end{aligned}$$

The correct option is (B)

64. Taking A.M. and G.M. of 7 numbers

$$\frac{a}{2}, \frac{a}{2}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}, \text{ we get}$$

$$\frac{2 \cdot \frac{a}{2} + 3 \cdot \frac{b}{3} + 2 \cdot \frac{c}{2}}{7} \geq \left\{ \left(\frac{a}{2}\right)^2 \left(\frac{b}{3}\right)^3 \left(\frac{c}{2}\right)^2 \right\}^{\frac{1}{7}}$$

$$\Rightarrow \frac{3}{7} \geq \left(\frac{a^2 b^3 c^2}{2^2 \cdot 3^3 \cdot 2^2} \right)^{\frac{1}{7}} \Rightarrow \frac{3^7}{7^7} \geq \frac{a^2 b^3 c^2}{2^2 \cdot 3^3 \cdot 2^2}$$

$$\Rightarrow a^2 b^3 c^2 \leq \frac{3^{10} \cdot 2^4}{7^7}$$

$$\therefore \text{greatest value of } a^2 b^3 c^2 = \frac{3^{10} \cdot 2^4}{7^7}$$

The correct option is (A)

65. Harmonic mean of a, b is $H = \frac{2ab}{a+b}$

$$\text{Geometric mean } G = \sqrt{ab}$$

$$\text{Given: } \frac{H}{G} = \frac{4}{5}, \text{ so } \frac{2\sqrt{ab}}{a+b} = \frac{4}{5}$$

$$\text{or, } \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$$

By componendo and dividendo

$$\frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})^2} = \frac{9}{1} \text{ or } \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{3}{1}$$

$$\text{Again, by componendo and dividendo } \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{3+1}{3-1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = 2 \text{ or } \frac{a}{b} = \frac{4}{1}$$

The correct option is (D)

66. We have, $S = \frac{n}{2} (a+l) \Rightarrow \frac{2S}{a+l} = n$ (1)

$$\text{Also, } l = a + (n-1)d \Rightarrow d = \frac{l-a}{n-1}$$

$$\begin{aligned} & = \frac{l-a}{\frac{2S}{a+l} - 1} \quad [\text{Using (1)}] \\ & = \frac{l^2 - a^2}{2S - (l+a)} \end{aligned}$$

$$\therefore k = 2S$$

The correct option is (B)

67. As $a_1, a_2, a_3, \dots, a_{n-1}, a_n$ are in A.P.,

$$d = a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1}$$

$$\sin d [\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \dots + \sec a_{n-1} \sec a_n]$$

$$= \frac{\sin(a_2 - a_1)}{\cos a_1 \cdot \cos a_2} + \frac{\sin(a_3 - a_2)}{\cos a_2 \cos a_3} + \dots + \frac{\sin(a_n - a_{n-1})}{\cos a_{n-1} \cos a_n}$$

$$= (\tan a_2 - \tan a_1) + (\tan a_3 - \tan a_2)$$

$$+ \dots + (\tan a_n - \tan a_{n-1})$$

$$= \tan a_n - \tan a_1$$

The correct option is (A)

68. We have, $t_r = \frac{(r-1)!}{(r+4)!}$ and $t_{r+1} = \frac{r!}{(r+5)!}$

$$\text{Now, } r t_r - (r+5)t_{r+1} = \frac{r!}{(r+4)!} - \frac{r!}{(r+4)!} = 0$$

$$\Rightarrow r t_r - (r+1)t_{r+1} = 4t_{r+1}$$

$$\Rightarrow 4 \sum_{r=1}^{n-1} t_{r+1} = \sum_{r=1}^{n-1} [r t_r - (r+1)t_{r+1}]$$

$$\Rightarrow 4(t_2 + t_3 + \dots + t_n) = 1t_1 - nt_n$$

$$\Rightarrow 4(t_1 + t_2 + \dots + t_n) = 5t_1 - nt_n = 5 \left(\frac{0!}{5!} \right) - \frac{n(n-1)}{(n+4)}$$

$$= \frac{1}{4!} - \frac{n!}{(n+4)}$$

$$\Rightarrow t_1 + t_2 + \dots + t_n = \frac{1}{4} \left[\frac{1}{4!} - \frac{n!}{(n+4)!} \right]$$

The correct option is (B)

69. $a_n = x^{1/2^n} + y^{1/2^n}$ and $b_n = x^{1/2^n} - y^{1/2^n}$

$$\text{Now, } a_n b_n = (x^{1/2^n} + y^{1/2^n})(x^{1/2^n} - y^{1/2^n})$$

$$\Rightarrow a_n b_n = (x^{1/2^n})^2 - (y^{1/2^n})^2$$

$$\Rightarrow a_n b_n = x^{1/2^{n-1}} - y^{1/2^{n-1}} = b_{n-1} \quad (1)$$

Now, $a_1 a_2 a_3 \dots a_n$

$$\begin{aligned} &= \left(\frac{a_1 a_2 a_3 \dots a_n}{b_n} \right) b_n \\ &= \frac{(a_1 a_2 a_3 \dots a_{n-1}) (a_n b_n)}{b_n} \\ &= \frac{a_1 a_2 a_3 \dots a_{n-1} b_{n-1}}{b_n} \quad \{\text{using (1)}\} \\ &= \frac{(a_1 a_2 a_3 \dots a_{n-2}) (a_{n-1} b_{n-1})}{b_n} \\ &= \frac{a_1 a_2 a_3 \dots a_{n-1} b_{n-2}}{b_n} \\ &\dots \dots \dots \\ &\dots \dots \dots \\ &= \frac{a_1 b_1}{b_n} = \frac{b_0}{b_n} = \frac{x-y}{b_n} \end{aligned}$$

The correct option is (C)

70. Since n is an odd integer $(-1)^{n-1} = 1$ and $n-1, n-3, n-5$ etc., are even integers. We have,

$$\begin{aligned} n^3 - (n-1)^3 + (n-2)^3 - (n-3)^3 + \dots + (-1)^{n-1} 1^3 \\ &= n^3 + (n-1)^3 + (n-2)^3 + \dots + 1^3 \\ &\quad - 2[(n-1)^3 + (n-3)^3 + \dots + 2^3] \\ &= n^3 + (n-1)^3 + (n-2)^3 + \dots + 1^3 \\ &\quad - 2 \times 2^3 \left[\left(\frac{n-1}{2} \right)^3 + \left(\frac{n-3}{2} \right)^3 + \dots + 1^3 \right] \\ &\quad [\because n-1, n-3 \text{ are even integers}] \\ &= \left[\frac{n(n+1)}{2} \right]^2 - 16 \left[\frac{1}{2} \left(\frac{n-1}{2} \right) \left(\frac{n-1}{2} + 1 \right) \right]^2 \\ &= \frac{1}{4} n^2 (n+1)^2 - 16 \frac{(n-1)^2 (n+1)^2}{16 \times 4} \\ &= \frac{1}{4} (n+1)^2 [n^2 - (n-1)^2] = \frac{1}{4} (n+1)^2 (2n-1). \end{aligned}$$

The correct option is (D)

71. We have,

$$\begin{aligned} a(n) &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n)-1} \\ &= 1 + \left(\frac{1}{2} + \frac{1}{3} \right) + \left(\frac{1}{4} + \dots + \frac{1}{7} \right) + \left(\frac{1}{8} + \dots + \frac{1}{15} \right) \\ &\quad + \dots + \left(\frac{1}{2^{n-1}} + \dots + \frac{1}{2^{n-1}} \right) \end{aligned}$$

$$\begin{aligned} &< 1 + \frac{2}{2} + \frac{4}{4} + \frac{8}{8} + \dots + \frac{2^{n-1}}{2^{n-1}} \\ &= 1 + \frac{1+\dots+1}{(n+1) \text{ times}} = n. \end{aligned}$$

Thus, $a(100) < 100$

Next, $a(n) = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4} \right) + \left(\frac{1}{5} + \dots + \frac{1}{8} \right) + \dots$

$$+ \frac{1}{2^{n-1}+1} + \dots + \frac{1}{2^n-1}$$

$$\begin{aligned} &> 1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \dots + \frac{2^{n-1}}{2^n} - \frac{1}{2^n} \\ &= 1 + \underbrace{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots + \frac{1}{2}}_{n \text{ times}} - \frac{1}{2^n} \\ &= \left(1 - \frac{1}{2^n} \right) + \frac{n}{2} \end{aligned}$$

Thus, $a(200) > \left(1 - \frac{1}{2^{100}} \right) + \frac{200}{2} > 100.$

The correct option is (D)

72. Let $\alpha = \frac{1}{a-d}, \beta = \frac{1}{a}$ and $\gamma = \frac{1}{a+d}$

Then, $\frac{1}{a-d} \cdot \frac{1}{a} \cdot \frac{1}{a+d} = -\frac{1}{3}$

and, $\frac{1}{a-d} \cdot \frac{1}{a} + \frac{1}{a+d} \cdot \frac{1}{a} + \frac{1}{a-d} \cdot \frac{1}{a+d} = -1$

$\Rightarrow (a-d)a(a+d) = -3$

and, $\frac{(a+d)+(a-d)+a}{(a-d)a(a+d)} = -1$

$\Rightarrow 3a = 3$ or $a = 1$ and $(1-d)(1+d) = -3$

or, $1-d^2 = -3$ or $d^2 = 4 \Rightarrow d = \pm 2.$

When $d = 2, \alpha = -1, \beta = 1, \gamma = \frac{1}{3}.$

When $d = -2, \alpha = \frac{1}{3}, \beta = 1, \gamma = -1.$

Therefore, $|\alpha - \gamma| = \frac{4}{3}$

The correct option is (C)

73. We have, $\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}$

$\Rightarrow \frac{x-a}{b} - \frac{b}{x-a} = \frac{a}{x-b} - \frac{x-b}{a}$

$\Rightarrow \frac{(x-a)^2 - b^2}{b(x-a)} = \frac{a^2 - (x-b)^2}{(x-b)a}$

$\Rightarrow \frac{(x-a-b)(x-a+b)}{b(x-a)} = -\frac{(x-b-a)(x-b+a)}{(x-b)a}$

$$\Rightarrow (x - a - b) [a(x - b)(x - a + b) + b(x - b + a)(x - a)] = 0$$

$$\Rightarrow x(x - a - b) [(a + b)x - (a^2 + b^2)] = 0$$

$$\Rightarrow x = 0, x = a + b \text{ or } x = \frac{a^2 + b^2}{a + b}$$

$$\text{Since } \frac{a^2 + b^2}{a + b} = a + b - \frac{2ab}{a + b} < a + b \quad (\because a, b > 0)$$

$$\text{we take } x_1 = a + b, x_2 = \frac{a^2 + b^2}{a + b} \text{ and } x_3 = 0.$$

$$\text{Since } x_1 - x_2 - x_3 = c, \text{ we get } a + b - \frac{a^2 + b^2}{a + b} = c$$

$$\Rightarrow \frac{2ab}{a + b} = c \Rightarrow a, b, c \text{ are in H.P.}$$

The correct option is (C)

74. The given product

$$= (-1)^n (x - 1)(2x - 1)(2^2x - 1)(2^3x - 1) \dots (2^n x - 1)$$

$$= (-1)^n \cdot 2^{1+2+\dots+n} \cdot (x - 1) \left(x - \frac{1}{2}\right) \left(x - \frac{1}{2^2}\right) \dots \left(x - \frac{1}{2^n}\right)$$

$$= (-1)^n \cdot 2^{\frac{n(n+1)}{2}} \cdot (x - 1) \left(x - \frac{1}{2}\right) \left(x - \frac{1}{2^2}\right) \dots \left(x - \frac{1}{2^n}\right)$$

\therefore coefficient of x^n

$$= (-1)^n \cdot 2^{\frac{n(n+1)}{2}} \cdot \left(-1 - \frac{1}{2} - \frac{1}{2^2} \dots \frac{1}{2^n}\right)$$

$$= (-1)^{2n+1} \cdot 2^{\frac{n(n+1)}{2}} \cdot \frac{\left(1 - \frac{1}{2^{n+1}}\right)}{\left(1 - \frac{1}{2}\right)} = (1 - 2^{n+1}) \cdot 2^{\frac{n(n-1)}{2}}$$

The correct option is (A)

75. Let $R = 0.272727\dots$

$$\Rightarrow 10^2 R = 27.2727\dots$$

$$\text{and, } 10^4 R = 2727.2727\dots$$

$$\Rightarrow (10^4 - 10^2) R = 2727 - 27$$

$$\therefore R = \frac{2700}{9900} = \frac{3}{11}$$

$$\text{Similarly, } 0.727272\dots = \frac{8}{11}$$

Since $0.272727\dots, x$ and $0.727272\dots$ are in H.P.

$$\Rightarrow \frac{3}{11}, x, \frac{8}{11} \text{ are in H.P.}$$

$$\Rightarrow x = \frac{2 \cdot \frac{3}{11} \cdot \frac{8}{11}}{\frac{3}{11} + \frac{8}{11}} = \frac{48}{121}$$

$\therefore x$ is rational.

The correct option is (A)

76. We have, $|a_i| = |a_{i-1} + 1|$

$$\Rightarrow a_i^2 = a_{i-1}^2 + 2a_{i-1} + 1$$

Putting $i = 1, 2, 3, \dots, n + 1$, we get

$$a_1^2 = 0$$

$$a_2^2 = a_1^2 + 2a_1 + 1$$

$$a_3^2 = a_2^2 + 2a_2 + 1$$

$$\vdots \quad \vdots \quad \vdots$$

$$a_n^2 = a_{n-1}^2 + 2a_{n-1} + 1$$

$$a_{n+1}^2 = a_n^2 + 2a_n + 1$$

$$\text{On adding, we get } \sum_{i=1}^{n+1} a_i^2 = \sum_{i=1}^n a_i^2 + 2 \sum_{i=1}^n a_i + n$$

$$\Rightarrow 2 \sum_{i=1}^n a_i = -n + a_{n+1}^2 \geq -n$$

$$\Rightarrow \frac{a_1 + a_2 + \dots + a_n}{n} \geq -\frac{1}{2} \Rightarrow x \geq -\frac{1}{2}$$

The correct option is (B)

77. Given: $a_1, a_2, a_3, \dots, a_n$ are in H.P.

$$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n} \text{ are in A.P.}$$

$$\Rightarrow \frac{a_1 + a_2 + a_3 + \dots + a_n}{a_1}, \frac{a_1 + a_2 + \dots + a_n}{a_2}, \dots,$$

$$\frac{a_1 + a_2 + \dots + a_n}{a_n}, \text{ are in A.P.}$$

$$\Rightarrow \frac{a_1 + a_2 + a_3 + \dots + a_n}{a_1} - 1, \frac{a_1 + a_2 + \dots + a_n}{a_2} - 1, \dots,$$

$$\frac{a_1 + a_2 + \dots + a_n}{a_n} - 1, \text{ are in A.P.}$$

$$\Rightarrow \frac{a_2 + a_3 + \dots + a_n}{a_1}, \frac{a_1 + a_3 + \dots + a_n}{a_2}, \dots,$$

$$\frac{a_1 + a_2 + \dots + a_{n-1}}{a_n} \text{ are in A.P.}$$

$$\Rightarrow \frac{a_1}{a_2 + a_3 + \dots + a_n}, \frac{a_2}{a_1 + a_3 + \dots + a_n}, \dots,$$

$$\frac{a_n}{a_1 + a_2 + \dots + a_{n-1}} \text{ are in H.P.}$$

The correct option is (C)

78. Let the three digits be a, ar, ar^2 .

Then, according to the hypothesis,

$$100a + 10ar + ar^2 + 792 = 100ar^2 + 10ar + a$$

$$\Rightarrow a(r^2 - 1) = 8$$

(1)

and, $a, ar + 2, ar^2$ are in A.P.
 then, $2(ar + 2) = a + ar^2$
 $\Rightarrow a(r^2 - 2r + 1) = 4$ (2)

Dividing (1) by (2),

then, $\frac{a(r^2 - 1)}{a(r^2 - 2r + 1)} = \frac{8}{4}$

$\Rightarrow \frac{(r+1)(r-1)}{(r-1)^2} = 2$

$\Rightarrow \frac{r+1}{r-1} = 2$

$\therefore r = 3 \therefore$ from (1), $a = 1$.

Thus, digits are 1, 3, 9 and so the required number is 931.

The correct option is (C)

79. The general term of the given sequence is

$$T_n = \frac{n^2}{500 + 3n^3}$$

then, $\frac{dT_n}{dn} = \frac{n(1000 - 3n^3)}{(500 + 3n^3)^2}$

For maximum or minimum of T_n

$$\frac{dT_n}{dn} = 0$$

$\therefore n = \left(\frac{1000}{3}\right)^{\frac{1}{3}}$

Now, $6 < \left(\frac{1000}{3}\right)^{\frac{1}{3}} < 7$

Hence, T_7 is largest term. So largest term in the given sequence is $\frac{49}{1529}$.

The correct option is (C)

80. Consider the equation

$$(x - 1)(x - 2)(x - 3) \dots (x - 100) = 0$$
 (1)

Its root are 1, 2, 3, ..., 100.

(1) is a polynomial equation in x of degree 100. Coefficient of $x^{100} = 1$.

Now, sum of the roots of equation (1); taken one at a time

i.e., $1 + 2 + 3 + \dots + 100 = (-1)^1 \frac{\text{coeff. of } x^{99}}{\text{coeff. of } x^{100}}$
 $= -\text{coeff. of } x^{99}$

$\therefore \text{coeff. of } x^{99} = -(1 + 2 + 3 + \dots + 100) = -\frac{100 \times 101}{2}$
 $= -5050$

Sum of the product of the roots 1, 2, 3, ..., 100 taken two at a time

$$= (-1)^2 \frac{\text{coeff. of } x^{98}}{\text{coeff. of } x^{100}} = \text{coeff. of } x^{98}$$

\therefore coeff. of $x^{98} =$ sum of the product of 1, 2, 3, ..., 100 taken two at a time

$$= \frac{1}{2} [(1 + 2 + 3 + \dots + 100)^2 - (1^2 + 2^2 + \dots + 100^2)]$$

$$= \frac{1}{2} \left[(5050)^2 - \frac{100 \times 101 \times 201}{6} \right]$$

$$= \frac{1}{2} [(5050)^2 - 338350] = 12582075.$$

The correct option is (C)

81. Let the sides of the triangle be a, ar, ar^2 .

If $r = 1$, then the three terms of G.P. will be a, a, a and hence an equilateral triangle will be formed.

Thus when $r = 1$, triangle will be formed (1)

If $r > 1$, then greatest side will be ar^2 and in this case triangle will be formed if

$$a + ar > ar^2 \Rightarrow r^2 - r - 1 < 0$$

$$\Rightarrow \frac{1 - \sqrt{5}}{2} < r < \frac{1 + \sqrt{5}}{2}$$

$$\Rightarrow r < \frac{1 + \sqrt{5}}{2} \quad [\because r > 1] \quad (2)$$

If $r < 1$, then greatest side will be a and triangle will be formed if

$$ar + ar^2 > a \Rightarrow r^2 + r - 1 > 0$$

$$\Rightarrow r < \frac{-1 - \sqrt{5}}{2} \text{ or } r > \frac{-1 + \sqrt{5}}{2}$$

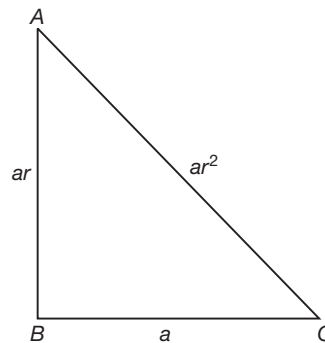
$$\Rightarrow \frac{\sqrt{5} - 1}{2} < r < 1. \quad [\because 0 < r < 1] \quad (3)$$

From (1), (2) and (3), possible values of r are given by

$$\frac{\sqrt{5} - 1}{2} < r < \frac{\sqrt{5} + 1}{2}.$$

The correct option is (B)

82. Let the sides of the right angled triangle be a, ar, ar^2 out of which ar^2 is the hypotenuse, then $r > 1$.



Now, $a^2 r^4 = a^2 + a^2 r^2$

or, $r^4 - r^2 - 1 = 0 \therefore r^2 = \frac{1 \pm \sqrt{5}}{2}$

$$\therefore r > 1 \therefore r^2 > 1 \therefore r^2 = \frac{1 + \sqrt{5}}{2}$$

Angle C is the greater acute angle

$$\text{Now, } \cos C = \frac{a}{ar^2} = \frac{1}{r^2} = \frac{1}{1 + \sqrt{5}}$$

The correct option is (A)

83. Here, the series is

$2 + (2 + 3) + (2 + 3 + 9) + (2 + 3 + 9 + 27) + \dots$
the difference of the consecutive terms being 3, 9, 27, ...

$$\begin{aligned} \therefore t_n &= 2 + 3 + 9 + 27 \dots \text{ to } n \text{ terms} \\ &= 2 + [3 + 9 + 27 + \dots \text{ to } (n-1) \text{ terms}] \\ &= 2 + \frac{3(1-3^{n-1})}{1-3} = 2 - \frac{3}{2} (1-3^{n-1}) \\ &= \frac{1}{2} + \frac{1}{2} \cdot 3^n; \end{aligned}$$

$$\begin{aligned} \therefore S_n &= \sum t_n = \frac{1}{2} \sum 1 + \frac{1}{2} \sum 3^n \\ &= \frac{1}{2} \cdot n + \frac{1}{2} (3 + 3^2 + 3^3 + \dots + 3^n) \\ &= \frac{n}{2} + \frac{1}{2} \cdot \frac{3(1-3^n)}{1-3} = \frac{n}{2} + \frac{3}{4} (3^n - 1). \end{aligned}$$

The correct option is (B)

84. Let the A.P. be $x, x + y, x + 2y \dots$

$$\text{Then, } a = x - (p-1)y, \quad (1)$$

$$b = x + (q-1)y \quad (2)$$

$$c = x + (r-1)y \quad (3)$$

$$\therefore b - c = (q - r)y \quad (4)$$

$$c - a = (r - p)y \quad (5)$$

$$a - b = (p - q)y \quad (6)$$

Let the G.P. be u, uv, uv^2, \dots

$$\text{Then, } a = uv^{p-1} \quad (7)$$

$$b = uv^{q-1}, \quad (8)$$

$$c = uv^{r-1}, \quad (9)$$

Now, $\log(a^{b-c} \cdot b^{c-a} \cdot c^{a-b})$

$$\begin{aligned} &= (b-c) \log a + (c-a) \log b + (a-b) \log c \\ &= (q-r)y \log(uv^{p-1}) + (r-p)y \log(uv^{q-1}) \\ &\quad + (p-q)y \log(uv^{r-1}), \text{ using (4), (5), (6),} \\ &\quad \quad \quad (7), (8), (9). \end{aligned}$$

$$\begin{aligned} &= y [(q-r) \{\log u + (p-1) \log v\} + (r-p) \{\log u \\ &\quad + (q-1) \log v\} + (p-q) \{\log u + (r-1) \log v\}] \\ &= y [\log u (q-r+r-p+p-q) + \log v \{(q-r)(p-1) \\ &\quad + (r-p)(q-1) + (p-q)(r-1)\}] \\ &= y [\log u \times 0 + \log v \times 0] = 0. \end{aligned}$$

$$\text{Then, } \log[a^{b-c} \cdot b^{c-a} \cdot c^{a-b}] = 0$$

$$\therefore a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1;$$

$$\therefore a^b b^c c^a = a^c b^a c^b.$$

The correct option is (C)

85. Let the $3n$ terms of G.P. are $a, ar, ar^2, \dots, ar^{n-1}, ar^n, ar^{n+1}, ar^{n+2}, \dots, ar^{2n-1}, ar^{2n}, ar^{2n+1}, ar^{2n+2}, \dots, ar^{2n-1}$

$$\text{Then, } S_1 = a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$S_2 = ar^n + ar^{n+1} + ar^{n+2} + \dots + ar^{3n-1}$$

$$= \frac{ar^n(1-r^n)}{1-r}$$

$$S_3 = ar^{2n} + ar^{2n+1} + ar^{2n+2} + \dots + ar^{3n-1}$$

$$= \frac{ar^{2n}(1-r^n)}{1-r}$$

$$\text{Now, } (S_2)^2 = a^2 r^{2n} \frac{(1-r^n)^2}{(1-r)^2}$$

$$= \frac{a(1-r^n)}{1-r} \cdot ar^{2n} \frac{(1-r^n)}{1-r} = S_1 S_3$$

Hence, S_1, S_2, S_3 are in G.P.

The correct option is (B)

86. Let $r > 1$. Then, $S_n = \frac{a(r^n - 1)}{r - 1}$

$$u_n = S_1 + S_2 + S_3 + \dots + S_n$$

$$= \frac{a(r-1)}{r-1} + \frac{a(r^2-1)}{r-1} + \frac{a(r^3-1)}{r-1} + \dots + \frac{a(r^n-1)}{r-1}$$

$$= \frac{a}{r-1} [(r + r^2 + r^3 + \dots + r^n) - n]$$

$$= \frac{a}{r-1} \left[\frac{r(r^n-1)}{r-1} - n \right]$$

$$\therefore r S_n + (1-r) u_n$$

$$= r \cdot \frac{a(r^n-1)}{r-1} + (1-r) \frac{ar(r^n-1)}{(1-r)^2} - \frac{an(1-r)}{r-1}$$

$$= -\frac{ar(r^n-1)}{1-r} + \frac{ar(r^n-1)}{1-r} + an = na$$

The correct option is (A)

87. We have, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

and, $\sin A = ka$ (k is a content)

$$\therefore \cot A = \frac{b^2 + c^2 - a^2}{2abck} \text{ and similarly, we have}$$

$$\cot B = \frac{a^2 + c^2 - b^2}{2abck}, \cot C = \frac{a^2 + b^2 - c^2}{2abck}$$

Given: $\frac{b^2 + c^2 - a^2}{2abck}, \frac{a^2 + c^2 - b^2}{2abck}, \frac{a^2 + b^2 - c^2}{2abck}$ are in A.P.

$\Rightarrow b^2 + c^2 - a^2, a^2 + c^2 - b^2, a^2 + b^2 - c^2$ are in A.P.

(Multiplying each term by $2abck$)

$\Rightarrow -2a^2, -2b^2, -2c^2$ are in A.P.

(Subtracting $a^2 + b^2 + c^2$ from each term)

$\Rightarrow a^2, b^2, c^2$ are in A.P. (dividing each term by -2)

The correct option is (A)

88. Let $S = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$ up to ∞

$$\text{Given: } \frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{2^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$$

$$= S + \frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots$$

$$= S + \frac{1}{2^4} \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \right]$$

$$= S + \frac{1}{2^4} \times \frac{\pi^4}{90}$$

$$\therefore S = \frac{\pi^4}{90} \left(1 - \frac{1}{16} \right) = \frac{\pi^4}{96}$$

The correct option is (B)

89. Let a and d respectively be the first term and common difference of the A.P.

$$\text{Given: } n = \frac{2mr}{m+r} \quad (1)$$

$$\text{and, } (a+nd)^2 = (a+md)(a+rd)$$

$$\Rightarrow \left(\frac{a}{d} + n \right)^2 = \left(\frac{a}{d} + m \right) \left(\frac{a}{d} + r \right)$$

$$\Rightarrow (x+n)^2 = (x+m)(x+r) \quad \left[\text{Putting } \frac{a}{d} = x \right]$$

$$\Rightarrow x^2 + 2nx + n^2 = x^2 + (m+r)x + mr$$

$$\Rightarrow (m+r-2n)x = n^2 - mr = n^2 - \frac{n(m+r)}{2}$$

[Using (1)]

$$= \frac{n}{2}(2n-m-r)$$

$\therefore x = -\frac{n}{2}$, which is the required ratio.

The correct option is (D)

90. Let the n numbers in G.P. be

$$a, ar, ar^2, \dots, ar^{n-1}$$

Thus, we have,

$$S_n = a \left(\frac{1-r^n}{1-r} \right)$$

Also,

$$S_n^2 = [a + ar + ar^2 + \dots + ar^{n-1}]^2$$

$$\Rightarrow a^2 \left(\frac{1-r^n}{1-r} \right)^2 = a^2 + (ar)^2 + (ar^2)^2 + \dots + (ar^{n-1})^2 + 2S$$

where S denotes the sum of the product of the terms of the G.P. taken two at a time.

$$\Rightarrow a^2 \left(\frac{1-r^n}{1-r} \right)^2 = a^2 \left(\frac{1-r^{2n}}{1-r^2} \right) + 2S$$

$$\Rightarrow S = \frac{a^2}{2} \left[\frac{(1-r^n)^2}{(1-r)^2} - \frac{1-r^{2n}}{1-r^2} \right]$$

$$= \frac{a^2}{2} \left(\frac{1-r^n}{1-r} \right) \left[\frac{1-r^n}{1-r} - \frac{1+r^n}{1+r} \right]$$

$$= S_n \times \frac{a}{2} \times \frac{(1-r^n)(1+r) - (1+r^n)(1-r)}{(1+r)(1-r)}$$

$$= S_n \times \frac{a}{2} \times \frac{2(r-r^n)}{(1+r)(1-r)}$$

$$= S_n \times \frac{r}{1+r} \times \frac{a(1-r^{n-1})}{1-r} = \left(\frac{r}{1+r} \right) S_n S_{n-1}$$

$$\therefore k = \frac{r}{r+1}$$

The correct option is (C)

91. Let the common difference of the given A.P. be t . Then,

$$d = a^2 + b^2 + c^2 \Rightarrow a + 3t = a^2 + (a+t)^2 + (a+2t)^2$$

$$\Rightarrow 5t^2 + 3(2a-1)t + 3a^2 - a = 0 \quad (1)$$

$$\therefore t \text{ is real } \Rightarrow D \geq 0$$

$$\Rightarrow 9(2a-1)^2 - 4(5)(3a^2-a) \geq 0$$

$$\Rightarrow 24a^2 + 16a - 9 \leq 0$$

$$\Rightarrow -\frac{1}{3} - \frac{\sqrt{70}}{12} < a < -\frac{1}{3} + \frac{\sqrt{70}}{12}$$

$$\Rightarrow a = -1, 0 \quad [\because a \text{ is integer}]$$

When $a = 0$, from (1), $t = 0, \frac{3}{5}$. Rejecting both these values since t must be non zero

$$\text{When, } a = -1, \text{ from (1), } t = 1, \frac{4}{5} \Rightarrow t = 1$$

$$\therefore a + b + c + d = -1 + 0 + 1 + 2 = 2$$

The correct option is (A)

$$\begin{aligned}
 92. \quad & 1 + \frac{3}{2} + \frac{5}{3} + \dots + \left(\frac{2n-1}{n}\right) \\
 &= (2-1) + \left(2 - \frac{1}{2}\right) + \left(2 - \frac{1}{3}\right) + \dots + \left(2 - \frac{1}{n}\right) \\
 &= 2n - \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}\right) = 2n - H_n
 \end{aligned}$$

The correct option is (B)

$$\begin{aligned}
 93. \quad & \text{We have, } a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{2n-1}^2 - a_{2n}^2 \\
 &= (a_1 - a_2) [a_1 + a_2 + a_3 + \dots + a_{2n}] \\
 &\quad [\because a_1 - a_2 = a_3 - a_4 = \dots = a_{2n-1} - a_{2n}] \\
 &= -d \times \frac{2n}{2} [2a_1 + (2n-1)d] \quad (1)
 \end{aligned}$$

where d is the common difference of the A.P.

$$\text{Since, } a_{2n} = a_1 + (2n-1)d$$

$$\Rightarrow d = \frac{a_{2n} - a_1}{2n-1}$$

Thus, we have from (1), required expression

$$\begin{aligned}
 &= \frac{a_1 - a_{2n}}{2n-1} \times n [2a_1 + a_{2n} - a_1] \\
 &= \frac{n(a_1 - a_{2n})(a_1 + a_{2n})}{2n-1} = \frac{n(a_1^2 - a_{2n}^2)}{2n-1}
 \end{aligned}$$

The correct option is (C)

$$94. \quad \text{We have, } a_{n+1} = \frac{1}{1-a_n}$$

$$\begin{aligned}
 \therefore \quad & a_2 = \frac{1}{1-a_1} \text{ and } a_3 = \frac{1}{1-a_2} = \frac{1}{1-\frac{1}{1-a_1}} \\
 &= \frac{1-a_1}{1-a_1-1} = \frac{1-a_1}{-a_1}
 \end{aligned}$$

$$\text{Since } a_3 = a_1, \therefore \frac{1-a_1}{-a_1} = a_1$$

$$\Rightarrow a_1^2 - a_1 + 1 = 0 \Rightarrow a_1 = -w \text{ or } -w^2$$

$$\begin{aligned}
 \text{Now, } \quad & a_5 = \frac{1}{1-a_4} = \frac{1}{1-\frac{1}{1-a_3}} = \frac{1-a_3}{1-a_3-1} \\
 &= \frac{1-a_1}{-a_1} = a_1 = a_3 \text{ and so on}
 \end{aligned}$$

$$\therefore a_1 = a_3 = a_5 = \dots = a_{2001}$$

$$\text{Thus, } (a_{2001})^{2001} = (-w)^{2001} \text{ or } (-w^2)^{2001} = -1$$

The correct option is (B)

$$95. \quad \text{Given: } b^2 = ac \quad (1)$$

$$\text{and, } 2 \log \left(\frac{3b}{5c}\right) = \log \left(\frac{5c}{a}\right) + \log \left(\frac{a}{3b}\right)$$

$$= \log \left(\frac{5c}{a} \cdot \frac{a}{3b}\right) = \log \left(\frac{5c}{3b}\right) = -\log \left(\frac{3b}{5c}\right)$$

$$\text{i.e., } 3 \log \left(\frac{3b}{5c}\right) = 0 \text{ or } b = \frac{5}{3}c \quad (2)$$

From (1) and (2), we have,

$$a = \frac{b^2}{c} = \frac{25c}{9}$$

Now, we have,

$$b + c = \frac{5c}{3} + c = \frac{8c}{3} < \frac{25c}{9} = a$$

and hence, a, b, c cannot form the sides of a triangle.

The correct option is (D)

$$96. \quad \text{Given: } b^2 = ac \quad (\because a, b, c \text{ are in G.P.})$$

$$\text{and, } 2(\log 2b - \log 3c) = \log a - \log 2b + \log 3c - \log a \quad (\because \text{given terms are in A.P.})$$

$$\Rightarrow \log \left(\frac{2b}{3c}\right)^2 = \log \left(\frac{3c}{2b}\right) \Rightarrow b = \frac{3c}{2}$$

$$\text{Now, } a = \frac{b^2}{c} = \frac{3b}{2} = \frac{9c}{4}$$

$\therefore a$ is the largest side

$$\text{Now, } \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{\frac{9c^2}{4} + c^2 - \frac{81}{16}c^2}{2 \times \frac{3}{2}c \times c} = \text{negative}$$

$\therefore A > 90^\circ$, \therefore triangle is obtuse.

The correct option is (C)

$$97. \quad \text{The middle term of the } 4n + 1 \text{ terms is the } (2n + 1) \text{th term. Let it be } m.$$

The middle term of $(2n + 1)$ terms is the $(n + 1)$ th term.

$$\text{Thus, the middle term of the A.P. is } m - (n + 1 - 1)2 = m - 2n$$

and the middle term of the G.P. is

$$= m \left(\frac{1}{2}\right)^{n+1-1} = \frac{m}{2^n}$$

According to the given condition, we have

$$m - 2n = \frac{m}{2^n}$$

$$\Rightarrow m = \frac{n \cdot 2^{n+1}}{2^n - 1}$$

The correct option is (B)

98. Let the common difference of the three A.P.s be d_1, d_2 and d_3
Then, we have

$$S_1 = \frac{n}{2}[2.1 + (n-1)d_1]$$

$$\Rightarrow d_1 = \frac{2(S_1 - n)}{n(n-1)} \quad (1)$$

Similarly, $d_2 = \frac{2(S_2 - n)}{n(n-1)} \quad (2)$

and, $d_3 = \frac{2(S_3 - n)}{n(n-1)} \quad (3)$

Since d_1, d_2 and d_3 are given to be in H.P, therefore,

$$\frac{1}{d_2} - \frac{1}{d_1} = \frac{1}{d_3} - \frac{1}{d_2}$$

$$\Rightarrow \frac{1}{S_2 - n} - \frac{1}{S_1 - n} = \frac{1}{S_3 - n} - \frac{1}{S_2 - n}$$

[Using results (1), (2), (3)]

$$\Rightarrow \frac{S_1 - S_2}{(S_2 - n)(S_1 - n)} = \frac{S_2 - S_3}{(S_2 - n)(S_3 - n)}$$

$$\Rightarrow \frac{S_1 - S_2}{S_1 - n} = \frac{S_2 - S_3}{S_3 - n} \Rightarrow n = \frac{2S_3S_1 - S_1S_2 - S_2S_3}{S_1 - 2S_2 + S_3}$$

The correct option is (C)

99. We have,

$$S_n = 1^3 + 3.2^3 + 3^3 + 3.4^3 + 5^3 + \dots$$

Let $n = 2m$. Then,

$$S_{2m} = (1^3 + 3^3 + 5^3 \dots \text{to } m \text{ terms}) + 3(2^3 + 4^3 + 6^3 + \dots \text{to } m \text{ terms})$$

$$= \{1^3 + 2^3 + 3^3 + 4^3 + \dots + (2m-1)^3 + (2m)^3\} - \{2^3 + 4^3 + \dots + (2m)^3\} + 3\{2^3 + 4^3 + 6^3 + \dots + (2m)^3\}$$

$$= \left[\frac{2m(2m+1)}{2} \right]^2 + 8 \times 2\{1^3 + 2^3 + 3^3 + \dots + m^3\}$$

$$= m^2(2m+1)^2 + 16 \frac{m^2(m+1)}{4}$$

$$= \frac{n^2(n^2 + 3n + 1)}{2} \quad [\text{Put } m = \frac{n}{2}]$$

The correct option is (B)

100. Since p, q, r are in A.P. we have

$$p - q = q - r = k \text{ (say)} \quad (1)$$

Given: $\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz}$

$$\Rightarrow \frac{\left(\frac{a}{x} - 1\right) - \left(\frac{a}{y} - 1\right)}{p - q} = \frac{\left(\frac{a}{y} - 1\right) - \left(\frac{a}{z} - 1\right)}{q - r}$$

(by componendo-dividendo)

$$\Rightarrow \frac{\frac{a}{x} - \frac{a}{y}}{x - y} = \frac{\frac{a}{y} - \frac{a}{z}}{y - z} \quad [\text{Using equation (1)}]$$

$$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{y} - \frac{1}{z} \Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$\therefore x, y, z$ are in H.P.

The correct option is (C)

101. We have,

$$S = 1 + (1+a)b + (1+a+a^2)b^2 + (1+a+a^2+a^3)b^3 + \dots \infty$$

$$= (1+b+b^2+\dots \infty) + a(b+b^2+b^3+\dots \infty) + a^2(b^2+b^3+\dots \infty) + \dots \infty$$

$$= \frac{1}{1-b} + \frac{ab}{1-b} + \frac{a^2b^2}{1-b} + \dots \infty = \frac{1}{(1-b)(1-ab)}$$

The correct option is (A)

102. We have,

$$b_n = (x^{2^{n-1}})^2 - (y^{2^{n-1}})^2$$

$$= (x^{2^{n-1}} - y^{2^{n-1}})(x^{2^{n-1}} + y^{2^{n-1}})$$

$$= b_{n+1} a_{n+1}$$

i.e., $a_n = \frac{b_{n-1}}{b_n} \quad (\text{Putting } n \text{ in place of } n-1)$

$$\therefore a_1, a_2, \dots, a_n = \frac{b_0}{b_1} \cdot \frac{b_1}{b_2} \cdot \frac{b_2}{b_3} \dots \frac{b_{n-1}}{b_n} = \frac{x-y}{b_n}$$

The correct option is (B)

103. Given: $a_1 + 5d = 2$

Let $y = a_1 a_4 a_5 = a_1(a_1 + 3d)(a_1 + 4d)$
 $= (2 - 5d)(2 - 2d)(2 - d)$

(Putting $a_1 = 2 - 5d$)

$$= 2(4 - 16d + 17d^2 - 5d^3)$$

The value of d at which y attains maxima is given by

$$\frac{dy}{dx} = 0 \quad (\text{by calculus})$$

$$\Rightarrow -16 + 34d - 15d^2 = 0$$

$$\Rightarrow d = \frac{34 \pm 14}{30} = \frac{2}{3}, \frac{8}{5}$$

Now, $\frac{d^2y}{dx^2} = 34 - 30d > 0$ for $d = \frac{2}{3}$
 < 0 for $d = \frac{8}{5}$

Hence, y is maximum for $d = \frac{8}{5}$

The correct option is (A)

104. Let t_n denotes the n th term of the sequence

$$1, 2, 4, 7$$

$$\text{Let } S = 1 + 2 + 4 + 7 + \dots + t_n$$

$$\text{Again, } S = 1 + 2 + 4 + \dots + t_{n-1} + t_n$$

On subtracting, we get

$$0 = 1 + [1 + 2 + 3 + \dots + (n-1) \text{ terms}] - t_n$$

$$\Rightarrow t_n = 1 + \frac{n(n-1)}{2} = \frac{n^2 - n + 2}{2}$$

which denotes the first term of the n th row which contains n terms in A.P., having common difference 1.

Hence, we have,

$$\begin{aligned} S &= \frac{n}{2} [2t_n + (n-1)1] \\ &= \frac{n}{2} [n^2 - n + 2 + n - 1] = \frac{n(n^2 + 1)}{2} \end{aligned}$$

The correct option is (C)

More than One Option Correct Type

105. Let the numbers be a and b . If H is the harmonic mean then

$$H = 4, 2A + G^2 = 27 \quad (1) \quad (\text{Given})$$

But we know, $AH = G^2$.

$$\therefore 4A = G^2; \therefore \text{from (1), } 2A + 4A = 27;$$

$$\therefore A = \frac{27}{6} = \frac{9}{2}$$

$$\therefore \frac{a+b}{2} = \frac{9}{2}, \quad \left\{ \because A = \frac{a+b}{2} \right\}$$

$$\therefore a + b = 9 \quad (2)$$

$$\begin{aligned} \text{Again, } H = 4 &= \frac{2ab}{a+b}; \therefore 4 = \frac{2ab}{9} \\ \therefore ab &= 18. \end{aligned} \quad (3)$$

From (2) and (3), we get

$$a - b = \sqrt{(a+b)^2 - 4ab} = \sqrt{9^2 - 4 \times 18} = \pm 3 \quad (4)$$

\therefore From (2) and (4), $2a = 9 \pm 3$ and $2b = 9 \mp 3$;

$\therefore a = 6, 3$ and $b = 3, 6$.

\therefore The numbers are 3 and 6.

The correct option is (C, D)

106. Since n th term of the first $(2n-1)$ terms is the middle term,
 $\therefore a$ is the A.M. (A); b is the G.M. (G); c is the H.M. (H) of the series, whose first and last terms are equal.

We know that $A \geq G > H$ and $AH = G^2$

$$\therefore a \geq b \geq c \text{ and } ac = b^2$$

The correct option is (B, D)

107. Since x_1, x_2, x_3 are in A.P. we may take $x_1 = \alpha - d, x_2 = \alpha$ and $x_3 = \alpha + d$.

Since x_1, x_2, x_3 are roots of

$$x^3 - x^2 + bx + \gamma = 0$$

$$\text{we have, } (\alpha - d) + \alpha + (\alpha + d) = 1 \quad (1)$$

$$(\alpha - d)\alpha + \alpha(\alpha + d) + (\alpha - d)(\alpha + d) = \beta \quad (2)$$

$$(\alpha - d)(\alpha)(\alpha + d) = -\gamma \quad (3)$$

$$\text{From (1), we get } 3\alpha = 1 \text{ or } \alpha = \frac{1}{3}$$

From (2), we get

$$\begin{aligned} 3\alpha^2 - d^2 &= \beta \\ \Rightarrow \beta &= 3 \left(\frac{1}{3} \right)^2 - d^2 = \frac{1}{3} - d^2 \leq \frac{1}{3}. \end{aligned}$$

$$\text{Thus, } \beta \in \left(-\infty, \frac{1}{3} \right].$$

From (3), we get

$$\begin{aligned} \alpha(\alpha^2 - d^2) &= -\gamma \\ \Rightarrow \frac{1}{3} \left(\frac{1}{9} - d^2 \right) &= -\gamma \end{aligned}$$

$$\Rightarrow \gamma = \frac{1}{3} d^2 - \frac{1}{27} \geq -\frac{1}{27}$$

$$\text{Thus, } \gamma \in \left[-\frac{1}{27}, \infty \right)$$

$$\text{Hence, } \beta \in \left(-\infty, \frac{1}{3} \right] \text{ and } \gamma \in \left[-\frac{1}{27}, \infty \right)$$

The correct option is (A, D)

108. As a, b, c are in A.P., $b = \frac{a+c}{2}$ (1)

$$\text{As } a^2, b^2, c^2 \text{ are in H.P., } b^2 = \frac{2a^2c^2}{a^2 + c^2}$$

$$\Rightarrow b^2(a^2 + c^2) = 2a^2c^2 \Rightarrow b^2\{(a+c)^2 - 2ac\} = 2a^2c^2$$

$$\Rightarrow b^2\{4b^2 - 2ac\} = 2a^2c^2, \quad \text{using (1)}$$

$$\Rightarrow b^2(2b^2 - ac) = a^2c^2; \Rightarrow 2b^4 - b^2ac - (ac)^2 = 0$$

$$\Rightarrow (2b^2 + ac)(b^2 - ac) = 0.$$

$$\therefore \text{ Either } 2b^2 + ac = 0 \text{ or } b^2 = ac;$$

$$\text{i.e., } b^2 = -\frac{ac}{2} \quad \therefore \left(\frac{a+c}{2} \right)^2 = ac$$

$$\text{i.e., } -, \frac{a}{2} b, c \text{ are in G.P. i.e., } (a-c)^2 = 0 \text{ i.e., } a = c;$$

or, $-\frac{c}{2}, b, a$, are in G.P. $\therefore b = \frac{a+c}{2} = \frac{a+a}{2} = a$

i.e., $a = b = c$.

The correct option is (A, B, C)

109. We have,

$$G = \sqrt{ab}, H = \frac{2ab}{a+b} \text{ and } G = 2H.$$

$$\therefore \sqrt{ab} = \frac{4ab}{a+b} \text{ or } a + b = 4\sqrt{ab} \text{ or } \sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = 4.$$

If $\frac{a}{b} = x^2$, then $x + \frac{1}{x} = 4$ or $x^2 + 1 = 4x$

or, $x^2 - 4x + 1 = 0$;

$$\therefore x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}.$$

Taking the positive sign,

$$\begin{aligned} x^2 &= (2 + \sqrt{3})^2 = \frac{(2 + \sqrt{3})^2}{4-3} \\ &= \frac{(2 + \sqrt{3})^2}{2^2 - (\sqrt{3})^2} = \frac{(2 + \sqrt{3})^2}{(2 + \sqrt{3})(2 - \sqrt{3})} \end{aligned}$$

$$\therefore \frac{a}{b} = \frac{2 + \sqrt{3}}{2 - \sqrt{3}}.$$

Similarly, taking the negative sign, we get $\frac{a}{b} = \frac{2 - \sqrt{3}}{2 + \sqrt{3}}$.

The correct option is (A, B)

110. $\therefore a, b, c$ are in G.P. $\therefore b^2 = ac$ (1)

Again, as x is A.M. between a and b ,

$$\therefore x = \frac{a+b}{2} \quad (2)$$

and, y is the A.M. between b and c .

$$\therefore y = \frac{b+c}{2} \quad (3)$$

Consider $\frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c}$

$$= 2 \left[\frac{a}{a+b} + \frac{c}{b+c} \right] \quad \text{[Using (2) and (3)]}$$

$$= 2 \left[\frac{ab+ac+ac+bc}{ab+ac+b^2+bc} \right]$$

$$= 2 \left[\frac{ab+ac+ac+bc}{ab+ac+ac+bc} \right] = 2 \quad \text{[Using (1)]}$$

Again, $\frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c}$

$$= 2 \left[\frac{b+c+a+b}{ab+ac+b^2+ac} \right]$$

$$= \frac{2}{b} \left[\frac{b^2 + bc + ab + b^2}{ab + ac + b^2 + bc} \right]$$

$$= \frac{2}{b} \left[\frac{ac + bc + ab + ac}{ab + ac + ac + bc} \right]$$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{2}{b}.$$

The correct option is (B, C)

111. We have,

$$\log x + \log x^{1/2} + \log x^{1/4} + \dots = y$$

$$\Rightarrow \left(1 + \frac{1}{2} + \frac{1}{4} + \dots \right) \log x = y$$

$$\Rightarrow \left(\frac{1}{1-1/2} \right) \log x = y$$

$$\Rightarrow 2 \log x = y \quad (1)$$

and, $\frac{1+3+5+\dots+(2y-1)}{4+7+10+\dots+(3y+1)} = \frac{20}{7 \log x}$

$$\Rightarrow \frac{y \left(\frac{1+2y-1}{2} \right)}{y \left(\frac{4+3y+1}{2} \right)} = \frac{20}{7 \left(\frac{y}{2} \right)} \quad \text{[Using (1)]}$$

$$\Rightarrow \frac{2y}{3y+5} = \frac{40}{7y} \Rightarrow 7y^2 - 60y - 100 = 0$$

$$\Rightarrow y = 10, -\frac{10}{7} \text{ and corresponding } x = 10^5, 10^{-5/7}.$$

The correct option is (A, B)

112. Let the G.P. be a, ar, ar^2, \dots

and let the A.P. be $a, r+a, r+2a, \dots$

Given: $a + ar = 9$ (1)

and, $\frac{10}{2} [2r + (10-1)a] = 155$

$$\Rightarrow 2r + 9a = 31 \quad (2)$$

Putting $a = \frac{9}{1+r}$ from equation (1) in equation (2), we have,

$$2r + \frac{81}{1+r} = 31 \Rightarrow 2r^2 - 29r + 50 = 0$$

$$\Rightarrow r = \frac{29 \pm 21}{4} = \frac{25}{2}, 2 \text{ and corresponding } a = \frac{2}{3}, 3$$

113. We have,

$$(1+2x^2+x^4) \left[1 + nx + \frac{n(n-1)}{2} x^2 + \frac{n(n-1)(n-2)}{6} x^3 + \dots \right]$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

Comparing the coefficients, we have

$$a_1 = n$$

$$a_2 = 2 + \frac{n(n-1)}{2}$$

$$\text{and, } a_3 = 2n + \frac{n(n-1)(n-2)}{6}$$

According to the given condition, we have $a_1 + a_3 = 2a_2$

$$\Rightarrow 3n + \frac{n(n-1)(n-2)}{6} = 4 + n(n-1)$$

$$\Rightarrow \frac{n(n-1)(n-2)}{6} = 6(n^2 - 4n + 4)$$

$$\Rightarrow n^3 - 3n^2 + 2n = 6n^2 - 24n + 24$$

$$\Rightarrow n^3 - 9n^2 + 26n - 24 = 0$$

$$\therefore n = 2, 3, 4$$

The correct option is (A, C, D)

114. Since $3(a^2 + b^2 + c^2 + 1) - 2(a + b + c + ab + bc + ca) = 0$

$$\therefore [(a-b)^2 + (b-c)^2 + (c-a)^2] + [(a-1)^2 + (b-1)^2 + (c-1)^2] = 0$$

$$\Rightarrow a-b = b-c = c-a = 0$$

$$\text{and, } a-1 = b-1 = c-1 = 0 \Rightarrow a = b = c = 1$$

$$\Rightarrow a, b, c \text{ are in } \mathbf{A.P.} \text{ and } \mathbf{G.P.} \text{ both. Also, they are in } \mathbf{H.P.}$$

The correct option is (A, B, C, D)

115. Since t_{912}, t_{951} and t_{480} are divisible by 3

\therefore none of them is prime.

For t_{91} , we have $t_{91} = \frac{1}{9} \underbrace{9 \cdot 9 \cdot \dots \cdot 9}_{91 \text{ times}}$

$$= \frac{1}{9} (10^{91} - 1) = \frac{1}{9} [(10^7)^{13} - 1]$$

$$= \left[\frac{(10^7)^{13} - 1}{10^7 - 1} \right] \left[\frac{10^7 - 1}{10 - 1} \right]$$

$$= [(10^7)^{12} + (10^7)^{11} + \dots + 10^7 + 1] \cdot$$

$$[10^6 + 10^5 + \dots + 10 + 1]$$

= (An integer) (An integer)

$\therefore t_{91}$ is not prime.

The correct option is (A, B, C, D)

Passage Based Questions

116. Here, $t_n = \frac{1}{1 + 2 + 3 + \dots + n} = \frac{1}{n(n+1)/2}$

$$= \frac{2}{n(n+1)} = 2 \frac{(n+1) - 1}{n(n+1)}$$

$$= 2 \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

Putting $n = 1, 2, 3, \dots, n$, we get

$$t_1 = 2 \left(\frac{1}{1} - \frac{1}{2} \right)$$

$$t_2 = 2 \left(\frac{1}{2} - \frac{1}{3} \right)$$

$$t_3 = 2 \left(\frac{1}{3} - \frac{1}{4} \right)$$

\vdots

$$t_n = 2 \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$\text{Adding, we get } S_n = 2 \left(1 - \frac{1}{n+1} \right) = \frac{2n}{n+1}$$

The correct option is (B)

117. $S_\infty = \lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \frac{2}{1 + 1/n} = \frac{2}{1 + 0} = 2.$

The correct option is (B)

118. n th term of the given series

$$= \frac{1}{(\text{nth term of } 1, 3, 5, \dots \text{ in A.P.})(\text{nth term of } 3, 5, 7, \dots \text{ in A.P.})}$$

$$\therefore T_n = \frac{1}{(1 + (n-1)2)(3 + (n-1)2)}$$

$$= \frac{1}{(2n-1)(2n+1)}$$

$$\Rightarrow T_n = \frac{1}{2} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

Putting $n = 1, 2, 3, \dots$

$$T_1 = \frac{1}{2} \left(\frac{1}{1} - \frac{1}{3} \right)$$

$$T_2 = \frac{1}{2} \left(\frac{1}{3} - \frac{1}{5} \right)$$

$$T_3 = \frac{1}{2} \left(\frac{1}{5} - \frac{1}{7} \right)$$

\vdots

$$T_n = \frac{1}{2} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

Adding vertically, we get

$$S_n = \frac{1}{2} \left(1 - \frac{1}{2n+1} \right) = \frac{1}{2} \frac{2n+1-1}{2n+1} = \frac{n}{2n+1}$$

$$\therefore S_n = \frac{n}{2n+1}$$

The correct option is (A)

119. We have, $S_n = \frac{n}{2n+1}$

Dividing numerator and denominator by n , we obtain

$$S_n = \frac{1}{2 + 1/n}. \text{ Now when } n \rightarrow \infty, \frac{1}{n} \rightarrow 0$$

$$\therefore S_\infty = \frac{1}{2+0} = \frac{1}{2}$$

The correct option is (C)

120. The series is an arithmetico-geometric series, since each term is formed by multiplying corresponding terms of the series

1, 3, 5, 7, ... which is an A.P.

and, $1, \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \dots$ which is a G.P.

Let S denote the sum to infinity of the given series, then

$$S = 1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \dots \quad (1)$$

Multiplying both sides by $\frac{1}{2}$, the common ratio of G.P.,

$$\text{we get } \frac{1}{2} S = \frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \frac{7}{2^4} + \dots \quad (2)$$

Subtracting (2) from (1), we get

$$\begin{aligned} \left(1 - \frac{1}{2}\right) S &= 1 + \left(\frac{3}{2} - \frac{1}{2}\right) + \left(\frac{5}{2^2} - \frac{3}{2^2}\right) + \left(\frac{7}{2^3} - \frac{5}{2^3}\right) + \dots \\ &= 1 + \left(1 + \frac{1}{2} + \frac{1}{2^2} + \dots\right) \end{aligned}$$

$$\frac{1}{2} S = 1 + \frac{1}{1 - \frac{1}{2}} = 1 + \frac{1}{\frac{1}{2}} = 1 + 2 = 3$$

$$\therefore S = 3 \times 2 = 6.$$

The correct option is (B)

121. Let $S = 3 + 5r + 7r^2 + \dots \infty \quad (1)$

Multiplying both sides of (1) by r , we get

$$rS = 3r + 5r^2 + 7r^3 + \dots \infty \quad (2)$$

Subtracting (2) from (1), we get

$$\begin{aligned} (1-r)S &= 3 + 2r + 2r^2 + 2r^3 + \dots \text{ to } \infty \\ &= 3 + 2r(1 + r + r^2 + \dots \infty) \\ &= 3 + 2r \times \frac{1}{1-r} = 3 + \frac{2r}{1-r} \end{aligned}$$

$$\therefore S = \frac{3}{1-r} + \frac{2r}{(1-r)^2}$$

But $S = \frac{44}{9}$ (Given), $\therefore \frac{44}{9} = \frac{3}{1-r} + \frac{2r}{(1-r)^2}$.

Multiplying both sides by $9(1-r)^2$, we get

$$\Rightarrow 44(1-r)^2 = 27(1-r) + 18r$$

$$\Rightarrow 44(1-2r+r^2) = 27-27r+18r$$

$$\Rightarrow 44r^2 - 79r + 17 = 0$$

$$\Rightarrow r = \frac{79 \pm \sqrt{(-79)^2 - 4 \times 44 \times 17}}{2 \times 44}$$

$$= \frac{79 \pm \sqrt{3249}}{88} = \frac{79 \pm 57}{88}$$

$$= \frac{79+57}{88}, \frac{79-57}{88}$$

$$= \frac{136}{88}, \frac{22}{88} = \frac{17}{11}, \frac{1}{4}$$

Now, $\frac{17}{11} > 1$ i.e., $|r| > 1$.

But $|r| < 1$, $\therefore r = \frac{1}{4}$.

The correct option is (A)

122. The given series is

$$3 + (3+d)\frac{1}{4} + (3+2d)\frac{1}{4^2} + \dots$$

$$S = 3 + (3+d)\frac{1}{4^2} + (3+2d)\frac{1}{4^3} + \dots \infty \quad (1)$$

Multiplying both sides by $1/4$, we get

$$\frac{1}{4} S = 3 \cdot \frac{1}{4} + (3+d)\frac{1}{4^2} + (3+2d)\frac{1}{4^3} + \dots \infty$$

Subtracting (2) from (1), we get

$$\left(1 - \frac{1}{4}\right) S = 3 + d \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots \infty\right)$$

$$\frac{3}{4} S = 3 + d \frac{1/4}{1-1/4} = 3 + \frac{d}{3} \therefore S = 4 + \frac{4}{9} d$$

But $S = \frac{44}{9}$ (Given)

$$\therefore \frac{44}{9} = 4 + \frac{4}{9} d$$

$$\Rightarrow \frac{4}{9} d = \frac{44}{9} - 4 = \frac{44-36}{9} = \frac{8}{9}$$

$$\therefore d = \frac{8}{9} \times \frac{9}{4} = 2$$

Hence, $d = 2$.

Aliter. $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Here, $a = 3$, $S_\infty = \frac{44}{9}$ (Given), $r = \frac{1}{4}$

$$\therefore \frac{44}{9} = \frac{3}{1-1/4} + \frac{d \cdot (1/4)}{(1-r)^2} = \frac{3}{3/4} + \frac{d/4}{(3/4)^2}$$

$$\therefore \frac{44}{9} - 4 = \frac{4}{9}d \therefore d = 2.$$

The correct option is (B)

123. We have, $3^{1/3} \cdot 9^{2/9} \cdot 27^{3/27} \cdot 81^{4/81} \dots$ to ∞
 $= 3^{\left(\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} \dots\right)}$

Let $S = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \dots$ to ∞

$$\frac{1}{3}S = \frac{1}{9} + \frac{2}{27} + \frac{3}{81} \dots \text{ to } \infty$$

On subtraction,

$$\frac{2}{3}S = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots \text{ to } \infty = \frac{1/3}{1-1/3} = \frac{1}{2};$$

$$\therefore S = \frac{3}{4}.$$

Hence the given product = $3^{3/4} = \sqrt[4]{27}$.

The correct option is (C)

124. and 125. Since $S_3 = \frac{S_1 + S_2}{2}$, $S_2 < S_3 < S_1$

Again, $S_4 = \frac{S_2 + S_3}{2} \Rightarrow S_2 < S_4 < S_3 < S_1$

Further $S_5 = \frac{S_4 + S_3}{2} \Rightarrow S_4 < S_5 < S_3 < S_1$

Continuing in this manner, we will get

$$S_2 < S_4 < S_6 < S_8 < \dots < S_7 < S_5 < S_3 < S_1$$

\Rightarrow Sequence $\{S_{2n}\}$ is increasing and $\{S_{2n+1}\}$ is decreasing

Thus, correct choices of are

(a) and (b) respectively. These sequences cannot be unbounded since all terms of the sequence lie between S_2 and S_1

The correct option is (A)

The correct option is (B)

126. $S_{2n+1} - S_{2n} = \frac{1}{2}(S_{2n} + S_{2n-1}) - S_{2n}$
 $= -\frac{1}{2}(S_{2n} - S_{2n-1})$
 $= \left(-\frac{1}{2}\right)^2 (S_{2n-1} - S_{2n-2})$ and so on.

Continuing in this manner, we get

$$S_{2n+1} - S_{2n} = \left(\frac{1}{2}\right)^{2n-1} (S_1 - S_2)$$

(A) This recursion relation shows that this choice is not true.

(C) is ruled out because $S_{2n+1} - S_{2n}$ is always positive

(D) is ruled out since S_{2n+1} is strictly greater than S_{2n} .

The correct option is (B)

127. From above,

$$S_{2n+1} - S_{2n} = \left(\frac{1}{2}\right)^{2n-1} (S_1 - S_2)$$

which approaches zero as $n \rightarrow \infty$

$$\Rightarrow \lim_{n \rightarrow \infty} S_{2n+1} = \lim_{n \rightarrow \infty} S_{2n} = l \text{ (say)}$$

$$2S_{n+1} + S_n = S_n + S_{n-1} + S_n = 2S_n + S_{n-1}$$

$$= 2S_{n-1} + S_{n-2} = 2S_2 + S_1$$

On taking limits, we get

$$2l + l = 2S_2 + S_1$$

$$\Rightarrow l = \frac{1}{3}(2S_2 + S_1)$$

The correct option is (C)

Match the Column Type

128.

I. We have, $a_n = S_n - S_{n-1}$ (1)

and, $a_{n-1} = S_{n-1} - S_{n-2}$ (2)

$$\therefore d = a_n - a_{n-1}$$

$$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$$

[From (1) and (2)]

$$= S_n - 2S_{n-1} + S_{n-2}.$$

The correct option is (C)

II. It is in A.P. for which $a = 20$, $d = 2$ $\frac{2}{3} = \frac{8}{3}$

$$\text{Now, } S_n > 1568 \Rightarrow \frac{n}{2} \left[40 + (n-1) \frac{8}{3} \right] > 1568$$

$$\Rightarrow \frac{n}{2} \cdot \frac{112 + 8n}{3} > 1568$$

$$\Rightarrow n^2 + 14n > \frac{6}{8} \times 1568 = 1176$$

$$\Rightarrow n^2 + 14n - 1176 > 0,$$

$$\text{or, } (n+42)(n-28) > 0$$

As n is positive, $n - 28 > 0$ i.e., $n > 28$

\therefore Minimum value of $n = 29$.

The correct option is (A)

III. Since $5^{1+x} + 5^{1-x}$, $\frac{a}{2}$, $25^x + 25^{-x}$ are in A.P., we have

$$2 \frac{a}{2} = 5^{1+x} + 5^{1-x} + 25^x + 25^{-x}$$

Now, put $5^x = t$ so that $t > 0$, we then have

$$a = 5t + \frac{5}{t} + t^2 + \frac{1}{t^2} = \left(t^2 + \frac{1}{t^2}\right) + 5\left(t + \frac{1}{t}\right)$$

$$\begin{aligned} \text{or, } a &= \left(t - \frac{1}{t}\right)^2 + 2 + 5\left[\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 2\right] \\ &= \left(t - \frac{1}{t}\right)^2 + 5\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 12 \geq 12. \end{aligned}$$

Thus, values of a are given by the inequality $a \geq 12$.

The correct option is (D)

IV. $\log_{2^{12}} a + \log_{2^{14}} a + \log_{2^{16}} a + \log_{2^{18}} a + \dots$

$$\begin{aligned} &= 2 \log_2 a + 4 \log_2 a + 6 \log_2 a + \dots + 40 \log_2 a \\ &= \log_2 a [2 + 4 + 6 + \dots + 40] \\ &= \frac{20}{2} (2 + 40) \log_2 a \\ &= 420 \log_2 a = 840 \\ &\Rightarrow \log_2 a = 2 \Rightarrow a = 4 \end{aligned}$$

The correct option is (B)

129. I. Let the G.P. be $1, r, r^2, r^3, \dots$

Given: $t_n = 2(t_{n+1} + t_{n+2} + t_{n+3} + \dots \text{ to } \infty)$

$$\text{or, } t_n = 2 \cdot \frac{t_{n+1}}{1-r}, \quad (\because \text{common ratio} = r)$$

$$\therefore \frac{1-r}{2} = \frac{t_{n+1}}{t_n}; \text{ or } \frac{1-r}{2} = r$$

$$\text{or, } 1-r = 2r; \therefore r = \frac{1}{3}.$$

The correct option is (D)

II. Let $S = \frac{2}{3} - \frac{5}{6} + \frac{2}{3} - \frac{11}{24} + \dots \text{ to } \infty$ (1)

Multiplying both sides by $-\frac{1}{2}$, the common ratio of G.P.

$$-\frac{1}{2} S = -\frac{2}{6} + \frac{5}{12} - \frac{8}{24} + \dots \text{ to } \infty \quad (2)$$

Subtracting (2) from (1), we have

$$\begin{aligned} \frac{3}{2} S &= \frac{2}{3} - \frac{3}{6} + \frac{3}{12} - \frac{3}{24} + \dots \text{ to } \infty \\ &= \frac{2}{3} - \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \dots \text{ to } \infty\right) \end{aligned}$$

$$= \frac{2}{3} - \frac{\frac{1}{2}}{1 - \left(-\frac{1}{2}\right)} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\therefore S = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

The correct option is (A)

III. Let $S = (1 + 3^{-1})(1 + 3^{-2})(1 + 3^{-4})(1 + 3^{-8}) \dots (1 + 3^{-2^n})$

$$\begin{aligned} \Rightarrow (1 - 3^{-1}) S &= (1 - 3^{-1})(1 + 3^{-1})(1 + 3^{-2})(1 + 3^{-4}) \dots (1 + 3^{-2^n}) \\ &= (1 - 3^{-8}) \dots (1 + 3^{-2^n}) \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{2}{3} S &= (1 - 3^{-2})(1 + 3^{-2})(1 + 3^{-4})(1 + 3^{-8}) \dots (1 + 3^{-2^n}) \\ &= (1 - 3^{-4})(1 + 3^{-4})(1 + 3^{-8}) \dots (1 + 3^{-2^n}) \\ &= (1 - 3^{-8})(1 + 3^{-8}) \dots (1 + 3^{-2^n}) \\ &= (1 - 3^{-2^n})(1 + 3^{-2^n}) = 1 - (3^{-2^n})^2 \\ &= 1 - 3^{-2^{n+1}}. \end{aligned}$$

$$\Rightarrow S = \frac{3}{2} (1 - 3^{-2^{n+1}})$$

$$\therefore \lim_{n \rightarrow \infty} S = \lim_{n \rightarrow \infty} \frac{3}{2} (1 - 3^{-2^{n+1}})$$

$$= \frac{3}{2} (1 - 0) = \frac{3}{2}.$$

The correct option is (B)

IV. $\sum_{r=1}^n \left(\sum_{m=1}^k m^2 \right) = \sum_{k=1}^n (1^2 + 2^2 + \dots + k^2)$

$$\begin{aligned} &= \frac{1}{3} \left\{ \frac{n(n+1)^2}{2} \right\} + \frac{1}{2} \left\{ \frac{n(n+1)(2n+1)}{6} \right\} \\ &\quad + \frac{1}{6} \left(\frac{n(n+1)}{2} \right) \end{aligned}$$

$$= \frac{1}{2} \{ n^4 + 4n^3 + 5n^2 + 2n \}$$

$$\therefore a = \frac{1}{12}, b = \frac{1}{3}, c = \frac{5}{12}, d = \frac{1}{6}, e = 0$$

So, $a + b + c + d + e = 1$

The correct option is (C)

130. I. Since a, b, c are in A.P.

$$\therefore b = \frac{a+c}{2}$$

$\therefore b, c, d$ are in G.P.

(1)

$$\therefore c^2 = bd \quad (2)$$

Also, $\because c, d, e$, are in H.P.

$$\therefore d = \frac{2ce}{c+e} \quad (3)$$

Substituting the values of b and d from (1) and (3) respectively in (2), we get

$$c^2 = \frac{a+c}{2} \cdot \text{or } c = \frac{e(a+c)}{c+e}$$

$$\Rightarrow c^2 + ce = ae + ce$$

$$\Rightarrow c^2 = ae, \text{ which shows that } a, c, e \text{ are in G.P.}$$

The correct option is (B)

II. $\because 2(y-a)$ is H.M. between $y-x$ and $y-z$

$$\therefore \frac{2}{2(y-a)} = \frac{1}{y-x} + \frac{1}{y-z}$$

$$= \frac{1}{(y-a)-(x-a)} + \frac{1}{(y-a)-(z-a)} \quad (1)$$

Let $x-a = X; y-a = Y; z-a = Z$

then (1) reduces to

$$\frac{1}{Y} = \frac{1}{Y-X} + \frac{1}{Y-Z} \Rightarrow Y^2 = ZX$$

$\therefore X, Y, Z$ are in G.P. or $x-a, y-a, z-a$ are in G.P.
The correct option is (B)

III. Let the numbers in H.P. be

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$$

Then, the numbers obtained by subtracting $\frac{1}{2(a+b)}$ are

$$\frac{1}{a} - \frac{1}{2(a+d)}, \frac{1}{a+d} - \frac{1}{2(a+d)}, \frac{1}{a+2d} - \frac{1}{2(a+d)}$$

$$\text{i.e., } \frac{2a+2d-a}{2a(a+d)}, \frac{1}{2(a+d)}, \frac{2a+2d-(a+2d)}{2(a+2d)(a+d)}$$

$$\text{i.e. } \frac{a+2d}{2a(a+d)}, \frac{1}{2(a+d)}, \frac{a}{2(a+2d)(a+d)},$$

Product of first and third

$$= \frac{a+2d}{2a(a+d)} \cdot \frac{a}{2(a+2d)(a+d)}$$

$$= \frac{1}{4(a+d)^2} = \left[\frac{1}{2(a+d)} \right]^2$$

\therefore New numbers are in G.P.

The correct option is (B)

IV. Solving the equation $ax^2 + 2bx + c = 0$, we get

$$x = \frac{-2b \pm \sqrt{4b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - ac}}{a} = -\frac{b}{a}$$

[$\because a, b, c$ are in G.P., $\therefore b^2 = ac$]

This is also root of

$$dx^2 + 2ex + f = 0$$

$$\therefore d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{a}\right) + f = 0$$

$$\Rightarrow db^2 - 2eba + a^2f = 0$$

$$\Rightarrow dac - 2eba + a^2f = 0 \quad [\because b^2 = ac]$$

Dividing throughout by a

$$\therefore dc - 2eb + af = 0 \Rightarrow 2eb = dc + af$$

Dividing both sides by b

$$\Rightarrow \frac{2e}{b} = \frac{dc + af}{b^2} \Rightarrow \frac{2e}{b} = \frac{dc + af}{ac} \quad [b^2 = ac]$$

$$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c} \Rightarrow \frac{e}{b} - \frac{d}{a} = \frac{f}{c} - \frac{e}{b}$$

$$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

The correct option is (A)

Assertion-Reason Type

131. Let $2n$ arithmetic means be $A_1, A_2, A_3, \dots, A_{2n}$ between a and b .

$$\text{Then, } A_1 + A_2 + A_3 + \dots + A_{2n} = \frac{a+b}{2} \times 2n$$

$$= \frac{13}{6} \times 2n = \frac{13n}{6}$$

Given, $A_1 + A_2 + A_3 + \dots + A_{2n} = 2n + 1$;

$$\therefore 2n + 1 = \frac{13n}{6}; \text{ or } 12n + 6 = 13n; \therefore n = 6.$$

\therefore The number of means $= 2n = 2 \times 6 = 12$.

The correct option is (A)

132. Since a and b are unequal,

$$\frac{a^2 + b^2}{2} > \sqrt{a^2 b^2} \quad [\text{A.M.} > \text{G.M. for unequal numbers}]$$

$$\Rightarrow a^2 + b^2 > 2ab$$

Similarly, $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$

Hence, $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$

$$\Rightarrow ab + bc + ca < 1$$

The correct option is (A)

133. $b = \text{H.M. of } a \text{ and } c > \text{A.M. of } a \text{ and } c = \frac{a+c}{2}$
 $\Rightarrow b > \frac{a+c}{2}$. Similarly, $c > \frac{b+d}{2}$. Adding we get

$$b + c > a + d.$$

The correct option is (A)

134. Let T_n be the n th term of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$$

Then, $T_n = \frac{n}{1+n^2+n^4} = \frac{n}{(1+n^2)^2 - n^2}$

$$= \frac{n}{(n^2+n+1)(n^2-n+1)}$$

$$= \frac{1}{2} \left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1} \right]$$

$$= \frac{1}{2} \left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)} \right]$$

Now,

$$\begin{aligned} \sum_{r=1}^n T_r &= \frac{1}{2} \left[\frac{1}{1} - \frac{1}{1+1.2} \right] + \frac{1}{2} \left[\frac{1}{1+1.2} - \frac{1}{1+2.3} \right] \\ &\quad + \frac{1}{2} \left[\frac{1}{1+2.3} - \frac{1}{1+3.4} \right] + \dots \\ &\quad + \frac{1}{2} \left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)} \right] \\ &= \frac{1}{2} \left[1 - \frac{1}{1+n(n+1)} \right] = \frac{n(n+1)}{2(n^2+n+1)}. \end{aligned}$$

The correct option is (A)

135. We know that sum of n A.M.'s between two quantities is equal to n times their single mean.

Now, x, y, z are three A.M.'s between a and b

$$x + y + z = 3 \left(\frac{a+b}{2} \right) = 15$$

$$\text{or, } a + b = 10 \quad (1)$$

a, x, y, z, b are in H.P.

$$\therefore \frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b} \text{ are in A.P.}$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{2} \left(\frac{1}{a} + \frac{1}{b} \right) = 3 \left(\frac{a+b}{2ab} \right)$$

$$\text{or, } \frac{5}{3} = \frac{3}{2ab} \cdot 10, \text{ by (1) } \therefore ab = 9 \quad (2)$$

Hence, a and b are the roots of

$$t^2 - 10t + 9 = 0$$

[Using (1) and (2)]

$$\Rightarrow t = 9, 1$$

$\therefore 9, 1$ are the required values of a and b .

The correct option is (A)

136. Since A. M. > G.M. for n distinct positive quantities, we have,

$$\frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n} > \left(1^3 \cdot 2^3 \cdot 3^3 \cdot \dots \cdot n^3 \right)^{1/n}$$

$$\Rightarrow \left(\frac{n(n+1)}{4} \right)^2 > (1 \cdot 2 \cdot 3 \cdot \dots \cdot n)^{3/n} = (n!)^{3/n}$$

$$\Rightarrow (n!)^3 < n^n \left(\frac{n+1}{2} \right)^{2n}$$

The correct option is (A)

Previous Year's Questions

137. $\therefore 1, \log_3 \sqrt{3^{1-x} + 2}, \log_3(4 \cdot 3^x - 1)$ are in AP

$$\therefore 2 \log_3(3^{1-x} + 2)^{1/2} = \log_3 3 + \log_3(4 \cdot 3^x - 1)$$

$$\Rightarrow \log_3(3^{1-x} + 2)^{1/2} = \log_3 3(4 \cdot 3^x - 1)$$

Let $3^x = t$, then

$$\frac{3}{t} + 2 = 12t - 3$$

$$\Rightarrow 12t^2 - 5t - 3 = 0$$

$$\Rightarrow (3t+1)(4t-3) = 0$$

$$\Rightarrow t = -\frac{1}{3}, \frac{3}{4}$$

$$\Rightarrow 3^x = \frac{3}{4} \text{ (neglecting the negative value)}$$

$$\Rightarrow \log_3 \left(\frac{3}{4} \right) = x$$

$$\Rightarrow x = \log_3 3 - \log_3 4$$

$$\Rightarrow x = 1 - \log_3 4$$

The correct option is (B)

138. The product $2^{1/4} \cdot 4^{1/4} \cdot 8^{1/16} \dots$

$$= 2^{1/4} \cdot 2^{2/8} \cdot 2^{3/16} \dots$$

$$= 2^{\frac{1}{4} \left[1 + \frac{2}{2} + \frac{3}{2^2} + \dots \right]}$$

$$= 2^{\frac{1}{4} \left[\frac{1}{1-\frac{1}{2}} + \frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} \right]}$$

$$= 2^{\frac{1}{4}[2+2]}$$

$$= 2$$

The correct option is (B)

139. Since fifth term of a GP = 2

$$\therefore ar^4 = 2$$

where a and r are the first term and the common ratio respectively of a GP.

Now required product

$$= a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$$

$$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$$

The correct option is (B)

140. Key Idea : The number of triangles those can be formed using n points = ${}^n C_3$

$$\therefore T_n = {}^n C_3$$

$$\therefore T_{n+1} - T_n = 21$$

$$\Rightarrow {}^{n+1} C_3 - {}^n C_3 = 21$$

$$\Rightarrow {}^n C_2 + {}^n C_3 - {}^n C_3 = 21$$

$$(\because {}^n C_2 + {}^n C_3 = {}^{n-1} C_3)$$

$$\Rightarrow {}^n C_2 = 21$$

$$\Rightarrow \frac{n(n-1)}{2} = 21 \Rightarrow n^2 - n - 42 = 0$$

$$\Rightarrow (n-7)(n+6) = 0$$

$$\Rightarrow n = 7 \quad (\because n \neq -6)$$

The correct option is (B)

141. $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \dots$

$$= 1 - \frac{1}{2} - \frac{1}{2} + \frac{1}{3} + \frac{1}{3} - \frac{1}{4} - \dots$$

$$= 1 - 2 \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots \right) \quad (\text{Rearranging the terms})$$

$$= 2 \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right) - 1$$

$$= 2 \log 2 - \log e$$

$$= \log \left(\frac{4}{e} \right)$$

The correct option is (D)

142. $f(1) = 7$

$$f(1+1) = f(1) + f(1)$$

$$\Rightarrow f(2) = 2 \times 7$$

$$\text{Also, } f(3) = 3 \times 7$$

$$\therefore \sum_{r=1}^n f(r) = 7(1+2+\dots+n)$$

$$= 7 \frac{n(n+1)}{2}$$

The correct option is (D)

143. Given $t_n = \sum_{r=0}^n \frac{r}{{}^n C_r}$

$$\text{Also, } t_n = \sum_{r=0}^n \frac{n-r}{{}^n C_{n-r}} = \sum_{r=0}^n \frac{n-r}{{}^n C_r} \quad (\because {}^n C_r = {}^n C_{n-r})$$

Adding above two equalities we write

$$2t_n = \sum_{r=0}^n \frac{r+n-r}{{}^n C_r} = \sum_{r=0}^n \frac{n}{{}^n C_r}$$

$$\Rightarrow t_n = \frac{n}{2} \sum_{r=0}^n \frac{1}{{}^n C_r} = \frac{n}{2} S_n \Rightarrow \frac{t_n}{S_n} = \frac{n}{2}$$

The correct option is (A)

144. Given that $T_m = \frac{1}{n} = a + (m-1)d$ (1)

$$\text{And } T_n = \frac{1}{m} = a + (n-1)d \quad (2)$$

$$\text{From (1) and (2) we get } a = \frac{1}{mn}, d = \frac{1}{mn}$$

$$\text{Hence, } a - d = 0$$

The correct option is (A)

145. If n is odd then $(n-1)$ is even and so the sum of odd terms

$$= \frac{(n-1)n^2}{2} + n^2 = \frac{n^2(n+1)}{2}$$

The correct option is (D)

$$146. x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$$

$$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$$

$$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$$

a, b, c are in A.P.

$$2b = a + c$$

$$2 \left(1 - \frac{1}{y} \right) = 1 - \frac{1}{x} + 1 - \frac{1}{y}$$

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\Rightarrow x, y, z \text{ are in H. P}$$

The correct option is (D)

147. Given condition implies that

$$\frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2} \Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

$$\frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q}$$

For $\frac{a_6}{a_{21}}$, $p = 11$, $q = 41$,

$$\frac{a_6}{a_{21}} = \frac{11}{41}$$

The correct option is (D)

148. Given that $\frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \dots = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$ (say)

Then $a_1 a_2 = \frac{a_1 - a_2}{d}$, $a_2 a_3 = \frac{a_2 - a_3}{d}$, ..., $a_{n-1} a_n = \frac{a_{n-1} - a_n}{d}$

$$\therefore a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n = \frac{a_1 - a_n}{d}$$

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$$\Rightarrow \frac{a_1 - a_n}{d} = (n-1)a_1 a_n$$

The correct option is (D)

149. Given that $ar^{n-1} = ar^n + ar^{n+1}$

$$\Rightarrow 1 = r + r^2$$

$$\therefore r = \frac{\sqrt{5} - 1}{2}$$

The correct option is (D)

150. Using the condition A.M. \geq G.M., we write

$$\frac{p^2 + q^2}{2} \geq pq$$

$$\Rightarrow pq \leq \frac{1}{2}$$

$$(p + q)^2 = p^2 + q^2 + 2pq$$

$$\Rightarrow p + q \leq \sqrt{2}$$

The correct option is (D)

151. Let a, ar, ar^2, ar^3 be the first four terms of a G.P., then

$$a + ar = 12 \quad (1)$$

$$ar^2 + ar^3 = 48 \quad (2)$$

dividing (2) by (1), we have

$$\frac{ar^2(1+r)}{a(r+1)} = 4$$

$$\Rightarrow r^2 = 4 \quad \text{if } r \neq -1$$

$$\therefore r = -2$$

also, $a = -12$ (using (1))

The correct option is (B)

152. Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{10}{3^4} + \dots$ (1)

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \dots$$
 (2)

Subtracting (2) from (1), we get

$$S \left(1 - \frac{1}{3}\right) = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \dots$$

$$\frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} \left(1 + \frac{1}{3} + \frac{1}{3^2} + \dots\right)$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} \left(\frac{1}{1 - \frac{1}{3}}\right) = \frac{4}{3} + \frac{4 \cdot 3}{3^2 \cdot 2} = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} \Rightarrow \frac{2}{3}S = \frac{6}{3}$$

$$\Rightarrow S = 3$$

The correct option is (B)

153. Till 10th minute number of counted notes = 1500

$$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$$

$$n^2 - 149n + 3000 = 0$$

$$n = 125, 24$$

$n = 125$ is not possible.

Total time = $24 + 10 = 34$ minutes.

The correct option is (A)

154. 1 2 3 4 5 6

$$200 \ 200 \ 200 \ 240 \ 280 \ \dots \dots \dots$$

Sum = 11040

$$120 + 80 + 160 + 40 + 200 + 240 + \dots = 11040$$

$$\frac{n}{2}[2a + (n-1)d] + 80 + 40 = 11040$$

$$\frac{n}{2}[240 + (n-1)40] = 10920$$

$$n[6 + n - 1] = 546$$

$$n(n + 5) = 546$$

$$n = 21$$

The correct option is (C)

155. Statement 1 has 20 terms whose sum is 8000

And statement 2 is true and supporting statement 1.

$$\therefore k^{\text{th}} \text{ bracket is } (k-1)^2 + k(k-1) + k^2 = 3k^2 - 3k + 1.$$

The correct option is (B)

156. $100(T_{100}) = 50(T_{50})$

$$\Rightarrow 2[a + 99d] = a + 49d$$

$$\Rightarrow a + 149d = 0$$

$$\Rightarrow T_{150} = 0$$

The correct option is (D)

157. $t_r = .7 + .77 + \dots + r$ times

$$= 7(10^{-1} + 10^{-2} + 10^{-3} + \dots + 10^{-r})$$

$$= \frac{7}{9}(1 - 10^{-r})$$

Now,

$$\begin{aligned} S_{20} &= \sum_{r=1}^{20} t_r = \frac{7}{9} \left(20 - \sum_{r=1}^{20} 10^{-r} \right) \\ &= \frac{7}{9} \left(20 - \frac{1}{9}(1 - 10^{-20}) \right) = \frac{7}{81}(179 + 10^{-20}) \end{aligned}$$

The correct option is (B)

158. $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$$2q = p + r$$

$$\Rightarrow -2(\alpha + \beta) = 1 + \alpha\beta$$

$$\Rightarrow -2 \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) = \frac{1}{\alpha\beta} + 1$$

$$\Rightarrow \frac{1}{\alpha\beta} = -9$$

Equation having roots α, β is $9x^2 + 4x - 1 = 0$

$$\alpha, \beta = \frac{-4 \pm \sqrt{16 + 36}}{2 \times 9}$$

$$|\alpha - \beta| = \frac{2\sqrt{13}}{9}$$

The correct option is (D)

159. Let numbers be a, ar, ar^2

$$\text{Now, } 2(2ar) = a + ar^2 \quad [a \neq 0]$$

$$\Rightarrow 4r = 1 + r^2$$

$$\Rightarrow r^2 - 4r + 1 = 0$$

$$\Rightarrow r = 2 \pm \sqrt{3}$$

$$r = 2 + \sqrt{3} \quad (\text{Positive value})$$

The correct option is (D)

160. $S = 10^9 + 2 \cdot 11^1 \cdot 10^8 + \dots + 10 \cdot 11^9$

$$\frac{11}{10} \cdot S = 11^1 \cdot 10^8 + \dots + 9 \cdot 11^9 + 11^{10}$$

$$\Rightarrow \frac{1}{10} S = 10^9 + 11^1 \cdot 10^8 + 11^2 \cdot 10^7 + \dots + 11^9 - 11^{10}$$

$$\Rightarrow \frac{1}{10} S = 10^9 \left(\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1} \right) - 11^{10}$$

$$\Rightarrow \frac{1}{10} S = 11^{10} - 10^{10} - 11^{10}$$

$$S = 10^{11}$$

$$S = 100 \cdot 10^9$$

$$\Rightarrow k = 100$$

The correct option is (C)

161. $t_r = \frac{\sum r^3}{\sum (2r-1)} = \frac{r^2(r+1)^2}{4r^2} = \frac{1}{4}(r+1)^2$

$$\text{Now, } S_9 = \frac{1}{4} \sum_{r=1}^9 (r+1)^2, \text{ let } t = r+1$$

$$= \frac{1}{4} \left(\sum_{t=1}^{10} t^2 - 1 \right) = 96$$

The correct option is (A)

162. Let 'a' be the first term and d be the common difference

$$2^{\text{nd}} \text{ term} = a + d, \quad 5^{\text{th}} \text{ term} = a + 4d,$$

$$9^{\text{th}} \text{ term} = a + 8d$$

$$\therefore \text{Common ratio} = \frac{a+4d}{a+d} = \frac{a+8d}{a+4d} = \frac{4d}{3d} = \frac{4}{3}$$

The correct option is (C)

163. Given series is $S = \frac{8^2}{5^2} + \frac{12^2}{5^2} + \frac{16^2}{5^2} + \dots$ 10 terms

$$= \frac{4^2}{5^2} (2^2 + 3^2 + 4^2 + \dots 10 \text{ terms})$$

$$= \frac{16}{25} \left(\frac{11 \cdot 12 \cdot 23}{6} - 1 \right) = \frac{16}{25} \times 505$$

$$\therefore m = 101$$

The correct option is (C)