CHAPTER 6

Determinants

Chapter Highlights

Determinants, Minors and cofactors, Expansion of a determinant of order three, Properties of determinants, Evaluation of determinants using elementary operations, Product of determinants of same order, Solution of linear equations by determinants

DETERMINANTS

A determinant is a pure number associated with a square matrix. Corresponding to each square matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

there is associated an expression, called the *determinant of A*, **denoted by det A** or $|A|$, written as

$$|A| = \det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

A matrix is an arrangement of numbers and it has no fixed value but a determinant is a number and it has a fixed value. A determinant having n rows and n columns is called a determinant of order n.

Determinant of a Square Matrix of Order 1

Let $A = [a_{11}]$ be a 1×1 matrix, then the determinant of A is the number a_{11} itself i.e., $|a_{11}| = a_{11}$.

Determinant of a Square Matrix of Order 2

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a 2×2 matrix, then

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\, a_{22} - a_{12}\, a_{21}$$

i.e., the determinant of a 2×2 matrix is obtained by taking the product of the entries on the main diagonal and subtracting from it the product of the entries in the other diagonal.

Illustration

1. $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 2 \times 5 - 4 \times 3 = 10 - 12 = -2.$

2. $\begin{vmatrix} x-1 & x+1 \\ x^2-x+1 & x+1 \end{vmatrix} = (x-1)(x+1) - (x+1)(x^2-x+1)$

$$= x^2 - 1 - (x^3 + 1) = x^2 - x^3 - 2$$

MINORS AND COFACTORS

Minor of an Element of a Determinant

If we take an element of the determinant and delete the row and the column containing that element, the determinant left is called the minor of that element. It is denoted by M_{ij}. For example, given the 3×3 determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Then the minor of a_{11} is $M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$; the minor of a_{12} is $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and so on.

Cofactor of an Element of a Determinant

The cofactor C_{ij} of an element a_{ij} (the element in the ith row and jth column) is defined as $C_{ij} = (-1)^{i+j} M_{ij}$. It is denoted by C_{ij}. Thus,

$$C_{ij} = \begin{cases} M_{ij}, & \text{when } i + j \text{ is even} \\ -M_{ij}, & \text{when } i + j \text{ is odd} \end{cases}$$

For example, the cofactor of a_{12} in the 3×3 determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\text{is } C_{12} = (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

EXPANSION OF A DETERMINANT OF ORDER THREE

Consider the following determinant:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

We can find the value of this determinant by various methods.

Method 1

Step 1: Write the elements of the first row with alter-na-tively positive and negative sign, the first element always has positive sign before it.
Step 2: Multiply each signed element by the determinant of second order obtained after deleting the row and the column in which that element occurs. i.e.,

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1 (a_2b_3 - a_3b_2)$$

For example,

$$\begin{vmatrix} 1 & -3 & 4 \\ 0 & 2 & 5 \\ -2 & 6 & 3 \end{vmatrix} = (1) \begin{vmatrix} 2 & 5 \\ 6 & 3 \end{vmatrix} + (-3)(-1) \begin{vmatrix} 0 & 5 \\ -2 & 3 \end{vmatrix} + (4) \begin{vmatrix} 0 & 2 \\ -2 & 6 \end{vmatrix}$$

$$= 1 (6 - 30) + 3 (0 + 10) + 4 (0 + 4)$$

$$= -24 + 30 + 16 = 22.$$

We can also expand the determinant along any row or any column. For example, if we had expanded the above determinant along the first column, then

$$\begin{vmatrix} 1 & -3 & 4 \\ 0 & 2 & 5 \\ -2 & 6 & 3 \end{vmatrix} = (1) \begin{vmatrix} 2 & 5 \\ 6 & 3 \end{vmatrix} + 0 + (-2) \begin{vmatrix} -3 & 4 \\ 2 & 5 \end{vmatrix}$$

$$= 1(6 - 30) - 2 (-15 - 8)$$

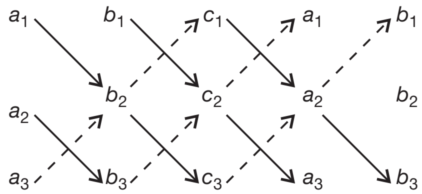
$$= -24 + 46 = 22, \text{ as before.}$$

Method 2: Using Sarrus Rule

The following diagram called *sarrus diagram*, enables us to write the value of the determinant of order 3 very conveni-entlty.

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ be a determinant of order 3.

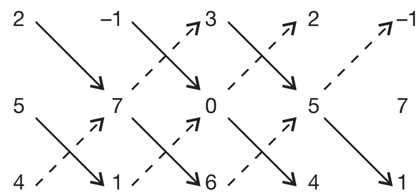
Write the elements as:



Multiply the elements joined by arrows. Assign the positive sign to an expression if it is formed by a downward arrow and negative sign to an expression if it is formed by an upward arrow. Note that the first two columns are repeated in the above table to complete the process. The value of the given determinant is

$$a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - a_3b_2c_1 - b_3c_2a_1 - c_3a_2b_1$$

For example, to evaluate $\begin{vmatrix} 2 & -1 & 3 \\ 5 & 7 & 0 \\ 4 & 1 & 6 \end{vmatrix}$, write the elements as



The value of the given determinant

$$= (2)(7)(6) + (-1)(0)(4) + (3)(5)(1) - (4)(7)(3) - (1)(0)(2) - (6)(5)(-1)$$

$$= 84 + 15 - 84 + 30 = 45$$

CAUTION

Sarrus rule does not work for determinants of order greater than 3.

Method 3: Using Cofactors

A determinant can also be evaluated by multiplying the entries of any row (or column) by their cofactors and summing the resulting products.

Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then

$$\Delta = a_1C_{11} + b_1C_{12} + c_1C_{13} = a_1M_{11} - b_1M_{12} + c_1M_{13}.$$

For example, $\begin{vmatrix} 1 & 2 & 0 & -1 \\ 3 & -1 & 4 & 1 \\ -2 & 0 & -3 & 3 \\ 4 & 3 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} -1 & 4 & 1 \\ 0 & -3 & 3 \\ 3 & 1 & 2 \end{vmatrix}$

$$-2\begin{vmatrix} 3 & 4 & 1 \\ -2 & -3 & 3 \\ 4 & 1 & 2 \end{vmatrix} + 0 + 1\begin{vmatrix} 3 & -1 & 4 \\ -2 & 0 & -3 \\ 4 & 3 & 1 \end{vmatrix}$$

$= 54 - 94 + 13 = 27.$

NOTE

- The value of determinant is same when expanded by any row or any column.
- The above method of expansion is general and is valid for determinant of any order.

TRICK(S) FOR PROBLEM SOLVING

- If a row or a column of a determinant consists of all zeros, the value of the determinant is zero.
- Always expand a determinant along a row or column with maximum number of zeros.
- If each element above or below the main diagonal of a determinant is zero, then the value of the determinant is the product of elements along the main diagonal.

CAUTION

Sarrus rule does not work for determinants of order greater than 3.

SOLVED EXAMPLES

1. If $\begin{vmatrix} x^3+4x & x+3 & x-2 \\ x-2 & 5x & x-1 \\ x-3 & x+2 & 4x \end{vmatrix} = ax^5 + bx^4 + cx^3 + dx^2$ $+ ex + f$, be an identity in x, where a, b, c, d, e, f are independent of x, then the value of f is

(A) 0 (B) 15
(C) 17 (D) None of these

Solution: (C)

We have, $\begin{vmatrix} 0 & 3 & -2 \\ -2 & 0 & -1 \\ -3 & 2 & 0 \end{vmatrix} = f$

(Putting $x = 0$ on both sides)

$\therefore f = 17$

2. If $\begin{vmatrix} \alpha & -\beta & 0 \\ 0 & \alpha & \beta \\ \beta & 0 & \alpha \end{vmatrix} = 0$ then

(A) α/β is one of the cube roots of unity
(B) α is one of the cube roots of unity
(C) β is one of the cube roots of unity
(D) None of these

Solution: (A)

We have, $\begin{vmatrix} \alpha & -\beta & 0 \\ 0 & \alpha & \beta \\ \beta & 0 & \alpha \end{vmatrix} = 0 \Rightarrow \alpha^3 - \beta^3 = 0$

$\Rightarrow \left(\dfrac{\alpha}{\beta}\right)^3 = 1 \Rightarrow \dfrac{\alpha}{\beta}$ is one of the cube roots of unity.

3. If a, b, c are different, then the value of x satisfying

$$\begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix} = 0 \text{ is}$$

(A) c (B) c
(C) b (D) 0

Solution: (D)

Since for $x = 0$, the determinant reduces to the determinant of a skew-symmetric matrix of odd order which is always zero. Hence, $\boldsymbol{x = 0}$ is the solution of the given equation.

4. If $\begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & 1-2\lambda & \lambda-4 \\ \lambda-2 & \lambda+4 & 3\lambda \end{vmatrix} = p\lambda^4+q\lambda^3+r\lambda^2+s\lambda+$ t be an identity in λ, where p, q, r, s and t are constants, then the value of t is

(A) 0 (B) 10
(C) – 10 (D) None of these

Solution: (B)
Putting $\lambda = 0$ in the given identity, we get

$$\begin{vmatrix} 0 & -1 & 3 \\ 1 & 1 & -4 \\ -2 & 4 & 0 \end{vmatrix} = t$$

$\Rightarrow \quad t = 0 + 1\,(0 - 8) + 3\,(4 + 2) = 10$

5. If $f(x)$ satisfies the equation

$$\begin{vmatrix} f(x-3) & f(x+4) & f[(x+1)(x-2)-(x-1)^2 \\ 5 & 4 & -5 \\ 5 & 6 & 15 \end{vmatrix} = 0$$

for all real x, then
(A) $f(x)$ is periodic with period 7
(B) $f(x)$ is periodic with period 1
(C) $f(x)$ is non-periodic
(D) $f(x)$ is periodic with no fundamental period

Solution: (A)
The given determinant $= 90f(x-3) - 10f(x+4) + 10f(x-3)$.
So, $f(x)$ satisfies the equation $f(x+4) = f(x-3)$.
Replacing x by $x+3$, we get $f(x+7) = f(x)$ for all x.
Hence, $f(x)$ is periodic with period 7.

6. If d is the determinant of a square matrix A of order n, then the determinant of its adjoint is

(A) d^n (B) d^{n-1}
(C) d^{m+1} (D) d

Solution: (B)
We have, $|\text{Adj.}\, A| = |A|^{n-1} = d^{n-1}$

PROPERTIES OF DETERMINANTS

Properties of determinants of order three only are stated below. However these properties hold for determinants of any order. These properties help a good deal in the evaluation of determinants.

1. The value of the determinant remains unchanged if rows are changed into columns and columns are changed into rows, i.e.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

2. If two adjacent rows (columns) of a determinant are interchanged, the value of the determinant so obtained is the negative of the value of the original determinant, i.e.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

3. If two rows or columns of a determinant are identical then its value is zero, i.e.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

4. If each element of a row or column of a determinant is multiplied by a constant k then the value of the new determinant is k times the value of the original determinant, i.e.,

$$\begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

5. If any two rows or columns of a determinant are proportional, then its value is zero, i.e.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} & ka_{12} & ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

6. If each element of a row (or column) of a determinant is the sum of two or more terms, then the determinant can be expressed as the sum of the two or more determ-inants, i.e.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+c_1 & a_{22}+c_2 & a_{23}+c_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ c_1 & c_2 & c_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

7. If each element of a row (column) of a determinant is multiplied by a constant k and then added to the corresponding elements of some other row (column), then the value of the determinant remains the same, i.e.,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31}+ka_{21} & a_{32}+ka_{22} & a_{33}+ka_{23} \end{vmatrix}$$

8. If each element of a row (column) of a determinant is zero, then its value is zero.

9. If the elements of a determinant that involve x are polynomials in x, and if the determinant is equal to zero when a is substituted for x, the $x - a$ is a factor of given determinant.

Illustration

let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$. If we put $a = b$ in Δ, then we have

$$\begin{vmatrix} 1 & 1 & 1 \\ b & b & c \\ b^2 & b^2 & c^2 \end{vmatrix} = 0$$

(first and second column are identical)

This implies that $(a - b)$ must be a factor of Δ. Similarly, $(b - c)$ and $(c - a)$ are also factors of Δ. Since the product of the diagonal elements of Δ is $1.b.c,^2$ which is a third degree expression, Δ is a polynomial of degree 3. But $(a - b)(b - c)(c - a)$ is a factor of Δ which is of degree 3 itself. Therefore, the only other factor of Δ can be a constant, say k.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = k(a - b)(b - c)(c - a)$$

In order to find the value of k, give values to a, b and c such that calculations are easy and the two sides do not vanish. For example, assume $a = 0$, $b = 1$, $c = 2$, we get

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{vmatrix} = k(0 - 1)(1 - 2)(2 - 0)$$

or $2 = 2k$ (on solving the determinant along first column)

Thus $k = 1$. Hence,

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a - b)(b - c)(c - a).$$

IMPORTANT POINTS

In general, if r rows (or r columns) become identical when a is substituted for x, then $(x - a)^{r-1}$ is a factor of given determinant.

10. The sum of the products of the elements of any row (or column) of a determinant with the corresponding co-factors is equal to the value of determinant, i.e., if

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \text{ then}$$

$$a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = \Delta \text{ and so on.}$$

11. The sum of the products of elements of any row (or column) of a determinant with the co-factors of the corresponding elements of any other row (or column) is zero, i.e., if

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \text{ then}$$

$$a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33} = 0 \text{ and so on.}$$

IMPORTANT POINTS

If $\Delta = |a_{ij}|$ is a determinant of order n, then the value of the determinant $|A_{ij}|$, where A_{ij} is the cofactor of a_{ij} is Δ^{n-1}.

For example, $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \begin{vmatrix} o & c & b \\ c & o & a \\ b & a & o \end{vmatrix}^2$.

EVALUATION OF DETERMINANTS USING ELEMENTARY OPERATIONS

To evaluate determinants of higher order, we should always try to introduce zeros at the maximum number of places in a particular row (column) by using the properties of the determinant. We denote the rows of the determinant by R_1, R_2, R_3, ... and columns by C_1, C_2, C_3, ...

We shall use the following notations to evaluate a determinant.

1. The operation of interchanging the ith row and jth row will be denoted by $R_i \leftrightarrow R_j$.

2. The operation of multiplying each element of the ith row by a number k will be denoted by $R_i \to kR_i$.
3. The operation of adding to each element of the ith row, k times the corresponding elements of the jth row ($j \neq i$) will be denoted by $R_i \to R_i + kR_j$.

Similar notations are used for operations on columns replacing R by C.

SOLVED EXAMPLES

7. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ if

(A) x, y, z are in A.P.
(B) x, y, z are in G.P.
(C) x, y, z are in H.P.
(D) xy, yz, zx are in A.P.

Solution: (B)

The given determinant

$$= \begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ -(xp^2+2yp+z) & 0 & 0 \end{vmatrix}$$

(Applying $R_3 \to R_3 - pR_1 - R_2$)

$$= -(xp^2+2yp+z)(xz-y^2)$$

$= 0$ if x, y, z are in G. P.

8. If T_p, T_q, T_r are pth, qth, rth terms of an A.P, then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ is equal to

(A) $p+q+r$ (B) 0
(C) -1 (D) 1

Solution: (B)

We have, $T_p = a + (p-1)d$, $T_q = a + (q-1)d$, $T_r = a + (r-1)d$, where a is the first term and d is the common difference.

$\therefore$ The given determinant

$$= \begin{vmatrix} a+(p-1)d & a+(q-1)d & a+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a & a \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} \text{ [Applying } R_1 \to R_1 - (R_2 - R_3)d]$$

$$= a\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

9. If $f(x) = \begin{vmatrix} \cos^2 x & \cos x \cdot \sin x & -\sin x \\ \cos x \sin x & \sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then for all x

(A) $f(x) = 0$ (B) $f(x) = 1$
(C) $f(x) = 2$ (D) None of these

Solution: (B)

We have,

$$f(x) = \begin{vmatrix} \cos^2 x & \cos x \cdot \sin x & -\sin x \\ \cos x \sin x & \sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & -\sin x \\ 0 & 1 & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$$

(Applying $C_1 \to C_1 - \sin x \cdot C_3$ and $C_2 \to C_2 + \cos x \cdot C_3$)

$$= \begin{vmatrix} 1 & 0 & -\sin x \\ 0 & 1 & \cos x \\ 0 & -\cos x & \sin^2 x \end{vmatrix}$$

(Applying $R_3 \to R_3 - \sin x \cdot R_1$)

$= \sin^2 x + \cos^2 x = 1$ for all x. (Expanding along C_1)

10. The value of the determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is

(A) independent of n
(B) independent of a
(C) independent of x
(D) None of these

Solution: (A)

We have,

$$\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

$$= \begin{vmatrix} a^2 - 2a\cos x + 1 & a & 1 \\ 0 & \cos(n+1)x & \cos(n+2)x \\ 0 & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$$

(Applying $C_1 \to C_1 + C_3 - 2\cos x\, C_2$)

$= (a^2 - 2a\cos x + 1)\sin x$ (Expanding along C_1),

11. If $f(x)$, $g(x)$ and $h(x)$ are three polynomials of degree 2 and $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$, then $\Delta(x)$ is a polynomial of degree

(A) 2 (B) 3
(C) at most 2 (D) at most 3

which is independent of n.

Solution: (C)

Let $f(x) = a_0x^2 + a_1x + a_2$

$g(x) = b_0x^2 + b_1x + b_2$

$h(x) = c_0x^2 + c_1x + c_2$

Then,

$$\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0x + a_1, & 2b_0x + b_1 & 2c_0x + c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$$

$$= x\begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0 & 2b_0 & 2c_0 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$$

$$= 0 + 2\begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ a_0 & b_0 & c_0 \end{vmatrix}$$

$= 2[(b_1c_0 - b_0c_1)f(x) - (a_1c_0 - a_0c_1)g(x) + (a_1b_0 - a_0b_1)h(x)]$

Hence, degree of $\Delta(x) \le 2$.

12. If $a \ne b \ne c$, one value of x which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0 \text{ is given by,}$$

(A) $x = a$ (B) $x = b$
(C) $x = c$ (D) $x = 0$

Solution: (D)

Let $\Delta = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$.

On putting $x = a$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & a-b \\ 2a & 0 & a-c \\ a+b & a+c & 0 \end{vmatrix} = (a+c)(a+b)(a-c)$$

Clearly $\Delta \ne 0$ on expansion along second column, so that $x = a$ does not satisfy the equation $\Delta = 0$. Similarly $x = b$ and $x = c$ also do not satisfy. Now, put $x = 0$, we get

$$\Delta = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$$

Hence, $\boldsymbol{x = 0}$ satisfies the equation $\Delta = 0$.

13. If $U_n = \begin{vmatrix} 1 & k & k \\ 2n & k^2+k+1 & k^2+k \\ 2n-1 & k^2 & k^2+k+1 \end{vmatrix}$

and $\sum_{n=1}^{k} U_n = 72$ then $k =$

(A) 8 (B) 9
(C) 6 (D) None of these

Solution: (A)

$$\sum_{n=1}^{k} U_n = \begin{vmatrix} \sum_{n=1}^{k} 1 & k & k \\ 2\sum_{n=1}^{k} n & k^2+k+1 & k^2+k \\ 2\sum_{n=1}^{k} n - \sum_{n=1}^{k} 1 & k^2 & k^2+k+1 \end{vmatrix}$$

$$= \begin{vmatrix} k & k & k \\ k(k+1) & k^2+k+1 & k^2+k \\ k^2 & k^2 & k^2+k+1 \end{vmatrix}$$

$$= \begin{vmatrix} k & 0 & k \\ k^2+k & 1 & k^2+k \\ k^2 & 0 & k^2+k+1 \end{vmatrix}$$

(Applying $C_2 \to C_2 - C_1$)

$= k(k^2 + k + 1) - k^3 = k(k+1) = 72$ (given)

$\Rightarrow k = 8$.

14. If $\Sigma\cos^2\alpha_1 = \Sigma\cos^2\beta_1 = \Sigma\cos^2\gamma_1 = 1$; $\Sigma\cos\alpha_1\cos\beta_1 = \Sigma\cos\beta_1\cos\gamma_1 = \Sigma\cos\gamma_1\cos\alpha_1 = 0$,

then the value of $\begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 \end{vmatrix}$ is equal to

(A) 0 (B) – 1
(C) 1 (D) None of these

Solution: (C)

$$\begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\gamma_3 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 \end{vmatrix}^2$$

$$= \begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\gamma_3 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 \end{vmatrix} \times \begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\gamma_3 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 \end{vmatrix}$$

$$= \begin{vmatrix} \Sigma\cos^2\alpha_1 & \Sigma\cos\alpha_1\cos\beta_1 & \Sigma\cos\alpha_1\cos\gamma_1 \\ \Sigma\cos\beta_1\cos\alpha_1 & \Sigma\cos^2\beta_1 & \Sigma\cos\beta_1\cos\gamma_1 \\ \Sigma\cos\alpha_1\cos\gamma_1 & \Sigma\cos\gamma_1\cos\beta_1 & \Sigma\cos^2\gamma_1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

15. If $b^2 - ac < 0$ and $a > 0$ then the value of the determinant

$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$ is

(A) positive (B) negative
(C) zero (D) $b^2 + ac$

Solution: (B)

We have,

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ ax+by & bx+cy & -(ax^2+2bxy+cy^2) \end{vmatrix}$$

(Applying $C_3 \to C_3 - xC_1 - y\,C_2$)

$= -(ax^2 + 2bxy + cy^2)(ac - b^2)$

$= \frac{1}{a}(b^2 - ac)[(ax+by)^2 + y^2(ac - b^2)] < 0$

($\because b^2 - ac < 0$ and $a > 0$)

16. If α, β and γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) q (B) 0
(C) p (D) $p^2 - 2q$

Solution: (B)

Since α, β, γ are the roots of the equation $x^3 + px + q = 0$

$\therefore \quad \alpha + \beta + \gamma = 0$

So, $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix}$

(Applying $C_1 \to C_1 + C_2 + C_3$)

$$= \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0.$$

17. If $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then maximum value of Δ is

(A) 10 (B) 14
(C) 1 (D) None of these

Solution: (A)

$$\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 0 & \sin\theta - 3\cos\theta & 0 \end{vmatrix}$$

(Applying $R_3 \to R_3 - R_1$)

$= -(\sin\theta - 3\cos\theta)(3\cos\theta - \sin\theta)$

$= (3\cos\theta - \sin\theta)^2$

But $-\sqrt{9+1} \le 3\cos\theta - \sin\theta \le \sqrt{9+1}$

Therefore, $(3\cos\theta - \sin\theta)^2 \le 10$

18. If the determinant

$\begin{vmatrix} a & b & 2a\alpha+3b \\ b & c & 2b\alpha+3c \\ 2a\alpha+3b & 2b\alpha+3c & 0 \end{vmatrix} = 0$, then

(A) a, b, c are in H.P.
(B) α is root of $4ax^2 + 12bx + 9c = 0$ or a, b, c are in G.P.
(C) a, b, c are in G.P.
(D) a, b, c are in A.P.

Solution: (B)

Operate $R_3 \to R_3 - 2\alpha R_1 - 3R_2$, we get

$$\begin{vmatrix} a & b & 2a\alpha + 3b \\ b & c & 2b\alpha + 3c \\ 0 & 0 & -4a\alpha^2 - 6b\alpha - 6b\alpha - 9c \end{vmatrix} = 0$$

$\Rightarrow (4a\alpha^2 + 12b\alpha + 9c)(ac - b^2) = 0$

$\Rightarrow \alpha$ is a root of $4ax^2 + 12bx + 9c = 0$ or a, b, c are in G. P.

19. If $D_1 = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix}$ and $D_2 = \begin{vmatrix} 1 & 1 & 1 \\ yz & xz & xy \\ x & y & z \end{vmatrix}$, then

(A) $D_1 = D_2$
(B) $D_1 = -D_2$
(C) $D_1 = -2D_2$
(D) $D_2 = 2D_1$

Solution: (A)

We have,

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ yz & xz & xy \\ x & y & z \end{vmatrix} = \frac{1}{xyz}\begin{vmatrix} x & y & z \\ xyz & xyz & xyz \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= \frac{xyz}{xyz}\begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ x^2 & y^2 & z^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix} = D_1.$$

20. The value of n for which the determinant

$\begin{vmatrix} {}^8C_3 & {}^9C_5 & {}^{10}C_7 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix}$ becomes zero is

(A) $n = 2$
(B) $n = 3$
(C) $n = 4$
(D) None of these

Solution: (C)

We have,

$$\begin{vmatrix} {}^8C_3 & {}^9C_5 & {}^{10}C_7 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix}$$

$$= \begin{vmatrix} {}^8C_3 & {}^9C_5 & {}^{10}C_5 \\ {}^9C_4 & {}^{10}C_6 & {}^{11}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix}$$

(Applying $R_2 \to R_2 + R_1$ and using ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$)

For $\boldsymbol{n} = \mathbf{4}$, R_2 and R_3 become identical and, therefore, the value of the determinant becomes zero.

21. The value of θ lying between $\theta = 0$ and $\theta = \frac{\pi}{2}$ and satisfying the equation

$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$ is

(A) $\frac{5\pi}{2}$
(B) $\frac{7\pi}{24}$ or $\frac{11\pi}{24}$
(C) $\frac{3\pi}{4}$
(D) $\frac{\pi}{24}$

Solution: (B)

Operate $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow \quad 1 + 4\sin 4\theta + \cos^2\theta + \sin^2\theta = 0$

$\Rightarrow \quad 2 + 4\sin 4\theta = 0$

$\Rightarrow \quad \sin 4\theta = -\frac{1}{2}$

$\Rightarrow \quad 4\theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6} \Rightarrow \theta = \frac{7\pi}{24}$ or $\frac{11\pi}{24}$.

22. If $\Delta a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-n \end{vmatrix}$, then $\sum_{a=1}^{n} \Delta a =$

(A) 0
(B) n
(C) a
(D) None of these

Solution: (A)

We have, $\displaystyle\sum_{a=1}^{n} \Delta_a = \begin{vmatrix} \sum_{a=1}^{n}(a-1) & n & 6 \\ \sum_{a=1}^{n}(a-1)^2 & 2n^2 & 4n-2 \\ \sum_{a=1}^{n}(a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$

$$= \begin{vmatrix} \frac{n(n-1)}{2} & n & 6 \\ \frac{n(n-1)(2n-1}{6} & 2n^2 & 4n-2 \\ \frac{n^2(n-1)^2}{4} & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \frac{n^2(n+1)}{2}\begin{vmatrix} 1 & 1 & 6 \\ \frac{2n-1}{3} & 2n & 4n-2 \\ \frac{n(n-1)}{2} & 3n^2 & 3n^2-3n \end{vmatrix}$$

$$= \frac{n^3(n-1)}{12}\begin{vmatrix} 1 & 1 & 6 \\ 2n-1 & 6n & 12n-6 \\ n-1 & 6n & 6n-6 \end{vmatrix}$$

$$= \frac{n^3(n-1)}{12}\begin{vmatrix} 1 & 1 & 0 \\ 2n-1 & 6n & 0 \\ n-1 & 6n & 0 \end{vmatrix} = 0$$

(Applying $C_3 \to C_3 - 6C_1$)

23. If the three linear equations $x + 4ay + az = 0$, $x + 3by + bz = 0$ and $x + 2cy + cz = 0$ have a non-trivial solution, then a, b, c are in

(A) $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ (B) $b^2 = ac$

(C) $2b = a + c$ (D) None of these

Solution: (A)

For a non-trivial solution, we must have,

$$\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 4a & a \\ 0 & 3b-4a & b-a \\ 0 & 2c-4a & c-a \end{vmatrix} = 0$$

(Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$)

$\Rightarrow (3b - 4a)(c - a) - (2c - 4a)(b - a) = 0$

$\Rightarrow bc + ab - 2ac = 0 \Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

24. The value of the determinant

$$\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix} \text{ is}$$

(A) $-5\sqrt{3}(5 - \sqrt{6})$ (B) $-5\sqrt{3}(5 + \sqrt{6})$

(C) $-5\sqrt{3}(\sqrt{6} - 5)$ (D) None of these

Solution: (A)

We have, $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$

$$= (\sqrt{5})^2 \begin{vmatrix} \sqrt{13}+\sqrt{3} & 2 & 1 \\ \sqrt{15}+\sqrt{26} & \sqrt{5} & \sqrt{2} \\ 3+\sqrt{65} & \sqrt{3} & \sqrt{5} \end{vmatrix} \text{ is}$$

(Taking $\sqrt{5}$ common from C_2 and C_3)

$$= 5\begin{vmatrix} -\sqrt{3} & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

(Applying $C_1 \to C_1 - \sqrt{3}\,C_2 - \sqrt{13}\,C_3$)

$= -5\sqrt{3}(5 - \sqrt{6})$ (Expanding along C_1)

25. If $\Delta_r = \begin{vmatrix} r-1 & n & 6 \\ (r-1)^2 & 2n^2 & 4n-2 \\ (r-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$, then $\sum_{r=1}^{n} \Delta_r$ is equal to

(A) 1 (B) 2

(C) 3 (D) 0

Solution: (D)

We have,

$$\sum_{r=1}^{n}\Delta_r = \begin{vmatrix} \sum_{r=1}^{n}(r-1) & n & 6 \\ \sum_{r=1}^{n}(r-1)^2 & 2n^2 & 4n-2 \\ \sum_{r=1}^{n}(r-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$$

[$\because$ the terms in C_1 are dependent on r whereas the terms in C_2 and C_3 are constant]

$$= \begin{vmatrix} \frac{1}{2}(n-1)n & n & 6 \\ \frac{1}{6}(n-1)n(2n-1) & 2n^2 & 4n-2 \\ \frac{1}{4}(n-1)^2n^2 & 3n^3 & 3n^2-3n \end{vmatrix}$$

$$= \frac{1}{12} n^2 (n-1) \begin{vmatrix} 6 & 1 & 6 \\ 2(2n-1) & 2n & 2(2n-1) \\ 3n(n-1) & 3n^2 & 3n(n-1) \end{vmatrix}$$

[Taking $\frac{1}{12}$ $n(n-1)$ common from C_1 and n common from C_2] = 0. [$\because$ C_1 and C_3 are identical].

26. If $\Delta_r = \begin{vmatrix} 2^{r-1} & x & 2^n - 1 \\ 2.3^{r-1} & y & 3^n - 1 \\ 4.5^{r-1} & z & 5^n - 1 \end{vmatrix}$, then $\sum_{r=1}^{n} \Delta_r$ is

(A) independent of x (B) independent of y
(C) independent of z (D) independent of n

Solution: (A, B, C, D)

We have,

$$\sum_{r=1}^{n} \Delta_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & x & 2^n - 1 \\ \sum_{r=1}^{n} 2.3^{r-1} & y & 3^n - 1 \\ \sum_{r=1}^{n} 4.5^{r-1} & z & 5^n - 1 \end{vmatrix}$$

$$= \begin{vmatrix} 2^n - 1 & x & 2^n - 1 \\ 3^n - 1 & y & 3^n - 1 \\ 5^n - 1 & z & 5^n - 1 \end{vmatrix}$$

$$\left(\because \sum_{r=1}^{n} \Delta_r = 1 + 2 + 2^2 + \ldots + 2^{n-1} = \frac{2^n - 1}{2 - 1}, = 2^n - 1, \right.$$

$$\text{similarly } \sum_{r=1}^{n} 2 \cdot 3^{r-1} = 2 \, \frac{3^n - 1}{3 - 1} = 3^n - 1$$

$$\left. \text{and } \sum_{r=1}^{n} 4 \cdot 5^{r-1} = 4 \cdot \frac{5^n - 1}{5 - 1} = 5^n - 1. \right)$$

$$= 0 \qquad (\because C_1 \text{ and } C_3 \text{ are identical})$$

$\therefore$ $\sum_{r=1}^{n} \Delta_r$ is independent of x, y, z and n.

27. If a, b, c are the pth, qth and rth terms respectively of a geometric progression, then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to

(A) 0 (B) 1
(C) – 1 (d) none of these

Solution: (A)

We have, $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$

$$= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$$

[Let A be the first term and R the common ratio of G.P., then

$a = T_p = AR^{p-1}$,

$\therefore$ $\log a = \log A + (p - 1) \log R$.

$b = T_q = AR^{q-1}$,

$\therefore$ $\log b = \log A + (q - 1) \log R$.

$c = T_r = AR^{r-1}$,

$\therefore$ $\log c = \log A + (r - 1) \log R$.]

$$= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$$

$$= 0 + \log R \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix}$$

(Applying $C_1 \to C_1 - C_2 + C_3$)

$= 0$.

28. If $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$ are three-digit numbers, each of which is divisible by k, then

$\Delta = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ is

(A) divisible by k (B) divisible by k^2
(C) divisible by $2k$ (D) None of these

Solution: (A)

Since $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$ are divisible by k, therefore

$$100A_1 + 10B_1 + C_1 = n_1k$$

$$100A_2 + 10B_2 + C_2 = n_2k$$

$$100A_3 + 10B_3 + C_3 = n_3k$$

where n_1, n_2, n_3 are integers.

Now $\Delta = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

$$= \begin{vmatrix} A_1 & B_1 & 100A_1 + 10B_1 + C_1 \\ A_2 & B_2 & 100A_2 + 10B_2 + C_2 \\ A_3 & B_3 & 100A_3 + 10B_3 + C_3 \end{vmatrix}$$

(Applying $C_3 \to C_3 + 10C_2 + 100C_1$)

$$= \begin{vmatrix} A_1 & B_1 & n_1k \\ A_2 & B_2 & n_2k \\ A_3 & B_3 & n_3k \end{vmatrix} = k \begin{vmatrix} A_1 & B_1 & n_1 \\ A_2 & B_2 & n_2 \\ A_3 & B_3 & n_3 \end{vmatrix} = k\Delta_1$$

$\Rightarrow \Delta$ is divisible by k

(Since elements of Δ_1 are integers, $\therefore$ Δ_1 is an integer)

29. The determinant $\begin{vmatrix} a^2 + 2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ is

(A) > 0 if $a > 1$ (B) $= 0$ if $a = 1$
(C) < 0 if $a < 1$ (D) All of these

Solution: (D)

Let $\Delta = \begin{vmatrix} a^2 + 2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$$= \begin{vmatrix} a^2 + 2a - 3 & 2a-2 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

(Applying $R_1 \to R_1 - R_3$, and $R_2 \to R_2 - R_3$)

$$= \begin{vmatrix} a^2 + 2a - 3 & 2a-2 \\ 2a-2 & a-1 \end{vmatrix}$$

(Expanding along C_3)

$$= \begin{vmatrix} (a+3)(a-1) & 2(a-1) \\ 2(a-1) & a-1 \end{vmatrix}$$

$$= (a-1)^2 \begin{vmatrix} a+3 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (a-1)^2 \cdot (a+3-4) = (a-1)^3.$$

Clearly, $\Delta > 0$ if $a > 1$; $\Delta = 0$ if $a = 1$ and $\Delta < 0$ if $a < 1$.

30. If A, B, and C are the angles of a triangle, then the value of the determinant

$\begin{vmatrix} -1 + \cos B & \cos C + \cos B & \cos B \\ \cos C + \cos A & -1 + \cos A & \cos A \\ -1 + \cos B & -1 + \cos A & -1 \end{vmatrix}$ is

(A) 0 (B) 1
(c) – 1 (D) 2

Solution: (A)

We have,

$$\begin{vmatrix} -1 + \cos B & \cos C + \cos B & \cos B \\ \cos C + \cos A & -1 + \cos A & \cos A \\ -1 + \cos B & -1 + \cos A & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$$

(Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$)

$$= \frac{1}{a} \begin{vmatrix} -a & \cos C & \cos B \\ a\cos C & -1 & \cos A \\ a\cos B & \cos A & -1 \end{vmatrix}$$

$$= \frac{1}{a} \begin{vmatrix} 0 & \cos C & \cos B \\ 0 & -1 & \cos A \\ 0 & \cos A & -1 \end{vmatrix}$$

(Applying $C_1 \to C_1 + b\,C_2 + c\,C_3$)

$= 0.$

31. If the three digit numbers $A28$, $3B9$ and $62C$, where A, B and C are integers between 0 and 9, are divisible by a fixed integer k, then the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is

(A) divisible by k (B) divisible by k^2
(C) divisible by $2k$ (D) None of these

Solution: (A)

Since A28, 3B9 and 62C are divisible by k

$\therefore$ $A28 = n_1k = 100A + 20 + 8$ (1)

$3B9 = n_2k = 300 + 10B + 9$ (2)

$62C = n_3k = 600 + 20 + C$ (3)

where n_1, n_2 and n_3 are integers.

Now, $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$

$$= \begin{vmatrix} A & 3 & 6 \\ 100A + 20 + 8 & 300 + 10B + 9 & 600 + 20 + C \\ 2 & B & 2 \end{vmatrix}$$

(Applying $R_2 \to R_2 + 100R_1 + 10R_3$)

$$= \begin{vmatrix} A & 3 & 6 \\ n_1k & n_2k & n_3k \\ 2 & B & 2 \end{vmatrix} \qquad \text{[Using (1), (2) and (3)]}$$

$$= k \begin{vmatrix} A & 3 & 6 \\ n_1 & n_2 & n_3 \\ 2 & B & 2 \end{vmatrix}, \text{ which is divisible by } k.$$

32. If $\begin{vmatrix} x+a & a^2 & a^3 \\ x+b & b^2 & b^3 \\ x+c & c^2 & c^3 \end{vmatrix} = 0$ and $a \neq b \neq c$ then x is equal to

(A) $abc/(ab + bc + ca)$
(B) $-abc/(ab + bc + ca)$
(C) $(ab + bc + ca)/(abc)$
(D) $-(ab + bc + ca)/(abc)$

Solution: (B)

We have, $\begin{vmatrix} x+a & a^2 & a^3 \\ x+b & b^2 & b^3 \\ x+c & c^2 & c^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & a^2 & a^3 \\ x & b^2 & b^3 \\ x & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow x \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow x\,(a-b)\,(b-c)\,(c-a)\,(ab+bc+ca) + abc\,(a-b)\,(b-c)\,(c-a) = 0$$

$$\Rightarrow \quad x = -abc/(ab+bc+ca) \qquad (\because\ a \neq b \neq c)$$

33. If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$,

then $\int_0^{\pi/2} f(x)\,dx$ is equal to

(A) $(15\pi + 32)/60$
(B) $-(15\pi + 32)/60$
(C) $(15\pi + 32)/4$
(D) None of these

Solution: (B)

We have,

$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & \sec^2 x + \dfrac{\cos x}{\sin^2 x} - \cos x \\ 0 & \cos^2 x - \cos^4 x & \operatorname{cosec}^2 x - \cos^4 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

(Applying $R_1 \to R_1 - R_3 \sec x$ and $R_2 \to R_2 - R_3 \cos^2 x$)
Expanding along C_1, we get

$$f(x) = -\cos^2 x \sin^2 x \left[\frac{1}{\cos^2 x} + \frac{\cos x}{\sin^2 x} - \cos x\right]$$

$$= -(\sin^2 x + \cos^3 x - \cos^3 x \sin^2 x)$$

$$= -(\sin^2 x + \cos^3 x - \cos^3 x + \cos^5 x)$$

$$= -\sin^2 x - \cos^5 x.$$

$$\therefore \quad \int_0^{\pi/2} f(x)\,dx = -\int_0^{\pi/2} \sin^2 x\,dx - \int_0^{\pi/2} \cos^5 x\,dx$$

$$= -\frac{1}{2}\cdot\frac{\pi}{2} - \frac{4\cdot 2}{5\cdot 3\cdot 1}$$

$$= \frac{-\pi}{4} - \frac{8}{15} = -(15\pi + 32)/60.$$

34. The value of the determinant of n^{th} order, being given by $\begin{vmatrix} x & 1 & 1 & \ldots \\ 1 & x & 1 & \ldots \\ 1 & 1 & x & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}$, is

(A) $(x-1)^{n-1}(x+n-1)$
(B) $(x-1)^n(x+n-1)$
(C) $(1-x)^{n-1}(x+n-1)$
(D) None of these

Solution: (A)

We have, $\begin{vmatrix} x & 1 & 1 & \ldots \\ 1 & x & 1 & \ldots \\ 1 & 1 & x & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}$

$$= \begin{vmatrix} x & 1 & 1 & \dots \\ (1-x) & (x-1) & 0 & \dots \\ (1-x) & 0 & (x-1) & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, \dots, R_n \to R_n - R_1$]

$$= x(x-1)^{n-1} + \underbrace{(x-1)^{n-1} + (x-1)^{n-1} + \dots + (x-1)^{n-1}}_{(n-1) \text{ times}}$$

(Expanding along R_1)

$$= x\,(x-1)^{n-1} + (x-1)^{n-1}\,[1 + 1 + \dots + (n-1) \text{ times}]$$

$$= (x-1)^{n-1}\,(x + n - 1).$$

PRODUCT OF DETERMINANTS OF SAME ORDER

Let $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix}$

Then row by row multiplication of Δ_1 and Δ_2 is given by,

$$\Delta_1 \times \Delta_2 = \begin{vmatrix} a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1 & a_1\alpha_2 + b_1\beta_2 + c_1\gamma_2 & a_1\alpha_3 + b_1\beta_3 + c_1\gamma_3 \\ a_2\alpha_1 + b_2\beta_1 + c_2\gamma_1 & a_2\alpha_2 + b_2\beta_2 + c_2\gamma_2 & a_2\alpha_3 + b_2\beta_3 + c_2\gamma_3 \\ a_3\alpha_1 + b_3\beta_1 + c_3\gamma_1 & a_3\alpha_2 + b_3\beta_2 + c_3\gamma_2 & a_3\alpha_3 + b_3\beta_3 + c_3\gamma_3 \end{vmatrix}$$

Multiplication can also be performed row by column; column by row or column by column as required in the problem.

Some Useful Determinants

Any of the following determinants can be used directly in solving problems:

1. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

2. $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

3. $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

4. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$

$$= -\frac{1}{2}[a+b+c][(a-b)^2 + (b-c)^2 + (c-a)^2]$$

TRICK(S) FOR PROBLEM SOLVING

If Δ' is the determinant obtained by replacing all the elements of determinant Δ of order n by their corresponding cofactors, then $\Delta' = \Delta^{n-1}$.

In particular,

If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta' = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

where $A_1, B_1, C_1, \dots$ are cofactors of $a_1, b_1, c_1, \dots$ etc., then $\Delta^1 = \Delta^2$.

SOLVED EXAMPLES

35. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then

(A) $\Delta_1 = 3(\Delta_2)^2$

(B) $\frac{d}{dx}\Delta_1 = 3\Delta_2$

(C) $\frac{d}{dx}\Delta_1 = 3\Delta_2^{\,2}$

(D) $\Delta_1 = 3(\Delta_2)^{3/2}$

Solution: (B)

$$\frac{d}{dx}\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ a & x & b \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ 0 & 1 & 0 \\ a & a & x \end{vmatrix} + \begin{vmatrix} x & b & b \\ a & x & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2.$$

36. If $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$, then the value of $5A + 4B + 3C + 2D + E$ is equal to

(A) –11 (B) 17
(C) –17 (D) 0

Solution: **(A)**

$$\text{Let } \Delta(x) = \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}$$

Then, $5A + 4B + 3C + 2D + E = \Delta(1) + \Delta'(1)$

Now, $\Delta(1) = 0$ as R_2 and R_3 are identical.

$$\Delta'(x) = \begin{vmatrix} 1 & 0 & 1 \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ 2x & 1 & 0 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\Delta'(1) = 0 + \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix} = -17 + 6 = -11$$

$\therefore \quad 5A + 4B + 3C + 2D + E = -11$

37. If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and

$$D' = \begin{vmatrix} a_1 + pb_1, & b_1 + qc_1 & c_1 + ra_1 \\ a_2 + pb_2 & b_2 + qc_2 & c_2 + ra_2 \\ a_3 + pb_3 & b_3 + qc_3 & c_3 + ra_3 \end{vmatrix}, \text{ then}$$

(A) $D' = D(1 + pqr)$
(B) $D' = D$
(C) $D' = D(1 - pqr)$
(D) $D' = D(1 + p + q + r)$

Solution: (A)

$$D' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + pqr \begin{vmatrix} b_1 & c_1 & a_1 \\ b_2 & c_2 & a_2 \\ b_3 & c_3 & a_3 \end{vmatrix}$$

(All other determinants will vanish)

$= (1 + pqr)D$

SOLUTION OF LINEAR EQUATIONS BY DETERMINANTS

1. Cramer's Rule: Solution of system of linear equations in two unknowns

The solution of the system of equations

$$a_1x + b_1y = c_1 \qquad a_2x + b_2y = c_2$$

is given by

$$x = \frac{D_1}{D} \text{ and } y = \frac{D_2}{D},$$

where

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

and $D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$, provided $D \neq 0$.

2. Cramer's Rule: Solution of system of linear equations in three unknowns

The solution of the system of equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

is given by

$$x = \frac{D_1}{D}, y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}, \text{ where}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}; D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}; D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix};$$

provided $D \neq 0$.

Conditions for Consistency

The following cases may arise:

1. If $D \neq 0$, then the system is consistent and has a unique solution, which is given by Cramer's rule:
$$x = \frac{D_1}{D}, y = \frac{D_2}{D}, z = \frac{D_3}{D}$$
2. If $D = 0$ and atleast one of the determinants D_1, D_2, D_3 is non-zero, the given system is inconsistent, i.e., it has no solution.

3. If $D = 0$ and $D_1 = D_2 = D_3 = 0$, then the system is consistent and dependent, and has infinitely many solutions.

Homogeneous and Non-homogeneous System

If $d_1 = d_2 = d_3 = 0$, then the system is said to be, *homogeneous*, otherwise it is called non-homogeneous.

If the system of equations is homogeneous, then $D_1 = D_2 = D_3 = 0$ (value of the determinant is zero, if one column has all elements = 0). Thus,

1. if $D \neq 0$, the system has only trivial solution ($x = y = z = 0$), and
2. if $D = 0$, the system has a non-trivial solution. In fact it has infinitely many solutions.

SOLVED EXAMPLES

38. If $x = cy + bz$, $y = az + cx$, $z = bx + ay$ where x, y, z are not all zero, then

(A) $a^2 + b^2 + c^2 + 2abc = 0$
(B) $a^2 + b^2 + c^2 - 2abc = 1$
(C) $a^2 + b^2 + c^2 + 2abc = 1$
(D) None of these

Solution: (C)

We have,
$$x - cy - bz = 0$$
$$cx - y + az = 0$$
$$bx + ay - z = 0$$

Since x, y, z are not all zero, so the system will have a non-trivial solution. Therefore,

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$$
$$\Rightarrow \quad a^2 + b^2 + c^2 + 2abc = 1$$

39. The value of λ for which the equations $x + y - 3 = 0$, $(1 + \lambda)x + (2 + \lambda)y - 8 = 0$, $x - (1 + \lambda)y + (2 + \lambda) = 0$ are consistent is

(A) 1 (B) 5/3
(C) – 5/3 (D) None of these

Solution: (A, C)

Here the equations are in two variables x and y. If they are consistent then the values of x and y obtained from first two equations should satisfy the third equation and hence $D = 0$. i.e.,

$$\Rightarrow \quad \begin{vmatrix} 1 & 1 & -3 \\ 1+\lambda & 2+\lambda & -8 \\ 1 & -1-\lambda & 2+\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & 0 & 0 \\ 1+\lambda & 1 & -5+3\lambda \\ 1 & -2-\lambda & 5+\lambda \end{vmatrix} = 0$$

(Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 + 3C_1$)

$$\Rightarrow \quad (5 + \lambda) + (2 + \lambda)(-5 + 3\lambda) = 0$$
$$\Rightarrow \quad 3\lambda^2 + 2\lambda - 5 = 0$$
$$\Rightarrow \quad (\lambda - 1)(3\lambda + 5) = 0$$
$$\Rightarrow \quad \lambda = 1. -\frac{5}{3}$$

40. Let λ and α be real. The set of all values of λ for which the system of linear equations

$$\lambda x + (\sin \alpha)y + (\cos \alpha)z = 0$$
$$x + (\cos \alpha)y + (\sin \alpha)z = 0$$
$$-x + (\sin \alpha)y - (\cos a)z = 0$$

has a non-trivial solution, is

(A) $[0, \sqrt{2}]$ (B) $[-\sqrt{2}, 0]$
(C) $[-\sqrt{2}, \sqrt{2}]$ (D) None of these

Solution: (C)

Since the system has a non-trivial solution,

$$\text{therefore} \begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda(-\cos^2 \alpha - \sin^2 \alpha) - (-\sin \alpha \cos \alpha - \sin \alpha \cos \alpha) - (\sin^2 \alpha - \cos^2 \alpha) = 0$

$\Rightarrow \quad -\lambda + \sin 2\alpha + \cos 2\alpha = 0 \Rightarrow \lambda = \sin 2\alpha + \cos 2\alpha$

$$\Rightarrow \quad \lambda = \sqrt{2} \cos\left(2\alpha - \frac{\pi}{4}\right)$$

Since $-1 \leq \cos\left(2\alpha - \frac{\pi}{4}\right) \leq 1 \ \forall \ \alpha \in R$

$\therefore \quad -\sqrt{2} \leq \lambda \leq \sqrt{2}$ *i.e.* $\lambda \in [-\sqrt{2}, \sqrt{2}]$.

EXERCISES

Single Option Correct Type

1. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then

(A) $\frac{b^2}{q^2} = \frac{ac}{pr}$ (B) $\frac{c^2}{r^2} = \frac{ab}{pq}$

(C) $\frac{a^2}{p^2} = \frac{bc}{qr}$ (D) None of these

2. a, b, c are in G.P. with common ratio r_1 and α, β, γ are in G.P. with common ratio r_2. If the equations $ax + \alpha y + z = 0$, $bx + \beta y + z = 0$, $cx + \gamma y + z = 0$ have only trivial solution, then

(A) $a, \alpha = 0$ (B) $r_1, r_2 = 1$

(C) $r_1, r_2 \neq 1$ (D) $r_1 = r_2$

3. If the value of a third order determinant is 11, then the value of the determinant formed by its cofactors will be

(A) 11 (B) 121

(C) 1331 (D) 14641

4. If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are respectively the p^{th}, q^{th} and r^{th} terms of an A.P., then the value of the determinant

$$\begin{vmatrix} {}^8C_3 & {}^9C_5 & {}^{10}C_7 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix}$$ is

(A) abc (B) pqr

(C) 0 (D) None of these

5. The value of the determinant

$$\begin{vmatrix} \sqrt{x}+\sqrt{y} & 2\sqrt{2} & \sqrt{z} \\ \sqrt{yz}+\sqrt{2x} & z & \sqrt{2z} \\ y+\sqrt{xz} & \sqrt{yz} & z \end{vmatrix};$$

where x, y, z are positive real numbers, is

(A) $z\,(\sqrt{2}y - z\sqrt{y})$ (B) $y\,(\sqrt{2}z - y\sqrt{z})$

(C) $x\,(\sqrt{2}y - z\sqrt{y})$ (D) None of these

6. Let $D_k = \begin{vmatrix} \alpha & \beta & \gamma \\ 2.3^k & 16.9^k & 26.27^k \\ (3^{10}-1) & 2(9^{10}-1) & (27^{10}-1) \end{vmatrix}$ then the value of $\sum_{k=1}^{10} D_k$ is

(A) $2(\alpha+\beta+\gamma)$ (B) $\alpha\beta+\alpha\gamma+\beta\gamma$

(C) $\alpha\beta\gamma$ (D) 0

7. If M is a 3×3 matrix, where $M'M = I$ and $\det(M) = 1$, then $\det(M - I) =$

(A) 1 (B) 0

(C) –1 (D) None of these

8. If $[x]$ denotes the greatest integer less than or equal to x, then the value of the determinant

$$\begin{vmatrix} [e] & [\pi] & [\pi^2-6] \\ [\pi] & [\pi^2-6] & [e] \\ [\pi^2-6] & [e] & [\pi] \end{vmatrix}$$, then

(A) –8 (B) 8

(C) 0 (D) None of these

9. If $a_i, b_i, c_i \in R$ $(i = 1, 2, 3)$ and $x \in R$ and

$$\begin{vmatrix} a_1+b_1x & a_1x+b_1 & c_1 \\ a_2+b_2x & a_2x+b_2 & c_2 \\ a_3+b_3x & a_3x+b_3 & c_3 \end{vmatrix} = 0$$, then

(A) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 4$ (B) $x = \pm 1$

(C) $x = 2$ (D) None of these

10. The value of the determinant

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$$ is

(A) 0 (B) $\sin\theta$

(C) $\cos\theta$ (D) independent of θ

11. If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+2 & n^2+n \\ 2k-1 & n^2 & n^2+n+2 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 48$, then n equals

(A) 4 (B) 6
(C) 8 (D) None of these

12. If A, B, C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0,$$

then the triangle is a/an

(A) equilaterral (B) isosceles
(C) right-angled triangle (D) any triangle

13. If a_0, a_1 a_2, a_3, a_4 are in A.P with the common difference d, the value of $\begin{vmatrix} a_1a_2 & a_1 & a_0 \\ a_2a_3 & a_2 & a_1 \\ a_3a_4 & a_3 & a_2 \end{vmatrix}$ is

(A) $2d^4$ (B) $2d^3$
(C) $2d^2$ (D) $2d$

14. If α, β, γ are different from and are the roots of $ax^3 + bx^2 + cx + d = 0$ and

$(\beta - \gamma)(\gamma - \alpha)(\alpha - \beta) = \dfrac{25}{2}$, then the determinant

$$\Delta = \begin{vmatrix} \dfrac{\alpha}{1-\alpha} & \dfrac{\beta}{1-\beta} & \dfrac{\gamma}{1-\gamma} \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix}$$ equals

(A) $\dfrac{25d}{2a}$ (B) $\dfrac{25d}{a}$

(C) $\dfrac{-25d}{a+b+c+d}$ (D) None of these

15. Let $\{\Delta_1, \Delta_2, \Delta_3, ..., \Delta_k\}$ be the set of third order determinants that can be made with the distinct nonzero real numbers $a_1, a_2, a_3, ..., a_9$. Then

(A) $k = 9!$ (B) $\sum_{i=1}^{k} \Delta_i = 0$

(C) at least one $\Delta_i = 0$ (D) None of these

16. If $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$, a, b being positive integers, then

(A) constant term of $f(x)$ is 4
(B) coefficieent of x in $f(x)$ is 0
(C) constant term in $f(x)$ is $a - b$
(D) constant term in $f(x)$ is $a + b$

17. If $P = \begin{bmatrix} \dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \\ -\dfrac{1}{2} & \dfrac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP'$, then $p'Q^{2005}P$ is

(A) $\begin{bmatrix} 1 & 1 \\ 2005 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

18. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix}$

$= (x-y)(y-z)(z-x), \left(\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}\right)$ then $n =$

(A) –2 (B) –1
(C) 0 (D) 1

19. If α, β, γ are the roots of the equation $ax^3 + bx^2 + c = 0$, then the value of the determinant $\begin{vmatrix} \alpha\beta & \beta\gamma & \gamma\alpha \\ \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix}$

(A) a (B) b
(C) 0 (D) c

20. If $p + q + r = 0 = a + b + c$, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

(A) 0 (B) $pq + qb + rc$
(C) 1 (D) None of these

21. A determinant of second order is made with the elements 0 and 1. The number of determinants with non-negative values is

(A) 3 (B) 10
(C) 11 (D) 13

22. If $f_j = \sum_{i=0}^{2} a_{ij}x^i$, $j = 1, 2, 3$ and if f_j', f_j'' denote $\dfrac{df_j}{dx}, \dfrac{d^2 f_j}{dx^2}$ respectively, then $g(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix}$ is

(A) a cubic in x (B) a quadratic in x
(C) linear in x (D) a constant

23. The value of the determinant

$$\Delta = \begin{vmatrix} 2a_1b_1 & a_1b_2 + a_2b_1 & a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 & 2a_2b_2 & a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 & a_3b_2 + a_2b_3 & 2a_3b_3 \end{vmatrix} \text{ is}$$

(A) 1 (B) –1
(C) 0 (D) $a_1a_2a_3b_1b_2b_3$

24. If $A + B + C = \pi$, $e^{i\theta} = \cos\theta + i\sin\theta$ and

$$z = \begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix} \text{ then}$$

(A) $Re(z) = 4$ (B) $Im(z) = 0$
(C) $Re(z) = -4$ (D) $Im(z) = -1$

25. If $x_i = a_i b_i c_i$, $i = 1,2,3$, are theree-digit positive integers such that each x_i is a multiple of 19, then for some integer n, $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ is given by

(A) $19n + 1$ (B) $19n + 2$
(C) $19n$ (D) $19n + 3$

26. If the system of equations $ax + by + c = 0$, $bx + cy + a = 0$, $cx + ay + b = 0$ has a solution then the system of equations

$(b + c)x + (c + a)y + (a + b)z = 0$
$(c + a)x + (a + b)y + (b + c)z = 0$
$(a + b)x + (b + c)y + (c + a)z = 0$ has

(A) only one solution
(B) no solution
(C) infinite number of solutions
(D) None of these

27. $(b + c)(y + z) - ax = b - c$,
$(c + a)(z + x) - by = c - a$,
$(a + b)(x + y) - cz = a - b$,
where $a + b + c \neq 0$, then $x =$

(A) $\dfrac{c-b}{a+b+c}$ (B) $\dfrac{a-c}{a+b+c}$
(C) $\dfrac{b-a}{a+b+c}$ (D) $\dfrac{1}{a+b+c}$

28. The equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + mz = n$ give infinite number of values of the triplet (x, y, z) if

(A) $m = 3, n \in R$ (B) $m = 3, n \neq 10$
(C) $m = 3, n = 10$ (D) None of these

29. If $x \neq 0$, $y \neq 0$, $z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$, then $x^{-1} + y^{-1} + z^{-1}$ is equal to

(A) –1 (B) –2
(C) –3 (D) None of these

30. If $\Delta(x) = \begin{vmatrix} x & 1+x^2 & x^3 \\ \log(1+x^2) & e^x & \sin x \\ \cos x & \tan x & \sin^2 x \end{vmatrix}$ then

(A) $\Delta(x)$ is divisible by x (B) $\Delta(x) = 0$
(C) $\Delta'(x) = 0$ (D) None of these

31. The number of values of k for which the linear equations

$4x + ky + 2z = 0$
$kx + 4y + z = 0$
$2x + 2y + z = 0$

possess a non-zero solution is

(A) 0 (B) 3 (C) 2 (D) 1

32. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to:

(A) –2 (B) 1 (C) 0 (D) –1

33. The value of the determinant $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & 10 \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$ is equal to:

(A) $5\sqrt{3}(\sqrt{6} - 5)$ (B) $5\sqrt{3}(\sqrt{6} - \sqrt{5})$
(C) $5(\sqrt{6} - 5)$ (D) $\sqrt{3}(\sqrt{6} - \sqrt{5})$

34. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(A) 2 (B) –1 (C) 0 (D) 1

35. Let a, b, c be such that $b(a+c) \neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$, then the value of '$n$' is

(A) zero (B) any even integer
(C) any odd integer (D) any integer

36. Let A be a 2×2 matrix
Statement-1: adj (adj A) = A
Statement-2: $|\text{adj } A| = |A|$
(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
(C) Statement-1 is true, Statement-2 is false
(D) Statement-1 is false, Statement-2 is true

37. If $a, b, c, d > 0$; $x \in R$ and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)\,x + b^2 + c^2 + d^2 \le 0$,
then $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$

(A) 1 (B) –1
(C) 0 (D) None of these

38. The value of the determinant
$$\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix} \text{ is}$$
(A) $-5\sqrt{3}\,(5-\sqrt{6})$ (B) $-5\sqrt{3}\,(5+\sqrt{6})$
(C) $-5\sqrt{3}\,(\sqrt{6}-5)$ (D) None of these

39. If $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$ are three three-digit numbers, each of which is divisible by k, then
$$\Delta = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} \text{ is}$$
(A) divisible by k (B) divisible by k^2
(C) divisible by $2k$ (D) None of these

40. If the three-digit numbers $A28$, $3B9$ and $62C$, where A, B and C are integers between 0 and 9, are divisible by a fixed integer k, then the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is

(A) divisible by k (B) divisible by k^2
(C) divisible by $2k$ (D) None of these

41. The value of the determinant of nth order, being given by $\begin{vmatrix} x & 1 & 1 & \dots \\ 1 & x & 1 & \dots \\ 1 & 1 & x & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$, is

(A) $(x-1)^{n-1}(x+n-1)$
(B) $(x-1)^n(x+n-1)$
(C) $(1-x)^{n-1}(x+n-1)$
(D) None of these

42. The value of the determinant
$$\begin{vmatrix} \sqrt{x}+\sqrt{y} & 2\sqrt{z} & \sqrt{z} \\ \sqrt{yz}+\sqrt{2x} & z & \sqrt{2z} \\ y+\sqrt{xz} & \sqrt{yz} & z \end{vmatrix};$$
where x, y, z are positive real numbers, is
(A) $z\,(\sqrt{2}y - z\sqrt{y})$ (B) $y\,(\sqrt{2}z - y\sqrt{z})$
(C) $x\,(\sqrt{2}y - z\sqrt{y})$ (D) None of these

43. If $f_j = \sum_{i=0}^{2} a_{ij}x^i$, $j = 1, 2, 3$ and if f_j', f_j'' denote $\dfrac{df_j}{dx}, \dfrac{d^2f_j}{dx^2}$ respectively, then $g(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix}$ is

(A) a cubic in x (B) a quadratic in x
(C) linear in x (D) a constant

44. If $x_i = a_i\,b_ic_i$, $i = 1,2,3$, are theree-digit positive integers such that each x_i is a multiple of 19, then for some integer n, $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ is given by

(A) $19n+1$ (B) $19n+2$
(C) $19n$ (D) $19n+3$

45. $(b+c)(y+z) - ax = b-c$,
$(c+a)(z+x) - by = c-a$,
$(a+b)(x+y) - cz = a-b$,
where $a+b+c \neq 0$, then $x =$

(A) $\dfrac{c-b}{a+b+c}$ (B) $\dfrac{a-c}{a+b+c}$

(C) $\dfrac{b-a}{a+b+c}$ (D) $\dfrac{1}{a+b+c}$

46. If $x \neq 0, y \neq 0, z \neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 0$,

then $x^{-1} + y^{-1} + z^{-1}$ is equal to

(A) –1 (B) –2
(C) –3 (D) None of these

47. If $2s = a + b + c$ and $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} =$

$k(s-a)(s-b)(s-c)$, then k is equal to

(A) 2 (B) $2s$
(C) $2s^2$ (D) $2s^3$

48. Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Let $s_n = \alpha^n + \beta^n$ for $n \geq 1$. Then, the value of the determinant $\begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix}$ is

(A) $\dfrac{(a+b+c)(b^2-4ac)}{a^4}$

(B) $\dfrac{(a+b+c)^2(b^2-4ac)}{a^4}$

(C) $\dfrac{(a+b+c)^2(b^2-4ac)}{a^2}$

(D) None of these

49. The value of the determinant $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}$ is

(A) $a^2 + b^2 + c^2$
(B) $abc(a+b+c)$
(C) $(a^2+b^2+c^2)(a+b+c)$
(D) None of these

50. If $\begin{vmatrix} \operatorname{cosec}\alpha & 1 & 0 \\ 1 & 2\operatorname{cosec}\alpha & 1 \\ 0 & 1 & 2\operatorname{cosec}\alpha \end{vmatrix} = \frac{1}{2}\left(z^3 + \frac{1}{z^3}\right)$,

then z is equal to

(A) $\sin \alpha/2$ (B) $\cos \alpha/2$
(C) $\tan \alpha/2$ (D) None of these

51. If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = k(a-b)(b-c)$

$(c-a)$, then k is equal to

(A) 4 (B) – 4
(C) 2 (D) – 2

52. The value of the determinant

$\begin{vmatrix} (a-a_1)^{-2} & (a-a_1)^{-1} & a_1^{-1} \\ (a-a_2)^{-2} & (a-a_2)^{-1} & a_2^{-1} \\ (a-a_3)^{-2} & (a-a_3)^{-1} & a_3^{-1} \end{vmatrix}$ is

(A) $\dfrac{a^2\Pi(a_i - a_j)}{\pi a_i \Pi(a-a_i)^2}$ (B) $\dfrac{-a^2\Pi(a_i - a_j)}{\Pi a_i \Pi(a-a_i)^2}$

(C) $\dfrac{\Pi a_i \Pi(a-a_i)^2}{a^2\Pi(a_i - a_j)}$ (D) $-\dfrac{\Pi a_i \Pi(a-a_i)^2}{a^2\Pi(a_i - a_j)}$

53. If $\begin{vmatrix} \frac{1}{a+x} & \frac{1}{b+x} & \frac{1}{c+x} \\ \frac{1}{a+y} & \frac{1}{b+y} & \frac{1}{c+y} \\ \frac{1}{a+z} & \frac{1}{b+z} & \frac{1}{c+z} \end{vmatrix} = \frac{P}{Q}$, where Q is the product of denominators, then P is equal to

(A) $(a-b)(b-c)(c-a)$
(B) $(x-y)(y-z)(z-x)$
(C) $(a-b)(b-c)(c-a)(x-y)(y-z)(z-x)$
(D) None of these

54. If a, b, c, d are the roots of the equation $\alpha x^4 + \beta x^3 + \gamma x^2 + \delta x + \xi = 0$, then the value of the determinant

$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix}$ is

(A) $\dfrac{\delta - \gamma}{\alpha}$ (B) $\dfrac{\xi - \delta}{\alpha}$

(C) $\dfrac{\alpha - \beta}{\alpha}$ (D) $\dfrac{\beta - \alpha}{\alpha}$

55. The value of the determinant $\begin{vmatrix} 0 & x & y & z \\ -x & 0 & c & b \\ -y & -c & 0 & a \\ -z & -b & -a & 0 \end{vmatrix}$ is

(A) $(ax + by + cz)^2$ (B) $(ax - by + cz)^2$
(C) $(ax + by - cz)^2$ (D) None of these

56. The value of the determinant

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} \text{ is}$$

(A) $a^2b^2c^2$ (B) $2a^2b^2c^2$
(C) $4a^2b^2c^2$ (D) None of these

57. If $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ and $g(x) = (c_1 - x)$ $(c_2 - x)(c_3 - x)$, then $f(0)$ is equal to

(A) $\dfrac{bg(a) - ag(b)}{(b-a)}$ (B) $\dfrac{bg(a) + ag(b)}{(b+a)}$

(C) $\dfrac{bg(a) - ag(b)}{(b+a)}$ (D) $\dfrac{bg(a) + ag(b)}{(b-a)}$

58. If $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$ $= (a^3+b^3+c^3+kabc)^2$, then k is equal to

(A) 2 (B) – 2
(C) 3 (D) – 3

59. The value of the determinant

is $\begin{vmatrix} \beta\gamma & \beta\gamma' + \beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha' + \gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta' + \alpha'\beta & \alpha'\beta' \end{vmatrix}$

(A) $(\alpha\beta' - \alpha'\beta)(\beta\gamma' - \beta'\gamma)(\gamma\alpha' - \gamma'\alpha)$
(B) $\alpha\beta\gamma(\alpha+\beta+\gamma)(\alpha'+\beta'+\gamma')$
(C) $\alpha'\beta'\gamma'(\alpha+\beta+\gamma)(\alpha'+\beta'+\gamma)$
(D) None of these

60. If $a \neq 0$, $a \neq 1$ and

$$\begin{vmatrix} x+1 & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{vmatrix} = a^3 + f(x)\cdot a(a^2+a+1), \text{ then}$$

(A) $f(x) = x$ (B) $f(x) = x^2$
(C) $f(x) = x^3$ (D) None of these

61. The value of the determinant

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} \text{ is}$$

(A) $(a^2+b^2+c^2)^3$
(B) $(ab+bc+ca)^3$
(C) $(a^2+b^2+c^2)(ab+bc+ca)^2$
(D) None of these

62. If $\begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix} = 0$ and $x\,(\neq 0) \in R$ then x is equal to

(A) $a^2+b^2+c^2$ (B) $-(a^2+b^2+c^2)$
(C) $2(a^2+b^2+c^2)$ (D) None of these

63. The values of m for which the system of equations $3x + my = m$ and $2x - 5y = 20$ has a solution satisfying the condition $x > 0$, $y > 0$, are

(A) $m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (0, \infty)$

(B) $m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (30, \infty)$

(C) $m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (0, 30)$

(D) None of these

64. If $a = \cos\theta + i\sin\theta$, $b = \cos 2\theta - i\sin 2\theta$, $c = \cos 3\theta + i\sin 3\theta$ and if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then θ is equal to

(A) $n\pi$ (B) $2n\pi$
(C) $(2n+1)\dfrac{\pi}{2}$ (D) None of these

65. The value of the determinant

$$\begin{vmatrix} \dfrac{1}{a} & \dfrac{1}{a(a+d)} & \dfrac{1}{(a+d)(a+2d)} \\ \dfrac{1}{a+d} & \dfrac{1}{(a+d)(a+2d)} & \dfrac{1}{(a+2d)(a+3d)} \\ \dfrac{1}{a+2d} & \dfrac{1}{(a+2d)(a+3d)} & \dfrac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

where $a, d > 0$, is

(A) $-\dfrac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$

(B) $\dfrac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$

(C) $\dfrac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)^2}$

(D) None of these

66. The value of the determinant

$$\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix} \text{ is}$$

(A) $2(ab+bc+ca)^3$ (B) $(ab+bc+ca)^3$
(C) $4(ab+bc+ca)^3$ (D) None of these

67. If the equations $(a+1)^3x+(a+2)^3y=(a+3)^3$, $(a+1)x+(a+2)y=a+3$, $x+y=1$ are consistent then a is equal to
(A) 1 (B) – 1
(C) 2 (D) – 2

68. If the system of equations $x\sin\alpha+y\sin\beta+z\sin\gamma=0$, $x\cos\alpha+y\cos\beta+z\cos\gamma=0$, $x+y+z=0$, where α, β, γ are angles of a triangle, have a non-trivial solution, then the triangle must be
(A) isosceles (B) equilateral
(C) right angled (D) None of these

69. If $x_1 \neq 0, x_2 \neq 0, x_3 \neq 0$, then the determinant

$$\begin{vmatrix} x_1+a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & x_2+a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & x_3+a_3b_3 \end{vmatrix} \text{ is equal to}$$

(A) $x_1x_2x_3\left(1+\frac{a_1b_1}{x_1}+\frac{a_2b_2}{x_2}+\frac{a_3b_3}{x_3}\right)$

(B) $-x_1x_2x_3\left(1+\frac{a_1b_1}{x_1}+\frac{a_2b_2}{x_2}+\frac{a_3b_3}{x_3}\right)$

(C) $x_1x_2x_3\left(1-\frac{a_1b_1}{x_1}-\frac{a_2b_2}{x_2}-\frac{a_3b_3}{x_3}\right)$

(D) None of these

70. If $\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix}=0$, then
(A) $a+d=0$
(B) $d=0$
(C) $d=0$ or $a+d=0$
(D) None of these

71. Let $\begin{vmatrix} x+3 & x+2 & (x+2)^3 \\ x+2 & x+3 & (x+2)^3 \\ (x+2)^3 & x+2 & x+3 \end{vmatrix}$ $=ax^7+bx^6+cx^5+dx^4+ex^3+fx^2+gx+h$ be an identity in x, where a, b, c, d, e, f, g, h are independent of x, then the value of g is
(A) – 213 (B) 213
(C) 0 (D) None of these

72. If $\begin{vmatrix} x^n & y^n & z^n \\ x^{n+2} & y^{n+2} & z^{n+2} \\ x^{n+3} & y^{n+3} & z^{n+3} \end{vmatrix}$

$=(x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ then

(A) $n=1$ (B) $n=-1$
(C) $n=2$ (D) $n=-2$

73. The value of the determinant

$$\begin{vmatrix} \sin\alpha\cos\beta & \cos\alpha\cos\beta & -\sin\alpha\sin\beta \\ \sin\alpha\sin\beta & \cos\alpha\sin\beta & \sin\alpha\cos\beta \\ \cos\alpha & -\sin\alpha & 0 \end{vmatrix} \text{ is}$$

(A) is independent of α
(B) independent of β
(C) independent of α and β
(D) None of these

74. If α, β, γ are the roots of the equation $x^3+px+q=0$, then the value of the determinant

$$\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix} \text{ is}$$

(A) p^2-2q
(B) $3pq$
(C) $p-q$
(D) None of these

75. The value of a determinant of third order whose all elements are 1 or – 1 is
(A) an even number
(B) an odd number
(C) a prime number
(D) cannot be determined

76. If square matrices A and B are such that $AA^\theta = A^\theta A$, $BB^\theta = B^\theta B$ and $AB^\theta = B^\theta A$, then $(AB)(AB)^\theta$ is equal to
(A) $B^\theta A^\theta AB$
(B) $BA^\theta AB$
(C) $BA^\theta AB^\theta$
(D) None of these

77. Let $\Delta(x) = \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E.$

Then, the value of $5A + 4B + 3C + 2D + E$ is equal to

(A) 9 (B) – 9 (C) 11 (D) – 11

78. If $\Delta_1 =$

$$\begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4z^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2z^3(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix}$$

and, $\Delta_2 = \begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix}$, then $\Delta_1\Delta_2 =$

(A) Δ^2_2 (B) Δ^3_2

(C) Δ^4_2 (D) None of these

79. If $abc = \gamma$, $A = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$ and $AA' = I$, then a, b, c are the roots of the equation.

(A) $x^3 \pm x^2 + \gamma = 0$

(B) $x^3 \pm 2x^2 + \gamma = 0$

(C) $x^3 \pm x^2 - \gamma = 0$

(D) $x^3 \pm 2x^2 - \gamma = 0$

80. If a, b, c are the sides of a triangle ABC such that

$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$, then ΔABC is

(A) a right angled triangle

(B) an isosceles triangle

(C) an equilateral triangle

(D) None of these

81. The set of equations : $\lambda x - y + (\cos\theta)z = 0$; $3x + y + 2z = 0$; $(\cos\theta)x + y + 2z = 0$, $0 \le \theta < 2\pi$, has non-trivial solutions.

(A) for no values of λ and θ

(B) for all values of λ and θ

(C) for all values of λ and only two values of θ

(D) for only one value of λ and all values of θ

More than One Option Correct Type

82. The value of λ for which the equations $x + y - 3 = 0$, $(1 + \lambda)x + (2 + \lambda)y - 8 = 0$, $x - (1 + \lambda)y + (2 + \lambda) = 0$ are consistent is

(A) 1 (B) 5/3

(C) – 5/3 (D) None of these

83. Let $\{\Delta_1, \Delta_2, \Delta_3, ..., \Delta_k\}$ be the set of third order determinants that can be made with the distinct non-zero real numbers $a_1, a_2, a_3, ..., a_9$. Then,

(A) $k = 9!$ (B) $\sum_{i=1}^{k} \Delta_i = 0$

(C) at least one $\Delta_i = 0$ (D) None of these

84. If $A + B + C = \pi$, $e^{i\theta} = \cos\theta + i\sin\theta$ and

$z = \begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}$ then

(A) $Re(z) = 4$ (B) $Im(z) = 0$

(C) $Re(z) = -4$ (D) $Im(z) = -1$

85. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax + B$, then

(A) $A = \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & 3 \\ 4 & 0 & 2 \end{vmatrix}$ (B) $B = \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & 3 \\ 4 & 0 & -1 \end{vmatrix}$

(C) $A = \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & -3 \\ 4 & 0 & 2 \end{vmatrix}$ (D) $B = \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & -3 \\ 4 & 0 & -1 \end{vmatrix}$

86. If $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$, $a \ne b \ne c$, then

(A) $x = 0$ if $b(a + c) \le ac$

(B) $x = \pm\sqrt{b(a+c)-ac}$ if $b(a + c) > ac$

(C) $x = 0, \pm\sqrt{b(a+c)-ac}$ if $b(a + c) > ac$

(D) None of these

87. If $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \begin{vmatrix} \alpha^2 & \beta^2 & \beta^2 \\ \beta^2 & \alpha^2 & \beta^2 \\ \beta^2 & \beta^2 & \alpha^2 \end{vmatrix}$,

then

(A) $\alpha^2 = a^2 + b^2 + c^2$ (B) $\beta^2 = ab + bc + ca$
(C) $\alpha^2 = ab + bc + ca$ (D) $\beta^2 = a^2 + b^2 + c^2$

88. The determinant $\begin{vmatrix} \sin x & \sin y & \sin z \\ \cos x & \cos y & \cos z \\ \cos^3 x & \cos^3 y & \cos^3 z \end{vmatrix}$; $0 < x, y,$ $z < \frac{\pi}{2}$, is equal to zero if

(A) $x = y$ (B) $y = z$
(C) $z = x$ (D) $x + y + z = \frac{\pi}{2}$

89. The value of the determinant

$\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$ is

(A) independent of θ for all $\lambda \in$ R
(B) independent of θ and α when $\lambda = 1$
(C) independent of θ and α when $\lambda = -1$
(D) None of these

90. The value of θ lying between $\theta = 0$ and $\theta = \frac{\pi}{2}$ and satisfying the equation

$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$ is

(A) $\frac{7\pi}{24}$ (B) $\frac{5\pi}{24}$
(C) $\frac{11\pi}{24}$ (D) $\frac{\pi}{24}$

91. If $a_n = \int_0^{\pi/2} \frac{1-\cos 2nx}{1-\cos 2x}\,dx$, then

(A) a_{n+1} is A.M. between a_n and a_{n+2}
(B) a_{n+1} is G.M between a_n and a_{n+2}
(C) a_{n+1} is H.M. between a_n and a_{n+2}
(D) $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = 0$

92. If α, β, γ are non-zero real numbers such that

$\begin{vmatrix} \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \\ \alpha\beta & \beta\gamma & \gamma\alpha \end{vmatrix} = 0$, then

(A) $\frac{1}{\gamma} + \frac{1}{\alpha\omega} + \frac{1}{\beta\omega^2} = 0$

(B) $\frac{1}{\beta} + \frac{1}{\alpha\omega} + \frac{1}{\gamma\omega^2} = 0$

(C) $\frac{1}{\beta} + \frac{1}{\gamma\omega} + \frac{1}{\alpha\omega^2} = 0$

(D) $(\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3 = 3\alpha^2\beta^2\gamma^2$

93. The positive integral solutions of the equation $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 30$ are

(A) (3, 1, 1) (B) (1, 3, 1)
(C) (1, 1, 3) (D) (– 1, 1, 3)

94. If $f(x) = \begin{vmatrix} e^x & \sin x & 1 \\ \cos x & \log(1+x^2) & 1 \\ x & x^2 & 1 \end{vmatrix} = a + bx + cx^2$, then

(A) $a = 0$ (B) $a = 1$
(C) $b = -1$ (D) $b = -2$

95. If maximum and minimum values of the determinant $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ are α and β, then

(A) $\alpha + \beta^{99} = 4$
(B) $\alpha^3 - \beta^{17} = 26$
(C) $(\alpha^{2n} - \beta^{2n})$ is always an even integer for $n \in N$
(D) a triangle can be constructed having its sides as $\alpha - \beta$, $\alpha + \beta$ and $\alpha + 3\beta$

Passage Based Questions

Passage I

Let $A = [a_{ij}]$ be an $n \times n$ matrix. The matrix $A - \lambda I$ is called the characteristics matrix of A, where λ is a scalar and I is the identity matrix. The determinant $|A - \lambda I|$ is a non-null polynomial of degree n in λ and is called the characteristic polynomial of A. The equation $|A - \lambda I| = 0$ is called the characteristic equation of A and its roots are called the characteristic roots or latent roots or eigen values of A. The set of all eigenvalues of the matrix A is called the spectrum of A. The product of the eigenvalues of a matrix A is equal to the determinant A.

96. The characteristic roots of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ are

(A) 1 (B) 2
(C) -2 (D) 3

97. The given values of the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ are

(A) $4, -2, -2$, (B) $-4, 2, -2$
(C) $-4, 2, 2$ (D) $4, -4, 2$

98. Which of the following statements are true?

If A is any $n \times n$ matrix and λ is a characteristic root of A, then

(A) A and A' have the same characteristic roots
(B) $k\lambda$ is a characteristic root of kA (k being scalar)
(C) λ^n is a characteristic root of A^n (n being positive integer)
(D) $\dfrac{1}{\lambda}$ is a characteristic root of A^{-1}

99. Which of the following statements are correct?

(A) If A, B are n rowed square matrices and A is non-singular, then $A^{-1}B$ and BA^{-1} has same char-acter-istic roots.
(B) If A and P are square matrices of same order and P is non-singular, then A and $P^{-1}AP$ have same characteristic roots.
(C) If A and B be two square matrices of same order, then AB and BA have same characteristic roots.
(D) All of these

Match the Column Type

100. If $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = px^5 + qx^4 + rx^3 + sx^2 + tx + w$, then

Column-I	Column-II
I. w is equal to	(A) 3
II. t is equal to	(B) 1
III. $p + r$ is equal to	(C) -1
IV. $q + s$ is equal to	(D) 0

Assertion-Reason Type

Instructions: *In the following questions an Assertion (A) is given followed by a Reason. (R). Mark your responses from the following options:*

(A) Assertion(A) is True and Reason(R) is True; Reason(R) is a correct explanation for Assertion(A)
(B) Assertion(A) is True, Reason(R) is True; Reason(R) is not a correct explanation for Assertion(A)
(C) Assertion(A) is True, Reason(R) is False
(D) Assertion(A) is False, Reason(R) is True

101. **Assertion:** If a, b, c are different, then the value of x satisfying $\begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix} = 0$ is 0

Reason: Determinant of a skew-symmetric matrix of odd order is zero.

102. **Assertion:** Let λ and α be real. The set of all values of λ for which the system of linear equations

$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$
$x + (\cos\alpha)y + (\sin\alpha)z = 0$
$-x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution, is

$[-\sqrt{2}, \sqrt{2}]$

Reason: The equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + n_3y + c_3z = 0$ have a non-trivial solution if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

103. **Assertion:** Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then $\frac{b^2}{q^2} = \frac{ac}{pr}$

Reason: The equations $a_1x + b_1y = 0$, $a_2x + b_2y = 0$ have a non-trivial solution if $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$.

104. **Assertion:** a, b, c are in G.P. with common ratio r_1 and α, β, γ are in G.P. with common ratio r_2. If the equations $ax + \alpha y + z = 0$, $bx + \beta y + z = 0$, $cx + \gamma y + z = 0$ have only trivial solution, then $r_1 \neq r_2$, $r_1, r_2 \neq 1$.

Reason: The equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$ have only trivial solution if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

105. **Assertion:** If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, then $abc > -8$

Reason: A. M. > G. M.

106. **Assertion:** If $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$, then $f(0) = \frac{b\,g(a) - a\,g(b)}{b-a}$, where $g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$

Reason: $f(x)$ is linear is x.

Previous Year's Questions

107. l, m, n are the pth, qth and rth term of an GP and all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals **[2002]**

(A) 3 (B) 2
(C) 1 (D) Zero

108. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then **[2002]**

(A) $x = 3, y = 1$
(B) $x = 1, y = 3$
(C) $x = 0, y = 3$
(D) $x = 0, y = 0$

109. If ($\omega \neq 1$) is a cubic root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$ equals **[2002]**

(A) Zero
(B) 1
(C) i
(D) ω

110. If the system of linear equations **[2003]**

$x + 2ay + az = 0$
$x + 3by + bz = 0$
$x + 4cy + cz = 0$

has a non-zero solution, then a, b, c

(A) are in A. P.
(B) are in G.P.
(C) are in H.P.
(D) satisfy $a + 2b + 3c = 0$

111. If 1, ω, ω^2 are the cube roots of unity, then $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ (where, n is not a multiple of 3) is equal to **[2003]**

(A) 0 (B) 1 (C) ω (D) ω^2

112. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the value of the determinant **[2004]**

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$

(A) 0 (B) –2 (C) 2 (D) 1

113. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is **[2007]**

(A) divisible by neither x not y
(B) divisible by both x and y
(C) divisible by x but not y
(D) divisible by y but not x

114. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to **[2008]**

(A) 2 (B) – 1 (C) 0 (D) 1

115. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? **[2008]**

(A) If detA = ± 1, then A^{-1} exists but all its entries are not necessarily integers
(B) If detA ≠ ± 1, then A^{-1} exists and all its entries are non-integers
(C) If detA = ± 1, then A^{-1} exists and all its entries are integers
(D) If detA = ± 1, then A^{-1} need not exist

116. Let a, b, c be such that $b(a + c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of 'n' is **[2009]**

(A) Zero
(B) any even integer
(C) any odd integer
(D) any integer

117. Consider the following system of linear equations: **[2010]**

$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$

The system has
(A) exactly 3 solutions
(B) a unique solution
(C) no solution
(D) infinite number of solutions

118. The number of values of k for which the homogeneous system of linear equations

$4x + ky + 2z = 0$; $kx + 4y + z = 0$; $2x + 2y + z = 0$

possess a non-zero solution is **[2011]**

(A) 2 (B) 1
(C) Zero (D) 3

119. Let P and Q be 3 by 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to **[2012]**

(A) –2 (B) 1 (C) 0 (D) –1

120. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to **[2013]**

(A) 11 (B) 5 (C) 0 (D) 4

121. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2,$$

then K isequal to **[2014]**

(A) $\alpha\beta$ (B) $\frac{1}{\alpha\beta}$ (C) 1 (D) –1

122. The set of all values of λ for which the system of linear equations **[2015]**

$2x_1 - 2x_2 + x_3 = \lambda x_1$
$2x_1 - 3x_2 + 2x_3 = \lambda x2_1$
$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,
(A) is a singleton.
(B) contains two elements.
(C) contains more than two elements.
(D) is an empty set.

123. The system of linear equations **[2016]**

$$x+\lambda y-z=0$$
$$\lambda x-y-z=0$$
$$x+y-\lambda z=0$$

has a non-trivial solution for:

(A) exactly three values of λ.
(B) infinitely many values of λ.
(C) exactly one value of λ.
(D) Exactly two values of λ.

ANSWER KEYS

Single Option Correct Type

1. (A)	**2.** (C)	**3.** (B)	**4.** (C)	**5.** (A)
6. (D)	**7.** (B)	**8.** (A)	**9.** (B)	**10.** (A, D)
11. (A)	**12.** (B)	**13.** (A)	**14.** (D)	**15.** (A, B)
16. (B)	**17.** (B)	**18.** (B)	**19.** (C)	**20.** (A)
21. (D)	**22.** (D)	**23.** (C)	**24.** (B, C)	**25.** (C)
26. (C)	**27.** (A)	**28.** (C)	**29.** (C)	**30.** (A)
31. (C)	**32.** (C)	**33.** (A)	**34.** (D)	**35.** (C)
36. (B)	**37.** (C)	**38.** (A)	**39.** (A)	**40.** (A)
41. (A)	**42.** (A)	**43.** (D)	**44.** (C)	**45.** (A)
46. (C)	**47.** (D)	**48.** (B)	**49.** (C)	**50.** (C)
51. (B)	**52.** (B)	**53.** (C)	**54.** (B)	**55.** (B)
56. (C)	**57.** (A)	**58.** (D)	**59.** (A)	**60.** (A)
61. (B)	**62.** (B)	**63.** (B)	**64.** (B)	**65.** (B)
66. (A)	**67.** (D)	**68.** (A)	**69.** (A)	**70.** (C)
71. (A)	**72.** (B)	**73.** (B)	**74.** (C)	**75.** (A)
76. (A)	**77.** (D)	**78.** (B)	**79.** (A)	**80.** (B)
81. (A)				

More than One Option Correct Type

82. (A, C)	**83.** (A, B)	**84.** (B, C)	**85.** (A, D)	**86.** (A, C)
87. (A, B)	**88.** (A, B, C, D)	**89.** (A, C)	**90.** (A, C)	**91.** (A, D)
92. (A, B, C, D)	**93.** (A, B, C)	**94.** (A, C)	**95.** (A, B, C)	

Passage Based Questions

96. (A, C, D)	**97.** (A)	**98.** (A, B, C, D)	**99.** (D)

Match the Column Type

100. I $\rightarrow$ (B), II $\rightarrow$ (A), III $\rightarrow$ (C), IV $\rightarrow$ (B)

Assertion-Reason Type

101. (A)	**102.** (A)	**103.** (A)	**104.** (A)	**105.** (A)
106. (A)				

Previous Year's Questions

107. (D)	**108.** (D)	**109.** (A)	**110.** (C)	**111.** (A)	**112.** (A)	**113.** (B)	**114.** (D)	**115.** (C)	**116.** (C)
117. (C)	**118.** (A)	**119.** (C)	**120.** (A)	**121.** (C)	**122.** (B)	**123.** (A)			

HINTS AND SOLUTIONS

Single Option Correct Type

1. Since α_1, α_2 and β_1, β_2 are the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively, therefore

$$\alpha_1 + \alpha_2 = \frac{-b}{a}, \alpha_1\alpha_2 = \frac{c}{a} \quad (1)$$

$$\text{and } \beta_1 + \beta_2 = \frac{-q}{p}, \beta_1\beta_2 = \frac{r}{p} \quad (2)$$

Since the given system of equations has a non-trivial solution

$$\therefore \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0 \text{ i.e., } \alpha_1\beta_2 - \alpha_2\beta_1 = 0$$

$$\text{or} \quad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \sqrt{\frac{\alpha_1\alpha_2}{\beta_1\beta_2}}$$

$$\Rightarrow \frac{pb}{qa} = \sqrt{\frac{pc}{ra}} \Rightarrow \frac{b^2}{q^2} = \frac{ac}{pr}$$

The correct option is (A)

2. Since a, b, c are in G. P. with common ratio r_1 and α, β, γ are in G. P. with common ratio r_2, therefore $a \neq 0$, $\alpha \neq 0$, $b = ar_1$, $c = ar_1^2$, $\beta = \alpha r_2$, $\gamma = \alpha r_2^2$

Also, the system of equations have only trivial solution, so

$$\begin{vmatrix} a & \alpha & 1 \\ b & \beta & 1 \\ c & \gamma & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow \begin{vmatrix} a & \alpha & 1 \\ ar_1 & \alpha r_2 & 1 \\ ar_1^2 & \alpha r_2^2 & 1 \end{vmatrix} \neq 0 \Rightarrow a\alpha \begin{vmatrix} 1 & 1 & 1 \\ r_1 & r_2 & 1 \\ r_1^2 & r_2^2 & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow a\alpha \begin{vmatrix} 1 & 0 & 0 \\ r_1 & r_2 - r_1 & 1 - r_1 \\ r_1^2 & r_2^2 - r_1^2 & 1 - r_1^2 \end{vmatrix} \neq 0$$

[Applying $C_2 \to C_2 - C_1$ $C_3 \to C_3 - C_1$]

$$\Rightarrow a\alpha(r_2 - r_1)(1 - r_1) \begin{vmatrix} 1 & 0 & 0 \\ r_1 & 1 & 1 \\ r_1^2 & r_2 + r_1 & 1 + r_1 \end{vmatrix} \neq 0$$

$$\Rightarrow a\alpha(r_2 - r_1)(1 - r_1)(1 - r_2) \neq 0$$

$$\Rightarrow r_1 \neq r_2, r_1 \neq 1, r_2 \neq 1$$

The correct option is (C)

3. We know that $\Delta^c = \Delta^{3-1} = \Delta^2 = (11)^2 = 121$.

The correct option is (B)

4. Since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are p^{th}, q^{th} and r^{th} terms of an A.P.

$$\Rightarrow \frac{1}{a} = A + (p-1)D$$

$$\frac{1}{b} = A + (q-1)D$$

$$\frac{1}{c} = A + (r-1)D$$

where A is the first term and D is the common difference.

$$\therefore \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = abc \begin{vmatrix} \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= abc \begin{vmatrix} A+(p-1)D & A+(q-1)D & A+(r-1)D \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= abc \begin{vmatrix} 0 & 0 & 0 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

[Applying $R_1 \to R_1 - (A - D)R_3 - DR_2$]

$$= 0$$

The correct option is (C)

5. We have,

$$\begin{vmatrix} \sqrt{x} + \sqrt{y} & 2\sqrt{z} & \sqrt{z} \\ \sqrt{yz} + \sqrt{2x} & z & \sqrt{2z} \\ y + \sqrt{xz} & \sqrt{yz} & z \end{vmatrix}$$

$$= z \begin{vmatrix} \sqrt{x} + \sqrt{y} & 2 & 1 \\ \sqrt{yz} + \sqrt{2x} & \sqrt{z} & \sqrt{2} \\ y + \sqrt{xz} & \sqrt{y} & \sqrt{z} \end{vmatrix}$$

(Taking $\sqrt{z}$ common from C_2 and C_3)

$$= z \begin{vmatrix} -\sqrt{y} & 2 & 1 \\ 0 & \sqrt{z} & \sqrt{2} \\ 0 & \sqrt{y} & \sqrt{z} \end{vmatrix}$$

(Applying $C_1 \to C_1 - \sqrt{y}\ C_2 - \sqrt{x}\ C_3$)

$$= -\sqrt{y} \cdot z(z - \sqrt{2y}) = z(\sqrt{2}\ y - z\sqrt{y}).$$

The correct option is (A)

6. We have,

$$\sum_{k=1}^{10} D_k = \begin{vmatrix} \alpha & \beta & \gamma \\ 2\sum_{k=1}^{10} 3^k & 16 \cdot \sum_{k=1}^{10} 9^k & 26 \cdot \sum_{k=1}^{10} 27^k \\ 3^{10} - 1 & 2(9^{10} - 1) & (27^{10} - 1) \end{vmatrix}$$

$$= \begin{vmatrix} \alpha & \beta & \gamma \\ 2\left(\frac{3^{10}-1}{3-1}\right) & 16\cdot\left(\frac{9^{10}-1}{9-1}\right) & 26\cdot\left(\frac{27^{10}-1}{27-1}\right) \\ 3^{10}-1 & 2(9^{10}-1) & (27^{10}-1) \end{vmatrix}$$

$$= \begin{vmatrix} \alpha & \beta & \gamma \\ 3^{10}-1 & 2(9^{10}-1) & (27^{10}-1) \\ 3^{10}-1 & 2(9^{10}-1) & (27^{10}-1) \end{vmatrix} = 0$$

($\because R_2$ and R_3 are identical)

The correct option is (D)

7. As, $M'M = I$ and $|M| = 1$

$\Rightarrow \quad |M'M| = |I|$ or $|M'M| = |M| \qquad$ (as $|I| = 1 = |M|$)

$\Rightarrow \quad |M'||M| - |M| = 0 \Rightarrow |M|(|M'| - 1) = 0$

$\Rightarrow \quad |M| = 0$ or $|M'| = 1$

$\Rightarrow \quad |M'| = 1 \qquad (\because |M| = 1)$

$\therefore \quad |M - I| = |M - I||M'| = |MM' - M'|$

$\qquad = |I - M'| = -|M' - I| = -|M - I|'$

$\Rightarrow \quad |M - I| + |M - I| = 0 \Rightarrow |M - I| = 0$

The correct option is (B)

8. Since $2 < e < 3$, $3 < \pi < 4$ and $3 < \pi^2 - 6 < 4$, the given determinant reduces to

$$\begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix} = -8$$

The correct option is (A)

9. Clearly $x = \pm 1$ satisfies the given equation.

The correct option is (B)

10. The given determinant

$$= \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ 2\sin\theta\cos\frac{2\pi}{3} & 2\cos\theta\cos\frac{2\pi}{3} & 2\sin 2\theta\cos\frac{4\pi}{3} \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix}$$

[Applying $R_2 \to R_2 + R_3$]

$$= \begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix} = 0,$$

which is independent of θ.

The correct option is (A, D)

11. $$\sum_{k=1}^{n} D_k = \begin{vmatrix} \sum_{k=1}^{n} 1 & n & n \\ \sum_{k=1}^{n} 2k & n^2+n+2 & n^2+n \\ \sum_{k=1}^{n} 2k-1 & n^2 & n^2+n+2 \end{vmatrix}$$

$$\Rightarrow \quad 48 = \begin{vmatrix} n & n & n \\ n^2+n & n^2+n+2 & n^2+n \\ n^2 & n^2 & n^2+n+2 \end{vmatrix}$$

$$= \begin{vmatrix} n & 0 & n \\ n^2+n & 2 & n^2+n \\ n^2 & 0 & n^2+n+2 \end{vmatrix} \quad C_2 \to C_2 - C_1$$

$= 2(n^3 + n^2 + 2n - n^3) = 2(n^2 + 2n)$

$\Rightarrow \quad 24 = n^2 + 2n \Rightarrow 25 = (n+1)^2$

$\Rightarrow n + 1 = 5 \qquad (\because n \in N)$

$\therefore \quad n = 4$

The correct option is (A)

12. The given determinant

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix}$$

Operate $R_3 \to R_3 + R_1 – R_2$, $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \therefore \Delta = 0 \Rightarrow (a-b)(b-c)(c-a) = 0$$

$\Rightarrow a = b$ or $b = c$ or $c = a$, i.e., the triangle is isosceles.

The correct option is (B)

13. The given determinant $= \begin{vmatrix} a_1a_2 & a_1 & a_0 \\ a_2 2d & d & d \\ a_3 2d & d & d \end{vmatrix}$

(Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$)

$$\begin{vmatrix} a_1a_2 & a_1 & a_0 \\ 2a_2 & 1 & 1 \\ 2a_3 & 1 & 1 \end{vmatrix} = d^2 \begin{vmatrix} a_1a_2 & a_1 & a_0 \\ 2a_2 & 1 & 1 \\ 2d & 0 & 0 \end{vmatrix} = d^2$$

(Applying $R_3 \to R_3 - R_2$)

$= 2d^3(a_1 – a_0) = 2d^4$

The correct option is (A)

14. Taking α, β, γ common from C_1, C_2, C_3 respectively, we get

$$\Delta = \alpha\beta\gamma\begin{vmatrix} \frac{1}{1-\alpha} & \frac{1}{1-\beta} & \frac{1}{1-\gamma} \\ 1 & 1 & 1 \\ \alpha & \beta & \gamma \end{vmatrix}$$

$$= \alpha\beta\gamma\begin{vmatrix} \frac{1}{1-\alpha} & \frac{1}{1-\beta}-\frac{1}{1-\alpha} & \frac{1}{1-\gamma}-\frac{1}{1-\alpha} \\ 1 & 0 & 0 \\ \alpha & \beta-\alpha & \gamma-\alpha \end{vmatrix}$$

(using $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$)

$$= \frac{\alpha\beta\gamma(-1)(\beta-\alpha)(\gamma-\alpha)}{(1-\alpha)(1-\beta)(1-\gamma)}\begin{vmatrix} 1-\gamma & 1-\beta \\ 1 & 1 \end{vmatrix}$$

$$= \frac{\alpha\beta\gamma(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}{(1-\alpha)(1-\beta)(1-\gamma)}$$

As α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$,

$ax^3 + bx^2 + cx^2 + d = a(x-\alpha)(x-\beta)(x-\gamma)$

and $\alpha\beta\gamma = -d/a$

Thus, $\Delta = \dfrac{(-d/a)(25/2)}{(a+b+c+d)/a} = -\dfrac{25d}{2(a+b+c+d)}$

The correct option is (D)

15. $\begin{pmatrix}\text{The number} \\ \text{of third-order} \\ \text{determinants}\end{pmatrix} = \begin{pmatrix}\text{The number of} \\ \text{arrangements of} \\ \text{nine different} \\ \text{numbers in} \\ \text{nine places}\end{pmatrix} = 9!$

Now $\sum_{i=1}^{k} \Delta_i = \Delta_1 + \Delta_2 + \Delta_3 + \ldots + \Delta_k$

$$\Rightarrow \sum_{i=1}^{k} \Delta_i = \begin{vmatrix} \Sigma a_i & \Sigma a_i & \Sigma a_i \\ \Sigma a_i & \Sigma a_i & \Sigma a_i \\ \Sigma a_i & \Sigma a_i & \Sigma a_i \end{vmatrix} = 0$$

The correct option is (A, B)

16. Let $\begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$

$= A + Bx + Cx^2 + \ldots.$

Putting $x = 0$, we get

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Now differentiating both sides w.r.t. x and putting $x = 0$, we get

$$B = \begin{vmatrix} a & 2b & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 2b \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2b & 0 & a \end{vmatrix} = 0$$

Hence, coefficient of x is 0.

The correct option is (B)

17. We have,

$$P'P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad P'P = I$ or $P' = P^{-1}$

As, $\quad Q = PAP'$

$\therefore \quad P'Q^{2005}P = P'[(PAP')(PAP') \ldots 2005 \text{ times}]P$

$= \underbrace{(P'P)A(P'P)A(P'P)\ldots(P'P)A(P'P)}_{2005 \text{ times}}$

$= IA^{2005} = A^{2005}$

Now, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \ldots A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$\therefore P'Q^{2005}P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

The correct option is (B)

18. The degree of LHS (determinant) in $x, y, z = 3n + 5$

The degree of the expression in the RHS $= 2 \Rightarrow 3n + 5 = 2 \Rightarrow n = -1$

The correct option is (B)

19. $\begin{vmatrix} \alpha\beta & \beta\gamma & \gamma\alpha \\ \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix} = (\alpha\beta + \beta r + r\alpha)\begin{vmatrix} 1 & \beta\gamma & \gamma\alpha \\ 1 & \gamma\alpha & \alpha\beta \\ 1 & \alpha\beta & \beta\gamma \end{vmatrix}$

(Applying $C_1 \to C_1 + C_2 + C_3$)

From the given equation, $\alpha\beta + \beta\gamma + \gamma\alpha = 0$.

Thus, the value of the given eterminat is 0.

The correct option is (C)

20. Since $p + q + r = 0 = a + b + c$ (given)

$\Rightarrow p^3 + q^3 + r^3 = 3pqr$ or $a^3 + b^3 + c^3 = 3\,abc$

Let $\Delta = \begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$

$\Rightarrow \Delta = pqr\,(a^3 + b^3 + c^3) - abc\,(p^3 + q^3 + r^3)$

$\therefore\ \Delta = pqr\,(3\,abc) - abc(3pqr) = 0$

The correct option is (A)

21. There are only three determinants of second order with negative value, $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$

Number of possible determinants with elements 0 and 1are $2^4 = 16$.

Therefore, number of determinants with non-negative values is 13.

The correct option is (D)

22. $g'(x) = g_1(x) + g_2(x) + g_3(x)$

where $g_1(x) = \begin{vmatrix} f_1' & f_2' & f_3' \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = 0 \qquad (\because R_1 \equiv R_2),$

$$g_2(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1'' & f_2'' & f_3'' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = 0 \qquad (\because R_2 \equiv R_3)$$

$$g_3(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1''' & f_2''' & f_3''' \end{vmatrix} = 0$$

($\because f_r''' = 0$ as each f_r is a quadratic in x)

Therefore, we have $g'(x) = 0 \Rightarrow g(x)$ is a constant.

The correct option is (D)

23. Let $\Delta = \begin{vmatrix} 2a_1b_1 & a_1b_2 + a_2b_1 & a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 & 2a_2b_2 & a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 & a_3b_2 + a_2b_3 & 2a_3b_3 \end{vmatrix}$

$$\therefore\quad \Delta = \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix}\begin{vmatrix} b_1 & a_1 & 0 \\ b_2 & a_2 & 0 \\ b_3 & a_3 & 0 \end{vmatrix} = 0$$

The correct option is (C)

24. $z = e^{iA} \cdot e^{iB} \cdot e^{iC} \begin{vmatrix} e^{iA} & e^{-i(C+A)} & e^{-i(B+A)} \\ e^{-i(C+B)} & e^{iB} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$

$$\Rightarrow z = -1 \begin{vmatrix} e^{iA} & -e^{iB} & -e^{ic} \\ -e^{iA} & e^{iB} & -e^{ic} \\ -e^{iA} & -e^{iB} & e^{ic} \end{vmatrix}$$

since $e^{i(A+B+C)} = e^{i\pi} = \cos\pi + i\sin\pi) = -1$

$$\Rightarrow \qquad z = -\begin{vmatrix} 0 & -2e^{iB} & 0 \\ -2e^{iA} & 0 & 0 \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$$

(Using $R_1 \to R_1 + R_3$, $R_2 \to R_2 + R_3$)

$\Rightarrow \qquad z = -2e^{iB}\,[-2\,e^{i\,(A+C)}]$

$\therefore \qquad z = 4\,e^{i(A+B+C)} = 4\,e^{i}\pi = -4$

The correct option is (B, C)

25. $\Delta =$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ (100a_1 + 10b_1 + C_1) & (100a_2 + 10b_2 + C_2) & (100a_3 + 10b_3 + C_3) \end{vmatrix}$$

(Using $R_3 \to R_3 + 100R_1 + 10R_2$)

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ x_1 & x_2 & x_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 19m_1 & 19m_2 & 19m_3 \end{vmatrix}$$

(where each $m_i \in N$)

$$= 19 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 19\,n$$

where $n = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ m_1 & m_2 & m_3 \end{vmatrix}$ is certainly an integer.

The correct option is (C)

26. For existence of a solution for the first system of equations

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

The second system will have a non-trivial solution if

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Now, $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

Remember that the existence of one non-trivial solution implies existence of infinite number of non-trivial solutions.

The correct option is (C)

27. Adding all three equations, we get

$(a + b + c)\,(x + y + z) = 0$

$\Rightarrow \qquad x + y + z = 0$ since, $a + b + c \neq 0$

From the first equation

$$(b + c)(-x) - ax = b-c$$

$$\therefore \qquad x = \frac{c-b}{a+b+c}$$

The correct option is (A)

28. Each of the first three options contains $m = 3$. When $m = 3$, the last two equations become $x + 2y + 3z = 10$ and $x + 2y + 3z = n$.

Obviously, when $n = 10$ these equations become the same. So, we are left with only two independent equations to find the values of the three unknowns. Consequently, there will be infinite solutions.

The correct option is (C)

29. $$\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = xyz\begin{vmatrix} 1+\frac{1}{x} & \frac{1}{x} & \frac{1}{x} \\ 1+\frac{1}{y} & 2+\frac{1}{y} & \frac{1}{y} \\ 1+\frac{1}{z} & 1+\frac{1}{z} & 3+\frac{1}{z} \end{vmatrix}$$

$$= xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 1 & 1 \\ 1+\frac{1}{y} & 2+\frac{1}{y} & \frac{1}{y} \\ 1+\frac{1}{z} & 1+\frac{1}{z} & 3+\frac{1}{z} \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$ and taking $\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{2}\right)$ common]

$$= xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 0 & 0 \\ 1+\frac{1}{y} & 1 & -1 \\ 1+\frac{1}{z} & 0 & 2 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$= 2xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ giving $x^{-1} + y^{-1} + z^{-1} = -3$

The correct option is (C)

30. Let $\Delta(x) = A + Bx + Cx^2 + Dx^3 + \ldots$

$$\Delta(0) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0 \Rightarrow A = 0$$

$$\Delta'(0) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 1$$

$\Rightarrow \Delta(x) = x + Cx^2 + Dx^3 + \ldots$

$\Rightarrow \Delta(x)$ is divisible by x

The correct option is (A)

31. For non-trivial solution of given system of linear equations

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $8 + k(2 - k) + 2(2k - 8) = 0$

$\Rightarrow$ $-k^2 + 6k - 8 = 0$

$\Rightarrow$ $k^2 - 6k + 8 = 0$

$\Rightarrow$ $k = 2, 4$

Clearly there exists two values of k.

The correct option is (C)

32. Subtracting $P^3 - P^2Q = Q^3 - Q^2P$

$$P^2(P - Q) + Q^2(P - Q) = 0$$

$$(P^2 + Q^2)(P - Q) = 0$$

If $|P^2 + Q^2| \neq 0$ then $P^2 + Q^2$ is invertible

$\Rightarrow$ $P - Q = 0$ contradiction

Hence, $|P^2 + Q^2| = 0$

The correct option is (C)

33. $$\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & 10 \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$$

$$= 5\begin{vmatrix} \sqrt{13} & 2 & 1 \\ \sqrt{26} & \sqrt{5} & \sqrt{2} \\ \sqrt{65} & \sqrt{3} & \sqrt{5} \end{vmatrix} + 5\begin{vmatrix} \sqrt{3} & 2 & 1 \\ \sqrt{15} & \sqrt{5} & \sqrt{2} \\ 3 & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$= 0 + 5\sqrt{3}\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix} = 5\sqrt{3}\begin{vmatrix} 1 & 1 & 1 \\ \sqrt{5} & 0 & \sqrt{2} \\ \sqrt{3} & 0 & \sqrt{5} \end{vmatrix}$$

$$= -5\sqrt{3}(5 - \sqrt{6}) = 5\sqrt{3}(\sqrt{6} - 5)$$

The correct option is (A)

34. The system of equations $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ have non-trivial solution if $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$\Rightarrow$ $1(1 - a^2) + c(-c - ab) - b(ca + b) = 0$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2abc = 1$

The correct option is (D)

35. $$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+1} \begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix}$$

$$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

This is equal to zero only if $n + 2$ is odd, i.e., n is odd integer.

The correct option is (C)

36. $|\text{adj } A| = |A|^{n-1} = |A|^{2-1} = |A|$

adj (adj A) $= |A|^{n-2} A = |A|^0 A = A$

The correct option is (B)

37. We have,

$(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$

$\Rightarrow\ (ax - b)^2 + (bx - c)^2 + (cx - d)^2 \le 0$

$\Rightarrow\ (ax - b)^2 + (bx - c)^2 + (cx - d)^2 = 0$

$\Rightarrow\ \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = x$

$\Rightarrow\ b^2 = ac$ or $2\log b = \log a + \log c$.

Now, $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = \begin{vmatrix} 130 & 54 & \log a + \log c \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$

[Apply $R_1 \to R_1 + R_3$]

$$= \begin{vmatrix} 0 & 0 & 0 \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = 0$$

[Apply $R_1 \to R_1 - 2R_2$]

The correct option is (C)

38. We have, $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$

$= (\sqrt{5})^2 \begin{vmatrix} \sqrt{13}+\sqrt{3} & 2 & 1 \\ \sqrt{15}+\sqrt{26} & \sqrt{5} & \sqrt{2} \\ 3+\sqrt{65} & \sqrt{3} & \sqrt{5} \end{vmatrix}$ is

[Taking $\sqrt{5}$ common from C_2 and C_3]

$$= 5 \begin{vmatrix} -\sqrt{3} & 2 & 1 \\ 0 & \sqrt{5} & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

[Applying $C_1 \to C_1 - \sqrt{3}\ C_2 - \sqrt{3}\ C_3$]

$= -5\sqrt{3}\,(5 - \sqrt{6})$ [Expanding along C_1].

The correct option is (A)

39. Since $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$ are divisible by k, therefore,

$100A_1 + 10B_1 + C_1 = n_1k$

$100A_2 + 10B_2 + C_2 = n_2k$

$100A_3 + 10B_3 + C_3 = n_3k$

where n_1, n_2, n_3 are integers.

Now, $\Delta = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

$$= \begin{vmatrix} A_1 & B_1 & 100A_1 + 10B_1 + C_1 \\ A_2 & B_2 & 100A_2 + 10B_2 + C_2 \\ A_3 & B_3 & 100A_3 + 10B_3 + C_3 \end{vmatrix}$$

[Applying $C_3 \to C_3 + 10C_2 + 100C_1$]

$$= \begin{vmatrix} A_1 & B_1 & n_1k \\ A_2 & B_2 & n_2k \\ A_3 & B_3 & n_3k \end{vmatrix} = k \begin{vmatrix} A_1 & B_1 & n_1 \\ A_2 & B_2 & n_2 \\ A_3 & B_3 & n_3 \end{vmatrix} = k\Delta_1$$

$\Rightarrow \Delta$ is divisible by k

[since elements of Δ_1 are integers, $\therefore\ \Delta_1$ is an integer].

The correct option is (A)

40. Since A28, 3B9 and 62C are divisible by k

$\therefore\ A28 = n_1k = 100A + 20 + 8$ (1)

$3B9 = n_2k = 300 + 10B + 9$ (2)

$62C = n_3k = 600 + 20 + C$ (3)

where n_1, n_2 and n_3 are integers.

Now, $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$

$$= \begin{vmatrix} A & 3 & 6 \\ 100A + 20 + 8 & 300 + 10B + 9 & 600 + 20 + C \\ 2 & B & 2 \end{vmatrix}$$

[Applying $R_2 \to R_2 + 100R_1 + 10R_3$]

$= \begin{vmatrix} A & 3 & 6 \\ n_1k & n_2k & n_3k \\ 2 & B & 2 \end{vmatrix}$ [Using (1), (2) and (3)

$= k \begin{vmatrix} A & 3 & 6 \\ n_1 & n_2 & n_3 \\ 2 & B & 2 \end{vmatrix}$, which is divisible by k.

The correct option is (A)

41. We have, $\begin{vmatrix} x & 1 & 1 & \dots \\ 1 & x & 1 & \dots \\ 1 & 1 & x & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$

$$= \begin{vmatrix} x & 1 & 1 & \dots \\ (1-x) & (x-1) & 0 & \dots \\ (1-x) & 0 & (x-1) & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ $\vdots \vdots \vdots$ $R_n \to R_n - R_1$]

$$= x(x-1)^{n-1} + \underbrace{(x-1)^{n-1} + (x-1)^{n-1} + \dots + (x-1)^{n-1}}_{(n-1) \text{ times}}$$

[Expanding along R_1]

$$= x\ (x-1)^{n-1} + (x-1)^{n-1}\,[1 + 1 + \dots + (n-1) \text{ times}]$$

$$= (x-1)^{n-1}\ (x + n - 1).$$

The correct option is (A)

42. We have,

$$\begin{vmatrix} \sqrt{x}+\sqrt{y} & 2\sqrt{z} & \sqrt{z} \\ \sqrt{yz}+\sqrt{2x} & z & \sqrt{2z} \\ y+\sqrt{xz} & \sqrt{yz} & z \end{vmatrix}$$

$$= z \begin{vmatrix} \sqrt{x}+\sqrt{y} & 2 & 1 \\ \sqrt{yz}+\sqrt{2x} & \sqrt{z} & \sqrt{2} \\ y+\sqrt{xz} & \sqrt{y} & \sqrt{z} \end{vmatrix}$$

[Taking $\sqrt{z}$ common from C_2 and C_3]

$$= z \begin{vmatrix} -\sqrt{y} & 2 & 1 \\ 0 & \sqrt{z} & \sqrt{2} \\ 0 & \sqrt{y} & \sqrt{z} \end{vmatrix}$$

[Applying $C_1 \to C_1 - \sqrt{y}\ C_2 - \sqrt{x}\ C_3$]

$$= -\sqrt{y} \cdot z\,(z - \sqrt{2y}\,) = z\,(\sqrt{2y}\ y - z\sqrt{y}\,).$$

The correct option is (A)

43. $g'(x) = g_1(x) + g_2(x) + g_3(x)$

where, $g_1(x) = \begin{vmatrix} f_1' & f_2' & f_3' \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = 0\ (\because R \equiv R_2)$,

$g_2(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1'' & f_2'' & f_3'' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = 0\ (\because R_2 \equiv R_3)$ and

$g_3(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1''' & f_2''' & f_3''' \end{vmatrix} = 0$

($\because$ $f_r''' = 0$ as each f_r is a quadratic in x)

All these $\Rightarrow g'(x) = 0 \Rightarrow g(x) =$ a constant.

The correct option is (D)

44. $\Delta =$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ (100a_1 + 10b_1 + C_1) & (100a_2 + 10b_2 + C_2) & (100a_3 + 10b_3 + C_3) \end{vmatrix}$$

(Using $R_3 \to R_3 + 100R_1 + 10R_2$)

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ x_1 & x_2 & x_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 19m_1 & 19m_2 & 19m_3 \end{vmatrix}$$

(where each $m_i \in N$)

$$= 19 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ m_1 & m_2 & m_3 \end{vmatrix} = 19\,n$$

where $n = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ m_1 & m_2 & m_3 \end{vmatrix}$ is certainly an integer.

The correct option is (C)

45. Adding all three equations, we get

$(a + b + c)\,(x + y + z) = 0$

$\Rightarrow\ x + y + z = 0$

since , $a + b + c \neq 0$.

From the first equation

$(b + c)\,(-x) - ax = b - c$

$\therefore\ x = \dfrac{c-b}{a+b+c}$

The correct option is (A)

46. $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = xyz \begin{vmatrix} 1+\frac{1}{x} & \frac{1}{x} & \frac{1}{x} \\ 1+\frac{1}{y} & 2+\frac{1}{y} & \frac{1}{y} \\ 1+\frac{1}{z} & 1+\frac{1}{z} & 3+\frac{1}{z} \end{vmatrix}$

$$= xyz\left(3 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) \begin{vmatrix} 1 & 1 & 1 \\ 1+\frac{1}{y} & 2+\frac{1}{y} & \frac{1}{y} \\ 1+\frac{1}{z} & 1+\frac{1}{z} & 3+\frac{1}{z} \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$= xyz\left(3 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) \begin{vmatrix} 1 & 0 & 0 \\ 1+\frac{1}{y} & 1 & -1 \\ 1+\frac{1}{z} & 0 & 2 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$]

$= 2xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ giving $x^{-1}+y^{-1}+z^{-1}=-3$

The correct option is (C)

47. Let $s-a=\alpha, s-b=\beta, s-c=\gamma$,

then $\beta+\gamma=2s-(b+c)=2s-(2s-a)=a$.

Similarly, $\gamma+\alpha=b$ and $\alpha+\beta=c$.

Also, $\alpha+\beta+\gamma=3s-(a+b+c)=3s-2s=s$.

Therefore, $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}$

$$= \begin{vmatrix} (\beta+\gamma)^2 & \alpha^2 & \alpha^2 \\ \beta^2 & (\gamma+\alpha)^2 & \beta^2 \\ \gamma^2 & \gamma^2 & (\alpha+\beta)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (\beta+\gamma)^2-\alpha^2 & 0 & \alpha^2 \\ \beta^2-(\gamma+\alpha)^2 & (\gamma+\alpha)^2-\beta^2 & \beta^2 \\ 0 & \gamma^2-(\alpha+\beta)^2 & (\alpha+\beta)^2 \end{vmatrix}$$

[Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$]

$$= \begin{vmatrix} (\beta+\gamma+\alpha)(\beta+\gamma-\alpha) & 0 & \alpha^2 \\ (\beta+\gamma+\alpha)(\beta-\gamma-\alpha) & (\gamma+\alpha+\beta)(\gamma+\alpha-\beta) & \beta^2 \\ 0 & (\gamma+\alpha+\beta)(\gamma-\alpha-\beta) & (\alpha+\beta)^2 \end{vmatrix}$$

$$= (\alpha+\beta+\gamma)^2 \begin{vmatrix} \beta+\gamma-\alpha & 0 & \alpha^2 \\ \beta-\gamma-\alpha & \gamma+\alpha-\beta & \beta^2 \\ 0 & \gamma-\alpha-\beta & (\alpha+\beta)^2 \end{vmatrix}$$

[Taking $\alpha+\beta+\gamma$ common from C_1 and C_2]

$$= (\alpha+\beta+\gamma)^2 \begin{vmatrix} \beta+\gamma-\alpha & 0 & \alpha^2 \\ \beta-\gamma-\alpha & \gamma+\alpha-\beta & \beta^2 \\ 2\alpha-2\beta & -2\alpha & 2\alpha\beta \end{vmatrix}$$

[Applying $R_3 \to R_3 - (R_1 + R_2)$]

$$= 2\,(\alpha+\beta+\gamma)^2 \begin{vmatrix} \beta+\gamma-\alpha & 0 & \alpha^2 \\ \beta-\gamma-\alpha & \gamma+\alpha-\beta & \beta^2 \\ \alpha-\beta & -\alpha & \alpha\beta \end{vmatrix}$$

[Taking 2 common from R_3]

$$= 2\,(\alpha+\beta+\gamma)^2 \begin{vmatrix} \beta+\gamma-\frac{\alpha^2}{\beta} & \frac{\alpha^2}{\beta} & 0 \\ -\gamma-\alpha+\frac{\beta^2}{\alpha} & \gamma+\alpha & 0 \\ \alpha-\beta & -\alpha & \alpha\beta \end{vmatrix}$$

[Applying $R_1 \to R_1 - \frac{\alpha}{\beta} R_3$ and $R_2 \to R_2 - \frac{\beta}{\alpha} R_3$]

$$= 2\alpha\beta\,(\alpha+\beta+\gamma)^2 \begin{vmatrix} \beta+\gamma-\frac{\alpha^2}{\beta} & \frac{\alpha^2}{\beta} \\ -\gamma-\alpha+\frac{\beta^2}{\alpha} & \gamma+\alpha \end{vmatrix}$$

[Expanding along C_3]

$$= 2\alpha\beta\,(\alpha+\beta+\gamma)^2 \begin{vmatrix} \beta+\gamma & \frac{\alpha^2}{\beta} \\ \frac{\beta^2}{\alpha} & \gamma+\alpha \end{vmatrix}$$

[Applying $C_1 \to C_1 + C_2$]

$= 2\alpha\beta\,(\alpha+\beta+\gamma)^2\,[(\beta+\gamma)(\gamma+\alpha)-ab]$

$= 2\alpha\beta\,(\alpha+\beta+\gamma)^2\,(g^2+a\gamma+b\gamma)$

$= 2ab\gamma(\alpha+\beta+\gamma)^3 = 2s^3\,(s-a)\,(s-b)\,(s-c)$.

$\therefore\ k = 2s^3$.

The correct option is (D)

48. Let $\Delta = \begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix}$

$$= \begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

[$S_n = \alpha^n + \beta^n$ for $n \ge 1$]

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = \Delta_1^2,$$

where, $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$.

Now, $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & \beta-1 \\ 1 & \alpha^2-1 & \beta^2-1 \end{vmatrix}$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$$= \begin{vmatrix} \alpha - 1 & \beta - 1 \\ \alpha^2 - 1 & \beta^2 - 1 \end{vmatrix} \quad \text{[Expanding along } R_1\text{]}$$

$$= (\alpha - 1)(\beta - 1) \begin{vmatrix} 1 & 1 \\ \alpha + 1 & \beta + 1 \end{vmatrix}$$

$$= (a\beta - (\alpha + \beta) + 1) \cdot (\beta - \alpha)$$

$$= (a\beta - (\alpha + \beta) + 1) \cdot \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \left(\frac{c}{a} + \frac{b}{a} + 1\right)\sqrt{\frac{b^2}{a^2} - \frac{4c}{a}}$$

$$\left[\begin{array}{l} \because \alpha, \beta \text{ are the roots of the equation } ax^2 + bx + c = 0, \\ \therefore \alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a} \end{array}\right]$$

$$= \frac{(a + b + c)\sqrt{b^2 - 4ac}}{a^2}$$

$$\therefore \quad \Delta = \Delta_1^2 = \frac{(a + b + c)^2 \times (b^2 - 4ac)}{a^4}$$

The correct option is (B)

49. We have, $\begin{vmatrix} a & b - c & c + b \\ a + c & b & c - a \\ a - b & a + b & c \end{vmatrix}$

$$= \frac{1}{abc} \begin{vmatrix} a^2 & b(b - c) & c(c + b) \\ a(a + c) & b^2 & c(c - a) \\ a(a - b) & b(a + b) & c^2 \end{vmatrix}$$

[Multiplying C_1, C_2 and C_3 by a, b and c, respectively]

$$= \frac{1}{abc} \begin{vmatrix} a^2 + b^2 + c^2 & b(b - c) & c(c + b) \\ a^2 + b^2 + c^2 & b^2 & c(c - a) \\ a^2 + b^2 + c^2 & b(a + b) & c^2 \end{vmatrix}$$

[Applying $C_1 \to C_1 + C_2 + C_3$]

$$= \frac{a^2 + b^2 + c^2}{abc} \begin{vmatrix} 1 & b(b - c) & c(c + b) \\ 1 & b^2 & c(c - a) \\ 1 & b(a + b) & c^2 \end{vmatrix}$$

[Taking $a^2 + b^2 + c^2$ common from C_1]

$$= \frac{a^2 + b^2 + c^2}{abc} \cdot bc \begin{vmatrix} 1 & b - c & c + b \\ 1 & b & c - a \\ 1 & a + b & c \end{vmatrix}$$

[Taking b and c common from C_2 and C_3, respectively,]

$$= \frac{a^2 + b^2 + c^2}{a} \begin{vmatrix} 1 & b - c & c + b \\ 0 & c & c - a \\ 0 & a + c & c \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$= \frac{a^2 + b^2 + c^2}{a} (-bc + a^2 + ab + ac + bc)$$

[Expanding along C_1]

$$= (a^2 + b^2 + c^2)(a + b + c).$$

The correct option is (C)

50. We have, $\begin{vmatrix} \operatorname{cosec}\alpha & 1 & 0 \\ 1 & 2\operatorname{cosec}\alpha & 1 \\ 0 & 1 & 2\operatorname{cosec}\alpha \end{vmatrix}$

$$= \begin{vmatrix} \operatorname{cosec}\alpha & 1 & 0 \\ 1 & 2\operatorname{cosec}\alpha - \sin\alpha & 1 \\ 0 & 1 & 2\operatorname{cosec}\alpha \end{vmatrix}$$

[Applying $R_2 \to R_2 - \sin\alpha R_1$]

$$= \operatorname{cosec}\alpha \begin{vmatrix} 2\operatorname{cosec}\alpha - \sin\alpha & 1 \\ 1 & 2\operatorname{cosec}\alpha \end{vmatrix}$$

[Expanding along R_1]

$$= \operatorname{cosec}\alpha(4\operatorname{cosec}^2\alpha - 2 - 1)$$

$$= \frac{1}{\sin\alpha}\left[\left(\frac{2}{\sin\alpha}\right)^2 - 3\right]$$

$$= \frac{1}{2}\left[\frac{\tan^2\alpha/2 + 1}{\tan\alpha/2}\right]\left[\left(\frac{\tan^2\alpha/2 + 1}{\tan\alpha/2}\right)^2 - 3\right]$$

$$= \frac{1}{2}\left(\tan\frac{\alpha}{2} + \cot\frac{\alpha}{2}\right)\left[\left(\tan\frac{\alpha}{2} + \cot\frac{\alpha}{2}\right)^2 - 3\right]$$

$$= \frac{1}{2}\left[\left(\tan\frac{\alpha}{2} + \cot\frac{\alpha}{2}\right)^3 - 3\tan\frac{\alpha}{2}\cot\frac{\alpha}{2}\left(\tan\frac{\alpha}{2} + \cot\frac{\alpha}{2}\right)\right]$$

$$= \frac{1}{2}\left(\tan^3\frac{\alpha}{2} + \cot^3\frac{\alpha}{2}\right)$$

$$\therefore z = \tan.\frac{\alpha}{2}$$

The correct option is (C)

51. We have, $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a + 1)^2 & (b + 1)^2 & (c + 1)^2 \\ (a - 1)^2 & (b - 1)^2 & (c - 1)^2 \end{vmatrix}$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_3$ and then taking 4 common from R_2]

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

[Applying $R_3 \to R_3 - (R_1 - 2R_2)$]

$$= 4\begin{vmatrix} a^2 & b^2 - a^2 & c^2 - a^2 \\ a & b-a & c-a \\ 1 & 0 & 0 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$$= 4\,(b-a)\,(c-a)\begin{vmatrix} a^2 & b+a & c+a \\ a & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= 4\,(b-a)\,(c-a)\,(b-c) = -4\,(a-b)\,(b-c)\,(c-a).$$

$\therefore \quad k = -4.$

The correct option is (B)

52. We have, $\begin{vmatrix} (a-a_1)^{-2} & (a-a_1)^{-1} & a_1^{-1} \\ (a-a_2)^{-2} & (a-a_2)^{-1} & a_2^{-1} \\ (a-a_3)^{-2} & (a-a_3)^{-1} & a_3^{-1} \end{vmatrix}$

$$= (a-a_1)^{-2}\,(a-a_2)^{-2}\,(a-a_3)^{-2}$$

$$\begin{vmatrix} 1 & (a-a_1) & a_1^{-1}(a-a_1)^2 \\ 1 & (a-a_2) & a_2^{-1}(a-a_2)^2 \\ 1 & (a-a_3) & a_3^{-1}(a-a_3)^2 \end{vmatrix}$$

$$= \frac{1}{\Pi(a-a_i)^2}\begin{vmatrix} 1 & (a-a_1) & a_1^{-1}(a-a_1)^2 \\ 0 & (a_1-a_2) & \dfrac{(a^2-a_1a_2)(a_1-a_2)}{a_1a_2} \\ 0 & (a_1-a_3) & \dfrac{(a^2-a_1a_3)(a_1-a_3)}{a_1a_3} \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$= \frac{1}{\Pi(a-a_i)^2}\begin{vmatrix} (a_1-a_2) & \dfrac{(a^2-a_1a_2)\,(a_1-a_2)}{a_1a_2} \\ (a_1-a_3) & \dfrac{(a^2-a_1a_3)\,(a_1-a_3)}{a_1a_3} \end{vmatrix}$$

[Expanding along C_1]

$$= \frac{(a_1-a_2)\,(a_1-a_3)}{\Pi(a-a_i)^2}\begin{vmatrix} 1 & \dfrac{a^2-a_1a_2}{a_1a_2} \\ 1 & \dfrac{a^2-a_1a_3}{a_1a_3} \end{vmatrix}$$

$$= \frac{(a_1-a_2)\,(a_1-a_3)\,a^2\,(a_2-a_3)}{a_1a_2a_3\,\Pi(a-a_i)^2}$$

$$= \frac{-a^2\,\Pi(a_i-a_j)}{\Pi a_i\,\Pi(a-a_i)^2}.$$

The correct option is (B)

53. We have, $\begin{vmatrix} \dfrac{1}{a+x} & \dfrac{1}{b+x} & \dfrac{1}{c+x} \\ \dfrac{1}{a+y} & \dfrac{1}{b+y} & \dfrac{1}{c+y} \\ \dfrac{1}{a+z} & \dfrac{1}{b+z} & \dfrac{1}{c+z} \end{vmatrix}$

$$= \begin{vmatrix} \dfrac{1}{a+x} & \dfrac{a-b}{(a+x)\,(b+x)} & \dfrac{a-c}{(a+x)\,(c+x)} \\ \dfrac{1}{a+y} & \dfrac{a-b}{(a+y)\,(b+y)} & \dfrac{a-c}{(a+y)\,(c+y)} \\ \dfrac{1}{a+z} & \dfrac{a-b}{(a+z)\,(b+z)} & \dfrac{a-c}{(a+z)\,(c+z)} \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$$= (a-b)\,(a-c)\begin{vmatrix} \dfrac{1}{a+x} & \dfrac{1}{(a+x)\,(b+x)} & \dfrac{1}{(a+x)\,(c+x)} \\ \dfrac{1}{a+y} & \dfrac{1}{(a+y)\,(b+y)} & \dfrac{1}{(a+y)\,(c+y)} \\ \dfrac{1}{a+z} & \dfrac{1}{(a+z)\,(b+z)} & \dfrac{1}{(a+z)\,(c+z)} \end{vmatrix}$$

$$= \frac{(a-b)\,(a-c)}{Q}\begin{vmatrix} (b+x)\,(c+x) & (c+x) & b+x \\ (b+y)\,(c+y) & (c+y) & b+y \\ (b+z)\,(c+z) & (c+z) & b+z \end{vmatrix}$$

$$= \frac{(a-b)\,(a-c)}{Q}\begin{vmatrix} (b+x)\,(c+x) & c+x & b+x \\ (y-x)\,(y+x+b+c) & y-x & y-x \\ (z-x)\,(z+x+b+c) & z-x & z-x \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$$= \frac{(a-b)\,(a-c)\,(y-x)\,(z-x)}{Q}$$

$$\begin{vmatrix} (b+x)\,(c+x) & c+x & b+x \\ y+x+b+c & 1 & 1 \\ z+x+b+c & 1 & 1 \end{vmatrix}$$

$$= \frac{(a-b)(a-c)(y-x)(z-x)}{Q}\begin{vmatrix} (b+x)(c+x) & c+x & b+x \\ y+x+b+c & 1 & 1 \\ z-y & 0 & 0 \end{vmatrix}$$

[Applying $R_3 \to R_3 - R_2$]

$$= \frac{(a-b)(a-c)(y-x)(z-x)(z-y)}{Q}\begin{vmatrix} c+x & b+x \\ 1 & 1 \end{vmatrix}$$

[Expanding along R_3]

$$= \frac{(a-b)(b-c)(c-a)(x-y)(y-z)(z-x)}{Q}$$

$\therefore P = (a-b)(b-c)(c-a)(x-y)(y-z)(z-x)$.

The correct option is (C)

54. We have, $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix}$

$$= abcd\begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} & \frac{1}{c} \\ \frac{1}{d} & \frac{1}{d} & \frac{1}{d} & 1+\frac{1}{d} \end{vmatrix}$$

[Dividing R_1, R_2, R_3 and R_4 by a, b, c and d respectively]

$$= abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \times \begin{vmatrix} 1 & 1 & 1 & 1 \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} & \frac{1}{c} \\ \frac{1}{d} & \frac{1}{d} & \frac{1}{d} & 1+\frac{1}{d} \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3 + R_4$ and taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$ common from R_1]

$$= abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \times \begin{vmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{b} & 1 & 0 & 0 \\ \frac{1}{c} & 0 & 1 & 0 \\ \frac{1}{d} & 0 & 0 & 1 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ and $C_4 \to C_4 - C_1$]

$$= abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

[Expanding along R_1]

$= abcd + (bcd + acd + abd + abc)$

$$= \frac{\xi}{\alpha} - \frac{\delta}{\alpha}$$

$\left[\begin{array}{l} \because a, b, c, d \text{ are roots of the equation} \\ \alpha x^4 + \beta x^3 + \gamma x^2 + \delta x + \xi = 0 \\ \therefore bcd + acd + abd + abc = -\frac{\delta}{\alpha} \\ \text{and } abcd = \frac{\xi}{\alpha} \end{array}\right]$

$$= \frac{\xi - \delta}{\alpha}.$$

The correct option is (B)

55. We have, $\begin{vmatrix} 0 & x & y & z \\ -x & 0 & c & b \\ -y & -c & 0 & a \\ -z & -b & -a & 0 \end{vmatrix}$

$$= \frac{1}{a}\begin{vmatrix} 0 & ax-by+cz & y & z \\ -x & 0 & c & b \\ -y & 0 & 0 & a \\ -z & 0 & -a & 0 \end{vmatrix}$$

[Applying $C_2 \to aC_2 - bC_3 + cC_4$]

$$= -\frac{(ax-by+cz)}{a}\begin{vmatrix} -x & c & b \\ -y & 0 & a \\ -z & -a & 0 \end{vmatrix}$$

[Expanding along C_2]

$$= \frac{(ax-by+cz)}{a^2}\begin{vmatrix} ax-by+cz & 0 & 0 \\ y & 0 & a \\ z & -a & 0 \end{vmatrix}$$

[Taking (– 1) common from C_1 and applying $R_1 \to aR_1 - bR_2 + cR_3$]

$= (ax - by + cz)^2$. [Expanding along R_1].

The correct option is (B)

56. We have,

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$$

$$= \frac{1}{abc}\begin{vmatrix} a(b^2+c^2) & ab^2 & ac^2 \\ a^2b & b(c^2+a^2) & bc^2 \\ a^2c & cb^2 & c(a^2+b^2) \end{vmatrix}$$

[Multiplying C_1, C_2 and C_3 by a, b and c, respectively]

$$= \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & b^2 & c^2 \\ a^2 & c^2+a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$$

[Taking a, b, c common from R_1, R_2 and R_3, respectively]

$$= \begin{vmatrix} 0 & b^2 & c^2 \\ -2c^2 & c^2+a^2 & c^2 \\ -2b^2 & b^2 & a^2+b^2 \end{vmatrix}$$

[Applying $C_1 \to C_1 - C_2 - C_3$]

$$= -2\begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & a^2 & 0 \\ b^2 & 0 & a^2 \end{vmatrix}$$

[Taking – 2 common from C_1 and then applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$]

$= -2\,[0 - b^2(a^2c^2) + c^2(-a^2b^2)] = 4a^2b^2c^2$.

The correct option is (C)

57. We have, $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ (1)

$$= \begin{vmatrix} x+c_1 & a-c_1 & 0 \\ x+b & c_2-b & a-c_2 \\ x+b & 0 & c_3-b \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$]

$$= x\begin{vmatrix} 1 & a-c_1 & 0 \\ 1 & c_2-b & a-c_2 \\ 1 & 0 & c_3-b \end{vmatrix} + \begin{vmatrix} c_1 & a-c_1 & 0 \\ b & c_2-b & a-c_2 \\ b & 0 & c_3-b \end{vmatrix}$$

So, $f(x)$ is linear.

Let $f(x) = Px + Q$,

then, $f(-a) = -aP + Q, f(-b) = -bP + Q$

$\therefore \quad f(0) = 0.P + Q = Q = \dfrac{b\,f(-a) - a\,f(-b)}{(b-a)}$ (2)

From (1), $f(-a) = \begin{vmatrix} c_1-a & 0 & 0 \\ b-a & c_2-a & 0 \\ b-a & b-a & c_3-a \end{vmatrix}$

$= (c_1 - a)(c_2 - a)(c_3 - a)$.

Similarly, $f(-b) = (c_1 - b)(c_2 - b)(c_3 - b)$

Also, $g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$

$\therefore \quad g(A) = f(-a)$ and $g(b) = f(-b)$

So, we get from (2), $f(0) = \dfrac{bg(a) - ag(b)}{(b-a)}$.

The correct option is (A)

58. We have,

$$\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$= [a(bc - a^2) + b(ac - b^2) + c(ab - c^2)]^2$

$= [a^3 + b^3 + c^3 - 3abc]^2$

$\therefore \quad k = -3$.

The correct option is (D)

59. We have, $\begin{vmatrix} \beta\gamma & \beta\gamma' + \beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha' + \gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta' + \alpha'\beta & \alpha'\beta' \end{vmatrix}$ m

$$= (\beta'\gamma')(\gamma'\alpha')(\alpha'\beta')\begin{vmatrix} \frac{\beta}{\beta'}\cdot\frac{\gamma}{\gamma'} & \frac{\beta}{\beta'}+\frac{\gamma}{\gamma'} & 1 \\ \frac{\gamma}{\gamma'}\cdot\frac{\alpha}{\alpha'} & \frac{\gamma}{\gamma'}+\frac{\alpha}{\alpha'} & 1 \\ \frac{\alpha}{\alpha'}\cdot\frac{\beta}{\beta'} & \frac{\alpha}{\alpha'}+\frac{\beta}{\beta'} & 1 \end{vmatrix}$$

[Taking $\beta'\gamma'$, $\gamma'\alpha'$ and $\alpha'\beta'$ common from R_1, R_2 and R_3 respectively]

$$= (\alpha'\beta'\gamma')^2\begin{vmatrix} \frac{\beta}{\beta'}\cdot\frac{\gamma}{\gamma'} & \frac{\beta}{\beta'}+\frac{\gamma}{\gamma'} & 1 \\ \frac{\gamma}{\gamma'}\left(\frac{\alpha}{\alpha'}-\frac{\beta}{\beta'}\right) & \left(\frac{\alpha}{\alpha'}-\frac{\beta}{\beta'}\right) & 0 \\ \frac{\beta}{\beta'}\left(\frac{\alpha}{\alpha'}-\frac{\gamma}{\gamma'}\right) & \left(\frac{\alpha}{\alpha'}-\frac{\gamma}{\gamma'}\right) & 0 \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$$= (\alpha'\beta'\gamma')^2\left(\frac{\alpha}{\alpha'}-\frac{\beta}{\beta'}\right)\left(\frac{\alpha}{\alpha'}-\frac{\gamma}{\gamma'}\right)$$

$$\begin{vmatrix} \frac{\beta}{\beta'}\cdot\frac{\gamma}{\gamma'} & \frac{\beta}{\beta'}+\frac{\gamma}{\gamma'} & 1 \\ \frac{\gamma}{\gamma'} & 1 & 0 \\ \frac{\beta}{\beta'} & 1 & 0 \end{vmatrix}$$

$$= (\alpha'\beta'\gamma')^2 \cdot \frac{(\alpha\beta' - \alpha'\beta)}{\alpha'\beta'} \cdot \frac{(\alpha\gamma' - \alpha'\gamma)}{\alpha'\gamma'} \left(\frac{\gamma}{\gamma'} - \frac{\beta}{\beta'}\right)$$

$$= (\alpha'\beta'\gamma')^2 \cdot \frac{(\alpha\beta' - \alpha'\beta)(\alpha\gamma' - \alpha'\gamma)(\gamma\beta' - \gamma'\beta)}{(\alpha'\beta'\gamma)^2}$$

$$= (a\beta' - \alpha'\beta)(a\gamma' - \alpha'\gamma)(\gamma\beta' - \gamma'\beta).$$

The correct option is (A)

60. We have,

$$\begin{vmatrix} x+1 & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{vmatrix} = \begin{vmatrix} x+1 & x & x \\ x+0 & x+a & x \\ x+0 & x & x+a^2 \end{vmatrix}$$

$$= \begin{vmatrix} x & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{vmatrix} + \begin{vmatrix} 1 & x & x \\ 0 & x+a & x \\ 0 & x & x+a^2 \end{vmatrix}$$

$$= \begin{vmatrix} x & x & x \\ 0 & a & 0 \\ 0 & 0 & a^2 \end{vmatrix} + \begin{vmatrix} 1 & x & x \\ 0 & x+a & x \\ 0 & x & x+a^2 \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ in first determinant]

$$= x \cdot a \cdot a^2 + 1 \begin{vmatrix} x+a & x \\ x & x+a^2 \end{vmatrix}.$$

[Expanding both the determinants along C_1]

$= xa^3 + [(x + a)(x + a^2) - x^2]$

$= xa^3 + a^2x + ax + a^3 = a^3 + xa(a^2 + a + 1).$

$\therefore \quad f(x) = x.$

The correct option is (A)

61. We have, $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$

$$= \frac{abc}{abc} \begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ca & -ab \end{vmatrix}$$

[Multiplying R_1, R_2, R_3 by a, b, c, respectively, and taking a, b, c common from C_1, C_2 and C_3, respectively]

$$= (ab + bc + ac)^2 \begin{vmatrix} -bc & 1 & 1 \\ ab+bc & -1 & 0 \\ ac+bc & 0 & -1 \end{vmatrix}$$

[Applying $C_3 \to C_3 - C_1$ and $C_2 \to C_2 - C_1$, and taking $(ab + ac + bc)$ common from C_2 and C_3]

$= (ab + bc + ac)^2 [1 \cdot (0 + ac + bc) - 1 \cdot (bc - ab - bc)]$

[Expanding along C_3]

$= (ab + bc + ac)^3.$

The correct option is (B)

62. We have, $\begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x+a^2 & b^2 & c^2 \\ a^2 & x+b^2 & c^2 \\ a^2 & b^2 & x+c^2 \end{vmatrix} = 0$$

[Taking a, b, c common from R_1, R_2, R_3, respectively and then multiplying columnwise]

$$\Rightarrow (x + a^2 + b^2 + c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & x+b^2 & c^2 \\ 1 & b^2 & x+c^2 \end{vmatrix} = 0$$

[Applying $C_1 \to C_1 + C_2 + C_3$ and taking $(x + a^2 + b^2 + c^2)$ common from C_1]

$$\Rightarrow (x + a^2 + b^2 + c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$\Rightarrow x^2 (x + a^2 + b^2 + c^2) = 0$

But $x \neq 0$, $\therefore x = -(a^2 + b^2 + c^2)$.

The correct option is (B)

63. We have,

$$D = \begin{vmatrix} 3 & m \\ 2 & -5 \end{vmatrix} = -15 - 2m,$$

$$D_1 = \begin{vmatrix} m & m \\ 20 & -5 \end{vmatrix} = -25m$$

and, $D_2 = \begin{vmatrix} 3 & m \\ 2 & 20 \end{vmatrix} = 60 - 2m.$

So, by Cramer's rule

$$x = \frac{D_1}{D} = \frac{-25m}{-15-2m} = \frac{25m}{15+2m}$$

and, $y = \dfrac{D_2}{D} = \dfrac{60-2m}{-15-2m} = \dfrac{2m-60}{2m+15}$

Since $x > 0 \Rightarrow \dfrac{25m}{15+2m} > 0$ i.e., $25m(2m + 15) > 0$

$\Rightarrow \quad m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (0, \infty)$ (1)

Also, $y > 0 \Rightarrow \dfrac{2m-60}{2m+15} > 0$ i.e., $(2m - 60)(2m + 15) > 0$

$\Rightarrow \quad m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (30, \infty)$ (2)

From (1) and (2), we get $m \in \left(-\infty, \dfrac{-15}{2}\right) \cup (30, \infty)$.

The correct option is (B)

64. We have, $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow \quad -\dfrac{1}{2}\,(a+b+c)\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$

$\Rightarrow$ Either $a + b + c = 0$ or $a = b = c$

If $a + b + c = 0$, then we must have

$\cos\theta + \cos 3\theta + \cos 2\theta = 0$

and, $\sin\theta + \sin 3\theta - \sin 2\theta = 0$

or, $\cos 2\theta\,(2\cos\theta + 1) = 0$

and, $\sin 2\theta\,(2\cos\theta - 1) = 0$

The above equations do not hold simultaneously because $\cos 2\theta = 0$ i.e., $\theta = \dfrac{\pi}{4}$ then second equation is not satisfied and if $2\cos\theta + 1 = 0$ or $\cos\theta = -\dfrac{1}{2}$ i.e, $\theta = \dfrac{2\pi}{3}$, then also second equation is not satisfied.

Therefore, the only possibility is $a = b = c$.

or, $e^{i}\theta = e^{-2i}\theta = e^{3i}\theta$ which is satisfied only when

$e^{i}\theta = 1$ i.e., $\cos\theta + i\sin\theta = 1$

$\therefore \quad \cos\theta = 1$ and $\sin\theta = 0$. $\therefore\ \theta = 2n\pi$.

The correct option is (B)

65. We have,

$$\begin{vmatrix} \dfrac{1}{a} & \dfrac{1}{a(a+d)} & \dfrac{1}{(a+d)(a+2d)} \\ \dfrac{1}{a+d} & \dfrac{1}{(a+d)(a+2d)} & \dfrac{1}{(a+2d)(a+3d)} \\ \dfrac{1}{a+2d} & \dfrac{1}{(a+2d)(a+3d)} & \dfrac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

$$= \frac{1}{a\,(a+d)^2\,(a+2d)^3\,(a+3d)^2\,(a+4d)} \times \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)(a+3d) & a+3d & a+d \\ (a+3d)(a+4d) & a+4d & a+2d \end{vmatrix}$$

$$= \frac{1}{a(a+d)^2\,(a+2d)^3\,(a+3d)^2\,(a+4d)} \begin{vmatrix} (a+d)(a+2d) & a+2d & a \\ (a+2d)\,2d & d & d \\ (a+3d)\,2d & d & d \end{vmatrix}$$

[Applying $R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1$]

$$= \frac{1}{a(a+d)^2\,(a+2d)^3\,(a+3d)^2\,(a+4d)} \begin{vmatrix} (a+d)(a+2d) & a+2d & -2d \\ (a+2d)\,2d & d & 0 \\ (a+3d)\,2d & d & 0 \end{vmatrix},\ C_3 \to C_3 - C_2$$

$$= \frac{-2d}{a(a+d)^2\,(a+2d)^3\,(a+3d)^2\,(a+4d)} \begin{vmatrix} (a+2d)2d & d \\ (a+3d)2d & d \end{vmatrix}$$

$$= \frac{-4d^3}{a(a+d)^2\,(a+2d)^3\,(a+3d)^2\,(a+4d)} \begin{vmatrix} a+2d & 1 \\ a+3d & 1 \end{vmatrix}$$

$$= \frac{4d^4}{a(a+d)^2\,(a+2d)^3\,(a+3d)^2\,(a+4d)}$$

The correct option is (B)

66. We have, $\begin{vmatrix} (b+c)^2 & c^2 & b^2 \\ c^2 & (c+a)^2 & a^2 \\ b^2 & a^2 & (a+b)^2 \end{vmatrix}$

$$= \frac{1}{a^2b^2c^2} \begin{vmatrix} a^2(b+c)^2 & a^2c^2 & a^2b^2 \\ c^2b^2 & b^2(c+a)^2 & a^2b^2 \\ b^2c^2 & a^2c^2 & c^2(a+b)^2 \end{vmatrix}$$

[Multiplying R_1, R_2 and R_3 by a^2, b^2 and c^2, respectively]

$$= \frac{1}{xyz} \begin{vmatrix} (y+z)^2 & y^2 & z^2 \\ x^2 & (z+x)^2 & z^2 \\ x^2 & y^2 & (x+y)^2 \end{vmatrix}$$

[Putting $bc = x$, $ca = y$ and $ab = z$]

$$= \frac{1}{xyz} \begin{vmatrix} (x+y+z)(y+z-x) & (x+y+z) & 0 \\ 0 & (x+y+z)(z+x-y) & (x+y+z)(z-x-y) \\ x^2 & y^2 & (x+y)^2 \end{vmatrix}$$

[Applying $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$]

$$= \frac{(x+y+z)^2}{xyz} \begin{vmatrix} y+z-x & y-z-x & 0 \\ 0 & z+x-y & (z-x-y) \\ x^2 & y^2 & (x+y)^2 \end{vmatrix}$$

$$= \frac{(x+y+z)^2}{xyz} \begin{vmatrix} y+z-x & y-z-x & 2(x-y) \\ 0 & z+x-y & -2x \\ x^2 & y^2 & 2xy \end{vmatrix}$$

[Applying $C_3 \to C_3 - C_1 - C_2$]

$$= \frac{(x+y+z)^2}{xyz} \begin{vmatrix} y+z-x & 0 & -2y \\ 0 & z+x-y & -2x \\ x^2 & y^2 & 2xy \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2$]

$$= \frac{(x+y+z)}{xy \cdot xyz} \begin{vmatrix} x(y+z-x) & 0 & -2xy \\ 0 & y(z+x-y) & -2xy \\ x^2 & y^2 & 2xy \end{vmatrix}$$

[Multiplying R_1, R_2 by x and y, respectively]

$$= \frac{(x+y+z)}{xy \cdot xyz} \begin{vmatrix} x(y+z) & y^2 & 0 \\ x^2 & y(z+x) & 0 \\ x^2 & y^2 & 2xy \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_3$, $R_2 \to R_2 + R_3$]

$$= \frac{(x+y+z)^2}{xy \cdot xyz} \cdot 2xy \begin{vmatrix} x(y+z) & y^2 \\ x & y\,(z+x) \end{vmatrix}$$

[Expanding along C_3]

$$= \frac{2(x+y+z)^2}{xy \cdot xyz} \cdot xy \begin{vmatrix} x(y+z) & y \\ x^2 & z+x \end{vmatrix}$$

$$= \frac{2(x+y+z)^2}{z} \begin{vmatrix} x+y+z & x+y+z \\ x & z+x \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2$]

$$= \frac{2(x+y+z)^3}{z} \begin{vmatrix} 1 & 1 \\ x & z+x \end{vmatrix} = 2\,(x+y+z)^3$$

$= 2\,(ab+bc+ca)^3$.

The correct option is (A)

67. The three equations in two unknowns will be consistent if

$$\begin{vmatrix} (a+1)^3 & (a+2)^3 & -(a+3)^3 \\ (a+1) & (a+2) & -(a+3) \\ 1 & 1 & -1 \end{vmatrix} = 0$$

i.e., $$\begin{vmatrix} (a+1)^3 & (a+2)^3 & (a+3)^3 \\ (a+1) & (a+2) & (a+3) \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x^3 & y^3 & z^3 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = 0$$

[where $x = a+1$, $y = a+2$ and $z = a+3$]

$\Rightarrow (x-y)(y-z)(z-x)(x+y+z) = 0$

$\Rightarrow (-1)(-1)(2)(3a+6) = 0 \Rightarrow a+2 = 0$ or $a = -2$.

The correct option is (D)

68. Since the given system has a non-trivial solution, therefore,

$$\begin{vmatrix} \sin\alpha & \sin\beta & \sin\gamma \\ \cos\alpha & \cos\beta & \cos\gamma \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \sin\alpha & \sin\beta - \sin\alpha & \sin\gamma - \sin\alpha \\ \cos\alpha & \cos\beta - \cos\alpha & \cos\gamma - \cos\alpha \\ 1 & 0 & 0 \end{vmatrix} = 0$$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$]

$$\Rightarrow \begin{vmatrix} \sin\alpha & 2\cos\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right) & 2\cos\left(\frac{\gamma+\alpha}{2}\right)\sin\left(\frac{\gamma-\alpha}{2}\right) \\ \cos\alpha & 2\sin\left(\frac{\beta+\alpha}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) & 2\sin\left(\frac{\gamma+\alpha}{2}\right)\sin\left(\frac{\alpha-\gamma}{2}\right) \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow 4\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\gamma-\alpha}{2}\right)$$

$$\left[\sin\left(\frac{\gamma+\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right) - \sin\left(\frac{\beta+\alpha}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right)\right] = 0$$

$$\Rightarrow -4\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\beta-\gamma}{2}\right)\sin\left(\frac{\gamma-\alpha}{2}\right) = 0$$

$$\Rightarrow \sin\left(\frac{\alpha-\beta}{2}\right) = 0$$

or $\sin\left(\frac{\beta-\gamma}{2}\right) = 0$ or $\sin\left(\frac{\gamma-\alpha}{2}\right) = 0$

$\Rightarrow \alpha - \beta = 0$ or $\beta - \gamma = 0$ or $\gamma - \alpha = 0$

$\Rightarrow \alpha = \beta$ or $\beta = \gamma$ or $\gamma = \alpha$

$\Rightarrow$ triangles must be isosceles.

The correct option is (A)

69. We have,

$$\begin{vmatrix} x_1 + a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & x_2 + a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & x_3 + a_3b_3 \end{vmatrix}$$

$$= \frac{1}{b_1^2} \begin{vmatrix} x_1 + a_1b_1 & a_1b_1b_2 & a_1b_1b_3 \\ a_2b_1 & b_1\,x_2 + a_2b_1b_2 & a_2b_1b_3 \\ a_3b_1 & a_3b_1b_2 & b_1x_3 + a_3b_1b_3 \end{vmatrix}$$

[Multiplying C_2 and C_3 by b_1]

$$= \frac{1}{b_1^2}\begin{vmatrix} x_1 + a_1b_1 & -b_2x_1 & -b_3x_1 \\ a_2b_1 & b_1x_2 & 0 \\ a_3b_1 & 0 & b_1x_3 \end{vmatrix}$$

[Applying $C_2 \to C_2 - b_2 C_1$ and $C_3 \to C_3 - b_3C_1$]

$$= \begin{vmatrix} x_1 + a_1b_1 & -b_2x_1 & -b_3x_1 \\ a_2 & x_2 & 0 \\ a_3 & 0 & x_3 \end{vmatrix}$$

$$= x_1x_2x_3 \begin{vmatrix} 1+\frac{a_1b_1}{x_1} & -b_2 & -b_3 \\ \frac{a_2}{x_2} & 1 & 0 \\ \frac{a_3}{x_3} & 0 & 1 \end{vmatrix}$$

[Taking x_1, x_2 and x_3 common from R_1, R_2 and R_3, respectively]

$$= x_1x_2x_3 \begin{vmatrix} 1+\frac{a_1b_1}{x_1}+\frac{a_2b_2}{x_2}+\frac{a_3b_3}{x_3} & 0 & 0 \\ \frac{a_2}{x_2} & 1 & 0 \\ \frac{a_3}{x_3} & 0 & 1 \end{vmatrix}$$

[Applying $R_1 \to R_1 + b_2R_2 + b_3R_3$]

$$= x_1x_2x_3 \cdot \left(1+\frac{a_1b_1}{x_1}+\frac{a_2b_2}{x_2}+\frac{a_3b_3}{x_3}\right)$$

The correct option is (A)

70. We have,

$$\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & d & d \\ a^2 & d(d+2a) & d(3d+2a) \\ 2a+3d & -d & -d \end{vmatrix} = 0$$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_2$]

$$\Rightarrow d^2 \begin{vmatrix} a & 1 & 1 \\ a^2 & d+2a & 3d+2a \\ 2a+3d & -1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow d^2 \begin{vmatrix} a & 1 & 0 \\ a^2 & d+2a & 2d \\ 2a+3d & -1 & 0 \end{vmatrix} = 0$$

[Applying $C_3 \to C_3 - C_2$]

$\Rightarrow d^2(3a+3d) = 0$ [Expanding along C_3]

$\Rightarrow d = 0$ or $a + d = 0$.

The correct option is (C)

71. Differentiating both sides of the given equation w.r.t. x, we get

$$\begin{vmatrix} 1 & 1 & 3(x+2)^2 \\ x+2 & x+3 & (x+2)^3 \\ (x+2)^3 & x+2 & x+3 \end{vmatrix} + \begin{vmatrix} x+3 & x+2 & (x+2)^3 \\ 1 & 1 & 3(x+2)^2 \\ (x+2)^3 & x+2 & x+3 \end{vmatrix} + \begin{vmatrix} x+3 & x+2 & (x+2)^3 \\ x+2 & x+3 & (x+2)^3 \\ 3(x+2)^3 & 1 & 1 \end{vmatrix}$$

$= (7ax^6 + 6bx^5 + 5cx^4 + 4dx^3 + 3ex^2 + 2fx + g)$.

Putting $x = 0$ on both sides, we get

$$\begin{vmatrix} 1 & 1 & 12 \\ 2 & 3 & 8 \\ 8 & 2 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 2 & 8 \\ 1 & 1 & 12 \\ 8 & 2 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 2 & 8 \\ 2 & 3 & 8 \\ 12 & 1 & 1 \end{vmatrix} = g$$

$\Rightarrow g = -189 + 75 - 99.$ $\therefore g = -213$.

The correct option is (A)

72. We have, $x^ny^nz^n \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$

$$= \frac{1}{xyz}(x-y)(y-z)(z-x)(xy+xz+yz)$$

$$\Rightarrow x^{n+1} \cdot y^{n+1} \cdot z^{n+1} \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$= (x-y)(y-z)(z-x)(xy+xz+yz)$

Since the degree of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$ is 5,

$\therefore$ degree of L.H.S. $= 3n + 3 + 5 = 3n + 8$.

Also, degree of R.H.S. $= 5$. $\therefore 3n + 8 = 5 \Rightarrow n = -1$.

The correct option is (B)

73. We have,

$$\begin{vmatrix} \sin\alpha\cos\beta & \cos\alpha\cos\beta & -\sin\alpha\sin\beta \\ \sin\alpha\sin\beta & \cos\alpha\sin\beta & \sin\alpha\cos\beta \\ \cos\alpha & -\sin\alpha & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & -\frac{\sin\alpha}{\sin\beta} \\ \sin\alpha\sin\beta & \cos\alpha\sin\beta & \sin\alpha\cos\beta \\ \cos\alpha & -\sin\alpha & 0 \end{vmatrix}$$

$\left[\text{Applying } R_1 \to R_1 - \frac{\cos\beta}{\sin\beta}R_2\right]$

$$= -\frac{\sin\alpha}{\sin\beta}\,(-\sin^2\alpha\sin\beta - \cos^2\alpha\sin\beta)$$

$= \sin\alpha$, which is independent of β.

The correct option is (B)

74. We have, $\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix}$

$$= \alpha\beta\gamma \begin{vmatrix} \frac{1}{\alpha}+1 & \frac{1}{\beta} & \frac{1}{\gamma} \\ \frac{1}{\alpha} & \frac{1}{\beta}+1 & \frac{1}{\gamma} \\ \frac{1}{\alpha} & \frac{1}{\beta} & \frac{1}{\gamma}+1 \end{vmatrix}$$

$$= \alpha\beta\gamma\left(1+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) \begin{vmatrix} 1 & \frac{1}{\beta} & \frac{1}{\gamma} \\ 1 & \frac{1}{\beta}+1 & \frac{1}{\gamma} \\ 1 & \frac{1}{\beta} & \frac{1}{\gamma}+1 \end{vmatrix}$$

[Applying $C_1 \to C_1 + C_2 + C_3$)

$$= \alpha\beta\gamma\left(1+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) \begin{vmatrix} 1 & \frac{1}{\beta} & \frac{1}{\gamma} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$= \alpha\beta\gamma\left(1+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) \text{ [Expanding along } C_1]$$

$$= \alpha\beta\gamma + (\beta\gamma + \alpha\gamma + \alpha\beta)$$

[Since α, β, γ are roots of $x^3 + px + q = 0$

$\therefore\ \alpha\beta + \alpha\gamma + \beta\gamma = p$ and $\alpha\beta\gamma = -q$]

$= -q + p$.

The correct option is (C)

75. Let $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ be any determinant of third order.

Then, $\Delta = \begin{vmatrix} 0 & a_2 - \frac{a_1}{b_1}b_2 & a_3 - \frac{a_1}{b_1}b_3 \\ 0 & b_2 - \frac{b_1}{c_1}c_2 & b_3 - \frac{b_1}{c_1}c_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$$\left[\text{Applying } R_1 \to R_1 - \frac{a_1}{b_1}R_2,\ R_2 \to R_2 - \frac{b_1}{c_1}R_3\right]$$

$$= c_1\left[\left(a_2 - \frac{a_1}{b_1}b_2\right)\left(b_3 - \frac{b_1}{c_1}c_3\right) - \left(a_3 - \frac{a_1}{b_1}b_3\right)\left(b_2 - \frac{b_1}{c_1}c_2\right)\right]$$

Since $a_1, a_2, a_3, b_1, b_2, b_3$ are 1 or – 1 (1)

$\therefore\ a_2, \frac{a_1}{b_1}, b_2, b_3, \frac{b_1}{c_1}, c_3, a_3, \frac{a_1}{b_1}\ b_3, b_2, \frac{b_1}{c_1}, c_2$ are 1 or – 1

$\therefore\ a_2, -\frac{a_1}{b_1}\ b_2, b_3 - \frac{b_1}{c_1}\ c_3, a_3 - \frac{a_1}{b_1}\ b_3, b_2 - \frac{b_1}{c_1}\ c_2$ are 2, – 2 or 0

$\therefore\ \left(a_2 - \frac{a_1}{b_1}b_2\right)\left(b_3 - \frac{b_1}{c_1}c_3\right) = 4, -4$ or $0 =$ an even number.

$\therefore$ From (1), $\Delta =$ an even number ($c_1 = 1$ or – 1).

The correct option is (A)

76. Since $\quad AB\theta = B\theta A\ \therefore\ (AB\theta)\theta = (B\theta A)\theta$

$\Rightarrow \quad (B\theta)\theta A\theta = A\theta(B\theta)\theta \Rightarrow BA\theta = A\theta B$

Now, $AB(AB)\theta = AB(B\theta A\theta) = A(BB\theta)\,A\theta$

$= A(B\theta B)A\theta = (AB\theta)\,(BA\theta)$

$= B\theta AA\theta B = B\theta A\theta AB$

The correct option is (A)

77. $\Delta'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$\therefore\quad 5A + 4B + 3C + 2D + E = \Delta(1) + \Delta'(1)$

But, $\Delta(1) = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 6 \end{vmatrix} = 0$ [$\because$ R_2, R_3 are identical]

$$\Delta'(x) = \begin{vmatrix} 1 & 0 & 1 \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ 2x & 1 & 0 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\therefore\quad \Delta'(1) = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 0 + \begin{vmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 0 & -1 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 0 & 0 & -6 \end{vmatrix}$$

$$= -17 - 6(1-2) = -17 + 6 = -11$$

$\therefore$ required value = – 11.

The correct option is (D)

78. The given determinant Δ_1 is obtained by replacing each element of Δ_2 by its co-factor respectively.

$\therefore\quad \Delta_1 = \Delta_2^{\,2}$

$\therefore\quad \Delta_1\Delta_2 = \Delta_2^{\,2}\Delta_2 = \Delta_2^{\,3}$.

The correct option is (B)

79. We have, $AA' = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}\begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix} = I$

$$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ac+ab+ac & ab+bc+ca \\ ca+ab+bc & a^2+b^2+c^2 & cb+ac+ba \\ ba+cb+ac & bc+ca+ab & a^2+b^2+c^2 \end{bmatrix} = I$$

$\therefore\ a^2+b^2+c^2 = 1$ and $ab+bc+ca = 0$

$\Rightarrow\ a+b+c = \pm 1$

Also, $abc = \gamma$

$\therefore$ a, b, c are the roots of the equation

$x^3 \pm x^2 + \gamma = 0.$

The correct option is (A)

80. We have, $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$

$$\Rightarrow\ 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$$

(Apply $R_2 \to R_2 - R_3$)

$$\Rightarrow \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1-2a & 1-2b & 1-2c \end{vmatrix} = 0 \qquad \text{(Apply } R_3 \to R_3 - R_1\text{)}$$

$$\Rightarrow \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow\ (a-b)(b-c)(c-a) = 0$

$\Rightarrow\ a = b$ or $b = c$ or $c = a$

The correct option is (B)

81. Determinant of coefficients $= \begin{vmatrix} \lambda & -1 & \cos\theta \\ 3 & 1 & 2 \\ \cos\theta & 1 & 2 \end{vmatrix}$

$= \cos\theta - \cos^2\theta + 6$ and this is positive for all θ since $|\cos\theta| \le 1$, the only solution is therefore the trivial solution.

The correct option is (A)

More than One Option Correct Type

82. Here, the equations are in two variables x and y. If they are consistent then the values of x and y obtained from first two equations should satisfy the third equation and hence $D = 0$. i.e.,

$$\Rightarrow \begin{vmatrix} 1 & 1 & -3 \\ 1+\lambda & 2+\lambda & -8 \\ 1 & -1-\lambda & 2+\lambda \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\lambda & 1 & -5+3\lambda \\ 1 & -2-\lambda & 5+\lambda \end{vmatrix} = 0$$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 + 3C_1$]

$\Rightarrow (5+\lambda) + (2+\lambda)(-5+3\lambda) = 0 \Rightarrow 3\lambda^2 + 2\lambda - 5 = 0$

$\Rightarrow (\lambda - 1)(3\lambda + 5) = 0 \Rightarrow \lambda = 1,\ -\dfrac{5}{3}.$

The correct option is (A, C)

83. $\begin{pmatrix} \text{The number} \\ \text{of third-order} \\ \text{determinants} \end{pmatrix} = \begin{pmatrix} \text{The number of} \\ \text{arrangements of} \\ \text{nine different} \\ \text{numbers in} \\ \text{nine places} \end{pmatrix} = 9!$

Now, $\sum_{i=1}^{k} \Delta_i = \Delta_1 + \Delta_2 + \Delta_3 + \ldots + \Delta_k$

$$\Rightarrow \sum_{i=1}^{k} \Delta_i = \begin{vmatrix} \Sigma a_i & \Sigma a_i & \Sigma a_i \\ \Sigma a_i & \Sigma a_i & \Sigma a_i \\ \Sigma a_i & \Sigma a_i & \Sigma a_i \end{vmatrix} = 0$$

The correct option is (A, B)

84. $z = e^{iA}.e^{iB}.e^{iC}\begin{vmatrix} e^{iA} & e^{-i(C+A)} & e^{-i(B+A)} \\ e^{-i(C+B)} & e^{iB} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$

$$\Rightarrow z = -1\begin{vmatrix} e^{iA} & -e^{iB} & -e^{ic} \\ -e^{iA} & e^{iB} & -e^{ic} \\ -e^{iA} & -e^{iB} & e^{ic} \end{vmatrix}$$

since $e^{i(A+B+C)} = e^{i}\pi = \cos\pi + \sin\pi) = -1$

$$\Rightarrow z = -\begin{vmatrix} 0 & -2e^{iB} & 0 \\ -2e^{iA} & 0 & 0 \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$$

(Using $R_1 \to R_1 + R_3$, $R_2 \to R_2 + R_3$)

$\Rightarrow z = -2e^{iB}\{-2e^{i(A+C)}\}$

$\therefore z = 4e^{i(A+B+C)} = 4e^{i}\pi = -4$

The correct option is (B, C)

85. We have, $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

$= \begin{vmatrix} 4 & 0 & 0 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

[Applying $R_1 \to R_1 + R_3 - R_2$]

$= \begin{vmatrix} 4 & 0 & 0 \\ 2x^2+2 & 3 & 3x-3 \\ x^2+4 & 0 & 2x-1 \end{vmatrix}$

[Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$]

$= \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & 3x-3 \\ 4 & 0 & 2x-1 \end{vmatrix}$

$\left[\text{Applying } R_2 \to R_2 - \frac{x^2}{2}R_1, R_3 \to R_3 - \frac{x^2}{4}R_1\right]$

$= \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & 3x \\ 4 & 0 & 2x \end{vmatrix} + \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & -3 \\ 4 & 0 & -1 \end{vmatrix}$

$= x\begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & 3 \\ 4 & 0 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & -3 \\ 4 & 0 & -1 \end{vmatrix}$

$= xA + B$

where, $A = \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & 3 \\ 4 & 0 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 4 & 0 & 0 \\ 2 & 3 & -3 \\ 4 & 0 & -1 \end{vmatrix}$.

The correct option is (A, D)

86. We have,

$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$

$\Rightarrow$ $(x-a)(x+b)(x-c) + (x-b)(x+a)(x+c) = 0$

[Expanding along R_1]

$\Rightarrow$ $2x(x^2 + ac - ab - bc) = 0$

$\Rightarrow$ $x = 0$ or $x^2 = b(a+c) - ac$.

If $b(a+c) > ac$, we have three roots $0, \pm\sqrt{b(a+c) - ac}$.

If $b(a+c) \le ac$, we have only one real root $x = 0$.

The correct option is (A, C)

87. Let $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Since $\Delta^c = \Delta^2$, where Δ^c is determinant of co-factors of Δ

$\Rightarrow \begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a^2+b^2+c^2 & ab+bc+ca & ac+ba+bc \\ ab+bc+ca & b^2+c^2+a^2 & bc+ac+ab \\ ca+ab+bc & bc+ac+ab & c^2+a^2+b^2 \end{vmatrix}$

$= \begin{vmatrix} \alpha^2 & \beta^2 & \beta^2 \\ \beta^2 & \alpha^2 & \beta^2 \\ \beta^2 & \beta^2 & \alpha^2 \end{vmatrix}$ where $a^2 = a^2 + b^2 + c^2$

and, $b^2 = ab + bc + ca$.

The correct option is (A, B)

88. We have, $\begin{vmatrix} \sin x & \sin y & \sin z \\ \cos x & \cos y & \cos z \\ \cos^3 x & \cos^3 y & \cos^3 z \end{vmatrix}$

$= \cos x \cos y \cos z \begin{vmatrix} \tan x & \tan y & \tan z \\ 1 & 1 & 1 \\ \cos^2 x & \cos^2 y & \cos^2 z \end{vmatrix}$

$= \cos x \cos y \cos z$

$\begin{vmatrix} \tan x & \tan y - \tan x & \tan z - \tan y \\ 1 & 0 & 0 \\ \cos^2 x & \cos^2 y - \cos^2 x & \cos^2 z - \cos^2 y \end{vmatrix}$

[Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$]

$= -\cos x \cos y \cos z$

$\begin{vmatrix} \tan y - \tan x & \tan z - \tan y \\ \cos^2 y - \cos^2 x & \cos^2 z - \cos^2 y \end{vmatrix}$

[Expanding along R_2]

$= \begin{vmatrix} \cos z \sin(x-y) & \cos x \sin(y-z) \\ \sin(x+y)\sin(x-y) & \sin(y+z)\sin(y-z) \end{vmatrix}$

$= \sin(x-y)\sin(y-z)\sin(z-x)\cos(x+y+z)$

Therefore, the given determinant is zero only when any two of x, y, z are equal or $x + y + z = \frac{\pi}{2}$.

The correct option is (A, B, C, D)

89. We have, $\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$

$$= \frac{1}{\sin\alpha\cos\alpha}\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Multiplying R_2 and R_3 by sin α and cosα, respectively]

$$= \frac{1}{\sin\alpha\cos\alpha}\begin{vmatrix} 0 & 0 & \cos 2\alpha+\sin^2\alpha+\lambda\cos^2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$= \frac{\cos 2\alpha+\sin^2\alpha+\lambda\cos^2\alpha}{\sin\alpha\cdot\cos\alpha}\begin{vmatrix} \sin\theta\sin\alpha & \cos\theta\sin\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha \end{vmatrix}$$

$$= (\cos^2\alpha + \lambda\cos^2\alpha)\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = (1+\lambda)\cos^2\alpha.$$

Therefore, the given determinant is **independent of θ for all real values of λ. Also, if $\lambda = -1$, then it is independent of θ and α.**

The correct option is (A, C)

90. We have,

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 2 & 1+\cos^2\theta & 4\sin 4\theta \\ 1 & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

[Applying $C_1 \to C_1 + C_2$]

$$\Rightarrow \begin{vmatrix} 2 & \cos^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$\Rightarrow$ $2 + 4\sin 4\theta = 0$ [Expanding along R_2]

$$\Rightarrow \sin 4\theta = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(\frac{-\pi}{6}\right)$$

$$\Rightarrow 4\theta = n\pi + (-1)^n\left(\frac{-\pi}{6}\right)$$

The values of θ lying between 0 and $\frac{\pi}{2}$ are $\frac{7\pi}{24}$ and $\frac{11\pi}{24}$ for $n = 1$ and 2.

The correct option is (A, C)

91. We have,

$$a_n + a_{n+2} = \int_0^{\pi/2} \frac{2-[\cos 2nx + \cos 2(n+2)x]}{1-\cos 2x}dx$$

$$= \int_0^{\pi/2} \frac{2-2\cos 2(n+1)x\cos 2x}{1-\cos 2x}dx$$

Also, $2\cdot a_{n+1} = 2\int_0^{\pi/2} \frac{1-\cos 2(n+1)x}{1-\cos 2x}dx$

$\therefore a_n + a_{n+2} - 2\cdot a_{n+1}$

$$= 2\int_0^{\pi/2} \frac{1-\cos 2(n+1)x\cdot\cos 2x - 1 + \cos 2(n+1)x}{1-\cos 2x}dx$$

$$= 2\int_0^{\pi/2} \frac{\cos 2(n+1)x\cdot(1-\cos 2x)}{1-\cos 2x}dx$$

$$= 2\int_0^{\pi/2} \cos 2(n+1)x\,dx$$

$$\therefore \quad a_n + a_{n+2} - 2a_{n+1} = 2\left[\frac{\sin 2(n+1)x}{2(n+1)}\right]_0^{\pi/2}$$

$$= \frac{1}{n+1}(0-0) = 0$$

$$\therefore \quad a_{n+1} = \frac{a_n + a_{n+2}}{2} \qquad (1)$$

$\Rightarrow$ a_{n+1} is the A. M. between a_n and a_{n+2}.

Now, $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a_1 & 2a_2 & a_3 \\ a_4 & 2a_5 & a_6 \\ a_7 & 2a_8 & a_9 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} a_1 & 2a_2-(a_1+a_3) & a_3 \\ a_4 & 2a_5-(a_4+a_6) & a_6 \\ a_7 & 2a_8-(a_7+a_9) & a_9 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1 - C_3$]

$$= \frac{1}{2}\begin{vmatrix} a_1 & 0 & a_3 \\ a_4 & 0 & a_6 \\ a_7 & 0 & a_9 \end{vmatrix} \qquad \text{[using (1)]}$$

$= 0.$

The correct option is (A, D)

92. We have,

$$\begin{vmatrix} \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \\ \alpha\beta & \beta\gamma & \gamma\alpha \end{vmatrix} = 0$$

$$\Rightarrow (\alpha\beta)^3 + (\beta\gamma)^3 + (\gamma\alpha)^3 - 3(\alpha\beta)(\beta\gamma)(\gamma\alpha) = 0$$

$$\left[\because \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow a^3 + b^3 + c^3 - 3\,abc = 0\right]$$

$\Rightarrow$ $(\alpha\beta + \beta\gamma\omega^2 + \gamma\alpha\omega)(\alpha\beta\omega + \beta\gamma\omega^2 + \gamma\alpha)(\alpha\beta\omega^2 + \beta\gamma\omega + \gamma\alpha) = 0$

$\Rightarrow$ $\alpha\beta + \beta\gamma\omega^2 + \gamma\alpha\omega = 0$ or $\alpha\beta\omega + \beta\gamma\omega^2 + \gamma\alpha = 0$

or, $\alpha\beta\omega^2 + \beta\gamma\omega + \gamma\alpha = 0$

$$\Rightarrow \frac{1}{\gamma} + \frac{1}{\alpha\omega} + \frac{1}{\beta\omega^2} = 0 \text{ or } \frac{1}{\gamma\omega^2} + \frac{1}{\alpha\omega} + \frac{1}{\beta} = 0$$

or, $\frac{1}{\gamma\omega} + \frac{1}{\alpha\omega^2} + \frac{1}{\beta} = 0.$

The correct option is (A, B, C, D)

93. We have, $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix}$

$$= \begin{vmatrix} x^3 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} + \begin{vmatrix} 1 & x^2y & x^2z \\ 0 & y^3+1 & y^2z \\ 0 & yz^2 & z^3+1 \end{vmatrix}$$

$$= x\begin{vmatrix} x^2 & x^2y & x^2z \\ y^2 & y^3+1 & y^2z \\ z^2 & yz^2 & z^3+1 \end{vmatrix} + (y^3+1)(z^3+1) - y^3z^3$$

[Applying $C_2 \to C_2 - yC_1$ and $C_3 \to C_3 - zC_1$]

$$= x\begin{vmatrix} x^2 & 0 & 0 \\ y^2 & 1 & 0 \\ z^2 & 0 & 1 \end{vmatrix} + y^3z^3 + z^3 + y^3 + 1 - y^3z^3$$

$$= x^3 + y^3 + z^3 + 1$$

Given: $x^3 + y^3 + z^3 + 1 = 30 \Rightarrow x^3 + y^3 + z^3 = 29$

Since $29 = 3^3 + 1 + 1$
$= 1^3 + 3^3 + 1^3$
$= 1^3 + 1^3 + 3^3$

$\therefore$ solutions are (3, 1, 1), (1, 3, 1) (1, 1, 3).

The correct option is (A, B, C)

94. Put $x = 0$ in the given determinant, we get $a = 0$. Differentiating $f(x)$ column by column, we get

$$f'(x) = \begin{vmatrix} e^x & \sin x & 1 \\ -\sin x & \log_e(1+x^2) & 1 \\ 1 & x^2 & 1 \end{vmatrix} + \begin{vmatrix} e^x & \cos x & 1 \\ \cos x & \frac{2x}{1+x^2} & 1 \\ x & 2x & 1 \end{vmatrix} + 0$$

$= b + 2cx$

Now, putting $x = 0$, we have

$$b = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -1$$

The correct option is (A, C)

95. $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$

$$= \begin{vmatrix} 2 & \cos^2 x & \sin 2x \\ 2 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix} \quad [\text{Apply } C_1 \to C_1 + C_2]$$

$$= \begin{vmatrix} 2 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

[Apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$= 2 + \sin 2x$

Since the maximum value of $\sin 2x$ is 1, and minimum value of $\sin 2x$ is (– 1). Therefore $\alpha = 3$, $\beta = 1$. Now, $\alpha - \beta = 2$, $\alpha + \beta = 4$ and $\alpha + 3\beta = 6$. Thus, $(\alpha - \beta) + (\alpha + \beta) = (\alpha + 3\beta)$. So, $\alpha - \beta$, $\alpha + \beta$, $\alpha + 3\beta$ cannot form a triangle. All other options are correct.

The correct option is (A, B, C)

Passage Based Questions

96. The characteristic equation of matrix A is

$$|A - \lambda I| = 0 \text{ or } \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 1-\lambda & 2 \\ 1 & 2 & 0-\lambda \end{vmatrix} = 0$$

Expanding the determinant from first row, we get

$(1 - \lambda)[(1 - \lambda)(0 - \lambda) - 4] + 2[0 \times 2 - 1(1 - \lambda)] = 0$

or, $(1 - \lambda)[l^2 - \lambda - 4] - 2 + 2\lambda = 0$

or, $-\lambda^3 + \lambda^2 + 4\lambda + \lambda^2 - \lambda - 4 - 2 + 2\lambda = 0$

or, $-\lambda^3 + 2\lambda^2 + 5\lambda - 6 = 0$

or, $\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$

or, $(\lambda - 1)(\lambda + 2)(\lambda - 3) = 0$

or, $\lambda = 1, -2, 3$

The characteristic roots of the given martix A are 1, – 2 and 3.

The correct option is (A, C, D)

97. The characteristic equation of the matrix A is

$$|A - \lambda I| = 0 \text{ or } \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{vmatrix} = 0$$

$$\text{or, } \begin{vmatrix} 1-\lambda & -3 & 0 \\ 3 & -5-\lambda & -2-\lambda \\ 6 & -6 & -2-\lambda \end{vmatrix} = 0 \qquad [C_3 \to C_3 + C_2]$$

Expanding the determinant, we get

$(1-\lambda)[(-5-\lambda)(-2-\lambda) + 6(-2-\lambda)]$

$+ 3[3(-2-\lambda) - 6(-2-\lambda)] = 0$

or, $(-\lambda + 1)[\lambda^2 + 7\lambda + 10 - 6\lambda - 12]$

$+ 3[-6 - 3\lambda + 12 + 6\lambda] = 0$

or, $(-\lambda + 1)[\lambda^2 + \lambda - 2] + 3[3\lambda + 6] = 0$

or, $-\lambda^3 - \lambda^2 + 2\lambda + \lambda^2 + \lambda - 2 + 9\lambda + 18 = 0$

or, $-\lambda^2 + 12\lambda + 16 = 0$ or $\lambda^3 - 12\lambda - 16 = 0$

$(\lambda - 4)(\lambda + 2)^2 = 0 \Rightarrow \lambda = 4, -2, -2$

$\therefore$ The characteristic roots of the given matrix A are 4, – 2, and – 2.

The correct option is (A)

98. (A) Since the determinant of a matrix is the same as that of its transpose.

$|A - \lambda I| = |(A - \lambda I)'|$

or, $|A - \lambda I| = |(A' - \lambda I)|$ $[(\lambda I)' = \lambda I' = \lambda I]$

Hence, A and A' have the same characteristic roots.

(B) Let λ be a characteristic root of A so that

$AX = lX$ (for $n \times 1$ matrix X)

$\Rightarrow k(AX) = k(lX)$

$\Rightarrow (kA)X = (k\lambda)X$

Hence, $k\lambda$ is a characteristic root of kA.

(C) $AX = \lambda X$

$\Rightarrow A(AX) = A(\lambda X)$ (Multiplying by A)

$\Rightarrow A^2X = \lambda(AX)$

$= \lambda(\lambda X)$ $(\because AX = lX)$

$= \lambda^2 X$

Thus, $A^2X = \lambda^2 X \Rightarrow \lambda^2$ is a characteristic root of A^2.

Repeating this process n times, we get

$A^nX = \lambda^n X$

Hence, λ^n is a characteristic root of A^n.

(D) Let $\lambda \neq 0$ be a characteristic root of a non-singular matrix A so that

$AX = \lambda X$ (for $n \times 1$ matrix X)

since A is non singular, A^{-1} exists,

$\Rightarrow A^{-1}(AX) = A^{-1}(\lambda X)$

$\Rightarrow (A^{-1}A)X = \lambda(A^{-1}X)$

$\Rightarrow IX = \lambda(A^{-1}X)$

$\Rightarrow X = \lambda(A^{-1}X)$

$\Rightarrow A^{-1}X = \frac{1}{\lambda}X$ $(\because \lambda \neq 0)$

Hence, λ^{-1} is a characteristic root of A^{-1}.

The correct option is (A, B, C, D)

99. (A) The characteristic equation of the matrix $A^{-1}B$ is

$|A^{-1}B - \lambda I| = 0$ (1)

$\Rightarrow |A|\,|A^{-1}B - \lambda I|\,|A^{-1}| = 0$

$\Rightarrow |A(A^{-1}B - \lambda I)A^{-1}| = 0$ $[\because |AB| = |A|\,|B|]$

$\Rightarrow |A(A^{-1}B)A^{-1} - A\lambda IA^{-1}| = 0$

$\Rightarrow |(AA^{-1})(BA^{-1}) - \lambda AA^{-1}| = 0$ $(AA^{-1} = 1)$

$\Rightarrow |I(BA^{-1}) - \lambda I| = 0$

$\Rightarrow |BA^{-1} - \lambda I| = 0$ (2)

From (1) and (2), it follows that $A^{-1}B$ and BA^{-1} have the same characteristic equation and hence the same char-acter-istic roots.

(B) The characteristic equation of the matrix $P^{-1}AP$ is

$|P^{-1}AP - \lambda I| = 0$ (1)

$\Rightarrow |P^{-1}AP - P^{-1}\lambda IP| = 0$ $(P^{-1}P = I)$

$\Rightarrow |P^{-1}(A - \lambda I)P| = 0$

$\Rightarrow |P^{-1}|\,|A - \lambda I|\,|P| = 0$

$\Rightarrow |P^{-1}P|\,|A - \lambda I| = 0$ $[\because |AB| = |A|\,|B|]$

$\Rightarrow |A - \lambda I| = 0$ (2) $[|P^{-1}P| = |I| = 1]$

From (1) and (2) it follows that the matrix A and $P^{-1}AP$ have the same characteristic equation and hence the same characteristic roots.

(C) $AB = IAB$

$= B^{-1}B(AB)$

$= B^{-1}(BA)B$

The characteristic roots of AB = characteristic roots of $B^{-1}(BA)B$ = characteristic roots of BA.

The correct option is (D)

Match the Column Type

100. **I.** Put $x = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = f \Rightarrow f = 1$

The correct option is (B)

II. Differentiate both the sides and put $x = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = e \Rightarrow e = 3$$

The correct option is (A)

III, IV. Put $x = 1$, then $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = p + q + r + s + t + w$

$\Rightarrow \quad 4 = p + q + r + s + 3 + 1$

$\Rightarrow \quad p + q + r + s = 0 \quad (1)$

Put $x = -1$, then

$$\begin{vmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -p + q - r + s - t + w$$

$= -p + q - r + s - 3 + 1$

$\Rightarrow -p + q - r + s = 2 \quad (2)$

Solving (1) and (2), $q + s = 1$ and $p + r = -1$.

The correct option is (C, B)

Assertion-Reason Type

101. Since for $x = 0$, the determinant reduces to the determinant of a skew-symmetric matrix of odd order which is always zero. Hence, $x = 0$ is the solution of the given equation.

The correct option is (A)

102. Since the system has a non-trivial solution,

therefore, $\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$

$\Rightarrow \lambda(-\cos^2\alpha - \sin^2\alpha) - (-\sin\alpha\cos\alpha - \sin\alpha\cos\alpha) - (\sin^2\alpha - \cos^2\alpha) = 0$

$\Rightarrow -\lambda + \sin 2\alpha + \cos 2\alpha = 0 \Rightarrow \lambda = \sin 2\alpha + \cos 2\alpha$

$\Rightarrow \lambda = \sqrt{2}\cos\left(2\alpha - \frac{\pi}{4}\right)$

Since $-1 \le \cos\left(2\alpha - \frac{\pi}{4}\right) \le 1 \ \forall \ \alpha \in R$

$\therefore \quad -\sqrt{2} \le \lambda \le \sqrt{2}$ i.e., $\lambda \in [-\sqrt{2}, \sqrt{2}]$.

The correct option is (A)

103. Since α_1, α_2 and β_1, β_2 are the roots of $ax^2 + bx + c = 0$ and $px^2 + \theta x + r = 0$ respectively, therefore,

$$\alpha_1 + \alpha_2 = \frac{-b}{a}, \ \alpha_1\alpha_2 = \frac{c}{a} \quad (1)$$

and, $$\beta_1 + \beta_2 = \frac{-q}{p}, \ \beta_1\beta_2 = \frac{r}{p} \quad (2)$$

Since the given system of equations has a non-trivial solution,

$\therefore \quad \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0$ i.e., $\alpha_1\beta_2 - \alpha_2\beta_1 = 0$

or, $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \sqrt{\frac{\alpha_1\alpha_2}{\beta_1\beta_2}}$

$\Rightarrow \frac{pb}{qa} = \sqrt{\frac{pc}{ra}} \Rightarrow \frac{b^2}{q^2} = \frac{ac}{pr}$.

The correct option is (A)

104. Since a, b, c are in G. P. with common ratio r_1 and α, β, γ are in G. P. with common ratio r_2, therefore $a \ne 0$, $\alpha \ne 0$, $b = ar_1$, $c = a r_1^2$, $\beta = ar_2$, $\gamma = \alpha r_2^2$

Also, the system of equations have only trivial solution, so

$$\begin{vmatrix} a & \alpha & 1 \\ b & \beta & 1 \\ c & \gamma & 1 \end{vmatrix} \ne 0$$

$$\Rightarrow \begin{vmatrix} a & \alpha & 1 \\ ar_1 & \alpha r_2 & 1 \\ ar_1^2 & \alpha r_2^2 & 1 \end{vmatrix} \ne 0 \Rightarrow a\alpha \begin{vmatrix} 1 & 1 & 1 \\ r_1 & r_2 & 1 \\ r_1^2 & r_2^2 & 1 \end{vmatrix} \ne 0$$

$$\Rightarrow a\alpha \begin{vmatrix} 1 & 0 & 0 \\ r_1 & r_2 - r_1 & 1 - r_1 \\ r_1^2 & r_2^2 - r_1^2 & 1 - r_1^2 \end{vmatrix} \ne 0$$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$]

$$\Rightarrow a\alpha(r_2 - r_1)(1 - r_1) \begin{vmatrix} 1 & 0 & 0 \\ r_1 & 1 & 1 \\ r_1^2 & r_2 + r_1 & 1 + r_1 \end{vmatrix} \ne 0$$

$\Rightarrow a\alpha(r_2 - r_1)(1 - r_1)(1 - r_2) \ne 0$

$\Rightarrow r_1 \ne r_2$, $r_1 \ne 1$, $r_2 \ne 1$.

The correct option is (A)

105. We have, $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 1 - a & b - 1 & 1 \\ 1 - ac & 1 - c & c \end{vmatrix}$

[Apply $C_1 \to C_1 - aC_3$, $C_2 \to C_2 - C_3$]

$= (1 - a)(1 - c) - (b - 1)(1 - ac)$

$= 1 - a - c + ac - b + abc + 1 - ac$

$= (2 + abc) - (a + b + c)$

Since the value of the given determinant is positive, therefore,

$abc + 2 > a + b + c$

Using A.M. > G.M., we have,

$a + b + c > 3(abc)^{1/3}$

i.e., $x^3 - 3x + 2 > 0$ [putting $(abc)^{1/3} = x$]

$\Rightarrow \quad (x-1)^2(x+2) > 0$

$\Rightarrow \quad x > -2$

i.e., $x^3 = abc > -8$.

The correct option is (A)

106. We have,

$$f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix} \quad (1)$$

$$= \begin{vmatrix} x+c_1 & a-c_1 & 0 \\ x+b & c_2-b & a-c_2 \\ x+b & 0 & c_3-b \end{vmatrix}$$

$[C_3 \to C_3 - C_2, C_2 \to C_2 - C_1]$

$$= x\begin{vmatrix} 1 & c-c_1 & 0 \\ 1 & c_2-b & a-c_2 \\ 1 & 0 & c_3-b \end{vmatrix} + \begin{vmatrix} c_1 & a-c_1 & 0 \\ b & c_2-b & a-c_2 \\ c & 0 & c_3-b \end{vmatrix}$$

which proves that $f(x)$ is linear in x.

Let $\quad f(x) = ax + \beta$

Then, $\quad f(-a) = -a\alpha + \beta \quad (2)$

$\quad f(-b) = -b\alpha + \beta \quad (3)$

and, $\quad f(0) = \beta = \dfrac{bf(-a) - af(-b)}{(b-a)} \quad (4)$

[from (2) and (3)]

From equation (1), we have,

$$f(-a) = \begin{vmatrix} c_1-a & 0 & 0 \\ b-a & c_2-a & 0 \\ b-a & b-a & c_3-a \end{vmatrix}$$

$$= (c_1-a)(c_2-a)(c_3-a)$$

Similarly, $\quad f(-b) = (c_1-b)(c_2-b)(c_3-b)$

Since, $\quad g(x) = (c_1-x)(c_3-x)(c_2-x)$,

therefore, we can see that

$g(A) = f(-a)$ and $g(b) = f(-b)$

Hence, from (4), we have,

$$f(0) = .\frac{bg(a) - ag(b)}{(b-a)}$$

The correct option is (A)

Previous Year's Questions

107. $\because l$, m and n are the pth, qth and rth term of an GP whose first term is A and common ratio is R.

$\therefore l = AR^{P-1}$

$\Rightarrow \log l = \log a + (p-1)\log R$

Similarly, $\log m = \log A + (q-1)\log R$, and

$\log n = \log A + (r-1)\log R$

Now $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$

$$= \begin{vmatrix} \log A + (p-1) & p & 1 \\ \log A + (q-1) & q & 1 \\ \log A + (r-1) & r & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix}$$

[Using the column transformation $C_1 \to C_2 - (C_3 \log A + (C_2 - C_3)\log R)$]

The correct option is (D)

108. Determinant $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & 1 \\ 20 & 3 & i \end{vmatrix}$

$$= \begin{vmatrix} 6i+4 & 0 & 0 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} (R_1 \to R_1 + R_2)$$

$$= (6i+4)\begin{vmatrix} 3i & -1 \\ 3 & i \end{vmatrix}$$

$= (6i+4)(3i^2+3)$

$= 0$

Put $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$

$\Rightarrow 0 + 0i = x + iy$

$\Rightarrow x = 0, y = 0$

The correct option is (D)

109. $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix} (R_1 \to R_1 + R_3)$

$$= \begin{vmatrix} 1-i & -1 & \omega^2-1 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$$

$= 0$ ($\because$ Two rows are identical).

The correct option is (A)

110. Coefficient determinant is given by

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow b = \dfrac{2ac}{a+c}$.

The correct option is (C)

111. $\Delta = \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \end{vmatrix}$

$= 0$

(Since $1 + w^n + w^{2n} = 0$, if n is not a multiple of 3 and so the first column is entirely zero)

The correct option is (A)

112. $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

$C_3 \to C_3 - C_2, C_2 \to C_3 - C_1$

$= \begin{vmatrix} \log a_n & \log r & \log r \\ \log a_{n+3} & \log r & \log r \\ \log a_{n+6} & \log r & \log r \end{vmatrix} = 0$ (where r is a common ratio).

(Since two columns are identical.)

The correct option is (A)

113. The determinant

$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy$

The correct option is (B)

114. The system of linear equations

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

have non-trivial solution if

$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow 1(1-a^2) + c(-c-ab) - b(ca+b) = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

The correct option is (D)

115. Each entry of A is integer, so the cofactor of every entry is an integer and hence each entry in the adjoint of matrix A is integer.

Now det$A = \pm 1$ and $A^{-1} = \dfrac{1}{\det(A)}$ (adj A) implies that all entries in A^1 are integers.

The correct option is (C)

116. LHS of the given equality is

$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$

$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$

$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+1} \begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix}$

$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & C+1 \end{vmatrix}$

This is equal to zero only if n + 2 is odd i.e. n is odd integer.

The correct option is (C)

117. $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0$

$D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$No solution.

The correct option is (C)

118. $\begin{vmatrix} 4k2 \\ k41 \\ 221 \end{vmatrix} = 0 \Rightarrow k^2 - 6k + 8 = 0 \Rightarrow k = 4, 2$

The correct option is (A)

119. $P^3 = Q^3$

$P^3 - P^2Q = Q^3 - Q^2P$

$P^2(P - Q) = Q^2(Q - P)$

$P^2(P - Q) + Q^2(P - Q) = O$

$(P^2 + Q^2)(P - Q) = O$

$\Rightarrow |P^2 + Q^2| = 0$

The correct option is (C)

120. $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

Since, $|Adj\ A| = |A|^2$

$|Adj\ A| = 16$

$1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 16.$

$2\alpha - 6 = 16.$

$2\alpha = 22.$

$\alpha = 11.$

The correct option is (A)

121. $\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & \beta-1 \\ 1 & \alpha^2-1 & \beta^2-1 \end{vmatrix}^2$$

$$= \left((\alpha-1)(\beta^2-1)-(\beta-1)(\alpha^2-1)\right)^2$$

$$= (\alpha-1)^2(\beta-1)^2(\alpha-\beta)^2 \Rightarrow k = 1$$

The correct option is (C)

122. Given system

$(2-\lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (2-\lambda)(3\lambda+\lambda^2-4)+2(-2\lambda+2)+1(4-3-\lambda)=0$

$\Rightarrow (2-\lambda)(\lambda^2+3\lambda-4)+4(1-\lambda+2)+(1-\lambda)=0$

$\Rightarrow (2-\lambda)(\lambda+4)(\lambda-1)+5(1-\lambda)=0$

$\Rightarrow (1-\lambda)((\lambda+4)(\lambda-2)+5)=0 \Rightarrow \lambda = 1,1,-3$.

The correct option is (B)

123. We have,

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = 0,1,-1$$

The correct option is (A)